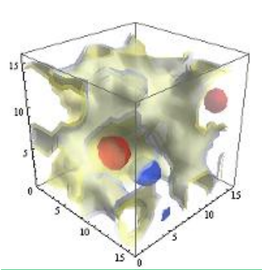
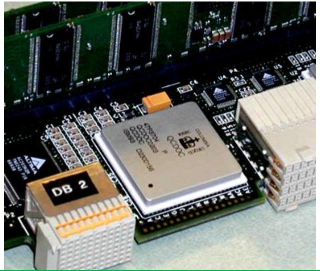
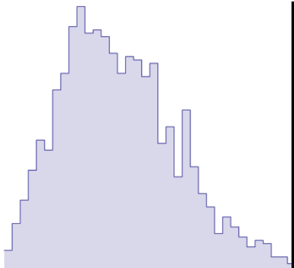
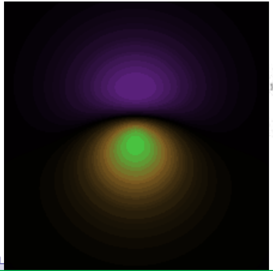
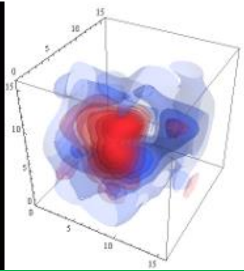
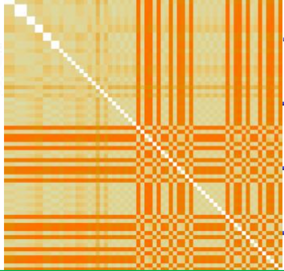
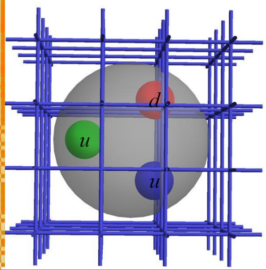
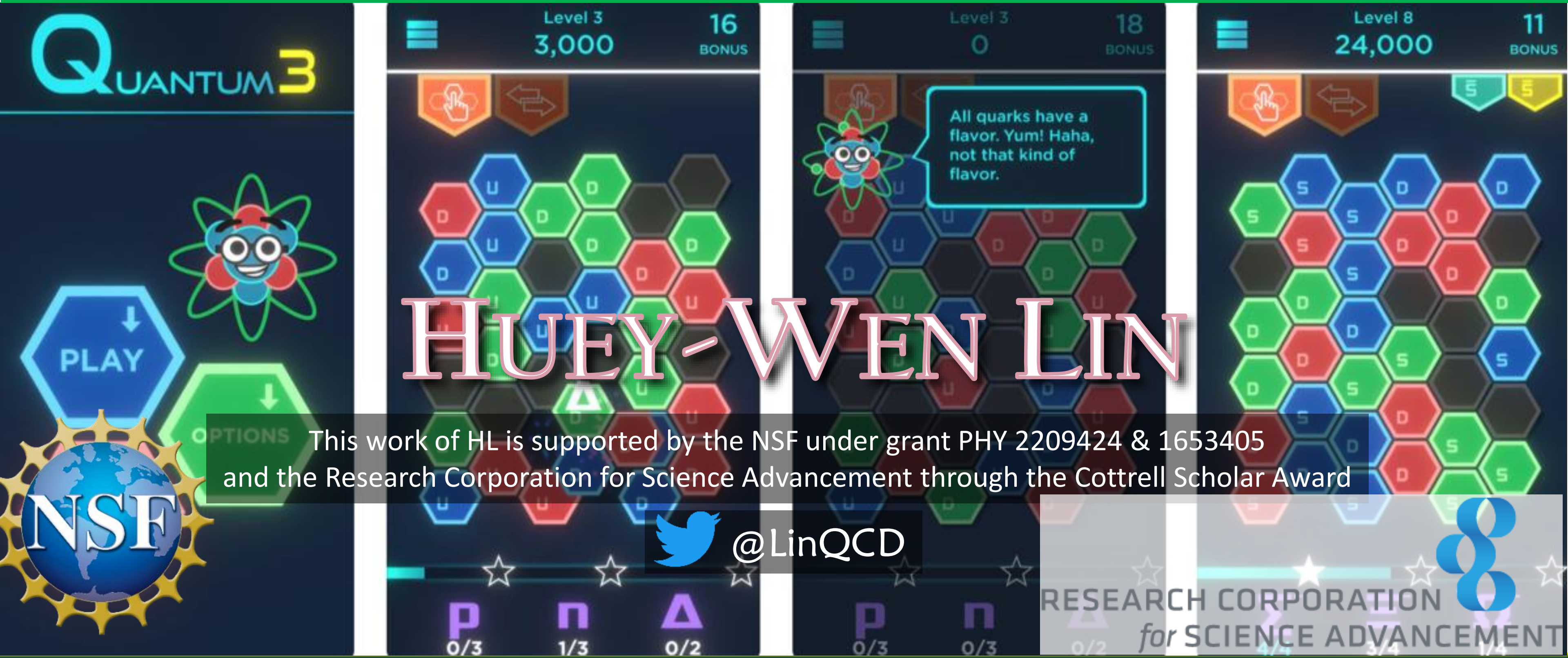








Mapping Nucleon and Meson Parton Distributions with Lattice QCD



Huey-Wen Lin

This work of HL is supported by the NSF under grant PHY 2209424 & 1653405
and the Research Corporation for Science Advancement through the Cottrell Scholar Award

@LinQCD

Outline

§ Recent Lattice *x*-dependent PDFs Progress

- Gluon PDFs for Nucleon, Pion and Kaons
- Strange PDFs and Impacts on Global Fits

* Isovector nucleon and pion PDFs likely being covered by other participants in this workshop

§ Generalized Parton Distributions at **Physical Pion Mass**

- Isovector nucleon unpolarized and polarized GPDs
- Preliminary results on Pion GPDs

Biased selected/highlighted results

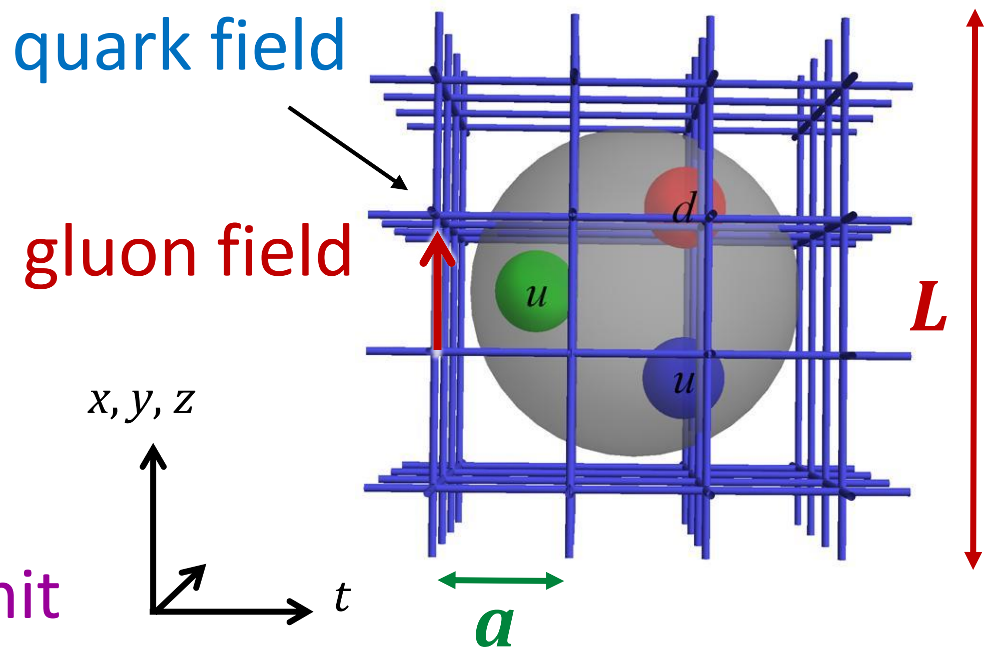
Lattice QCD in a Nutshell

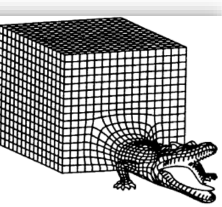
- Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field theories
- Physical observables are calculated from the path integral

$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{iS(\bar{\psi},\psi,A)}\, O(\bar{\psi},\psi,A)$$

in **Euclidean** space

- Quark mass parameter (described by m_π)
- Impose a UV cutoff
  discretize spacetime
- Impose an infrared cutoff
  finite volume





- Recover physical limit

$$m_\pi \to m_\pi^{\text{phys}},\ a \to 0,\ L \to \infty$$

Direct x-Dependent Structure

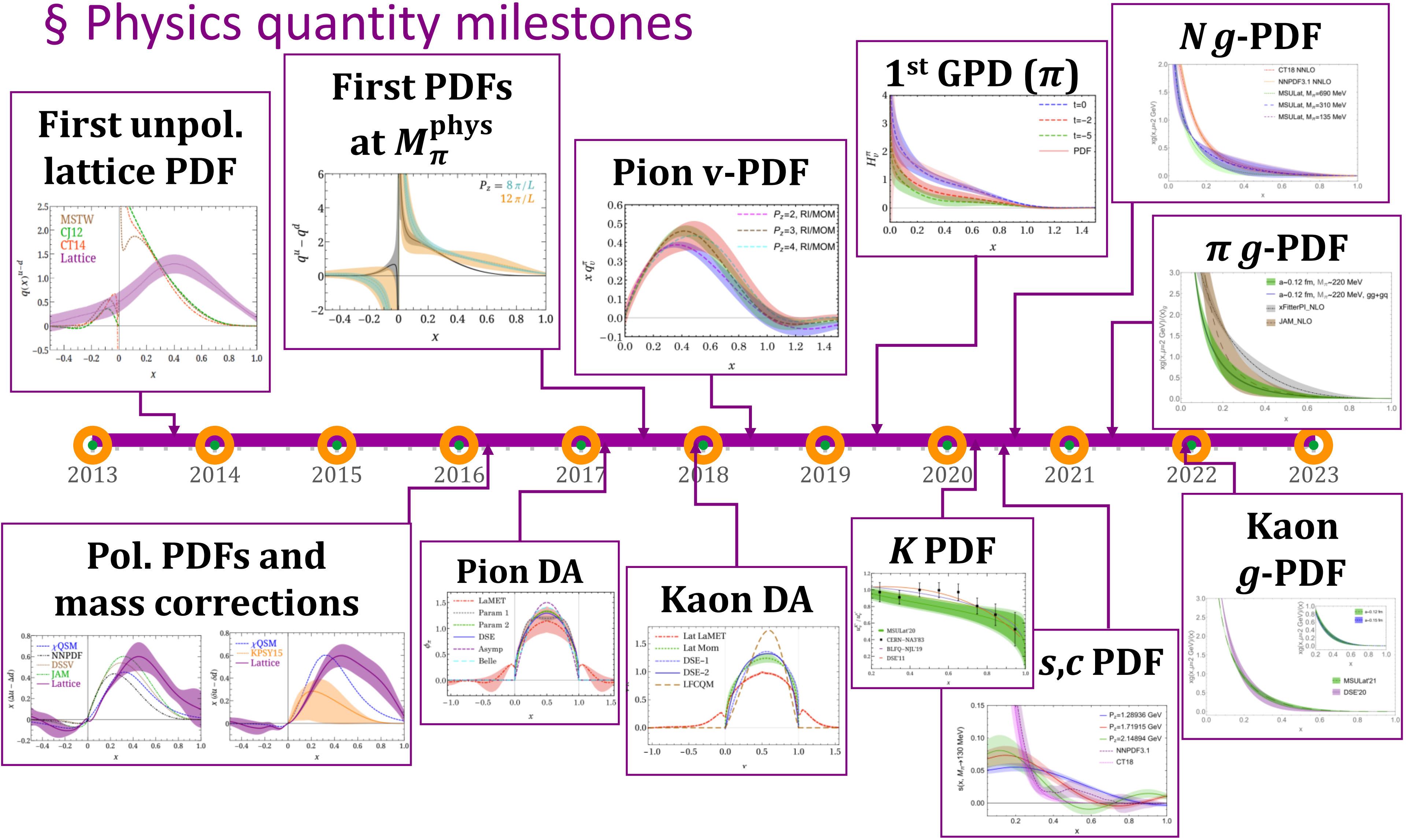
§ Longstanding obstacle to lattice calculations!

Quantities that can be calculated on the lattice today = $\sum$ Wanted PDFs, GPDs, etc. × pQCD-calculated kernel

- **Quasi-PDF**/large-momentum effective theory (LaMET) (X. Ji, 2013; See 2004.03543 for review)
- **Pseudo-PDF** method: differs in FT (A. Radyushkin, 2017)
- Lattice cross-section method (**LCS**) (Y Ma and J. Qiu, 2014, 2017)
- Hadronic tensor currents (Liu et al., hep-ph/9806491, … 1603.07352)
- Euclidean correlation functions (RQCD, 1709.04325)
- …

Lattice Parton Calculations

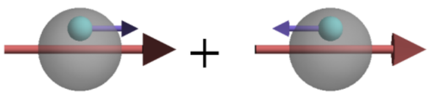
§ Physics quantity milestones

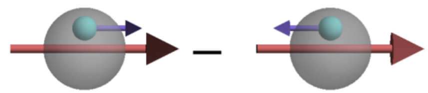


Lattice Example Results

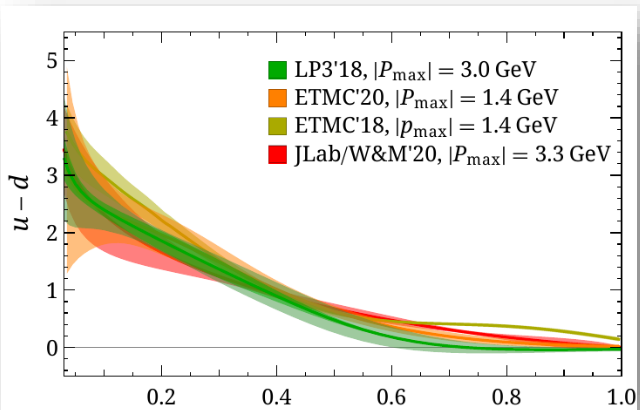
§ Summary of physical pion mass PDFs results

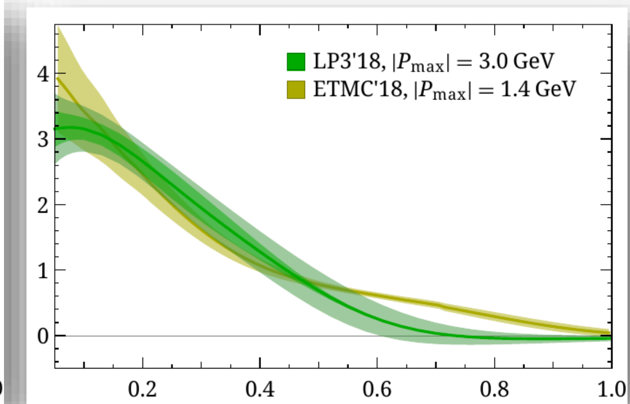
unpolarized | longitudinally polarized | transversely polarized

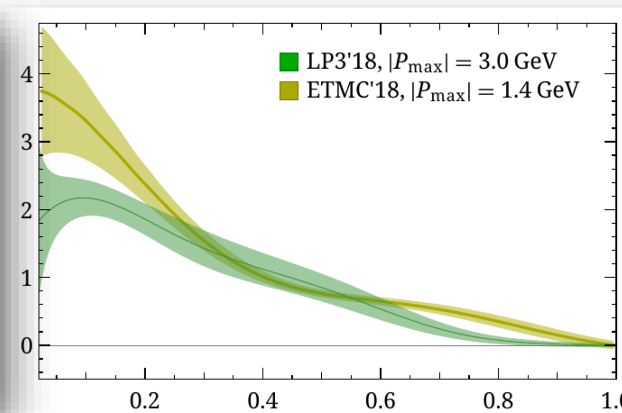


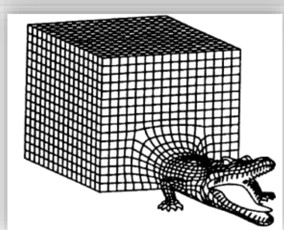


$u(x) - d(x)$ | $\Delta u(x) - \Delta d(x)$ | $\delta u(x) - \delta d(x)$









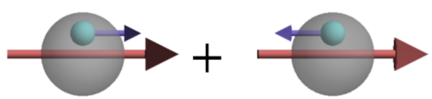
Finite volume,
Discretization,
…

2006.08636 (PDFLattice2019)

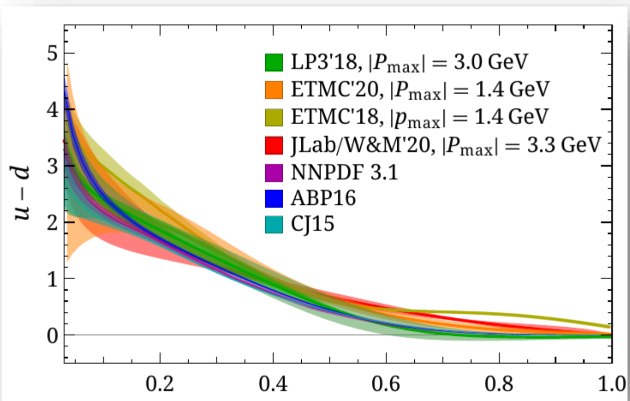
Lattice Example Results

§ Summary of physical pion mass PDFs results

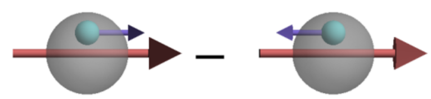
unpolarized



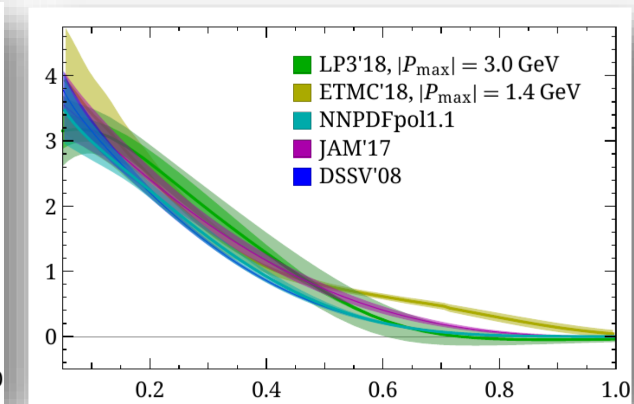
$u(x) - d(x)$



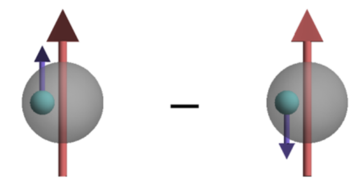
longitudinally polarized



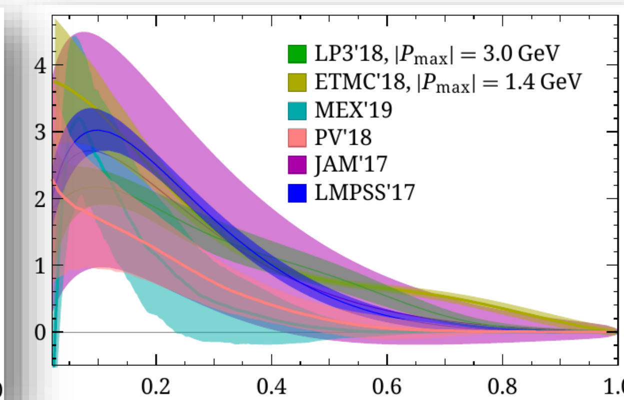
$\Delta u(x) - \Delta d(x)$

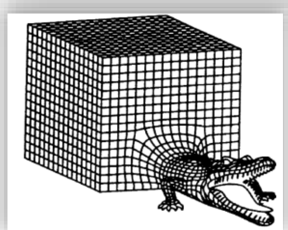


transversely polarized



$\delta u(x) - \delta d(x)$





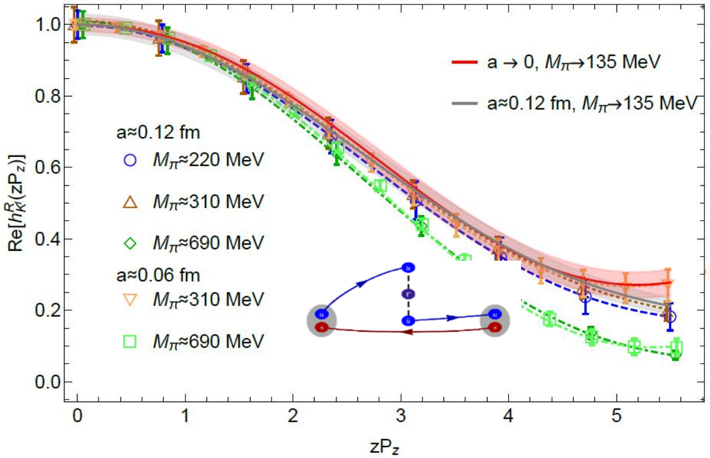
Finite volume,
Discretization,
…

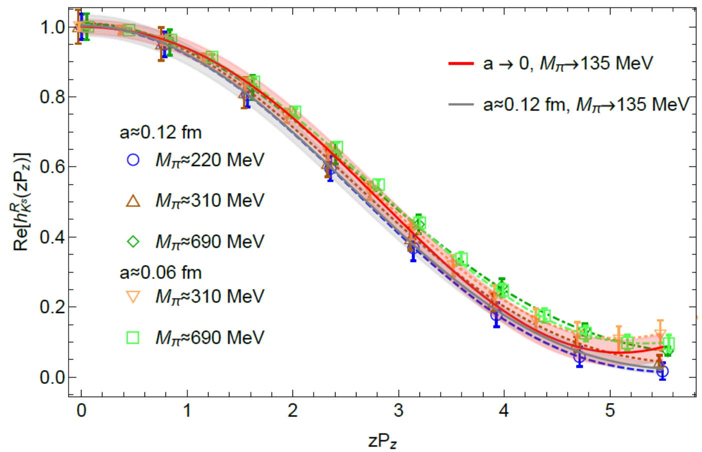
2006.08636 (PDFLattice2019)

See backup slides for calculations to remove these systematics

Meson Valence-quark PDFs

§ Pion/Kaon PDFs using quasi-PDF in the continuum limit

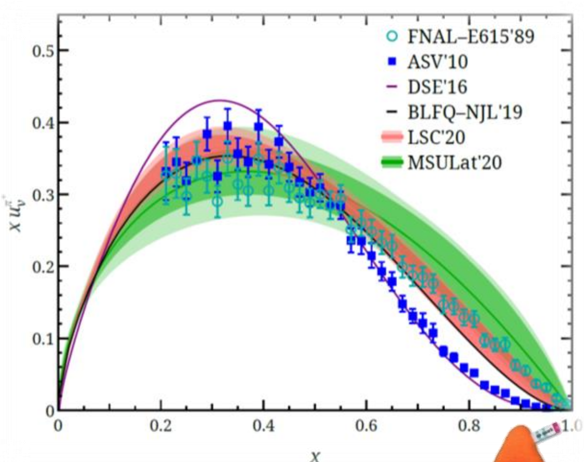


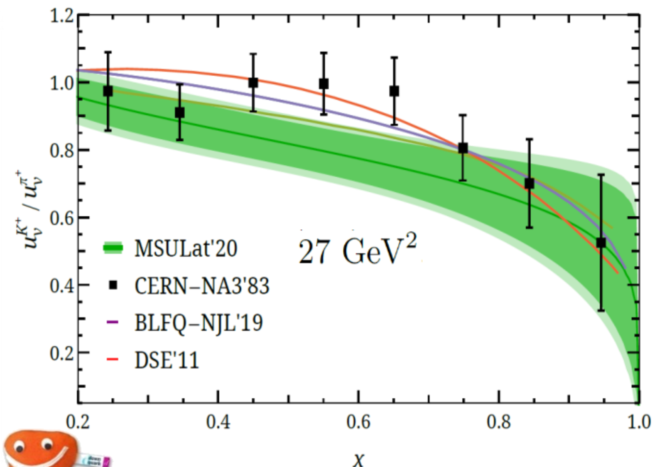


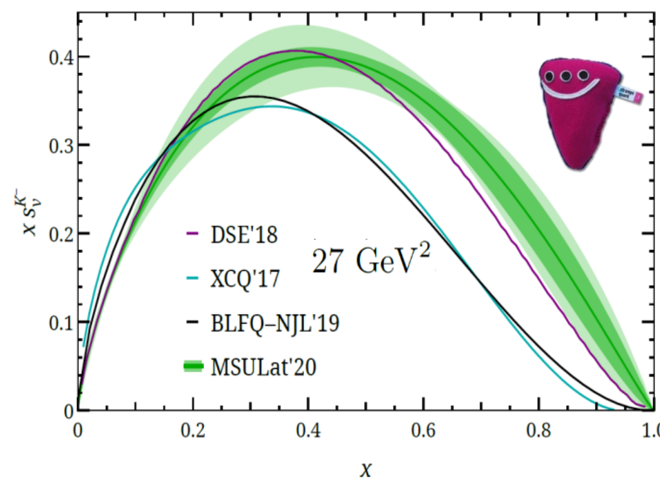
Quantities that can be calculated on the lattice

Wanted PDFs, GPDs, etc…

MSULat, 2003.14128







Gluon, Strange and Charm PDFs

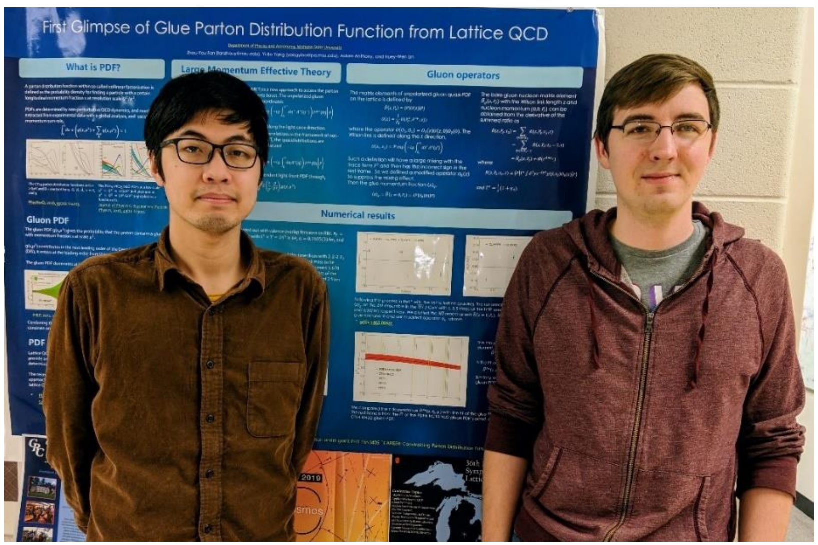
Biased selected/highlighted results

Nucleon Gluon PDF (2018)

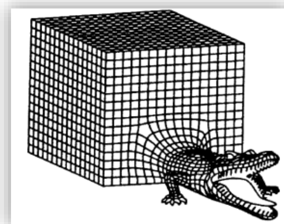
§ Pioneering first glimpse into gluon PDF using LaMET

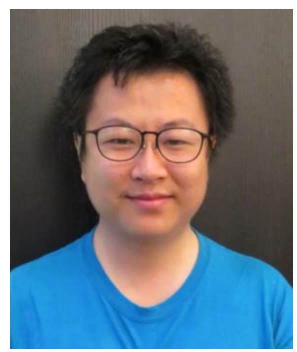
- Lattice details: overlap/2+1DWF, 0.16fm, 340-MeV sea pion mass
- Promising results using coordinate-space comparison, but signal does not go far in z
- Hard numerical problem to be solved

Fan et al, Phys.Rev.Lett. 121, 242001 (2018)

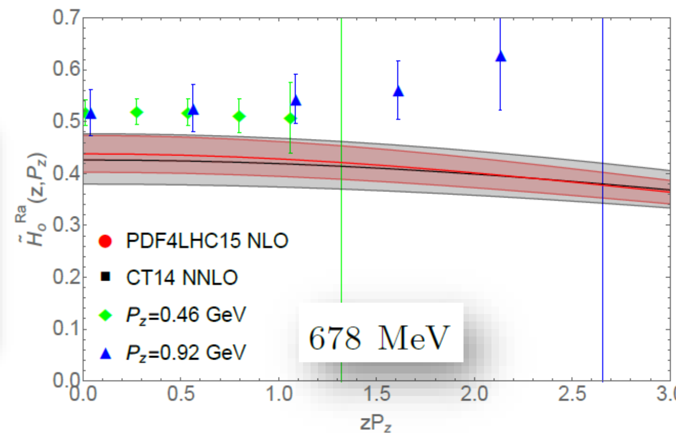


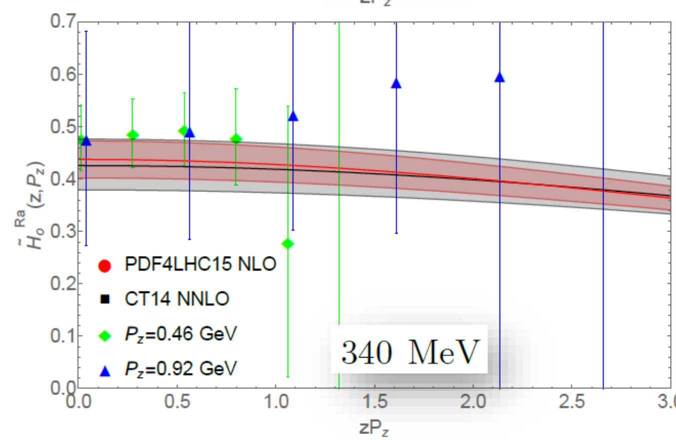
G: Zhouyou Fan G: Adam Antony





P: Yi-Bo Yang





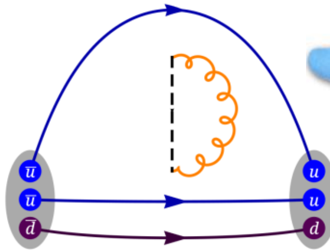
iCER@MSU is crucial for earlier code development and completion of this work

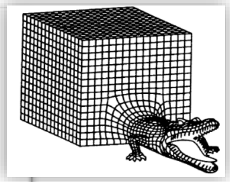
Nucleon Gluon PDF (2020)

§ Gluon PDF using pseudo-PDF

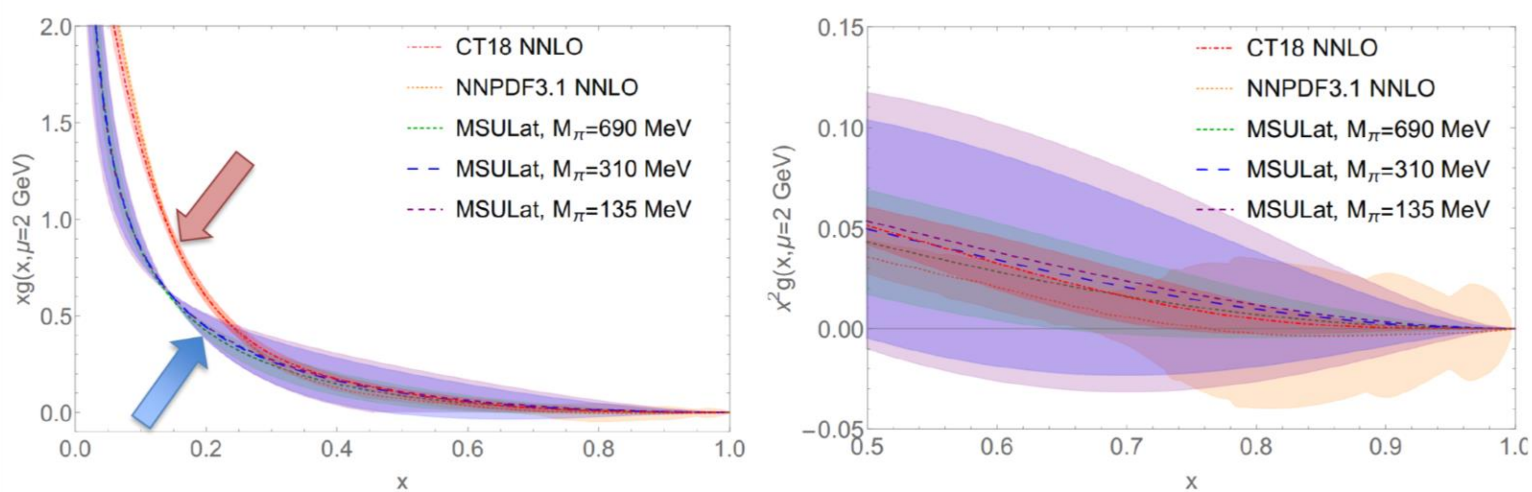
- Lattice details: clover/2+1+1 HISQ 0.12 fm, 310-MeV sea pion
- Study strange/light-quark

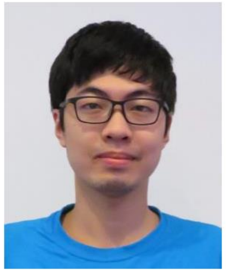
Z. Fan. et al (MSULat), 2007.16113





The comparison of the reconstructed unpolarized gluon PDF from the function form with CT18 NNLO and NNPDF3.1 NNLO gluon unpolarized PDF at $\mu = 2\ GeV$ in the $\overline{\rm MS}$ scheme.





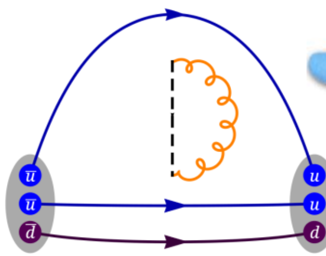
G: Zhouyou Fan

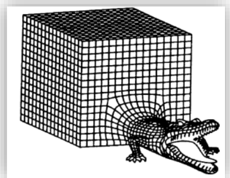
Slide by Zhouyou Fan@DNP2020

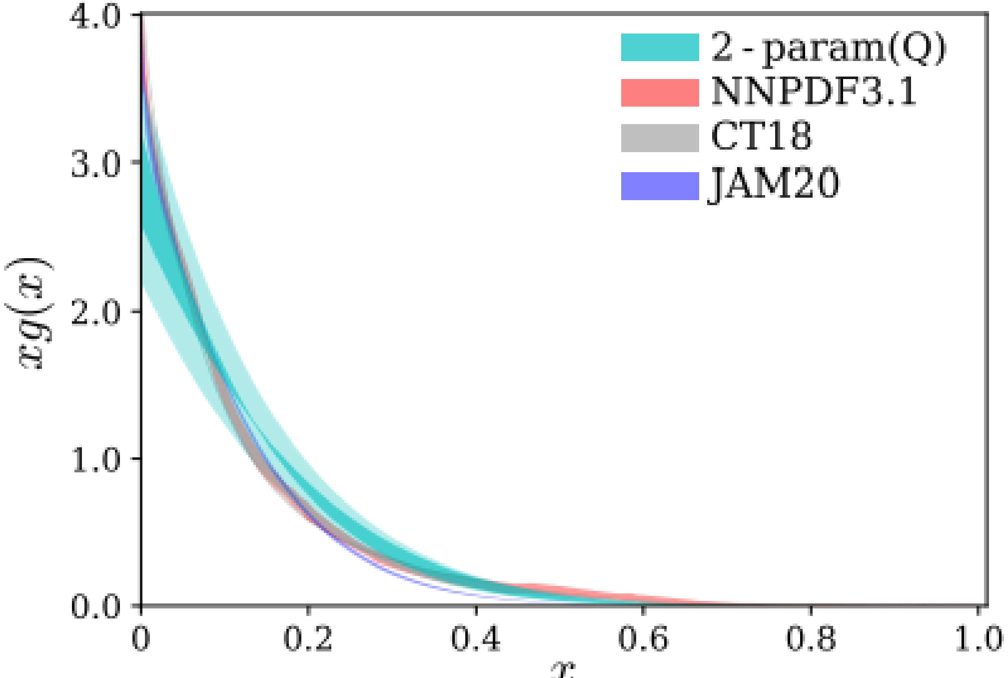
Nucleon Gluon PDF (2021)

§ Gluon PDF using pseudo-PDF

- Lattice details: 2+1 clover, 0.09 fm, 360-MeV sea pion T. Khan et al. (HadStruc), 2107.08960
- Use many nucleon Interpolating operators to improve signal with larger boosted momentum state



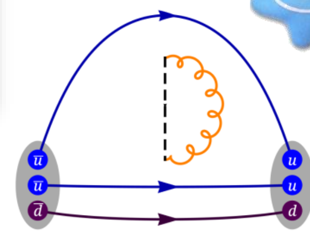




T. Khan et al. (HadStruc), 2107.08960

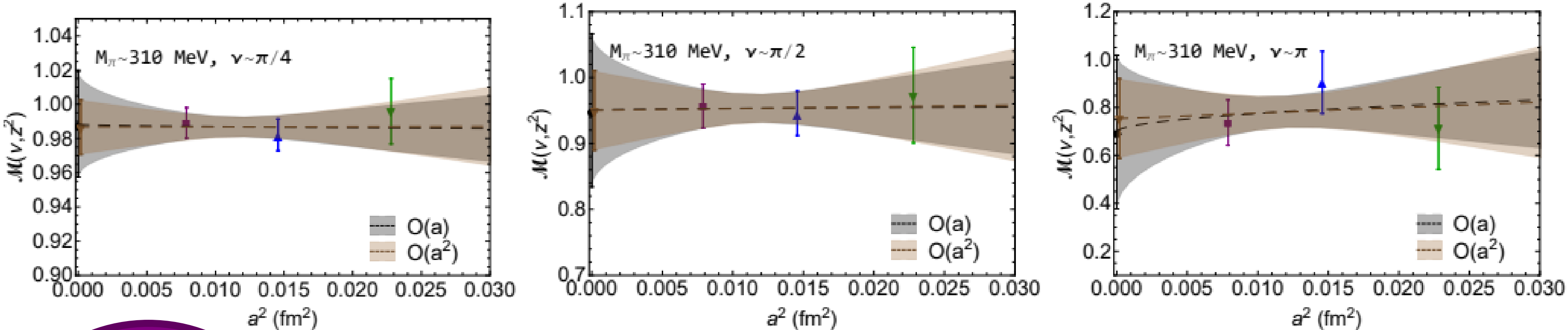
Nucleon Gluon PDF (2022)

§ Continuum Gluon PDF w/ pseudo-PDF



- 2+1+1 HISQ {0.09, 0.12, 0.15} fm, [220,310,700]-MeV pion, 10^5-10^6 statistics

Z. Fan, W. Good, HL (MSULat), 2210.09985



Quantities that can be calculated on the lattice

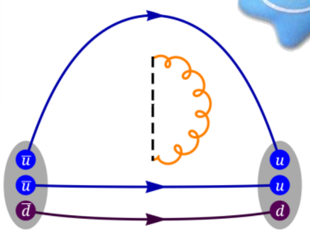
G: Bill Good

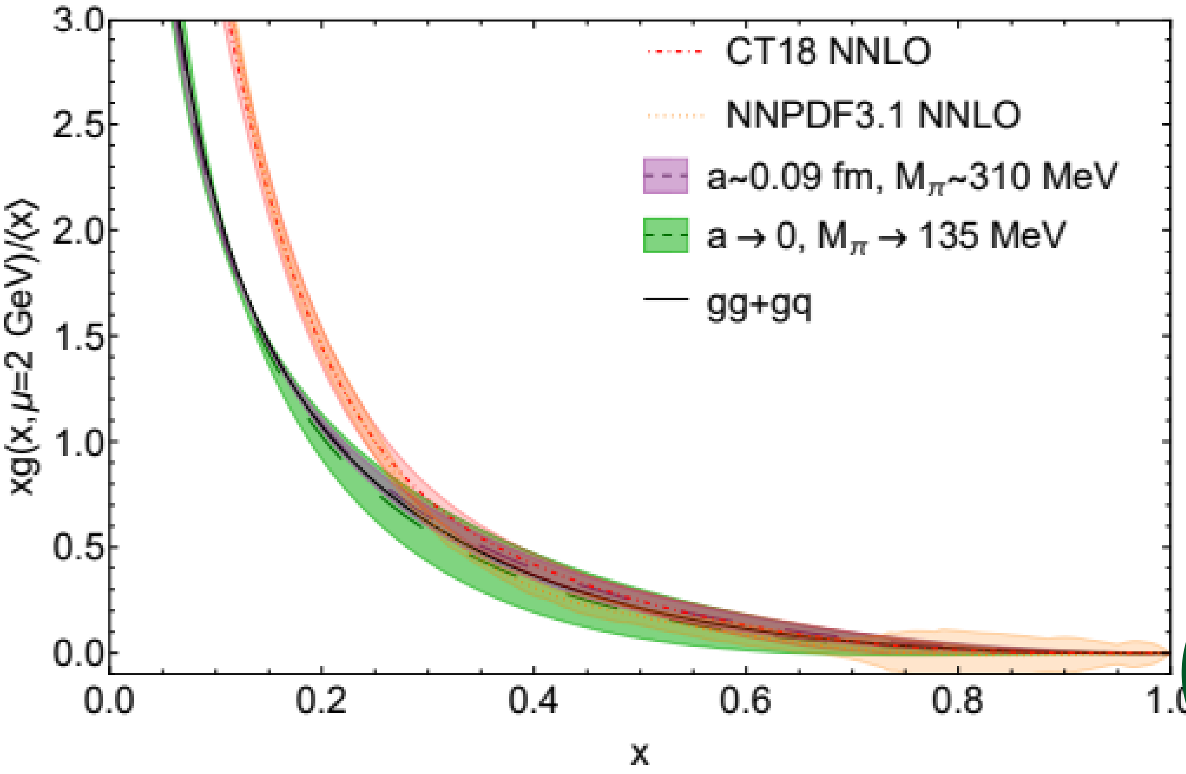
Nucleon Gluon PDF (2022)

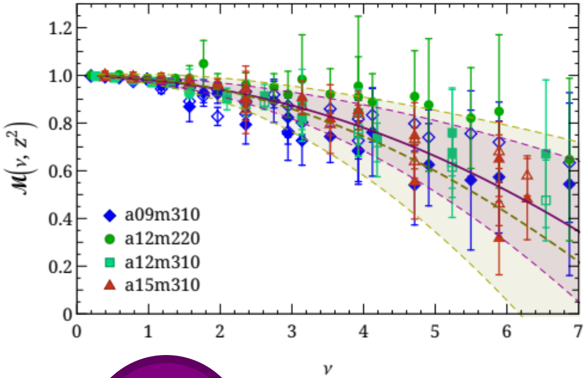
§ Continuum Gluon PDF w/ pseudo-PDF

- 2+1+1 HISQ {0.09, 0.12, 0.15} fm, [220,310,700]-MeV pion, 10^5-10^6 statistics

Z. Fan, W. Good, HL (MSULat), 2210.09985







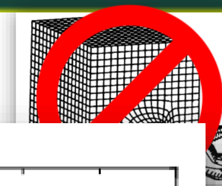
Quantities that can be calculated on the lattice

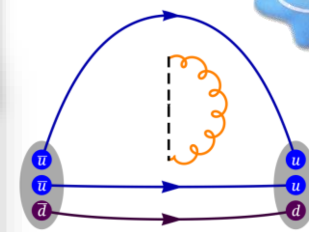
Wanted PDFs, GPDs, etc...

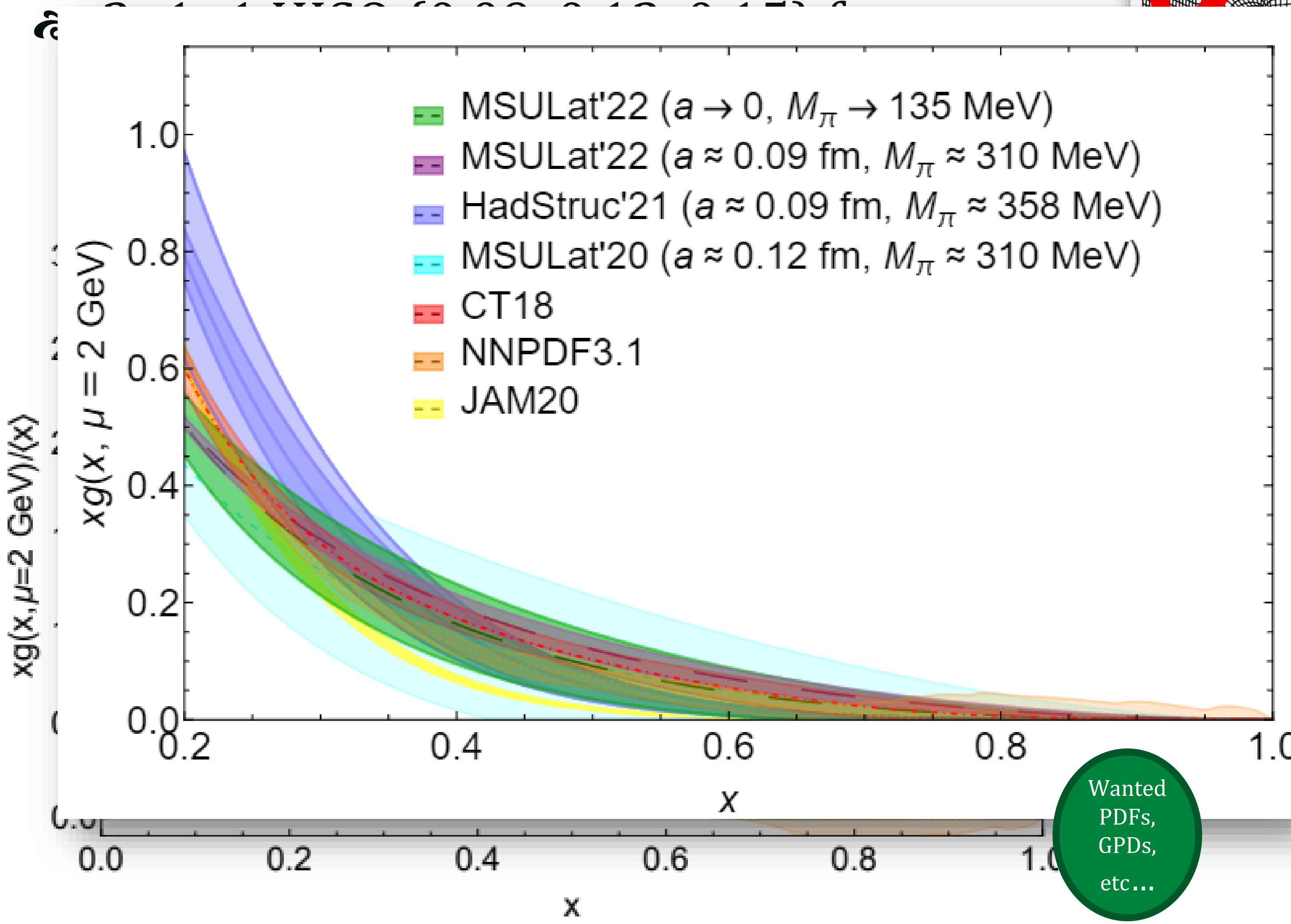
G: Bill Good

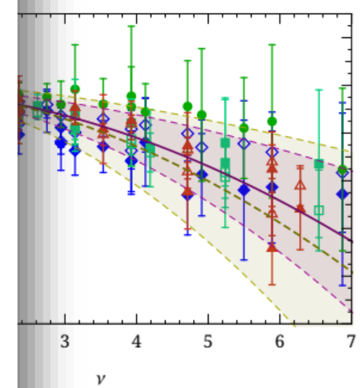
Nucleon Gluon PDF (2022)

§ Continuum Gluon PDF w/ pseudo-PDF







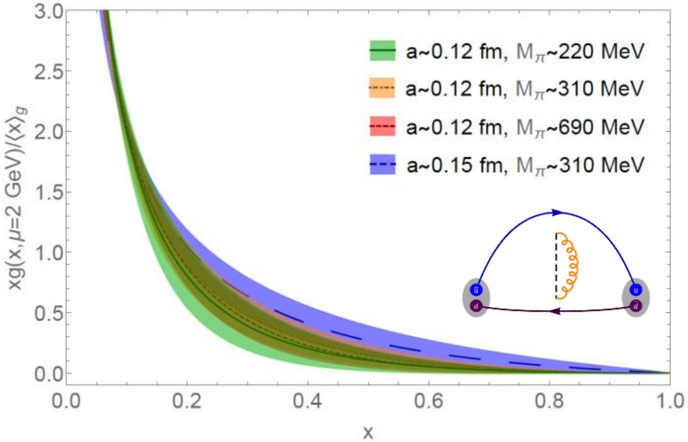


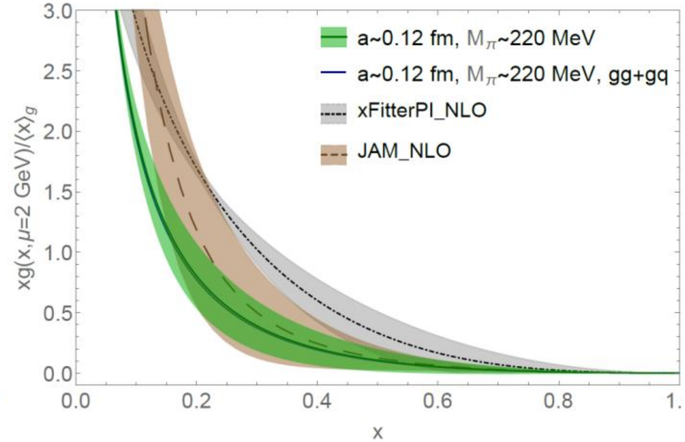
Wanted PDFs, GPDs, etc…

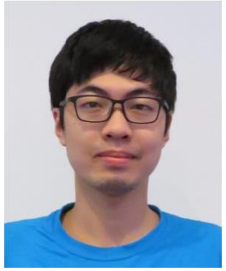
G: Bill Good

Meson Gluon PDFs

§ First pion and kaon gluon PDFs using pseudo-PDF



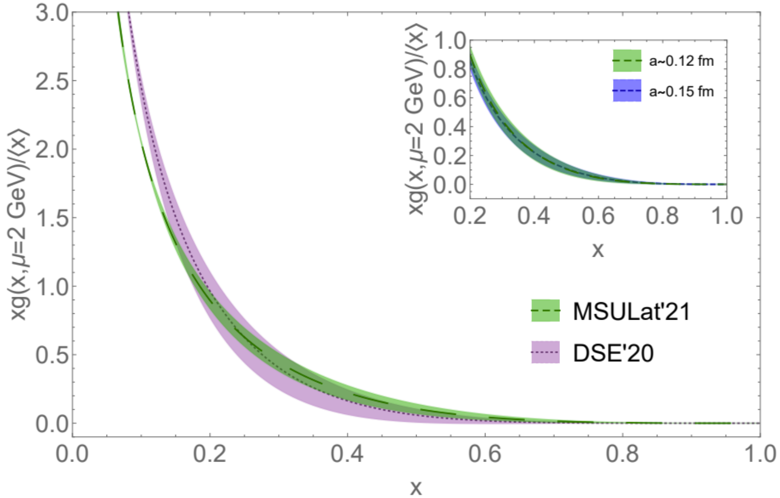




G: Zhouyou Fan

2104.06372, Fan et al (MSULat)

Wanted PDFs, GPDs, etc…



G: Alejandro Salas-Chavira

2112.03124 , Salas-Chavira et al (MSULat)

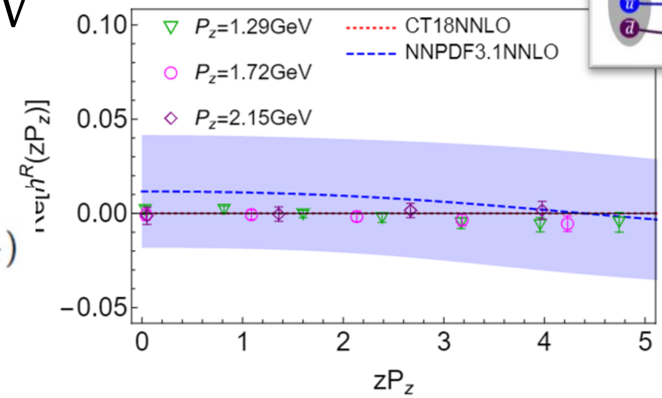
First Lattice Strange PDF

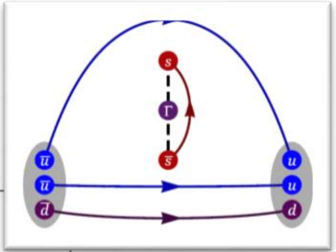
§ Results by MSULat/quasi-PDF method

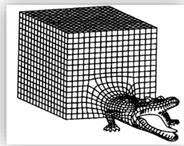
- Clover on 2+1+1 HISQ, 0.12-fm 310-MeV QCD vacuum
- Extrapolated to $M_\pi \approx 140$ MeV

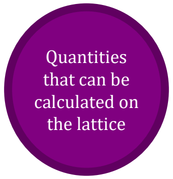
2005.01124, R. Zhang et al (MSULat)

$$\mathrm{Re}[h(z)] \propto \int dx\,(s(x) - \bar{s}(x))\cos(xzP_z)$$









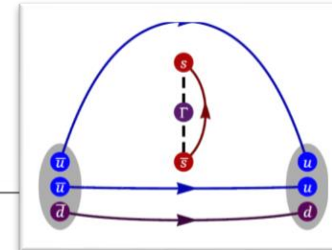
First Lattice Strange PDF

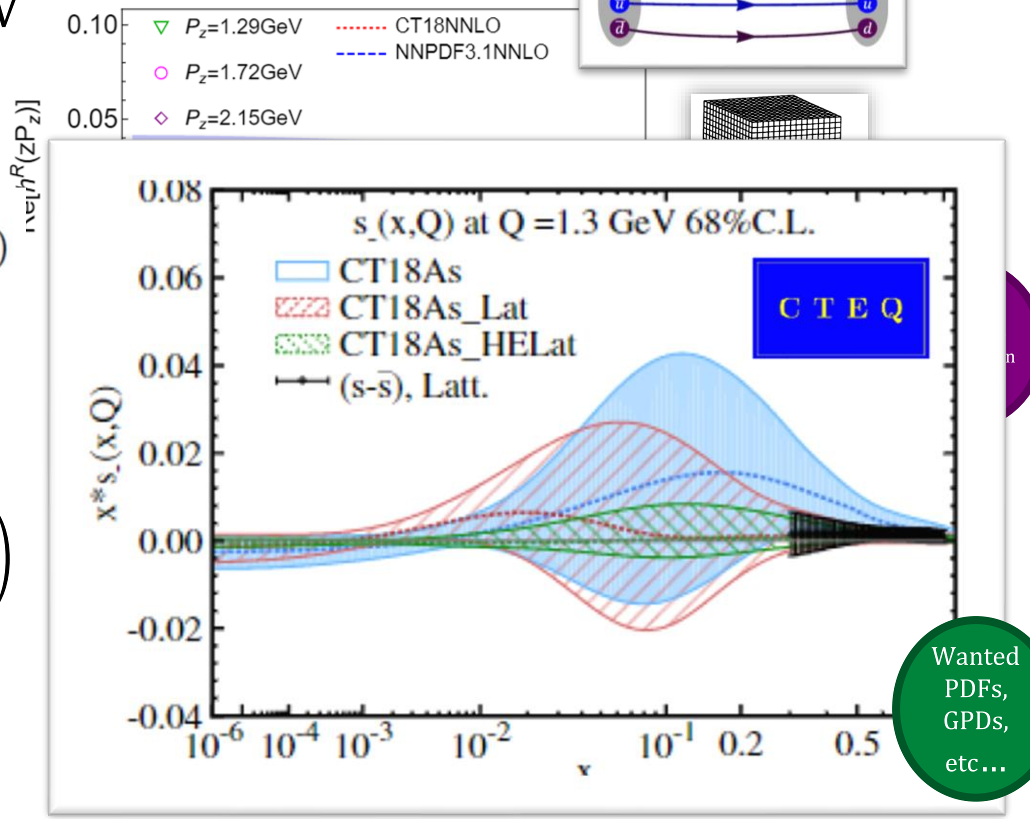
§ Results by MSULat/quasi-PDF method

- Clover on 2+1+1 HISQ, 0.12-fm 310-MeV QCD vacuum
- Extrapolated to $M_\pi \approx 140$ MeV

2005.01124, R. Zhang et al (MSULat)

$$\mathrm{Re}[h(z)] \propto \int dx\,(s(x) - \bar{s}(x)) \cos(xzP_z)$$





Wanted PDFs, GPDs, etc…

§ From quasi-PDF to PDF

$$\tilde{f}_q(x, P_z) = \int_{-1}^{1} \frac{dy}{|y|} f_q(y) C_{q/q}(x, y, P_z, \mu) + O\left(\frac{\Lambda_{\mathrm{QCD}}^2}{x^2 P_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(1-x)^2 P_z^2}\right)$$

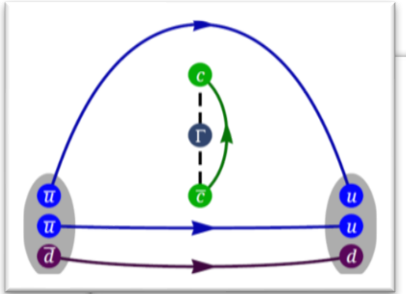
T. Hou, HL, M. Yan, C. Yuan, 2204.07944

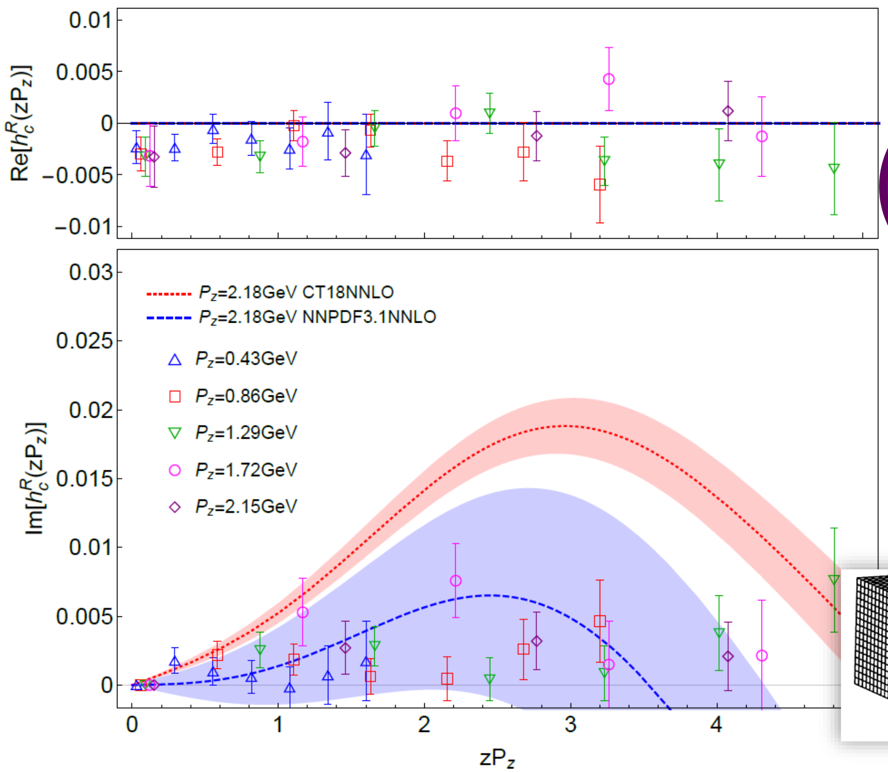
§ The strangeness asymmetry $s(x,Q) - \bar{s}(x,Q)$ at $x > 0.2$ is difficult to measure, but can be predicted in lattice QCD

First Lattice Charm PDF

- Large uncertainties in global PDFs
- Results by MSULat/quasi-PDF method
  - Clover on 2+1+1 HISQ 0.12-fm 310-MeV QCD vacuum

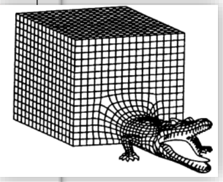
2005.01124, R. Zhang et al (MSULat)





- suggest a symmetric $c - \bar{c}$ distribution
- much smaller than strange PDF

Quantities that can be calculated on the lattice



First Lattice Charm PDF

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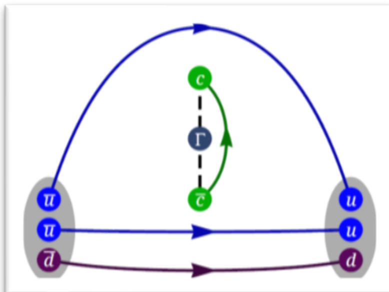

- suggest a symmetric $c - \bar{c}$ distribution
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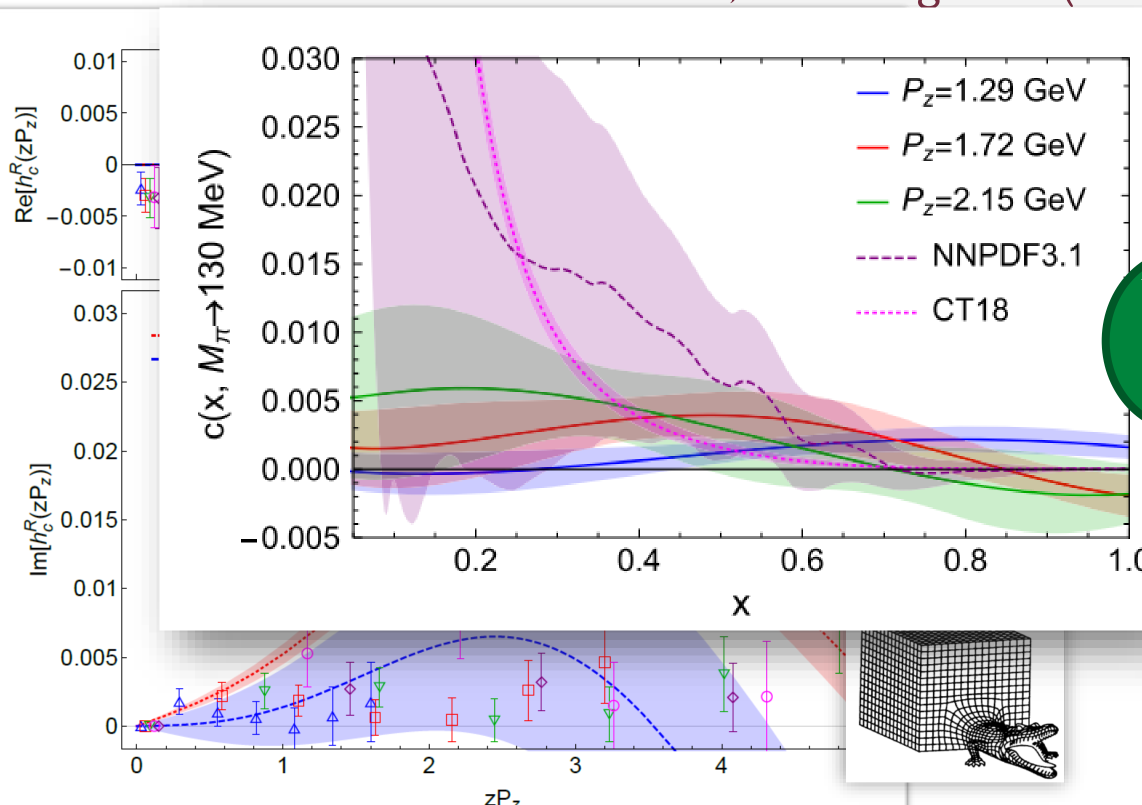
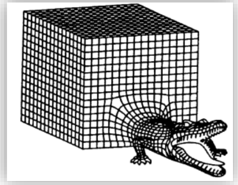


Wanted PDFs, GPDs, etc…

Generalized Parton Distributions

Single-ensemble result

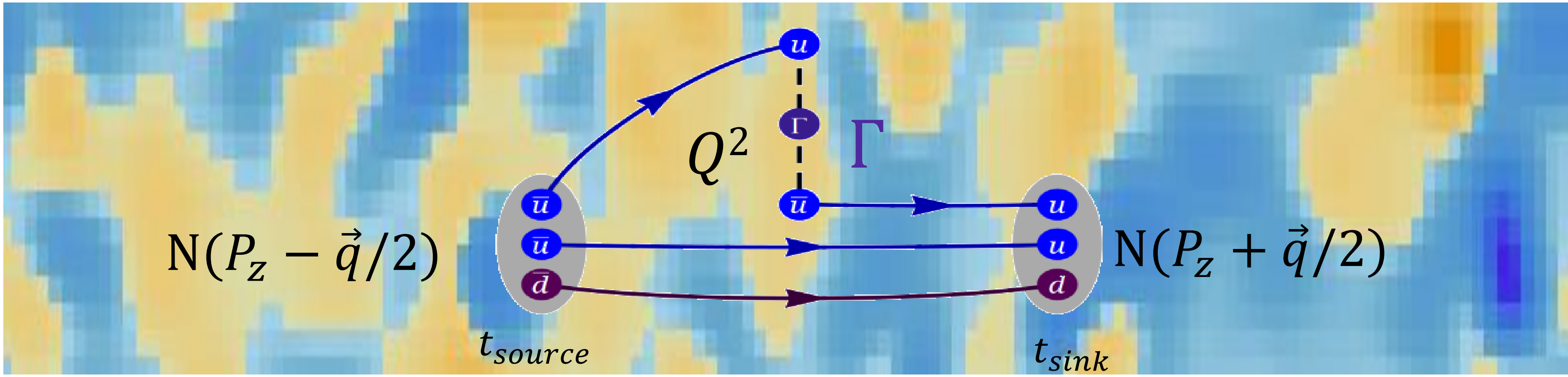


finite-volume,
discretization,
heavy quark mass,
...

Biased selected/highlighted results

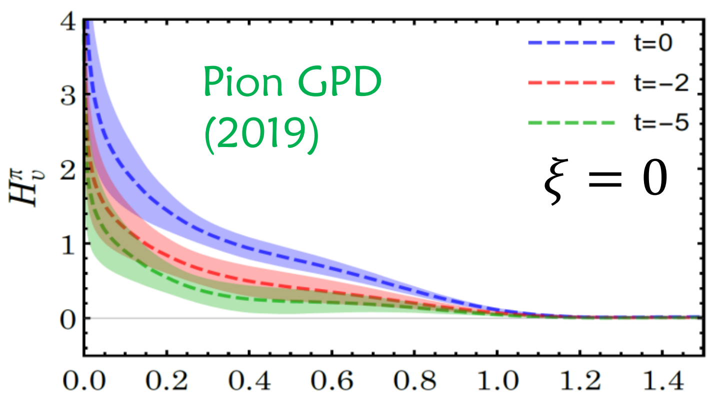
Generalized Parton Distributions

§ On the lattice, one needs to calculate the following

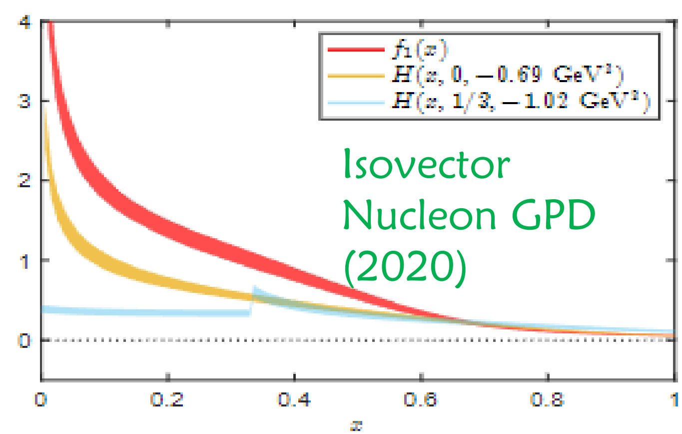


See earlier lattice talks for new setup for GPD calculations

§ Heavy pion-mass results



J. Chen, HL, J. Zhang, 1904.12376

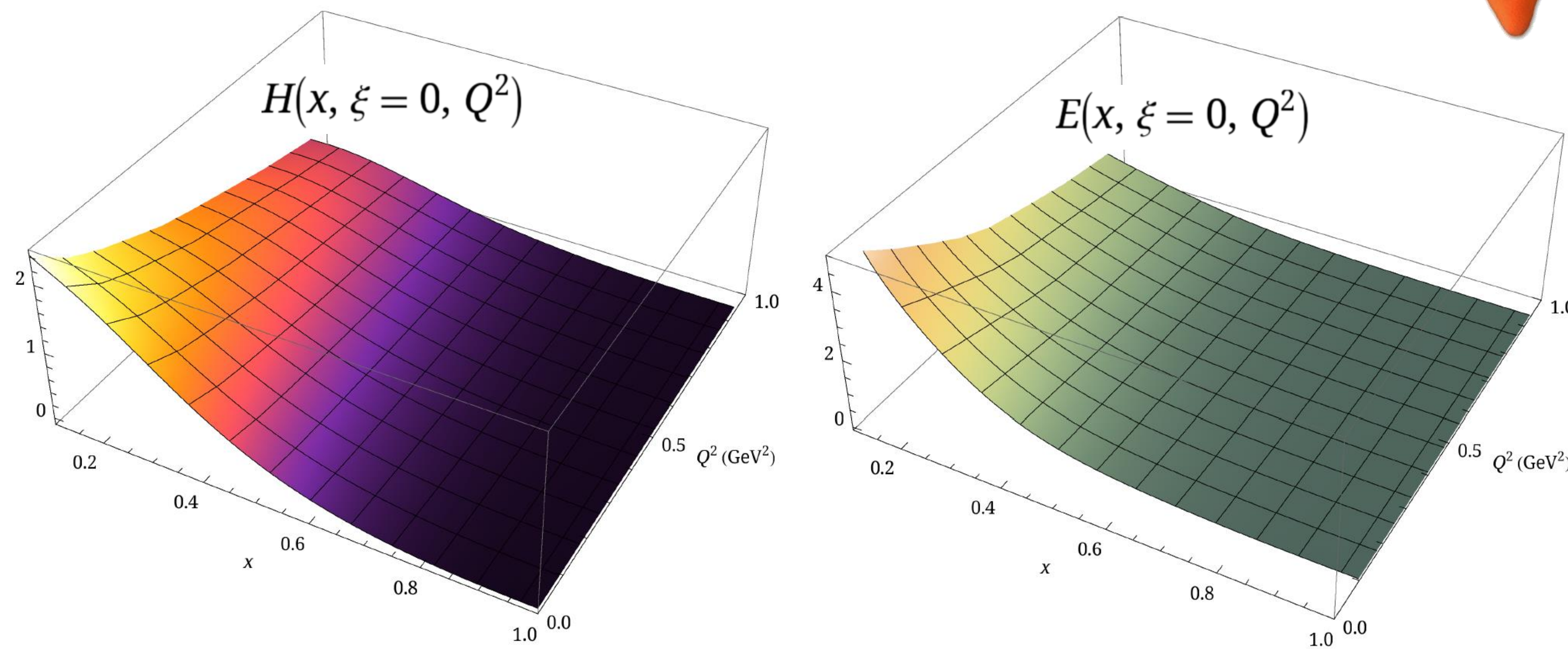


C. Alexandrou et al, 2008.10573

Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at **physical pion mass**

- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results

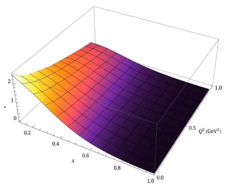


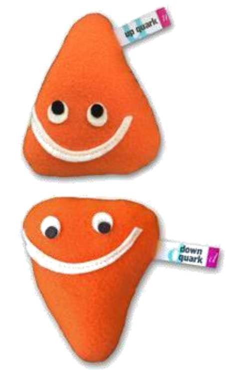
HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Isovector Nucleon GPDs

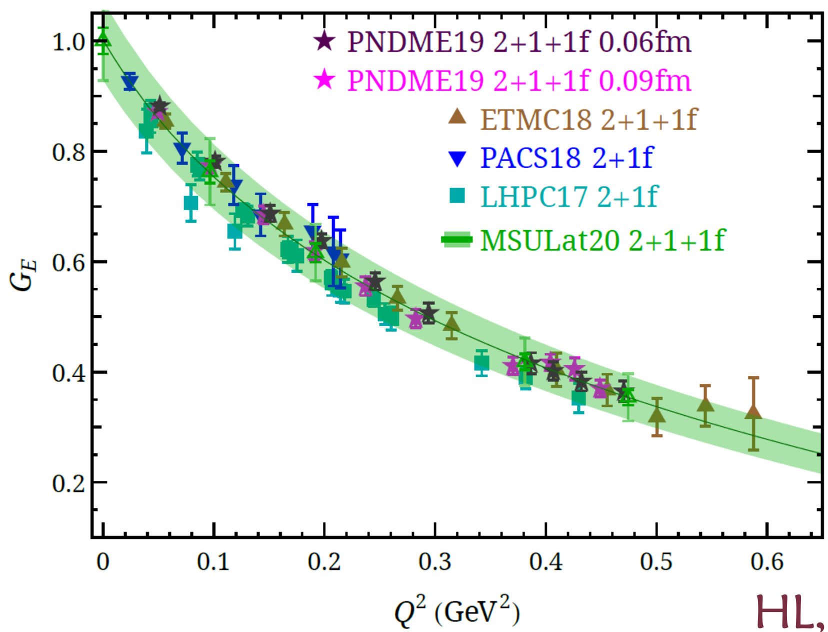
§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ (MSULat)
  0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results

$$\int_{-1}^{+1} dx\, x^{n-1}$$  $$= \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i A^q_{ni}(t) + (-2\xi)^n C^q_{n0}(t)\Big|_{n\ \text{even}}$$



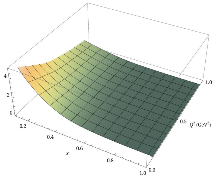
$n = 1$



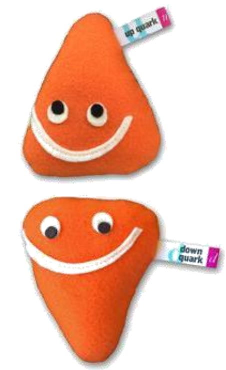
HL, Phys.Rev.Lett. 127 (2021) 18, 182001

§ Nucleon GPD using quasi-PDFs at physical pion mass

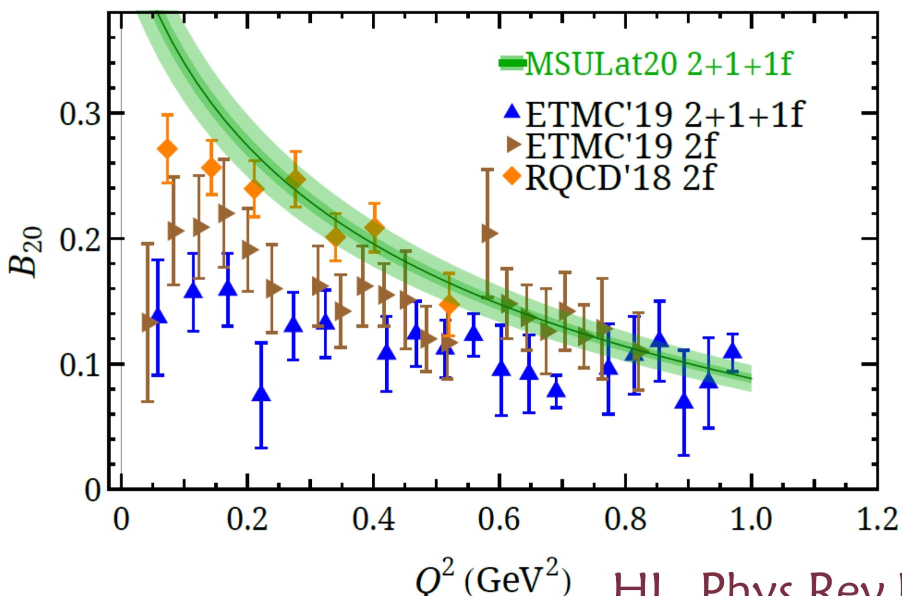
- Lattice details: clover/2+1+1 HISQ (MSULat)
  0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results

$$\int_{-1}^{+1} dx\, x^{n-1}$$


$$= \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i B_{ni}^q(t) - (-2\xi)^n C_{n0}^q(t) \Big|_{n\ \text{even}}$$



$n = 2$

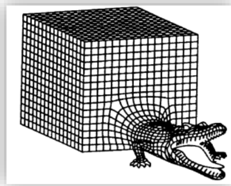


HL, Phys.Rev.Lett. 127 (2021) 18, 182001

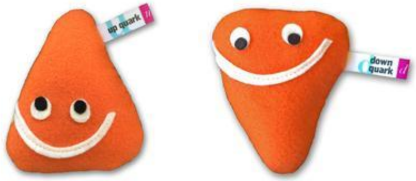
Nucleon Tomography

§ Nucleon GPD using quasi-PDFs at physical pion mass

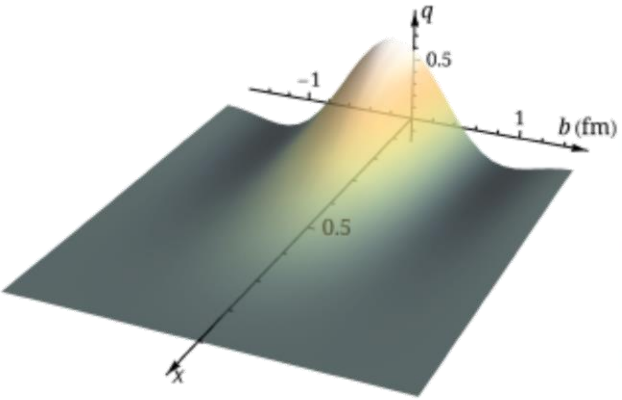
- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results

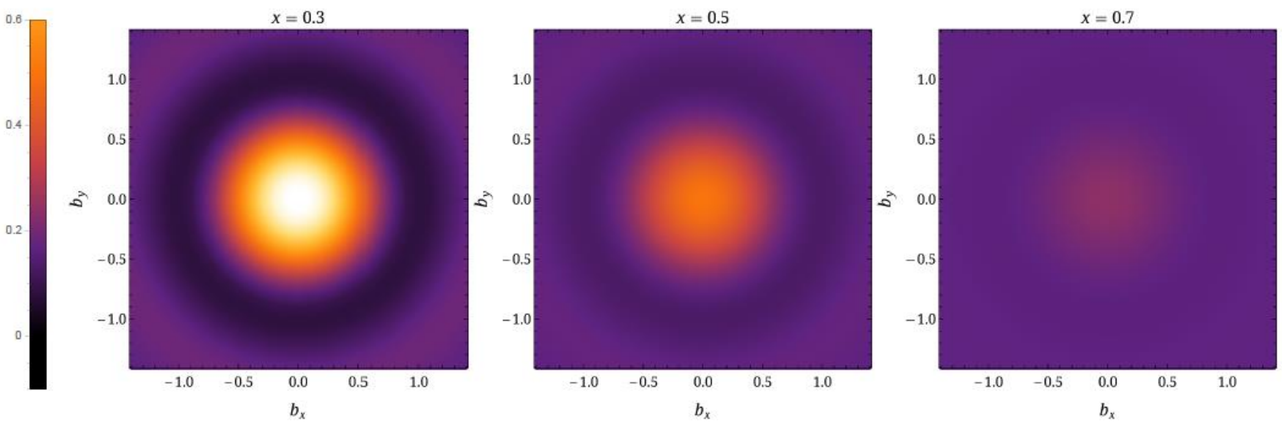


finite-volume,
discretization,



$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x, \xi = 0, t = -\vec{q}^2) e^{i\vec{q}\cdot\vec{b}}$$





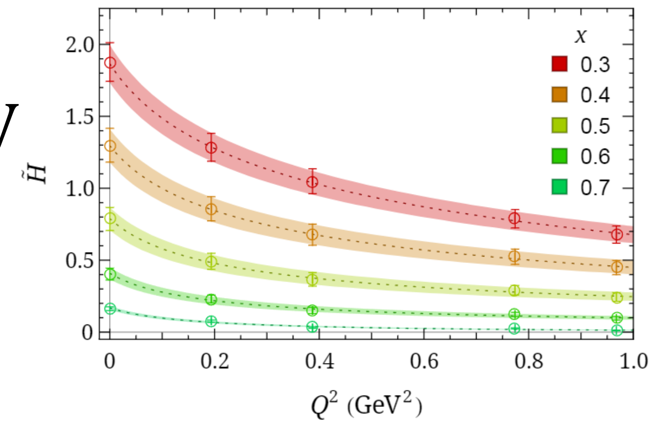
HL, Phys.Rev.Lett. 127 (2021) 18, 182001

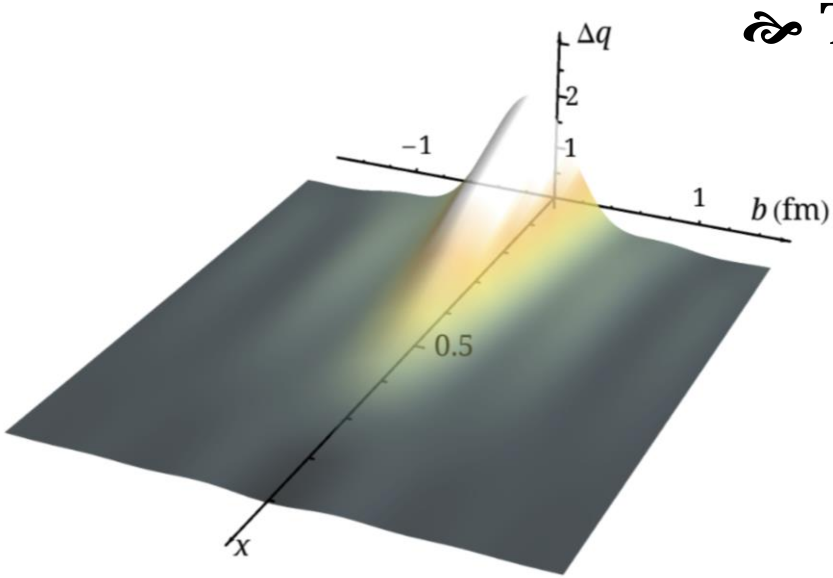
Nucleon Polarized GPDs

§ Helicity GPD ($\widetilde{H}$)using quasi-PDFs at **physical pion mass**

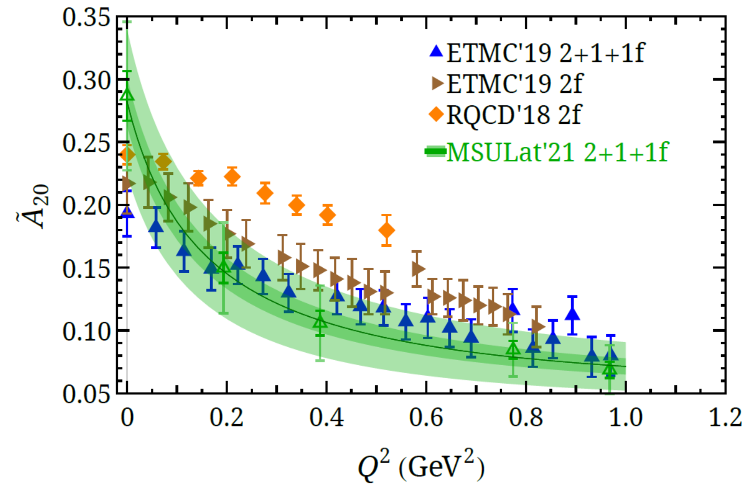
- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results

  HL (MSULat), Phys.Lett.B 824 (2022) 136821





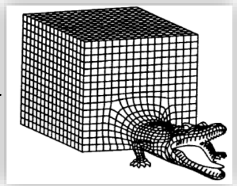
- Take the integral to form moments;
  e.g. $n = 2$



Valence-Quark Pion GPD

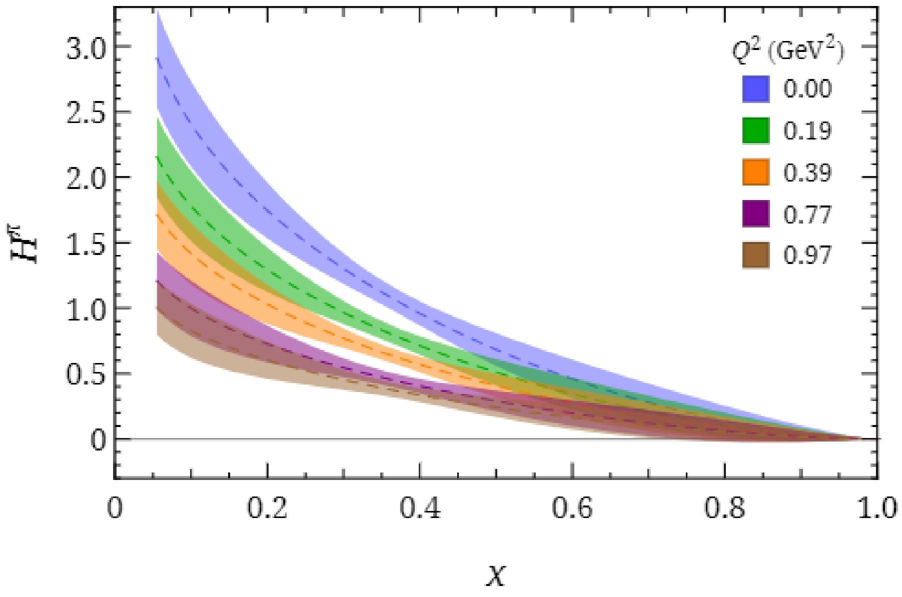
§ Pion GPD (H^π) using quasi-PDFs at physical pion mass

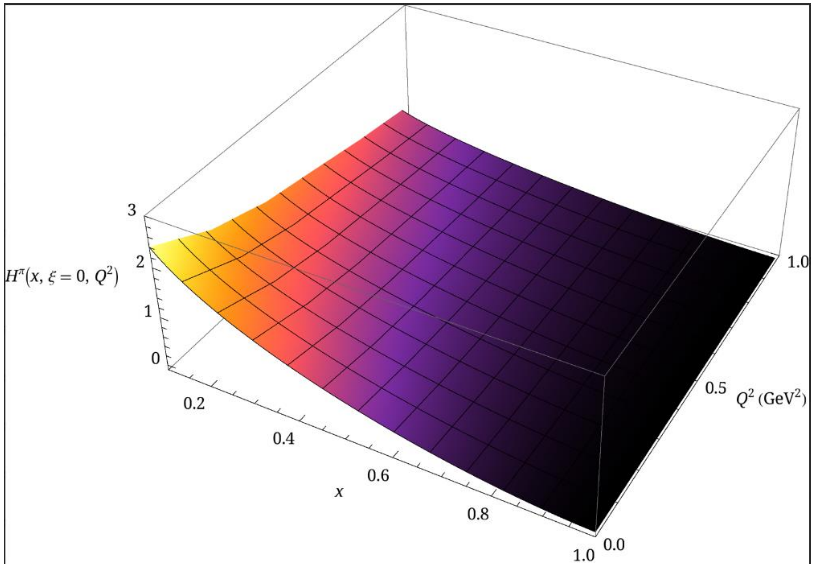
- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV
- $\xi = 0$ valence-quark Pion GPD results



finite-volume,
discretization,

MSULat, Preliminary





Valence-Quark Pion GPD

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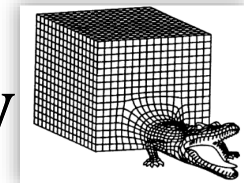
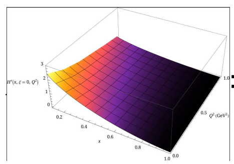
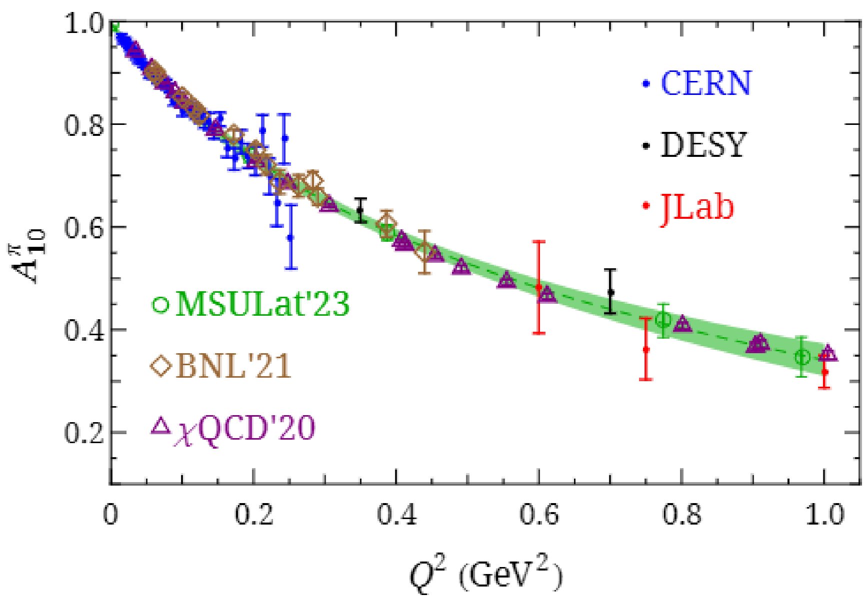


finite-volume, discretization,

$$\int_{-1}^{+1} dx\, x^{n-1}$$  $$= A^\pi_{ni}(t)$$



MSULat, **Preliminary**

Pion Tomography

§ Nucleon GPD using quasi-PDFs at physical pion mass

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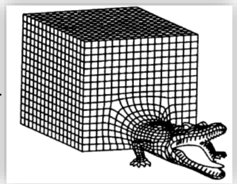
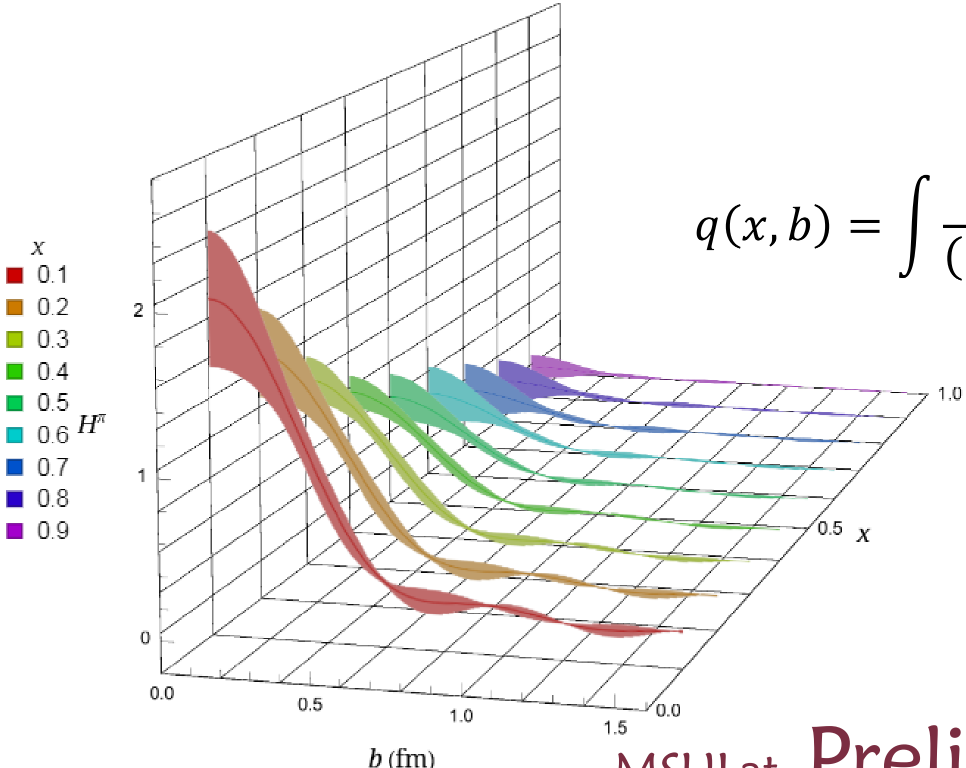
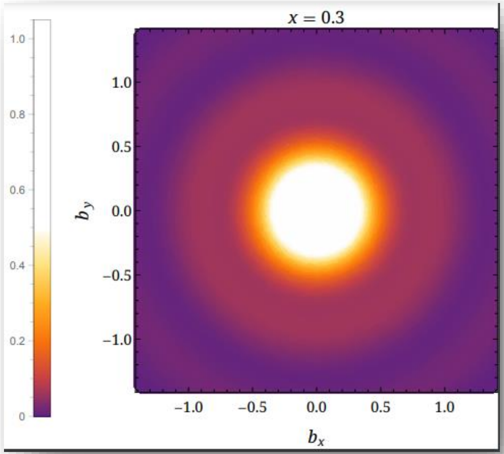


finite-volume, discretization,



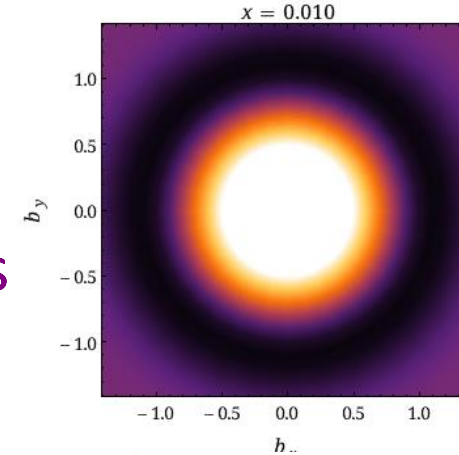
$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x,\xi=0,t=-\vec{q}^2) e^{i\vec{q}\cdot\vec{b}}$$



MSULat, Preliminary

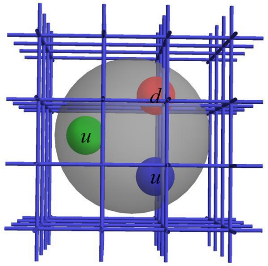
Summary and Outlook

- Exciting era using LQCD to study hadron structure
- Overcoming longstanding limitations of moment method
  - Bjorken-*x* dependence of parton distributions are widely studied
  - More study of systematics planned for the near future



- Precision and progress are limited on resources
  - Challenges = new opportunities quantities
- In the future

Theory Input → **Global Analysis of PDFs/GPDs** ← **Exp't Input** +



Thanks to Raza Sabbir Sufian, Sungwoo Park and Swagato Mukherjee for plots used in this talk

The work of HL is sponsored by NSF CAREER Award under grant PHY 1653405 & RCSA Cottrell Scholar Award
Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices & USQCD/NSF/DOE for computational resources

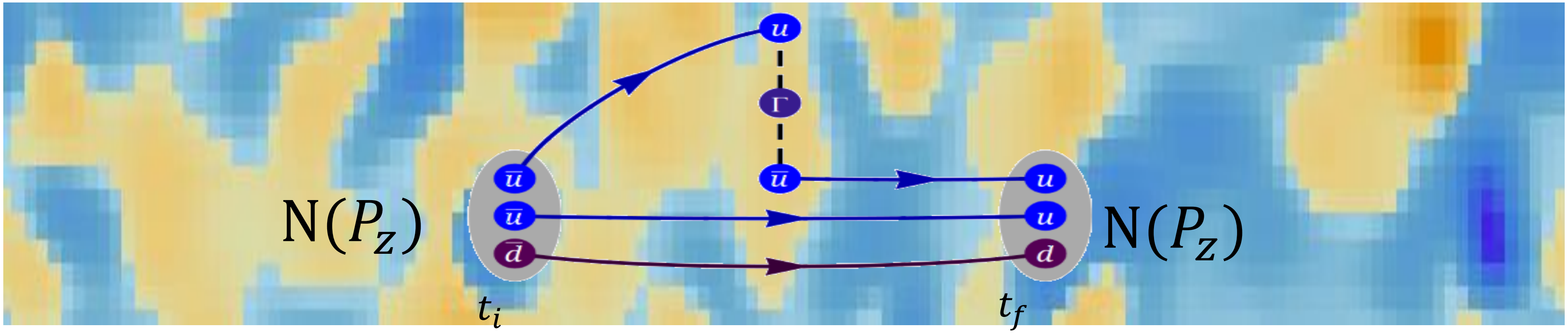
Backup Slides

Bjorken-x Dependent Hadron Structure

Biased selected/highlighted results

Lattice Parton Method

§ Large-momentum effective theory (LaMET)/quasi-PDF
(X. Ji, 2013; See 2004.03543 for review)



§ Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\Gamma \exp\left(-ig\int_0^z dz' A_z(z')\right)\psi(0) \right| P \right\rangle$$

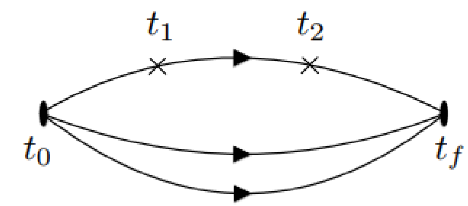
§ Recover true distribution (take Pz → ∞ limit)

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{M_N^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{(xP_z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P_z)^2}\right)$$

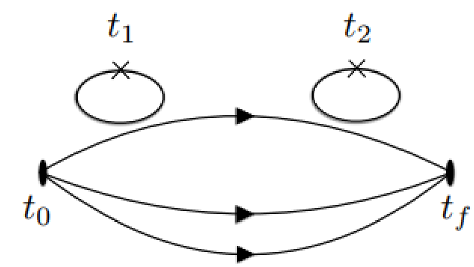
X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664

Lattice Parton Method

§ Short-distance factorization (SDF)



- **pseudo-PDF** method (A. Radyushkin, 2017)
- Hadronic tensor currents
  (Liu et al., hep-ph/9806491, … 1603.07352)



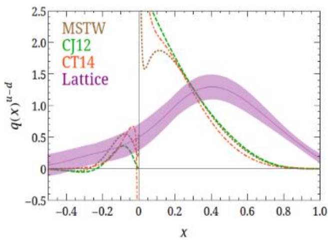
- Lattice cross-section method (LCS)
  (Y Ma and J. Qiu, 2014, 2017)
- Euclidean correlation functions
  (RQCD, 1709.04325)
- Compton amplitude approach (QCDSF, 1703.01153)

Quantities that can be calculated on the lattice today = ∑ Wanted PDFs, GPDs, etc. × pQCD-calculated kernel

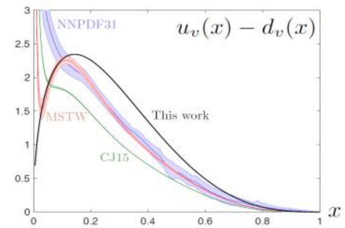
Lattice Parton Calculations

§ Rapid developments!

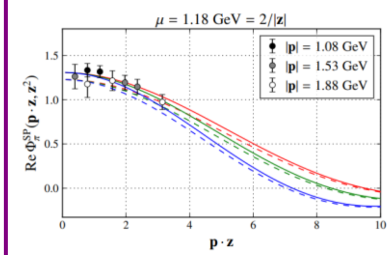
First unpol. PDF lattice calculation (2013)



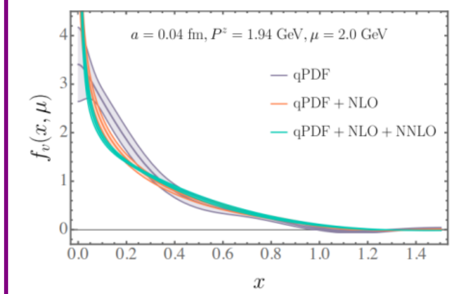
First lattice pseudo-PDFs (2017)



Euclidean correlation functions (2018)

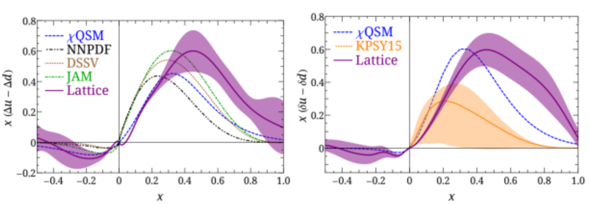


1st NNLO PDF (2022)

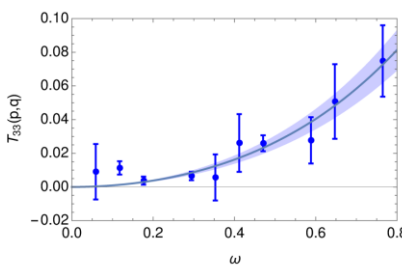


2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023

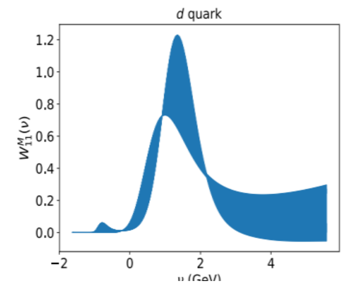
Pol. PDFs and mass corrections (2016)



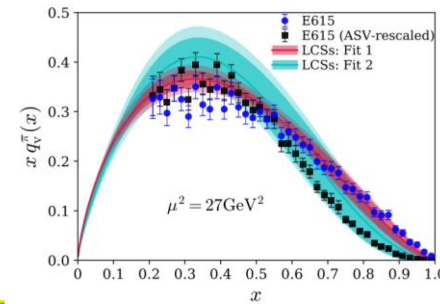
Compton amplitude (2017)



Hadronic tensor (2019)

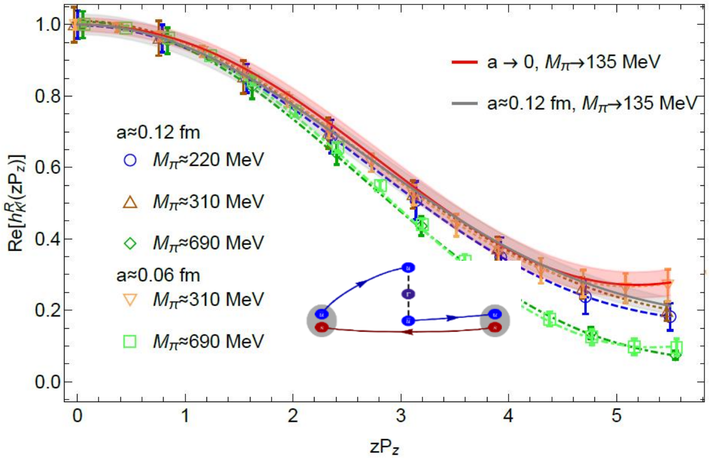


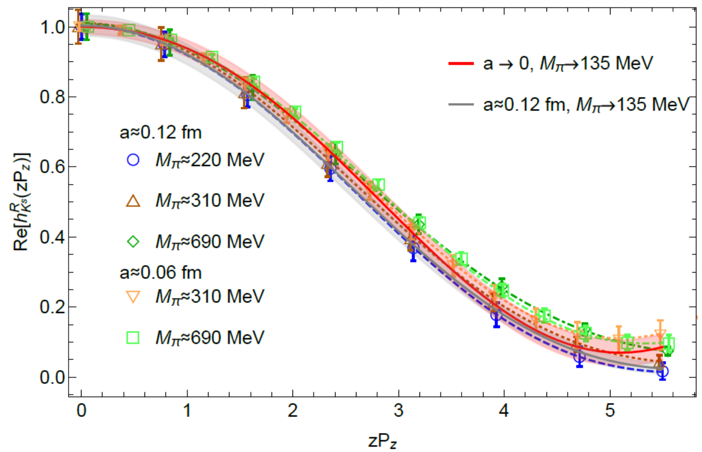
LCS (2020)



Meson Valence-quark PDFs

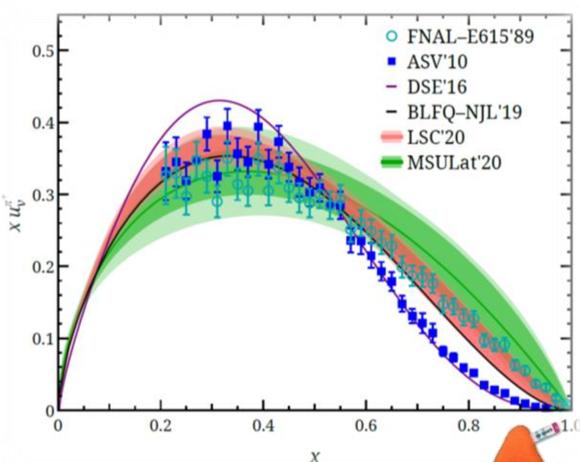
§ Pion/Kaon PDFs using quasi-PDF in the continuum limit

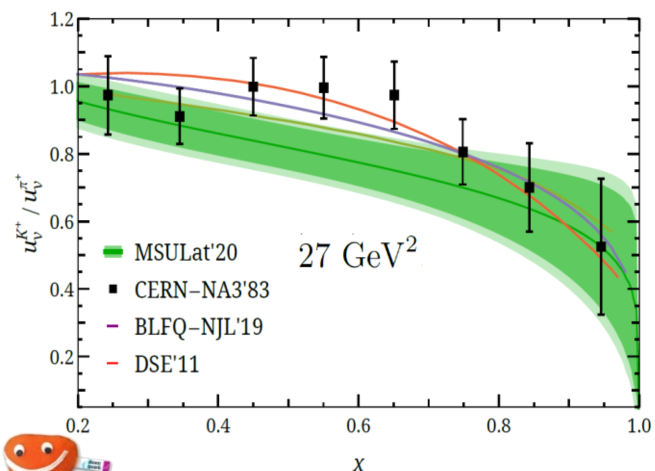


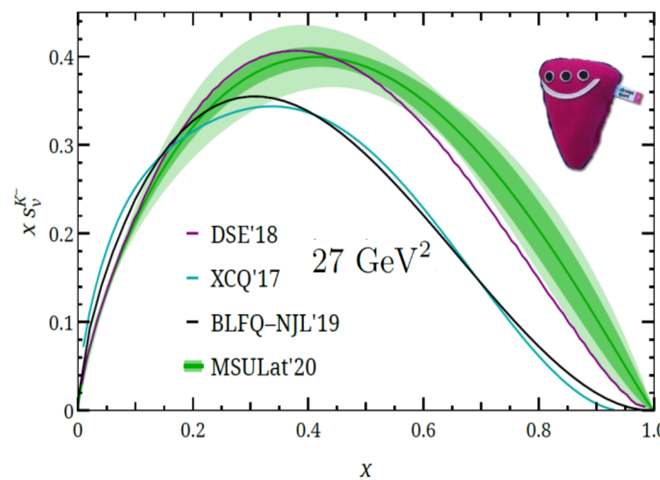


Quantities that can be calculated on the lattice

Wanted PDFs, GPDs, etc…

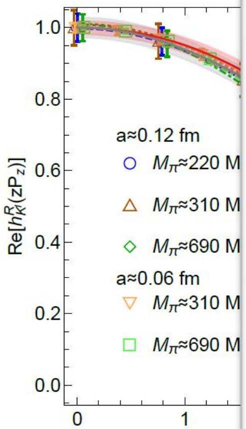




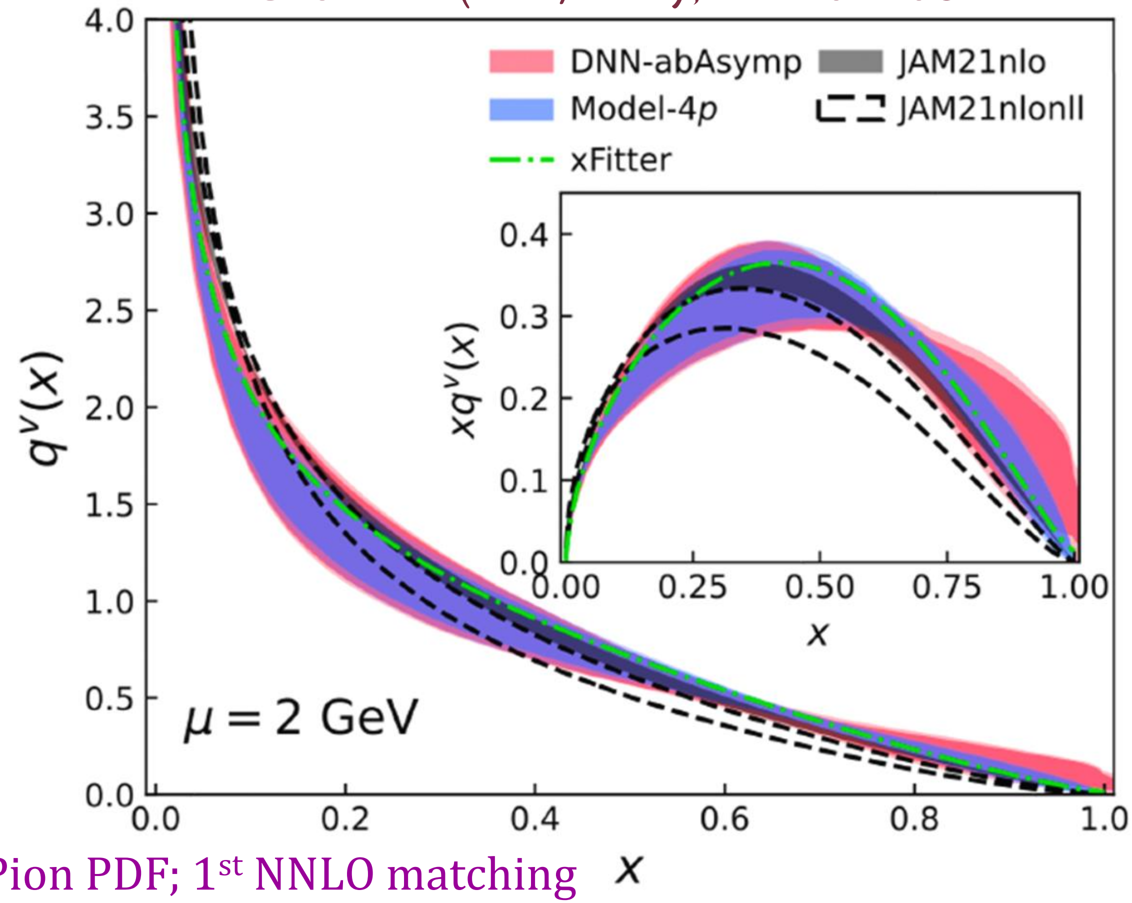


Meson Valence-quark PDFs

§ Pion/Ka… …um limit



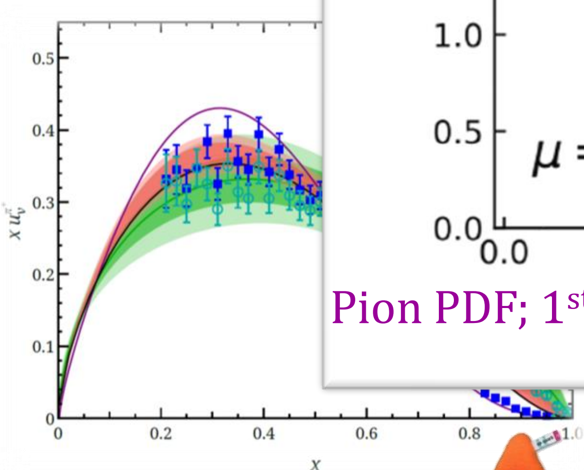
X. Gao et al (BNL/ANL), 2112.02208

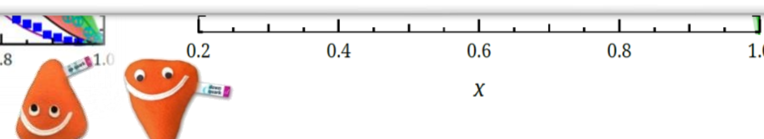


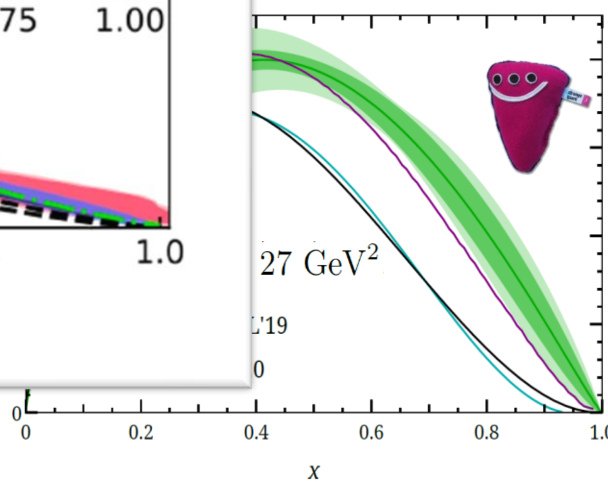
Pion PDF; 1st NNLO matching

Quantities that can be calculated on the lattice

Wanted PDFs, GPDs, etc…







First Continuum PDF

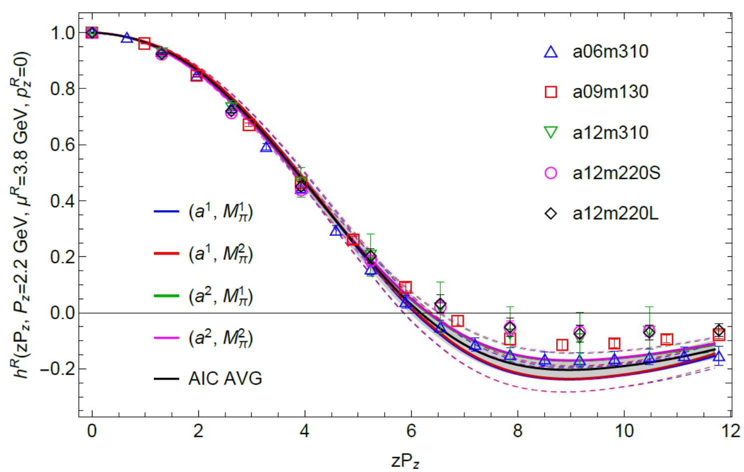
§ Nucleon PDFs using quasi-PDFs in the continuum limit

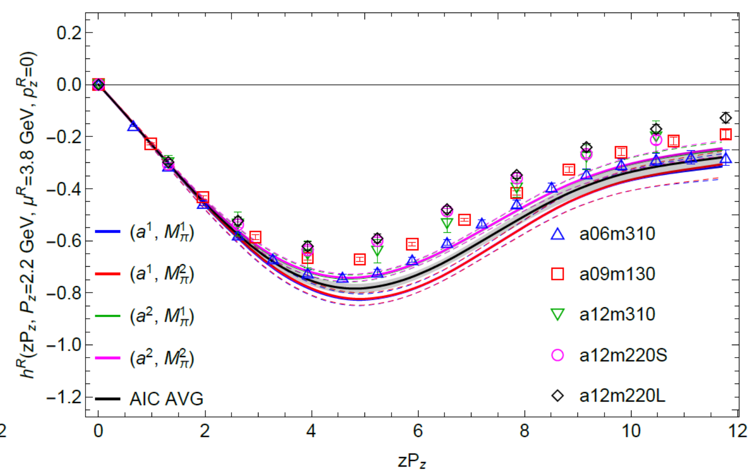
- Lattice details: clover/2+1+1 HISQ (MSULat)
  - $a \approx \{0.06, 0.09, 0.12\}$ fm,
  - $M_\pi \in \{135, 220, 310\}$-MeV pion,
  - $M_\pi L \in \{3.3, 5.5\}$.
  - $P_z \approx 2$ GeV

2011.14971, HL et al (MSULat)

- Naïve extrapolation to physical-continuum limit

Quantities that can be calculated on the lattice





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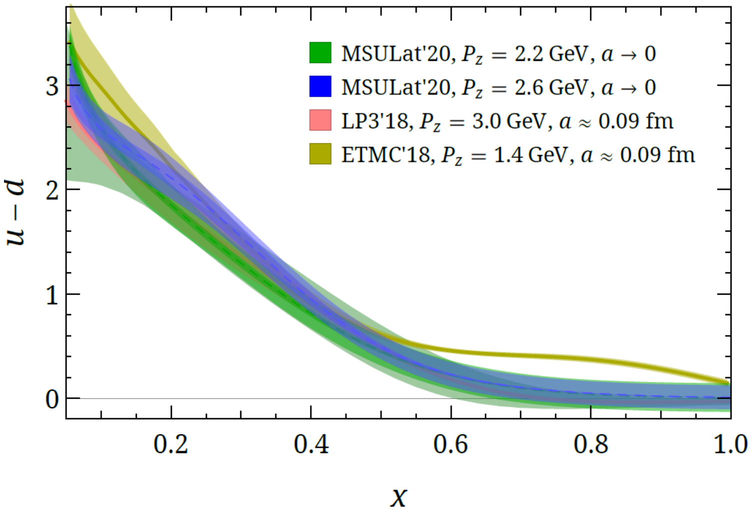
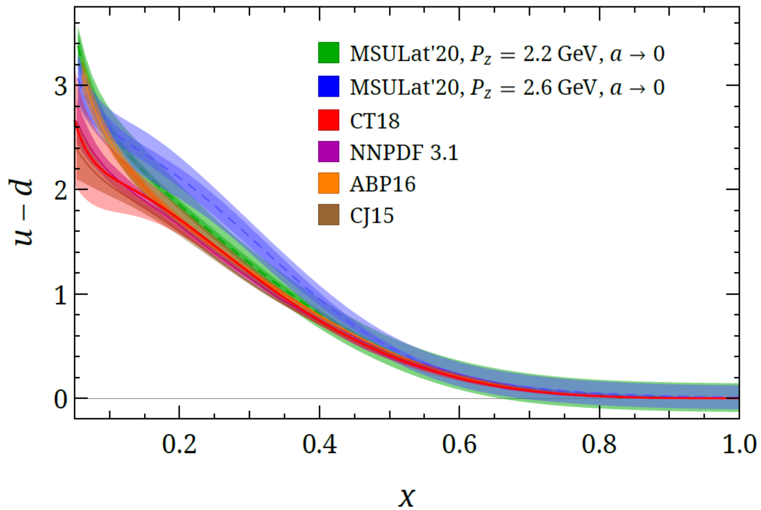
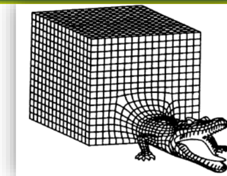
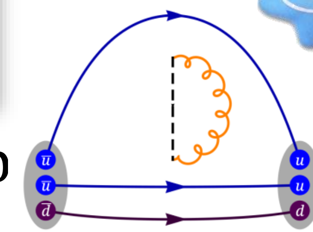




First Gluon Helicity Distribution

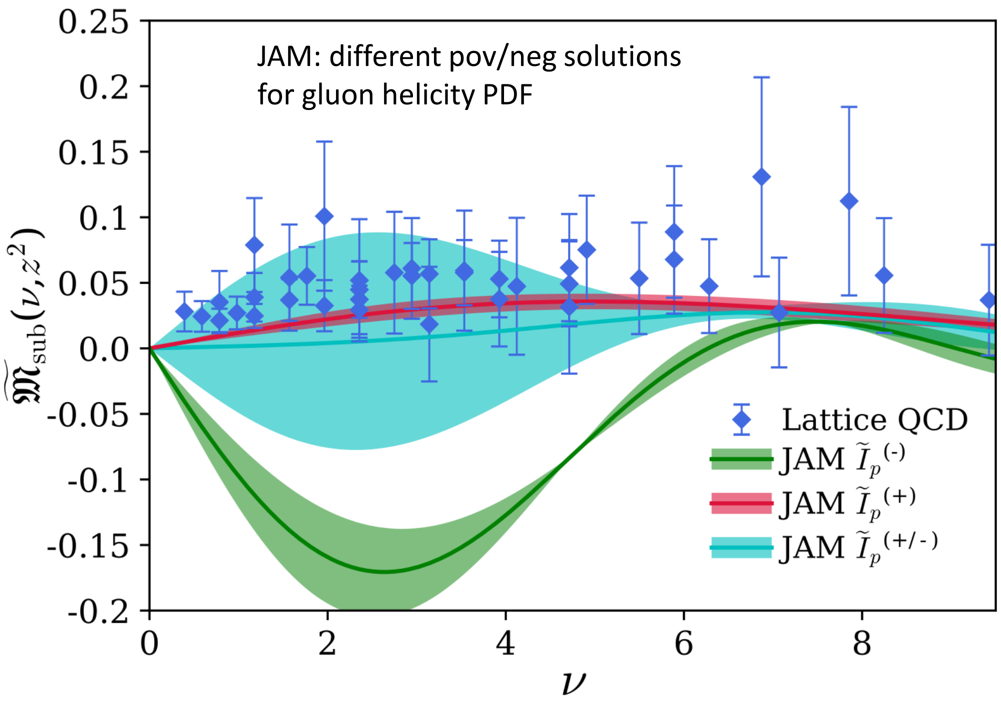
§ Helicity Gluon PDF w/ pseudo-PDF

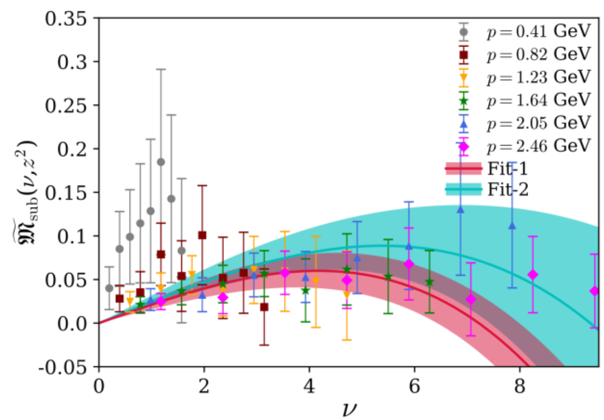




- 2+1 clover 0.09 fm, 360-MeV pion
- Lattice QCD hints at a positive gluon helicity distrib nucleon

C. Egerer el al (HadStruc), 2207.08733





Quantities that can be calculated on the lattice

Plots by Raza Sabbir Sufian

Lattice Progress & Challenges

- § Exploratory study on charm and gluon PDFs
- § Many approaches are moving to the NNLO level
  - Expect to see more improved lattice calculations
- § Beyond the standard twist-2 collinear PDFs
  - Generalized parton distributions (GPDs) for the pion and unpolarized/polarized nucleon
  - Transverse-momentum– dependent distributions (TMDs)
    - Collins-Soper kernel, soft function and wavefunctions
  - Twist-3 PDFs and GPDs

For more details and references, refer to 2202.07193

- § Challenges ahead for precision PDFs
  - Need large boost mom., better signal-to-noise, inverse problems in PDF extraction in SDF, more computational resources, etc.

Physical-continuum lattice charges/moments

Biased selected/highlighted results

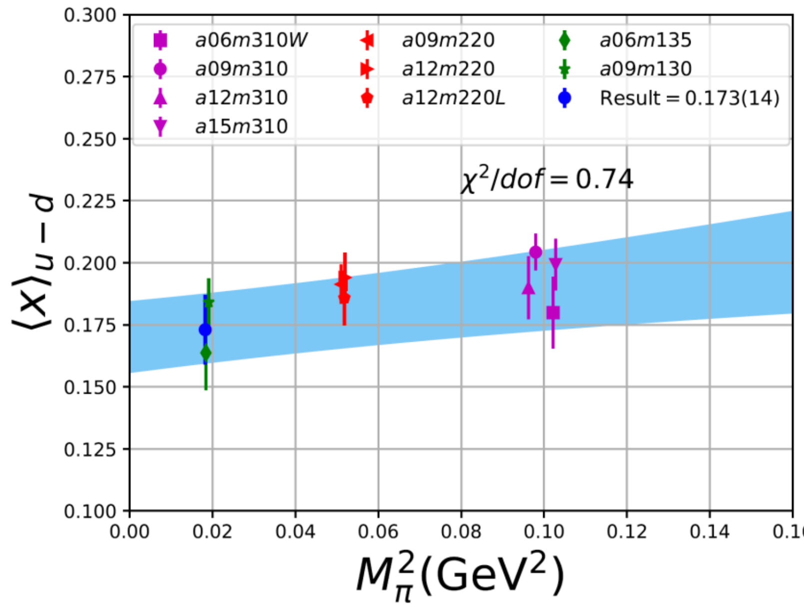
Moments of PDFs

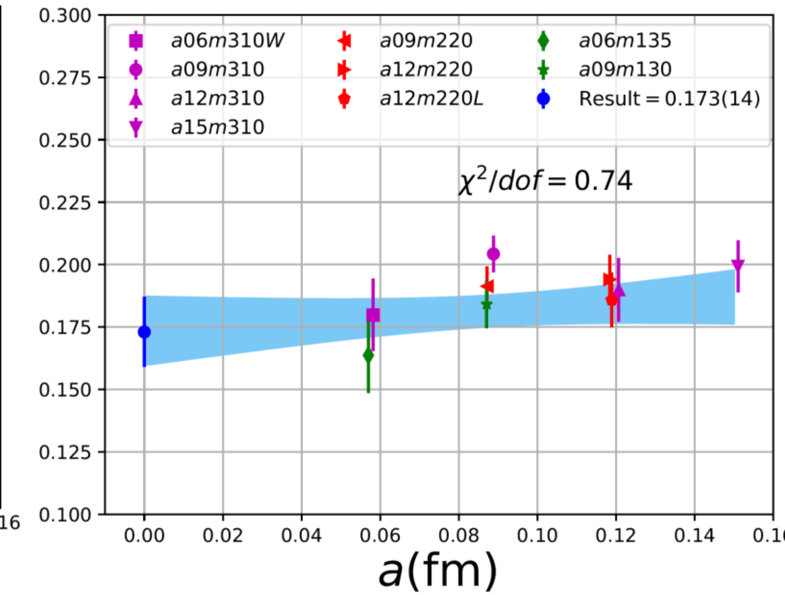
- First moments are most commonly done 

- State-of-the art example
  - Extrapolate to the physical limit

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

Santanu Mondal et al (PNDME collaboration), 2005.13779



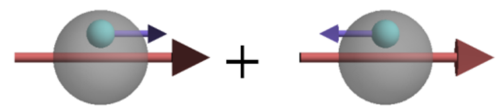


- Usually more than one LQCD calculation
  - Sometimes LQCD numbers do not even agree with each other…

Moments of PDFs

- PDG-like rating system or average
- LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- Lattice QCD/global fit status

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



LatticePDF Report, 1711.07916, 2006.08636

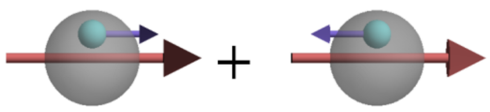
Moment	Collaboraton	Reference	N_f	DE	CE	FV	RE	ES		Value	Global Fit
$\langle x\rangle_{u^+-d^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.171(18)	0.161(18)
	PNDME 20	(Mondal *et al.*, 2020)	2+1+1	★	★	★	★	★		0.173(14)(07)	
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★		$0.180(25)(^{+14}_{-6})$	
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.151(28)(29)	
	RQCD 18	(Bali *et al.*, 2019b)	2	★	★	○	★	★		0.195(07)(15)	
$\langle x\rangle_{u^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.359(30)	0.353(12)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.307(30)(18)	
$\langle x\rangle_{d^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.188(19)	0.192(6)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.160(27)(40)	
$\langle x\rangle_{s^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.052(12)	0.037(3)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.051(26)(5)	
$\langle x\rangle_g$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.427(92)	0.411(8)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.482(69)(48)	
	χQCD 18a	(Yang *et al.*, 2018a)	2+1	■	★	★	★	■		0.47(4)(11)	

** No quenching effects are seen.

Moments of PDFs

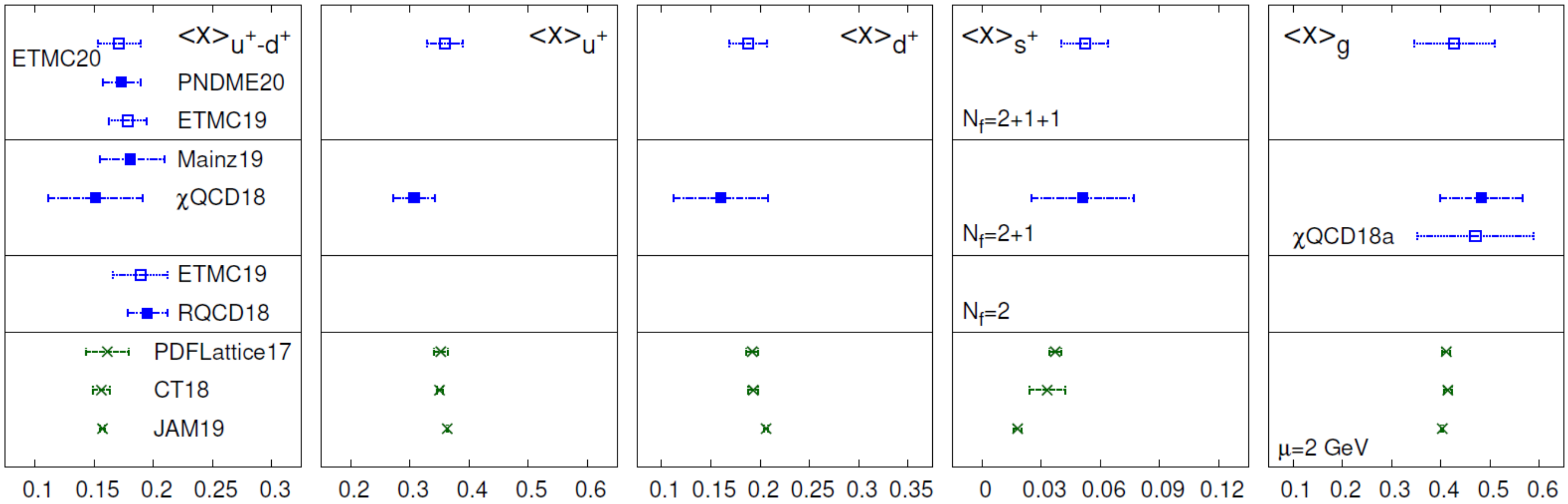
- § PDG-like rating system or average
- § LatticePDF Workshop
  - Lattice representatives came together and devised a rating system

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



- § Lattice QCD/global fit status

LatticePDF Report, 1711.07916, 2006.08636



Moments of PDFs

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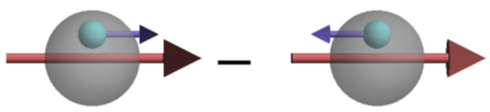
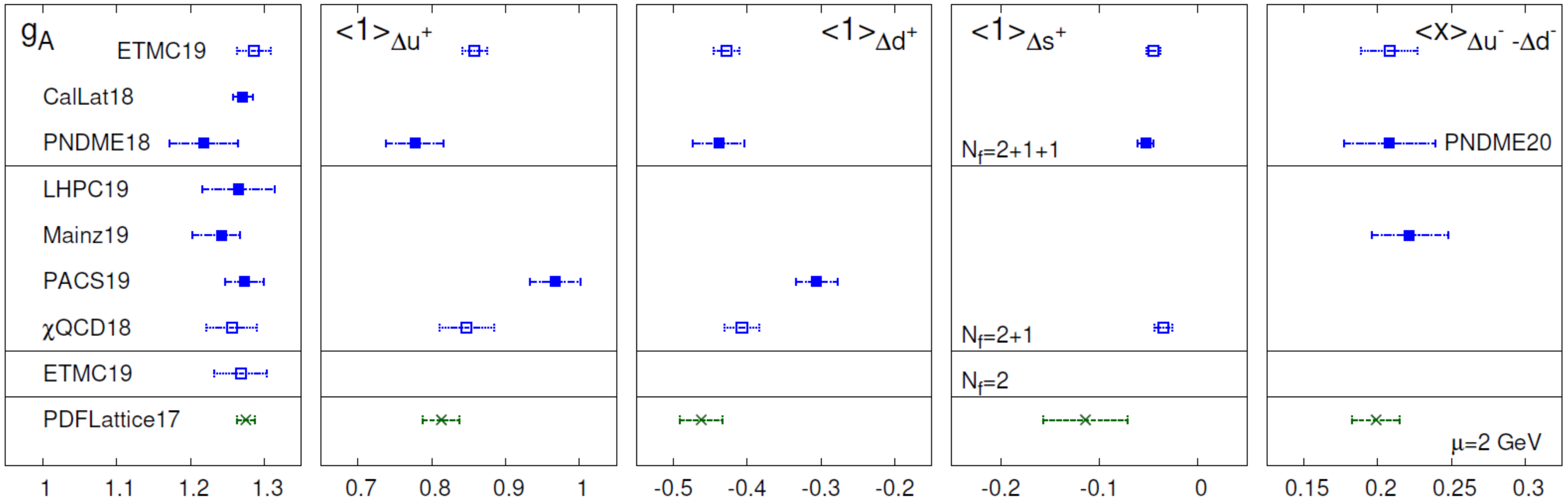
- § Recent lattice QCD/global fit status

$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$



LatticePDF Report, 1711.07916,2006.08636



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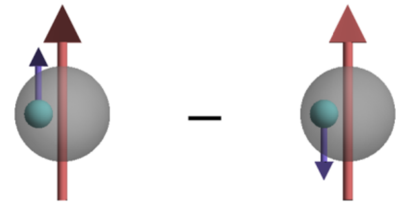

LatticePDF Report, 1711.07916, 2006.08636

Moment	Collaboration	Reference	N_f	DE	CE	FV	RE	ES		Value	Global Fit
g_T	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.926(32)	0.10 — 1.1
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.989(32)(10)	
	χQCD 20	(Horkel *et al.*, 2020)	2+1	■	★	○	★	★	†	1.096(30)	
	LHPC 19	(Hasan *et al.*, 2019)	2+1	○	★	○	★	★	*	0.972(41)	
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★		$0.965(38)(^{+13}_{-41})$	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		1.08(3)(3)(9)	
	ETMC 19	(Alexandrou *et al.*, 2019b)	2	■	★	○	★	★	**	0.974(33)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		1.004(21)(02)(19)	
	RQCD 14	(Bali *et al.*, 2015)	2	○	★	★	★	■		1.005(17)(29)	
$\langle 1\rangle_{\delta u^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.716(28)	-0.14 — 0.91
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.784(28)(10)	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		0.85(3)(2)(7)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		0.782(16)(2)(13)	
$\langle 1\rangle_{\delta d^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.210(11)	-0.97 — 0.47
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.204(11)(10)	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.24(2)(0)(2)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.219(10)(2)(13)	
$\langle 1\rangle_{\delta s^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.0027(58)	N/A
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.0027(16)	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.012(16)(8)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.00319(69)(2)(22)	

Moments of PDFs

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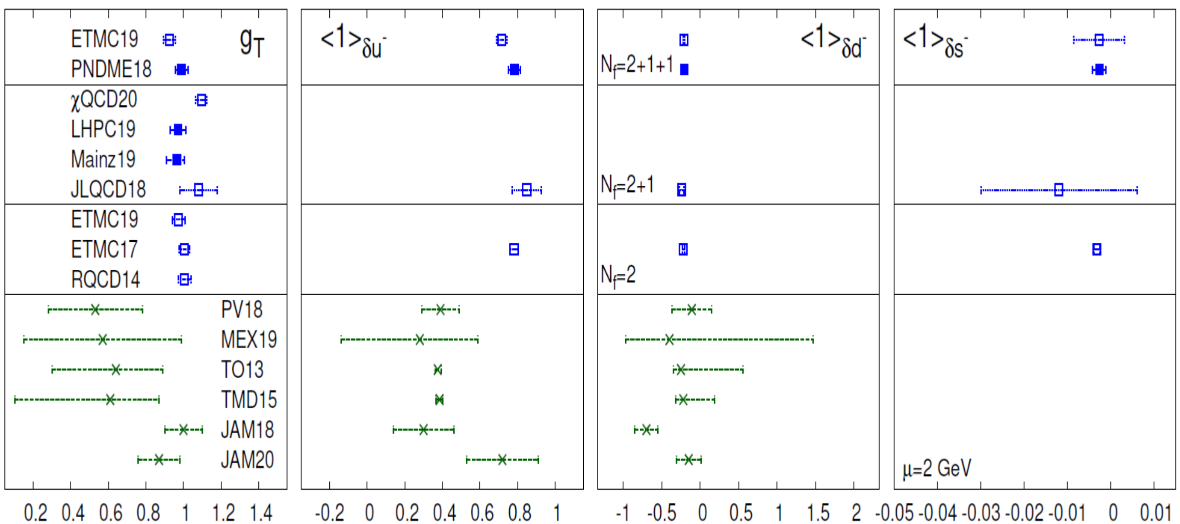
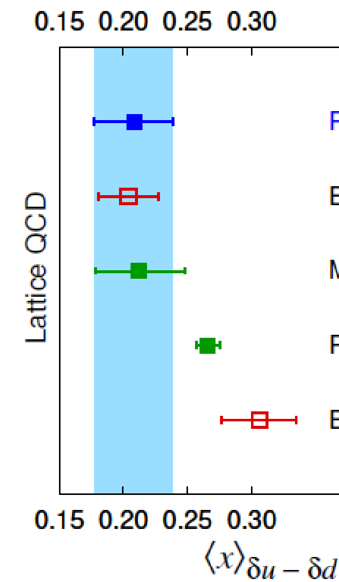
$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1}\delta q(x)$$

LatticePDF Report, 1711.07916, 2006.08636



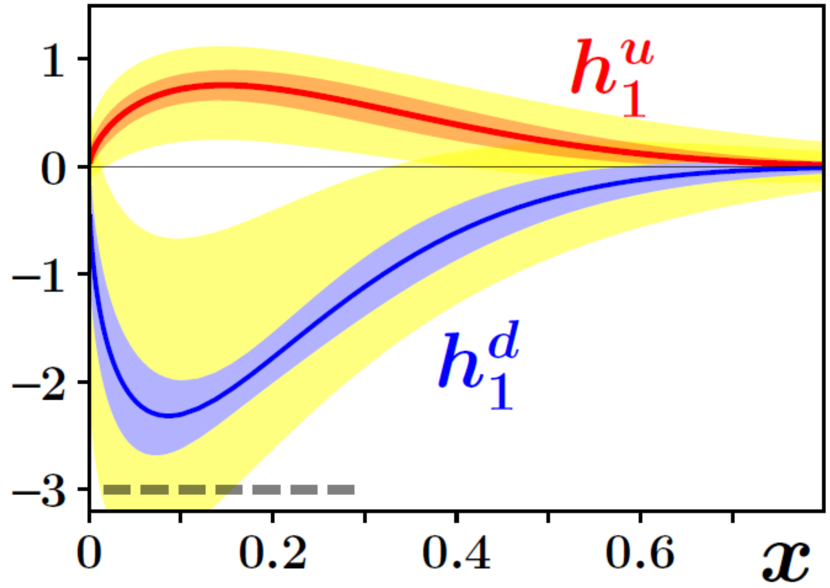


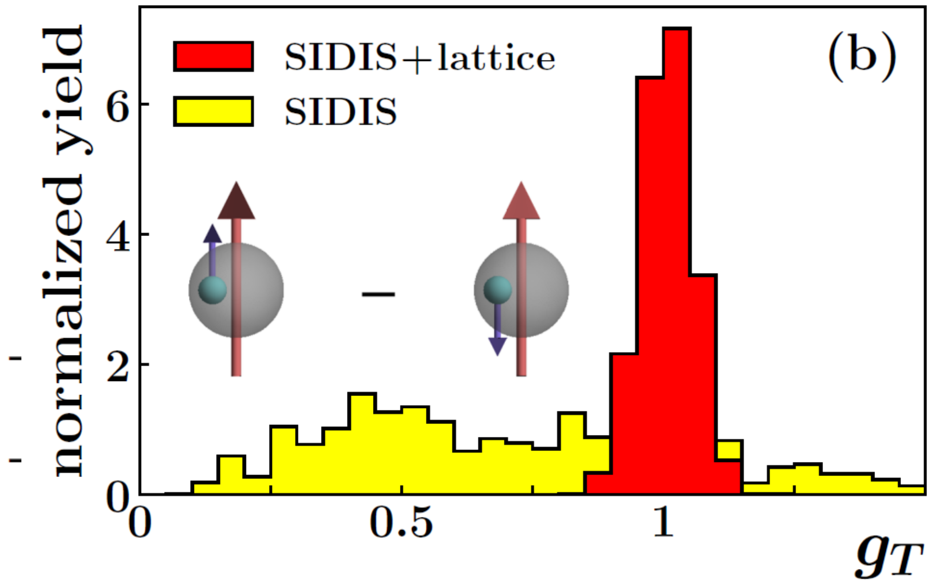
S. Mondal et al
2005.13779

From Charges to PDFs

§ Improved transversity distribution with LQCD g_T

- Global analysis with 12 extrapolation forms: $g_T = 1.006(58)$
- Use to constrain the global-analysis fits to SIDIS $\pi^\pm$ production data from proton and deuteron targets





Lin, Melnitchouk, Prokudin, Sato, 1710.09858, Phys. Rev. Lett. 120, 152502 (2018)

Global Analysis

§ Experiments cover diverse kinematics of parton variables

- Global analysis takes advantage of all data sets

Theory Input → **Global Analysis of PDFs** ← **Exp't Input**

§ Some choices made for the analysis

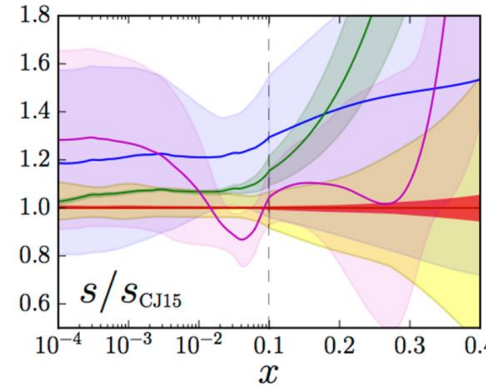
- Choice of data sets and kinematic cuts
- Strong coupling constant $\alpha_s(M_Z)$
- How to parametrize the distribution

$$xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2}P(x)$$

- Assumptions imposed

SU(3) flavor symmetry, charge symmetry, strange and sea distributions

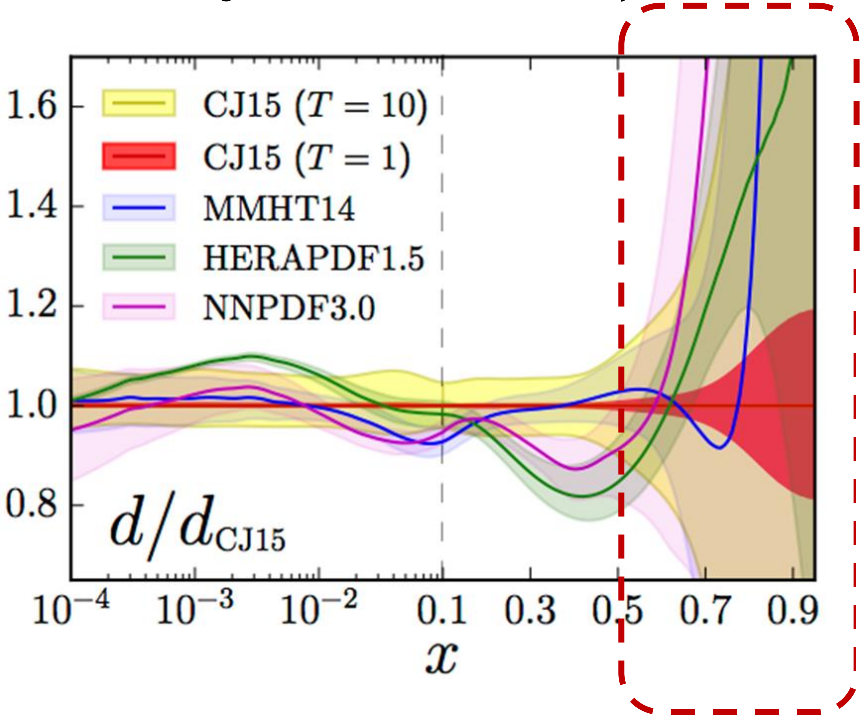
$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

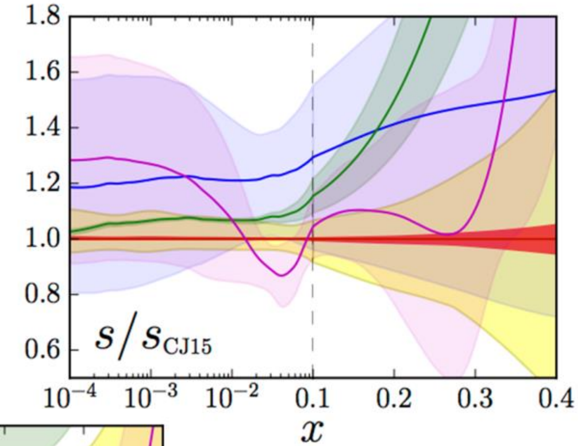


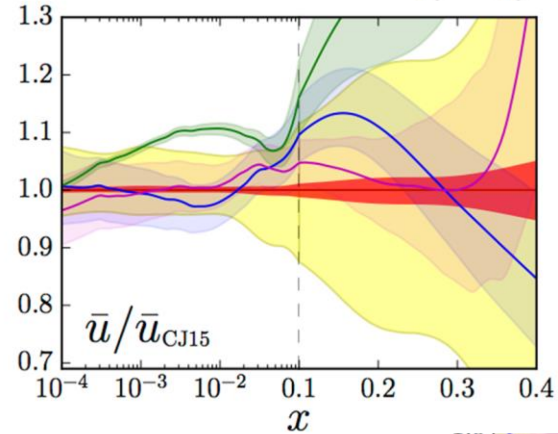
CTEQ-JLAB

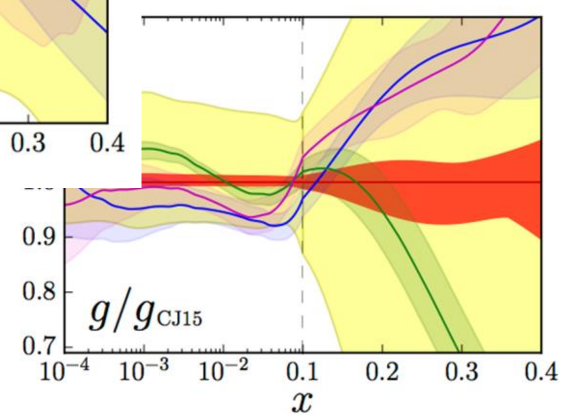
Global Analysis

- § Discrepancies appear when data is scarce
- § Many groups have tackled the analysis
  - CTEQ, MSTW, ABM, JR, NNPDF, etc.









CTEQ-JLAB
https://www.jlab.org/theory/cj/