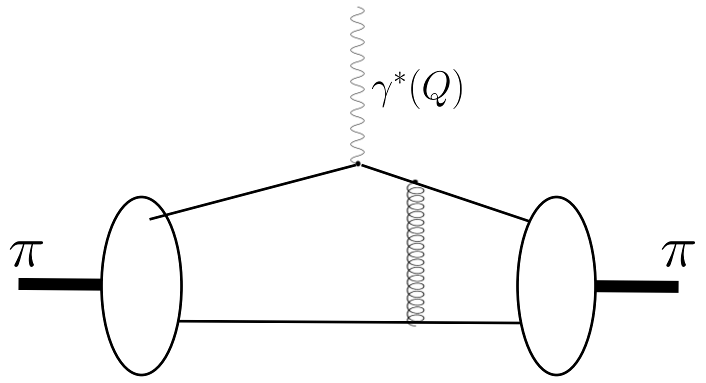
Pion Distribution Amplitude From Lattice QCD using Pseudo-Distributions

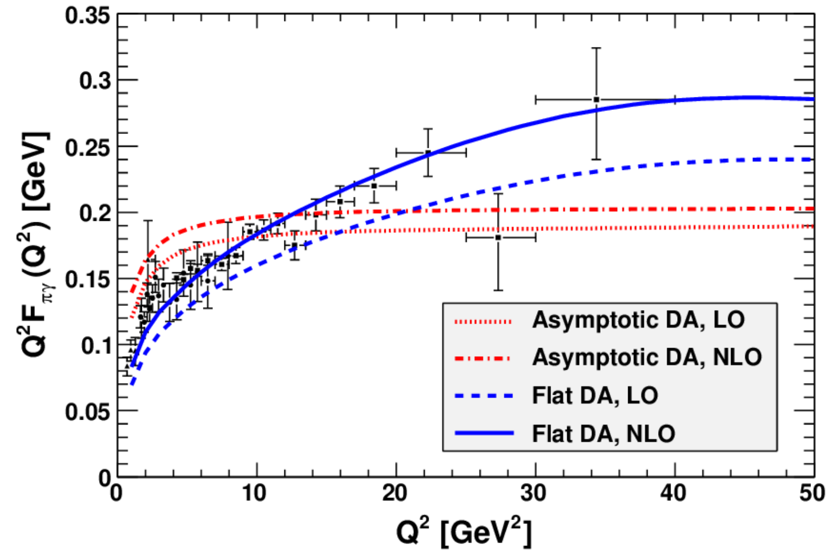
Daniel Kovner (W&M)

On behalf of the HadStruc Collaboration

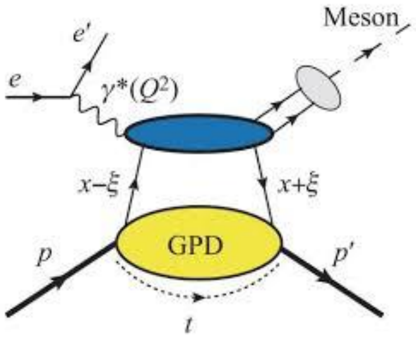
Exclusive Processes in QCD



- Electromagnetic form factor <==> charge radius
- Distribution Amplitude (DA): hadron-to-vacuum matrix element. "Longitudinal part of Wave Function".
- pQCD: $\phi(x, \mu \to \infty) = 6x(1-x)$

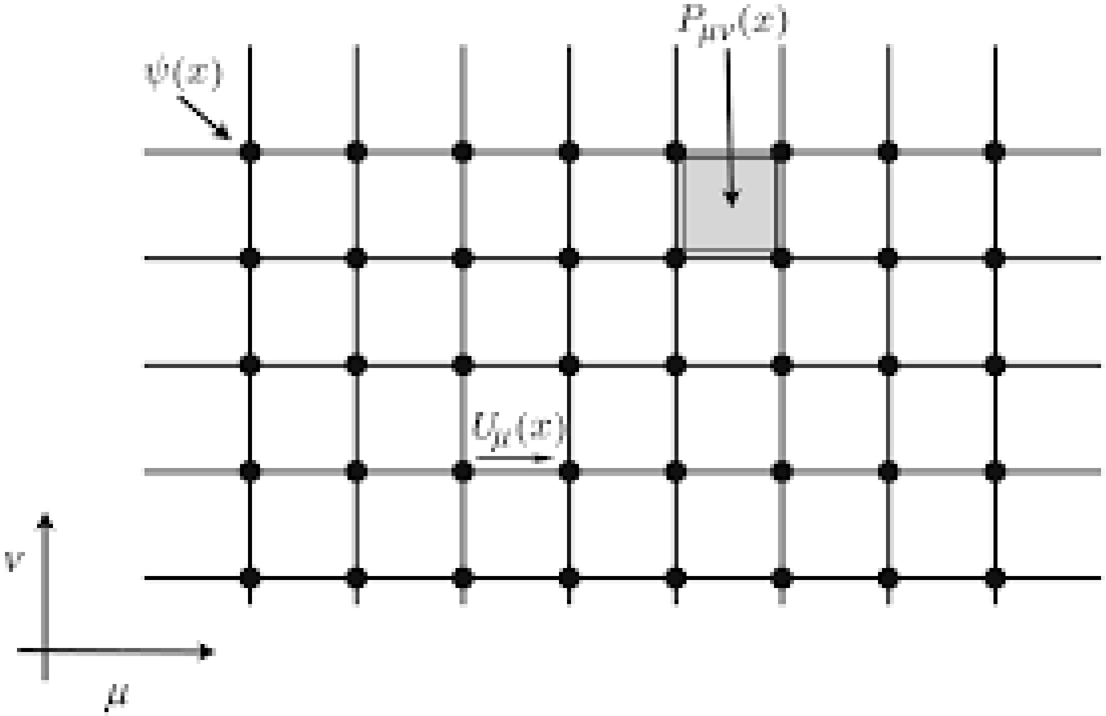


- Tension between experiments and pQCD
- Deeply Virtual Meson Production: DAs required to constrain GPDs.



Lattice QCD

- *Wick Rotate to Imaginary Time*
- D.O.F. are fermions and gauge links.
- Path Integral ==> Partition Function, amenable to numerical methods
- Various choices of lattice actions
- Some side effects may include:
  - Lattice Spacing
  - Matrix inversions are hard ==>work at heavier pion masses and extrapolate to physical point
  - Finite Volume
  - Rotational Symmetry reduction to hypercubic group
  - Chiral symmetry breaking at finite lattice spacing



[DOI: 10.1007/978-3-030-67235-5_1]

$$U_\mu(x) = \exp\Big(iaA_\mu(x)\Big)$$

$$\frac{1}{Z}\int \mathcal{D}[\overline{\psi},\psi,U]\ \exp\left(-S[\overline{\psi},\psi,U]\right)\mathcal{O}(\overline{\psi},\psi,U)$$

$$\approx \frac{1}{N}\sum_{i=1}^{N}\mathcal{O}(\overline{\psi}_i,\psi_i,U_i)$$

~Distributions

$$M^\alpha(p,z) = \langle 0|\overline{\psi}(0)\gamma^\alpha\gamma^5 W[0,z]\psi(z)|\pi(p)\rangle$$

$$M^\alpha(p,z) = 2p^\alpha \mathcal{M}(\nu, z^2) + z^\alpha \mathcal{N}(\nu, z^2)$$

$\nu = p \cdot z$ "Ioffe Time"

- Euclidean time ==> light-like separations not available!
- Approach: Space-like matrix element with same IR behavior as LCDA.

- Quasi-DA/LaMET: large momentum matching

$$\overline{\phi}(x, P_z) = \int \frac{d\nu}{2\pi}\, e^{ix\nu}\mathcal{M}_R\left(\nu, \frac{\nu^2}{P_z^2}\right)$$

$$\overline{\phi}(x) = \int_0^1 Z^{-1}(x, y, P_z, \mu)\phi(y, \mu)\, dy$$

$$+\mathcal{O}\left(\frac{\Lambda^2_{QCD}}{P_z^2}\right)$$

[J. Zhang, et. al. Phys. Rev. D 95, 094514 (2017)]

- Pseudo-DA: short distance matching

$$\Phi(x, z^2) = \int \frac{d\nu}{2\pi} e^{ix\nu}\mathcal{M}_R(\nu, z^2)$$

$$\mathcal{M}_R(\nu, z^2) = \int_0^1 R^{-1}(x, \nu, z^2\mu^2,)\phi(x, \mu^2)\, dx$$

$$+\mathcal{O}(z^2\Lambda^2_{QCD})$$

[A.Radyushkin, Phys. Rev. D 100, 116011 (2019)]

Pseudo-Distributions

$$M^\alpha(p,z) = \langle 0|\overline{\psi}(0)\gamma^\alpha\gamma^5 W[0,z]\psi(z)|\pi(p)\rangle$$

$$M^\alpha(p,z) = 2p^\alpha \mathcal{M}(\nu,z^2) + z^\alpha \mathcal{N}(\nu,z^2)$$

$\nu = p\cdot z$ "Ioffe Time"

- Caveat(s):
  - Center operator at origin for convenience

$$\widetilde{\mathcal{M}}(\nu,z^2) = e^{-\frac{i\nu}{2}}\mathcal{M}(\nu,z^2)$$

  - Results in a REAL matrix element <==> symmetric DA
  - UV divergence from Wilson Line: must renormalize!
  - Use RGI ratio:

$$\widetilde{\mathfrak{M}}(\nu,z^2) = \frac{\widetilde{\mathcal{M}}(\nu,z^2)}{\widetilde{\mathcal{M}}(\nu',z^2)}$$

[X. Gao, Phys.Rev.D 102 (2020)]

$$\widetilde{\mathfrak{M}}(\nu,z^2) = \int_0^1 K^{-1}(x,\nu,z^2\mu^2)\phi(x,\mu)\,dx \;+\mathcal{O}(z^2\Lambda_{QCD}^2)$$

[A.Radyushkin, Phys. Rev. D 100, 116011 (2019)]

Matrix Element Extraction

E5 CLS ensemble: [G. Engel et. al. Phys. Rev. D 91, 054505 (2015)]

- 2 Flavour
- O(a) improved Wilson fermion action

N_s^3 x N_t	Ncf	a(fm)	m_π(MeV)	F_π(MeV)
32^3 x 64	999	0.0652(6)	440(5)	115.2(6)

$$C^{\Gamma\gamma}_{\mathbf{p},\mathbf{z}}(t) = \langle \sum_{\mathbf{x},\mathbf{y}} e^{-i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \mathcal{O}^{\Gamma}_{\mathbf{x},\mathbf{z}}(t) \overline{\mathcal{O}}^{\gamma}_{\mathbf{y}}(0) \rangle$$

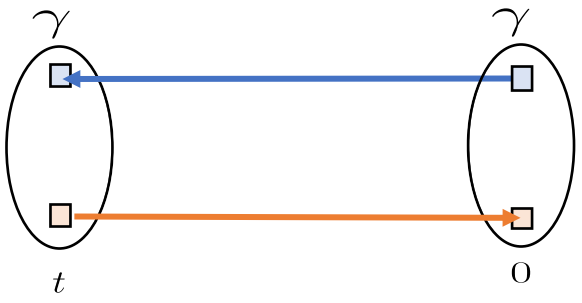
$$C^{\Gamma\gamma}_{\mathbf{p},\mathbf{z}}(t) = \sum_n \frac{M^{\alpha}_n(p,z) Z^*_n(\mathbf{p})}{2E_n(\mathbf{p})} e^{-E_n(\mathbf{p})t}$$

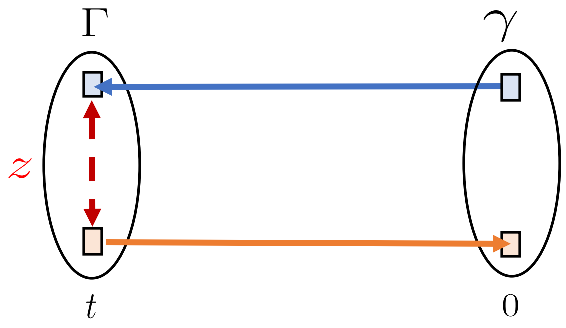
$$\gamma \in \{\gamma^4\gamma^5, \gamma^5\} \qquad \Gamma = \gamma^3$$

$$\mathcal{O}^{\gamma}_{\mathbf{x}}(t) = \overline{d}(\mathbf{x},t)\gamma u(\mathbf{x},t) \qquad \mathcal{O}^{\Gamma}_{\mathbf{x}\mathbf{z}}(t) = \overline{d}(\mathbf{x},t)\Gamma\gamma^5 W[\mathbf{x},\mathbf{z}]u(\mathbf{x}+\mathbf{z},t)$$

$$R^{\Gamma,\gamma}_{p,z}(t) = \frac{C^{\Gamma,\gamma}_{\mathbf{p},\mathbf{z}}(t)}{C^{\Gamma,\gamma}_{\mathbf{p},\mathbf{0}}(t)}$$

$$\sim \frac{M^{\alpha}(p,z)}{M^{\alpha}(p,0)} + A_{\mathbf{p},\mathbf{z}} e^{-\Delta E_{\mathbf{p}} t}$$



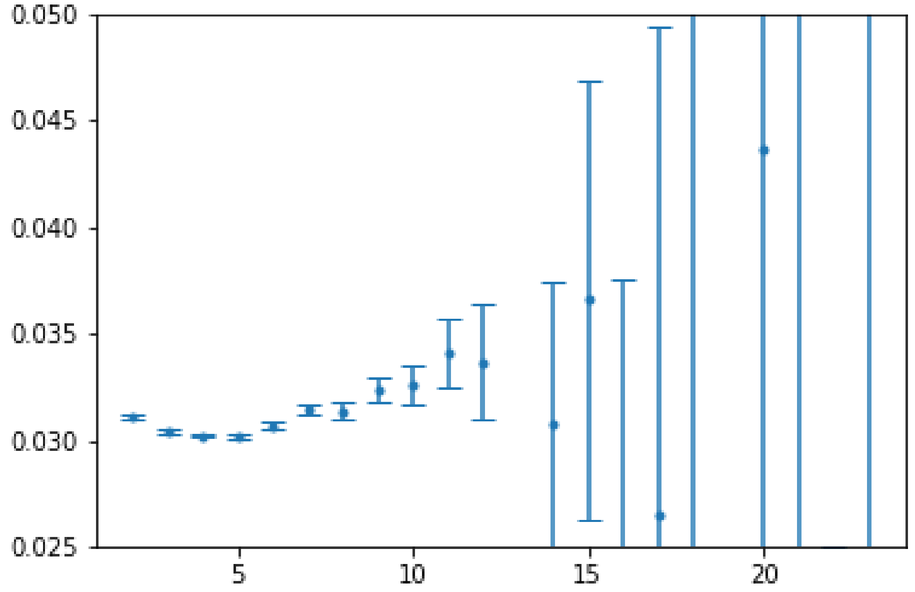


Matrix Element Extraction

$$R^{\Gamma,\gamma}_{\mathbf{p},\mathbf{z}}(t) = M^{\Gamma}_{\mathbf{p},\mathbf{z}} + A^{\gamma}_{\mathbf{p},\mathbf{z}} e^{-\Delta E_{\mathbf{p}} t}$$

- Fit with care:
  - Excited-state contamination at early times.
  - Noise for moderate-to-late times.
- Treatment of fit-range systematics:
  - Removed data points are now parameters
  - Marginalizing out these parameters results in weighted averages over models.

[W. Jay, E. Neil, Phys. Rev. D 103, 114502 (2021)]

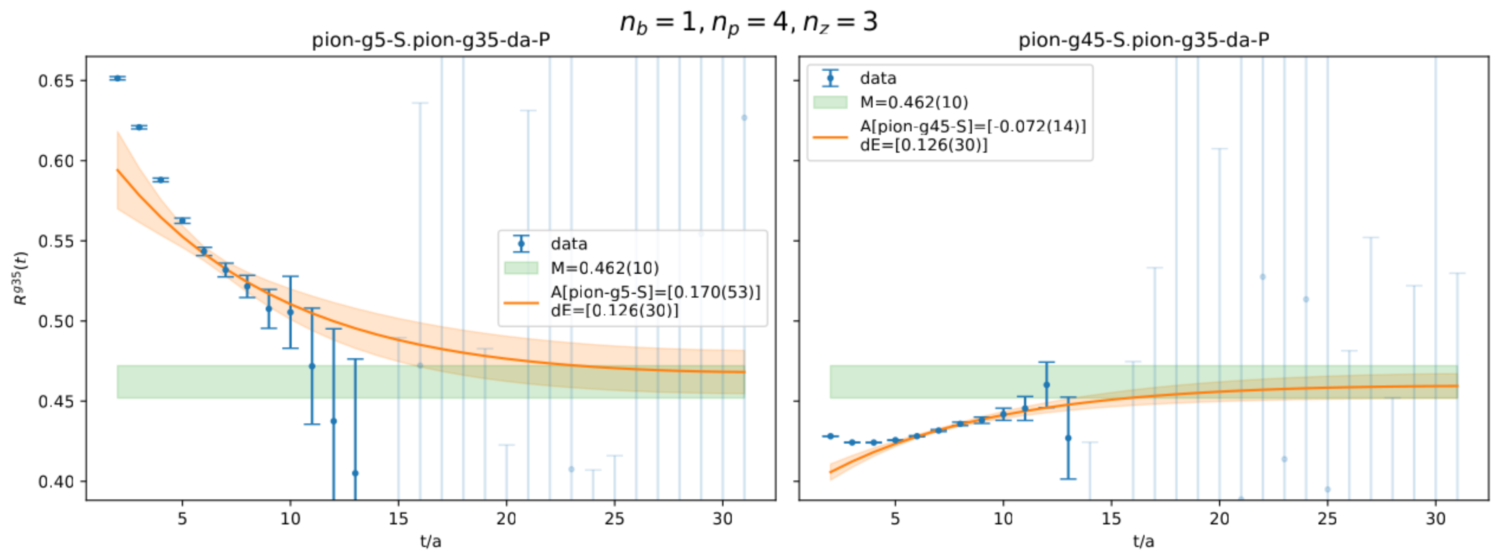


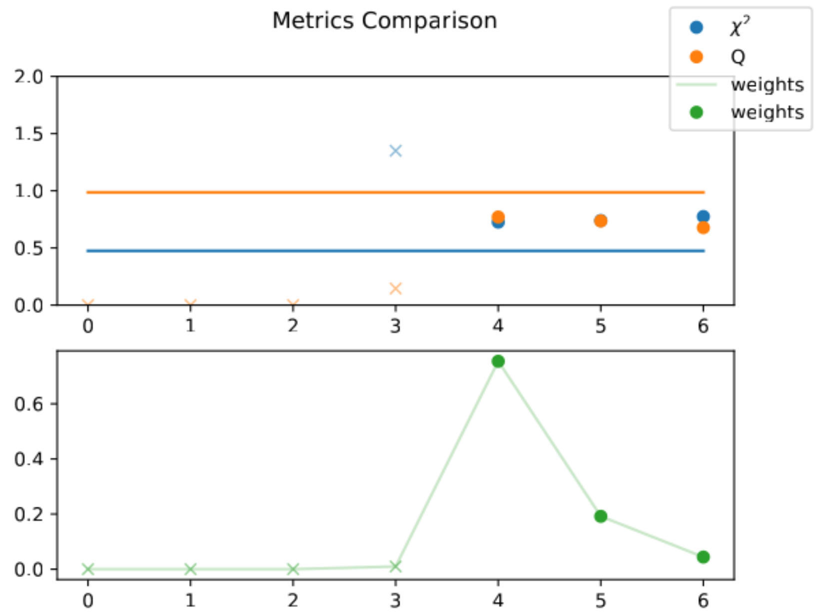
$$\langle f(\theta)\rangle = \sum_i f(\theta_i^*)\mathrm{pr}(M_i|y)$$

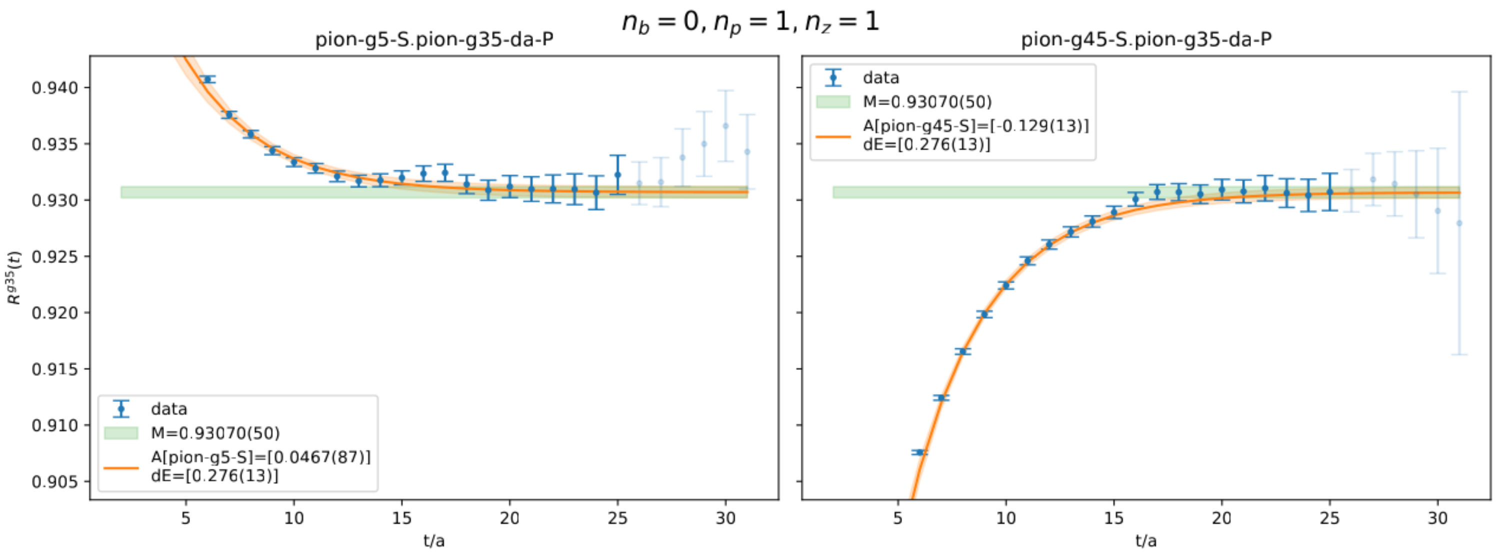
$$\mathbf{C}_{\theta\phi} = \langle\theta\phi\rangle - \langle\theta\rangle\langle\phi\rangle$$

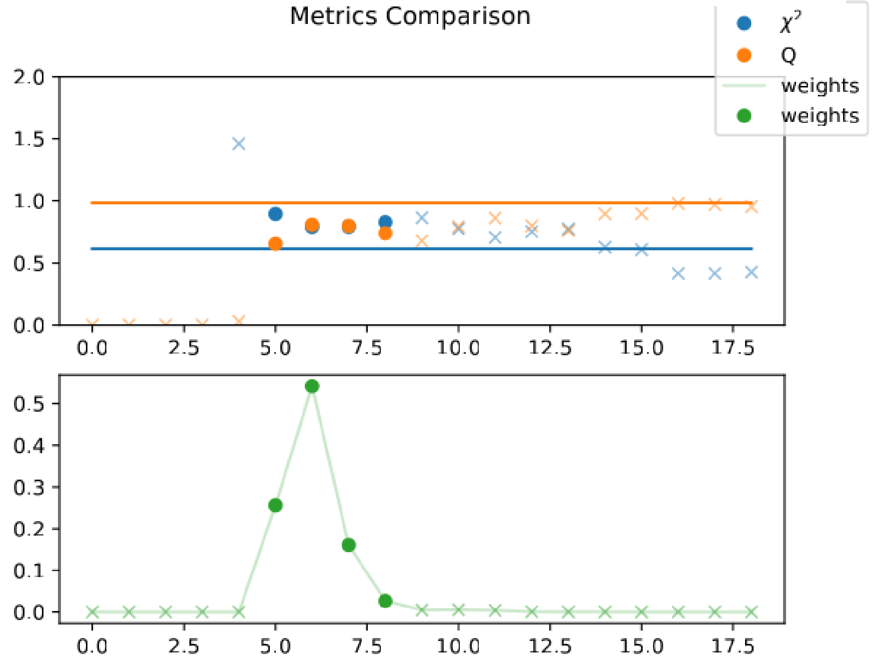
$$-2\ln \mathrm{pr}(M_i|y) = \chi^2_{aug}(y_{keep}) + 2(k + N_{cut})$$

Matrix Element Extraction









PRELIMINARY

Matrix Element Extracted

PRELIMINARY

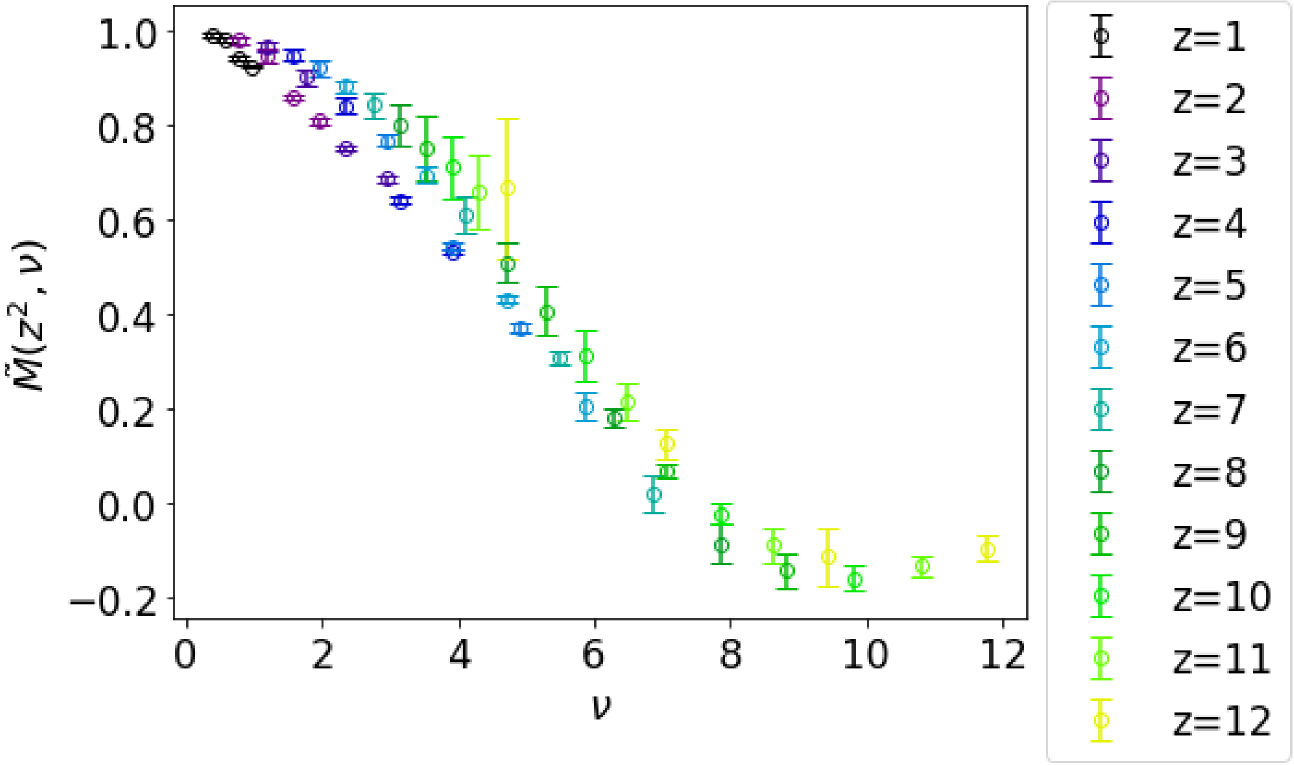
- Inverting factorization relation directly is ill-defined ==> physically motivated parametrization (WIP)

$$\widetilde{\mathfrak{M}}(\nu, z^2) = \int_0^1 K^{-1}(x, \nu, z^2\mu^2)\phi(x, \mu)\, dx$$

$$\alpha_s(\mu) \ln{(z^2\mu^2)} V^{\mathrm{ERBL}}(x, \nu)$$

$$\alpha_s(\mu) V^{\overline{\mathrm{MS}}}(x, \nu)$$

$$+\mathcal{O}(z^2\Lambda_{QCD}^2)$$



Conclusions and Future Work (WIP)

- DA is relevant for nuclear physics @ JLab and EIC, its importance precludes lattice calculations.
- Estimating systematic uncertainties is important: BMA handles this in a rigorous quantitative way.
- TODO: Need to remove lattice regulator as best as we can:
  - Repeat calculation at several lattice spacings, fit away lattice spacing effects. (Ongoing Effort)
  - Control excited state contamination: Distillation and GEVP.
  - Use of BMA to handle model dependence in the approach to the inverse problem.

Thank you!

Acknowledgements

- *D.Kovner acknowledges financial support from The Gordon and Betty Moore Foundation and the American Physical Society to present this work at the GHP 2023 workshop.*