SHEDDING LIGHT ON SHADOW GENERALIZED PARTON DISTRIBUTIONS (GPDS)

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Introduction

- Generalized Parton Distributions (GPDs) contain information about many hadron properties:
  - 3D structure
  - Spin sum
  - Pressure and shear force distributions
- Goal:
  - Perform a global fit of GPDs using the Jefferson Lab Angular Momentum (JAM) methodologies.
- Obstacle:
  - Shadow GPDs (SGPDS) (Bertone, et. al. Phys.Rev.D 103 (2021) 11, 114019):
    - There is an infinite number of functions that can give the same CFF.

GPDs

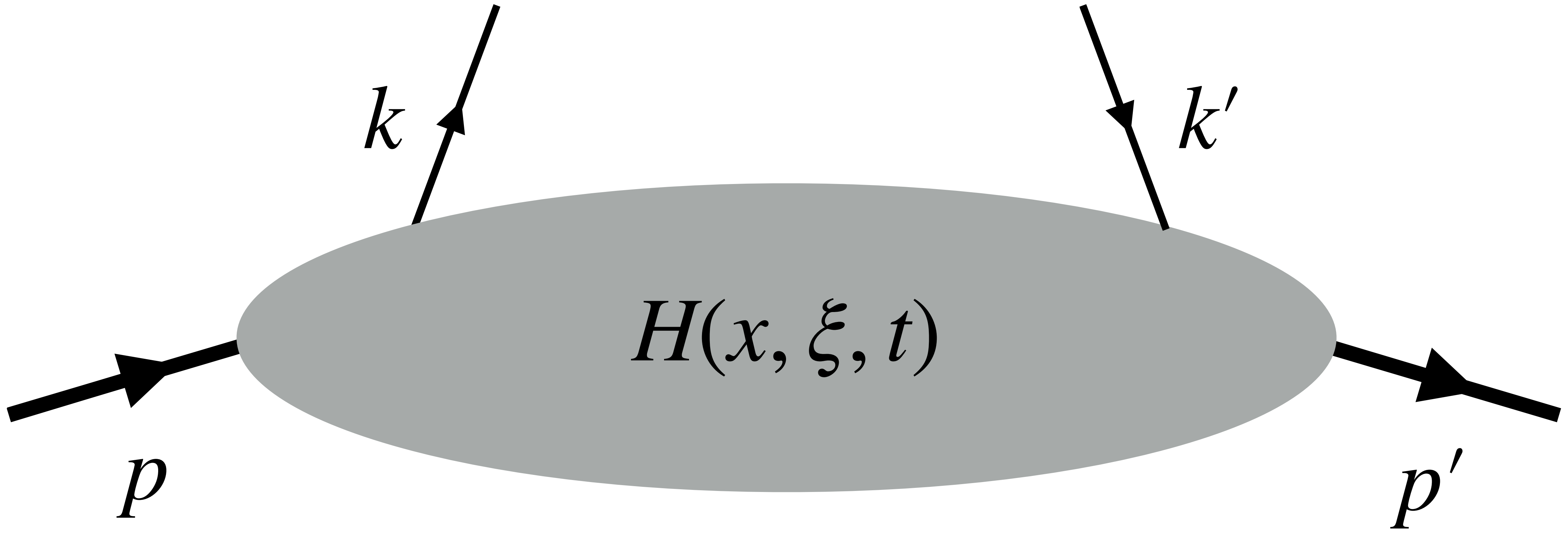
- Definition:

$$P\cdot n\int\frac{\mathrm{d}\lambda}{2\pi}\,e^{ixP\cdot n\lambda}\left\langle p'\left|\bar\psi^q(-\tfrac{1}{2}\lambda n)\,\not{n}\,\psi^q(\tfrac{1}{2}\lambda n)\right|p\right\rangle=\bar u(p')\left[\;H^q(x,\xi,t;\mu^2)\,\not{n}+\;E^q(x,\xi,t;\mu^2)\,\frac{i\sigma^{n\Delta}}{2M}\right]u(p),$$

$$n_\mu n_\nu\int\frac{\mathrm{d}\lambda}{2\pi}\,e^{ixP\cdot n\lambda}\left\langle p'\left|G^{\mu\alpha}(-\tfrac{1}{2}\lambda n)\,G_\alpha{}^{\nu}(\tfrac{1}{2}\lambda n)\right|p\right\rangle=\bar u(p')\left[x\,H^g(x,\xi,t;\mu^2)\,\not{n}+x\,E^g(x,\xi,t;\mu^2)\,\frac{i\sigma^{n\Delta}}{2M}\right]u(p),$$

- Functions of x, ξ, and t:

$$x=\frac{k^+ + k'^+}{p^+ + p'^+}\quad \xi=\frac{p'^+ - p^+}{p^+ + p'^+}\quad t=(p'-p)^2$$



GPDs

- Properties:
  - Polynomiality:

$$\int_{-1}^{1} \mathrm{d}x\, x^s H^a(x,\xi,t;\mu^2) = \sum_{i=0\,(\text{even})}^{s} (2\xi)^i A^a_{s+1,i}(t,\mu^2) + \mathrm{mod}(s,2)\,(2\xi)^{s+1} C^a_{s+1}(t,\mu^2),$$

$$\int_{-1}^{1} \mathrm{d}x\, x^s E^a(x,\xi,t;\mu^2) = \sum_{i=0\,(\text{even})}^{s} (2\xi)^i B^a_{s+1,i}(t,\mu^2) - \mathrm{mod}(s,2)\,(2\xi)^{s+1} C^a_{s+1}(t,\mu^2),$$

  - Forward Limit ($\xi, t \to 0$):

$$H^q(x,0,0) = q(x)\,\Theta(x) - \bar{q}(-x)\,\Theta(-x),$$

$$2\,H^g(x,0,0) = g(x)\,\Theta(x) - g(-x)\,\Theta(-x),$$

  - Evolution:
    - GPDs change with the energy scale in accordance with evolution equations of the general form:

$$\frac{\mathrm{d}H^a(x,\xi,t)}{\mathrm{d}\ln Q^2} = \int \mathrm{d}x P^a(x,\xi) H^a(x,\xi,t;Q_0^2)$$

The Inverse Problem

- Deeply virtual Compton scattering:
  - Compton Form Factors:

$$\mathcal{H}(\xi, t, Q^2) = \int_{-1}^{1} dx \sum_a C^a(x, \xi, Q^2, \mu^2)\, H^a(x, \xi, t; \mu^2)$$

- x-dependence is lost in the integration:
  - There is an infinite number of functions that can give the same CFF.

Shadow GPDs

- The difference between one of the multiple solutions to the inverse problem and the true GPD:

$$F_S^a(x, \xi; \mu^2) = F_F^a(x, \xi; \mu^2) - F_T^a(x, \xi; \mu^2)$$

- Can rule out any F_F^a that do not satisfy the properties of GPDs, therefore SGPDs:
  - Must satisfy polynomiality
  - Zero contribution to CFF:

$$\sum_a C^a(x, \xi, Q^2, \mu^2) \otimes F_S^a(x, \xi; \mu^2) = 0$$

  - Forward Limit: $H_S^a(x,0,0) = 0$

Evolution and SGPDs

- Example SGPDs explored in Bertone, et. al. Phys.Rev.D 103 (2021) 11, 114019 give very small but non-zero CFF after evolution to a different energy scale.
- SGPDs can be multiplied by any factor and the result would still be a SGPD at the input scale
- Non-zero CFF after evolution would be multiplied by this factor
- Data spanning a range of energy scales would give a limit to the possible scaling factors

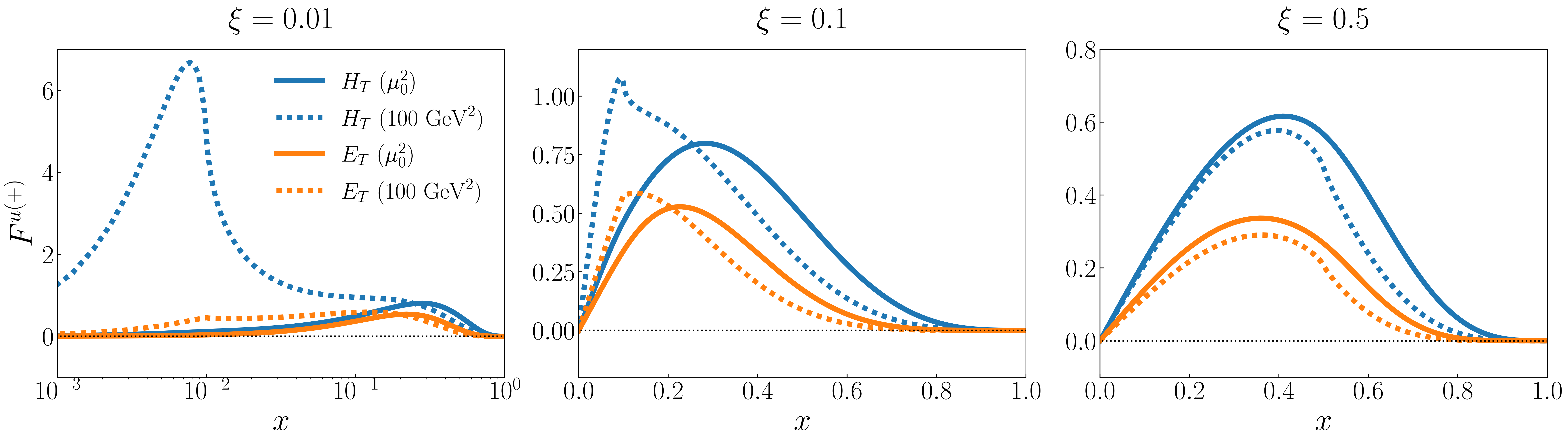
Evolution and SGPDs

- In this work:
  - Generate simulated CFF data spanning a range of energy scales and skewness using a model
  - Calculate how this data constrains a Monte Carlo sampling of SGPDs

“True” GPDs

- Use VGG model as a proxy for the “true” GPD:
  - Vanderhaeghen, et. al., Phys. Rev. Lett. 80, 5064 (1998)
  - Vanderhaeghen, et. al., Phys. Rev. D 60, 094017 (1999)
  - Goeke, et. al., Prog. Part. Nucl. Phys. 47, 401 (2001)
  - Guidal, et. al., Phys. Rev. D 72, 054013 (2005)
- Use PDFs from JAM20-SIDIS (EM, et. al., Phys. Rev. D 104, 016015 (2021))

“True” GPDs



Calculating Shadow GPDs

- Start from a double distribution (DD):

$$F_{DD}(\alpha, \beta) = \sum_{m \text{ even}, n \text{ odd}}^{m+n \leq N} c_{mn} \alpha^m \beta^n$$

- SGPD is a Radon transform of the DD:

$$H_S(x, \xi) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(x - \beta - \alpha\xi) F_{DD}(\alpha, \beta)$$

- This guarantees the SGPDs satisfy polynomiality

Calculating Shadow GPDs

- SGPD conditions give a set of equations that can be solved for the unknowns (c_{mn})
  - For a given N there are more unknown coefficients than constraining equations:
    - Assign random values to enough randomly selected coefficients to reduce the number of unknowns so that the equations can be solved
  - Use $N = 27$
  - SGPDs give zero contribution to the CFF at next-to-leading order

Calculating Shadow GPDs

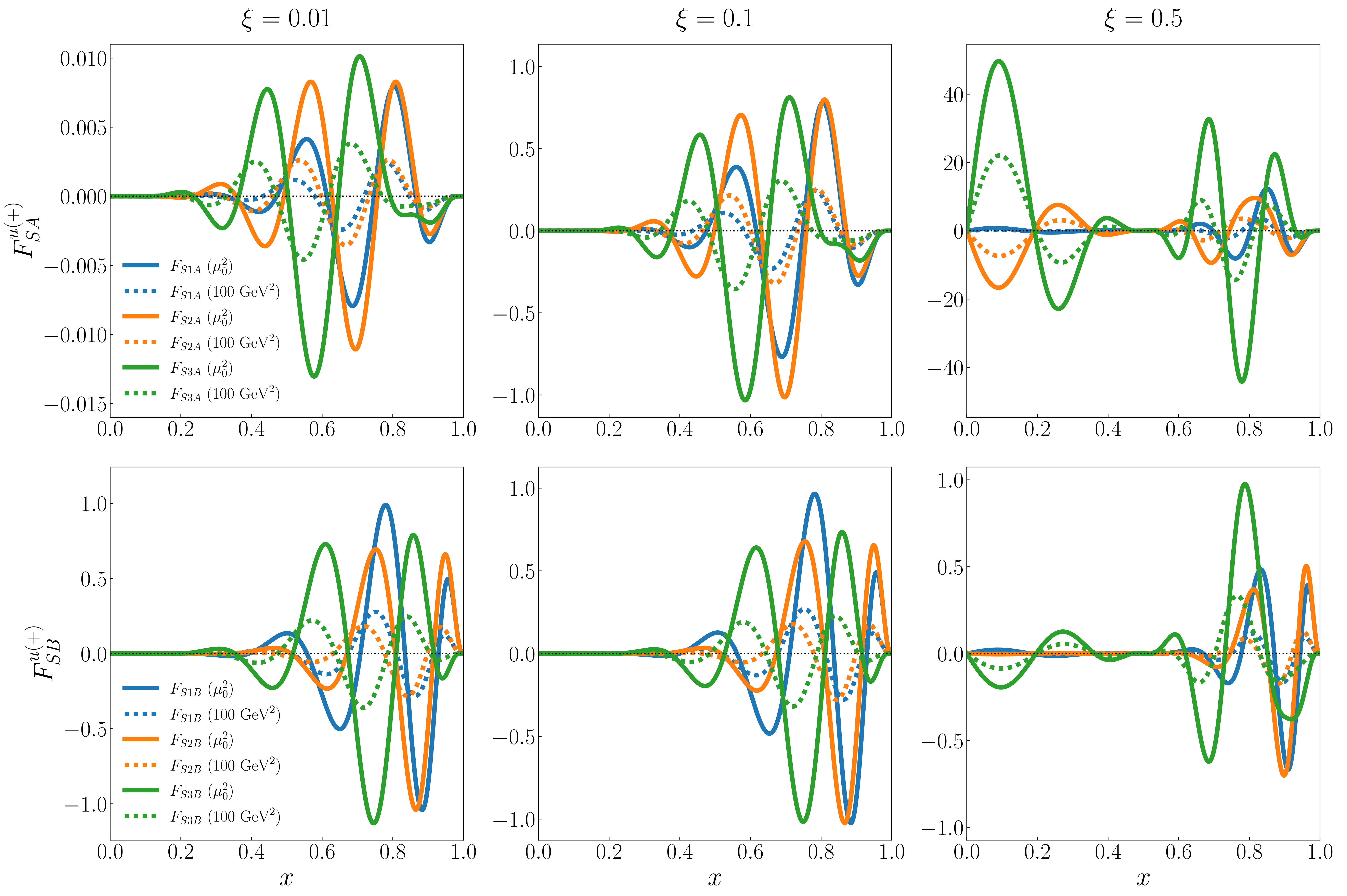
- For SGPDs derived this way we can impose the forward limit in two ways:
  - Type A:
    - Consistent with Bertone, et. al. Phys.Rev.D 103 (2021) 11, 114019:

$$H_S^{u(+)}(x,0;\mu_0)) = 0$$

  - Type B:
    - Could also multiply F_{DD} by a function of t that is zero when $t = 0$

$$H_S^{u(+)}(x,0;\mu_0)) \neq 0$$

Example Shadow GPDs



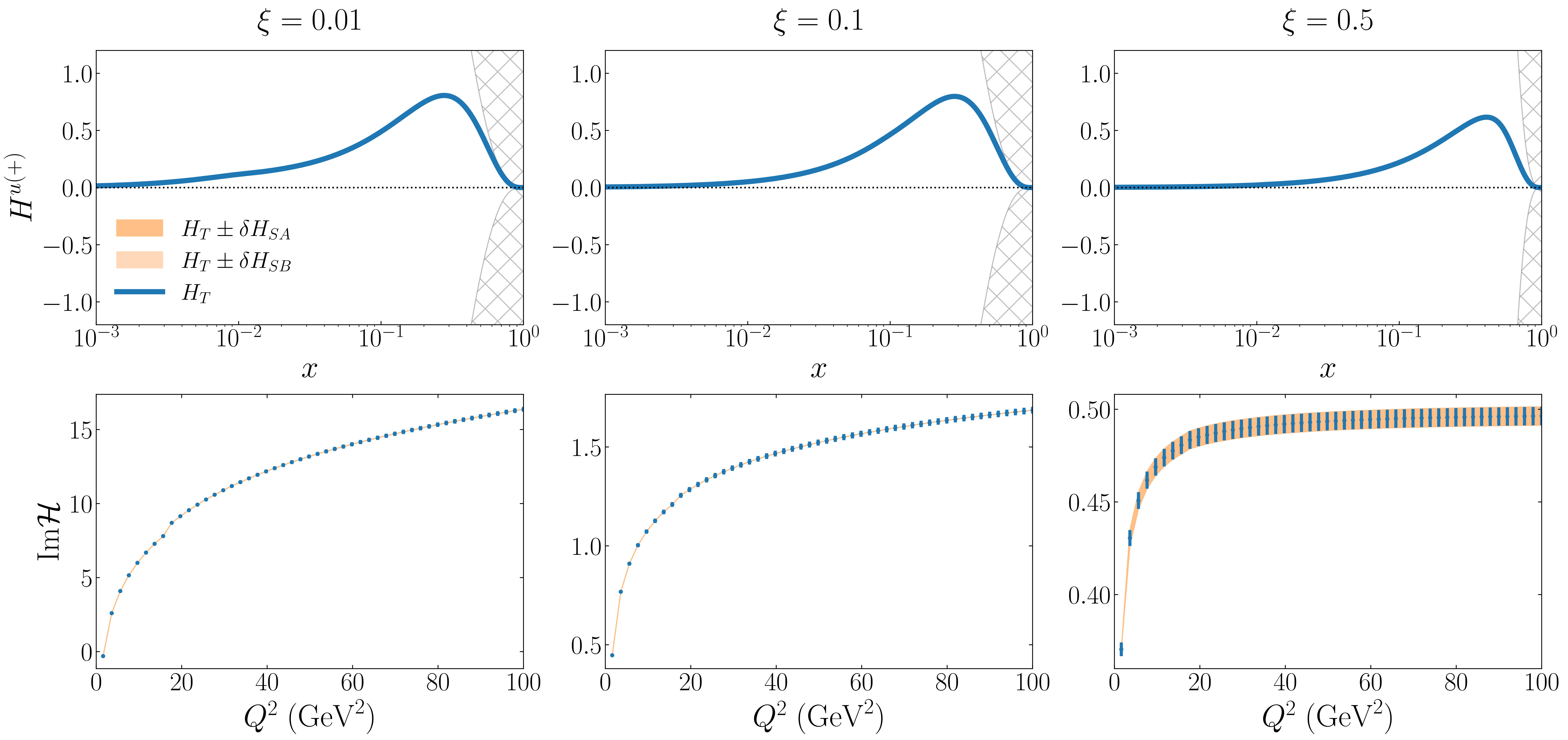
Exploring SGPDs and Evolution

- Use Monte Carlo sampling to generate replicas that are linear combinations of three SGPDs:

$$H^{u(+)}(x,\xi;\mu^2,\lambda) = H_T^{u(+)}(x,\xi;\mu^2) + \lambda_1 H_{S1}^{u(+)}(x,\xi;\mu^2) + \lambda_2 H_{S2}^{u(+)}(x,\xi;\mu^2) + \lambda_3 H_{S3}^{u(+)}(x,\xi;\mu^2)$$

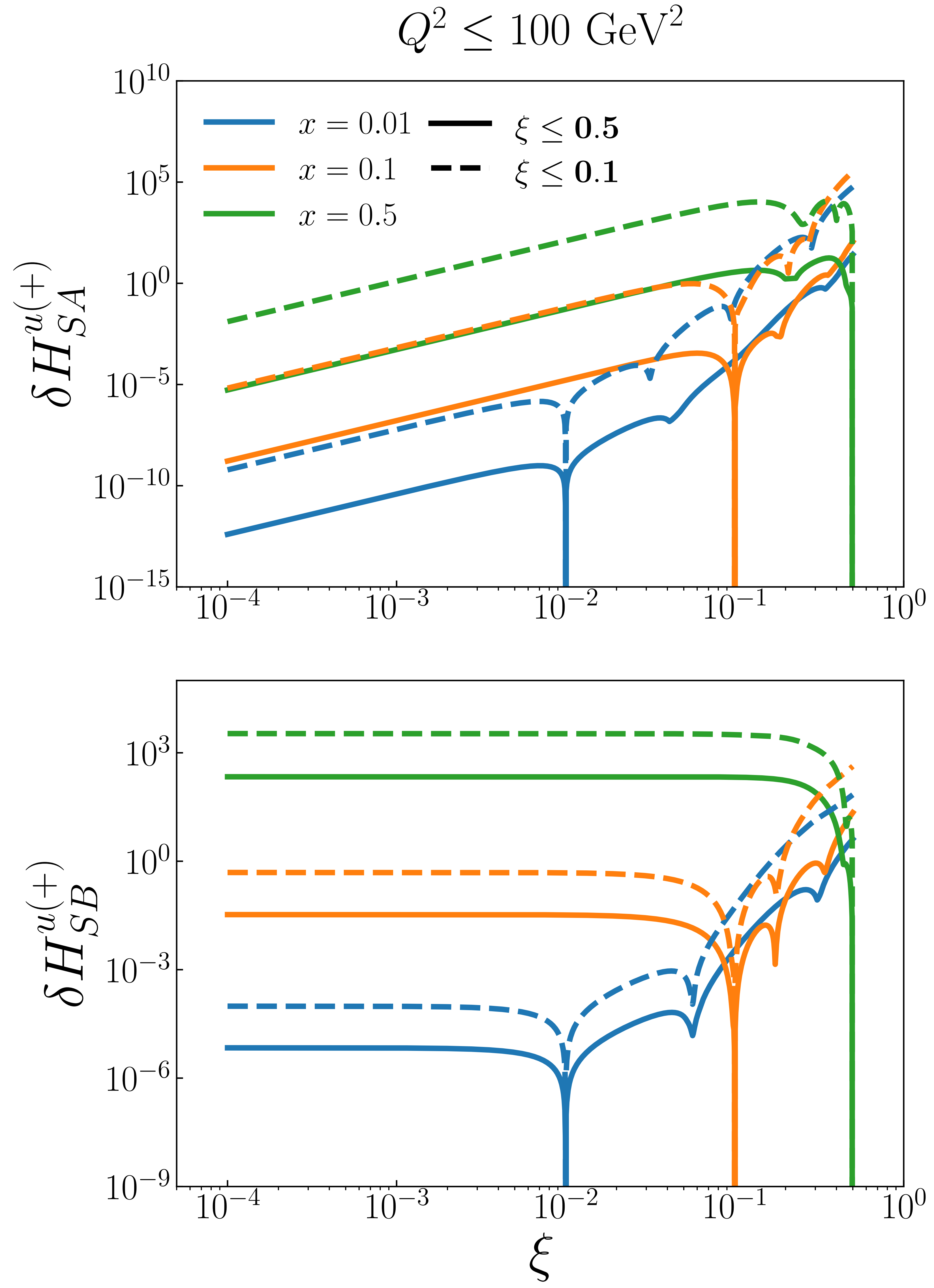
  - Randomly select the scaling factors until we get 10000 replicas that all give CFFs that are within 1% of the simulated data from the model.
  - Plot the region δH_S: Outer boundary of all 10000 replicas

Exploring SGPDs and Evolution



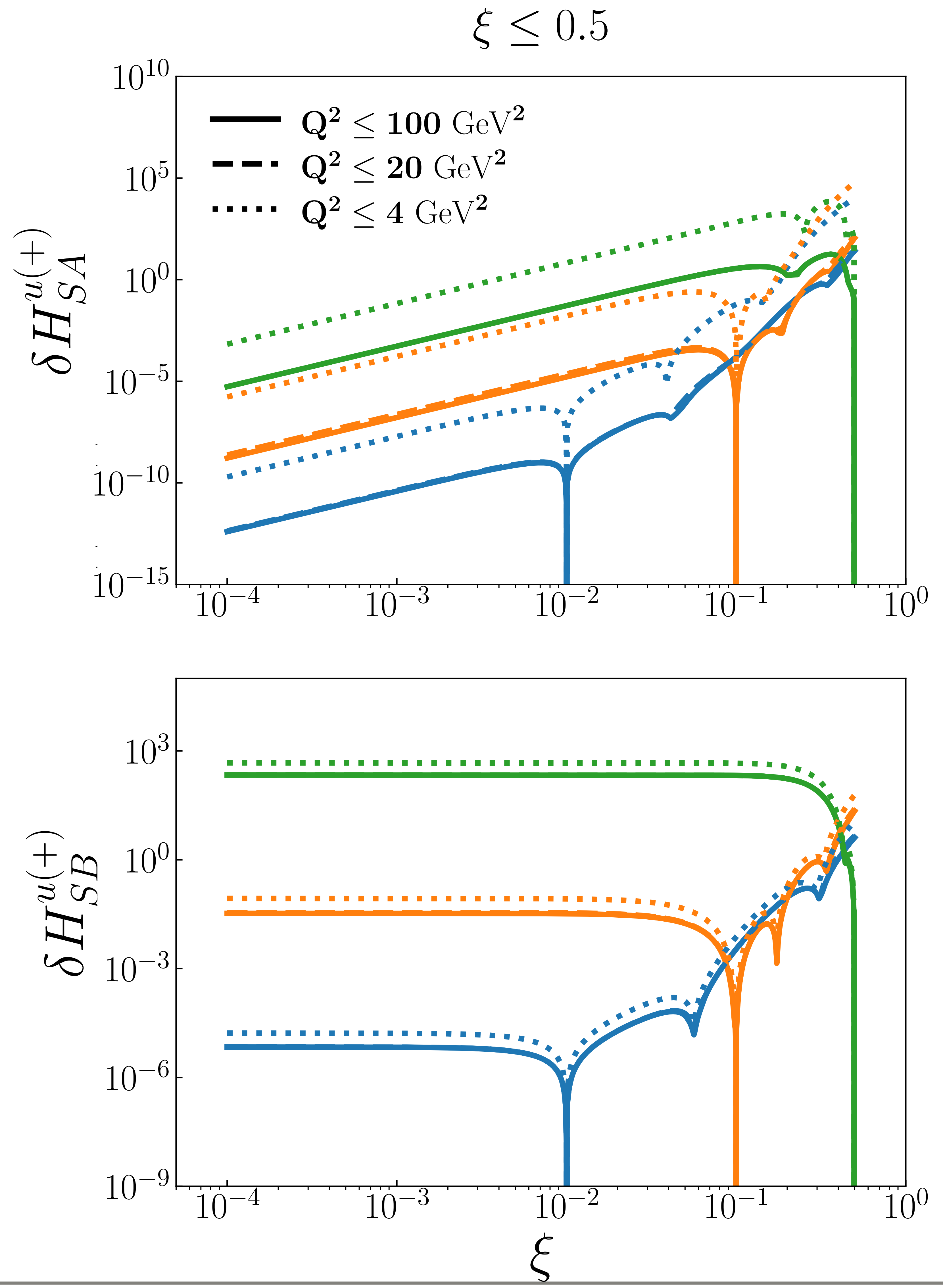
Exploring SGPDs and Evolution

- Inclusion of higher ξ data leads to better constraint of SGPDs at smaller ξ
  - True over the full range of x when $H_S^{u(+)}(x,0;\mu_0)) = 0$
  - Only true for low x when $H_S^{u(+)}(x,0;\mu_0)) \neq 0$



Exploring SGPDs and Evolution

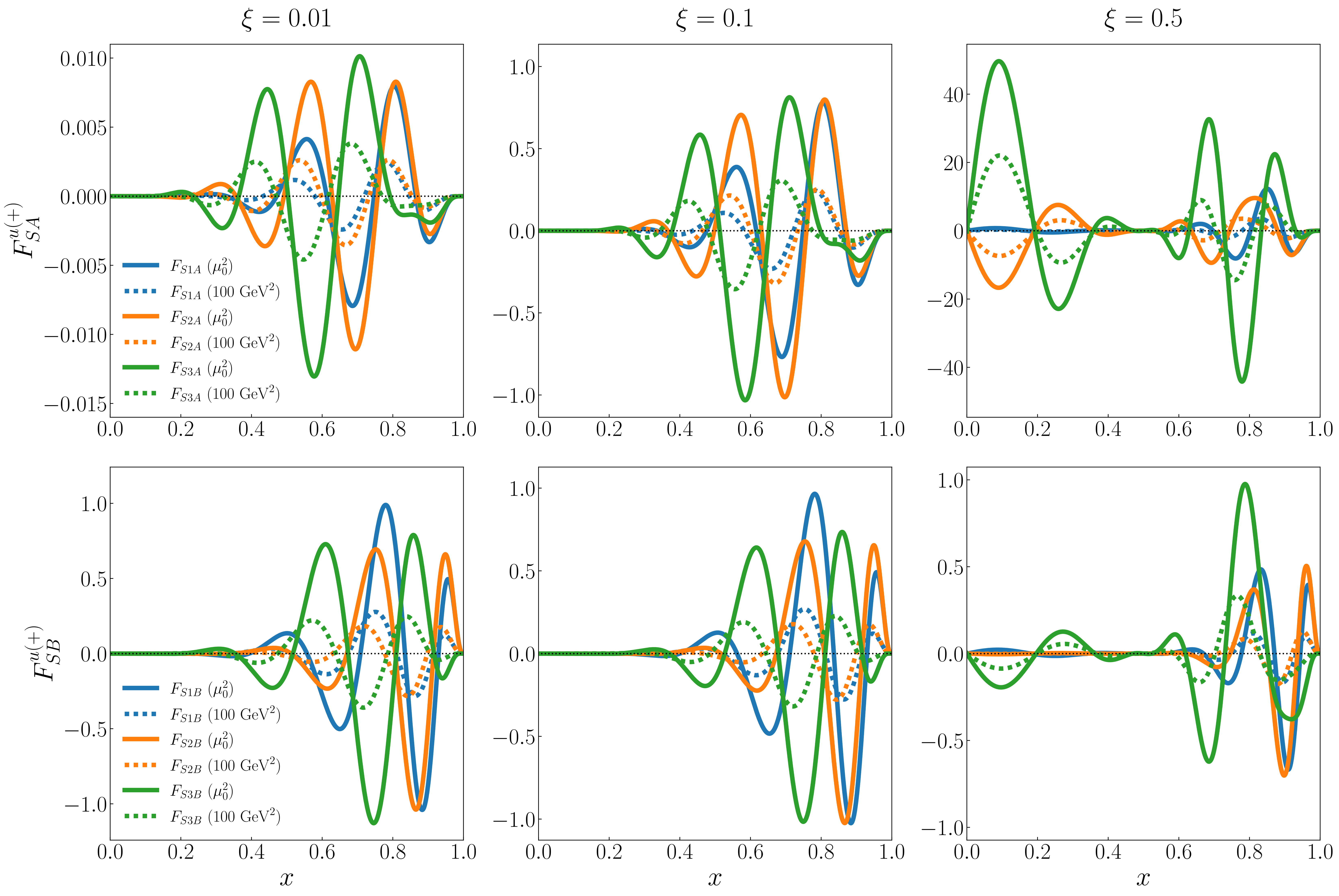
- Some range of Q^2 is necessary for evolution to constrain the SGPDs but a large range is not as necessary as having large ξ data.



Exploring SGPDs and Evolution

- The trend of larger ξ data leading to better constrained SGPDs at smaller ξ is a direct result of the ξ dependence of the SGPDs
  - Independent of the model used as a proxy for the “true” GPD
  - Independent of the chosen uncertainty

Example Shadow GPDs



Conclusions

- Conclusions:
  - For the SGPDs that have been explored here:
    - Data spanning a range of Q^2 at larger ξ leads to the SGPDs being better constrained at lower ξ at least in the range of low x
    - These findings are independent of the model used as the proxy for the “true” GPD.
  - The SGPDs explored are only a small sampling of all possible SGPDs:
    - At this point we cannot generalize these results to all SGPDs
    - Data spanning a range of Q^2 at larger ξ is a necessary but possibly not sufficient condition for extracting GPDs from DVCS data.