Quantum Computations for Field Theory Models in Hadron Physics and GPDs

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- Abstract

- Testing detailed predictions of QCD and searching for phenomena Beyond the Standard Model at the LHC and the EIC requires knowing **spin dependent Parton Distribution Functions for quarks and gluons**. For some observables Generalized or Transverse Momentum pdf's are needed. Calculating these distributions from QCD, ab initio, is prohibitively resource intensive and depends on non-perturbative techniques. **Simulation on a quantum computer of quantum field theories** offers a new way to investigate properties of the fundamental constituents of matter. We develop quantum simulation algorithms based on the **light-front formulation** of relativistic field theories, beginning with Yukawa theories in 1+1D and 2+1D. We compute pdf's and GPD's for a model of pion-like mesons and begin quark-diquark model of baryons.

- *Phys.Rev.A* 103 (2021) 6, 062601;
- *Entropy* 23 (2021) 5, 597;
- *Phys.Rev.A* 105 (2022) 3, 032418
- *arXiv:2211.07826 hep-th. Gustin & Goldstein*

Hugo Beauchemin, Gary Goldstein, Peter Love, Will Kirby, Michael Kreshchuk
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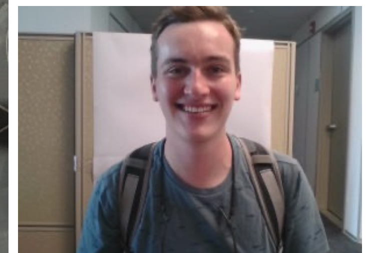

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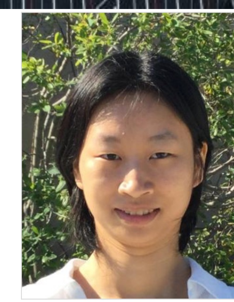


+ James Vary, Shaoyang Jia,
Mengyao Huang. Iowa State U

Key ideas

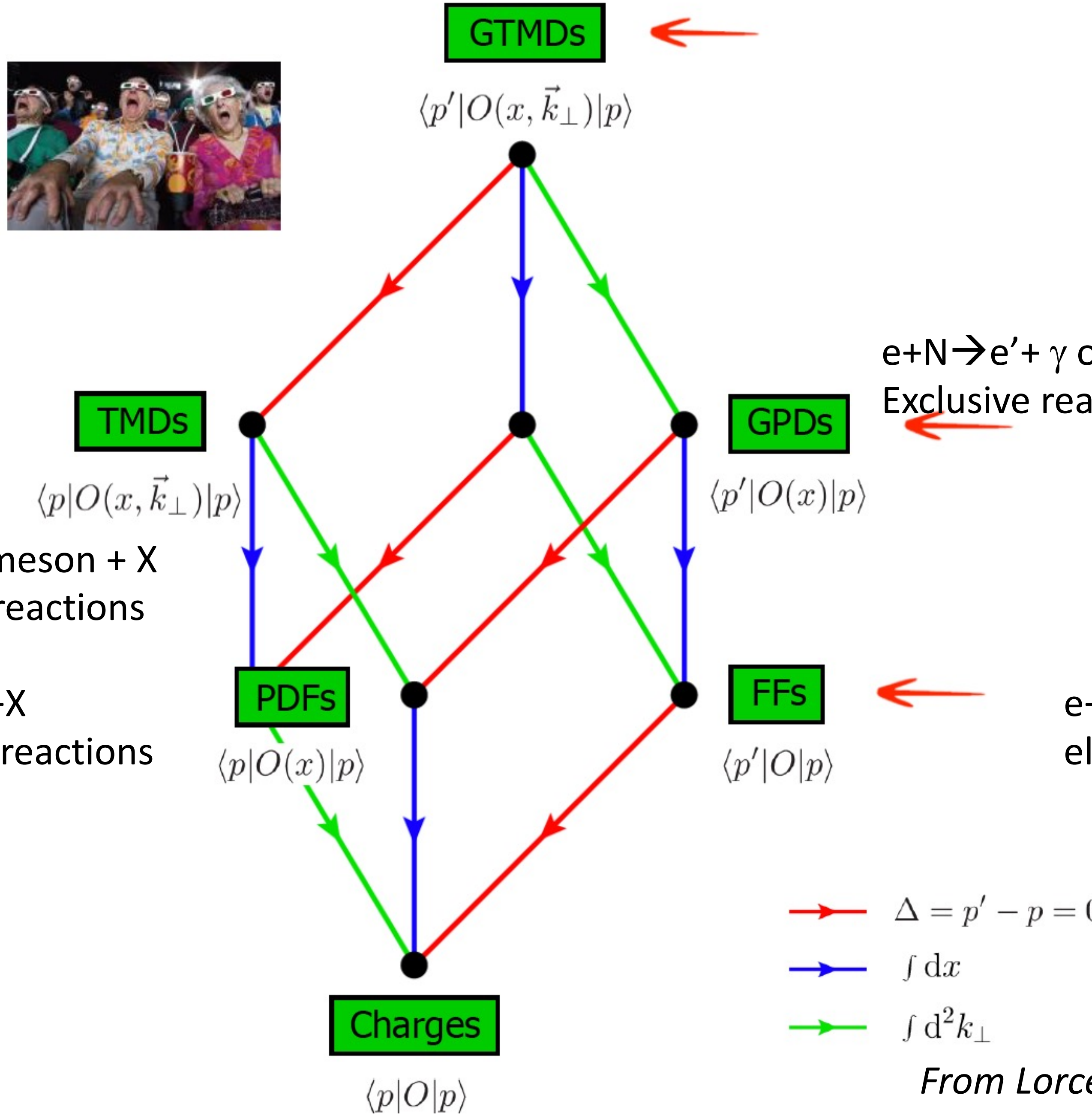
- Light cone quantization or infinite momentum frame – see Pauli and Brodsky for implementation
- "In the Fock-space representation, the light-cone Hamiltonian becomes block di-agonal, characterized by a new dynamic quantum number, the harmonic resolution K. K is closely related to the light-cone momentum, when the theory is defined with periodic boundary conditions in the light cone spatial coordinates. For each fixed value of IC, the Fock-space dimension in the block is finite, and finite matrices can be diagonalized numerically with unlimited precision. Eventually, the resulting field-theoretical many-body problem in one space and one time dimension becomes much simpler than its nonrelativistic and noncovariant approximation."
- This K dependence corresponds to discrete quantization – momenta in a box DLCQ
- Then each discrete Fock state has **qubit** correspondence

Goals of Particle and Nuclear Physics

- What can we know about the ***structure*** of hadrons, especially protons and neutrons?
- What can we know about the origin of Mass & Spin?
- QCD is the complete theory of the strong interactions among quarks and gluons
- Strong coupling, non-Abelian gauge theory, non-linear
- Perturbative at short range – high E –

  asymptotic freedom
- Non-perturbative at long range – "low" E ~ 1 GeV

  infrared slavery   Confinement?
- Model QFT calculations: Yukawa front form construction accommodates generalized parton distributions? No gluons, but . . .
- NJL model for pions with front form construction



e+N→e'+ γ or meson + N'
Exclusive reactions

e+N→e'+ γ or meson + X
Semi-inclusive reactions

e+N→e'+X
Inclusive reactions

e+N→e'+N'
elastic

From Lorce SPIN2018

Field theories & quantization

- Quantum field theories
- Functions in space & time
  - Relativistic covariance
  - Commuting and Anticommuting functions
  - Source charges and currents => A_μ ($\boldsymbol{r}$, t) or ***E*** & ***B*** waves
  - Approximation techniques: multipoles, Green functions, . . .
  - FINITE VOLUME space & numerical solutions => Lattice field theory enterprise
- ***Quantizing***: field operators with non-zero commutators => QED – charges & photon fields U(1) abelian gauge covariance
- With all the complications of singularities, renormalization, low energy photon emission regulation . . . Infinite degrees of freedom . . . α_E (Q^2) small
- **QCD chromoelectric & chromomagnetic fields - $SU(3)_{color}$** gauge covariance - α_S (Q^2) large!
- **Start with simpler quantum field theories, but non-linear interactions => non-perturbative solutions**
- Preliminary: Single scalar boson field theory: ϕ^4 (see Preskill et al.; Vary, et al.)
- Scalar boson and fermion Yukawa interaction – Tufts QC group
- Nambu Jona-Lasinio effective theory with confinement – Tufts + Iowa State
- aDS/CFT - Tufts + Iowa State + UCBerkeley = NuHaQ
- GPDs for mesonic system – C.Gustin and GG

Light-front quantization
1+1d Yukawa model

- Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}\mathrm{m}_B^2\phi^2 + i\overline{\psi}\gamma^\mu\partial_\mu\psi - \mathrm{m}_F\overline{\psi}\psi - \lambda\phi\overline{\psi}\psi$$

- Scalar boson ϕ + spin ½ fermion ψ
- Light-front coordinates $x^{\pm} = x^0 \pm x^1$
- **Quantize** in 1d box *L* with cutoff Λ:

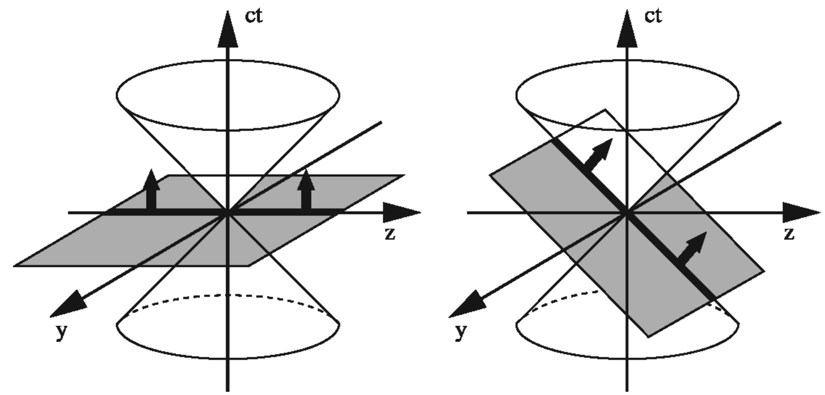
$$\mathrm{p}_\mathrm{n}^+ = \frac{2\pi}{L}\mathrm{n}\ , \qquad \mathrm{p}_\mathrm{n}^- = \frac{\mathrm{m}^2}{\mathrm{p}_\mathrm{n}^+}\ , \qquad \mathrm{n} = 1, 2, 3\ldots, \Lambda$$

- *Eigenstates* of Hamiltonian are bound states of ϕ & ψ *quanta.*

Quantum Field Theory in the Front Form

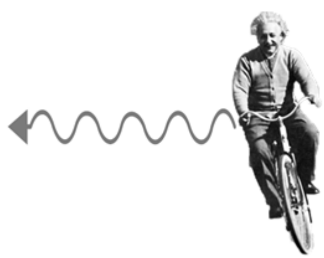
The "light-cone time" x^+ and "light-cone distance" x^-:

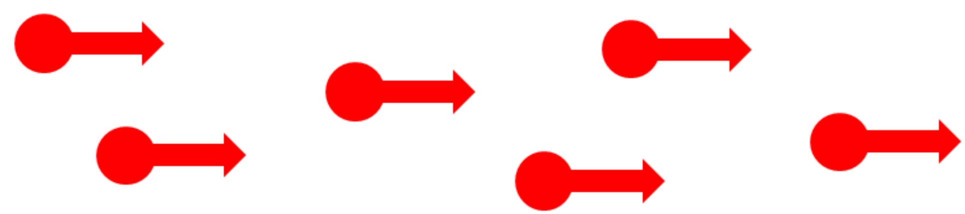
$$x^{\pm} = x^0 \pm x^1 \ . \qquad (2)$$



The instant form The front form

From the point of view of a massless particle moving, say, to the **left**, all the massive particles move to the **right**:





All the light-cone momenta of massive particles are **positive**.

Slide: Michael Kreshchuk, Tufts

Decompose scalar and fermions and quantize

$$\phi(x^+, x^-) = \sum_{\mathsf{n}=1}^{\Lambda} \frac{1}{\sqrt{4\pi \mathsf{n}}} \left(a_{\mathsf{n}} \mathrm{e}^{-i \mathsf{p}_{\mathsf{n}}^{\mu} x_{\mu}} + a_{\mathsf{n}}^{\dagger} \mathrm{e}^{i \mathsf{p}_{\mathsf{n}}^{\mu} x_{\mu}} \right) ,$$

$$\psi^{(+)}(x^+, x^-) = \frac{u}{\sqrt{2L}} \sum_{\mathsf{n}=1}^{\Lambda} \left(b_{\mathsf{n}} \mathrm{e}^{-i \mathsf{p}_{\mathsf{n}}^{\mu} x_{\mu}} + d_{\mathsf{n}}^{\dagger} \mathrm{e}^{i \mathsf{p}_{\mathsf{n}}^{\mu} x_{\mu}} \right) ,$$

$$\mathsf{p}_{\mathsf{n}}^{+} = \frac{2\pi}{L} \mathsf{n} , \qquad \mathsf{p}_{\mathsf{n}}^{-} = \frac{\mathrm{m}^2}{\mathsf{p}_{\mathsf{n}}^{+}} , \qquad \mathsf{n} = 1, 2, 3 \ldots , \Lambda \quad ,$$

$$[a_{\mathsf{m}}, a_{\mathsf{n}}^{\dagger}] = \delta_{\mathsf{mn}}, \ \{b_{\mathsf{m}}, b_{\mathsf{n}}^{\dagger}\} = \delta_{\mathsf{mn}}, \ \{d_{\mathsf{m}}, d_{\mathsf{n}}^{\dagger}\} = \delta_{\mathsf{mn}}$$

Digitizing 1+1 QFT on light front

- Complete commuting set of "observables"

  $P^{\pm} = E \pm P$ & "*Charge*" $= Q$

  define K & H via $P^{+} = (2\pi/L)$ ***K*** & $P^{-} = (L/2\pi)$ ***H***

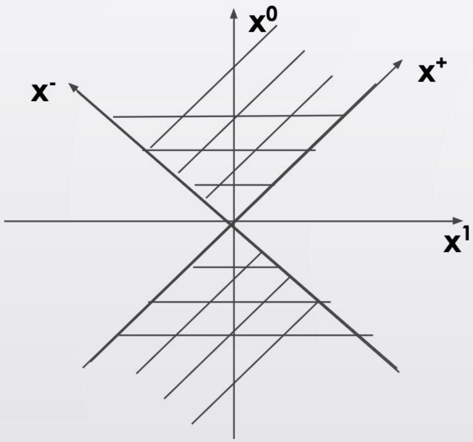
  ➔ $M^2 = E^2 - P^2 = P^+ P^- = K\,H$

- K or ***Harmonic resolution*** plays role of number operator in simulations of quantum chemistry
- H ~ Hamiltonian (Pauli &Brodsky PRD32,1993&2001 (1985))

DLCQ

Discretized light-cone quantization (DLCQ)

H. C. Pauli and S. J. Brodsky, PRD 32, 1993 (1985), T. Eller, H. C. Pauli, and S. J. Brodsky, PRD 35, 1493 (1987), A. Harindranath and J. P. Vary, PRD 36, 1141 (1987)



$x^+ = x^0 + x^1$ **light-front time**

$x^- = x^0 - x^1$ **light-front longitudinal coordinate**

$k^+ = k^0 + k^1$ **light-front momentum**

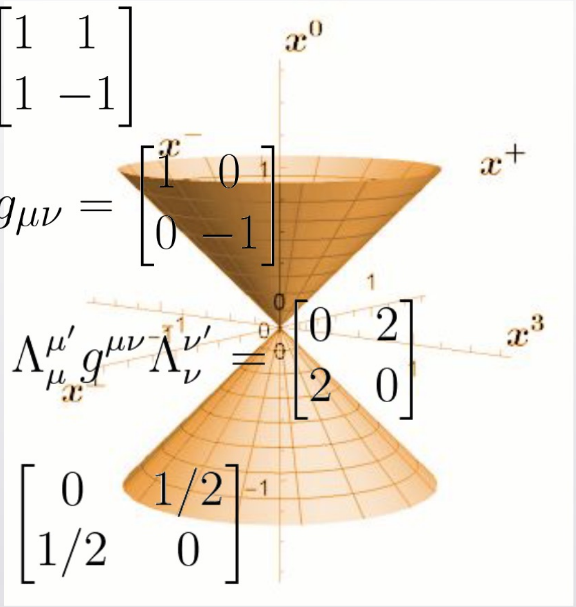
$k^- = k^0 - k^1$ **light-front energy**

$$\Lambda^{\mu'}_{\mu} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$g^{\mu\nu} = g_{\mu\nu} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$g^{\mu'\nu'} = \Lambda^{\mu'}_{\mu} g^{\mu\nu} \Lambda^{\nu'}_{\nu} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

$$g_{\mu'\nu'} = \begin{bmatrix} 0 & 1/2 \\ 1/2 & 0 \end{bmatrix}$$



$$k \cdot x = k^0 x^0 - k^1 x^1 = \frac{1}{2} k^+ x^- + \frac{1}{2} k^- x^+$$

$$= \frac{1}{2} k^+ (x^0 - x^1) + \frac{1}{2} k^- (x^0 + x^1) = \frac{1}{2}(k^+ + k^-) x^0 - \frac{1}{2}(k^+ - k^-) x^1$$

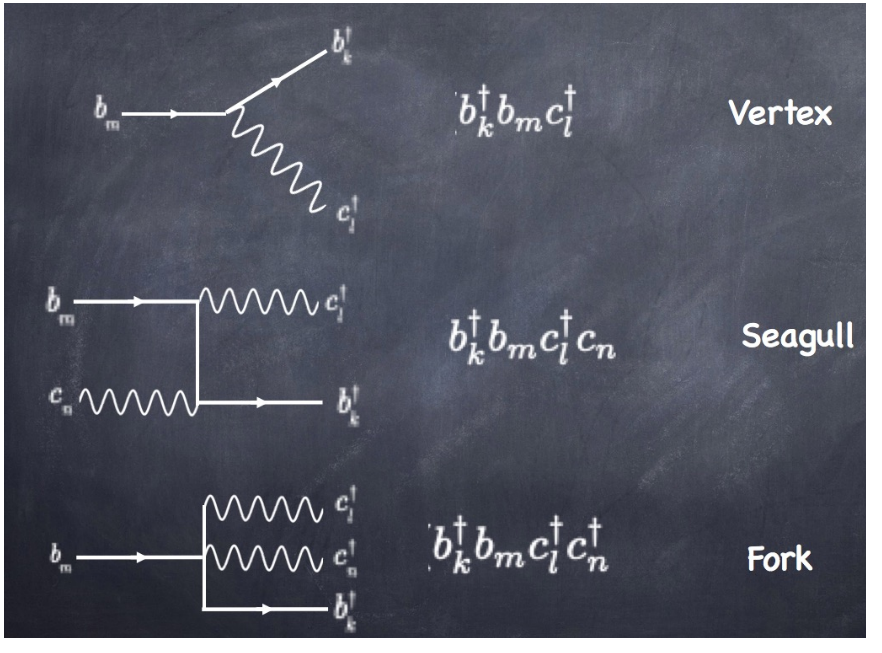
Slide: Mengyao Huang - ISU

Defining DLFQ - continued

- *H , K , Q (charge)* now decomposed in creation & annihilation operators

$$Q = \sum_{\mathsf{n}} (b^\dagger_{\mathsf{n}} b_{\mathsf{n}} - d^\dagger_{\mathsf{n}} d_{\mathsf{n}}) \ , \qquad K = \sum_{\mathsf{n}} \mathsf{n} (a^\dagger_{\mathsf{n}} a_{\mathsf{n}} + b^\dagger_{\mathsf{n}} b_{\mathsf{n}} + d^\dagger_{\mathsf{n}} d_{\mathsf{n}})$$

- $H = H_M + H_V + H_S + H_F$ *(mass, vertex, `seagull', `fork')*



Block structure of DLFQ solutions

- Fock space elements (like *orbital occupancies in Qchem*)

  fermionic, antifermionic, bosonic d.o.f.'s

$$|\{\widehat{n}_j, \widehat{w}_j\}\rangle = |{n_1}^{w_1}, {n_2}^{w_2}, \ldots, {n_N}^{w_N}; {\overline{n}_1}^{\overline{w}_1}, {\overline{n}_2}^{\overline{w}_2}, \ldots, \overline{n}_{\overline{N}}^{\overline{w}_{\overline{N}}}; \widetilde{n}_1^{\widetilde{w}_1}, \widetilde{n}_2^{\widetilde{w}_2}, \ldots, \widetilde{n}_{\widetilde{N}}^{\widetilde{w}_{\widetilde{N}}}\rangle \ ,$$

$$n_j, \overline{n}_j, \widetilde{n}_j = 1, 2, \ldots, \Lambda \quad , \qquad w_j, \overline{w}_j \in \{0, 1\} \ , \qquad 0 \leq \widetilde{w}_j \leq \lfloor \Lambda / \widetilde{n}_j \rfloor \ .$$

n_j and bar(n_j) and tilde(n_j) ∝ momentum states

with occupation numbers w_j 's 0 or 1 for fermions

Fixing K , Q leaves block structure – diagonalizing at fixed M^2

$$M^2|\Psi_{K,s}\rangle = KH|\Psi_{K,s}\rangle = (M_{K,s})^2|\Psi_{K,s}\rangle$$

Encoding for quantum computer qubits

- Direct-direct: Light front Fock states to qubit string
- Direct-compact – to minimize qubit number
- compact mapping stores only momentum modes with nonzero occupancies:

$$|(\widehat{n}_1, \widehat{w}_1), (\widehat{n}_2, \widehat{w}_2), \ldots\rangle$$

- For such an encoding, the number of qubits scales as O(√K log K)

Mapping	Qubit number, Q	Hamiltonian locality	Hamiltonian sparsity
Direct-Direct	$O(K \log K)$	$O(\log K)$	N/A
Direct-Compact	$O(K)$	$O(\log K)$	N/A
Compact	$O(\sqrt{K} \log K)$	N/A	$O(K^2)$

Block structure

$$M^2|\Psi_{K,s}\rangle = KH|\Psi_{K,s}\rangle = \left(M_{K,s}\right)^2|\Psi_{K,s}\rangle$$

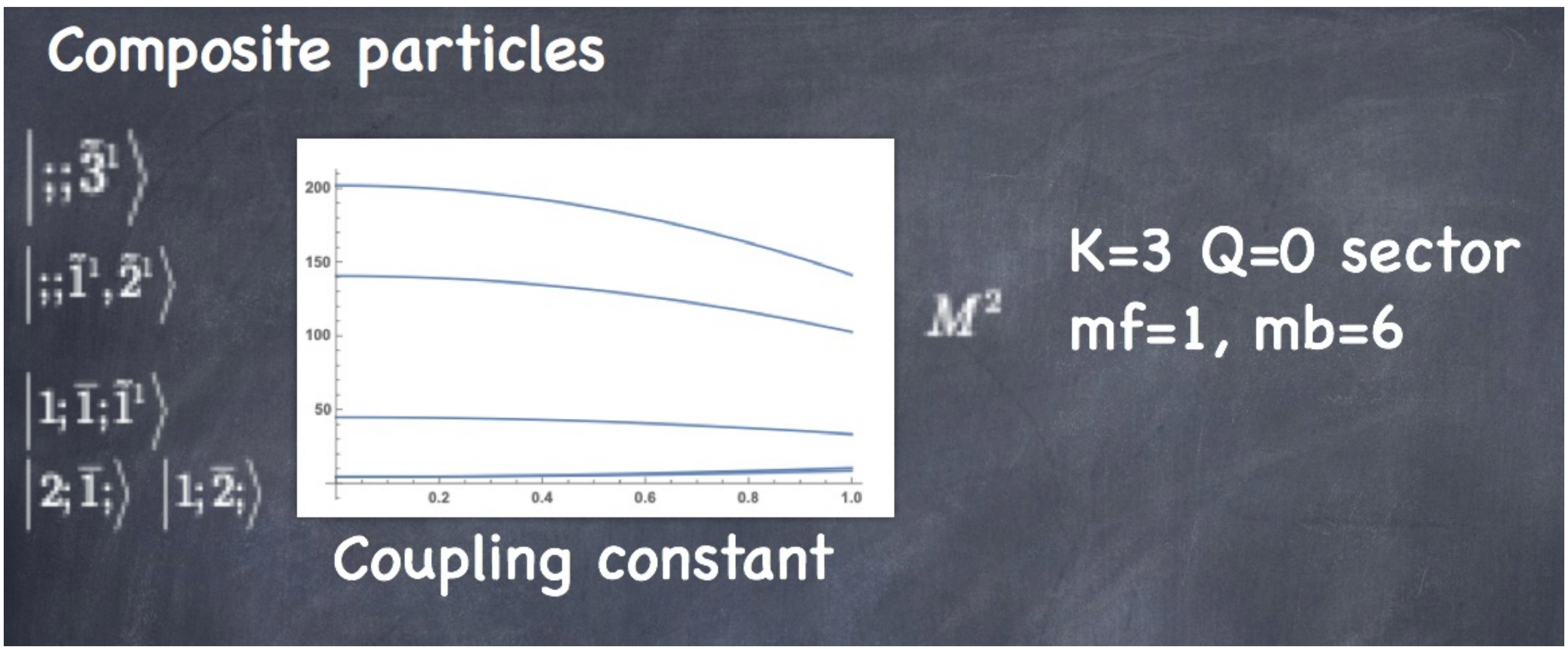
$$H_M = \sum_n \frac{1}{n}\left[a_n^\dagger a_n(m_B^2 + g^2\alpha_n) + b_n^\dagger b_n(m_F^2 + g^2\beta_n) + d_n^\dagger d_n(m_F^2 + g^2\gamma_n)\right]$$

- Increasing K → more bound states with higher resolution
- each state s=s* appears at some K_{s*} & is in all $K>K_{s*}$

K	Fermion States	Boson States	$Q=0$	$Q=1$	$Q=2$
2	$\|1;\bar{1}\rangle, \|2;\rangle$	$\|\tilde{1}^2\rangle, \|\tilde{2}^1\rangle$	$\|1;\bar{1};\rangle, \|;;\tilde{1}^2\rangle, \|;;\tilde{2}^1\rangle$	$\|1;;\tilde{1}^1\rangle, \|2;;\rangle$	
3	$\|2,1;\rangle, \|2;\bar{1}\rangle$ $\|1;\bar{2}\rangle, \|3;\rangle$	$\|\tilde{1}^3\rangle, \|\tilde{1}^1;\tilde{2}^1\rangle$ $\|\tilde{3}^1\rangle$	$\|;;\tilde{1}^3\rangle, \|;;\tilde{1}^1,\tilde{2}^1\rangle, \|;;\tilde{3}^1\rangle$ $\|1;\bar{1};\tilde{1}\rangle, \|2;\bar{1};\rangle, \|1;\bar{2};\rangle$	$\|2;\bar{1};\rangle, \|3;;\rangle$ $\|1;;\tilde{1}^2\rangle, \|1;;\tilde{2}\rangle$	$\|2,1;;\rangle$

Some composite particles . . .

- See our arXiV: 2002.04016
- also see early papers Pauli & Brodsky PRD32, 2001 (1985)



Extracting parton distribution functions

- $f_\ell(x) = f_\ell(p_n{}^+/P^+) = f_\ell(n/K) = \langle \Psi_K | \mathcal{N}_\ell | \Psi_K \rangle$

$$\mathcal{N}_f(\mathsf{n}/K) = b_\mathsf{n}^\dagger b_\mathsf{n} \ , \qquad \mathcal{N}_a(\mathsf{n}/K) = d_\mathsf{n}^\dagger d_\mathsf{n} \ , \qquad \mathcal{N}_b(\mathsf{n}/K) = a_\mathsf{n}^\dagger a_\mathsf{n}$$

- numbers for different parton species

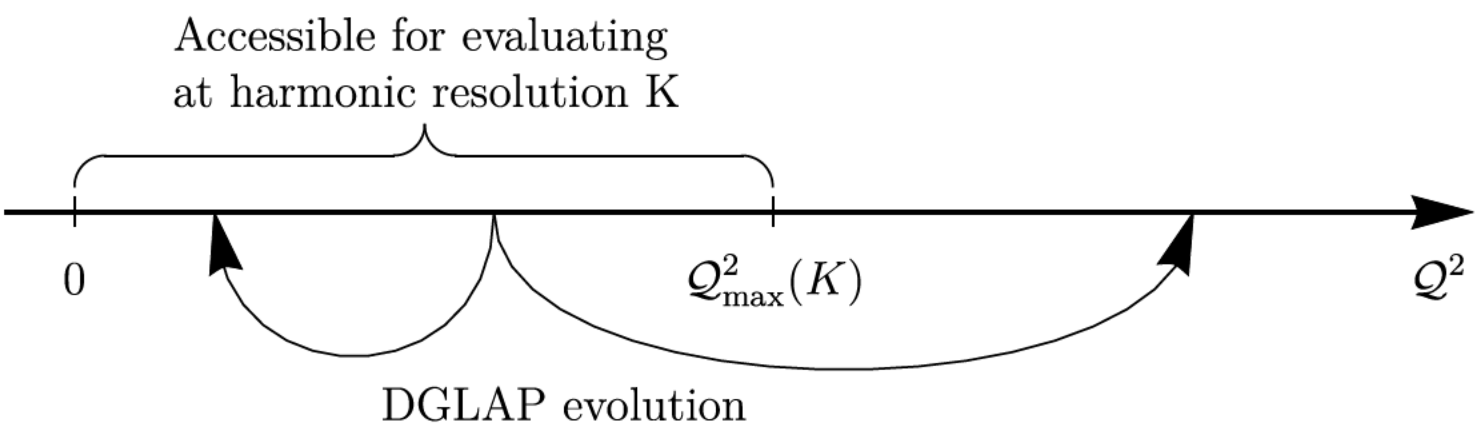


Figure 1: At fixed harmonic resolution K, one can calculate PDFs up to the energy scale $\mathcal{Q}^2_{\max}(K)$. Once calculated at some energy scale, the PDFs can be evolved according to the DGLAP equations.

Examples of pdf's

- Q^2 dependences via

$$P^+ P^-_{\text{free}} = \left(\sum_j \widehat{w}_j \widehat{p}_j^+\right)\left(\sum_j \widehat{w}_j \widehat{p}_j^-\right) = K\left(\sum_j \widehat{w}_j \frac{\mathrm{m}_j^2}{\widehat{n}_j}\right) \leq Q^2$$

- Truncated bound states at scale Q^2

$$f_\ell(\mathsf{n}/K, \mathcal{Q}) = \langle \Psi_K^{(\mathcal{Q})} | \mathcal{N}_\ell | \Psi_K^{(\mathcal{Q})} \rangle$$

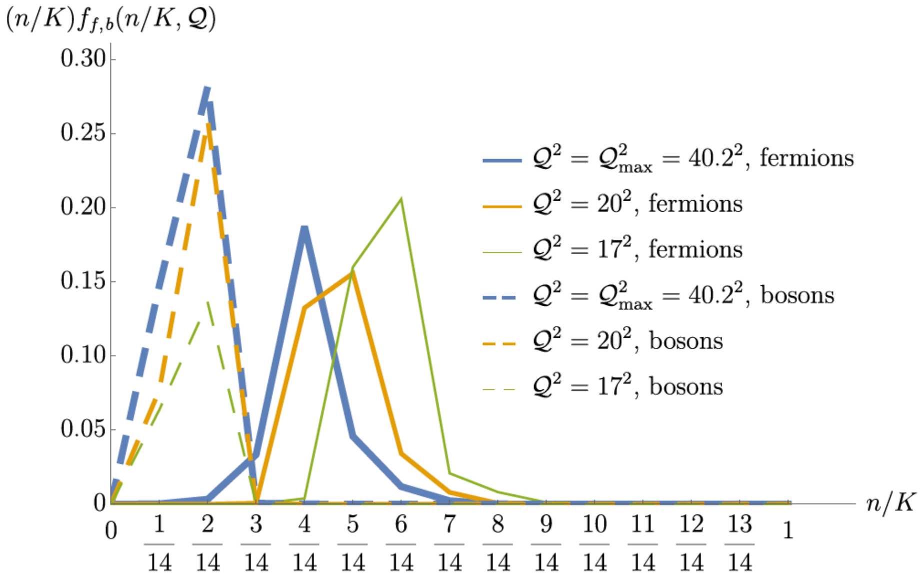


Figure 2: Bosonic and fermionic parton distribution functions for the model (1), as defined in eq. (10), evaluated for harmonic resolution $K = 14$. The values of parameters are chosen as in [114]: $\widetilde{m}_B = 6.7$, $\widetilde{m}_F = 1$, $\lambda = 1$, $\Lambda = 2048$. Shown for the $M = 18.96$ eigenstate with different values of momentum cut-off: $\mathcal{Q}^2 = \mathcal{Q}^2_{\max}, 20^2, 17^2$, where $\mathcal{Q}^2_{\max} = 40.2^2$. The choice $\mathcal{Q}^2 = \mathcal{Q}^2_{\max}$ corresponds to taking all the Fock states from the $K = 14$ sector into account.

QFT → QComputer

- Map bosonic d.o.f.'s straightforward each *p-mode* has qubit register assigned
- fermionic mapping → single qubit for each d.o.f.
- Anticommuting schemes "Jordan-Wigner" or "Bravyi-Kitaev" or . . .
- Partitioning Hilbert Space for *K* into blocks of charge Q – bosons $\{(\widetilde{n}_j, \widetilde{w}_j) | 1 \leq j \leq \widetilde{N} | \sum_j \widetilde{w}_j \widetilde{n}_j = K\}$
- With fermions

$$\dim \mathcal{D}_{K,Q} \geq \dim \mathcal{D}_{K-Q(Q+1)/2,0} \geq p\left(K - Q(Q+1)/2\right)$$

Some Features of the model

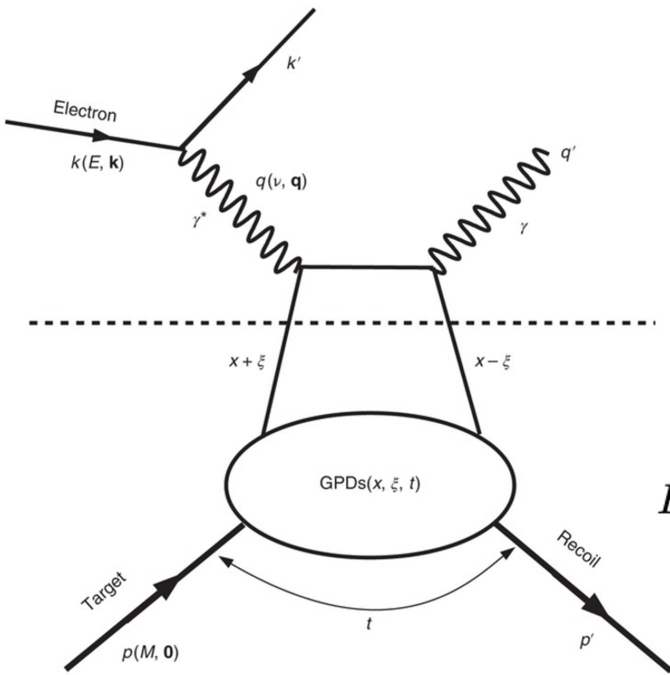
- 1+1 d Yukawa theory is a first step toward 3+1 d gauge theories. Confining (e.g. Brodsky, Pauli, Pinsky)
- C.f. Schwinger model 1+1d QED
- Scalar boson – fermion interaction
- Can extract "wavefunctions", form factors, pdf's, boson &/or fermion correlators
- Set up `mapping' DLCQ Fock states to qubits
- Counting conventional vs. qcomputer needed resources

$$K = \sum_{n} n(a_n^\dagger a_n + b_n^\dagger b_n + d_n^\dagger d_n)$$

- q-chemistry no.ops & orbitals ←→ qubit measurement operators

The Generalized Parton Distribution Function (GPD)

Carter Gustin & GG



- The GPD represents the probability amplitude to find a parton with a fraction of the total light-front momentum after an interaction

$$F^q = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P'|\bar{q}(-\frac{z}{2})\gamma^+ q(\frac{z}{2}))|P\rangle \,|_{z^+=\boldsymbol{z}=0}$$

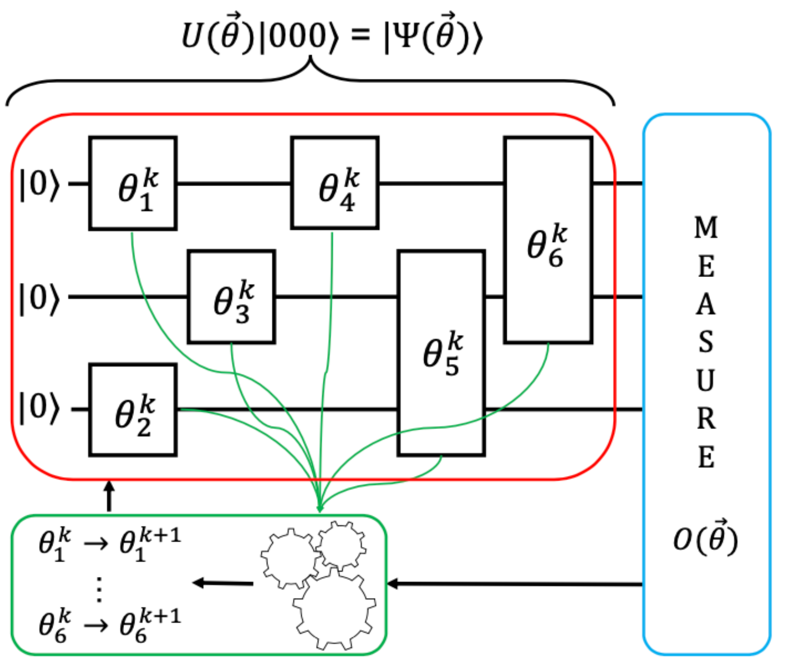
$$x = \frac{n}{K} \qquad t = (P - P')^2$$

https://www.nature.com/articles/s41567-019-0774-3

Variational Quantum Eigensolver (VQE)

- In order to calculate the GPD, we need ground bound states of the Hamiltonian.
- Use VQE which is a hybrid-classical algorithm that finds the minimum eigenvalue of an operator (generally the Hamiltonian)
- VQE exploits the variational principle: $E_0 \leq \langle \Psi(\vec{\theta}) | H | \Psi(\vec{\theta}) \rangle$

Variational Quantum Eigensolver (VQE)



Quantum Computational Chemistry, McArdle et. al.

Results for $\pi^{+\text{ or }-}$ in valence qqbar in 2+1d Yukawa and DLCQ

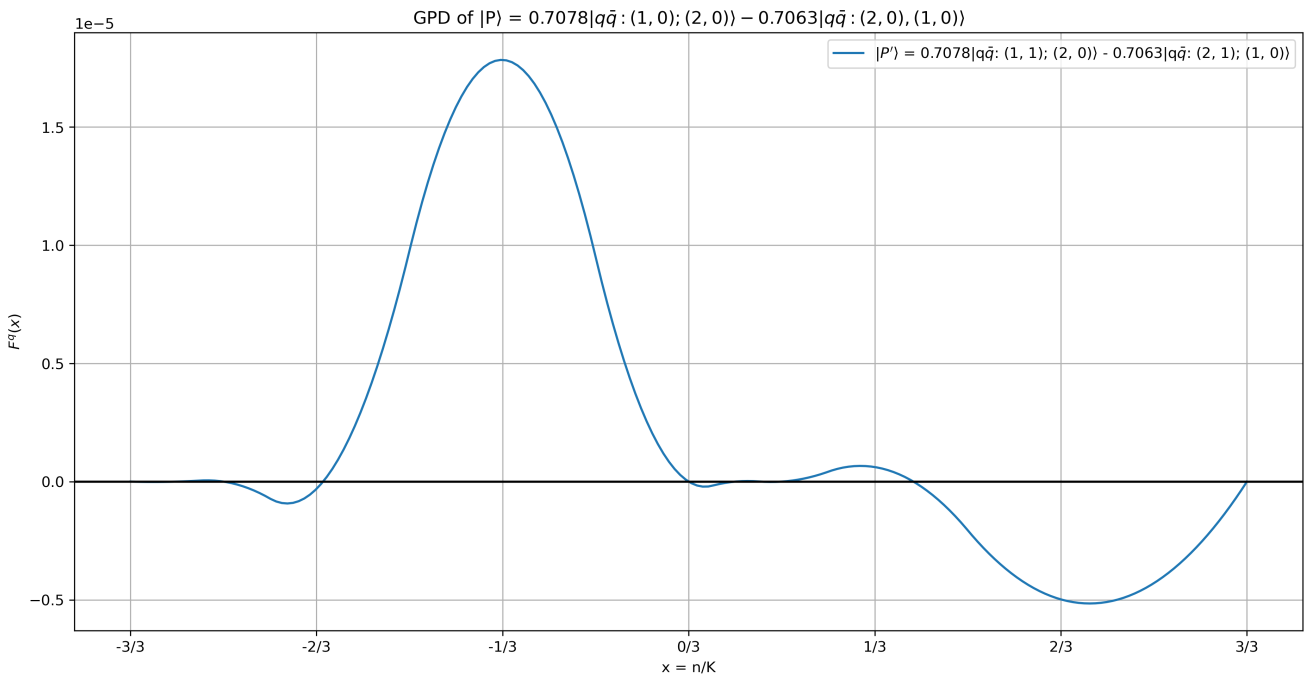
- Parameters: $P^+ = P'^+ = 3, P_\perp = 0, P'_\perp = 1$
- Ground bound state via VQE:

$$|P\rangle = 0.7078|q\bar{q}: (1,0), (2,0)\rangle - 0.7063|q\bar{q}: (2,0), (1,0)\rangle$$

- Look at $|P'\rangle$ such that the fermion gains an increased unit of transverse momentum:

$$|P'\rangle = 0.7078|q\bar{q}: (1,1), (2,0)\rangle - 0.7063|q\bar{q}: (2,1), (1,0)\rangle$$

Results



GPD of |P⟩ = 0.7078|qq̄ : (1, 0); (2, 0)⟩ − 0.7063|qq̄ : (2, 0), (1, 0)⟩
1e−5
|P′⟩ = 0.7078|qq̄: (1, 1); (2, 0)⟩ - 0.7063|qq̄: (2, 1); (1, 0)⟩
1.5
1.0
0.5
0.0
−0.5
F^q(x)
-3/3
-2/3
-1/3
0/3
1/3
2/3
3/3
x = n/K

Simulation and Measurements (2002.04016, 2105.10941)

Product formulas ("trotterization") do not work with compact encoding → **sparse methods** → optimal in both qubits&gates.

$$O_F \left|x, i\right\rangle = \left|x, y_i\right\rangle, \tag{6}$$

$$O_H \left|x, y, 0\right\rangle = \left|x, y, H_{xy}\right\rangle. \tag{7}$$

Example: *parton distribution functions* (PDFs) — momentum distributions of quarks and gluons inside a hadron.

$$f(x) = f(n/K) = \left\langle\Psi_K \middle| N \middle|\Psi_K\right\rangle, \tag{8}$$

$$(0 < x \leq 1),$$

$$N(n/K) = a_n^\dagger a_n. \tag{9}$$

Backup slides

Using BLFQ for NJL to use NISQ

- **Basis light front quantization**
- Nambu-Jona-Lasinio effective field theory
- Noisy Intermediate Scale Quantum era
  - What about error correction? Grows with qubits . . .
- NJL provides quark+antiquark mesons at valence level
- Exact solutions for H_0 with some truncated Fock states.
- Use **Variational Quantum Eigensolver** to find bound states (both conventional and quantum computers)

NISQ

- John Preskill (quant-ph > arXiv:1801.00862)
- Noisy Intermediate-Scale Quantum (NISQ) technology will be available in the near future. Quantum computers with 50-100 qubits may be able to perform tasks which surpass the capabilities of today's classical digital computers, but noise in quantum gates will limit the size of quantum circuits that can be executed reliably. NISQ devices will be useful tools for exploring many-body quantum physics, and may have other useful applications, but the 100-qubit quantum computer will not change the world right away --- we should regard it as a significant step toward the more powerful quantum technologies of the future. Quantum technologists should continue to strive for more accurate quantum gates and, eventually, fully fault-tolerant quantum computing.

NJL effective field theory & QC

- Consider the dynamics of valence quarks for light mesons on the light front - the Hamiltonian
- Hamiltonian includes kinetic energy, confinement potential in longitudinal and transverse directions
- the Nambu–Jona-Lasinio interaction for the chiral interactions among quarks.
- Limit to valence Fock sector of mesons while working with relative momentum variables.
- The dependence of the light-front wave functions for these valence quarks on the relative momentum is expanded in terms of orthonormal basis functions.
- After implementing finite cut-offs in this expansion, the light-front Hamiltonian becomes a Hermitian matrix in the basis representation. We use the scheme of Jia & Vary (Phys. Rev. C, 99:035206, 3 2019)
- fixes our model parameters at each choice of basis cut-offs.
- run the VQE minimization on the IBM Vigo machine to calculate the squared pion mass. Using the resulting wave function, we calculate squared mass, decay constant, mass radius, electromagnetic form factor, and charge radius of the pion.
- Pdf's for nucleon? See Vary, et al. arXiv preprint arXiv:2112.01927 (2021). Valence quarks ..
- GPDs and TMDs in progress for pions and diquark model of nucleons

NJL details . . .

- Hamiltonian $H_0 + H_{int}^{eff}$

$$H_0 = \frac{(\vec{\kappa}^{\perp})^2 + \mathbf{m}^2}{x} + \frac{(\vec{\kappa}^{\perp})^2 + \overline{\mathbf{m}}^2}{1-x} + b^4 x(1-x)\vec{r}_{\perp}^2 - \frac{b^4}{(\mathbf{m}+\overline{\mathbf{m}})^2}\partial_x x(1-x)\partial_x$$

$$H_{\rm int}^{\rm eff} = H_{{\rm NJL},\pi}^{\rm eff} = \int {\rm d}x^- \int {\rm d}\vec{x}^{\perp}\,(-\frac{G_\pi P^+}{2}) \times \left[\left(\overline{\psi}\psi\right)^2 + \left(\overline{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right]\,, \tag{4}$$

Here ψ is the fermion field operator, G_π is the NJL coupling constant, and P^+ is the total light-front longitudinal momentum of the system. We then expand eq. (4) into relevant combinations of ladder operators for the quark fields. In the basis representation, this term further takes the form of a hermitian matrix, the elements of which can be calculated analytically [43].

Basis functions

- For H_0 there is transverse momentum & confined longitudinal motion

$$\psi_{rs}(x, \vec{\kappa}^{\perp}) = \sum_{nml} \psi_{nmlrs}\, \phi_{nm}\left(\frac{\vec{\kappa}^{\perp}}{\sqrt{x(1-x)}}; b\right) \chi_l(x)$$

where ψ_{nmlrs} is the expansion coefficient, ϕ_{nm} is a 2-dimensional (2D) harmonic oscillator (HO) eigenfunction, and χ_l is the longitudinal basis function. Here r and s are the spin indices of the quark and the anti-quark. Each term in eq. (5) is an eigenfunction of H_0 in eq. (3). Explicitly, ϕ_{nm} is defined as

$$\phi_{nm}\left(\vec{q}^{\perp}; b\right) = \frac{1}{b}\sqrt{\frac{4\pi n!}{(n+|m|)!}}\left(\frac{|\vec{q}^{\perp}|}{b}\right)^{|m|} \times \exp\left(-\frac{\vec{q}^{\perp 2}}{2b^2}\right) \times L_n^{|m|}\left(\frac{\vec{q}^{\perp 2}}{b^2}\right) \exp^{im\varphi}, \quad (6)$$

with $\tan(\varphi) = q^2/q^1$ and $L_n^{|m|}$ being the associated Laguerre function. The parameter b sets the scale

J = 0 states

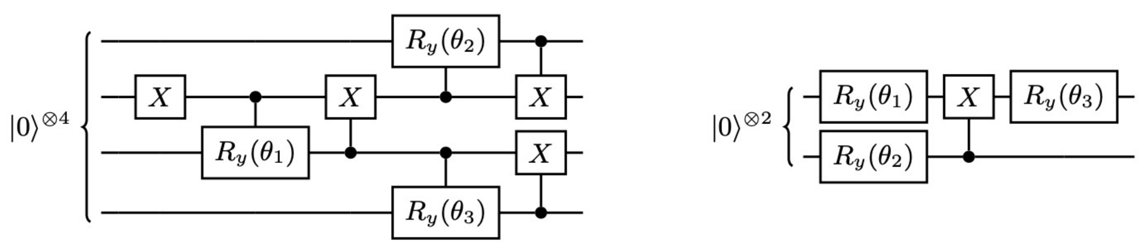
Some NJL Results

$$h_{ij} = H^{\text{BLFQ}}$$
$$= \begin{pmatrix} 640323 & 139872 & -139872 & -107450 \\ 139872 & 346707 & 174794 & 139872 \\ -139872 & 174794 & 346707 & -139872 \\ -107450 & 139872 & -139872 & 640323 \end{pmatrix}, \quad (39)$$

in units of MeV^2. The two lowest eigenvalues correspond to π and ρ meson squared masses: the ground state is $(0.34, -0.62, -0.62, 0.34)^T$, with $m_\pi^2 = 139.6^2\ \text{MeV}^2$.

$\mathbf{m}$	$\overline{\mathbf{m}}$	κ	G_π	$N_{\max}$	$M_{\max}$	$L_{\max}$
337.01 MeV	337.01 MeV	227.00 MeV	250.785 GeV^{-2}	0	2	0

Table III. Model parameters for the BLFQ-NJL model.



(a) (b)

Figure 3. Ansatz circuits for preparing an arbitrary superposition of single-particle Fock states with real coefficients. For the direct encoding (a), we use a generalization of a circuit from [71] for preparation of W_N states. For the binary encoding (b), we use arbitrary state preparation, with all single qubit rotations replaced by $R_y(\theta)$ gates, where $R_y(\theta)$ denotes a single-qubit rotation through an angle θ about the y-axis.

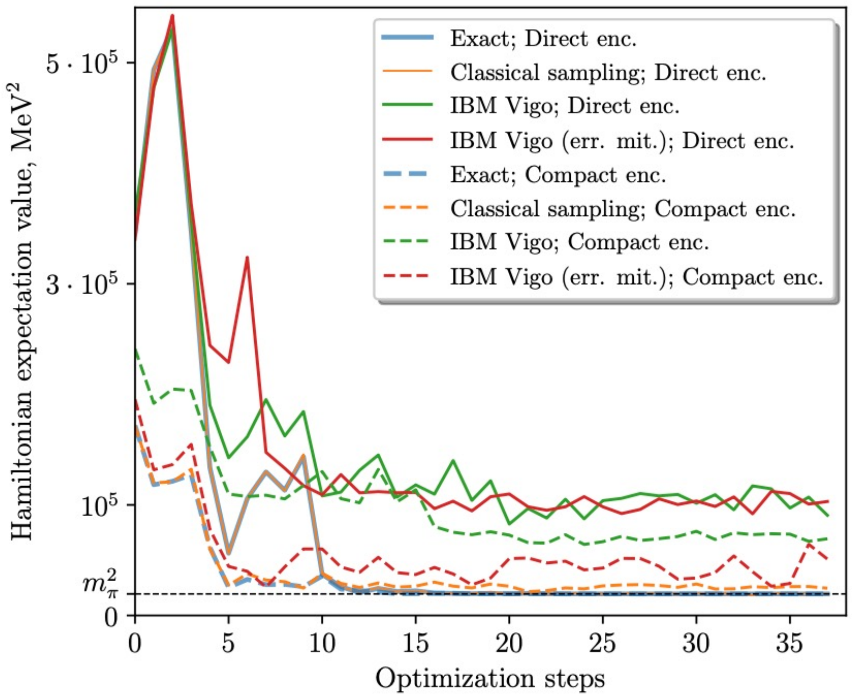


Figure 5. The results of the VQE minimization algorithm in the compact and direct encodings. These were obtained from 8192 samples per term on IBM Vigo machine, with and without measurement error mitigation.

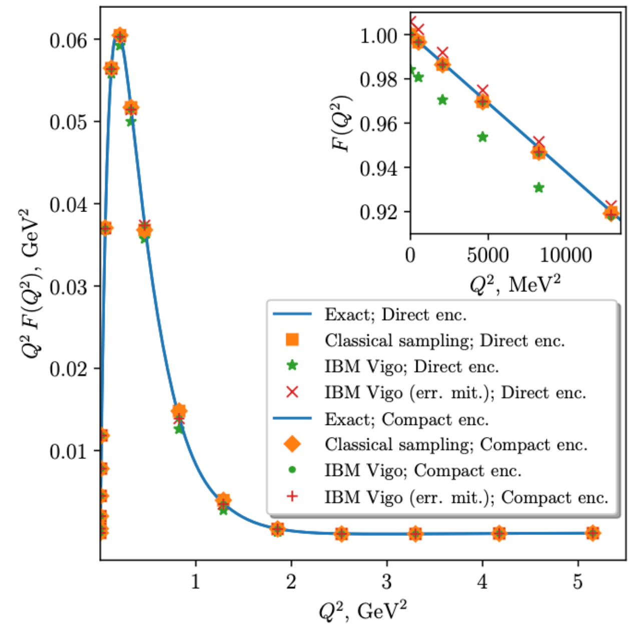


Figure 6. Pion elastic form factor, as defined in eq. (27). Pion elastic form factor is used to calculate the charge radius, obtaining the values given in Tab. IV (charge radius is defined in eq. 28). Datapoints for the quantum simulation on the IBM Vigo processor used 8192 samples per term, with and without measurement error mitigation. The results measured on the quantum computer are in good agreement with the exact ones due to the strong contribution to the measurement operators from the identity term.

Masses & Form Factor

Summary & future

- For 1+1 d Yukawa
- Obtained block diagonalized l.f. wavefunctions
- Developed mapping 1+1 d Yukawa occupation number states to qubit registers
- pdf's involve measurements of occupation numbers
- Extended to form factors and decay constants
- NJL model & pion structure
- GPDs and TMDs in progress for pions and fermion+boson model of nucleons
- Next QCD in 3+1 d with helicity & color d.o.f.'s
- Pdf's for nucleon? See Vary, et al. arXiv preprint arXiv:2112.01927 (2021). Valence quarks ..
- Quark+diquark fields to model nucleon with simple interaction

Particulars of 1+1D l.c. formulation

- Metric: $g_{00} = -g_{11} = 1, g_{01} = g_{10} = 0$
- $g^{++} = g^{--} = 0$; $g^{+-} = g^{-+} = 2$
- $\gamma^0 = \sigma_3$, $\gamma^1 = i\sigma_2$
- Independent fields ϕ & $\psi^{(+)}$ with = $\psi^{(\pm)} = \Lambda^{(\pm)} \psi$

$$\Lambda^{(\pm)} = \tfrac{1}{4} \gamma^{\pm} \gamma^{\mp}$$

- Box quantization:

$$\phi(x^+, x^-) = \sum_{\mathsf{n}=1}^{\Lambda} \frac{1}{\sqrt{4\pi \mathsf{n}}} \left(a_{\mathsf{n}} \mathrm{e}^{-i \mathsf{p}_{\mathsf{n}}^{\mu} x_{\mu}} + a_{\mathsf{n}}^{\dagger} \mathrm{e}^{i \mathsf{p}_{\mathsf{n}}^{\mu} x_{\mu}} \right)$$

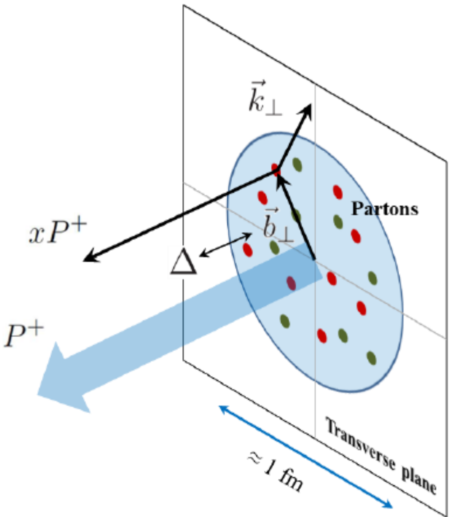
$$\psi^{(+)}(x^+, x^-) = \frac{u}{\sqrt{2L}} \sum_{\mathsf{n}=1}^{\Lambda} \left(b_{\mathsf{n}} \mathrm{e}^{-i \mathsf{p}_{\mathsf{n}}^{\mu} x_{\mu}} + d_{\mathsf{n}}^{\dagger} \mathrm{e}^{i \mathsf{p}_{\mathsf{n}}^{\mu} x_{\mu}} \right)$$

$$[a_{\mathsf{m}}, a_{\mathsf{n}}^{\dagger}] = \delta_{\mathsf{mn}}, \ \{b_{\mathsf{m}}, b_{\mathsf{n}}^{\dagger}\} = \delta_{\mathsf{mn}}, \ \{d_{\mathsf{m}}, d_{\mathsf{n}}^{\dagger}\} = \delta_{\mathsf{mn}}$$

Impact-parameter distributions

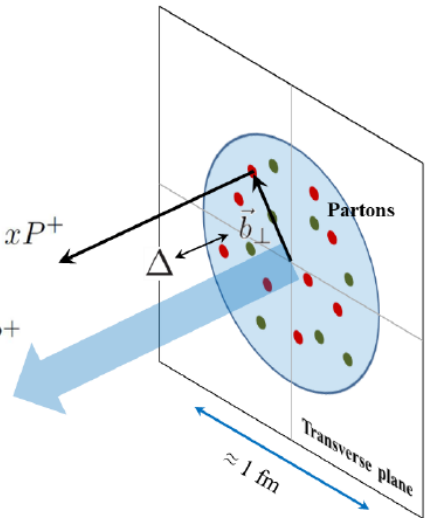
GTMDs

Complicated hard exclusive processes ?



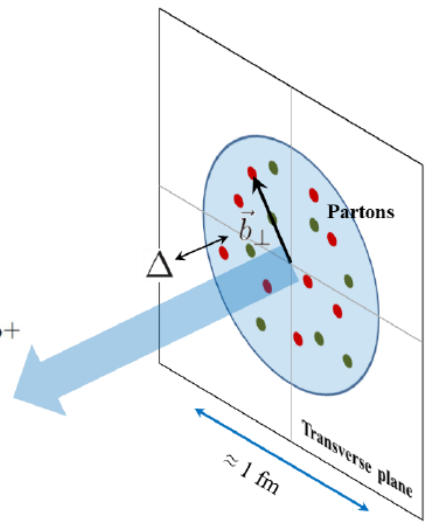
GPDs

Hard exclusive processes



FFs

Elastic scattering



From Lorce SPIN2018

Computation sizes

- One example of advantage of l.f. formulation
- Encoding l.f. Fock states → higher dim QFT's
- Qubit scaling increases *O*(K) vs. equal time quantization.
- For 20^3 grid in l.f. method mom-space with n_f=5 & n_c=3, the upper bound equation → 1360 qubits << 4×10^5 in some estimates (e.g. Lamm, et al. arXiV:1908.10439)

Time evolution at constant harmonic resolution & state preparation

- Goal of simulation algorithm to prepare eigenstates of interacting field theory $\mathcal{L}$ for *K & charge Q*

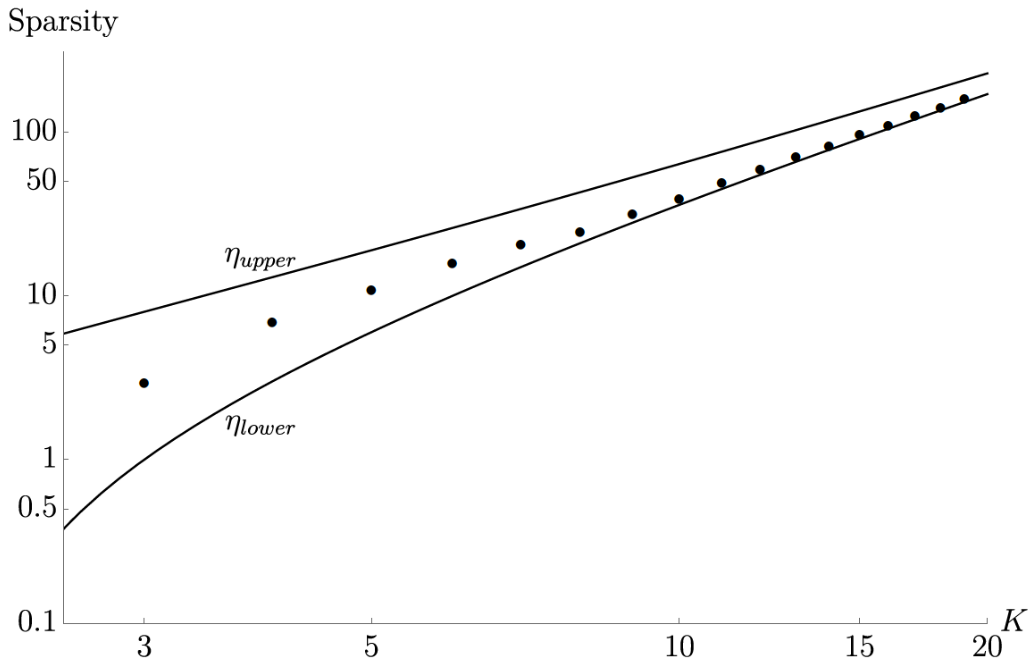


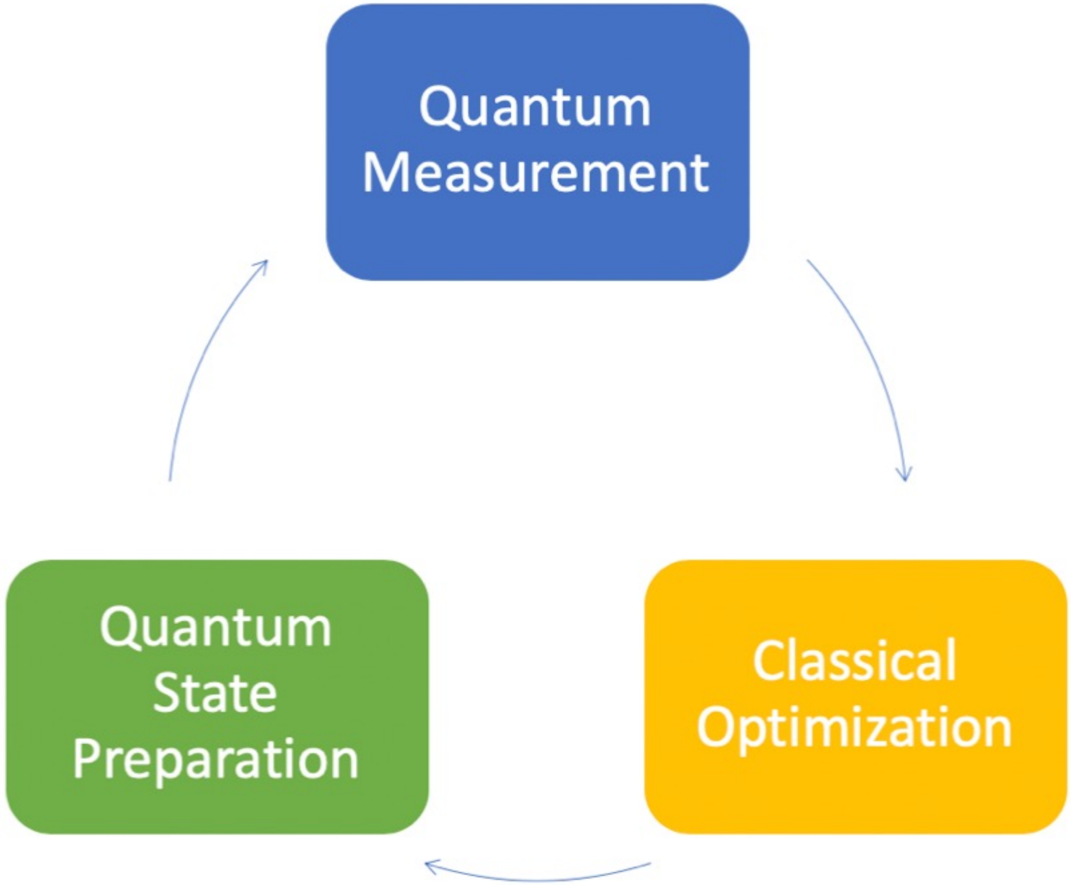
Figure 3: Hamiltonian sparsity vs. K. The curves label the upper and lower bounds on the sparsity, while the data points mark the exact sparsities for $K = 3, 4, \ldots, 19$. The upper and lower bounds are given by $\eta_{\text{upper}} = \frac{1}{2}K^2 + \frac{3}{2}K - 1$ and $\eta_{\text{lower}} = \frac{1}{2}K^2 - \frac{3}{2}K + 1$ (derived in App. A.2).

The Steps of VQE

VQE Inputs: Hamiltonian in terms of Paulis P_k, parameterized ansatz circuit which outputs ansatz state $|\psi(\theta)\rangle$

VQE Outputs: Approximate ground state and ground state energy

1. Encode Hamiltonian $H = \sum_k \alpha_{P_k} P_k$
2. Quantum: Prepare ansatz state $|\psi(\theta)\rangle$
3. Quantum: Sample $\langle\psi(\theta)| P_k |\psi(\theta)\rangle$ for all k (eigenvalues ± 1)
4. Classical: Add up $\langle H\rangle = \sum_k \alpha_{P_k} \langle\psi(\theta)| P_k |\psi(\theta)\rangle$
5. Classical: Change $\theta \to \theta'$ to lower $\langle H\rangle$
6. Both: Repeat Steps (2)-(5) to find lowest $\langle H\rangle$
7. Both: Stop when converged



- Thanks Ken Robbins

Beginning QCD 3+1D simulation

- Upper bound on number of required qubits to store l.f. wavefunctions

$$Q \leq \underbrace{2K}_{\substack{\text{number of}\\ \text{occupied}\\ \text{fermion/antifermion}\\ \text{modes}}} \Big[\underbrace{\underbrace{\lceil \log_2 K \rceil + 2\lceil \log_2 \Lambda_\perp \rceil}_{\text{momentum}} + \underbrace{1}_{\text{helicity}} + \underbrace{\lceil \log_2 n_f \rceil}_{\text{flavors}} + \underbrace{\lceil \log_2 n_c \rceil}_{\text{colors}}}_{\text{fermion/antifermion mode quantum numbers}} \Big]$$

$$+ \underbrace{K}_{\substack{\text{number of}\\ \text{occupied}\\ \text{boson modes}}} \Big[\underbrace{\underbrace{\lceil \log_2 K \rceil + 2\lceil \log_2 \Lambda_\perp \rceil}_{\text{momentum}} + \underbrace{\lceil \log_2 K \rceil}_{\text{occupancy}} + \underbrace{1}_{\text{helicity}} + \underbrace{\lceil \log_2 (n_c^2 - 1) \rceil}_{\text{colors}}}_{\text{boson mode quantum numbers}} \Big]$$

Time evolution & state preparation

2.3 Time evolution at constant harmonic resolution

The goal of our simulation algorithm is first to prepare the eigenstates of the interacting quantum field theory described by Lagrangian given in eq. (1). In each sector of fixed harmonic resolution K and charge Q, the lowest mass-energy particle is a physical particle of the theory. We then aim to perform measurements on the state to determine properties of these composite particles such as PDFs and form factors.

State preparation is a basic element of any quantum simulation algorithm. In this section we give bounds on the cost in terms of quantum gates required to evolve a state in a subspace of fixed harmonic resolution K for time t, to precision ϵ. We use the methods of [17, 13, 136], which are optimal in all relevant parameters.

Sparse Hamiltonians may be specified efficiently by two oracles: functions that can be called to give the defining information for the Hamiltonian. In App. C we give details of implementing two oracles needed by the methods of [17, 13]. The first is O_F — an oracle that enumerates the positions of non-zero entries of the Hamiltonian in a given row. O_F is defined in App. C.1 where we show that the cost of O_F for the compact mapping is $O(\sqrt{K}\log K)$.The second is O_H, an oracle that computes the value of a nonzero entry to p bits of precision given its indices. O_H is defined in App. C.2 where we show that the cost of O_H for the compact mapping is $O\big(K\log K + \mathrm{p}^2\log \mathrm{p}\big)$.

Using Theorem 1 from [13], simulation of time evolution for time t under a Hamiltonian on n qubits of sparsity d and maximum matrix element $||H||_{\max}$ to precision ϵ is given in terms of the parameter $\tau = d||H||_{\max}t$. The number of calls to O_H and O_F is

$$O\left(\tau\frac{\log\tau/\epsilon}{\log\log\tau/\epsilon}\right), \tag{18}$$

and an additional

$$O\left(\tau[n+\log^{5/2}(\tau/\epsilon)]\frac{\log\tau/\epsilon}{\log\log\tau/\epsilon}\right) \tag{19}$$

gates are required.

To simulate time evolution in a subspace of constant harmonic resolution K for time t in the compact mapping we have $n = O(\sqrt{K}\log K)$, $||H||_{\max} = O(K\log K/\Lambda)$, $d = O(K^2)$ and hence $\tau = O(tK^3\log K/\Lambda)$. The number of oracle calls required is then $\widetilde{O}(tK^3)$, and the number of gates required for this number of calls is $\widetilde{O}(tK^4)$ if p is polylogarithmic in K. The number of additional gates required is $\widetilde{O}(tK^{7/2})$ and so the overall simulation cost up to logarithmic factors is $\widetilde{O}(tK^4)$.

Something more about errors . . .

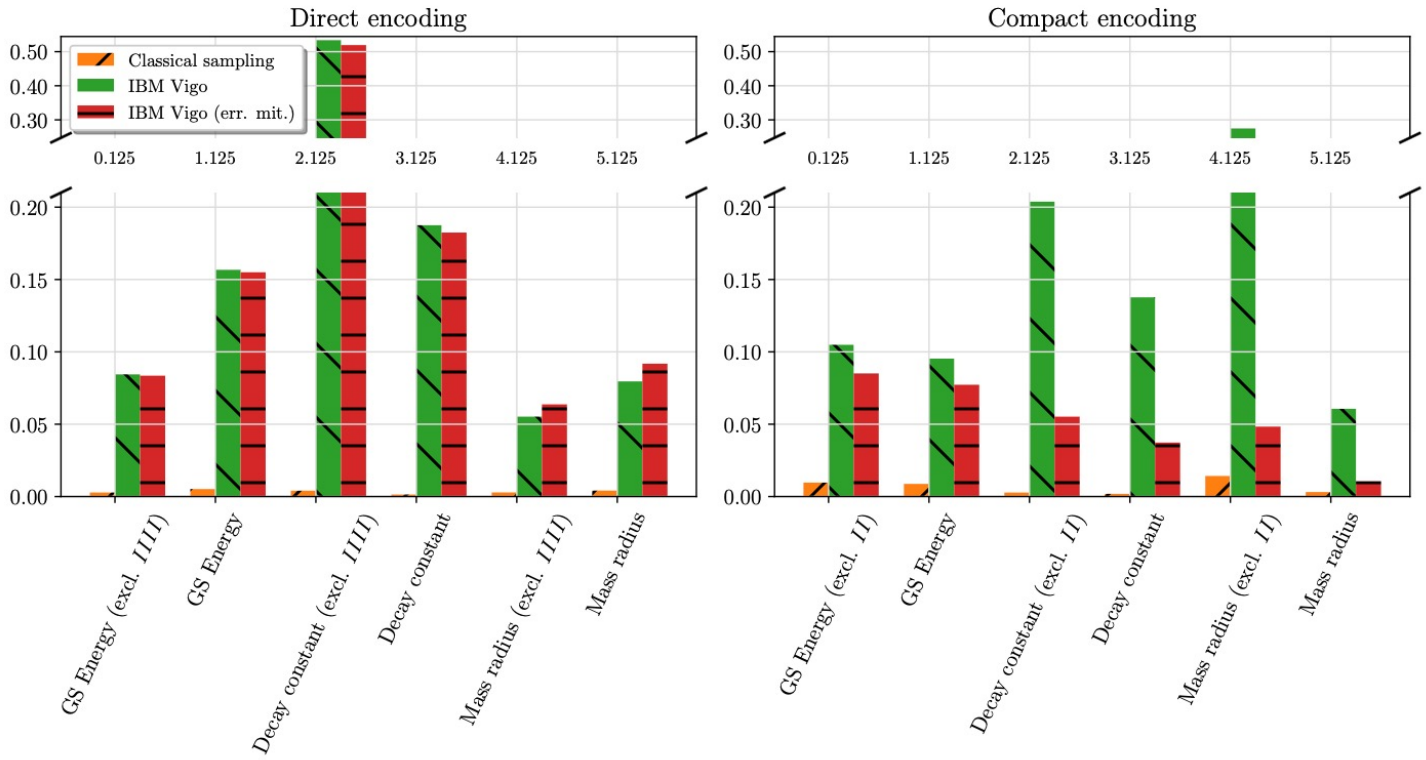


Figure 7. Relative errors in estimates of various observables. These were obtained from 8192 samples per term on IBM Vigo machine, with and without measurement error mitigation. Physically significant observables have a significant contribution from the constant term in their multi-qubit representation. Observables are shown with and without the contribution of the constant term. For the GS energy, the error was calculated relative to the second lowest eigenvalue, m_ρ^2. For the compact encoding, measurement error mitigation consistently improves the results.

Gates & errors (K. Robbins)

Noisy Intermediate Scale Quantum (NISQ)

A **NISQ** device makes mistakes (noisy) but does not have the overhead to correct those mistakes (intermediate-scale)