Pion and kaon form factors from lattice QCD

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Extended Twisted Mass Collaboration

APS

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Motivation

- Understanding structure of pion and kaon important for describing QCD dynamics
- Useful in studying SU(3) symmetry breaking
- Pion and kaon less studied than proton
- Most experimental, theoretical, and lattice studies on pion form factor
- Important to understand pion and kaon from first principles

PHYSICAL REVIEW D **103,** 014508 (2021)

Mellin moments $\langle x \rangle$ and $\langle x^2 \rangle$ for the pion and kaon from lattice QCD

Constantia Alexandrou,[1,2] Simone Bacchio,[1,2] Ian Cloët,[3] Martha Constantinou,[4] Kyriakos Hadjiyiannakou,[1,2] Giannis Koutsou,[2] and Colin Lauer[3,4]

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PHYSICAL REVIEW D **104,** 054504 (2021)

Pion and kaon $\langle x^3 \rangle$ from lattice QCD and PDF reconstruction from Mellin moments

Constantia Alexandrou,[1,2] Simone Bacchio,[2] Ian Cloët,[3] Martha Constantinou,[4] Kyriakos Hadjiyiannakou,[1,2] Giannis Koutsou,[2] and Colin Lauer[3,4]

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PHYSICAL REVIEW D **105,** 054502 (2022)

Scalar, vector, and tensor form factors for the pion and kaon from lattice QCD

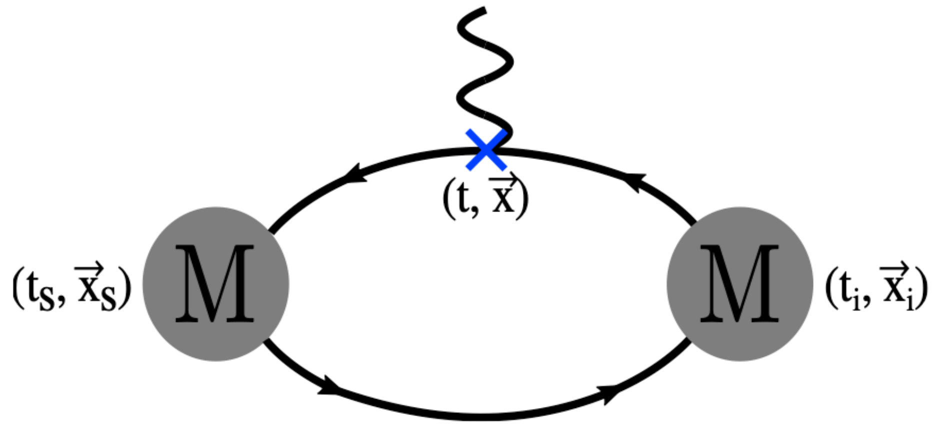
Constantia Alexandrou,[1,2] Simone Bacchio,[2] Ian Cloët,[3] Martha Constantinou,[4] Joseph Delmar,[4] Kyriakos Hadjiyiannakou,[1,2] Giannis Koutsou,[2] Colin Lauer,[4] and Alejandro Vaquero[5]

(ETM Collaboration)

Theoretical Setup

- Form factors obtained from matrix elements of ultra-local operators

$$\mathcal{O}^f_S = \bar{\psi}\hat{1}\psi \quad \mathcal{O}^f_V = \bar{\psi}\gamma^\mu\psi \quad \mathcal{O}^f_T = \bar{\psi}\sigma^{\mu\nu}\psi$$



- Matrix elements decompose to usual form factors

$$\langle M(p')|\mathcal{O}^f_V|M(p)\rangle = -i\frac{2P^\mu}{\sqrt{4E(p)E(p')}}F^{M,f}_V$$

$$\langle M(p')|\mathcal{O}^f_S|M(p)\rangle = \frac{1}{\sqrt{4E(p)E(p')}}F^{M,f}_S$$

$$\langle M(p')|\mathcal{O}^f_T|M(p)\rangle = i\frac{P^\mu Q^\nu - P^\nu Q^\mu}{m_M\sqrt{4E(p)E(p')}}F^{M,f}_T$$

- Two methods to extract ground-state contribution
  - fit plateau region to a constant value
  - perform a two-state fit on three-point function
- Form factors are frame independent
- Non-perturbative renormalization ($\overline{\mathrm{MS}}$ at 2 GeV)

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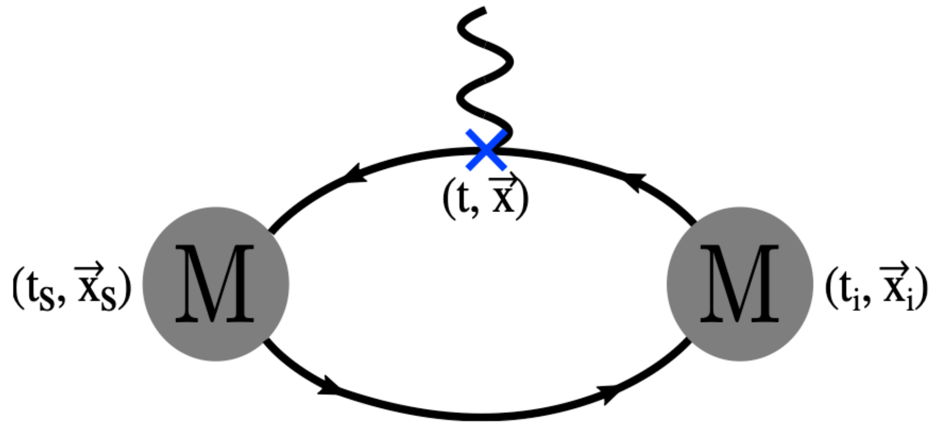

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$F_T(0)$ cannot be accessed directly

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Lattice Details

- Two ensembles of twisted-clover fermions and Iwasaki improved gluons [C. Alexandrou et al., PRD 104, 074520 (2021)]

Parameters

Ensemble	β	a(fm)	$L^3 \times T$	N_f	m_π(MeV)	L(fm)
cA211.30.32	1.726	0.094	$32^3 \times 64$	$2+1+1$	265	3.0
cB211.25.48	1.778	0.079	$48^3 \times 96$	$2+1+1$	250	3.79

- Boosted frame gives access to a denser range of -t

$$-t = Q^2_{boosted} = \vec{q}^2 - (E(p') - E(p))^2$$

- Kinematic frame: $\vec{p}' = \frac{2\pi}{L}(\pm 1, \pm 1, \pm 1), \quad \vec{p} = \vec{p}' - \vec{q}$

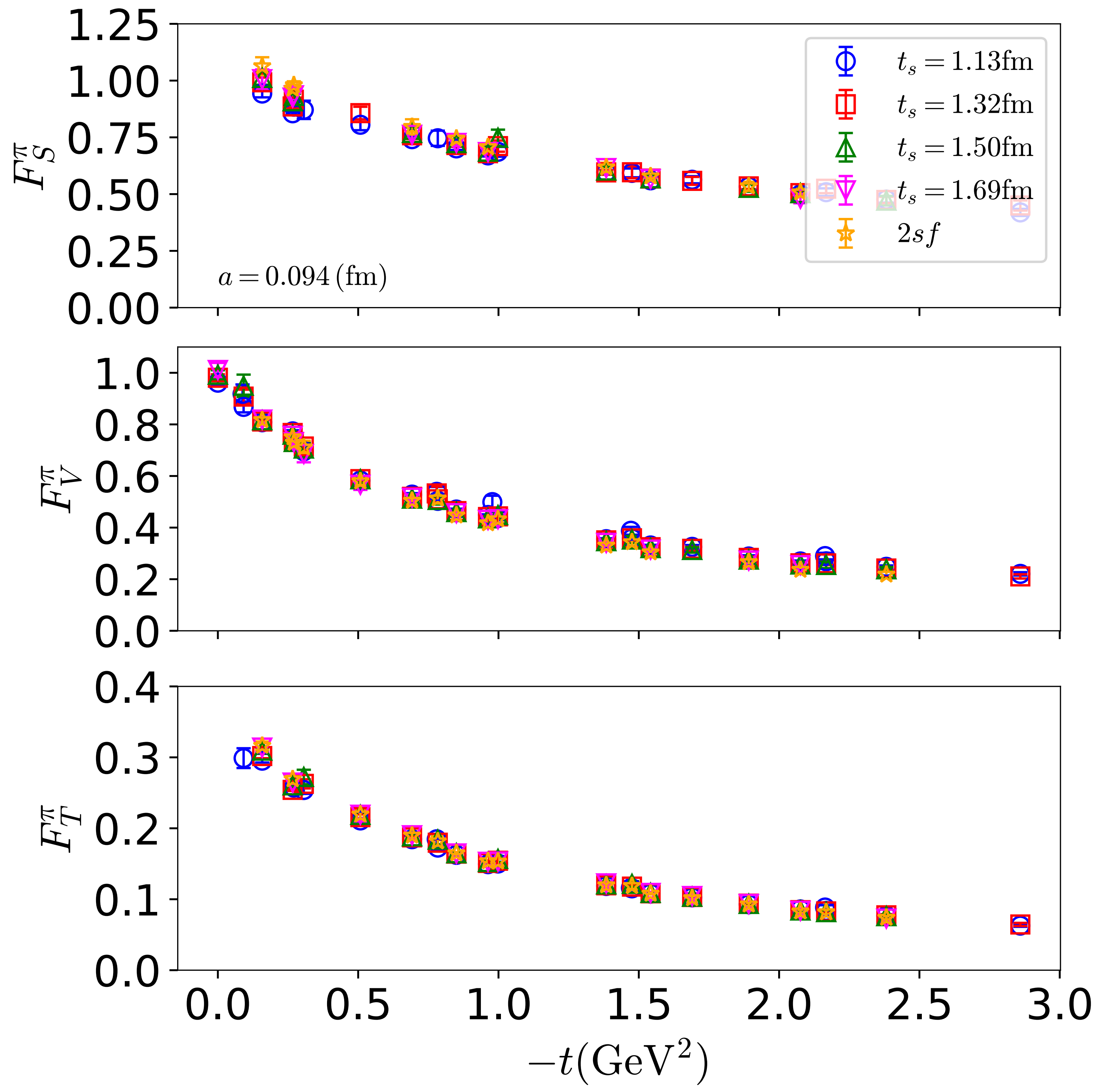
(matrix element for all values of $\vec{q}$ are obtained at once (sequential method))

$$\vec{q} = \frac{2\pi}{L}(\pm n_x, \pm n_y, \pm n_z) \quad n_x, n_y, n_z \in [0,8]$$

Statistics

Ensemble	Frame	$\vec{p}\,(2\pi/L)$	t_s/a	t_s (fm)	confs	src pos.	Total
cA211.30.32	R	$(0,0,0)$	12, 14, 16, 18, 20, 24	1.13, 1.32, 1.50, 1.69, 1.88, 2.256	122	16	1,952
cA211.30.32	B	$(\pm1,\pm1,\pm1)$	12, 14, 16, 18	1.13, 1.32, 1.50, 1.69	122	136	132,736
cB211.25.48	B	$(\pm1,\pm1,\pm1)$	14, 16, 18, 20	1.11, 1.26, 1.42, 1.58	45	6	2,160

Pion Form Factors



- Increase of statistical error is not linear with $\vec{q}$ $(-t = \vec{q}^2 - (E(p') - E(p))^2)$
  → a careful analysis required to select data points with controlled uncertainties
- Excited state effects suppressed ~0.5 GeV^2 for tensor and ~1.0 GeV^2 for scalar

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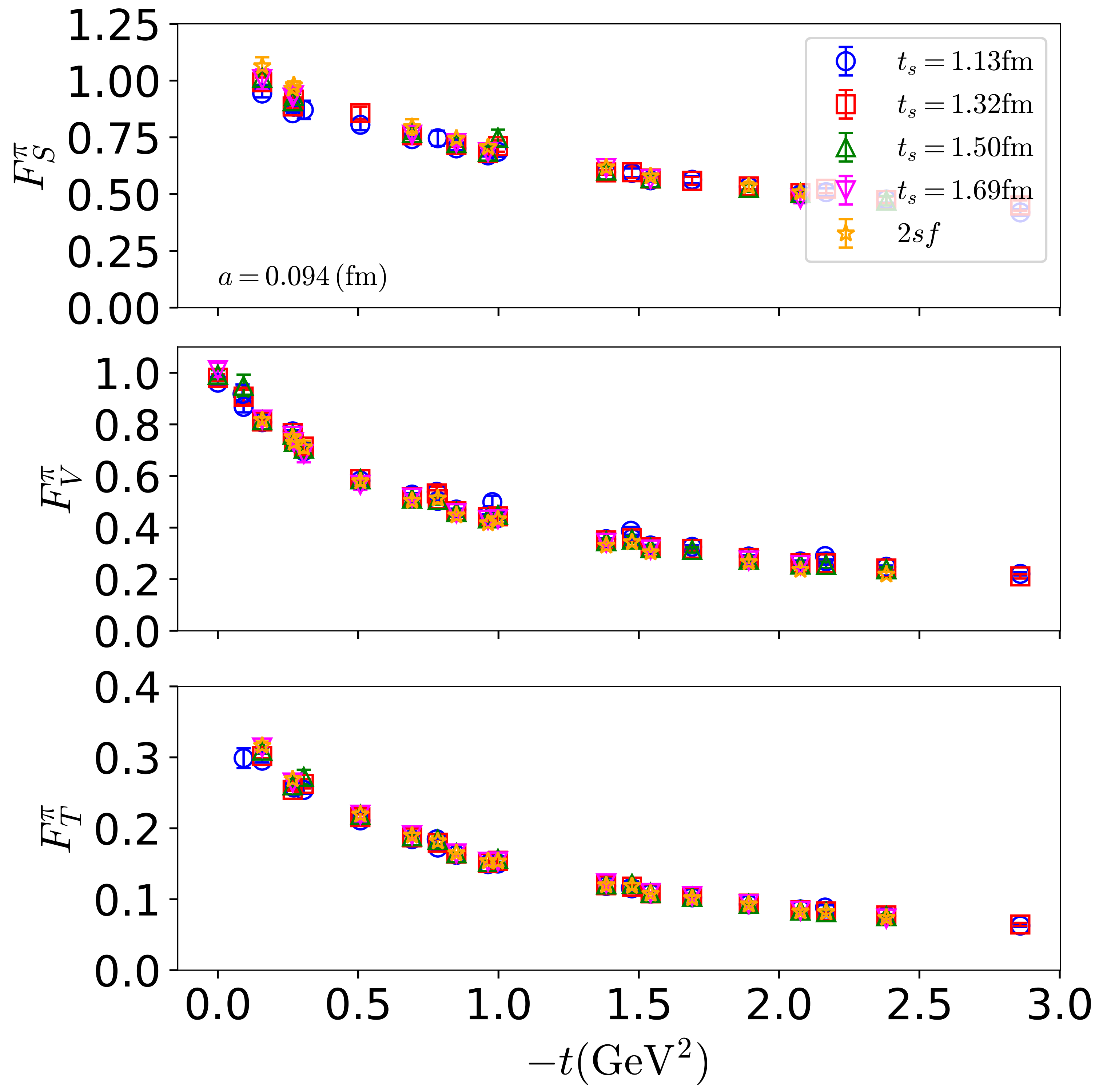

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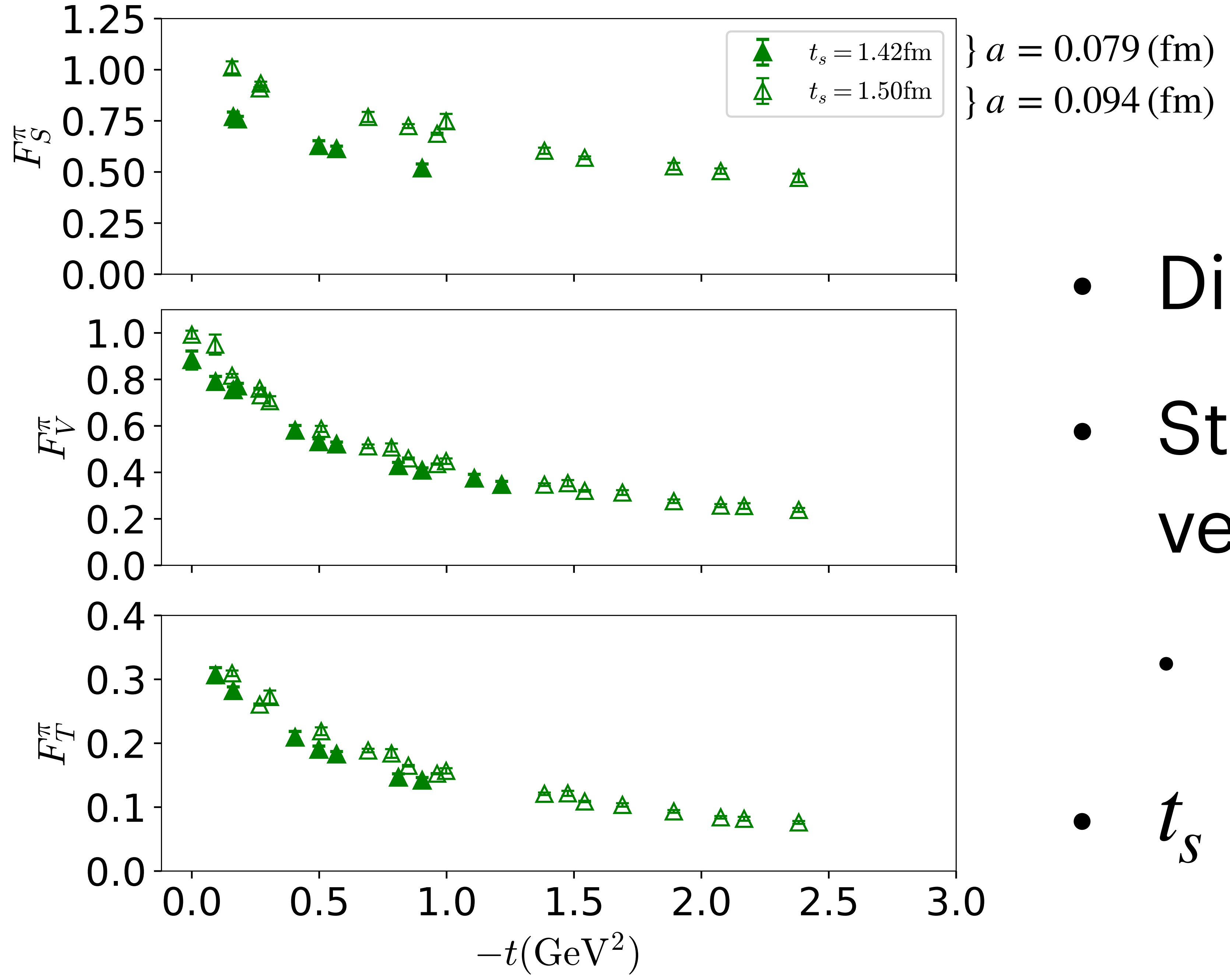
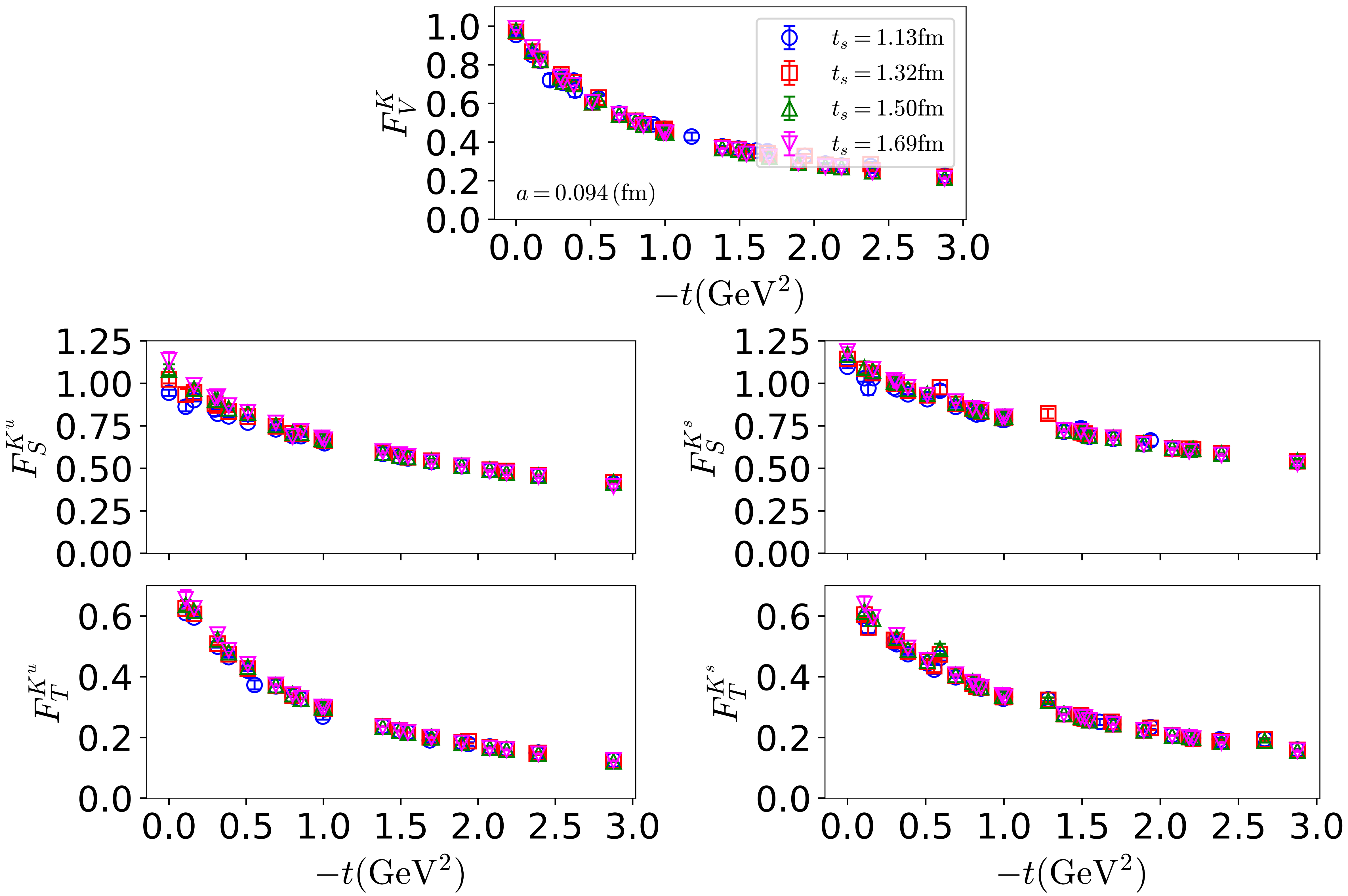


- Discretization effects non-negligible
- Statistics for $a = 0.079$ fm ensemble very small
  - Pion particularly effected
- $t_s \sim\ 1.6$ fm available and under analysis

Kaon Form Factors



- Outliers in small t_s
- Excited-state effects primarily in scalar and tensor
- Good signal up to about 3.0 GeV^2

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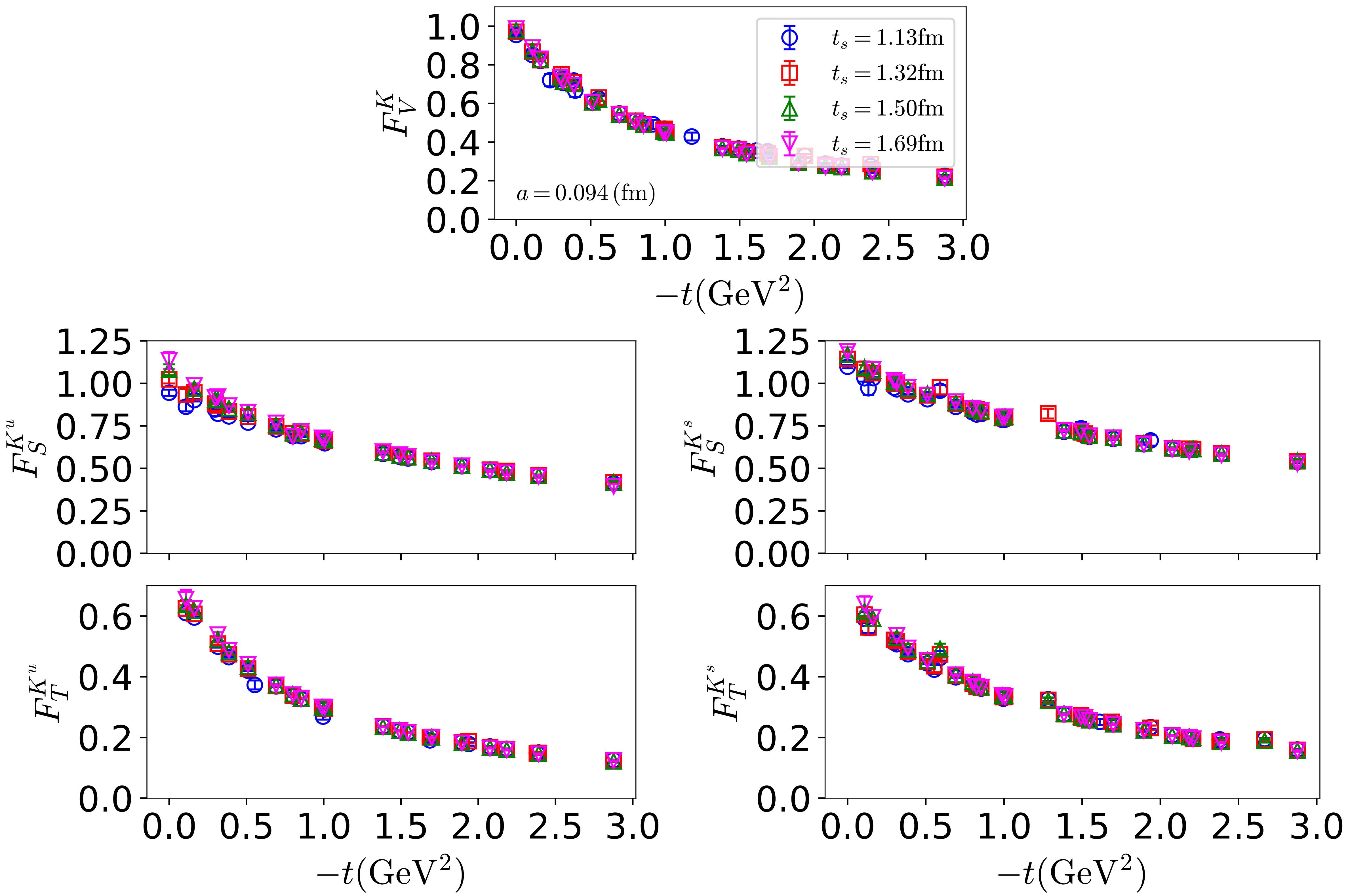

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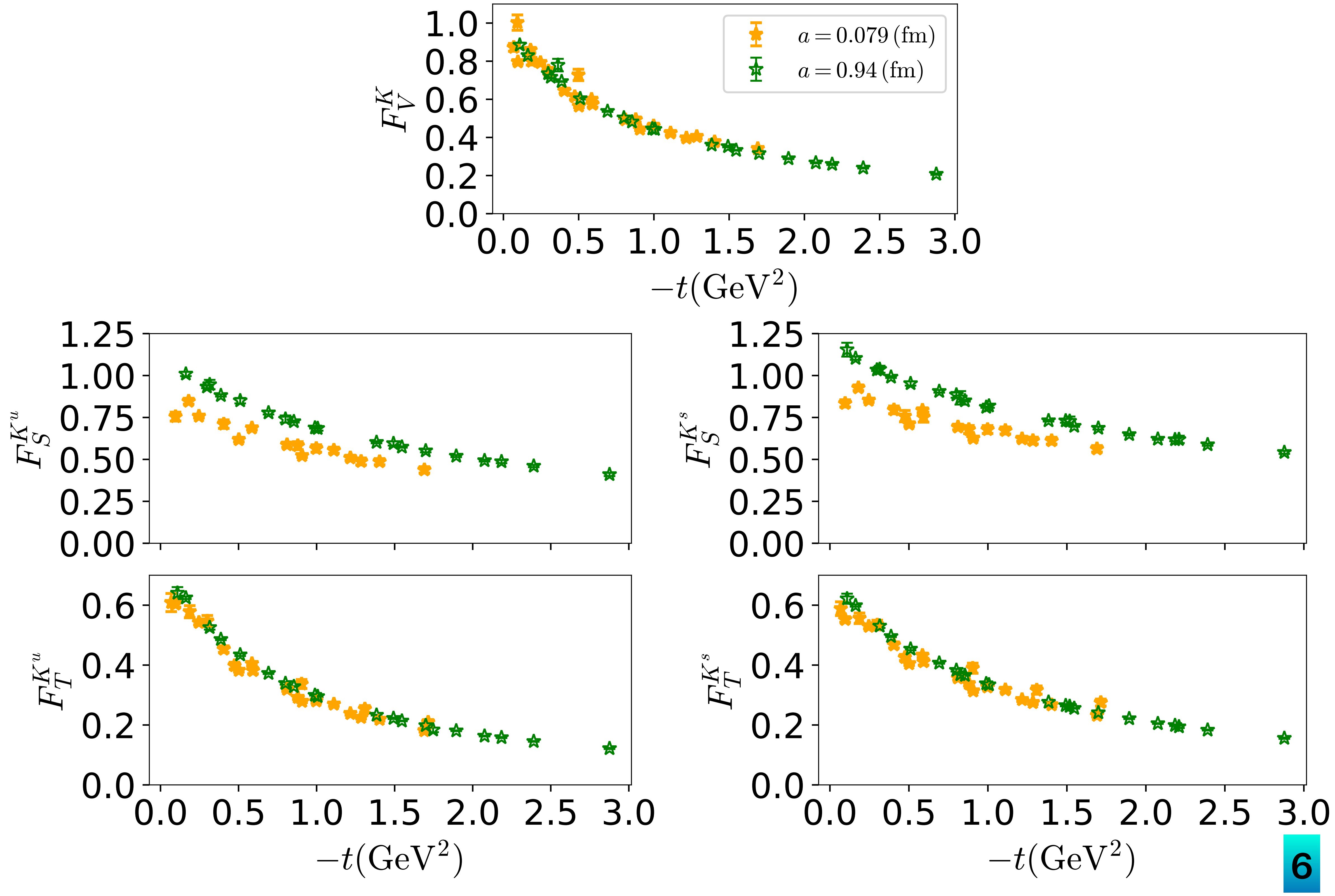

- Discretization effects apparent in scalar
- Good signal up to about 1.75 GeV^2 for new ensemble

Parameterization

- Form factors can be used to extract important physical quantities, most notably the radii
- We parameterize the -t dependence using the monopole Ansatz depicted by the Vector Meson Dominance Model (VMD)

$$F_\Gamma(Q^2) = \frac{F_\Gamma(0)}{1 + \frac{Q^2}{M_\Gamma^2}}$$

- Radius defined as $$\langle r^2 \rangle_\Gamma = -\frac{6}{F_\Gamma(0)} \frac{\partial F_\Gamma(Q^2)}{\partial Q^2}\bigg|_{Q^2=0} = \frac{6}{M_\Gamma^2}$$

- Examined several values of -t included in fit (~0.5 GeV^2, ~1 GeV^2, …)
- A-ensemble results

$$\langle r^2\rangle_s^\pi = 0.280(28)\,\text{fm} \quad \langle r^2\rangle_V^\pi = 0.317(22)\,\text{fm} \quad \langle r^2\rangle_T^\pi = 0.360(41)\,\text{fm} \quad \langle r^2\rangle_S^{K^u} = 0.138(3)\,\text{fm} \quad \langle r^2\rangle_S^{K^s} = 0.0982(19)\,\text{fm} \quad \langle r^2\rangle_V^{K} = 0.309(3)\,\text{fm} \quad \langle r^2\rangle_T^{K^u} = 0.445(5)\,\text{fm} \quad \langle r^2\rangle_T^{K^s} = 0.279(3)\,\text{fm}$$

- B-ensemble results (preliminary)

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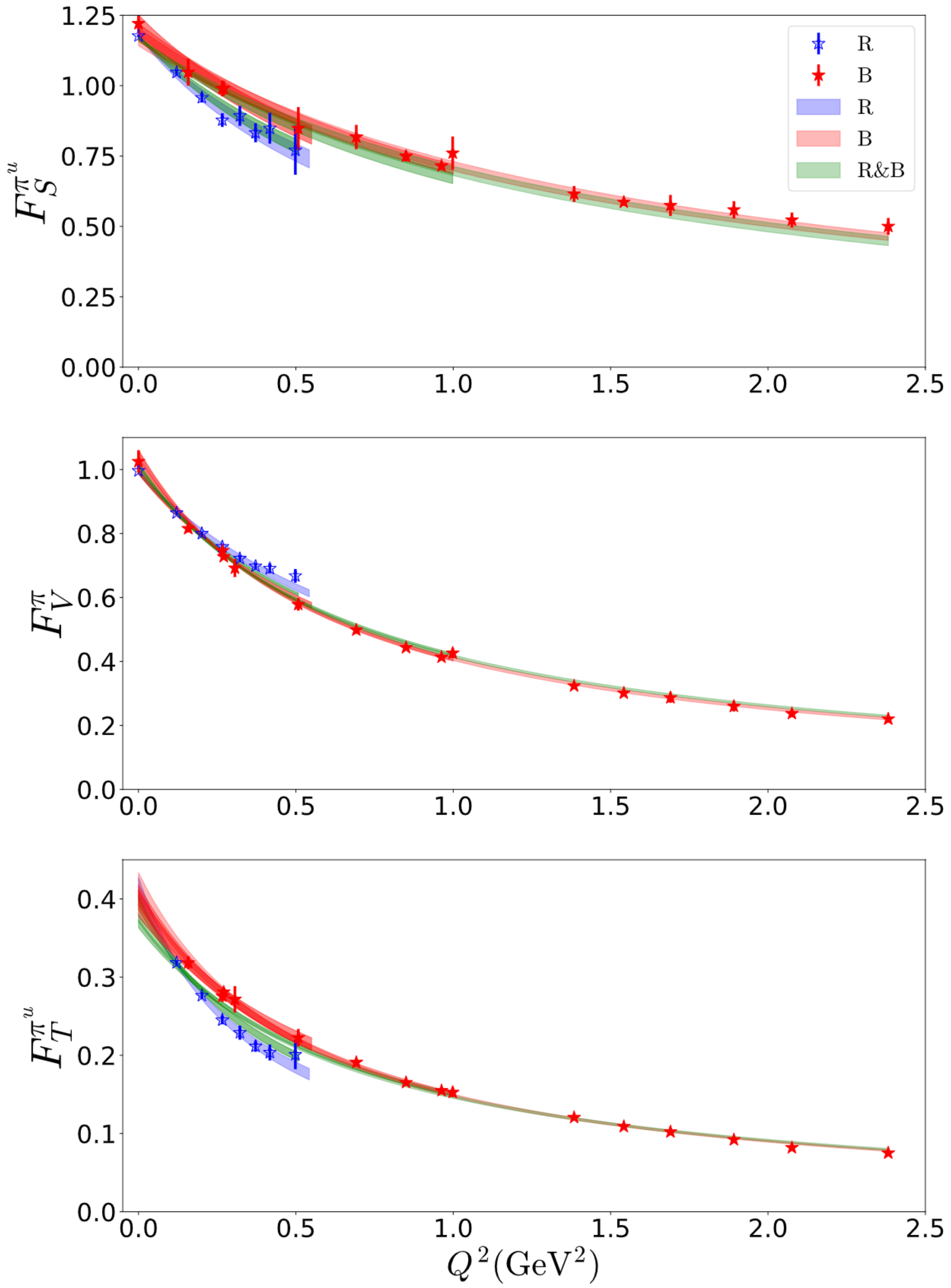

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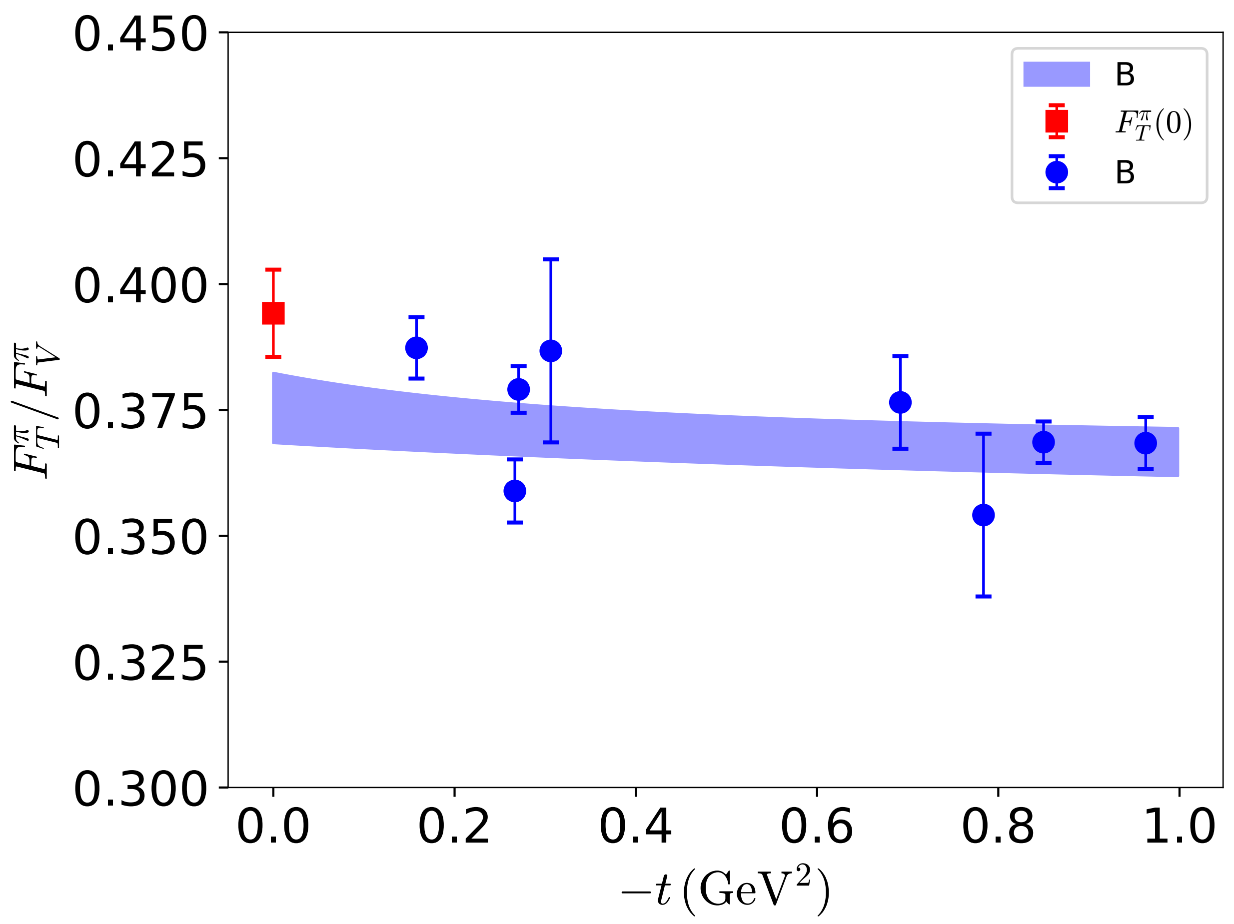
Tensor Anomalous Magnetic Moment κ_T



- $\kappa_T = F_T(0)$ must be extracted from parameterized lattice data

$$F_\Gamma(Q^2) = \frac{F_\Gamma(0)}{1 + \frac{Q^2}{M_\Gamma^2}}$$

- Ratio of F_T/F_V expected to be nearly constant due to elastic unitarity relation (below 1 GeV^2) [M. Hoferichter et al., PRL122 , 122001 (2019)]
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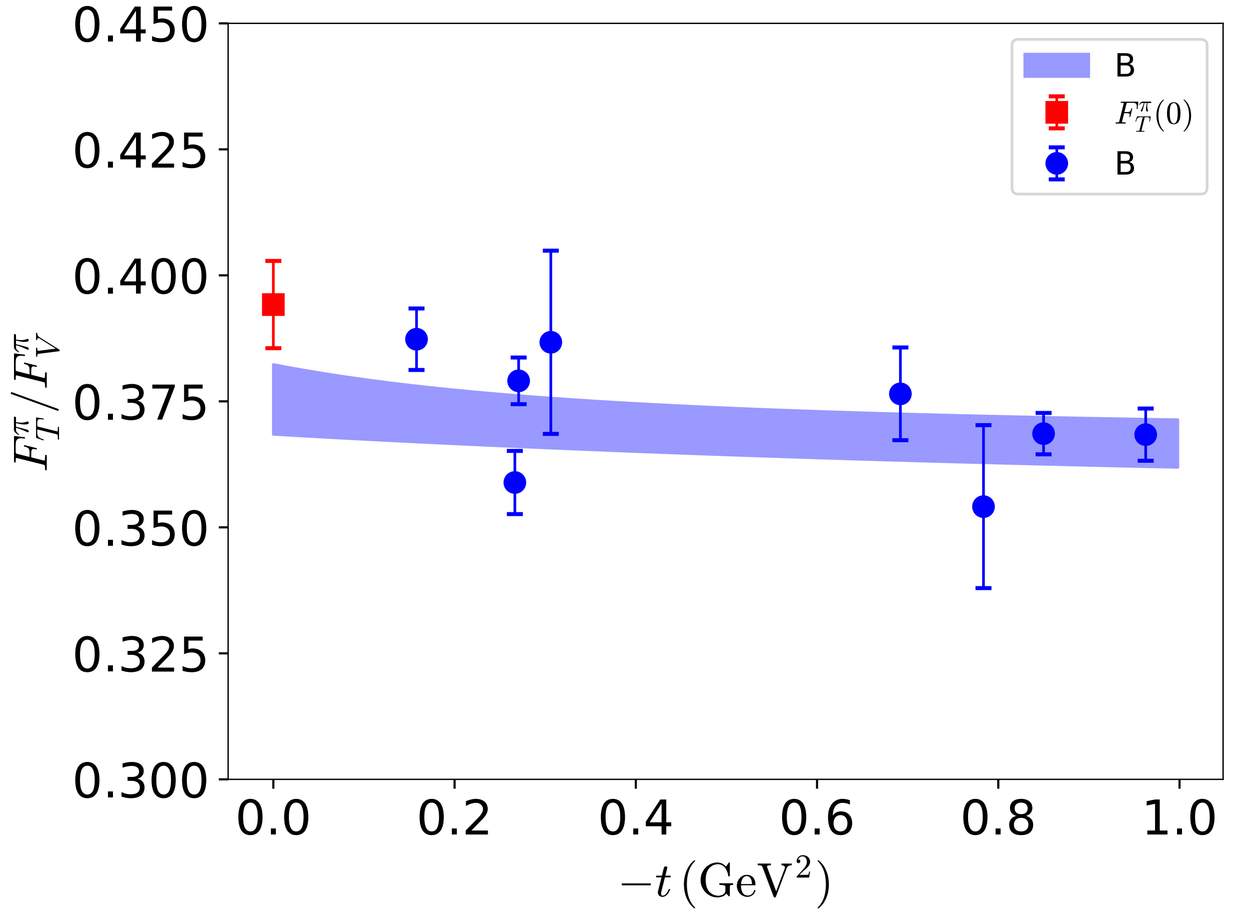

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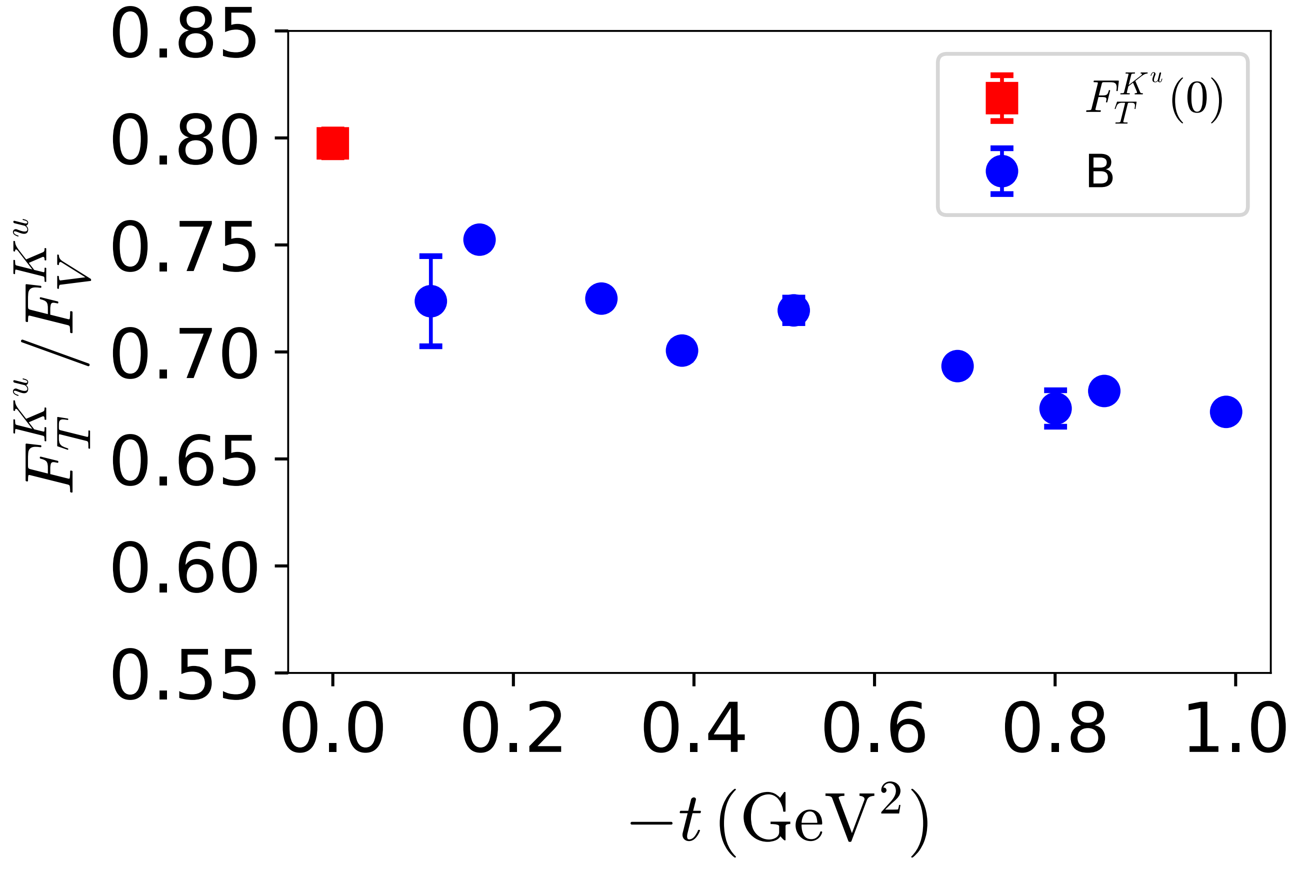

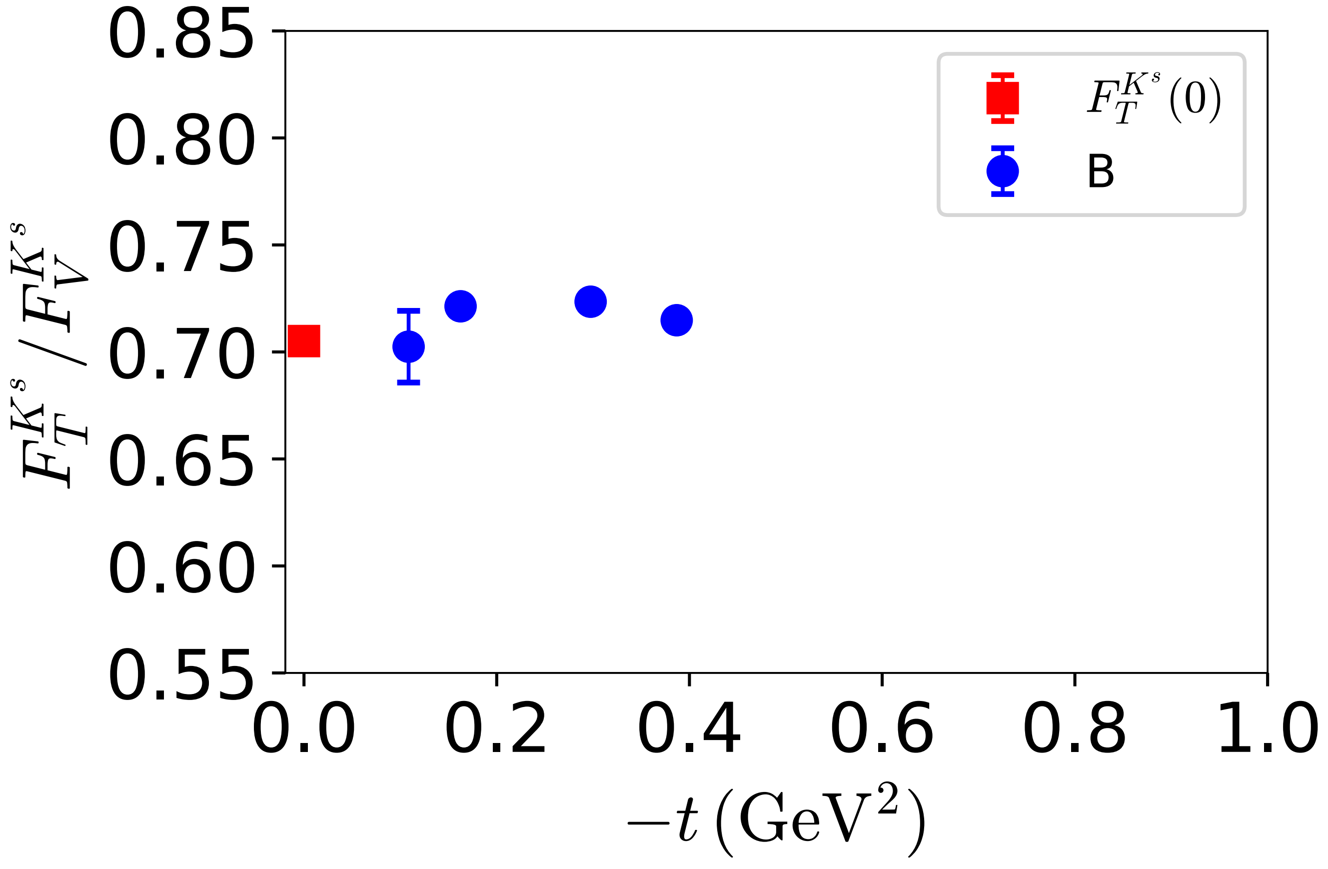

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- Slope still smaller but not insignificant
- Behavior believed to be due to SU(3) symmetry breaking

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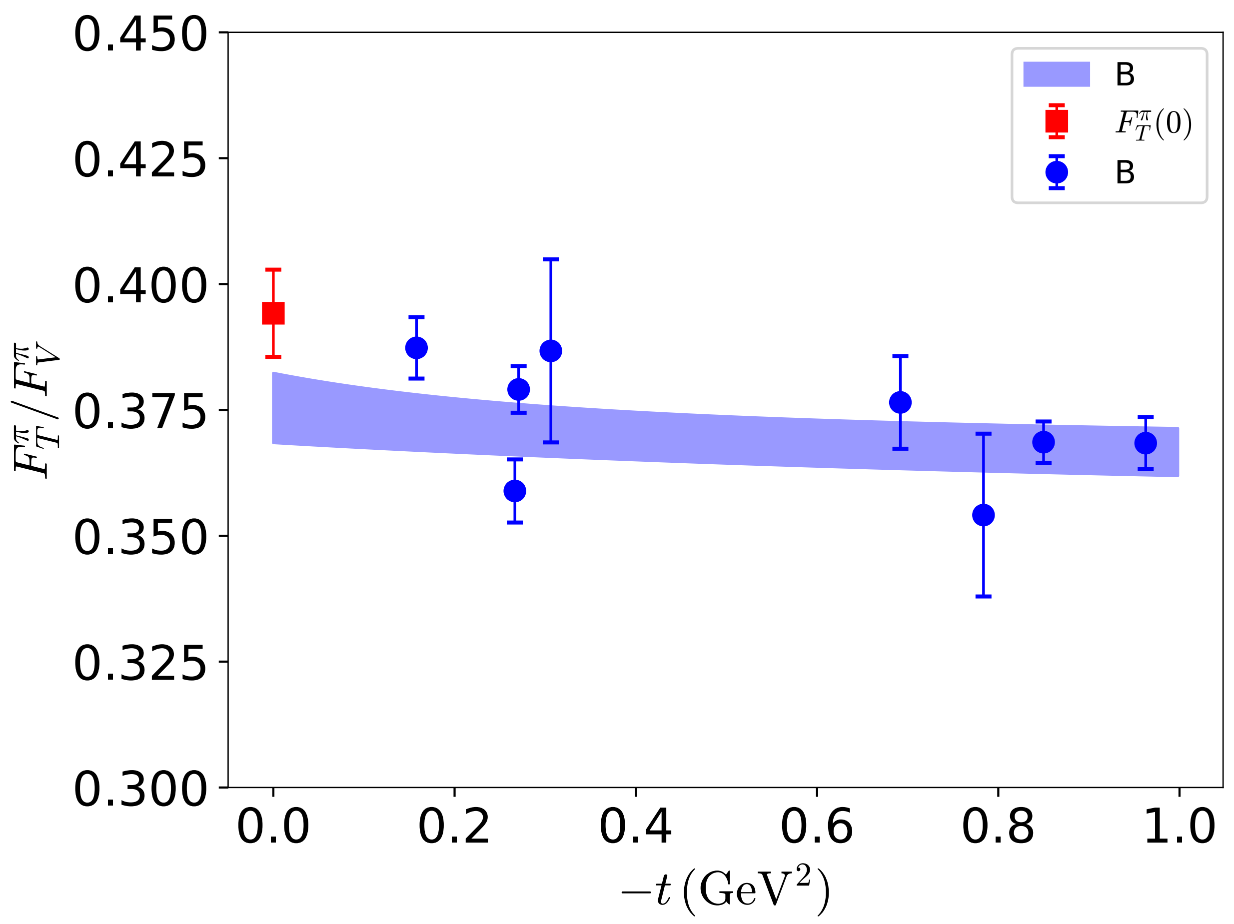

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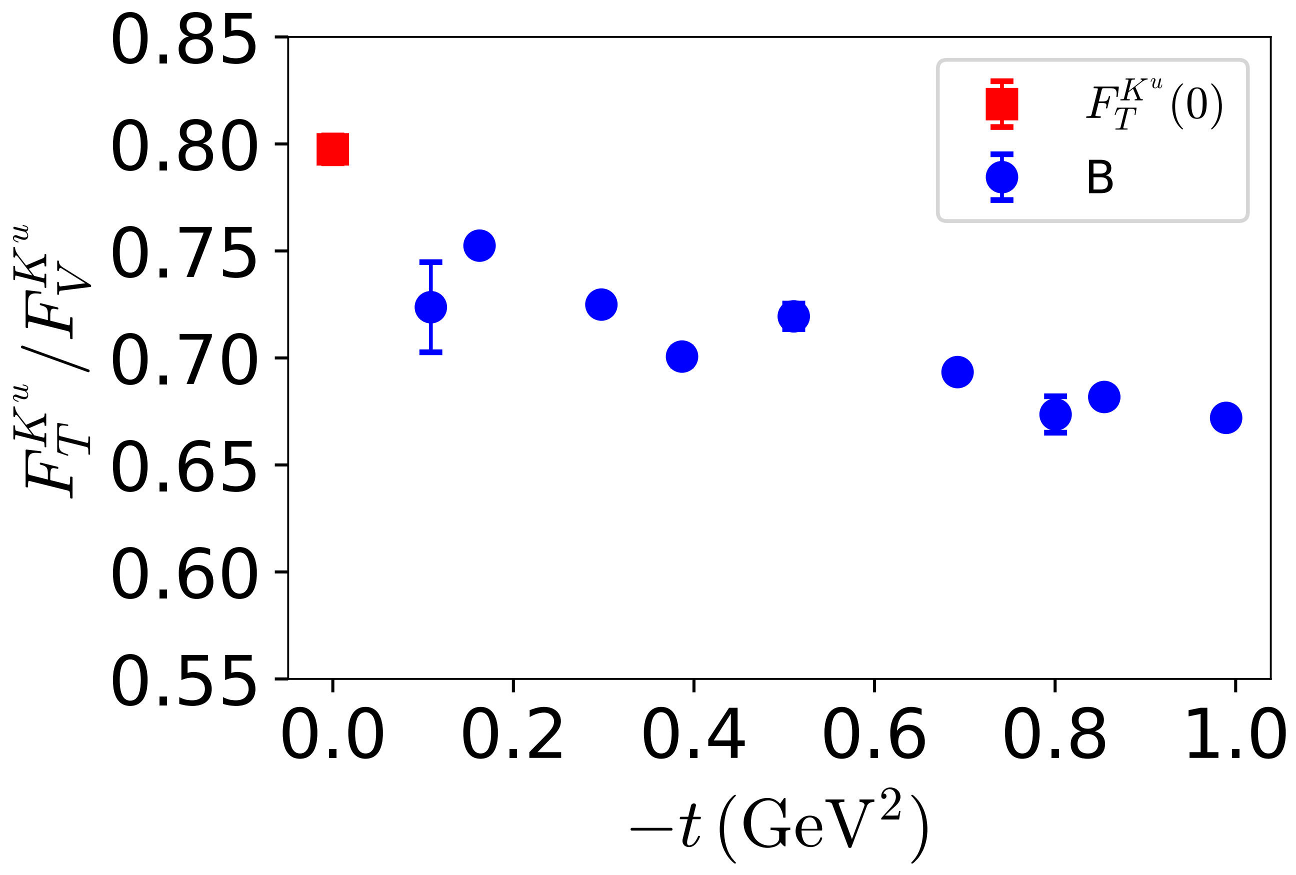

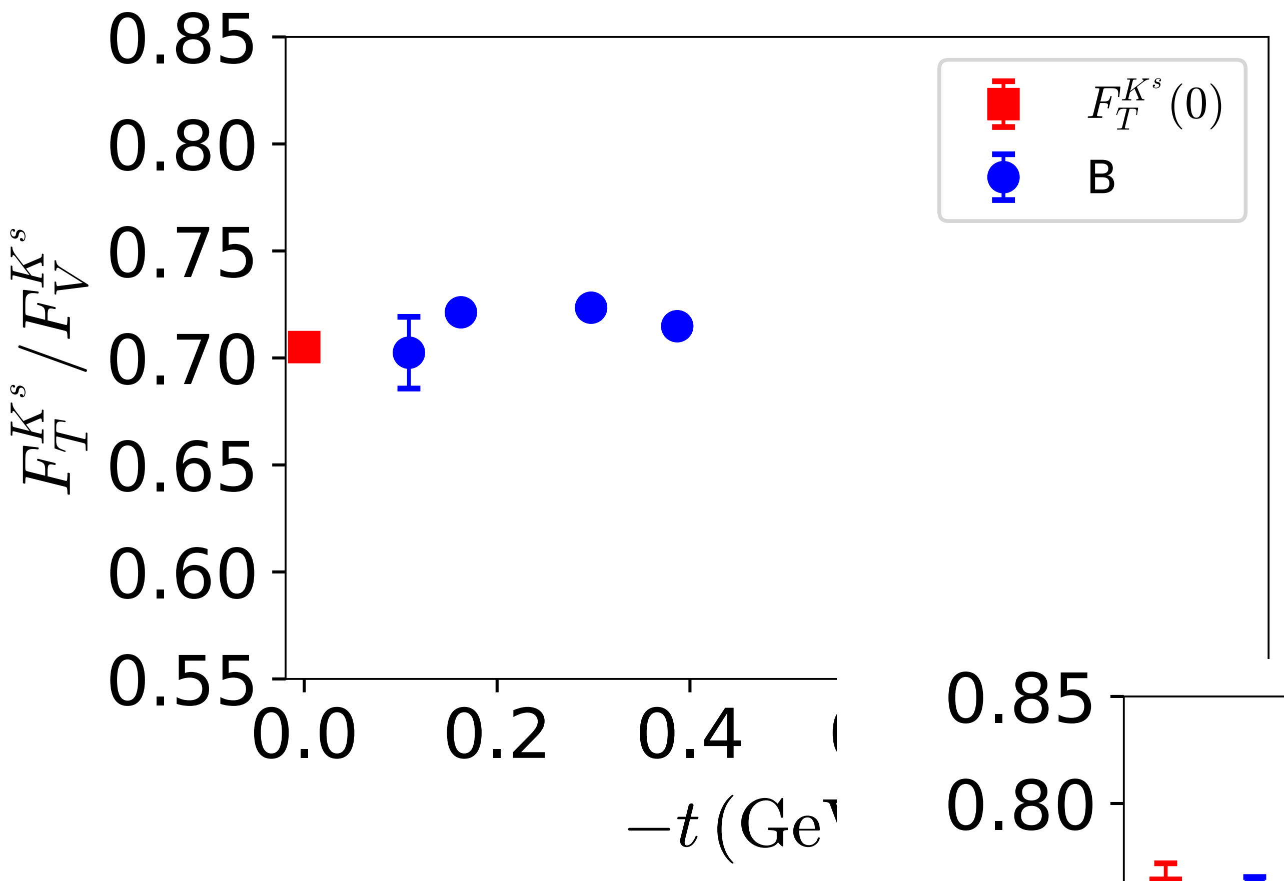

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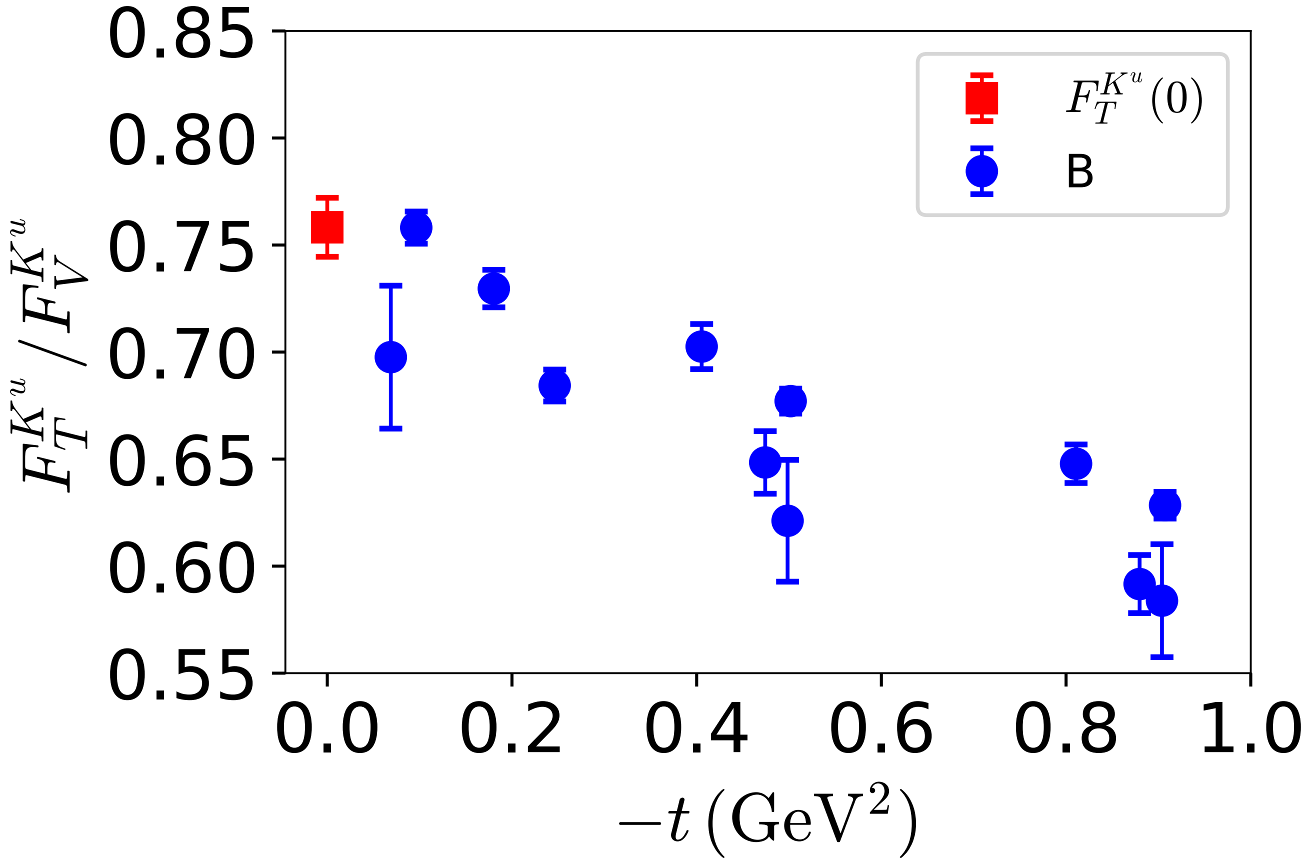
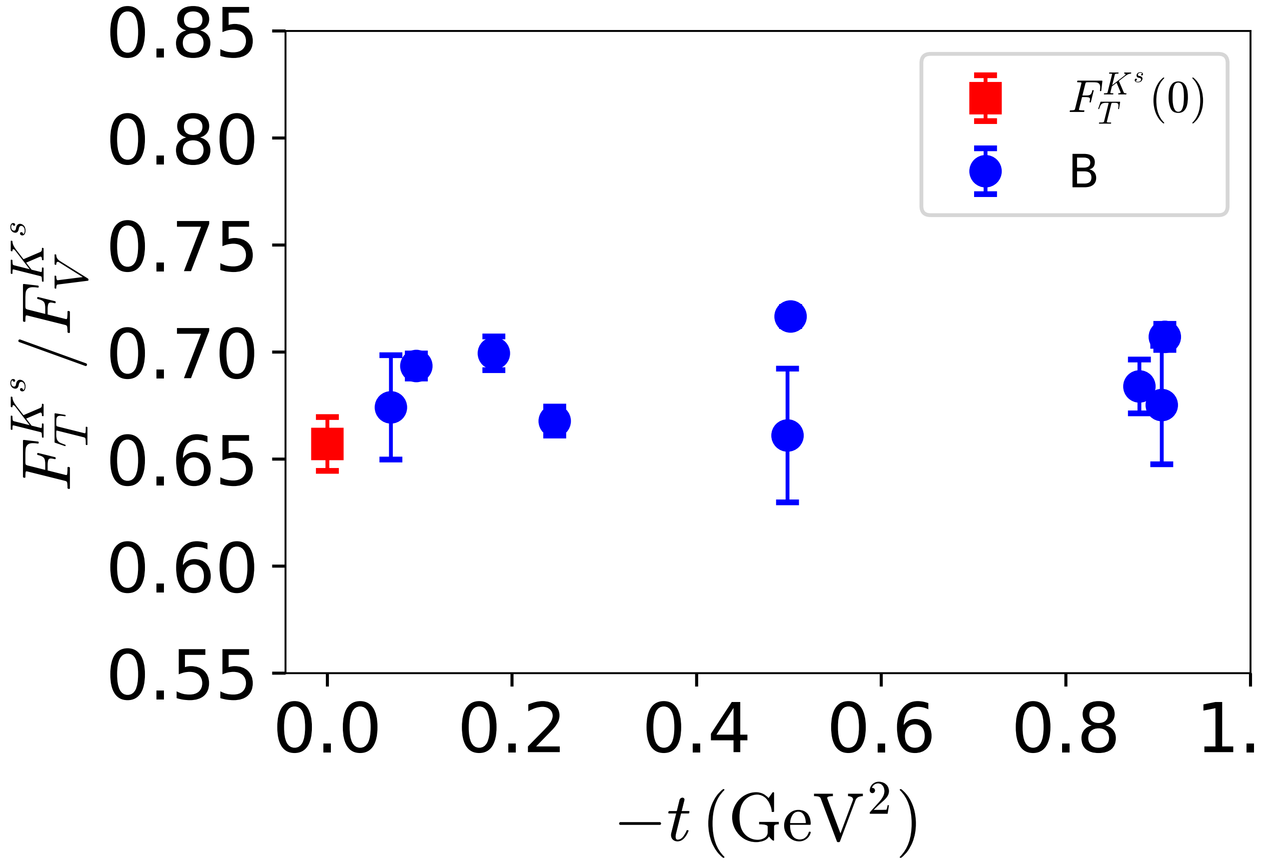
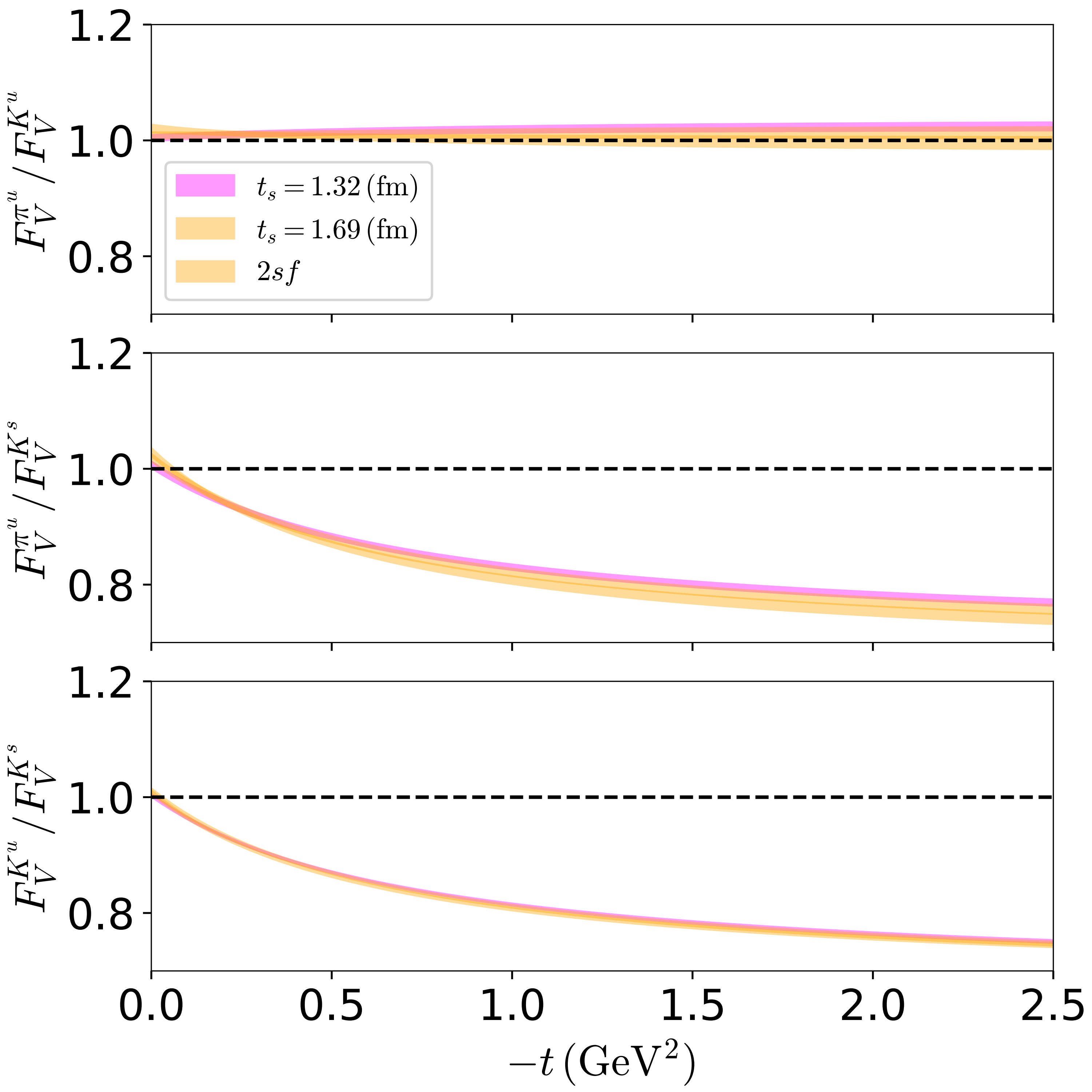




- Similar picture for A-ensemble

SU(3) Flavor Symmetry Breaking



- Ratios of fitted data
- Up quark equivalent between mesons
- SU(3) breaking effects up to 20% between up and strange
- Similar picture for scalar and tensor

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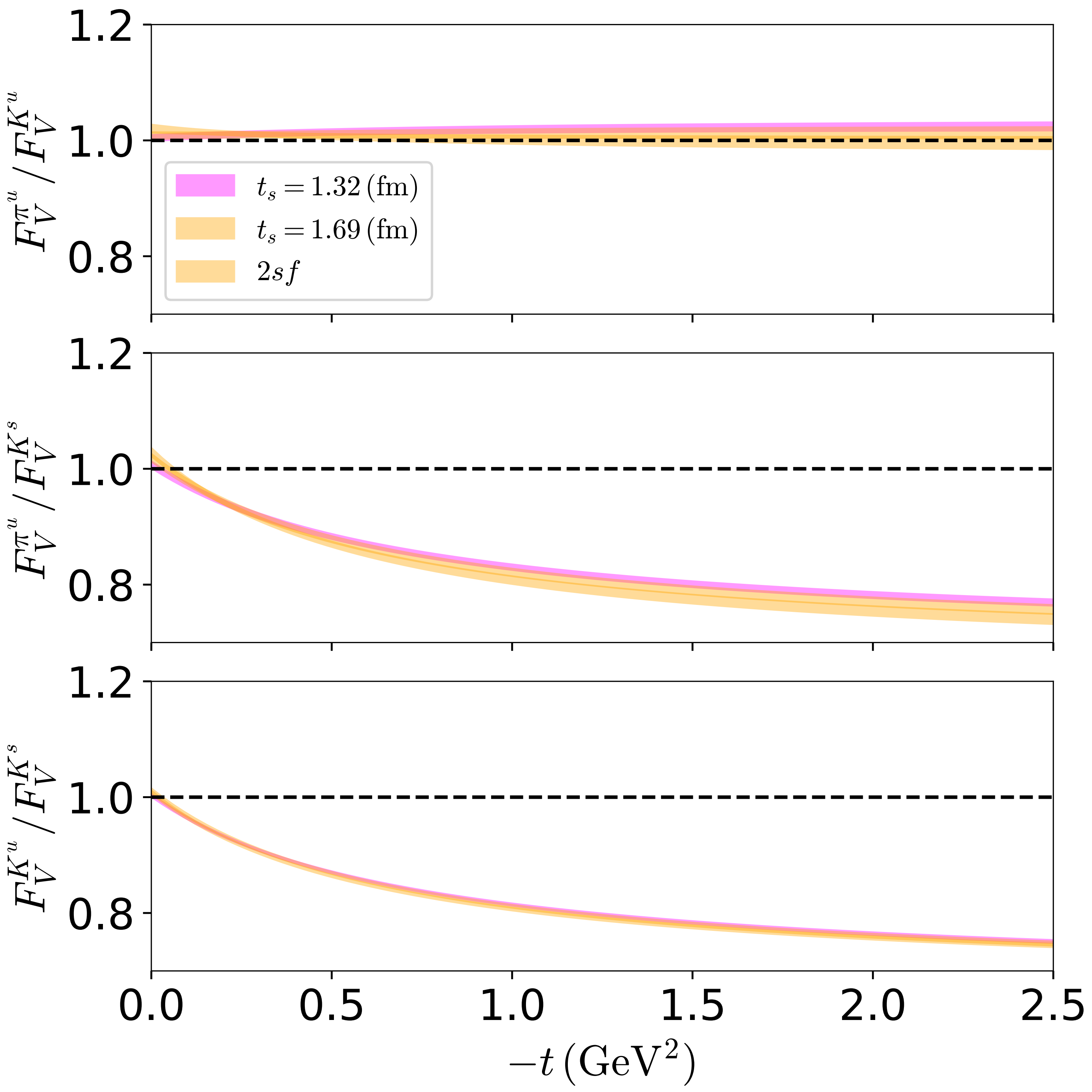

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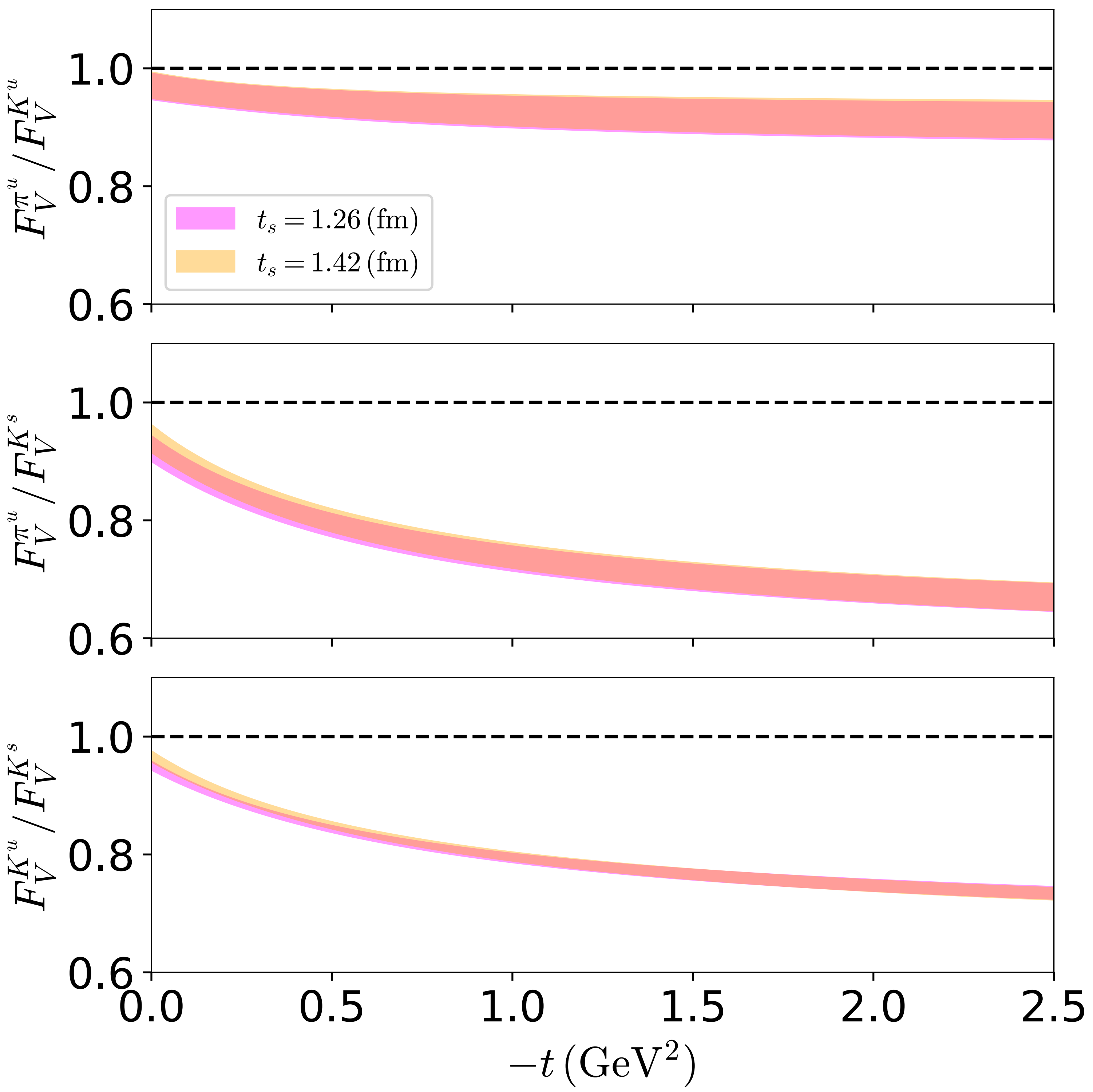

- Errors more significant than other ensemble (most likely due to statistics)
- Insignificant effects from t_s choice

Transverse Spin

- Vector and tensor form factors can be used to probe transverse spin structure
- Form factors Fourier transformed into impact parameter-space using continuum parametrization
- Density of transversely polarized quarks defined as

$$\rho(b_\perp, s_\perp) = \frac{1}{2}\left[\frac{M_V^2 F_V(0)}{2\pi} K_0\left(M_V \sqrt{b_\perp^2}\right) + \frac{s_\perp^i \epsilon^{ij} b_\perp^j}{m} \frac{M_T^3 F_T(0)}{4\pi\sqrt{b_\perp^2}} K_{-1}\left(M_T \sqrt{b_\perp^2}\right)\right]$$

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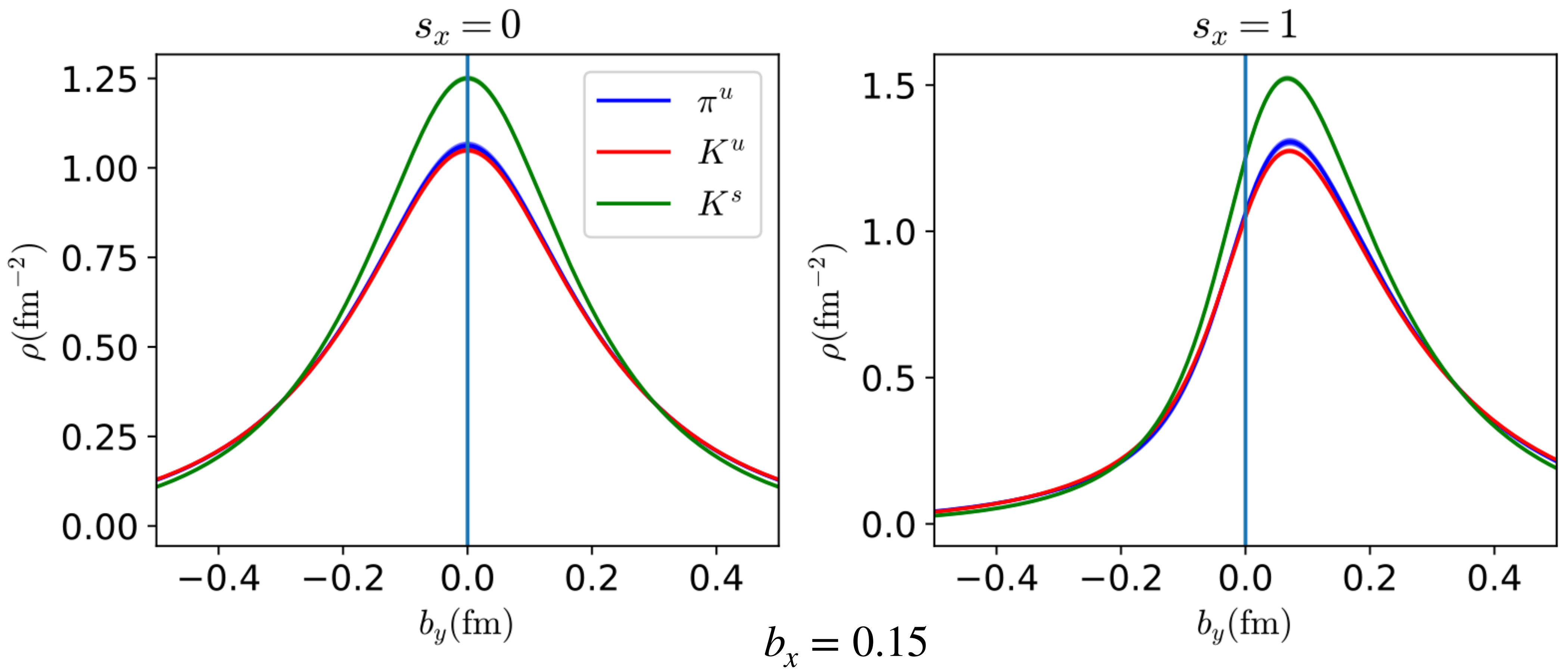

$b_x = 0.15$

- Unpolarized plot symmetric in b_x and b_y
- Polarized plot asymmetric in b_y (peak around 0.07 fm)

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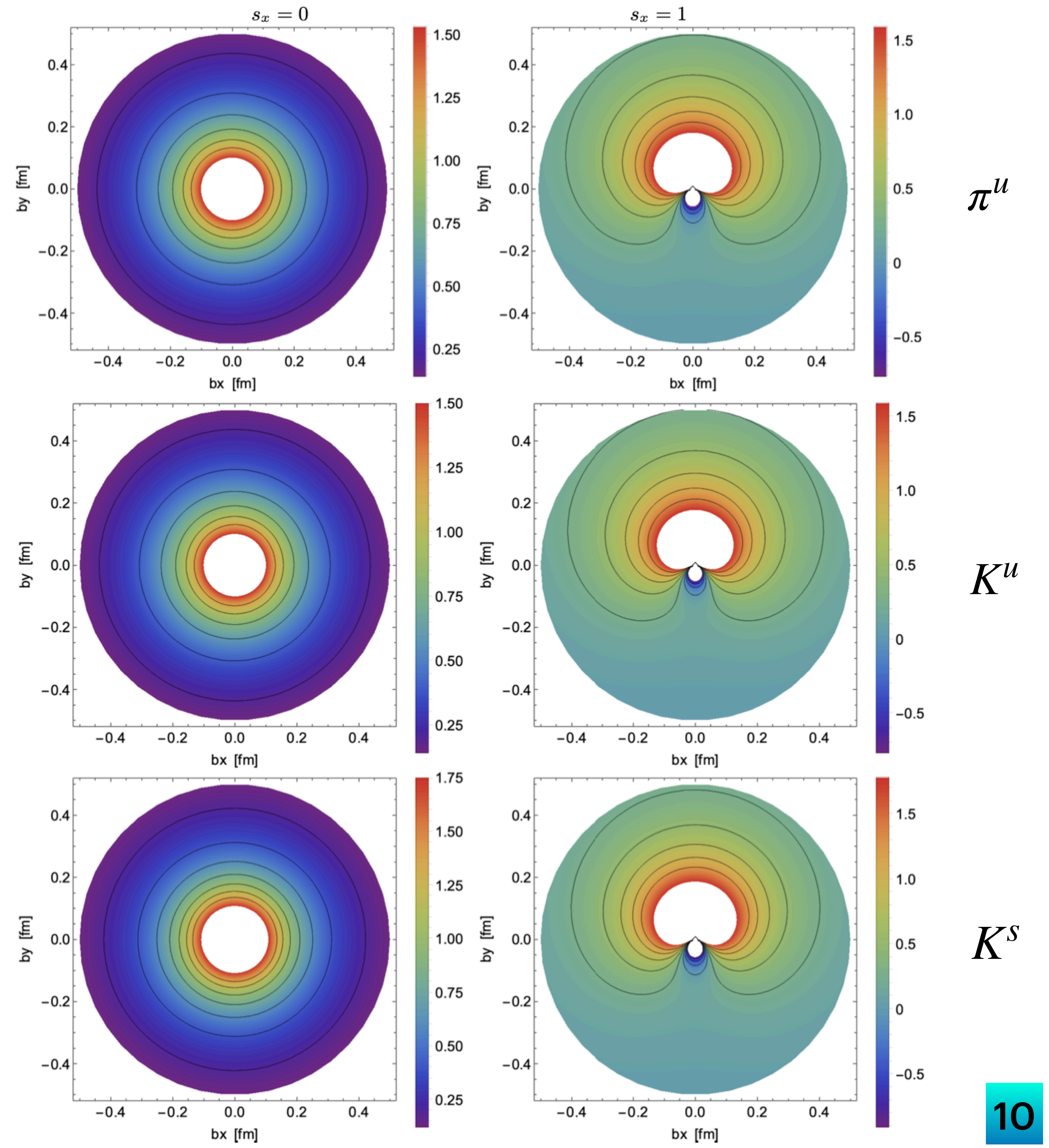
  - Density plots show distortion for polarized quarks
  - No qualitative difference between up and strange quark



Conclusions

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- Finite-a effects more prominent in scalar form factor
- Role of up quark similar in pion and kaon
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Future Work

- Kinematic setup designed to access generalized form factors up to 3-derivative operator
- Higher statistics and study of systematic uncertainties required for new ensemble
- Extension of analysis to $\langle x^n \rangle$ and generalized form factors
- Addition of two ensembles with finer lattice spacing for continuum limit

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