Isoscalar Parton Distribution with Disconnected Diagrams

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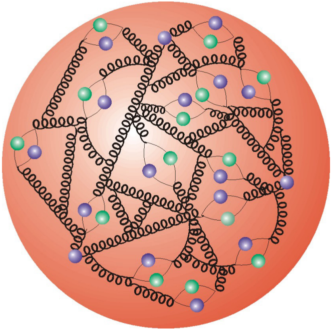
Theoretical Overview - PDFs

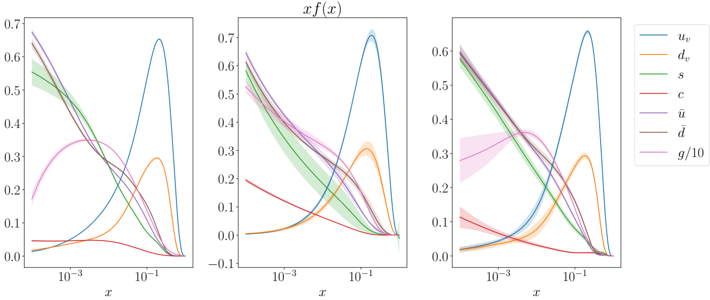
- At low energies, non-perturbative effects give rise to confinement
- What is the internal structure of these particles?
- How can we describe each parton's contribution to the macroscopic properties?



Parton Distribution Function (PDF)

Connection to Phenomenology

- PDF fits can be used to benchmark lattice calculations[1]
- Lattice calculations can be used in constraining global analysis PDFs[2]
- The isoscalar contribution is important in determining the up and down quark contributions independently



(Left) MSHT20 Eur. Phys. J. C 81 (2021) 4, 341. (Right) NNPDF4.0 Eur. Phys. J. C 82 (2022) 5, 428
(Center) JAM20 Phys. Rev. D 104 (2021) 1, 016015.

[1] Lin et al., "Parton distributions and lattice QCD calculations: A community white paper".

[2] Bringewatt et al., "Confronting lattice parton distributions with global QCD analysis".

Extraction from the Lattice

- PDFs are defined with light-like separations, $z^2 = 0$.
- Lattice is Euclidean
- Proposal to use space-like separations, with the Quasi-PDF approach.[3]

$$\mathcal{M}^{\alpha}(z,p) = \langle p| \, \overline{\psi}(z)\gamma^{\alpha} W(z,0)\psi(0) \, |p\rangle$$

These Matrix Elements can be related back to the PDFs[4]

$$\tilde{q}(x,\mu,p_z) = \int \frac{dz}{4\pi} \, e^{ixp_z z} \, \mathcal{M}(p,z)$$

$$\tilde{q}(y,p) = \int_{-1}^{1} \frac{dy}{|y|} \, Z\left(\frac{x}{y}\right) q(y) + \mathcal{O}\left(\frac{M_N^n}{(p_z)^n}\right) + \mathcal{O}\left(\frac{\Lambda_{QCD}^n}{(p_z)^n}\right)$$

[3] Ji, "Parton Physics on a Euclidean Lattice".

[4] Xiong, Luu, and Meißner, *Quasi-Parton Distribution Function in Lattice Perturbation Theory*.

Pseudo-PDF approach

Another approach is using the Pseudo-PDF[5]

$$\mathcal{M}^{\alpha}(\nu, z^2) = \langle p| \overline{\psi}(z)\gamma^{\alpha} W(z,0)\psi(0) |p\rangle$$

Expressing $\mathcal{M}$ as a function of the Lorentz invariants, ν and z^2.

By forming a double ratio[6], with local matrix elements and zero-momentum, z-dependence is reduced and has a well defined continuum limit.

$$\mathfrak{M}(\nu, z^2) = \left(\frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(\nu, z^2)|_{z=0}} \right) \Bigg/ \left(\frac{\mathcal{M}(\nu, z^2)|_{p=0}}{\mathcal{M}(\nu, z^2)|_{z,p=0}} \right)$$

The Ioffe Time Distribution is multiplicatively renormalizable[7]

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[5] Radyushkin, "Quasi-parton distribution functions, momentum distributions, and pseudo-parton distribution functions".

[6] Orginos et al., *Lattice QCD exploration of pseudo-PDFs*.

[7] Ishikawa et al., "Renormalizability of quasiparton distribution functions".

Gluon Matching Formula

The quark singlet channel and gluon channel mix in their evolution[8]

$$\begin{bmatrix} Q(\nu,\mu_2^2) \\ \nu G(\nu,\mu_2^2) \end{bmatrix} = \begin{bmatrix} Q(\nu,\mu_1^2) \\ \nu G(\nu,\mu_1^2) \end{bmatrix} - \frac{\alpha_s}{2\pi} ln \frac{\mu_2^2}{\mu_1^2} \int_0^1 du \begin{bmatrix} C_F K_{QQ}(u) & N_f K_{QG} \\ C_F K_{GQ}(u) & N_C K_{GG} \end{bmatrix} \begin{bmatrix} Q(u\nu,\mu_1^2) \\ \nu G(u\nu,\mu_1^2) \end{bmatrix}$$

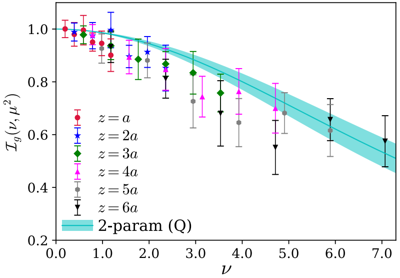
Calculations to date, ignore the mixing but estimate errors from the missing piece.

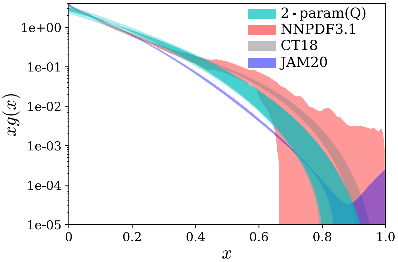
[8] Braun, Górnicki, and Mankiewicz, "Ioffe-time distributions instead of parton momentum distributions in the description of deep inelastic scattering".

Unpolarized Gluon PDF

Recent Extractions of Gluon PDFs from lattice[9]

- Calculations need higher statistics
- Gluon can improve signal-to-noise by applying gradient flow
- Extrapolated to $\tau \to 0$

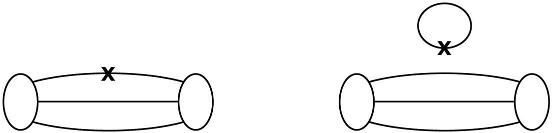




- Good agreement with global fits

[9] Khan et al., "Unpolarized gluon distribution in the nucleon from lattice quantum chromodynamics".

Isoscalar Quark Contribution



Wick Contractions of quarks

- In order to determine the contribution of this diagram, we consider the difference between just the connected piece and the combined (connected & disconnected) piece.

$$\mathcal{M} = \sum_{sc} \langle \bar{\psi}\bar{\psi}\bar{\psi} |\, \bar{\psi}_z^{sc} W(z,0) \psi_0^{sc} \,| \psi\psi\psi \rangle$$

- The disconnected loop corresponds to a Wick Contraction from the inner quark bilinear

$$\sum_{sc} \overline{\psi}_z^{sc} W(z,0)\psi_0^{sc} \to Tr[WD_{0z}^{-1}]$$

Estimation Techniques

Distillation:

- Smearing long time practice in LQCD
- Projects operators onto low modes (ground state)
- Separation of quark propagators and hadron interpolating fields
- Many correlation functions at fixed cost
- Statistical accuracy can be enhanced[10]

Deflation:

- Multigrid approach
- Low singular values dominate variance of the trace estimation
- Speed up over plain Hutchinson by up to 2 orders of magnitude[11]

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[10] Peardon et al., "Novel quark-field creation operator construction for hadronic physics in lattice QCD".

[11] Romero, Stathopoulos, and Orginos, "Multigrid deflation for Lattice QCD".

Estimating the Trace

- A^{-1} is not known explicitly
- Using Hutchinson Method

$$Tr(A^{-1}) \approx \frac{1}{s}\sum_{i=1}^{s} z_i^T A^{-1} z_i,$$

$$Var(z^T A^{-1} z) = 2(||A^-1||_F^2 - \sum_{i=1}^{N}(A_{i,i})^2)$$

Rademacher distribution

- Elements of z_i are chosen to be ± 1 randomly

$$z_1 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \quad z_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$$\langle z_i^{(k)} z_i^{(k`)} \rangle = \delta_{k,k`}$$

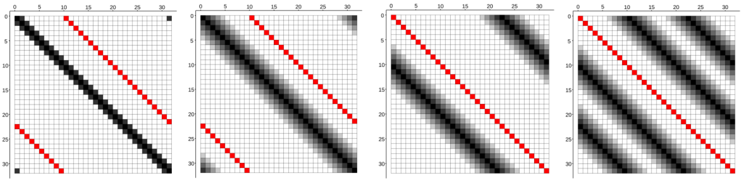
Coloring

- Process of separating z_i to cover independent elements (probing)
- Estimation of block diagonal A^{-1}
- Corresponds to setting elements of z_i's to 0.

[11] Hutchinson, "A stochastic estimator of the trace of the influence matrix for laplacian smoothing splines".

Coloring Algorithm

- Neumann series suggest A^{-1} dominated by low powers of A
- The disconnected diagram requires the trace of an off diagonal A^{-1} that corresponds to a displacement, d.
- This is equivalent to a trace of a shifted (permuted) matrix, PA^{-1}, so that the required off-diagonal elements are on the main diagonal.

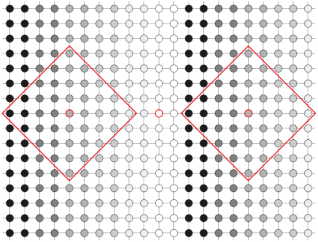


(a) Matrix A, 1D torus (b) Matrix of A^4 (c) Displace by 10 (d) Symmetrized

Producing PA^{-1} for trace estimation[12]

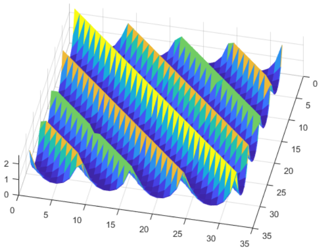
[12]Switzer et al., *Probing for the Trace Estimation of a Permuted Matrix Inverse Corresponding to a Lattice Displacement*.

Coloring Algorithm (cont.)

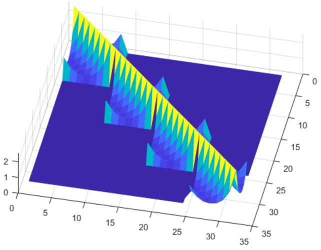


By considering the structure of the permuted matrix, variances are reduced

- The method computes a coloring of the points where neighbors of x are points within a distance-k around centers $x \pm d$.



Permuted into color-blocks



Block-diagonal A^{-1} after probing

[12] Switzer et al., Probing for the Trace Estimation of a Permuted Matrix Inverse Corresponding to a Lattice Displacement.

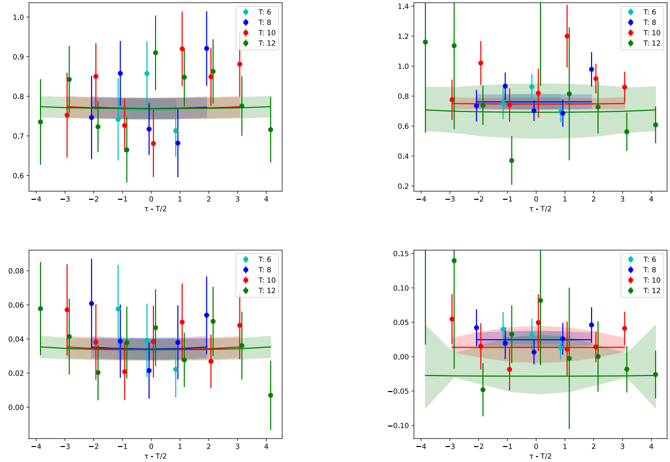
Fitting Procedure

$$\frac{C_{3pt}(T,\tau,z,p)}{C_{3pt}(T,\tau,z,p)} \approx \mathcal{M}_{00} + \tilde{\mathcal{M}}_{11} e^{-\Delta E_p T} + \tilde{\mathcal{M}}_{01}\left(e^{-\Delta E_p \tau} + e^{-\Delta E_p (T-\tau)}\right)$$

$p = 1$ $\qquad\qquad$ $p = 3$

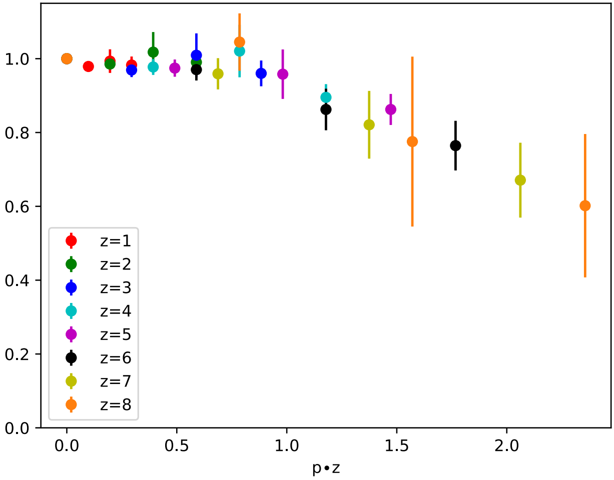
$z = 1$

$z = 8$



$m_\pi = 358(3)$ $\qquad$ $V = 32^3 x64$ $\qquad$ $\beta = 6.3$

Pseudo Ioffe Time Distribution



- Currently working to extend p
- Increasing statistics

Conclusion

- Disconnected Diagrams are very noisy
- Must take into account structure of the Matrix for Trace Estimation
- Need much higher statistics to accurately determine
- Currently working on increasing statistics to improve estimations

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