TMDs & factorization at sub-leading power “twist-3”

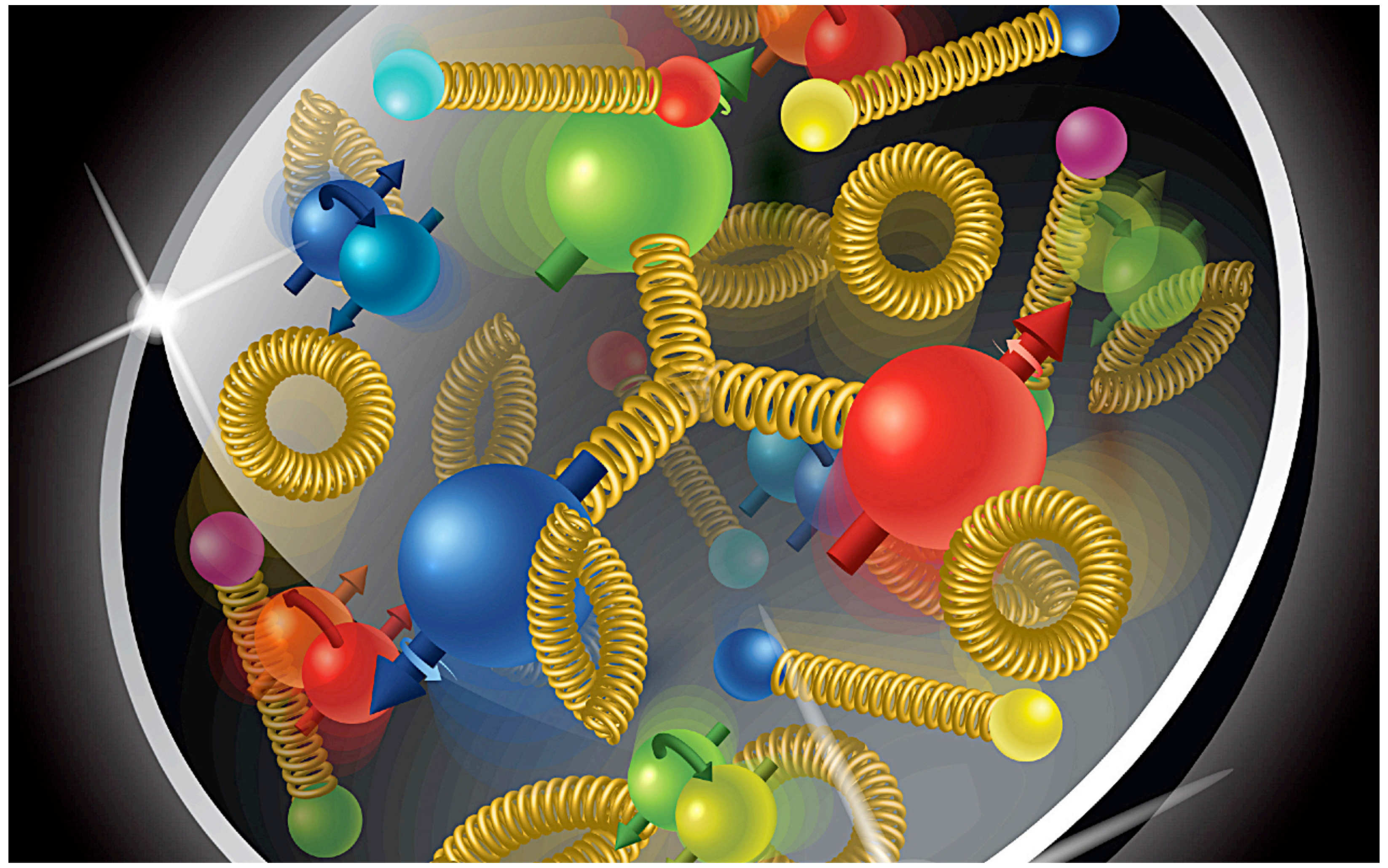
Leonard Gamberg

w/ Zhongbo Kang, Ding-Yu Shao, John Terry, Fany Zhao

The 10th Biennial Workshop of the APS Topical Group on Hadronic Physics (GHP2023)

APS Topical Group on Hadronic Physics GHP

Wed Apr 12th - Fri 14th



PennState Berks

TMD Collaboration

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Motivation discussion/preamble

We explore sub-leading power Λ_{QCD}/Q TMDs in the context of factorization theorem

- NLP factorization based on *"TMD formalism"*
  —extension of tree level Amsterdam formalism and beyond leading order
  CSS, Abyat Rogers, Boer Pijlman Mulders-framework
- Revisit matching: Consider consistency of matching to collinear factorization
  see Bacchetta, Boer, Diehl, Mulders JHEP 2008 also in context of EOMs
- Focus on Cahn effect & matching related to early picture of importance intrinsic $\mathbf{k}_T$
- *"Intrinsic" subleading TMDs related thru EOM in terms "kinematic" & "dyanmical"*
- See recent work:
  MIT group, Gao, Ebert, Stewart JHEP 2022
  Vladimirov & Rodini JHEP 2022
  Gamberg, Kang, Shao, Terry, Zhao arXiv: e-Print:221.13209
  See also Ch. 10 TMD handbook, e-Print:2304.03302 [hep-ph]

However, its an old subject in QCD ...

Importance of *SLP* TMDs

- Importance of SLP TMD *observables* underscored by observation that while they are suppressed by *M/Q* wrt LP observables, they are not small, especially in the kinematics of fixed-target experiments

- Their understanding is required for a complete description of SIDIS, DY & e+e- …

- They may are relevant for a proper extraction of the leading-power effects from data.

- NLP/SLP TMDs can be as sizable as leading-power TMDs in some situations, particularly when *Q* is not that large.

- They are of interest in their own right as they, offer a mechanism to probe physics of quark-gluon-quark correlations, which provide novel information about the partonic structure of hadrons, and are largely unexplored.
    - Such correlations may be considered quantum interference effects, and they could be related to average transverse forces acting on partons inside (polarized) hadrons as well as other phenomena.

Also, experimental information from SIDIS on effects related to subleading TMDs is available already.
In the future, the EIC with its *large* kinematical coverage will be ideal for making further groundbreaking progress in this area.

Important Literature (incomplete)

F. Rivindal PLB 1973
Cahn PLB 1978 (response to Georgi Politzer PRL 1978)
R. Tangerman, P. Mulders hep-ph/9408305 [hep-ph] (1994)
P.Mulders, R. Tangerman, NPB 461(1996)
L. Gamberg, D Hwang, A Metz, M. Schlegel, Phys.Lett.B 639 (2006), rapidity div. @tw3-factorization endangered
D. Boer, P. Mulders, Phys.Rev.D 57 (1998)
A.Bacchetta, D. Boer, M. Diehl, P. Mulders JHEP (2008) factorization at NLP consistency checks on matching
A.P. Chen, J.P. Ma, Phys. Lett. B 768 (2017) 380, arXiv:1610 .08634.
I. Feige, D.W. Kolodrubetz, I. Moult, I.W. Stewart, J. High Energy Phys. 11 (2017) 142
I. Balitsky, A. Tarasov, J. High Energy Phys. 07 (2017) 095
I. Balitsky, A. Tarasov, J. High Energy Phys. 05 (2018) 150
M.A. Ebert, I. Moult, I.W. Stewart, F.J. Tackmann, G. Vita, H.X. Zhu, J. High Energy Phys. 12 (2018) 084
M.A. Ebert, I. Moult, I.W. Stewart, F.J. Tackmann, G. Vita, H.X. Zhu, J. High Energy Phys. 04 (2019) 123
Moult, I.W. Stewart, G. Vita, arXiv:1905 .07411, 201
A. Bacchetta Bozzi, Echevarria, Pisano, Prokudin, Radici, Physics Letters B 797 (2019) 134850
A.Vladimirov Moos, Scimemi, & S.Rodini JHEP 2022
M. Ebert A. Gao I. Stewart JHEP 06 (2022)
S. Rodini, A. Vladimirov JHEP 08 (2022)
L.Gamberg, Z.Kang, D.Shao, J.Terry, F.Zhao arXiv: e-Print:221.13209
I. Balitsky, JHEP 03 (2023)

Challenges of *SLP/NLP* TMDs

However Theory for SLP TMD observables is challenging in comparison to the current state-of-the-art of leading power observables.
Treatments in the literature are mostly limited to a tree-level formalism until recently early studies beyond tree level can be found in ***Bacchetta et al. 2008, Chen et al. 2017***

More recently results beyond *LO,*
Vladimirov, Rodini, Moos, Scimemi 2002 BFM
Gao Stewart Ebert 2002 SCET,
Gamberg, Kang, Shao, Terry, Zhao, 2002 CSS factorization
Balitsky 2023 rapidity only TMD evolution

Various sources for power suppressed terms have been identified and discussed in the literature from
Tree level Studies, Mulders, Tangerman (1996), Bacchetta et al. (2008)

- This includes corrections associated to kinematic prefactors involving contractions between the leptonic and hadronic tensors, referred to as ***kinematic power corrections***.

- Another type involve subleading terms in quark-quark correlators involving Dirac structures that differ from LP ones which are sometimes called ***intrinsic power corrections****—most familiar e.g. Cahn function* $f^{\perp}(x, k_T)$

- Another from hadronic matrix elements of (interaction dependent) quark-gluon-quark operators, referred to *qgq* correlators for short referred to as ***dynamic power corrections***.

- *In arXiv: e-Print:221.13209 w*e present a systematic procedure for stress testing TMD factorization for DY & SIDIS at NLP using CSS formalism which addresses disagreements in the literature

Why TMDs @ twist-3 ➡ NLP

Some History-context

- **Georgi Politzer, PRL 1978**
  QCD analysis of *hard gluon* radiation in SIDIS to predict absolute value of $\boldsymbol{P}_T$ in SIDIS & the angular distribution relative to lepton scattering plane $\langle \cos \phi \rangle$
  "Measurement of $\langle \cos \phi \rangle$ provide very *clean test of the perturbative predictions of QCD*"

- **Cahn, PLB 1978, also earlier Ravndal, PLB 1972**
  "Critique of the parton model calculation of azimuthal dependence in leptoproduction**"**,
  emphasize importance *intrinsic* $\boldsymbol{k}_T$ …
- "We conclude that the azimuthal dependence in vector exchange interactions is inevitable since the partons have transverse momentum as a consequence of being confined and such dependence certainly does not require and special mechanism like gluon bremstrahlung
  "…The results **(G&P78)** can doubt on the utility of such azimuthal asymmetry as a clean test of quantum chromodynamics"
  "They (G&P78) ***suggested that this was a clean test of QCD since such effects would not arise simply as a result of the limited transverse momentum associated with confined quarks*"**

Clean tests of QCD

PHYSICAL REVIEW LETTERS

VOLUME 40 | 2 JANUARY 1978 | NUMBER 1

Clean Tests of Quantum Chromodynamics in μp Scattering

Howard Georgi
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

and

H. David Politzer
California Institute of Technology, Pasadena, California 91125
(Received 25 October 1977)

Hard gluon bremsstrahlung in μp scattering produces final-state hadrons with a large component of momentum transverse to the virtual-photon direction. Quantum chromodynamics can be used to predict not only the absolute value of the transverse momentum, but also its angular distribution relative to the muon scattering plane. The angular correlations should be insensitive to nonperturbative effects.

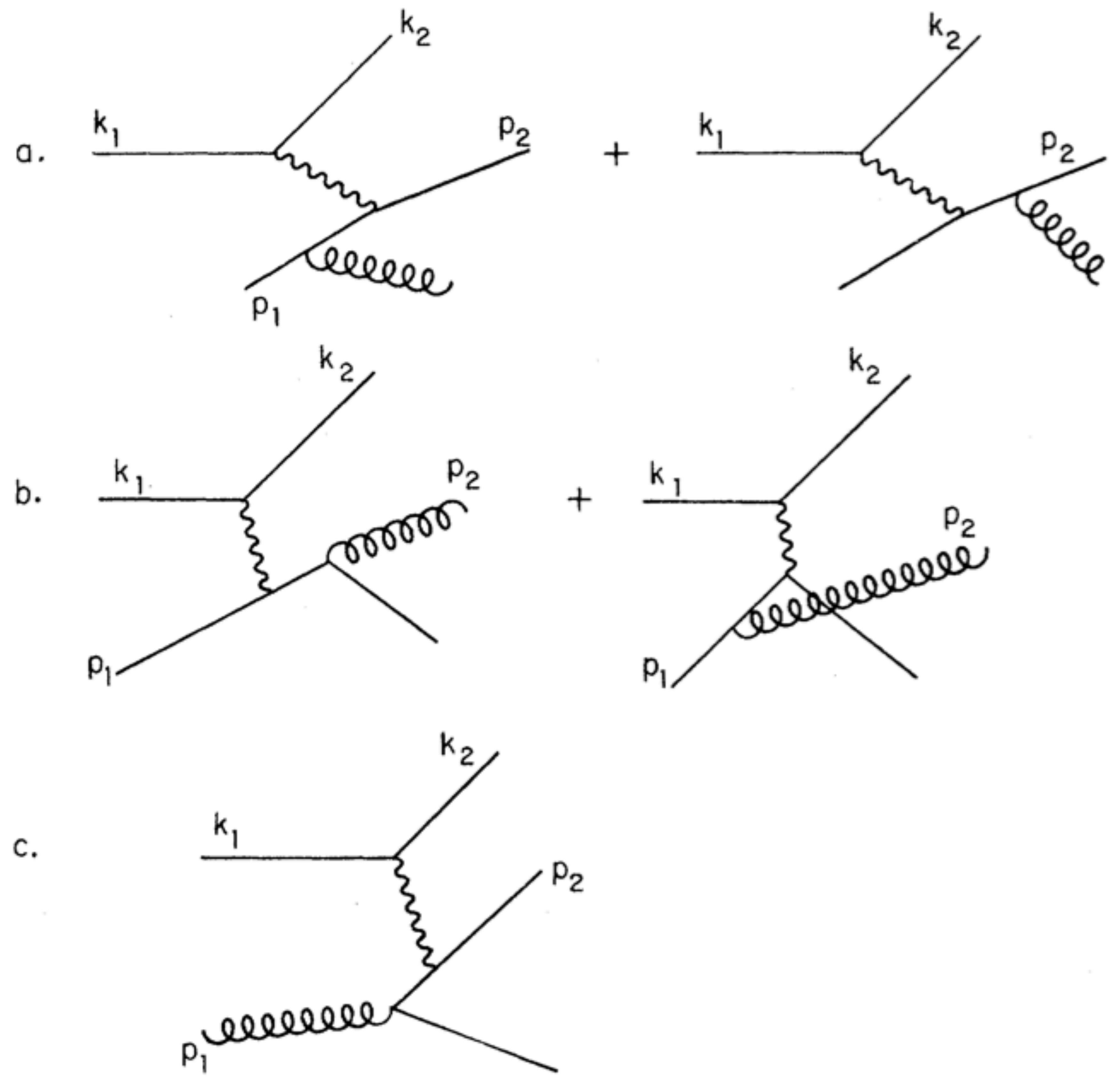


FIG. 1. Diagrams contributing to semi-inclusive μ-parton scattering to first order in α_s. k (p) denotes muon (parton) momentum. The wavy line is a virtual photon. The curly line is a gluon.

Cahn intrinsic k_T

Volume 78B, number 2,3 PHYSICS LETTERS 25 September 1978

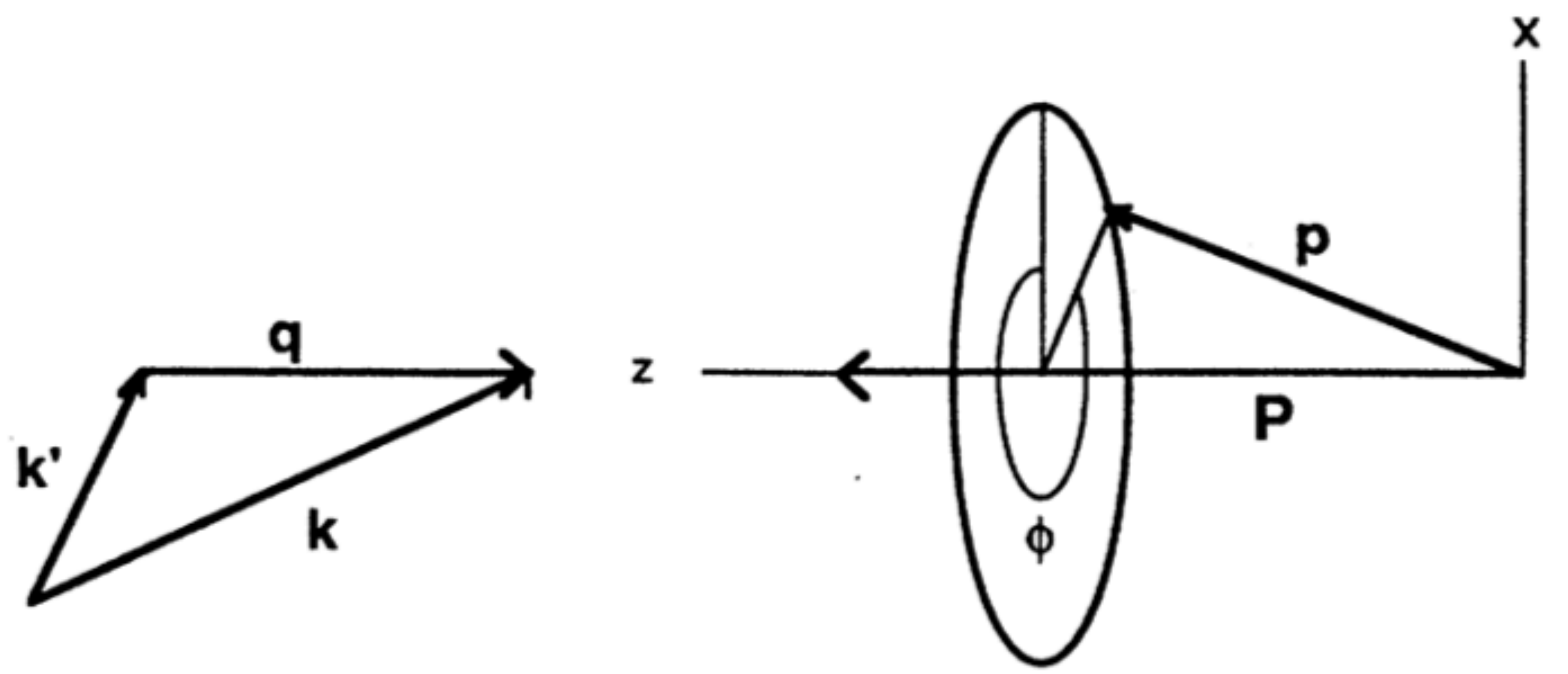
AZIMUTHAL DEPENDENCE IN LEPTOPRODUCTION: A SIMPLE PARTON MODEL CALCULATION☆

Robert N. CAHN
Department of Physics, University of Michigan, Ann Arbor, MI 48109, USA

Received 5 June 1978

Semi-inclusive leptoproduction, $\ell + p \to \ell' + h + X$, is considered in the naive parton model. The scattered parton shows an azimuthal asymmetry about the momentum transfer direction. Simple derivations for the effects in ep, νp and $\bar{\nu}$p scattering are given. Reduction of the asymmetry due to fragmentation of partons into hadrons is estimated. The results cast doubt on the utility of such azimuthal asymmetry as a clean test of quantum chromodynamics.

Cahn intrinsic k_T



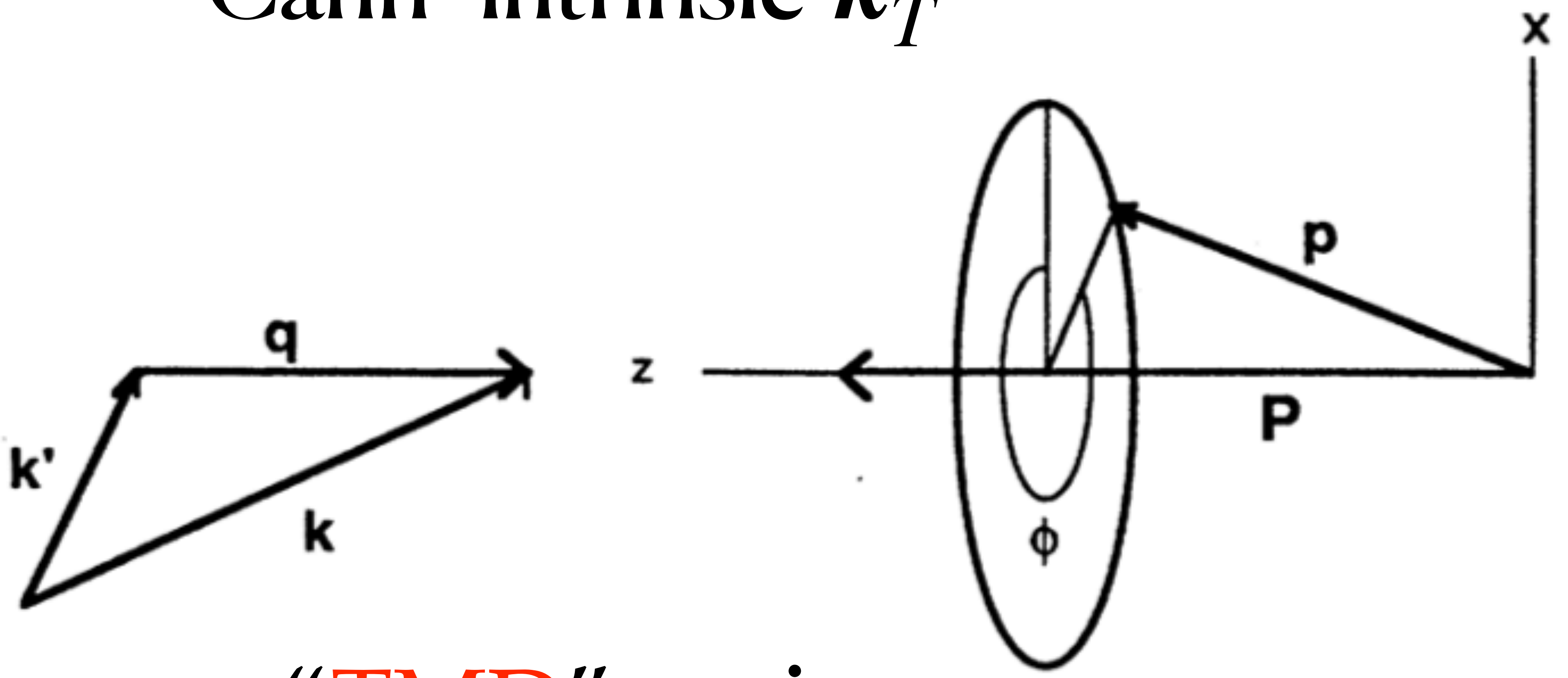
Simple parton model argument allowing for transverse momentum
in Mandelstam variables...

$$\sigma_{ep} \propto \hat{s}^2 + \hat{u}^2 \propto \left[1 - \frac{2p_\perp}{Q}\sqrt{1-y}\,\cos\phi\right]^2 + (1-y)^2\left[1 - \frac{2p_\perp}{Q\sqrt{1-y}}\cos\phi\right]^2$$

$$\langle \cos\phi \rangle_{ep} = -\left[\frac{2p_\perp}{Q}\right]\frac{(2-y)\sqrt{1-y}}{1+(1-y)^2}$$

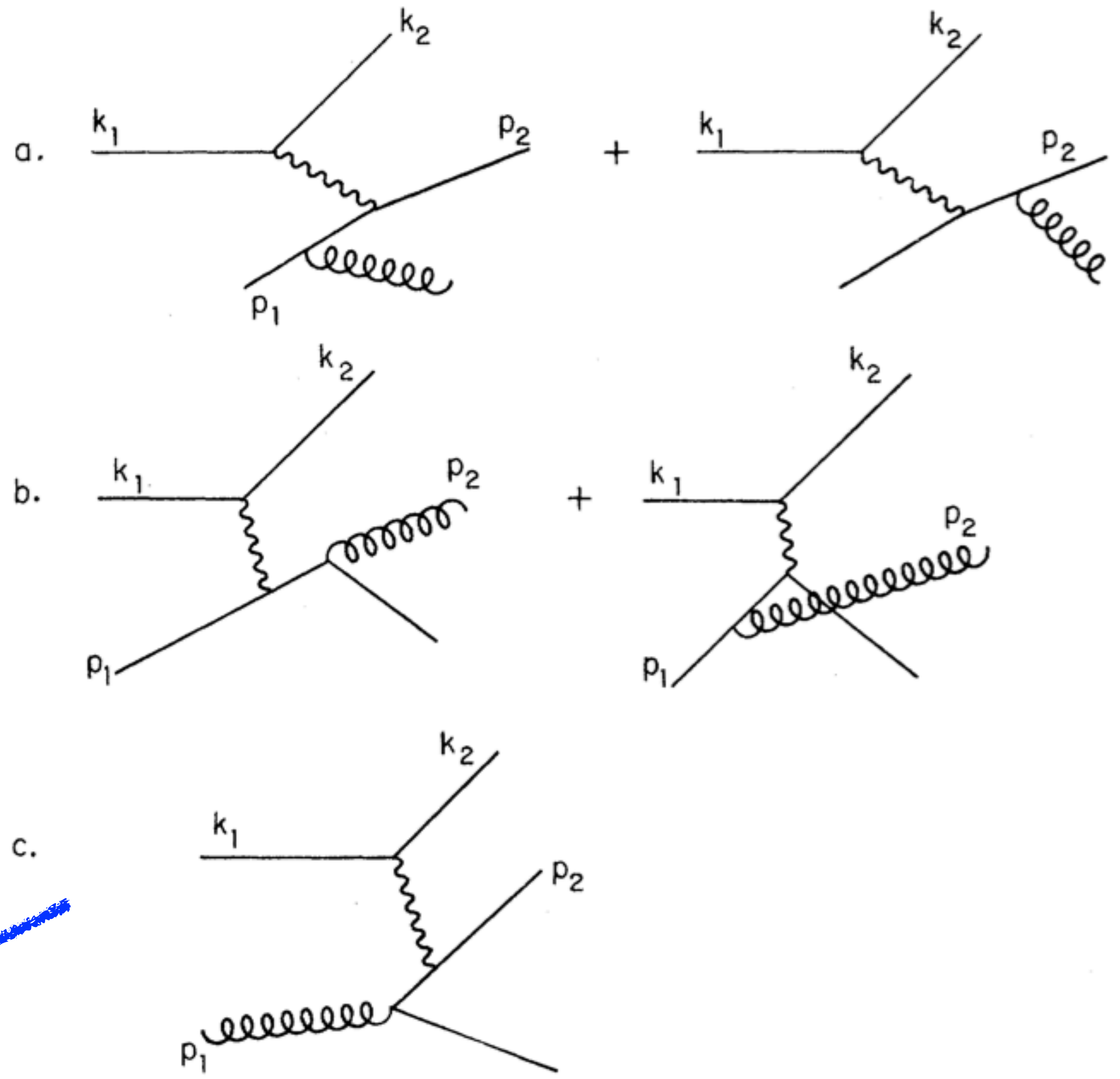
Two mechanisms ?

Cahn intrinsic $\boldsymbol{k}_T$



- "TMD" region

$(p_T \sim k_T) \sim q_T \ll Q$



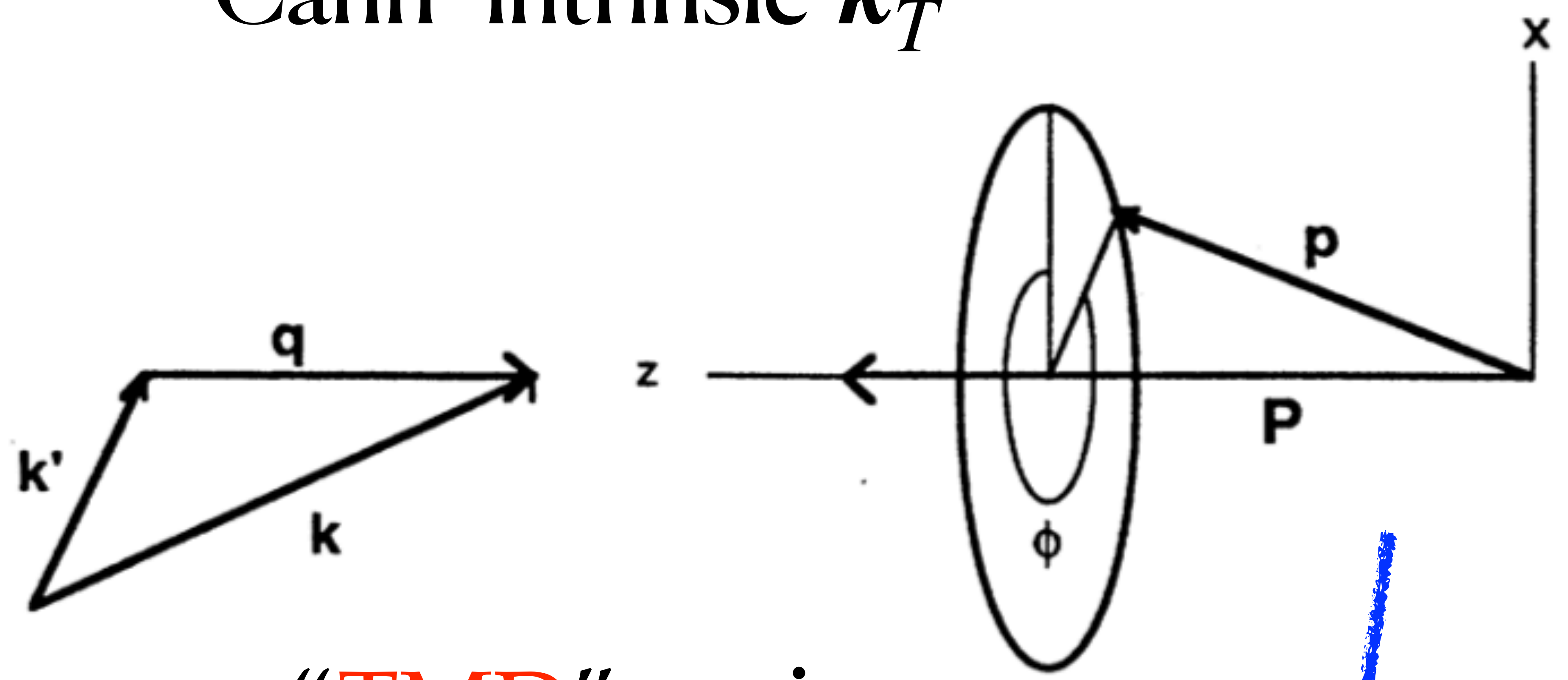
$$\frac{d^5\sigma}{dx_{bj}\, dQ^2\, dz_f\, dq_T^2\, d\phi} = \frac{\alpha_e^2 \alpha_s}{8\pi x_{bj}^2 S_{ep}^2 Q^2} \sum_k \mathcal{A}_k \int_{x_{\min}}^{1} \frac{dx}{x} \int_{z_f}^{1} \frac{dz}{z} [f \otimes D \otimes \hat{\sigma}_k] \times \delta\left(\frac{q_T^2}{Q^2} - \left(\frac{1}{\hat{x}} - 1\right)\left(\frac{1}{\hat{z}} - 1\right)\right)$$

- "Collinear " region

$\Lambda_{qcd} \ll q_T \sim Q$

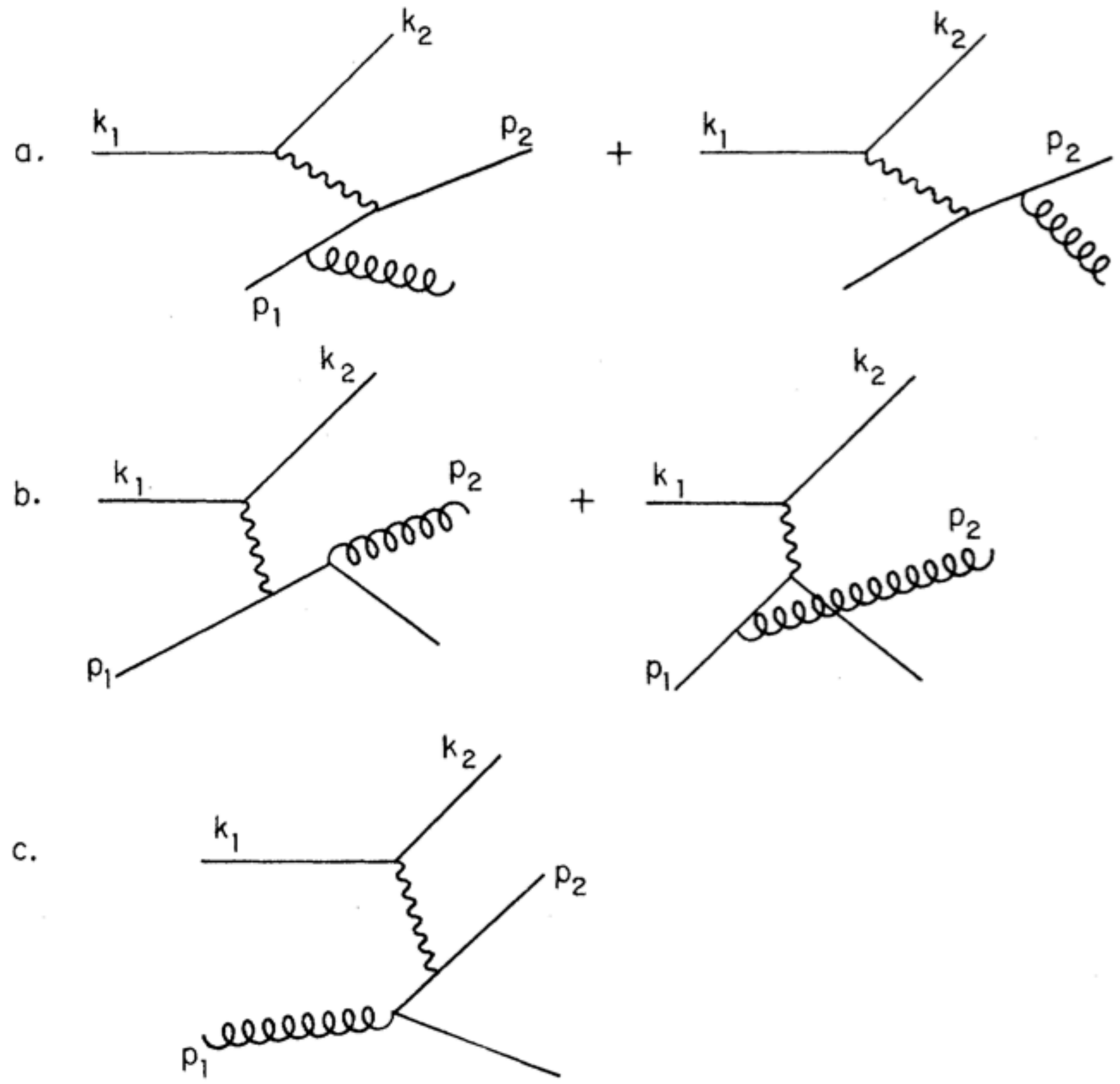
Two mechanisms ?

Cahn intrinsic $\boldsymbol{k}_T$



- "TMD" region

$(p_T \sim k_T) \sim q_T \ll Q$



- "Collinear" region

$\Lambda_{qcd} \ll q_T \sim Q$

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}\Big\}$$

e.g. $F_{UU}^{\cos\phi_h} \approx \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\,f_1 D_1\right]$

P.Mulders, R. Tangerman, NPB (1996), Bacchetta et al. JHEP 2007

Overview comments Matching

Matching studies in CSS related approaches

…

NPB Collins & Soper(1982), & Sterman 1985

NPB (1991) Arnold, Kauffman

PRD (1998) Nadolsky Stump Yuan

PRL (2001) Qiu, Zhang

PRD (2003) Berger, Qiu

NPB (2006) Bozzi, Catani, DeFlorian, Grazzini …

NPB (2006) Y. Koike, J. Nagashima, W. Vogelsang

JHEP (2008) Bacchetta et al.

arXiv (2014) Sun, Isacson, Yuan-CP,Yuan-F

JHEP (2015) Boglione, Hernandez, Melis Prokudin

PRD (2016) Collins, Gamberg, Prokudin, Rogers, Sato, Wang

PLB (2018) Gamberg , Metz, Pitonyak, Prokudin

PLB (2018) Echevarria, Kasemets, Lansberg, Pisano, Signori

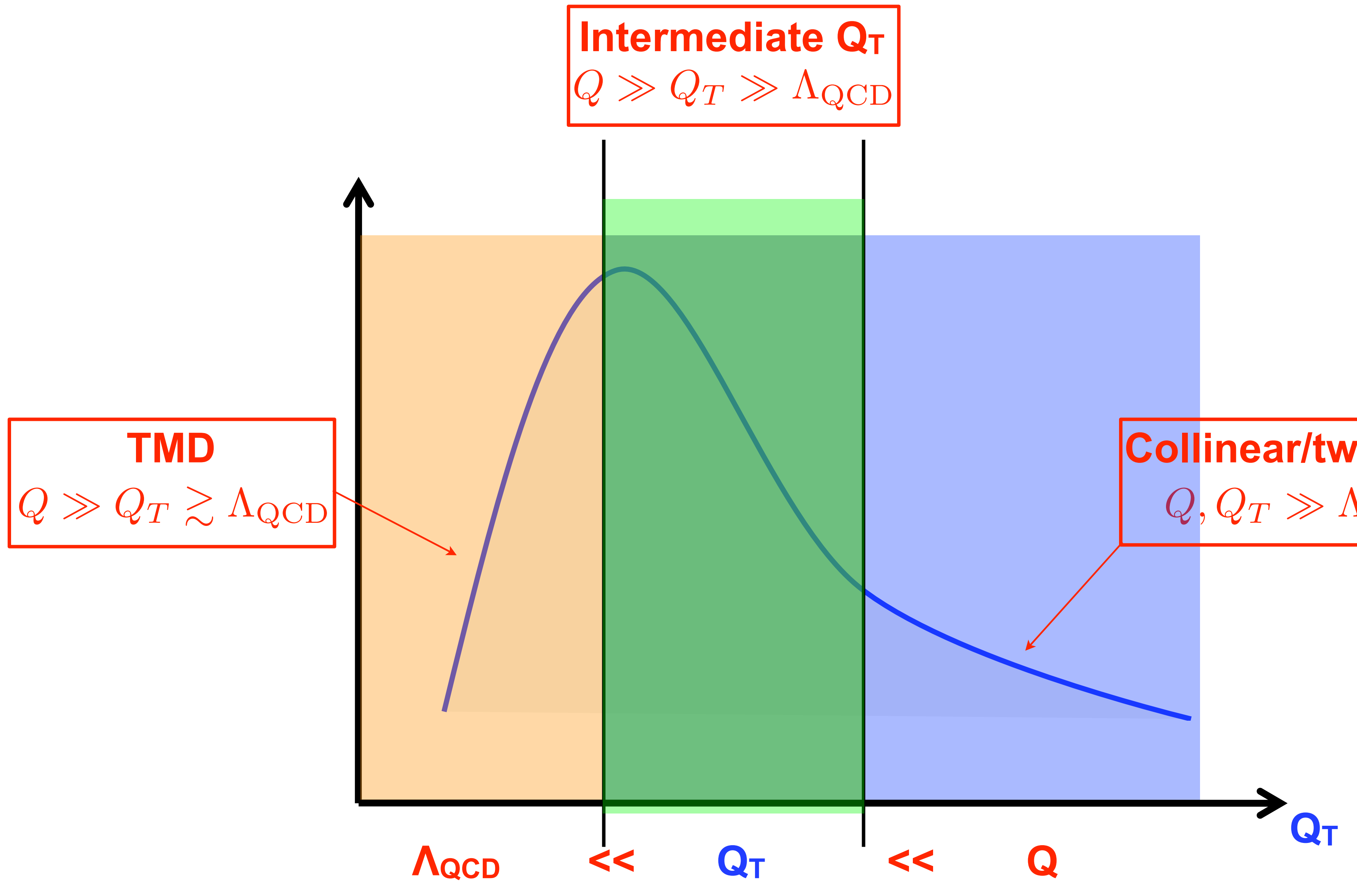
EJPA (2018) Scimemi, Vladimirov

JHEP 05 (2019) Scimemi , Tarasov, Vladimirov….

Series of papers on matching TMD and collinear ETQS transv. Spin

Ji, Qiu, Vogelsang, Yuan PRL PRD 2006, …

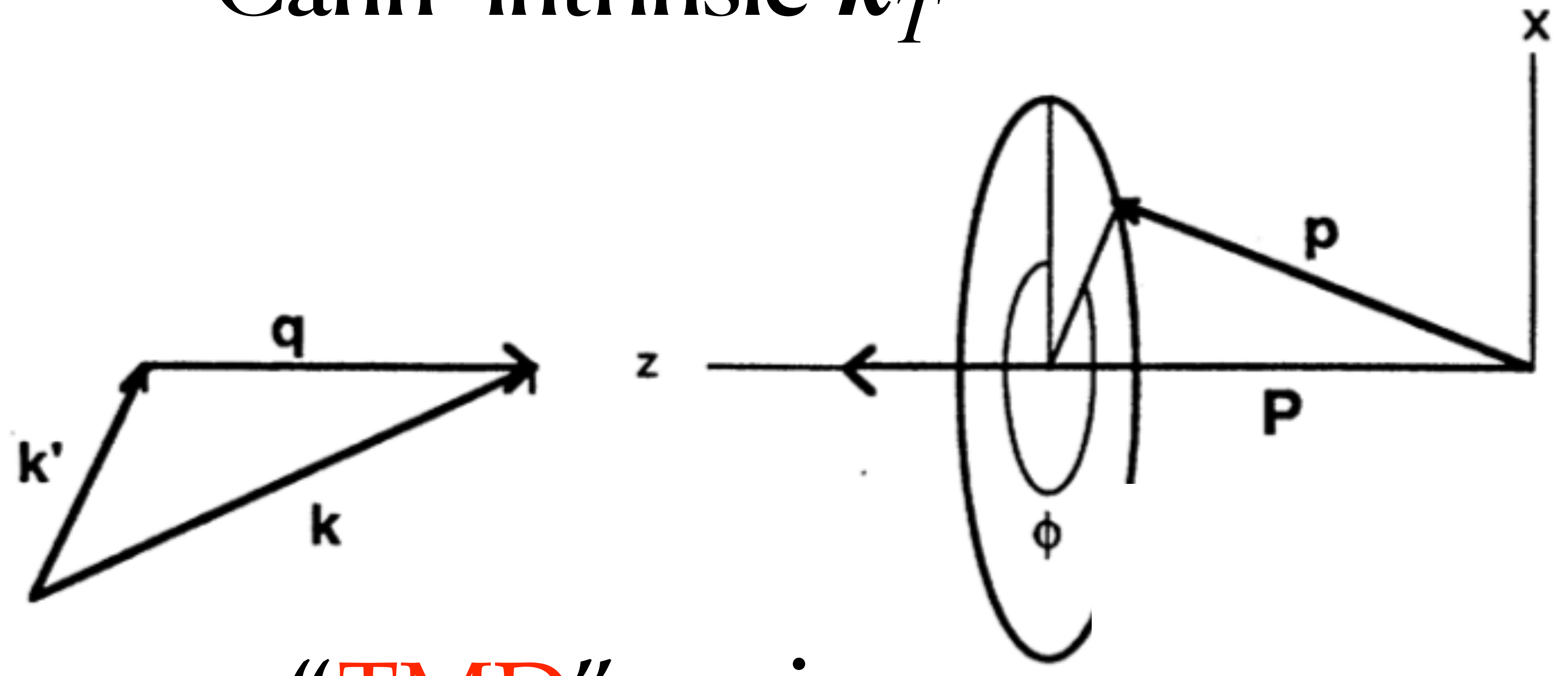
Kang, Xiao, Yuan PRL 2011



A comprehensive study of matching the hi & low Q_T in the overlap region in SIDIS was carried out by JHEP (2008) Bacchetta et al.
where attention was given to azimuthal and polarization dependence

"mis"-Matches Factorization @ sub-leading power

Cahn intrinsic $\boldsymbol{k_T}$



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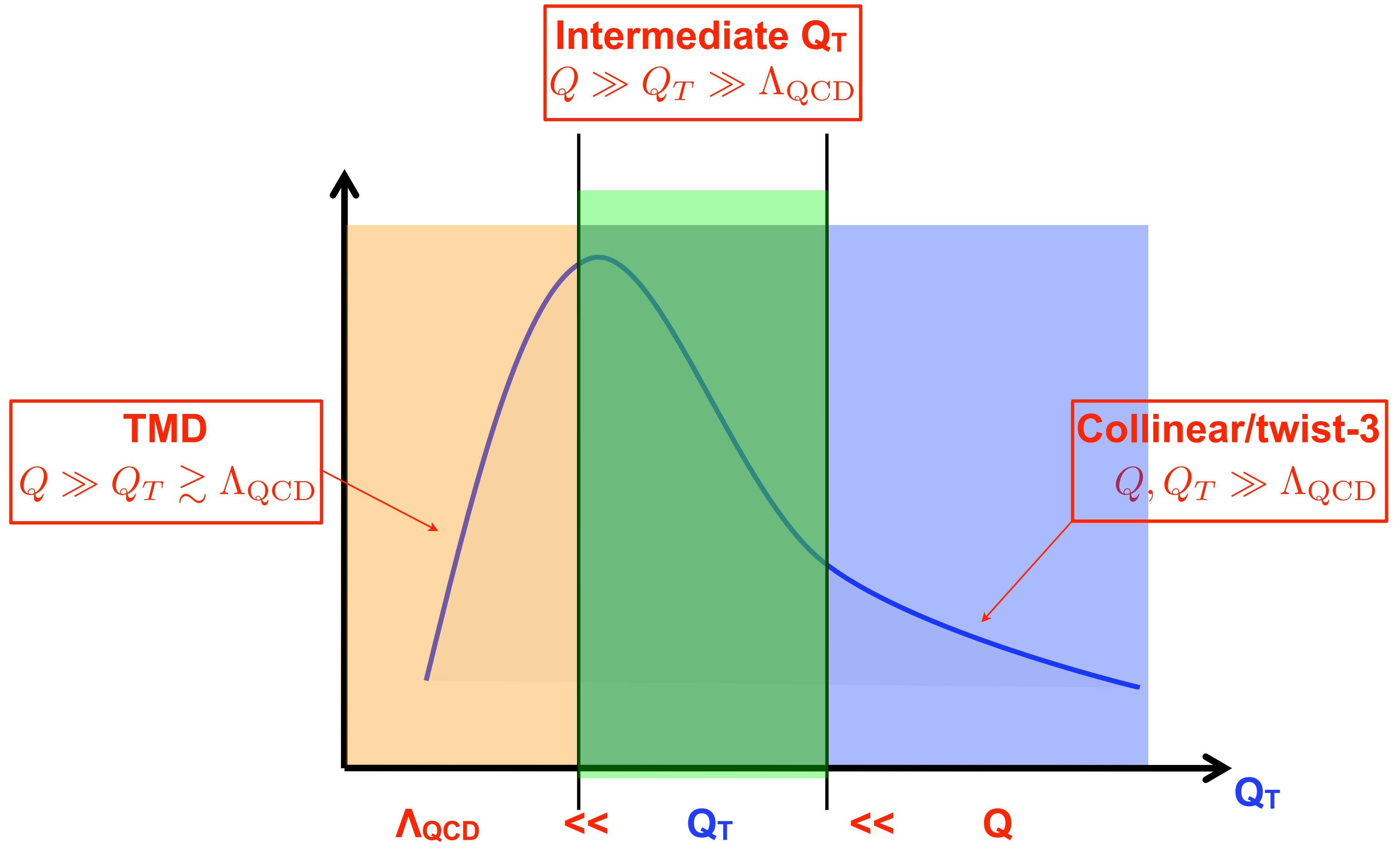

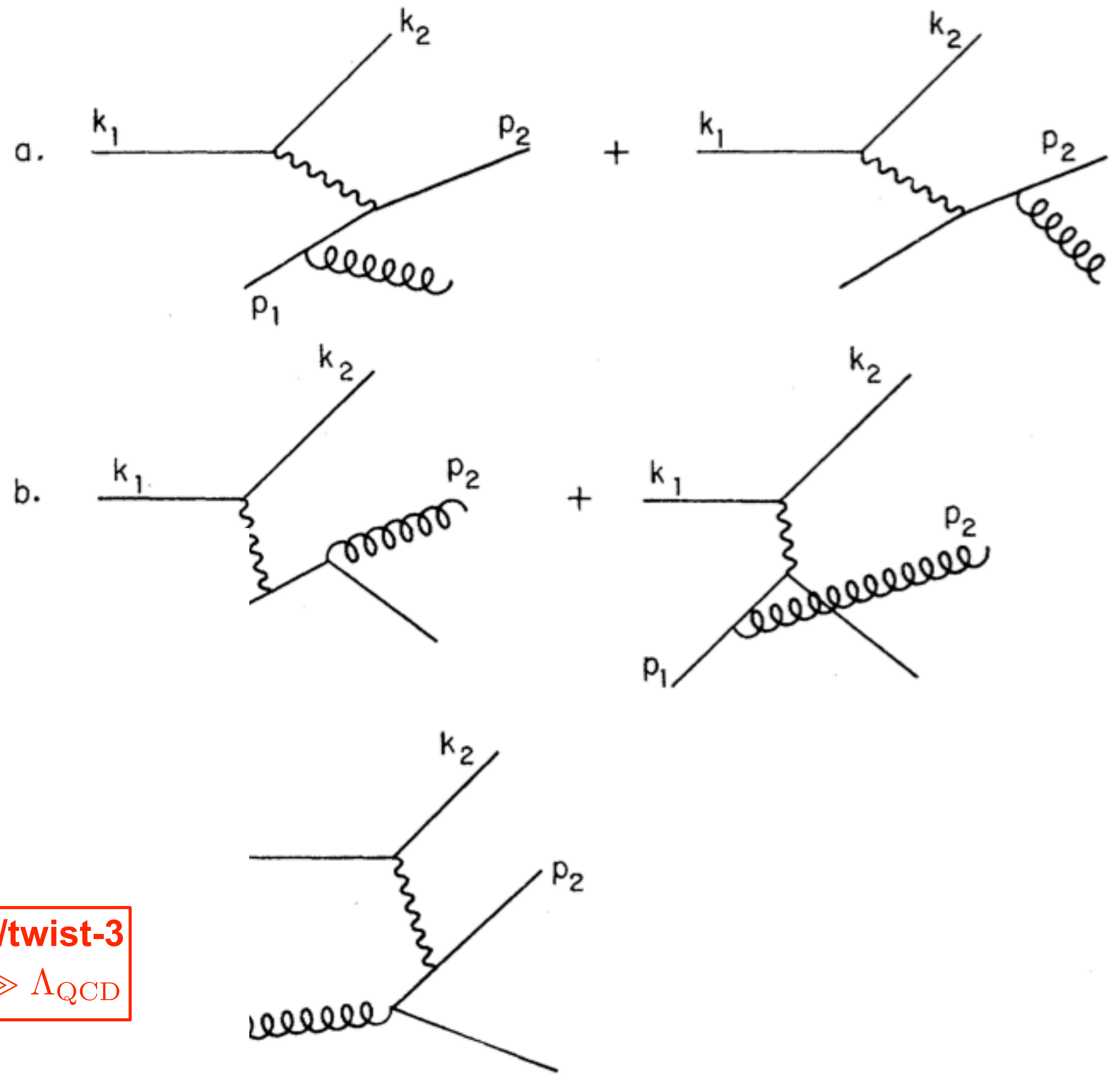

"Collinear" region

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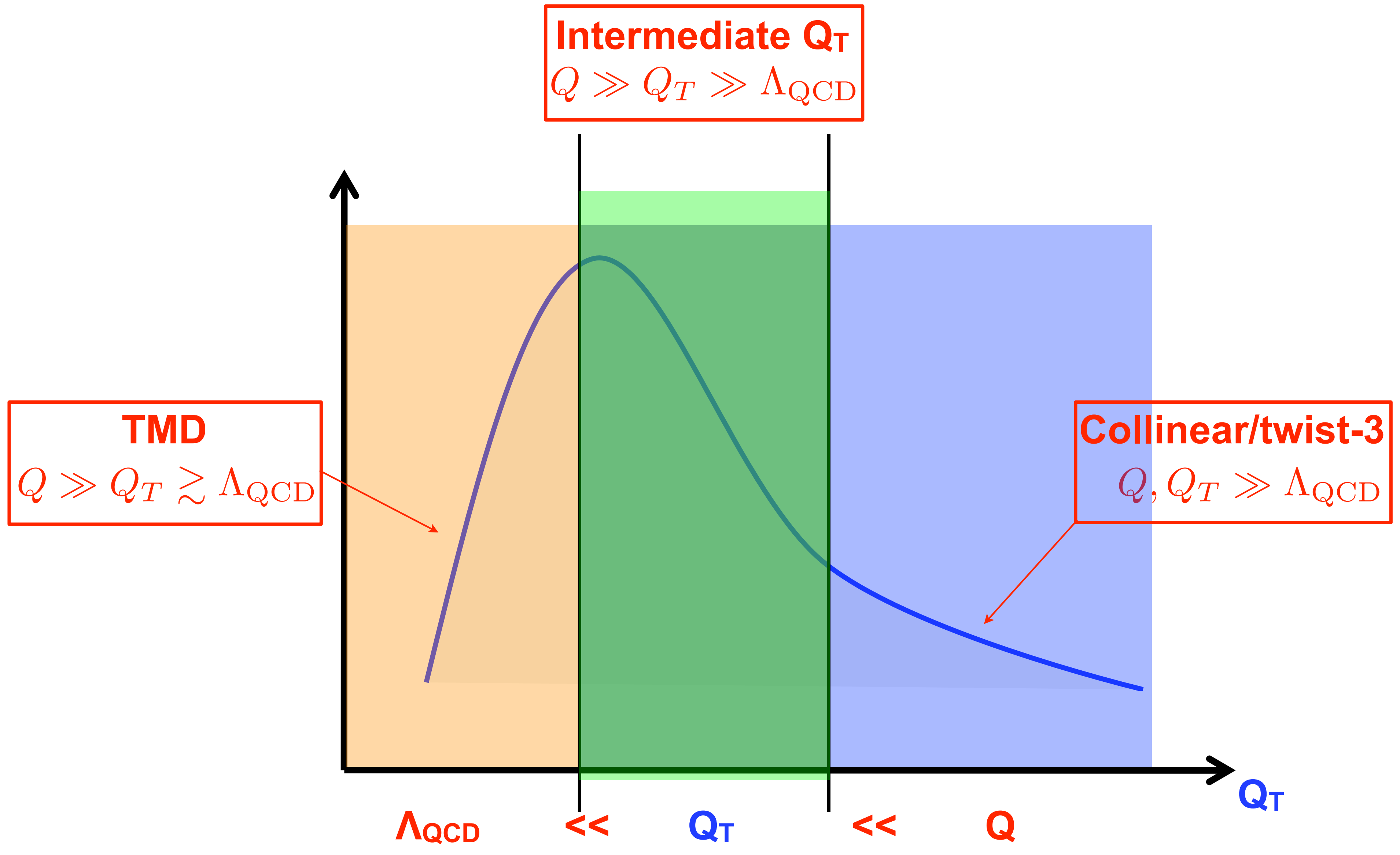
“mis”-Matches Factorization @ sub-leading power



A.Bacchetta, D. Boer, M. Diehl, P. Mulders JHEP (2008) mis-match/inconsistency breakdown of factorization at NLP?

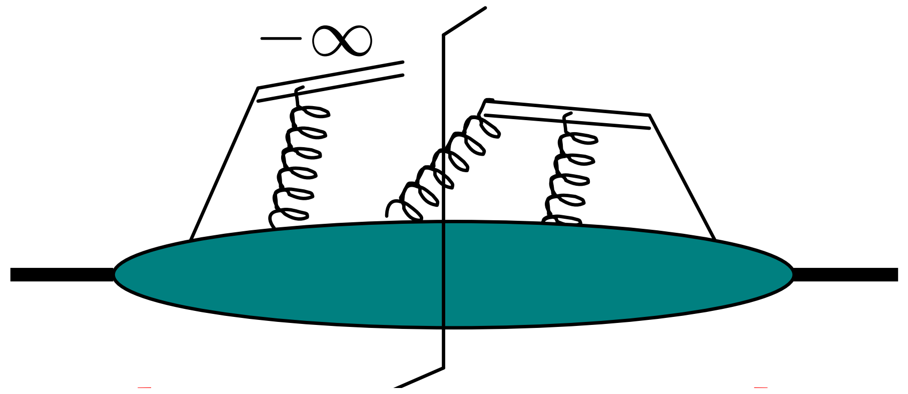
$$\langle \cos\phi \rangle = \frac{\int d\sigma^{(1)} \cos\phi}{\int d\sigma^{(1)}}$$

Attempt to address in
A. Bacchetta Bozzi, Echevarria, Pisano, Prokudin, Radici, Physics Letters B 797 (2019)

"mis"-Matches Factorization @ sub-leading power

$$\langle \cos\phi \rangle = \frac{\int d\sigma^{(1)} \cos\phi}{\int d\sigma^{(1)}}$$

Solution Bacchetta et al. is to introduce soft factor subtraction on TMD as in Collins 2011



$$\tilde{f}^{\rm sub}_{j/H}(x,b_T;\mu,y_n) = \lim_{\substack{y_A\to+\infty \\ y_B\to-\infty}} \underbrace{\tilde{f}^{\rm unsub}_{j/H}(x,b_T;\mu,y_P-y_B)} \sqrt{\frac{\tilde{S}(b_T;y_A,y_n)}{\tilde{S}(b_T;y_A,y_B)\tilde{S}(b_T;y_n,y_B)}} \times UV_{renorm}$$

$$\Updownarrow$$

$$\tilde{f}^{\rm unsub}_{j/H}(x,b_T;\mu,y_P-y_B) = \int \frac{db^-}{2\pi} e^{-ixP^+b^-} \langle P|\bar{\psi}(0)\gamma^+\mathcal{U}_{[0,b]}\psi(b)|P\rangle\Big|_{b^+=0}$$

JCC Soft factor further "repartitioned"
This is done to

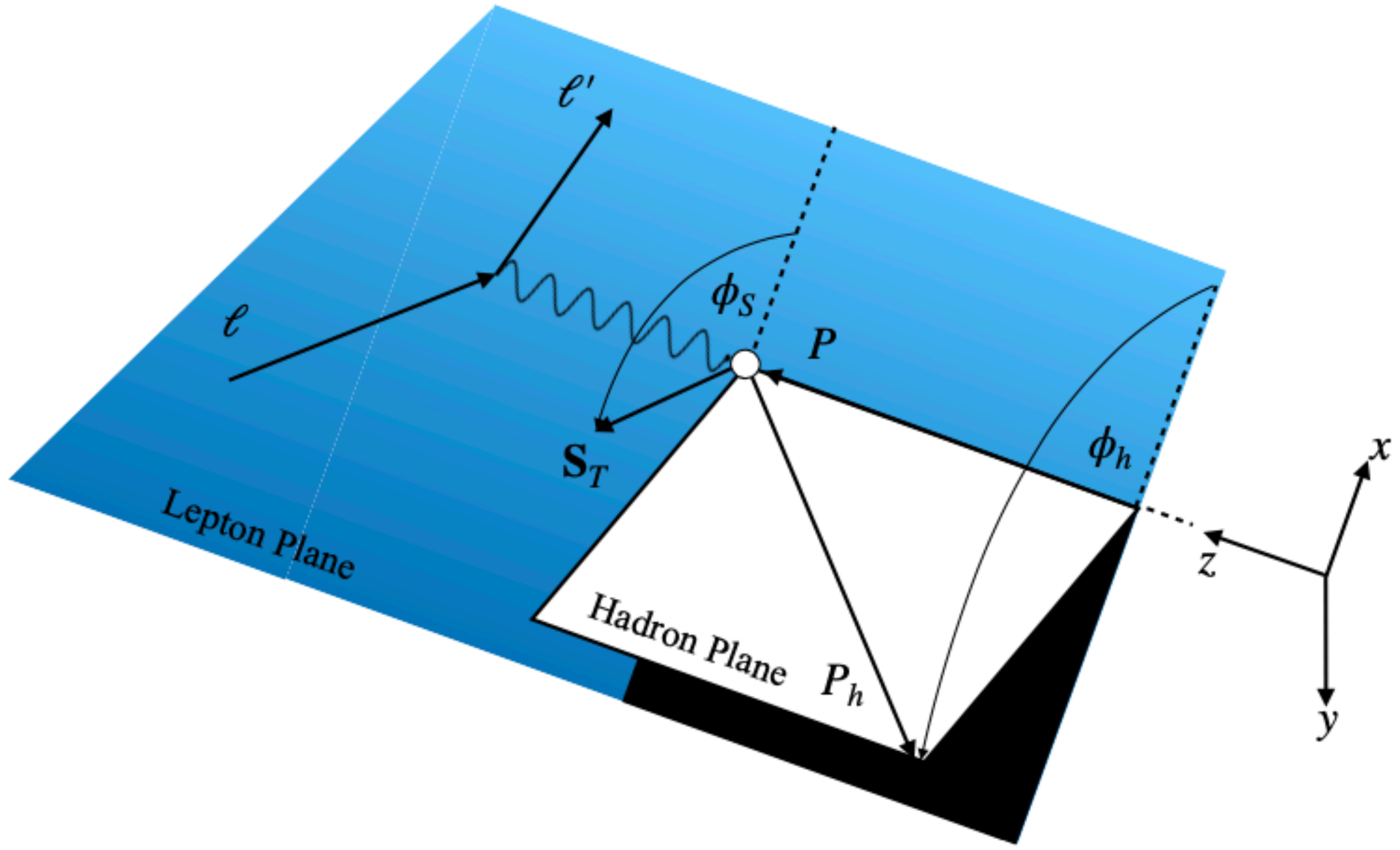
1) cancel LC divergences in "unsubtracted" TMDs
2) separate "right & left" movers i.e. full factorization
3) remove double counting of momentum regions

Factorization at sub-leading power … revisit Tree level

- "TMD" region $(p_T \sim k_T) \sim q_T \ll Q$

- Factorization beyond leading order and leading power via Collins, Aybat & Rogers 2011
- To do this at sub-leading power; revisit tree level build RG consistency
- Develop RG and rapidity renormalization group Eqs. CS equation
- The SIDIS cross section in the hadronic Breit frame

$$\frac{d\sigma}{dx\,dy\,d\Psi\,dz\,d^2P_{h\perp}} = \kappa\frac{\alpha_{\rm em}^2}{4Q^4}\frac{y}{z}L_{\mu\nu}W^{\mu\nu}$$



Factorization at sub-leading power ... revisit Tree level

- "TMD" region $(p_T \sim k_T) \sim q_T \ll Q$

$$W_{\mu\nu} = \frac{1}{(2\pi)^4} \sum_X \int d^4x\, e^{-iqx} \langle P | J^\dagger_\mu(0) | h, X\rangle \langle h, X | J_\nu(x) | P\rangle ,$$

$$J_\mu(x) = J^{(2)}_\mu(x) + J^{(3)}_\mu(x)$$

Consider 2 parton quark current with longitudinal gluon to make gauge invariant

$$W^{(2)}_{\mu\nu} = \frac{1}{(2\pi)^4} \int d^4x\, e^{-iqx} \left\langle P_1 , P_2 \middle| J^{(2)}_\mu(0)\, J^{(2)\,\dagger}_\nu(x) \middle| P_1 , P_2 \right\rangle$$

$$W^{(2)}_{\mu\nu} = \frac{1}{N_c} \sum_q e_q^2 \int d^2\boldsymbol{k}_{1\perp}\, d^2\boldsymbol{k}_{2\perp}\, \delta^{(2)}\left(\boldsymbol{q}_\perp - \boldsymbol{k}_{1\perp} - \boldsymbol{k}_{2\perp}\right) \times \mathrm{Tr}\left[\Phi_{q/P_1}\left(x_1, \boldsymbol{k}_{1\perp}, \boldsymbol{S}_1\right) \gamma^\mu \Phi_{\bar{q}/P_2}\left(x_2, \boldsymbol{k}_{2\perp}, \boldsymbol{S}_2\right) \gamma^\nu\right]$$

$$\Phi_{q/P_1\, jj'}(x, \boldsymbol{k}_\perp, \boldsymbol{S}) = \int \frac{d^4\xi}{(2\pi)^3} e^{ik\cdot\xi}\, \delta\left(\xi^+\right) \left\langle P, \boldsymbol{S} \middle| \bar{\psi}^c_{j'}(0)\, \mathcal{U}^{\bar{n}}_{\llcorner}(0)\, \mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\xi)\, \psi^c_j(\xi) \middle| P, \boldsymbol{S} \right\rangle$$

gauge invariant

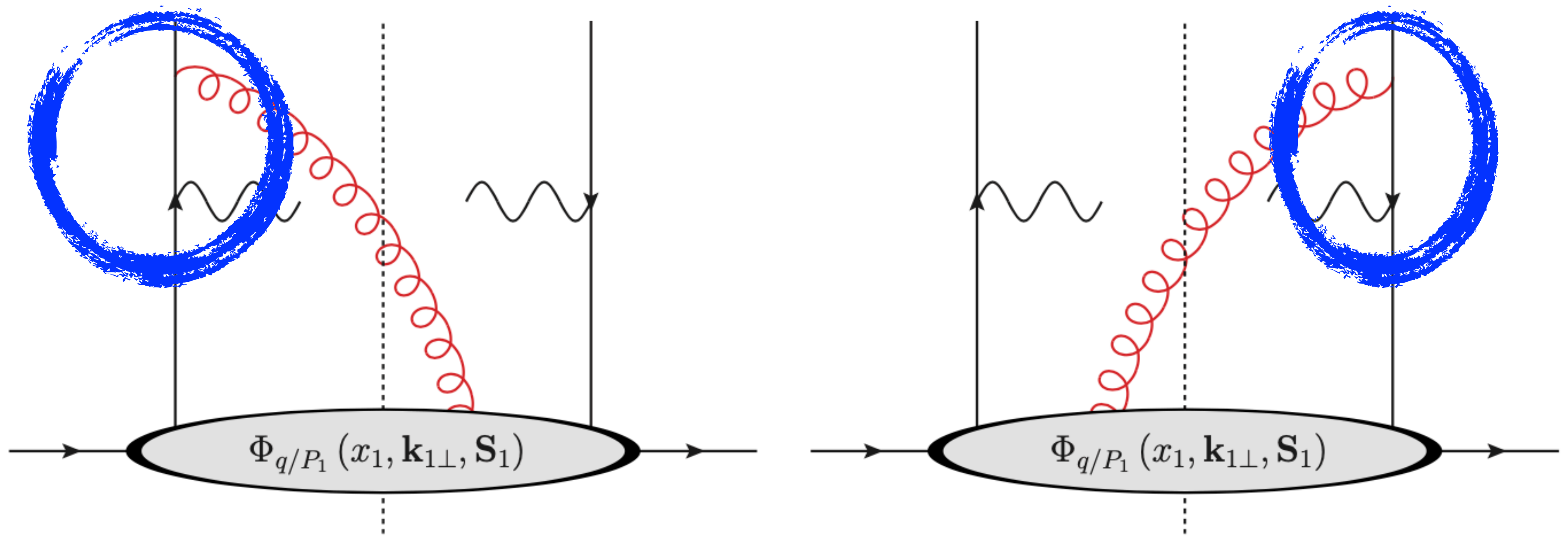
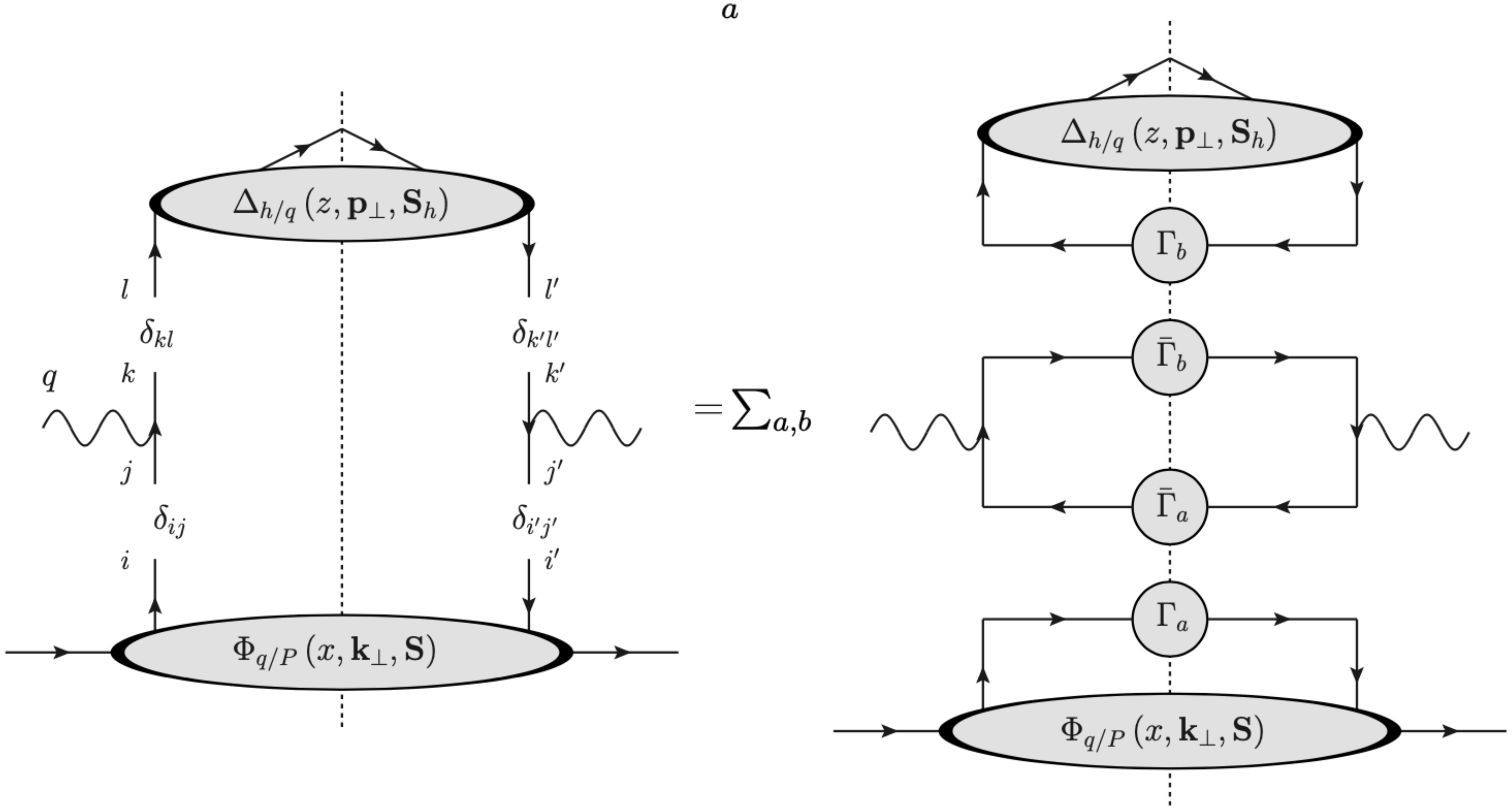


Figure 2. Representation of how the Wilson lines enter for the quark-quark correlation function. Left: The $\bar{n}$ Wilson line associated with $\mathcal{U}^{\bar{n}}_{\llcorner}(0)$. Right: The $\bar{n}$ Wilson line associated with $\mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\xi)$.

Factorization at sub-leading power revisit Tree level Fierz decomposition

The two parton hadronic tensor can be organized in terms of the contributions at a given twist by performing a Fierz decomposition of the quark lines.

Fierz decomposition of the 2 parton correlation function $\delta_{ij}\delta_{j'i'} = \sum_a \Gamma^a_{ii'}\,\bar{\Gamma}^a_{j'j}$ This decomposition is demonstrated in Fig.



Factorized !!

$$W^{(2)}_{\mu\nu} = \frac{1}{N_c}\sum_{a,b} \mathrm{Tr}\left[\gamma^\mu\,\bar{\Gamma}^a\,\gamma^\nu\,\bar{\Gamma}^b\right] \mathcal{C}^{\mathrm{DIS}}\left[\Phi^{[\Gamma^a]}(x, \boldsymbol{k}_\perp, \boldsymbol{S})\;\Delta^{[\Gamma^b]}(z, \boldsymbol{p}_\perp, \boldsymbol{S}_h)\right],$$

Factorization at sub-leading power … revisit Tree level

$$\Phi_{q/P_1\, jj'}(x, \boldsymbol{k}_\perp, \boldsymbol{S}) = \int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\, \delta(\xi^+) \left\langle P, \boldsymbol{S} \left| \bar{\psi}^c_{j'}(0)\, \mathcal{U}^{\bar{n}}_{\llcorner}(0)\, \mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\xi)\, \psi^c_j(\xi) \right| P, \boldsymbol{S} \right\rangle$$

ψ^c are quark fields with the momentum scaling $k^\mu \sim Q\,(1, \lambda^2, \lambda)$ $\quad \lambda = q_\perp/Q \quad$ k^μ are $(n\cdot k, \bar{n}\cdot k, \boldsymbol{k}_\perp)$

- To separate the contributions of hadronic tensor at LP & SLP, employ light-cone projections of the Dirac fields, called "good" and "bad (power surpressed)" $\lambda = q_\perp/Q$ light-cone components

$$\psi^c = \chi^c + \phi^c$$

$$\chi^c(x) = \frac{\bar{\not{n}}\not{n}}{4}\psi^c(x)\,, \qquad \varphi^c(x) = \frac{\not{n}\bar{\not{n}}}{4}\psi^c(x)$$

- Upon expressing ψ^c in terms of ϕ^c and χ^c in the correlation function, four field configurations enter into the position space matrix elements,

2 good twist 2 — 1 good 1 bad twist 3 — 2 bad twist 4

$$\langle P, \boldsymbol{S}|\bar{\chi}^c_{j'}\, \chi^c_j|P, \boldsymbol{S}\rangle,\ \langle P, \boldsymbol{S}|\bar{\varphi}^c_{j'}\, \chi^c_j|P, \boldsymbol{S}\rangle,\ \langle P, \boldsymbol{S}|\bar{\chi}^c_{j'}\, \varphi^c_j|P, \boldsymbol{S}\rangle,\ \text{and}\ \langle P, \boldsymbol{S}|\bar{\varphi}^c_{j'}\, \varphi^c_j|P, \boldsymbol{S}\rangle$$

Factorization at sub-leading power … revisit Tree level

- In the formulation of the cross section/hadronic tensor in terms of the correlation function, traces of the quark correlation functions with the Γ^a operators entered, $\Phi^{\Gamma^a}(x_1, \boldsymbol{k}_T, \boldsymbol{S}) \equiv \mathrm{Tr}\left[\Phi(x_1, \boldsymbol{k}_T, \boldsymbol{S})\, \Gamma^a\right]$

  Due to the idempotence of the projection operators, $\chi^c(x) = \frac{\bar{\not n}\not n}{4}\psi^c(x)\,, \qquad \varphi^c(x) = \frac{\not n\bar{\not n}}{4}\psi^c(x)$

  each Γ^a operator will be associated with a particular field configuration, and thus a particular twist which we organize in table

Twist 2	Twist 3	Twist 4
$\frac{1}{2}\not n\ ,\ \frac{1}{4}\bar{\not n}$	$\frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}\bar{\not n}\ ,\ \frac{1}{4}\not n$
$\frac{1}{2}\not n\gamma^5,\ \frac{1}{4}\gamma^5\bar{\not n}$	$\frac{1}{2}\gamma^5,\ \frac{1}{2}\gamma^5$	$\frac{1}{2}\bar{\not n}\gamma^5,\ \frac{1}{4}\gamma^5\not n$
$\frac{i}{2}\sigma^{k+}\gamma^5,\ \frac{i}{4}\gamma^5\sigma_{-k}$	$\frac{1}{2}\gamma^k,\ \frac{1}{2}\gamma_k$	$\frac{i}{2}\sigma^{k-}\gamma^5\ ,\ \frac{i}{4}\gamma^5\sigma_{+k}$
	$\frac{1}{2}\gamma^k\gamma^5,\ \frac{1}{2}\gamma^5\gamma_k$	
	$\frac{i}{2}\sigma^{kl}\gamma^5,\ \frac{i}{4}\gamma^5\sigma_{lk}$	
	$\frac{i}{4}\sigma^{+-}\gamma^5,\ \frac{i}{4}\gamma^5\sigma_{+-}$	

By organizing the operators by their twists,
we arrive at the well known expression for the LP and NLP correlation functions

$$\Phi^{(2)}_{q/P}(x,\boldsymbol{k}_\perp,\boldsymbol{S}) = \left(f_1 - \frac{\epsilon^{ij}_\perp k_{\perp i} S_{\perp j}}{M} f^\perp_{1T}\right)\frac{\not{\bar{n}}}{4} + \left(\lambda g_{1L} - \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_\perp}{M} g_{1T}\right)\frac{\gamma^5 \not{\bar{n}}}{4}$$
$$+ \left(S^i_\perp h_1 + \frac{\lambda k^i_\perp}{M} h^\perp_{1L} - \frac{\epsilon^{ij}_\perp k_{\perp j}}{M} h^\perp_1 - \frac{k^i_\perp k^j_\perp - \frac{1}{2} k^2_\perp g^{ij}_\perp}{M^2} S_{\perp j} h^\perp_{1T}\right)\frac{i\gamma^5 \sigma_{-i}}{4}$$

Twist 2	Twist 3	Twist 4
$\frac{1}{2}\not{n}\ ,\ \frac{1}{4}\not{\bar{n}}$	$\frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}\not{\bar{n}}\ ,\ \frac{1}{4}\not{n}$
$\frac{1}{2}\not{n}\gamma^5,\ \frac{1}{4}\gamma^5\not{\bar{n}}$	$\frac{1}{2}\gamma^5,\ \frac{1}{2}\gamma^5$	$\frac{1}{2}\not{\bar{n}}\gamma^5,\ \frac{1}{4}\gamma^5\not{n}$
$\frac{i}{2}\sigma^{k+}\gamma^5,\ \frac{i}{4}\gamma^5\sigma_{-k}$	$\frac{1}{2}\gamma^k,\ \frac{1}{2}\gamma_k$	$\frac{i}{2}\sigma^{k-}\gamma^5\ ,\ \frac{i}{4}\gamma^5\sigma_{+k}$
	$\frac{1}{2}\gamma^k\gamma^5,\ \frac{1}{2}\gamma^5\gamma_k$	
	$\frac{i}{2}\sigma^{kl}\gamma^5,\ \frac{i}{4}\gamma^5\sigma_{lk}$	
	$\frac{i}{4}\sigma^{+-}\gamma^5,\ \frac{i}{4}\gamma^5\sigma_{+-}$	

$$\Phi^{(3)}_{q/P}(x,\boldsymbol{k}_\perp,\boldsymbol{S}) = \frac{M}{P^+}\Bigg[\left(e - \frac{\epsilon^{ij}_\perp k_{\perp i} S_{\perp j}}{M} e^\perp_T\right)\frac{1}{2} - i\left(\lambda_g e_L - \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_\perp}{M} e_T\right)\frac{\gamma^5}{2}$$
$$+ \left(\frac{k^i_\perp}{M} f^\perp - \epsilon^{ij}_\perp S_{\perp j} f'_T - \frac{\epsilon^{ij}_\perp k_{\perp j}}{M}\left(\lambda_g f^\perp_L - \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_\perp}{M} f^\perp_T\right)\right)\frac{\gamma_i}{2}$$
$$+ \left(g'_T S^i_\perp - \frac{\epsilon^{ij}_\perp k_{\perp j}}{M} g^\perp + \frac{k^i_\perp}{M}\left(\lambda_g g^\perp_L - \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_\perp}{M} g^\perp_T\right)\right)\frac{\gamma^5 \gamma_i}{2}$$
$$+ \left(\frac{S^i_\perp k^j_\perp}{M} h^\perp_T\right)\frac{i\gamma^5 \sigma_{ji}}{4} + \left(h + \lambda_g h_L - \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_\perp}{M} h^\perp\right)\frac{i\gamma^5 \sigma_{+-}}{4}\Bigg]$$

Tree level factorization sub-leading power intrinsic TMDs

$\Phi(x, \boldsymbol{k}_T)$

SIDIS tree-level diagrams relevant for sub-leading-power observables
"intrinsic"

Subleading Quark TMDPDFs

Nucleon Polarization	Quark Chirality: Chiral Even	Quark Chirality: Chiral Odd
U	$f^{\perp}, g^{\perp}$	e, h
L	$f_L^{\perp}, g_L^{\perp}$	e_L, h_L
T	$f_T, f_T^{\perp}, g_T, g_T^{\perp}$	$e_T, e_T^{\perp}, h_T, h_T^{\perp}$

- ✦Mulders Tangerman NPB1995
- ✦Goeke Metz Schlegel PLB 2005
- ✦Bacchetta et al 2007 JHEP

$$\begin{aligned}\Phi^{(3)}(x,\boldsymbol{k}_T,\boldsymbol{S}) = \frac{M}{P^+}\Bigg[&\left(e - \frac{\epsilon_T^{\rho\sigma} k_{T\rho} S_{T\sigma}}{M} e_T^{\perp}\right)\frac{1}{2} - i\left(\lambda_g e_L - \frac{\boldsymbol{k}_T\cdot\boldsymbol{S}_T}{M} e_T\right)\frac{\gamma^5}{2}\\
&+\left(\frac{k_T^k}{M} f^{\perp} - \epsilon_T^{kl} S_{Tl} f_T' - \frac{\epsilon_T^{kl} k_{Tl}}{M}\left(\lambda_g f_L^{\perp} - \frac{\boldsymbol{k}_T\cdot\boldsymbol{S}_T}{M} f_T^{\perp}\right)\right)\frac{\gamma_k}{2}\\
&+\left(g_T' S_T^k - \frac{\epsilon_T^{kl} k_{Tl}}{M} g^{\perp} + \frac{k_T^k}{M}\left(\lambda_g g_L^{\perp} - \frac{\boldsymbol{k}_T\cdot\boldsymbol{S}_T}{M} g_T^{\perp}\right)\right)\frac{\gamma^5\gamma_k}{2}\\
&+\left(\frac{S_T^k k_T^l}{M} h_T^{\perp}\right)\frac{i\gamma^5\sigma_{lk}}{4} + \left(h + \lambda_g h - \frac{\boldsymbol{k}_T\cdot\boldsymbol{S}_T}{M} h^{\perp}\right)\frac{i\gamma^5\sigma_{+-}}{4}\Bigg]\end{aligned}$$

Factorization at sub-leading power … 3 partons

- "TMD" region

$$W_{\mu\nu} = \frac{1}{(2\pi)^4} \sum_X \int d^4x\, e^{-iqx} \langle P | J^\dagger_\mu(0) | h, X\rangle \langle h, X |\, J_\nu(x) | P\rangle\,,$$

$$J_\mu(x) = J^{(2)}_\mu(x) + J^{(3)}_\mu(x)$$

Consider 3 partons entering from one hadron

$$W^{(3)}_{\mu\nu} = \frac{1}{(2\pi)^4} \int d^4x\, e^{-iqx} \left\langle P_1\,, P_2 \left| \left(J^{(3)}_\mu(0)\, J^{(2)\,\dagger}_\nu(x) + J^{(2)}_\mu(0)\, J^{(3)\,\dagger}_\nu(x) \right) \right| P_1\,, P_2 \right\rangle$$

$$W^{(3)}_{\mu\nu} = -\frac{1}{N_c C_F} \sum_q e_q^2 \int d^2\boldsymbol{k}_\perp\, d^2\boldsymbol{p}_\perp\, \delta^{(2)}\left(\boldsymbol{q}_\perp + \boldsymbol{k}_\perp + \boldsymbol{p}_\perp/z\right)$$
$$\times \left[\int dk_g^+ \,\mathrm{Tr}\left[\Phi^i_{A\,q/P_1}(x, x_g, \boldsymbol{k}_\perp, \boldsymbol{S})\gamma^\mu \Delta_{h/q}(z, \boldsymbol{p}_\perp, \boldsymbol{S}_h)\gamma_i \frac{\not{p} - \not{k}_g}{(p-k_g)^2 + i\epsilon}\gamma^\nu \right] \right.$$
$$\left. + \int dp_g^- \,\mathrm{Tr}\left[\Delta^i_{A\,h/q}(z, z_g, \boldsymbol{p}_\perp, \boldsymbol{S}_h)\gamma^\nu \frac{\not{k} - \not{p}_g}{(k-p_g)^2 + i\epsilon}\gamma_i \Phi_{q/P}(x, \boldsymbol{k}_\perp, \boldsymbol{S})\gamma^\mu \right] + \text{h.c.} \right]$$

Similar Frierzing algorithm
Get *factorized* Hadronic tensor

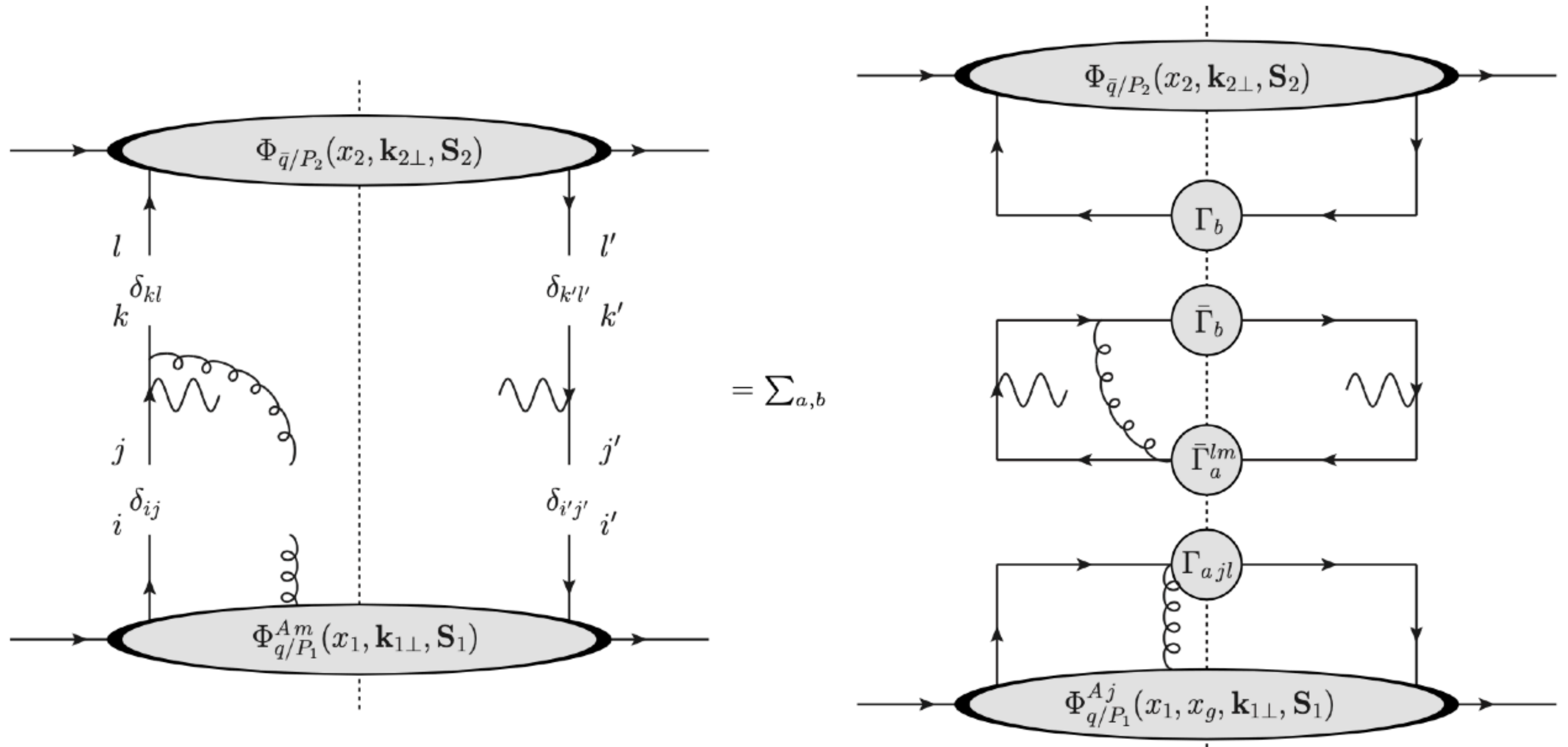


FIG. 4. Fierz decomposition of the dynamic sub-leading contribution to the cross section. In this graph, m represents a transverse Lorentz index.

DY/SIDIS tree-level diagrams relevant for sub-leading-power observables.
diagrams "*dynamical*" *qgq* contributions with

Tree level factorization sub-leading power

$$\Phi^{\rho}_{A}(x, x_g, \mathbf{k}_\perp, \mathbf{S})$$

SIDIS tree-level diagrams relevant for sub-leading-power observables. diagrams "*dynamical*" contributions with

Subleading Quark-Gluon-Quark TMDPDFs

Nucleon Polarization		Quark Chirality: Chiral Even	Quark Chirality: Chiral Odd
	U	$\tilde{f}^\perp, \tilde{g}^\perp$	$\tilde{e}\,, \tilde{h}$
	L	$\tilde{f}_L^\perp, \tilde{g}_L^\perp$	$\tilde{e}_L, \tilde{h}_L$
	T	$\tilde{f}_T, \tilde{f}_T^\perp, \tilde{g}_T, \tilde{g}_T^\perp$	$\tilde{e}_T, \tilde{e}_T^\perp, \tilde{h}_T, \tilde{h}_T^\perp$

Generalization of

- ✦Mulders Tangerman NPB1995
- ✦ Boer Pijlman Mulders NPB 2003
- ✦Bacchetta et al 2007 JHEP

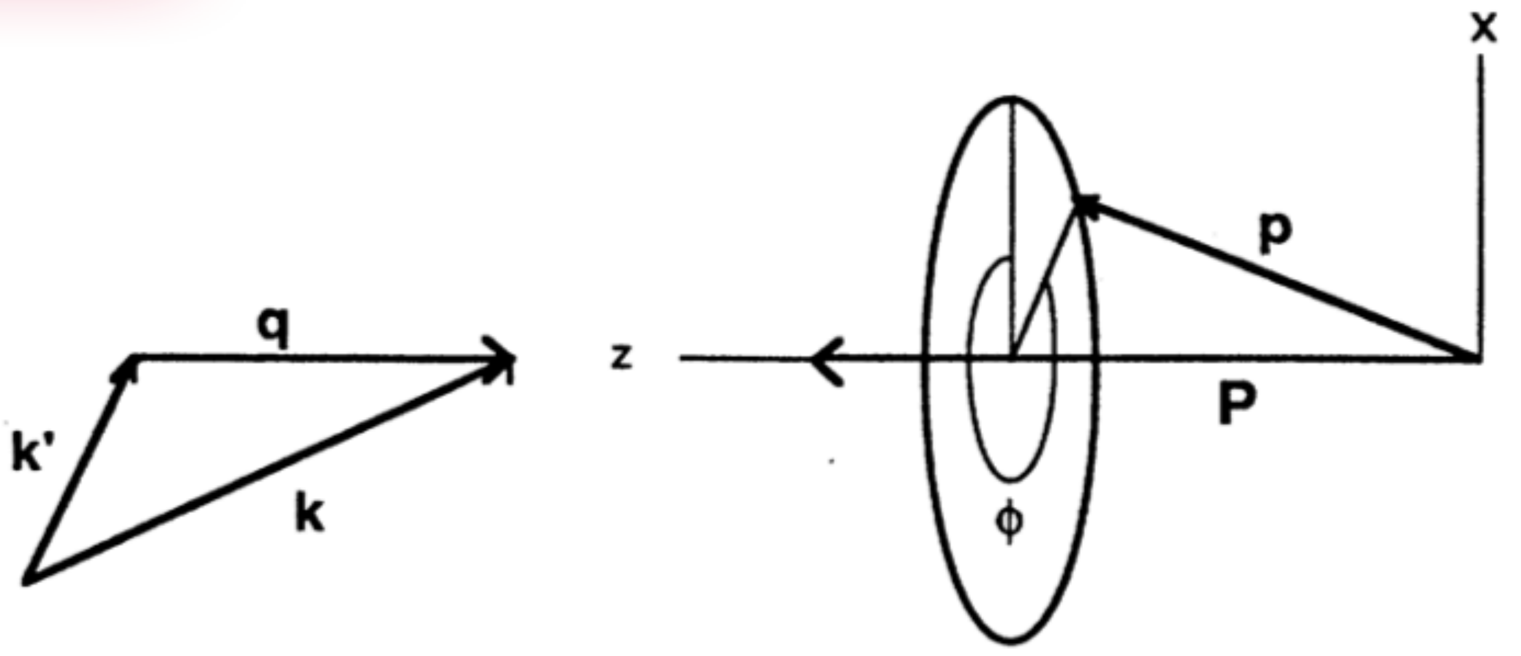
$$\begin{aligned}
&x_g P^+ \Phi^i_A(x, x_g, \boldsymbol{k}_\perp, \boldsymbol{S}) = \\
&\frac{xM}{2}\Bigg\{\Bigg[\left(\tilde{f}^\perp - i\tilde{g}^\perp\right)\frac{k^i_\perp}{M} - \left(\tilde{f}'_T + i\tilde{g}'_T\right)\epsilon_{\perp\, jl} S^l_\perp \\
&\quad - \left(\lambda\tilde{f}^\perp_L - \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp}{M}\tilde{f}^\perp_T\right)\frac{\epsilon_{\perp\, jl}k^l_\perp}{M} - i\left(\lambda\tilde{g}^\perp_L - \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp}{M}\tilde{g}^\perp_T\right)\frac{\epsilon_{\perp\, jl}k^l_\perp}{M}\Bigg]\left(g^{ij}_\perp - i\epsilon^{ij}_\perp\gamma_5\right) \\
&\quad - \left[\left(\lambda\tilde{h}^\perp_L - \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp}{M}\tilde{h}^\perp_T\right) + i\left(\lambda\tilde{e}^\perp_L - \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp}{M}\tilde{e}^\perp_T\right)\right]\gamma^i_\perp\gamma_5
\end{aligned}$$

Tree level factorization sub-leading power

Combining these contributions and multiplying by leptonic tensor
get factorized Cahn and more ....

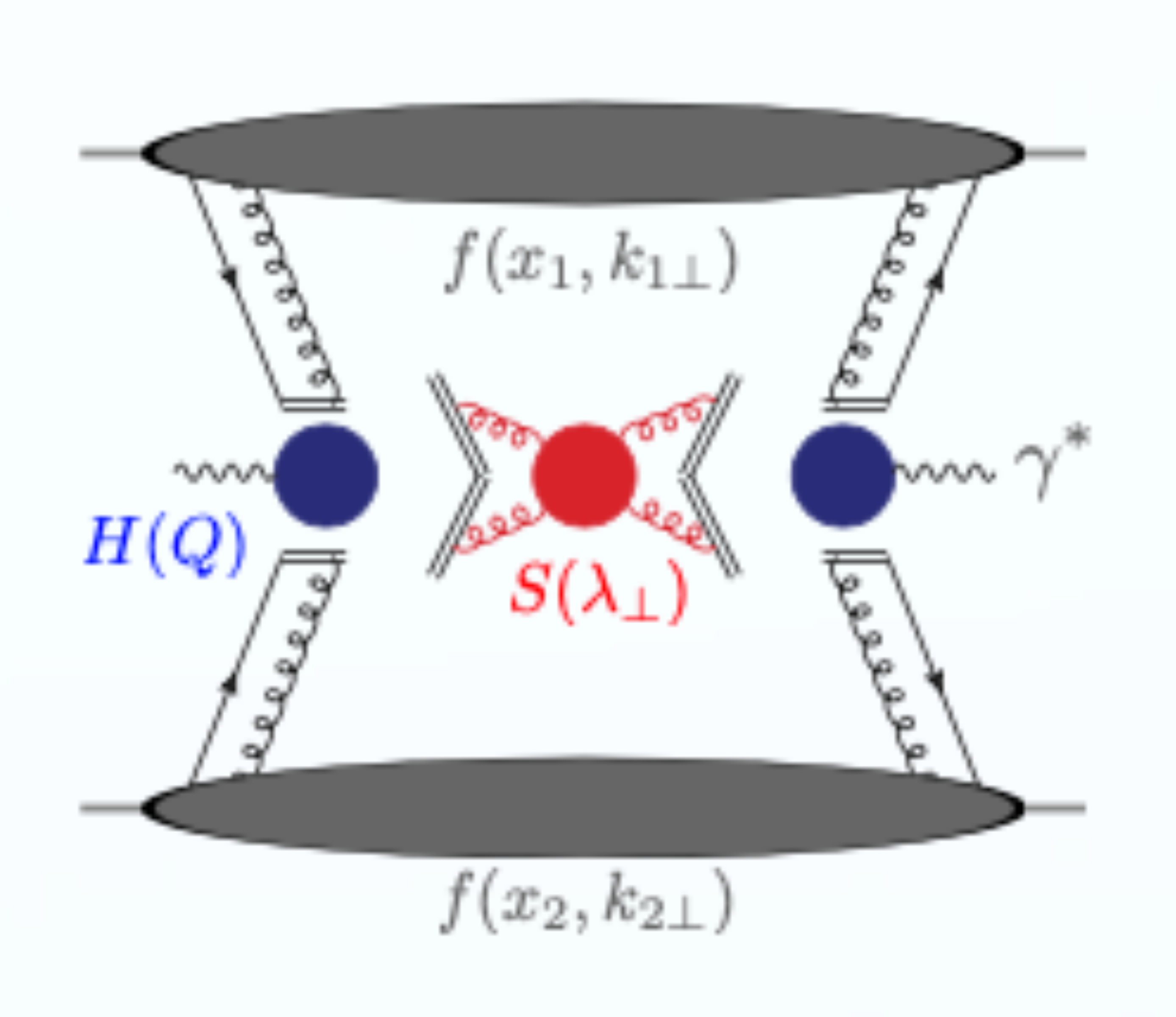
$$F^3_{\rm DIS}(x,z,\boldsymbol{P}_{h\perp}) = \mathcal{C}^{\rm DIS}\left[\frac{q_\perp}{Q} f_1\, D_1\right] - \mathcal{C}^{\rm DIS}\left[\left(x\,\frac{\boldsymbol{k}_\perp\cdot\hat{x}}{Q} f^\perp\right) D_1 - f_1\left(\frac{\boldsymbol{p}_\perp\cdot\hat{x}}{zQ} D^\perp\right)\right]$$
$$-\int\frac{dx_g}{x_g}\mathcal{C}^{\rm DIS}_{\rm dyn\,x_g}\left[\left(x\,\frac{\boldsymbol{k}_\perp\cdot\hat{x}}{Q}\tilde{f}^\perp\right) D_1\right] + \int\frac{dz_g}{z_g}\mathcal{C}^{\rm DIS}_{\rm dyn\,z_g}\left[f_1\left(\frac{\boldsymbol{p}_\perp\cdot\hat{x}}{zQ}\tilde{D}^\perp\right)\right],$$

Cahn intrinsic $\boldsymbol{k}_T$



Gamberg, Kang, Shao, Terry, Zhao arXiv: e-Print:221.13209

TMD factorization at NLO and NLP



$q_T \sim k_T \ll Q$

TMD Factorization

- *Collins Soper Sterman NPB 1985*
- *Ji Ma Yuan PRD PLB …2004, 2005*
- *Aybat Rogers PRD 2011*
- *Collins 2011 Cambridge Press*
- *Echevarria, Idilbi, Scimemi JHEP 2012, …*
- *SCET Becher & Neubert, 2011 EJPC*

$$\frac{d\sigma^W}{dQ^2\, dx_F\, dp_T^2} = \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2}\, e^{i\boldsymbol{p}_T\cdot\boldsymbol{b}_T}\tilde{W}(x_F, b_T, Q)$$

$$\tilde{W}(x_F, b_T, Q) = \sum_j H_{j\bar{j}}^{\rm DY}(Q, \mu, a_s(\mu))\tilde{f}_{j/A}(x_A, b_T; \zeta_A, \mu)\tilde{f}_{\bar{j}/B}(x_B, b_T; \zeta_B, \mu)$$

NLO and NLP

$$
\begin{aligned}
F^{3}_{\mathrm{DIS}}(x, z, \boldsymbol{P}_{h\perp}) = {} & H^{\mathrm{LP}}_{\mathrm{DIS}}(Q;\mu)\, \mathcal{C}^{\mathrm{DIS}}\left[\frac{q_\perp}{Q} f_1\, D_1\, \mathcal{S}^{\mathrm{LP}}\right] \\
& - H^{\mathrm{kin}}_{\mathrm{DIS}}(Q;\mu)\, \mathcal{C}^{\mathrm{DIS}}\left[\left(\frac{\boldsymbol{k}_\perp \cdot \hat{x}}{Q} f_1\, D_1 - \frac{\boldsymbol{p}_\perp \cdot \hat{x}}{Q} f_1\, D_1\right) \mathcal{S}^{\mathrm{kin}}\right] \\
& - 2\int \frac{dx_g}{x_g} H^{\mathrm{dyn}}_{\mathrm{DIS}}(x_g, Q;\mu)\, \mathcal{C}^{\mathrm{DIS}}_{\mathrm{dyn}\,\mathrm{x_g}}\left[x\, \frac{\boldsymbol{k}_\perp \cdot \hat{x}}{Q} \tilde{f}^{\perp}\, D_1\, \mathcal{S}^{\mathrm{dyn}}\right] \\
& + 2\int \frac{dz_g}{z_g} H^{\mathrm{dyn}}_{\mathrm{DIS}}(z_g, Q;\mu)\, \mathcal{C}^{\mathrm{DIS}}_{\mathrm{dyn}\,\mathrm{z_g}}\left[\frac{\boldsymbol{p}_\perp \cdot \hat{x}}{zQ} f_1\, \tilde{D}^{\perp} \mathcal{S}^{\mathrm{dyn}}\right],
\end{aligned}
$$

where H^{kin} and $\mathcal{S}^{\mathrm{kin}}$ are the kinematic sub-leading hard and soft functions.

Ingredients hard factor

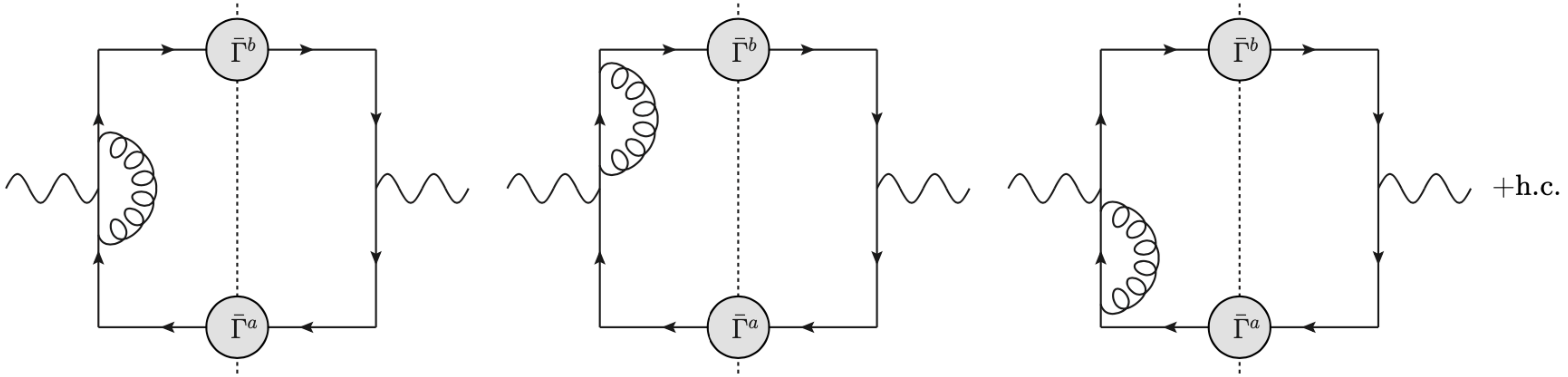
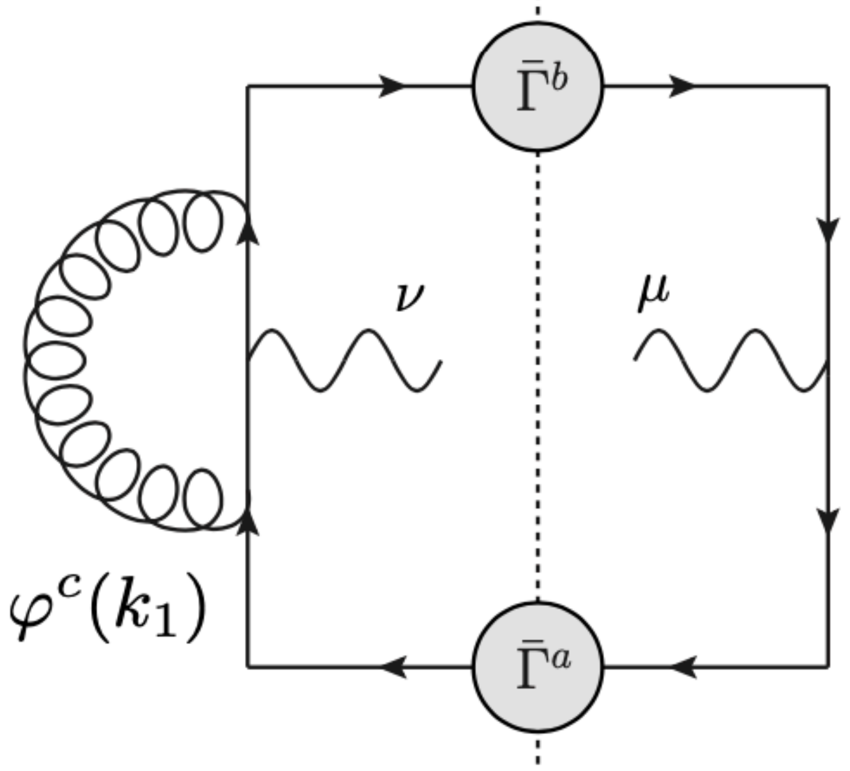


FIG. 9. The one loop diagrams for the hard contribution to the SIDIS cross section: the vertex correction (left), the self-energy diagram (middle and right), as well as their hermitian conjugates (denoted as "h.c.").

NLO and NLP hard factor

The hard region

The hard contribution can be obtained using the DY form factor $\gamma^\nu \to F^\nu(Q;\mu)$



$$
\begin{aligned}
F^\nu(Q;\mu) = {} & \gamma^\nu \left(1 + \frac{1}{2\epsilon} - L_Q\right) + \left(\frac{2}{\epsilon} - 4L_Q + 3\right)\frac{\not{n}\gamma^\nu\not{\bar{n}}}{4} \\
& + \left(-\frac{1}{\epsilon^2} - 2L_Q^2 + \frac{2}{\epsilon}L_Q - \frac{1}{\epsilon} + 2L_Q + \frac{\pi^2}{12} - 3\right)\frac{\not{\bar{n}}\gamma^\nu\not{n}}{4} + \left(2L_Q - \frac{1}{\epsilon} - 1\right)\frac{\not{\bar{n}}n^\nu}{4} \\
& + \left(4L_Q - \frac{2}{\epsilon} - 3\right)\frac{\not{n}\bar{n}^\nu}{4} + \left(2L_Q - \frac{1}{\epsilon} - 1\right)\frac{\not{n}n^\nu}{Q^2} + \left(4L_Q - \frac{2}{\epsilon} - 3\right)\frac{\not{\bar{n}}n^\nu}{4},
\end{aligned}
$$

Double pole vanishes for one amplitude

The insertion of a sub-leading operator alters the divergences

$$
\hat{H}^{\mathrm{LP}}(Q;\mu) = 1 + \frac{\alpha_s C_F}{2\pi}\left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 4L_Q^2 + \frac{4L_Q}{\epsilon} + 6L_Q + \frac{7\pi^2}{6} - 8\right]
$$

$J^\nu = \bar{\chi}^{\bar{c}}(x)\gamma_\perp^\nu\chi^c(x) + \text{conjugate}$

$$
\hat{H}^{\mathrm{NLP}}(Q;\mu) = 1 + \frac{\alpha_s C_F}{2\pi}\left[-\frac{1}{\epsilon^2} - \frac{2}{\epsilon} - 2L_Q^2 + \frac{2L_Q}{\epsilon} + 4L_Q + \frac{7\pi^2}{12} - 5\right]
$$

$J^\nu = \bar{\chi}^{\bar{c}}(x)\gamma^\nu\frac{\not{n}\not{\bar{n}}}{4}\varphi^c(x) + \text{perms}$

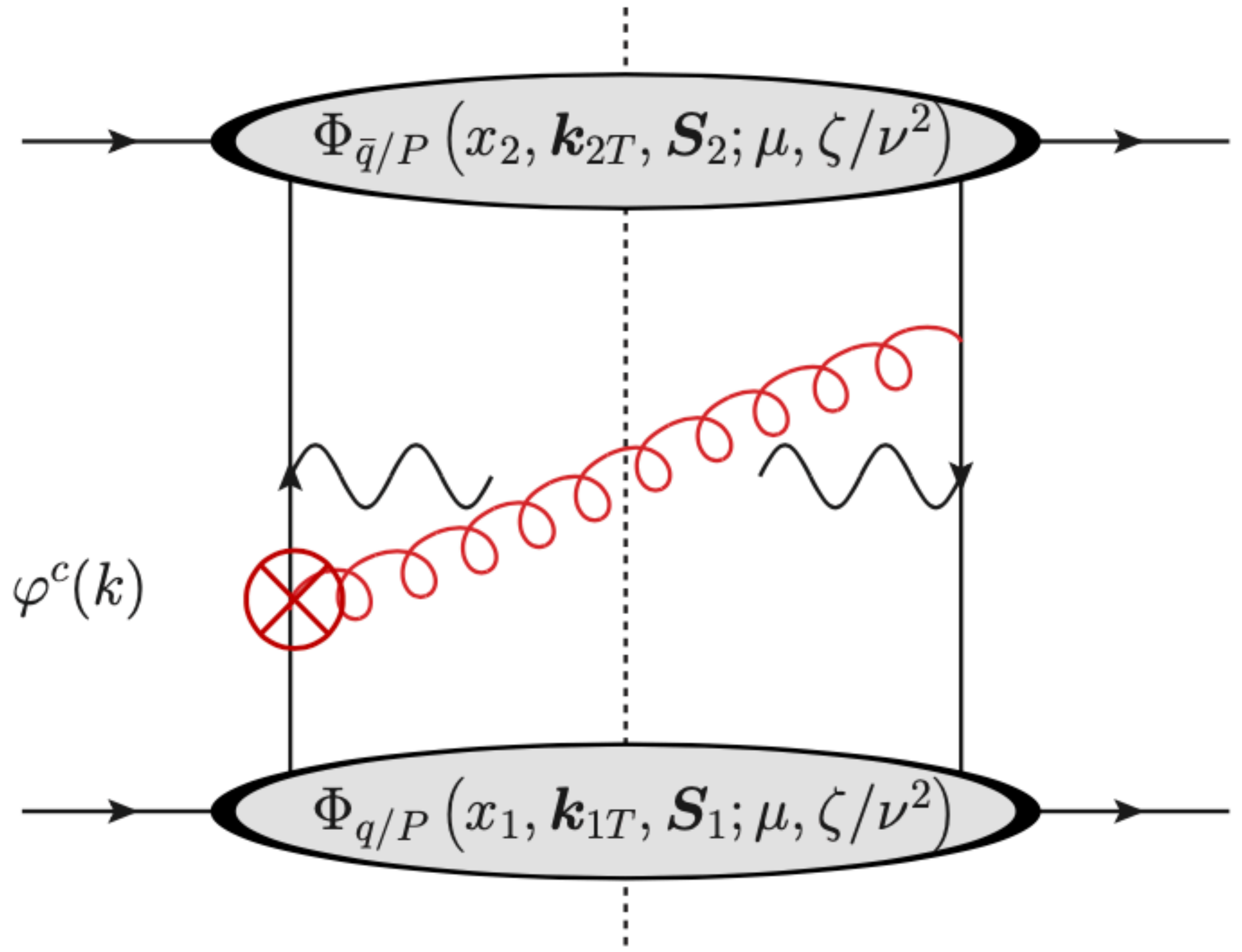
Double poles differ from those at LP. Issue enters due to current.

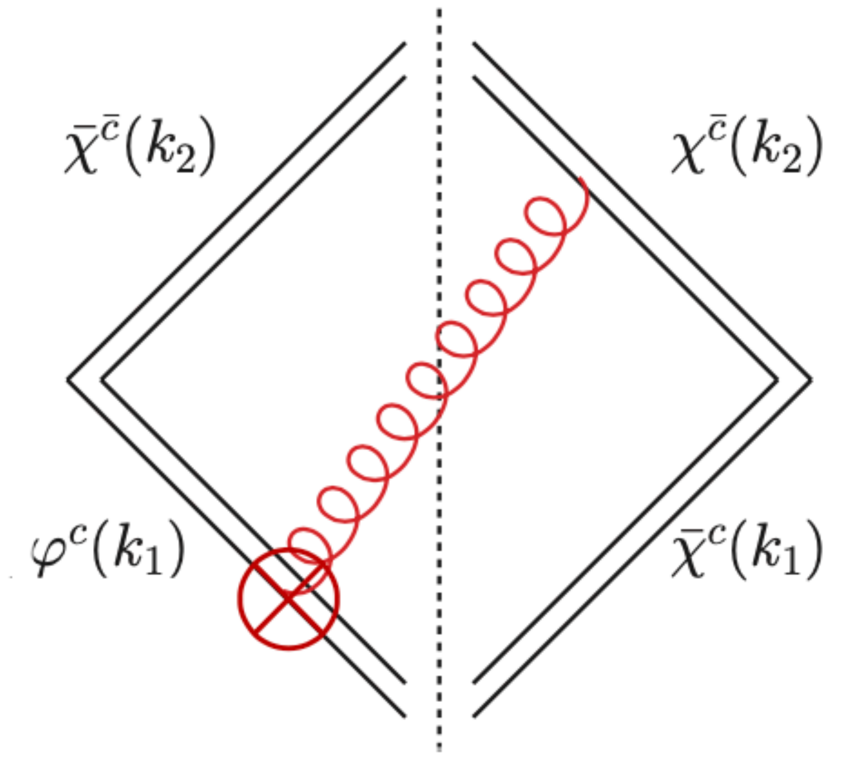
Thanks to John Terry for slide

NLO and NLP

The soft region

The soft function is generated through the emissions of soft gluons in the partonic cross section





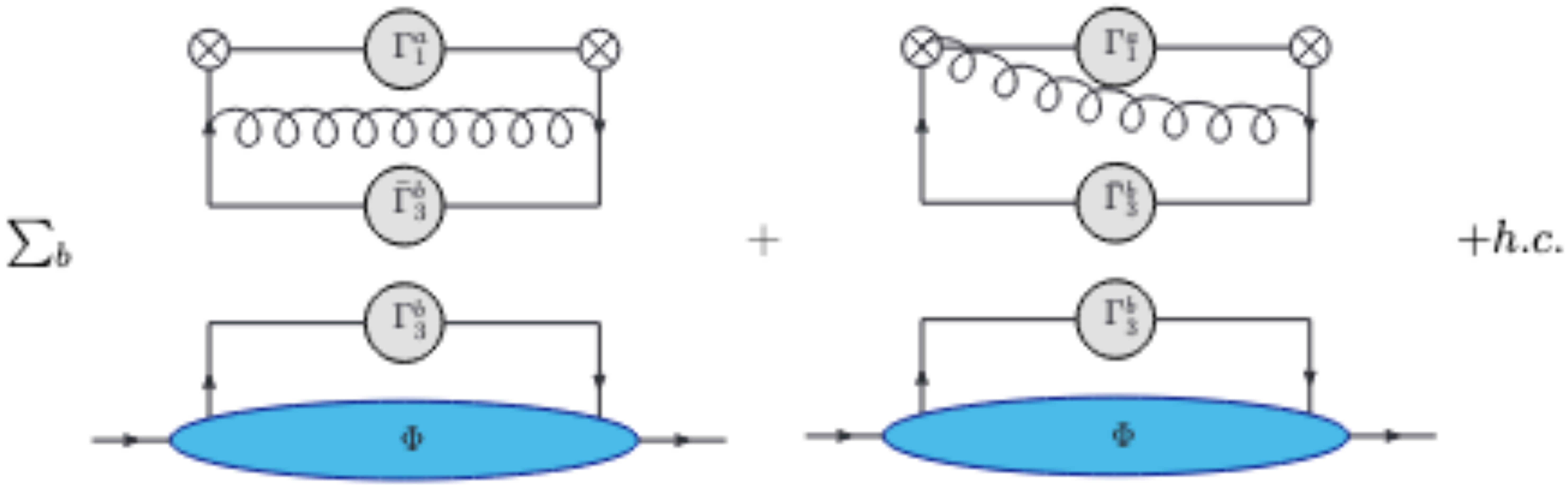
Soft emission from the sub-leading fields vanishes. NLO+NLP soft function is half the LP one

$$\Gamma^{\mu}_{\mathcal{S}\,\mathrm{int}} = \frac{1}{2}\Gamma^{\mu}_{\mathcal{S}\,\mathrm{LP}}\,, \qquad \Gamma^{\nu}_{\mathcal{S}\,\mathrm{int}} = \frac{1}{2}\Gamma^{\nu}_{\mathcal{S}\,\mathrm{LP}}$$

Double poles match what is required for RG consistency from the hard contribution.

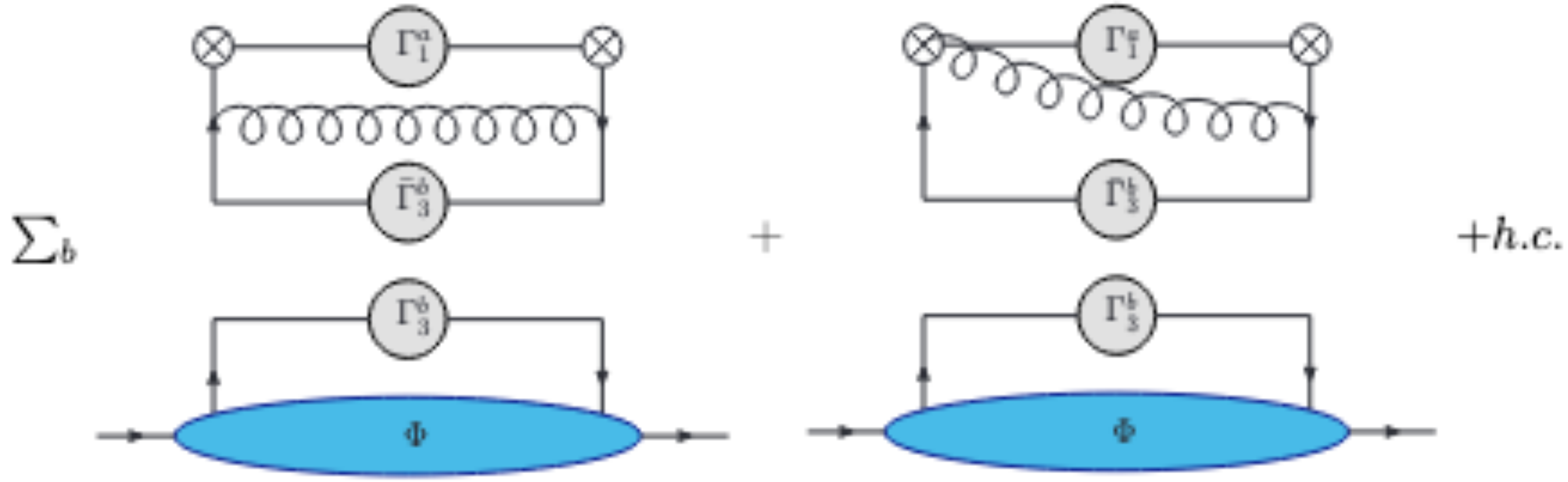
Thanks to John Terry for slide

Look under the hood rapidity and UV subtracted TMDs
Collins-Soper Equations determine rapidity and UV anomalous dimensions



To carry out analysis for intrinsic sub-leading distributions, we re-express quark fields in correlator in terms of good and bad field components $\psi(x) = \chi(x) + \phi(x)$
Impacts calculation of the gauge links and soft factors at NLP

As a consequence rapidity and UV subtracted TMDs obey Collins-Soper Equations & we can determine rapidity and UV anomalous dimensions

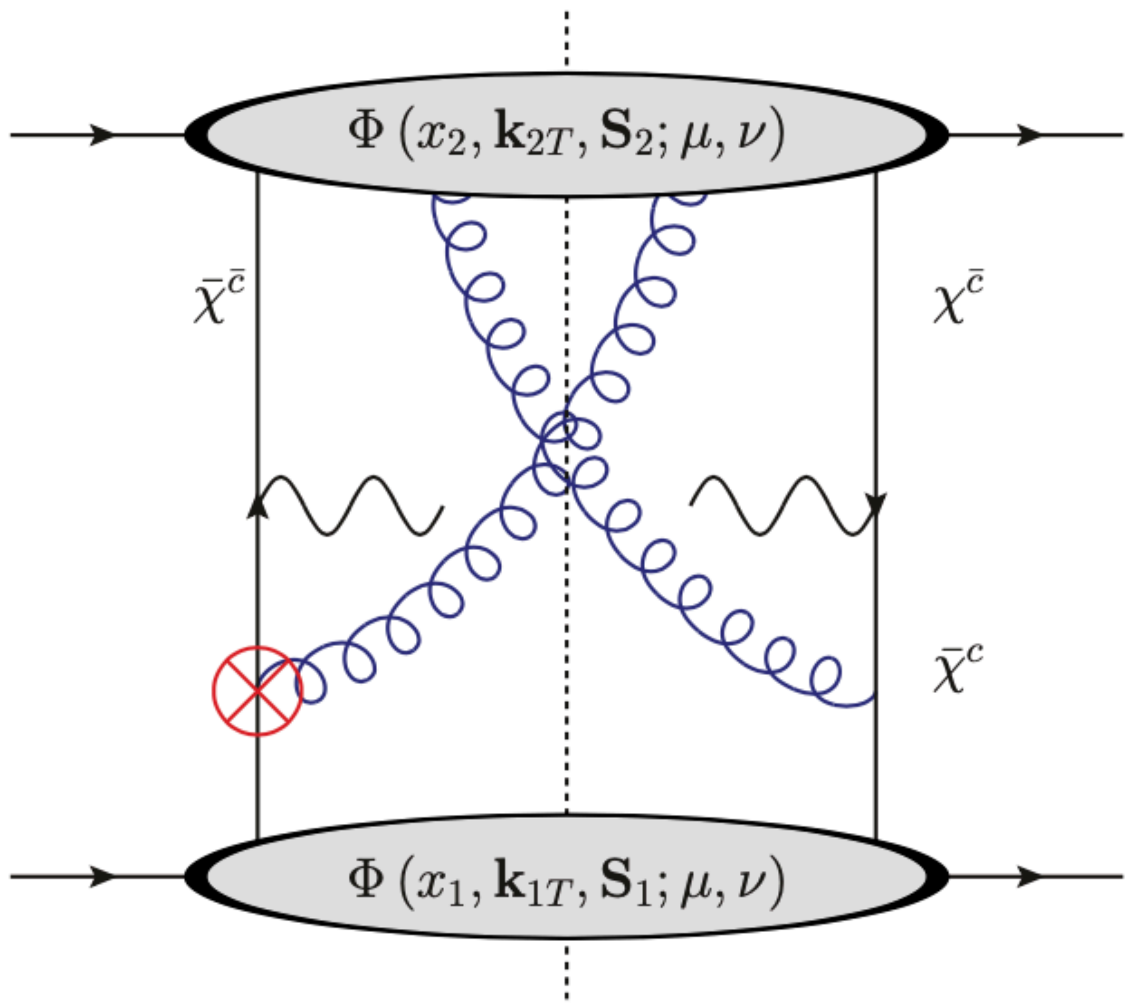


$$\frac{\partial}{\partial \ln \nu} \begin{bmatrix} \Phi^{[\not{n}]} \\ \Phi^{[\not{n}\gamma^5]} \\ \Phi^{[i\sigma^{k'+}\gamma^5]} \\ \Phi^{[1]} \\ \Phi^{[\gamma^5]} \\ \Phi^{[\gamma^{k'}]} \\ \Phi^{[\gamma^{k'}\gamma^5]} \\ \Phi^{[i\sigma^{k'l'}\gamma^5]} \\ \Phi^{[i\sigma^{+-}\gamma^5]} \end{bmatrix} = \frac{\alpha_s C_F}{2\pi} \mathbf{\Gamma}^{\nu} \begin{bmatrix} \Phi^{[\not{n}]} \\ \Phi^{[\not{n}\gamma^5]} \\ \Phi^{[i\sigma^{k+}\gamma^5]} \\ \Phi^{[1]} \\ \Phi^{[\gamma^5]} \\ \Phi^{[\gamma^{k}]} \\ \Phi^{[\gamma^{k}\gamma^5]} \\ \Phi^{[i\sigma^{kl}\gamma^5]} \\ \Phi^{[i\sigma^{+-}\gamma^5]} \end{bmatrix}$$

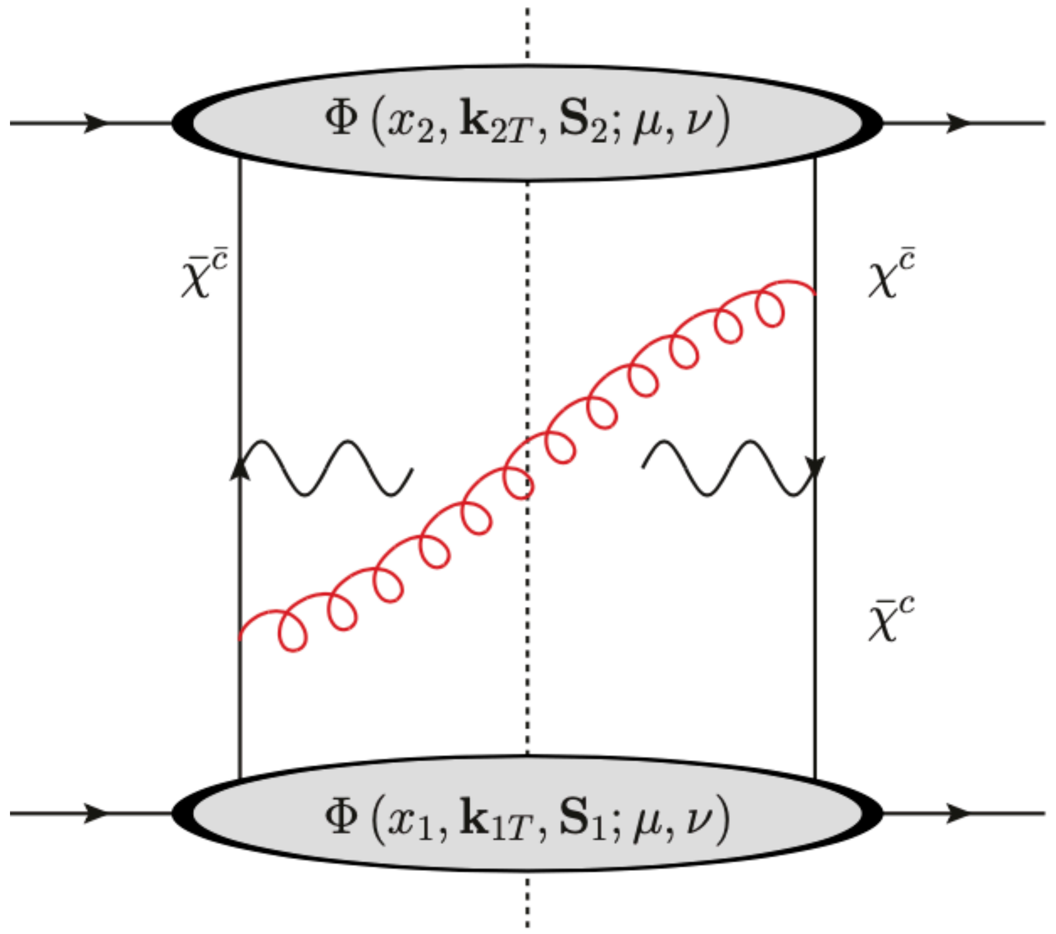
$$\frac{\partial}{\partial \ln \mu} \begin{bmatrix} \Phi^{[\not{n}]} \\ \Phi^{[\not{n}\gamma^5]} \\ \Phi^{[i\sigma^{k'+}\gamma^5]} \\ \Phi^{[1]} \\ \Phi^{[\gamma^5]} \\ \Phi^{[\gamma^{k'}]} \\ \Phi^{[\gamma^{k'}\gamma^5]} \\ \Phi^{[i\sigma^{k'l'}\gamma^5]} \\ \Phi^{[i\sigma^{+-}\gamma^5]} \end{bmatrix} = \frac{\alpha_s C_F}{2\pi} \mathbf{\Gamma}^{\mu} \begin{bmatrix} \Phi^{[\not{n}]} \\ \Phi^{[\not{n}\gamma^5]} \\ \Phi^{[i\sigma^{k+}\gamma^5]} \\ \Phi^{[1]} \\ \Phi^{[\gamma^5]} \\ \Phi^{[\gamma^{k}]} \\ \Phi^{[\gamma^{k}\gamma^5]} \\ \Phi^{[i\sigma^{kl}\gamma^5]} \\ \Phi^{[i\sigma^{+-}\gamma^5]} \end{bmatrix}$$

RG consistency established

$$\frac{d\sigma}{d\ln\nu} = 0 \quad \& \quad \frac{d\sigma}{d\ln\mu} = 0$$



$$\gamma^{\mu}_{H_{NL}} + \gamma^{\mu}_{S_{NLP}} + \gamma^{\mu}_{f_1^{kin}} + \gamma^{\mu}_{f^{\perp}} = 0$$



$$\gamma^{\nu}_{S_{NLP}} + \gamma^{\nu}_{f_1^{kin}} + \gamma^{\nu}_{f^{\perp}} = 0$$

The diagrams which give rise to the soft function at NLO+LP in Drell-Yan.

Summary

- We have revisited TMD factorization beyond leading power and beyond leading order in terms of intrinsic TMDs
- We are able to establish RG consistency and consistency of the EOM beyond leading order
- In doing so, we provide the basis for improved phenomenology of one the earliest observables used to study the intrinsic 3-D momentum structure of the nucleon—important observables for EIC study of nucleon
- Comparison of the work of Bacchetta et al. 2019 & Chen 2017 and others Vladimirov et al. and Gao/Stewart

Extras Data General features

$$\frac{\mathrm{d}\sigma^{ep\to ehX}}{\mathrm{d}\phi} = \mathcal{A} + \mathcal{B}\cos\phi + \mathcal{C}\cos 2\phi + \mathcal{D}\sin\phi + \mathcal{E}\sin 2\phi$$

EMC collaboration Phys. Lett. B 130 (1983) 118, & Z. Phys. C 34 (1987) 277

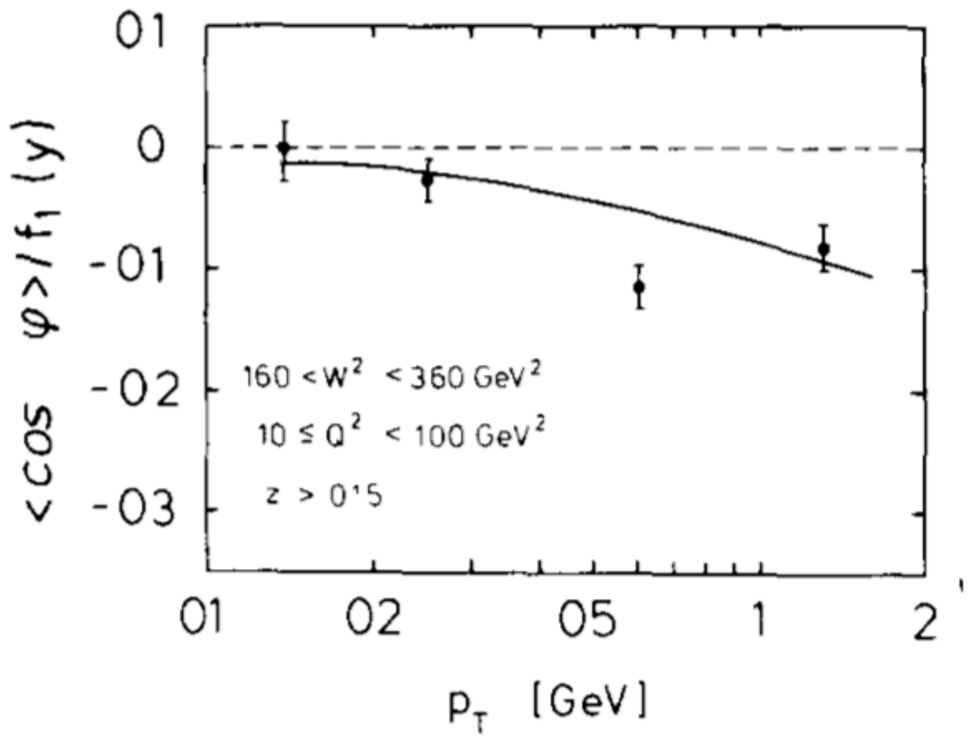
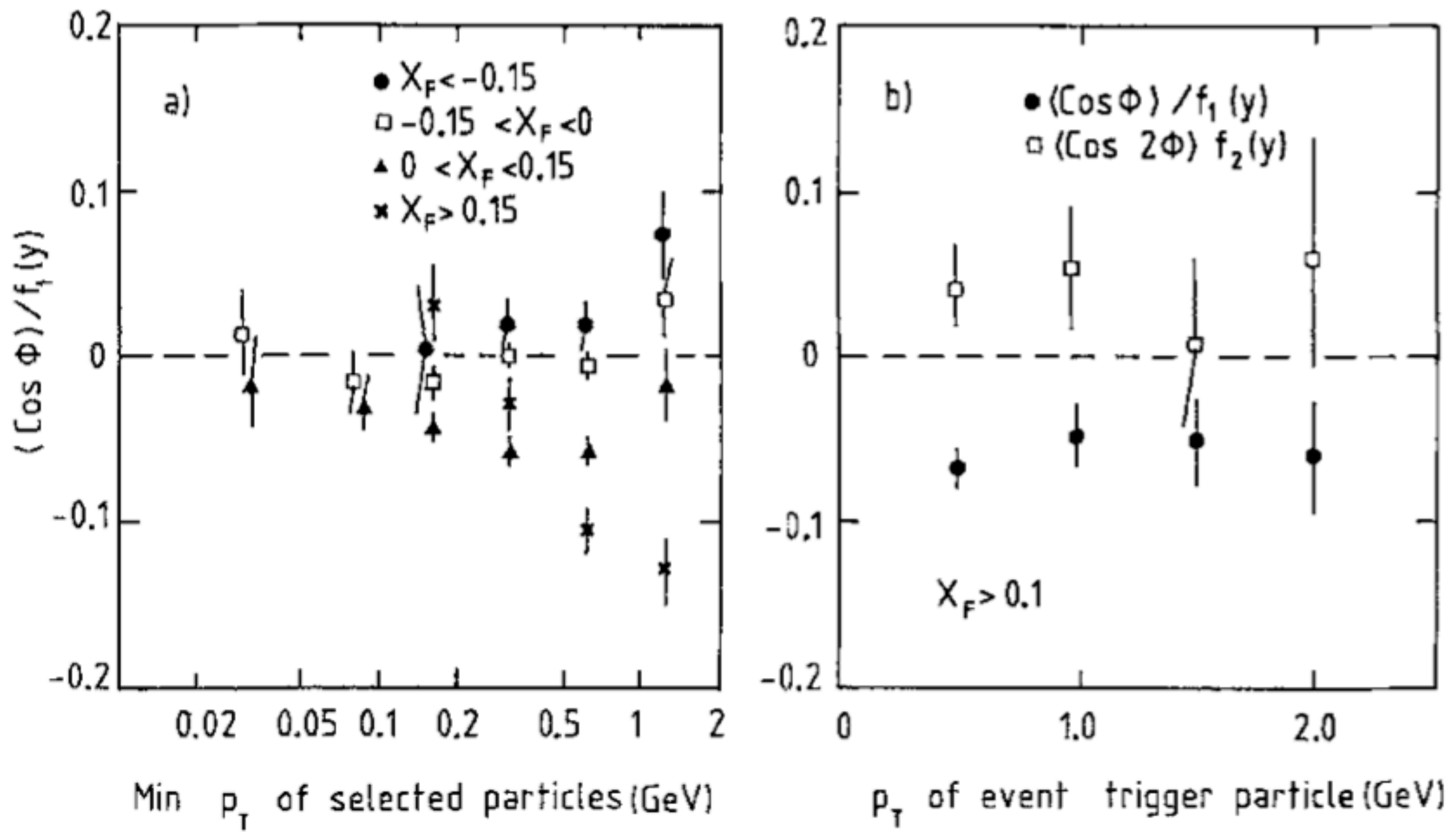
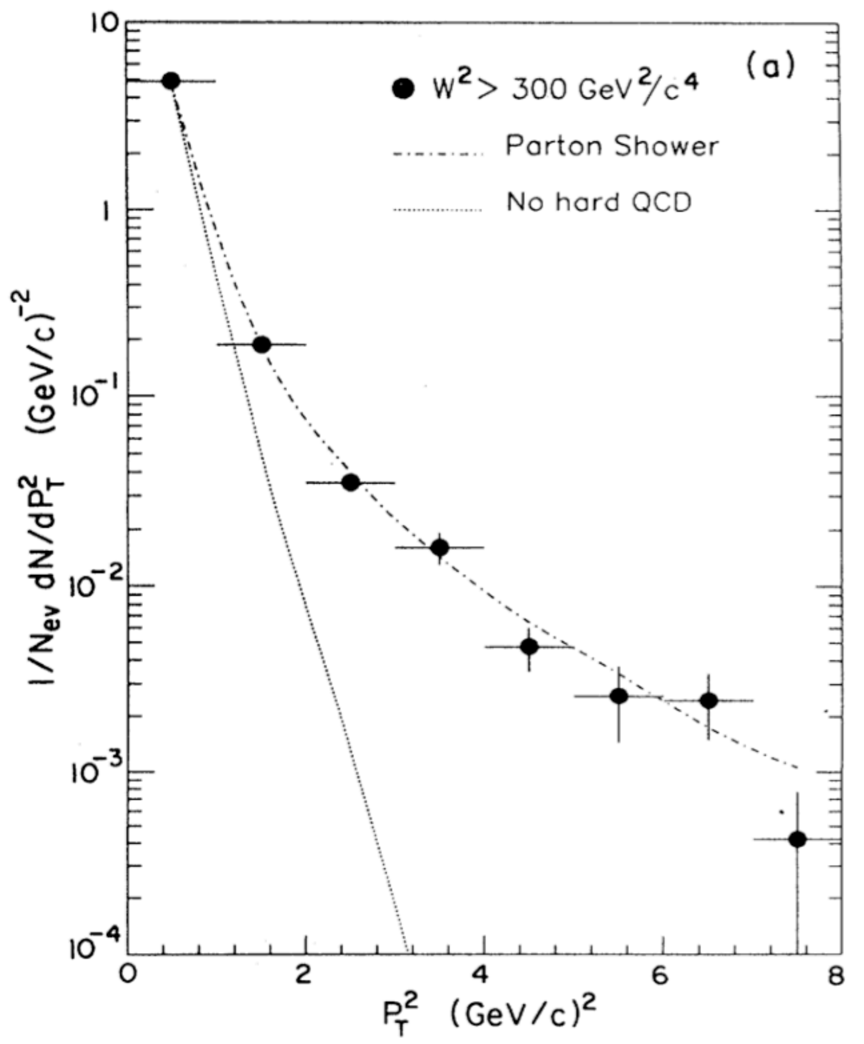


Fig 4 p_T dependence ($p_T > 50$ MeV) of cos φ moment for $160 \leqslant W^2 < 360$ GeV^2, $Q^2 > 10$ GeV^2 and $z > 0.15$ compared with model calculations described in ref [8] (statistical errors on model curve from Monte Carlo ±0 03 not shown)

M. Arneodo et al.: Measurement of Hadron Azimuthal Distributions



E665 Phys. Rev. D 48 (1993) 5057



Phys. Lett. B 481 (2000)

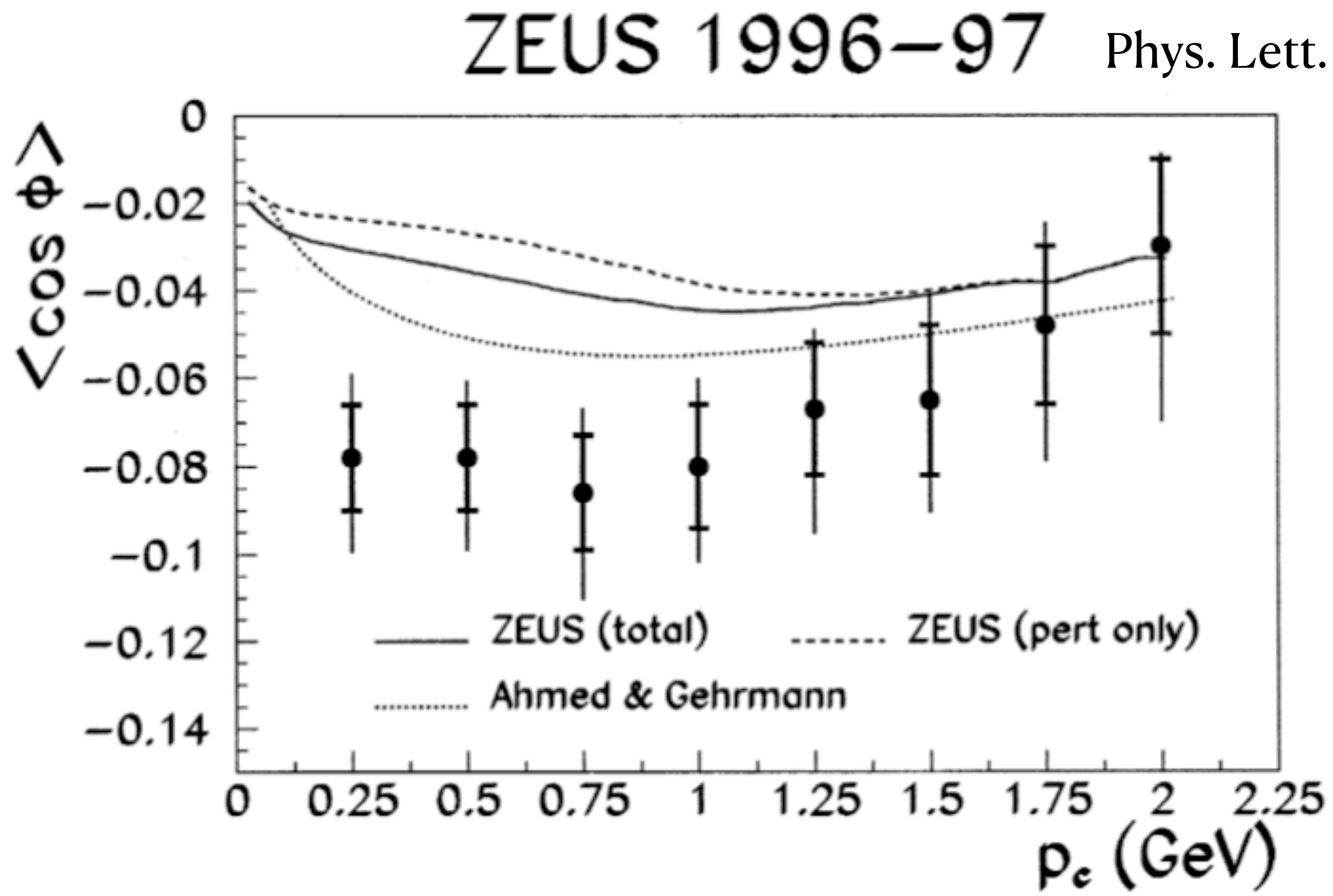
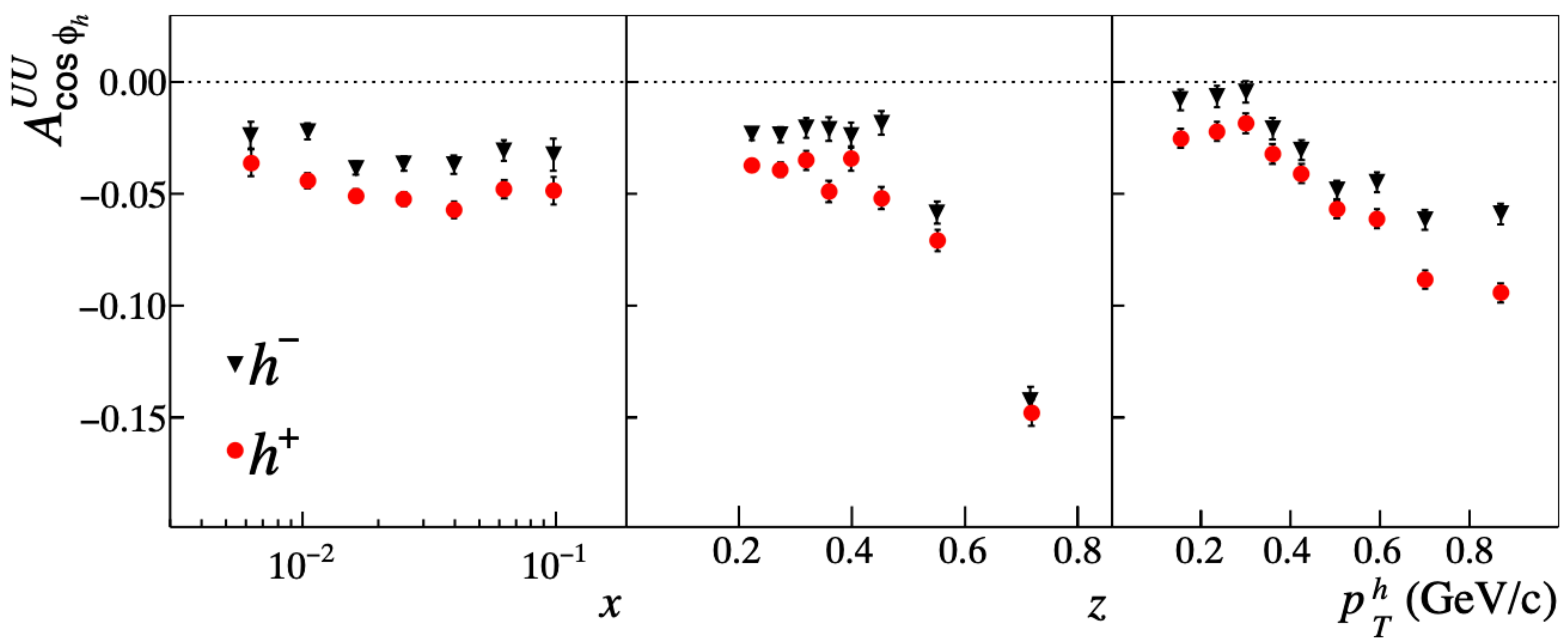


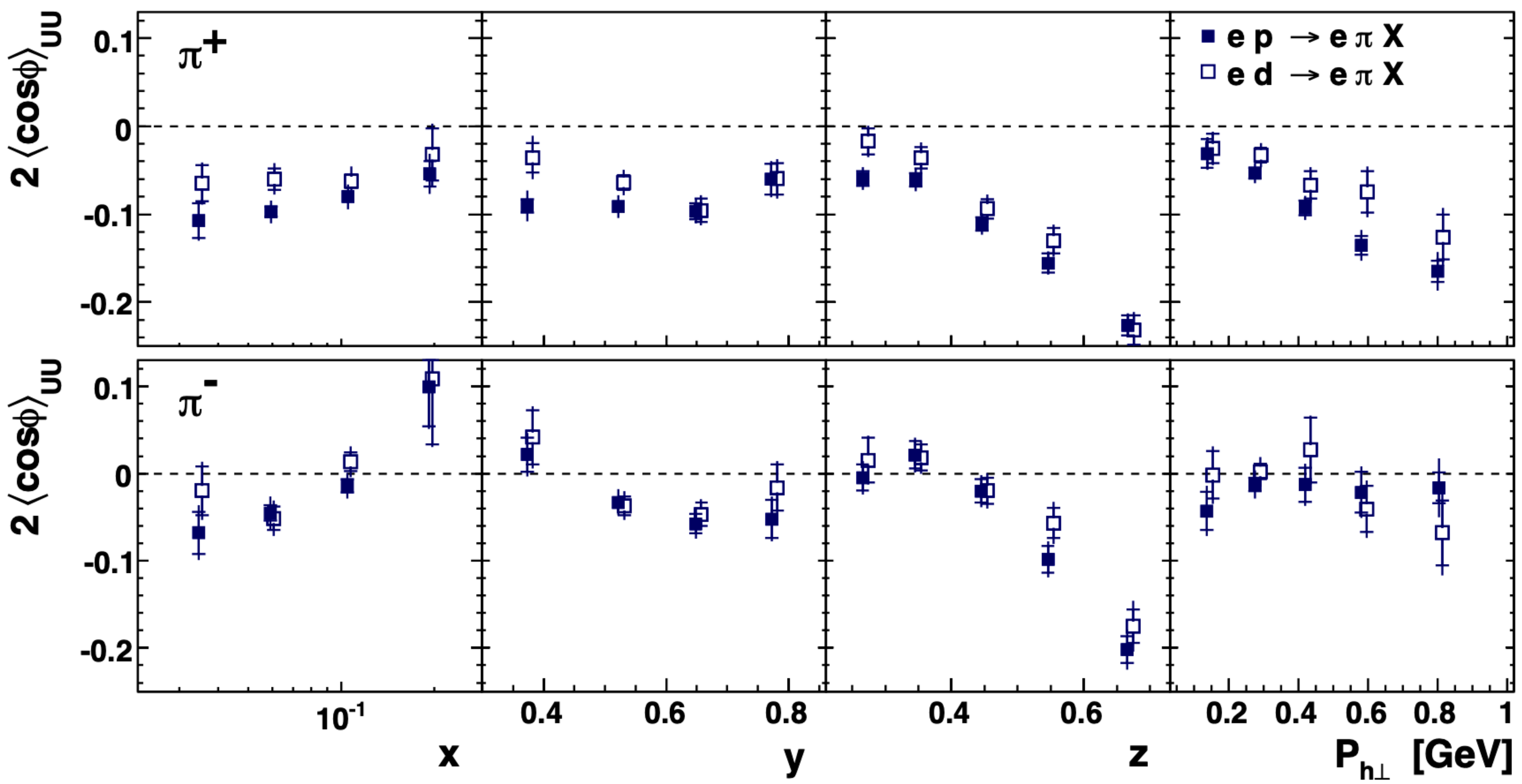
Fig. 4. The values of $\langle \cos\phi \rangle$ and $\langle \cos 2\phi \rangle$ are shown as a function of p_c in the kinematic region $0.01 < x < 0.1$ and $0.2 < y < 0.8$ for charged hadrons with $0.2 < z_h < 1.0$. The inner error bars represent the statistical errors, the outer are statistical and systematic errors added in quadrature. The lines are the LO predictions from ZEUS with perturbative and non-perturbative contributions (full line), ZEUS with the perturbative contribution only (dashed line) and Ahmed & Gehrmann (dotted line – see text

$$\frac{d\sigma^{ep\to ehX}}{d\phi} = \mathcal{A} + \mathcal{B}\cos\phi + \mathcal{C}\cos 2\phi + \mathcal{D}\sin\phi + \mathcal{E}\sin 2\phi$$

COMPASS, Nucl. Phys. B 886 (2014) 1046



HERMES, Phys. Rev. D 87 (2013) 012010



Parton model pheno studies

$$\langle \cos\phi \rangle = \frac{\int d\sigma^{(0)} \cos\phi + \int d\sigma^{(1)} \cos\phi}{\int d\sigma^{(0)} + \int d\sigma^{(1)}}$$

Larger than 2 GeV

$$\langle \cos\phi \rangle = \frac{\int d\sigma^{(1)} \cos\phi}{\int d\sigma^{(1)}}$$

Chay, S.D. Ellis, Stirling, Phys. Lett. B (1991)

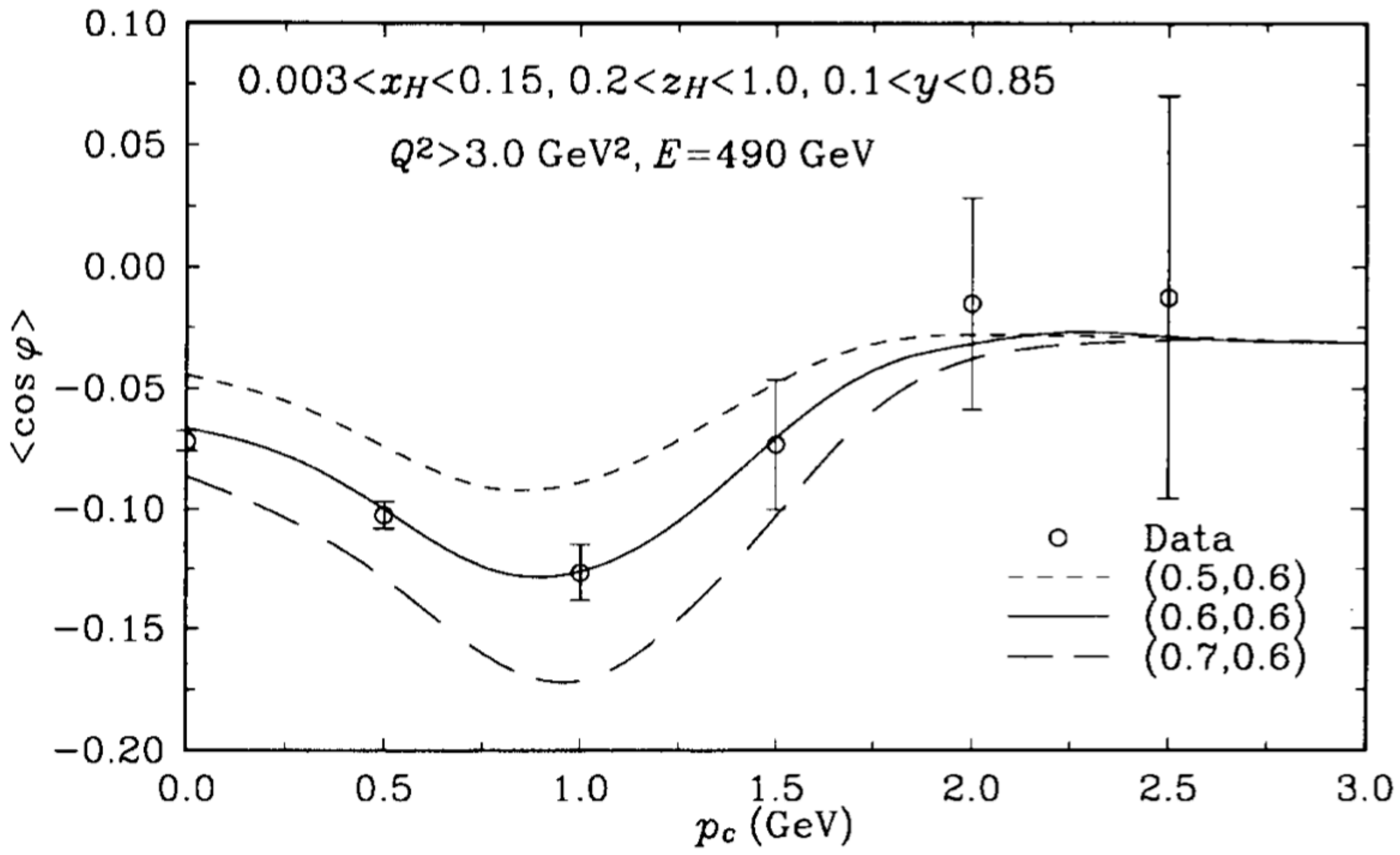
Oganessyan, Avakian, Bianchi, EPJC (1998)

$$\int d\sigma^{(0)} = 2\pi \frac{\alpha^2}{Q^2} \sum_j Q_j^2 F_j(x_H) D_j(z_H) \exp\left(-\frac{p_c^2}{b^2 + z_H^2 a^2}\right) \times \left\{ \frac{1+(1-y)^2}{y} + 4\frac{1-y}{yQ^2}\left[\frac{a^2 b^2}{b^2 + z_H^2 a^2} + \left(\frac{z_H a^2}{b^2 + z_H^2 a^2}\right)^2 (p_c^2 + b^2 + z_H^2 a^2)\right]\right\}$$

$$\int d\sigma^{(1)} \cos\phi = \int d^2 P_T \cos\phi \frac{d\sigma}{dx_H\, dy\, dz_H\, d^2 P_T} = \frac{8}{3}\frac{\alpha_s \alpha^2}{Q^2}\frac{(2-y)\sqrt{1-y}}{y}\int_{x_H}^1 \frac{dx}{x}\int_{z_H}^1 \frac{dz}{z}\sum_j Q_j^2 (A_j + B_j + C_j)$$

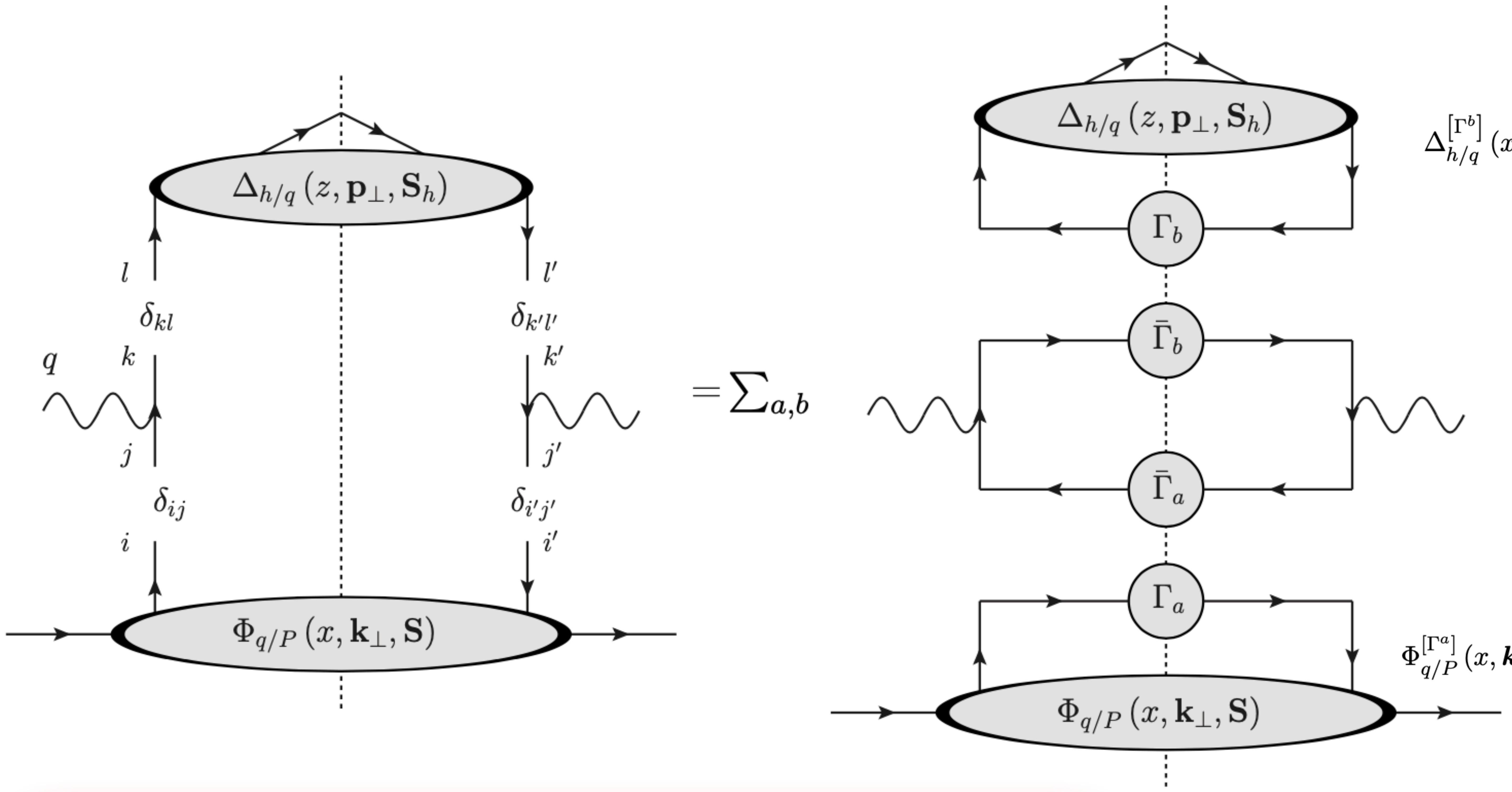
$$A_j = -\sqrt{\frac{xz}{(1-x)(1-z)}}\,[xz + (1-x)(1-z)]\, F_j\left(\frac{x_H}{x}, Q^2\right) D_j\left(\frac{z_H}{z}, Q^2\right)$$

Etc. ...



$\langle \cos\phi \rangle$ as a function of transverse momentum cutoff non-perturbative Cahn-like effect negligible at large values of p_c because assumed intrinsic transverse momentum in distribution and fragmentation functions are too small to produce $P_T > p_c$ (data E665 Fermi-lab).

Factorization at sub-leading power revisit Tree level Fierz decomposition



$$\Delta_{h/q}^{[\Gamma^b]}(x_1, \boldsymbol{p}_\perp, \boldsymbol{S}_h) = \mathrm{Tr}\left[\Delta_{h/q}(z, \boldsymbol{p}_\perp, \boldsymbol{S}_h)\,\Gamma^b\right]$$

$$\Phi_{q/P}^{[\Gamma^a]}(x, \boldsymbol{k}_\perp, \boldsymbol{S}) = \mathrm{Tr}\left[\Phi_{q/P}(x, \boldsymbol{k}_\perp, \boldsymbol{S})\,\Gamma^a\right]$$

Factorized !!

$$W_{\mu\nu}^{(2)} = \frac{1}{N_c}\sum_{a,b}\mathrm{Tr}\left[\gamma^\mu\,\bar{\Gamma}^a\,\gamma^\nu\,\bar{\Gamma}^b\right]\mathcal{C}^{\mathrm{DIS}}\left[\Phi^{[\Gamma^a]}(x, \boldsymbol{k}_\perp, \boldsymbol{S})\;\Delta^{[\Gamma^b]}(z, \boldsymbol{p}_\perp, \boldsymbol{S}_h)\right]$$

$$\mathcal{C}^{\mathrm{DIS}}[A\,B] = \sum_q e_q^2 \int d^2\boldsymbol{k}_\perp\, d^2\boldsymbol{p}_\perp\, \delta^{(2)}(\boldsymbol{q}_\perp + \boldsymbol{k}_\perp + \boldsymbol{p}_\perp/z) \;\times A_{q/P}(x, \boldsymbol{k}_\perp, \boldsymbol{S})\, B_{h/q}(z, \boldsymbol{p}_\perp, \boldsymbol{S}_h)$$

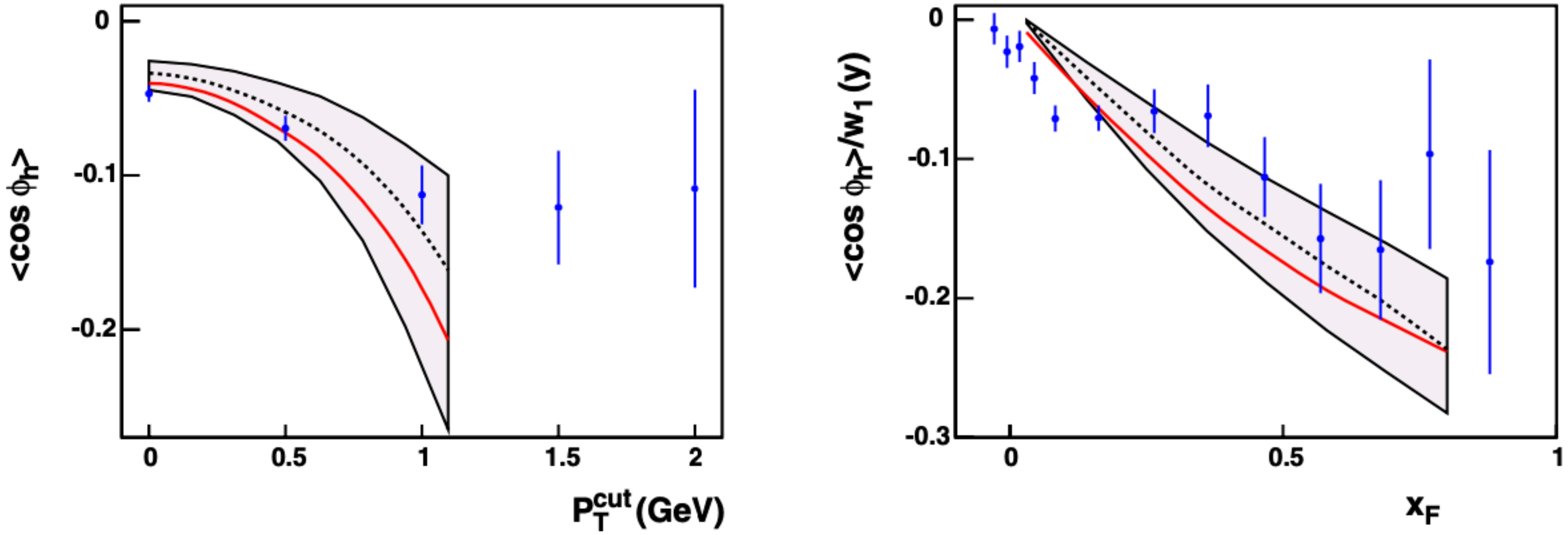
$$W^{\mu\nu}_{\{2,3\,\mathrm{intrinsic}\}} = \frac{1}{N_c}\sum_{a1,a2}\sum_q e_q^2 \int d^2k_{1T}\, d^2k_{2T}\, \delta^{(2)}(\boldsymbol{q}_T - \boldsymbol{k}_{1T} - \boldsymbol{k}_{2T})$$

$$\times\,\mathrm{Tr}\left[\gamma^\mu\,\bar{\Gamma}_1^{a1}\,\gamma^\nu\,\bar{\Gamma}_1^{a2}\right]\,\Phi^{[\Gamma^{a1}]}(x_1, \boldsymbol{k}_{1T}, \boldsymbol{S}_1)\;\bar{\Phi}^{[\Gamma^{a2}]}(x_2, \boldsymbol{k}_{2T}, \boldsymbol{S}_2)\,.$$

Role of Cahn effect in SIDIS from TMD framework
Modeling tree level result comparing w/ E665 data

Anselmino, Boglione,D'Alesio, Kotzinian, Murgia, Prokudin

PHYSICAL REVIEW D **71,** 074006 (2005)



$$F_{UU}^{\cos\phi_h} \approx \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\, f_1 D_1\right].$$

$$\frac{d^5\sigma^{\ell p\to\ell h X}}{dx_B\, dQ^2\, dz_h\, d^2\boldsymbol{P}_T} \simeq \sum_q \frac{2\pi\alpha^2 e_q^2}{Q^4} f_q(x_B) D_q^h(z_h)\left[1+(1-y)^2 - 4\frac{(2-y)\sqrt{1-y}\langle k_\perp^2\rangle z_h P_T}{\langle P_T^2\rangle Q}\cos\phi_h\right]\frac{1}{\pi\langle P_T^2\rangle} e^{-P_T^2/\langle P_T^2\rangle},$$