Polarized and unpolarized gluon distributions in the nucleon from Lattice QCD and machine learning

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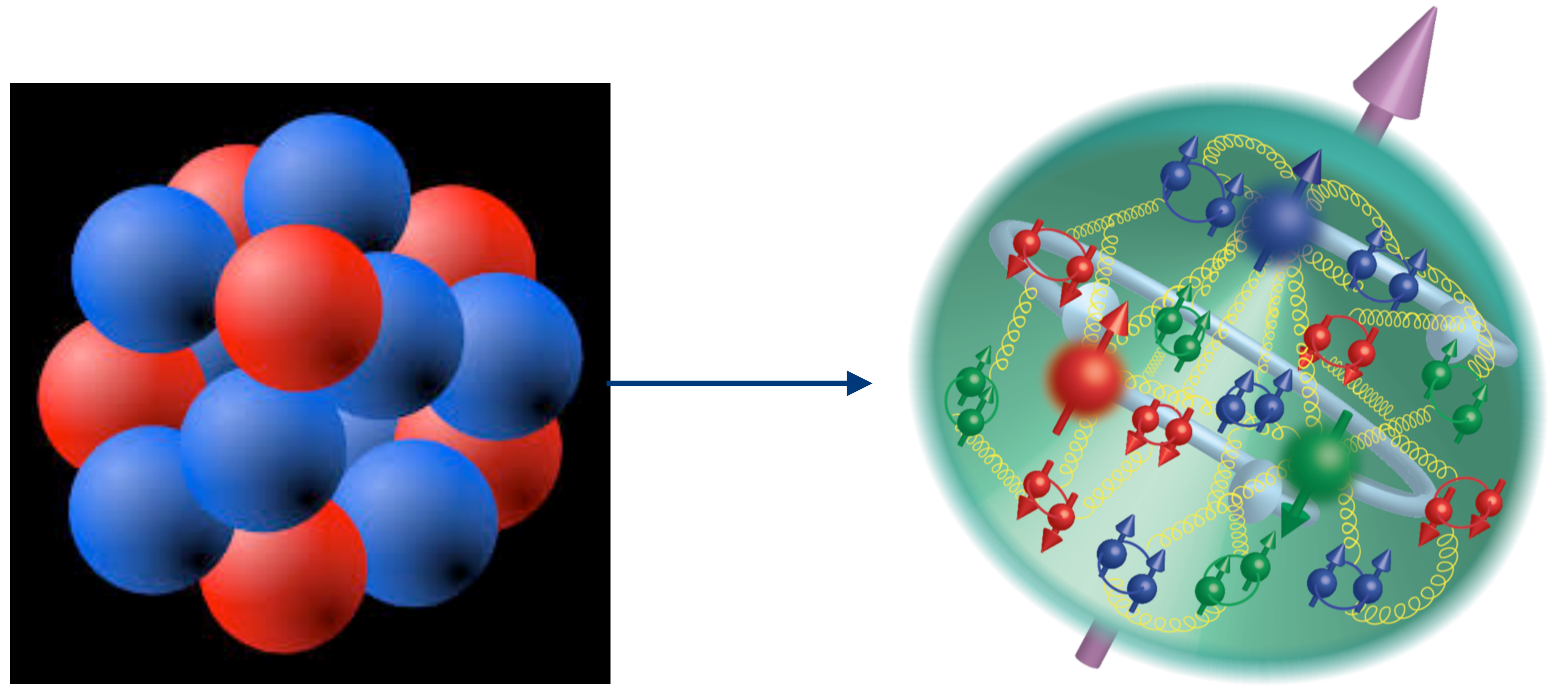
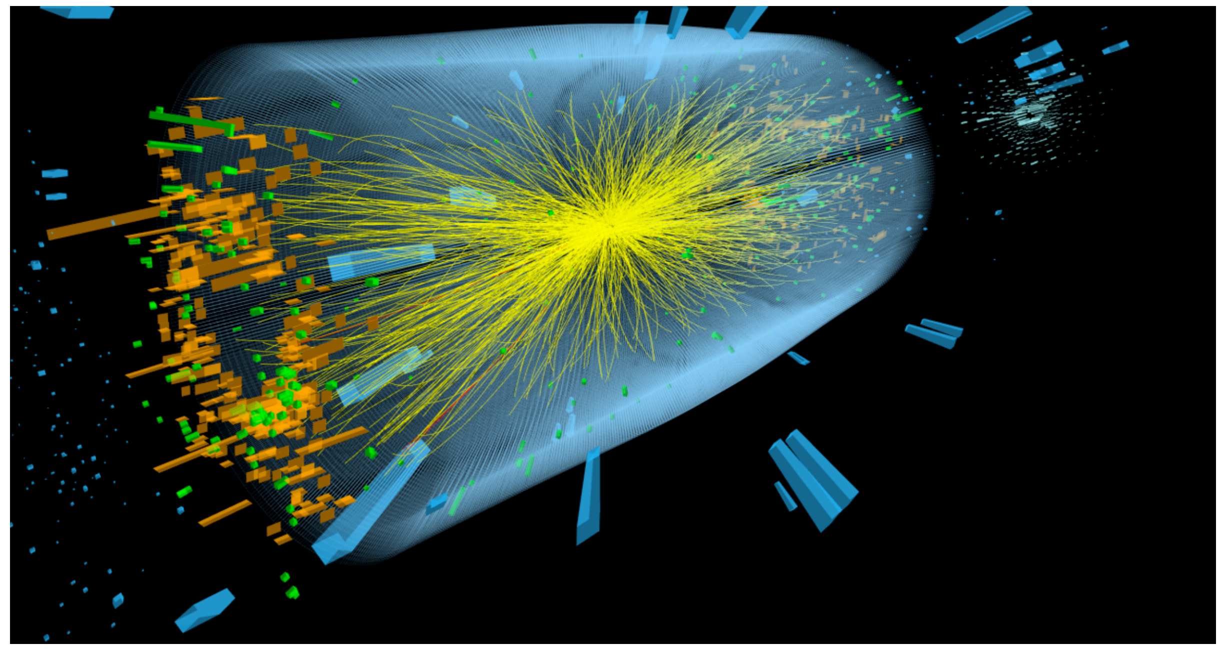
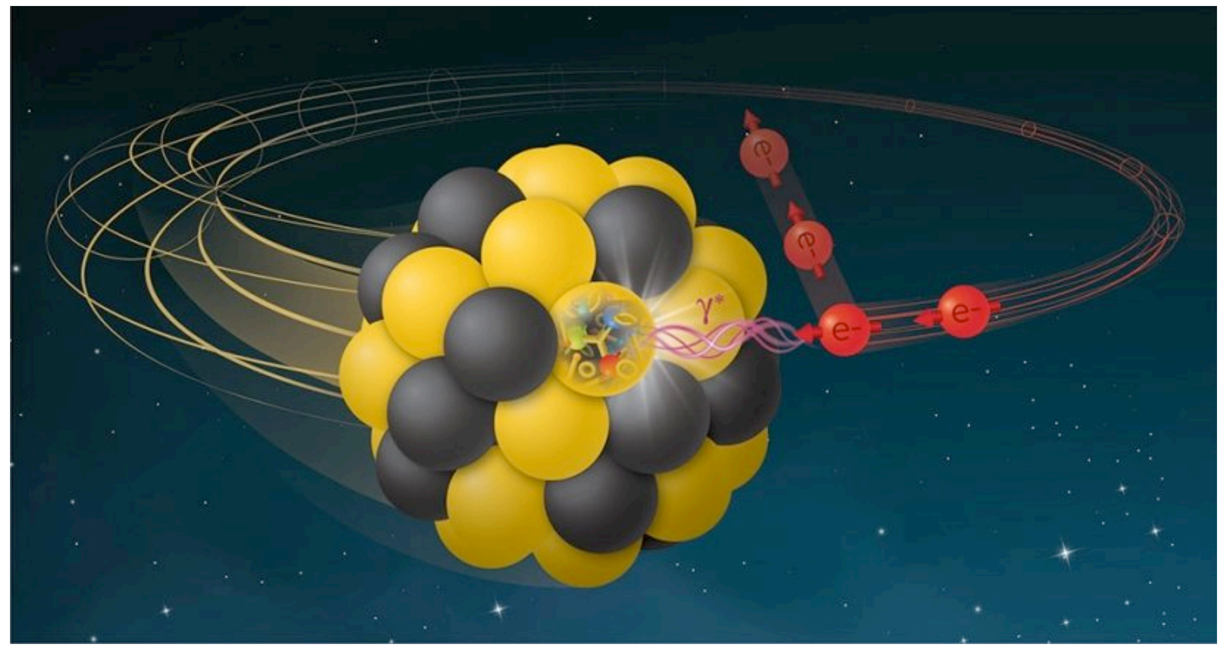
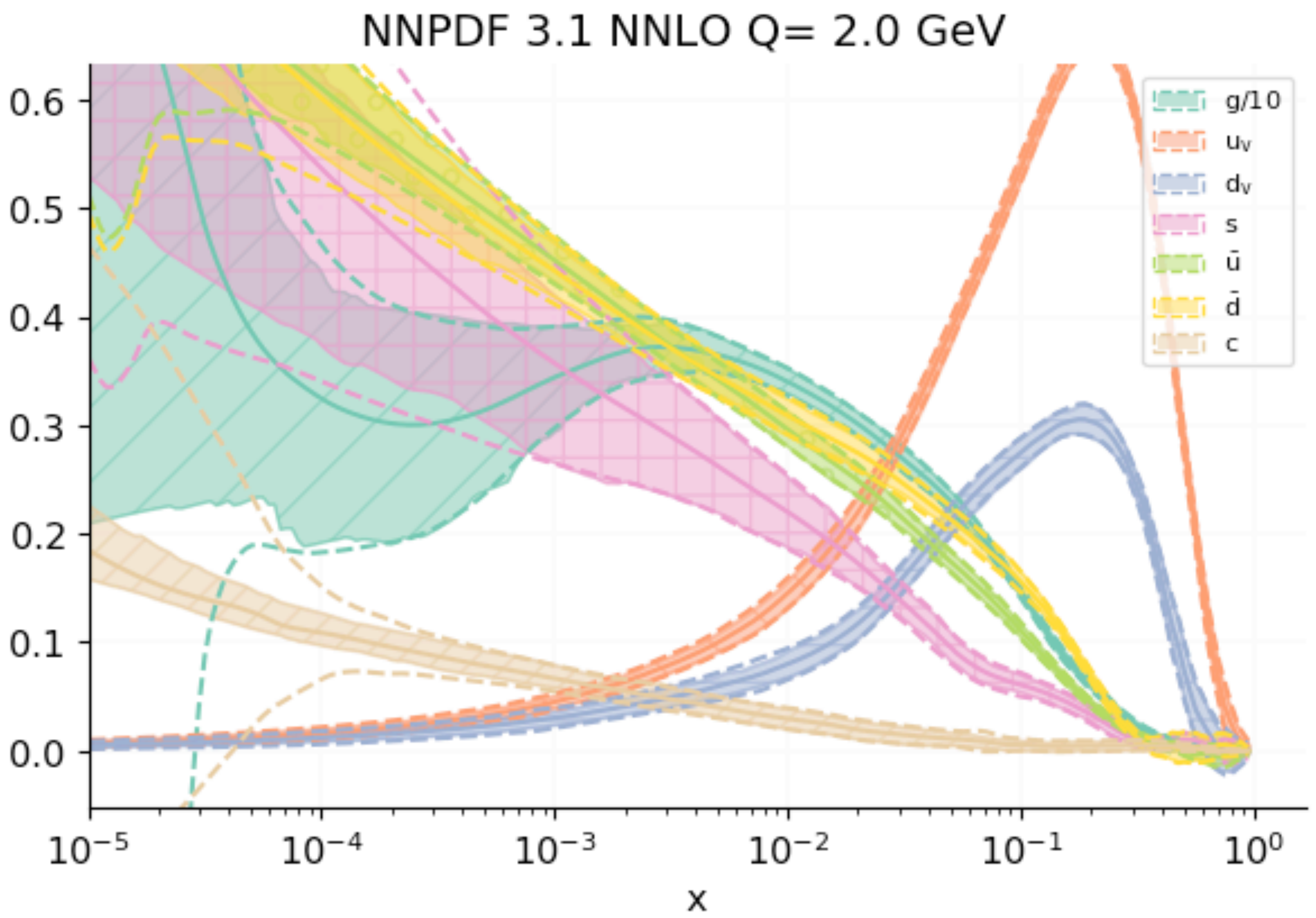
Nonperturbative distributions of quarks & gluons (PDFs)

- “Seeing” internal structure of nucleon without seeing quarks & gluons?





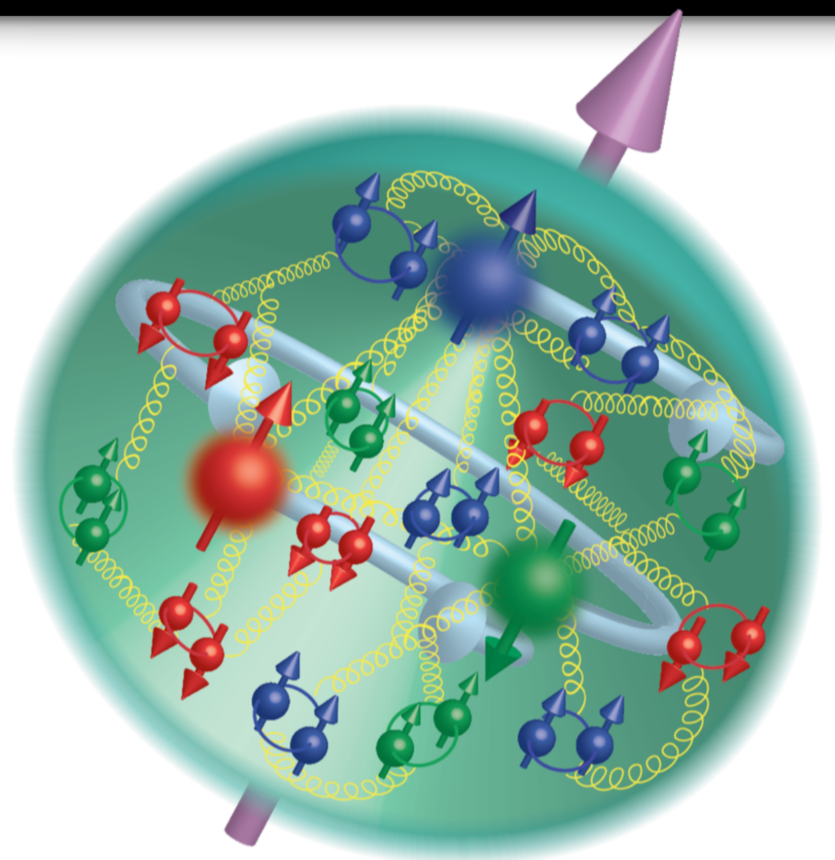




Parton distribution functions (PDFs)

- Parton distribution functions are universal properties of a hadron
  - predict/describe outcome of different experiments (e.g. @LHC, EIC, …)
  - governs nonperturbative properties of hadrons

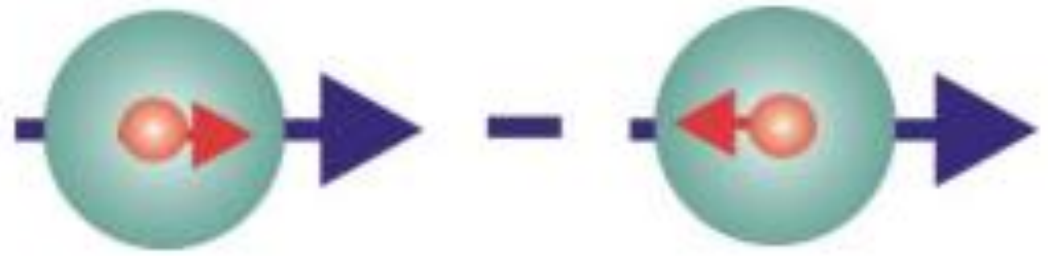
Gluon helicity distribution & origin of proton spin

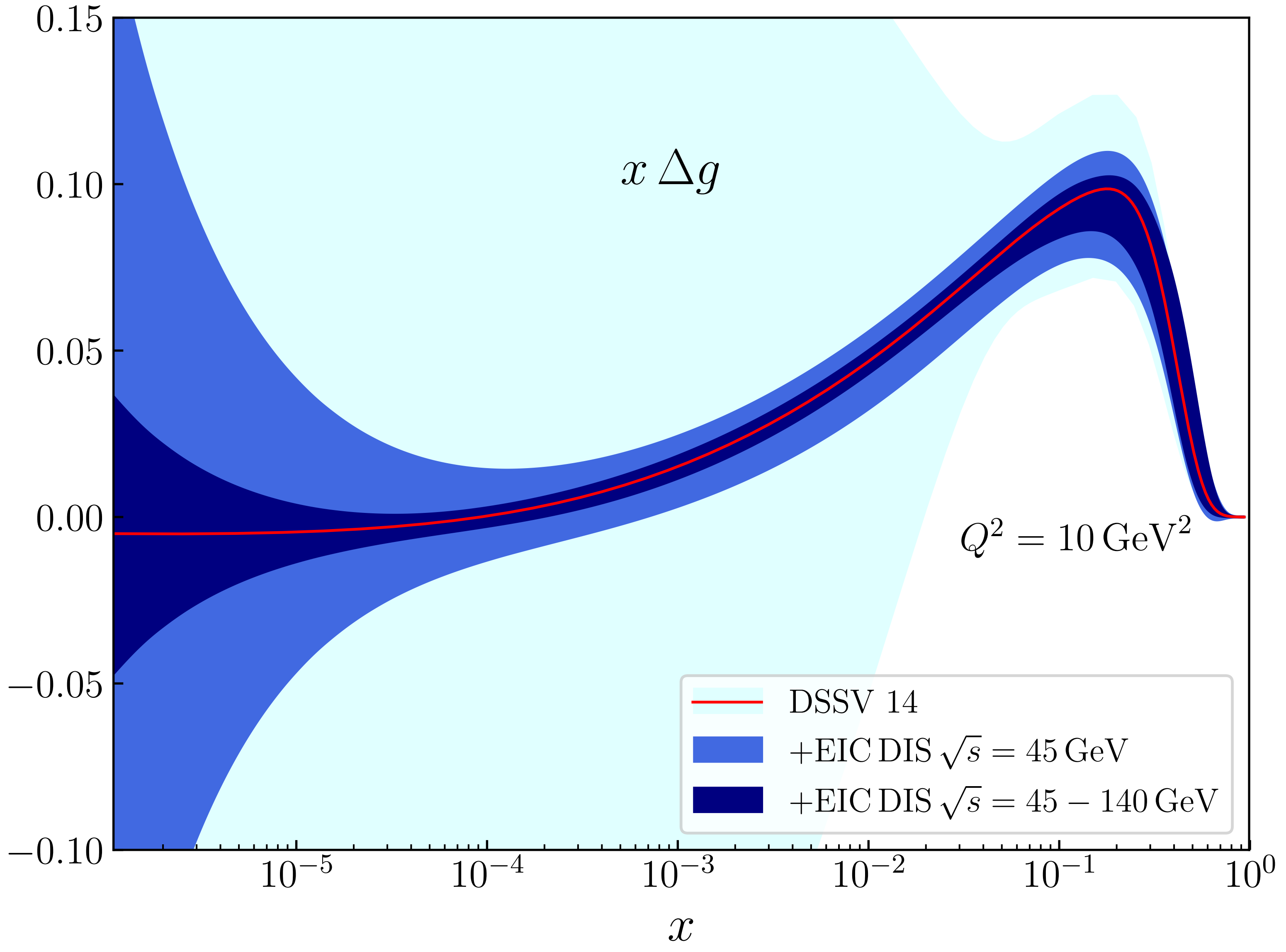


$$\frac{1}{2} = \text{spin of all quarks} \sim 30\% + \text{glue spin} + \text{quark orbital angular momentum} + \text{gluon orbital angular momentum}$$

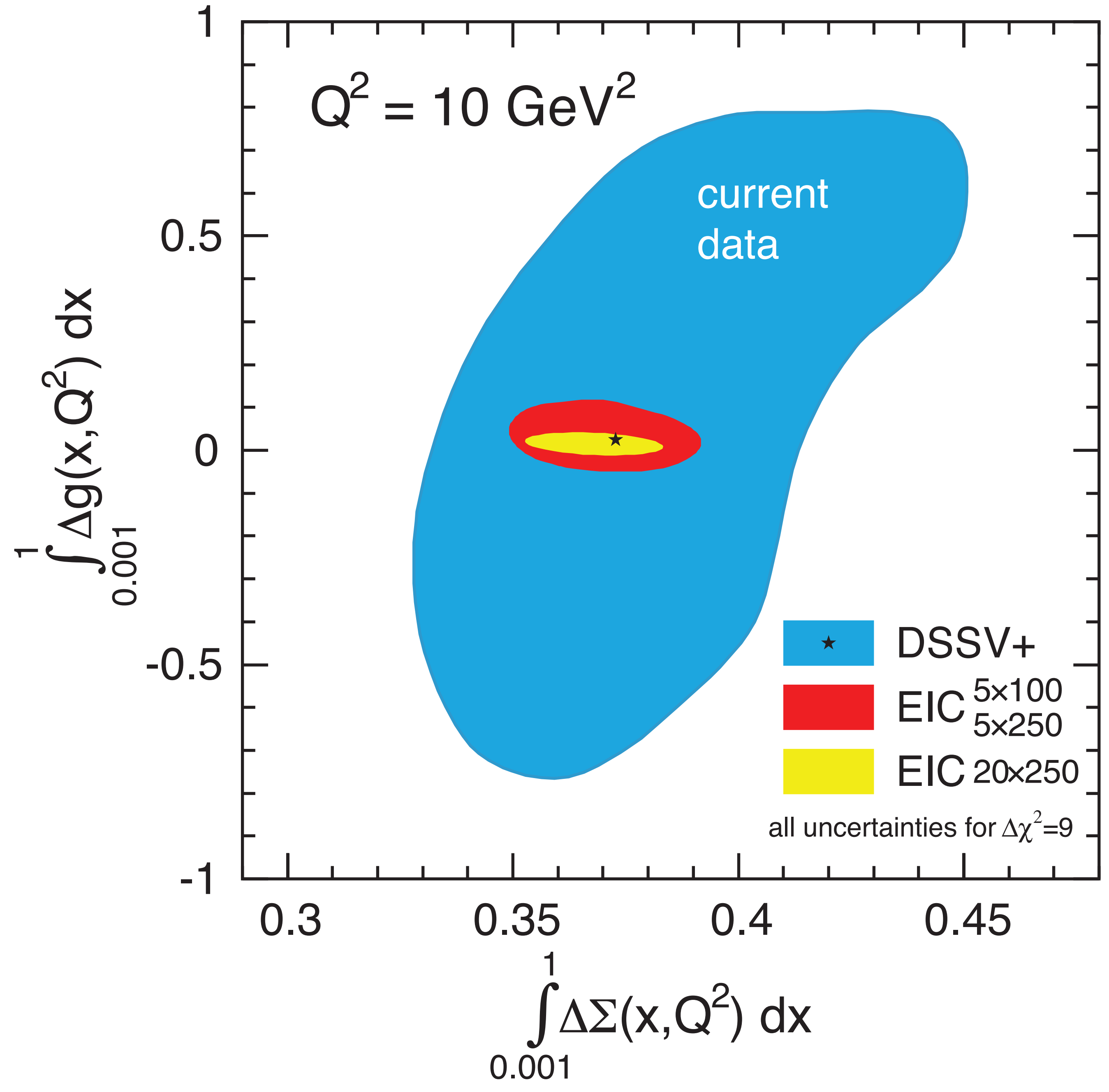
Jaffe & Manohar [1990]

- Gluon helicity distribution is not constrained from experimental data





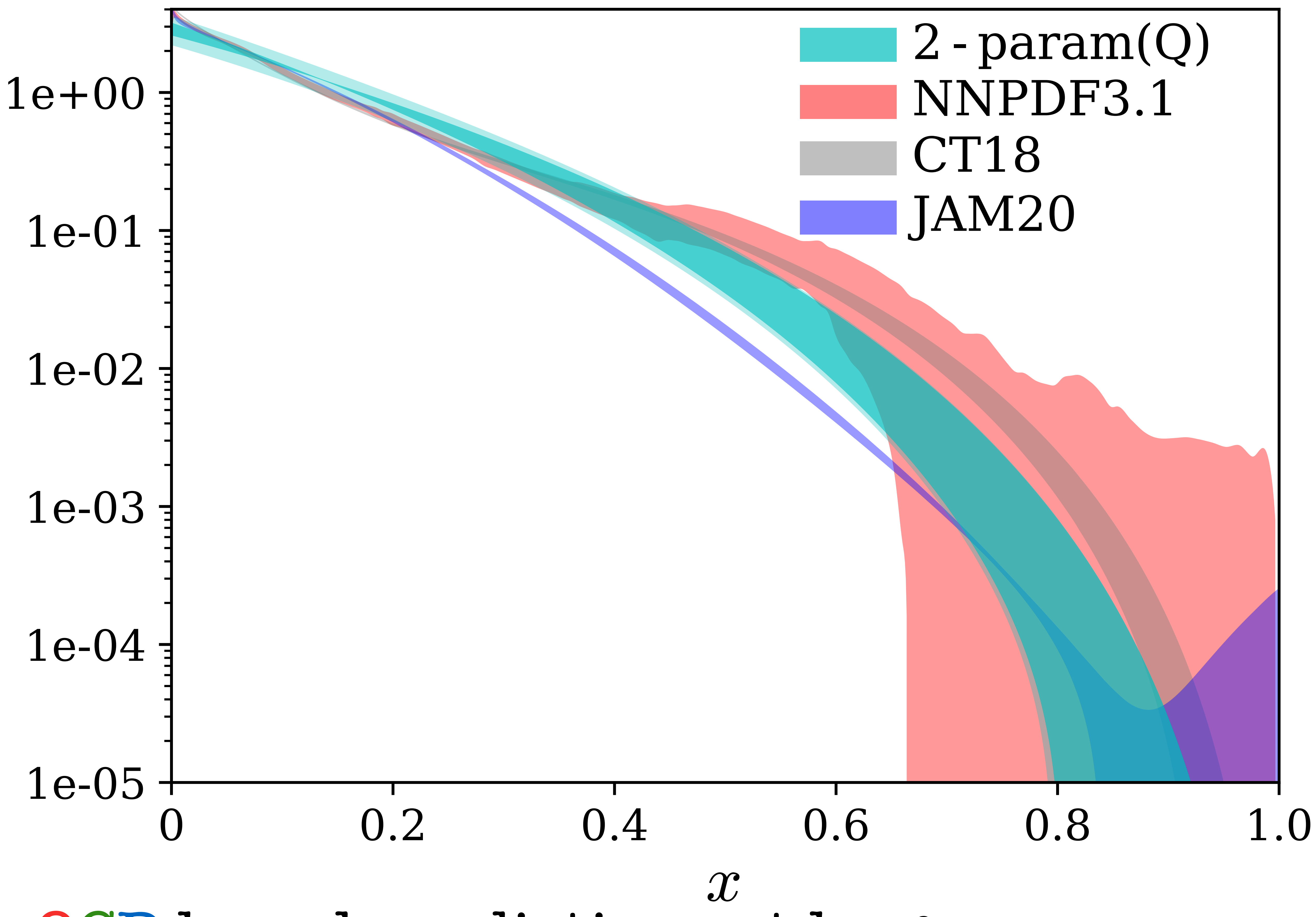
Impact of projected EIC data (EIC Yellow Report)



EIC white paper: EPJA (2016)

Status of unpolarized gluon distribution

- Noticeable differences in unpolarized gluon PDF between global fits

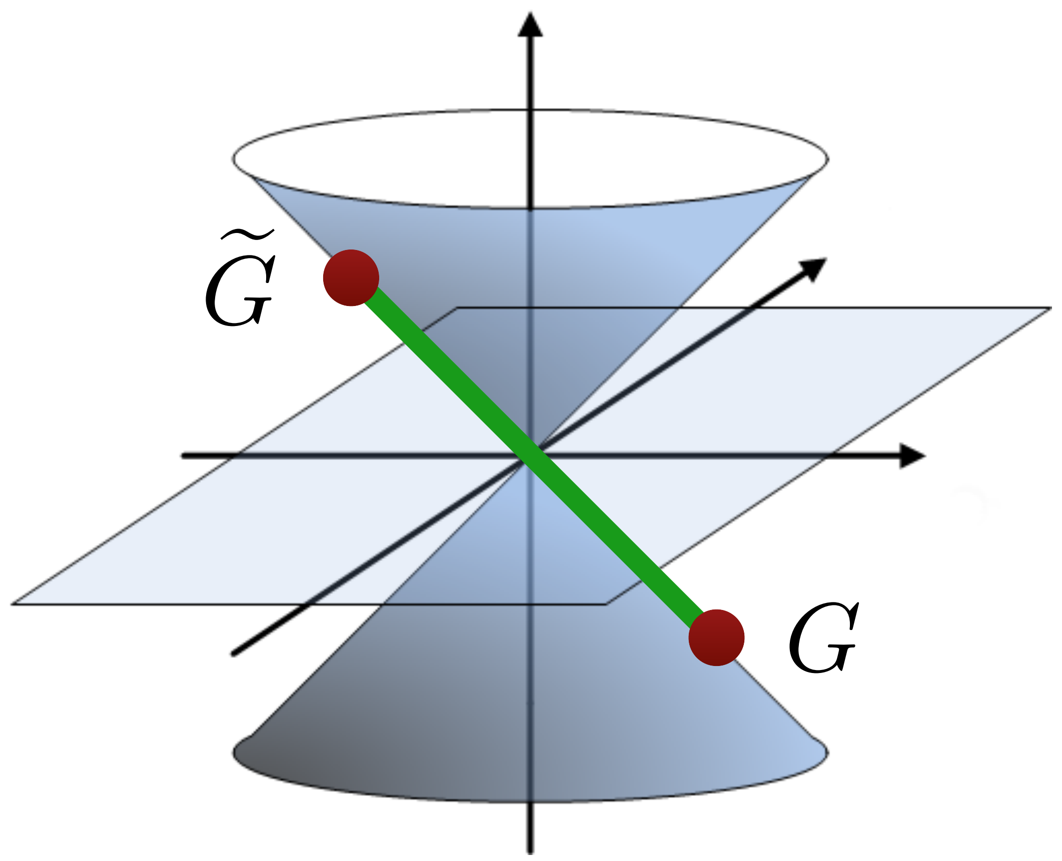


Khan, **RSS**, et al (HadStruc Collab) (PRD 2021)

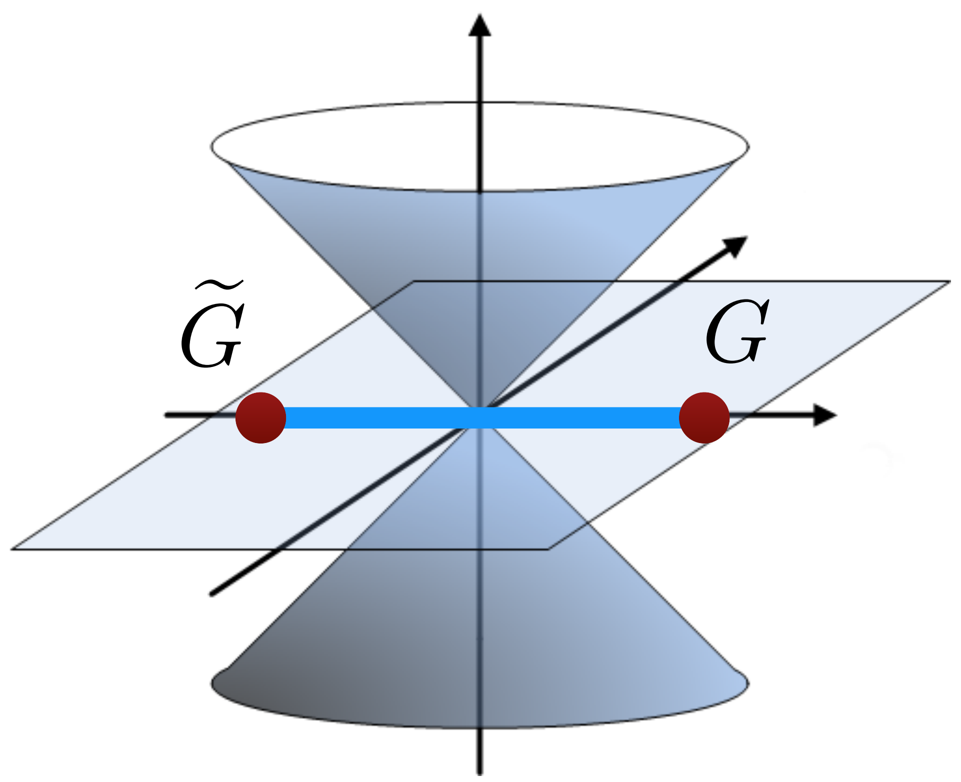
- Perturbative QCD based predictions at large x :
  - to which degree they hold?
  - modification due to nonperturbative effects?

LQCD formalism for calculating gluon PDFs

Light cone



Lattice



- For unpolarized gluon PDF: $M_{\mu\alpha;\lambda\beta}(z,p) \equiv \langle p|\, G_{\mu\alpha}(z)\, W[z,0]\, G_{\lambda\beta}(0)\, |p\rangle$

- For polarized gluon PDF: $\Delta M_{\mu\alpha;\lambda\beta}(z,p,s) = \langle p,s|\, G_{\mu\alpha}(z)\, W[z,0]\widetilde{G}_{\lambda\beta}(0)\, |p,s\rangle$

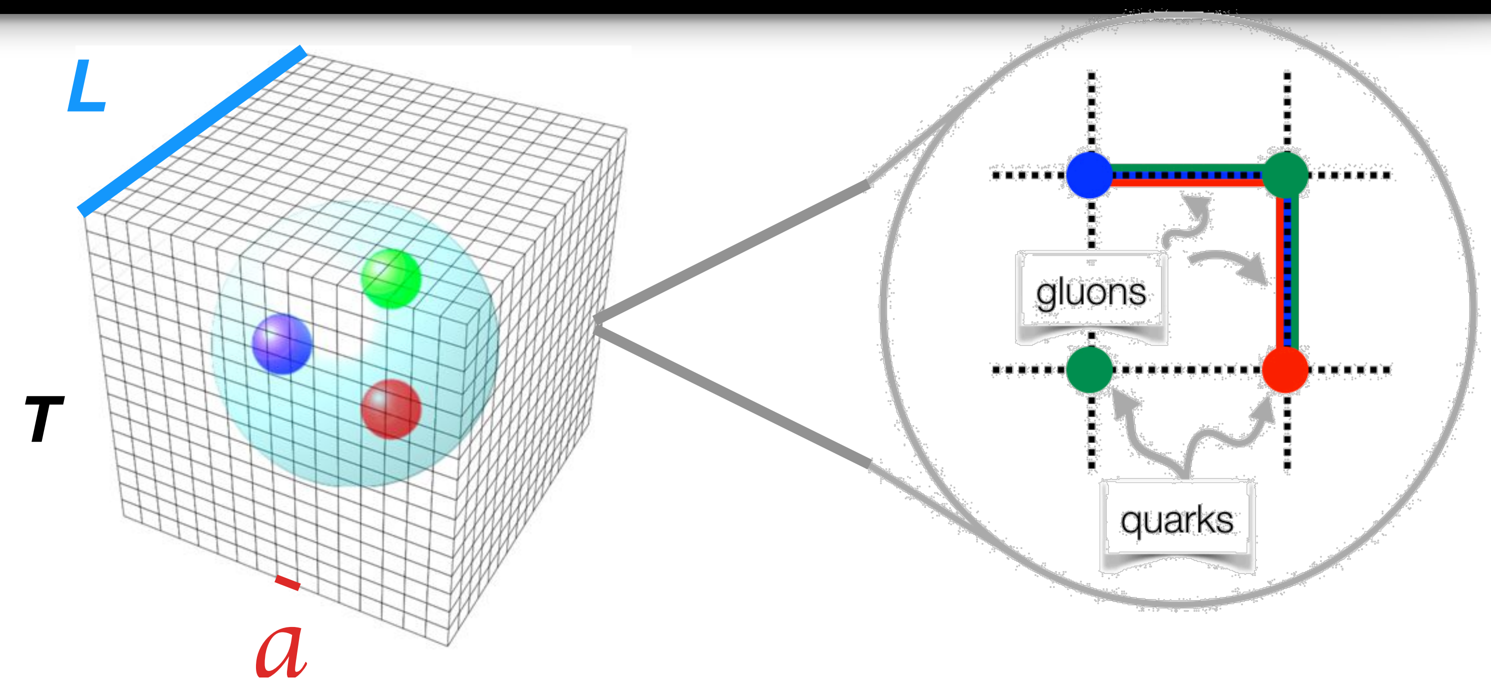
  - Quasi-PDFs/LaMET (Ji [PRL 2013])
  - Pseudo-PDFs (Radyushkin [PRD 2017])

X. Ji [PRL 2013]

- Appropriate combination for gluon helicity distribution calculation

$$\Delta\mathcal{M}_{00}(z,p_z) \equiv \Delta M_{0i;0i}(z,p_z) + \Delta M_{ij;ij}(z,p_z) \qquad (i,j = x,y)$$

Balitsky et al [JHEP 2022]



- Lattice details:

$L \times T = 32^3 \times 64 \qquad a \approx 0.094\,\text{fm} \qquad m_\pi = 358\,\text{MeV}$

- Matrix elements are multiplicatively renormalizable

Zhang, et al [PRL 2019], Li, et al [PRL 2019]

- Renormalization: $$\Delta\mathfrak{M}(z, p_z) \equiv i\frac{[\Delta\mathcal{M}_{00}(z, p_z)/p_z p_0]/Z_{\rm L}(z/a_L)}{\mathcal{M}_{00}(z, p_z = 0)/m_p^2}$$

Radyushkin [PRD 2017]
Balitsky, et al [JHEP 2022]

- Write renormalized LQCD matrix elements in terms of Lorentz invariant variables
  - z^2 and Ioffe time, $\omega = p_z z$

Braun, et al [PRD 1995]

$$\Delta\mathfrak{M}_g(\omega, z^2) \xrightarrow[\text{matching}]{\text{perturbative}} \Delta\mathcal{I}_g(\omega, \mu) = \frac{i}{2}\int_{-1}^{1} dx\, e^{-ix\omega} x\Delta g(x, \mu)$$

Saalfeld, et al [EPJ1998]

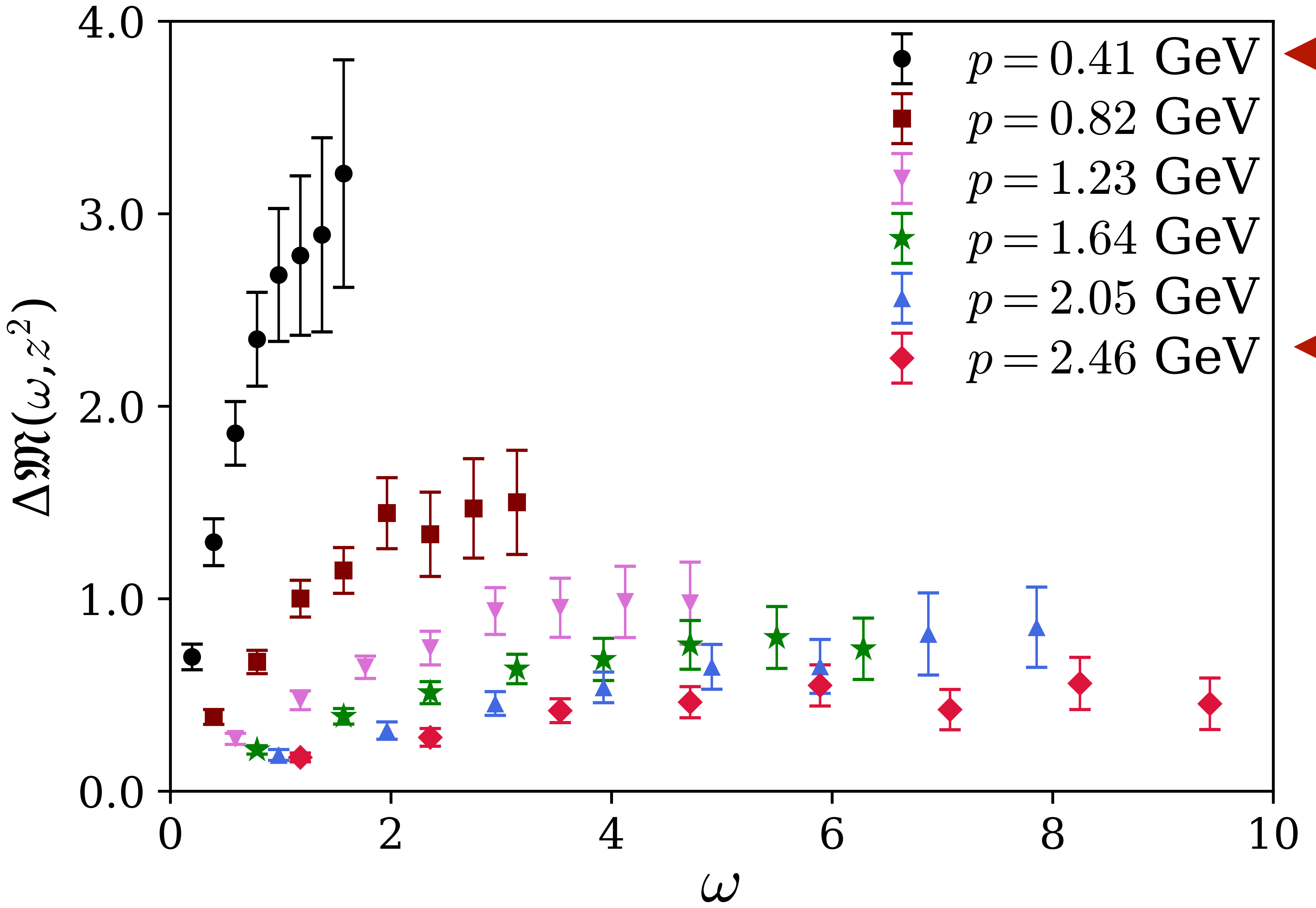
LQCD matrix elements for polarized gluon distribution

- What we want for the light-cone Ioffe-time distribution:

$$\Delta\mathcal{I}_g(\omega,\mu) \equiv i[\Delta\mathcal{M}_{sp}^{(+)}(\omega,\mu) - \omega\Delta\mathcal{M}_{pp}(\omega,\mu)]$$

- What we get from the lattice calculation:

$$\Delta\mathfrak{M}(\omega,z^2) = [\Delta\mathcal{M}_{sp}^{(+)}(\omega,z^2) - \omega\Delta\mathcal{M}_{pp}(\omega,z^2)] - \frac{m_p^2}{p_z^2}\omega\Delta\mathcal{M}_{pp}(\omega,z^2)$$



$m_p^2/p^2 \sim 7.3$

$m_p^2/p^2 \sim 0.2$

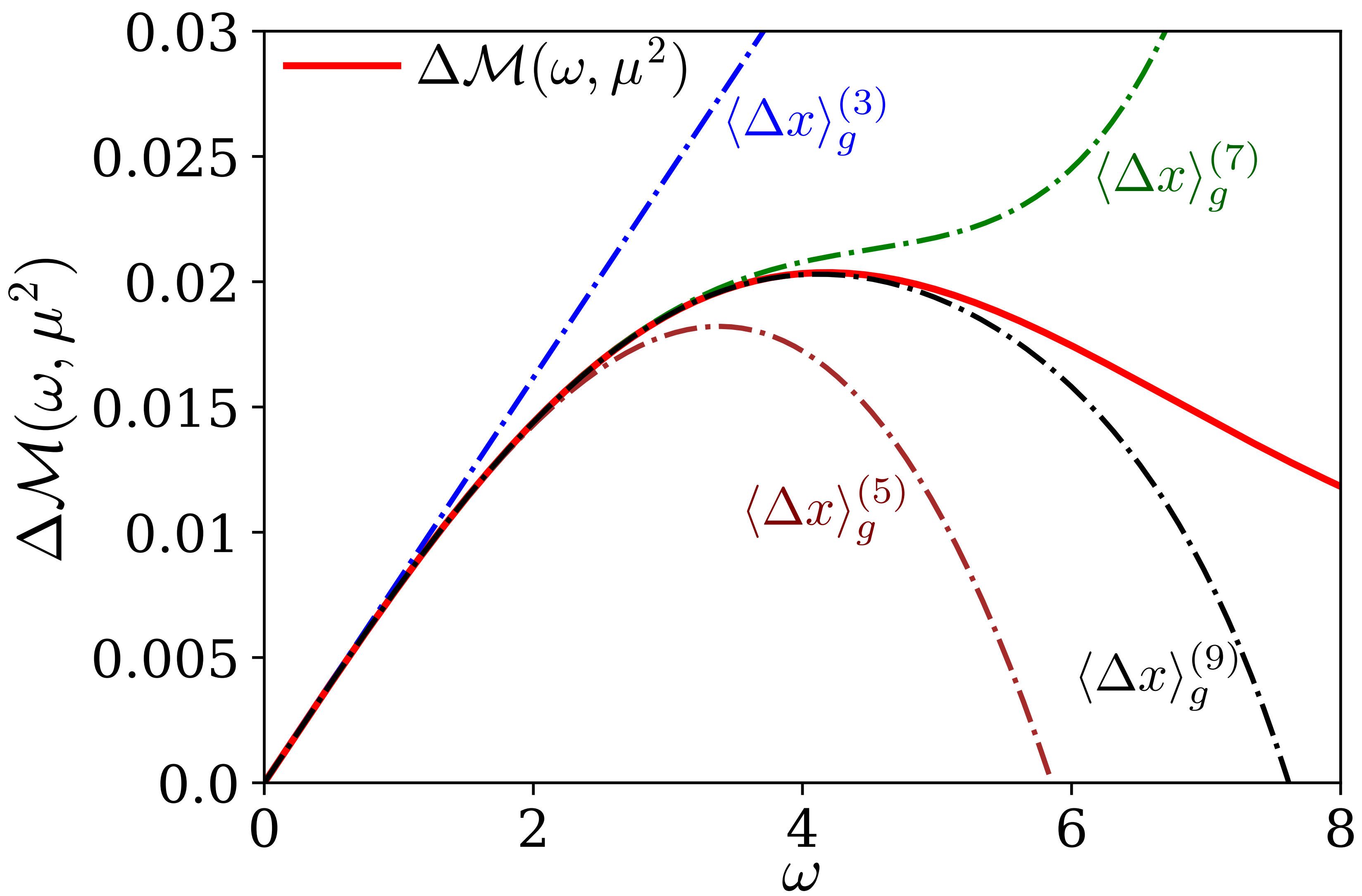
grows with ω

Isolating gluon helicity Ioffe-time distribution from LQCD data

- Correction through fits using moments
  - $\Delta\mathcal{M}_{sp}^{(+)}$: odd in ω
  - $\Delta\mathcal{M}_{pp}$: even in ω

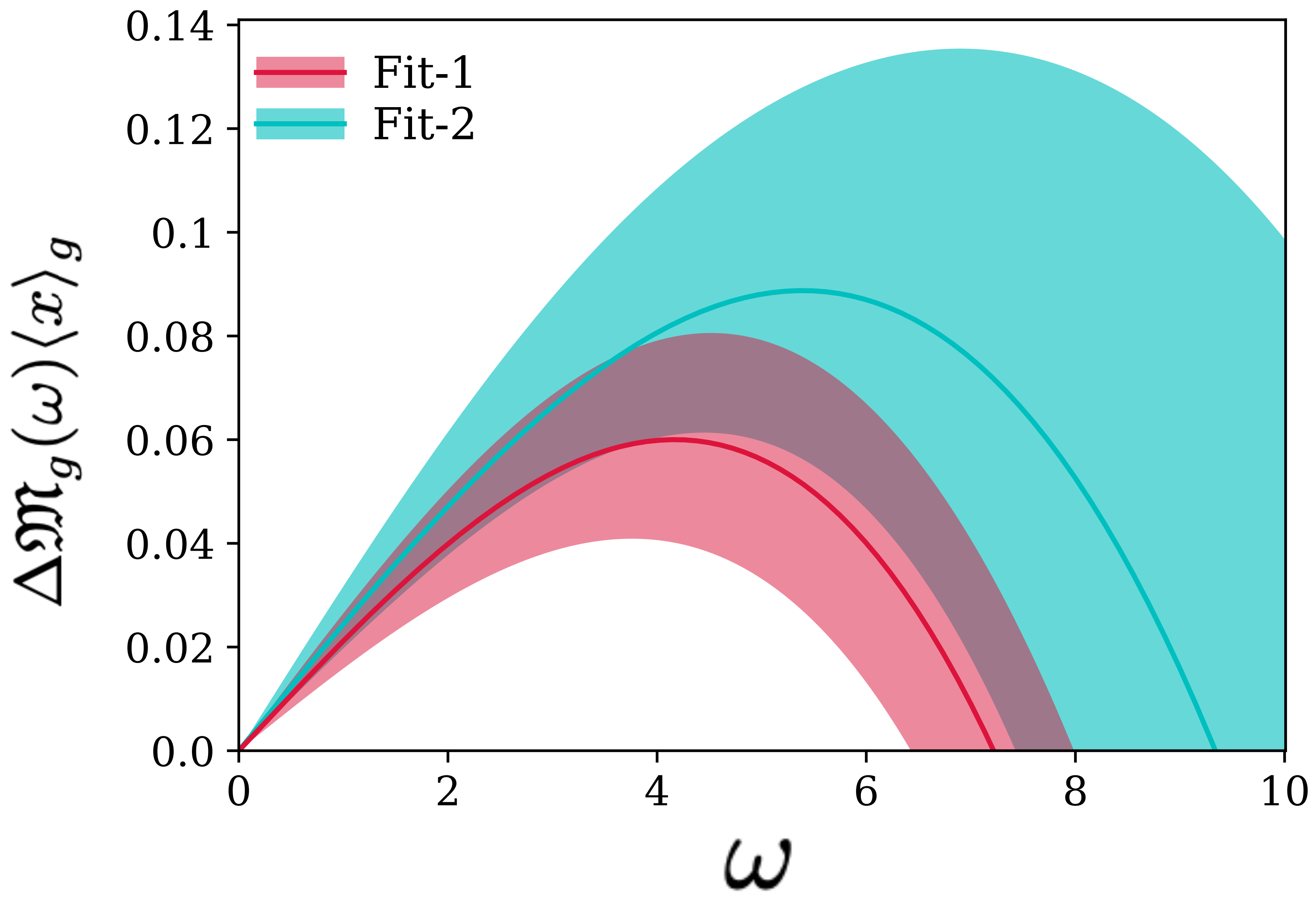
$$\Delta\mathfrak{M}(\omega) = \sum_{i=0} \frac{(-1)^i}{(2i+1)!} a_i \omega^{2i+1} + \omega \frac{m_p^2}{p_z^2} \sum_{j=0} \frac{(-1)^j}{(2j)!} b_j \omega^{2j}$$

- Truncation dependent & limited by lattice data

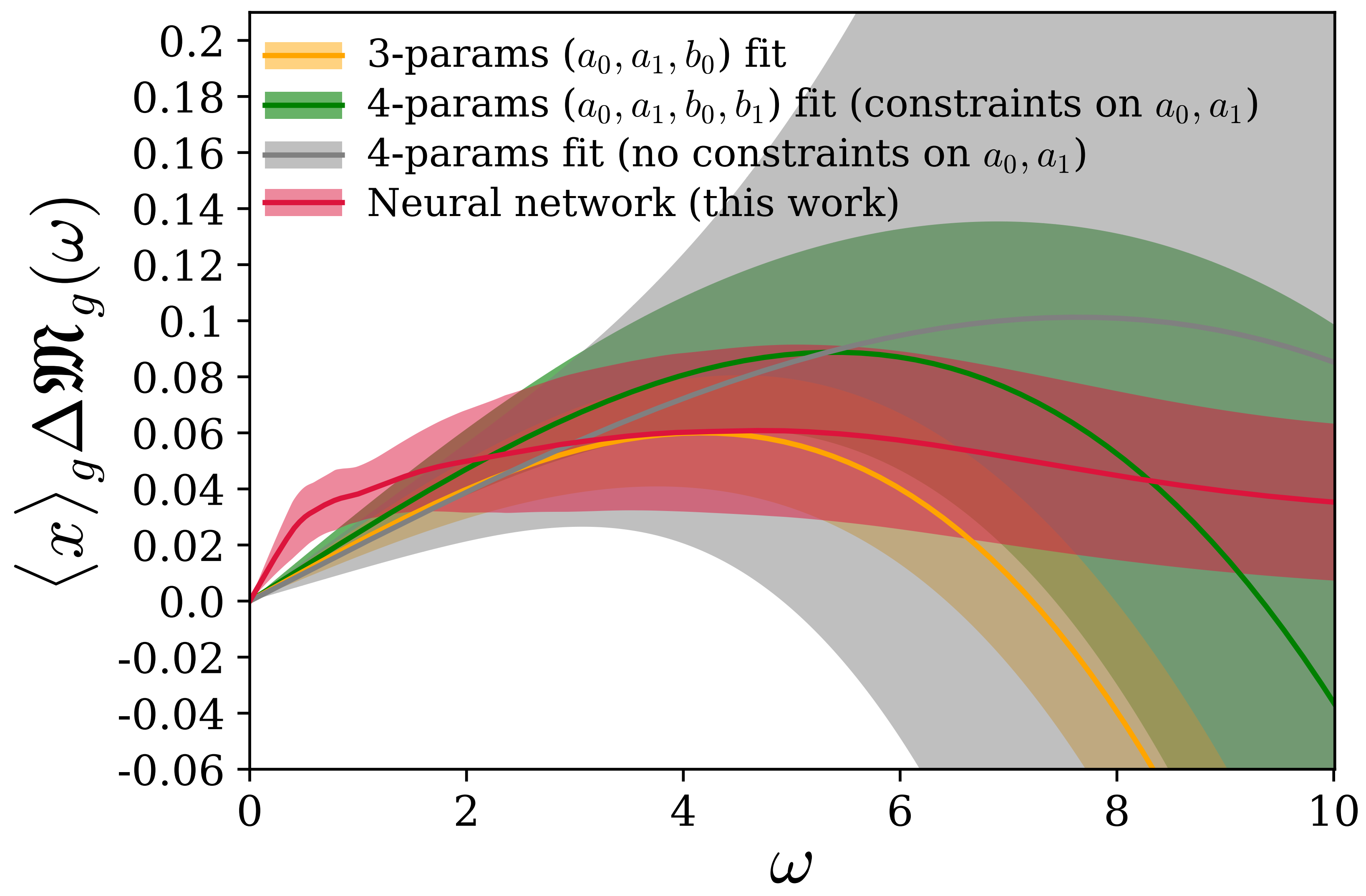


***RSS*, Liu, Paul**
PRD 2021

Saalfeld, et al [EPJ1998]



***RSS*, Khan, Karthik, et al**
(HadStruc Collaboration)
PRD 2022



Khan, Liu, RSS
[arXiv: 2211.15587]

Isolating gluon helicity Ioffe-time distribution from LQCD data

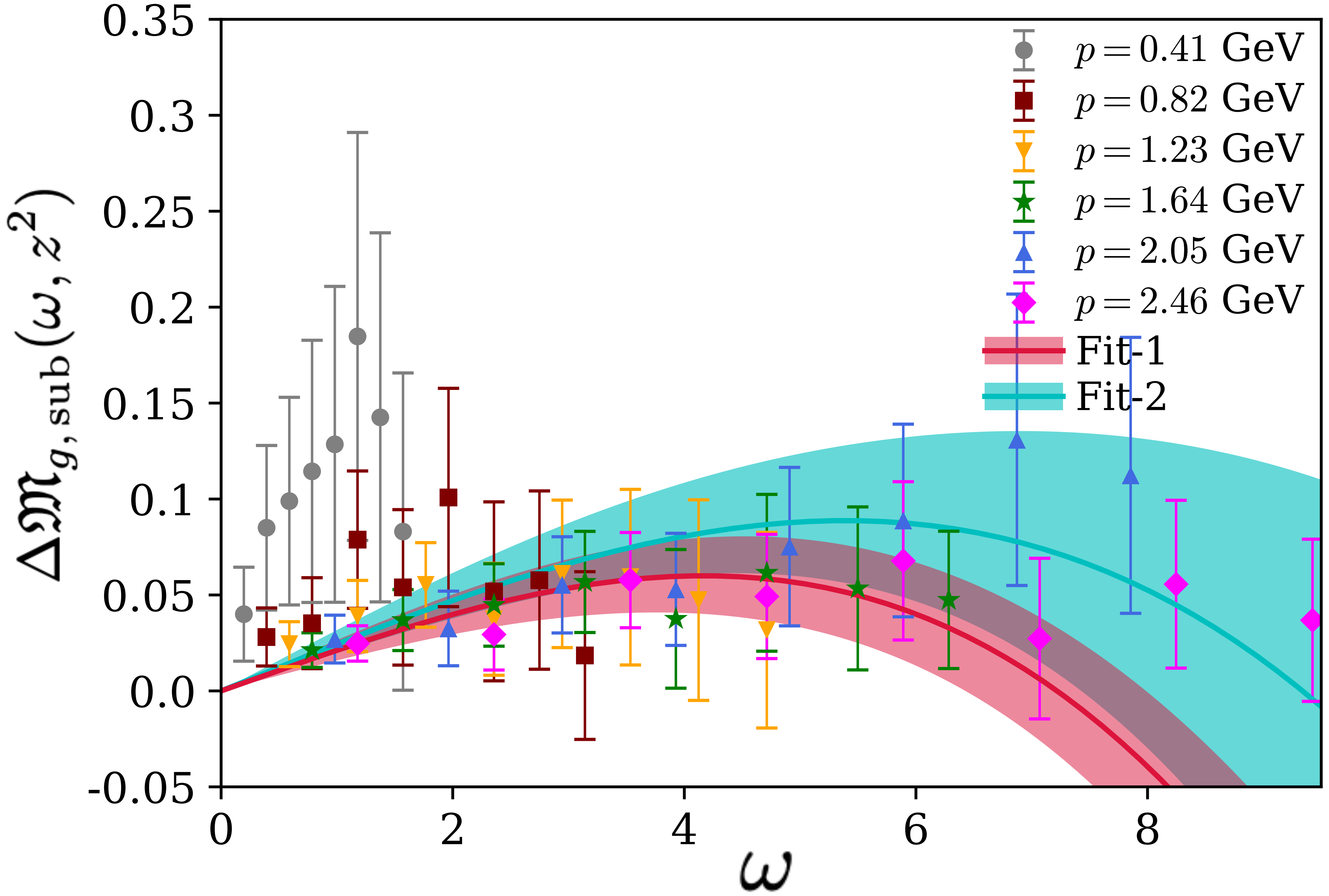
- Correction by subtracting zero momentum matrix elements

$$\Delta M_{0i;0i}(z,p) + \Delta M_{ij;ij}(z,p) = -2p_z p_0 \Delta\mathcal{M}^{(+)}_{sp}(\omega, z^2) + 2p_0^3 z \Delta\mathcal{M}_{pp}(\omega, z^2)$$

non-vanishing at $p_z = 0$

- Proposed subtraction :

$$\Delta\mathfrak{M}_{g,\,\mathrm{sub}}(\omega, z^2) = \Delta\mathcal{M}^{(+)}_{sp}(\omega, z^2) - \omega\Delta\mathcal{M}_{pp}(\omega, z^2) - \omega\frac{m_p^2}{p_z^2}[\Delta\mathcal{M}_{pp}(\omega, z^2) - \Delta\mathcal{M}_{pp}(\omega = 0, z^2)]$$

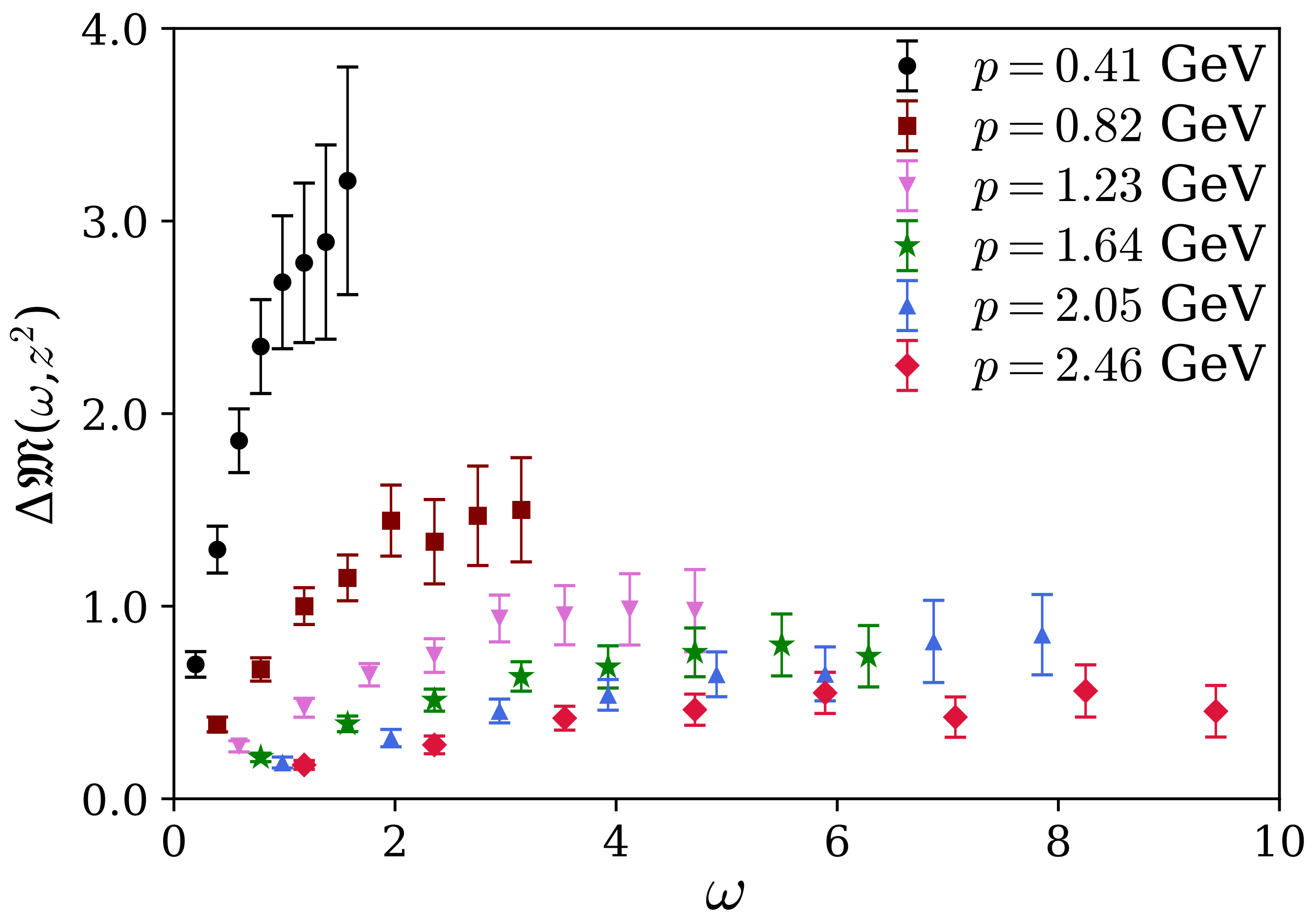


***RSS*, Khan, Karthik, et al (HadStruc Collaboration) [PRD 2022]**

- Lattice discrete momenta are related by

$$p_n = 2\pi n/(La)$$

- Isolate momentum-independent contamination term:



$$p_k^2 \Delta\mathfrak{M}(\omega)\Big|_{p_k} = p_k^2[\Delta\mathcal{M}_{sp}^{(+)}(\omega) - \omega\Delta\mathcal{M}_{pp}(\omega)] - m_p^2\omega\Delta\mathcal{M}_{pp}(\omega)$$

- Two different p data sets are related by

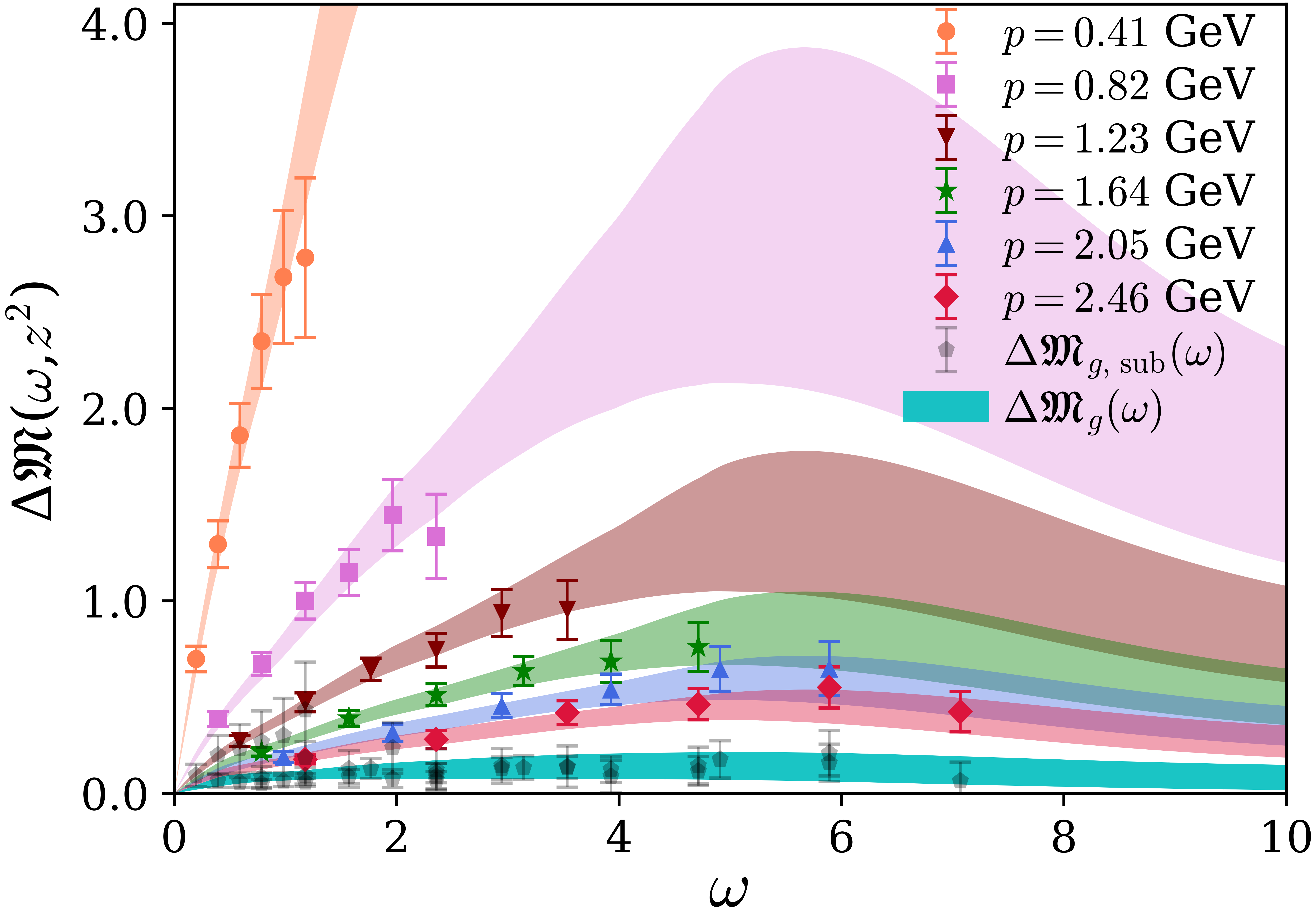
$$\Delta\mathfrak{M}_g(\omega) \equiv \Delta\mathcal{M}_{sp}^{(+)}(\omega) - \omega\Delta\mathcal{M}_{pp}(\omega) = \frac{r^2\Delta\mathfrak{M}(\omega)\Big|_{p_k} - \Delta\mathfrak{M}(\omega)\Big|_{p_l}}{r^2 - 1} \qquad \boxed{r = k/l}$$

  - **Main regulator for the neural network**

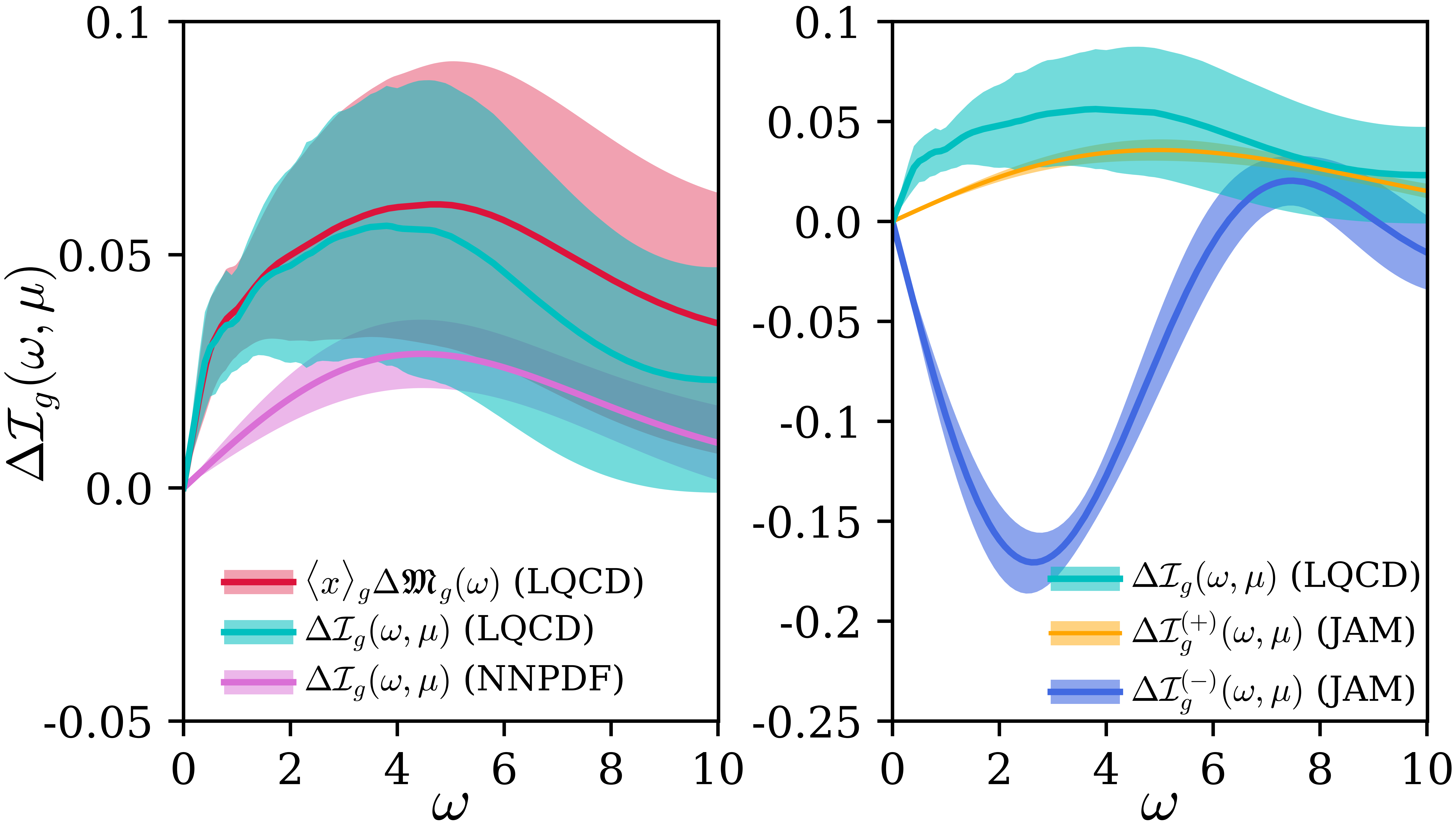
Khan, Liu, RSS [arXiv: 2211.15587]

- Eliminate the contamination term (connect two data sets) using neural network analysis

- Simultaneous fit to $\Delta\mathfrak{M}(\omega, z^2)$ and $\Delta\mathfrak{M}_{g,\,\mathrm{sub}}(\omega, z^2)$



- Ruling out negative gluon helicity PDF in moderate to large x-region



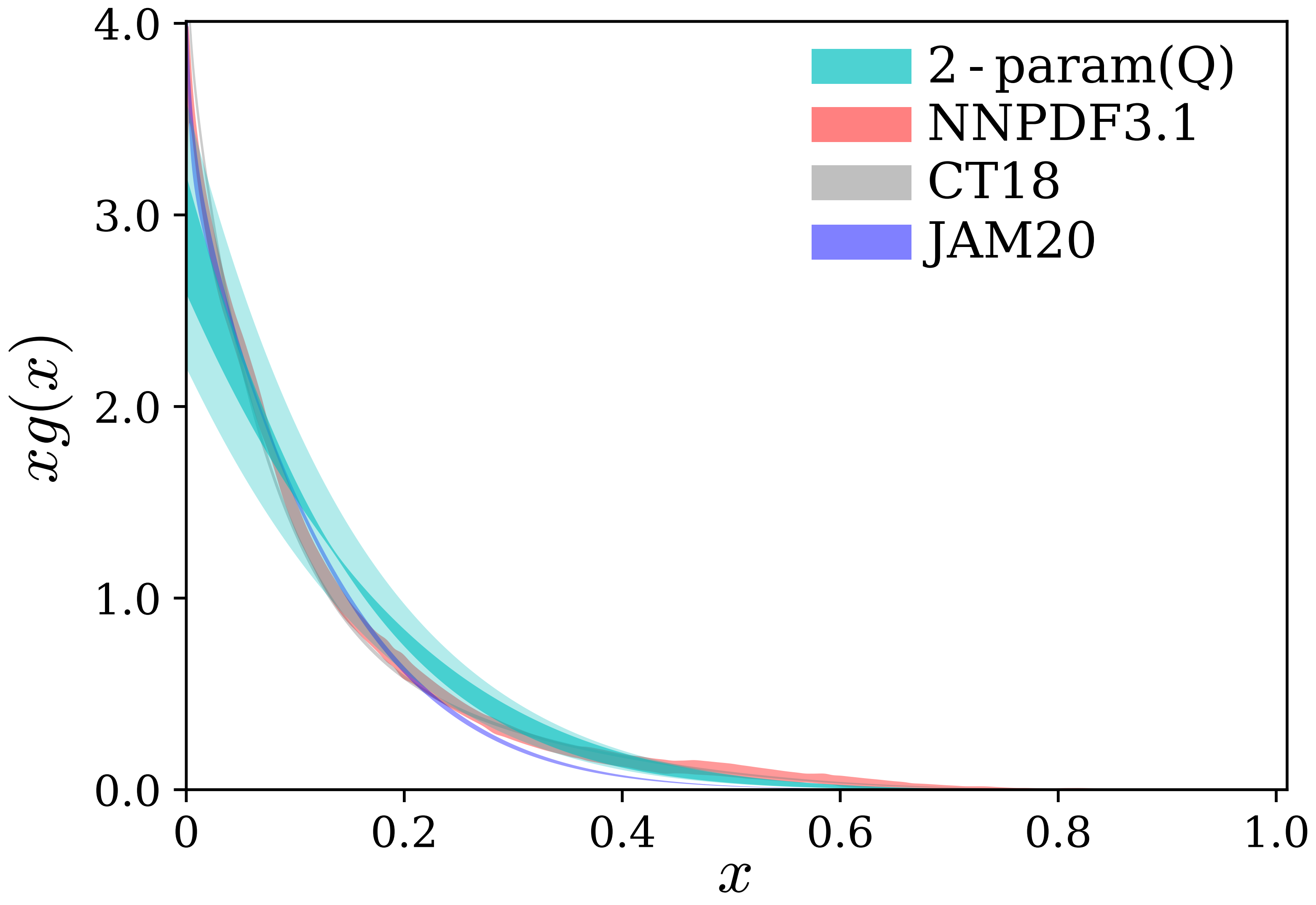
Khan, Liu, RSS [arXiv: 2211.15587]

Zhou. et al (JAM Collab) [PRD 2022]

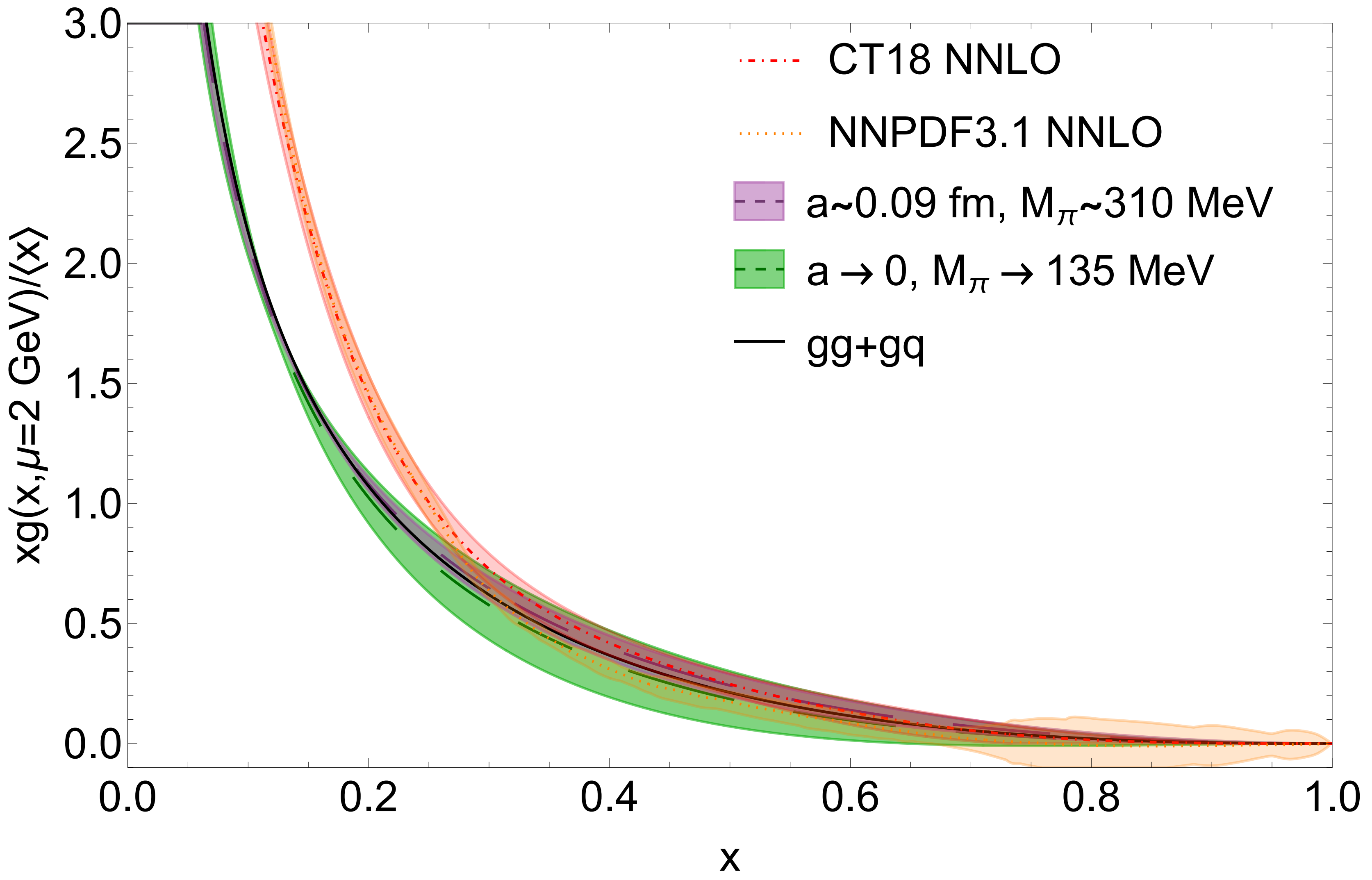
A big challenge: how to extract PDF from LQCD data

- Consider case for unpolaorized gluon PDF

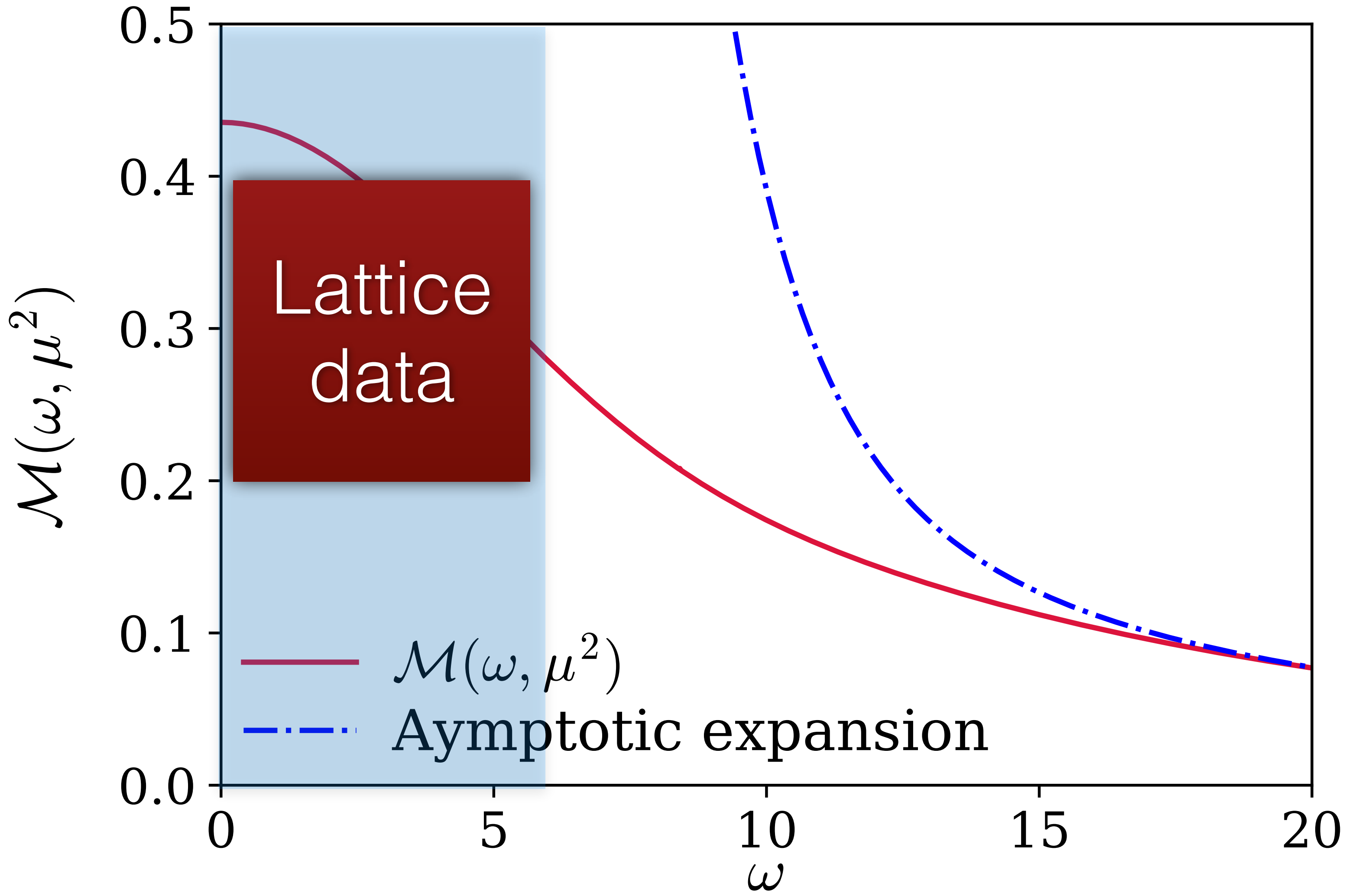
$$xg(x) = \frac{x^{\alpha}(1-x)^{\beta}(1+\cdots)}{B(\alpha+1,\beta+1)+\cdots}$$



Khan, **RSS**, et al (HadStruc Collab) (PRD 2021)



Fan, Good, Lin : arXiv: 2210.09985



RSS, Liu, Paul [PRD 2021]

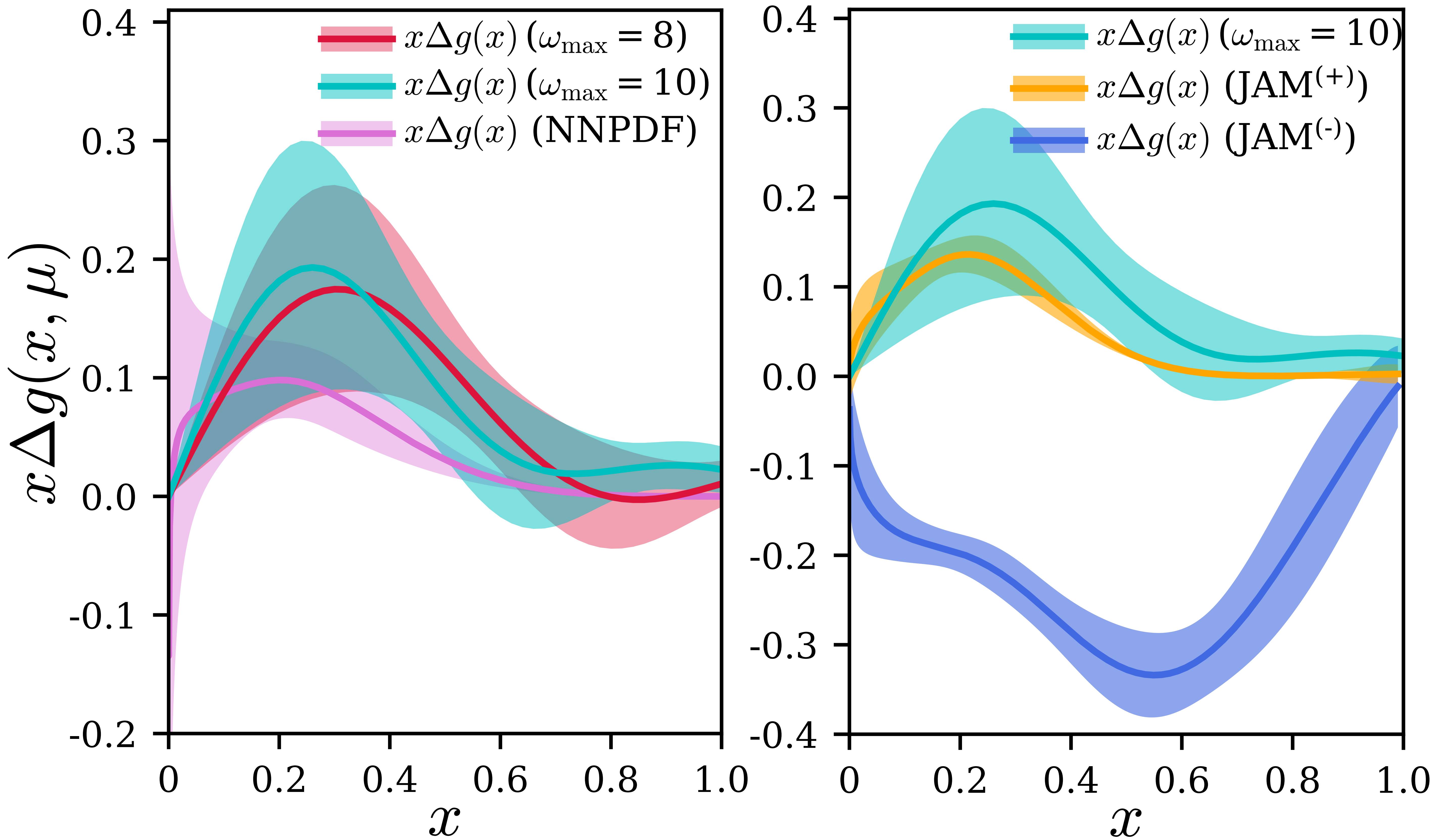
- Results can be very different from lattice data sets that appear similar
- *Different ML methods being implemented for extrapolations of unpolarized gluon data*

Extraction of PDF from LQCD data

- Avoid fitting lattice data with limited and biased functional forms
  [*not the case for global fits (e.g. CTEQ and others) with ample experimental data sets*]

$$x\Delta g(x,\mu) = \frac{2}{\pi}\int_0^\infty d\omega \;\; \sin(x\omega) \;\; \Delta\mathcal{I}_g(\omega,\mu)$$

- ▶ Accuracy depends on maximum ω



Khan, Liu, RSS [arXiv: 2211.15587]

Summary & Outlook

- With increased precision, LQCD can constrain unpolarized gluon distribution in the moderate-to-large x-region

- LQCD tends to rule out negative gluon polarization in the nucleon

  - Gluon contribution to proton spin & x-dependent helicity distribution from LQCD

- Physics informed machine learning methods can be useful to predict lattice data outside accessible range (future applications to expose higher twist effects)

Thank you!