Potential Model Calculations of Proton Mass Radius with Constrained Charge Radius

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EXOHAD

EXOTIC HADRONS TOPICAL COLLABORATION

Charge Radius

0.831 fm	(PRad 2019)
0.833 fm	(Bezginov 2019)
0.834 fm	(Beyer 2017)
0.841 fm	(Antognini 2013)
0.842 fm	(CREMA 2010)
0.848 fm	(Grinin 2020)
0.875 fm	(Zhan 2011)
0.877 fm	(Fleurbaey 2018)
0.879 fm	(Bernauer 2010)

Mass Radius

0.5 fm	(Kharzeev 2021)
0.5 fm	(Giacalone 2022)
0.5—0.75 fm	(Duran 2023)
0.7—0.8 fm	(Mamo 2021)
0.8—0.9 fm	(Parkkila 2021,2022)
1.0—1.1 fm	(JETSCAPE 2021)

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$$\gamma p \rightarrow J/\psi p$$

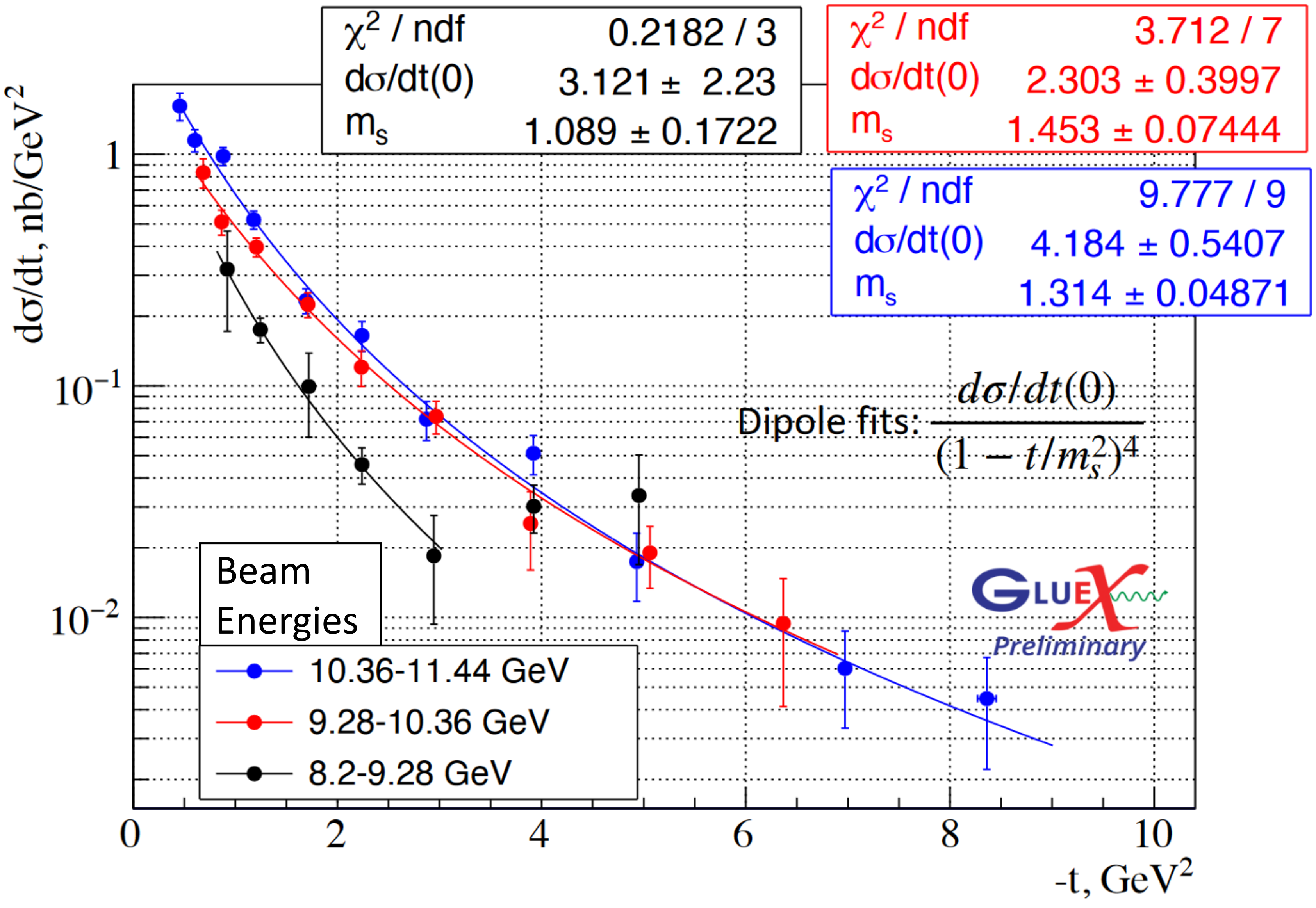
$$G(t) = \frac{M}{\left(1 - \frac{t}{m_s^2}\right)^2}$$

$$\langle R_m^2 \rangle = \frac{6}{M}\frac{dG}{dt}\bigg|_{t=0} = \frac{12}{m_s^2}$$

r_m = 0.518 +/− 0.019 fm

r_m = 0.471 +/− 0.024 fm

r_m = 0.630 +/− 0.100 fm



Quark Potential Models

- ✓One-gluon exchange at short range
- ✓Linear confinement at long range
- ✓Accurate for calculating ground state properties
- ✓Easy to visualize energy distribution

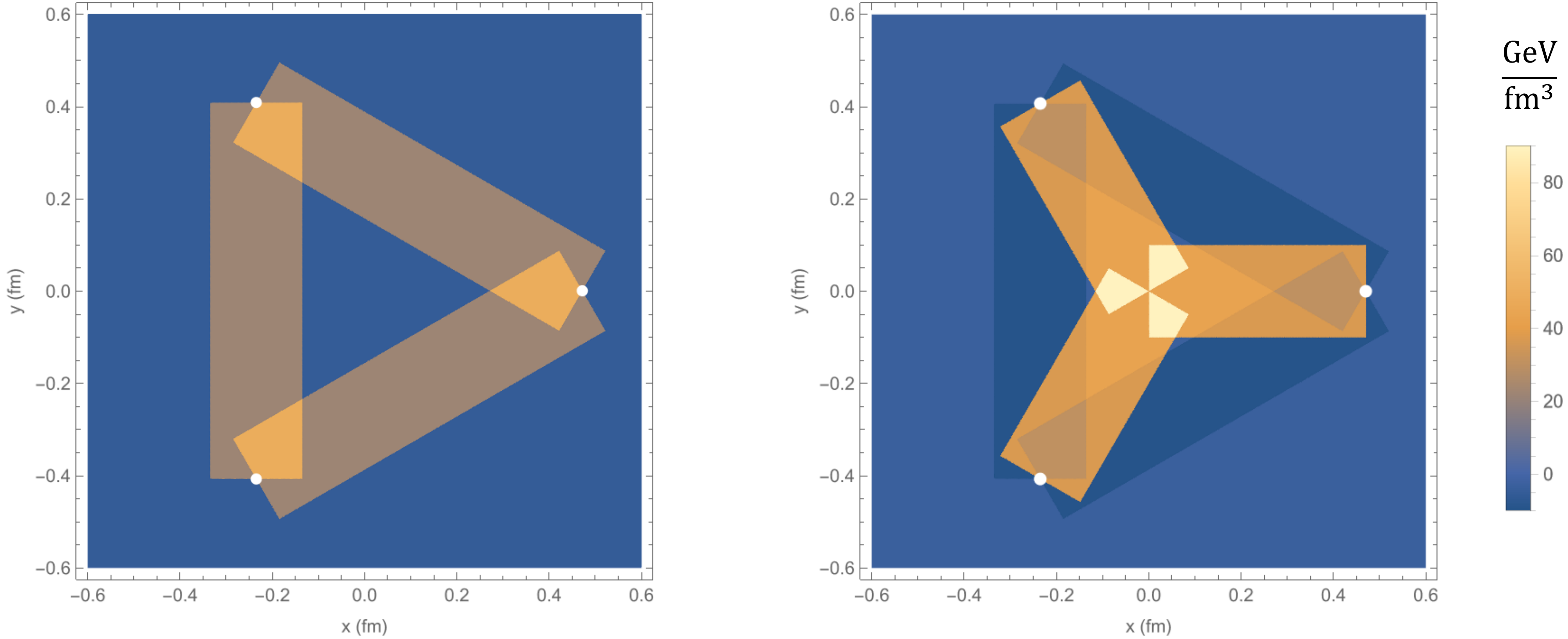
$$\left(\sum_i m_i + T + V\right)\psi = M\psi$$

$$V_{q\bar{q}} = -\frac{\kappa}{r} + \sigma r - V_0$$

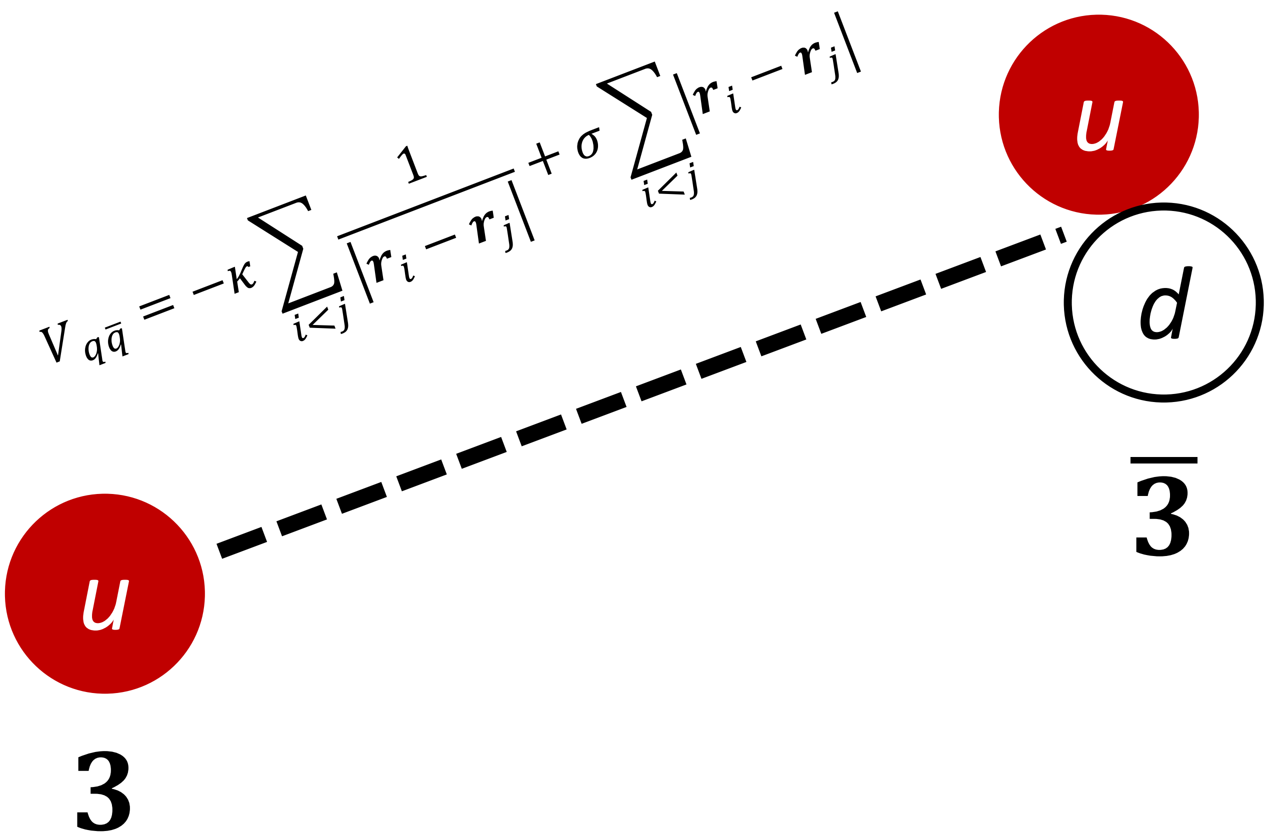
$$V_{qq} = \frac{1}{2} V_{q\bar{q}}$$

$$V_{\Delta} = -\frac{\kappa}{2}\sum_{i<j}\frac{1}{|\boldsymbol{r}_i - \boldsymbol{r}_j|} + \frac{\sigma}{2}\sum_{i<j}|\boldsymbol{r}_i - \boldsymbol{r}_j|$$

$$V_{\mathrm{Y}} = -\frac{\kappa}{2}\sum_{i<j}\frac{1}{|\boldsymbol{r}_i - \boldsymbol{r}_j|} + \sigma \min_{\boldsymbol{P}} \sum_{i}|\boldsymbol{r}_i - \boldsymbol{P}|$$



Diquark Clustering



Relativistic Corrections

Spin-Spin: $\frac{8\alpha_s}{9m^2r^2}(\boldsymbol{S}_1 \cdot \boldsymbol{S}_2)\delta(r)$

Spin-Orbit: $\left(\frac{2\alpha_s}{m^2r^3} - \frac{\sigma}{2m^2r}\right)(\boldsymbol{L} \cdot \boldsymbol{S})$

Tensor: $\frac{\alpha_s}{3m^2r^3}S_{12}$

Dynamical: $\sqrt{p^2 + m^2}$

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Can mostly be absorbed by the offset V_0!

$$\left\langle \frac{p^2/2m}{m} \right\rangle \sim \frac{1}{2}$$

Foldy-Wouthuysen (FW) Transform

$$H_D = \beta m + \mathcal{E} + \mathcal{O} \rightarrow \beta\sqrt{m^2 + \mathcal{O}^2} = H_{FW}$$

✓ H_{FW} is energy-separating

✓ FW variables satisfy Poincaré algebra ($\{\,\cdot,\cdot\,\} \rightarrow -i[\,\hat{\cdot},\hat{\cdot}\,]$)

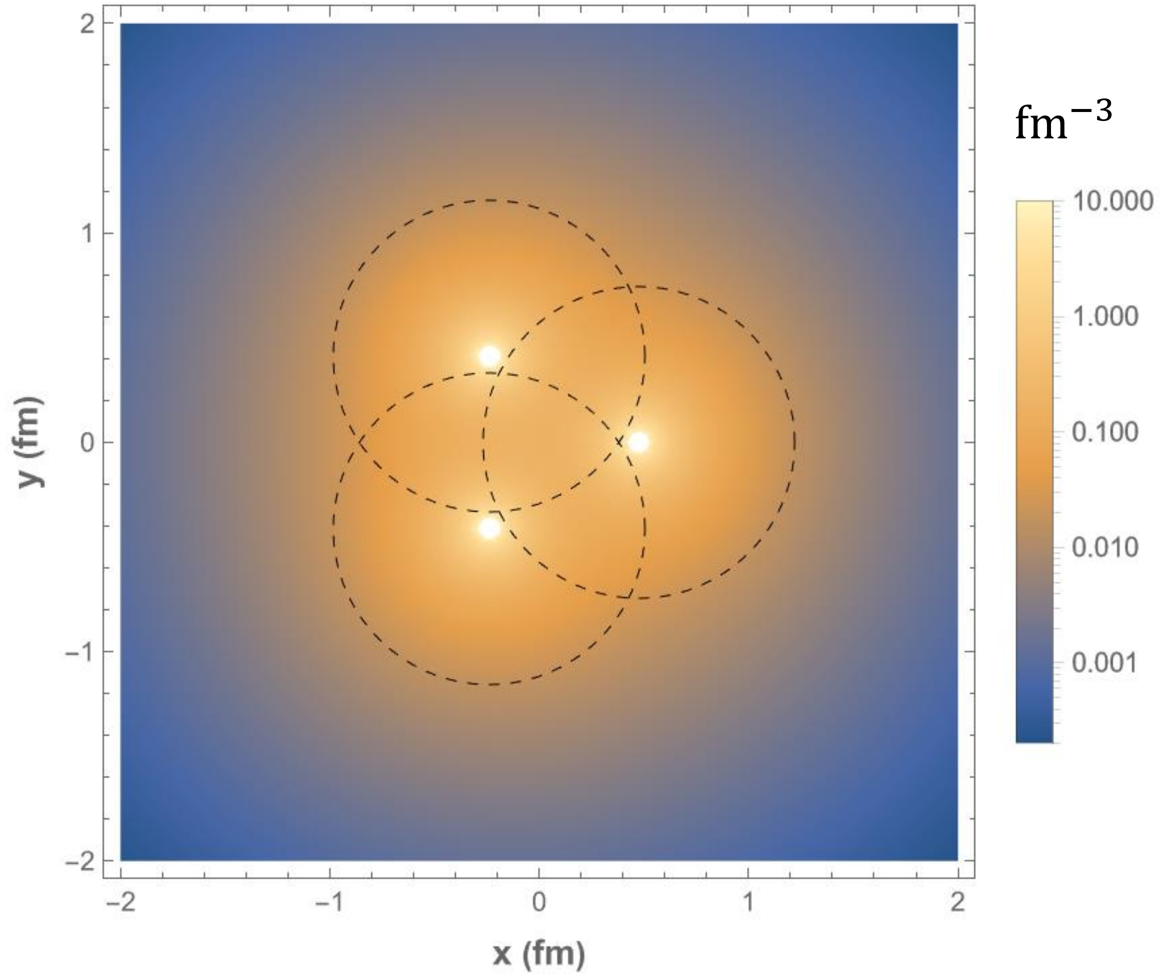
Necessary to write relativistic quantum wave theory in FW representation *before* taking Schrödinger approximation to maintain probabilistic interpretation of the wave function

Foldy-Wouthuysen Smearing

$$\psi_{FW}(\boldsymbol{r}) = \int d^3r' \, K(\boldsymbol{r}, \boldsymbol{r}')\psi_D(\boldsymbol{r}')$$

$$K(\boldsymbol{r}, \boldsymbol{r}') \neq \delta(\boldsymbol{r} - \boldsymbol{r}')$$

- Point-like objects smeared over a radius $\sim 1/m$
- May be significant overlap between smeared distributions!

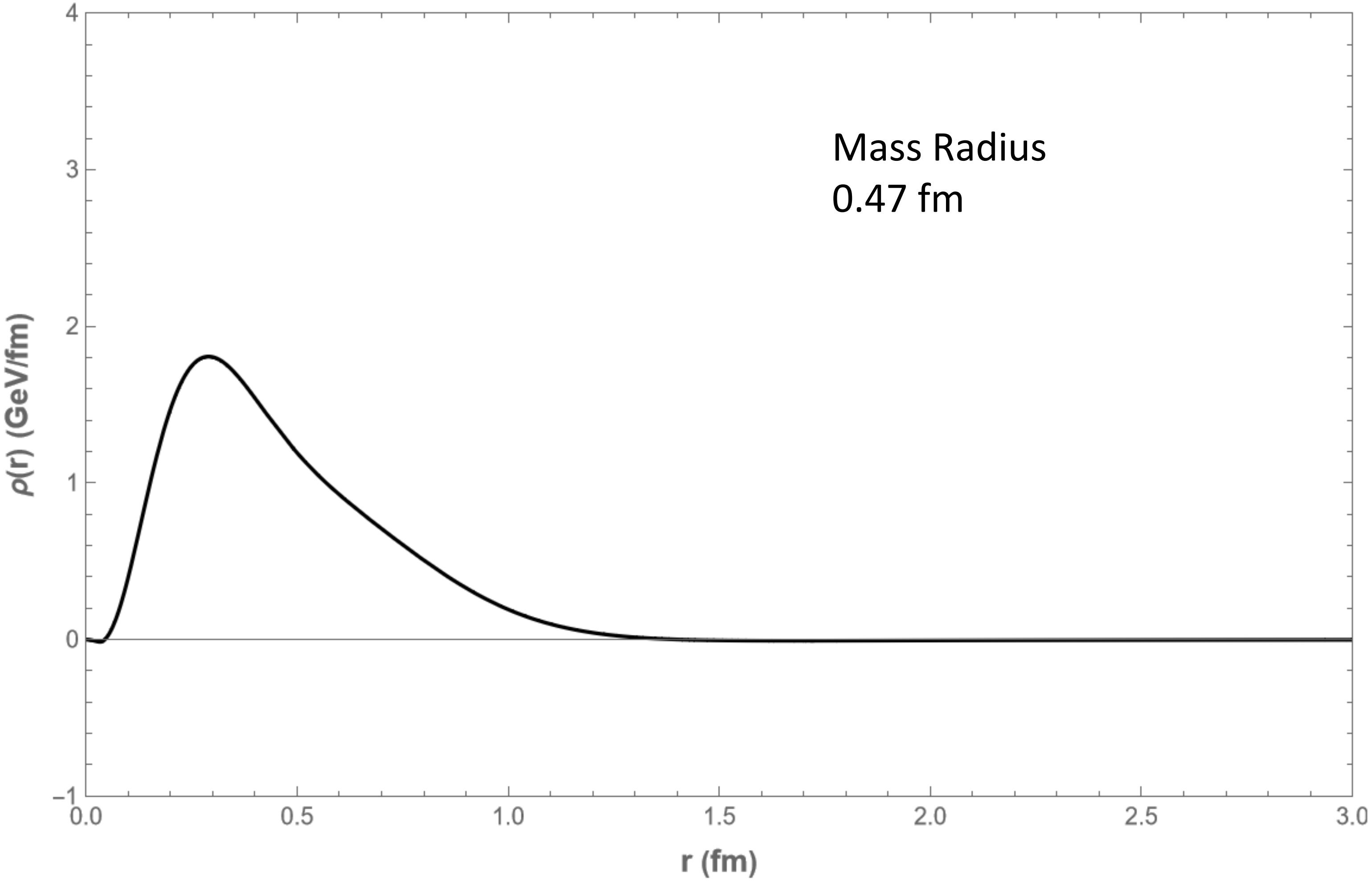


Δ-Model Parameters

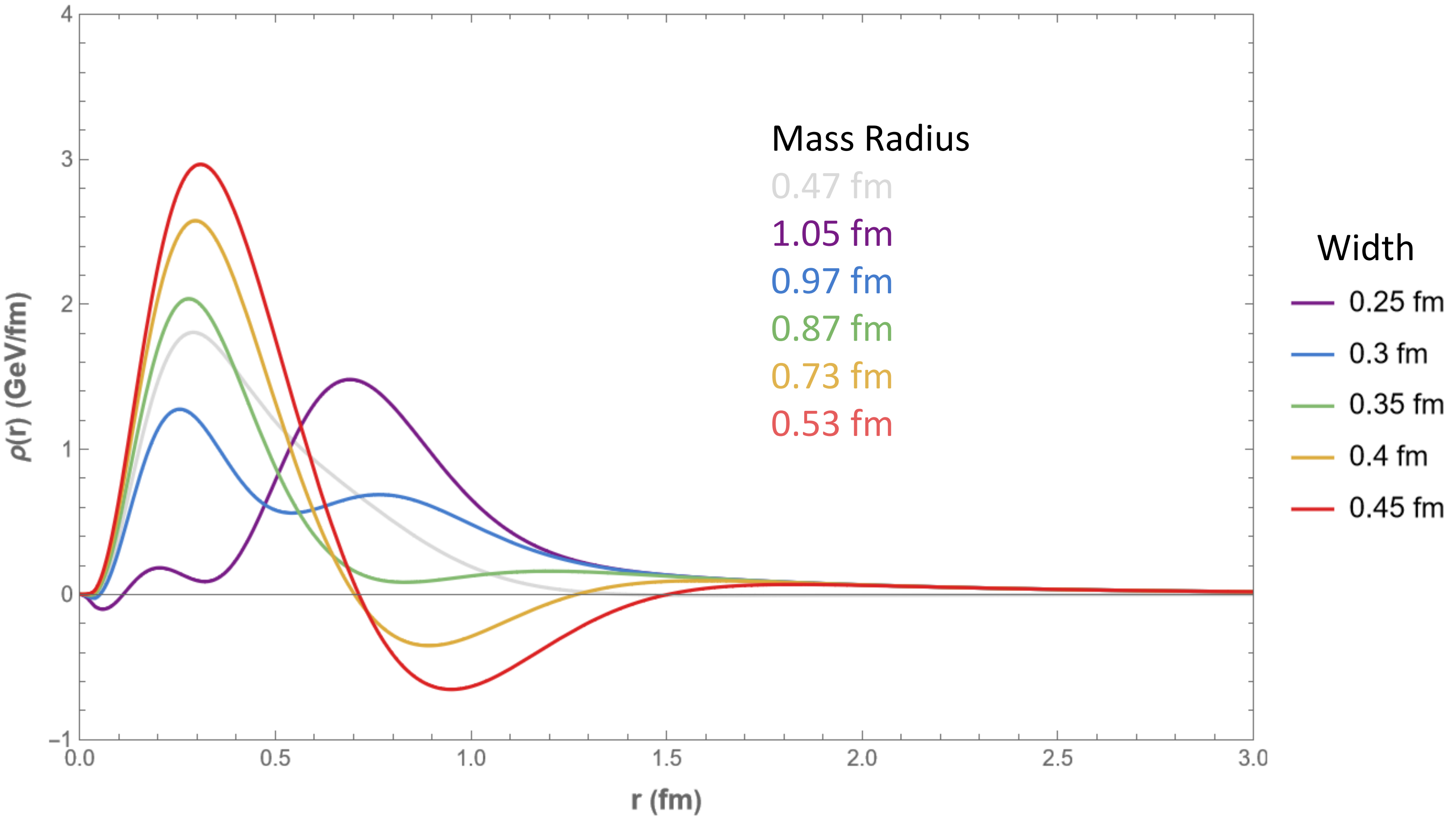
	With Smearing	Without Smearing
m	$0.265\ \text{GeV}$	$0.230\ \text{GeV}$
κ	0.42	0.10
σ	$0.208\ \text{GeV}^2$	$0.069\ \text{GeV}^2$
V_0	$1.2\ \text{GeV}$	$0.5\ \text{GeV}$

Constrained wave function without smearing *too extended*,
inconsistent with standard model parameters,
charge radius fixed at 0.84 fm

Δ-Model Quark-Distributed Offset



Δ-Model Gaussian Offset

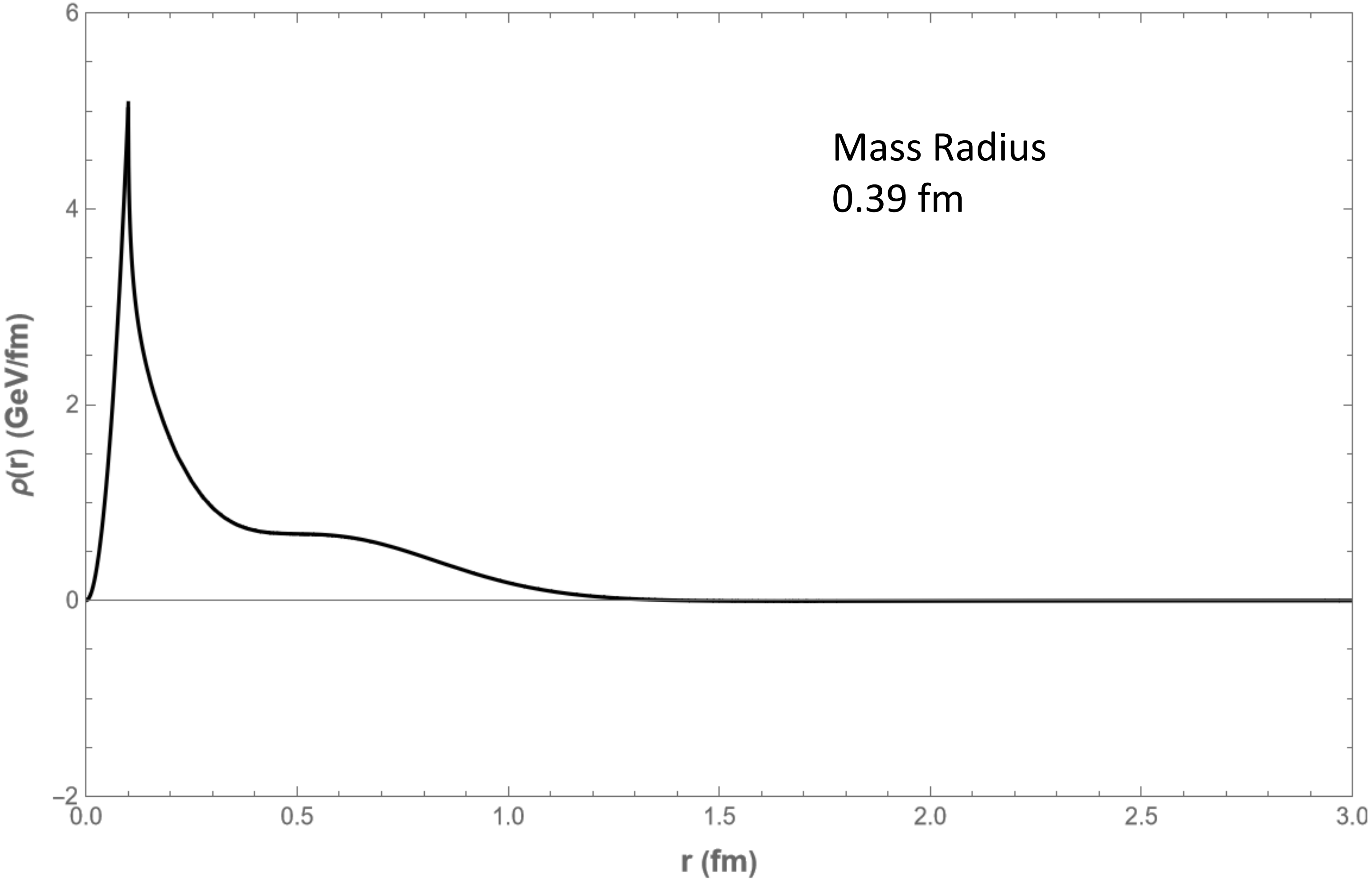


Y-Model Parameters

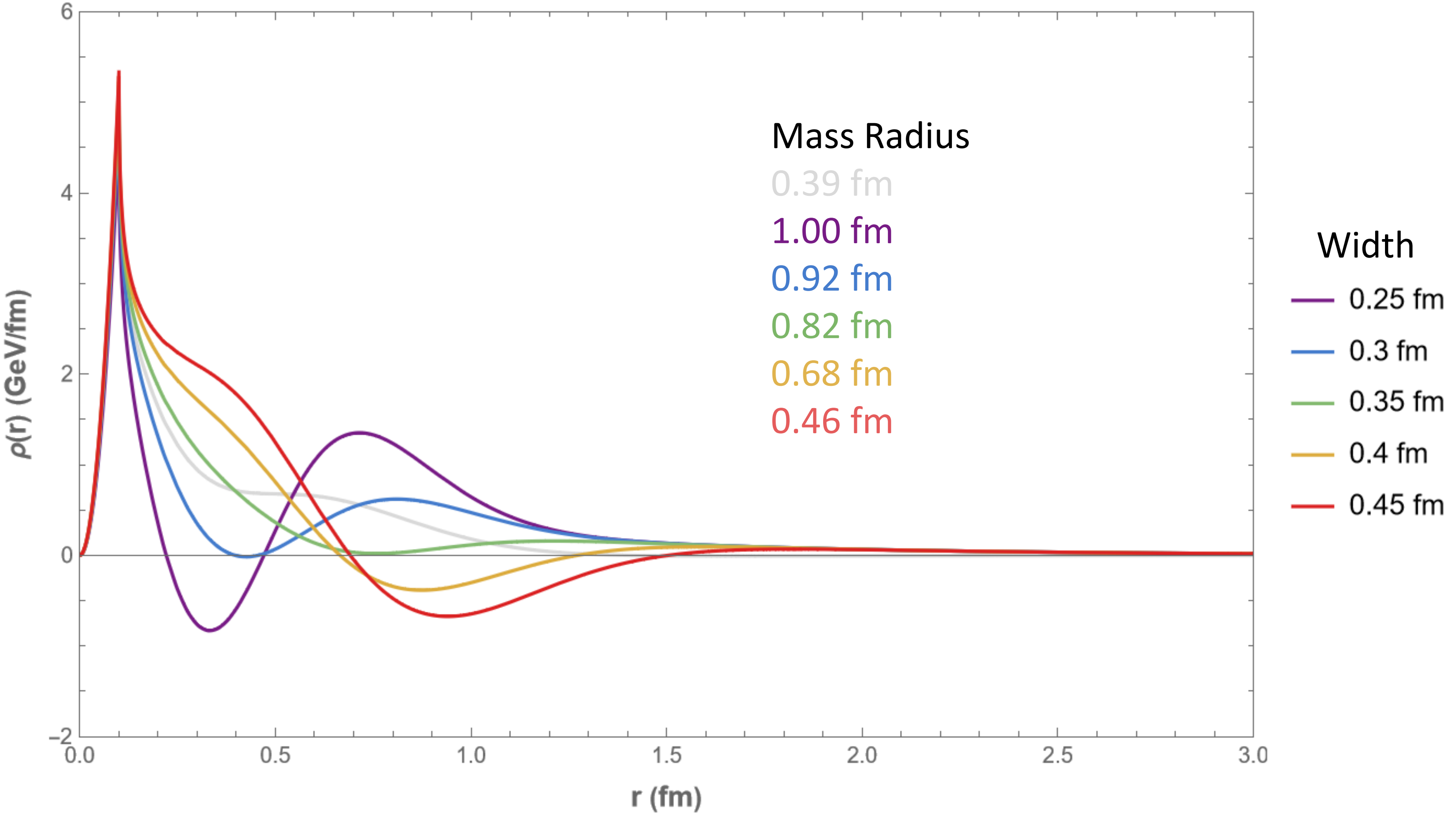
	With Smearing	**Without Smearing**
m	$0.265\ \mathrm{GeV}$	$0.230\ \mathrm{GeV}$
κ	0.42	0.10
σ	$0.180\ \mathrm{GeV}^2$	$0.060\ \mathrm{GeV}^2$
V_0	$1.2\ \mathrm{GeV}$	$0.5\ \mathrm{GeV}$

Charge radius fixed at 0.84 fm

Y-Model Quark-Distributed Offset



Y-Model Gaussian Offset



Summary

- Proton mass radius is poorly understood
- Ground state properties of proton well-described by Schrödinger potential model
- Charge radius used to constrain Hamiltonian parameters
- Foldy-Wouthuysen smearing crucial for fixing realistic light quark parameters
- Ambiguity remains in how offset energy is distributed

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