NC STATE UNIVERSITY

Center for Frontiers in Nuclear Science

Quark and Gluon Helicity Evolution at Small x

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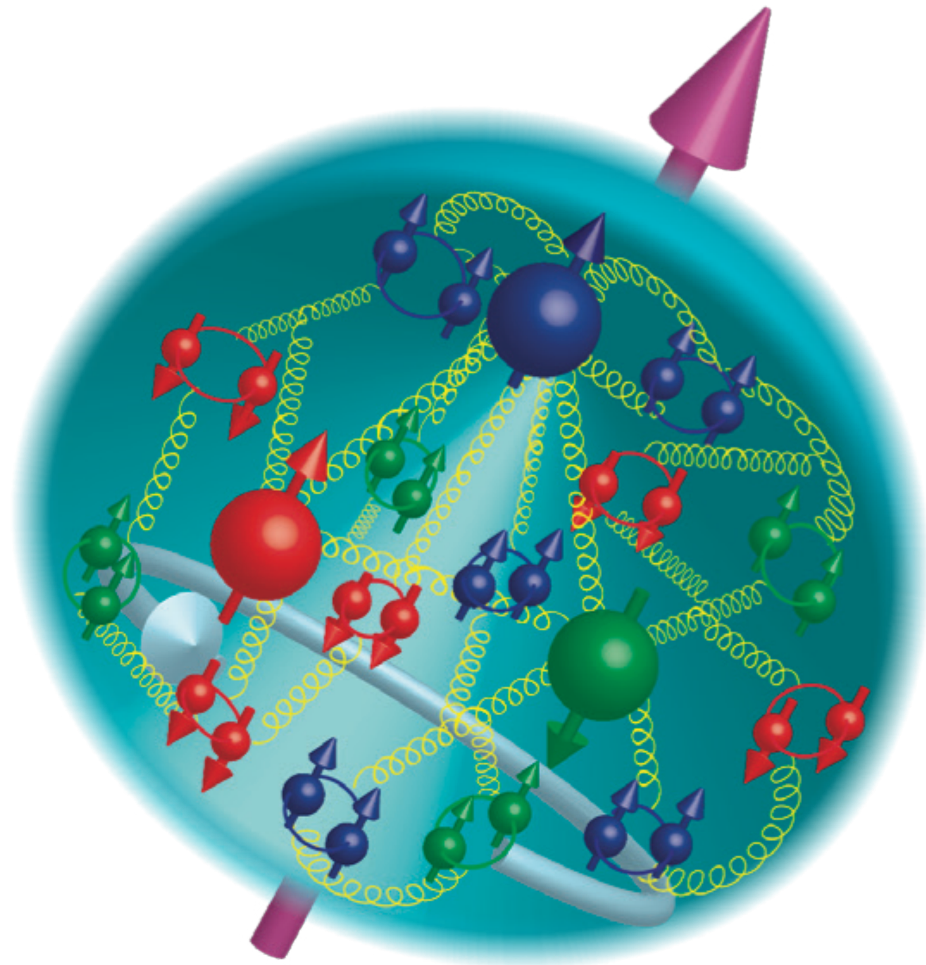
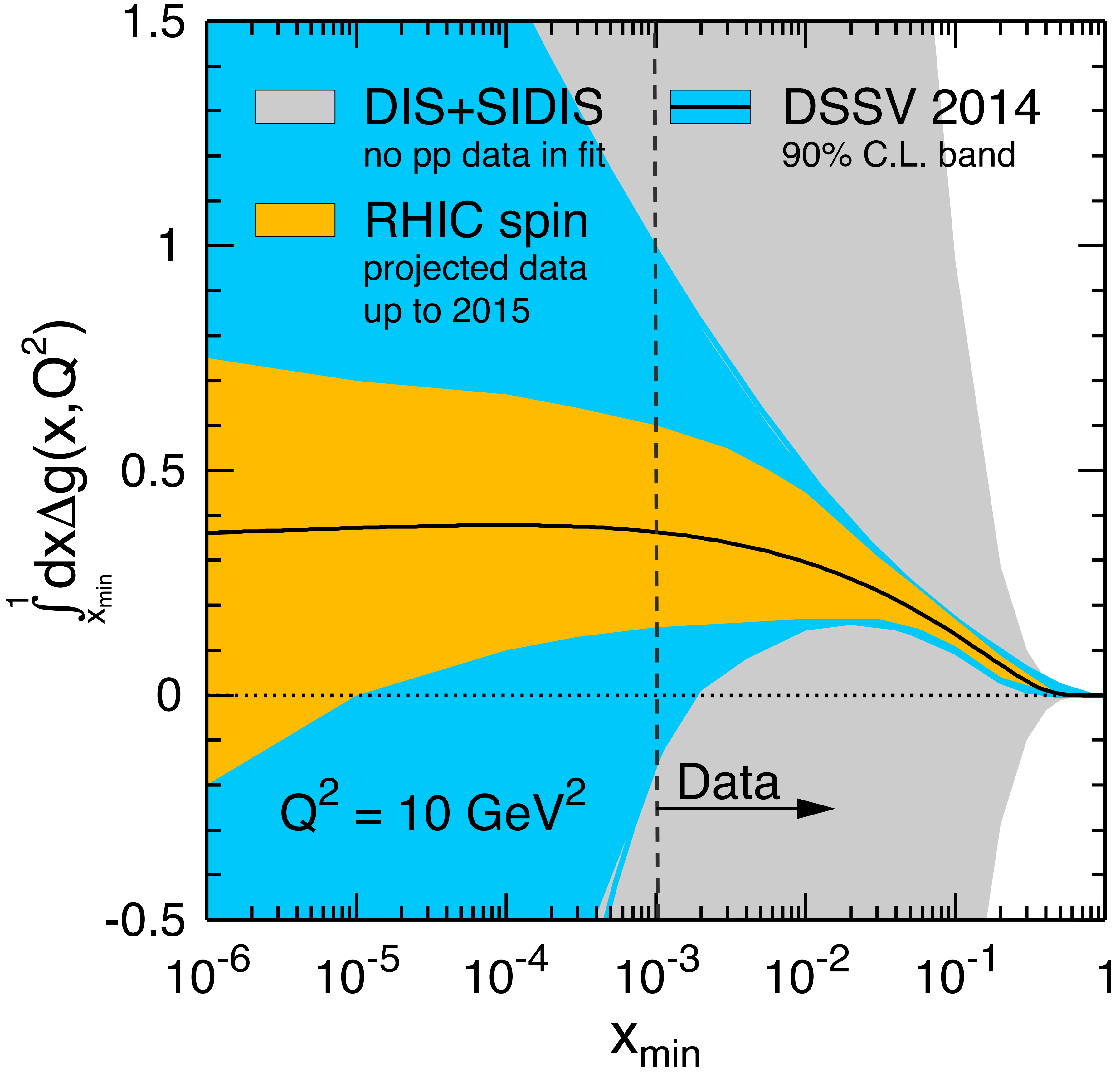
Based on
Florian Cougoulic, Yuri Kovchegov, Andrey Tarasov, Yossathorn Tawabutr, JHEP 07 (2022) 095

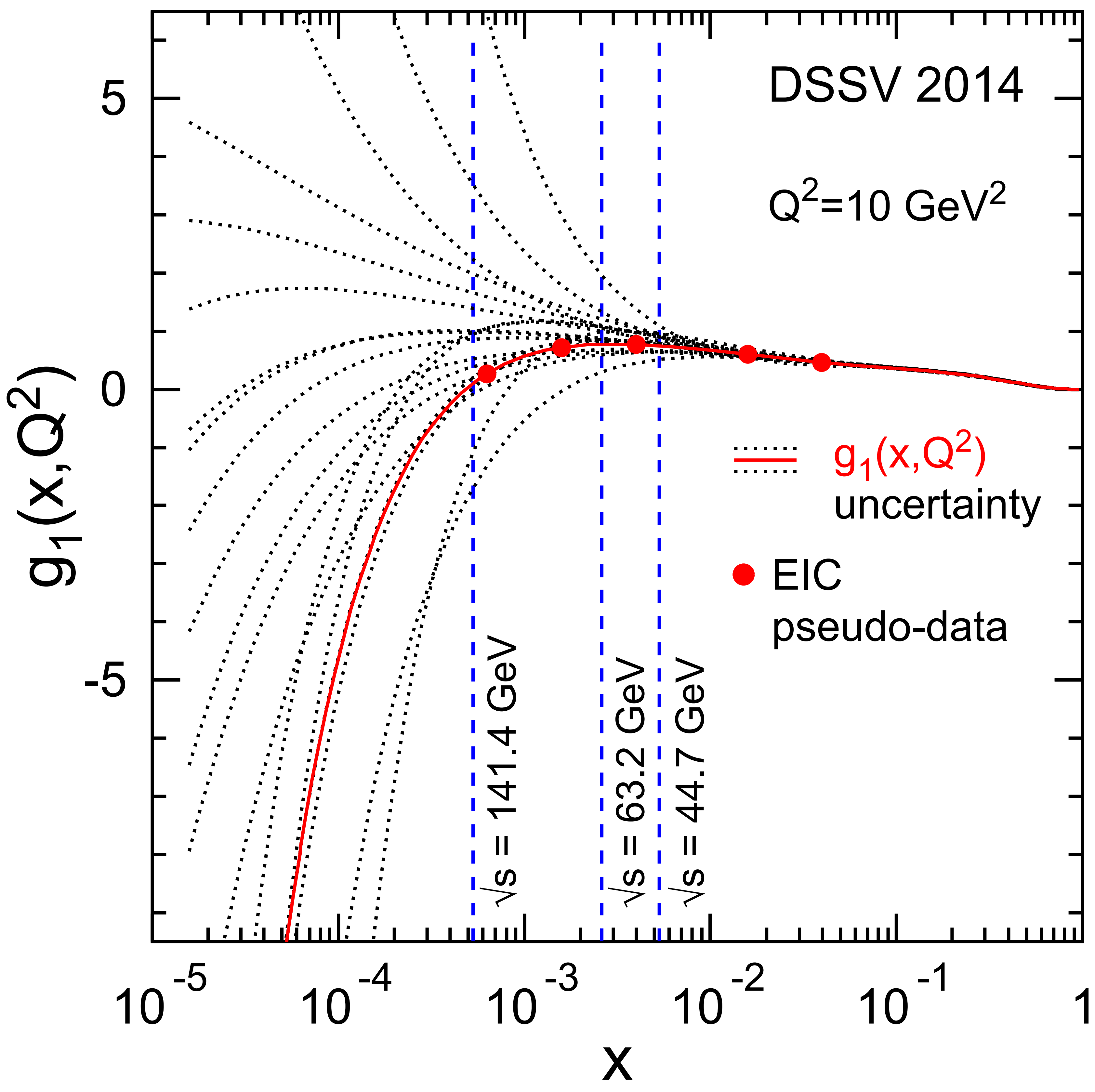
GHP2023, April 14, 2023

Proton helicity structure



The fundamental properties of hadrons, and in particular its spin, are defined by the complex dynamics of quarks and gluons which form a strongly bonded many-body parton system. This dynamics in the context of spin dependent observables is not well understood (spin puzzle, large uncertainties at small-x etc.)





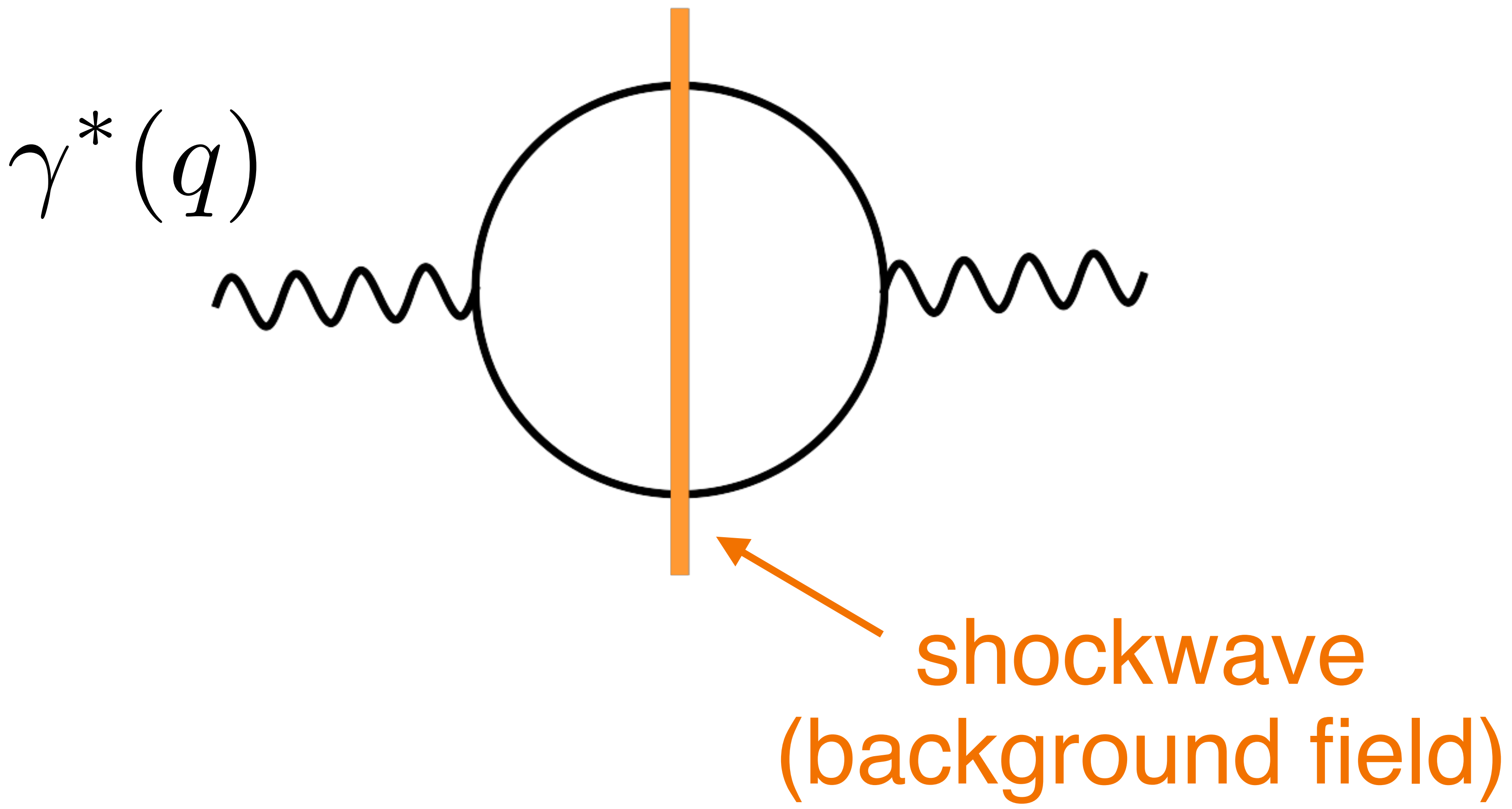
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

There is a lot of interest to study quark and gluon helicity evolution at small x

D. De Florian, R. Sassot, M. Stratmann, W. Vogelsang, PRL 113 (2014)

Helicity and sub-eikonal corrections at small-x

The analysis is non-trivial since it requires calculation of the sub-eikonal corrections at small-x. Indeed, in the leading (eikonal) approximation:



Impact factor

$$\sigma \propto \int \frac{d^2p_\perp}{4\pi^2} I(p_\perp, q_\perp) Tr\{U(p_\perp)U^\dagger(q_\perp - p_\perp)\}$$

Balitsky (1996)

Light-cone Wilson lines contain no information on the proton helicity

In the CGC EFT the shockwave background field has an infinitesimally small support and doesn't have the transverse component:

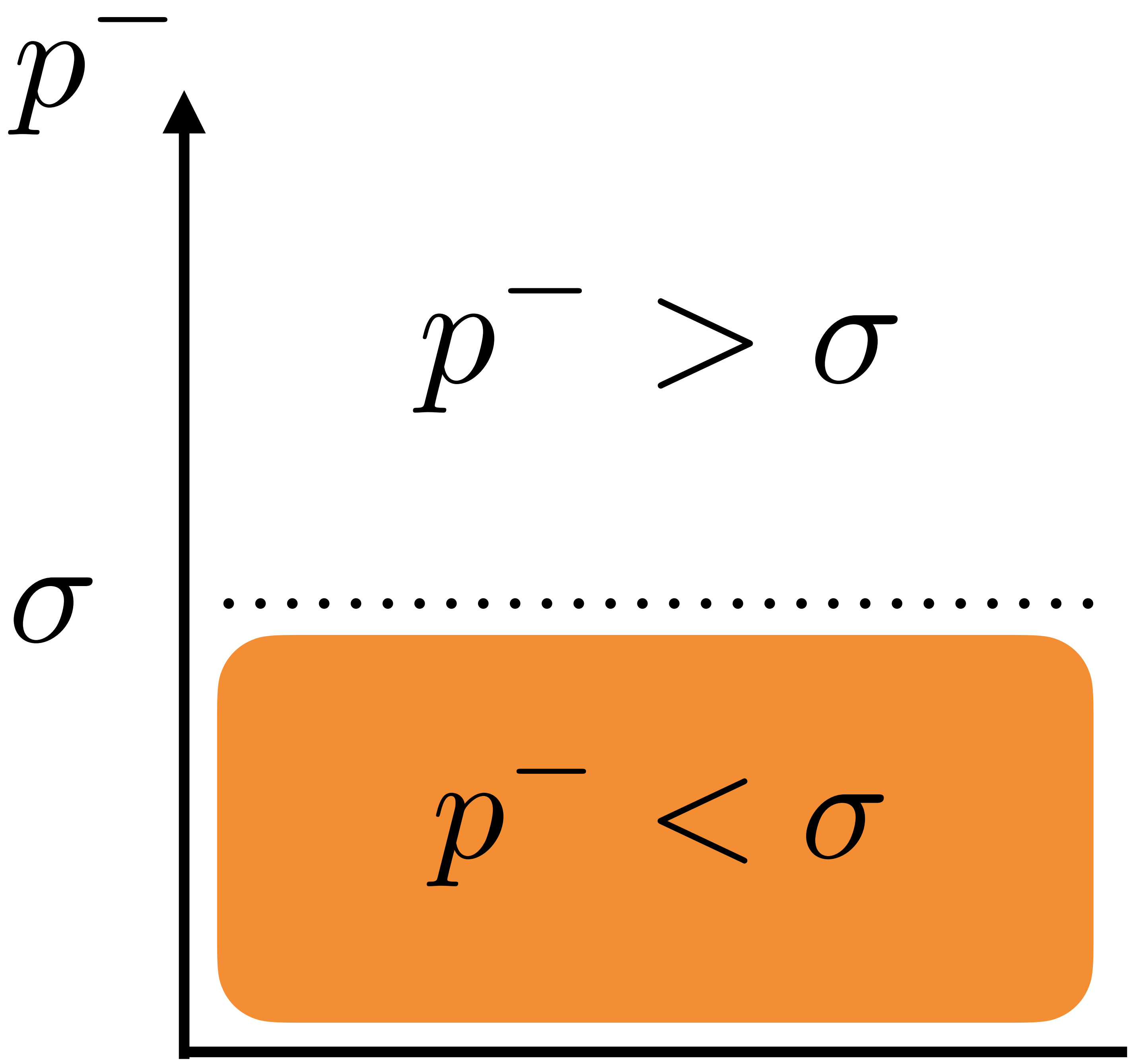
$$A^+_{\rm cl}(x) = -\frac{1}{\partial^2_\perp}\rho(x_\perp)\delta(x^-); \quad A^-_{\rm cl}(x) = A^i_{\rm cl}(x) = 0$$

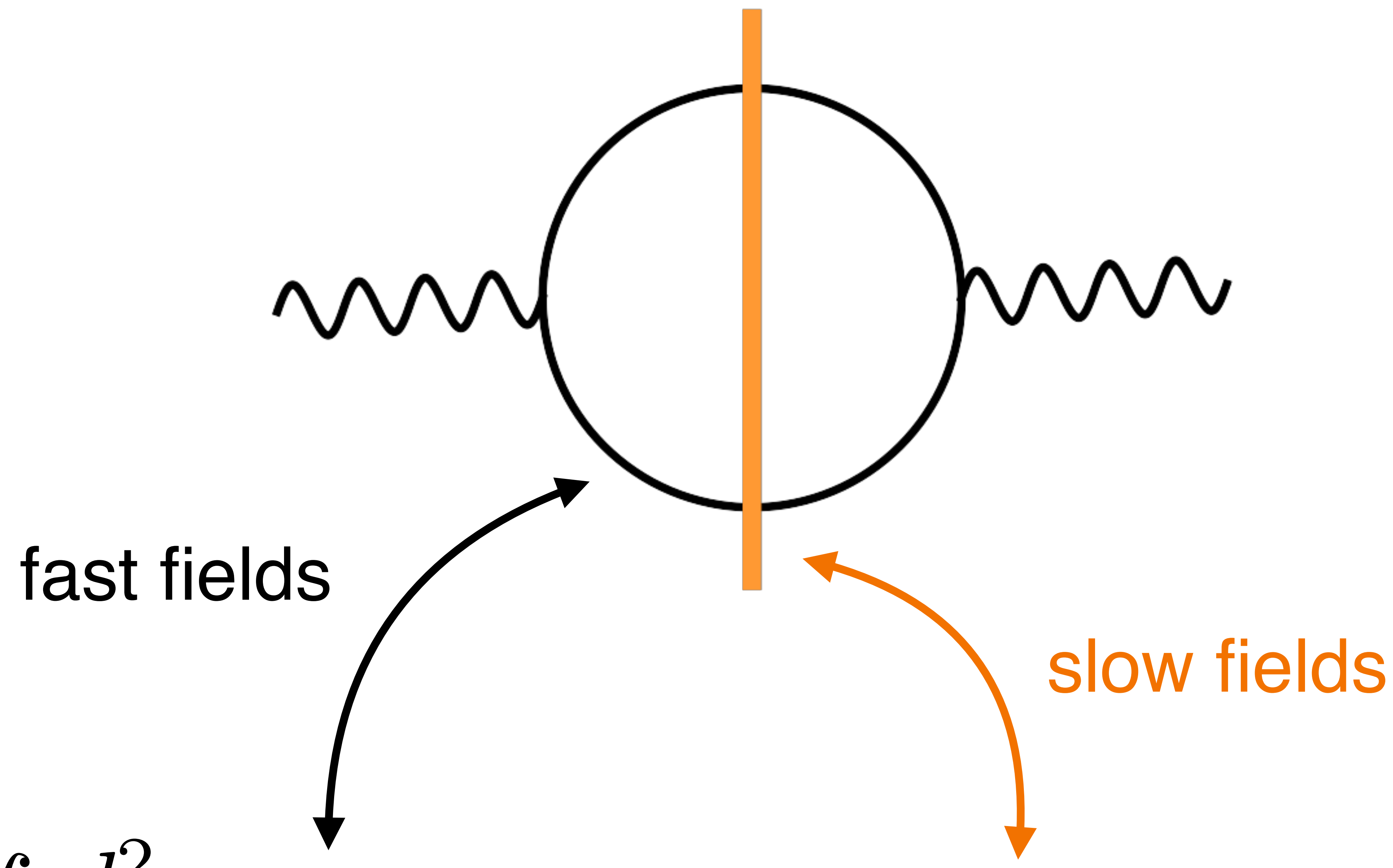
McLerran, Venugopalan (1994)

To include spin effects one has to take into account sub-eikonal corrections, related both to the A_i component and non-zero width of the shock-wave

Factorization scheme

To define the structure of the sub-eikonal correction one first needs to define the factorization scheme. Since we work in the small-x limit we use the rapidity factorization, where all fields are divided based on the value of the p^- component





$$\sigma \propto \int \frac{d^2 p_\perp}{4\pi^2} I(p_\perp, q_\perp) Tr\{U(p_\perp) U^\dagger(q_\perp - p_\perp)\}$$

Impact factor can be calculated by explicit integration over fast fields ($p^- > \sigma$), while slow fields ($p^- < \sigma$) are fixed and give rise to the operator

Background field method

The separation of fields into "slow" and "fast" can be formally done in the background field method. We start with a matrix element of an arbitrary operator:

$$\langle P_1|\mathcal{O}|P_2\rangle = \int \mathcal{D}A \int \mathcal{D}\psi\, \Psi^*_{P_1}(\vec{A}(t_f),\psi(t_f))\,\mathcal{O}(A,\psi)\,\Psi_{P_2}(\vec{A}(t_i),\psi(t_i))e^{iS_{QCD}(A,\psi)}$$

Abbott (1981)

and separate fields into fast ("quantum") and slow ("background"):

$$A_\mu \to A^{\rm q}_\mu + A^{\rm bg}_\mu, \quad \psi \to \psi^{\rm q} + \psi^{\rm bg}$$

as a result in the matrix element one can separate integrations over quantum and background fields:

$$\langle P_1|\mathcal{O}|P_2\rangle = \int \mathcal{D}A^{\rm bg} \int \mathcal{D}\psi^{\rm bg}\, \Psi^*_{P_1}(\vec{A}^{\rm bg}(t_f),\psi^{\rm bg}(t_f))\,\tilde{\mathcal{O}}(A^{\rm bg},\psi^{\rm bg},\sigma)\,\Psi_{P_2}(\vec{A}^{\rm bg}(t_i),\psi^{\rm bg}(t_i))e^{iS_{QCD}(A^{\rm bg},\psi^{\rm bg})}$$

we fix background fields

where

and integrate over quantum fields

$$\tilde{\mathcal{O}}(A^{\rm bg},\psi^{\rm bg},\sigma) = \int \mathcal{D}A^{\rm q} \int \mathcal{D}\psi^{\rm q}\,\mathcal{O}(A^{\rm q}+A^{\rm bg},\psi^{\rm q}+\psi^{\rm bg})e^{iS_{bQCD}(A^{\rm q},\psi^{\rm q};A^{\rm bg},\psi^{\rm bg})}$$

Propagators in the background field

Our goal is to perform integration over quantum fields which in general gives the following result

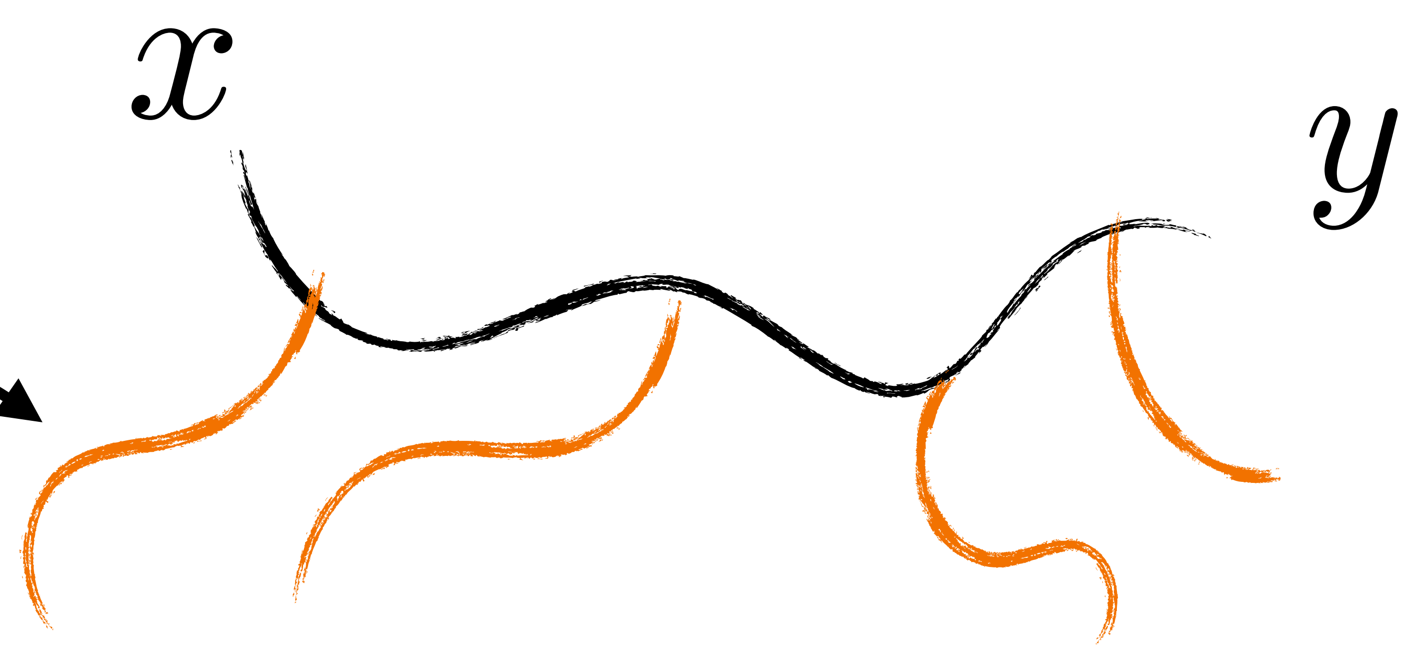
$$\tilde{\mathcal{O}}(A^{\rm bg}, \psi^{\rm bg}, \sigma) = \sum_i C_i(\sigma) \otimes \mathcal{V}_i(A^{\rm bg}, \psi^{\rm bg}, \sigma)$$

Integration over quantum fields generates propagators in the background field.

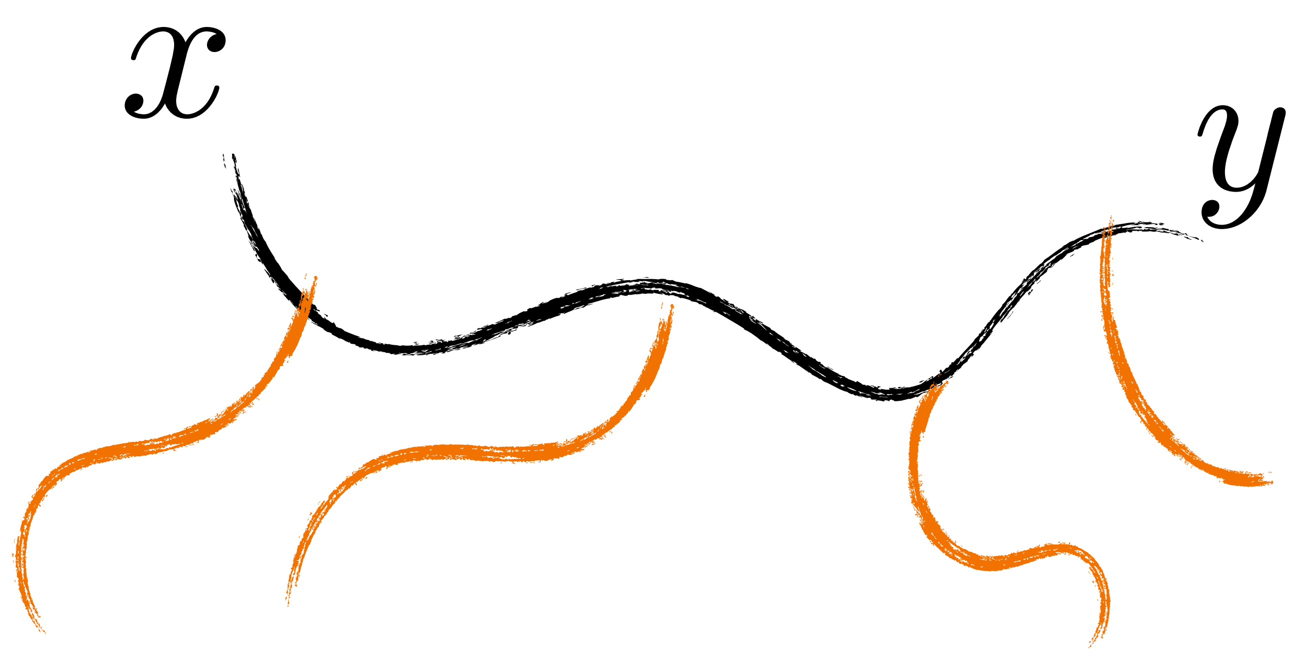
Example: propagator of a scalar particle
in a background field (scalar QED):

$$(x|\frac{1}{P^2 + i\epsilon}|y) = (x|\frac{1}{p^2 + g\{p^\mu, A_\mu(x)\} + g^2 A^\mu(x) A_\mu(x) + i\epsilon}|y)$$

background fields



Quark propagator in the background field



Quark propagator in the background field:

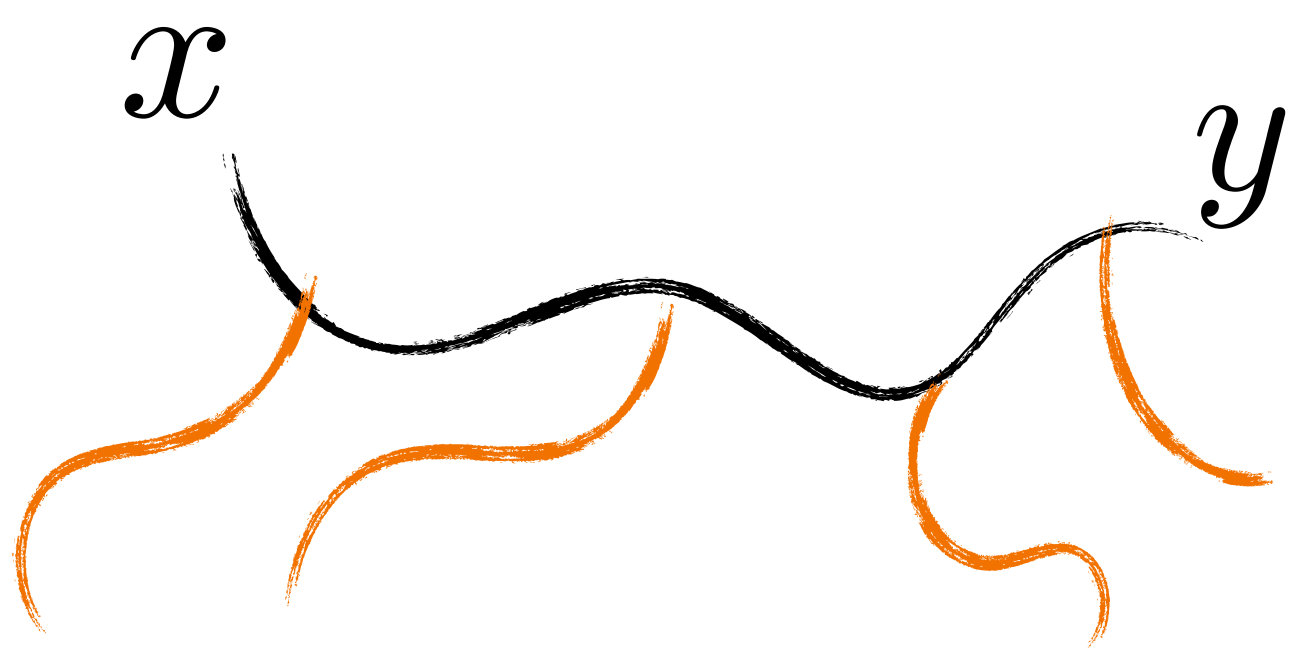
$$\mathrm{T}\left[\psi(x)\bar{\psi}(y)\right]_A = (x|\frac{i}{\not{P}+i\epsilon}|y) = (x|\not{P}\frac{i}{P^2+\frac{g}{2}\sigma^{\mu\nu}F_{\mu\nu}+i\epsilon}|y)$$

scalar phase, leads to DGLAP

genuine helicity dependent contribution, leads to F_{12}

Both terms contain contribution of the transverse component of the background field A_i

Quark propagator in the background field



Quark propagator in the background field:

$$\mathrm{T}\left[\psi(x)\bar{\psi}(y)\right]_A = (x|\frac{i}{\not{P} + i\epsilon}|y) = (x|\not{P}\frac{i}{P^2 + \frac{g}{2}\sigma^{\mu\nu}F_{\mu\nu} + i\epsilon}|y)$$

scalar phase, leads to DGLAP

genuine helicity dependent contribution, leads to F_{12}

Worldline (functional integral) representation of the propagator:

$$(x|\frac{1}{\not{P} + i\epsilon}|y) = -i\mathcal{N}^{-1}\int_0^\infty dT \int_{x(0)=y}^{x(T)=x} \mathcal{D}x(\tau)\left(\frac{1}{2}\not{\dot{x}} + \not{A}\right)e^{-i\int_0^T d\tau \frac{1}{4}\dot{x}^2} P\exp\left(ig\int_0^T d\tau(\dot{x}^\mu A_\mu + \frac{1}{2}\sigma^{\mu\nu}F_{\mu\nu})\right)$$

Scalar phase, Wilson line factor, for the finite size of the shock-wave contain effects related to the deviation from the light-cone trajectory

Eikonal expansion of the gluon propagator (axial gauge)

We construct a general expression for the gluon propagator in the background field:

describes interaction with the background field

$$\mathrm{T}\left[C^a_\mu(x) C^b_\nu(y)\right] = -\frac{1}{2\pi}\int\limits_0^\infty \frac{dp^-}{2p^-} e^{-ip^-(x-y)^+}$$

$$\times\, (x_\perp|(g_{\mu i} - \frac{n_\mu}{p^-}p_i)^{ac} e^{-i\frac{p_\perp^2}{2p^-}x^-}\, \mathcal{G}^{ij}(\infty,-\infty) e^{i\frac{p_\perp^2}{2p^-}y^-} (g_{j\nu} - p_j\frac{n_\nu}{p^-})^{db}|y_\perp) + \ldots$$

The general form of the propagators has to be simplified. We construct an eikonal expansion in the shock-wave approximation of the propagators which is suited to the rapidity factorization

Eikonal expansion of the gluon propagator (axial gauge)

Eikonal contribution

There are different operators at the sub-eikonal level

F_{12} terms

$$\mathcal{G}^{ij}(\infty,-\infty) = g^{ij}U + \frac{g^{ij}s}{2P^+p^-}U^{\mathrm{q}[2]} + \frac{i\epsilon^{ij}s}{2P^+p^-}U^{\mathrm{pol}[1]}$$

$$- \frac{igg^{ij}}{2p^-}p^k \int_{-\infty}^{\infty} dz^- z^- U[\infty, z^-]\mathcal{F}_{-k}U[z^-,-\infty] - \frac{igg^{ij}}{2p^-} \int_{-\infty}^{\infty} dz^- z^- U[\infty, z^-]\mathcal{F}_{-k}U[z^-,-\infty]p^k$$

$$+ \frac{ig^2g^{ij}}{2p^-} \int_{-\infty}^{\infty} dz_1^- \int_{-\infty}^{z_1^-} dz_2^- (z_1^- - z_2^-)U[\infty, z_1^-]\mathcal{F}_{-k}U[z_1^-, z_2^-]\mathcal{F}_{-k}U[z_2^-,-\infty] + O\Big(\frac{1}{(p^-)^2}\Big).$$

new terms

Sub-eikonal corrections are suppressed by $1/p^-$

see also Kovchegov, Pitonyak, Sievert (2017)
Altinoluk, Armesto, Beuf, Martínez, Salgado (2014)
Chirilli (2019)
Balitsky, Tarasov (2015)

Helicity evolution and sub-eikonal operators

We find that only two operators contribute to the helicity evolution. There is a genuine helicity dependent operator (e.g. see the corresponding term in the quark propagator) - polarized Wilson lines

$$V_x^{\mathrm{G}[1]} = \frac{i\, g\, P^+}{s} \int\limits_{-\infty}^{\infty} dx^- V_x[\infty, x^-]\, F^{12}(x^-, x_\perp)\; V_x[x^-, -\infty],$$

Note that this operator doesn't contribute in the collinear limit

$$V_x^{\mathrm{q}[1]} = \frac{g^2 P^+}{2\, s} \int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^- V_x[\infty, x_2^-]\, t^b\, \psi_\beta(x_2^-, x_\perp)\, U_x^{ba}[x_2^-, x_1^-]\; \left[\gamma^+ \gamma^5\right]_{\alpha\beta}\; \bar{\psi}_\alpha(x_1^-, x_\perp)\, t^a\, V_x[x_1^-, -\infty]$$

This operators generate a polarized dipole amplitude

$$Q_{10}(\sigma) \equiv \frac{1}{2N_c} \left\langle\!\!\left\langle \mathrm{T\,tr}\left[V_0\, V_1^{\mathrm{pol}[1]\,\dagger}\right] + \mathrm{T\,tr}\left[V_1^{\mathrm{pol}[1]}\, V_0^\dagger\right] \right\rangle\!\!\right\rangle(\sigma) \qquad V_x^{\mathrm{pol}[1]} = V_x^{\mathrm{G}[1]} + V_x^{\mathrm{q}[1]}$$

Helicity evolution which includes this operator has been studied before (Kovchegov, Pitonyak, Sievert 2016-2019), however the result didn't match the DGLAP evolution (Bartels, Ermolaev and Ryskin 1996).

Helicity evolution and sub-eikonal operators

We find another operator at the sub-eikonal level which generates the DGLAP evolution. This operator comes from the scalar phase in the propagator when we expand it onto the light-cone direction. In particular at small-x this operator describes corrections due to non-zero width of the shock-wave.

$$ig \int_{-\infty}^{\infty} dz^- \, z^- V_x[\infty, z^-] \, F_{-k} \, V_x[z^-, -\infty]$$

This operator in turn generates a new type of the polarized dipole amplitude:

$$G^i_{10}(\sigma) \equiv \frac{igP^+}{2sN_c} \left\langle\!\!\left\langle \mathrm{T}\,\mathrm{tr}\left[V_0^\dagger \int_{-\infty}^{\infty} dz^- \, z^- V_1[\infty, z^-] \, F^{+i} \, V_1[z^-, -\infty] \right] + \mathrm{c.c.} \right\rangle\!\!\right\rangle(\sigma)$$

Helicity evolution and sub-eikonal operators

$$G^i_{10}(\sigma) \equiv \frac{igP^+}{2sN_c} \left\langle\!\!\left\langle \mathrm{T}\,\mathrm{tr} \left[V_0^\dagger \int\limits_{-\infty}^{\infty} dz^- \, z^- V_1[\infty, z^-] \, F^{+i} \, V_1[z^-, -\infty] \right] + \mathrm{c.c.} \right\rangle\!\!\right\rangle(\sigma)$$

This operator is related to the Jaffe-Manohar polarized gluon distribution which satisfies the DGLAP evolution. It can by obtained by expanding the exponential factor in the definition of the JM operator:

$$\int\limits_{-\infty}^{\infty} dz^- e^{ixP^+ z^-} V_1[\infty, z^-] \, F^{+i}(z^-, x_1) \, V_1[z^-, -\infty]$$

$$= - \int\limits_{-\infty}^{\infty} dz^- V_1[\infty, z^-] \, \partial^i A^+ \, V_1[z^-, -\infty] + ixP^+ \boxed{\int\limits_{-\infty}^{\infty} dz^- \, z^- V_1[\infty, z^-] \, F^{+i} \, V_1[z^-, -\infty]} + \ldots$$

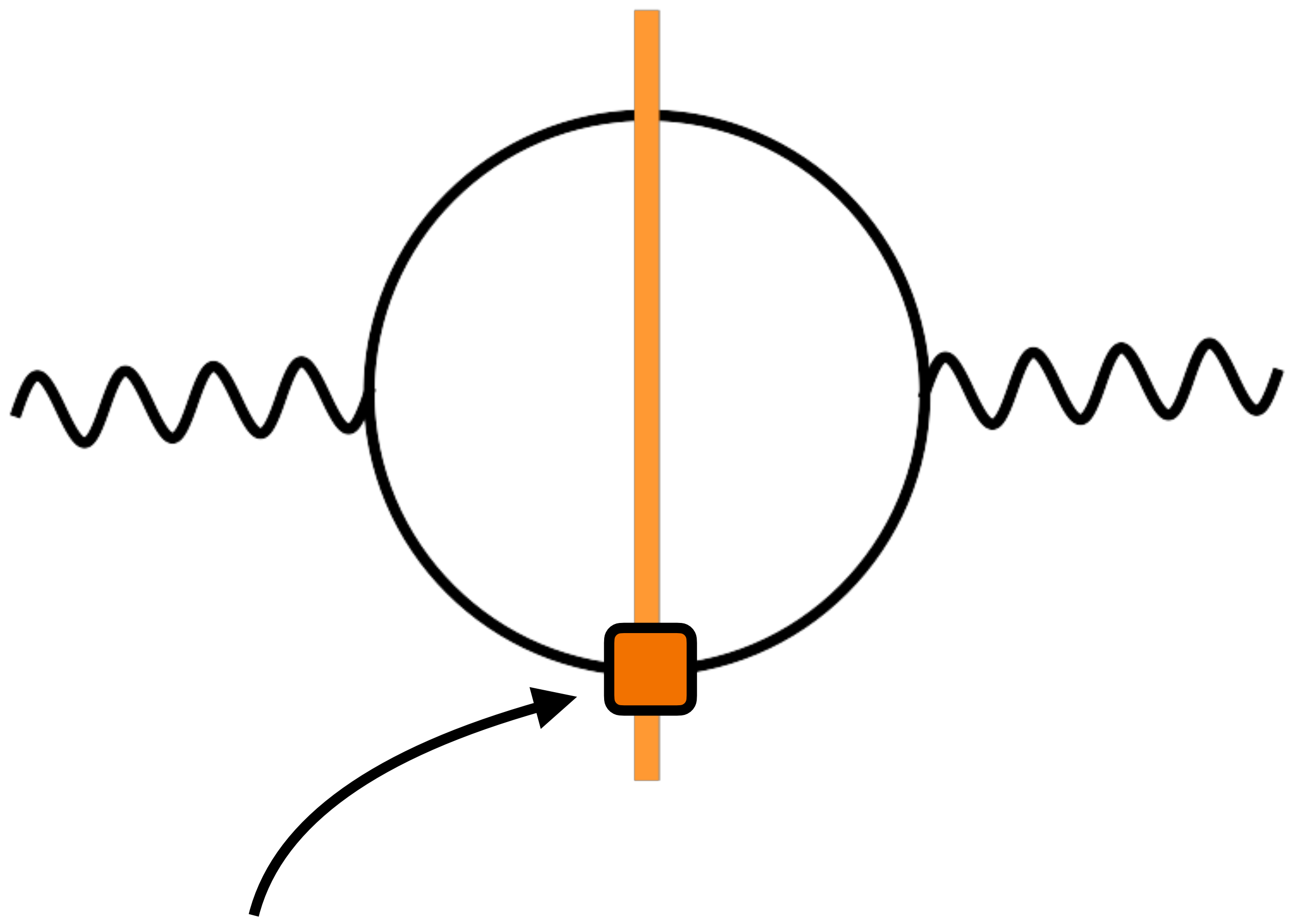
The operator can be also rewritten as

$$ig \int\limits_{-\infty}^{\infty} dz^- \, z^- V_x[\infty, z^-] \, F_{-k} \, V_x[z^-, -\infty] = \frac{1}{2} \int\limits_{-\infty}^{\infty} dz^- V_x[\infty, z^-] \left[D_k - \overleftarrow{D}_k \right] V_x[z^-, -\infty]$$

Sub-eikonal corrections and helicity dependent observables

$$\sigma^{\gamma^* p} \propto -\sum_f \frac{N_c\, Z_f^2}{4\pi^4} \int d^2 x_{10} \int\limits_{\Lambda^2/s}^{1} \frac{dz}{z} \Big\{ 2\left[z^2 + (1-z)^2\right] a_f^2 \,\left[K_1(x_{10}\, a_f)\right]^2 \, G_2(x_{10}^2, zs)$$

$$+ \left[(1-2z)\, a_f^2 \,\left[K_1(x_{10}\, a_f)\right]^2 - m_f^2 \,\left[K_0(x_{10}\, a_f)\right]^2\right] Q(x_{10}^2, zs) \Big\}$$



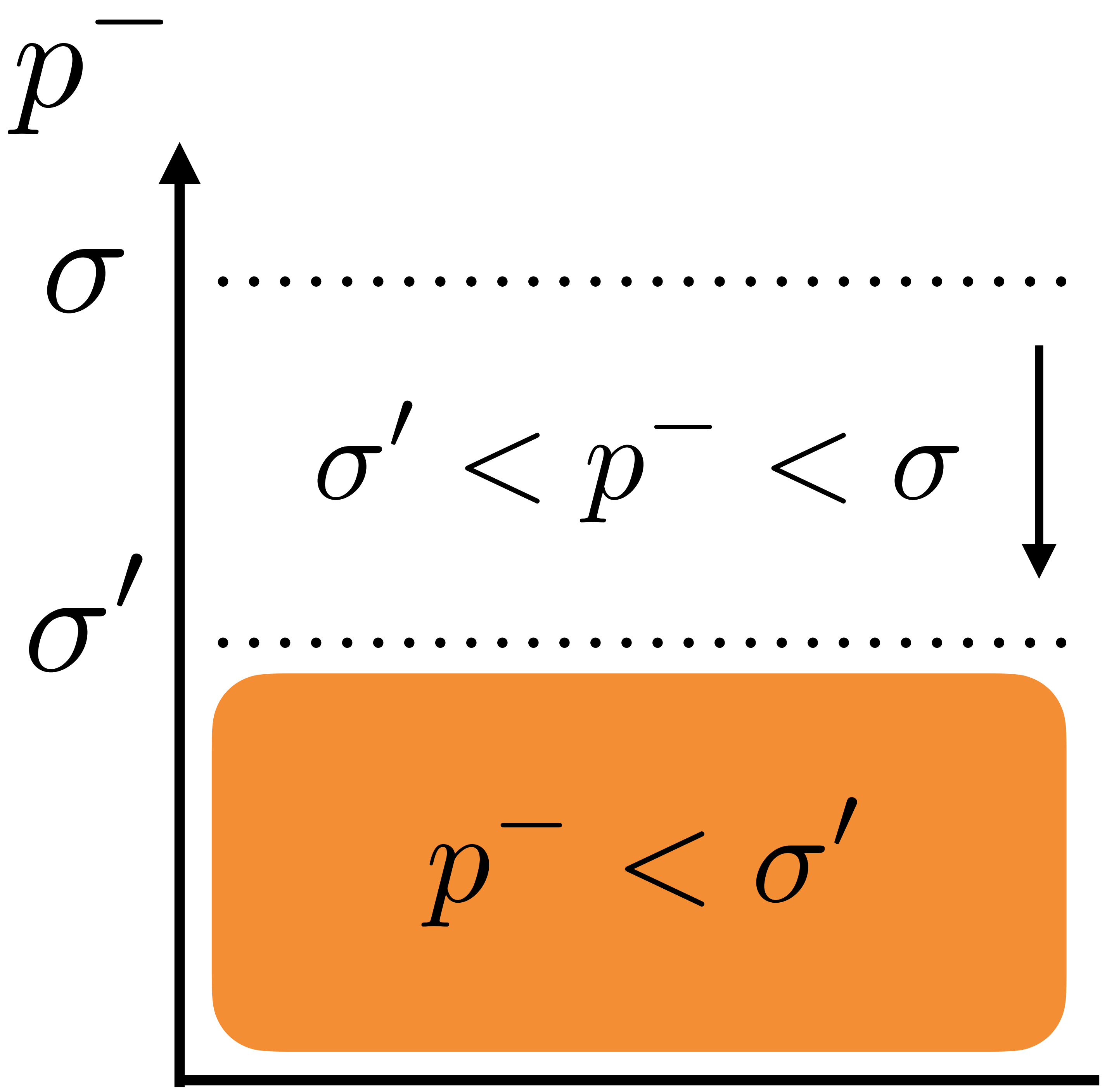
Helicity dependent interaction via sub-eikonal operators

where dipole amplitudes are integrated over impact parameter:

$$\int d^2\left(\frac{x_1 + x_0}{2}\right) G_{10}^i(zs) = (x_{10})_\perp^i \, G_1(x_{10}^2, zs) + \epsilon^{ij}\, (x_{10})_\perp^j \, G_2(x_{10}^2, zs)$$

$$\int d^2\left(\frac{x_0 + x_1}{2}\right) Q_{10}(zs) = Q(x_{10}^2, zs)$$

Evolution in the rapidity factorization approach



Introduce a new scale σ' and redefine the background fields as

$$A_\mu^{\mathrm{bg}} \to \hat{A}_\mu^{\mathrm{q}} + \hat{A}_\mu^{\mathrm{bg}}, \quad \psi^{\mathrm{bg}} \to \hat{\psi}^{\mathrm{q}} + \hat{\psi}^{\mathrm{bg}}$$

Perform integration over new quantum fields in

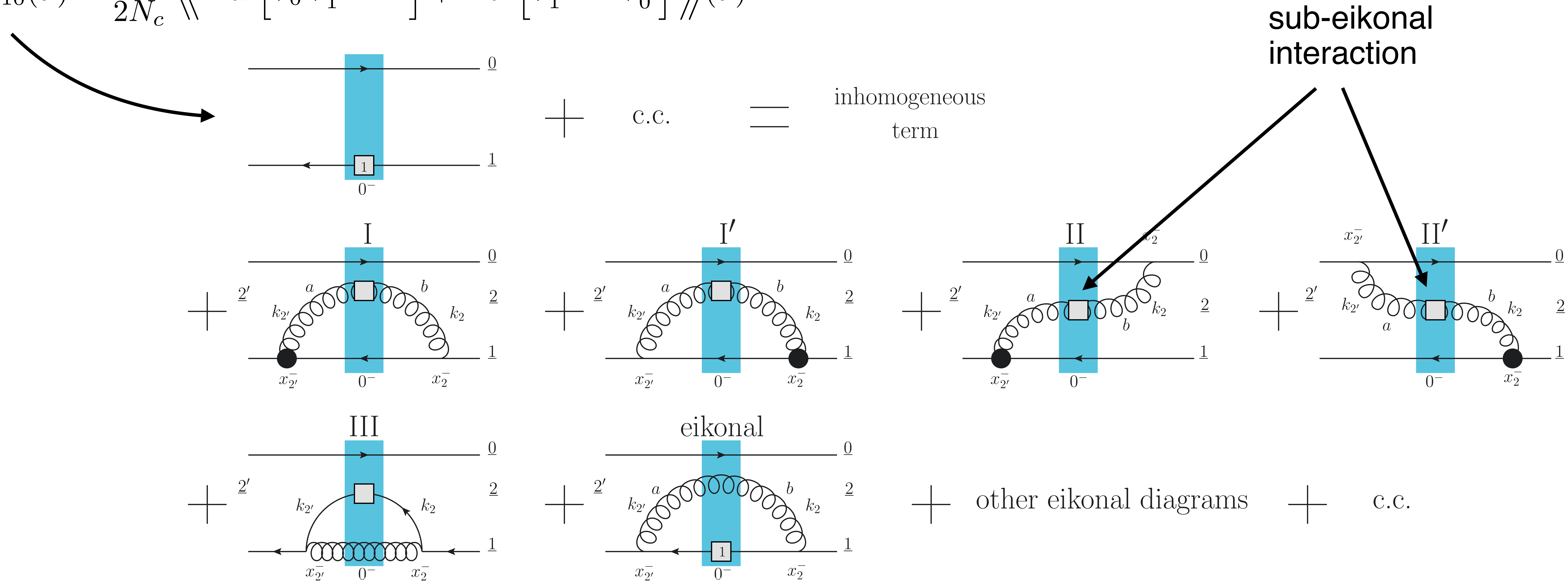
$$\mathrm{T}\left[\mathcal{V}_i(A^{\mathrm{bg}}, \psi^{\mathrm{bg}}, \sigma)\right]$$
$$\equiv \int \mathcal{D}\hat{A}^{\mathrm{q}} \int \mathcal{D}\hat{\psi}^{\mathrm{q}}\, \mathcal{V}_i(\hat{A}^{\mathrm{q}} + \hat{A}^{\mathrm{bg}}, \hat{\psi}^{\mathrm{q}} + \hat{\psi}^{\mathrm{bg}}, \sigma) e^{iS_{bQCD}(\hat{A}^{\mathrm{q}}, \hat{\psi}^{\mathrm{q}}; \hat{A}^{\mathrm{bg}}, \hat{\psi}^{\mathrm{bg}})}$$

The result of integration yields an evolution equation of the following form

$$\mathrm{T}\left[\mathcal{V}_i(A^{\mathrm{bg}}, \psi^{\mathrm{bg}}, \sigma)\right] = \int_{\sigma'}^{\sigma} \frac{dp^-}{p^-} \sum_j \mathcal{K}_{ij} \otimes \mathcal{V}_j(\hat{A}^{\mathrm{bg}}, \hat{\psi}^{\mathrm{bg}}, \sigma')$$

Evolution diagrams

$$Q_{10}(\sigma) \equiv \frac{1}{2N_c} \left\langle\!\!\left\langle \mathrm{T\,tr}\left[V_0\, V_1^{\mathrm{pol}[1]\,\dagger}\right] + \mathrm{T\,tr}\left[V_1^{\mathrm{pol}[1]}\, V_0^{\dagger}\right] \right\rangle\!\!\right\rangle(\sigma)$$



Similar diagrams for

$$G^i_{10}(\sigma) \equiv \frac{igP^+}{2sN_c} \left\langle\!\!\left\langle \mathrm{T\,tr}\left[V_0^{\dagger} \int\limits_{-\infty}^{\infty} dz^-\, z^-\, V_1[\infty, z^-]\, F^{+i}\, V_1[z^-, -\infty]\right] + \mathrm{c.c.} \right\rangle\!\!\right\rangle(\sigma)$$

Evolution equations

We construct evolution equations for the polarized dipole amplitudes.

$$
\begin{aligned}
&\frac{1}{2N_c}\left\langle\!\!\left\langle \mathrm{T}\,\mathrm{tr}\left[V_0\, V_1^{\mathrm{pol}[1]\,\dagger}\right] + \mathrm{c.c.}\right\rangle\!\!\right\rangle(\sigma) = \frac{1}{2N_c}\left\langle\!\!\left\langle \mathrm{T}\,\mathrm{tr}\left[V_0\, V_1^{\mathrm{pol}[1]\,\dagger}\right] + \mathrm{c.c.}\right\rangle\!\!\right\rangle_0(\sigma) \\
&+ \frac{\alpha_s N_c}{2\pi^2}\int\limits_{\sigma'}^{\sigma}\frac{dp^-}{p^-}\int d^2x_2\Bigg\{\left[\frac{1}{x_{21}^2} - \frac{x_{21}\cdot x_{20}}{x_{21}^2 x_{20}^2}\right]\frac{1}{N_c^2}\left\langle\!\!\left\langle \mathrm{tr}\left[t^b V_0\, t^a\; V_1^\dagger\right](U_2^{\mathrm{pol}[1]})^{ba} + \mathrm{c.c.}\right\rangle\!\!\right\rangle(\sigma') \\
&+ \left[2\epsilon^{ij}\frac{x_{21}^j}{x_{21}^4} - \frac{\epsilon^{ij}(x_{21}^j + x_{20}^j)}{x_{21}^2 x_{20}^2} - \frac{2x_{20}\times x_{21}}{x_{21}^2 x_{20}^2}\left(\frac{x_{21}^i}{x_{21}^2} - \frac{x_{20}^i}{x_{20}^2}\right)\right]\frac{1}{N_c^2}\left\langle\!\!\left\langle \mathrm{tr}\left[t^b V_0\, t^a\; V_1^\dagger\right](U_2^{i\mathrm{G}[2]})^{ba} + \mathrm{c.c.}\right\rangle\!\!\right\rangle(\sigma')\Bigg\} \\
&+ \frac{\alpha_s N_c}{4\pi^2}\int\limits_{\sigma'}^{\sigma}\frac{dp^-}{p^-}\int\frac{d^2x_2}{x_{21}^2}\left\{\frac{1}{N_c^2}\left\langle\!\!\left\langle \mathrm{tr}[V_0 t^a V_2^{\mathrm{pol}[1]\dagger} t^b] U_1^{ba}\right\rangle\!\!\right\rangle(\sigma') + 2\frac{\epsilon^{ij}x_{21}^j}{x_{21}^2}\frac{1}{N_c^2}\left\langle\!\!\left\langle \mathrm{tr}[t^b V_0\, t^a\, V_2^{i\mathrm{G}[2]\dagger}] U_1^{ba}\right\rangle\!\!\right\rangle(\sigma') + \mathrm{c.c.}\right\} \\
&+ \frac{\alpha_s\, N_c}{2\pi^2}\int\limits_{\sigma'}^{\sigma}\frac{dp^-}{p^-}\int d^2x_2\,\frac{x_{10}^2}{x_{21}^2\, x_{20}^2}\left\{\frac{1}{N_c^2}\left\langle\!\!\left\langle \mathrm{tr}\left[t^b\, V_0\, t^a\, V_1^{\mathrm{pol}[1]\,\dagger}\right]\, U_2^{ba}\right\rangle\!\!\right\rangle(\sigma') - \frac{C_F}{N_c^2}\left\langle\!\!\left\langle \mathrm{tr}\left[V_0\, V_1^{\mathrm{pol}[1]\,\dagger}\right]\right\rangle\!\!\right\rangle(\sigma') + \mathrm{c.c.}\right\}
\end{aligned}
$$

where $V_z^{i\,\mathrm{G}[2]} \equiv \dfrac{P^+}{2s}\displaystyle\int\limits_{-\infty}^{\infty} dz^-\, V_z[\infty, z^-]\,\left[D^i(z^-, z_\perp) - \overleftarrow{D}^i(z^-, z_\perp)\right]\, V_z[z^-, -\infty]$

which gives rise to the dipole amplitude G_{10}^i. The evolution equations contain mixing between two types of operators.

Evolution equations in the large N_c limit

We obtain a closed system of DLA evolution equations for helicity at large N_c (four equations in total)

Cougoulic, Kovchegov, Tarasov, Tawabutr (2022)

$$G(x_{10}^2, zs) = G^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{sx_{10}^2}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z's}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\Gamma(x_{10}^2, x_{21}^2, z's) + 3\, G(x_{21}^2, z's) + 2\, G_2(x_{21}^2, z's) + 2\, \Gamma_2(x_{10}^2, x_{21}^2, z's) \right]$$

$$\Gamma(x_{10}^2, x_{21}^2, z's) = G^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{sx_{10}^2}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z''s}}^{\min\left[x_{10}^2, x_{21}^2 \frac{z'}{z''}\right]} \frac{dx_{32}^2}{x_{32}^2} \left[\Gamma(x_{10}^2, x_{32}^2, z''s) + 3\, G(x_{32}^2, z''s) + 2\, G_2(x_{32}^2, z''s) + 2\, \Gamma_2(x_{10}^2, x_{32}^2, z''s) \right]$$

+ two similar equations for G_2 and Γ_2

where amplitudes G, Γ, G_2 and Γ_2 parametrize operators with dipole amplitudes G_{10}^i and Q_{10}.

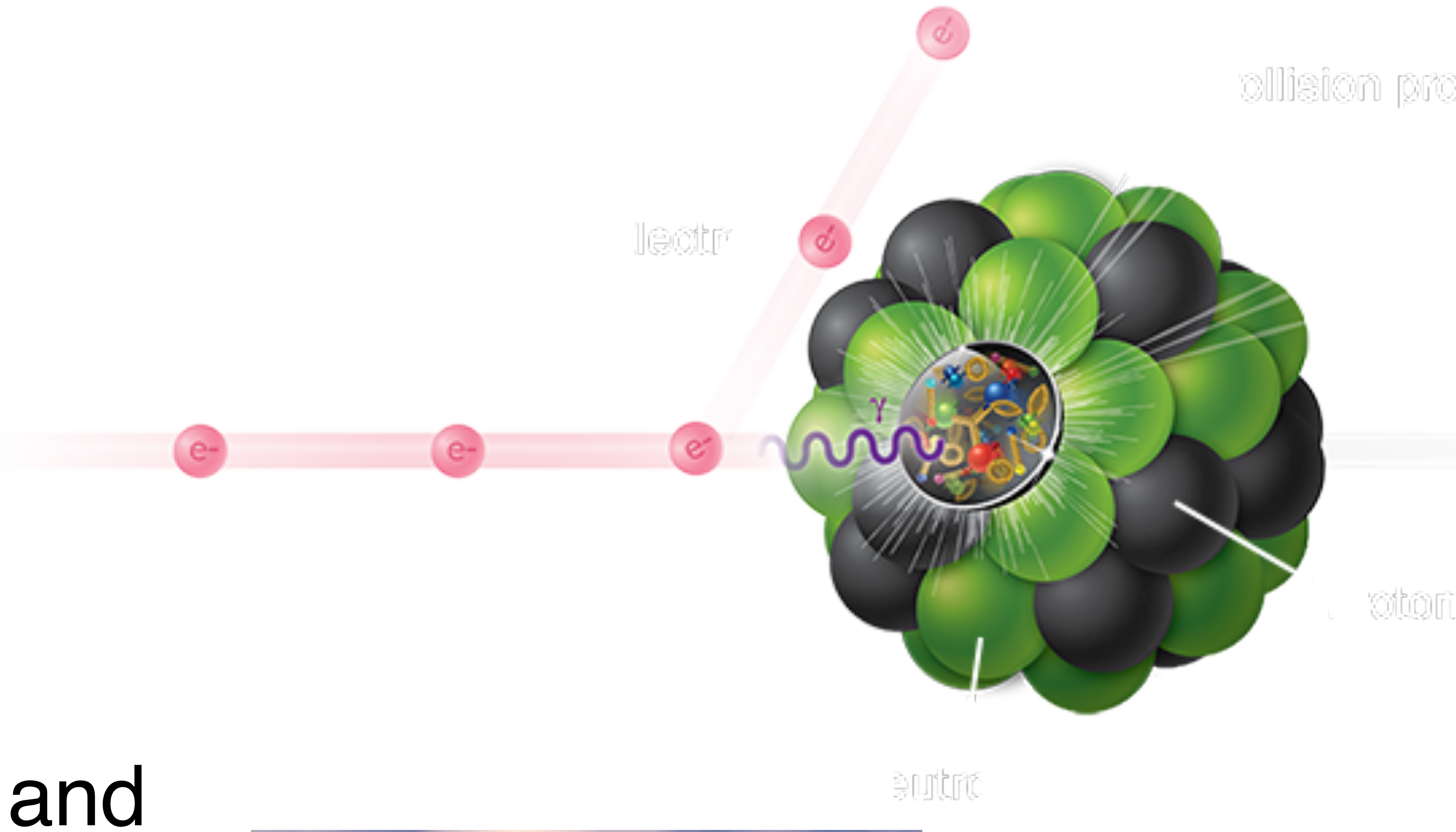
Evolution equations in the large N_c limit

One can construct a numerical solution of these equations, which leads to a result which is consistent with the small-x DGLAP evolution and is in complete agreement with the result obtained in the infrared evolution equations (IREE) approach (see Bartels, Ermolaev and Ryskin 1996)

$$\Delta\Sigma(x, Q^2) \sim \Delta G(x, Q^2) \sim \left(\frac{1}{x}\right)^{3.66\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

Summary

- We consider the problem of small Bjorken-x evolution of the gluon and flavor-singlet quark helicity distributions in the shock-wave formalism
- We obtain a complete set of the sub-eikonal corrections relevant to the small-x helicity evolution
- We find that the evolution contains not only fields strength operator F_{12} and quark axial current $\bar{\psi}\gamma^+\gamma_5\psi$, but also a sub-eikonal operator $D^i - \overleftarrow{D}^i$
- The operator $D^i - \overleftarrow{D}^i$ is related to the Jaffe-Manohar polarized gluon distribution and has a meaning of the sub-eikonal (covariant) phase
- We construct novel evolution equations mixing all three operators
- We also construct closed double-logarithmic evolution equations in the large-N_c and large-N_c & N_f limits



Thank you for your attention!