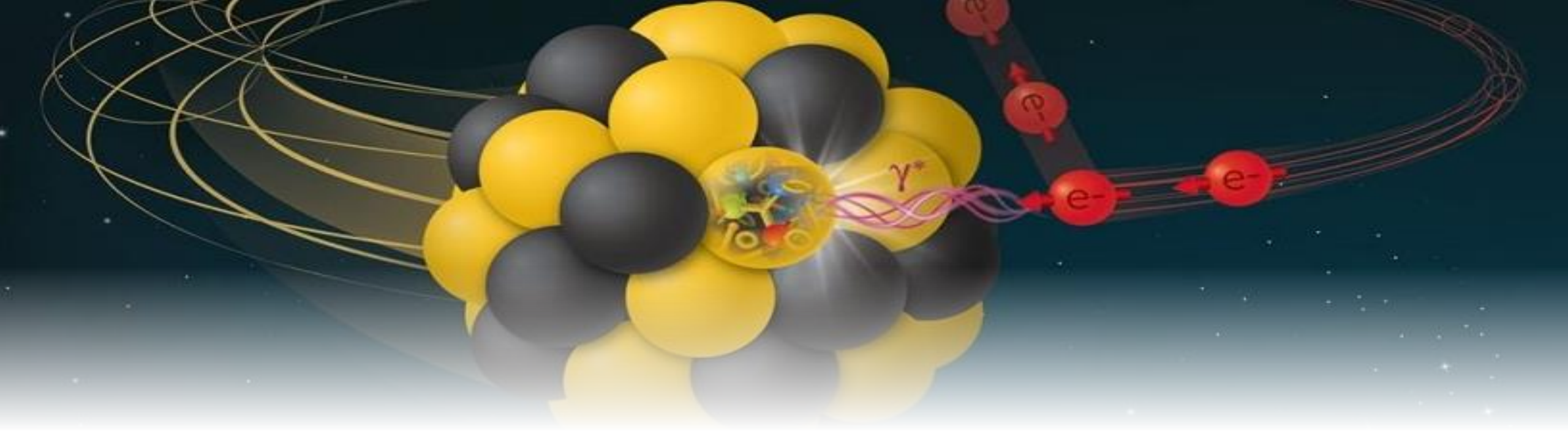


GPD through Universal Moment Parameterization (GUMP)

— Global DVCS analysis with quark GPDs

Yuxun Guo

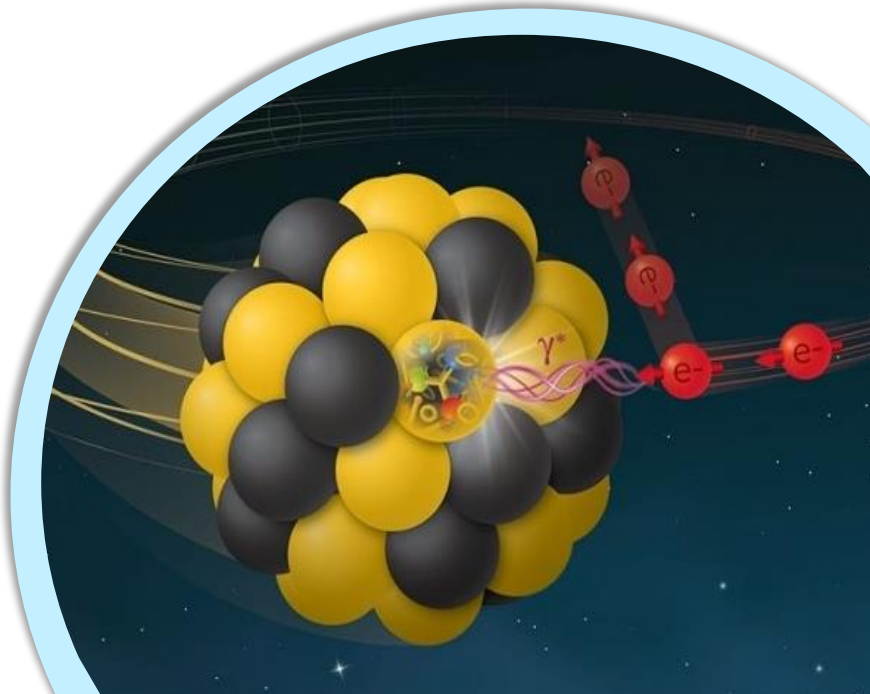
University of Maryland, College Park

10th workshop of APS GHP

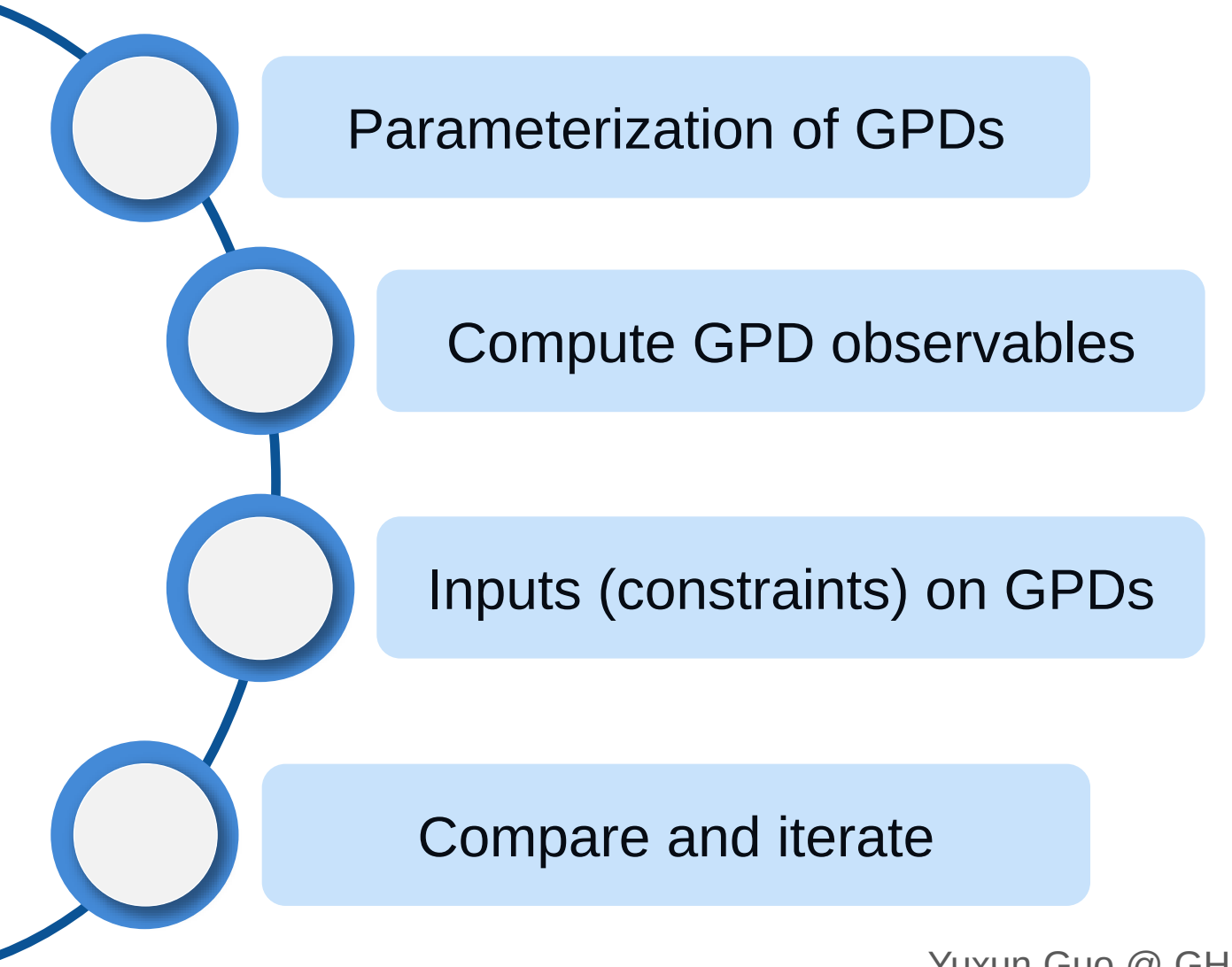
Apr. 12th, 2023

Outline

- Intro: GPD global analysis and GUMP
- Experimental and lattice inputs
- Extracted quantities: CFFs and GPDs
- Summary and outlook



General strategy of GPD global analysis



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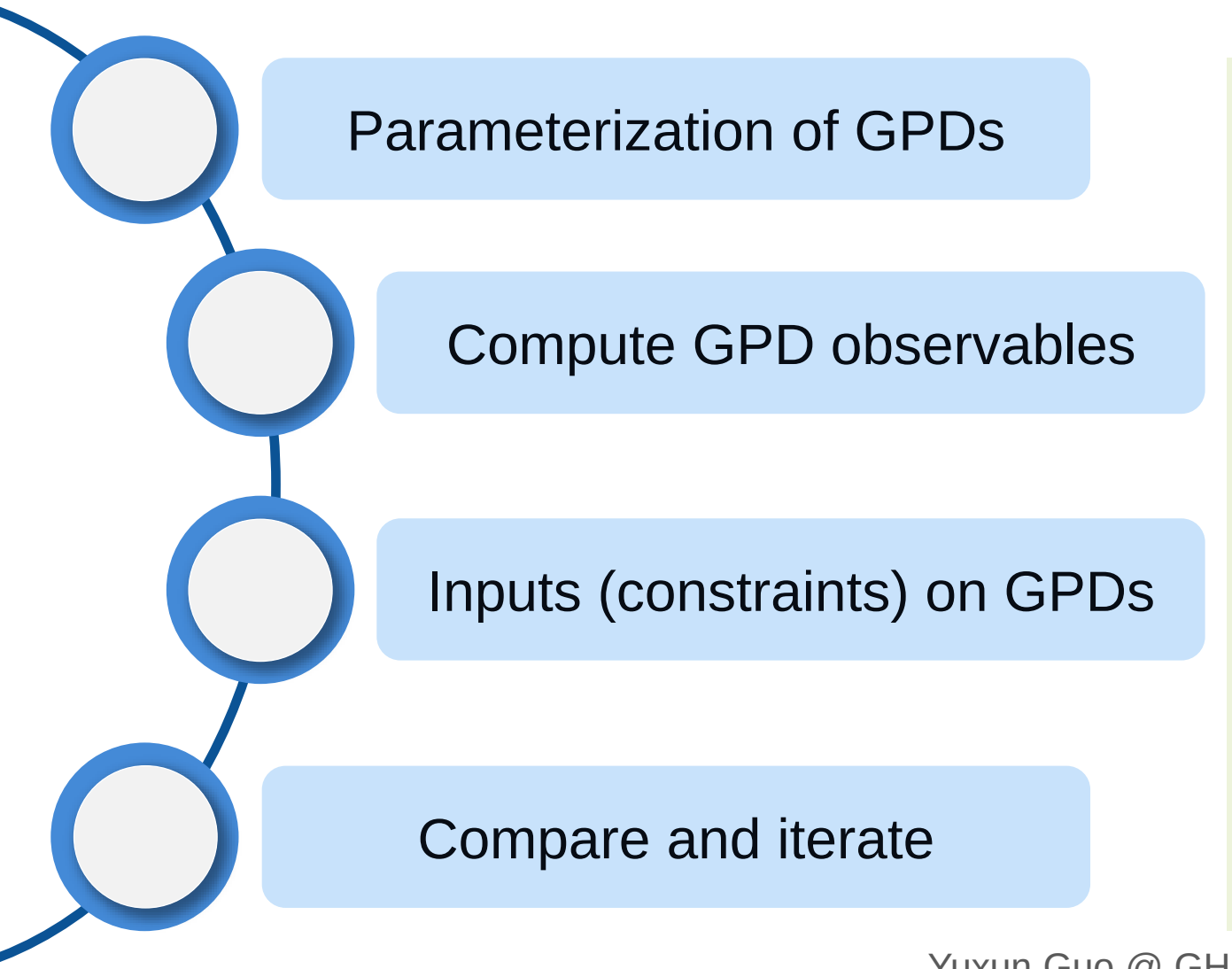

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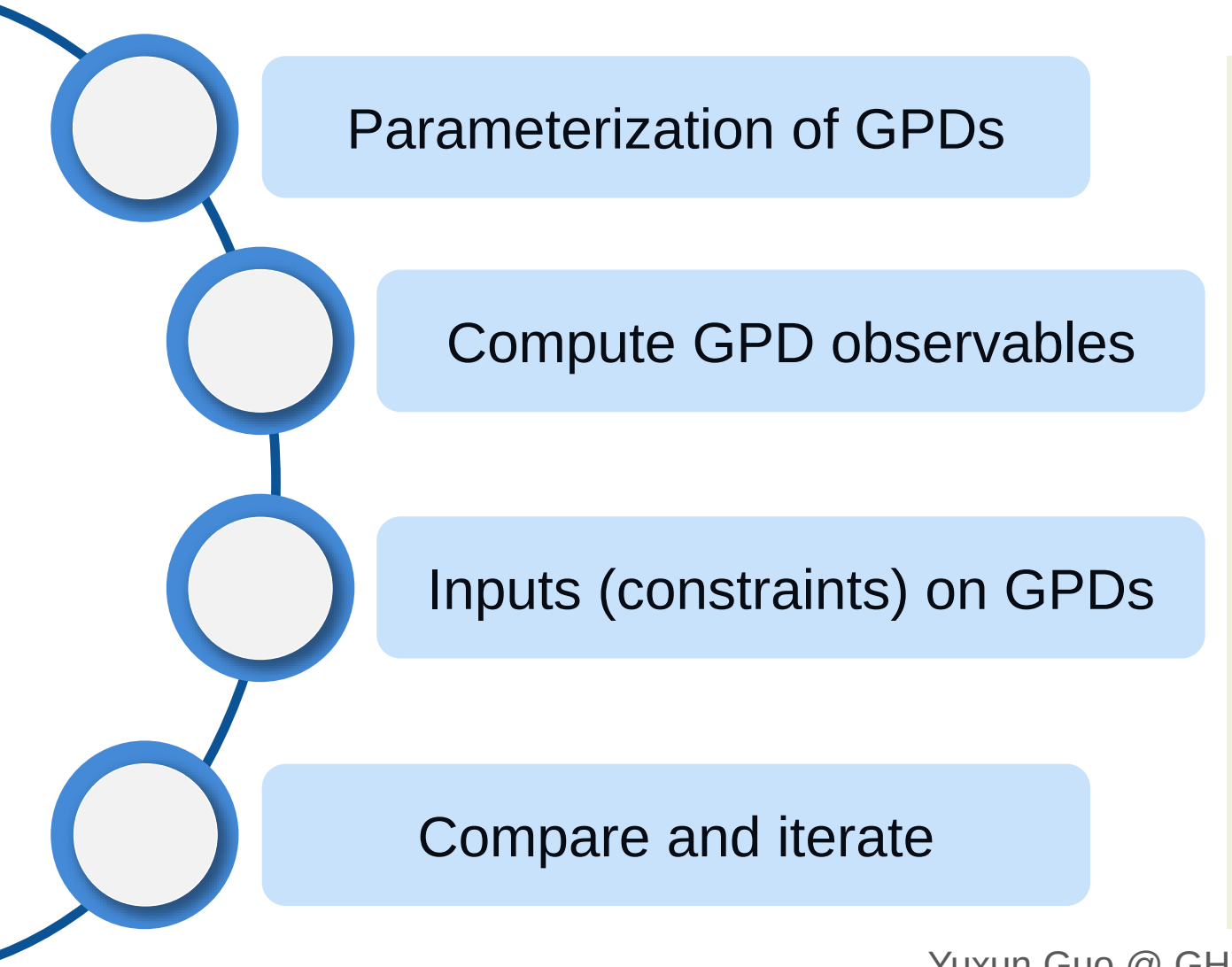

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- GPDs are 3D whereas PDFs are 1D

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- Parameterization of GPDs
- Compute GPD observables
- Inputs (constraints) on GPDs
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$$\langle \bar{\psi} \not{n} \psi \rangle \sim \bar{u}(P', S') \left[\not{n} H(x, \xi, t) + \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} E(x, \xi, t) \right] u(P, S)$$

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- Mixed in the amplitude

$$F_{UU} \propto 4 \Bigg[(1 - \xi^2) \left(\mathcal{H}^* \mathcal{H} + \widetilde{\mathcal{H}}^* \widetilde{\mathcal{H}} \right) - \frac{t}{4M^2} \left(\mathcal{E}^* \mathcal{E} + \xi^2 \widetilde{\mathcal{E}}^* \widetilde{\mathcal{E}} \right) - \xi^2 \left(\mathcal{E}^* \mathcal{E} + (\mathcal{E}^* \mathcal{H} + \mathcal{H}^* \mathcal{E}) + (\widetilde{\mathcal{E}}^* \widetilde{\mathcal{H}} + \widetilde{\mathcal{H}}^* \widetilde{\mathcal{E}}) \right) \Bigg],$$

Parameterization of GPD

The conformal moment parameterization of GPD is helpful

$$F(x,\xi,t) = \sum_{j=0}^{\infty} (-1)^j p_j(x,\xi) \mathcal{F}_j(\xi,t)$$

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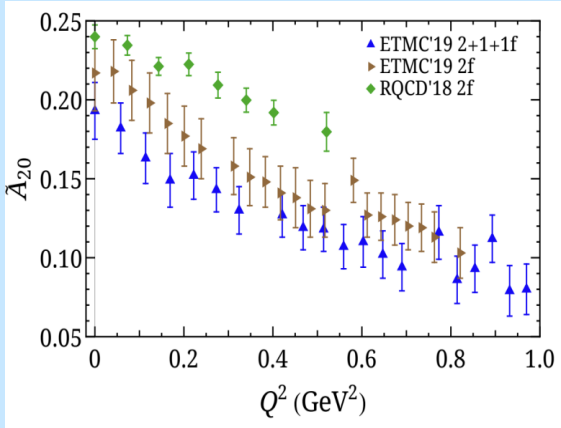

M. Constantinou et. al. Prog. Part. Nucl. Phys. 121 103908 (2021)

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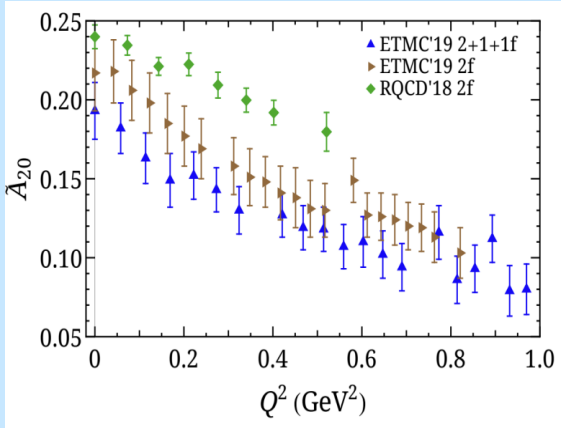

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- Lattice form factors and GPDs from a single group.

C. Alexandrou et. al. Phys. Rev. Lett. 125 26, 262001 (2020)
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Caveat: ansatz and empirical constraints

Even so, the GPDs are still far from being fully determined!

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Ansatz for GPDs: $\mathcal{F}_{j,k}(t) = N_k B(j+1-\alpha_{i,k}, 1+\beta_k) \frac{j+1-k-\alpha_k}{j+1-k-\alpha_k(t)} \beta(t)$

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Empirical constraints:

GPDs species and flavors	Fully parameterized	GPDs linked to	Proportional constants
H_{u_V} and $\widetilde{H}_{u_V}$	✔	-	-
E_{u_V} and $\widetilde{E}_{u_V}$	✔	-	-
H_{d_V} and $\widetilde{H}_{d_V}$	✔	-	-
E_{d_V} and $\widetilde{E}_{d_V}$	✘	E_{u_V} and $\widetilde{E}_{u_V}$	$R_{d_V}^{E/\widetilde{E}}$
$H_{\bar{u}}$ and $\widetilde{H}_{\bar{u}}$	✔	-	-
$E_{\bar{u}}$ and $\widetilde{E}_{\bar{u}}$	✘	$H_{\bar{u}}$ and $\widetilde{H}_{\bar{u}}$	$R_{\text{sea}}^{E/\widetilde{E}}$
$H_{\bar{d}}$ and $\widetilde{H}_{\bar{d}}$	✔	-	-
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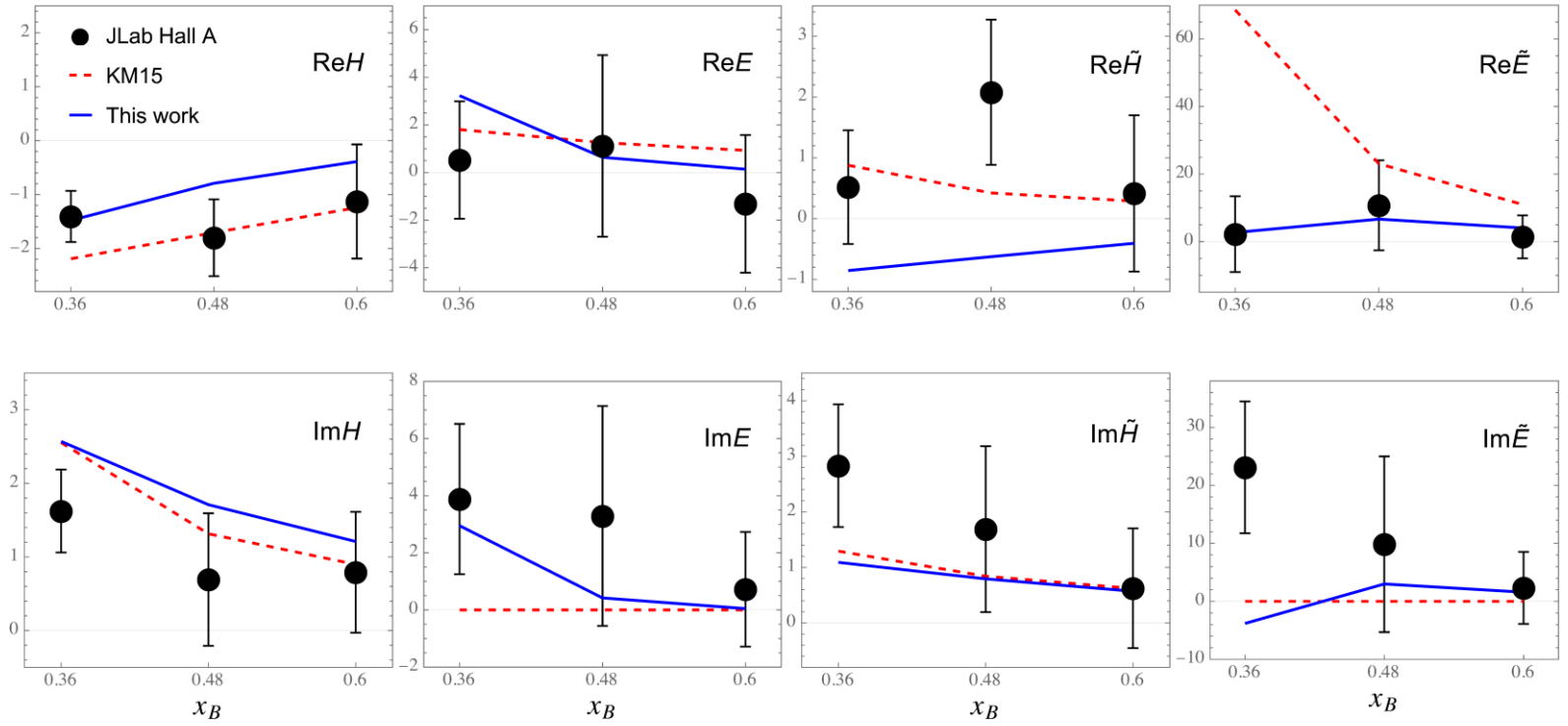
Table 1: A summary of how each GPDs with different species and flavors are parameterized respectively. Fully parameterized GPDs are expressed in terms of eq. (2.6), whereas the other GPDs are linked to the fully parameterized GPDs with proportional constants.

Extracted CFFs and GPDs

The extracted CFFs are generally close to the local extracted values



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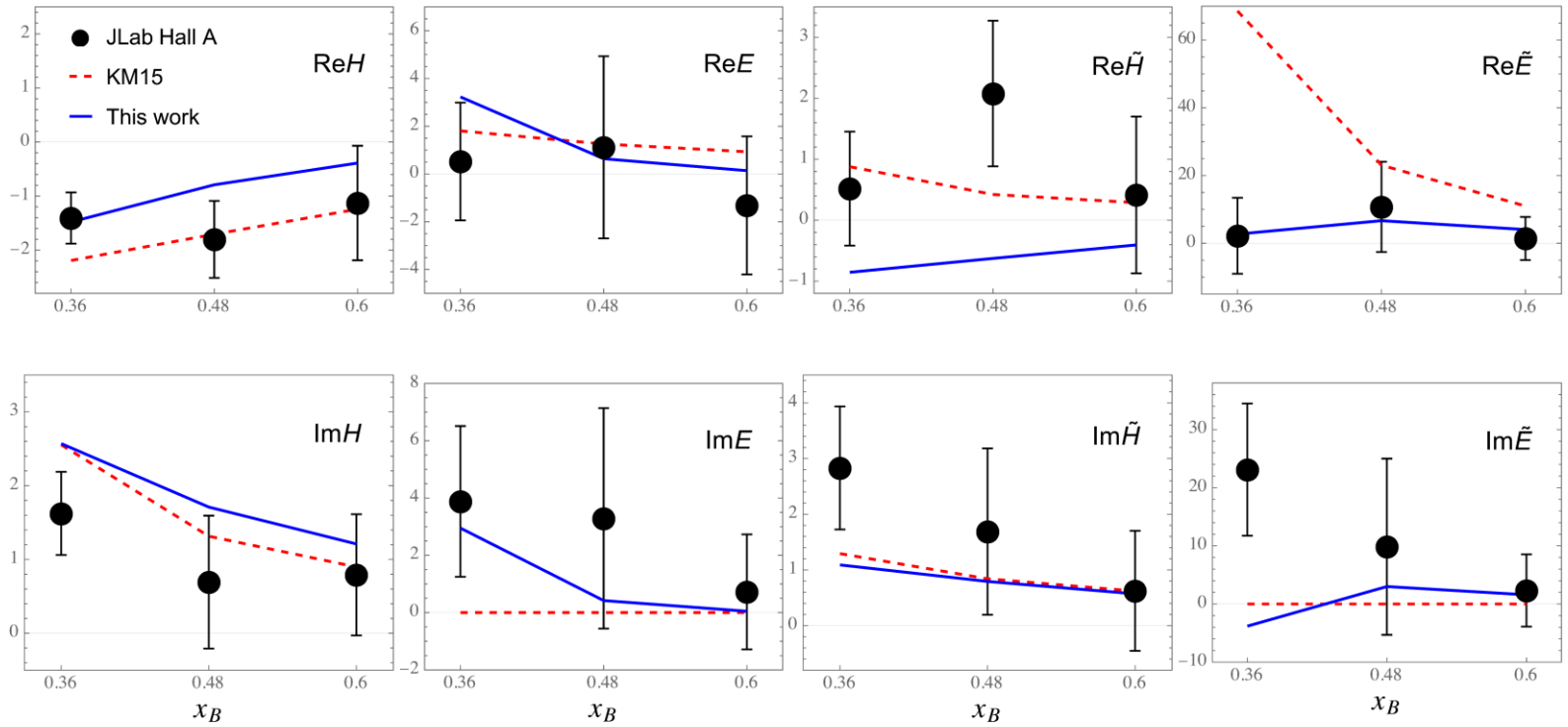
Y. Guo et. al. [2302.07279]



There are degeneracy in CFFs themselves

– quadratic equations have multiple solutions K. Shiells et. al. JHEP 08 048(2022)

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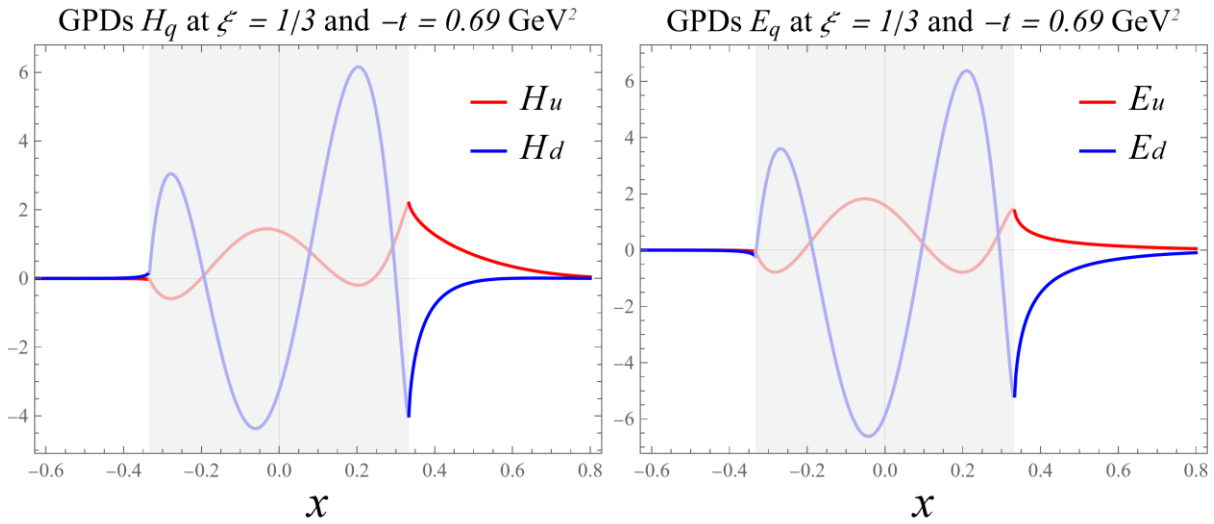

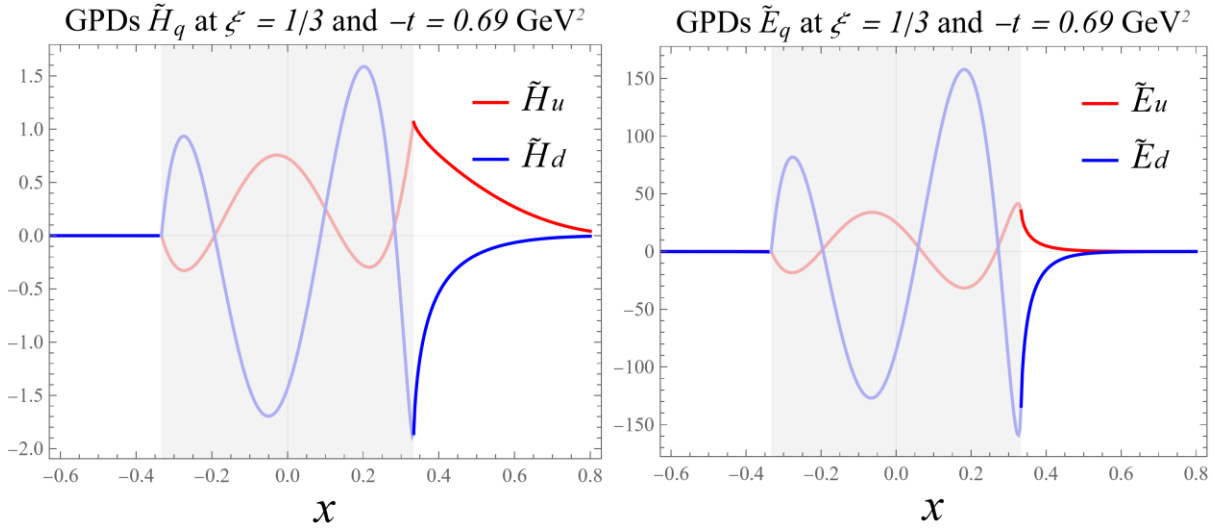

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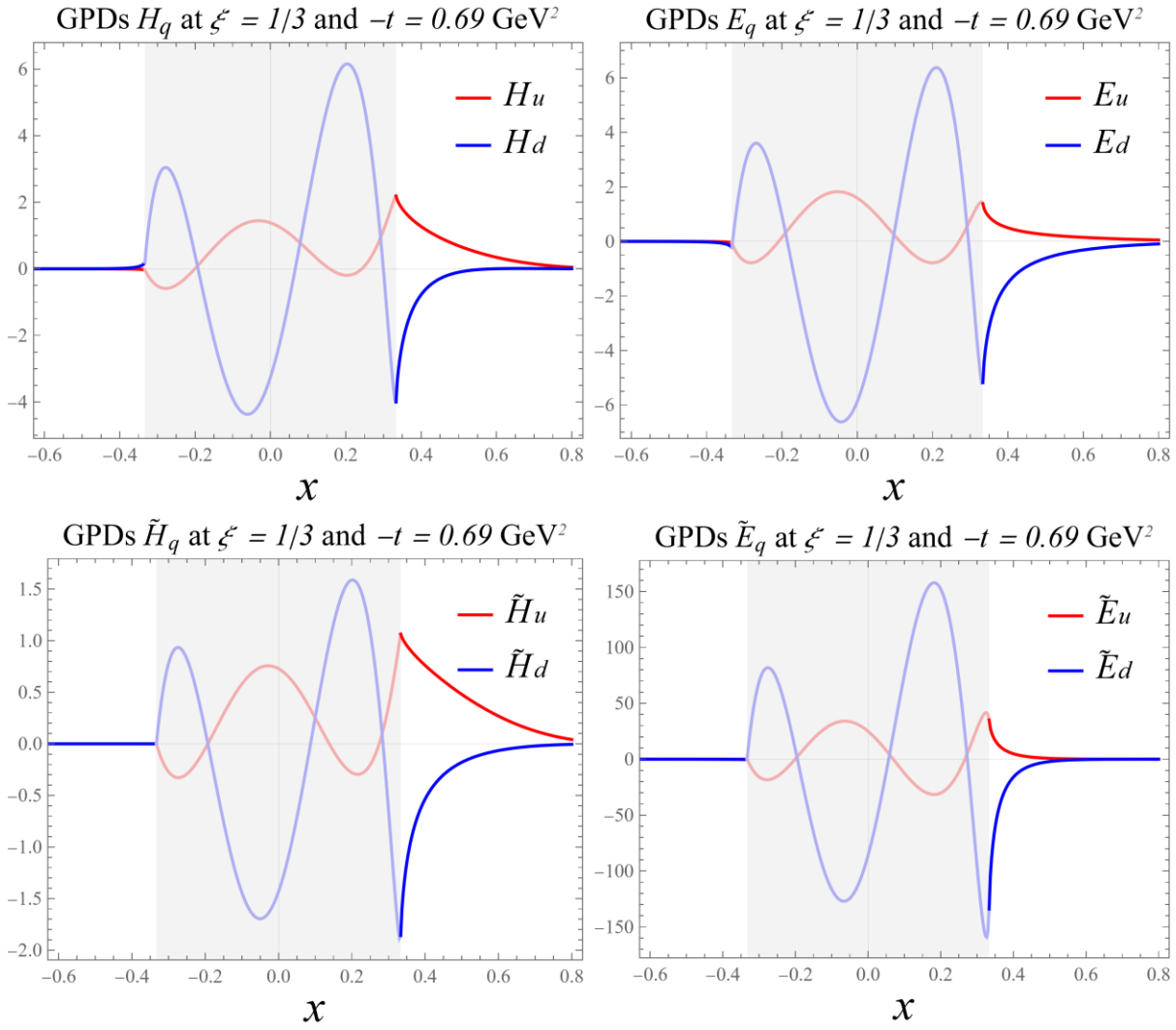

The wiggling behavior is an artifact of the partial wave construction

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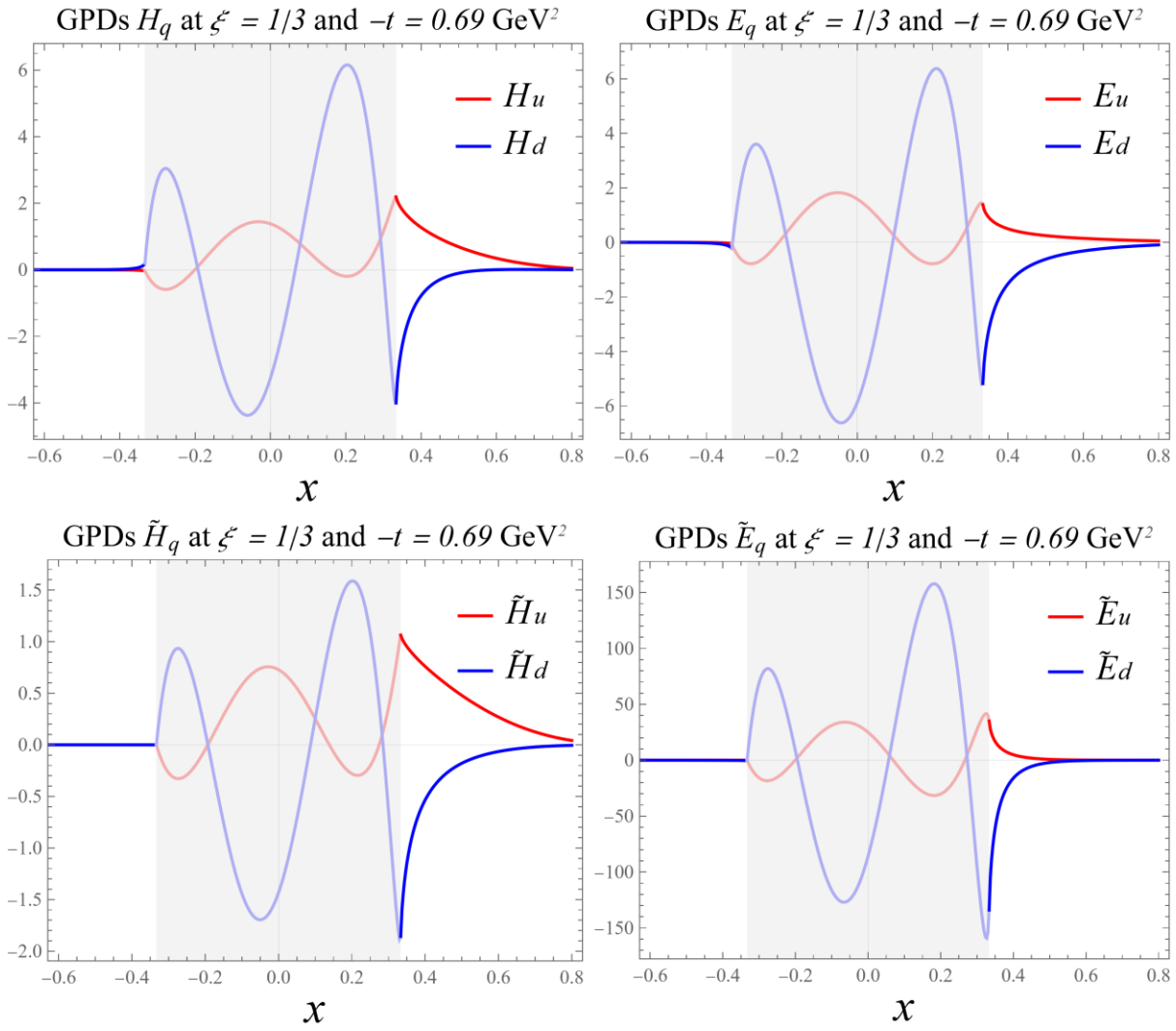

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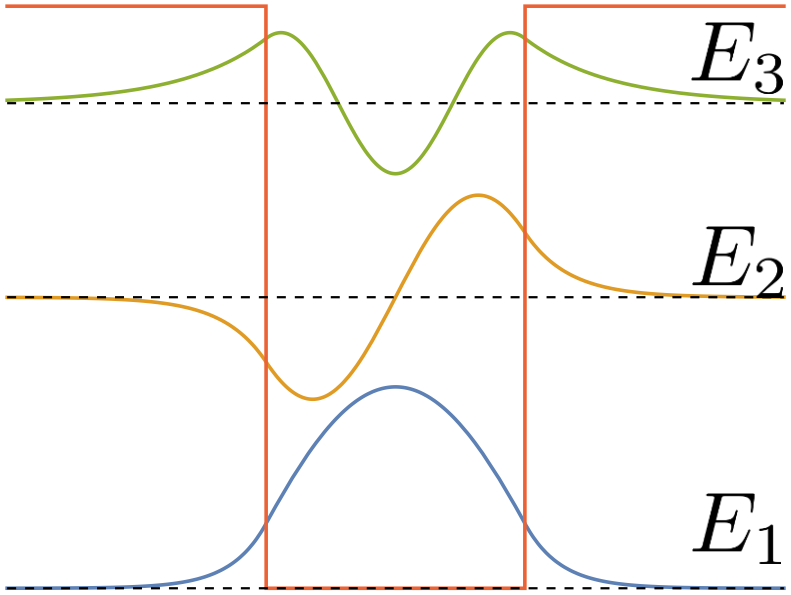

GPDs in the DA-like regions

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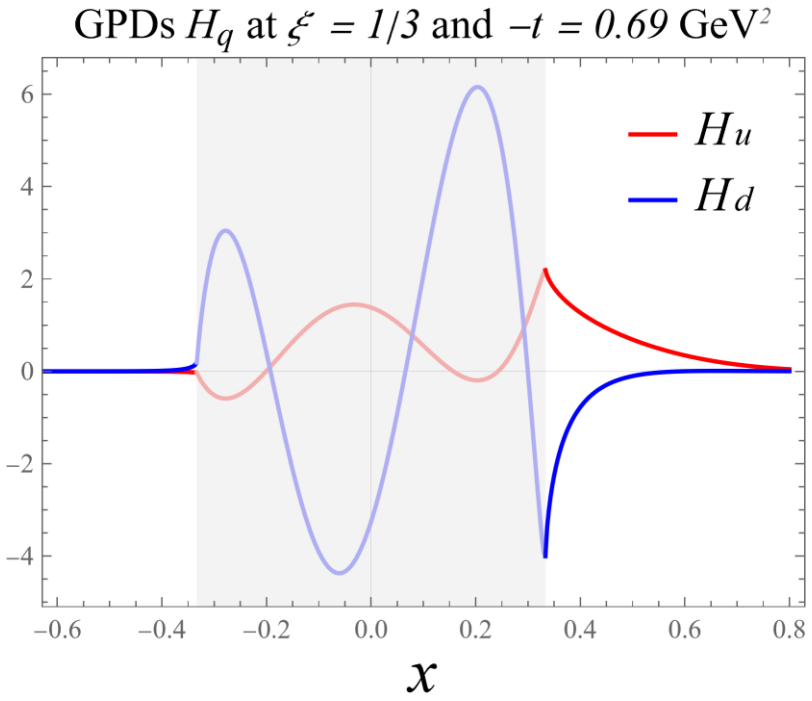

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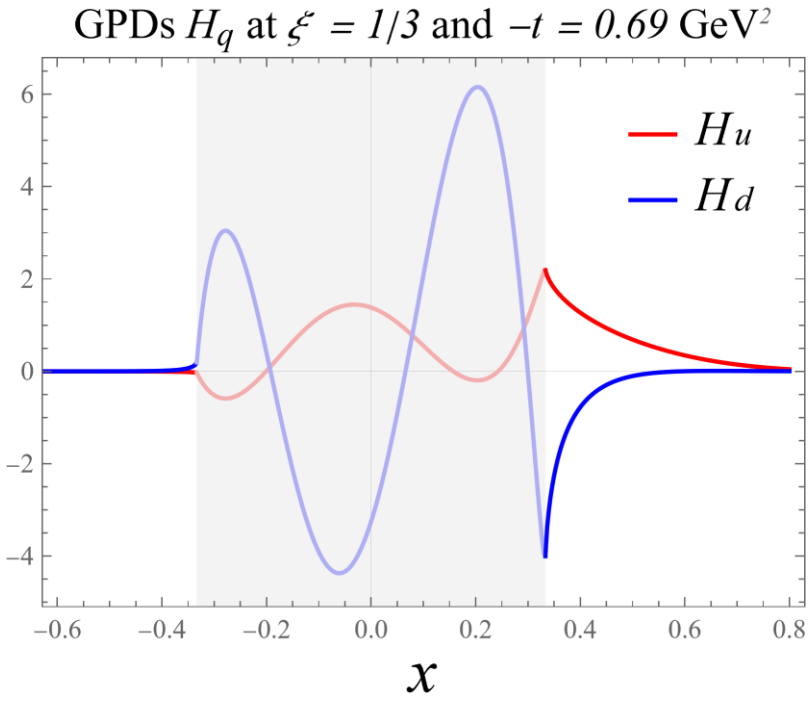

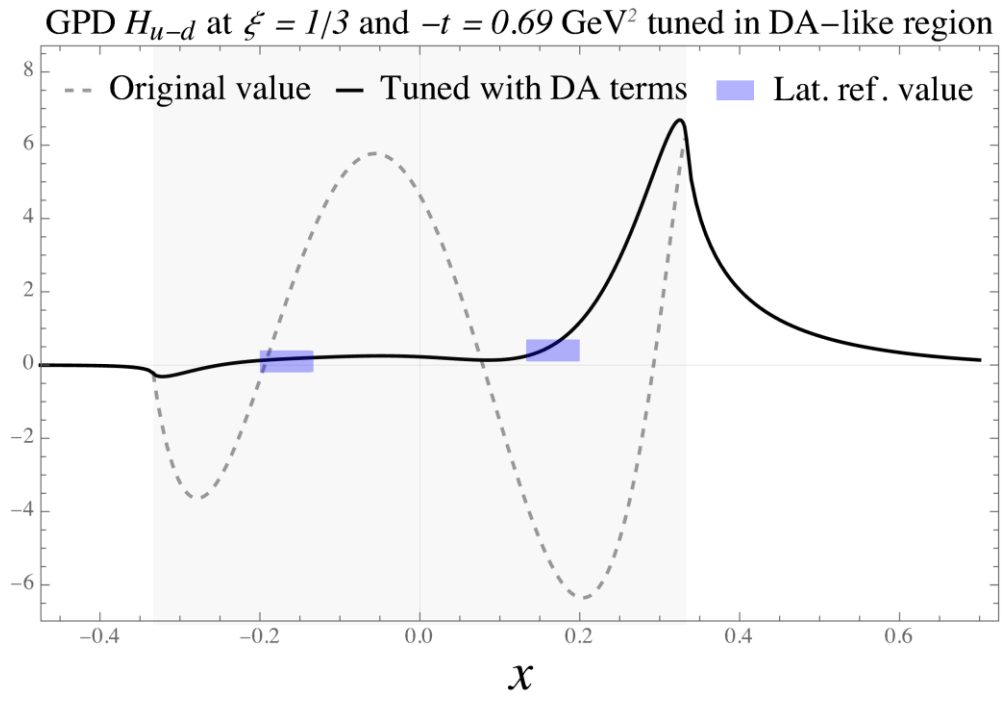

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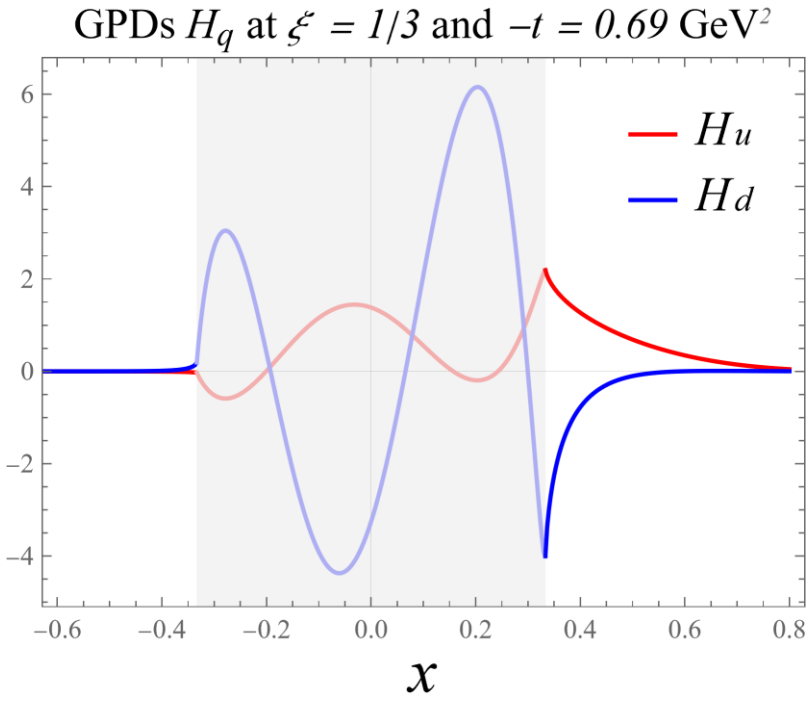
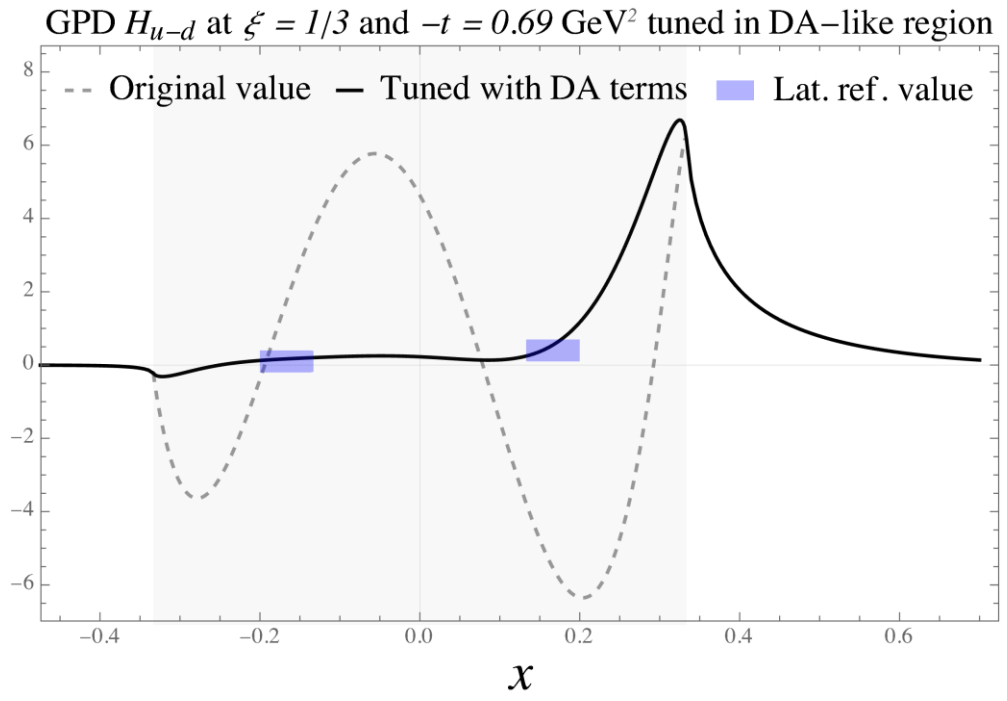
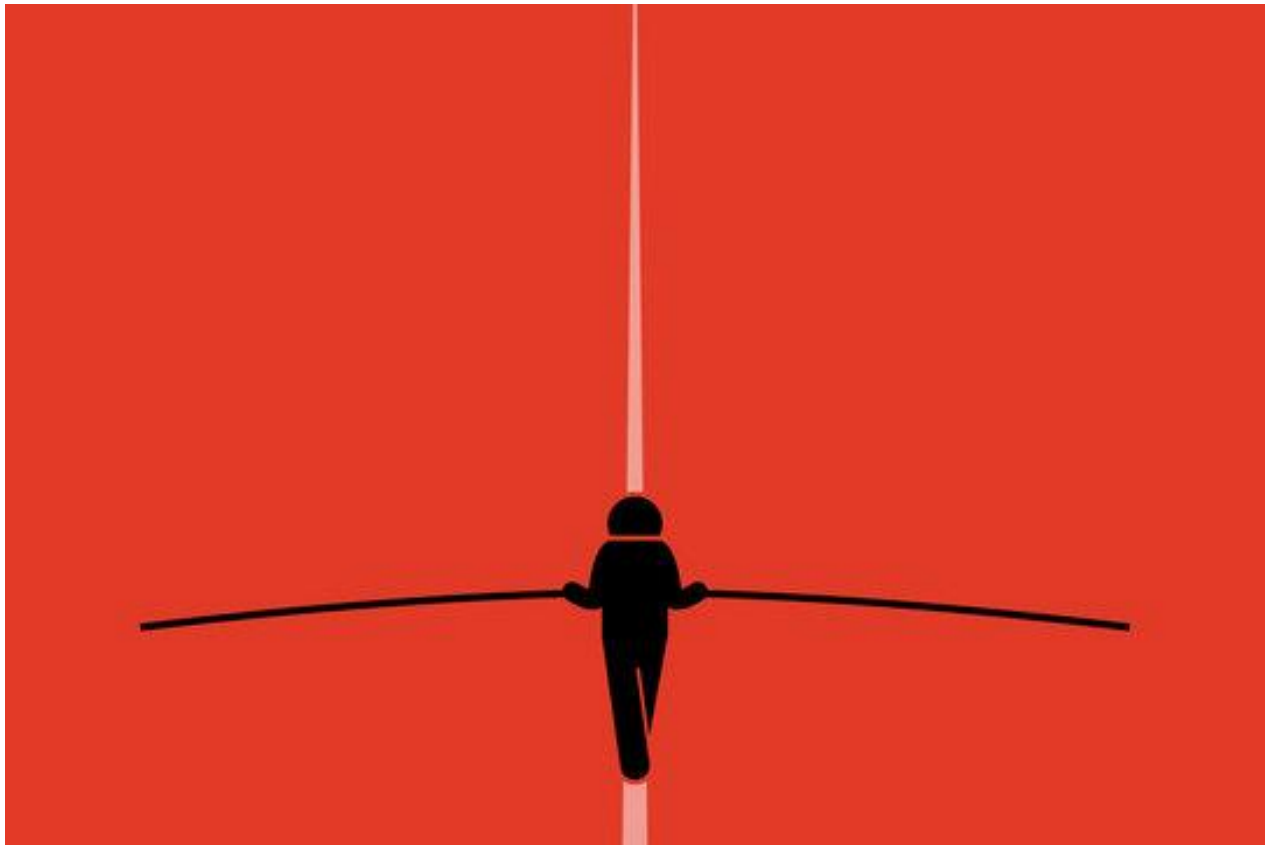




Extra inputs crucial to determine the shape of GPDs in the middle regions.

Challenge of GPD extraction

ERBL $x = \xi$ DGLAP



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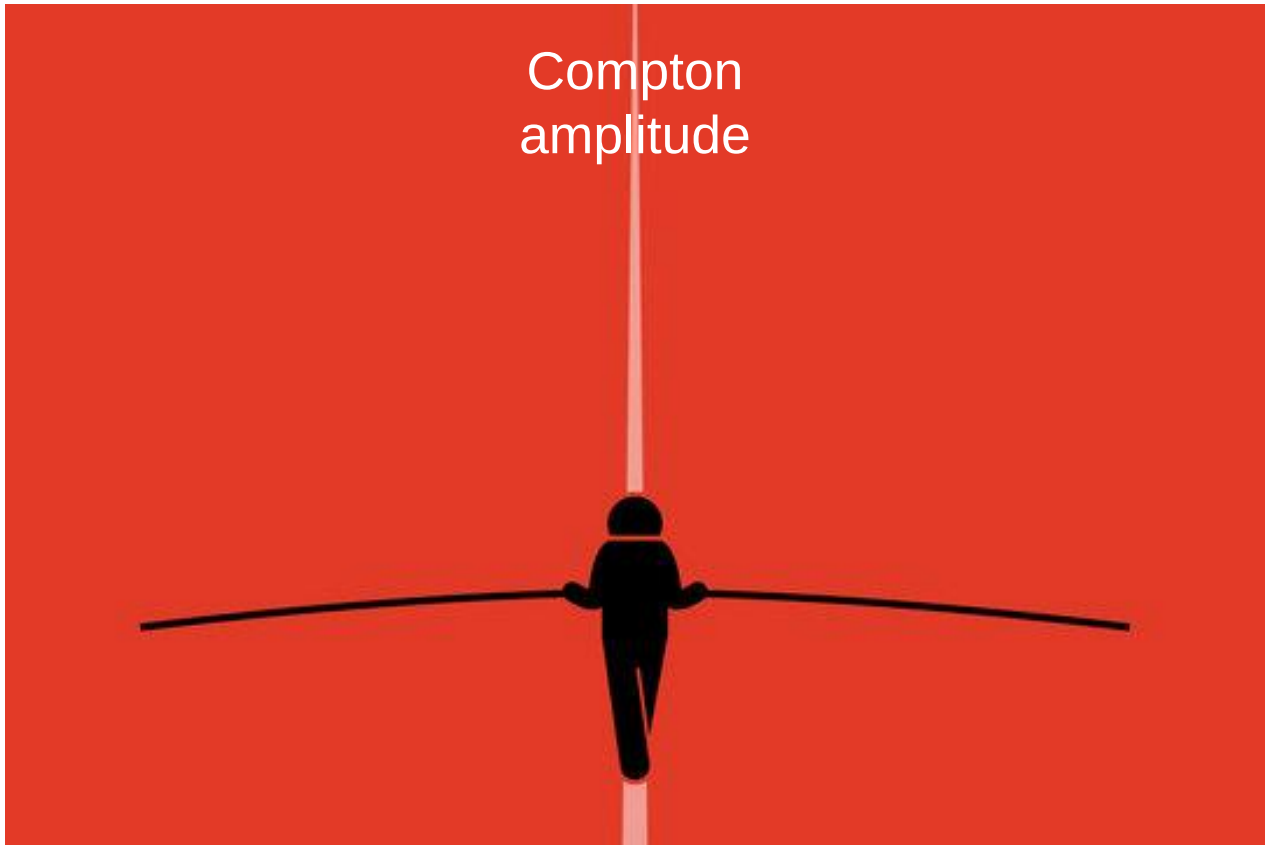

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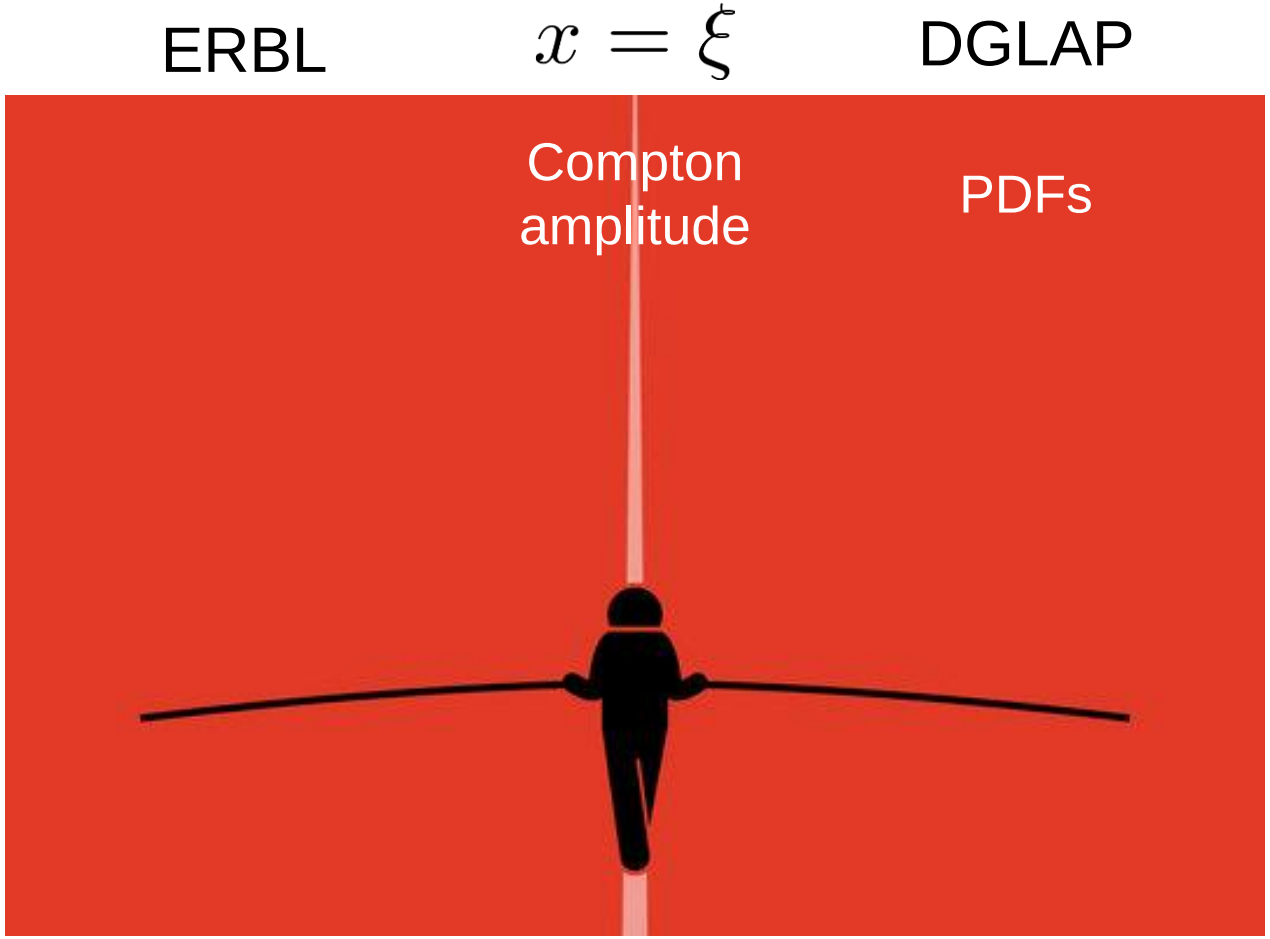

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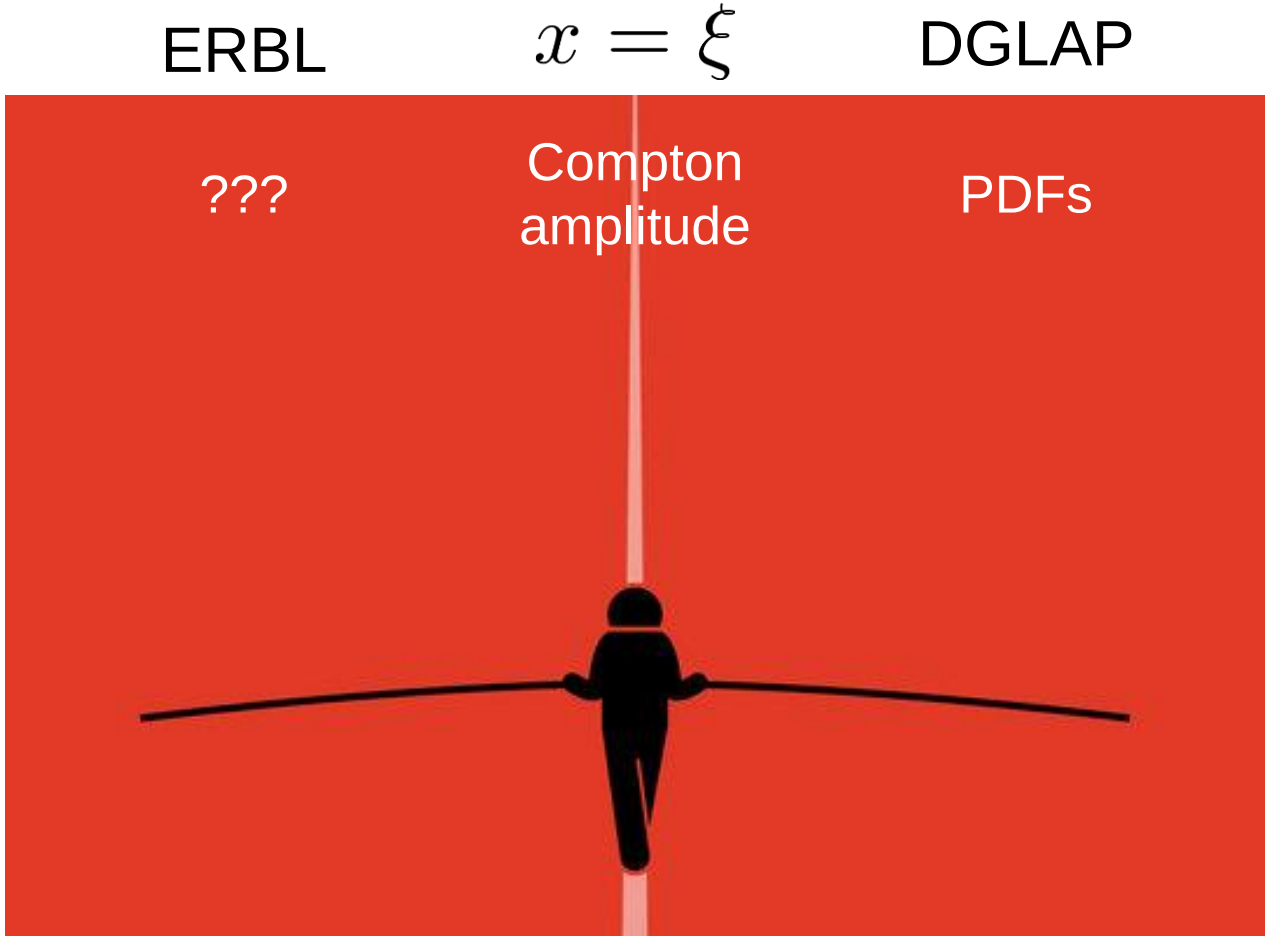

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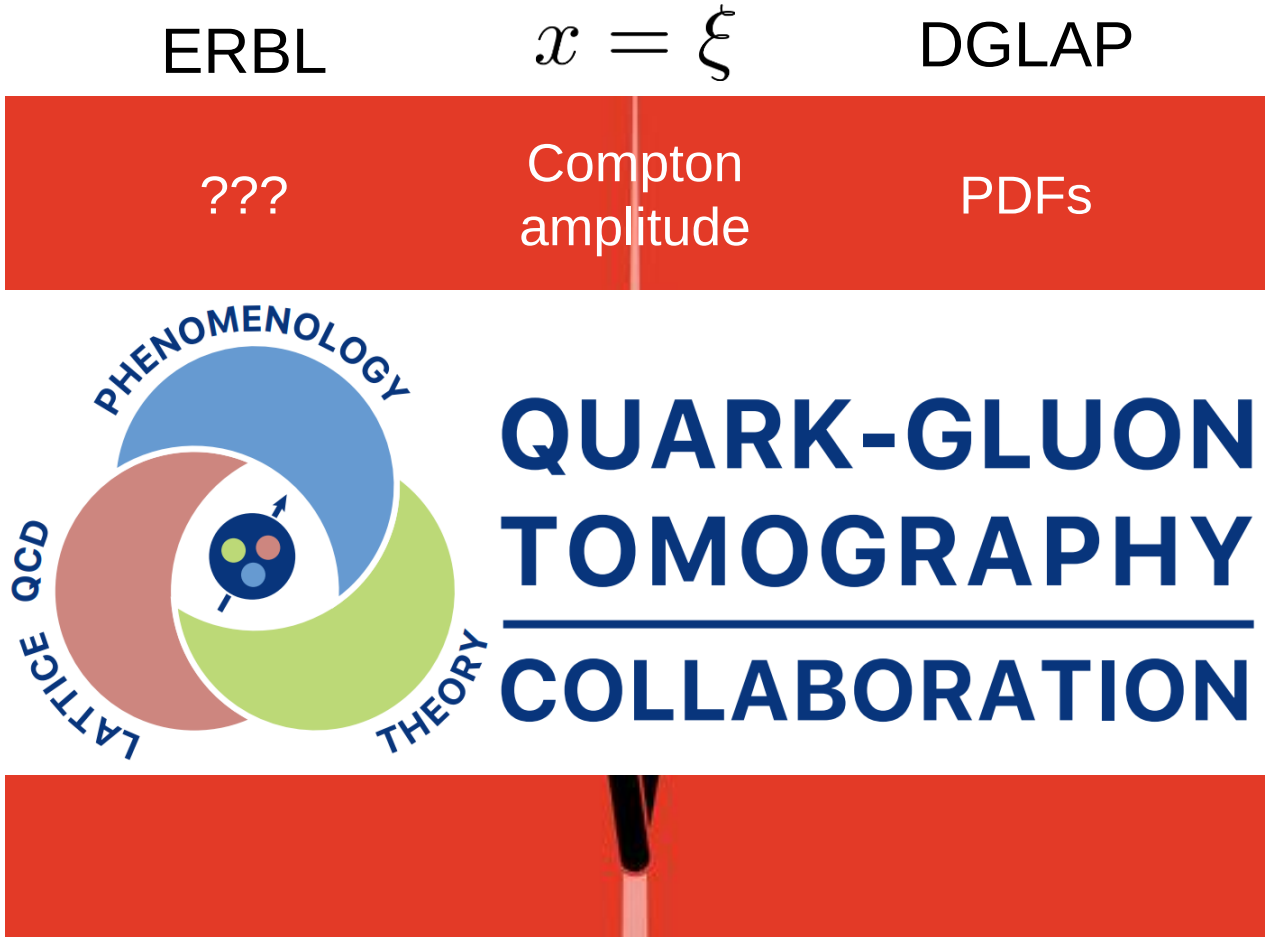

Summary and outlook

Summary

- GPDs reveals the nucleon 3D structures including mass and spin.
- Inputs from both experiment and lattice are necessary for determination of GPDs
- Global analysis program by parameterization moments of GPDs.

Outlook

- Global fitting with more data inputs (Hopefully all existing data)
- Extend to other processes that can probe GPDs
- Higher order corrections and more quark flavor

Thank you!