Proton GPDs from lattice QCD with novel methods

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In collaboration with:

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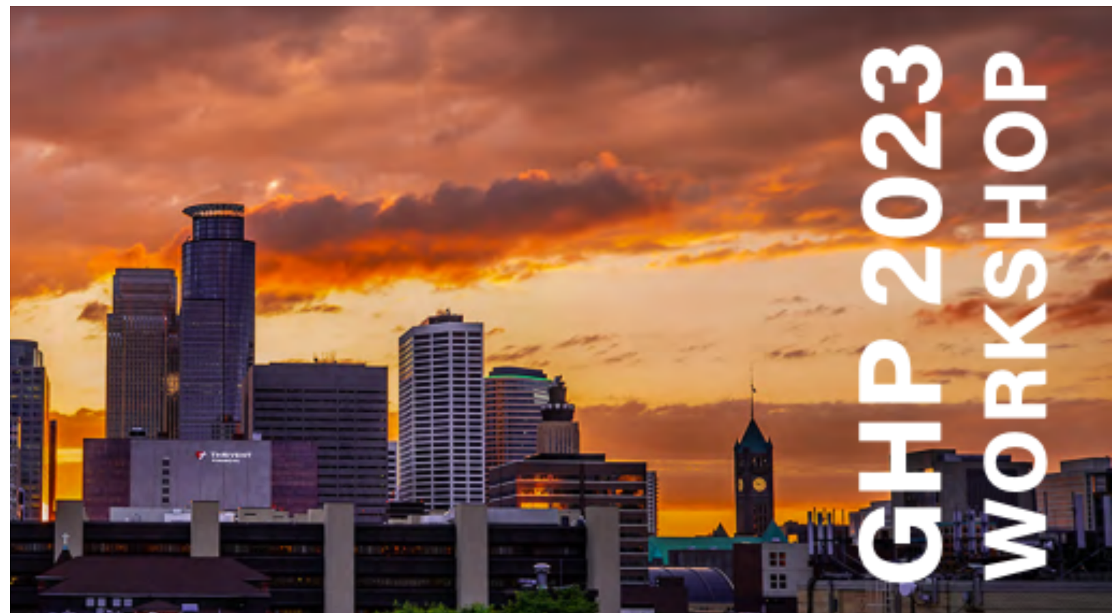

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Lattice QCD calculations complement the theoretical and experimental efforts

Theoretical Setup

❖ GPDs defined from off-forward matrix elements of non-local operators on the light-cone

$$F^{[\gamma^+]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle p';\lambda' | \bar{\psi}(-\tfrac{z}{2})\,\gamma^+\,\mathscr{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2}) | p;\lambda\rangle \Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

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❖ Parameterization in two leading twist GPDs

$$F^{[\gamma^+]}(x,\Delta;\lambda,\lambda') = \frac{1}{2P^+}\bar{u}(p',\lambda')\left[\gamma^+ H(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}E(x,\xi,t)\right]u(p,\lambda)$$

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No finite mixing on the lattice
[Constantinou & Panagopoulos (2017)]

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A NEW IDEA

Develop a different parameterization to access GPDs from LQCD

Do this for a broad range of $-t$ and ξ with realistic computational resources

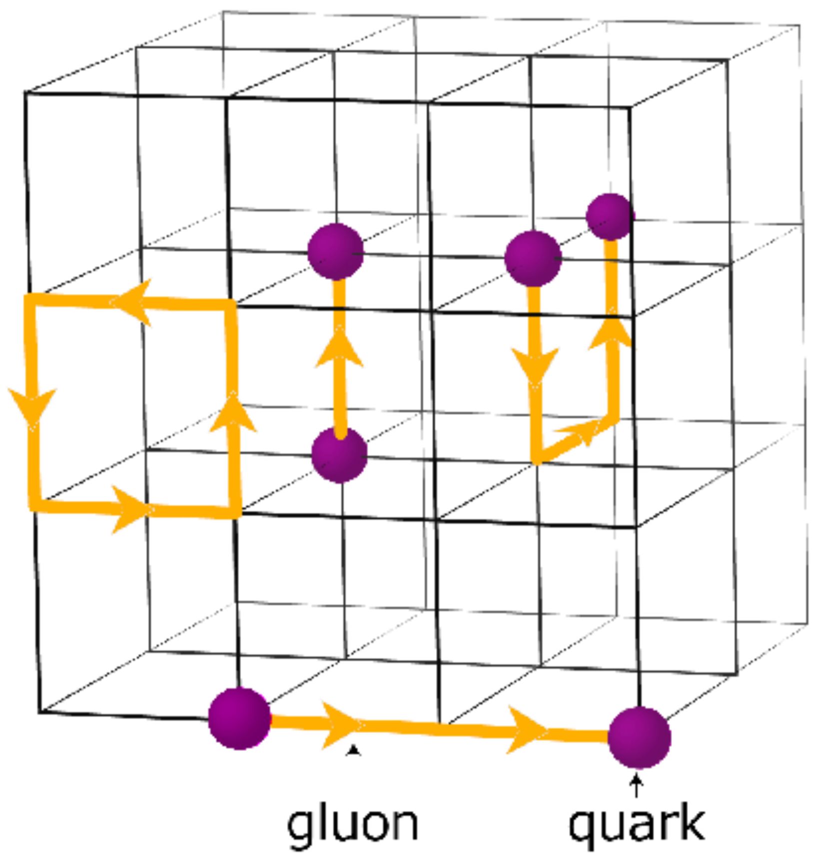
• *Phys.Rev.D* 106 (2022) 11, 114512 • e-Print: 2209.05373

GPDs from Lattice QCD

❖ Direct access to partonic distributions impossible in LQCD:

❖ PDFs/GPDs/TMDs are defined on the light cone, that is: $t^2 - \vec{r}^2 = 0$

❖ LQCD is a Euclidean formulation (Wick rotation, $t \to i\tau$) and light cone: $\tau^2 + \vec{r}^2 = 0$



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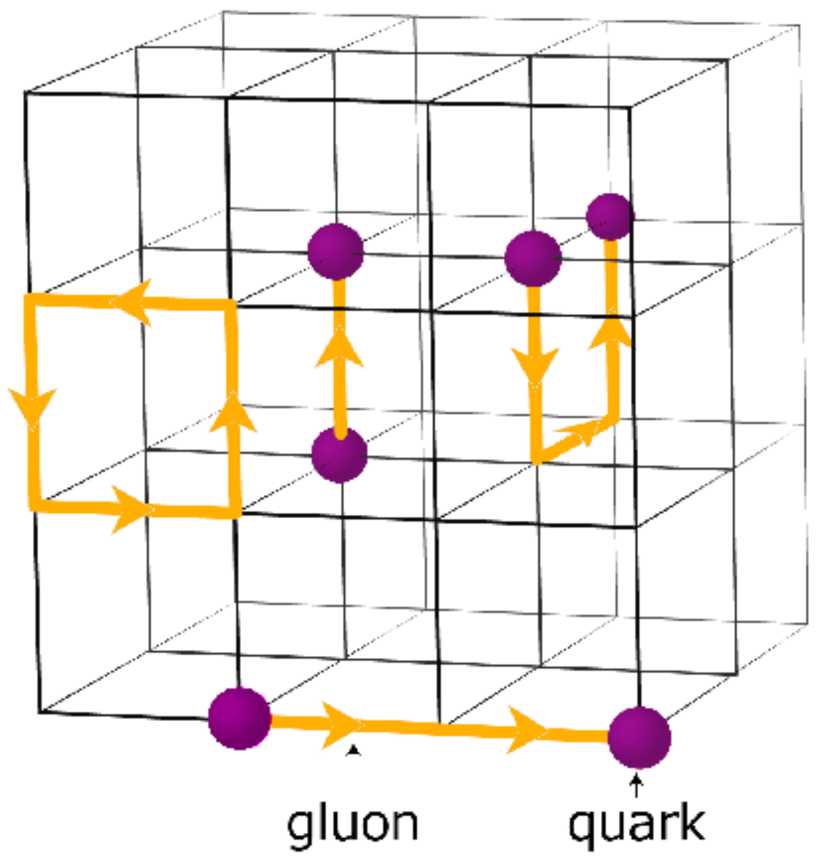

❖ GPD access in Lattice QCD:

- Mellin moments (generalized form factors)

$$\langle x^{n-1} \rangle = \int_{-1}^{+1} x^{n-1} f(x)\, dx$$

- **Novel methods (LaMET, pseudo-ITD, and many more)**

[Cichy & Constantinou, Adv.High Energy Phys. 2019 (2019) 3036904]

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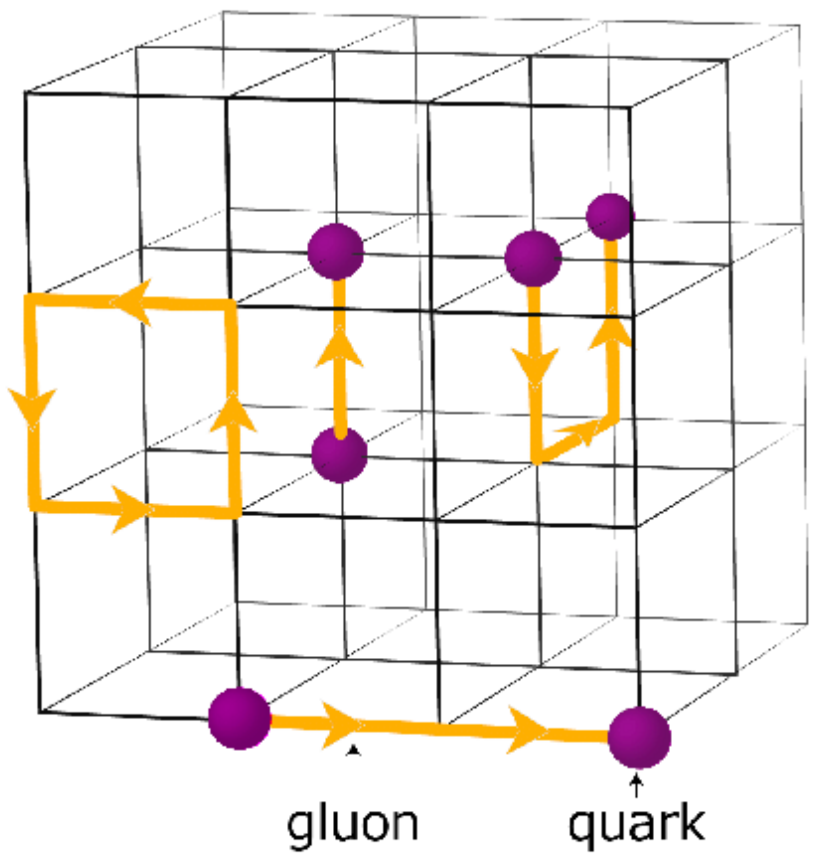

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❖ Calculation of quasi-GPD in Lattice QCD is very challenging

- Matrix elements of non-local operators (partons spatially separated)
- Hadron states with momentum boost
- renormalization prescriptions have limitations and may bring systematic uncertainties
- introduction of momentum transfer increases noise

→ A lot of computing time

Frame Dependence and Calculations

Almost all of the work in the literature uses the symmetric (Breit) frame.

Here, asymmetric kinematic frame: $\overrightarrow{P_i} = P_3\,\hat{z} - \overrightarrow{\Delta}, \qquad \overrightarrow{P_f} = P_3\,\hat{z},$

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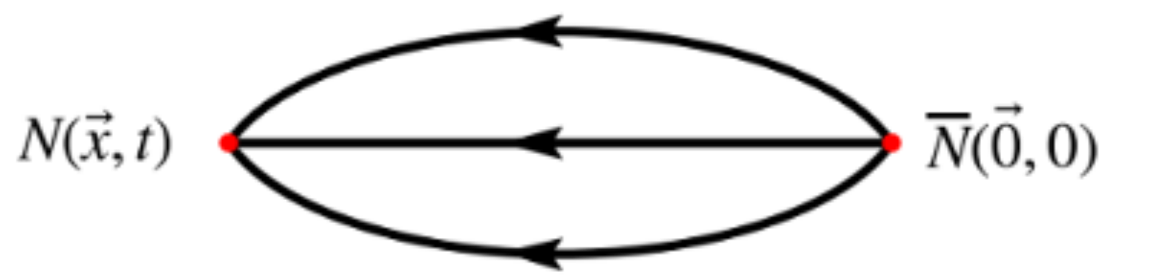
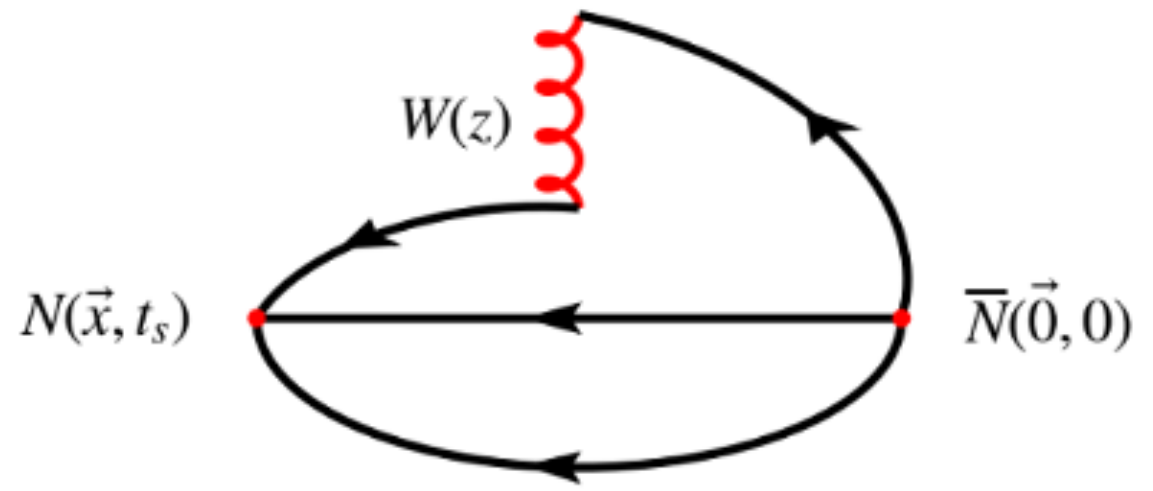
Necessary Steps

1. Calculation of appropriate ratio of the 3-point and 2-point correlation functions:

$$R = \frac{C^{3pt}(t_s, t, p_i, p_f)}{C^{2pt}(t_s, p_f)} \sqrt{\frac{C^{2pt}(t_s - t, p_i) C^{2pt}(t, p_f) C^{2pt}(t_s, p_f)}{C^{2pt}(t_s - t, p_f) C^{2pt}(t, p_i) C^{2pt}(t_s, p_i)}}$$



$\langle N(P) \,|\, N(P) \rangle$



$\langle N(P_f) \,|\, \bar{\Psi}(z)\, \Gamma\, \mathscr{W}(z,0) \Psi(0) \,|\, N(P_i) \rangle$

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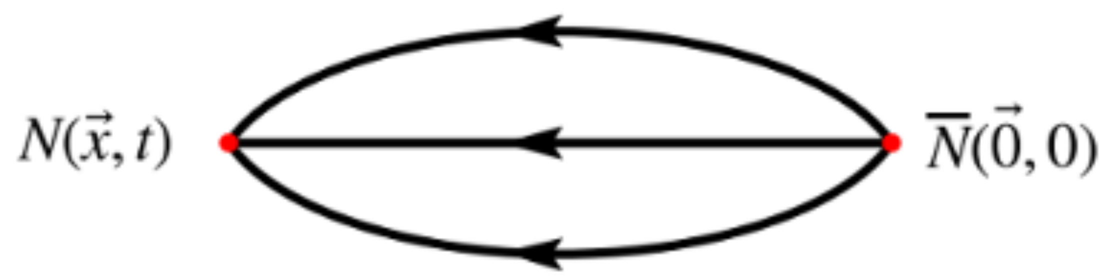

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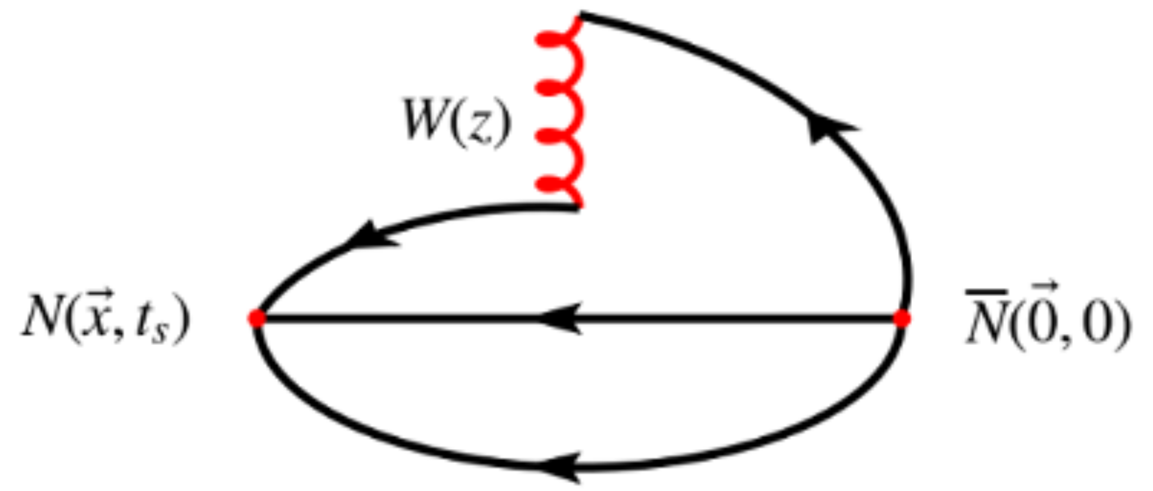

$\langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle$

2. Apply a single-state fit (plateau) to get the ground state of the matrix elements, Π_i^a

$$\longrightarrow F^\mu(z, P, \Delta) = \bar{u}(p_f, \lambda') \left[\frac{P^\mu}{m} A_1 + m z^\mu A_2 + \frac{\Delta^\mu}{m} A_3 + i m \sigma^{\mu z} A_4 + \frac{i\sigma^{\mu\Delta}}{m} A_5 + \frac{P^\mu i \sigma^{z\Delta}}{m} A_6 + m z^\mu i \sigma^{z\Delta} A_7 + \frac{\Delta^\mu i \sigma^{z\Delta}}{m} A_8 \right] u(p_i, \lambda)$$

[Bhattacharya et al., arXiv:2209.05373]
[Bhattacharya et al., (2022)]

$\longrightarrow$ Dependent upon 8 linearly-independent Lorentz invariant amplitudes!

$$A_i(z \cdot P, z \cdot \Delta, \Delta^2, z^2)$$

(Based on the idea of: S. Meissner, A. Metz, M. Schlegel, JHEP08(2009)056)

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8. Apply matching formalism

Decomposition

- Spin 1/2 particles:
  (4 operators: $\gamma^0, \gamma^1, \gamma^2, \gamma^3$) x (4 parity projectors: unpolarized/polarized proton) = 16 matrix element

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- Exploitation of different kinematics and symmetry properties of A_i to increase statistics.
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Example: Asymmetric Frame

$$(\pm\Delta_1,\ \pm\Delta_2,\ 0), (\pm\Delta_2,\ \pm\Delta_1,\ 0)$$

$$\Pi_0^a(\Gamma_1) = iK\left(\frac{(E_f+E_i)P_3\Delta_2}{8m^3}A_1 + \frac{(E_f-E_i)P_3\Delta_2}{4m^3}A_3 + \frac{(E_f+m)\Delta_2}{2m}zA_4 - \frac{(E_f+E_i+2m)P_3\Delta_2}{4m^3}A_5 - \frac{E_f(E_f+E_i)(E_f+m)\Delta_2}{4m^3}zA_6 - \frac{E_f(E_f-E_i)(E_f+m)\Delta_2}{2m^3}zA8\right)$$

$$\Pi_0^a(\Gamma_2) = iK\left(-\frac{(E_f+E_i)P_3\Delta_1}{8m^3}A_1 - \frac{(E_f-E_i)P_3\Delta_1}{4m^3}A_3 - \frac{(E_f+m)\Delta_1}{2m}zA_4 + \frac{(E_f+E_i+2m)P_3\Delta_1}{4m^3}A_5 + \frac{E_f(E_f+E_i)(E_f+m)\Delta_1}{4m^3}zA_6 + \frac{E_f(E_f-E_i)(E_f+m)\Delta_1}{2m^3}zA8\right)$$

❖ Kinematically equivalent matrix elements can be averaged

Decomposition

Symmetric Frame

$$\Pi_0^s(\Gamma_2) = iK\left(-\frac{EP_3\Delta_1}{4m^3}A_1 + \frac{(E+m)P_3\Delta_1}{2m^3}A_5 + \frac{E(P_3^2+m(E+m))\Delta_1}{2m^3}zA_6\right)$$

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Asymmetric Frame

$$\Pi_0^a(\Gamma_2) = iK\left(-\frac{(E_f+E_i)P_3\Delta_1}{8m^3}A_1 - \frac{(E_f-E_i)P_3\Delta_1}{4m^3}A_3 - \frac{(E_f+m)\Delta_1}{2m}zA_4 + \frac{(E_f+E_i+2m)P_3\Delta_1}{4m^3}A_5 + \frac{E_f(E_f+E_i)(E_f+m)\Delta_1}{4m^3}zA_6 + \frac{E_f(E_f-E_i)(E_f+m)\Delta_1}{2m^3}zA8\right)$$

$$\Pi_1^a(\Gamma_1) = K\left(-\frac{P_3\Delta_1\Delta_2}{8m^3}A_1 + \frac{P_3\Delta_1\Delta_2}{4m^3}A_3 + \frac{P_3\Delta_1\Delta_2}{4m^3}A_5 + \frac{E_f(E_f+m)\Delta_1\Delta_2}{4m^3}zA6 - \frac{E_f(E_f+m)\Delta_1\Delta_2}{2m^3}zA_8\right)$$

$$\Pi_1^a(\Gamma_3) = K\left(\frac{P_3\Delta_2}{4m}zA_4 + \frac{(E_f+m)\Delta_2}{2m^2}A_5\right)$$

Decomposition

Symmetric Frame

$$\Pi_0^s(\Gamma_2) = iK\left(-\frac{EP_3\Delta_1}{4m^3}A_1 + \frac{(E+m)P_3\Delta_1}{2m^3}A_5 + \frac{E(P_3^2+m(E+m))\Delta_1}{2m^3}zA_6\right)$$

$$\Pi_1^s(\Gamma_1) = K\left(\frac{P_3\Delta_1\Delta_2}{4m^3}A_3 + \frac{\Delta_1\Delta_2}{8m}zA_4 - \frac{(P_3^2+m(E+m))\Delta_1\Delta_2}{2m^3}zA_8\right)$$

$$\Pi_1^s(\Gamma_3) = K\frac{(E+m)\Delta_2}{2m^2}A_5$$

Asymmetric Frame

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★ Π_μ and kinematic coefficients depend on the frame, but A_i are frame invariant ★

Setup

Lattice Setup

- $N_f = 2 + 1 + 1$ twisted mass fermions & clover term (ETMC)
- Iwasaki gluons $\beta = 1.778$
- Lattice spacing $a \approx 0.0934$ fm
- $32^3 \times 64$ fm
- $m_\pi \approx 260$ MeV

frame	P_3 [GeV]	$\mathbf{\Delta}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	731	16	23392
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
symm	±1.67	(±2,0,0), (0,±2,0)	0.69	0	8	294	32	75264
symm	±1.25	(±2, ±2, 0)	1.39	0	16	224	8	28672
symm	±1.25	(±4,0,0), (0,±4,0)	2.76	0	8	329	32	84224
asymm	±1.25	(±1,0,0), (0,±1,0)	0.17	0	8	271	8	17344
asymm	±1.25	(±1, ±1, 0)	0.33	0	16	194	8	12416
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asymm	±1.25	(±1,±2,0), (±2,±1,0)	0.80	0	16	194	8	12416
asymm	±1.25	(±2,±2,0)	1.16	0	16	194	8	24832
asymm	±1.25	(±3,0,0), (0,±3,0)	1.37	0	8	271	8	17344
asymm	±1.25	(±1,±3,0), (±3,±1,0)	1.50	0	16	194	8	12416
asymm	±1.25	(±4,0,0), (0,±4,0)	2.26	0	8	271	8	17344

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Asymmetric frame done in groups of 2 runs!
Much faster than symmetric frame!

Symmetric frame calculations are done individually!
Computationally expensive!

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Different calculations

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Various $-t$ values simulated

Matrix Elements

Asymmetric Frame $-t = 0.64\ GeV^2$

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No symmetry properties ($z \cdot P, z \cdot \Delta, \Delta^2, z^2$) in asymmetric frame!

Amplitudes

For $(\Delta, 0, 0)$:

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$$A_1 = \frac{2m^2}{E_f(E_i+m)K}\Pi_0^a(\Gamma_0) + i\frac{2(E_f-E_i)P_3m^2}{E_f(E_f+m)(E_i+m)\Delta K}\Pi_0^a(\Gamma_2) + \frac{2(E_i-E_f)P_3m^2}{E_f(E_f+E_i)(E_f+m)(E_i+m)K}\Pi_1^a(\Gamma_2)$$

$$+i\frac{2(E_i-E_f)m^2}{E_f(E_i+m)\Delta K}\Pi_1^a(\Gamma_0) + \frac{(E_i-E_f)P_3m^2}{E_f(E_f+E_i)(E_f+m)(E_i+m)K}\Pi_2^a(\Gamma_1) + \frac{2(E_f-E_i)m^2}{E_f(E_i+m)\Delta K}\Pi_2^a(\Gamma_3)$$

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$$A_1^*(-z\cdot P, z\cdot\Delta, \Delta^2, z^2) = A_1(z\cdot P, z\cdot\Delta, \Delta^2, z^2) \qquad A_5^*(-z\cdot P, z\cdot\Delta, \Delta^2, z^2) = A_5(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$$

$$-A_2^*(-z\cdot P, z\cdot\Delta, \Delta^2, z^2) = A_2(z\cdot P, z\cdot\Delta, \Delta^2, z^2) \qquad -A_6^*(-z\cdot P, z\cdot\Delta, \Delta^2, z^2) = A_6(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$$

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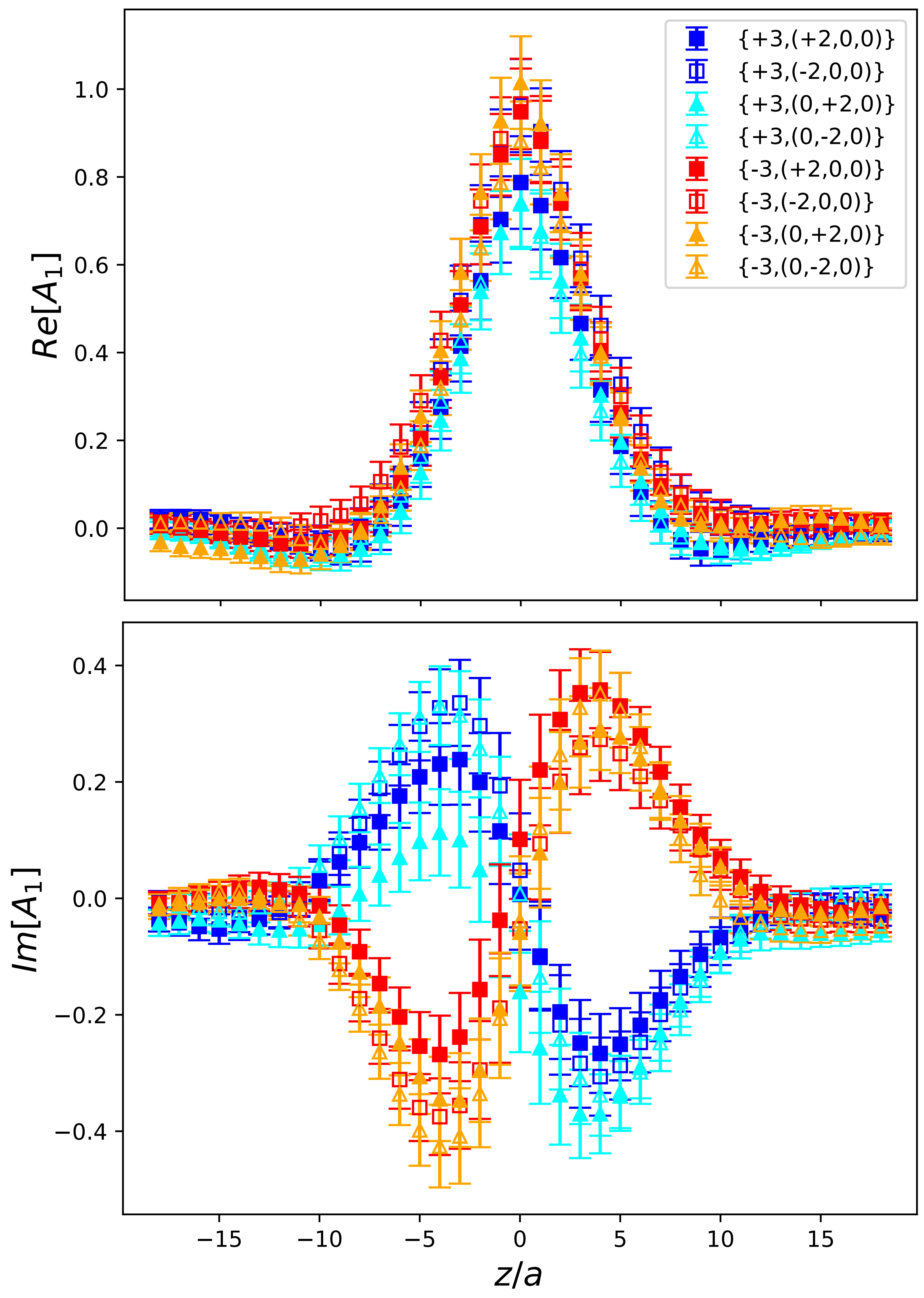
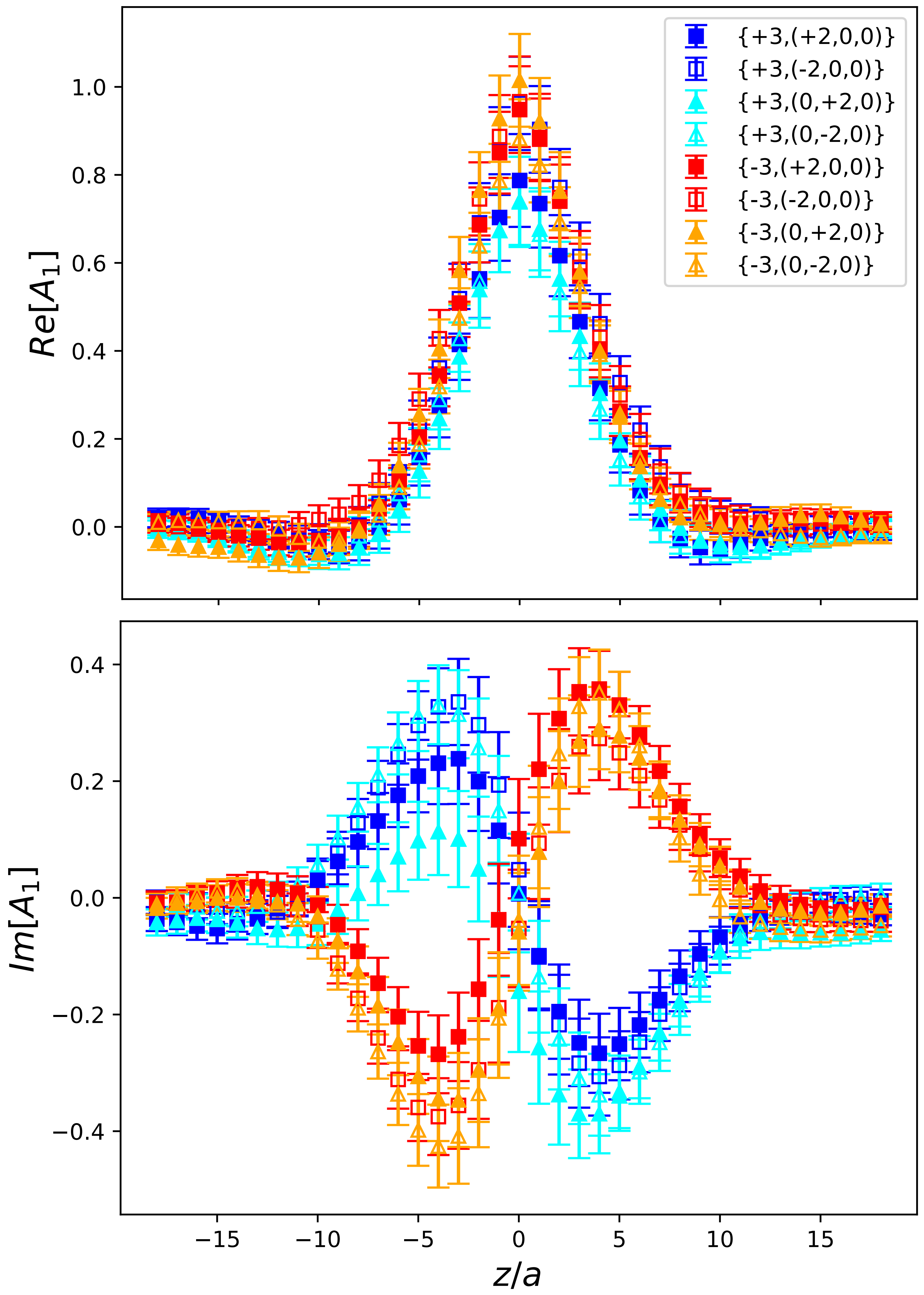


Symmetry in Amplitudes



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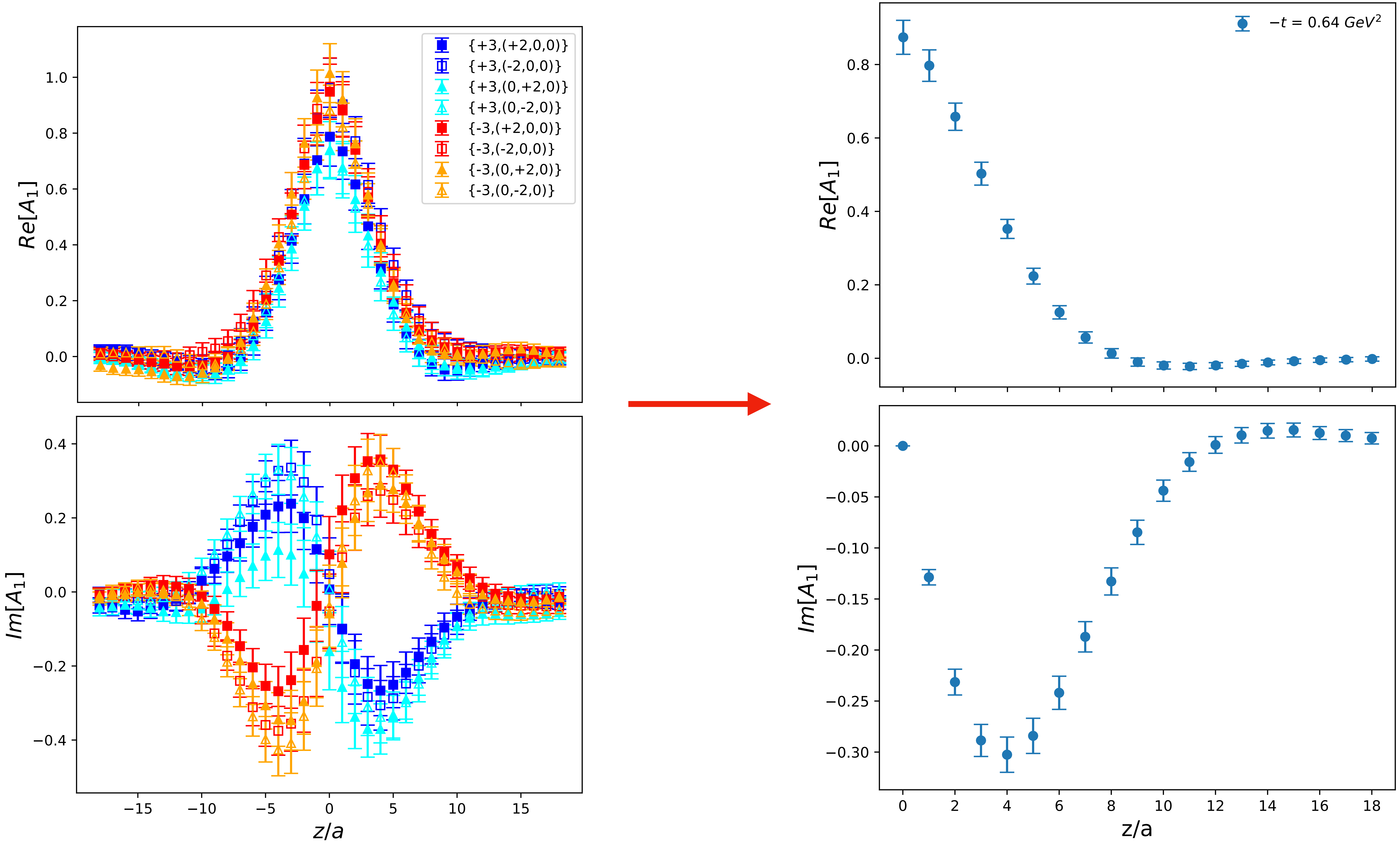
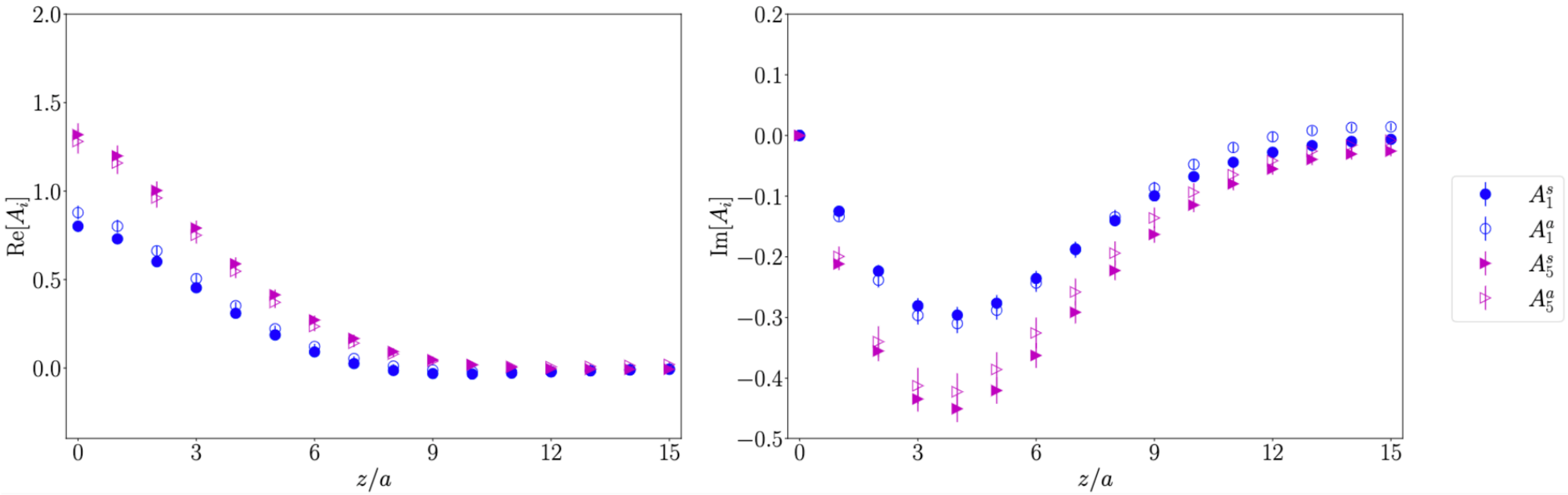


Agreement Between Frames



❖ A_1 and A_5 are the dominant contributions

❖ Full agreement in two frames for both the real and imaginary parts for A_1 and A_5

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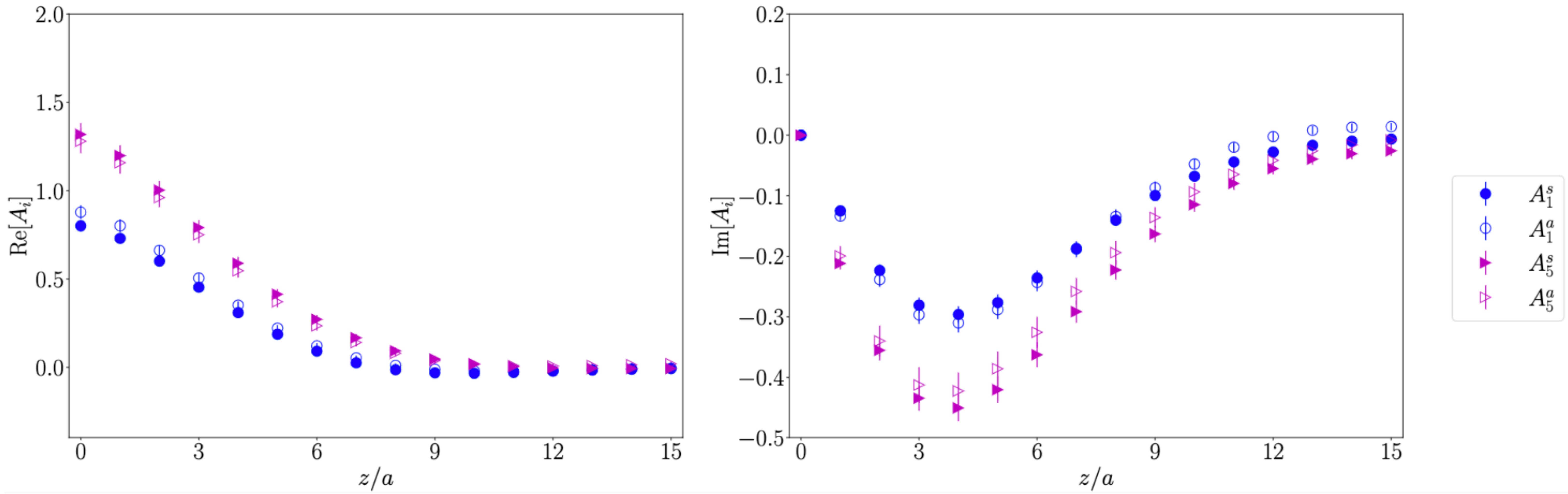

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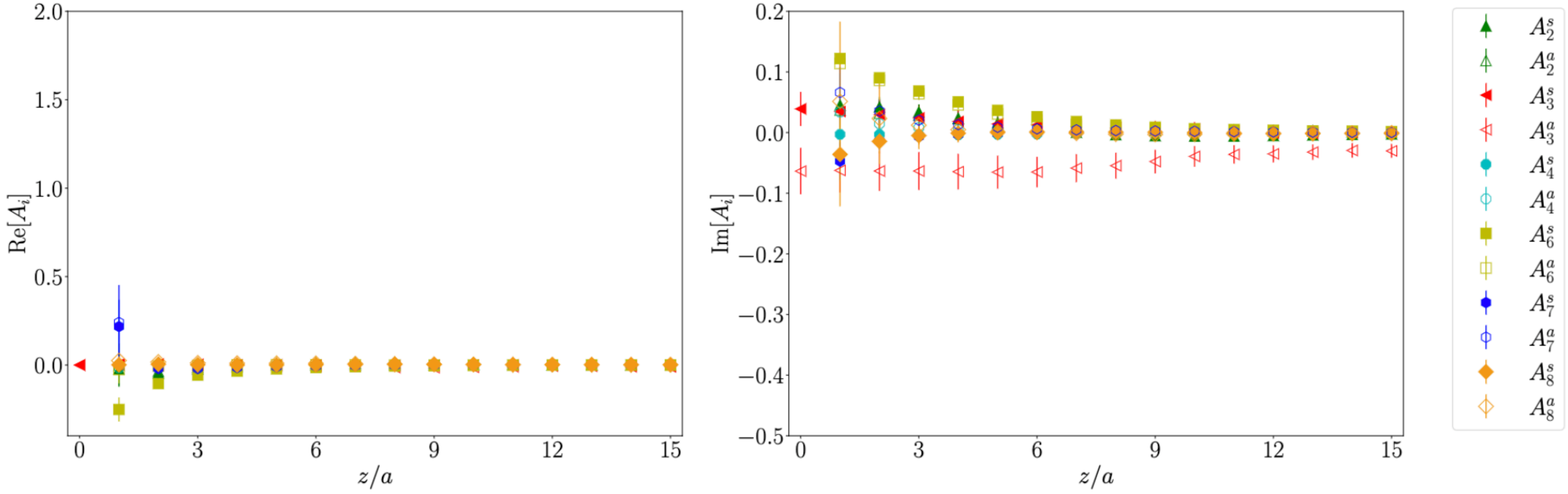


- ❖ Remaining A_i are suppressed (at least for this kinematic setup and for $\xi = 0$)
- ❖ Some A_i may be exactly zero for $\xi = 0$

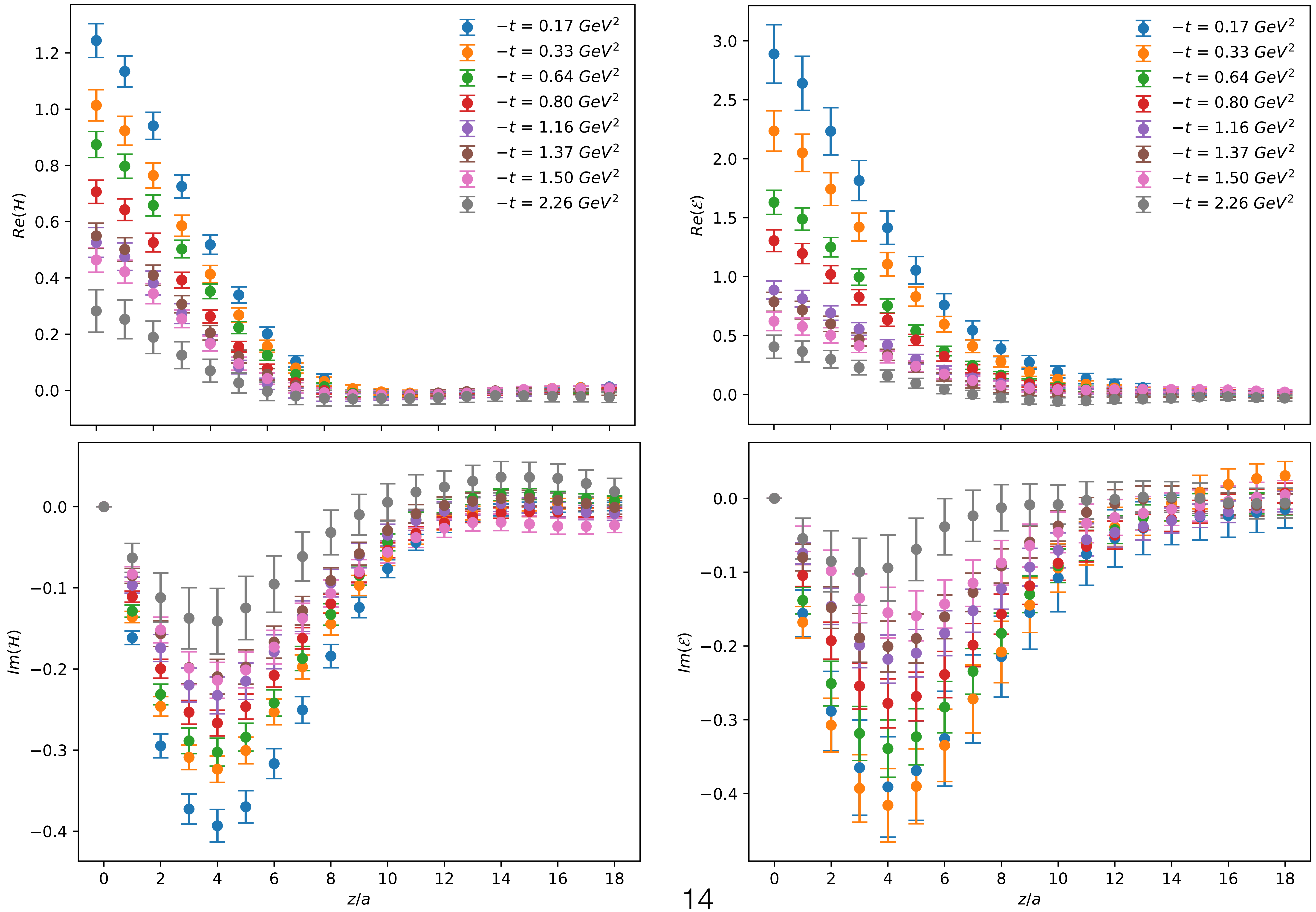
Quasi-GPDs

❖ We build the quasi-GPDs (coordinate space) by mapping with the A_i

❖ $\xi = 0$

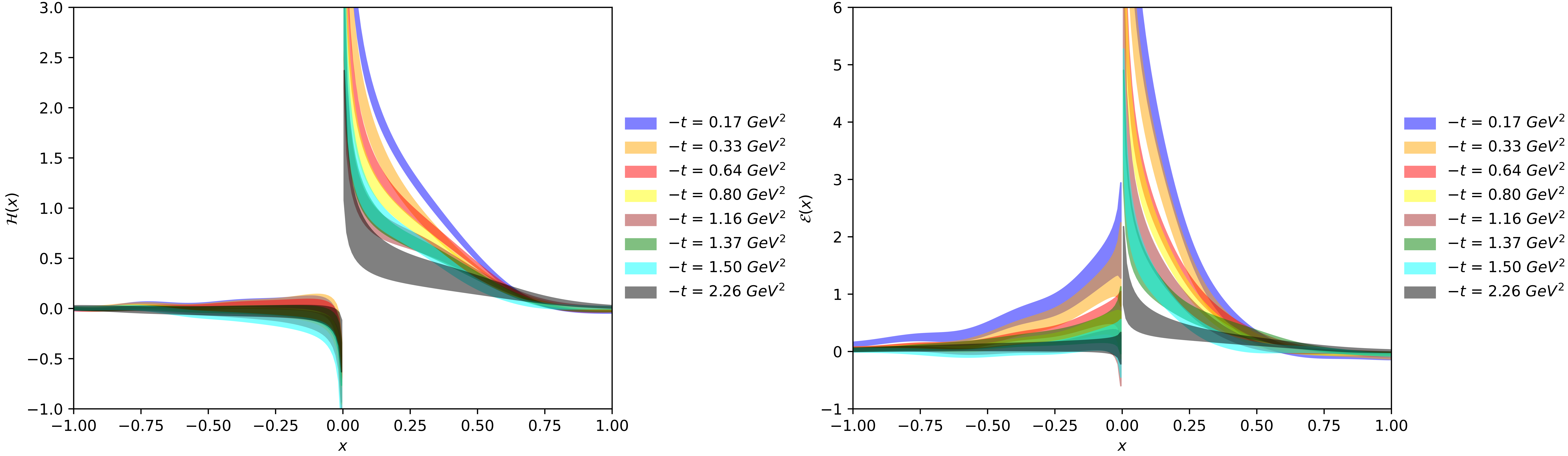
$\mathcal{H}(A_i^{s/a}; z) = A_1$

$\mathcal{E}(A_i^{s/a}; z) = -A_1 + 2A_5 + 2P_3 z A_6$



H and E GPDs

- ❖ Reconstruction of x-dependence using Backus-Gilbert **[Backus, Gilbert, (1968)]**
- ❖ 1-loop matching (same as PDF for zero skewness) **[Liu et al., (2019)]**



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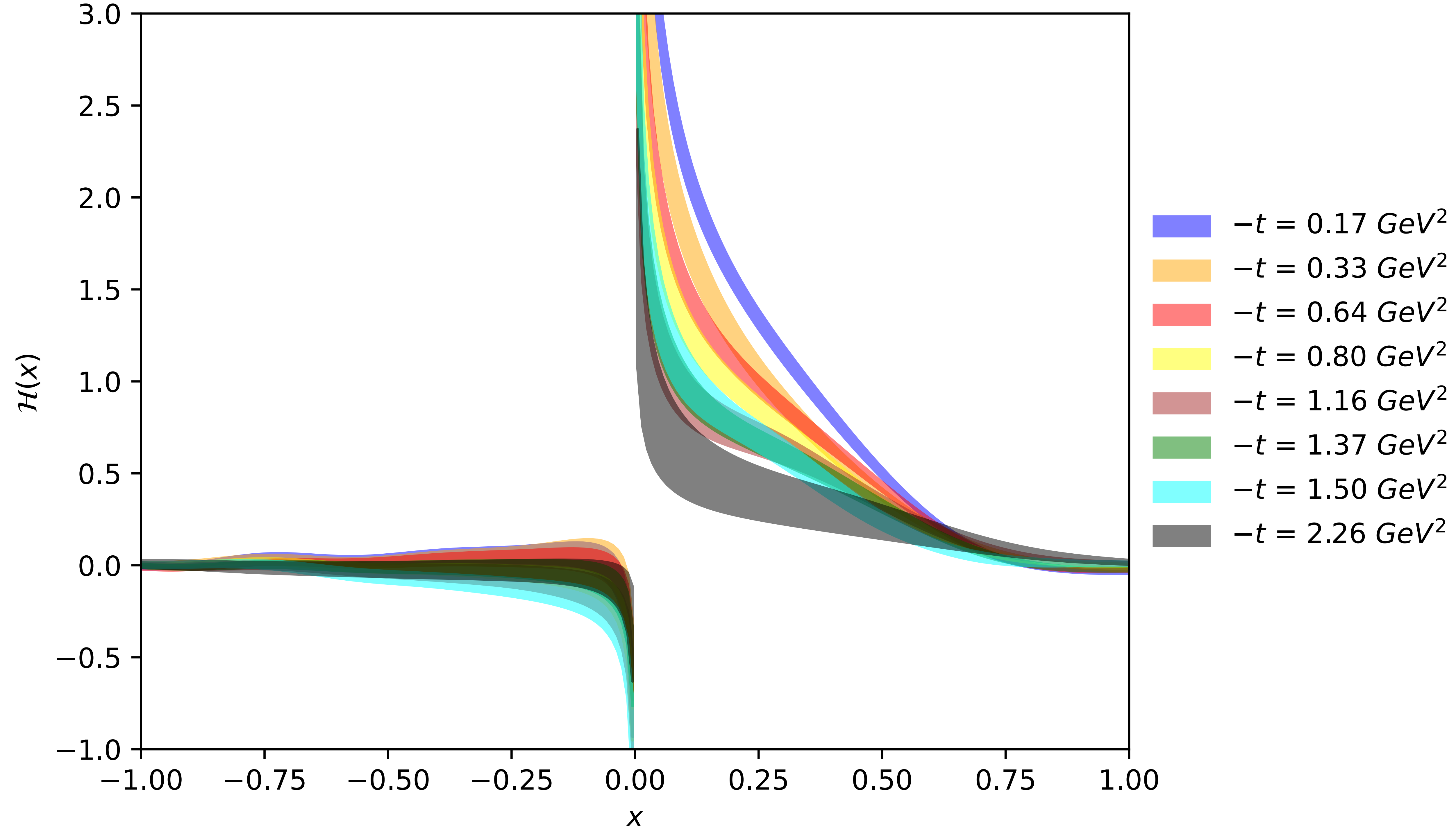

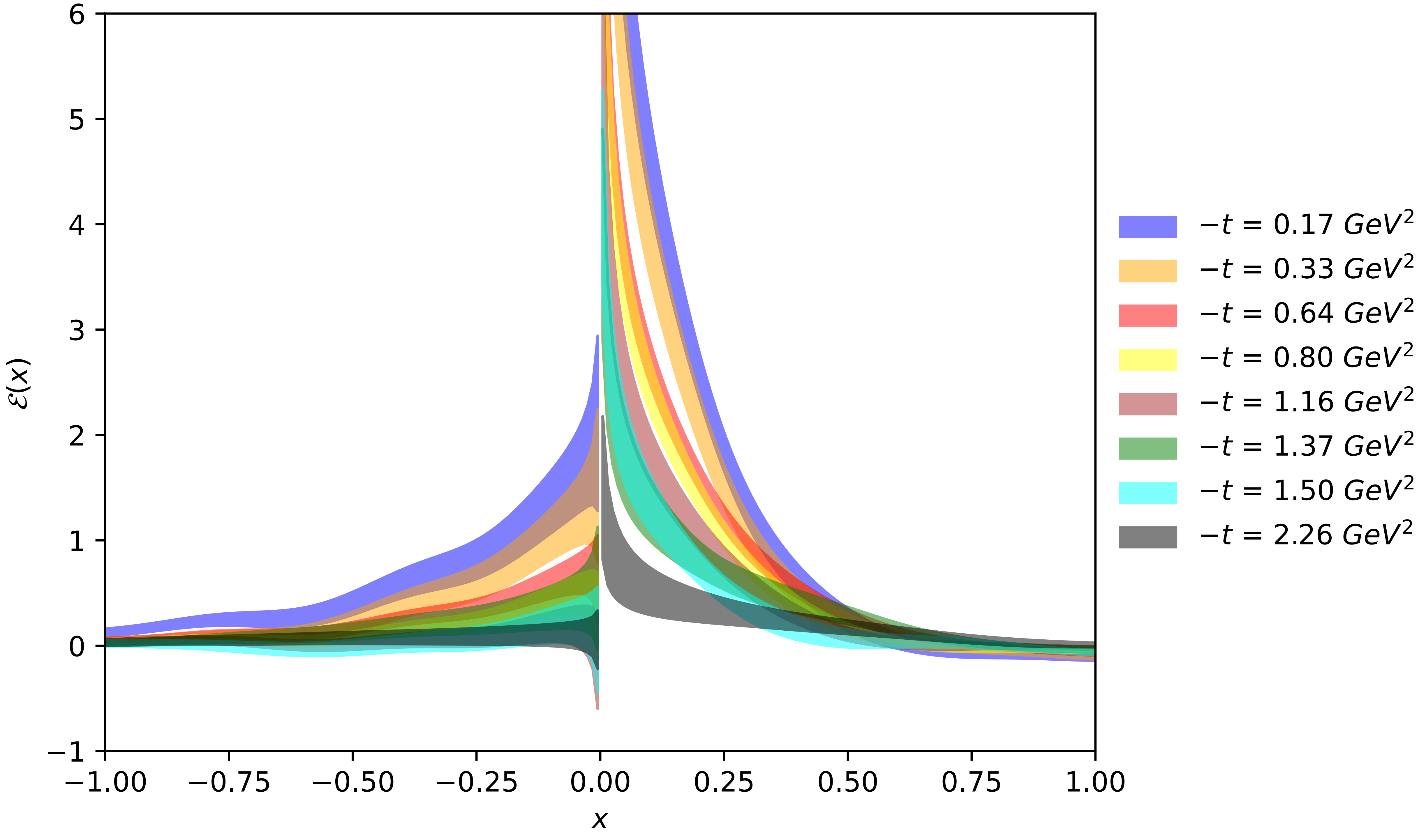

❖ GPDs decay with increase of momentum transfer

❖ High values of $-t$ have increased systematic uncertainties

Helicity GPDs

❖ Similar formalism has been developed for the helicity case

❖ Two quasi-GPDs: $\tilde{\mathscr{H}}$ and $\tilde{\mathscr{E}}$

❖ At $\xi = 0$, we cannot extract $\tilde{\mathscr{E}}$

❖ Like the unpolarized case, we exploit asymmetric frame calculation to extract several $-t$

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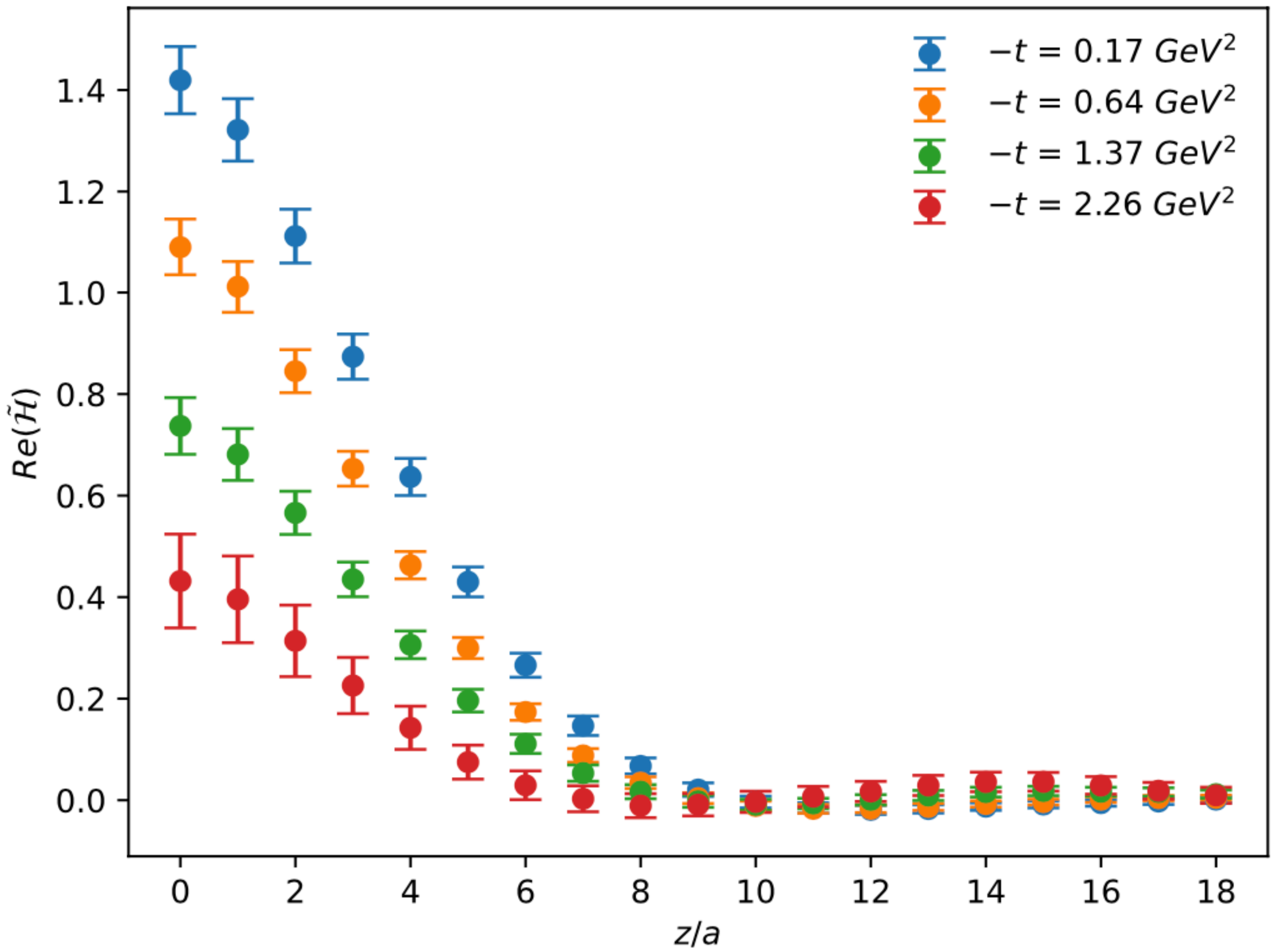
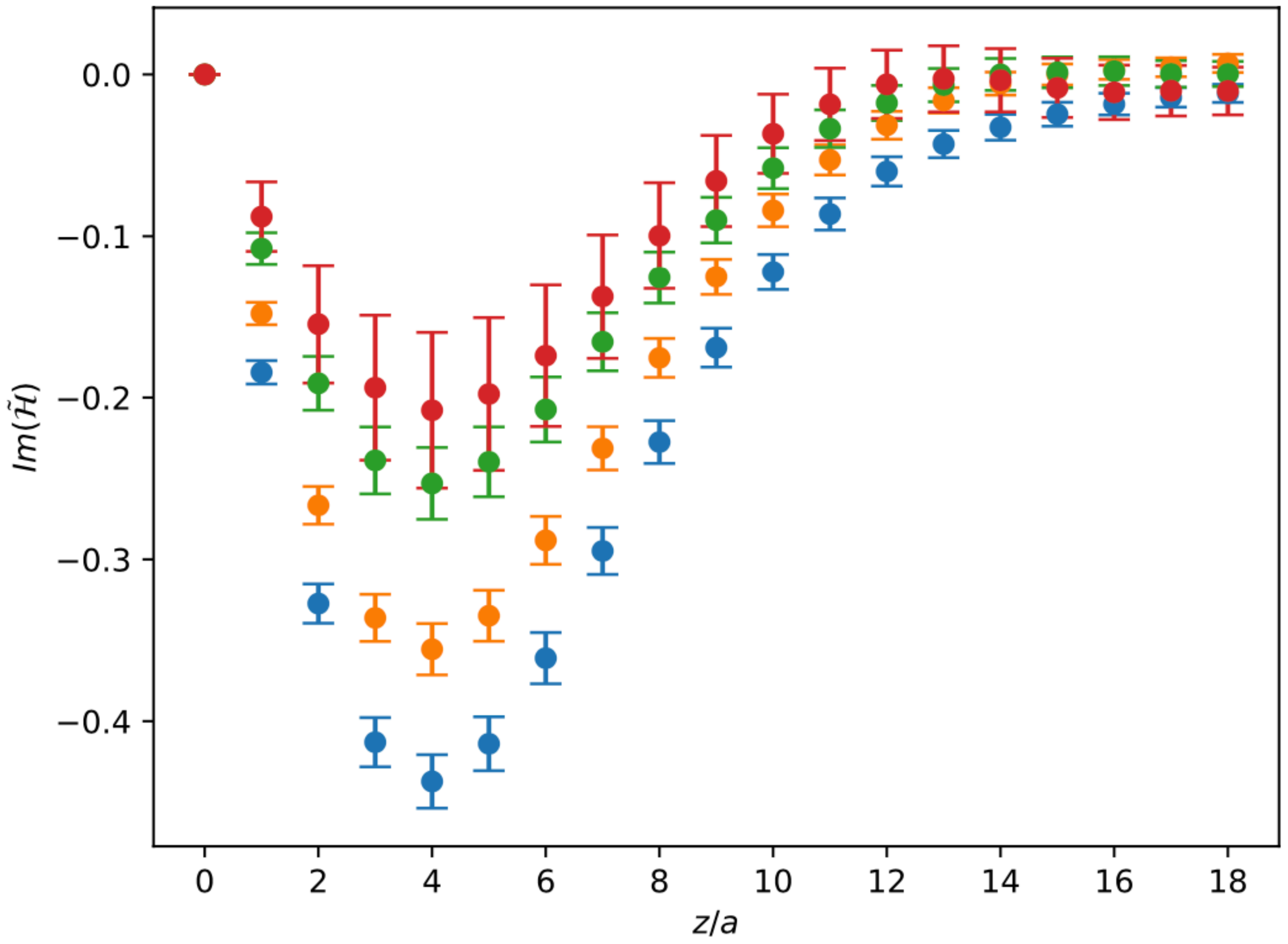
Preliminary





Summary

- ❖ New method to parameterize the MEs into Lorentz invariant amplitudes
- ❖ Method has great advantages
  - ❖ Access to a broad range of $-t$ and ξ
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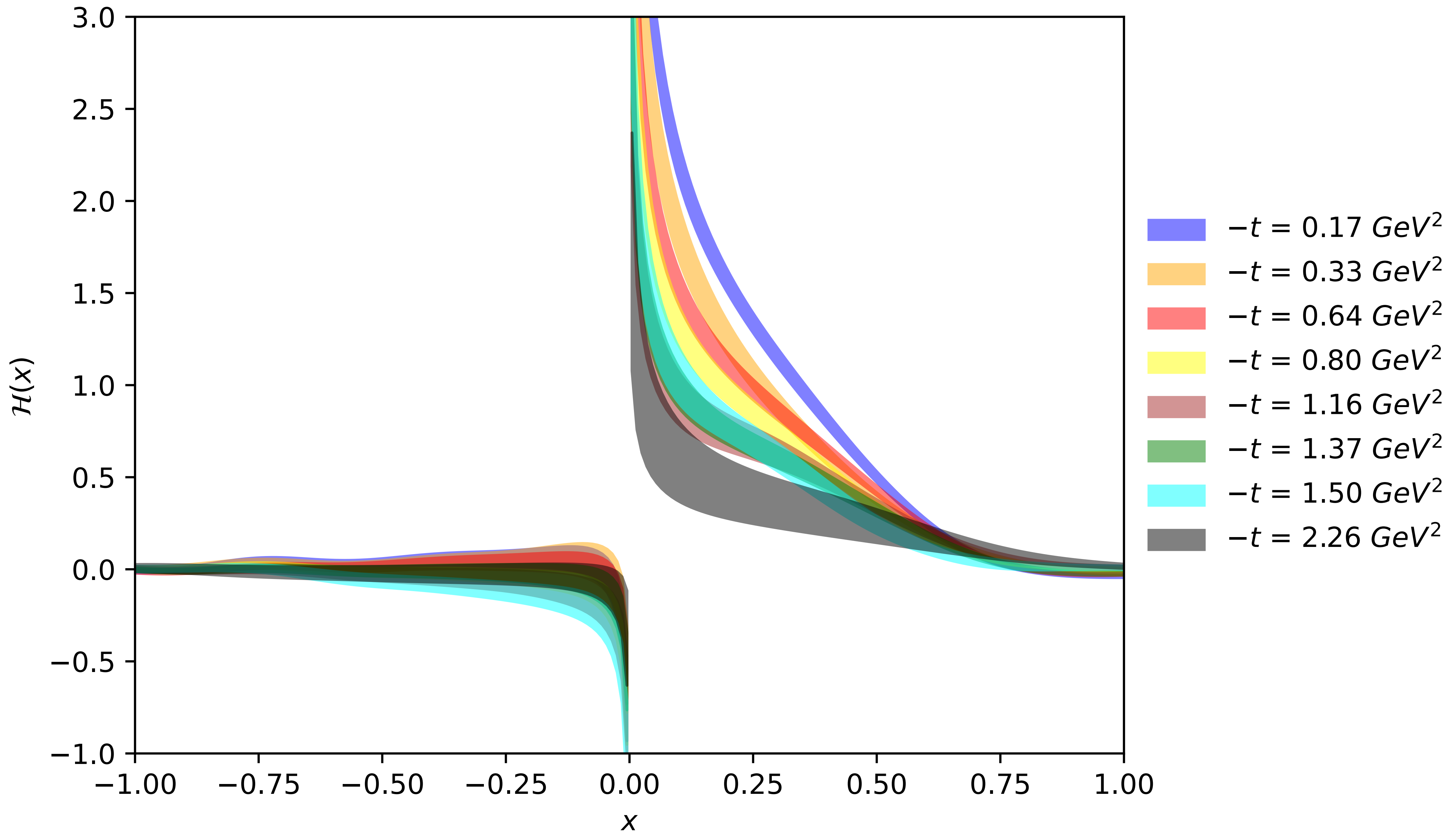

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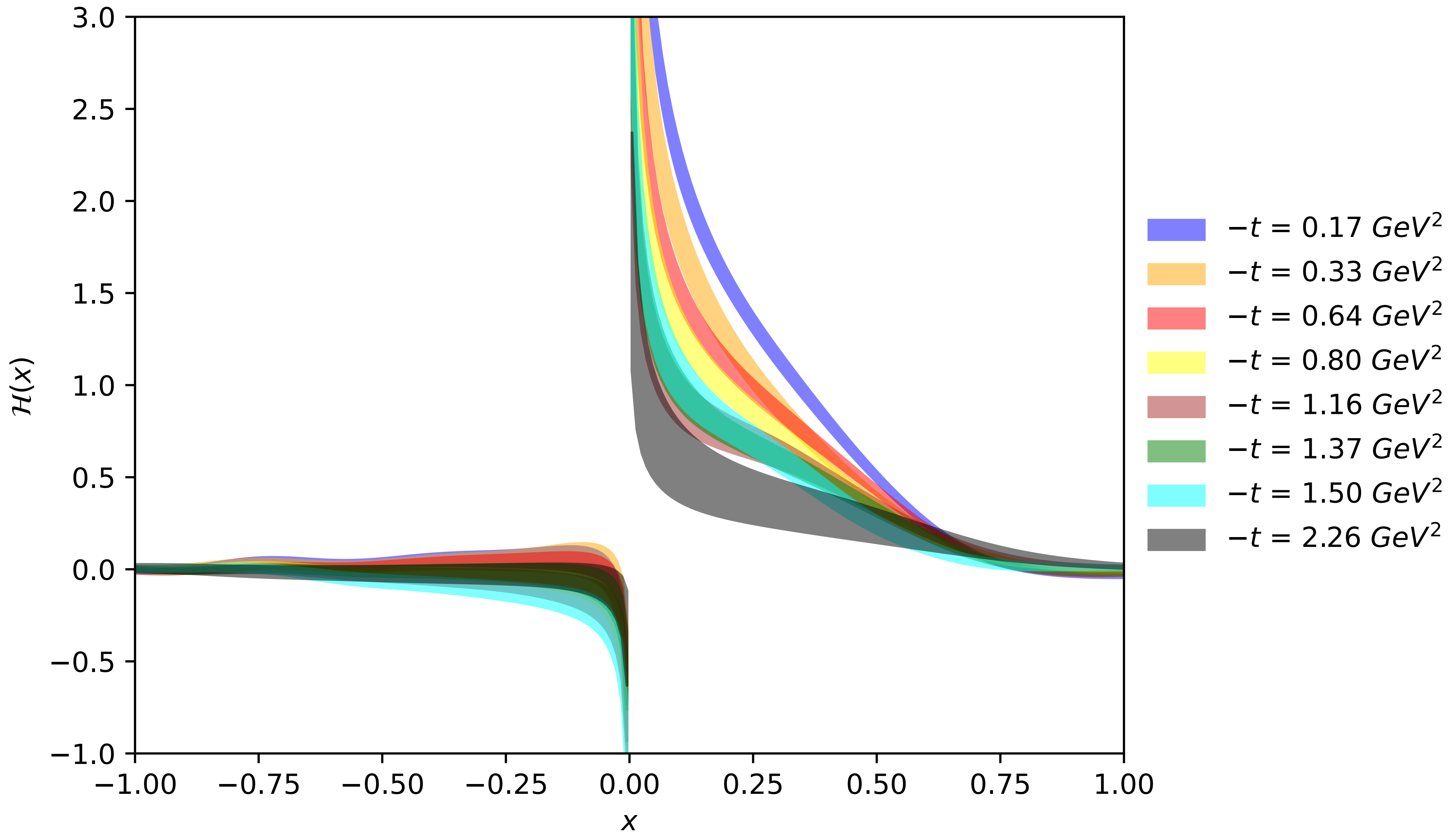

Thank you!

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