Towards accessing $\gamma^\star\gamma^\star \to \pi\pi$ from lattice QCD

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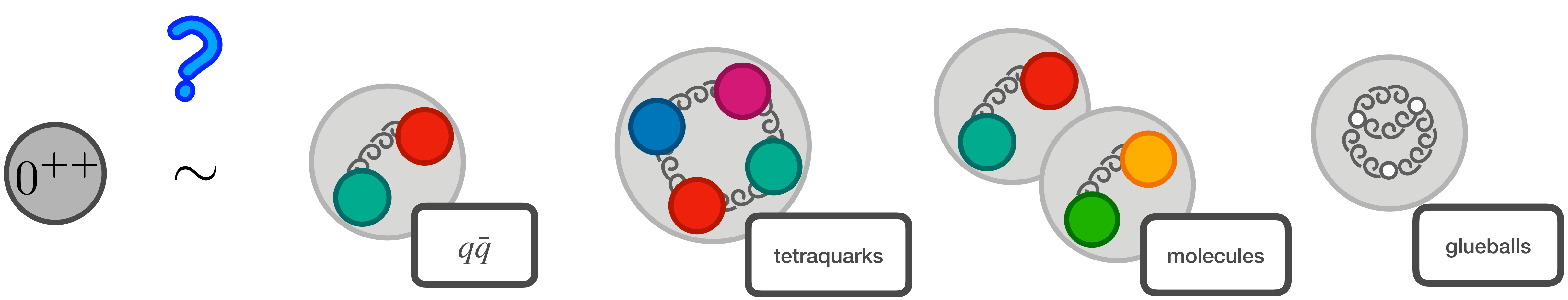
Few-Hadron Matrix Elements from QCD

What is the nature of hadrons in terms of their quarks and gluons in QCD?

- Can we discriminate against preferred compositions?
- Can we explore these features for resonances?

scalar mesons

$$[\pi\pi]_{J^{PC}=0^{++}}$$



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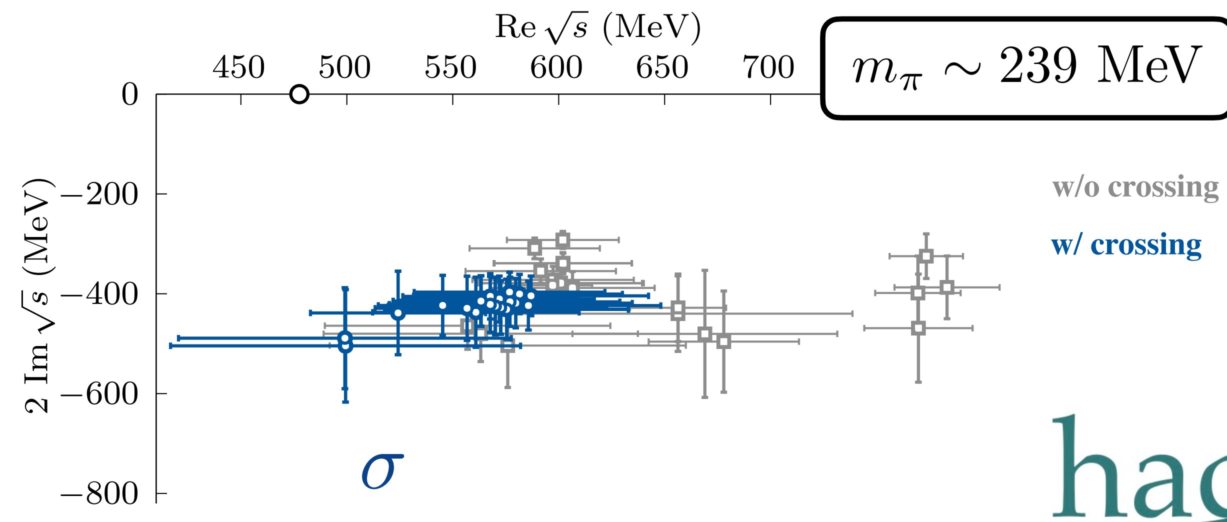

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A. Rodas, J. Dudek, R. Edwards, arXiv:2304.03762

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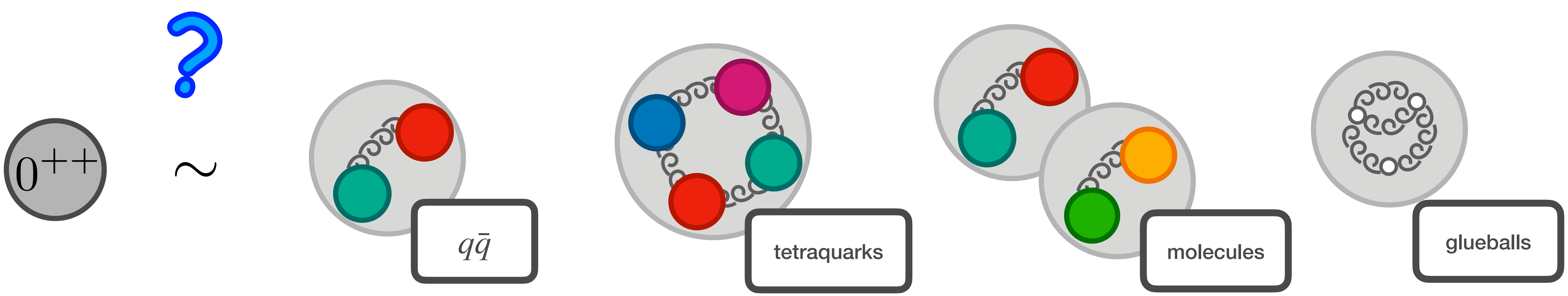

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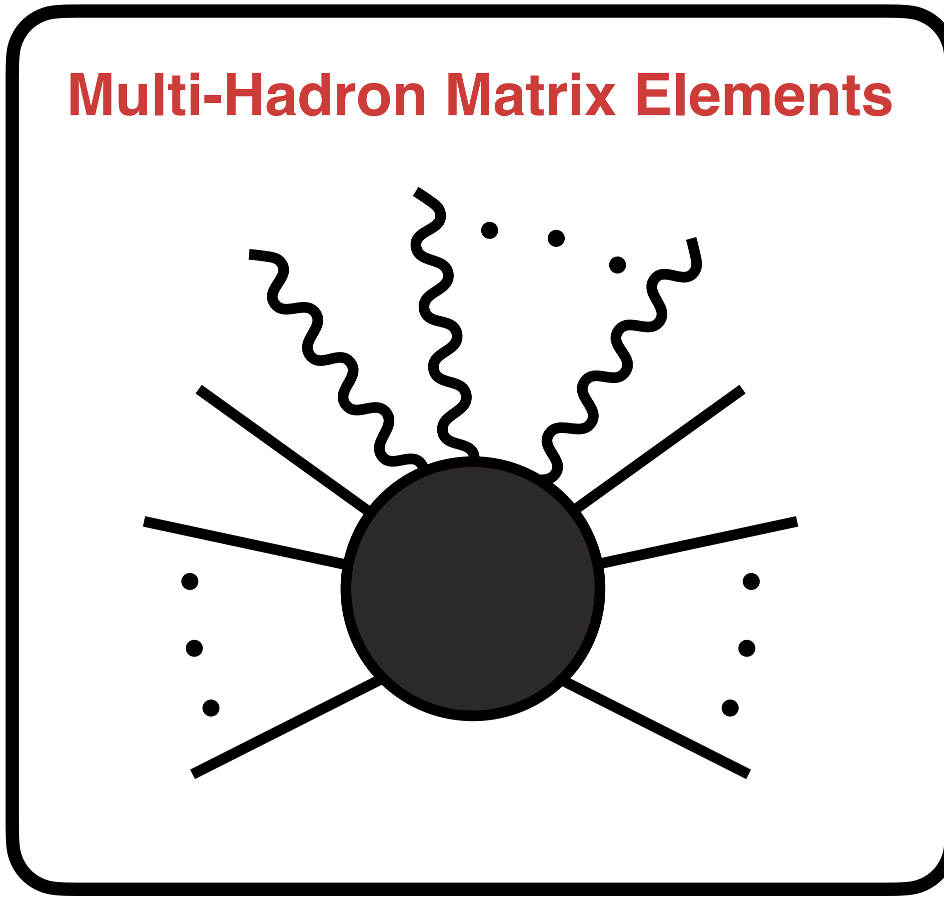

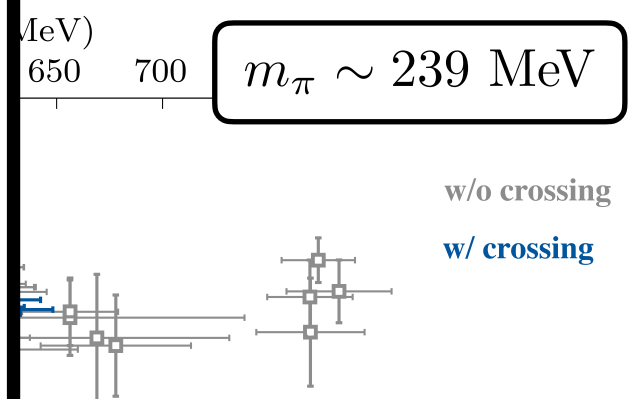

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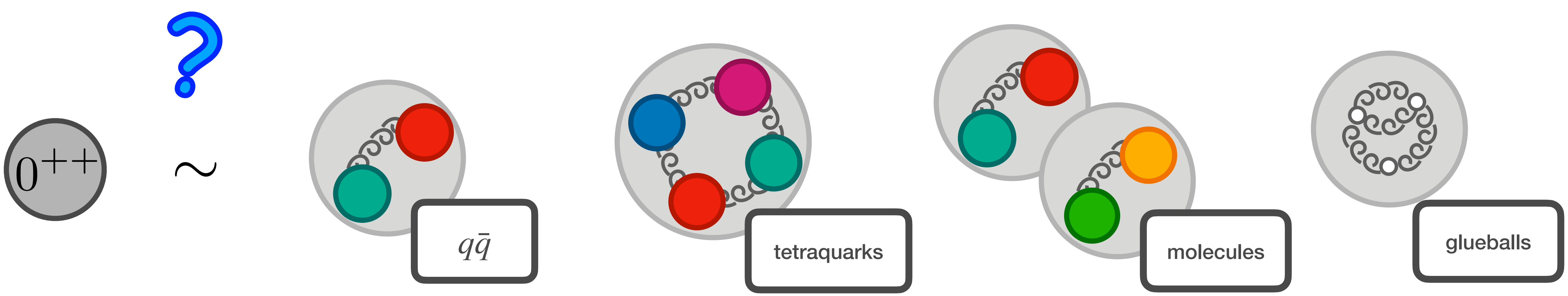
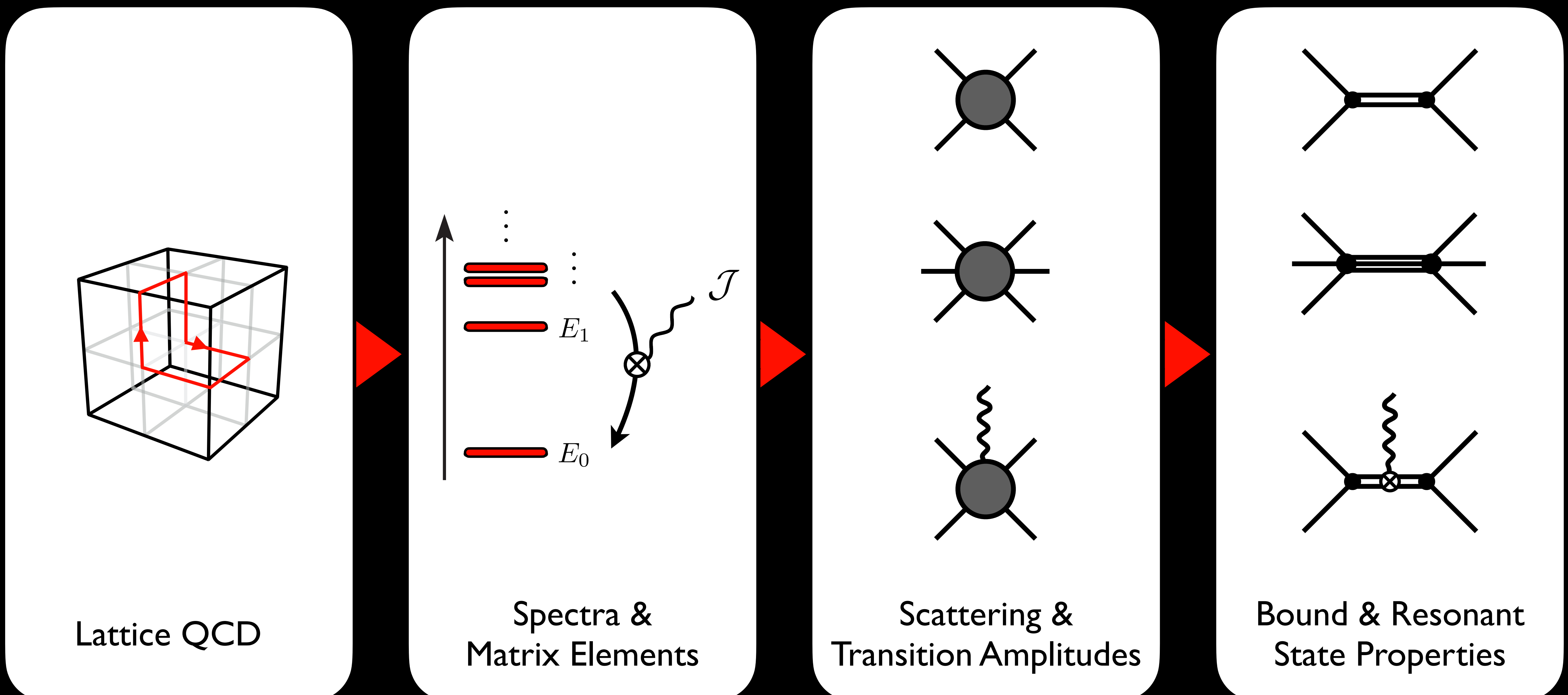


Few-Hadron Matrix Elements from QCD

A path toward understanding the substructure is to probe a hadron

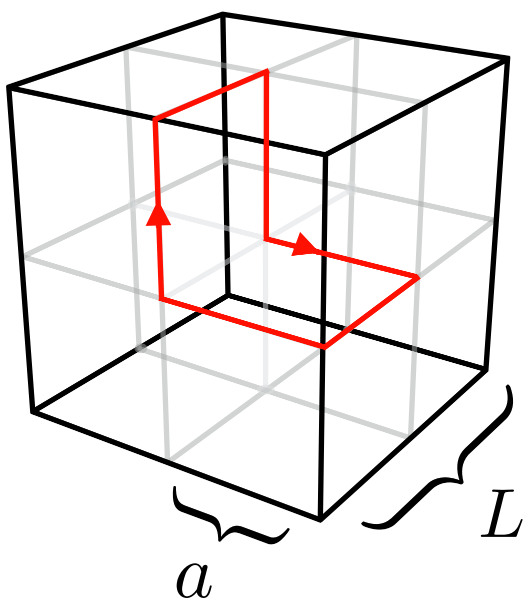
- Connect QCD to few-hadron matrix elements
- Tools: *Lattice QCD*, *Scattering Theory*, & *Effective Field Theory*



Connecting Resonances to QCD

Lattice QCD offers a systematic approach to compute hadrons from QCD

- Numerically evaluate QCD path integral via Monte Carlo sampling



$$Z_{\mathrm{QCD}} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_\mu \, e^{iS_{\mathrm{QCD}}(\psi, \bar{\psi}, A_\mu)}$$

- *Euclidean spacetime,* $t \to -i\tau$
- *Finite volume,* L
- *Discrete spacetime,* a
- *Heavier than physical quark mass,* $m > m_{\mathrm{phys.}}$

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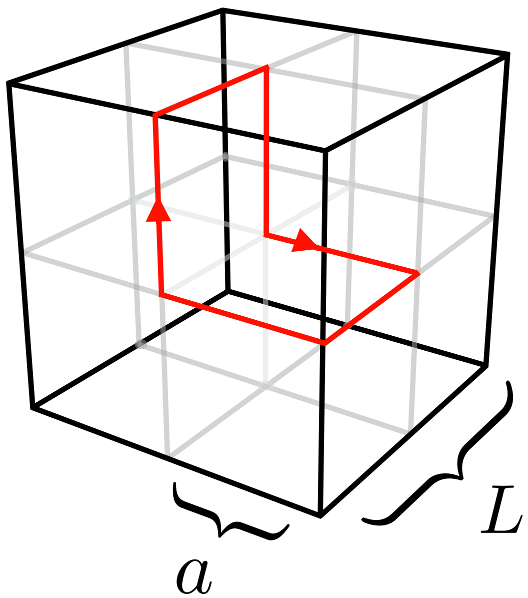

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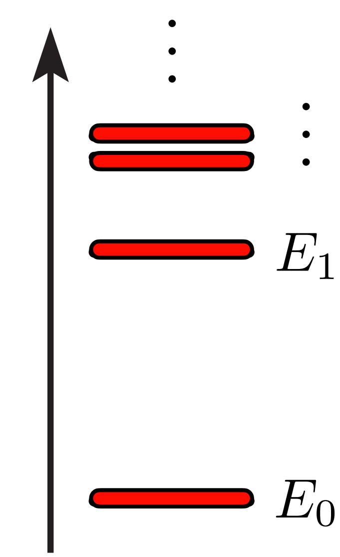
Correlation functions yield discrete spectrum

$$\langle \mathcal{O}(\tau) \mathcal{O}^\dagger(0) \rangle = \sum_{\mathfrak{n}} \left| \langle 0 | \, \mathcal{O} \, | \mathfrak{n} \rangle \right|^2 \, e^{-E_{\mathfrak{n}} \tau}$$



Connecting Resonances to QCD

Exploit *S* matrix unitarity to construct analytic amplitude

If we restrict our energies of interest…
…can formally isolate long-range from short-distance behavior

known (*on-shell kinematic functions*)

unknown (*encapsulating strong dynamics*)

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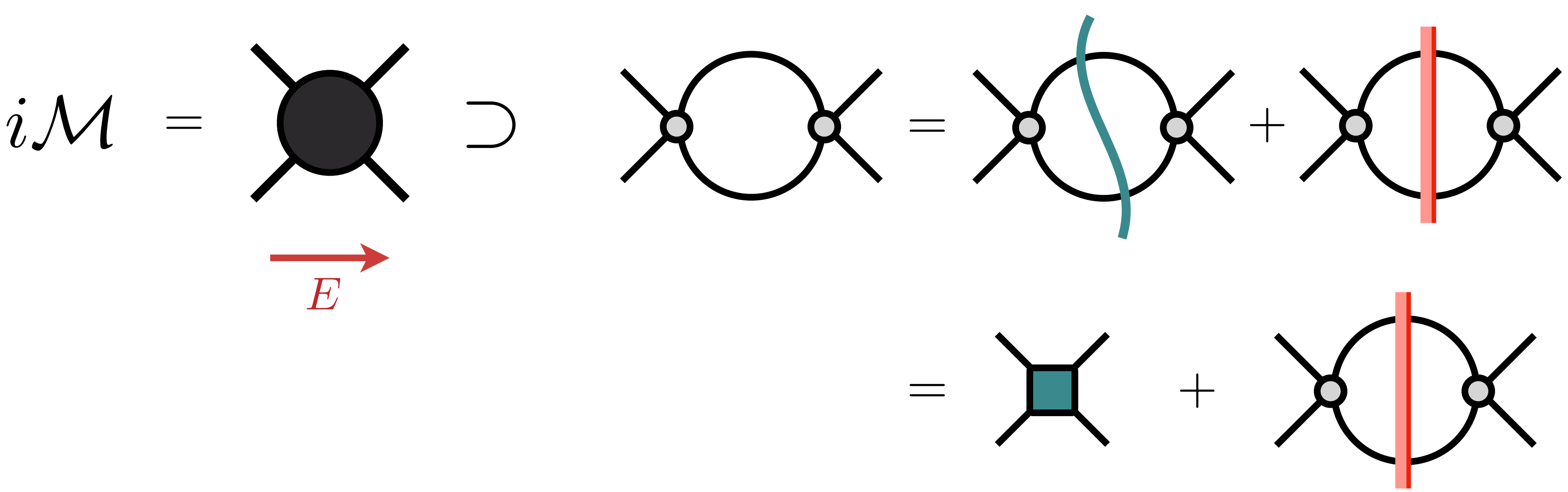

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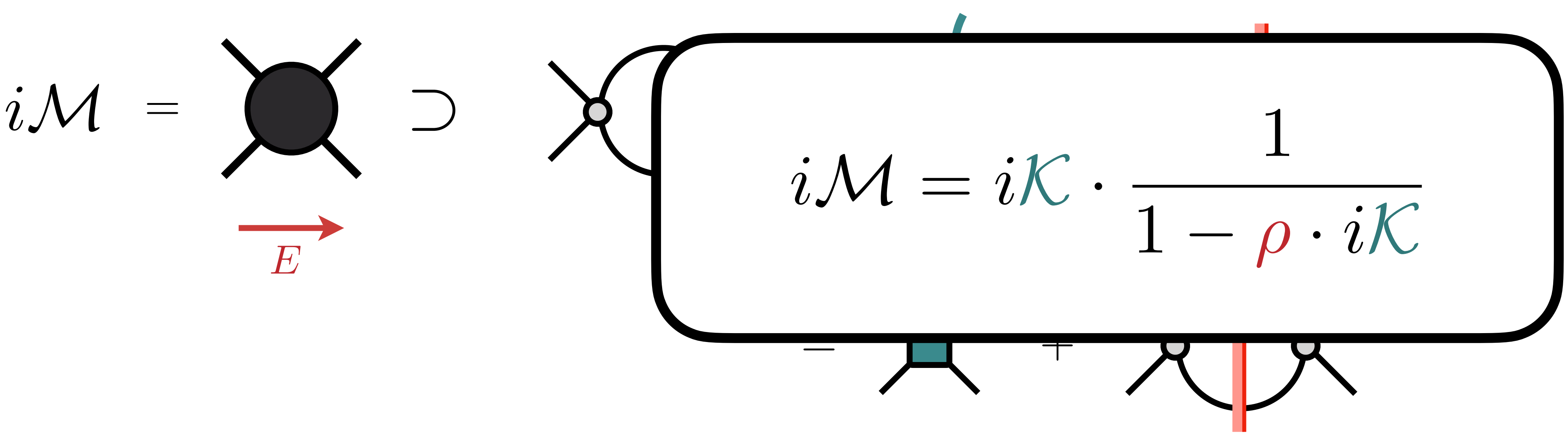
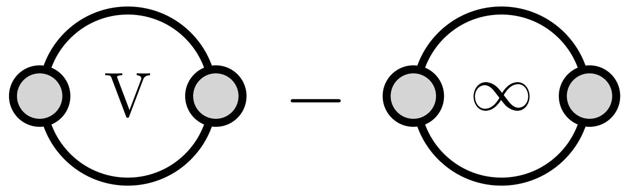


$$i\mathcal{M} = i\mathcal{K} \cdot \frac{1}{1 - \rho \cdot i\mathcal{K}}$$

Connecting Resonances to QCD

Connect finite-volume observables to amplitudes via non-perturbative mappings

$$\mathcal{C}_L - \mathcal{C}_\infty \supset$$



$$\sim \left[\frac{1}{L^3} \sum_{\mathbf{k}} - \int_{\mathbf{k}} \right] \frac{i}{E - 2\omega_{\mathbf{k}}}$$

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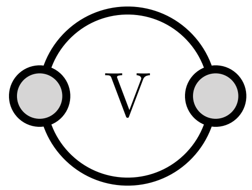
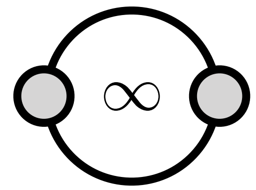
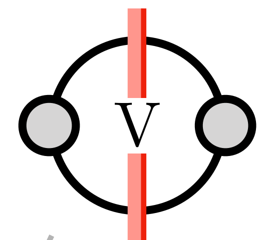


$$-$$



$$\sim \left[\frac{1}{L^3} \sum_{\mathbf{k}} - \int_{\mathbf{k}} \right] \frac{i}{E - 2\omega_{\mathbf{k}}} =$$



$$+ \mathcal{O}(e^{-mL})$$

$$\rho + \mathcal{F}_L$$

Geometric function
— characterizes finite-volume distortions

Poisson Summation Formula

$$\left[\frac{1}{L^3} \sum_{\mathbf{k}} - \int_{\mathbf{k}} \right] f(\mathbf{k}) = \mathcal{O}(e^{-mL})$$

Connecting Resonances to QCD

Connect finite-volume observables to amplitudes via non-perturbative mappings

$$\mathcal{C}_L - \mathcal{C}_\infty \sim \sum_{\mathfrak{n}} \frac{\mathcal{R}_L}{E - E_{\mathfrak{n}}} + \mathcal{O}(e^{-mL})$$

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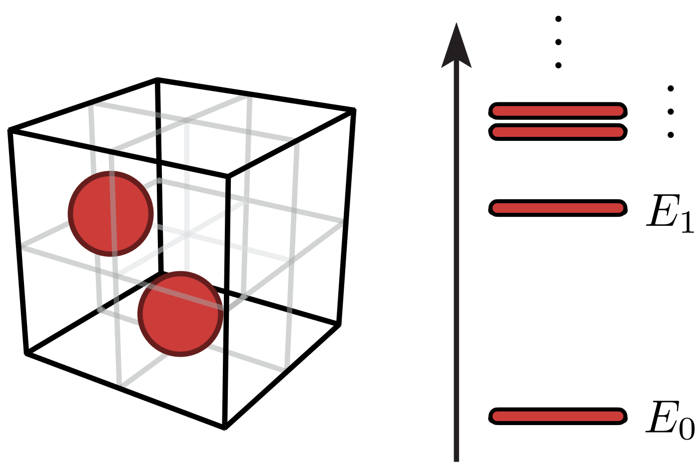
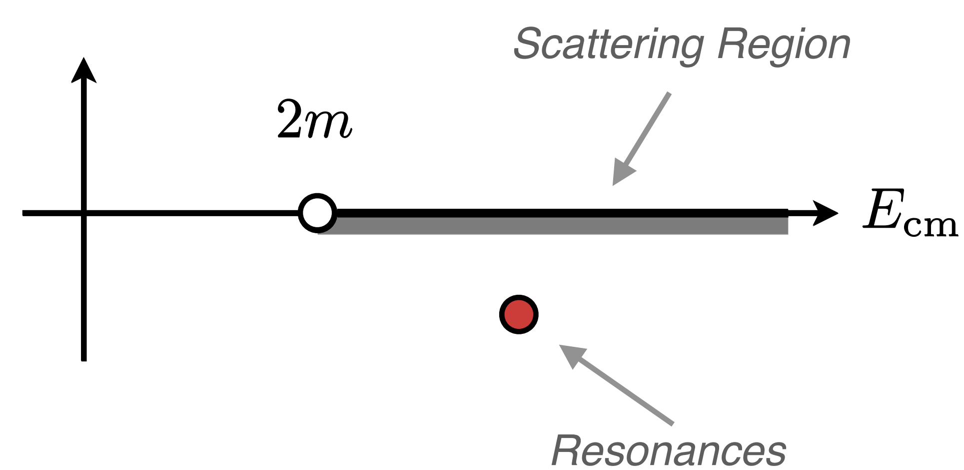


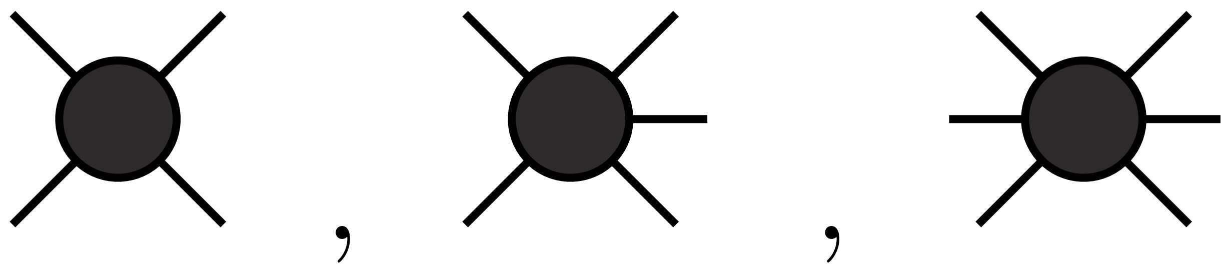


M. Lüscher
Commun.Math.Phys. **105**, 153 (1986)
Nucl.Phys. **B354**, 531 (1991)

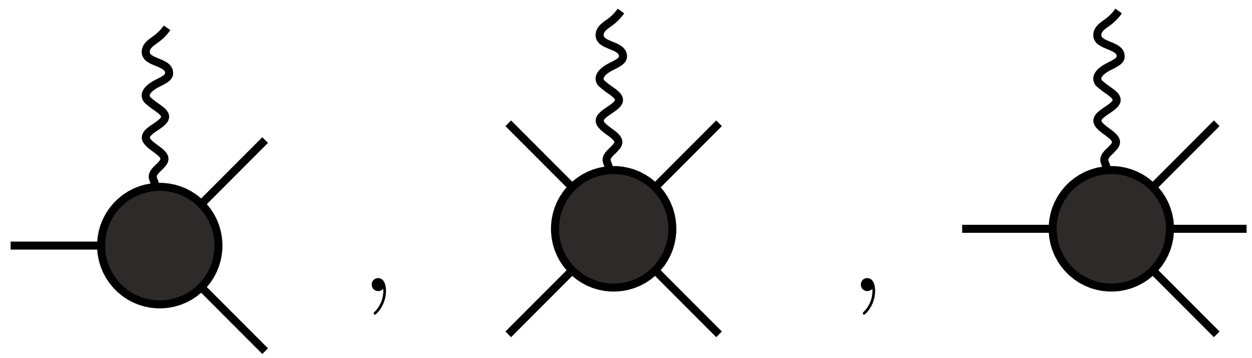
Many others…

Few-Hadron Matrix Elements

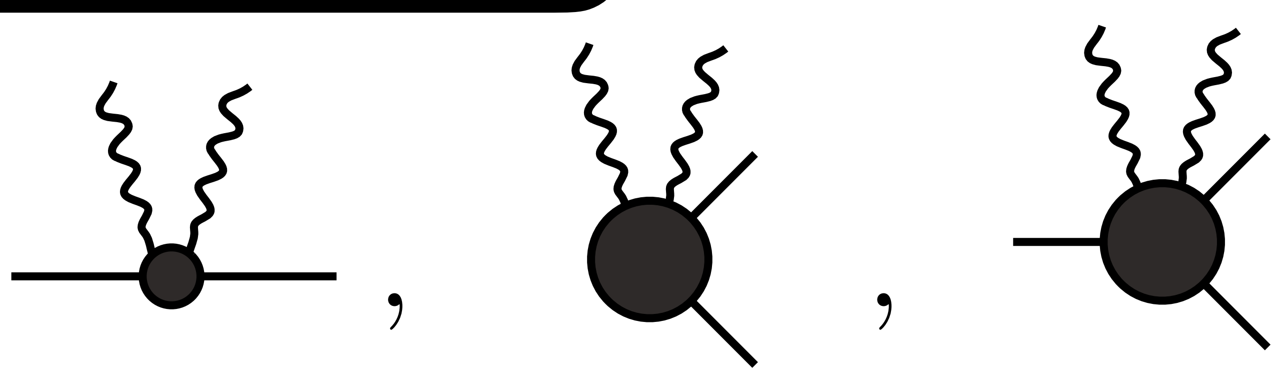
Hadronic scattering



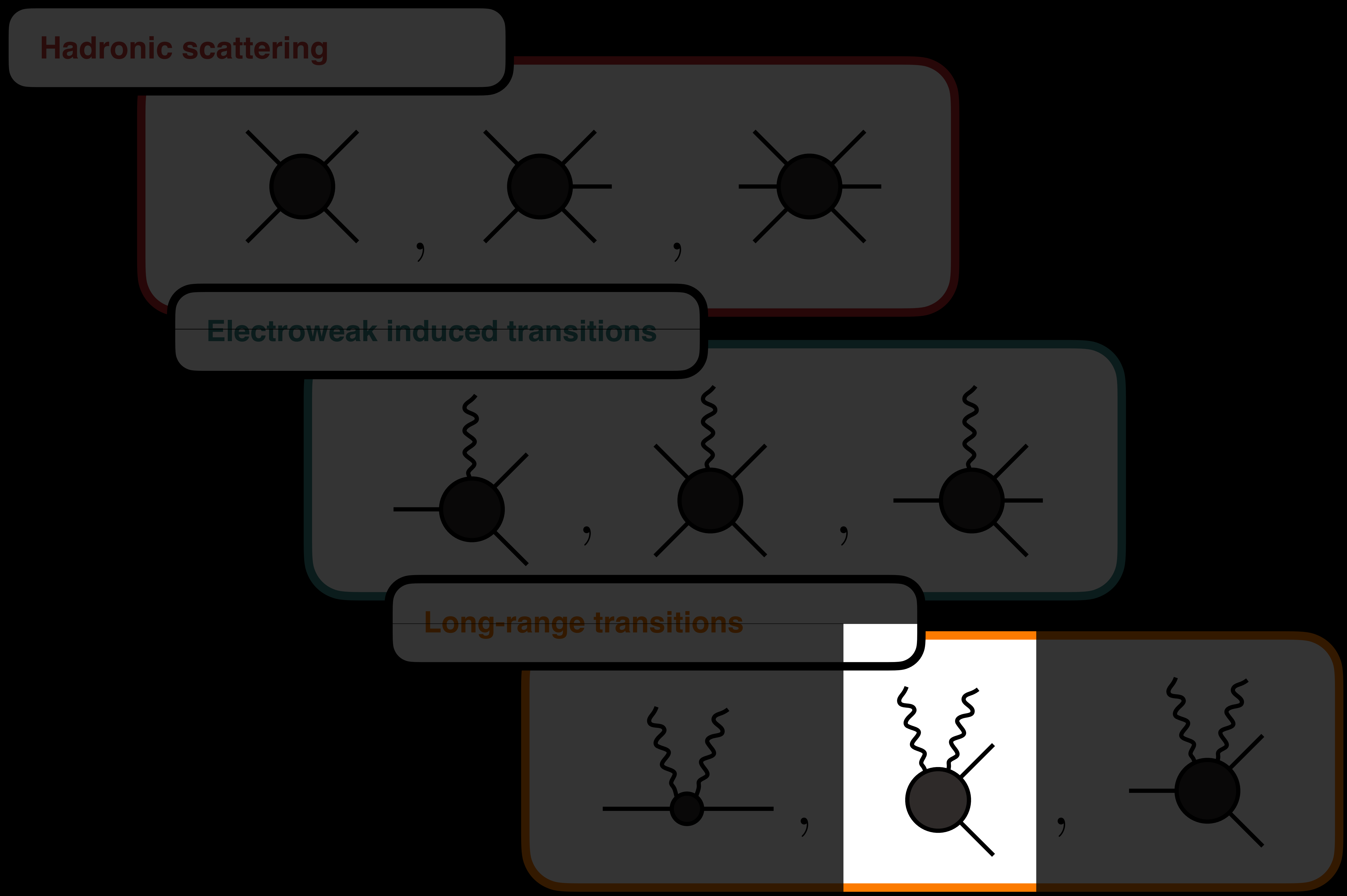
Electroweak induced transitions



Long-range transitions



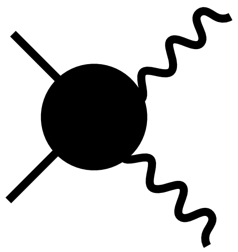
Few-Hadron Matrix Elements



Two-current systems

Coupling two currents to hadronic systems

- Restrict kinematic region to two-pions — Isolate singularity structure



$$= \quad \mathcal{T}^{\mu\nu} = i \int \mathrm{d}^4 x \, e^{-iq\cdot x} \, \langle \pi\pi | \, \mathsf{T} \mathcal{J}^{\mu}_{\mathsf{M}}(t) \mathcal{J}^{\nu}(0) \, | \Omega \rangle_{\infty}$$

F. Ortega-Gama, K. Sherman, AJ, R. Briceño,
Phys. Rev. D **105** (2022)

AJ, R. Briceño, A. Rodas, J. Guerrero
Phys. Rev. D **107** (2023)

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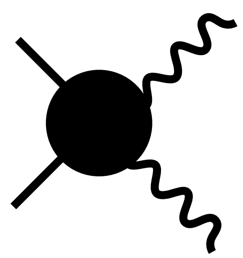

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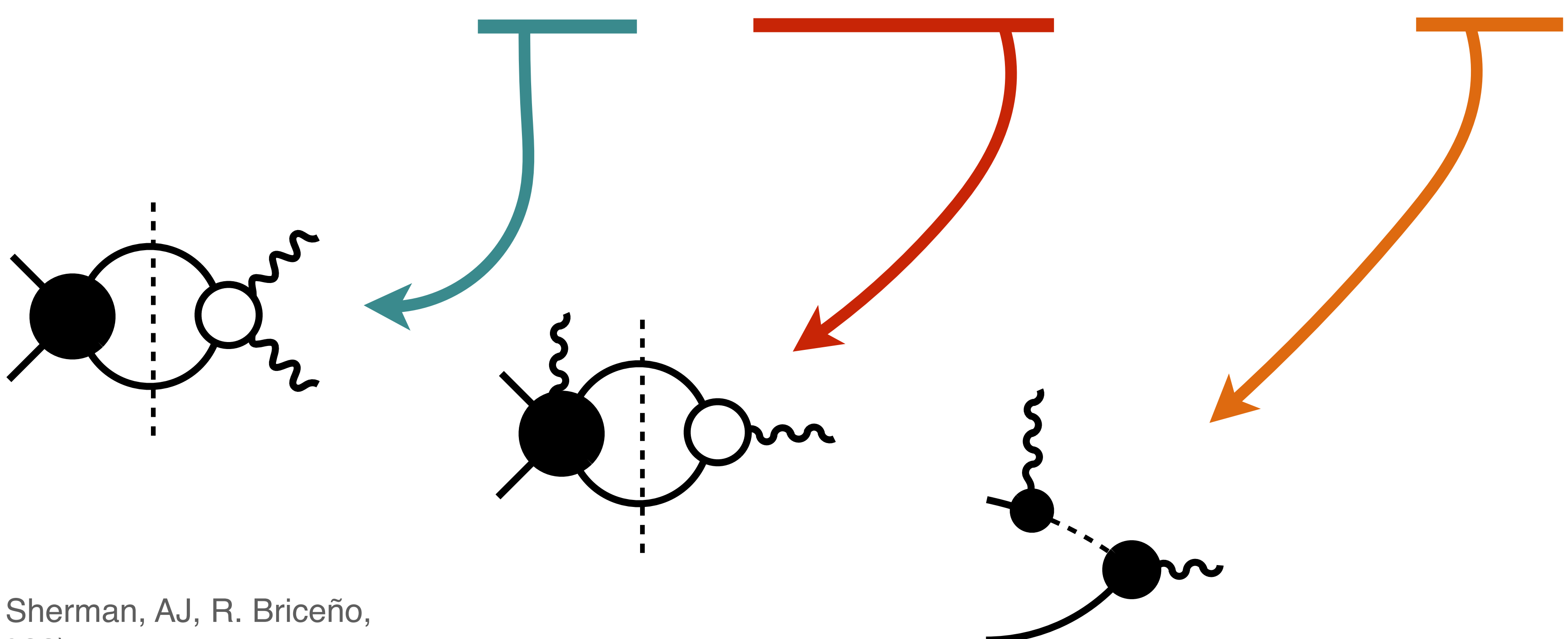
$$= \cdots$$

$$= \mathcal{M} \cdot \mathcal{B}^{\mu\nu} + \mathcal{W}^{\mu}_{\mathrm{df}} \cdot \mathcal{A}^{\nu} + \mathcal{W}^{\nu}_{\mathrm{df}} \cdot \mathcal{A}^{\mu} + \mathrm{poles}$$



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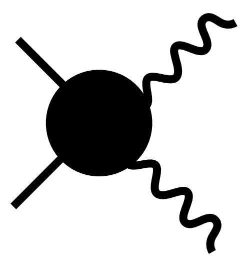

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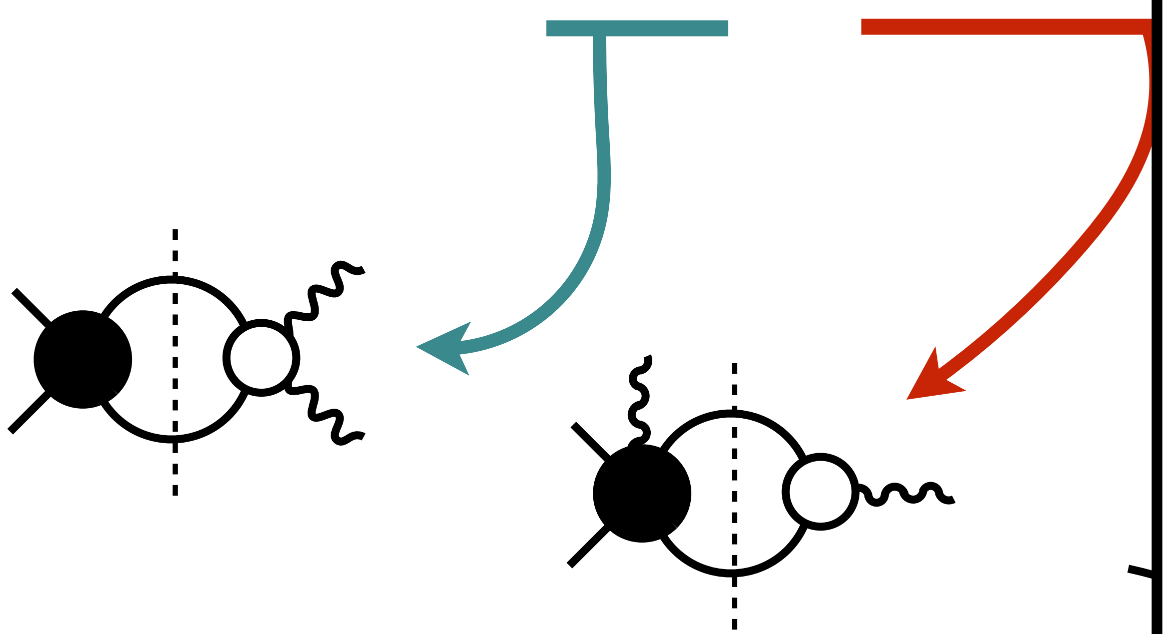

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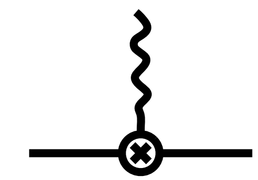
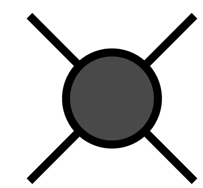
Computable from Lattice QCD

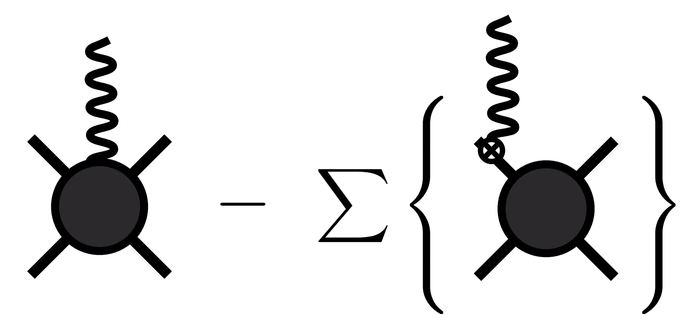
Single hadron form-factors

$f \; =$ 

Hadronic scattering amplitude

$\mathcal{M} \; =$ 

Local transition amplitude

$\mathcal{W}_{\mathrm{df}} \; =$ 

AJ, R. Briceño, F. Ortega-Gama, K. Sherman,
Phys. Rev. D **103** 114512 (2021)

A. Baroni, R. Briceño, M. Hansen, F. Ortega-Gama,
Phys. Rev. D **100** 034511 (2019)

Two-current systems

Coupling two currents to hadronic systems

- Non-local matrix elements — Need to correct for Euclidean/Minkowski time

$$M_L^{\mu\nu}(\tau) = \int_L \mathrm{d}^3\mathbf{x}\, e^{iq\cdot x} \, \langle \pi\pi, L| \, \mathsf{T}_{\mathsf{E}} \mathcal{J}^{\mu}_{\mathsf{E}}(\tau, \mathbf{x}) \mathcal{J}^{\nu}(0) \, |\Omega\rangle_L$$

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$$\mathcal{T}^{\mu\nu}(\tau) \sim \int \mathrm{d}\tau e^{-q\tau}\, M_L^{\mu\nu}(\tau) \sim \int_0^\infty \mathrm{d}\tau\, e^{(E_{\pi\pi} - (q_0 + E_{\mathfrak{n}}))\tau}\, c_{\mathfrak{n}}^{\mu\nu} \xrightarrow[E_{\pi\pi} > q_0 + E_{\mathfrak{n}}]{} \infty$$

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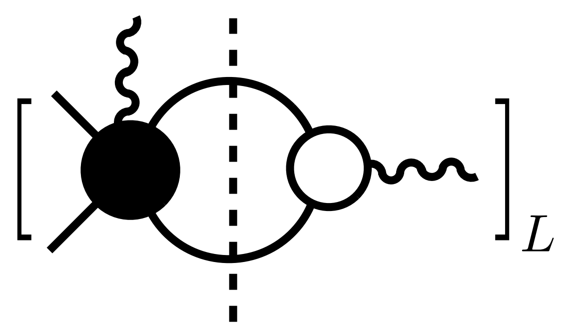

Euclidean and finite-volume artifacts are directly intertwined, and are due to intermediate states going on-shell

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Two-current systems

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- Non-local matrix elements — Need to correct for Euclidean/Minkowski time

(1) Define subtracted function — safe integration

$$\Delta M_L^{\mu\nu,>}(\tau) \sim M_L^{\mu\nu}(\tau) - \sum_{\mathfrak{n}}^{N} e^{(E_{\pi\pi}-(q_0+E_{\mathfrak{n}}))\tau} c_{\mathfrak{n}}$$

(2) Reconstruct FV poles

$$\mathcal{T}_L^{\mu\nu,>}(\tau) \sim \sum_{\mathfrak{n}}^{N} \frac{c_{\mathfrak{n}}}{E_{\mathfrak{n}} - (E_{\pi\pi} - q_0)}$$

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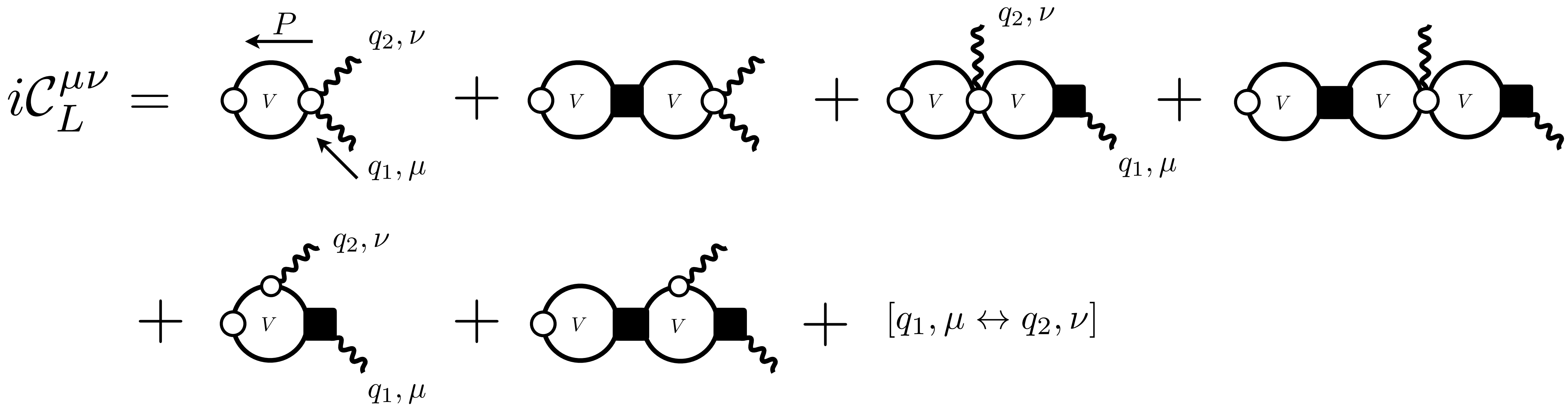
(3) Remove FV artifacts



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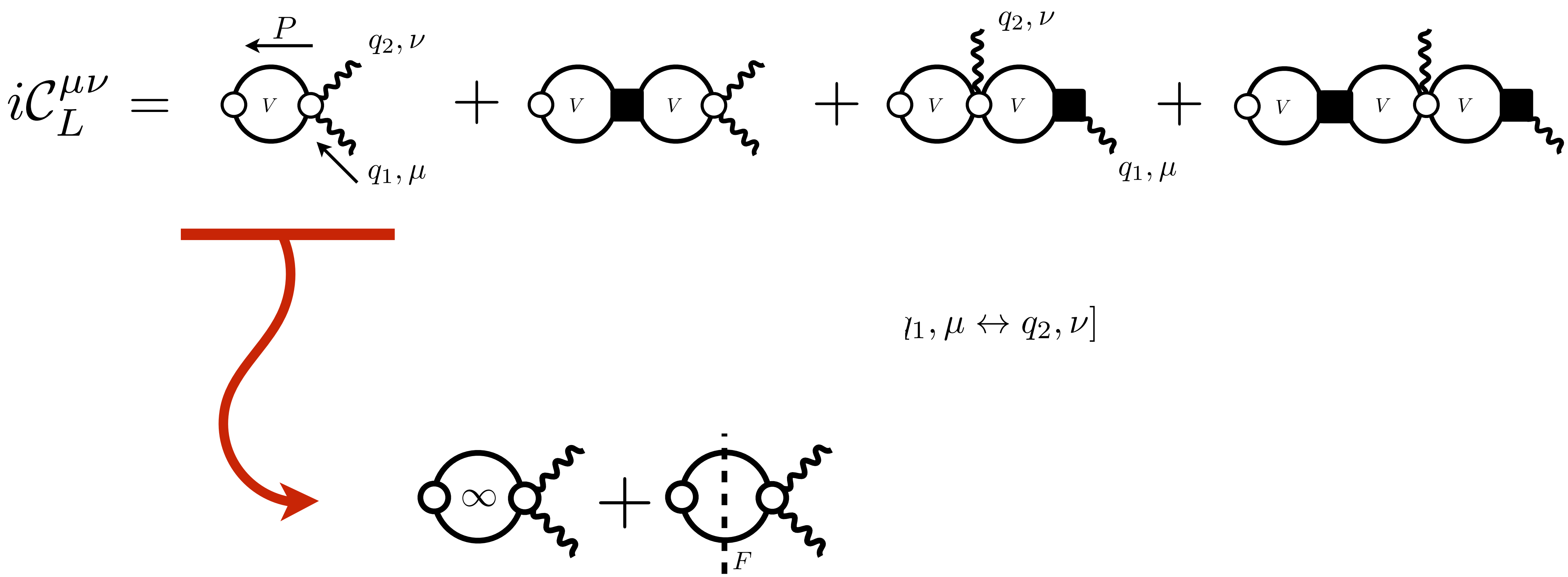

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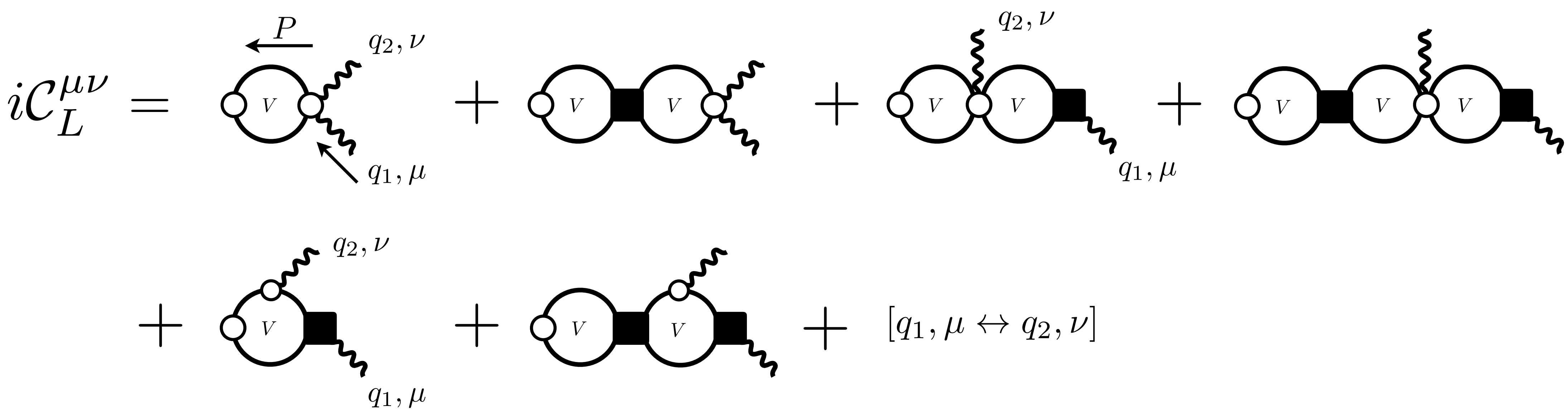

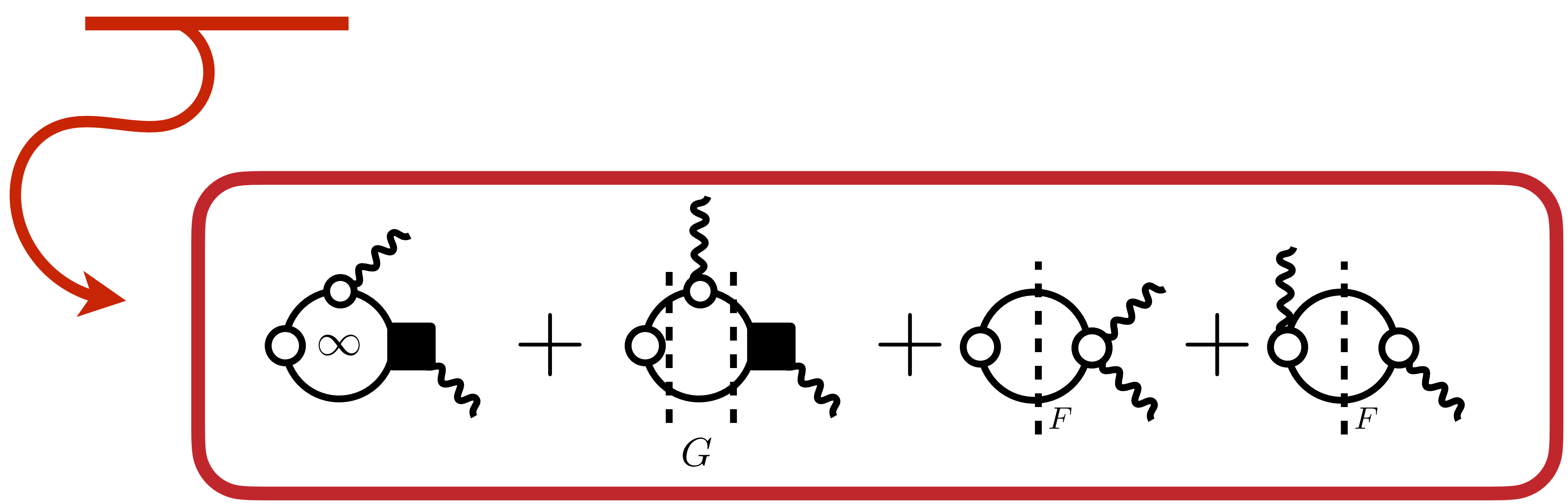

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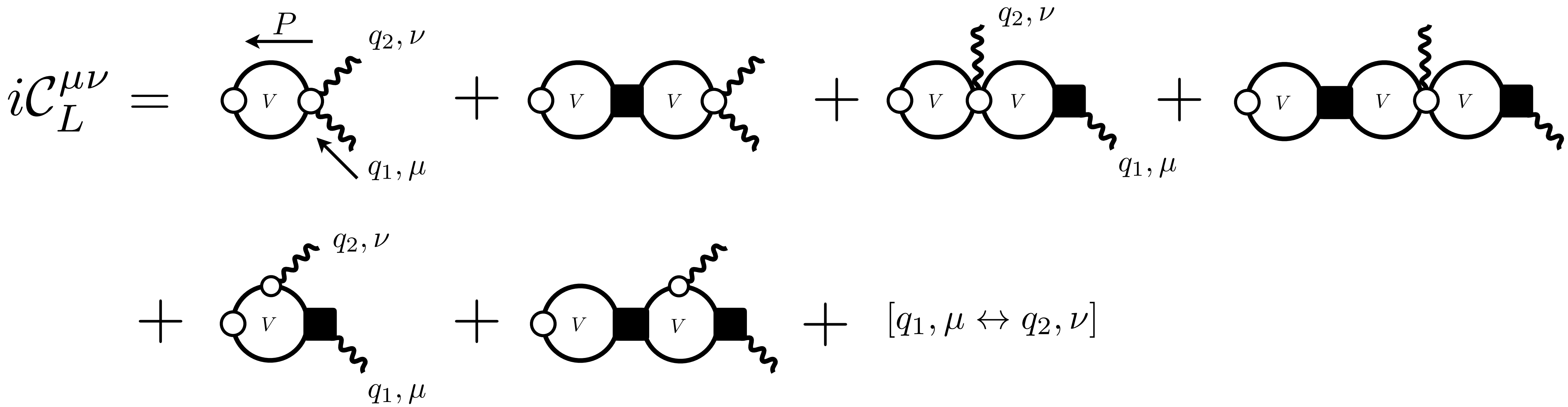

$$= -L^3 \frac{Z_n}{E - E_n} \sqrt{\frac{\mathcal{R}}{2E_n L^3}} \cdot \left(\mathcal{T}_{\rm df}^{\mu\nu} - \Delta\mathcal{T}_{L,\rm df}^{\mu\nu} \right)$$

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Two-current systems

Coupling two currents to hadronic systems

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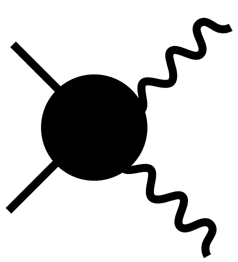
(4) Combine all pieces

$$\sqrt{\frac{\mathcal{R}}{2E_{\pi\pi}L^3}} \cdot \mathcal{T}_{\mathrm{df}}^{\mu\nu} = \int d\tau \; e^{\omega\tau} \, M_L^{\mu\nu,>}(\tau) + \left[\mathcal{T}_L^{\mu\nu,<} + \sqrt{\frac{\mathcal{R}}{2E_{\pi\pi}L^3}} \cdot \Delta\mathcal{T}_{L,\mathrm{df}}^{\mu\nu} \right]$$

$$\mathcal{T}_{\mathrm{df}}^{\mu\nu} = \mathcal{M} \cdot \mathcal{B}^{\mu\nu} + \mathcal{W}_{\mathrm{df}}^{\mu} \cdot \mathcal{A}^{\nu} + \mathcal{W}_{\mathrm{df}}^{\nu} \cdot \mathcal{A}^{\mu}$$

$$\mathcal{T}^{\mu\nu} = \mathcal{T}_{\mathrm{df}}^{\mu\nu} + \text{poles} \quad =$$



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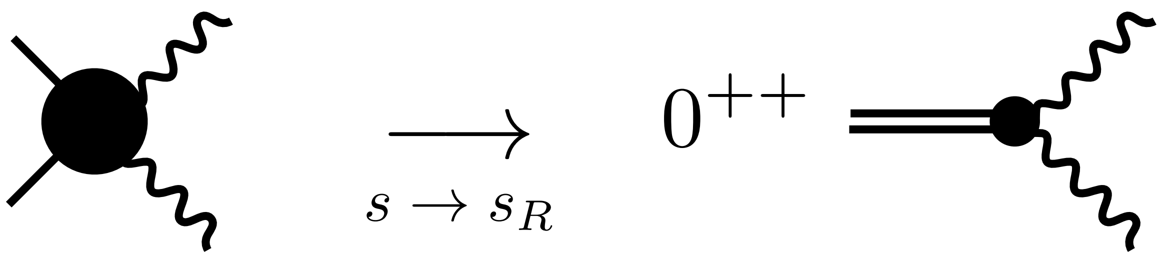
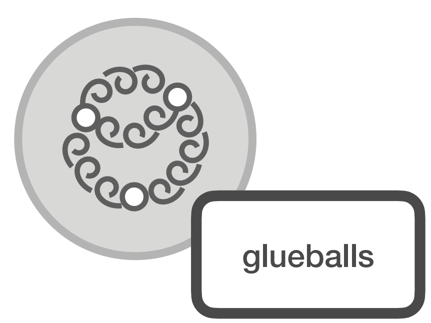
Two-current systems

Coupling two currents to hadronic systems

- Sensitivity to potential glueball components

(5) Analytically continue to resonance pole





Access to two-photon resonance coupling

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Summary

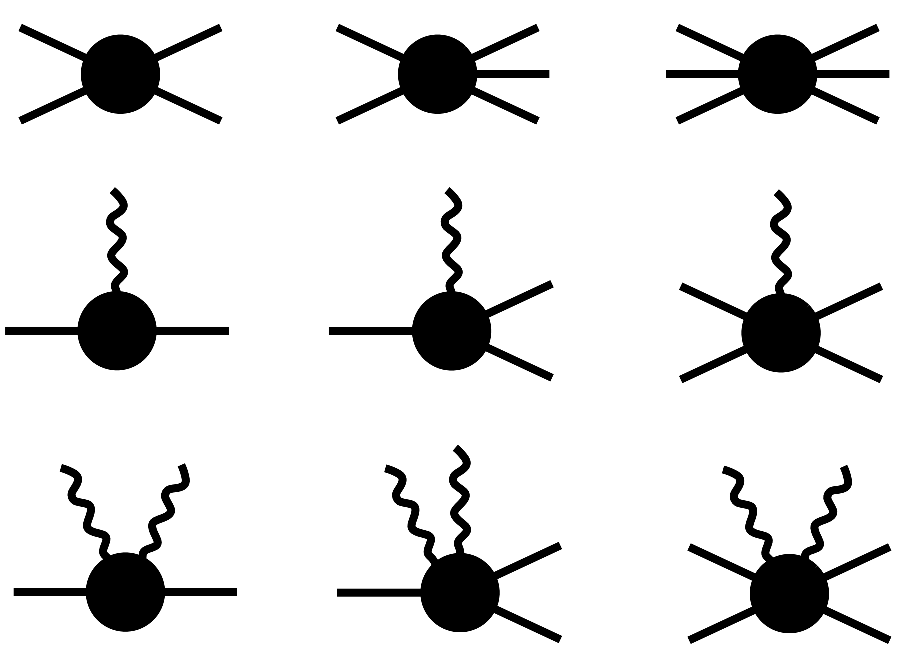
Few-body interactions play a key role in many outstanding problems in nuclear & hadron physics

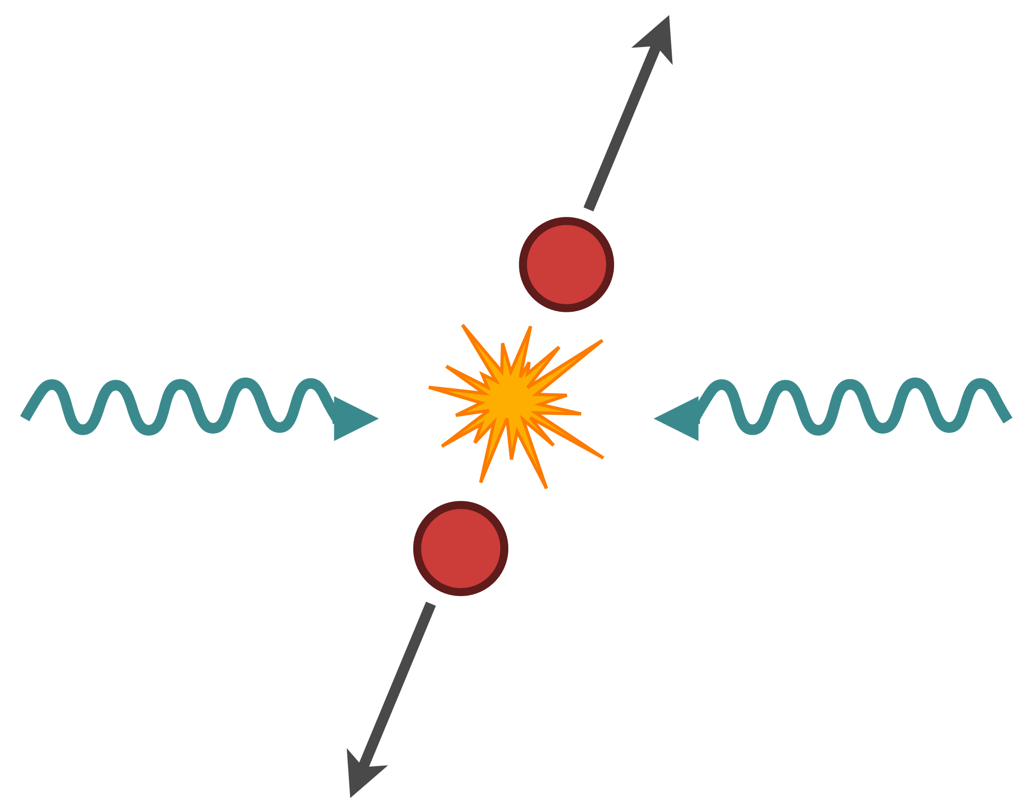
Lattice QCD, EFTs, & Scattering theory combined provide useful tools to extract physics from QCD

- Rapid development in formalisms relating lattice QCD observables to amplitudes
- Scattering phenomenology is advancing in tandem

Latest developments in three-body scattering & two-body matrix elements

- First applications appearing in literature
- Can address increasingly complicated processes





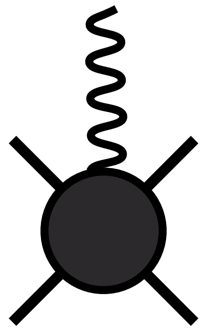
Much more to come!

AJ acknowledges financial support from The Gordon and Betty Moore Foundation and the American Physical Society to present this work at the GHP 2023 workshop.

Resonance Structure & Electroweak Probes

Scattering theory of $\mathbf{2} + \mathcal{J} \to \mathbf{2}$ amplitudes

$$i\mathcal{W} \;=$$



AJ, R. Briceño, F. Ortega-Gama, K. Sherman,
Phys. Rev. D **103** 114512 (2021)

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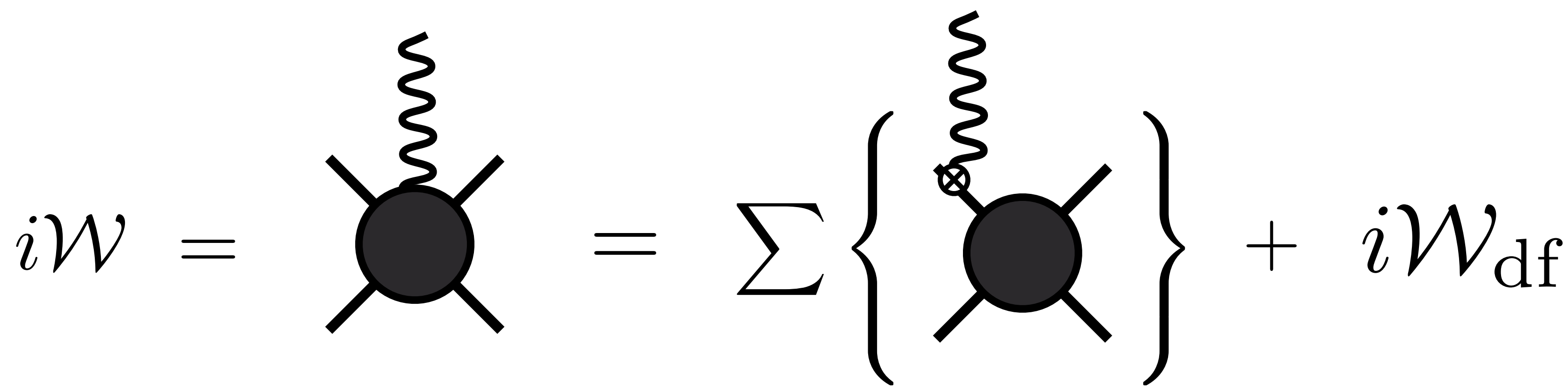

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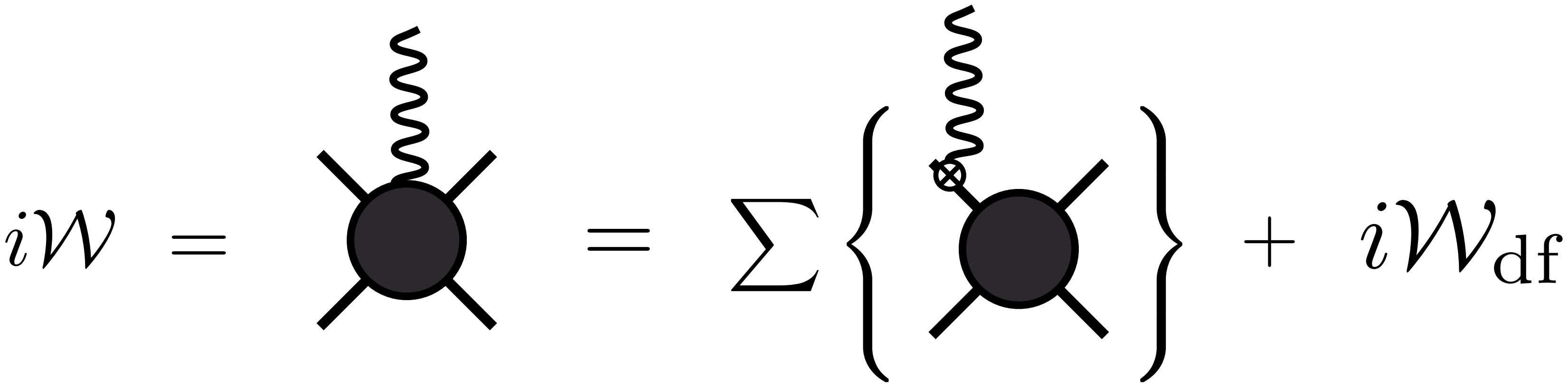

After considerable manipulations…

$$\mathcal{W}_{\mathrm{df}} = \mathcal{M} \cdot (\mathcal{A} + f \cdot \mathcal{G}) \cdot \mathcal{M}$$

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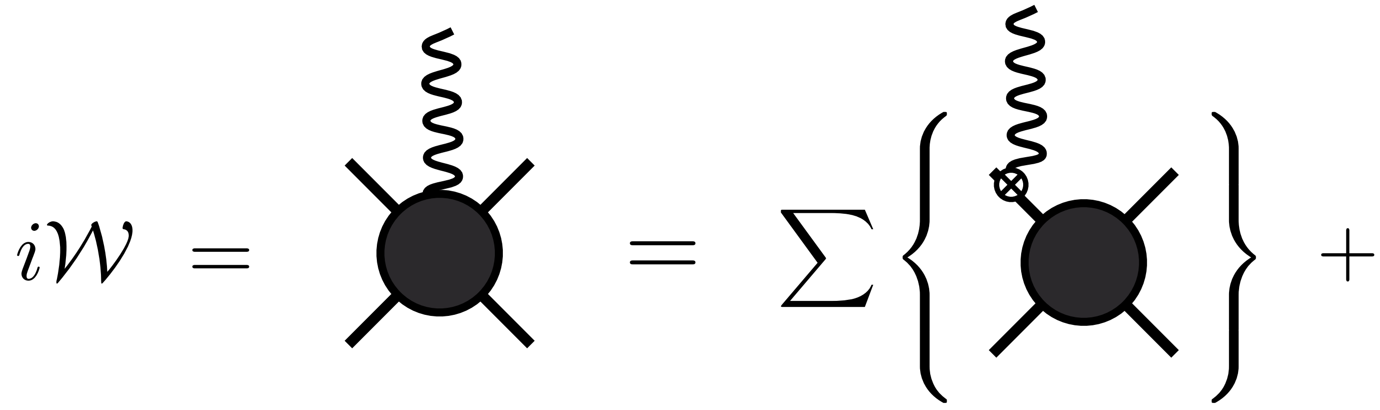


After considerable manipulations…

$$\mathcal{W}_{\rm df} = \mathcal{M} \cdot (\mathcal{A} + f \cdot \mathcal{G}) \cdot \mathcal{M}$$

Unknown short-distance function

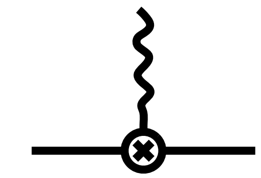
— Constrain using Lattice QCD

— Constrained by Ward-Takahashi identity

Computable from Lattice QCD

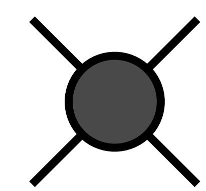
Single hadron form-factors

f =



Hadronic scattering amplitude

$\mathcal{M}$ =

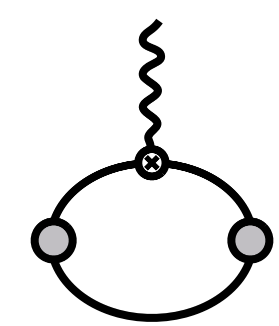


Known Kinematics

Triangle diagram

Contains normal and anomalous singularities from intermediate on-shell particles

$\mathcal{G}$ =



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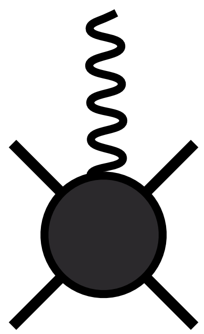
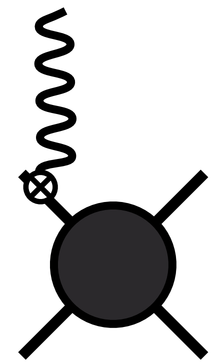


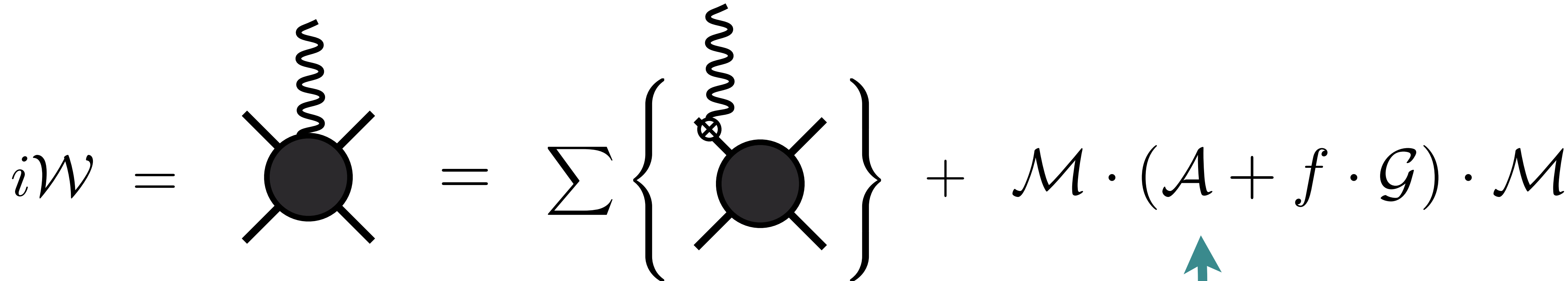
$$= \sum \Big\{$$



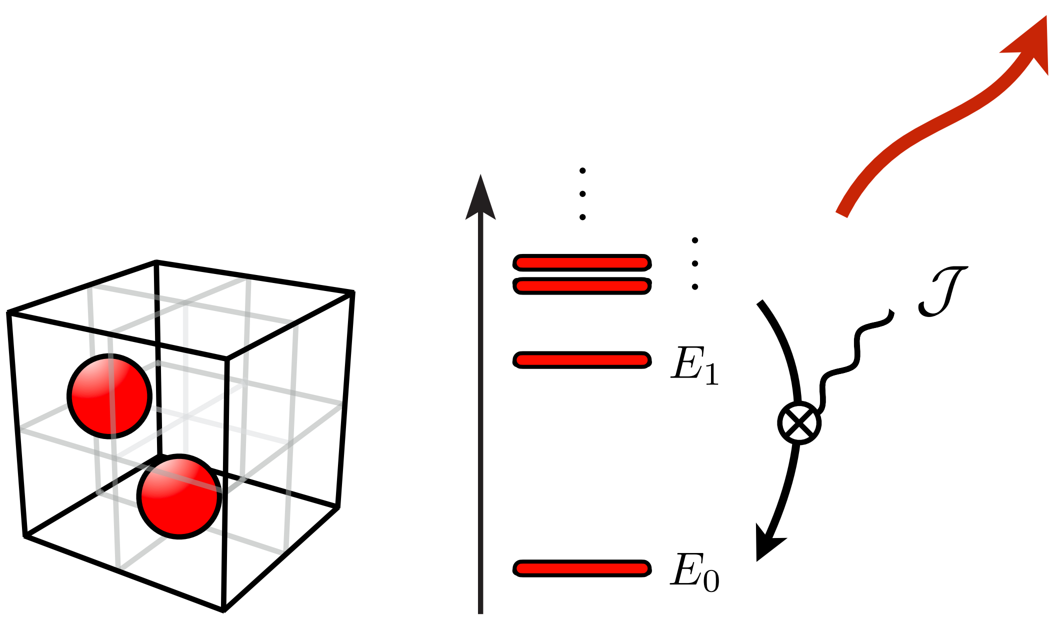
$$\Big\} + \mathcal{M} \cdot (\mathcal{A} + f \cdot \mathcal{G}) \cdot \mathcal{M}$$

Resonance Structure & Electroweak Probes

Scattering theory of $\mathbf{2} + \mathcal{J} \to \mathbf{2}$ amplitudes



$$\langle \mathbf{2} | \, \mathcal{J} \, | \mathbf{2} \rangle_L \sim \sqrt{\mathcal{R}_L} \cdot (\mathcal{A} + f \cdot \mathcal{G}_L) \cdot \sqrt{\mathcal{R}_L}$$



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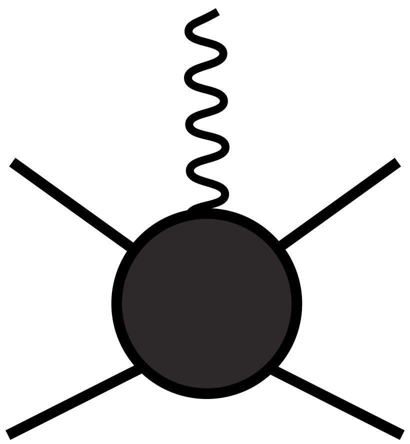
A. Baroni, R. Briceño, M. Hansen, F. Ortega-Gama,
Phys. Rev. D **100** 034511 (2019)

Resonance Structure & Electroweak Probes

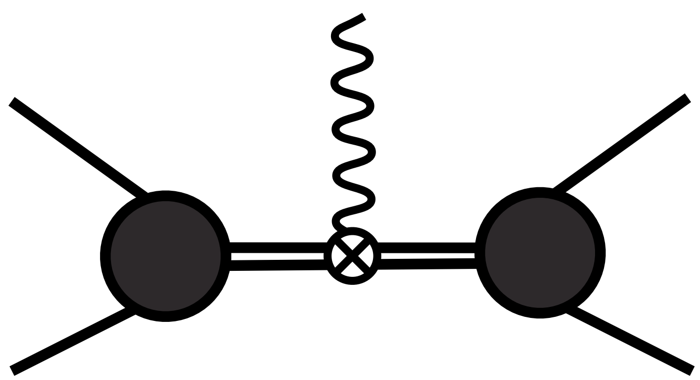
Scattering theory of $\mathbf{2} + \mathcal{J} \to \mathbf{2}$ amplitudes

- First class of resonance structure functions — form factors

$i\mathcal{W} =$



$\longrightarrow$



$$\sim \frac{g_f}{E_f - E_R} \cdot F_R(Q^2) \cdot \frac{g_i}{E_i - E_R}$$

AJ, R. Briceño, F. Ortega-Gama, K. Sherman,
Phys. Rev. D **103** 114512 (2021)

AJ, R. Briceño, M. Hansen,
Phys. Rev. D **100** 114505 (2019)

AJ, R. Briceño, M. Hansen,
Phys. Rev. D **101** 094508 (2020)

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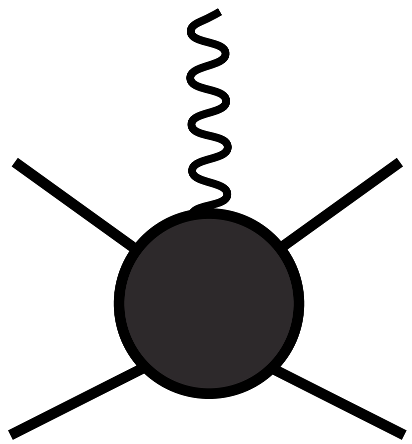

Fix *K* matrix from Lattice QCD

Determine poles and couplings

Compute single hadron matrix element / form factor

Fix *A* matrix from Lattice QCD

$$F_R(Q^2) = g_i g_f \left(\mathcal{A} + f \cdot \mathcal{G}\right)\Big|_{E_f = E_i = E_R}$$

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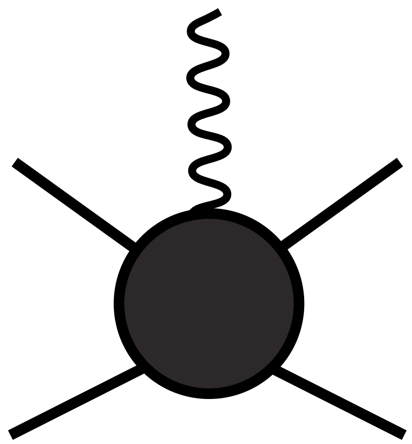
$i\mathcal{W} =$ 

Access to resonance charge radii

$$\langle r^2 \rangle = -6 \, \frac{\mathrm{d}}{\mathrm{d}Q^2} F(Q^2) \Big|_{Q^2=0}$$

Fix *K* matrix from Lattice QCD

Determine poles and couplings

Compute single hadron matrix element / form factor

Fix *A* matrix from Lattice QCD

$$F_R(Q^2) = g_i g_f \left(\mathcal{A} + f \cdot \mathcal{G} \right) \Big|_{E_f = E_i = E_R}$$

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Resonance Structure & Electroweak Probes

First class of structural observables of unstable hadrons

- Potential to discriminate compact / extended objects