Extracting the σ resonance from first-principles QCD

GHP 2023

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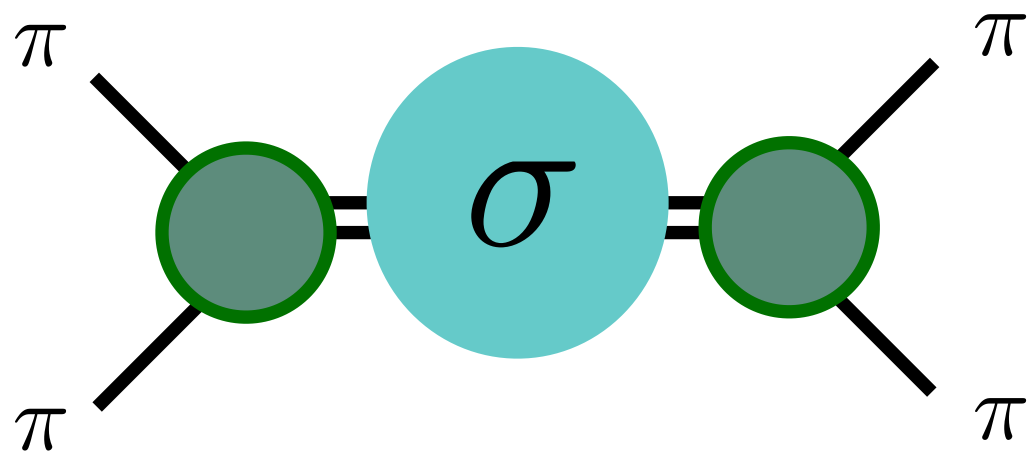
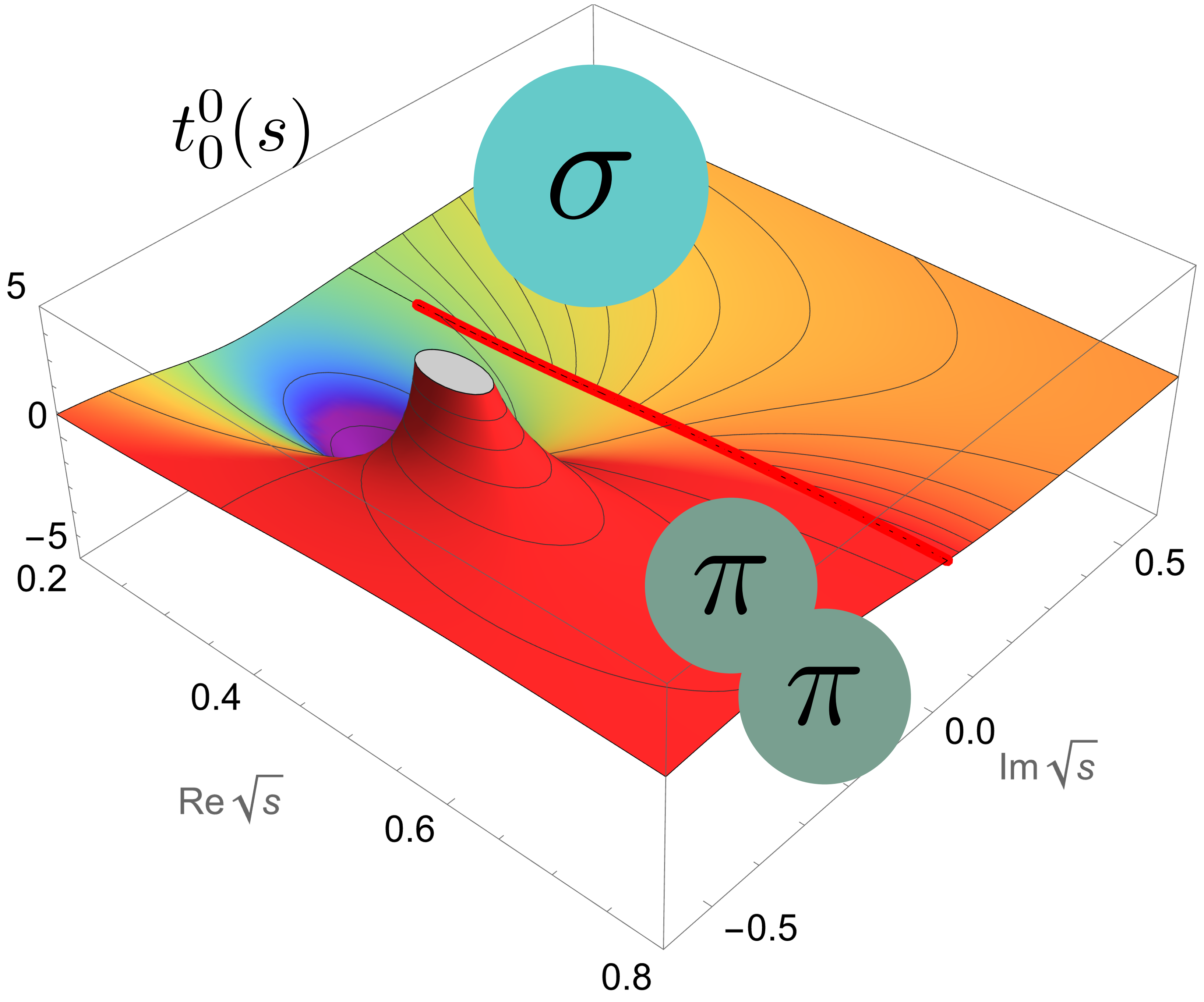
hadspec

*GHP 2023 presentation thanks to financial support from The Gordon and Betty Moore Foundation and the American Physical Society.

Light Scalars: the σ





Lightest resonance in QCD

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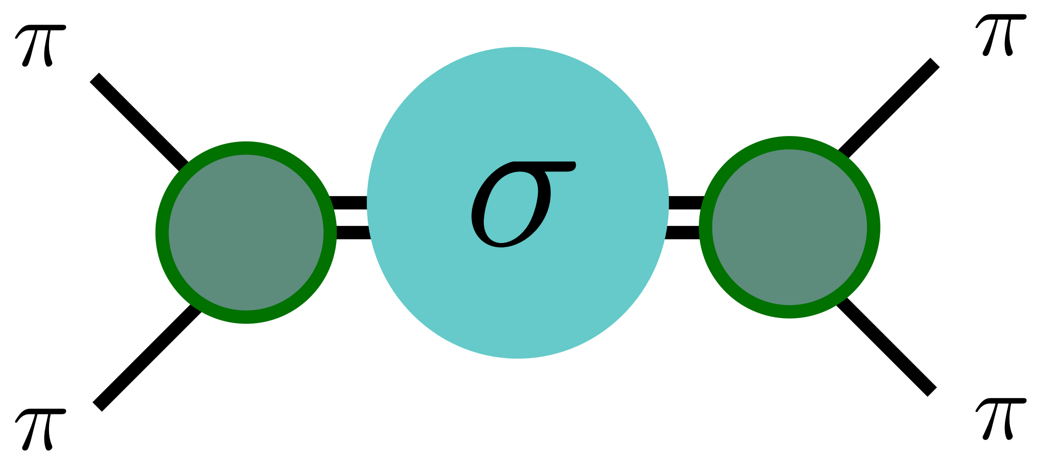

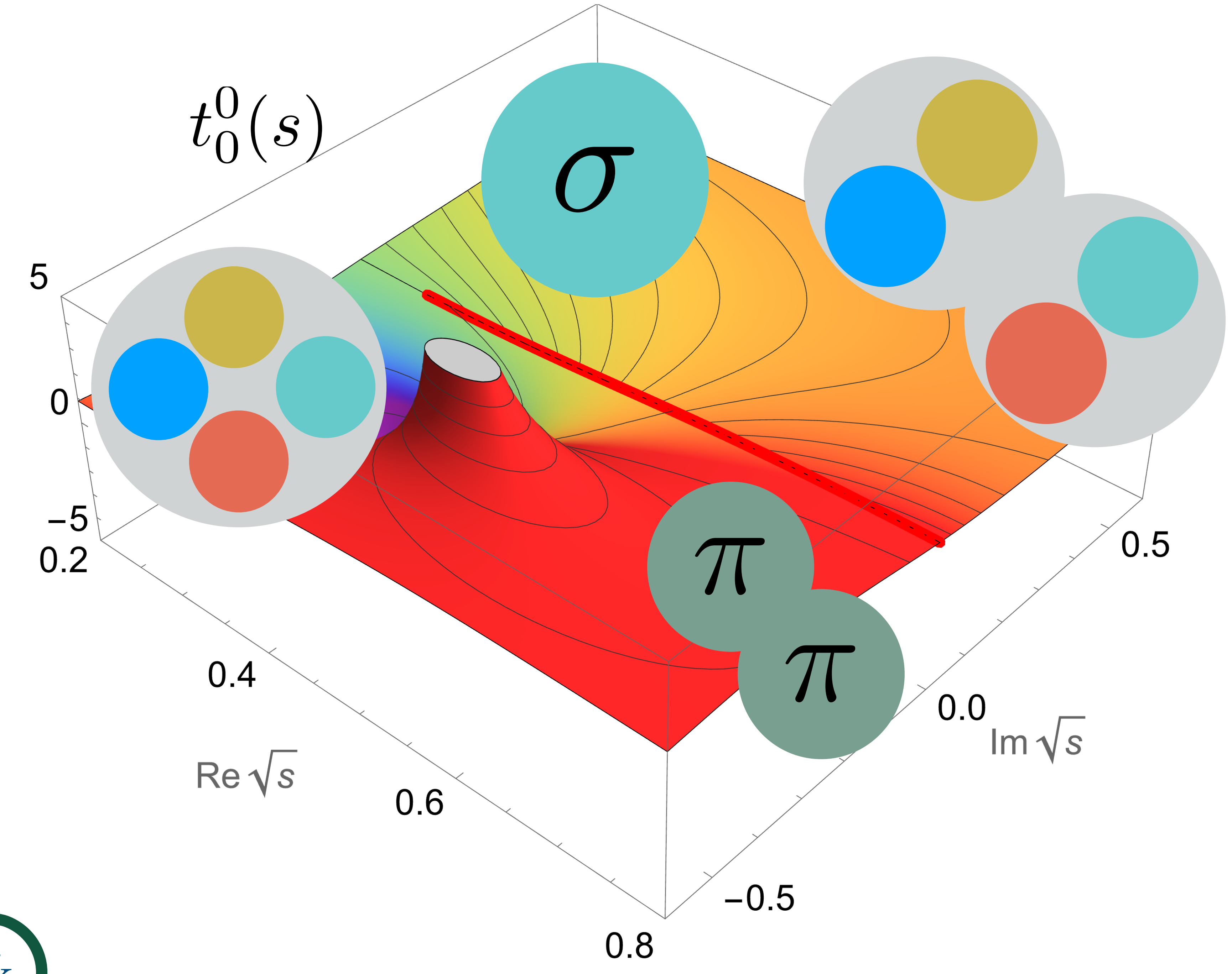

Lightest resonance in QCD

Extremely broad → extremely short-lived

Correlated with chiral symmetry-breaking phenomena

Not well-understood → new observables ?? *A. W. Jackura's talk*

Input to hadron physics observables

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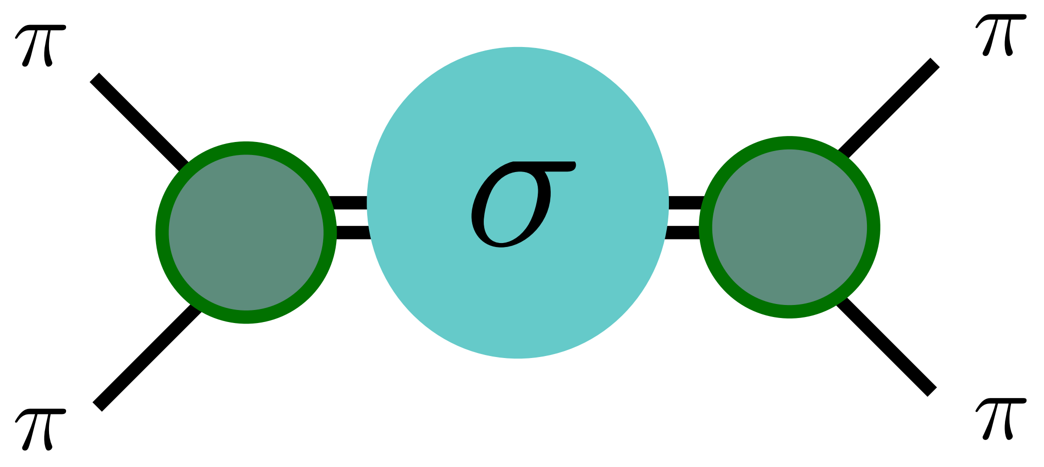

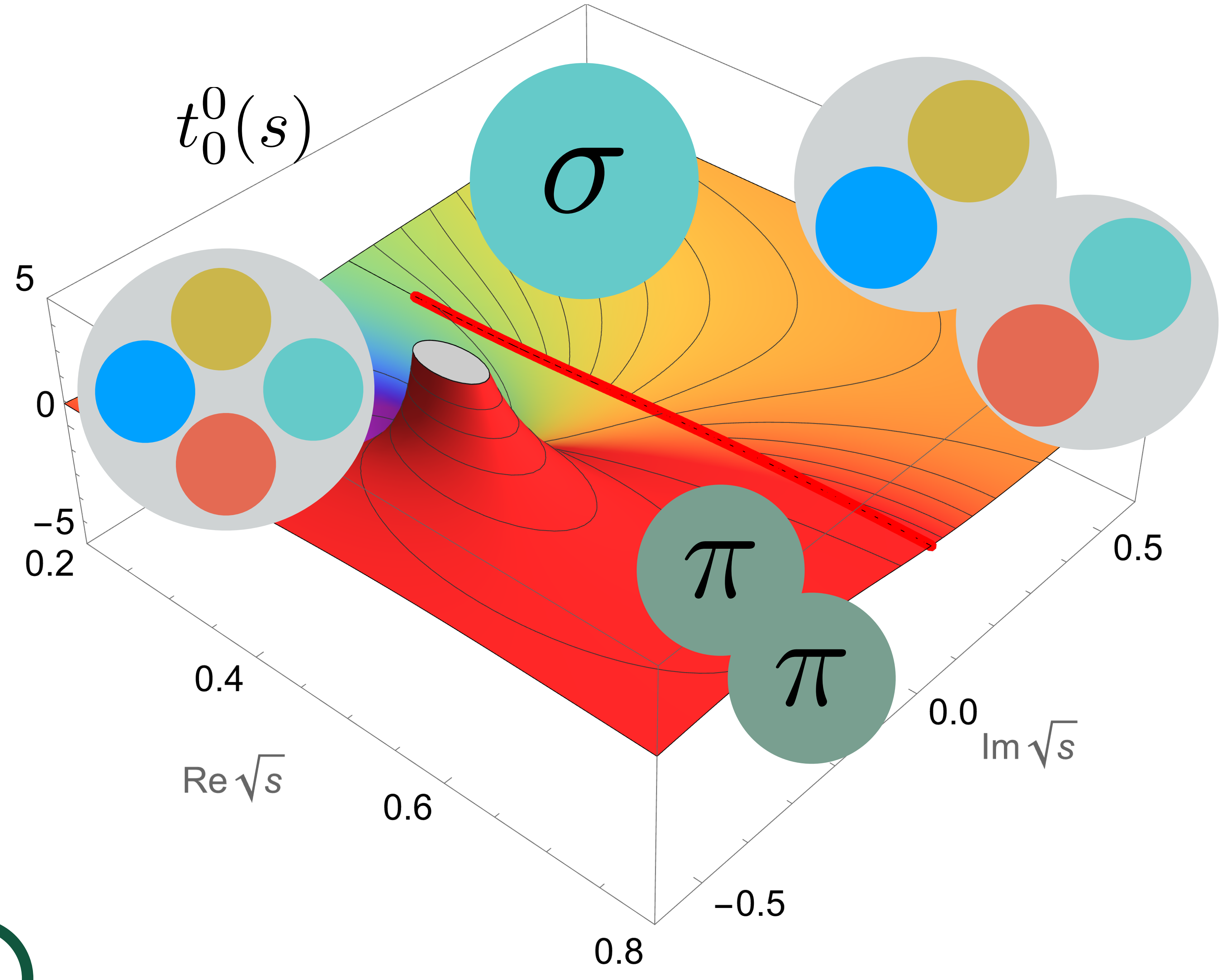

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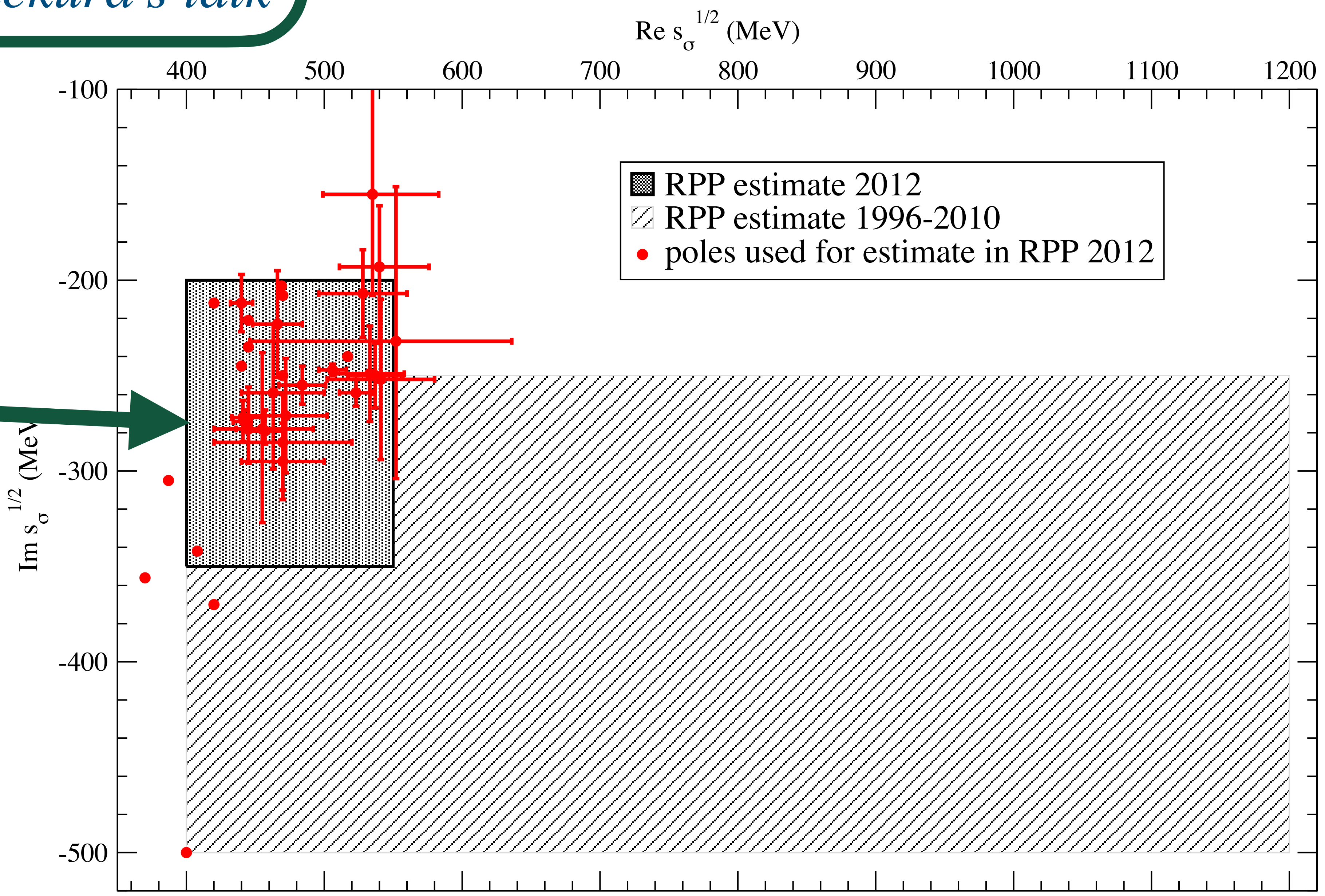
Very challenging experimental extraction



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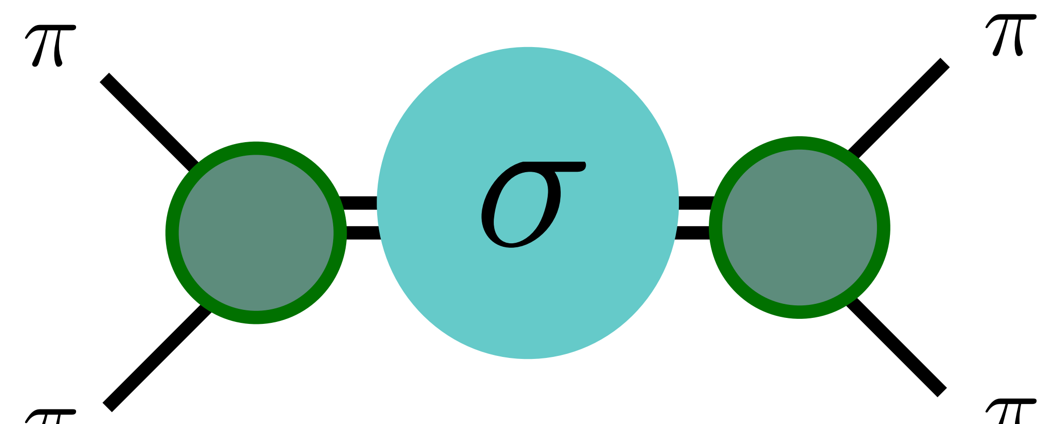

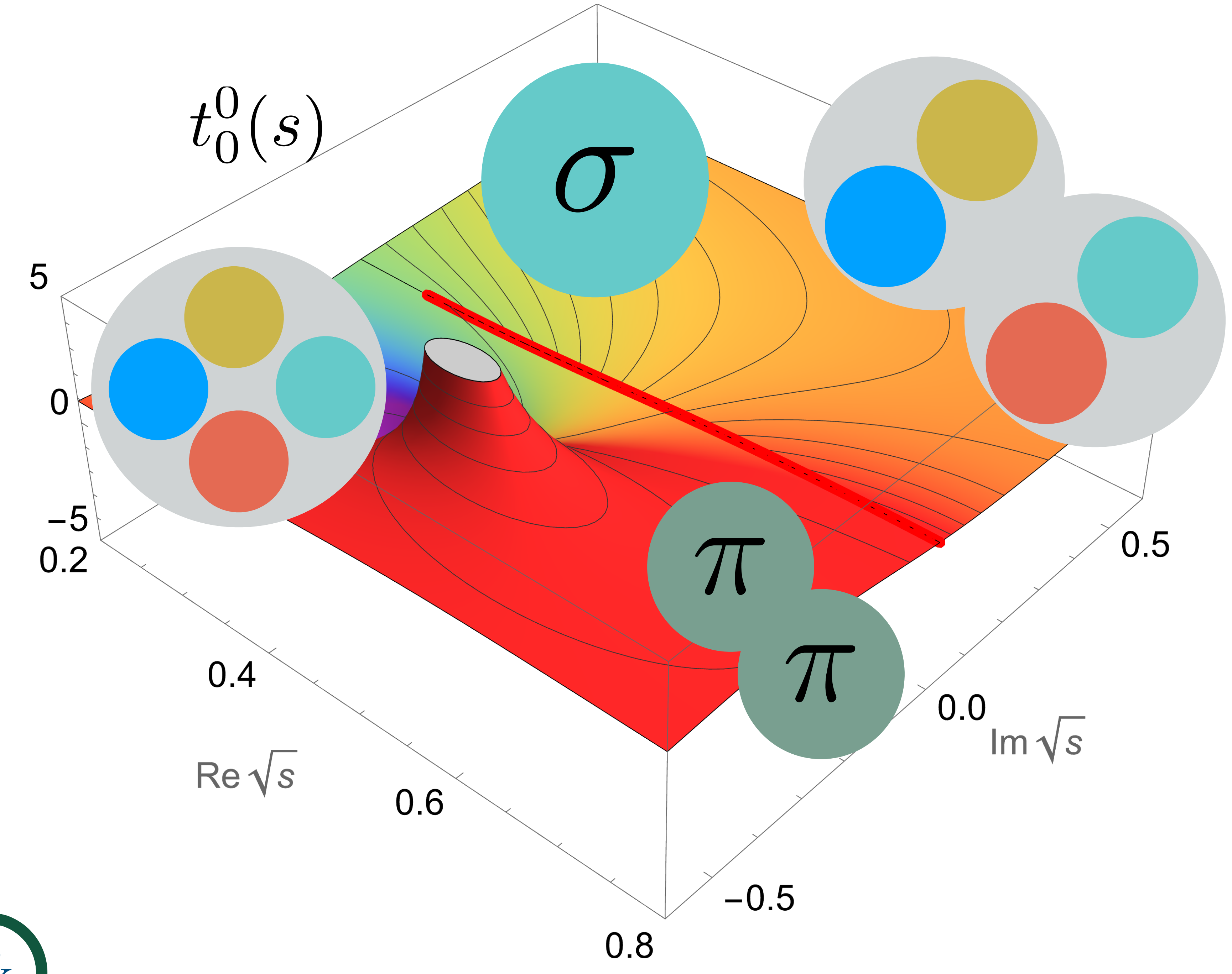

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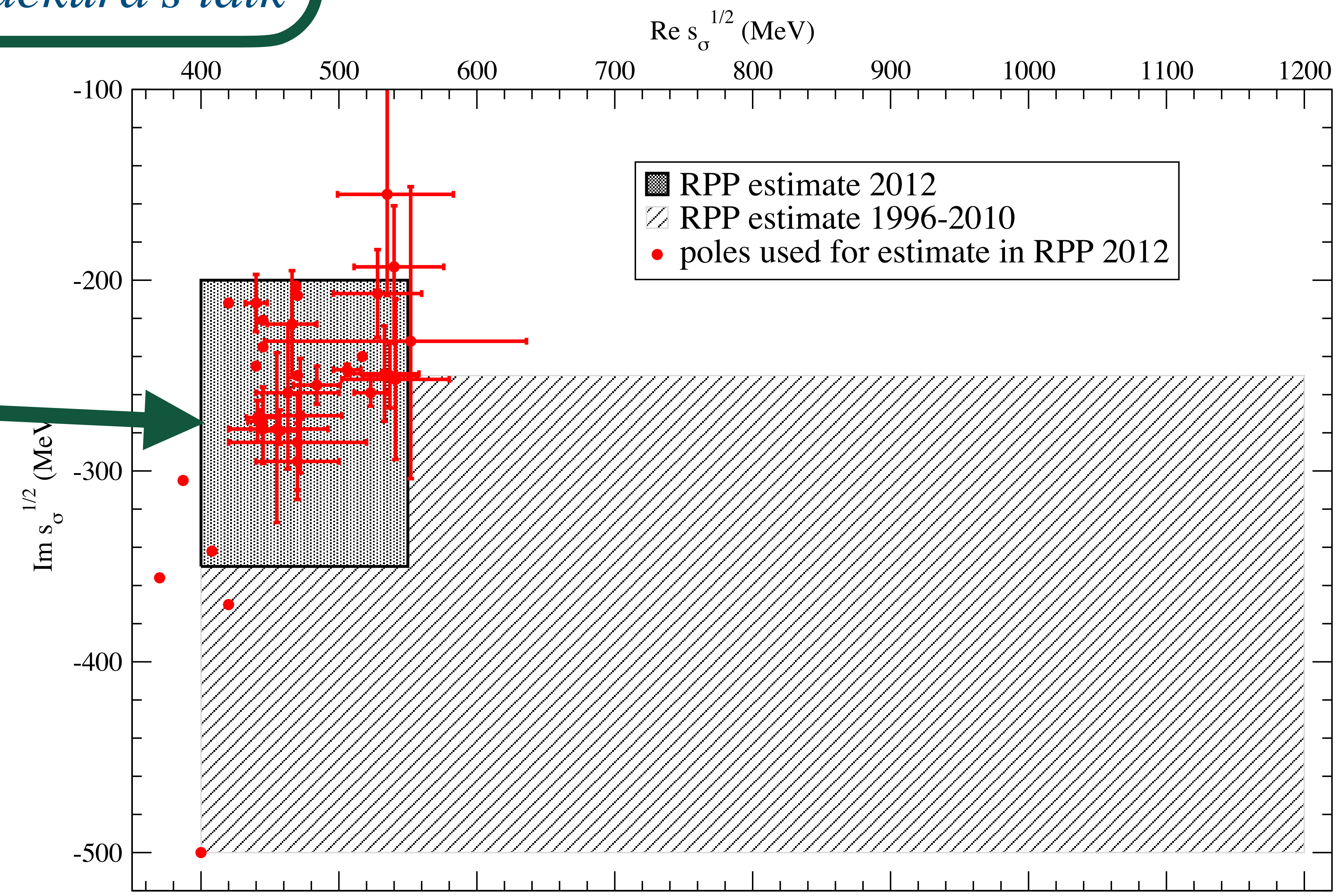


What happens for Lattice QCD ??

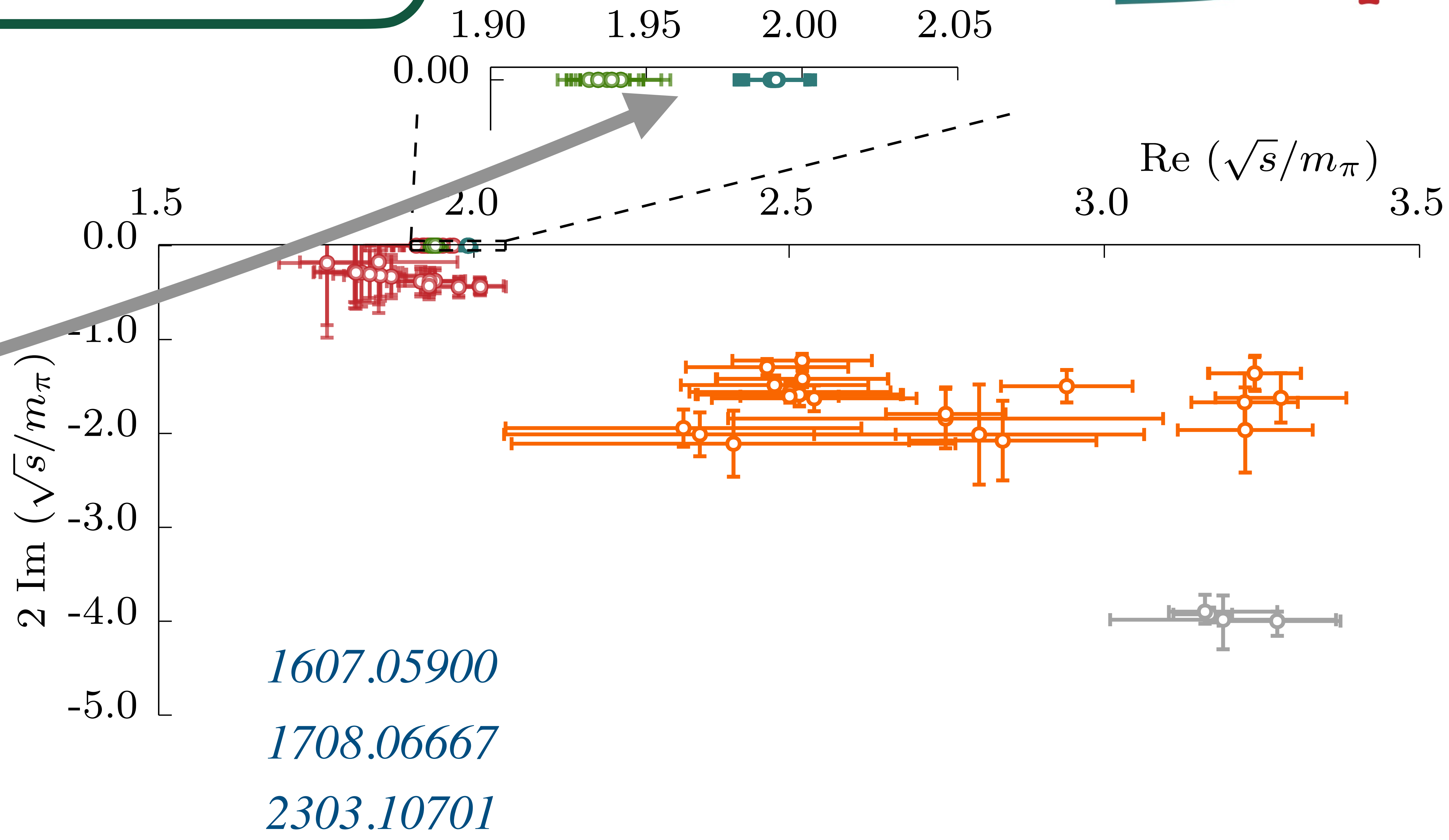
Light Scalars: the σ

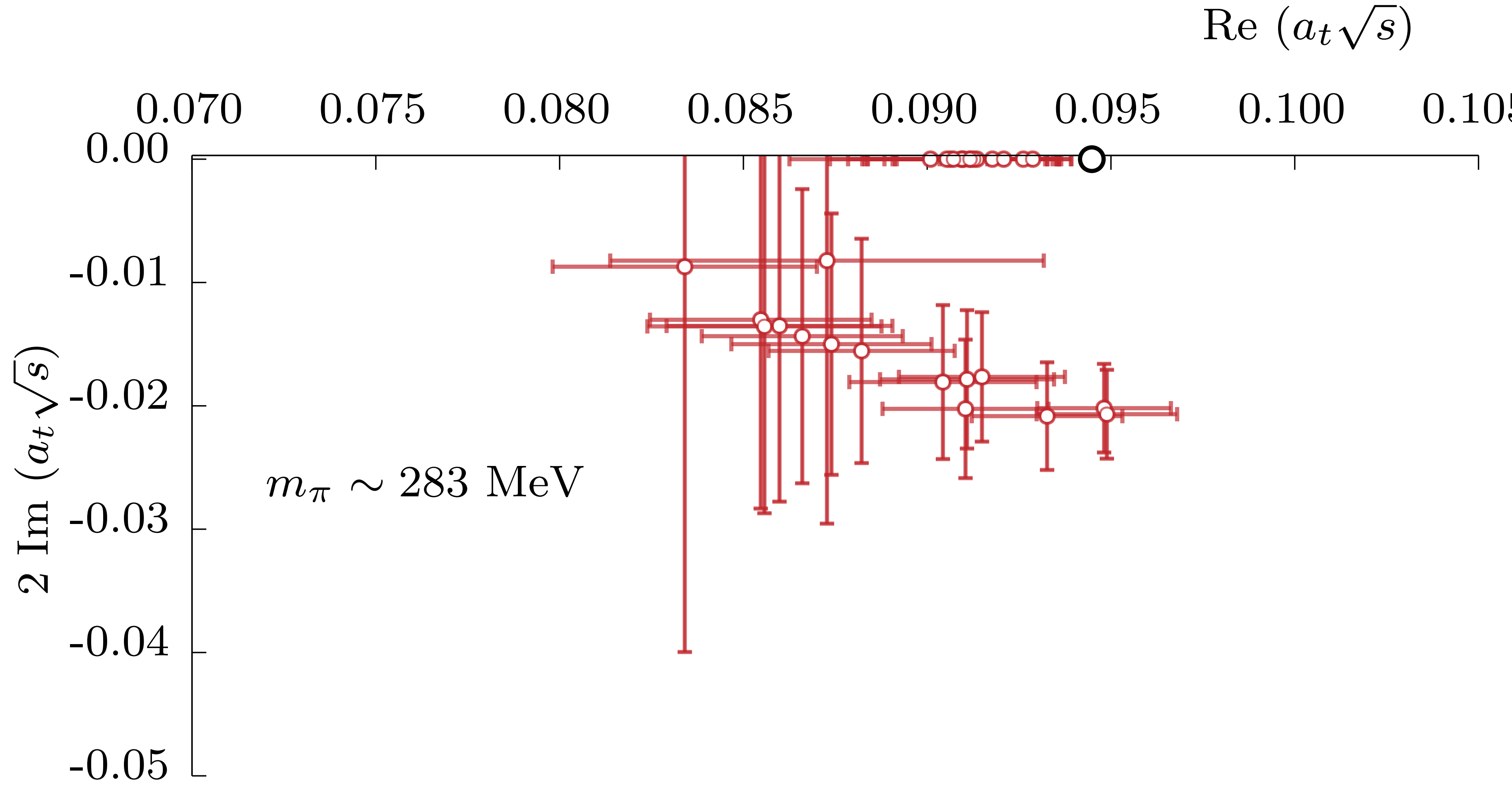
Other works

16010.10070 *1803.02897*

Stable and "easy" to extract at higher masses

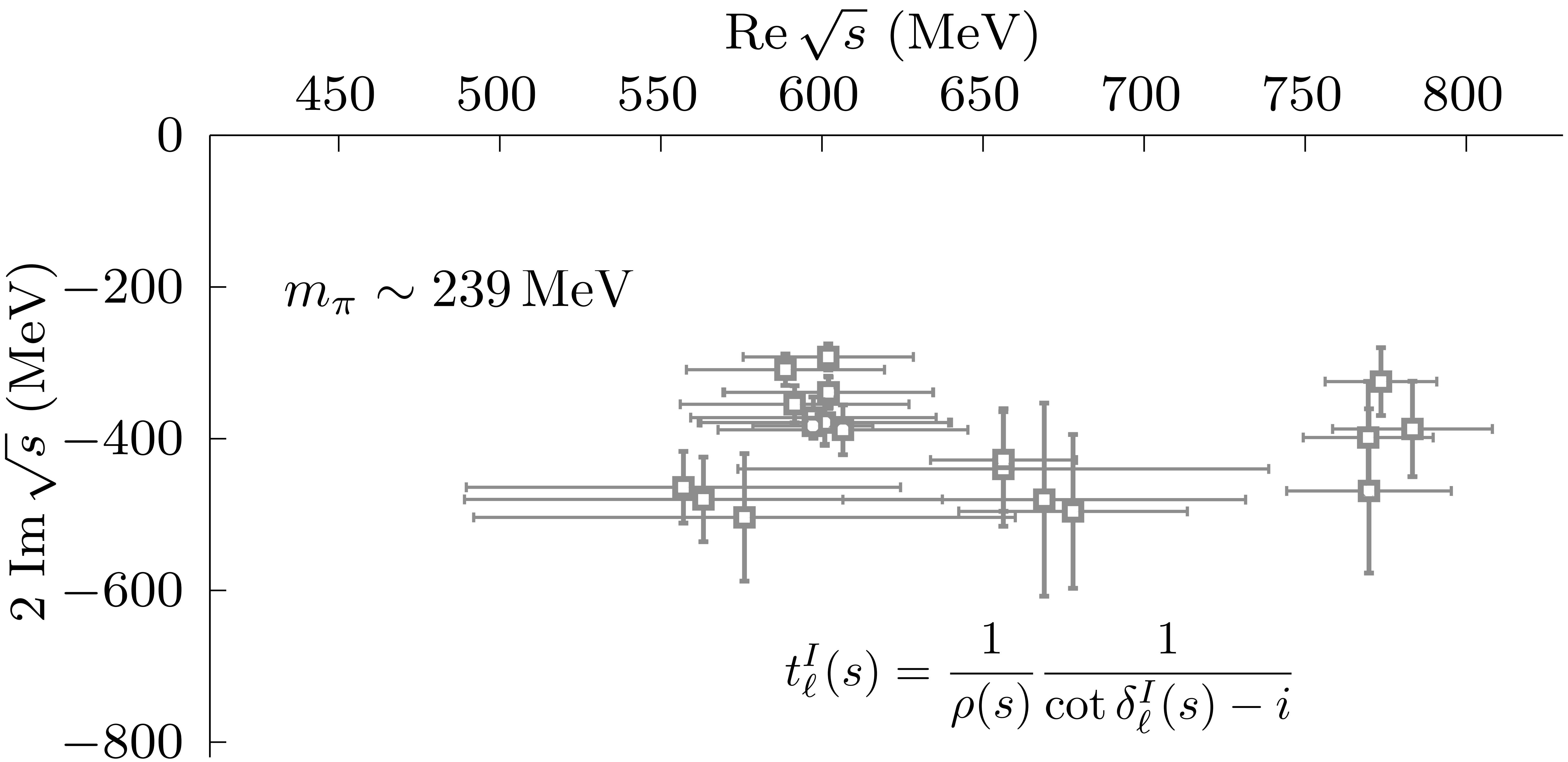
- $m_\pi \sim 391\, MeV \rightarrow$ **Stable**
- $m_\pi \sim 330\, MeV \rightarrow$ **Stable**
- $m_\pi \sim 239\, MeV \rightarrow$ **Broad resonance**
- $m_\pi \sim 283\, MeV \rightarrow$ **??**





Light Scalars: the σ

Total error becomes really large when decreasing the pion mass



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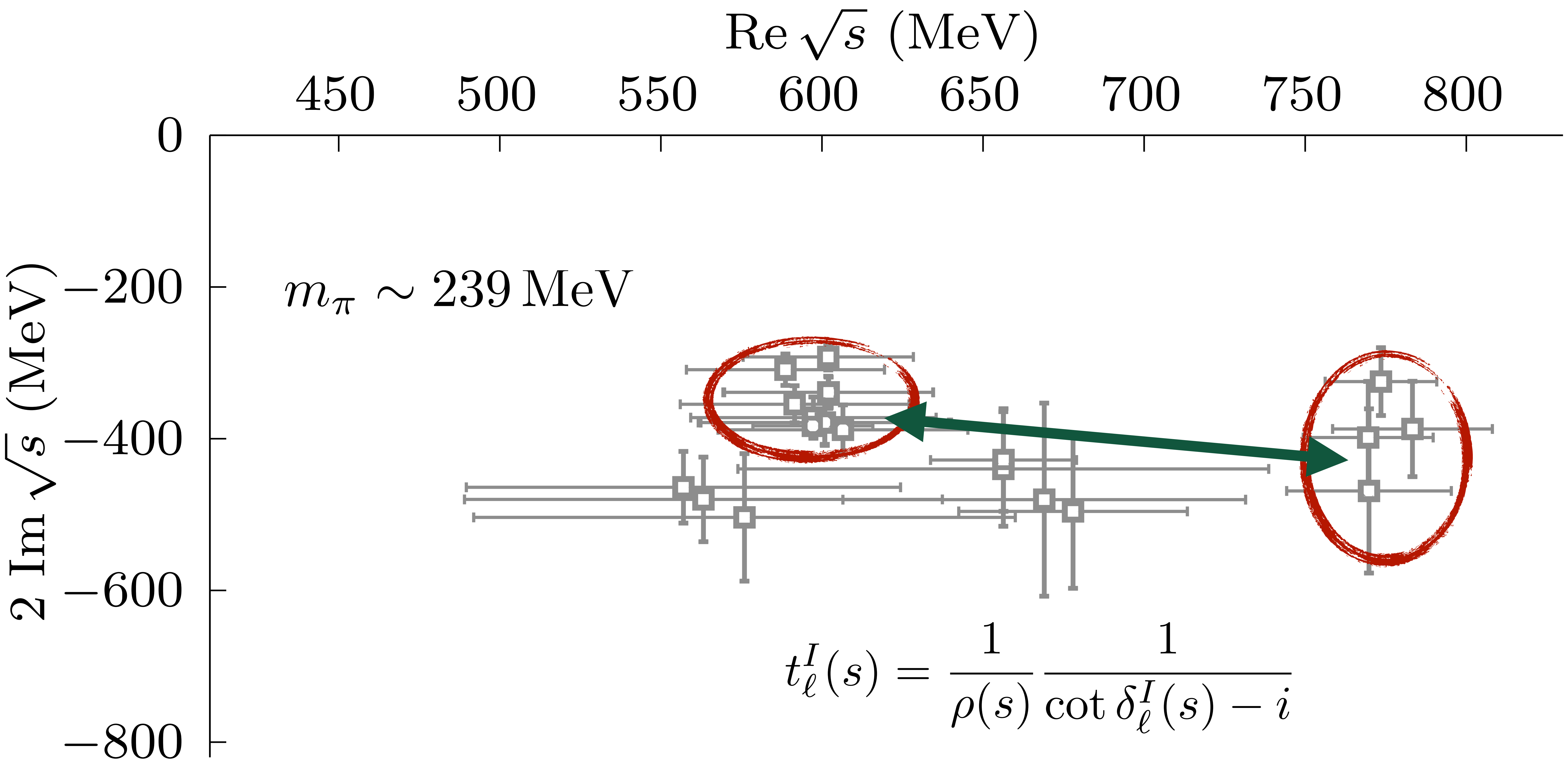

Models are incompatible with one another

Light Scalars: the σ

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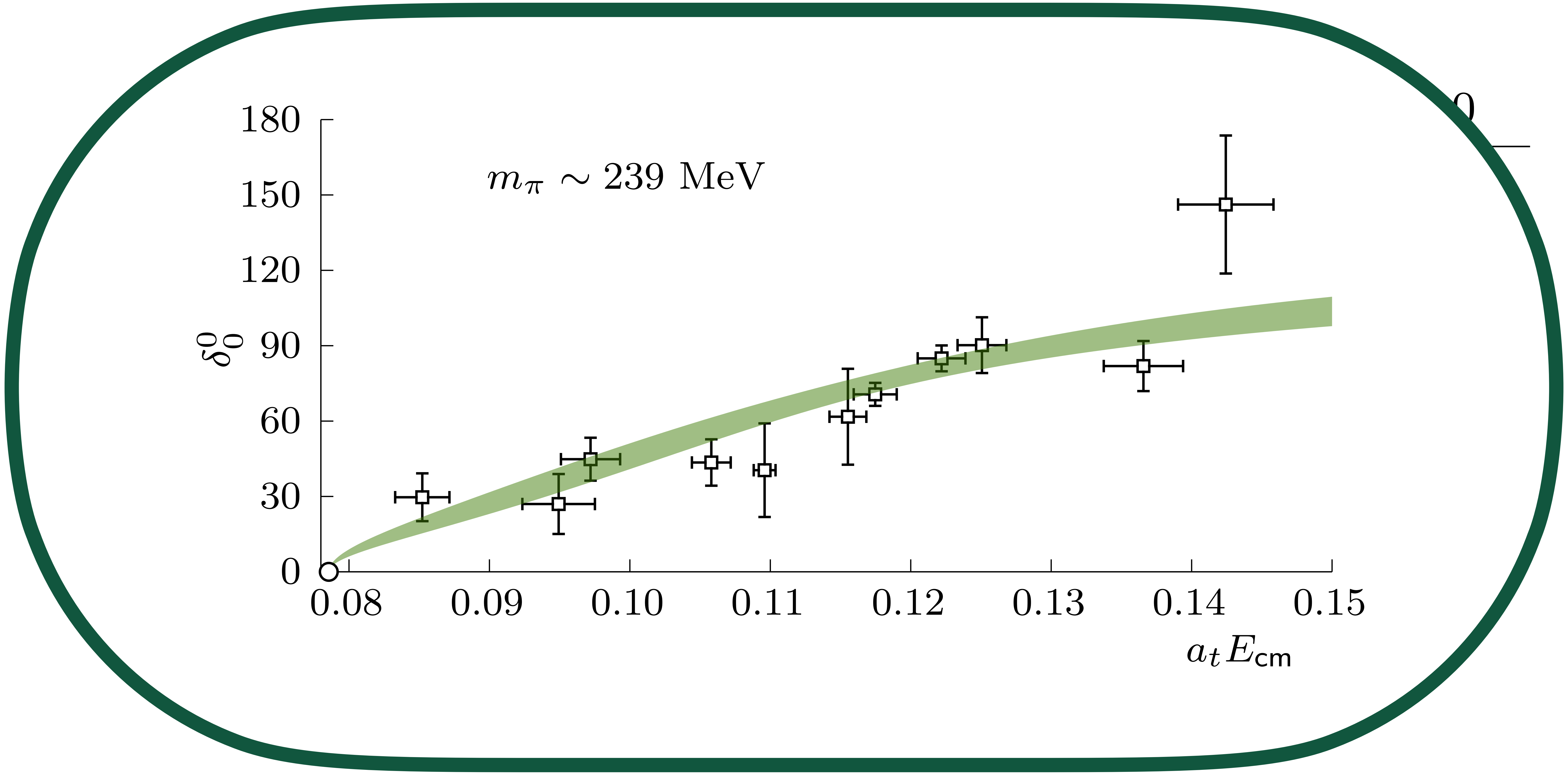

But data is precise !!

Light Scalars: the σ

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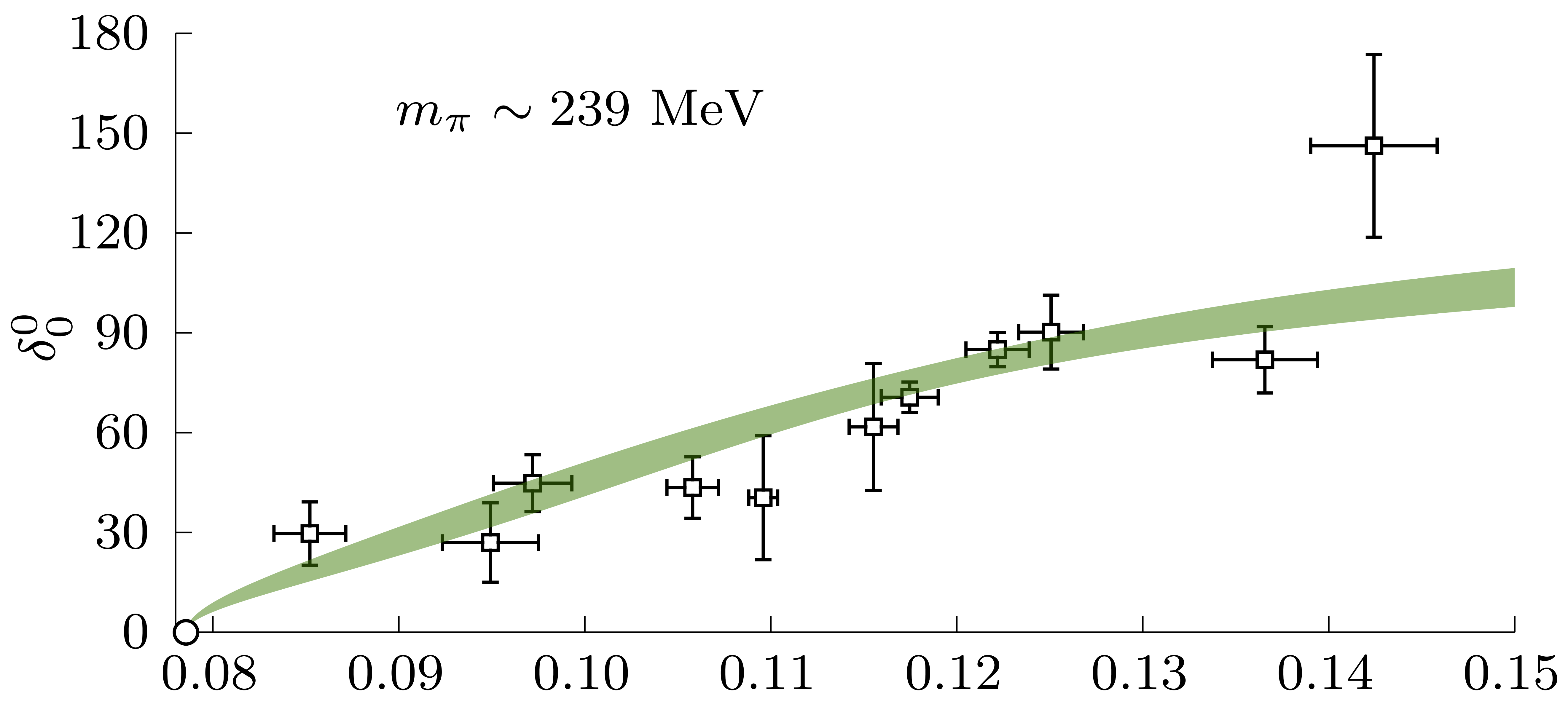
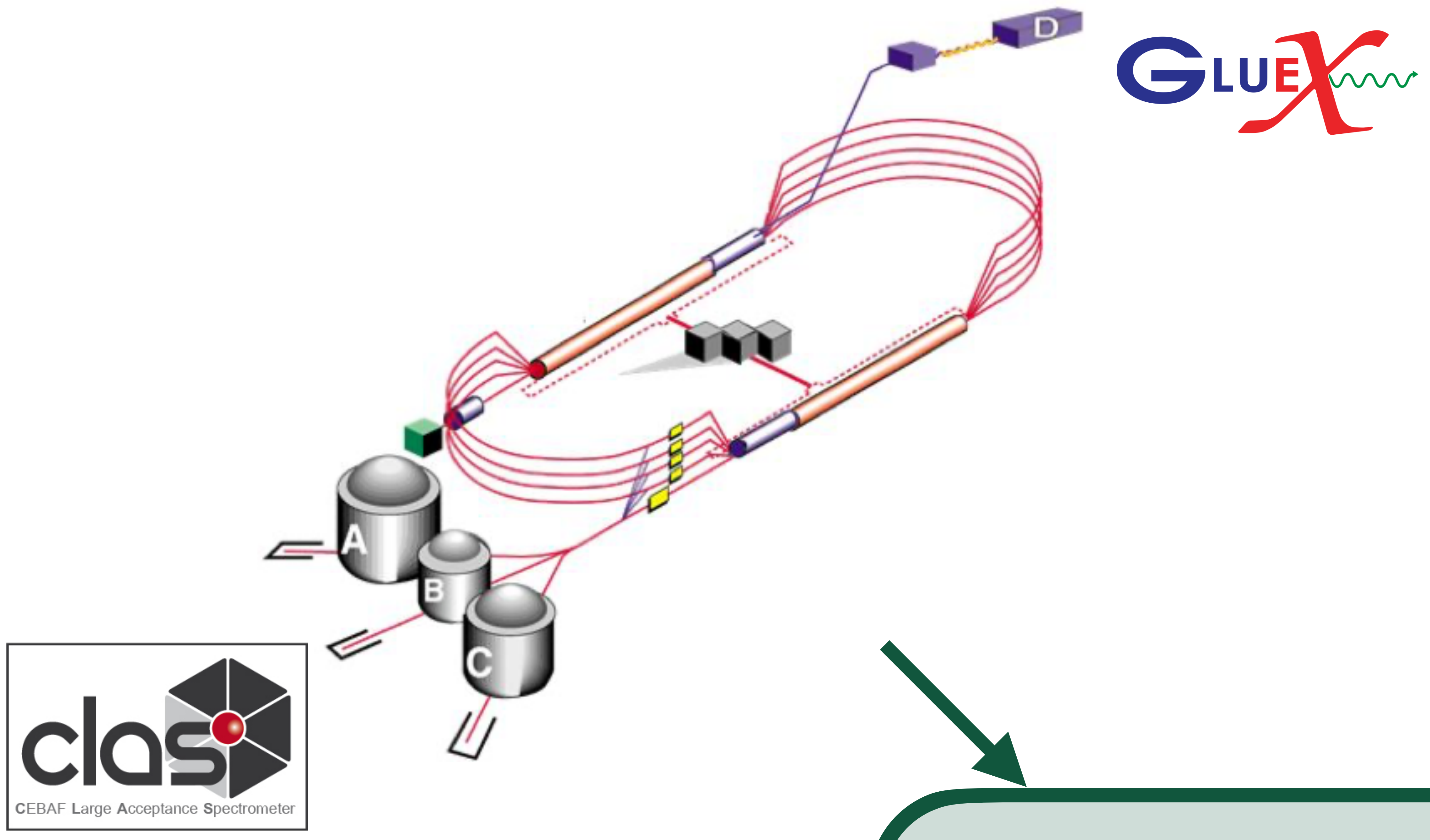
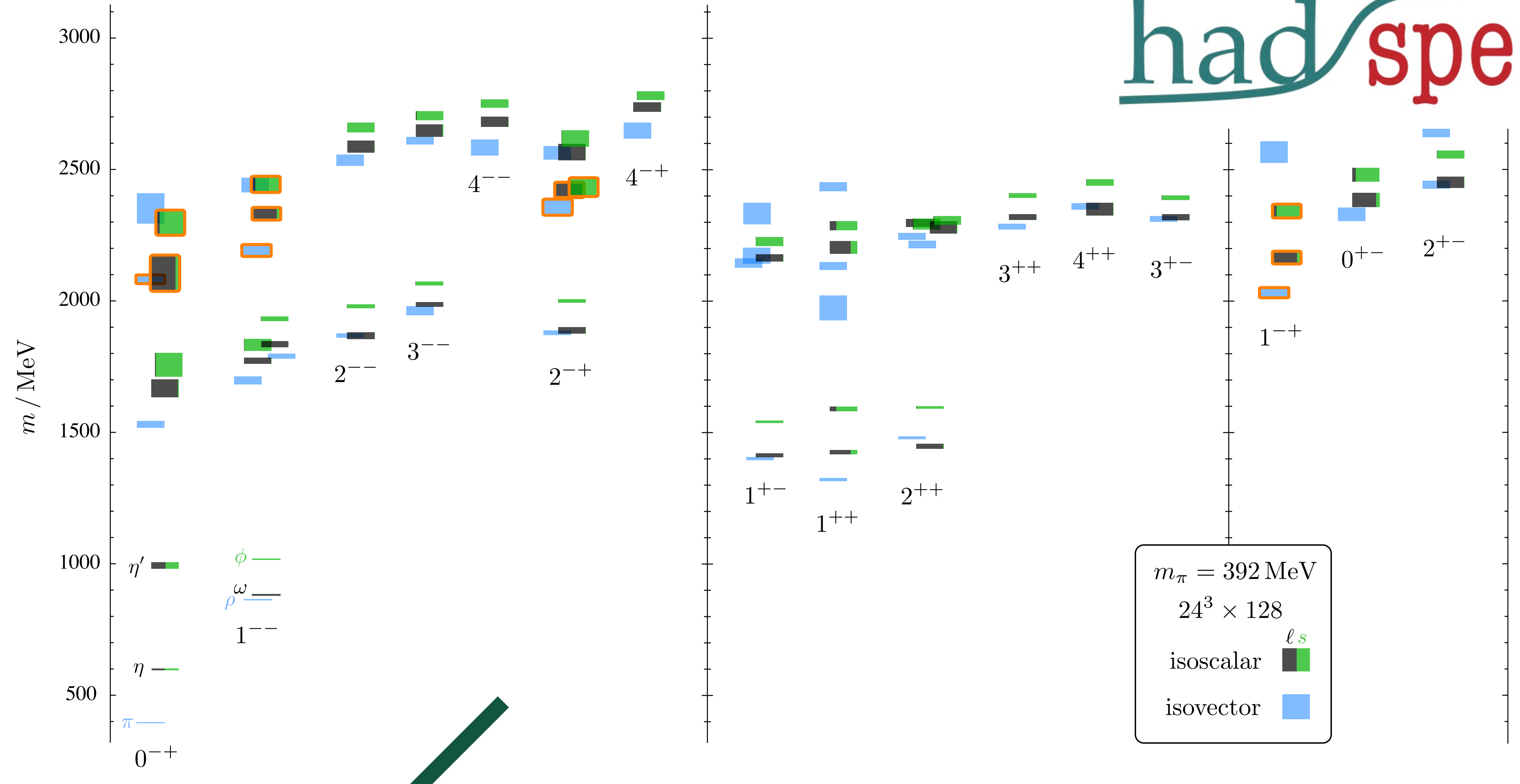


σ very challenging when reducing the pion mass

But data is precise !!

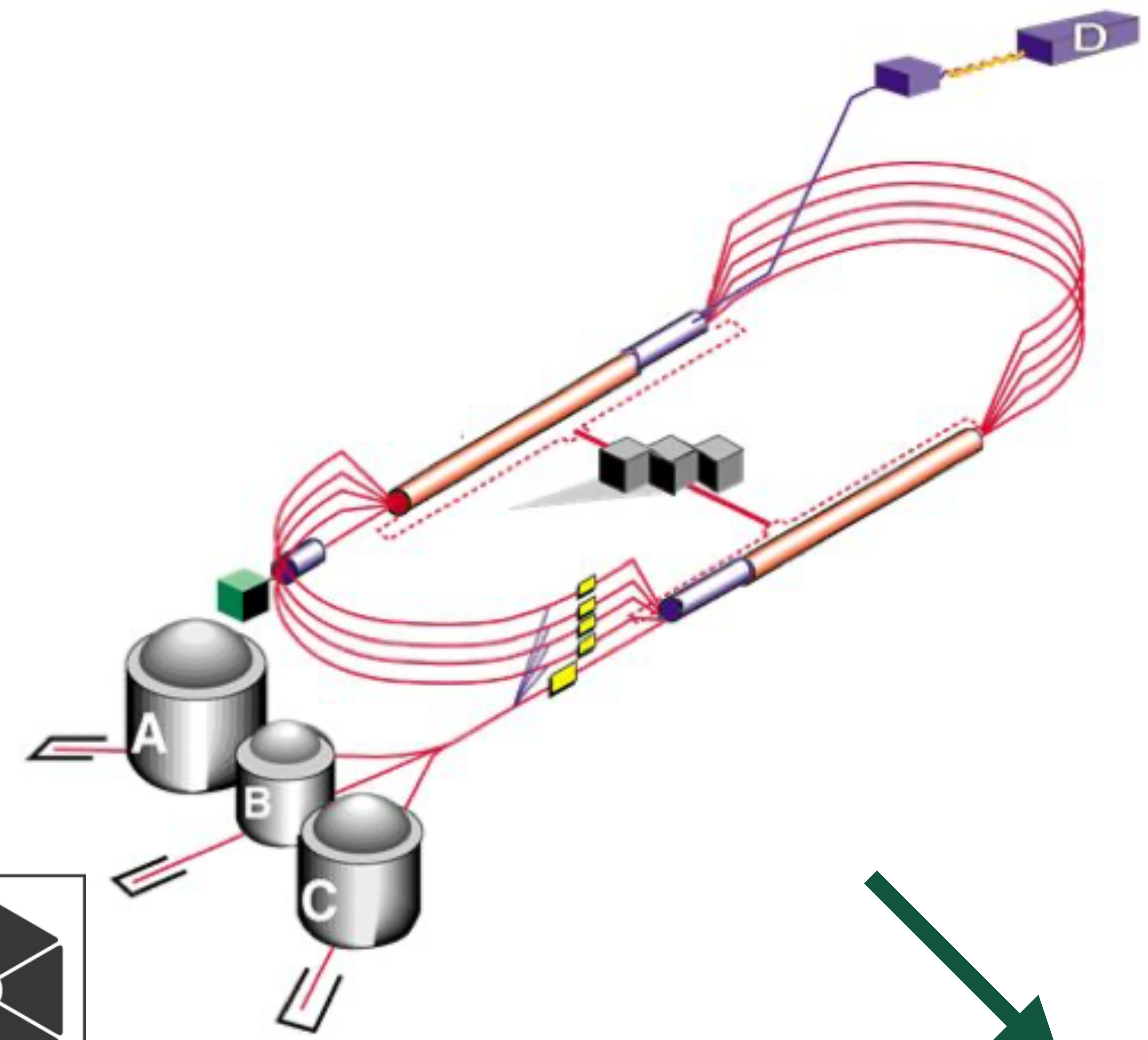


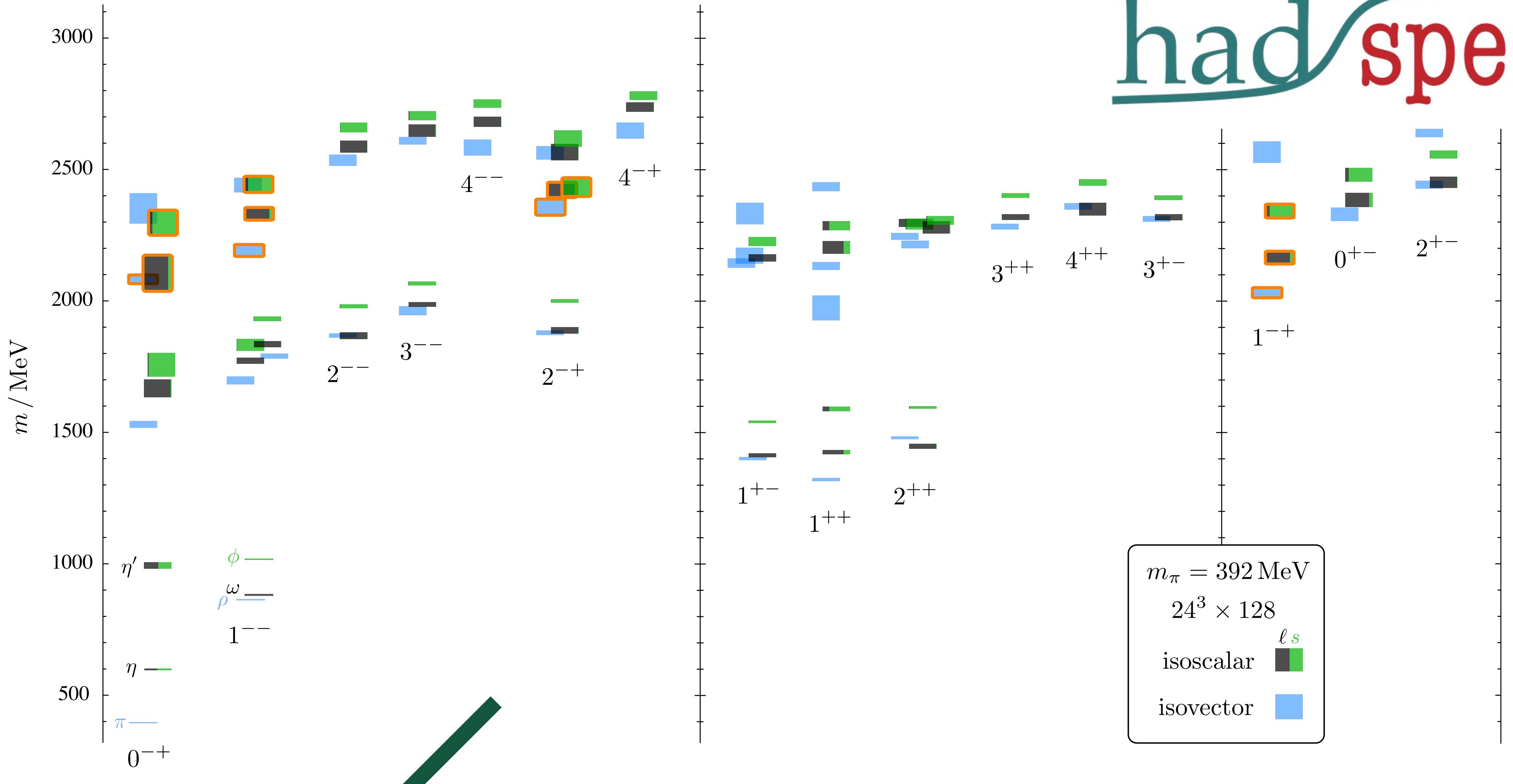


Amplitude analyses

- Unitarity
- Analiticity
- Crossing

Observables





Amplitude analyses

- Unitarity
- Analiticity
- Crossing

Sometimes → Analiticity

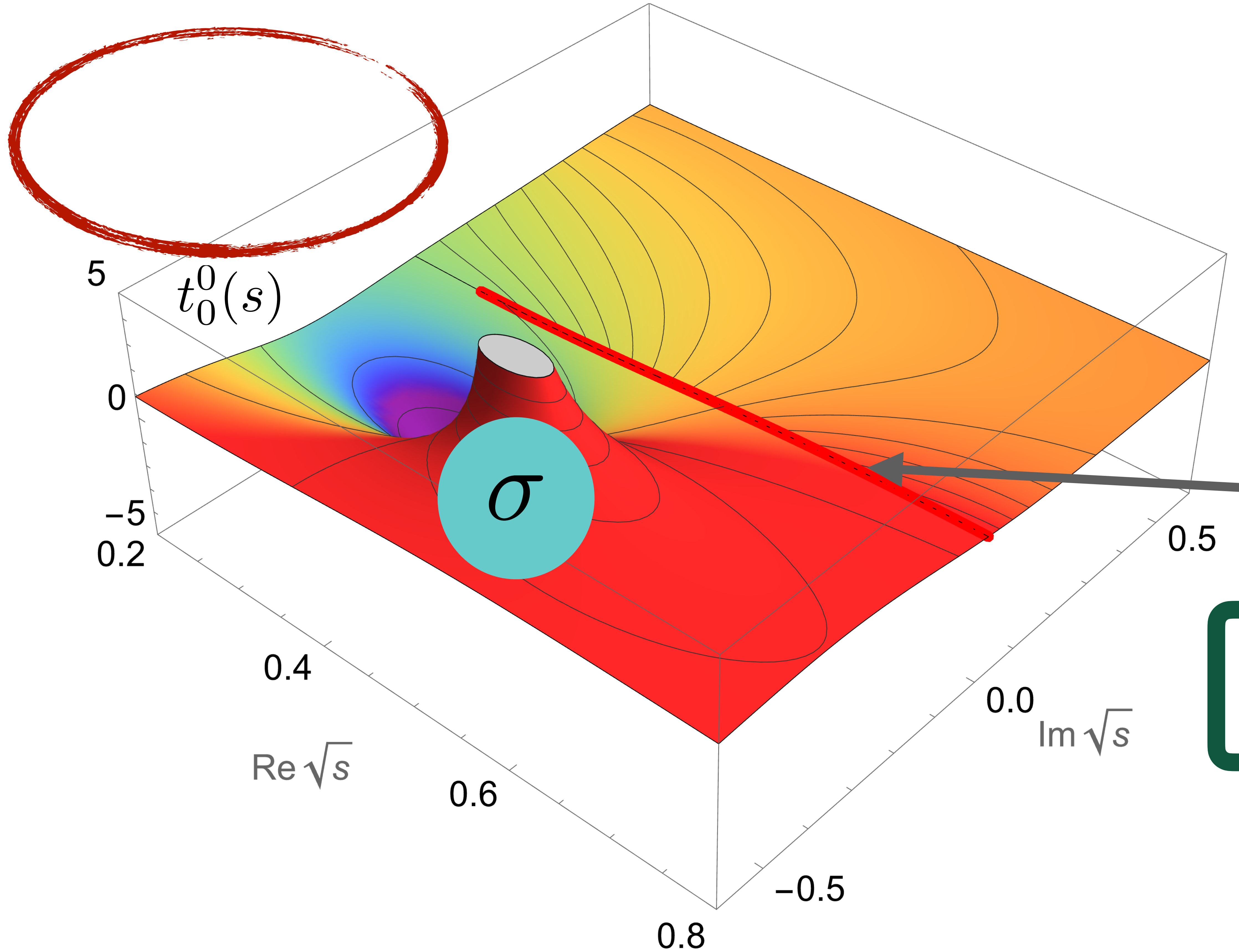
"Always" → Unitarity

$$t^I_\ell(s) = \frac{1}{\rho(s)} \frac{1}{\cot \delta^I_\ell(s) - i}$$

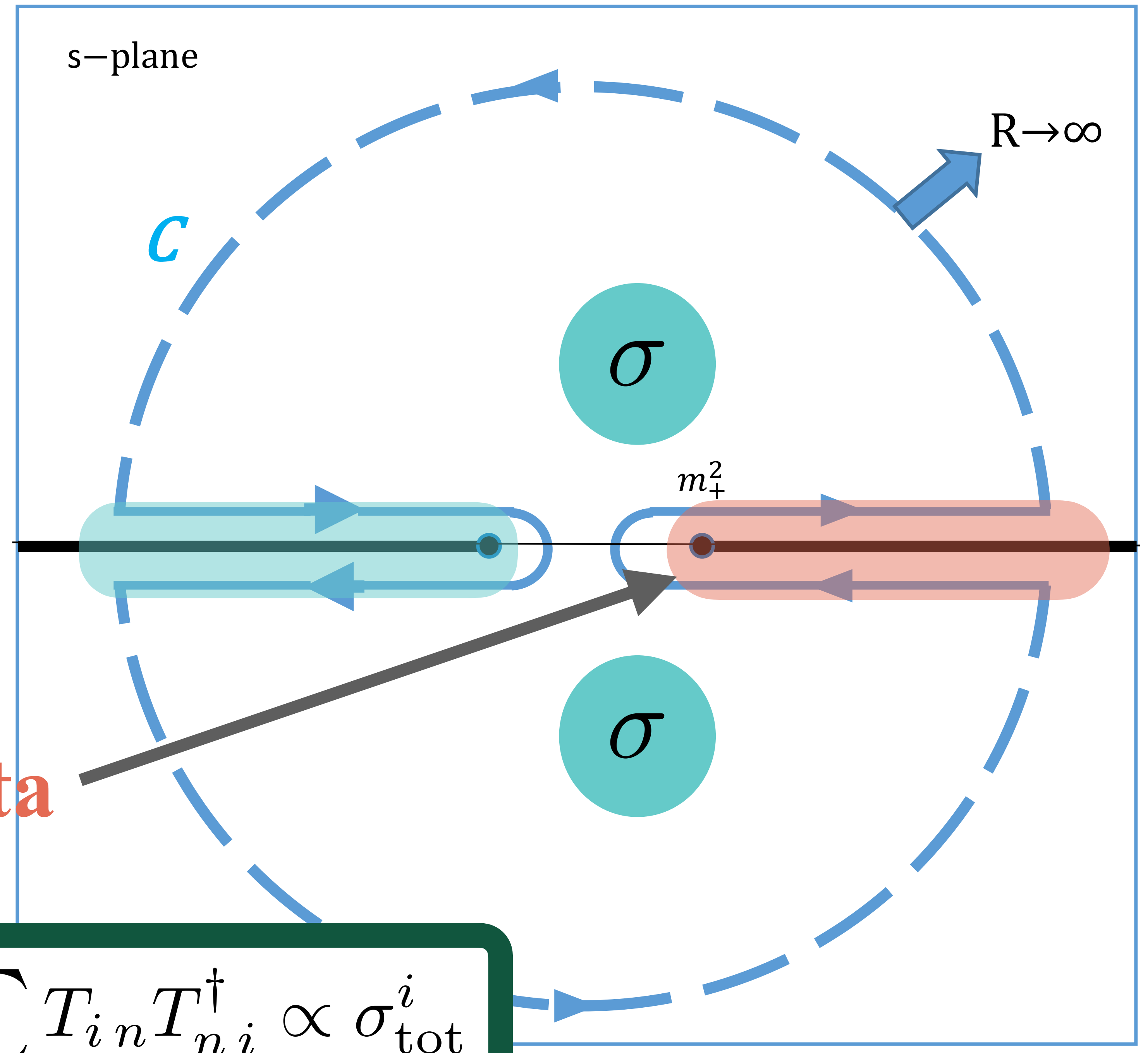
"Never" → Crossing

Observables

Crossing

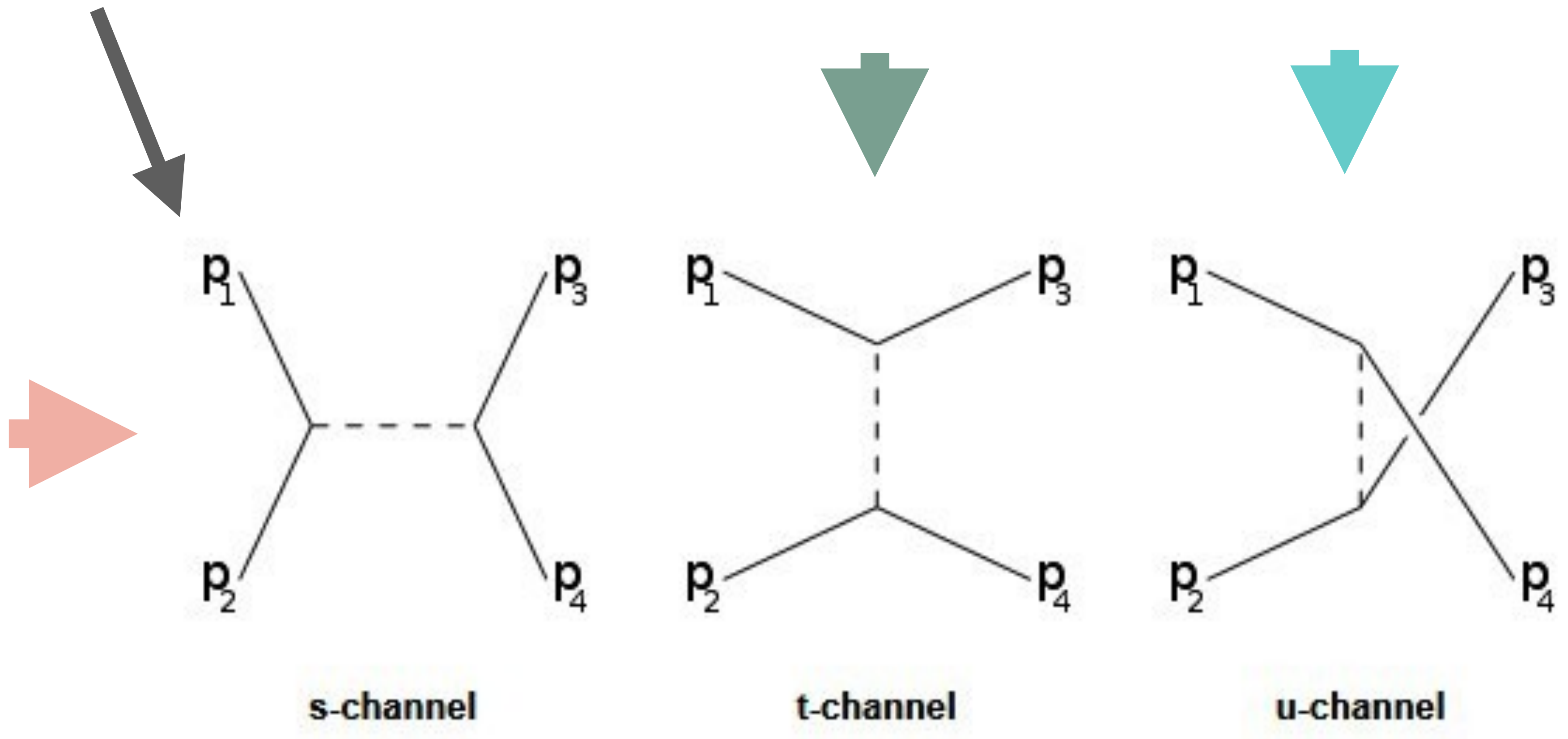


$t_0^0(s)$



Data

$$\text{Im}\, T_{i\,i}(s, t=0) \propto \sum_n T_{i\,n} T^\dagger_{n\,i} \propto \sigma^i_{\text{tot}}$$



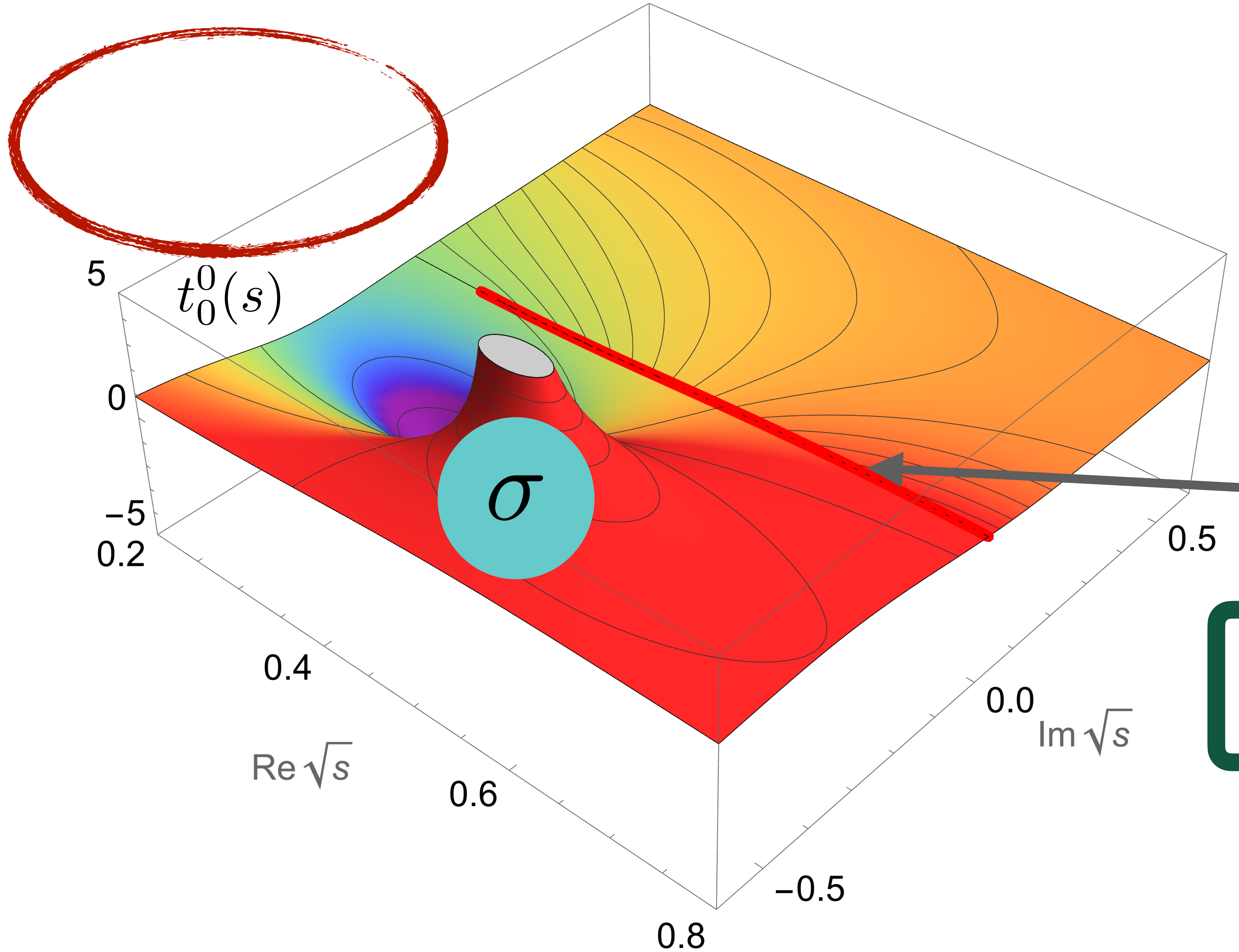
Particles and anti-particles are related

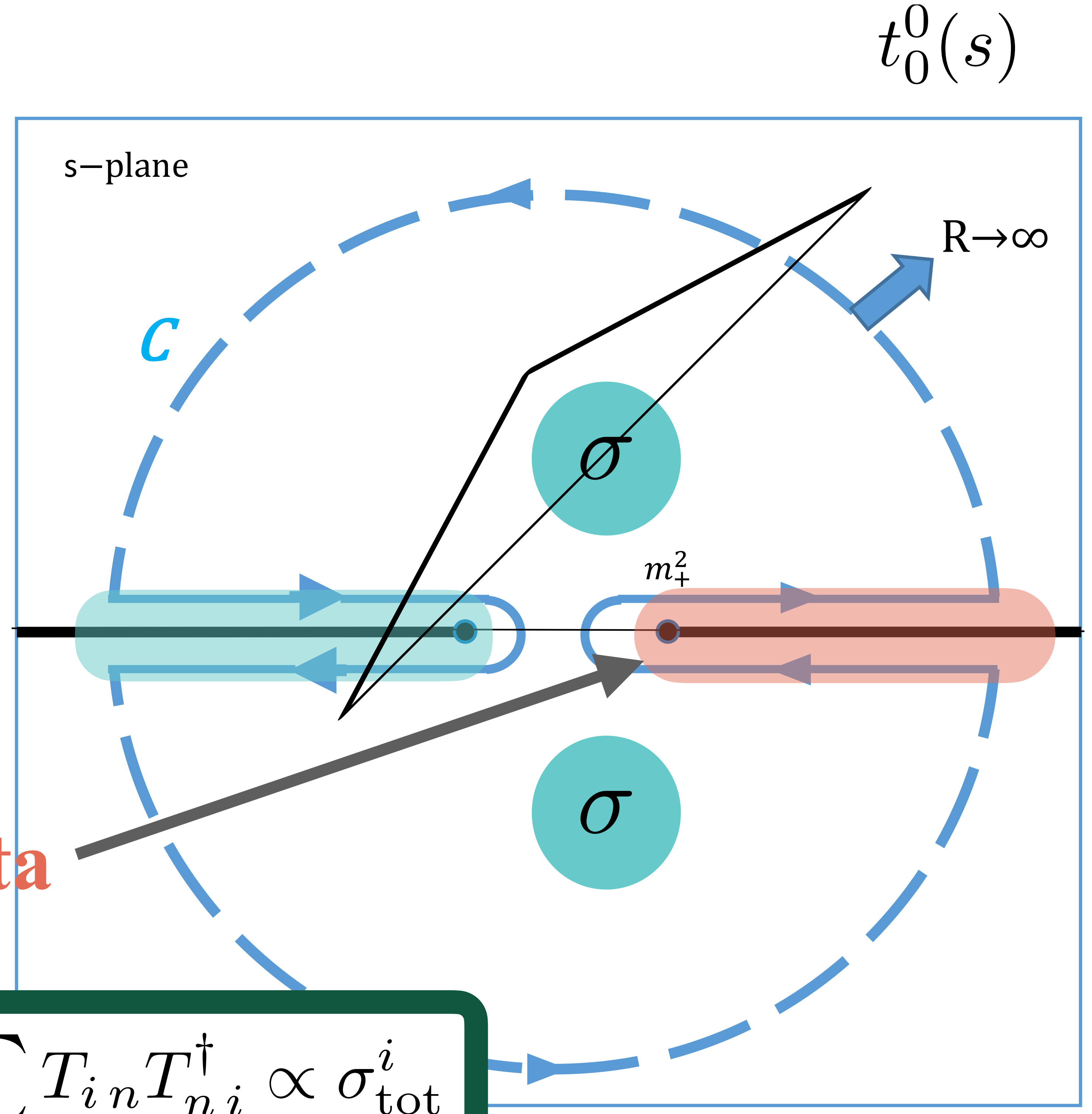
s-channel

t-channel

u-channel

Crossing





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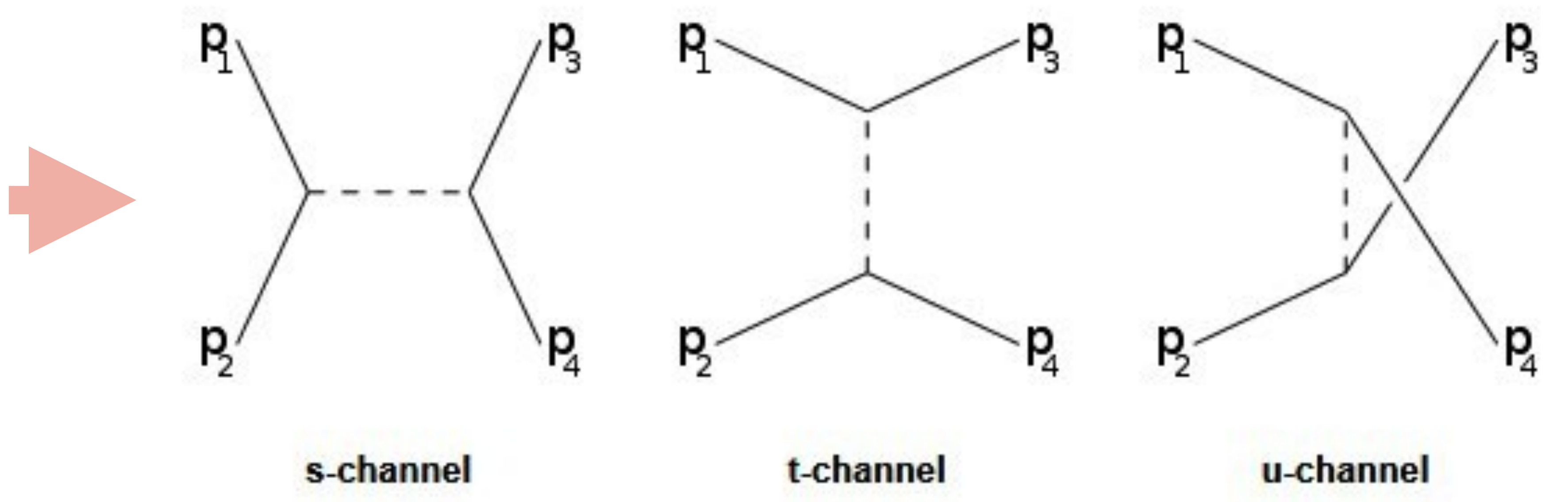
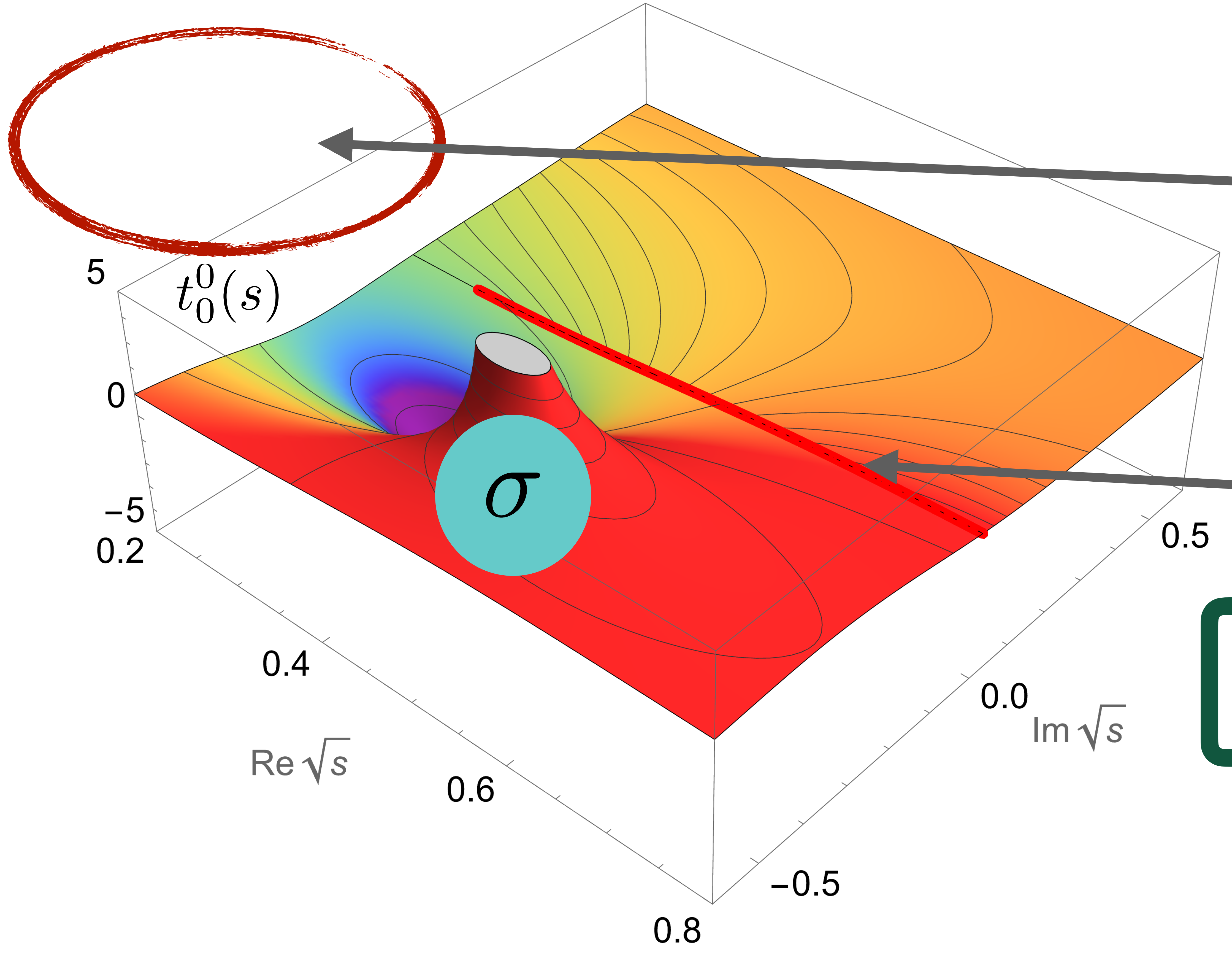
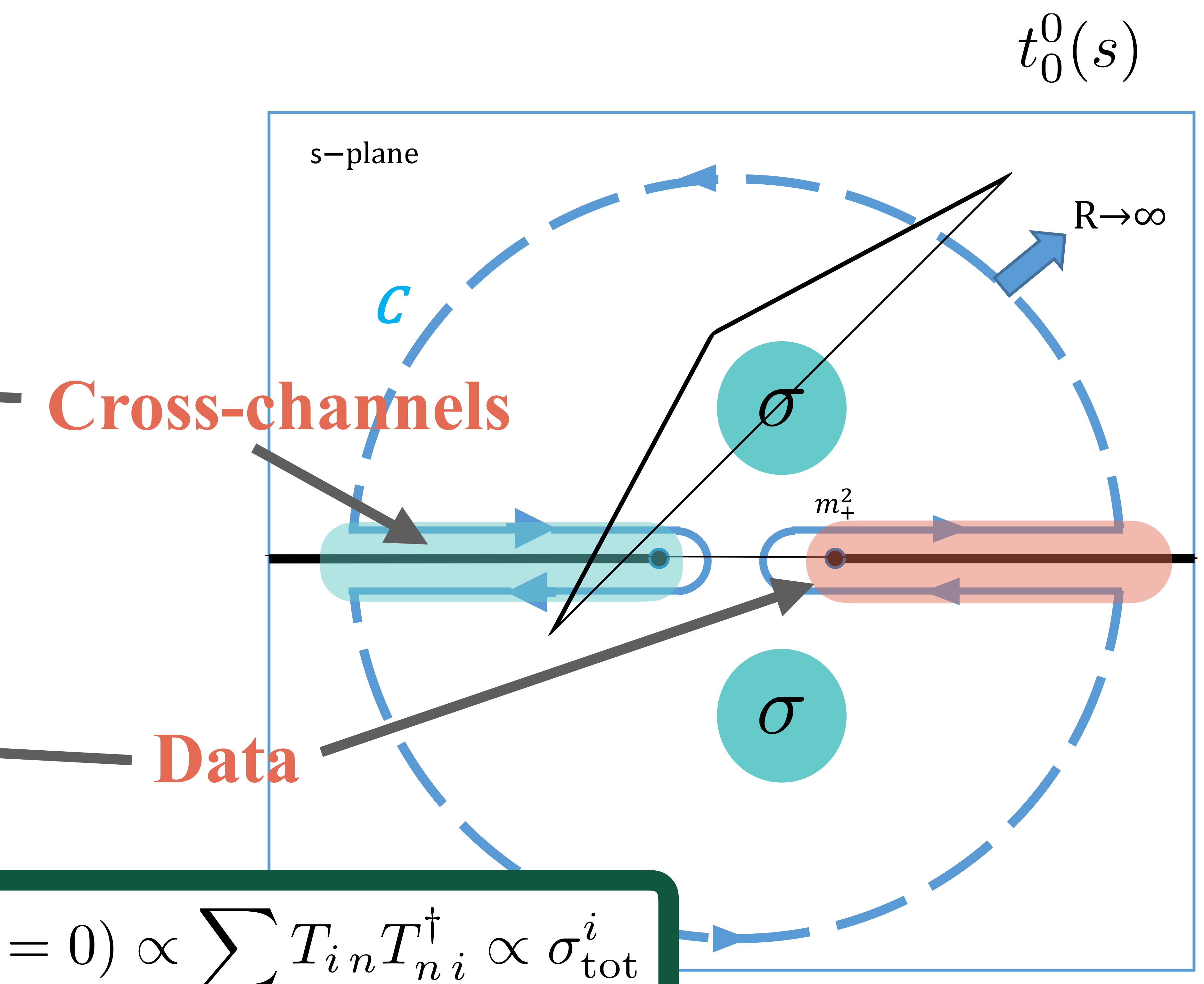


Crossing





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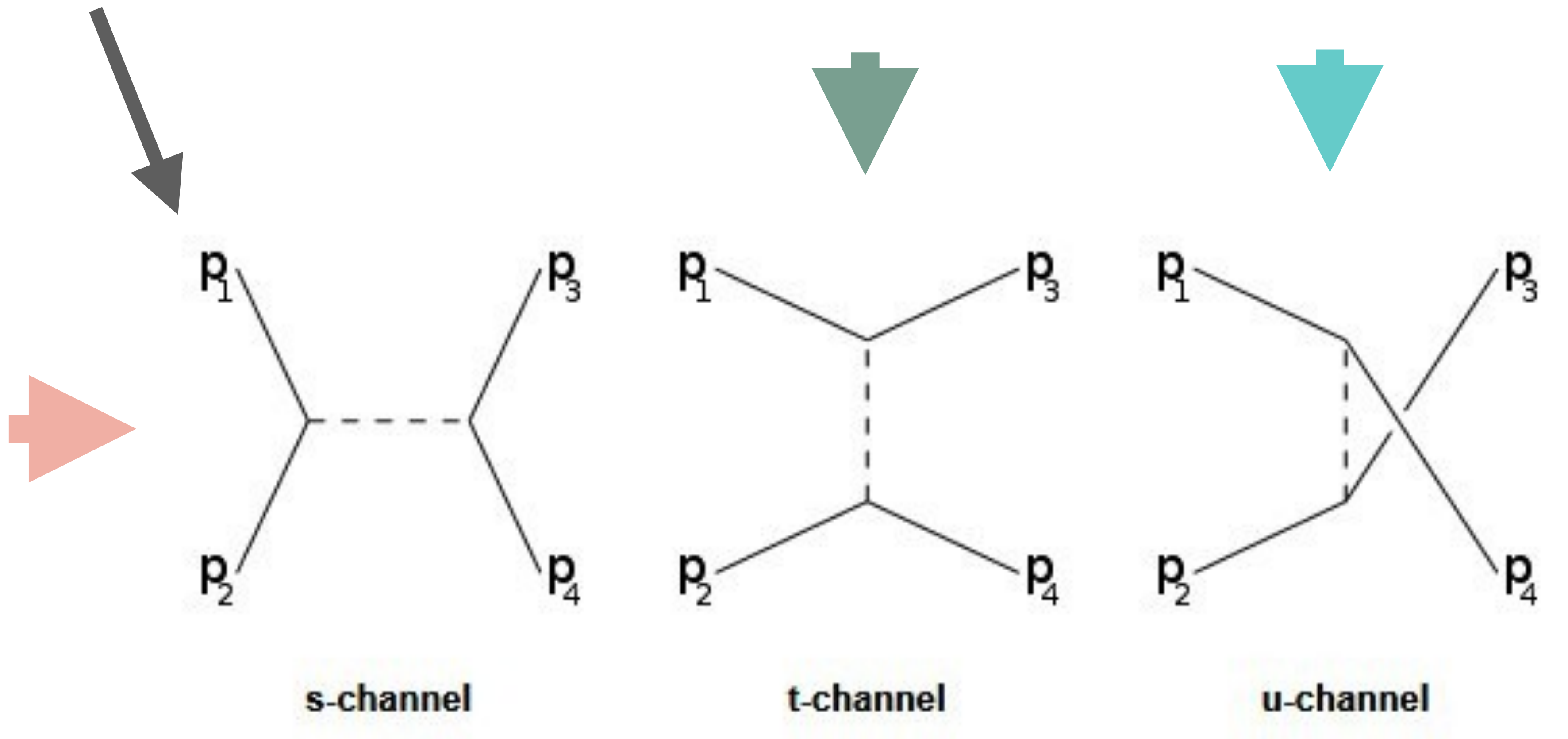
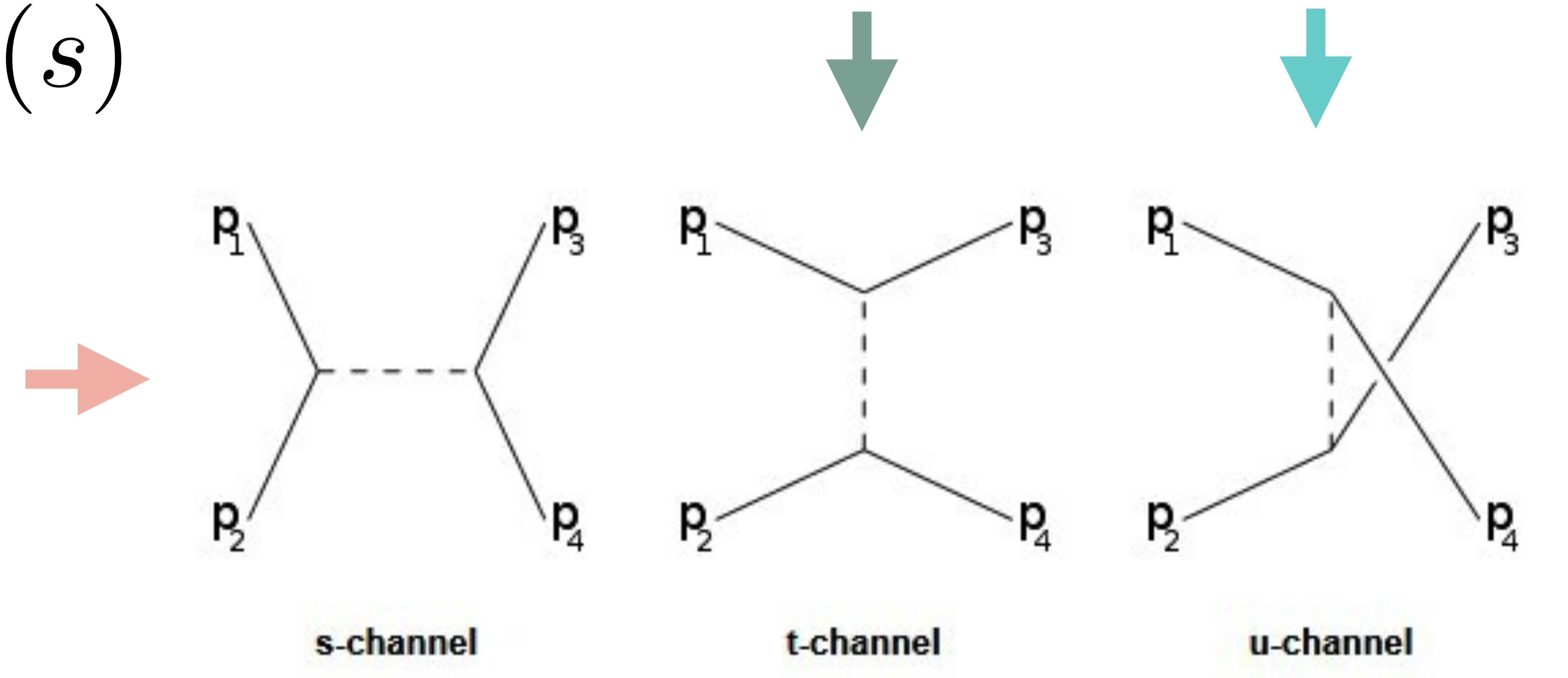


Crossing

$t_0^0(s)$



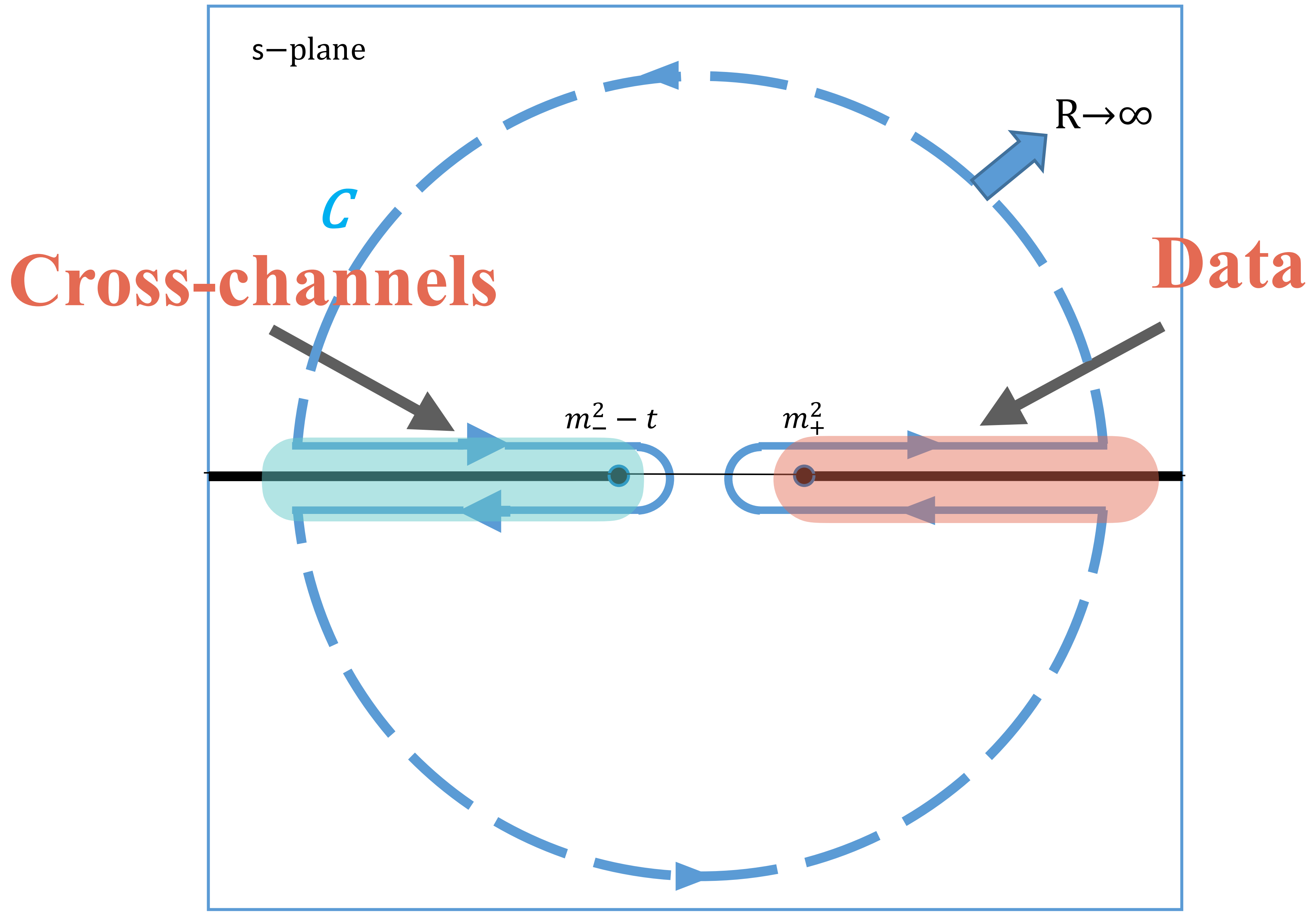
We need crossing!! Perhaps implemented analytically

Dispersion relations

Causality ↔ Analiticity

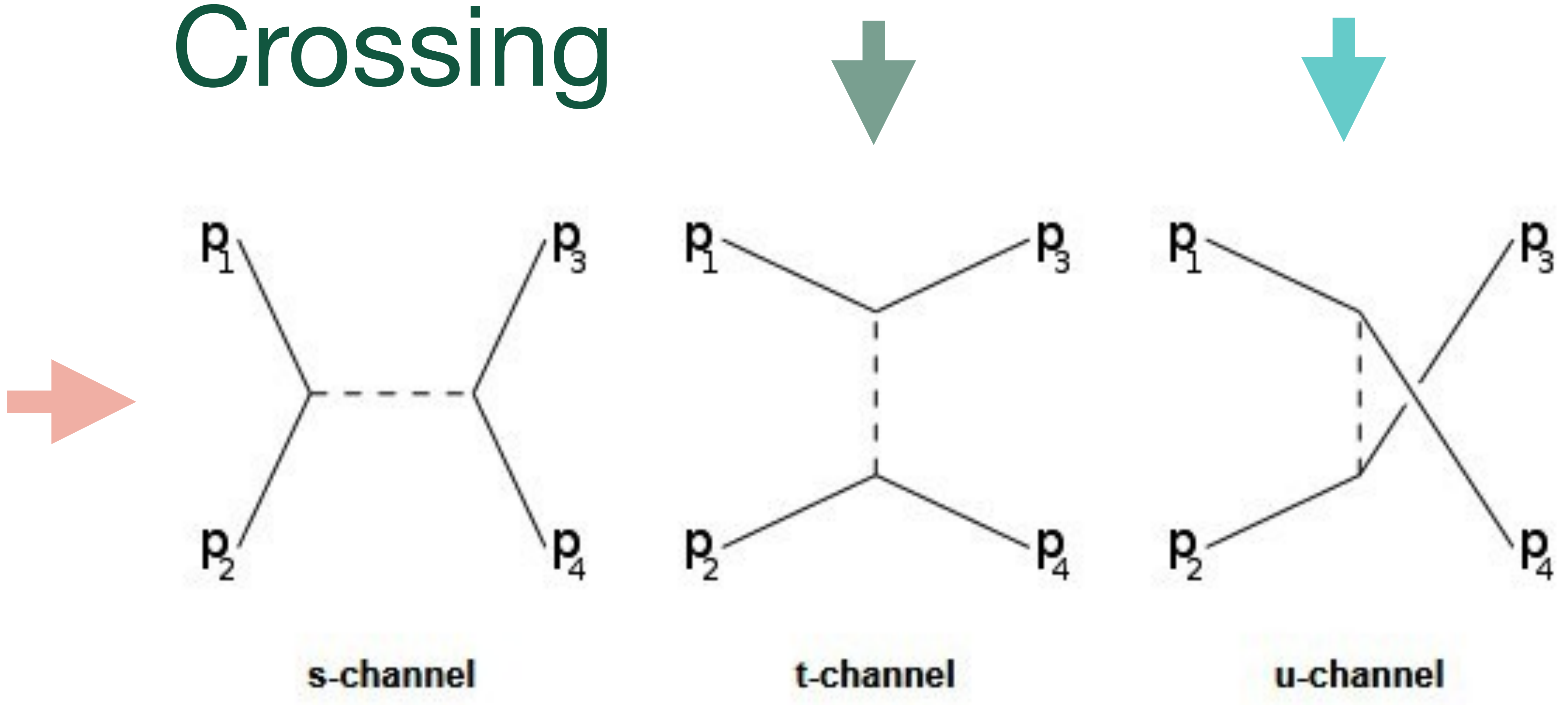
Built using Cauchy's theorem

$$t(z) = \oint_C \frac{t(z')}{z'-z} dz'$$



They can implement both analyticity AND crossing

Crossing



Partial wave dispersion relations

Amplitudes are decomposed in partial waves

$$T^I(s,t) = 32\pi \sum_\ell (2\ell+1) P_\ell(\cos\theta) t_\ell^I(s)$$

Fit to the data

Fit → In

DR → Out

Dispersive's result

$$\tilde{t}_\ell^I(s) = \tau_\ell^I(s) + \sum_{I',\ell'} \int_{4m_\pi^2}^\infty ds' \, K_{\ell\ell'}^{II'}(s',s) \, \mathrm{Im}\, t_{\ell'}^{I'}(s')$$

The most well-known are the ROY eqs., for example, for $I=\ell=0$ they look

Roy PLB (1971)

$$\tilde{t}_0^0(s) = a_0^0 + \frac{1}{12 m_\pi^2} \left(2a_0^0 - 5a_0^2\right)(s - 4m_\pi^2) + \sum_{I',\ell'} \int_{4m_\pi^2}^\infty ds' \, K_{0\ell'}^{0I'}(s',s) \, \mathrm{Im}\, t_{\ell'}^{I'}(s')$$

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DR → Out

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"Model-independent"

- [x] **Obtain your DRs** ⟷ **Crossing+analyticity**
- [ ] **Use all PWs available** ⟷ **Necessary Input**
- [ ] **Make** *Fit → In* *DR → Out* **compatible** ⟷ **Unitarity**

Use all PWs available

Scalar $\ell = 0$ waves dominate the DRs

But we extracted/fitted several waves

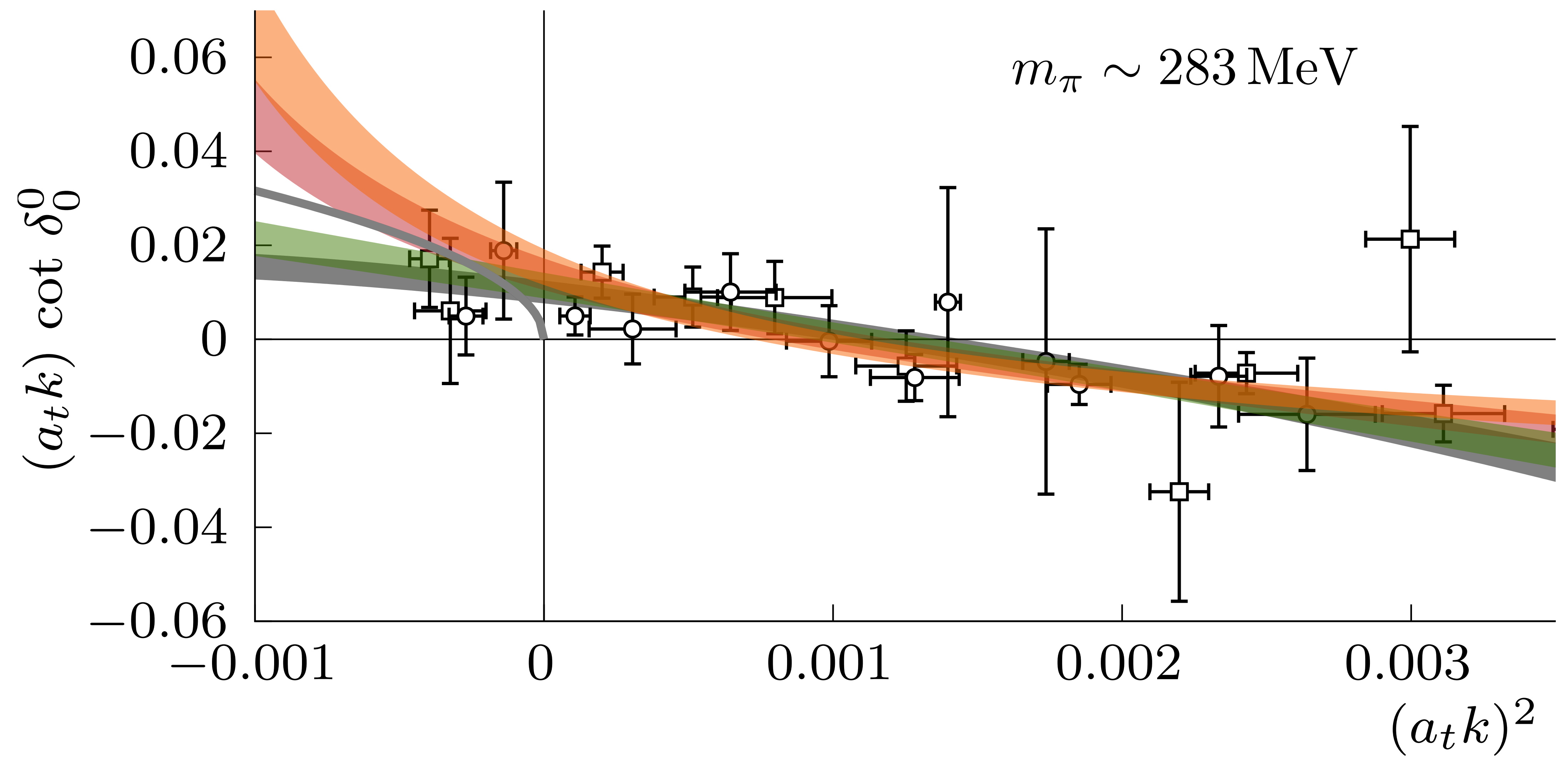
Every band is a different model fit

$$t_\ell^I(s) = \frac{1}{\rho(s)} \frac{1}{\cot \delta_\ell^I(s) - i}$$

Large SL spreads at threshold

$$k \cot \delta_0^I(s) \sim 1/a_0^I$$

$\ell = 0,\ I = 0\ \pi\pi$



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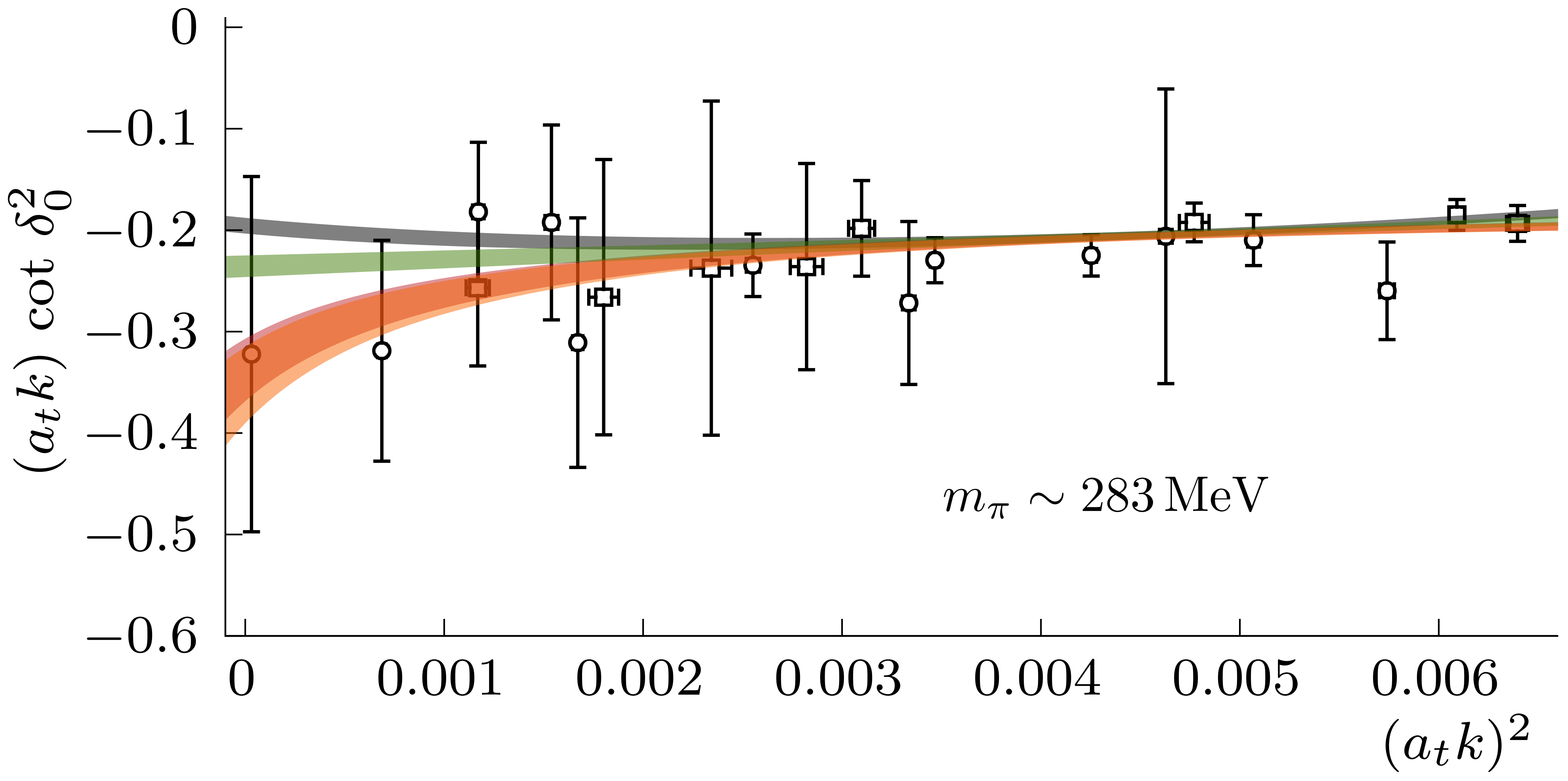
$\ell = 0,\ I = 2\ \pi\pi$



Fit → In

DR → Out

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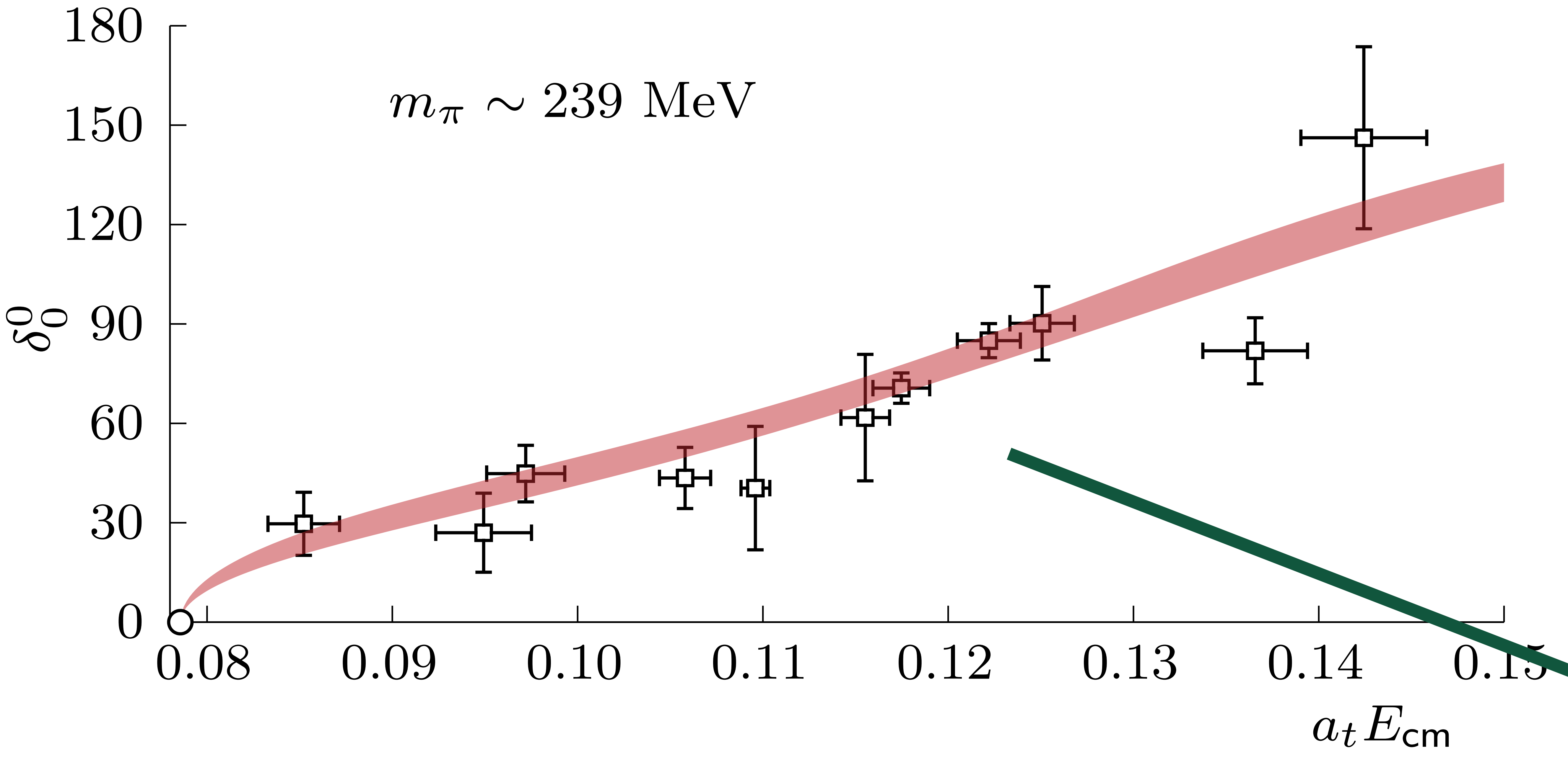
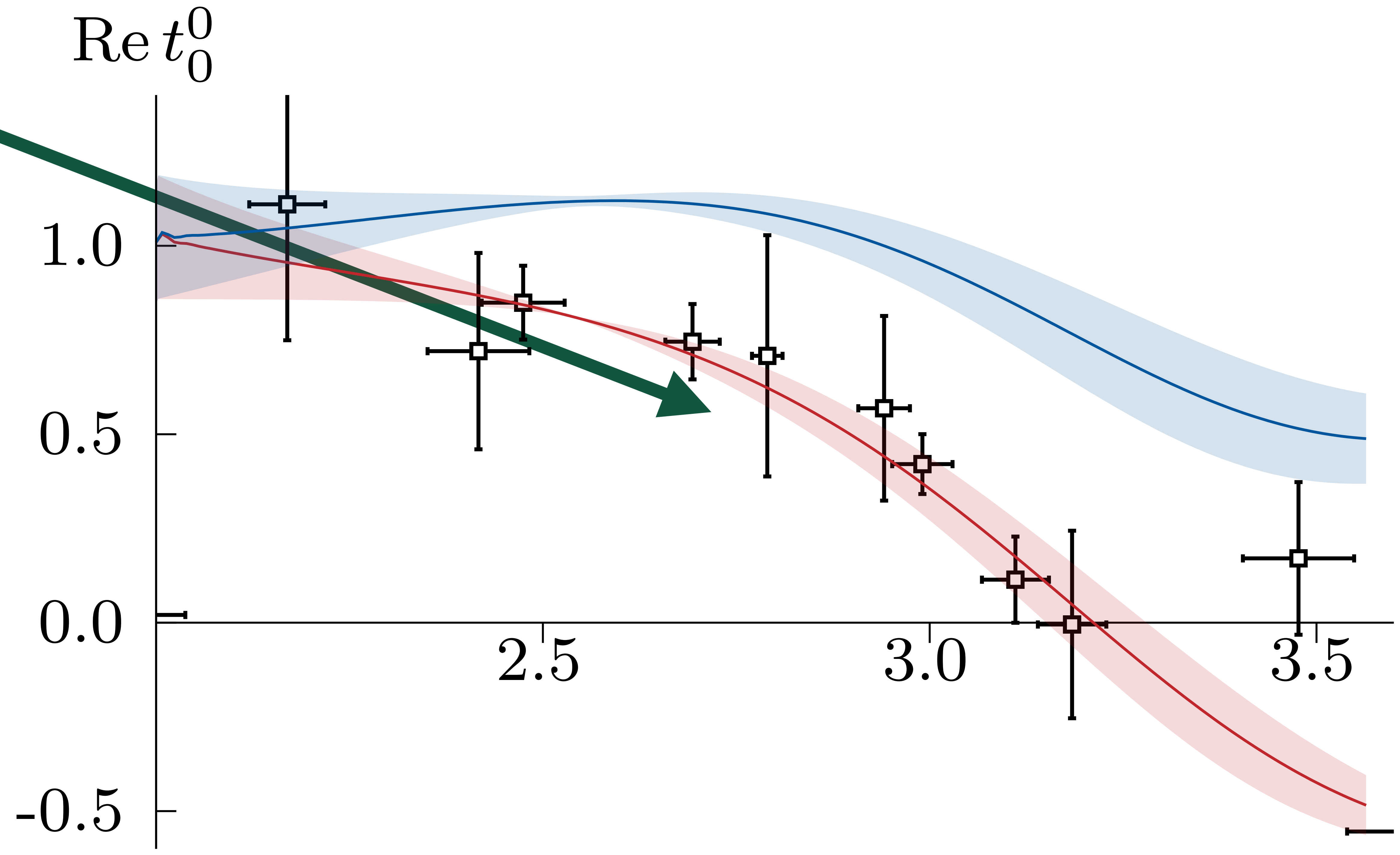
Make *Fit → In* *DR → Out* compatible



Model 1



Make *Fit → In* *DR → Out* compatible

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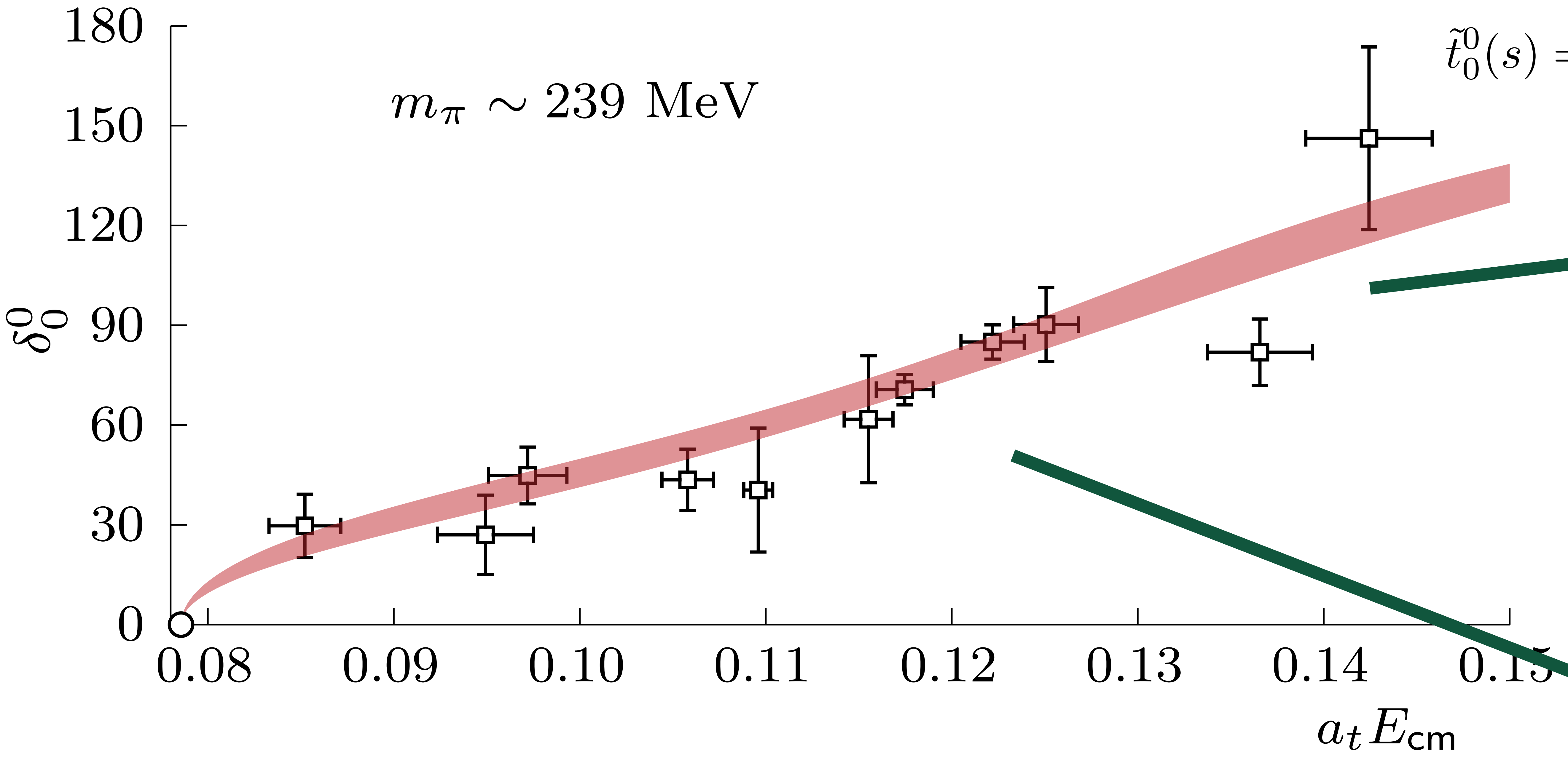
$$\tilde{t}_0^0(s) = a_0^0 + \frac{1}{12m_\pi^2}\left(2a_0^0 - 5a_0^2\right)\left(s - 4m_\pi^2\right) + \sum_{I',\ell'}\int_{4m_\pi^2}^{\infty} ds' K_{0\ell'}^{0I'}(s',s)\,\mathrm{Im}\,t_{\ell'}^{I'}(s')$$



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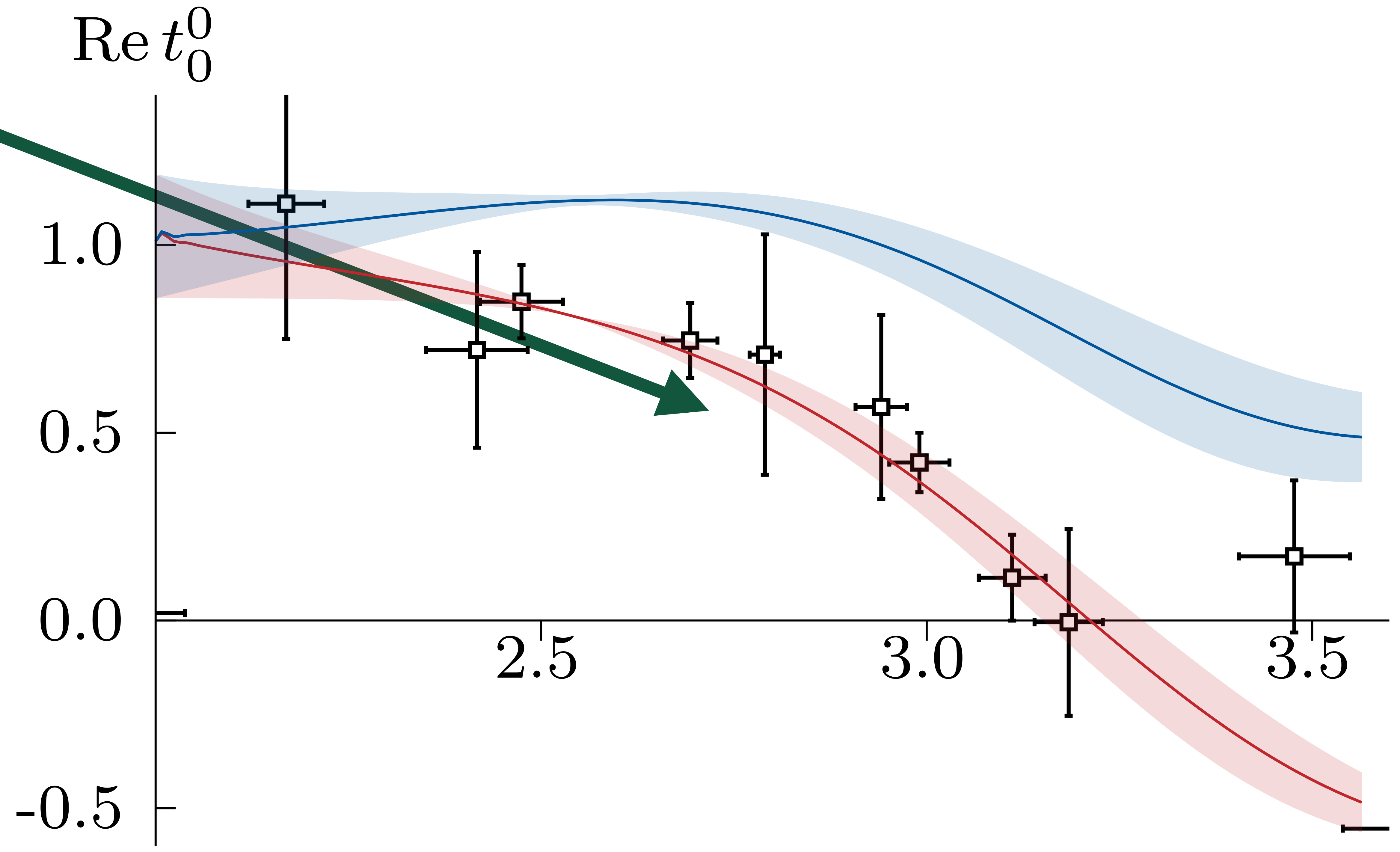

Make *Fit → In* *DR → Out* **compatible**

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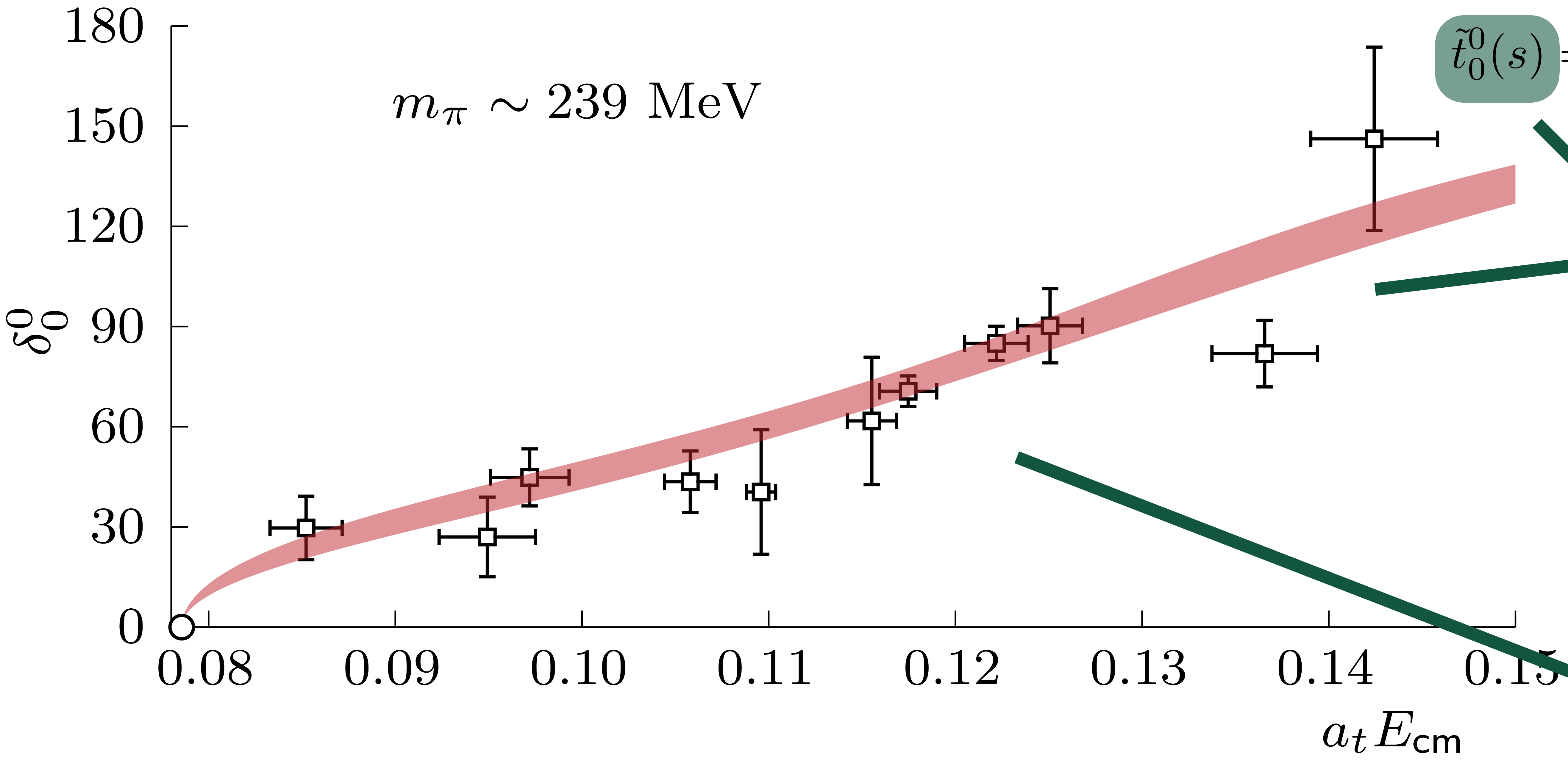

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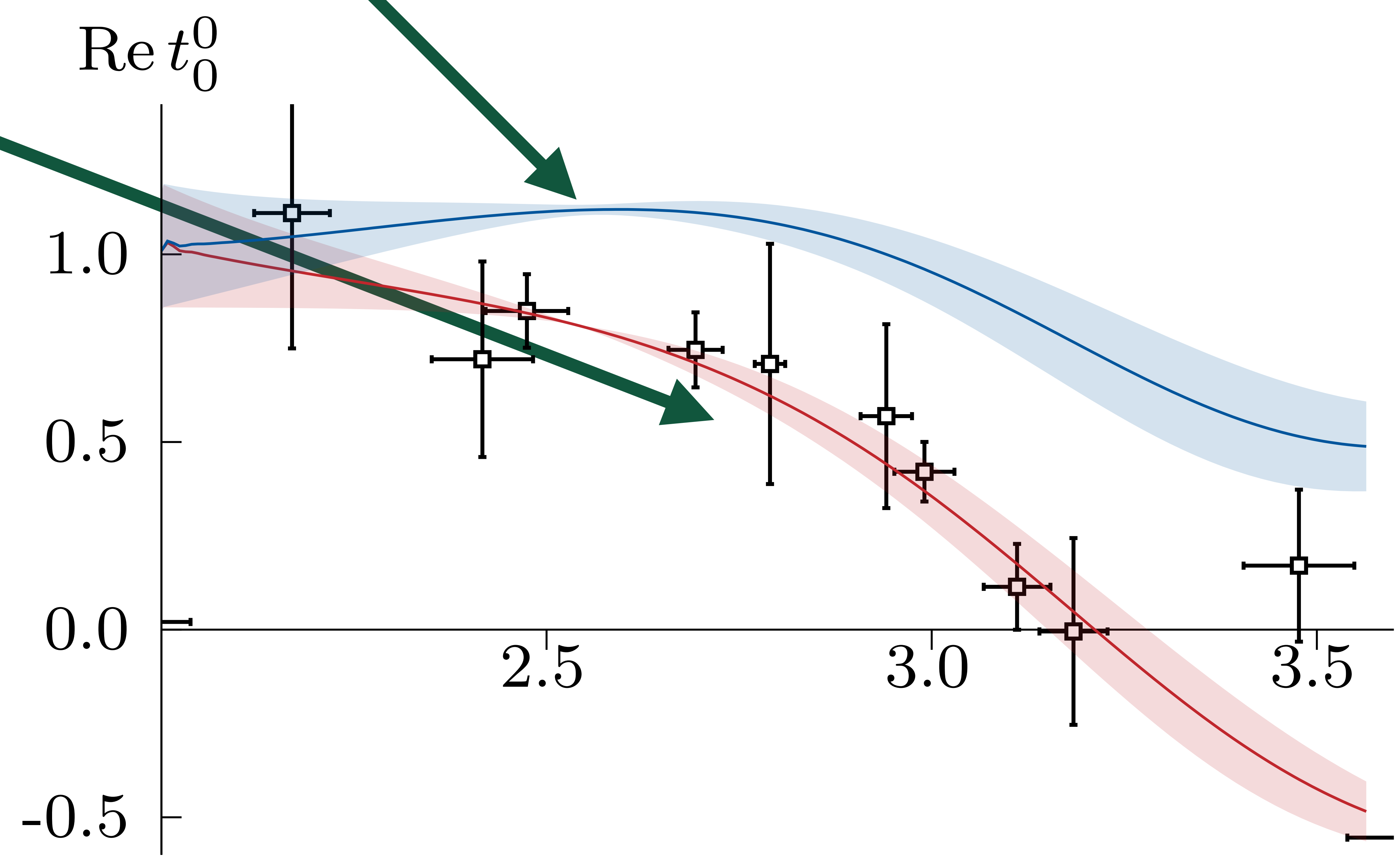

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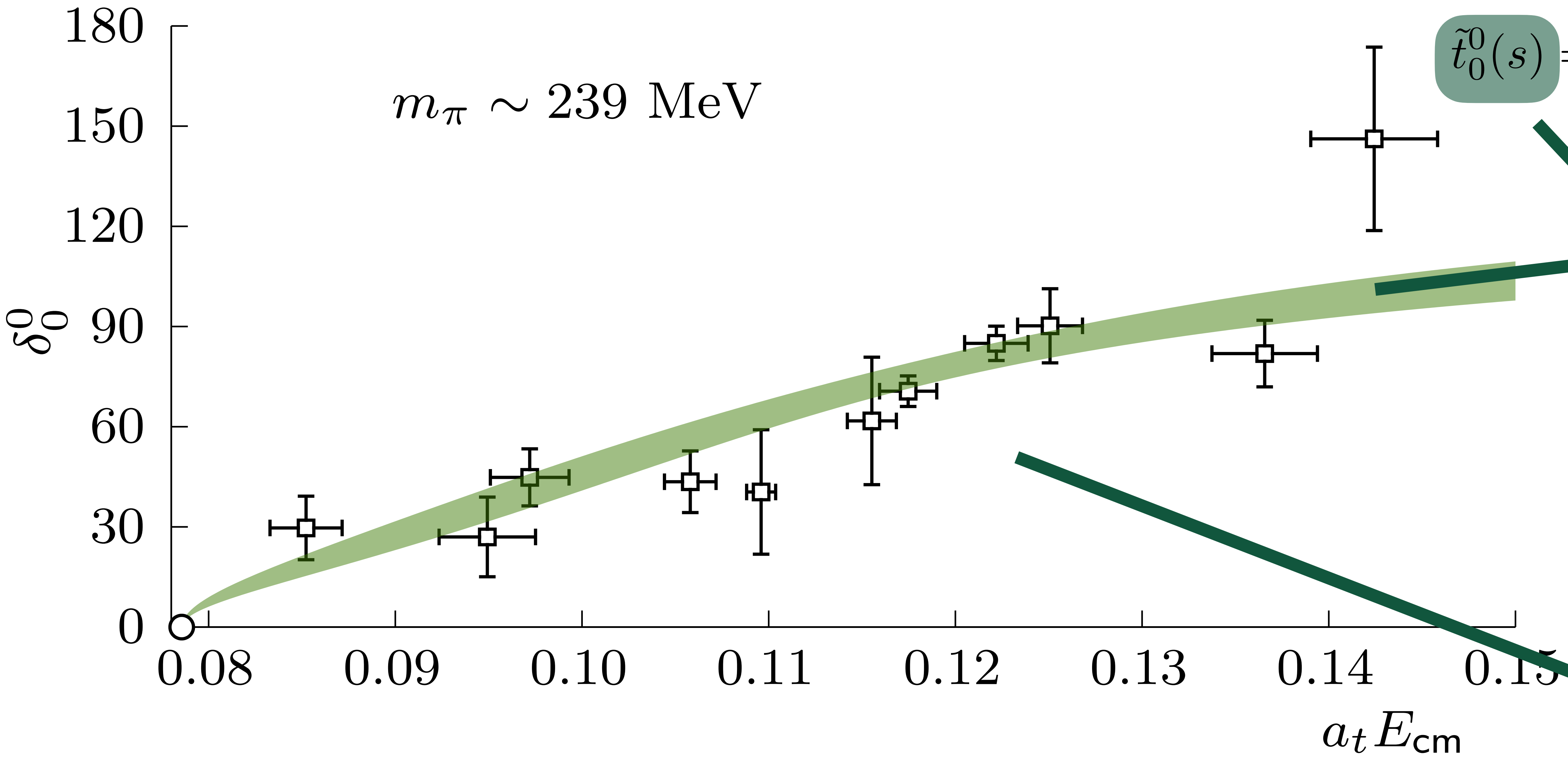
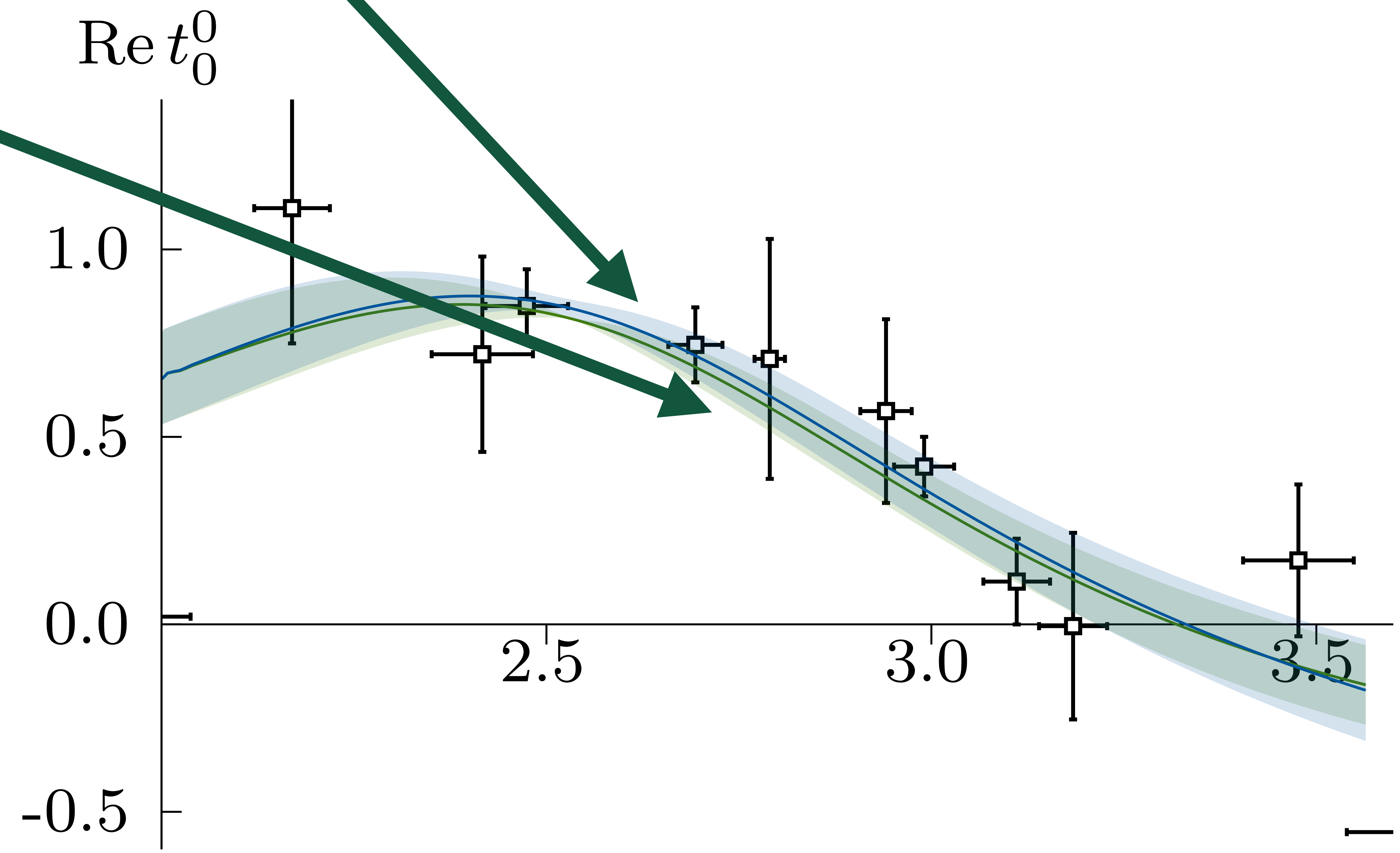


Model 2



Make *Fit → In* *DR → Out* compatible

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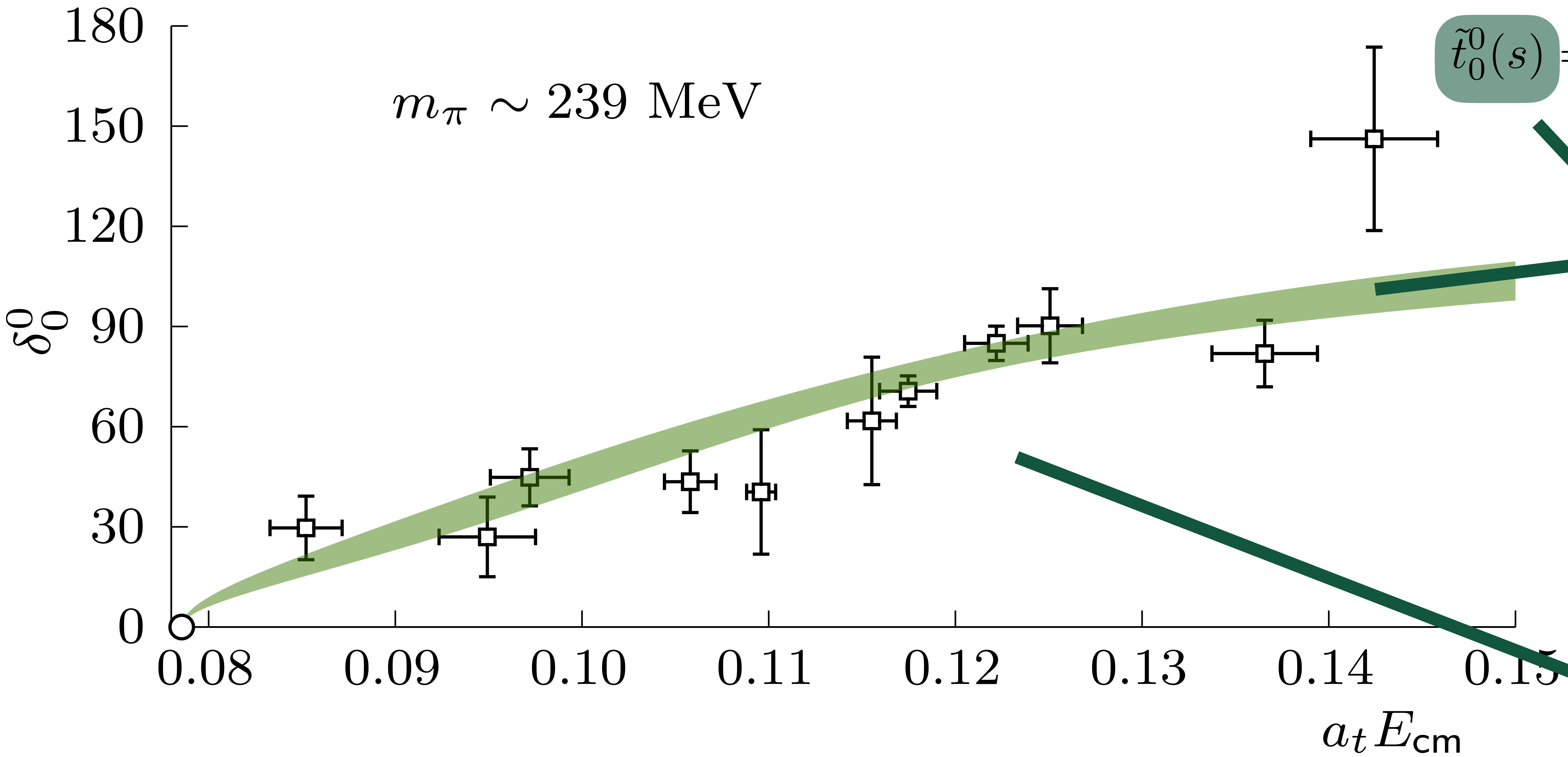

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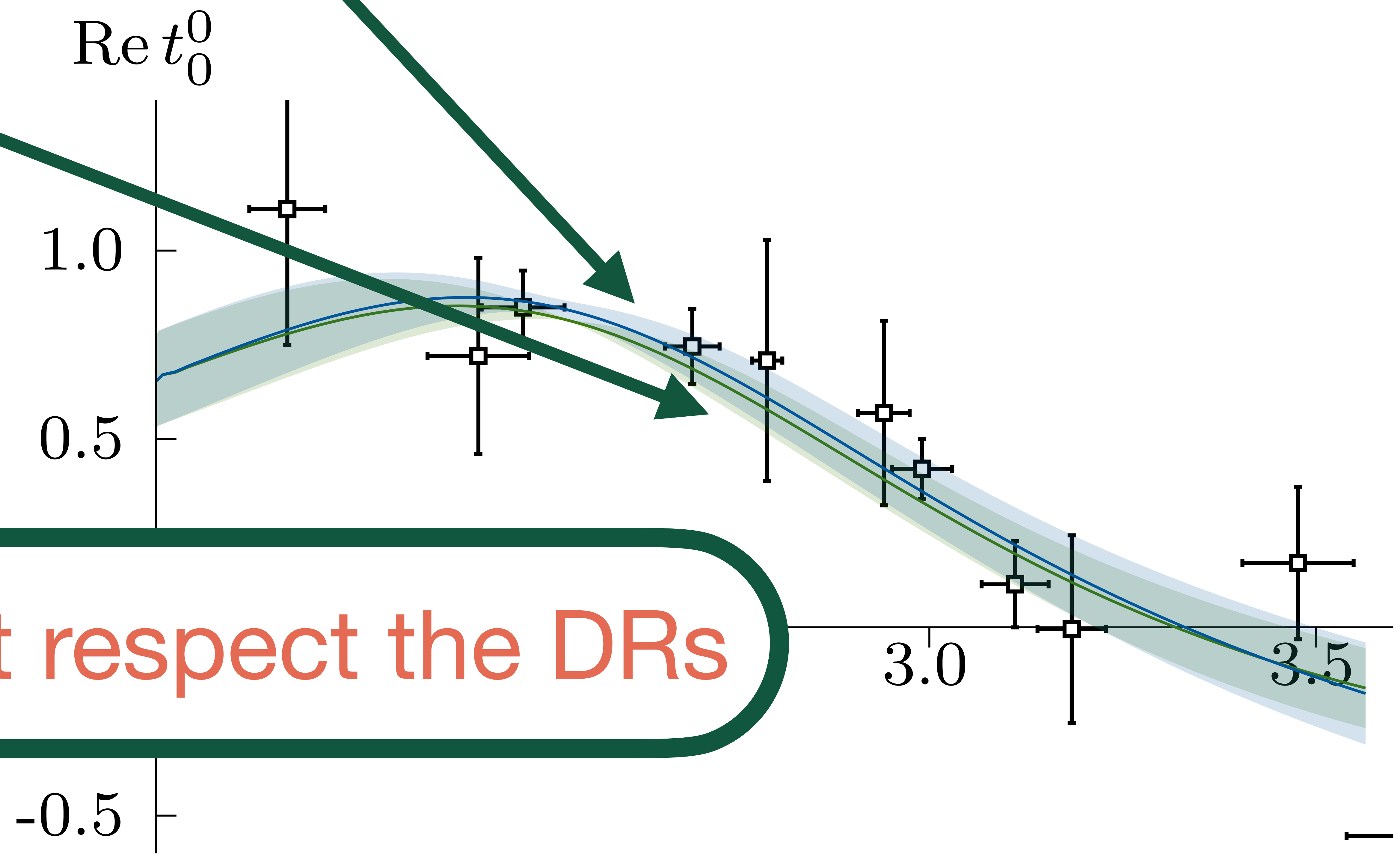
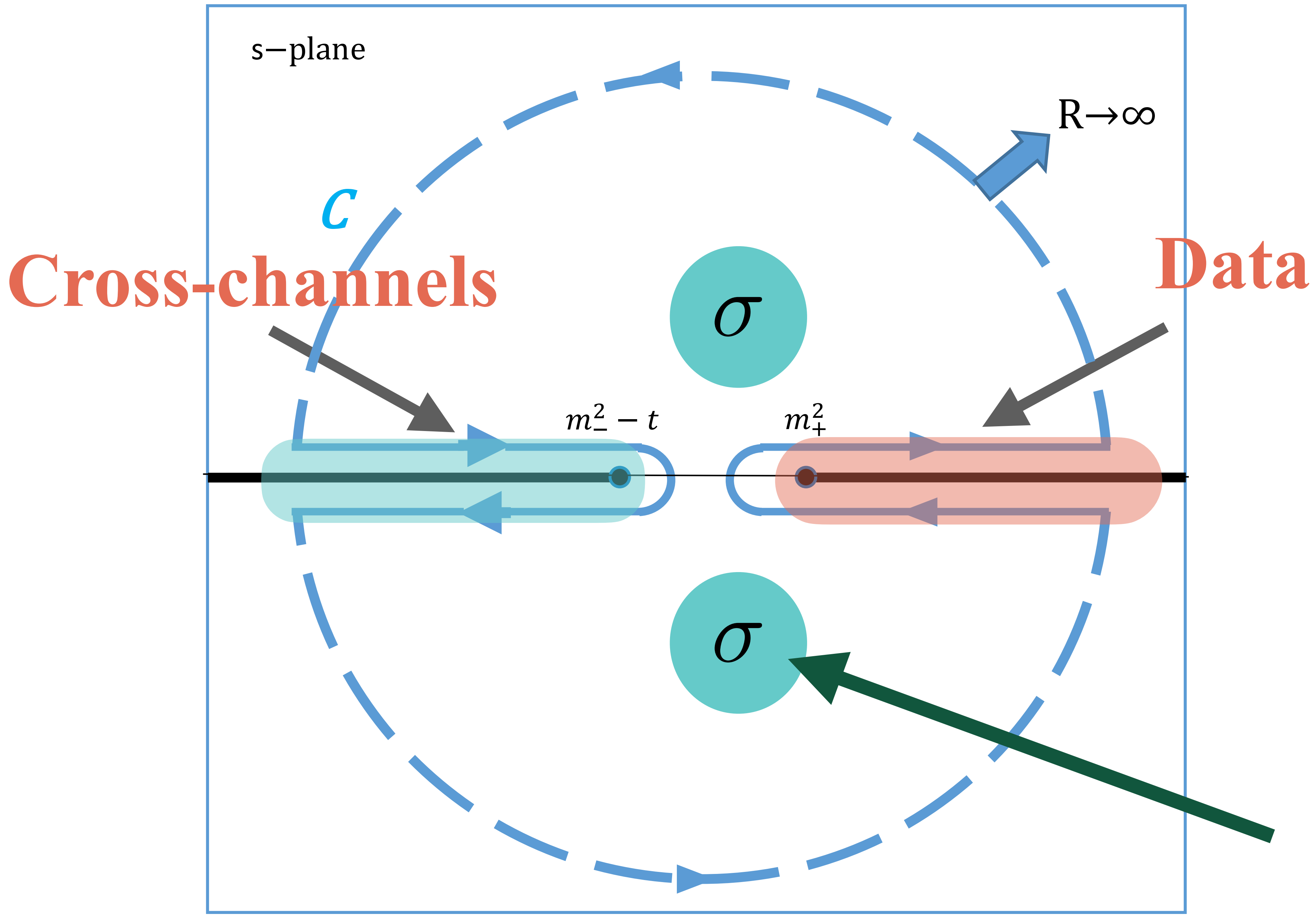


We select those models that respect the DRs

Outside the physical region

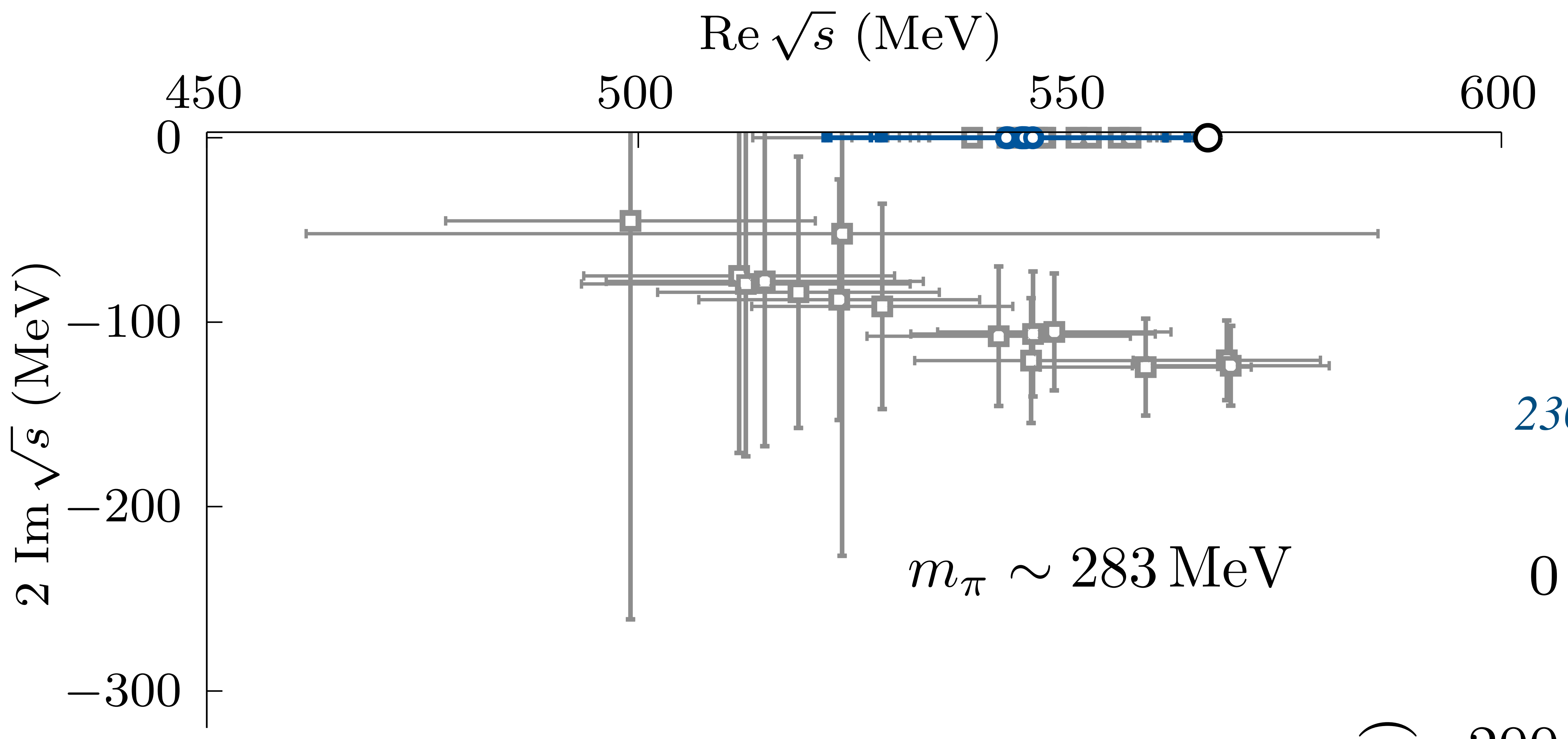


Both sides are good now

What happens everywhere else??

What happens here??

Dispersive σ



No tension anymore

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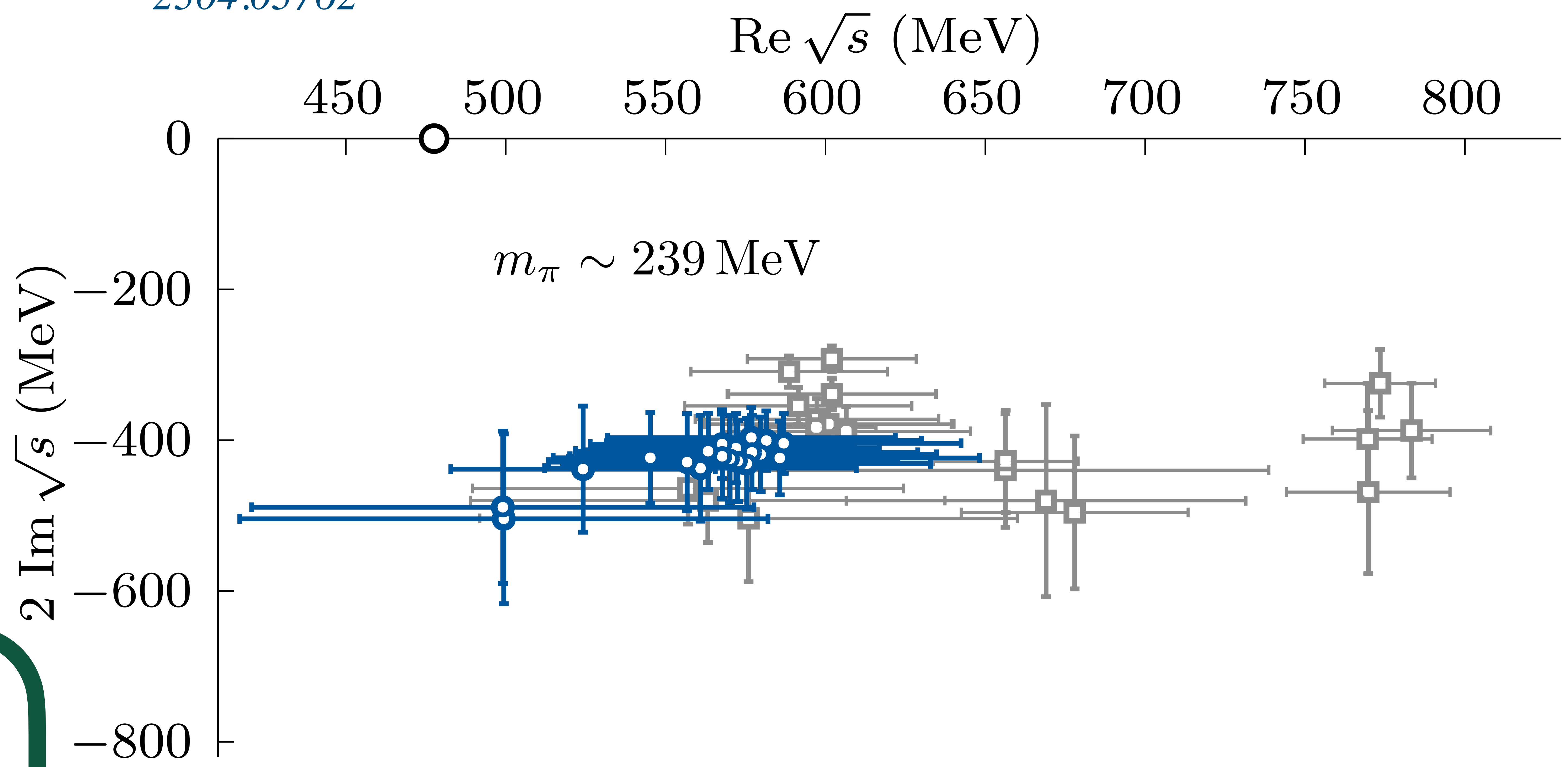
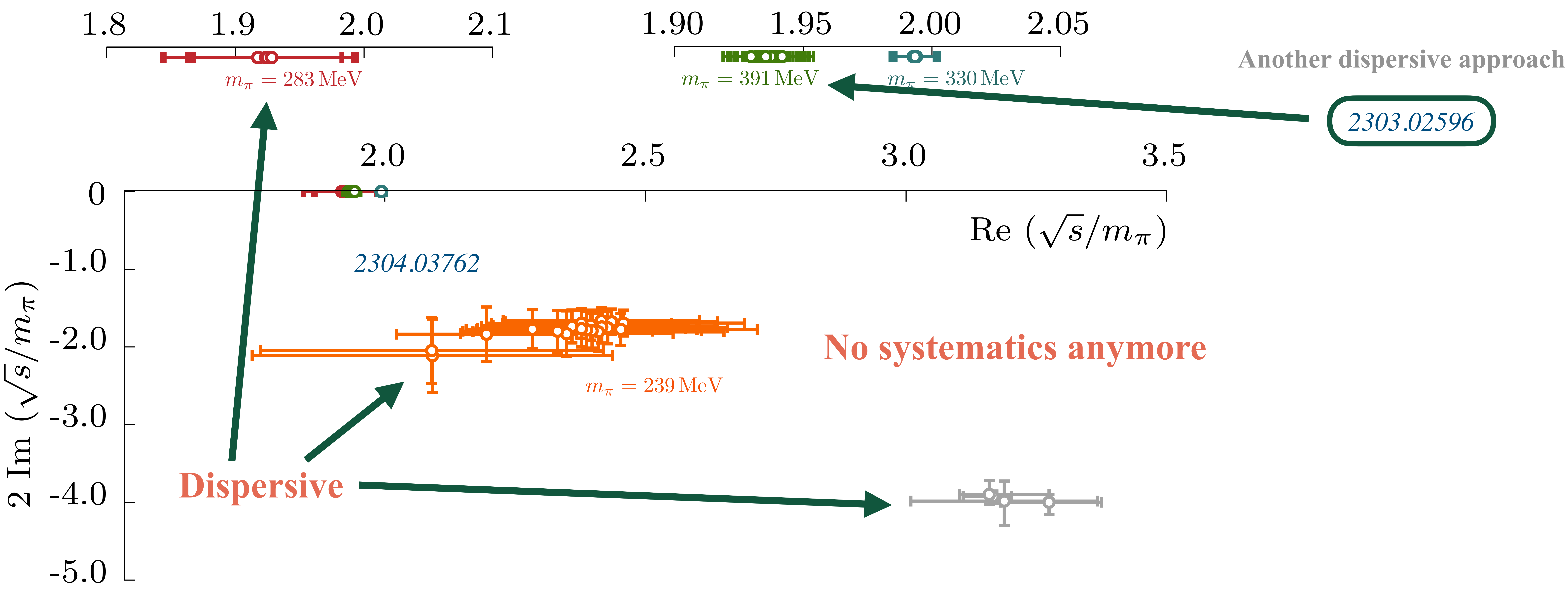


We traded statistical uncertainty increase by large systematic reduction

Dispersive σ



We traded statistical uncertainty increase by large systematic reduction

Summary and outlook

First-principles extraction of a broad resonance directly from QCD

The lighter the π, the more relevant this approach is

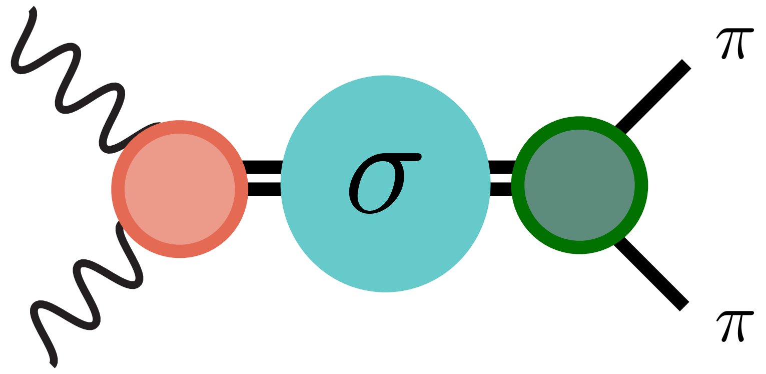
Better constraints over scattering lengths

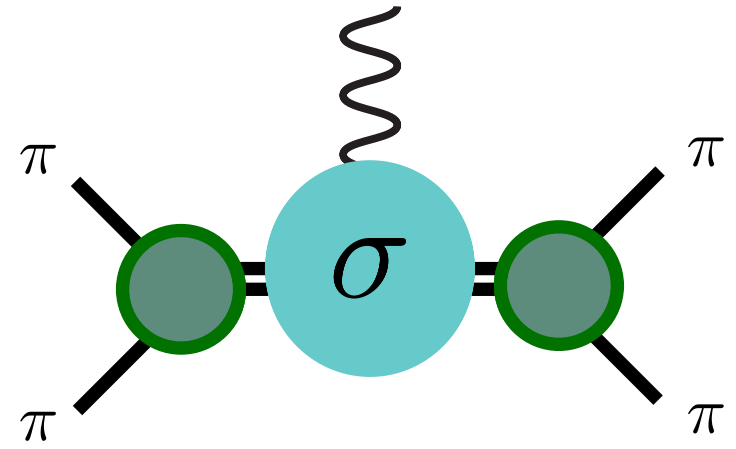
Future

Include second, larger volume for the lighter pion mass

Extract the $f_0(980)$??

Study new observables ?? *A. W. Jackura's talk*





Thank you!!

**GHP 2023 presentation thanks to financial support from The Gordon and Betty Moore Foundation and the American Physical Society.*

$I = 2\ \pi\pi$

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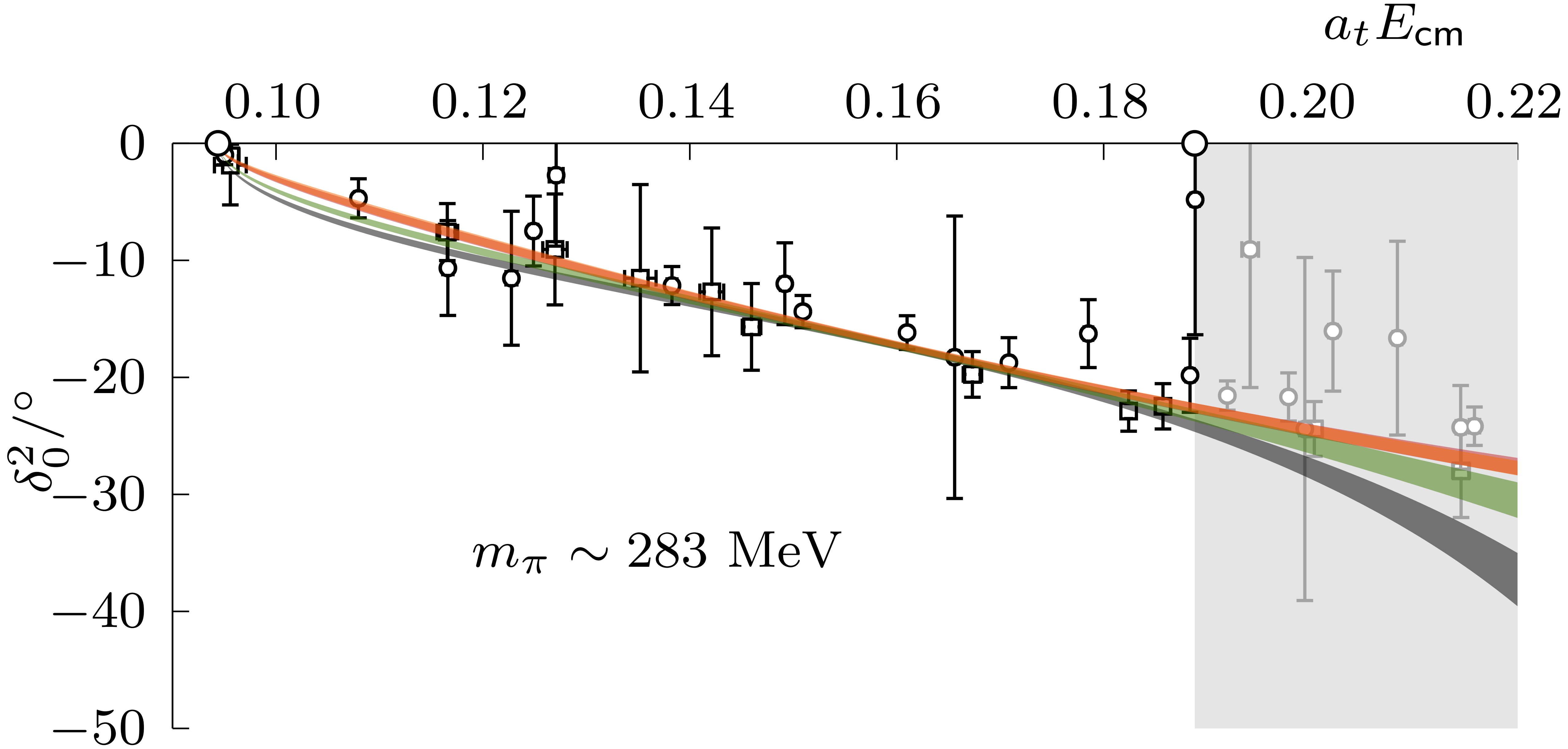
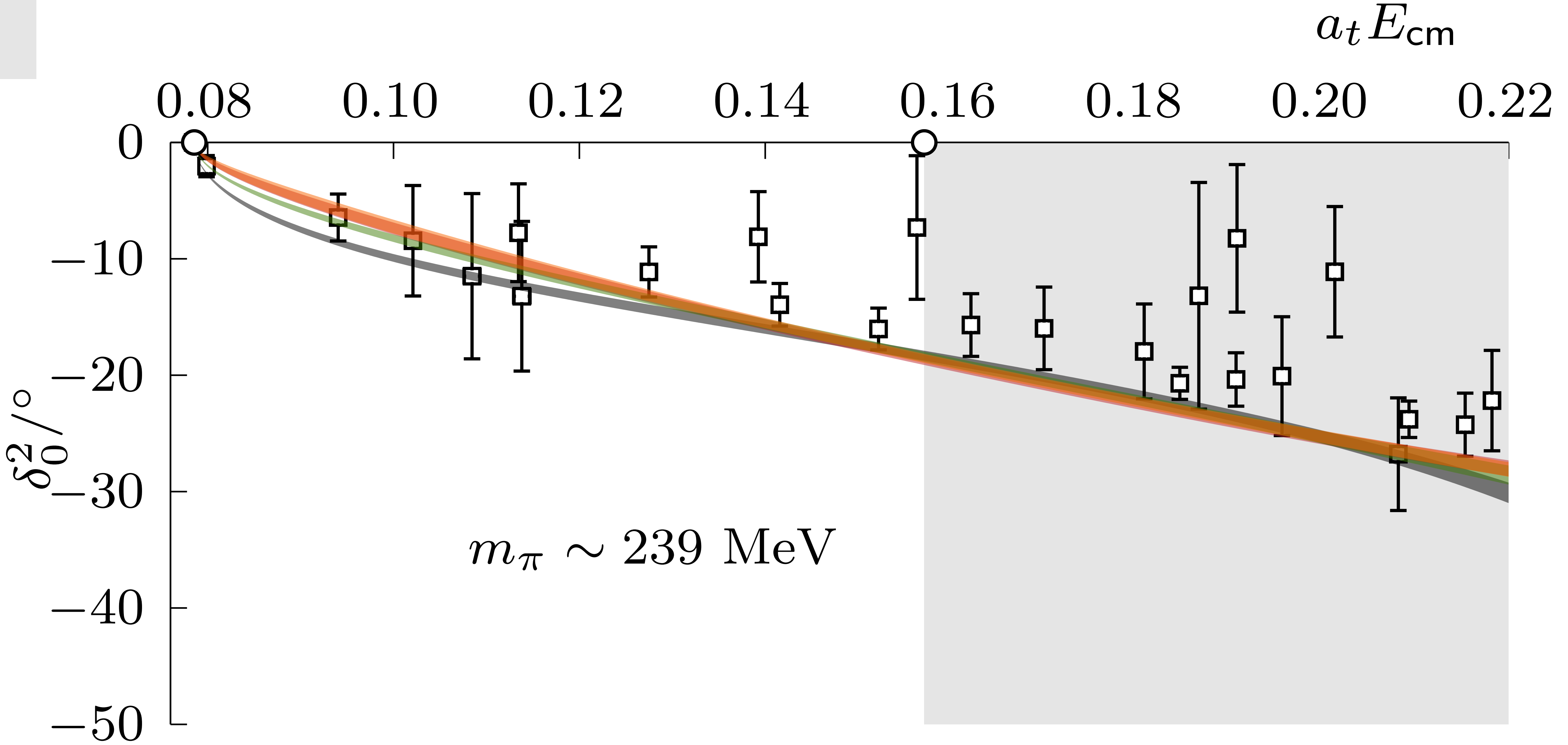


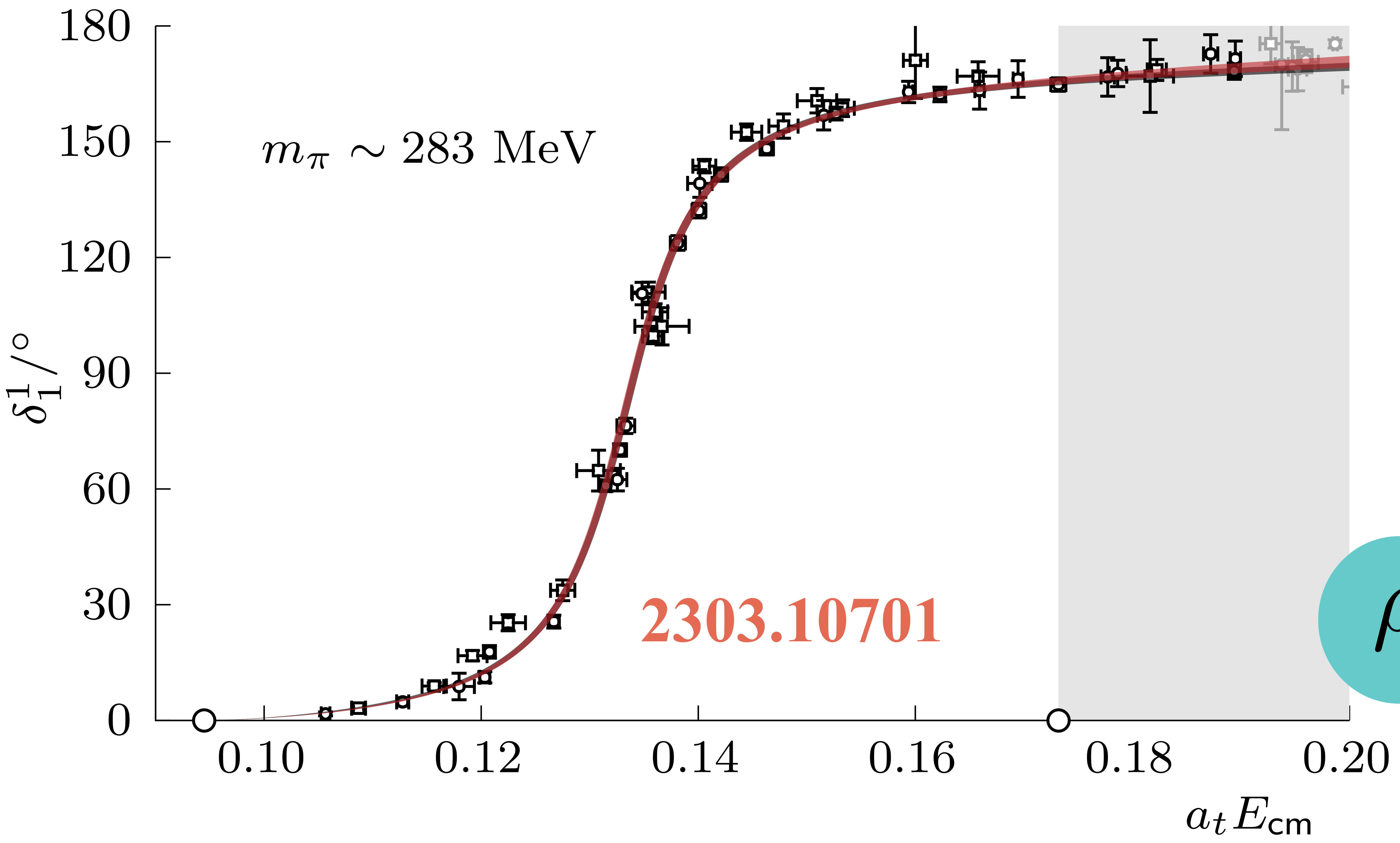
Percent error for $\delta(s)$

10+ parameterizations

Systematic spread at threshold



$I = 1\ \pi\pi$



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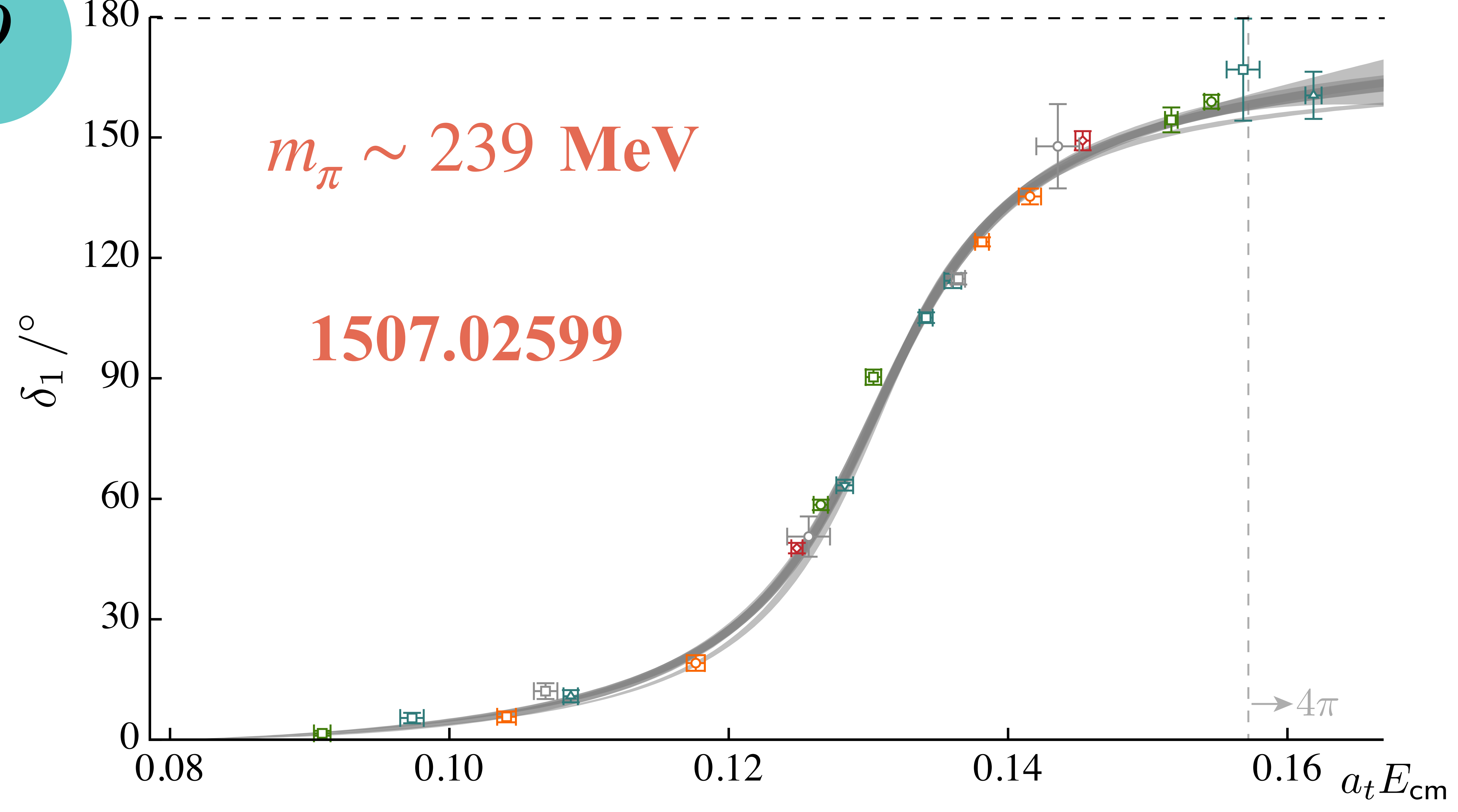
Percent error for $\delta(s)$

ρ



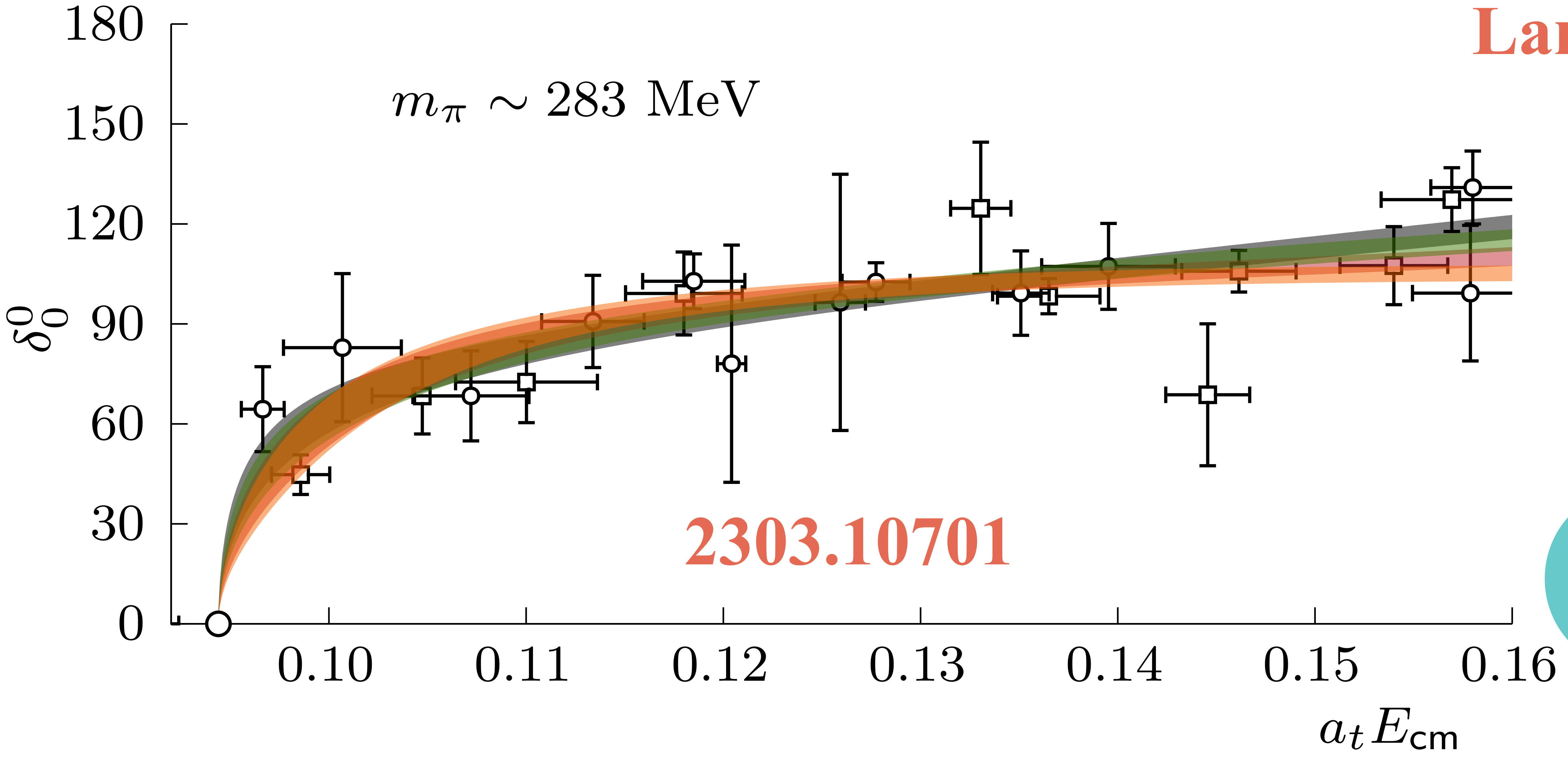
$m_\pi \sim 239$ **MeV**

1507.02599

Around 10 parameterizations

Very consistent amplitude fits

$I = 0\ \pi\pi$

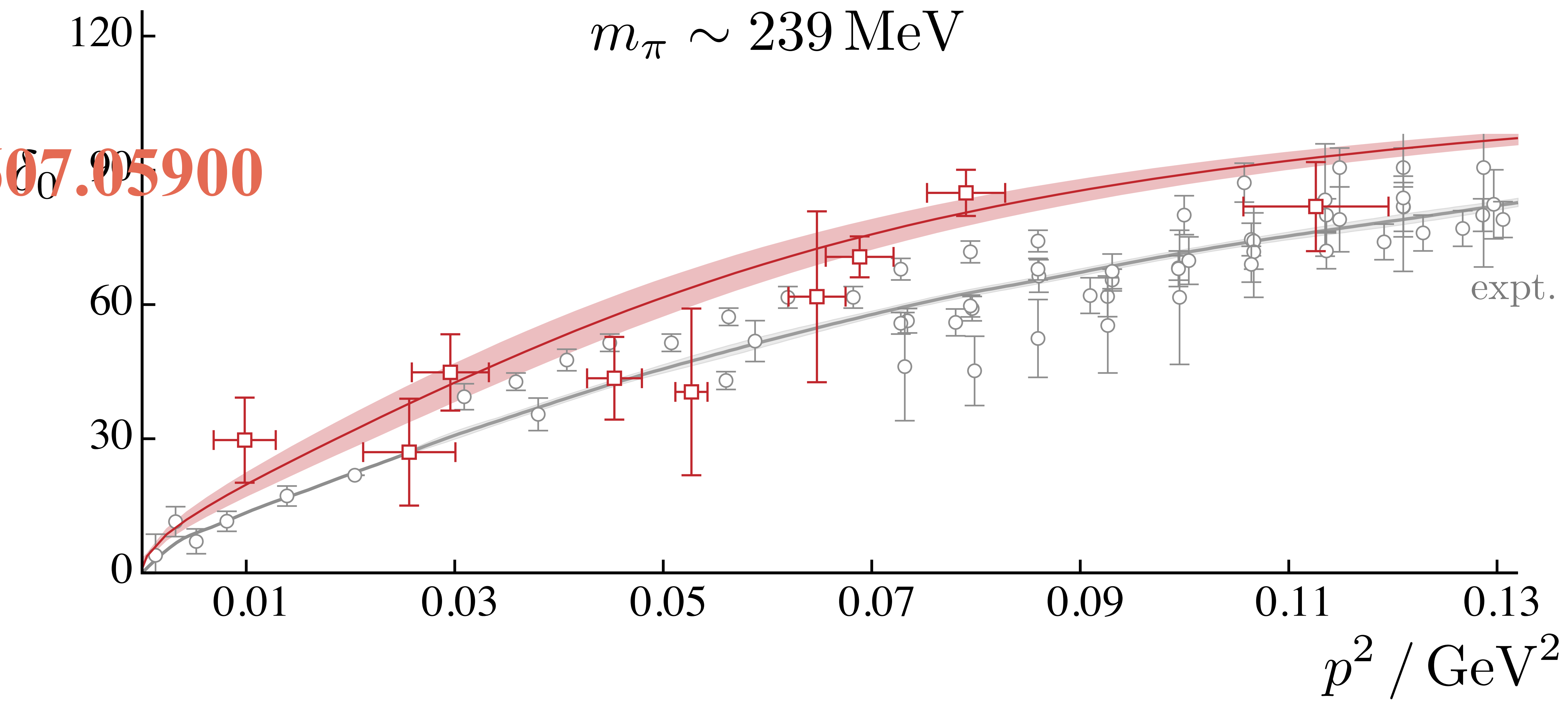


2303.10701

Large derivative at threshold

$\delta(s) = \pi/2$ is far from threshold

σ



1607.05900

Over 20 parameterizations

Smaller derivative at threshold

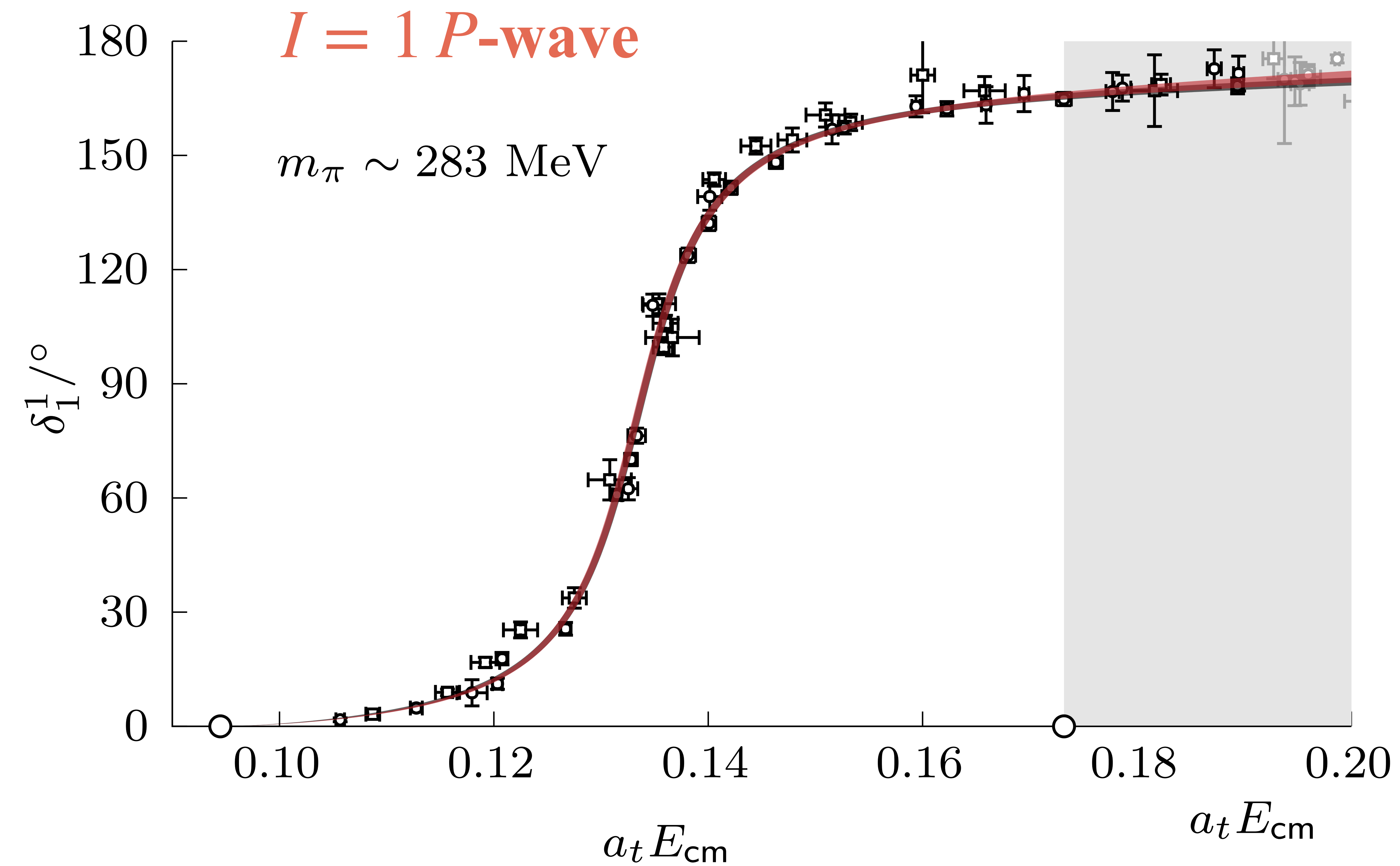
Permutations

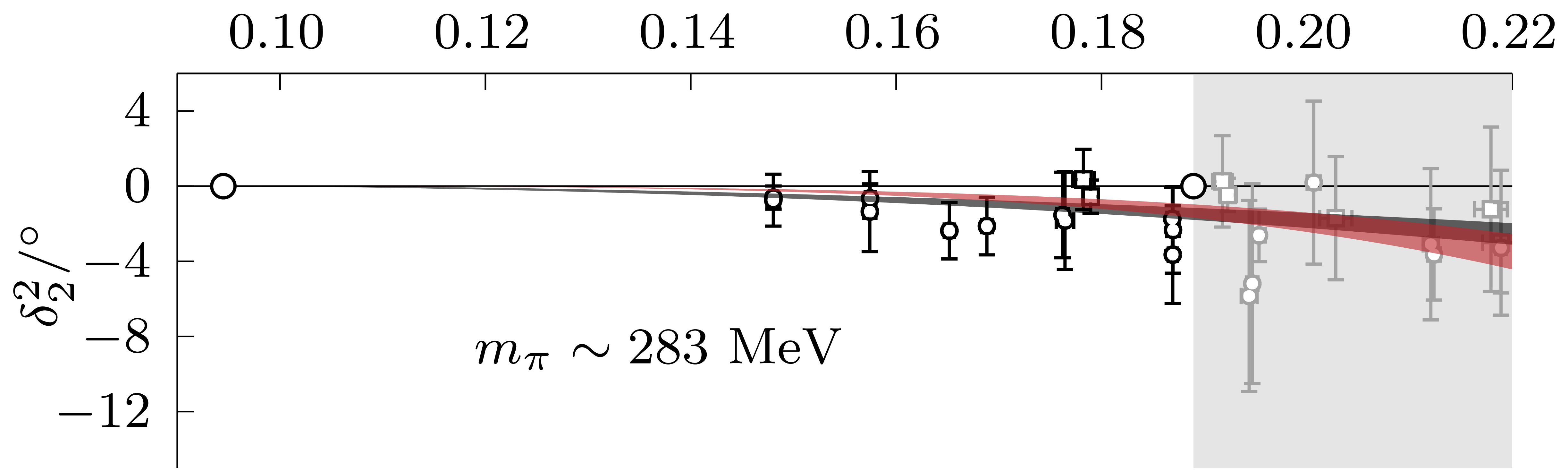
$$\sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds'\, K_{\ell\ell'}^{II'}(s',s)\, \mathrm{Im}\, t_{\ell'}^{I'}(s')$$

For ℓ_{max} partial waves

$$N_I \ell_{max} N_{params} \sim 10^5$$

We can fix most





$I = 2\ D$-wave

- Make *Fit → In* *DR → Out* **compatible** ⟷ **Unitarity**

$$\left[d^2\right]_\ell^I \equiv \sum_{i=1}^{N_{\text{smpl}}} \left(\frac{\text{Re}\,\tilde{t}_\ell^I(s_i) - \text{Re}\,t_\ell^I(s_i)}{\Delta\left[\text{Re}\,\tilde{t}_\ell^I(s_i) - \text{Re}\,t_\ell^I(s_i)\right]} \right)^2$$

- Make *DR → Out* **and data compatible** ⟷ **Lattice QCD data description**

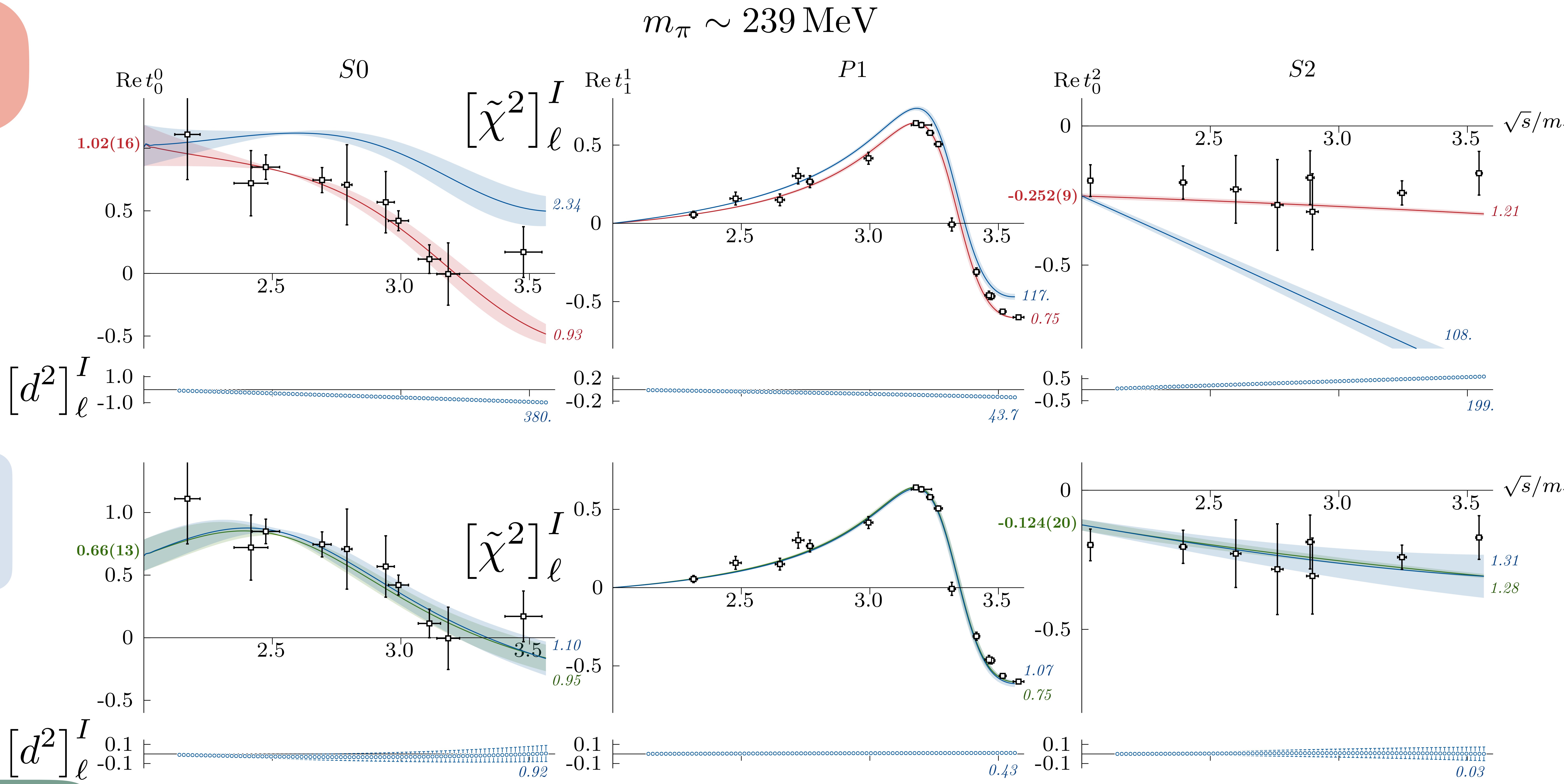
$$\left[\tilde{\chi}^2\right]_\ell^I \equiv \sum_{i,j=1}^{N_{\text{lat}}} \left(\frac{\mathfrak{f}_i - \text{Re}\,\tilde{t}_\ell^I(s_i)}{\Delta_i} \right) \text{corr}\left(\mathfrak{f}_i, \mathfrak{f}_j\right) \left(\frac{\mathfrak{f}_j - \text{Re}\,\tilde{t}_\ell^I(s_j)}{\Delta_j} \right)$$

Tests: good vs bad

Bad fit combination

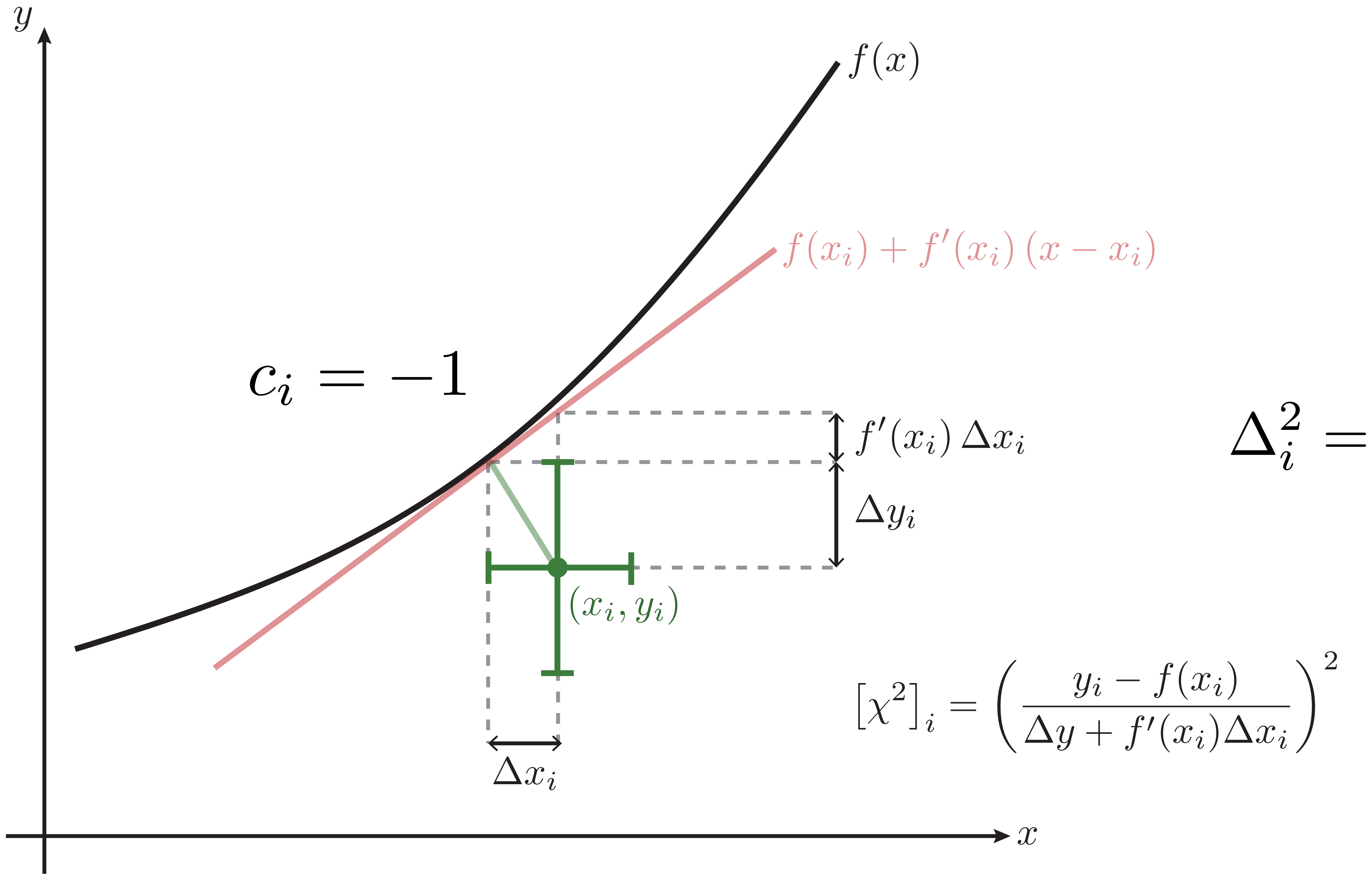
Dispersive output

Good fit combination



Make $DR \to Out$ and data compatible ⟷ Lattice QCD data description

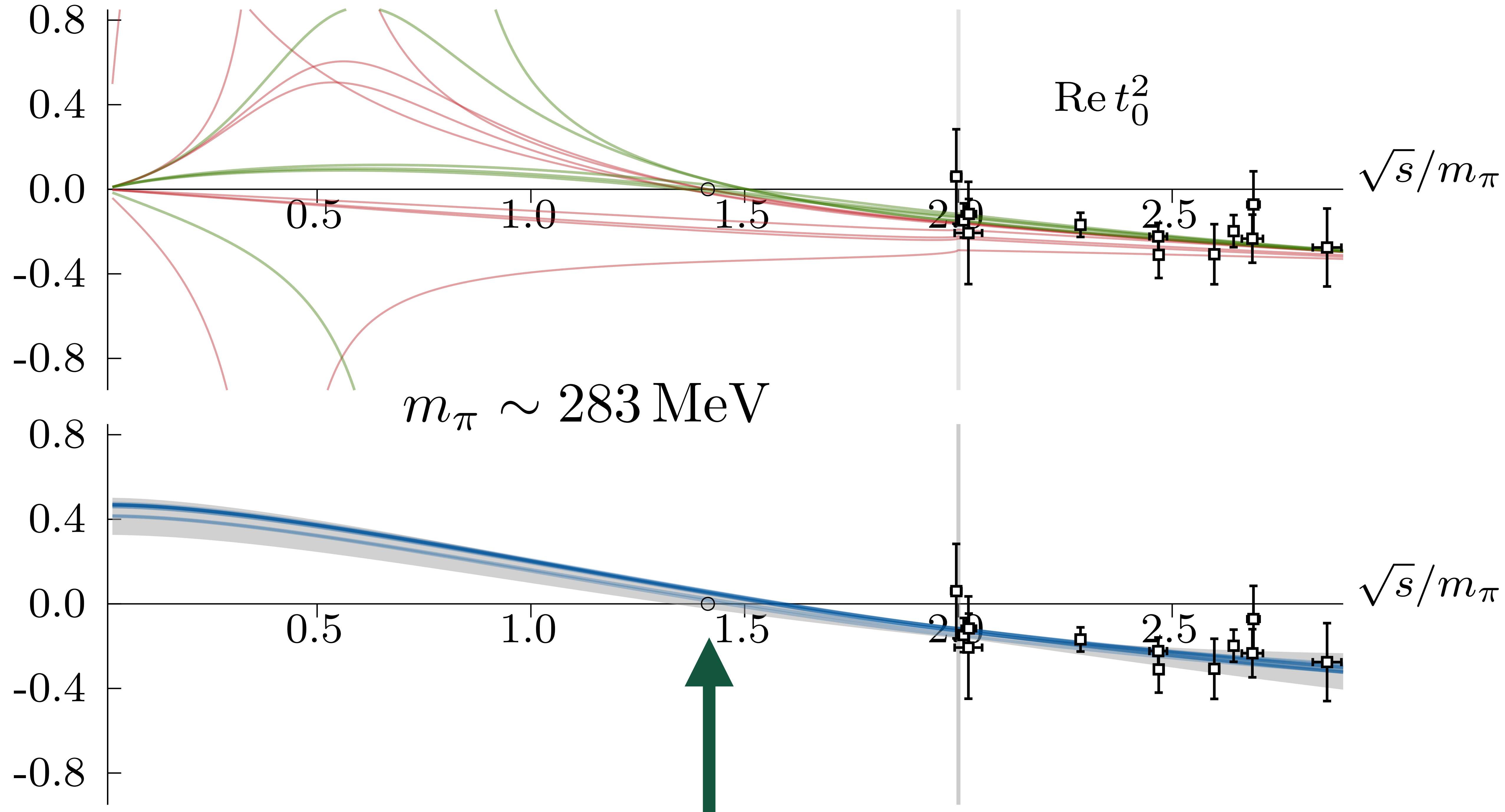
$$[\tilde{\chi}^2]^I_\ell \equiv \sum_{i,j=1}^{N_{\rm lat}} \left(\frac{\mathfrak{f}_i - \mathrm{Re}\,\tilde{t}^I_\ell(s_i)}{\Delta_i} \right) \mathrm{corr}(\mathfrak{f}_i, \mathfrak{f}_j) \left(\frac{\mathfrak{f}_j - \mathrm{Re}\,\tilde{t}^I_\ell(s_j)}{\Delta_j} \right)$$



$$\Delta_i^2 = \begin{pmatrix} \Delta\mathfrak{f}_i & \frac{d\tilde{f}^I_\ell(s_i)}{dE_i}\Delta E_i \end{pmatrix} \begin{pmatrix} 1 & -c_i \\ -c_i & 1 \end{pmatrix} \begin{pmatrix} \Delta\tilde{f}_i \\ \frac{d\tilde{f}^I_\ell(s_i)}{dE_i}\Delta E_i \end{pmatrix}$$

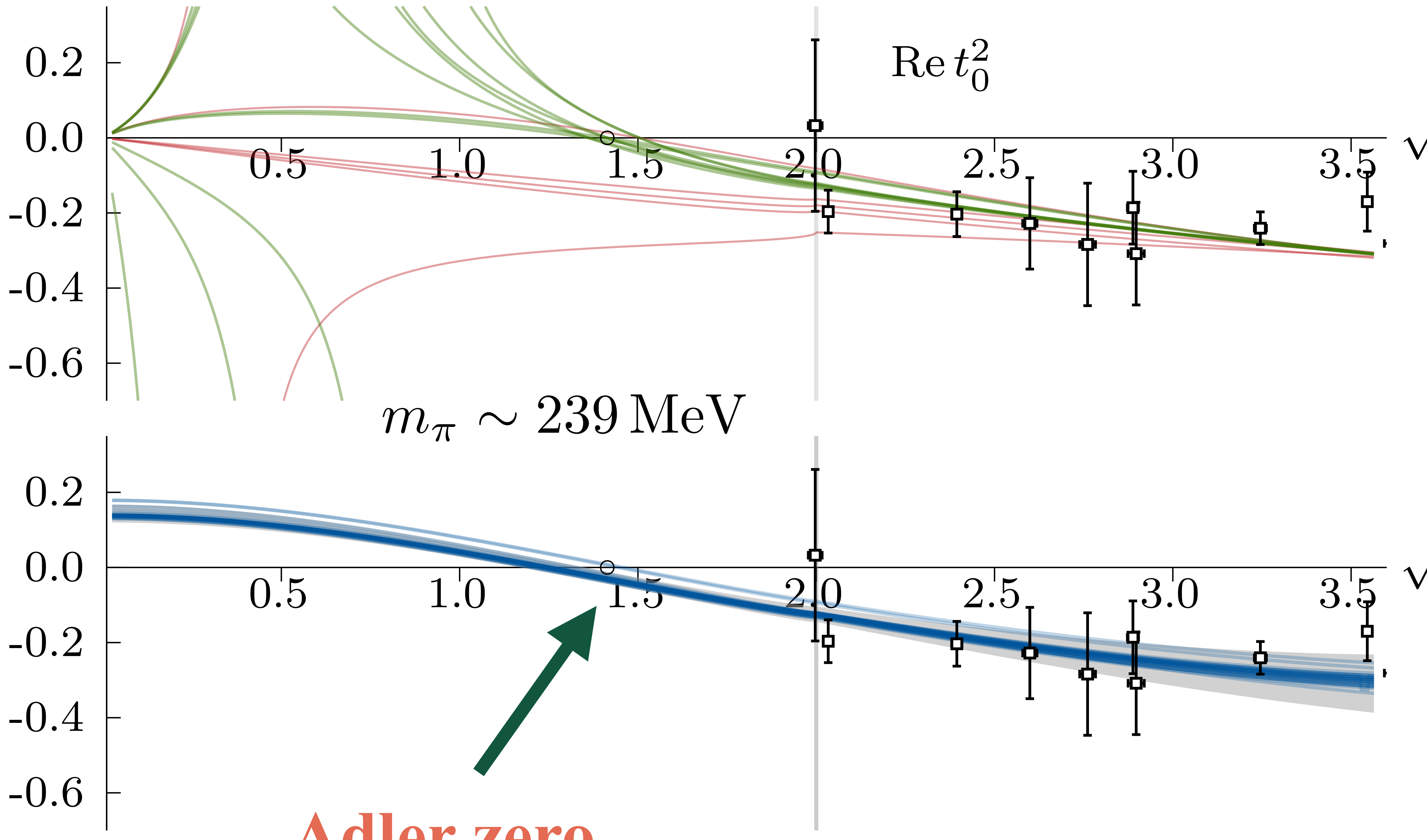
Sub-threshold

Very “stable” for $I = 2$ $\pi\pi$



Adler zero

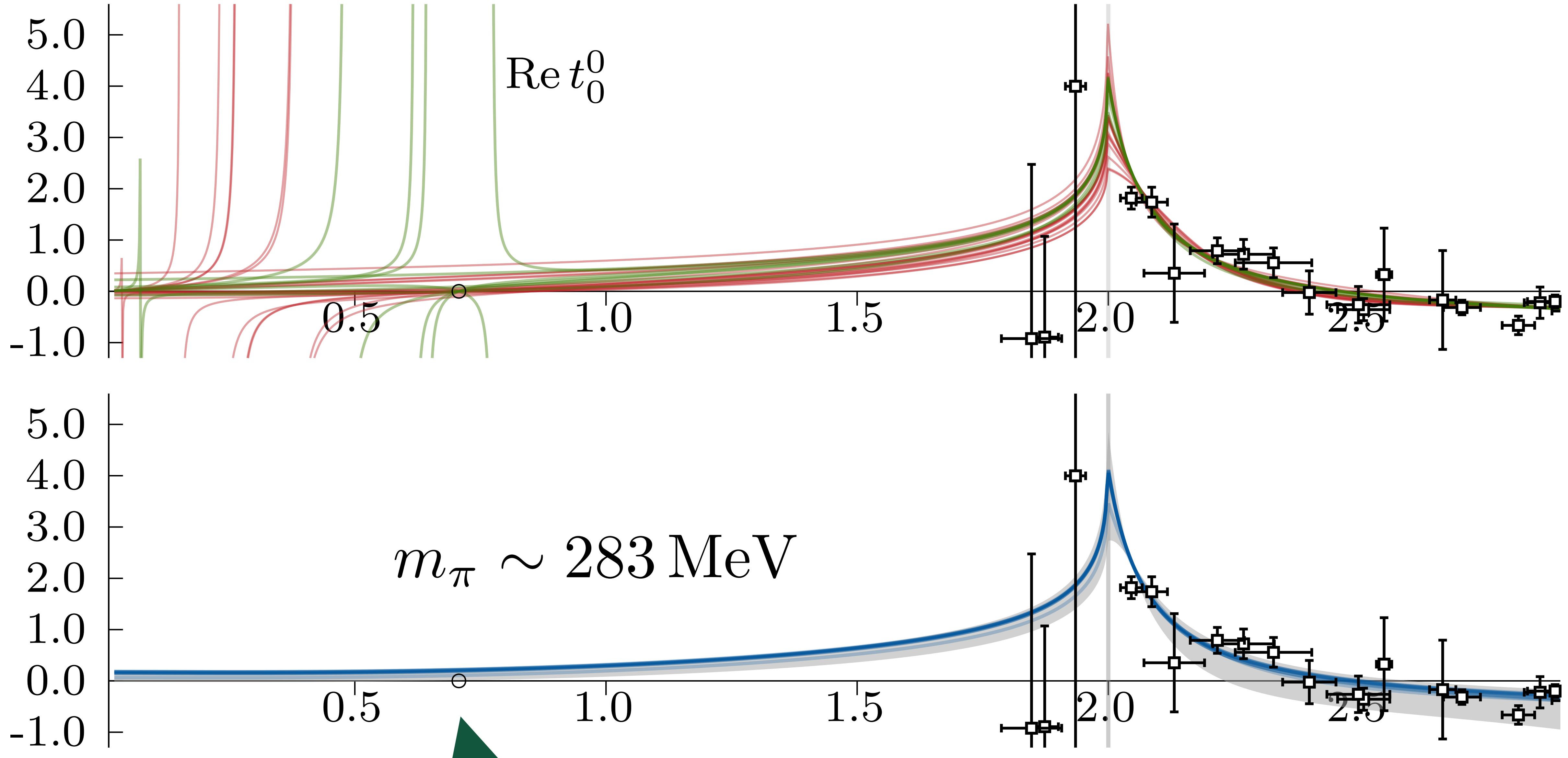
Even “bad” DRs produce Adler zeroes for I=2



Adler zero

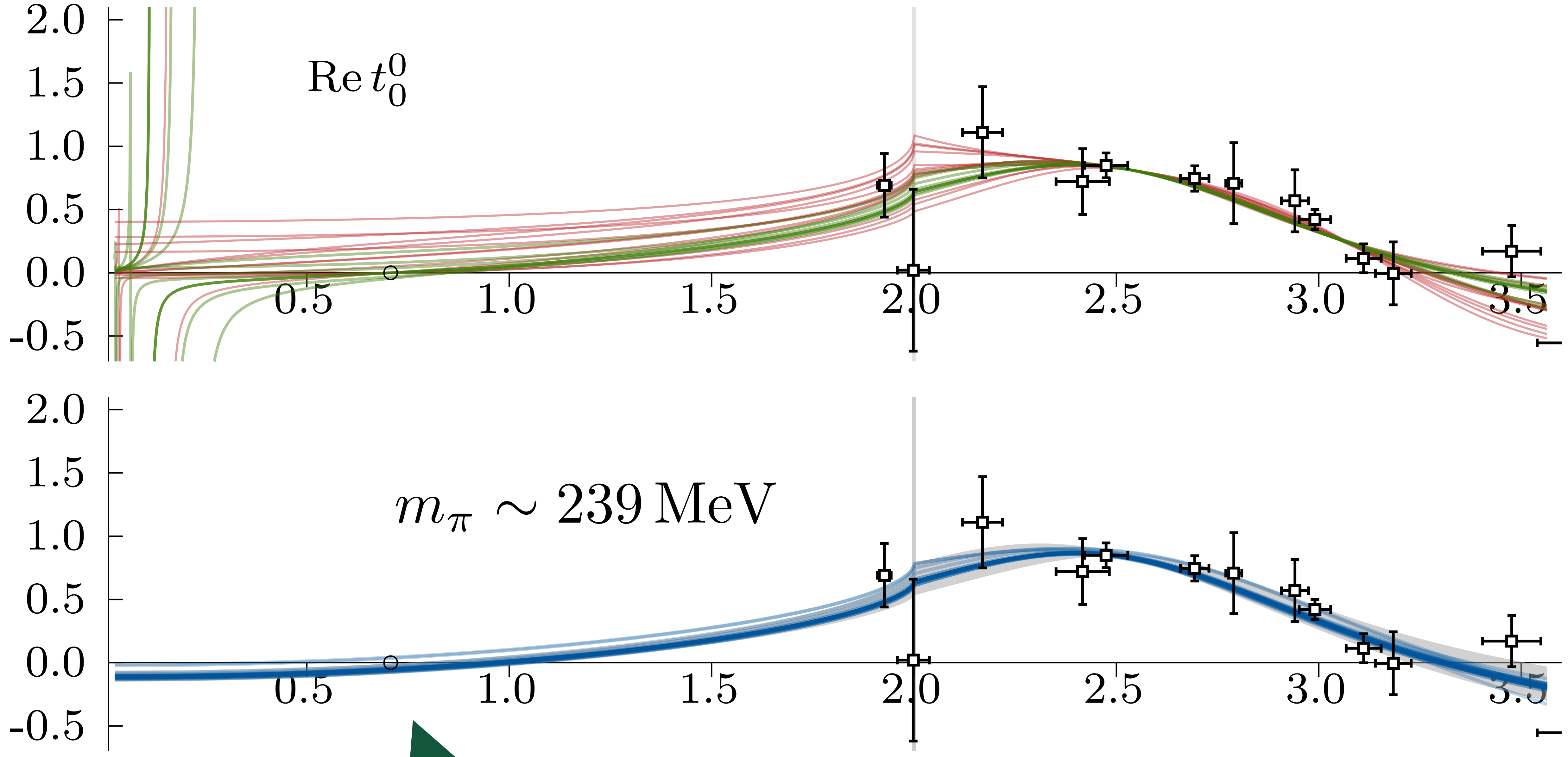
Sub-threshold

No good DR produces an $I = 0$ $\pi\pi$ Adler zero for the heavier mass



NO Adler zero

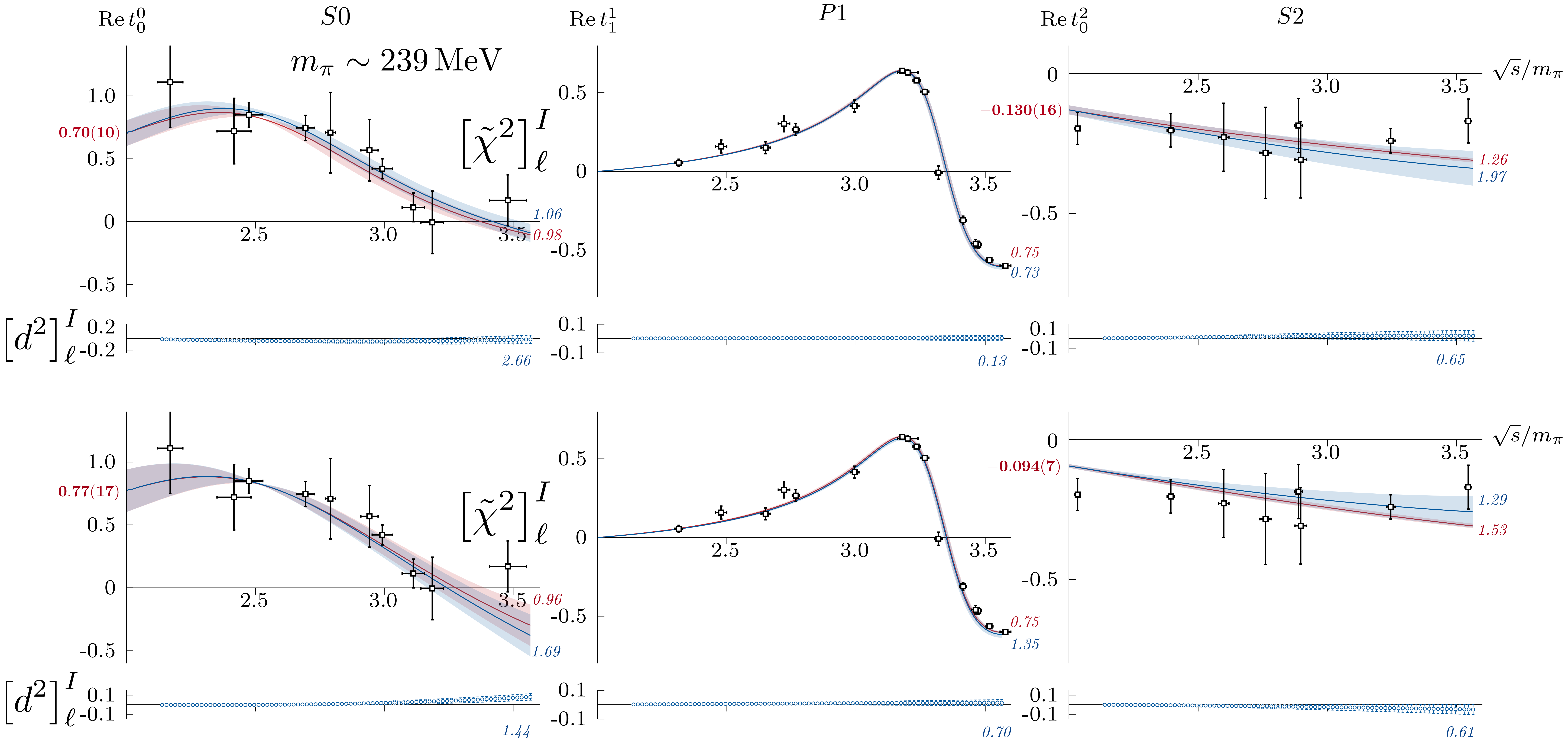
All good DRs produce an $I = 0$ $\pi\pi$ Adler zero for the lighter mass



Adler zero

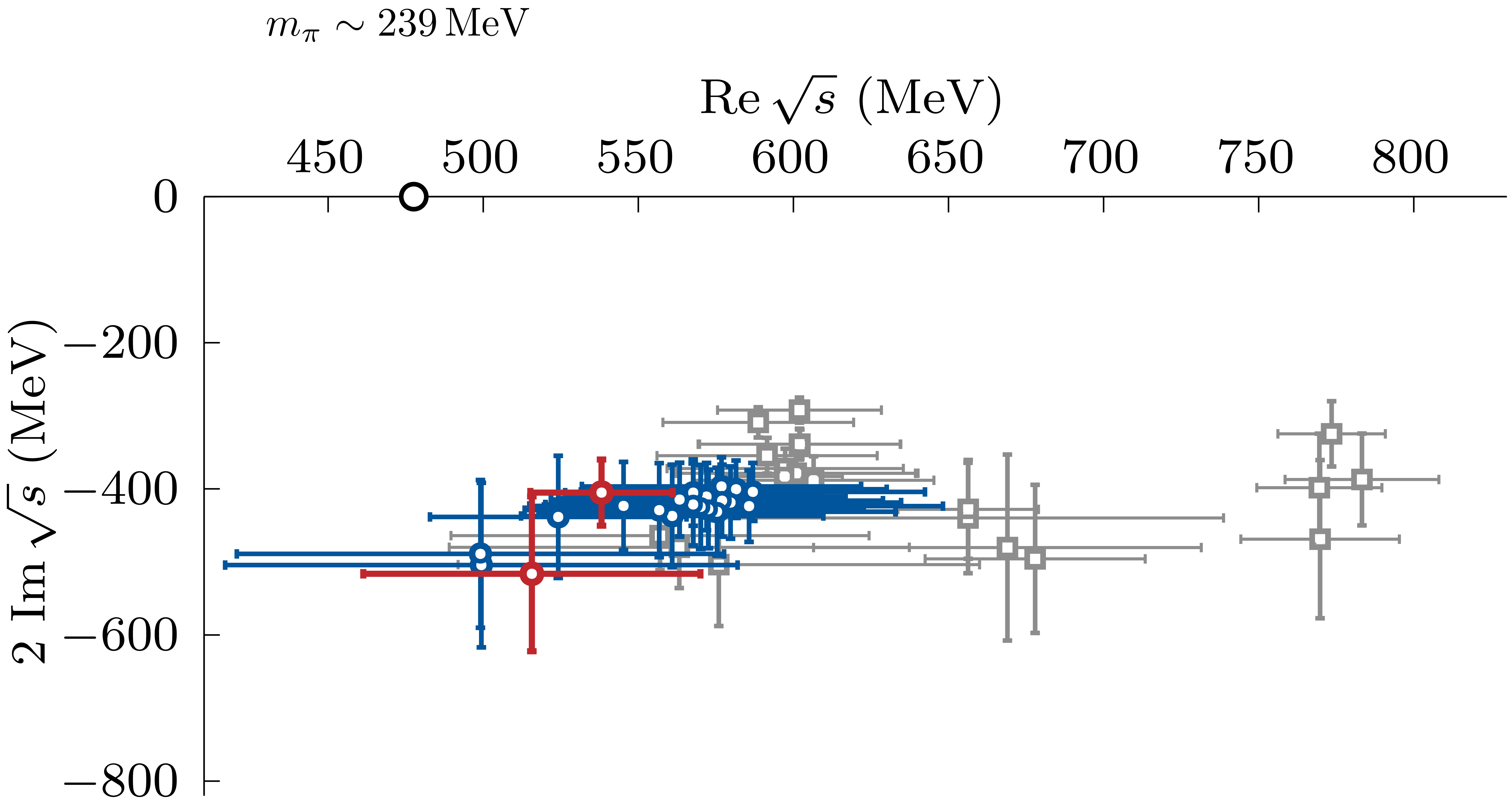
Ok but not great

Visually, they describe the data and fit, but they are not perfect



Ok but not great

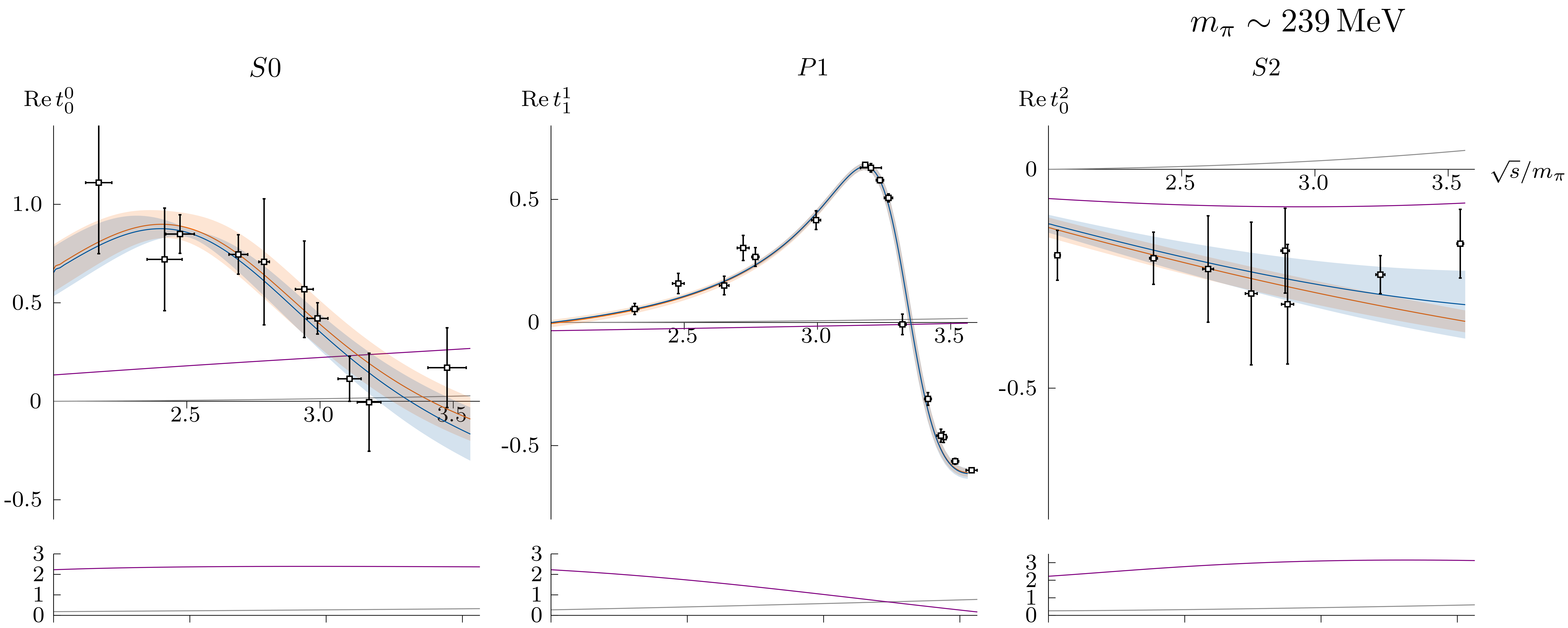
Visually, they describe the data and fit, but they are not perfect



GKPY vs ROY

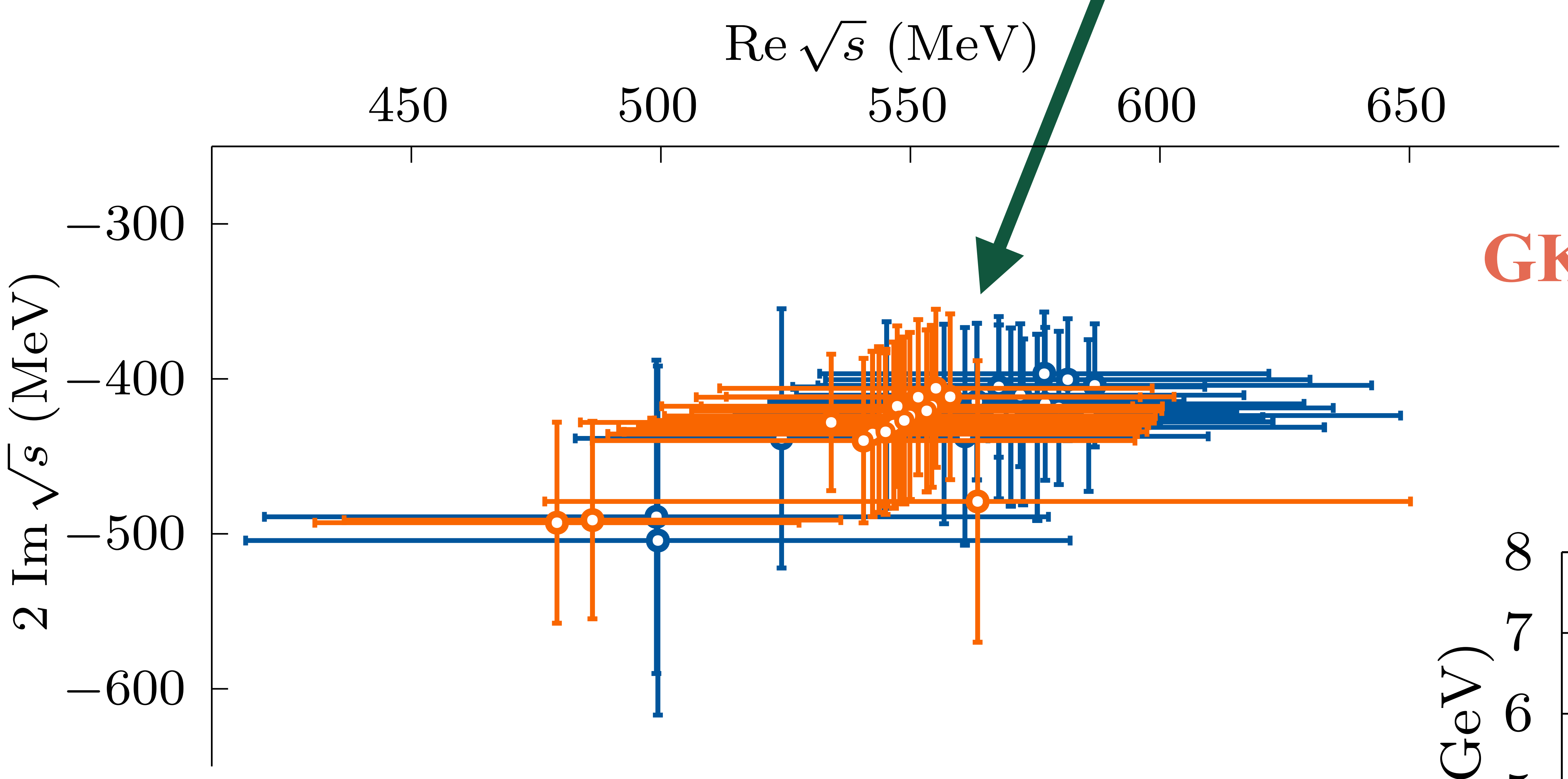
GKPY: Minimally subtracted → one less subtraction than ROY

For our analysis, Regge contribution too large for d^2

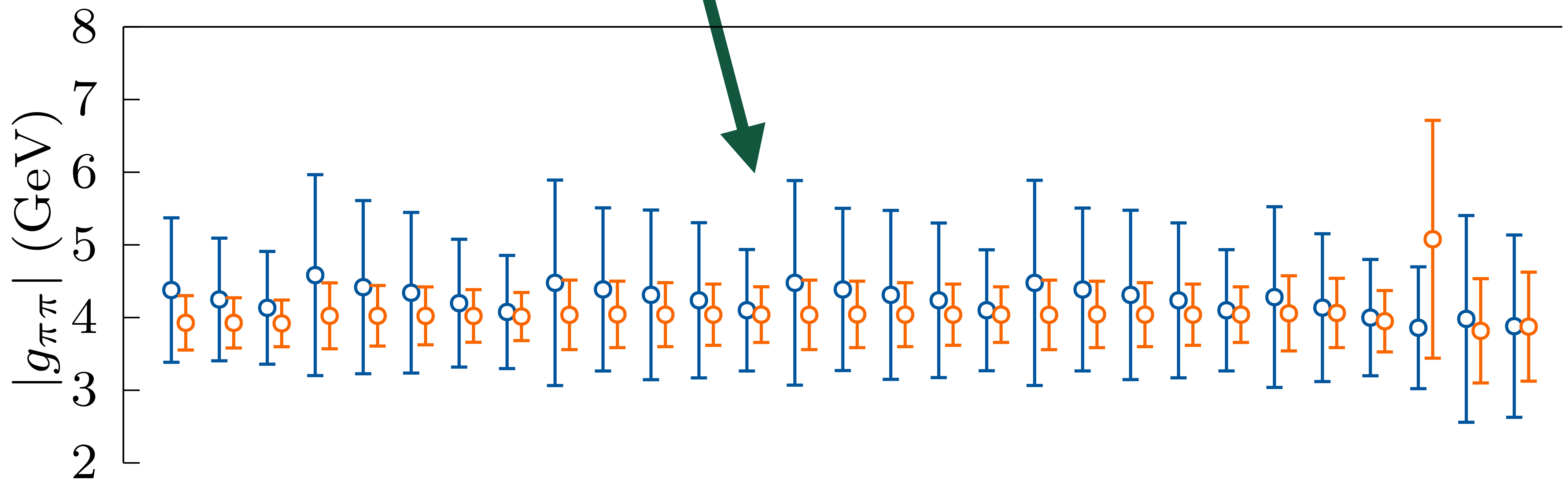


GKPY vs ROY

However, pole extraction is more accurate in most cases, particularly for the coupling



GKPY produces $\sim 40\,\%$ uncertainty in most cases



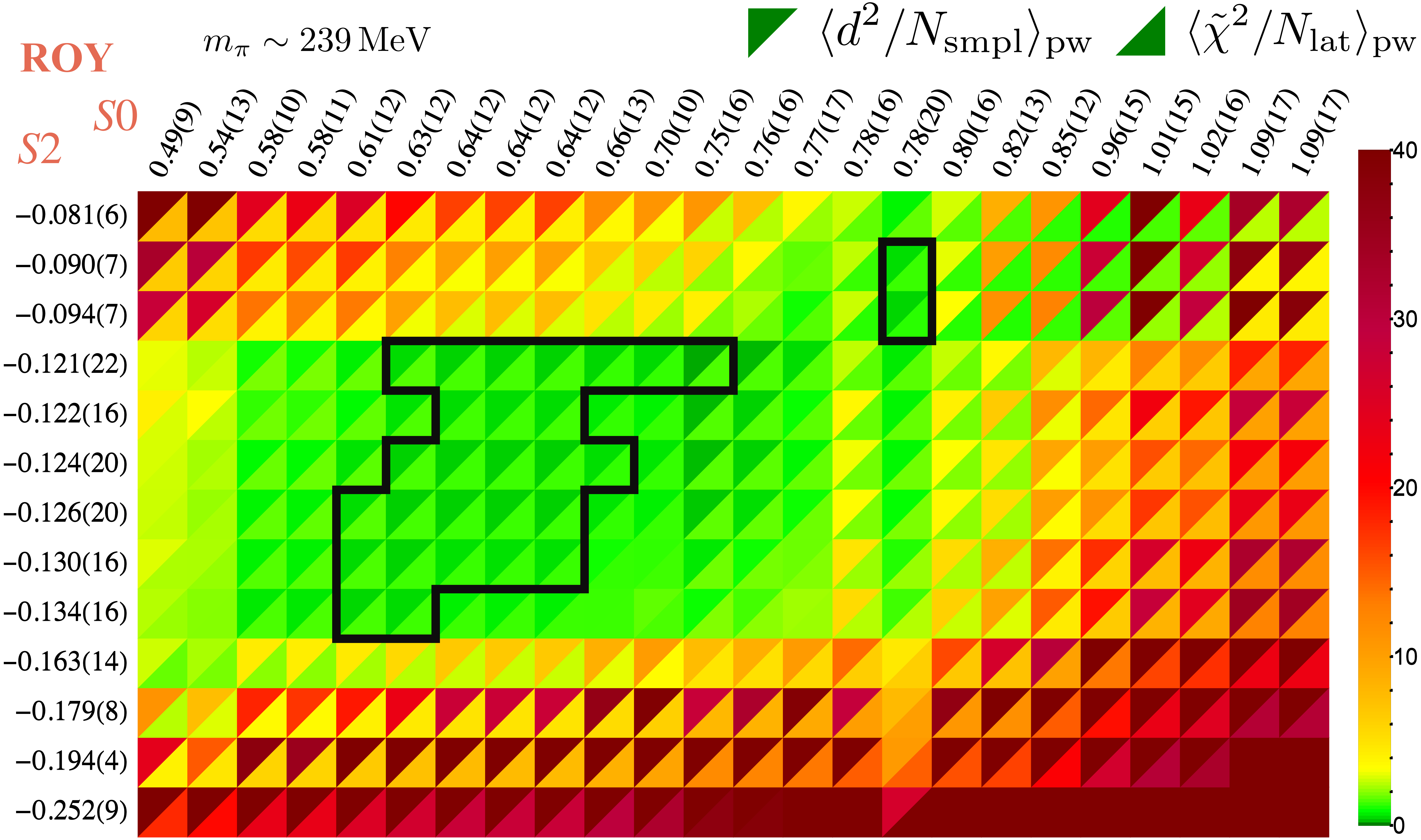
Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

$m_\pi \sim 239\,\text{MeV}$

Black **ROY**

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$



Scattering plane

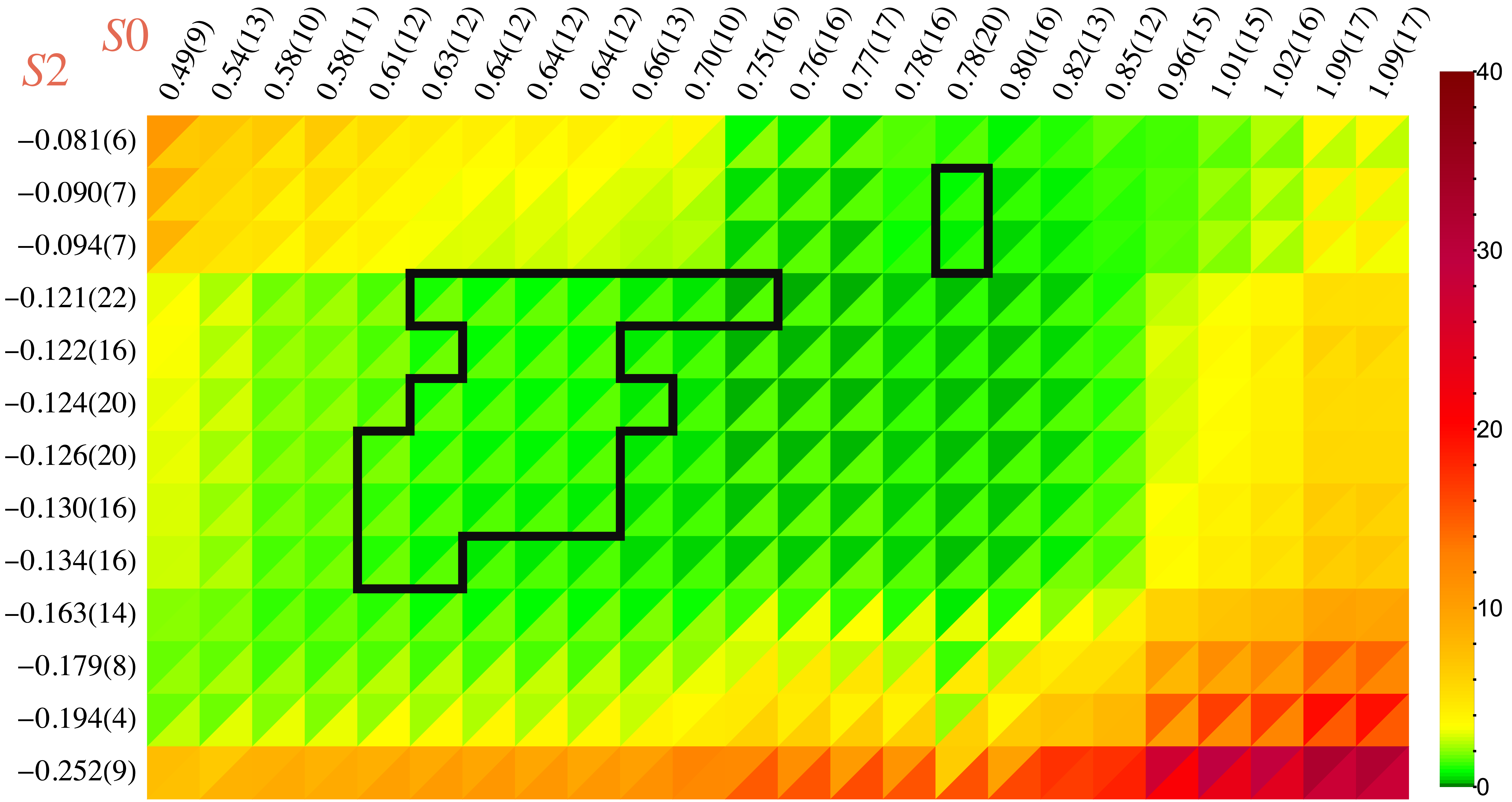
$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

Black **GKPY**

$m_\pi \sim 239\,\text{MeV}$

$\langle d^2/N_{\text{smpl}} \rangle_{\text{pw}}$ $\langle \tilde{\chi}^2/N_{\text{lat}} \rangle_{\text{pw}}$

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$



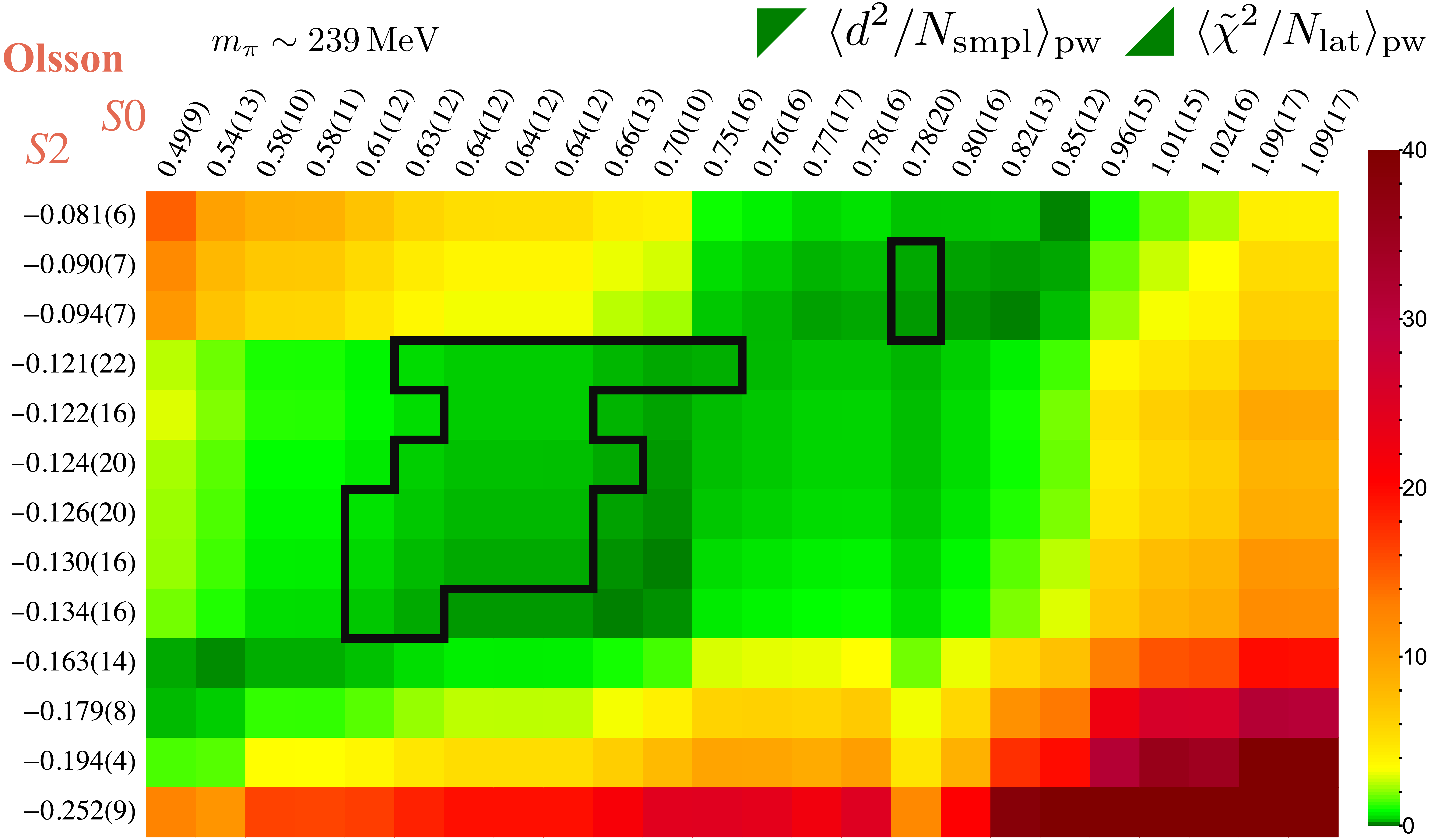
Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

$m_\pi \sim 239\,\text{MeV}$

Black **Olsson**

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$



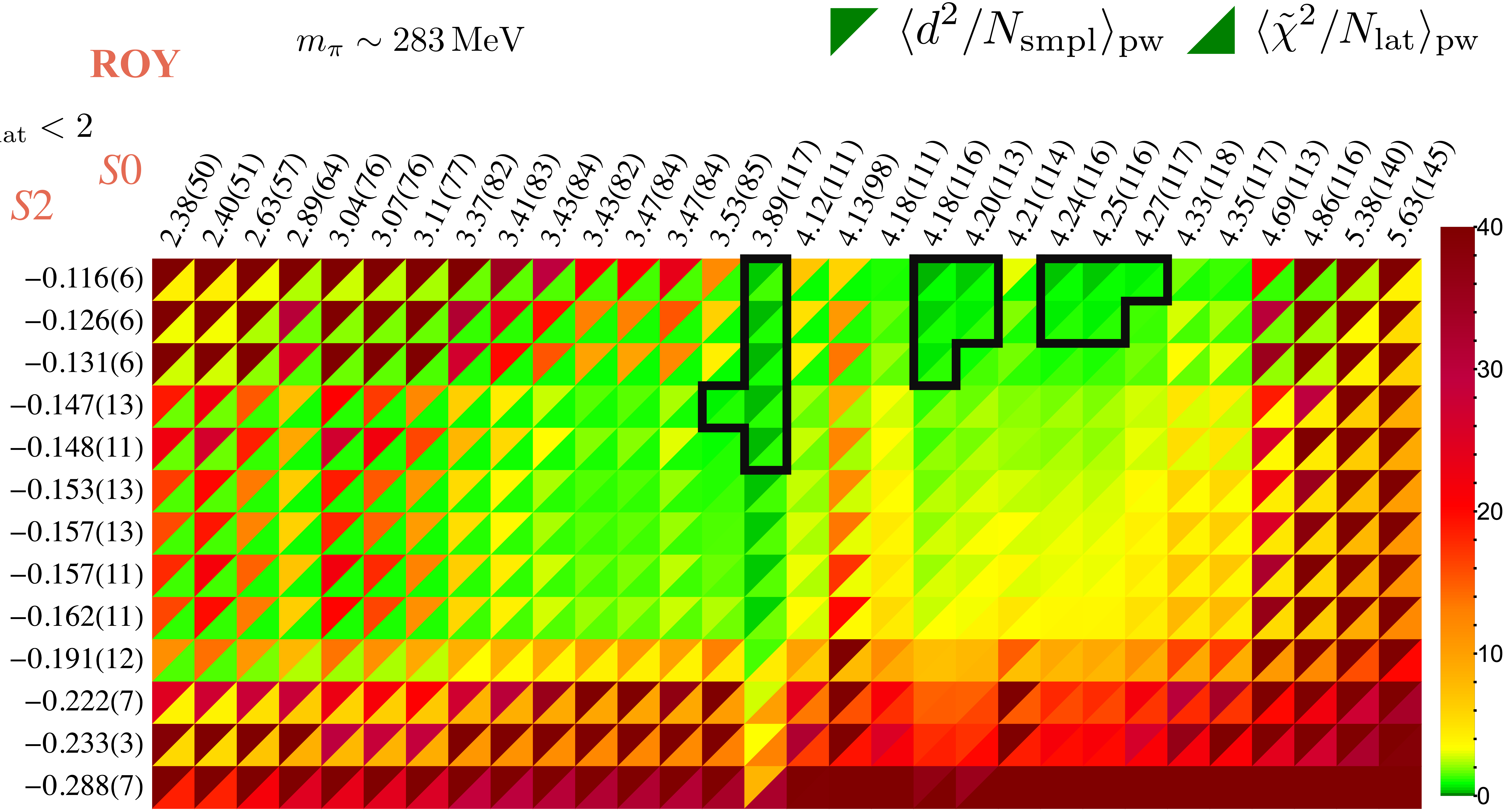
Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

$m_\pi \sim 283\,\text{MeV}$

Black **ROY**

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$



Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

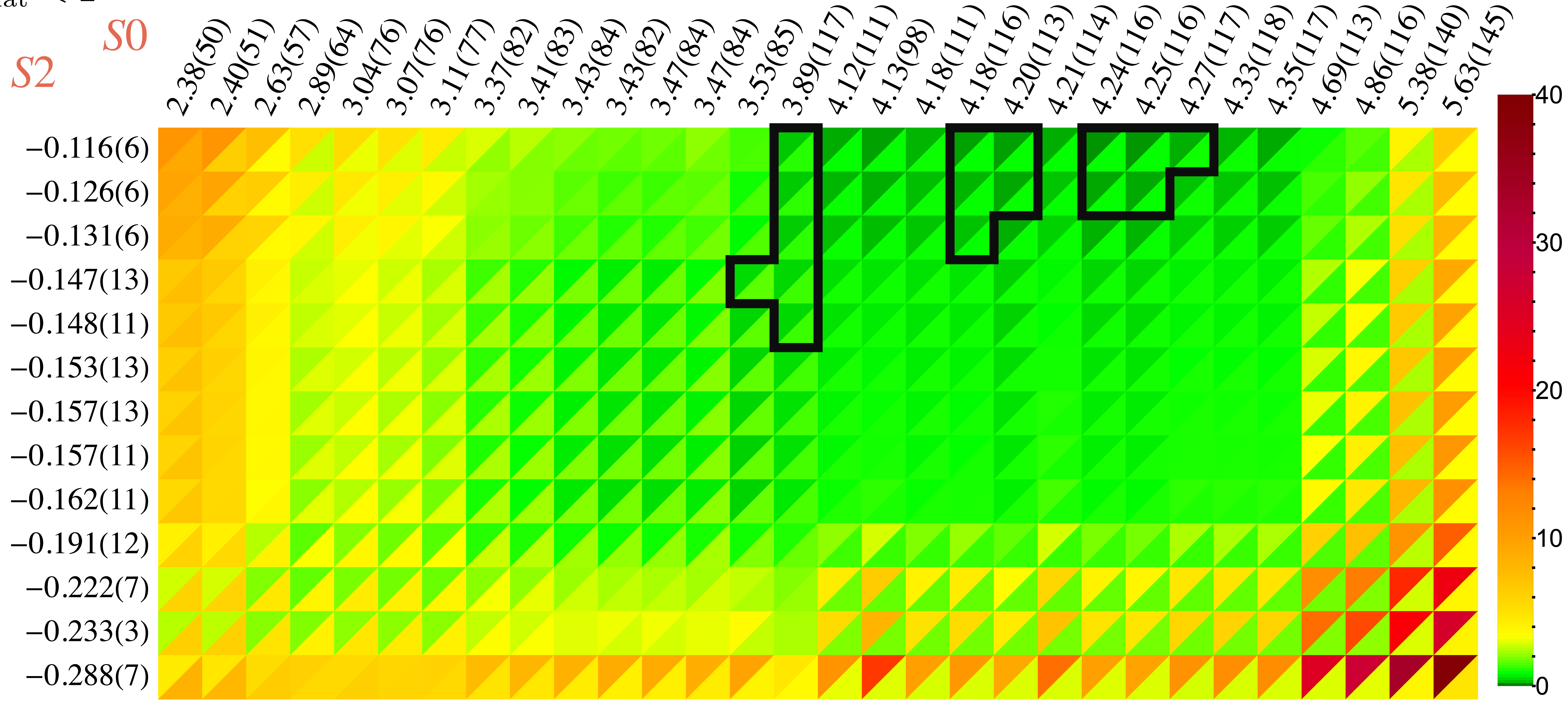
$m_\pi \sim 283\,\text{MeV}$

◤ $\langle d^2/N_{\text{smpl}} \rangle_{\text{pw}}$ ◢ $\langle \tilde{\chi}^2/N_{\text{lat}} \rangle_{\text{pw}}$

Black

GKPY

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$

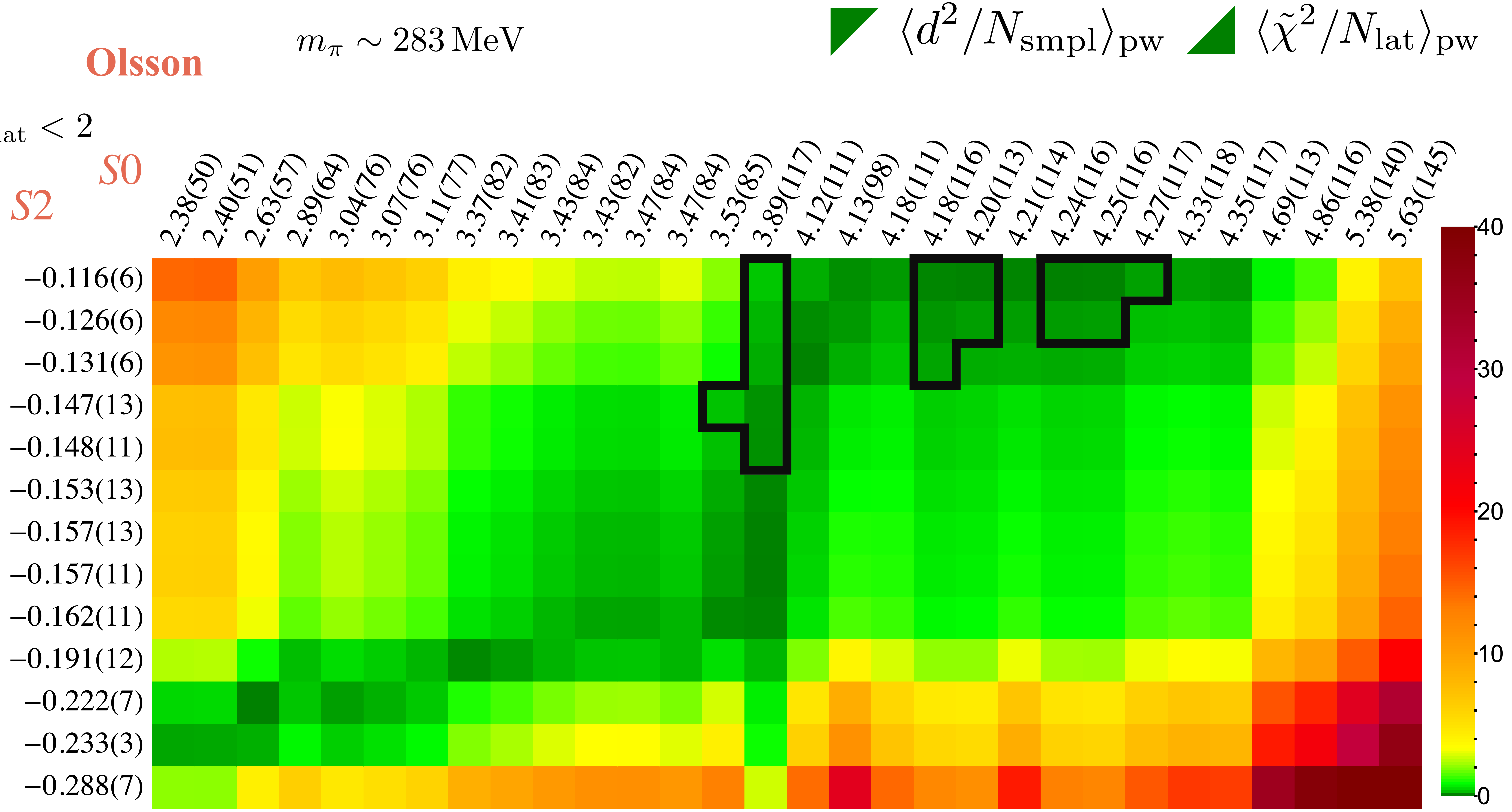


Scattering plane

$N_I \ell_{\max} N_{\text{params}} \sim 300 - 400$

$m_\pi \sim 283\,\text{MeV}$

$d^2/N_{\text{smpl}} < 1, \quad \tilde{\chi}^2/N_{\text{lat}} < 2$



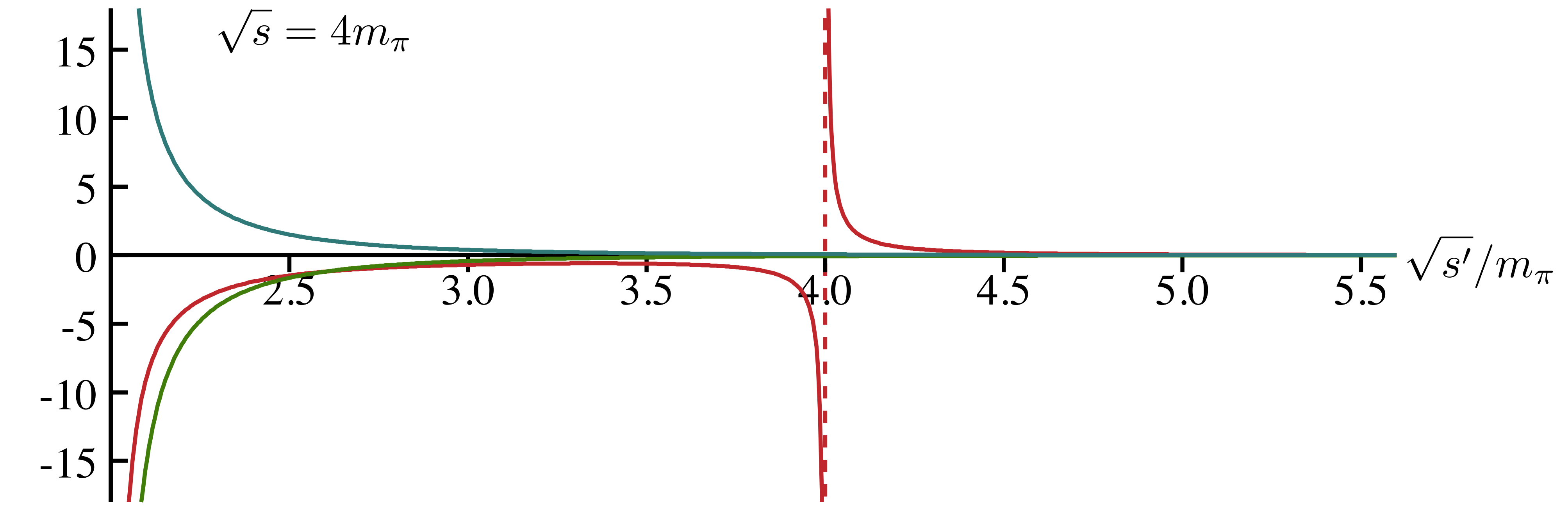
The good

Fit → In

DR → Out

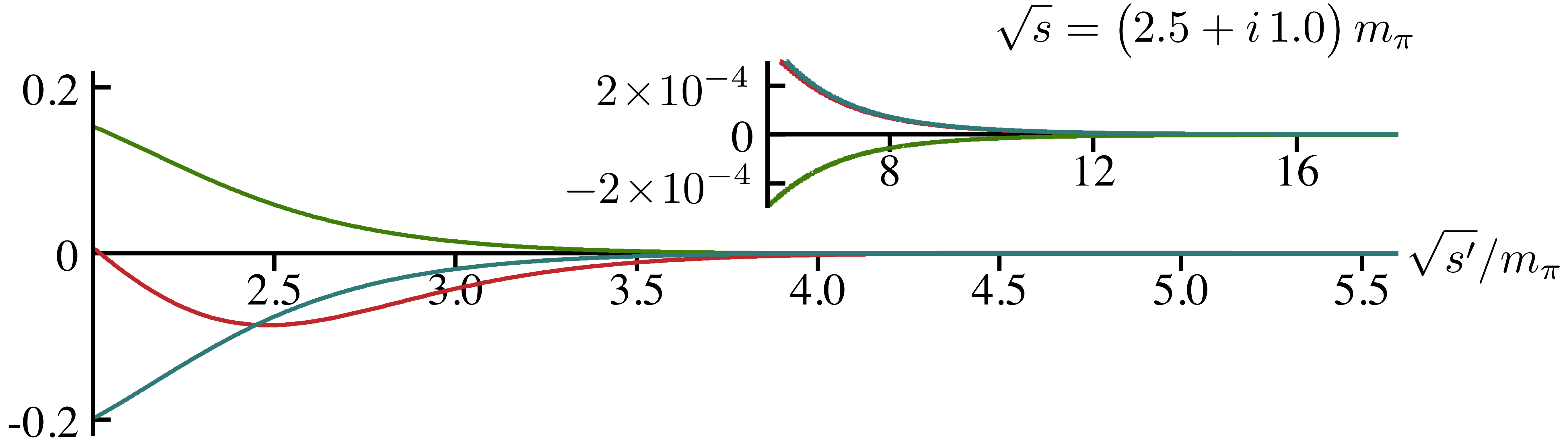
$$\tilde{t}^I_\ell(s) = \tau^I_\ell(s) + \sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' \, K^{II'}_{\ell\ell'}(s',s) \, \mathrm{Im}\, t^{I'}_{\ell'}(s')$$

Real s



Smeared over a large energy region

Complex s



An ϵ on the real axis $\to \epsilon'$ in the complex plane

The bad

Not happening

$$\sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' \, K_{\ell\ell'}^{II'}(s',s) \, \mathrm{Im}\, t_{\ell'}^{I'}(s')$$

Partial waves

Extrapolated

$$\int_{4m_\pi^2}^{\infty} = \int_{4m_\pi^2}^{s_{max}} + \int_{s_{max}}^{\infty} = \int_{4m_\pi^2}^{s_{fit}} + \int_{s_{fit}}^{s_{max}} + \int_{s_{max}}^{\infty}$$

Regge

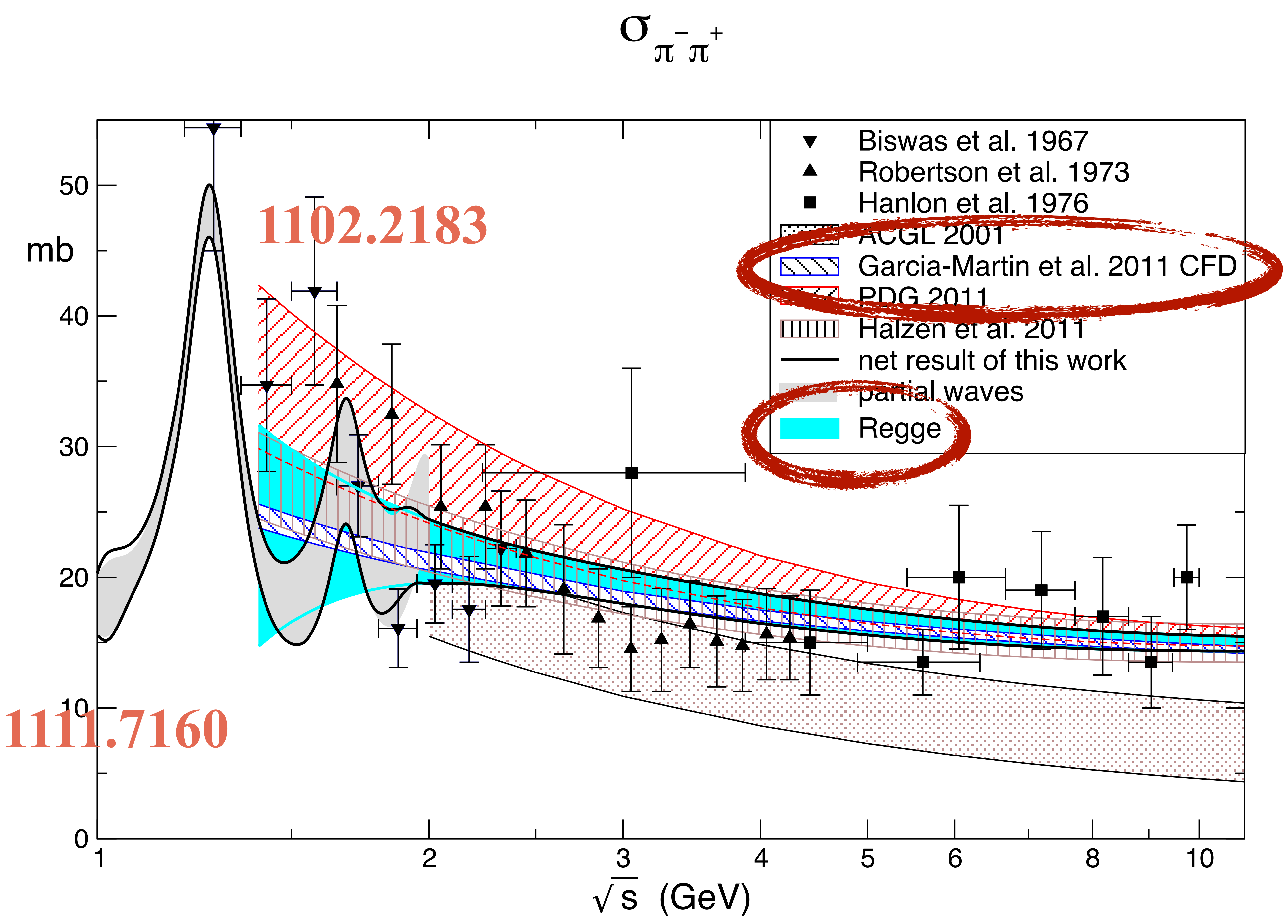
- Regge must be extrapolated from phys. m_π
- Regge is wrong below $a_t m_\pi \sim 0.22 - 0.25$

The Regge

Regge must be extrapolated from phys. m_π

$\mathbb{P} \to$ gluon exchanges $\to$ constant over m_q

$\rho, f_2 \to$ resonances, not constant $\to \lambda \sim \Gamma/M$

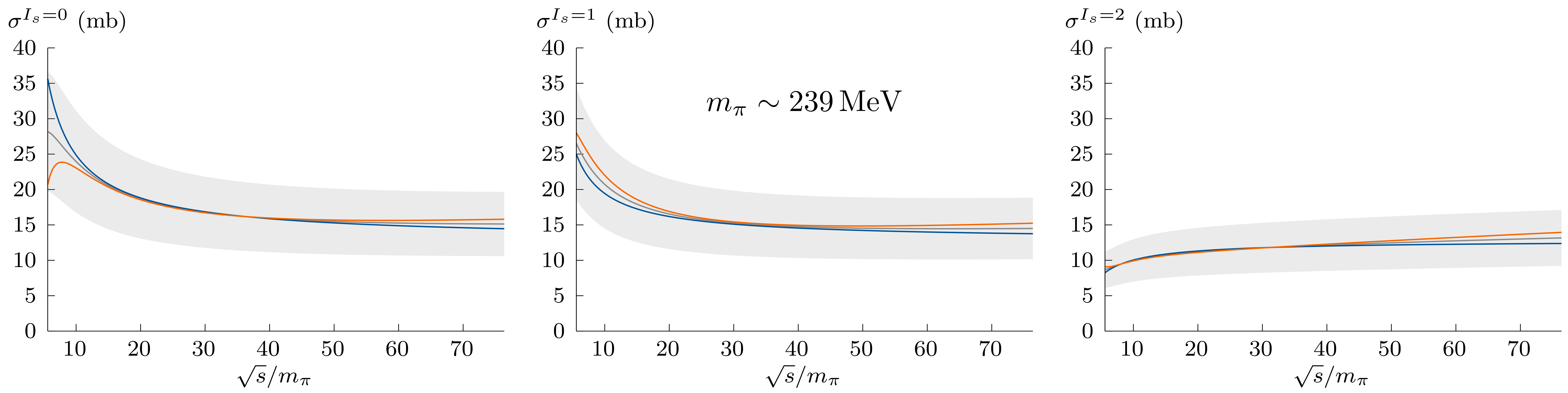


Our $F_{Regge} = \dfrac{F_{Regge1} + F_{Regge2}}{2}$

Big uncertainty $\Delta F_{Regge} = 0.3\, F_{Regge}$

Regge

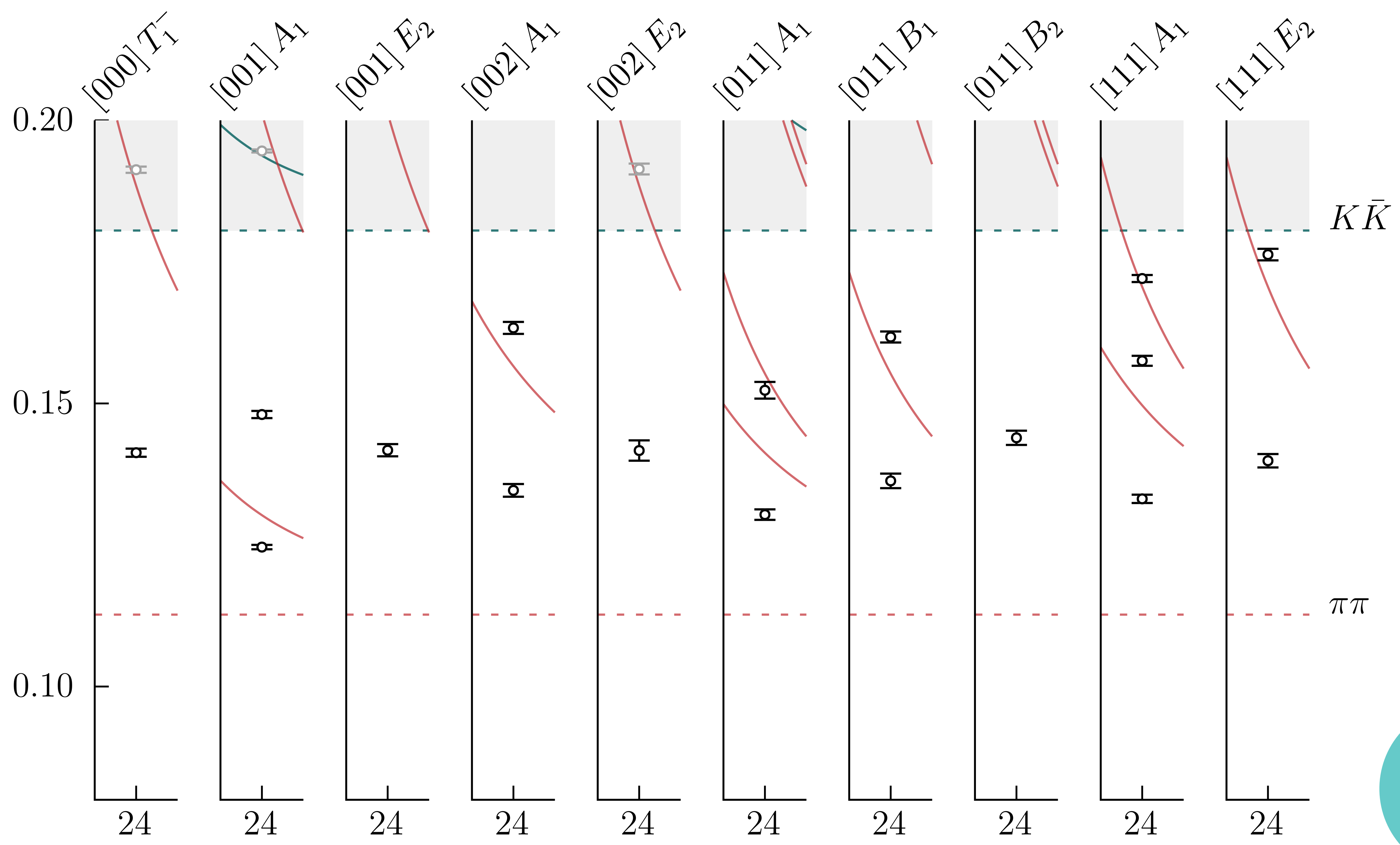
Regge must be extrapolated from phys. m_π



Our $F_{Regge} = \dfrac{F_{Regge1} + F_{Regge2}}{2}$

Big uncertainty $\Delta F_{Regge} = 0.3\, F_{Regge}$

$I = 1 \; \pi\pi$

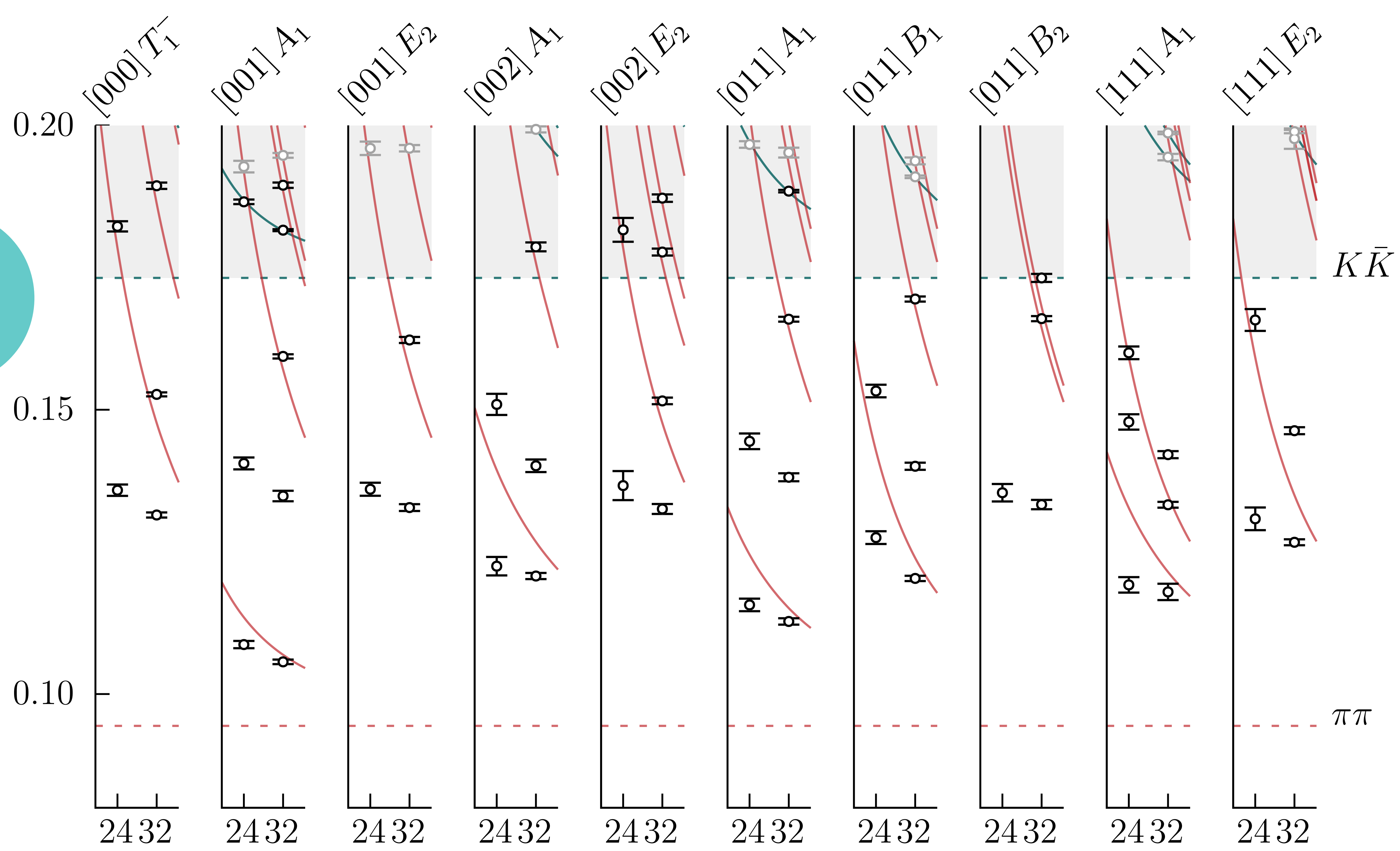


$m_\pi \sim 330\,\mathrm{MeV}$

Similar spectrum to previous masses

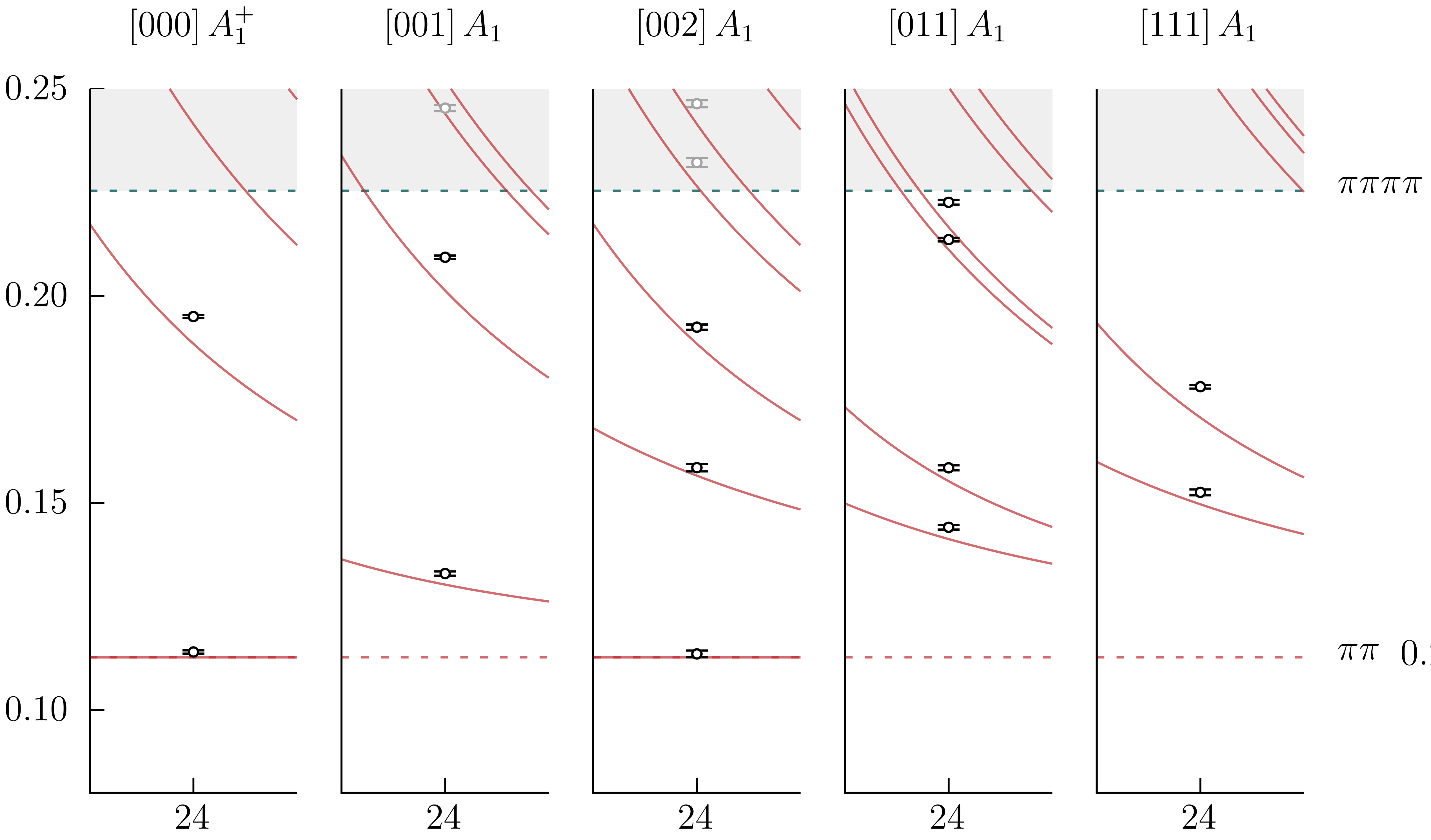
ρ

$m_\pi \sim 283\,\mathrm{MeV}$



Allows us to study the ρ resonance m_q dependence

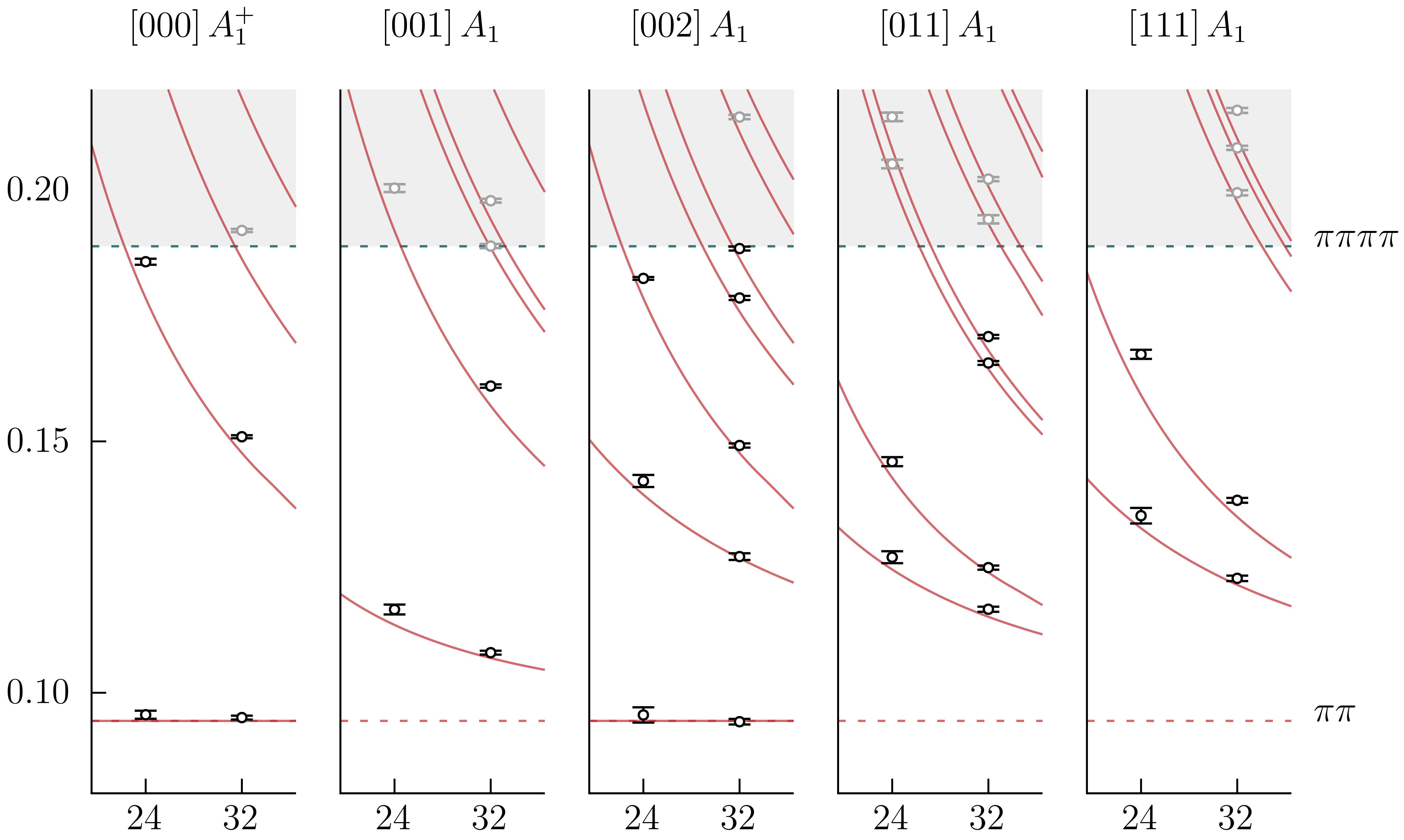
$I = 2\ \pi\pi$



$m_\pi \sim 330\,\text{MeV}$

Similar spectrum to previous masses

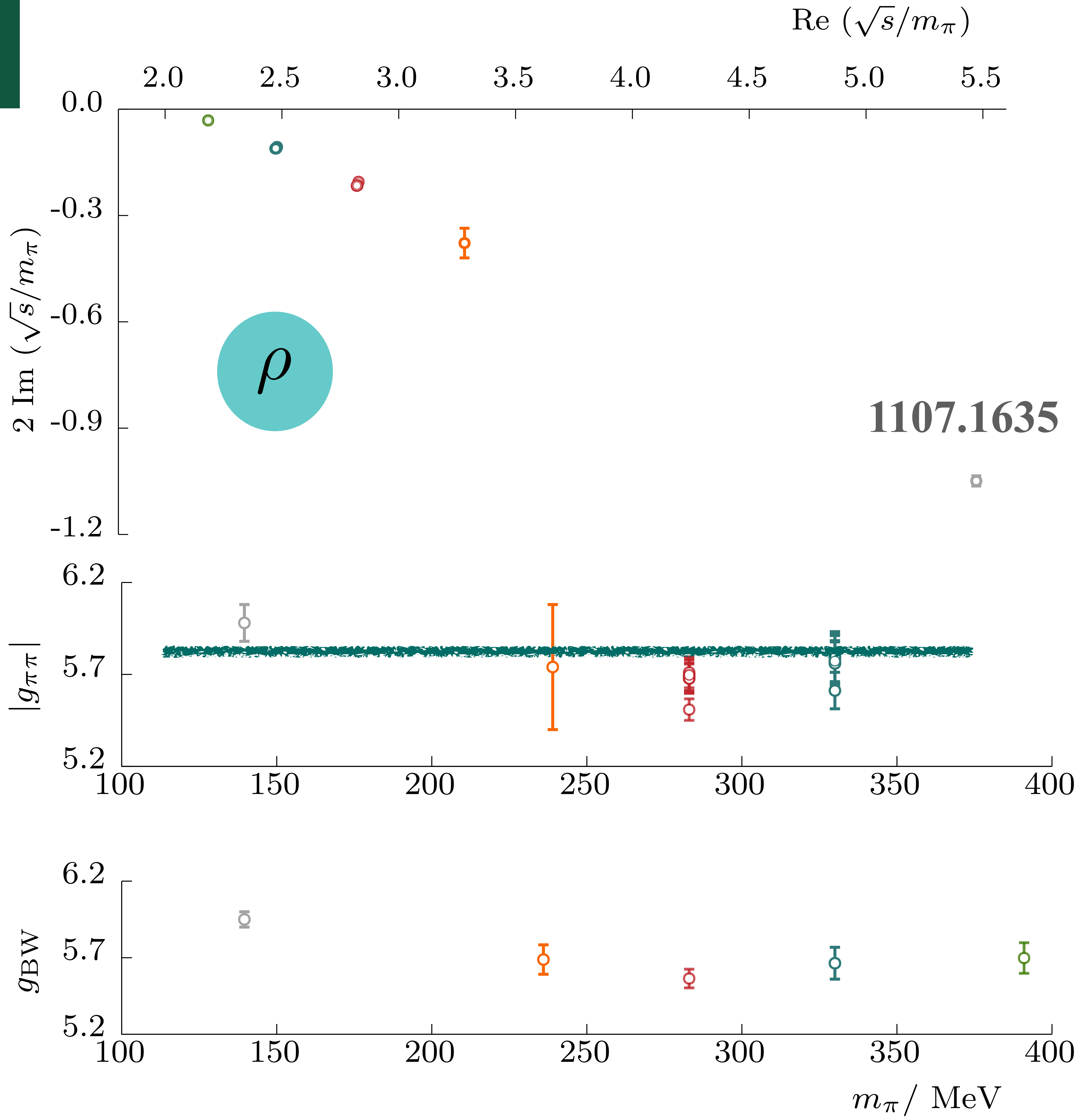
$m_\pi \sim 283\,\text{MeV}$



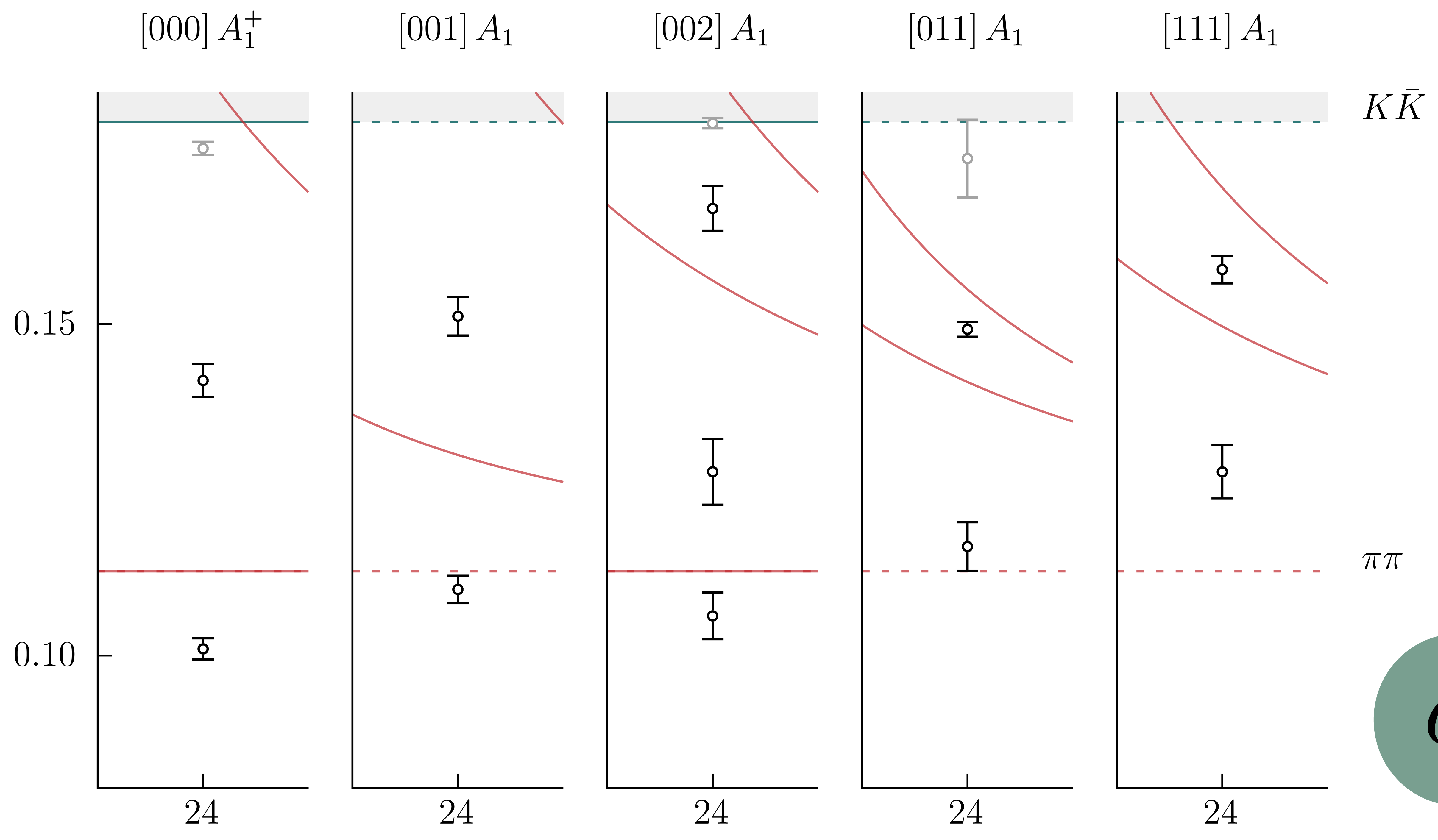
$I = 1 \ \pi\pi$

Ordinary m_q dependence

***g* constant**



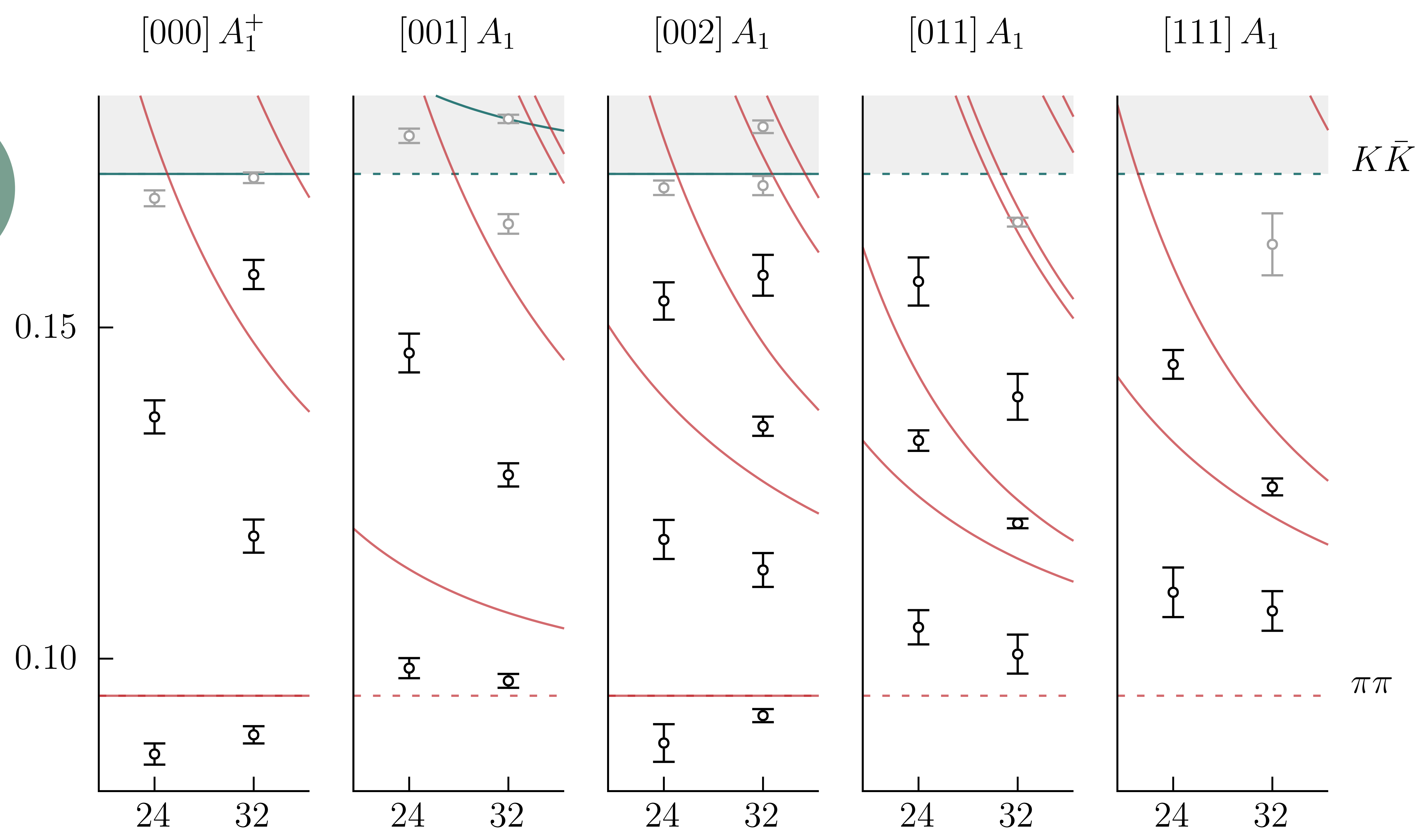
$I = 0\ \pi\pi$



$m_\pi \sim 330\,\text{MeV}$

Similar spectrum to previous masses

$m_\pi \sim 283\,\text{MeV}$



σ

Over 60 "elastic" levels for I=0

$I = 0\ \pi\pi$

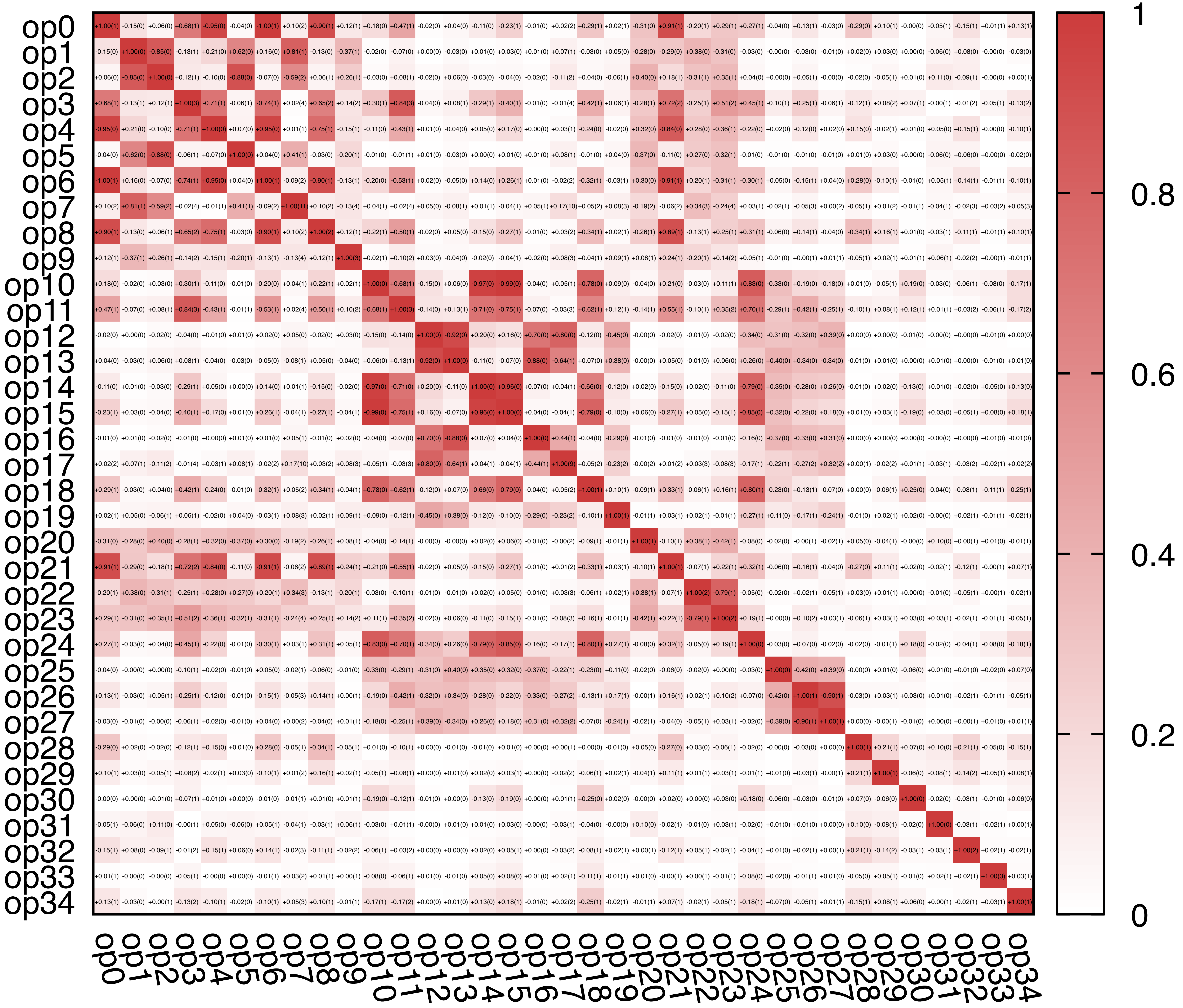
Many ops. for a good GEVP

Distillation → 0905.2160

Time src avg. correlations

Some highly correlated

More than a few relevant ops.



$I = 0\ \pi\pi$

Many fits for a good E_n

Many fits for diff

t_0

t_{min}, t_{max}

N_{exp}**(1-2)**

Model averaging technique

2008.01069

2208.13755