The Non – Perturbative Structure of Pion

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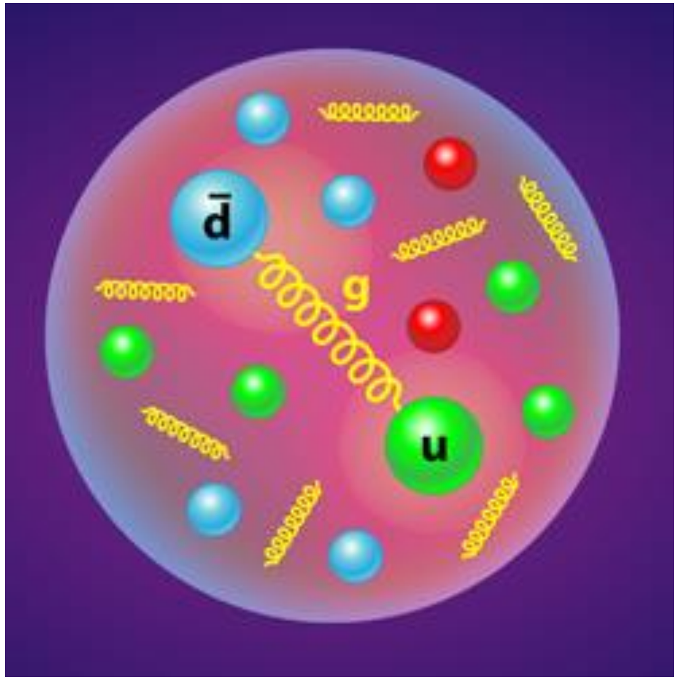
Pion



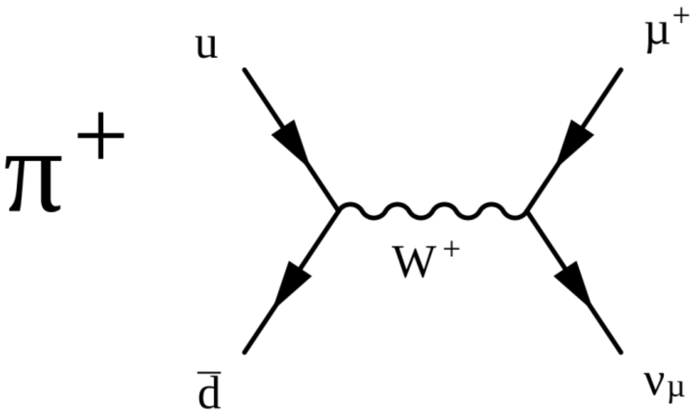
- Lightest strongly interacting bound system of valence quark and anti-quark, sea quarks, and gluons ($\sim 140\ MeV$).
- Role in chiral symmetry breaking of QCD as a pseudo – Goldstone boson.
- Short lifetime : π^+, π^- $(2.6 \times 10^{-8}\ s)$ decay electroweakly and π^0 $(8.5 \times 10^{-17} s)$ decay electromagnetically .
- Therefore, no fixed target experiments with pions.

$\pi^+ \rightarrow \mu^+ + \nu_\mu$

$\pi^- \rightarrow \mu^- + \overline{\nu}_\mu$

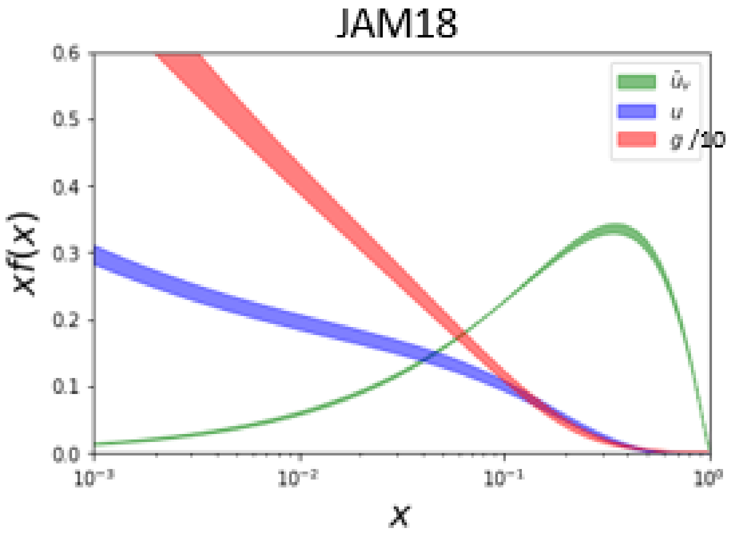
$\pi^0 \rightarrow 2\gamma.$

$\pi^0 \rightarrow \gamma + e^- + e^+.$



Partonic Structure of Pions

- Historically, the first DY processes also demonstrated that pions have a partonic picture similar to baryons.
- One wants to understand the partonic structure of the pion by extracting the PDFs:
- Drell Yan Process (DY)
- Leading Neutron and Sullivan Process (LN)



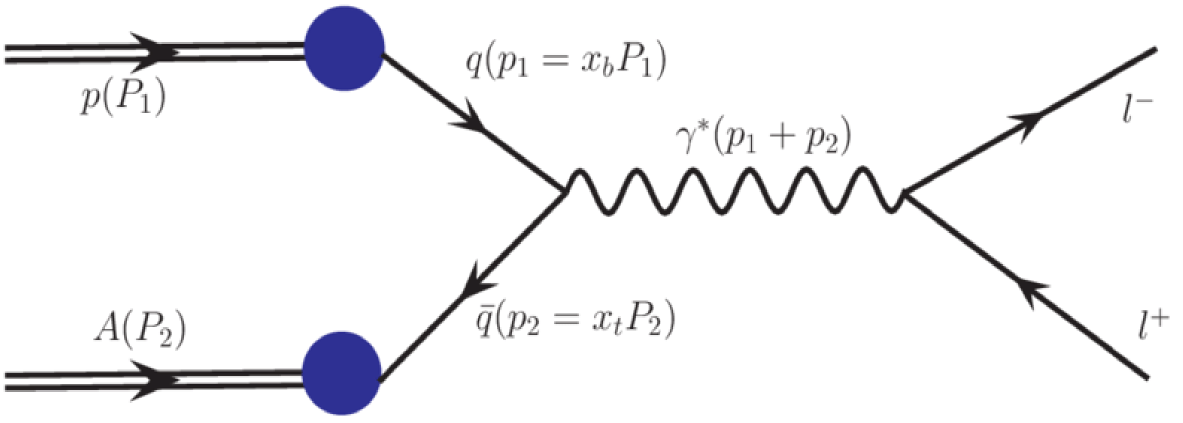
Drell Yan (DY) Process

- Hadron – Hadron collision

Pion : $\pi A \rightarrow \mu^{+}\mu^{-}X$

- Non – perturbative, universal PDFs $f_i^{\pi}(x_\pi)$ & $f_j^{A}(x_A)$
- Hard Coefficients C_{ij} (factorization scale)

$$\frac{d^2\sigma}{dQ^2dY} = \frac{4\pi\alpha^2}{9Q^2S}\sum_{i,j}\int_{x_\pi}^{1}\frac{d\hat{x}_\pi}{\hat{x}_\pi}\int_{x_A}^{1}\frac{d\hat{x}_A}{\hat{x}_A} \times C_{ij}(\hat{x}_\pi,\hat{x}_A,x_\pi,x_A,Q/\mu) f_i^{\pi}(\hat{x}_\pi,\mu) f_j^{A}(\hat{x}_A,\mu).$$

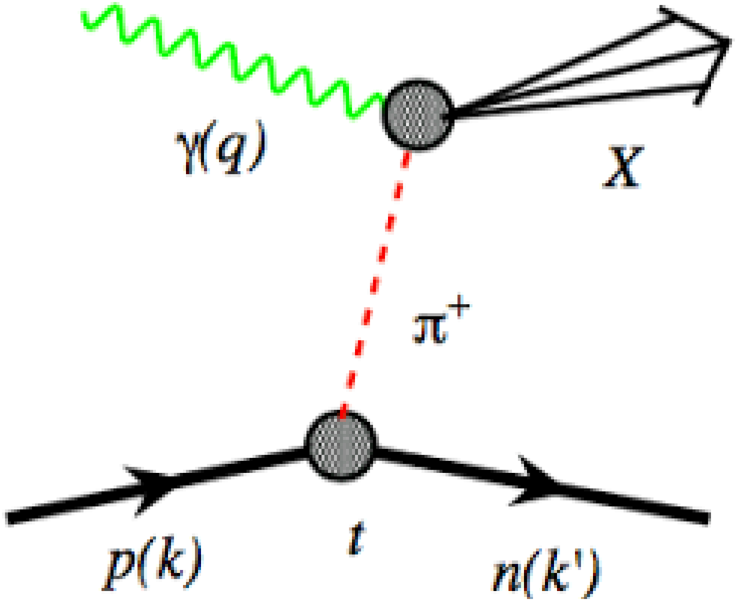


$$x_\pi = \sqrt{\frac{Q^2}{s}}e^{Y}, \text{ and } x_A = \sqrt{\frac{Q^2}{s}}e^{-Y}$$

Invariant Mass of Lepton Pair (time − like) $Q^2 = x_\pi x_A s$

Leading Neutron (LN) and Sullivan Process

- Electron – Proton scattering : $e \ + p \rightarrow n + e' + X$
- p becomes n and near on-shell mass pion at low $|t|$ & $x_L \rightarrow 1$
- The n and e is detected , while the pion is shattered.



$$\frac{d^3\sigma}{dx dQ^2 dx_L} = \frac{4\pi\alpha^2}{xQ^4}(1 - y + \frac{y^2}{2})F_2^{LN(3)}(x, Q^2, x_L)$$

$$F_2^{LN(3)}(x, Q^2, x_L) = 2f_{\pi n}^{(on)}(\bar{x}_L)F_2^{\pi}(x_\pi, Q^2)$$

$$F_2^{\pi}(x_\pi, Q^2) = \sum_i \int_{x_\pi}^{1} d\hat{x} H_i^{DIS}(\hat{x}, \mu^2, Q^2) f_{i/\pi}(x_\pi \ \hat{x}, \mu^2)$$

$$t = (p - p_n)^2 \qquad x_L = 1 - \frac{q\cdot(p-p_n)}{q\cdot p} \approx \frac{E_n}{E_P}$$

$$x = \frac{Q^2}{2p\cdot q} \qquad y = \frac{p\cdot q}{p\cdot l} \approx \frac{Q^2}{xs};$$

$$\bar{x}_L = 1 - x_L \qquad x_\pi = \frac{x}{\bar{x}_L}$$

High x_π Debate

- For high x, Brodsky – Farrar Counting rule:
  $f_V(x) \sim (1-x)^{\beta_V}$ $\beta_V = 2$ Pion

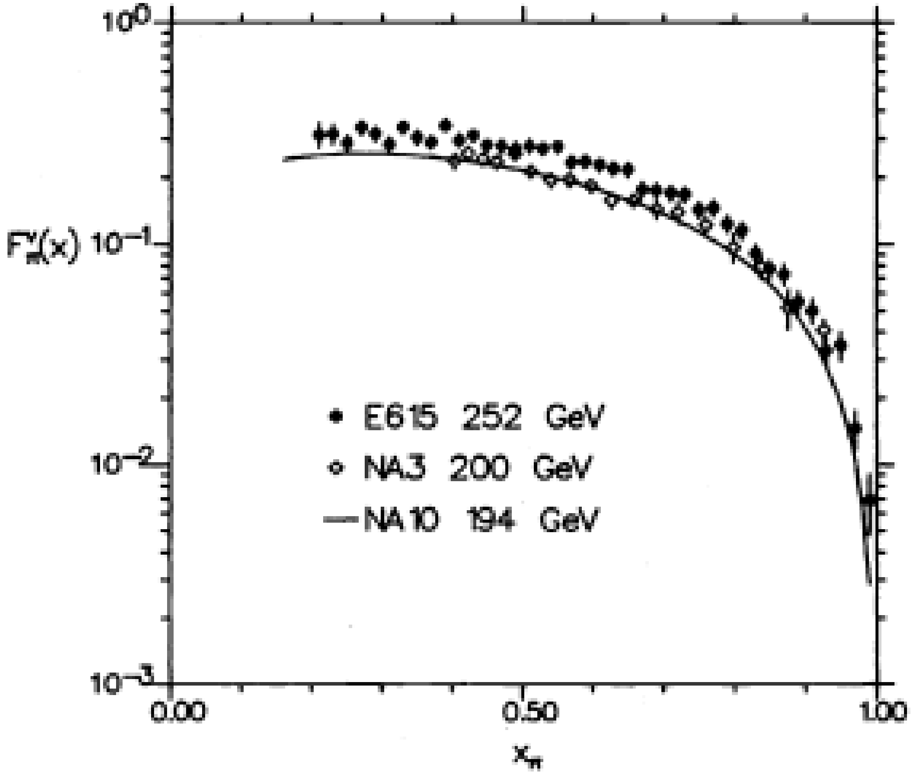
$$\beta_V = 2n - 1 + 2|\delta\lambda|$$

(pQCD) (1975 & 1979)

E. L. Berger and S. J. Brodsky, Physical Review Letters42, 940 (1979).

- FermiLab DY data for the pion $\beta_V = 1.2$ for LO analysis (1989) J. S. Conway et al., Phys. Rev. D 39, 92 (1989)
- NLO analysis increased $\beta_V = 1.5$ (2005)

Phys. Rev. Lett. **105**, 252003



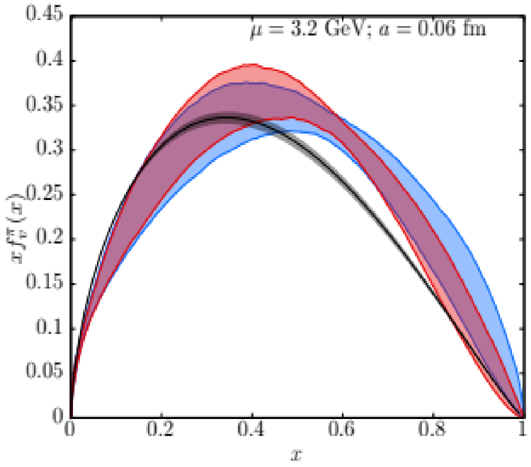
Recent Developments

- Extracted Pion PDF in DY and Sullivan process using Bayesian Monte Carlo Global Analysis – JAM18 $\beta_V = 1$

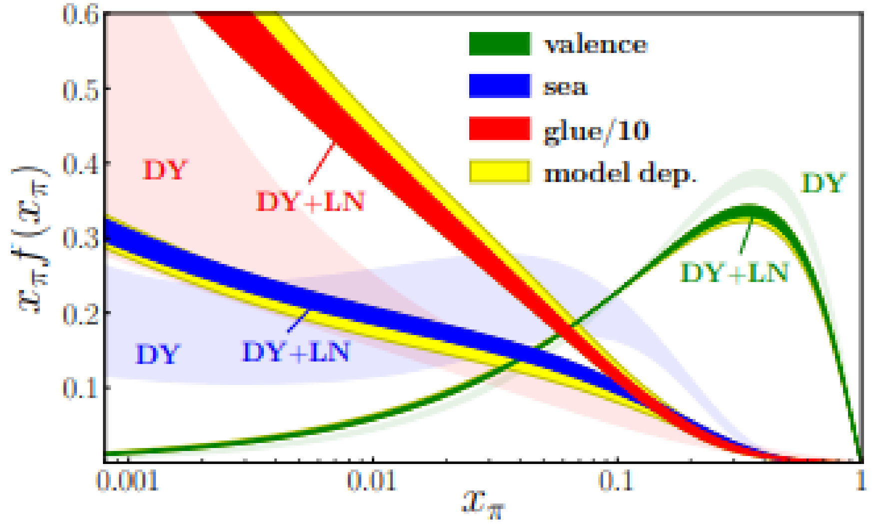
  arXiv: 1804.01965.

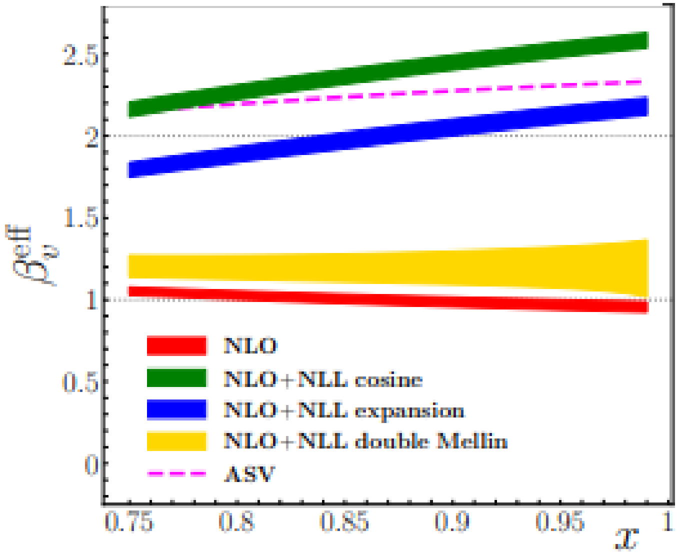
- JAM 2021 analysis – applied threshold resummation on Drell-Yan (NLO, NLL) $\beta_V \approx 1$ to > 2.5
- *Lattice QCD* - pion PDF through of pseudo-/quasi-PDFs.
- $\beta_V = 1$ or $\beta_V = 2$ is debated.

Phys. Rev. D **102**, 094513



JAM18





arXiv: 2108.05822

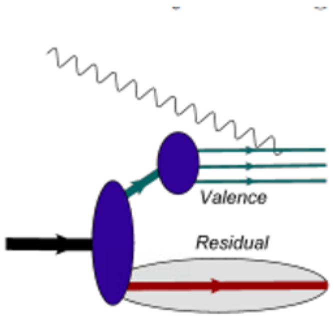
Goals

- We want to model the pion PDF using our model.

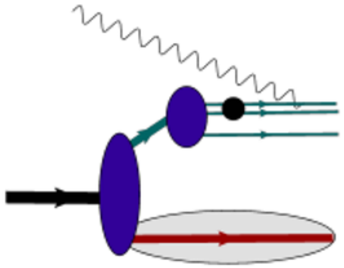
- For the nucleon, there are three mechanisms contributing to valence PDFs:

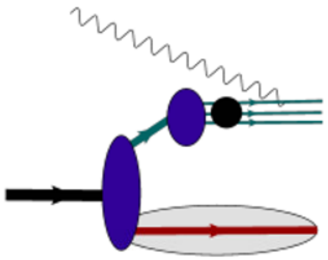
  1. mean field
  2. 2q short-range correlations,
  3. 3q short-range correlations.

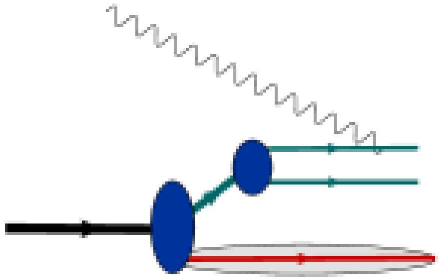
- For the pion, we model the soft contribution and hard $q\bar{q}$ short-range correlations.

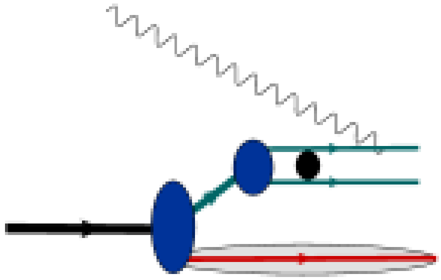


Mean Field


2q Short Range Correlation

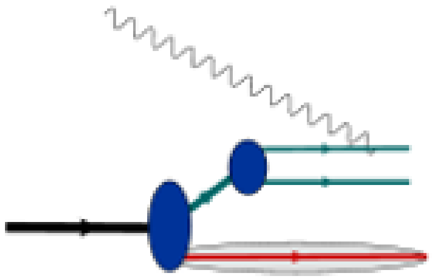

3q Short Range Correlation


Soft Contribution

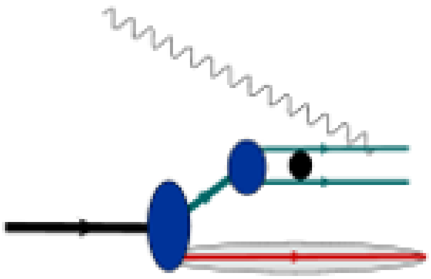

$q\bar{q}$ Short -Range Correlations

Residual Field Model - Pion

- Our goal to describe valence PDFs in the region : 0.1 < x < 1 as effective fermions whose number is conserved. (DIS)
- The model is based on assumption of pion transition into the valence $q\bar{q}$ cluster and residual system.
- In which 2 main mechanisms define the PDFs in the region of 0.1 < x < 1
- 2 main contributions
  I. *Soft contribution*
  II. Hard $q\bar{q}$ *correlation contribution*

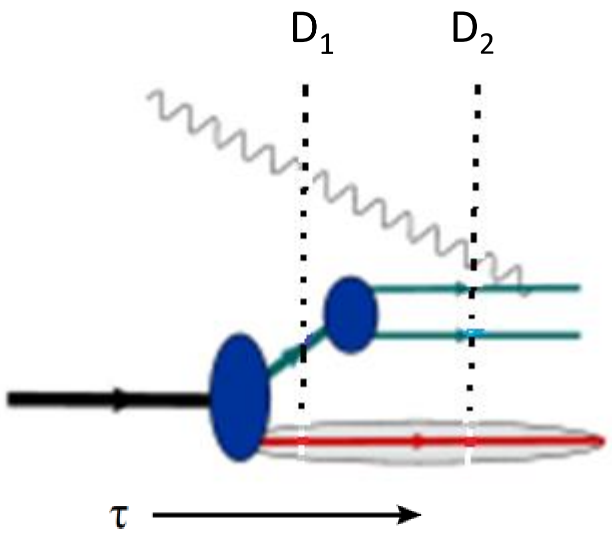


Soft Contribution



$q\bar{q}$ Short -Range Correlations

Pion Residual Model



- Calculating PDF using Effective LF diagrammatic method (LO).
- Charge symmetry – $f_V(\mathrm{x}, \mathrm{Q}^2) = u_V^{\pi^+}(x, Q^2) = d_V^{\pi^-}(x, Q^2)$

$$\Psi(\{x_i, \mathbf{k}_{i,\perp}\}_{i=1,2,R}) = \psi_{qq}(\{x_i, \mathbf{k}_{i,\perp}\}_{i=1,2})\psi_{VR}(x_R, \mathbf{k}_{R,\perp})$$

$$\tilde{\mathbf{k}}_{i,\perp} = \mathbf{k}_{i,\perp} - \beta_i \mathbf{k}_{V,\perp} \quad \text{(i=1,2)}$$

$$\begin{aligned} f_V(x_B, Q^2) &= \int^{Q^2} \frac{dx_1 d^2\mathbf{k}_{1,\perp}}{16\pi^3 x_1} \frac{dx_2 d^2\mathbf{k}_{2,\perp}}{16\pi^3 x_2} \frac{dx_R d^2\mathbf{k}_{R,\perp}}{16\pi^3 x_R} \delta(x_1 - x_B) \\ &\times 16\pi^3 \delta\left(1 - \sum_{i=1,2,R} x_i\right) \delta^{(2)}\left(\sum_{i=1,2,R} \mathbf{k}_{i,\perp}\right) \\ &\times |\Psi(\{x_i, \mathbf{k}_{i,\perp}\}_{i=1,2,R})|^2 \end{aligned} \quad (6)$$

$$f_V(x_B, Q^2) = \int_0^{1-x_B} \frac{dx_R}{(16\pi^3)^2 x_B(1 - x_B - x_R)x_R} \int_0^{Q^2} d^2\tilde{\mathbf{k}}_{1,\perp} d^2\tilde{\mathbf{k}}_{2,\perp} \delta^{(2)}\left(\sum_{i=1,2} \tilde{\mathbf{k}}_{i,\perp}\right) |\psi_{q\bar{q}}(\{x_i, \tilde{\mathbf{k}}_{i,\perp}\}_{i=1,2})|^2 \int_0^{Q^2} d^2\mathbf{k}_{R,\perp} |\psi_{VR}(x_R, \mathbf{k}_{R,\perp})|^2$$

Modeling Non – Perturbative Valence $q\bar{q}$ Lightfront Wavefunction

- Relativistic mutually coupled LF Relativistic Harmonic Oscillator for the valence $q\bar{q}$.
- Setting the quark masses is the same.

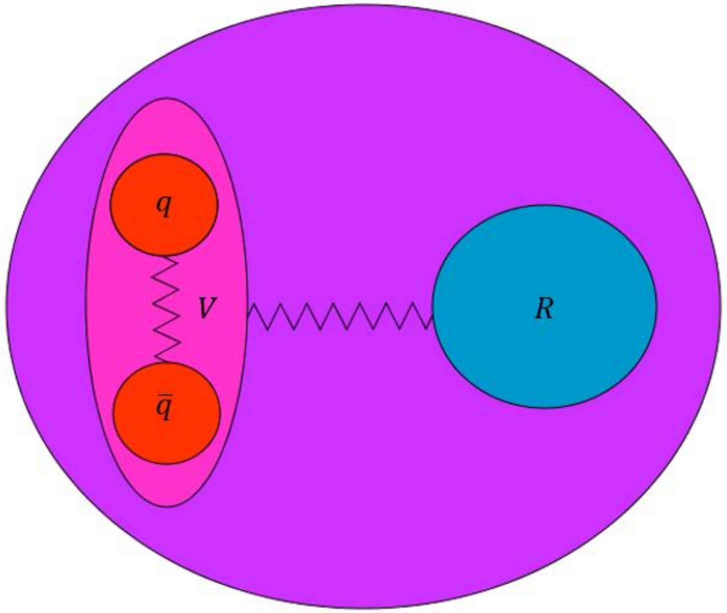
$$\psi_{qq}(\{x_i, \mathbf{k}_{i,\perp}\}_{i=1,2}) = \sqrt{16\pi^3 m_\pi}\Lambda_V \exp\left[-\frac{B_V}{2}k_{12}^2\right]\sqrt{x_2}$$

$$k_{12}^2 = \frac{\left(s_{12}-(m_1-m_2)^2\right)\left(s_{12}-(m_1+m_2)^2\right)}{4s_{12}}$$

$$s = \sum_{i=1,2}\frac{\tilde{k}_{i,\perp}^2+m^2}{\beta_i}$$

$$x_1 = x_B,\ \beta_1 = \frac{x_1}{x_V} = \frac{x_B}{1-x_R} \text{ and } \beta_2 = \frac{x_2}{x_V} = \frac{1-x_B-x_R}{1-x_R}$$

$$\psi_{qq}(\{x_i, \mathbf{k}_{i,\perp}\}_{i=1,2}) = \tilde{\Lambda}_V \exp\left[-\frac{B_V}{8}\sum_{i=1,2}\frac{\tilde{k}_{i,\perp}^2+m_i^2}{\beta_i}\right]\sqrt{x_2}$$



Modeling VR Lightfront Wavefunction

- Relativistic LF Harmonic Oscillator

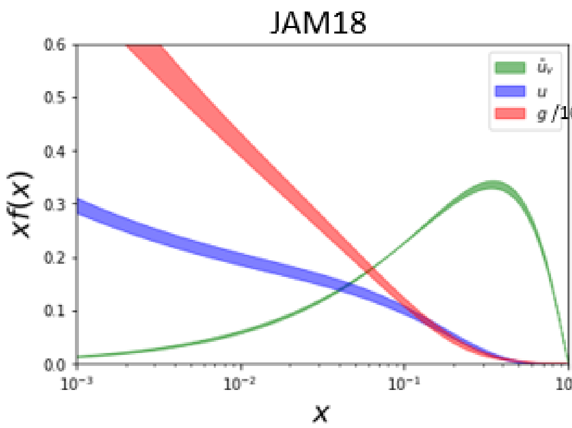
$$\psi^{Rel}_{VR}(\{x_i, \mathbf{k}_{i,\perp}\}_{i=V,R}) = \tilde{A}_R \exp\left[-\frac{B_R}{2} k^2_{VR}\right] \sqrt{x_R}$$

$$k^2_{VR} = \frac{(s_{VR} - (m_V - m_R)^2)(s_{VR} - (m_V + m_R)^2)}{4 s_{VR}}$$

- Assume the R system is the same at all x.
- However, the masses aren't equal. ($m_R \neq m_V$)
- Non-relativistic LF Harmonic Oscillator wavefunction

$$\psi^{NR}_{VR}(\{x_i, \mathbf{k}_{i,\perp}\}_{i=V,R}) = \tilde{A}_R e^{-\frac{B_R}{2}((m_R x_R - m_R)^2 + k^2_{R,\perp})} \sqrt{x_R}$$

PDF calculation



- Using Relativistic Harmonic Oscillator at m=0 and large Q^2

$$f_V^{Rel}(x_B,Q^2) = \mathcal{N}\int_0^{1-x_B} \frac{(1-x_B-x_R)}{(1-x_R)^2}\int_0^{\infty} e^{-B_R k_{12}^2} dk_{R,\perp}^2 \qquad \mathcal{N}^{Rel} = \frac{4\pi^2|\tilde{A}_V\tilde{A}_R|^2}{B_V(16\pi^3)^2}.$$

- Using Non - Relativistic Harmonic Oscillator at m=0 and large Q^2.

$$f_V^{NR}(x_B) = \mathcal{N}^{NR}\int_0^{1-x_B} dx_R \frac{(1-x_B-x_R)}{(1-x_R)^2} \times \exp\left[-B_R m_\pi^2 (x_R - m_R/m_\pi)^2\right] \qquad \mathcal{N}^{NR} = \frac{4\pi^2|\tilde{A}_V\tilde{A}_R|^2}{B_V B_R(16\pi^3)^2}.$$

- Using saddle point approx., $xf^{NR}{}_V(x,Q^2) \sim x_B(1-x_B - \frac{m_R}{m_\pi})$ *with* peak $x_p^{NR} \approx \frac{1}{2}\left(1-\frac{m_R}{m_\pi}\right)$

Fitting Results

Model has parameters that characterizes non – perturbative wave-function of the residual system.

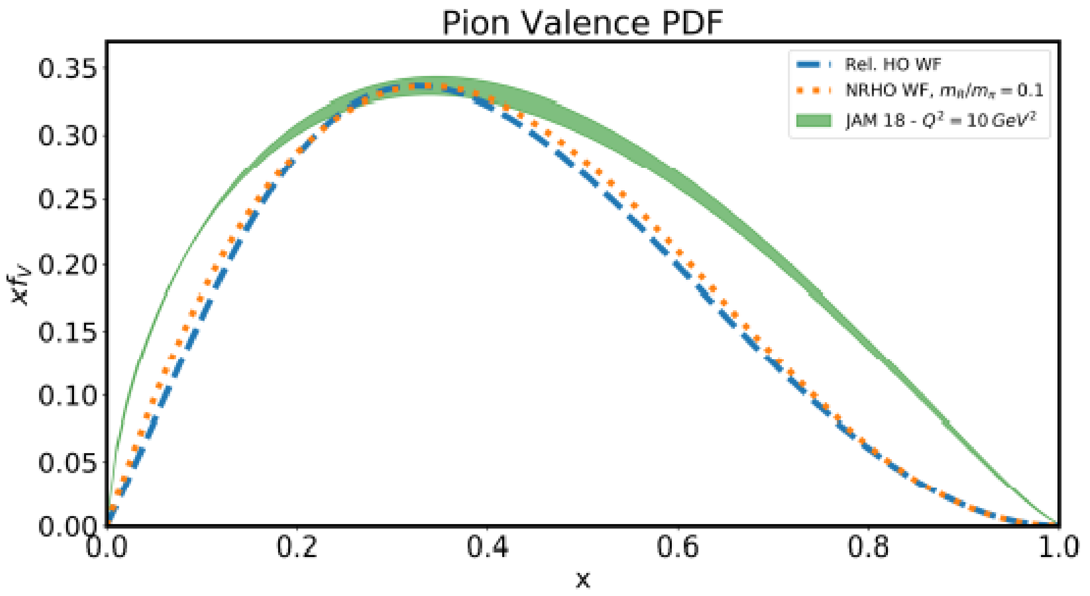
Fit to parameters to match the peak position and height.

Model Fit

- Peak matching- insensitive to the high x structure.

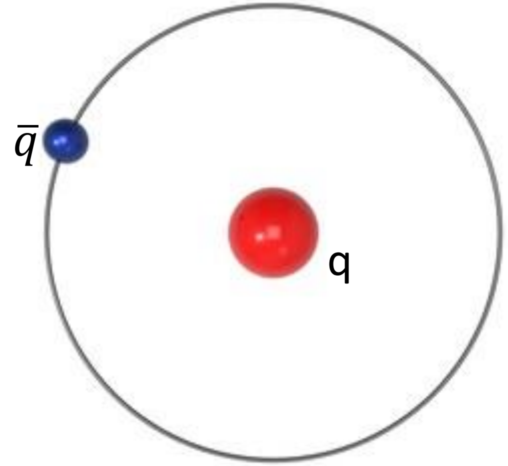
High x structure:

- JAM 18: β_V = 1
- Soft contribution result: β_V = 2



Positronium-like Model

- Perhaps the LF Harmonic Oscillator this not give the best shape for the xf_V where we only fit it to the peak.
- Alternative model to the pion's valence quark and antiquark as a positronium-like model (EM interaction) to perhaps obtain a better shape of xf_V.



$q\bar{q}$ Positronium – like Model

Positronium-like Model – Brodsky-Lepage LFWF

- We applied the same procedure of the Residual Field model (m = 0 and $Q^2 \to \infty$) to the positronium-like LFWF for the soft contribution.

$$\psi(r) = \frac{1}{\sqrt{\pi}a^{\frac{3}{2}}} e^{-\frac{r}{a}} \qquad \int_{\mathbb{R}^3} |\psi(r)|^2 d^3r = 1$$

Fourier Transform of $\psi(r)$

$$\phi(k) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \psi(r) e^{-i\mathbf{k}\cdot\mathbf{r}} d^3r$$

$$\phi(k) = \frac{2\sqrt{2}a^{\frac{3}{2}}}{\pi(1+a^2k^2)^2}$$

LFWF for Coulomb interaction BL

$$|\psi_{LF}(\beta_1, \beta_2, k_\perp)|^2 = \frac{16\pi^3 E_k |\psi_{NR}(k)|^2}{2\beta_2} \qquad E_k = \sqrt{m^2 + k_\perp^2 + k_z^2}$$

$$\beta_1 = \frac{k_z + E_k}{2E_k}, \ \beta_2 = \frac{E_k - k_z}{2E_k} \qquad \int \frac{|\psi_{LF}|^2 d\beta_1 d^2k_\perp}{16\pi^3 \beta_1} = 1 \qquad d\beta_1 = \frac{2\beta_1\beta_2}{E_k} dk_z$$

$$|\psi_{LF}(\beta_1, \beta_2, k_\perp)|^2 = \frac{16\pi^3 E_k |\frac{2\sqrt{2}a^{\frac{3}{2}}}{\pi(1+a^2k^2)^2}|^2}{2\beta_2} \qquad k_{12}^2 = \frac{\tilde{k}_\perp^2}{4\beta_1\beta_2}$$

Insert into PDF

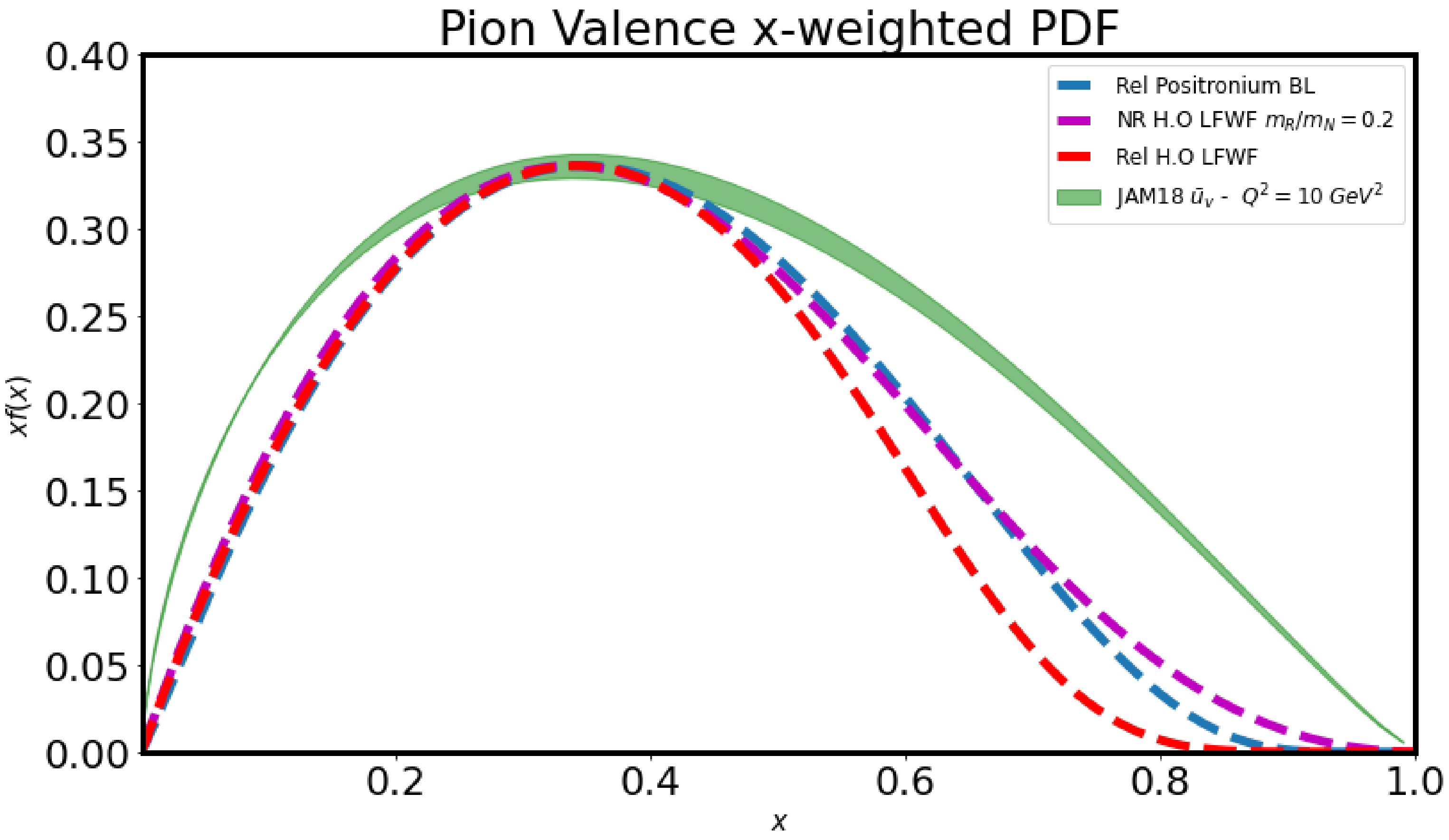
$$f_V(x_B, Q^2) = \int_0^{1-x_B} \frac{dx_R}{(16\pi^3)^2 x_B(1-x_B-x_R)x_R} \int_0^{Q^2} d^2\tilde{\mathbf{k}}_{1,\perp} d^2\tilde{\mathbf{k}}_{2,\perp} \delta^{(2)}\left(\sum_{i=1,2} \tilde{\mathbf{k}}_{i,\perp}\right)$$
$$|\psi_{q\bar{q}}(\{x_i, \tilde{\mathbf{k}}_{i,\perp}\}_{i=1,2})|^2 \int_0^{Q^2} d^2\mathbf{k}_{R,\perp} |\psi_{VR}(x_R, \mathbf{k}_{R,\perp})|^2$$

$$\int^{Q^2\to\infty} d^2\tilde{\mathbf{k}}_\perp |\psi_{q\bar{q}}(\{x_i, \tilde{\mathbf{k}}_{i,\perp}\}_{i=1,2})|^2 = 16\pi^3 \beta_1$$

$$f_V^{Rel}(x_B) = \int_0^{1-x_B} \frac{dx_R}{(16\pi^3)^2 x_B(1-x_B-x_R)x_R} \left(16\pi^3\beta_1\right)\left(|\tilde{A}_R|^2 x_R \pi \int_0^{Q^2\to\infty} e^{-B_R k_{VR}^2} dk_{R,\perp}^2\right)$$
$$= \frac{A_R^2}{16\pi^2} \int_0^{1-x_B} \frac{dx_R}{(1-x_B-x_R)(1-x_R)} \int_0^\infty e^{-B_R k_{VR}^2} dk_{R,\perp}^2$$

Conclusion

- We want to model the pion PDF using our model, the residual field model.
- Looking at soft part and modeling the LF wavefunction, we get the $(1-x)^2$ high x structure of the pion PDF.
- We also look at positronium-like PDF and we observe some sensitivity to the soft structure of the $q\bar{q}$ system (at the peak) compared to the harmonic – oscillator like model.



References

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Christopher Leon (Florida Intl. U.), Misak Sargsian (Florida Intl. U.)

arxiv:2012.14030

- A Novel Feature of Valence Quark Distributions in Hadrons

Christopher Leon (Florida Intl. U.), Misak M. Sargsian (Florida Intl. U.), Frank Vera (Florida Intl. U.)

arxiv:2003.12902

- The Pion Valence Structure in the Residual Field Model

Christopher Leon, Joseph Maerovitz, Misak Sargsian (Current Work)

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Thank You

Deep Inelastic Scattering

Now using light-cone (LC) coordinates with the 4-momenta and four-products

$$k^\mu = (k^+, k^-, \mathbf{k}_\perp) \quad k^\pm = E \pm k^z \quad \mathbf{k}_\perp = (k^x, k^y) \quad k_1 \cdot k_2 = \frac{1}{2}k_1^- k_2^+ + \frac{1}{2}k_1^+ k_2^- - \mathbf{k}_{1,\perp} \cdot \mathbf{k}_{2,\perp}$$

The four-momenta of the pion, p_π^μ and the virtual photon, q^μ are: $p_\pi^\mu = (p_\pi^+, \frac{m_\pi^2}{p_\pi^+}, \mathbf{0}_\perp)$ $q^\mu = (0, \frac{2p_\pi \cdot q}{p_\pi^+}, \mathbf{q}_\perp)$

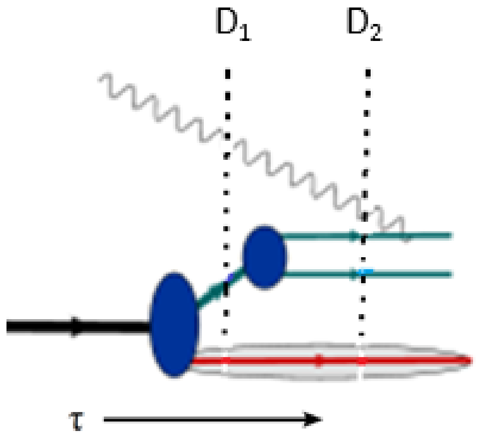
The important kinematical condition of the chosen reference frame

$$p_\pi^+ \gg m_\pi, k_i^-, k_{i,\perp}$$

$$Q^2 = -q^2 = |q_\perp|^2 \qquad x_B = \frac{Q^2}{2p_N \cdot q}$$

The pion nucleonic tensor $$W_\pi^{\mu\nu} = \frac{F_2(x_B, Q^2)}{m_\pi(p_\pi \cdot q)}\left(p_\pi^\mu - \frac{p_\pi \cdot q}{q^2}q^\mu\right)\left(p_\pi^\nu - \frac{p_\pi \cdot q}{q^2}q^\nu\right)$$

The Stucture Function $$F_2(x_B, Q^2) = \frac{m_\pi Q^2}{2x_B(p_\pi^+)^2} W^{++} \qquad F_2(x, Q^2) = \sum_i e_i^2 x f_i(x, Q^2)$$



Soft Contribution

We have a pion that transitions in a valence system containing a quark and anti - quark and residual system. To calculate this amplitude, we apply effective light-front diagrammatic rules which results as:

$$A^\mu = \sum_{h_V, h_1} \frac{1}{k_V^+} \frac{1}{k_1^+} \frac{\bar{u}(k_1', h_1')(ie_1\gamma^\mu)u(k_1, h_1)\bar{u}(k_1, h_1)v(k_2, h_2)\Gamma^{V\to q\bar{q}}\chi_V \chi_V^\dagger \chi_R^\dagger \Gamma^{\pi\to VR}}{D_1 D_2} \qquad (71)$$

Effective Light Front Diagrammatic Rules

- Given for processes order in light-cone time $\tau = t + z$.
- Intermediate states get a LF energy denominator. $\frac{1}{\mathcal{D}} = \frac{1}{\sum_{init.} p^- - \sum_{interm.} p^- + i\epsilon}$
- Every vertex in the diagram gets effective transition factor Γ from particle A to n – constituents.
- .For a transition of particle A to n – constituents, the LF wave function is defined as:

$$\psi(\{x_i, k_{i,\perp}, h_i\}_i^n) = \frac{\prod_{i=1}^{n} \chi_{fi}(x_i, k_{i,\perp}, h_i) \Gamma \chi_A(p_A, h_A)}{p_A^+ \mathcal{D}},$$

The denominator for pion splitting to the residual and valence parts is $D_1 = p_\pi^- - k_R^- - k_V^- = \frac{1}{p_\pi^+}(m_\pi^2 - \frac{k_{R,\perp}+m_R^2}{x_R} - \frac{k_{V,\perp}+m_V^2}{x_V})$

The denominator for valence splitting to the quark and anti - quark of the Pion is $D_2 = k_V^- - k_1^- + k_2^- = \frac{1}{k_V^+}(m_V^2 + k_{V,\perp}^2 - \frac{k_{1,\perp}^2+m_1^2}{\beta_1} - \frac{k_{2,\perp}^2+m_2^2}{\beta_2})$

; $\beta_1 = \frac{k_1^+}{k_V^+}$ and, $\beta_2 = \frac{k_2^+}{k_V^+}$. $x_R = \frac{k_R^+}{p_N^+}$ and $x_V = \frac{k_V^+}{p_N^+}$.

scattering amplitude

$$A^\mu = \sum_{h_V,h_1} \frac{1}{x_V}\frac{1}{\beta_1} \frac{\bar{u}(k_1',h_1')(ie_1\gamma^\mu)u(k_1,h_1)\bar{u}(k_1,h_1)v(k_2,h_2)\Gamma^{V\to q\bar{q}}\chi_V\chi_V^\dagger\chi_R^\dagger\Gamma^{\pi\to VR}}{(m_\pi^2 - \frac{k_{R,\perp}+m_R^2}{x_R} - \frac{k_{V,\perp}+m_V^2}{x_V})(m_V^2 + k_{V,\perp}^2 - \frac{k_{1,\perp}^2+m_1^2}{\beta_1} - \frac{k_{2,\perp}^2+m_2^2}{\beta_2})}$$

$$\psi_{VR}(x_V, \mathbf{k}_{R,\perp}, x_R, \mathbf{k}_{V,\perp}) = \frac{\chi_V^\dagger\chi_R^\dagger\Gamma^{\pi\to VR}}{m_\pi^2 - \frac{k_{V,\perp}^2+m_V^2}{x_V} - \frac{k_{R,\perp}^2+m_R^2}{x_R}}$$

$$A^\mu = \sum_{h_1,h_V} \bar{u}(k_1',h_1')(ie_1\gamma^\mu)u(k_1,h_1) \quad \frac{\psi_{VR}(x_V,\mathbf{k}_{V,\perp},x_R,\mathbf{k}_{R,\perp})}{x_V}\frac{\psi_{q\bar{q}}(\beta_1,\beta_2,\mathbf{k}_{1,\perp},\mathbf{k}_{2,\perp},h_1,h_2)}{\beta_1}$$

$$\psi_{q\bar{q}}(x_1,x_2,\mathbf{k}_{1,\perp},\mathbf{k}_{2,\perp}) = \frac{\bar{u}(k_1,h_1)\Gamma^{V\to q\bar{q}}\chi_V v(k_2,h_2)}{m_V^2 + k_{V,\perp}^2 - \sum_{i=1}^2 \frac{k_{i,\perp}^2+m_i^2}{\beta_i}}$$

The pion nucleonic tensor $W_\pi^{\mu\nu}$ for the case of the final state is the ongoing valence quark and residual system, in the leading order.

$$W_\pi^{\mu\nu} = \frac{1}{4\pi m_\pi}\sum_{q,h_i}\int \delta(k_R^2 - m_R^2)\frac{d^4k_R}{(2\pi)^3}\delta(k_1'^2 - m_1^2)\frac{d^4k_1'}{(2\pi)^3}\delta(k_2^2 - m_2^2)\frac{d^4k_2}{(2\pi)^3} \quad (2\pi)^4\delta^{(4)}(p_\pi + q - k_1' - k_2 - k_R)A^{\mu\dagger}A^\nu$$

$$\delta(k_i^2 - m_i^2)d^4k = \frac{dx_i d^2\mathbf{k}_{i,\perp}}{2x_i}\Big|_{k_i^- = \frac{k_{i,\perp}^2+m_i^2}{x_i p_\pi^+}} \qquad \delta^{(4)}(p_\pi + q - k_1' - k_2 - k_R) \approx \frac{x_1}{p_\pi\cdot q}\delta(1 - x_1 - x_2 - x_R)\delta(x_1 - x_B)\delta^{(2)}(\mathbf{k}_{1,\perp} + \mathbf{k}_{2,\perp} + \mathbf{k}_{R,\perp})$$

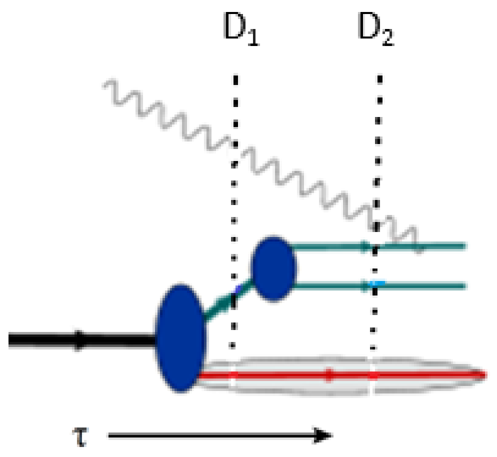
$$W_\pi^{\mu\nu} = \frac{1}{4\pi m_\pi}\sum_{q,h_i}\int \frac{dx_R d^2\mathbf{k}_{R,\perp}}{2x_R(2\pi)^3}\frac{dx_1 d^2\mathbf{k}_{1,\perp}}{2x_1(2\pi)^3}\frac{dx_2 d^2\mathbf{k}_{2,\perp}}{2x_2(2\pi)^3}(2\pi)^4\frac{x_1}{p_\pi\cdot q}\delta(1 - x_1 - x_2 - x_R)\delta(x_1 - x_B)\delta^{(2)}(\mathbf{k}_{1,\perp} + \mathbf{k}_{2,\perp} + \mathbf{k}_{R,\perp})A^{\mu\dagger}A^\nu$$

$$A^+ = 2\sum_{h_V}(ie_1)x_1 p_\pi^+ \frac{\psi_{VR}(x_V,\mathbf{k}_{V,\perp},x_R,\mathbf{k}_{R,\perp})}{x_V}\frac{\psi_{q\bar{q}}(\beta_1,\beta_2,\mathbf{k}_{1,\perp},\mathbf{k}_{2,\perp},h_1,h_2)}{\beta_1}$$

$$W_\pi^{++} = \frac{1}{m_\pi} \sum_{q,h_i,h_V} \int [dx][d^2\mathbf{k}_\perp] \frac{x_1}{p_\pi \cdot q} \delta(x_1 - x_B) e_1^2 p_\pi^{+2} |\psi_{VR}(x_V, \mathbf{k}_{V,\perp}, x_R, \mathbf{k}_{R,\perp})|^2 \ |\psi_{q\bar{q}}(\beta_1, \beta_2, \mathbf{k}_{1,\perp}, \mathbf{k}_{2,\perp}, h_1, h_2)|^2$$

$$F_2(x_B, Q^2) = \sum_{q,h_i,h_V} \int [dx][d^2\mathbf{k}_\perp] e_1^2 x_1 \delta(x_1 - x_B) |\psi_{VR}(x_V, \mathbf{k}_{V,\perp}, x_R, \mathbf{k}_{R,\perp})|^2 \ |\psi_{q\bar{q}}(\beta_1, \beta_2, \mathbf{k}_{1,\perp}, \mathbf{k}_{2,\perp}, h_1, h_2)|^2$$

$$F_2(x_B, Q^2) = \frac{m_\pi Q^2}{2x_B (p_\pi^+)^2} \ W^{++} \qquad F_2(x, Q^2) = \sum_i e_i^2 x f_i(x, Q^2)$$



Calculating pion PDF using Effective Light-Front diagrammatic method (LO)

$$f_V(x_B, Q^2) = \int \delta(1 - x_1 - x_2 - x_R) \frac{dx_R}{x_R} \frac{dx_1}{x_1} \frac{dx_2}{x_2} 16\pi^3 \delta^{(2)}(\mathbf{k}_{1,\perp} + \mathbf{k}_{2,\perp} + \mathbf{k}_{R,\perp})$$

$$\frac{d^2\mathbf{k}_{R,\perp}}{16\pi^3} \frac{d^2\mathbf{k}_{1,\perp}}{16\pi^3} \frac{d^2\mathbf{k}_{2,\perp}}{16\pi^3} \delta(x_1 - x_B) |\psi_{VR}(x_V, \mathbf{k}_{V,\perp}, x_R, \mathbf{k}_{R,\perp})|^2 |\psi_{q\bar{q}}(\beta_1, \beta_2, \mathbf{k}_{1,\perp}, \mathbf{k}_{2,\perp}, h_1, h_2)|^2$$

$$\tilde{\mathbf{k}}_{i,\perp} = \mathbf{k}_{i,\perp} - \beta_i \mathbf{k}_{V,\perp} \qquad (i=1,2)$$

Charge symmetry – $f_V(\mathrm{x}, \mathrm{Q}^2) = u_V^{\pi^+}(x, Q^2) = d_V^{\pi^-}(x, Q^2)$

$$f_V(x_B, Q^2) = \int_0^{1-x_B} \frac{dx_R}{(16\pi^3)^2 x_1 x_2 x_R}$$
$$\times \int^{Q^2} d^2\tilde{\mathbf{k}}_{1,\perp} d^2\tilde{\mathbf{k}}_{2,\perp} \delta^{(2)}\left(\sum_{i=1,2} \tilde{\mathbf{k}}_{i,\perp}\right) |\psi_{qq}(\{x_i, \tilde{\mathbf{k}}_{i,\perp}\}_{i=1,2})|^2$$
$$\times \int^{Q^2} d^2\mathbf{k}_{R,\perp} |\psi_{VR}(x_R, \mathbf{k}_{R,\perp})|^2$$