Phenomenology of proton and deuteron deeply virtual exclusive scattering

APS GHP Meeting

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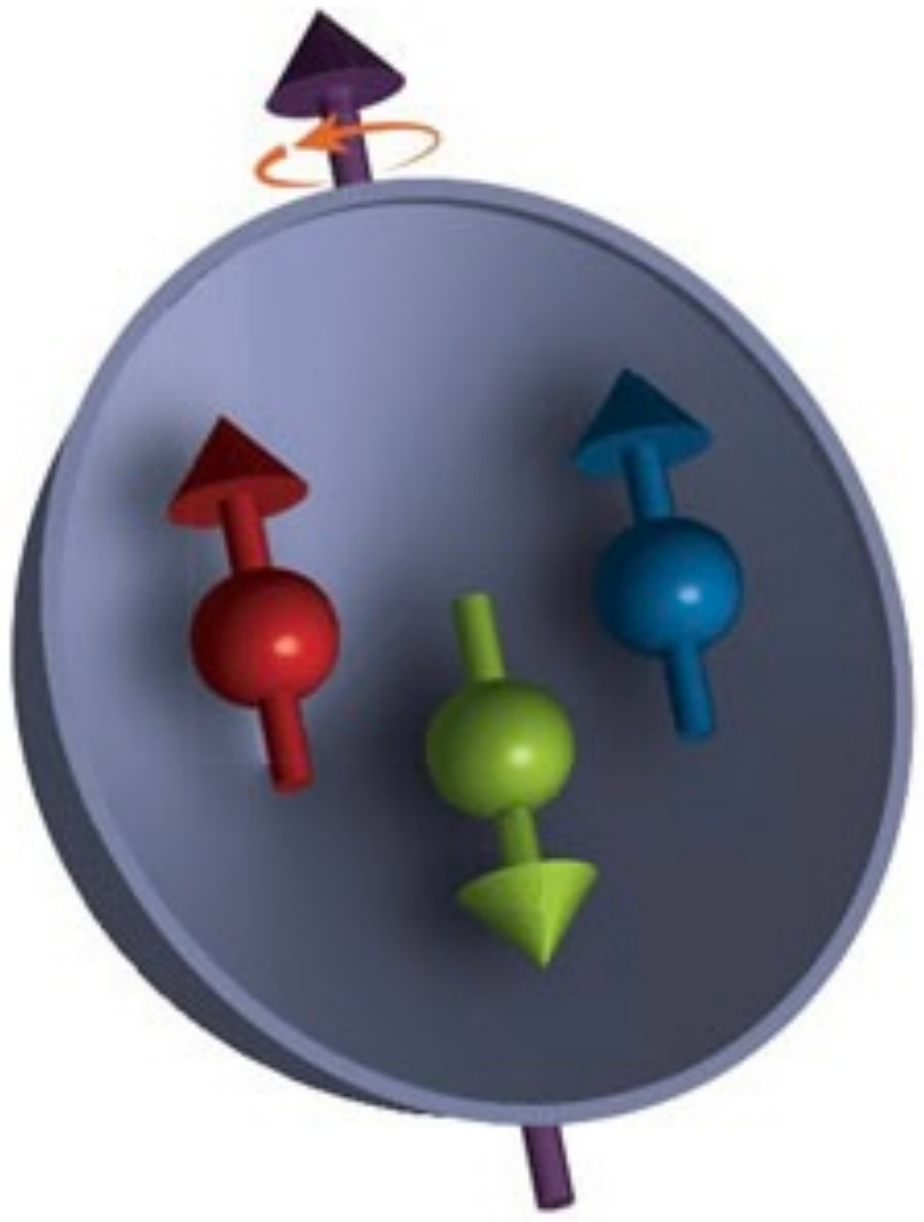
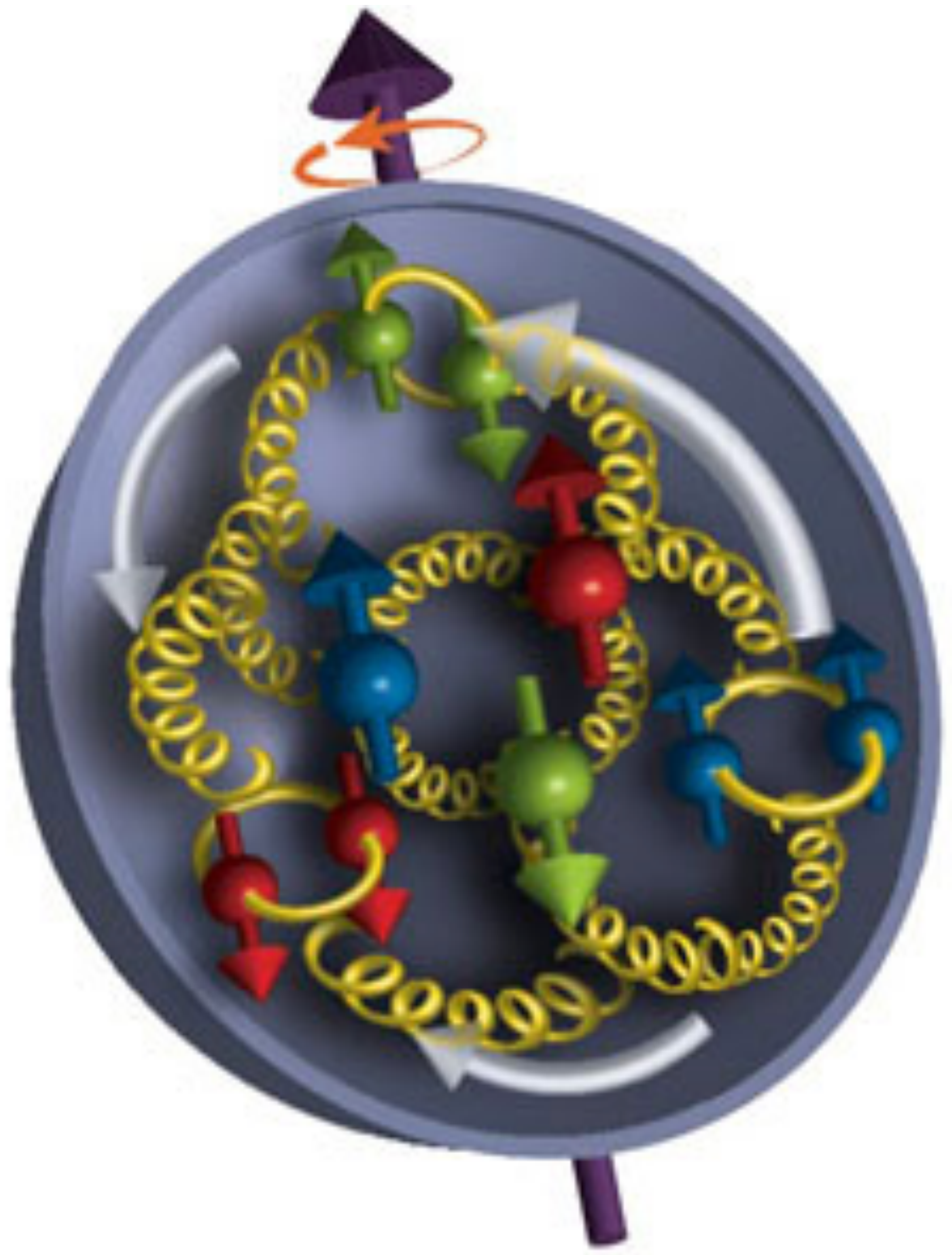
April 12, 2023

Proton spin



Naive parton model does not explain total proton spin



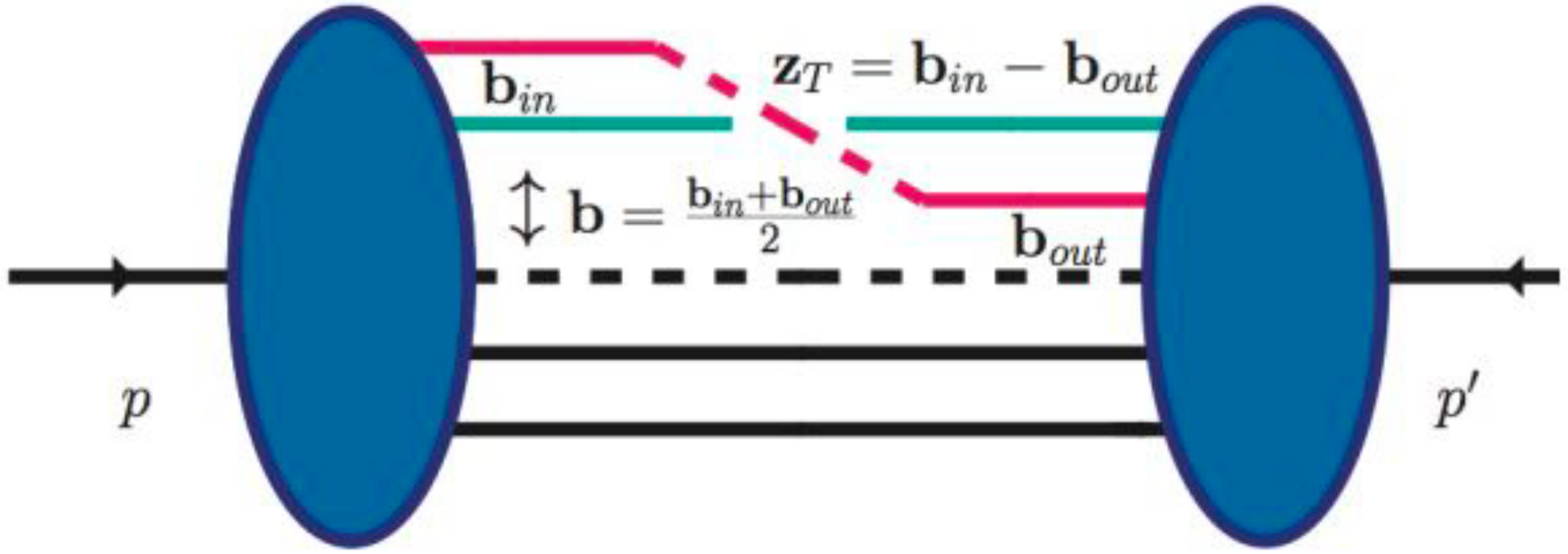
Significant contribution to proton spin comes from orbital dynamics

Orbital dynamics encoded in Generalized Parton Distributions (GPDs)

Generalized Parton Distributions

Parton orbital angular momentum as quantum correlation functions

$$F^{[\Gamma]}_{\Lambda,\Lambda'}(x,\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle p',\Lambda' \middle| \bar{\psi}\left(-\frac{z}{2}\right) \Gamma \mathcal{W}\left(-\frac{z}{2},\frac{z}{2} \mid n\right) \psi\left(\frac{z}{2}\right) \middle| p,\Lambda \right\rangle \Big|_{z=z_T=0}$$



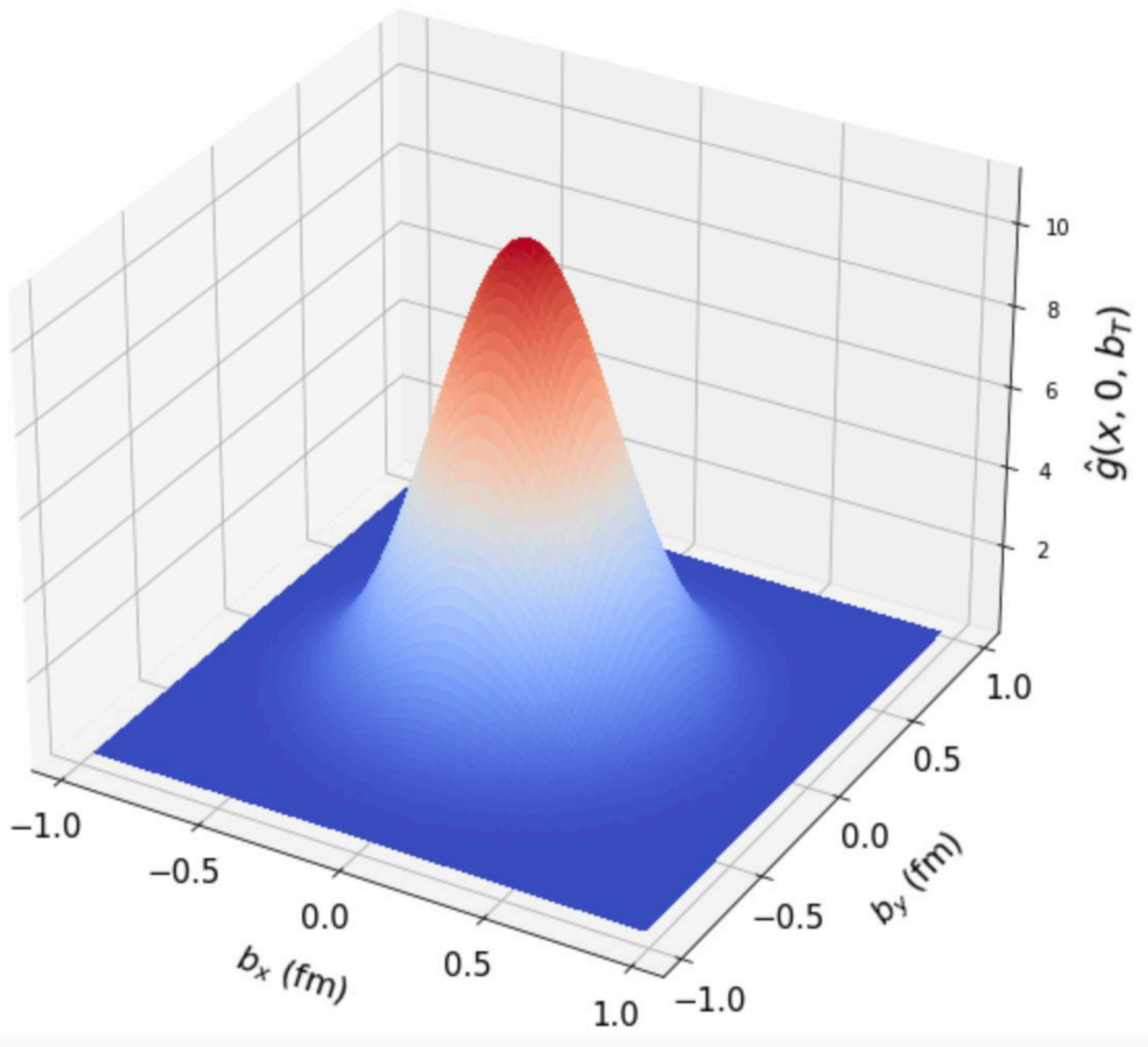
A. Rajan, M. Engelhardt, S. Liuti **PRD 98 (2018)**

$$t = (p' - p)^2$$

$$\xi = \frac{(p'-p)^+}{(p'+p)^+}$$

X. Ji **PRL. 78 (1997)**
A. Radyushkin **PRD. 56 (1997)**
D. Muller, et. al. **(1994)**
M. Diehl **Phys. Rep. (2003)**

Generalized Parton Distributions

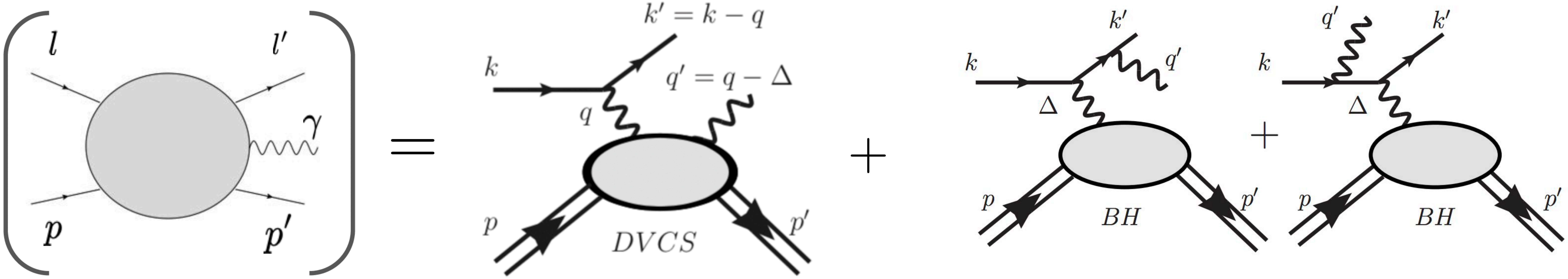


Impact parameter distribution function A.K.A **one body density distribution** in the transverse plane.

M. Burkardt **Int. J. Mod. Phys. A (2003)**
B. Kriesten, P. Velie, E. Yeats, F.Y. Lopez, S. Liuti **PRD 105 (2022)**

Deeply Virtual Compton Scattering

DVCS is known to probe Generalized Parton Distributions.



$$\sigma = \sigma_{BH} + \sigma_{DVCS} + \sigma_{\mathcal{I}}$$

We focus on these

X. Ji **PRD 55 (1997)**
B. Kriesten, S. Liuti, et. al. **PRD 101 (2020)**

DVCS Phenomenology

$$\varepsilon_\mu^{\Lambda_\gamma^*}(hadron) = e^{-i\Lambda_\gamma^*\phi}\varepsilon_\mu^{\Lambda_\gamma^*}(lepton)$$

$$\frac{d^5\sigma_{DVCS}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \Gamma\left|T_{DVCS}\right|^2$$

$$= \frac{\Gamma}{Q^2(1-\epsilon)}\Big\{F_{UU,T} + \epsilon F_{UU,L} + \epsilon\cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{\epsilon(\epsilon+1)}\cos\phi F_{UU}^{\cos\phi}$$

$$+ (2h)\sqrt{2\epsilon(1-\epsilon)}\sin\phi F_{LU}^{\sin\phi}$$

$$+ (2\Lambda)\Big[\sqrt{\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi} + \epsilon\sin 2\phi F_{UL}^{\sin 2\phi}$$

$$+ (2h)\left(\sqrt{1-\epsilon^2}F_{LL} + 2\sqrt{\epsilon(1-\epsilon)}\cos\phi F_{LL}^{\cos\phi}\right)\Big]$$

$$+ (2\Lambda_T)\Big[\sin(\phi-\phi_S)\left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)}\right)$$

$$+ \quad \epsilon\sin(\phi+\phi_S)F_{UT}^{\sin(\phi+\phi_S)} + \epsilon\sin(3\phi-\phi_S)F_{UT}^{\sin(3\phi-\phi_S)}$$

$$+\sqrt{2\epsilon(1+\epsilon)}\left(\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi-\phi_S)F_{UT}^{\sin(2\phi-\phi_S)}\right)\Big]$$

$$+(2h)(2\Lambda_T)\Big[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S)F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)}\cos\phi_S F_{LT}^{\cos\phi_S}$$

$$+\sqrt{2\epsilon(1-\epsilon)}\cos(2\phi-\phi_S)F_{LT}^{\cos(2\phi-\phi_S)}\Big]\Big\}$$

B. Kriesten, S. Liuti, et. al. **PRD 101 (2020)**

BH-DVCS interference term

$$\frac{d^5\sigma_{\mathcal{I}}}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = e_l \frac{\Gamma}{Q^2|t|} \left\{F^{\mathcal{I}}_{UU} + (2h)F^{\mathcal{I}}_{LU} + (2\Lambda)F^{\mathcal{I}}_{UL} + (2h)(2\Lambda)F^{\mathcal{I}}_{LL} + (2\Lambda_T)\, F^{\mathcal{I}}_{UT} + (2h)\,(2\Lambda_T)\, F^{\mathcal{I}}_{LT}\right\}$$

$$F_{UU}^{\mathcal{I},tw2} = \underline{A^{\mathcal{I}}_{UU}}\,\Re e\,(F_1\mathcal{H} + \tau F_2\mathcal{E}) + \underline{B^{\mathcal{I}}_{UU}}\,G_M\,\Re e(\mathcal{H}+\mathcal{E}) + \underline{C^{\mathcal{I}}_{UU}}\,G_M\,\Re e\widetilde{\mathcal{H}}$$

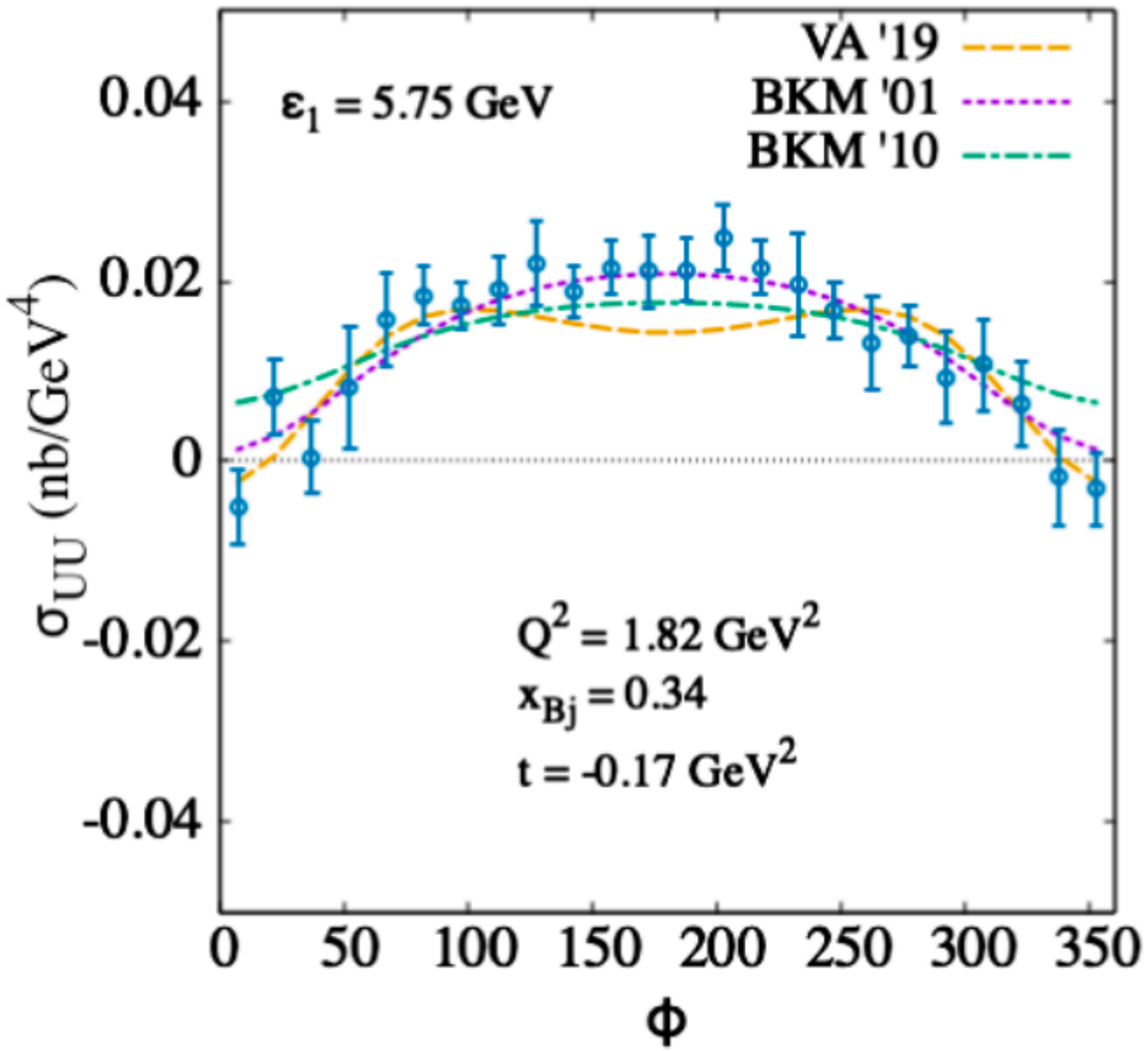
Proportional to cosϕ

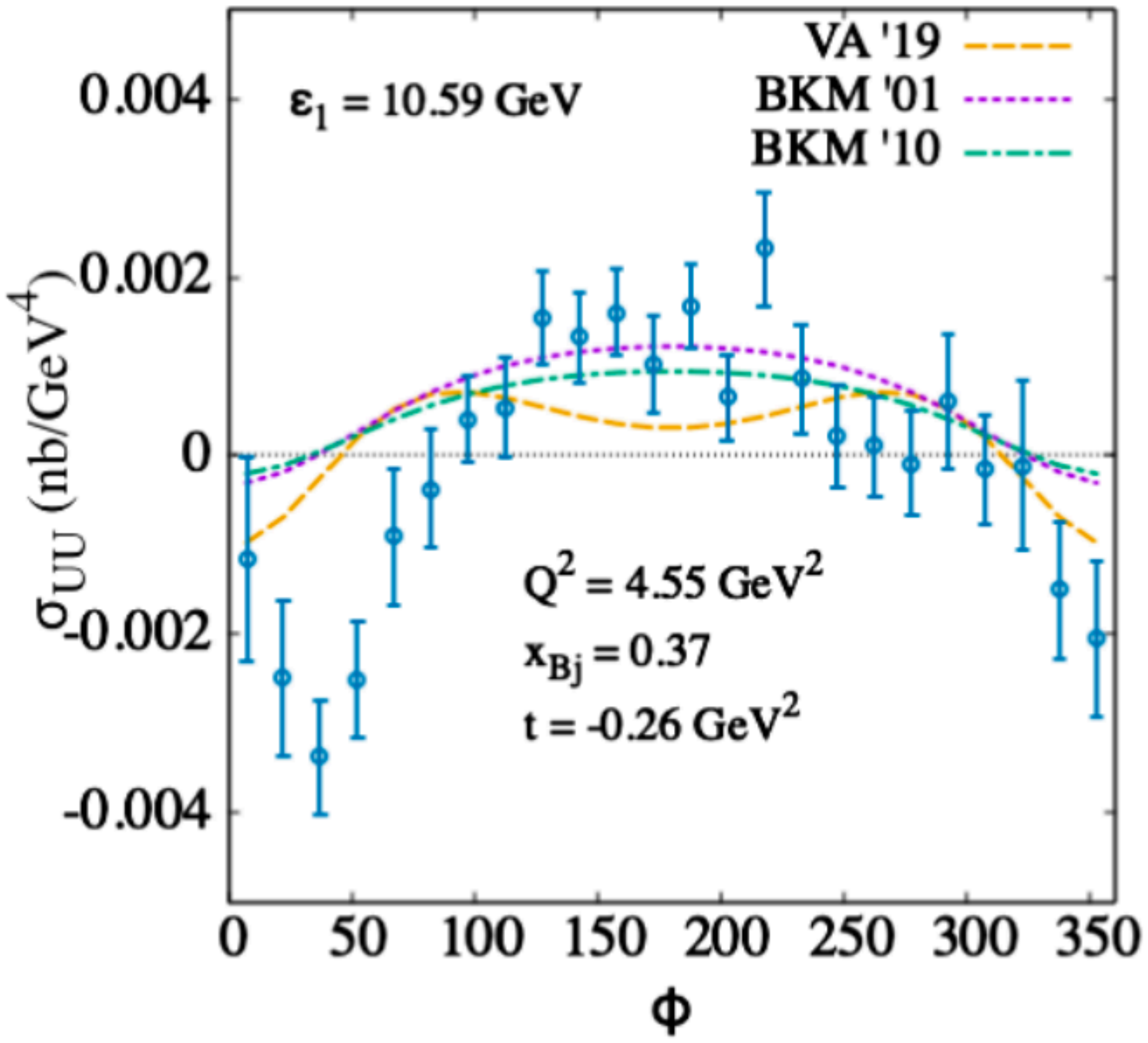
$$F_{UU}^{\mathcal{I},tw3} = \Re e\left\{\underline{A^{(3)\mathcal{I}}_{UU}}\left[F_1\left(2\widetilde{\mathcal{H}}_{2T} + \mathcal{E}_{2T}\right) + F_2\left(\mathcal{H}_{2T} + \tau\widetilde{\mathcal{H}}_{2T}\right)\right]\right.$$
$$\left. + \underline{B^{(3)\mathcal{I}}_{UU}}\,G_M\,\widetilde{E}_{2T} + \underline{C^{(3)\mathcal{I}}_{UU}}\,G_M\left[2\xi H_{2T} - \tau\left(\widetilde{E}_{2T} - \xi E_{2T}\right)\right]\right\}$$

Not proportional to cosϕ

B. Kriesten, S. Liuti, et. al. **PRD 101 (2020)**

Formalism comparison on phase structure



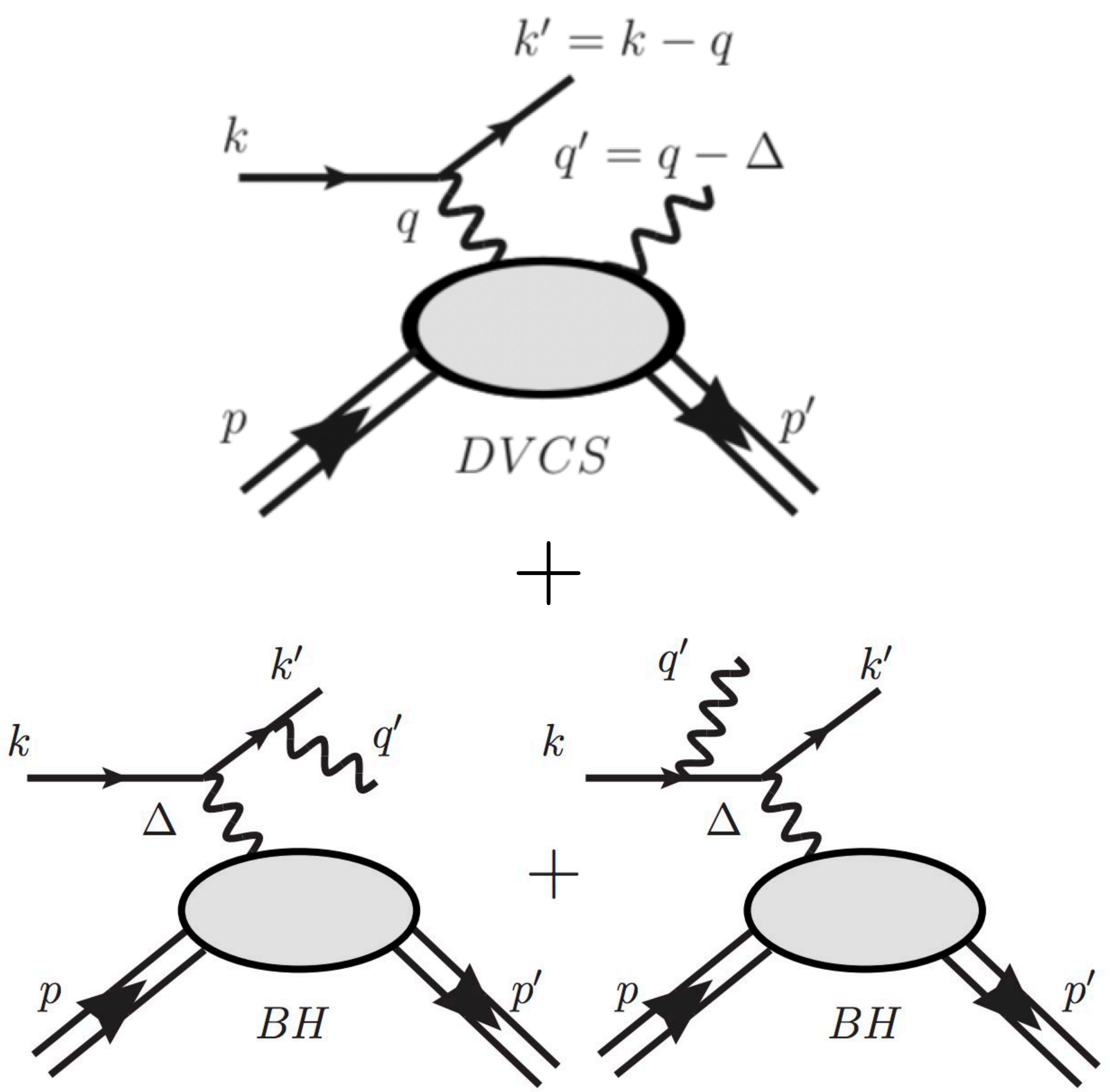


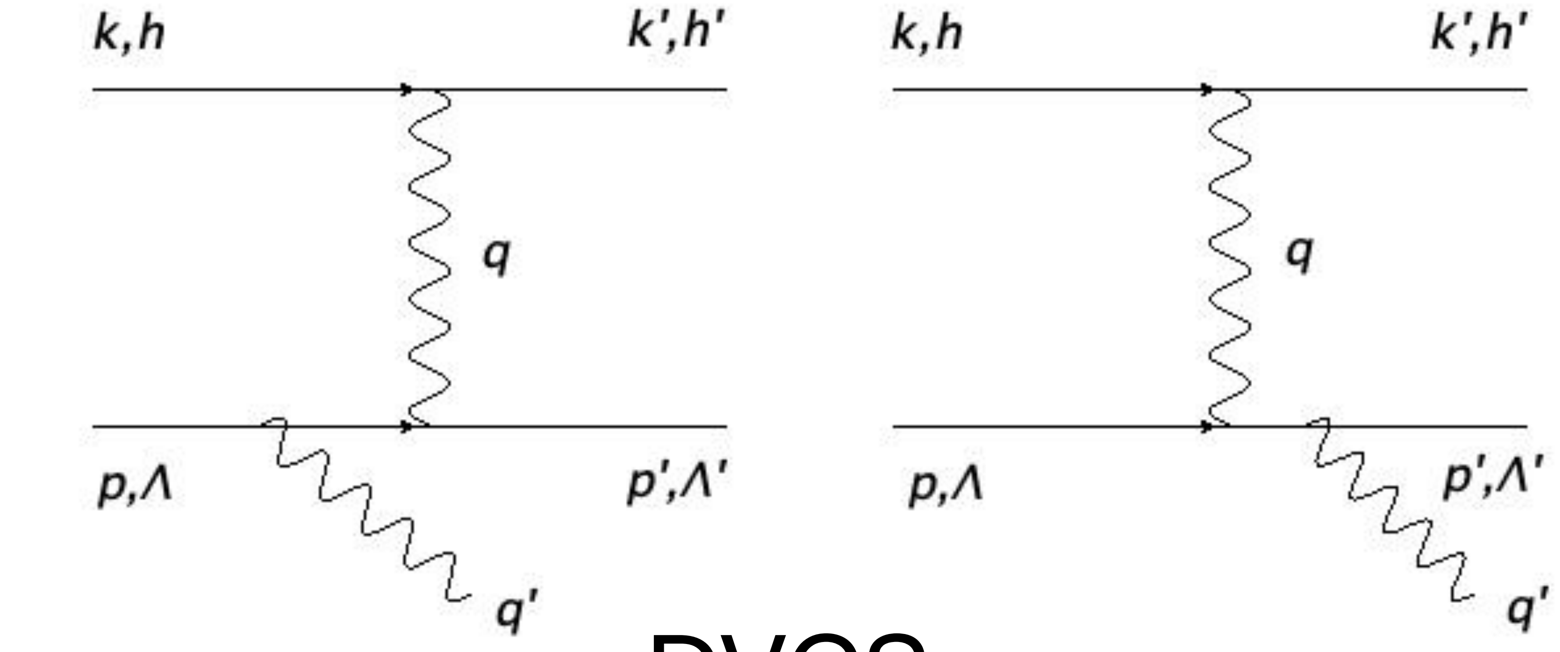
B. Kriesten, S. Liuti, et. al. **PRD 101 (2020)**
A. V. Belitsky, D. Muller, A. Kirchner **Nuclear Physics B 629 (2002)**
A. V. Belitsky, D. Muller **PRD 82 (2010)**

To recapitulate…

- Proper understanding of phase leads to better separation of twist-2 and twist-3 terms and target mass corrections.
- Different formalisms disagree on phase structure.
- Is there a way to distinguish using data?

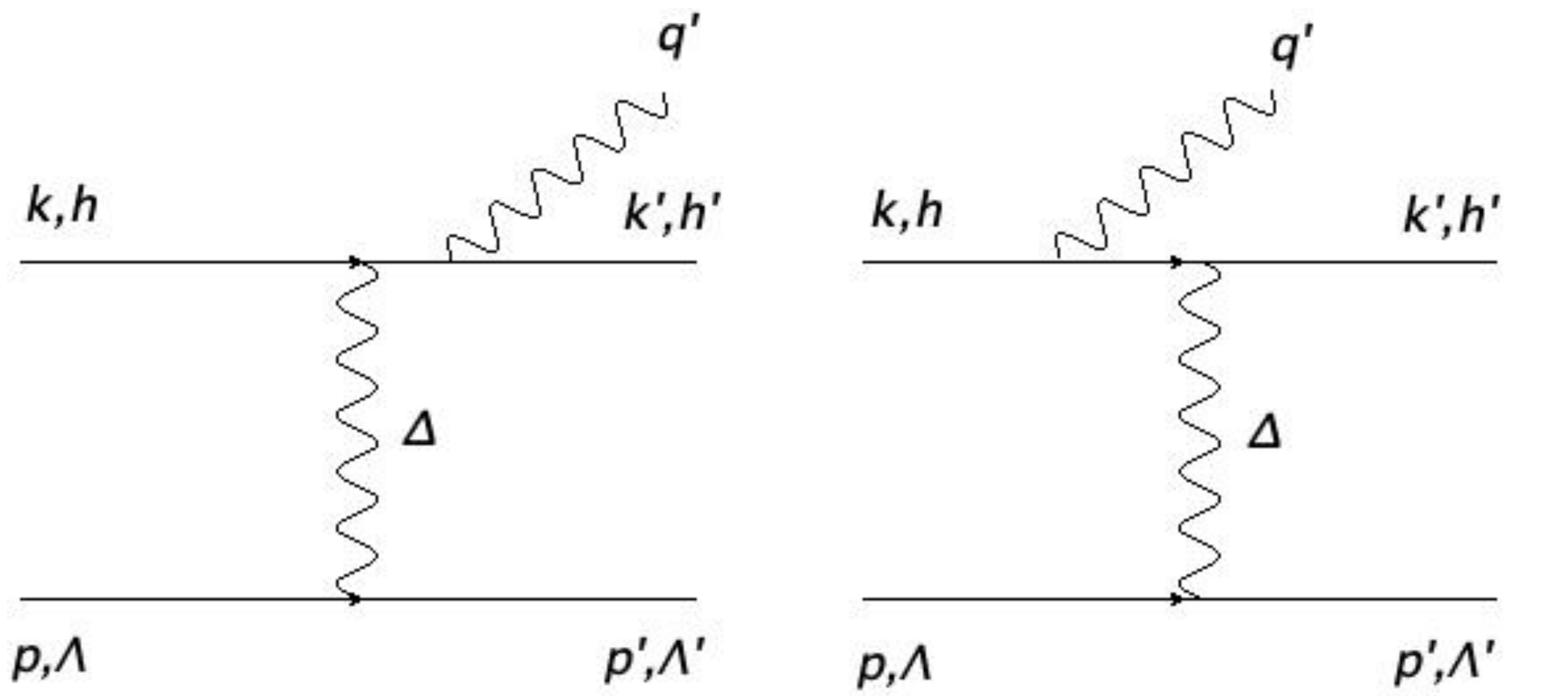
Strip non-essential elements





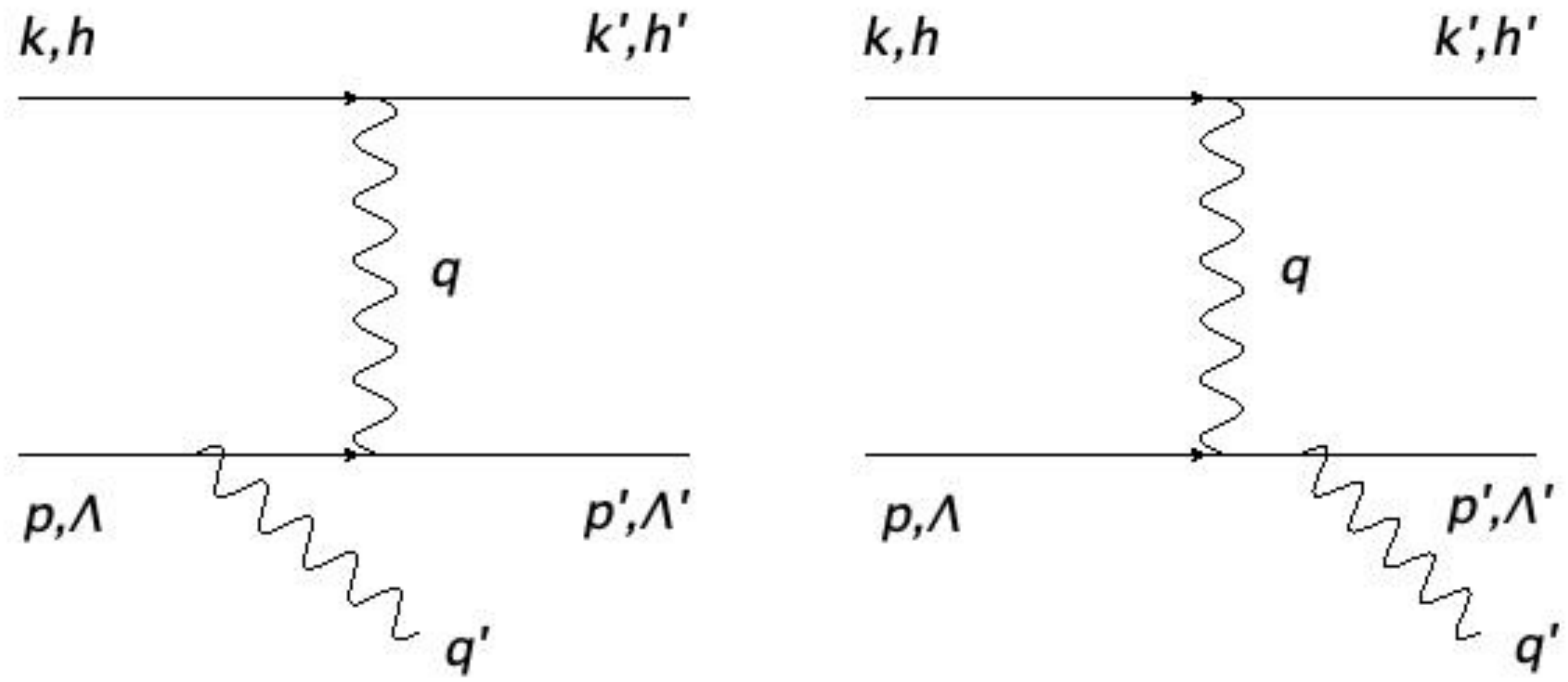
DVCS

+



BH

Pointlike proton



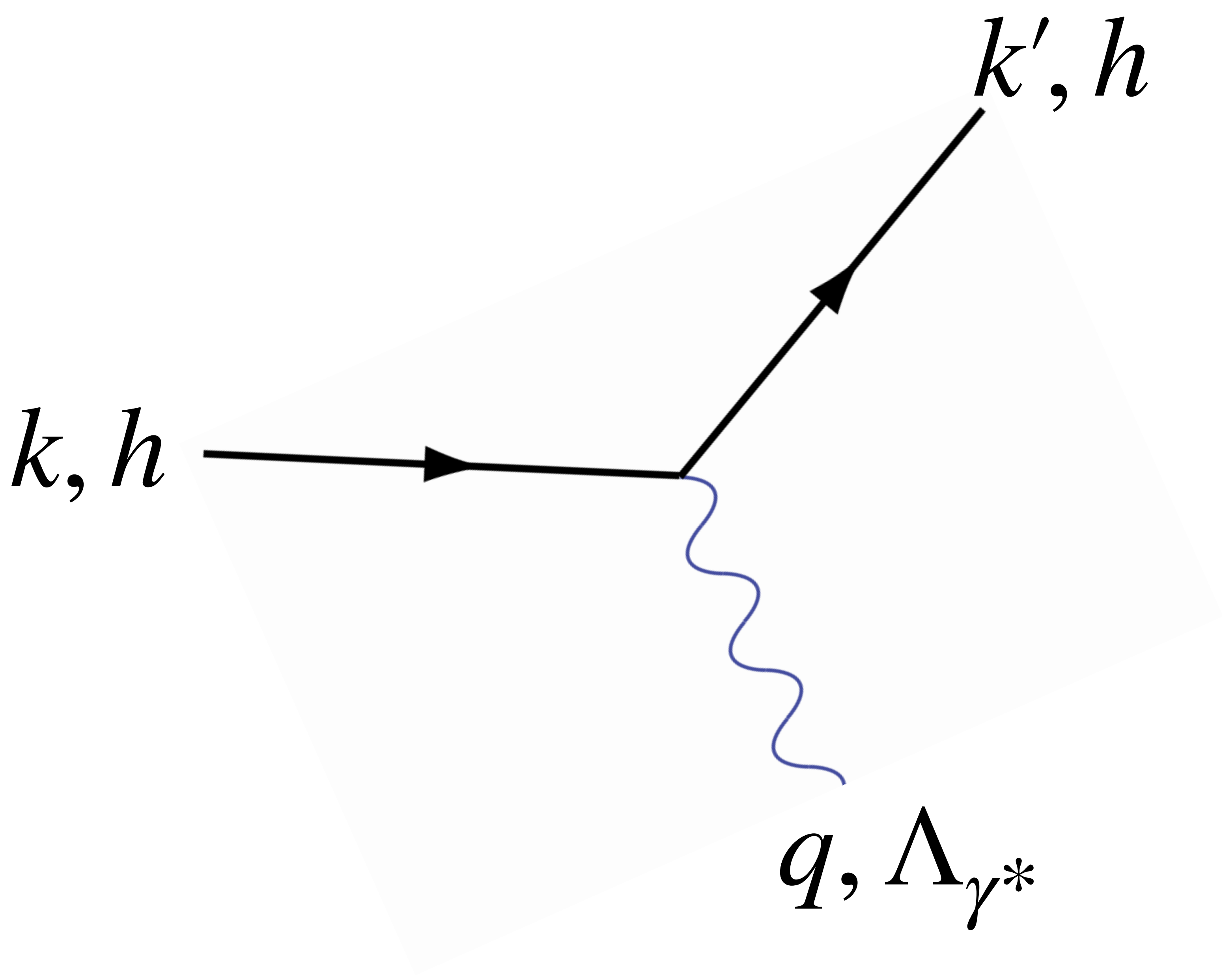
Virtual photon propagator

$$T_{DVCS} = \frac{e^3}{q^2}\bar{u}\left(k', h'\right)\gamma^{\mu} u(k,h) g_{\mu\nu} \bar{u}\left(p', \Lambda'\right)\left(\frac{\gamma^{\alpha}(\not{p} + \not{q} + M)\gamma^{\nu}}{(p+q)^2 - M^2} + \frac{\gamma^{\nu}\left(\not{p} - \not{q}' + M\right)\gamma^{\alpha}}{\left(p - q'\right)^2 - M^2}\right) u(p,\Lambda)\varepsilon_{\alpha}^{*\Lambda_{\gamma'}}\left(q'\right)$$

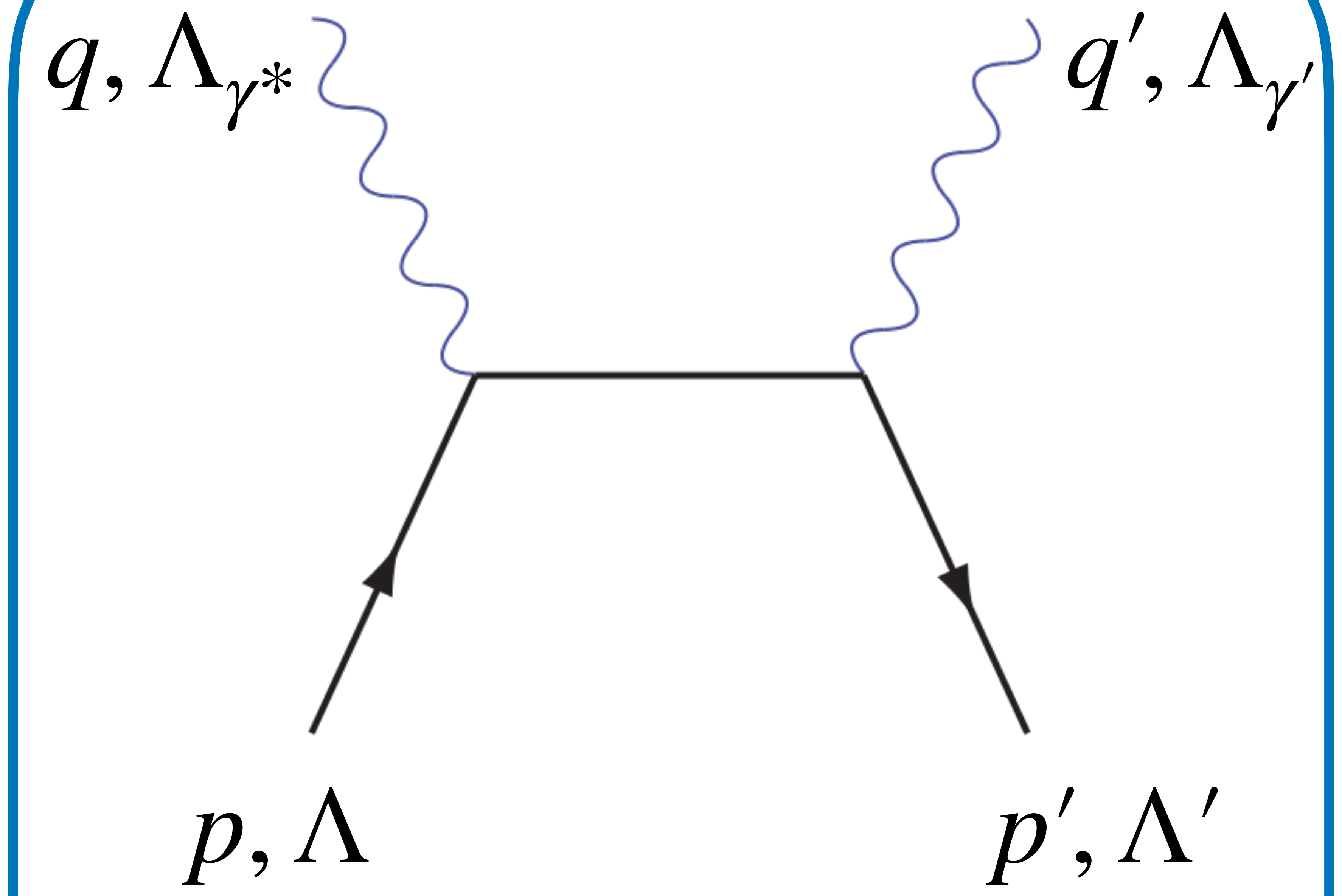
$$-g_{\mu\nu} = \sum_{\Lambda_{\gamma^*} = \pm 1, 0} (-1)^{\Lambda_{\gamma^*}+1} \varepsilon_{\mu}^{*\Lambda_{\gamma^*}}(q) \varepsilon_{\nu}^{\Lambda_{\gamma^*}}(q)$$

$$T_{DVCS,\Lambda\Lambda'}^{h\Lambda'_{\gamma}} = \sum_{\Lambda_{\gamma^*}} (-1)^{\Lambda_{\gamma^*}+1} A_h^{\Lambda_{\gamma^*}}\left(k, k', q\right) f_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}\Lambda'_{\gamma}}\left(q, p, q', p'\right)$$

DVCS Helicity structure



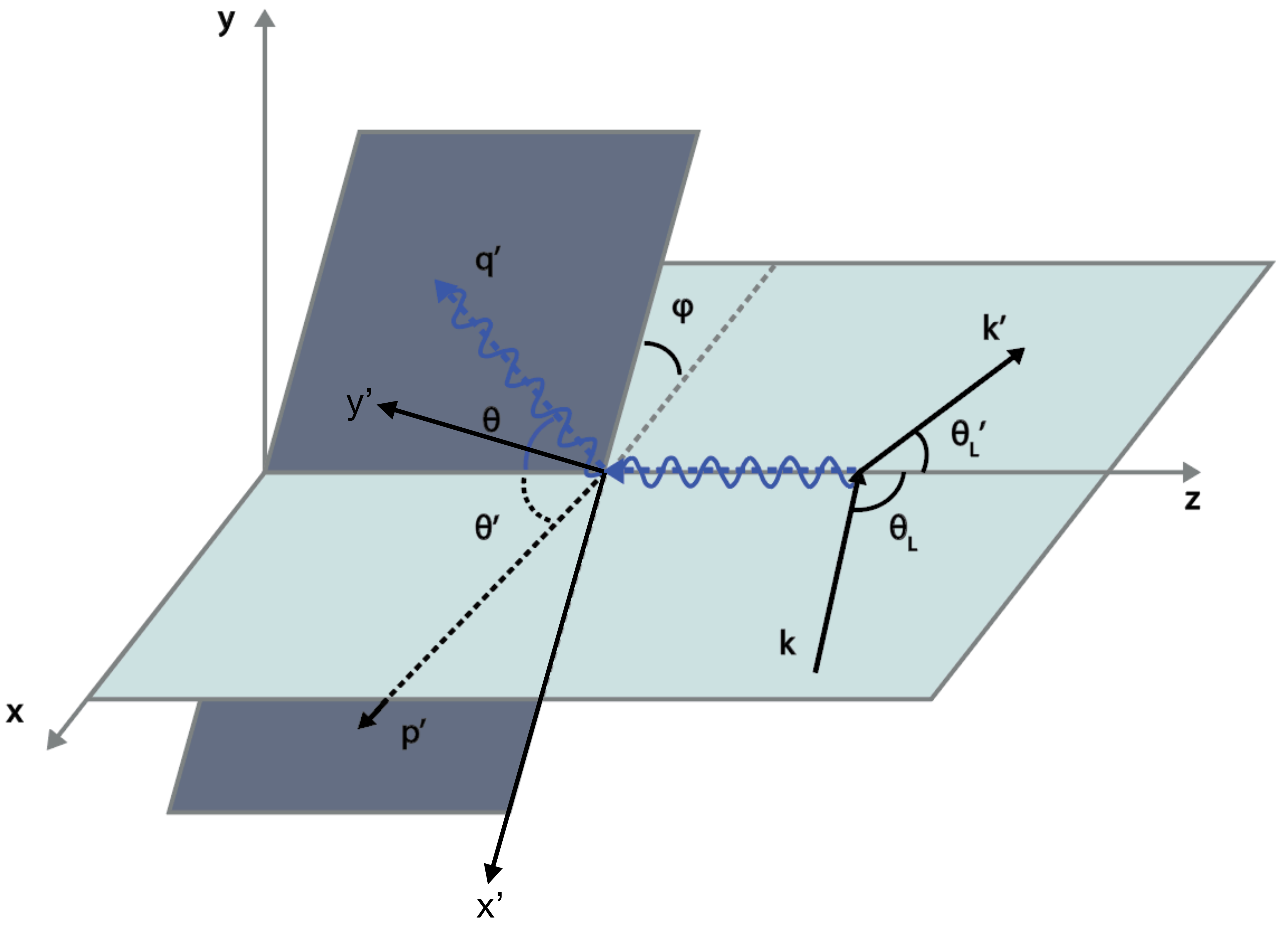
$$A_h^{\Lambda_{\gamma^*}}(k, k', q)$$



$$f_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}\Lambda'_{\gamma}}(q, p, q', p')$$

More interested in this piece

Setting up the kinematics



Virtual photon along z-axis means

$$\varepsilon_\nu^{\Lambda_{\gamma^*}=\pm1}(q) = \frac{1}{\sqrt{2}}(0, \pm1, -i, 0)$$

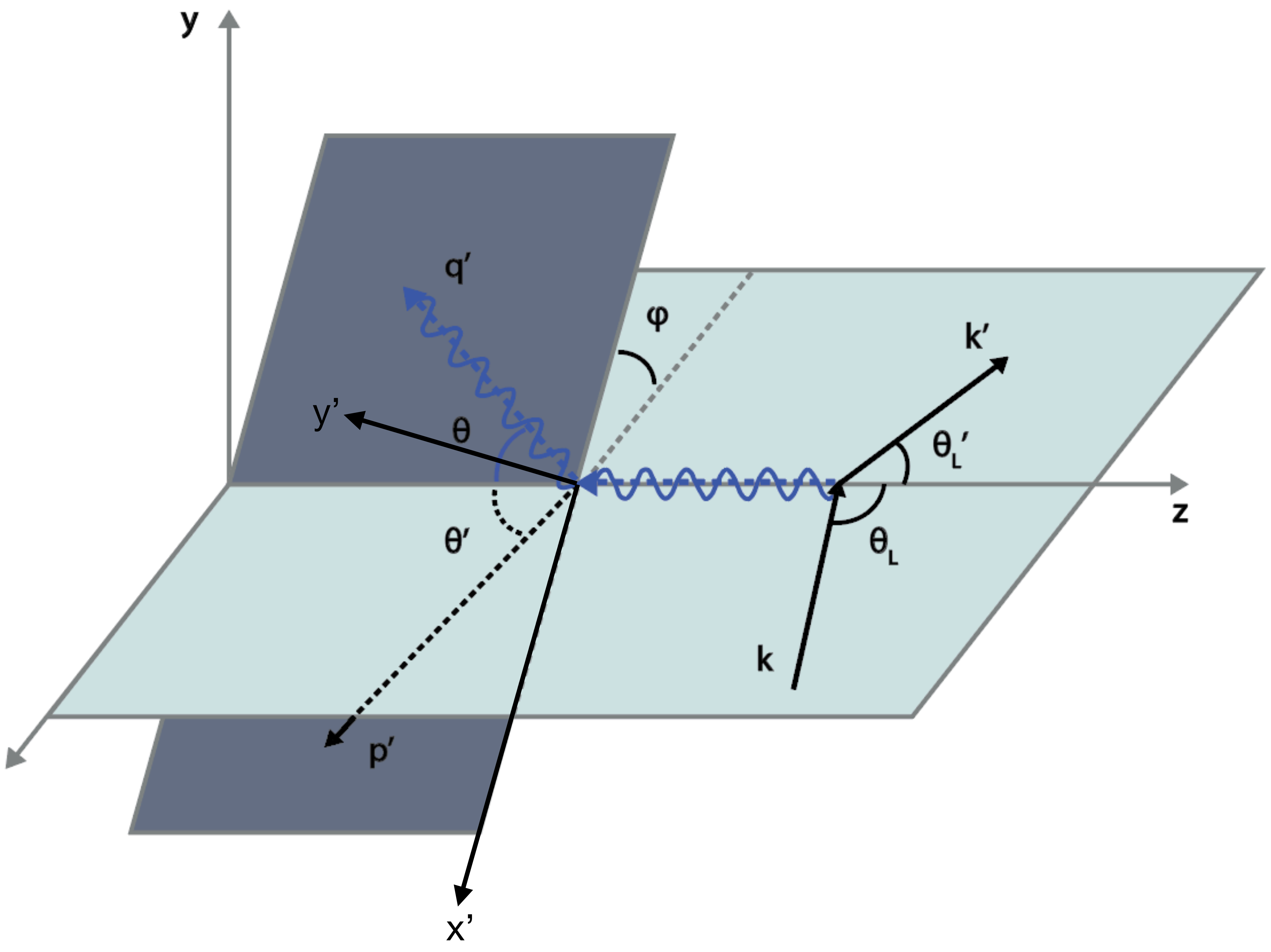
$$\varepsilon_\nu^{\Lambda_{\gamma^*}=0}(q) = \frac{1}{Q}\left(\sqrt{\nu^2+Q^2}, 0, 0, \nu\right)$$

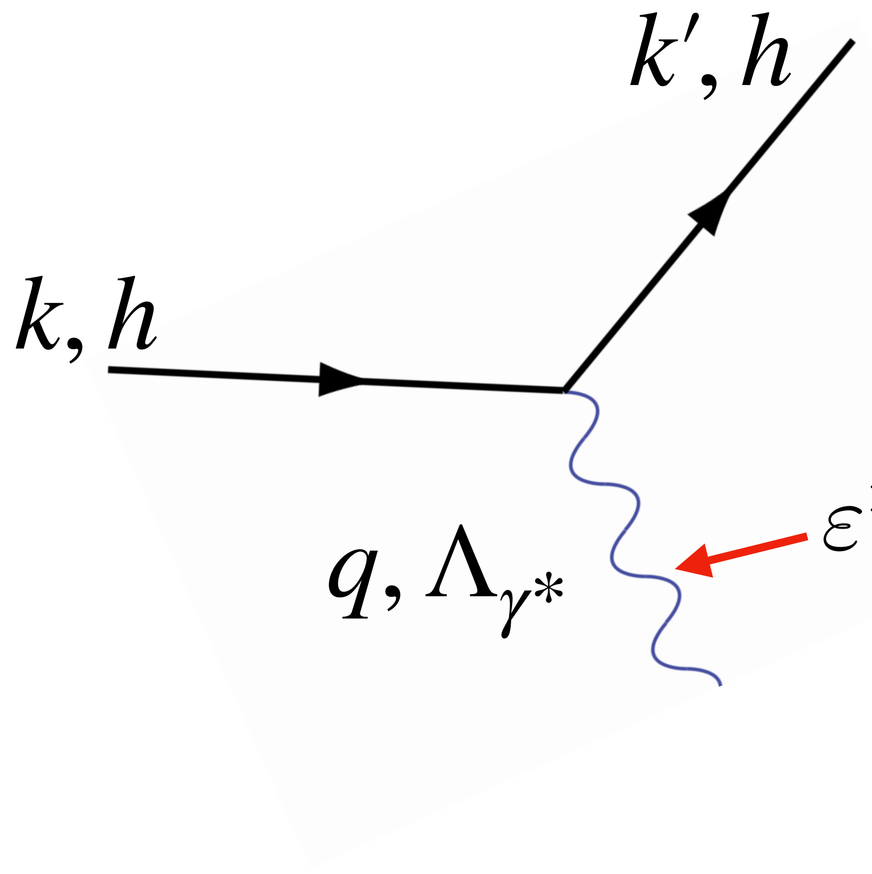
Easier to separate out leading twist contributions

Lepton plane (x,y,z) and Hadron plane (x',y',z') is rotated with respect to it

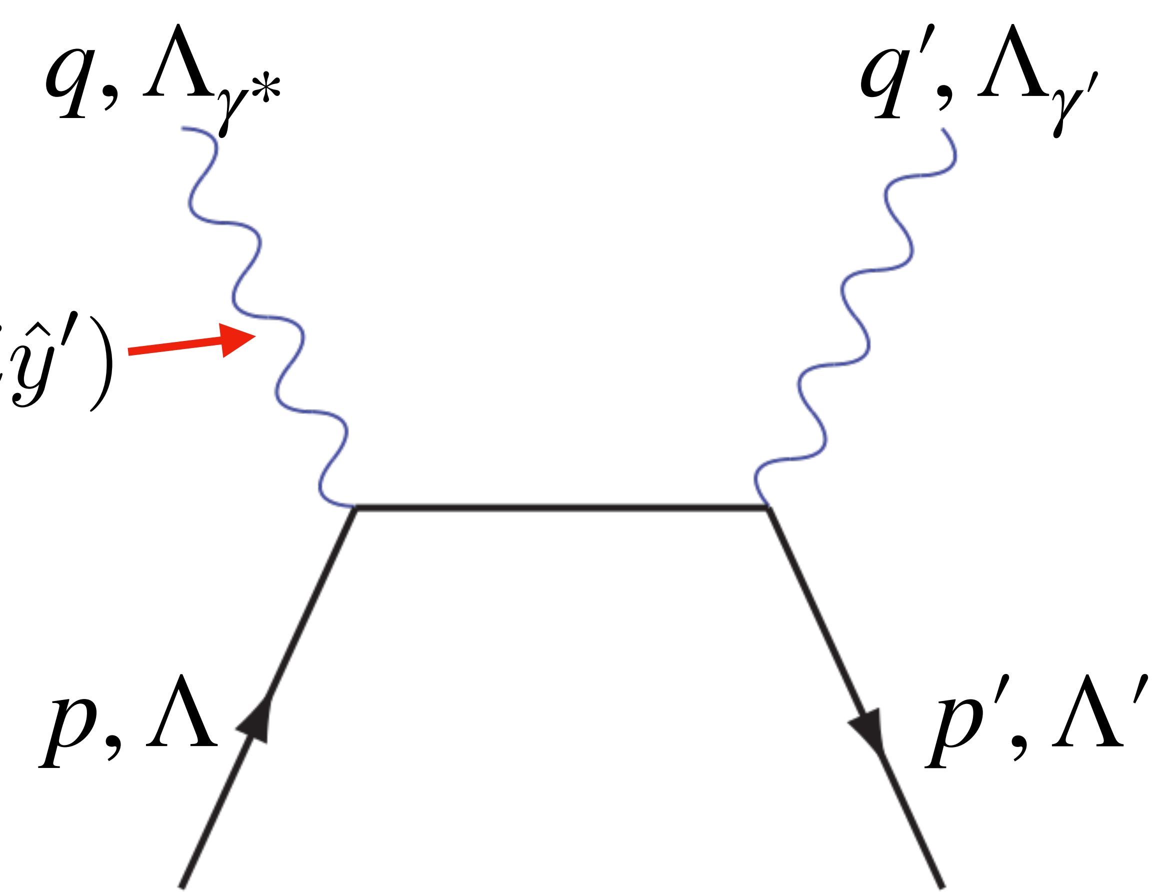
$$\varepsilon_\mu^{\Lambda_\gamma^*}(hadron) = e^{-i\Lambda_\gamma^*\phi}\, \varepsilon_\mu^{\Lambda_\gamma^*}(lepton)$$

Why we put in the phase?





$$\varepsilon^{\pm}(lepton) = \frac{1}{\sqrt{2}}(\pm\hat{x} - i\hat{y})$$



$$\varepsilon^{\pm}(hadron) = \frac{1}{\sqrt{2}}(\pm\hat{x}' - i\hat{y}')$$

We are doing the calculation in two different planes.
The rotation must be accounted for by the extra phase factor.

Phase difference is a signature of quantum interference

- For high energy QED Bremsstrahlung processes (e.g., $e^+e^- \to \gamma\gamma\gamma$ or $e^+e^- \to \mu^+\mu^-\gamma$), it is convenient to define two different planes for outgoing real photons, which leads to a phase difference.

  P. De Causmaecker, R. Gastmans, W. Troost, Tai Tsun Wu **Nuclear Physics B206 (1982)**

- In SIDIS, you also see a phase difference due to quantum interference, but this is governed by QCD.

  S. Brodsky, D. Hwang, I. Schmidt **Phys. Lett. B 530 (2002)**
  D. Sivers **PRD 41 (1990)**

Quantum interference and ϕ dependence

$$f_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}\Lambda_{\gamma'}} \propto e^{-i\Lambda_{\gamma^*}\phi}$$

$$\sigma_{h\Lambda} = \sum_{\Lambda'_\gamma,\Lambda'} \sum_{\Lambda^{(1)}_{\gamma^*}} \left[A_h^{\Lambda^{(1)}_{\gamma^*}} f_{\Lambda,\Lambda'}^{\Lambda^{(1)}_{\gamma^*},\Lambda'_\gamma} \right]^* \sum_{\Lambda^{(2)}_{\gamma}} A_h^{\Lambda^{(2)}_{\gamma^*}} f_{\Lambda,\Lambda'}^{\Lambda^{(2)}_{\gamma^*},\Lambda'_\gamma}$$

$$= \sum_{\Lambda'_\gamma,\Lambda'} \left(A_h^1 f_{\Lambda,\Lambda'}^{1,\Lambda'_\gamma} + A_h^{-1} f_{\Lambda,\Lambda'}^{-1,\Lambda'_\gamma} + A_h^0 f_{\Lambda,\Lambda'}^{0,\Lambda'_\gamma} \right)^* \left(A_h^1 f_{\Lambda,\Lambda'}^{1,\Lambda'_\gamma} + A_h^{-1} f_{\Lambda,\Lambda'}^{-1,\Lambda'_\gamma} + A_h^0 f_{\Lambda,\Lambda'}^{0,\Lambda'_\gamma} \right)$$

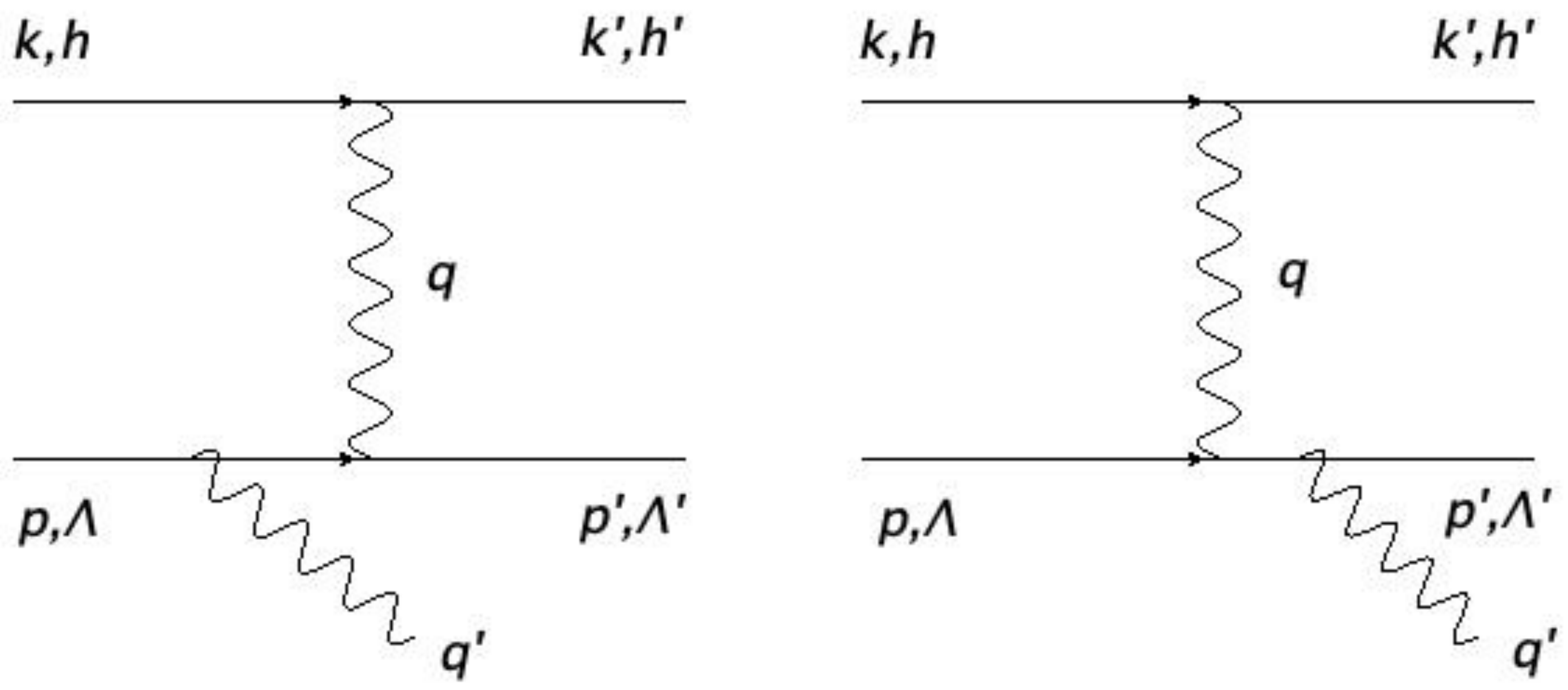
$$= \sum_{\Lambda'_\gamma,\Lambda'} \left(A_h^1\right)^2 \left| f_{\Lambda,\Lambda'}^{1,\Lambda'_\gamma} \right|^2 + \left(A_h^{-1}\right)^2 \left| f_{\Lambda,\Lambda'}^{-1,\Lambda'_\gamma} \right|^2 + \left(A_h^0\right)^2 \left| f_{\Lambda,\Lambda'}^{0,\Lambda'_\gamma} \right|^2$$

$$+ A_h^1 A_h^0 \left[\left(f_{\Lambda,\Lambda'}^{1,\Lambda'_\gamma} \right)^* f_{\Lambda,\Lambda'}^{0,\Lambda'_\gamma} + \left(f_{\Lambda,\Lambda'}^{0,\Lambda'_\gamma} \right)^* f_{\Lambda,\Lambda'}^{1,\Lambda'_\gamma} \right] + A_h^{-1} A_h^0 \left[\left(f_{\Lambda,\Lambda'}^{-1,\Lambda'_\gamma} \right)^* f_{\Lambda,\Lambda'}^{0,\Lambda'_\gamma} + \left(f_{\Lambda,\Lambda'}^{0,\Lambda'_\gamma} \right)^* f_{\Lambda,\Lambda'}^{-1,\Lambda'_\gamma} \right] \quad \cos(\phi) \text{ terms}$$

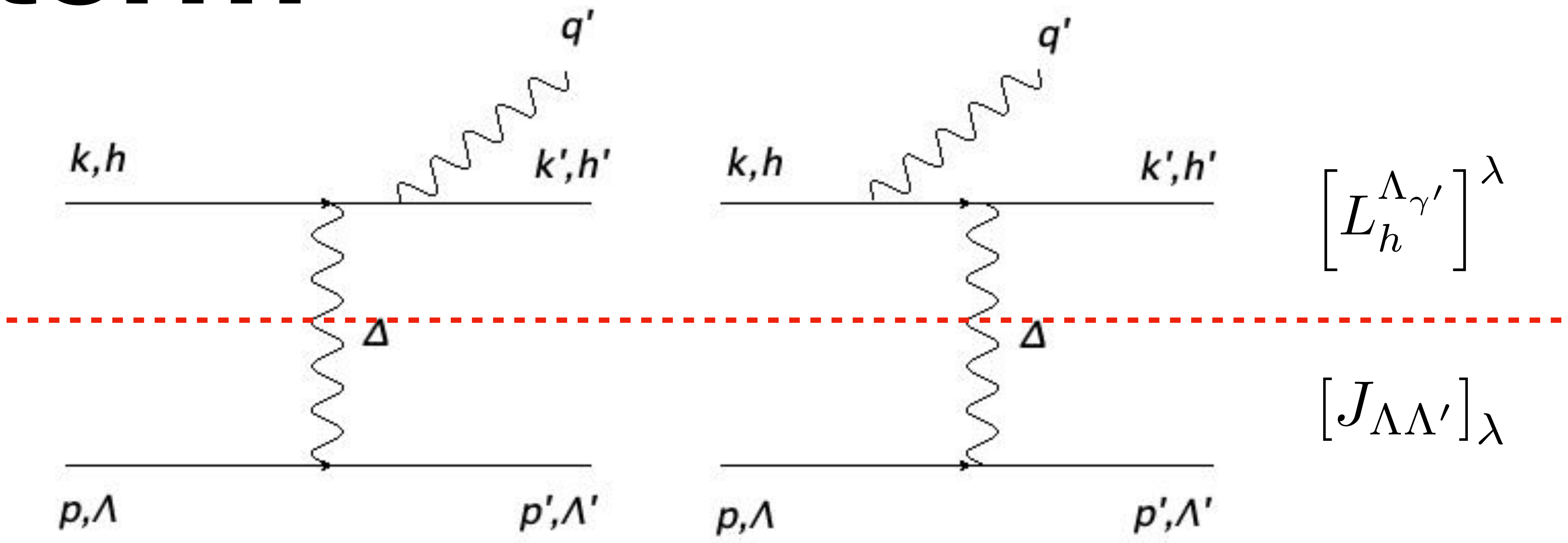
$$+ A_h^1 A_h^{-1} \left[\left(f_{\Lambda,\Lambda'}^{1,\Lambda'_\gamma} \right)^* f_{\Lambda,\Lambda'}^{-1,\Lambda'_\gamma} + \left(f_{\Lambda,\Lambda'}^{-1,\Lambda'_\gamma} \right)^* f_{\Lambda,\Lambda'}^{1,\Lambda'_\gamma} \right] . \quad \cos(2\phi) \text{ terms}$$

ϕ dependence due to quantum interference between transverse and longitudinal virtual photons.

Phase in interference term



DVCS



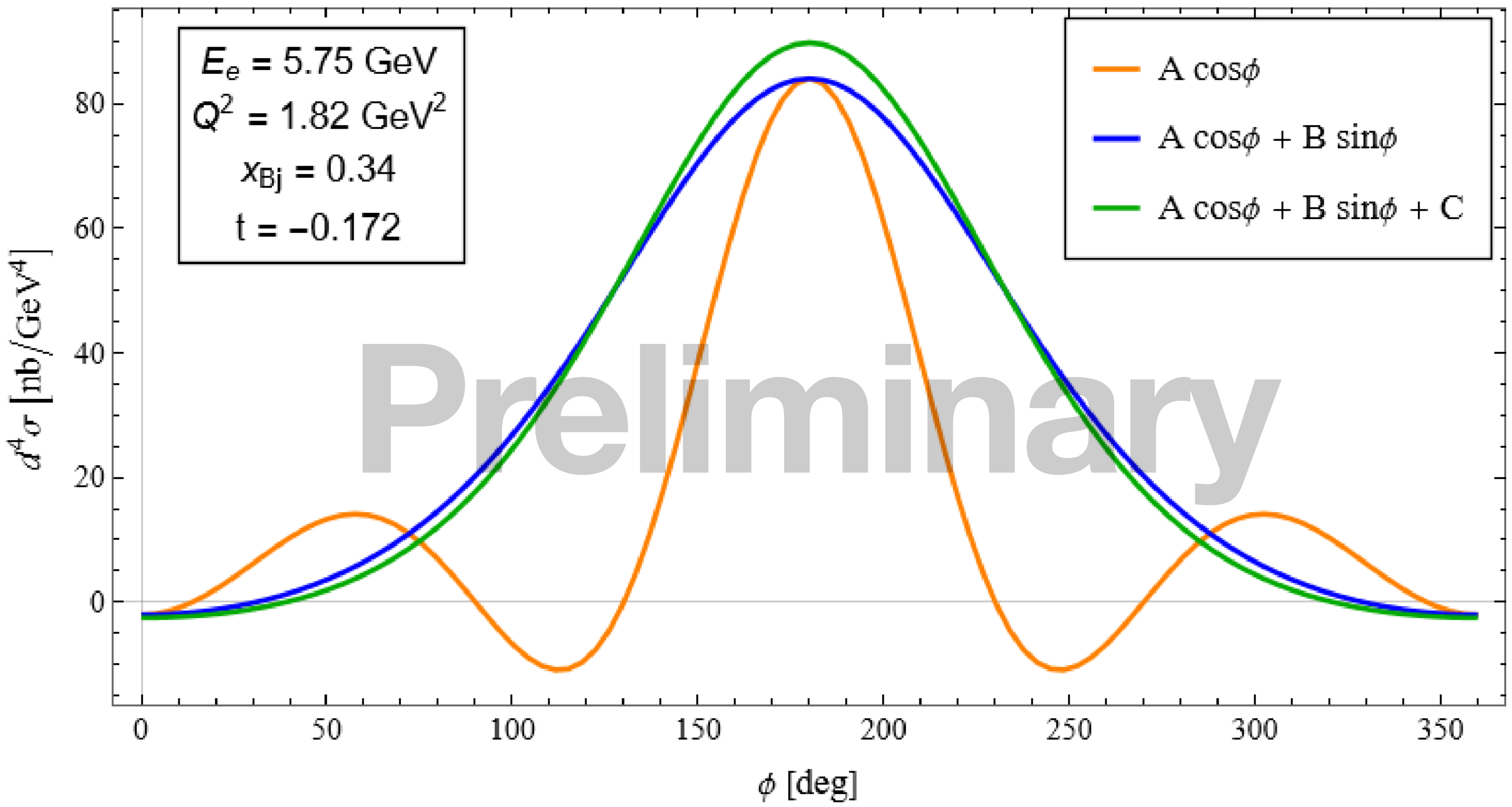
BH

$$T^*_{BH} T_{DVCS} = \frac{e^6}{\Delta^2} \sum_{\Lambda_{\gamma^*}} (-1)^{\Lambda_{\gamma^*}+1} A_h^{\Lambda_{\gamma^*}}(k,k',q)\, \boxed{f_{\Lambda\Lambda'}^{\Lambda_{\gamma^*}\Lambda_{\gamma'}}(q,p,q',p')} \left[L_h^{\Lambda_{\gamma'}}(k,k',q,q')\right]^\lambda [J_{\Lambda\Lambda'}(p,p')]^*_\lambda$$

$\propto e^{-i\Lambda_{\gamma^*}\phi}$

Unpolarized interference term proportional to $\cos(\phi)$ (Kriesten et al. 2020)

Interference term in pointlike case



Pointlike case has $sin(\phi)$ term…

Conjecture: $sin(\phi)$ terms proportional to masses.

Summary and Outlook

- A good understanding of the phase structure of the cross section is crucial for separating out twist-2 and twist-3 contributions and target mass corrections.
- To hone-in on the phase structure, we calculated the cross section as if the proton was a pointlike particle.
- This extends to deeply virtual exclusive processes for the deuteron or any other targets.
- **Beyond phase structure**
  - This sort of calculation is useful to relate and translate between different formalisms.
  - This can also be useful to study the phase structure of parity violating pieces in Bethe-Heitler, which can be a possible avenue for independent measurement of the W mass.