

BASICS OF FACTORIZATION IN A SCALAR YUKAWA FIELD THEORY

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BASED ON [arXiv:2212.00757 [hep-ph]] by
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GOAL

- ❑ To stress-test the limits of factorization
- ❑ Examine the general properties of pdfs

DIS Factorization

Parton model $F_1(x_B; Q^2) \rightarrow \int \frac{d\xi}{\xi} \hat{F}_1(x_B/\xi; Q^2) f(\xi)$

Renormalizable QFT ... $F_1(x_B; Q^2) \rightarrow \int \frac{d\xi}{\xi} \hat{F}_1[x_B/\xi, \mu^2/Q^2; g(\mu)] f[\xi; \mu/\Lambda; g(\mu)] + \mathcal{O}(\frac{m^2}{Q^2})$

- Do the naïve expectations from a parton model framework hold for the theories that require renormalization?
- How important are the higher twists?
-

HOW?

□ Using Yukawa theory

QCD

- Non-perturbative
- Confinement
- Asymptotic freedom
- Soft gluons
- Light-cone divergences
- **Renormalizable**

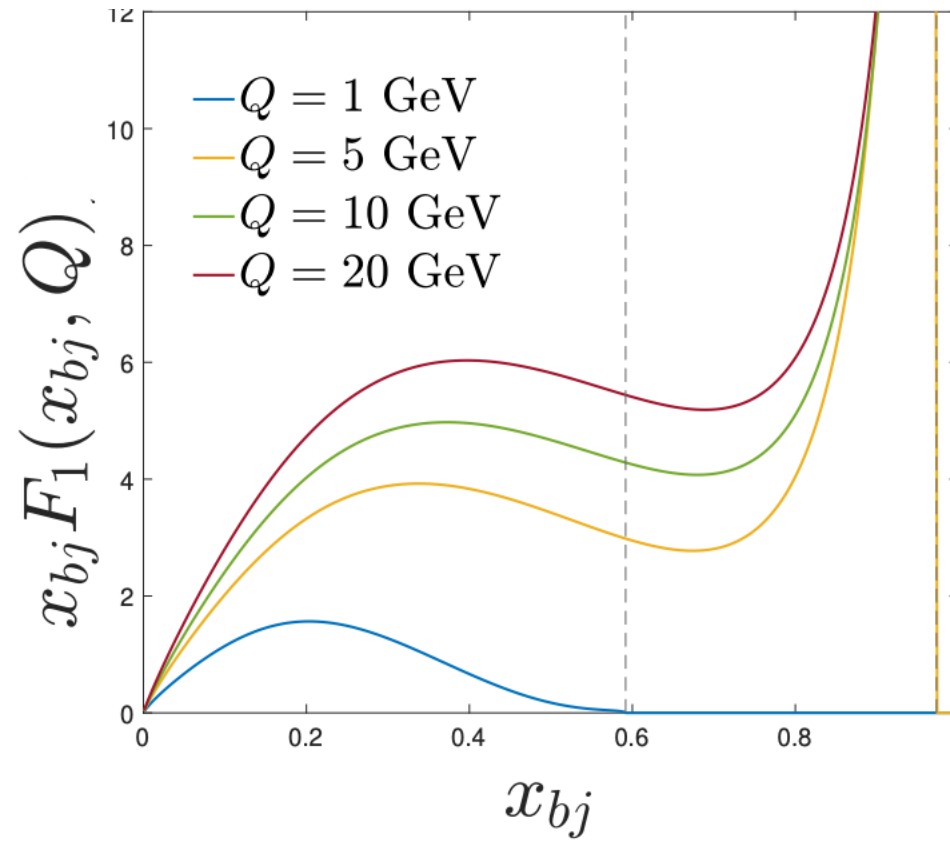
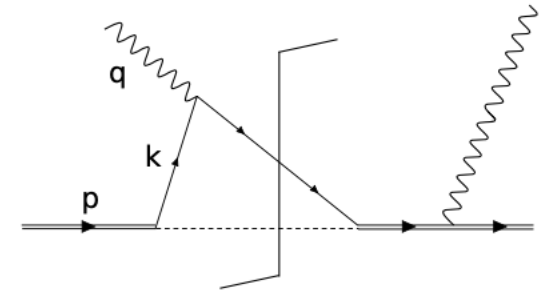
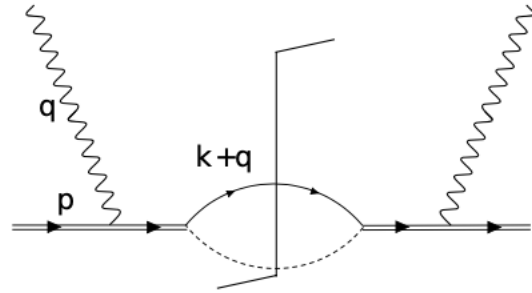
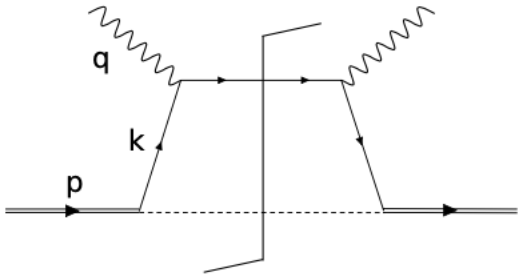
Yukawa

- Perturbative
- Hadron state is not a bound state
- All interactions are pointlike
- **Renormalizable**

- Yukawa is simpler than QCD but still useful for highlighting general but subtle properties of parton densities and for examining the limits of factorization while side stepping issues like confinement, large coupling, gauge invariance, and other complicating features of QCD.
- It is possible to compare the unfactorized results with standard collinear and TMD factorization treatments.

THE EXACT/UNAPPROXIMATED STRUCTURE FUNCTIONS IN YUKAWA MODEL

$$\mathcal{L}_{\text{int}} = -\lambda \bar{\Psi}_N \psi_q \phi + \text{H.C.}$$

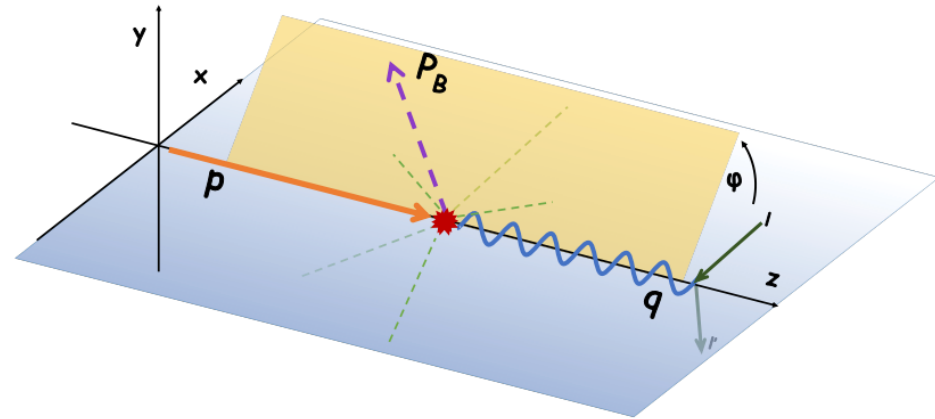
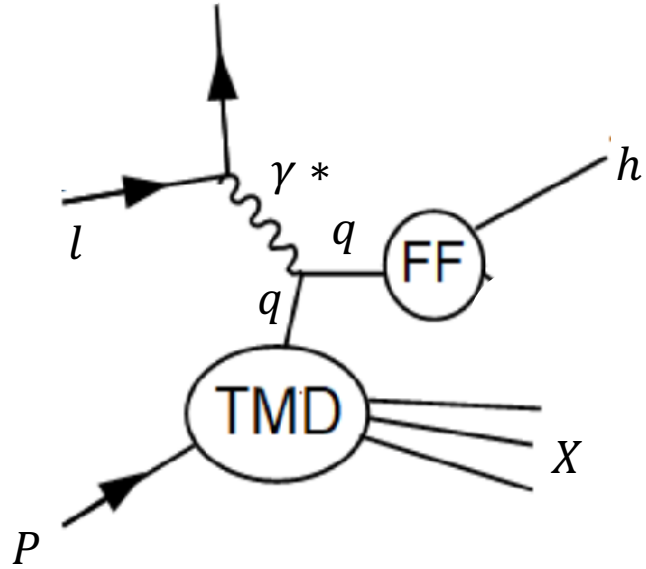


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[arXiv:2212.00757](https://arxiv.org/abs/2212.00757) [hep-ph]

$$x_{\text{max}} = \frac{Q^2}{[(m_q + m_s)^2 - m_p^2 + Q^2]}$$

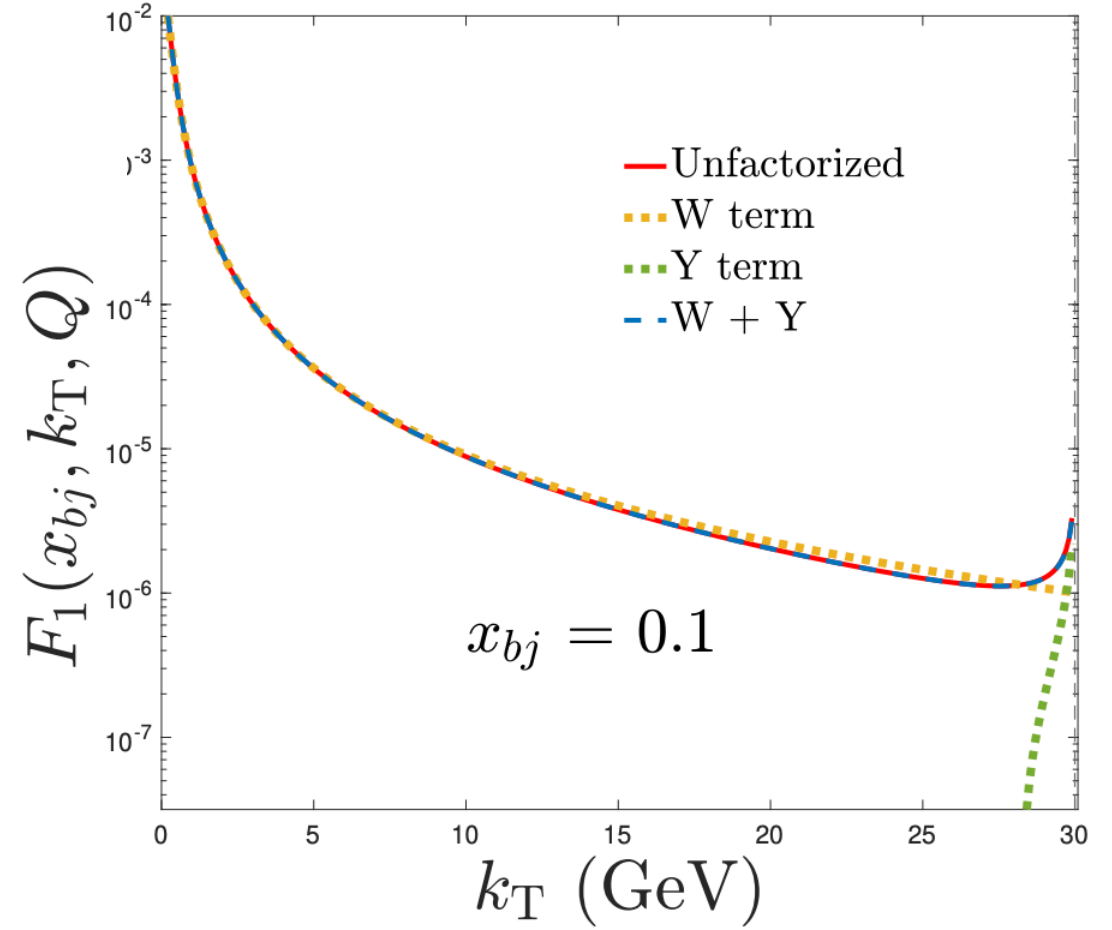
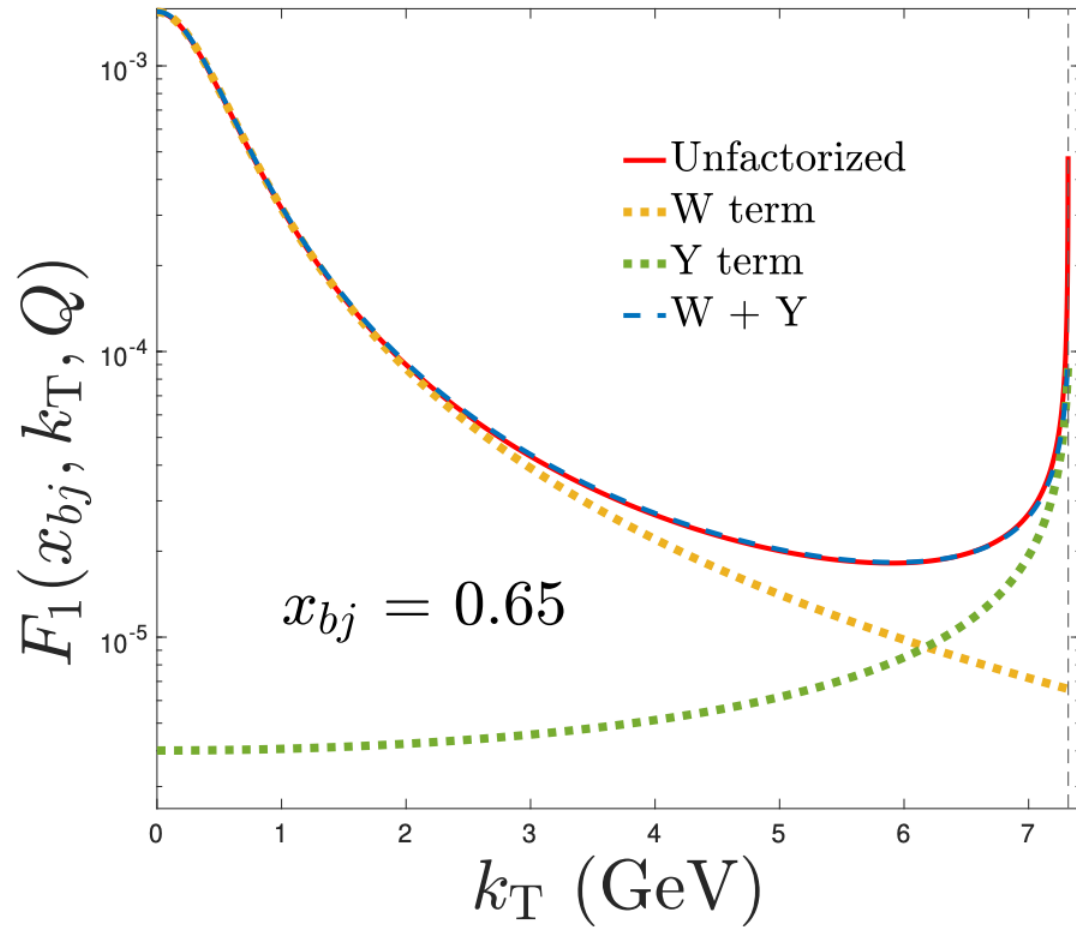
FACTORIZED RESULTS: SIDIS and TMD Factorization



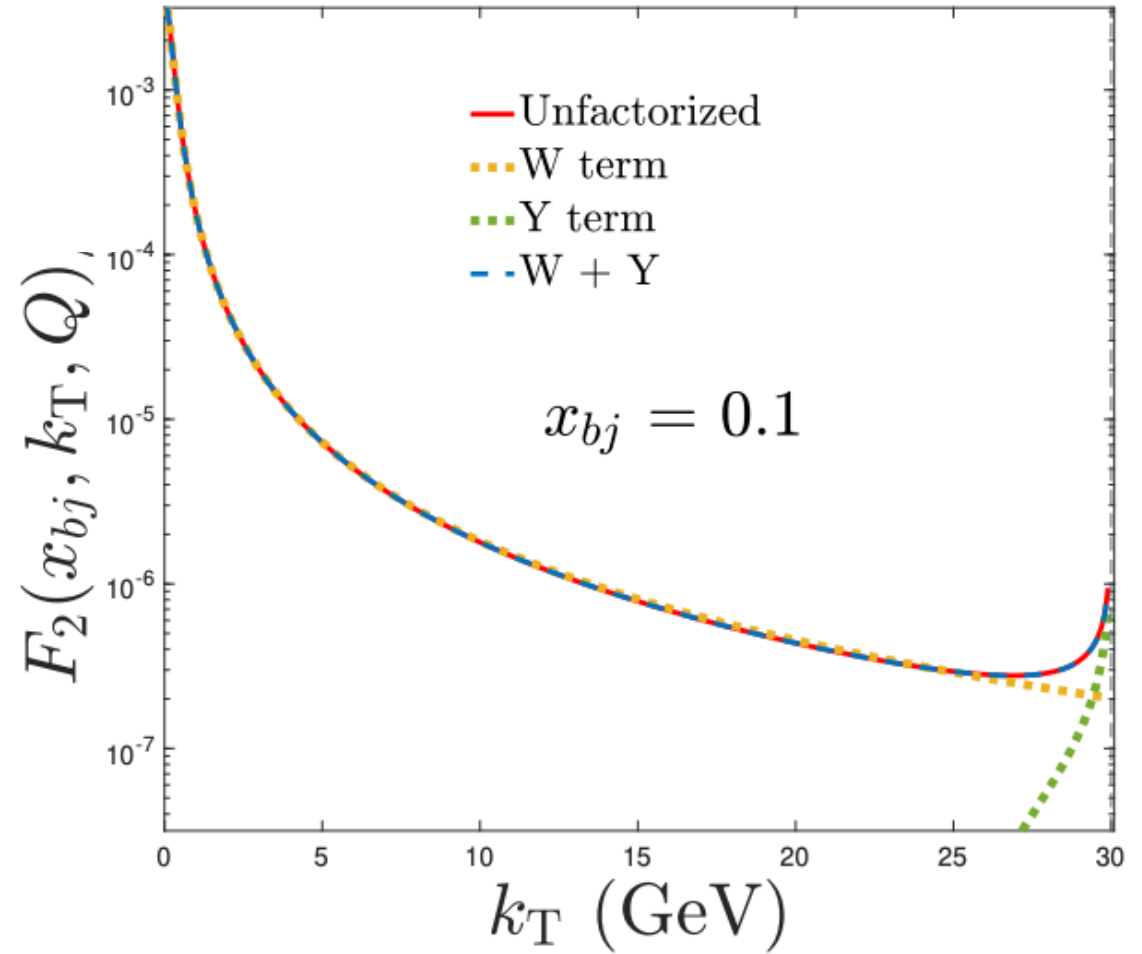
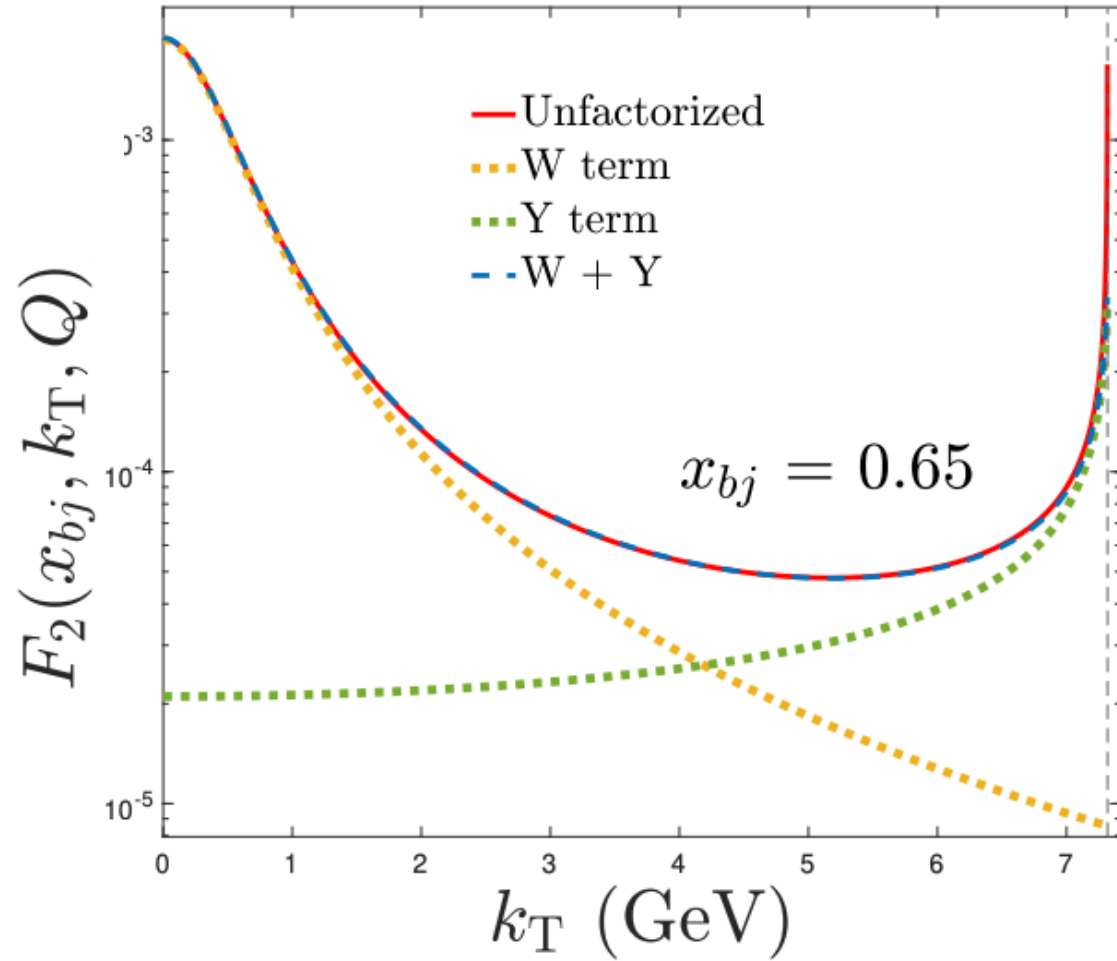
Two types of factorization schemes!

$$\begin{aligned}
 F_1(x_{bj}, Q, \mathbf{k}_T) &= T_{\text{small}} F_1(x_{bj}, Q, \mathbf{k}_T) + T_{\text{large}} [F_1(x_{bj}, Q, \mathbf{k}_T) - T_{\text{small}} F_1(x_{bj}, Q, \mathbf{k}_T)] + \mathcal{O}\left(\frac{k_T^2}{Q^2} \times \frac{m^2}{k_T^2}\right) \cdot \\
 &= \text{W-term} + \text{Y-term} + \mathcal{O}\left(\frac{m^2}{Q^2}\right).
 \end{aligned}$$

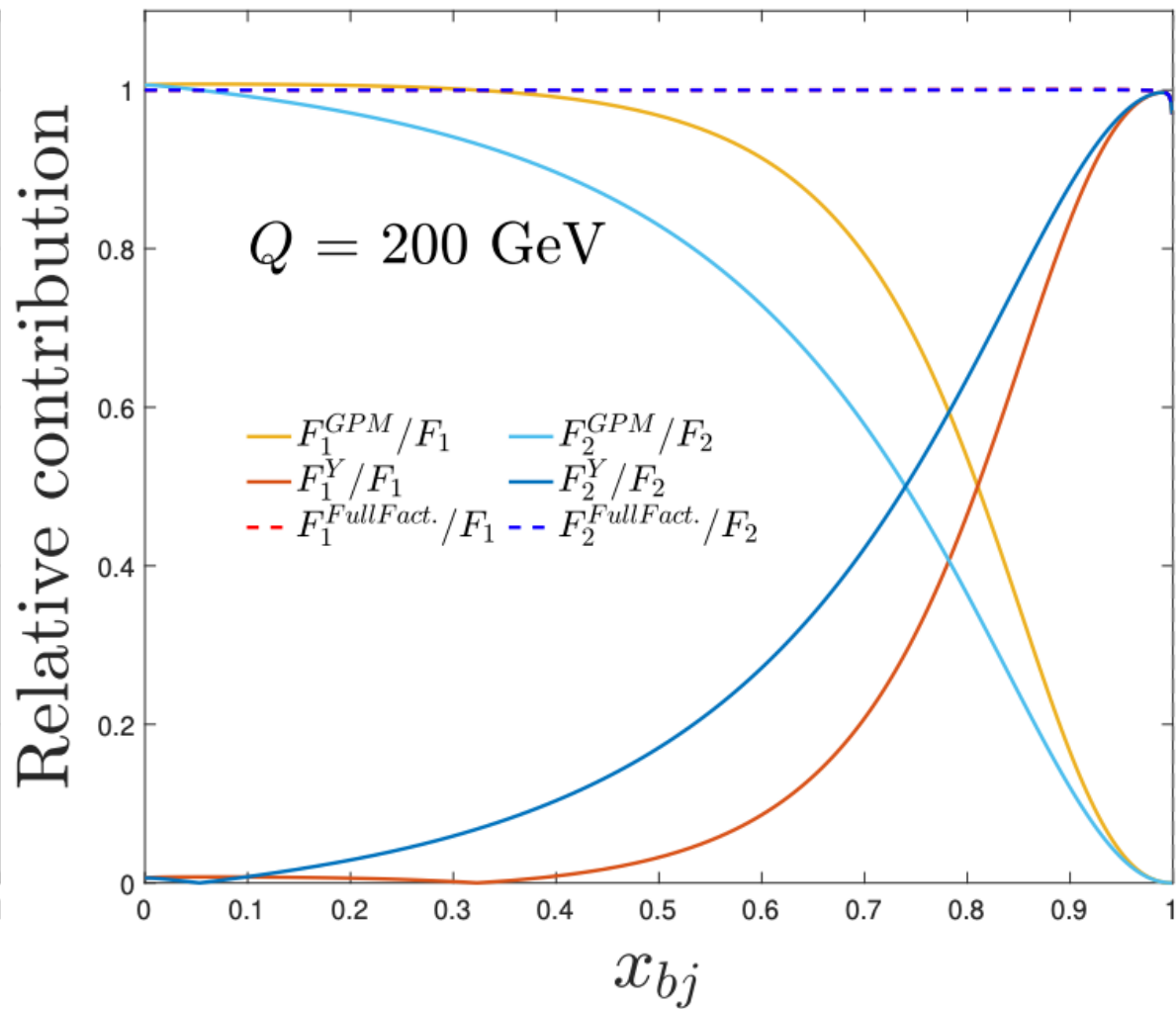
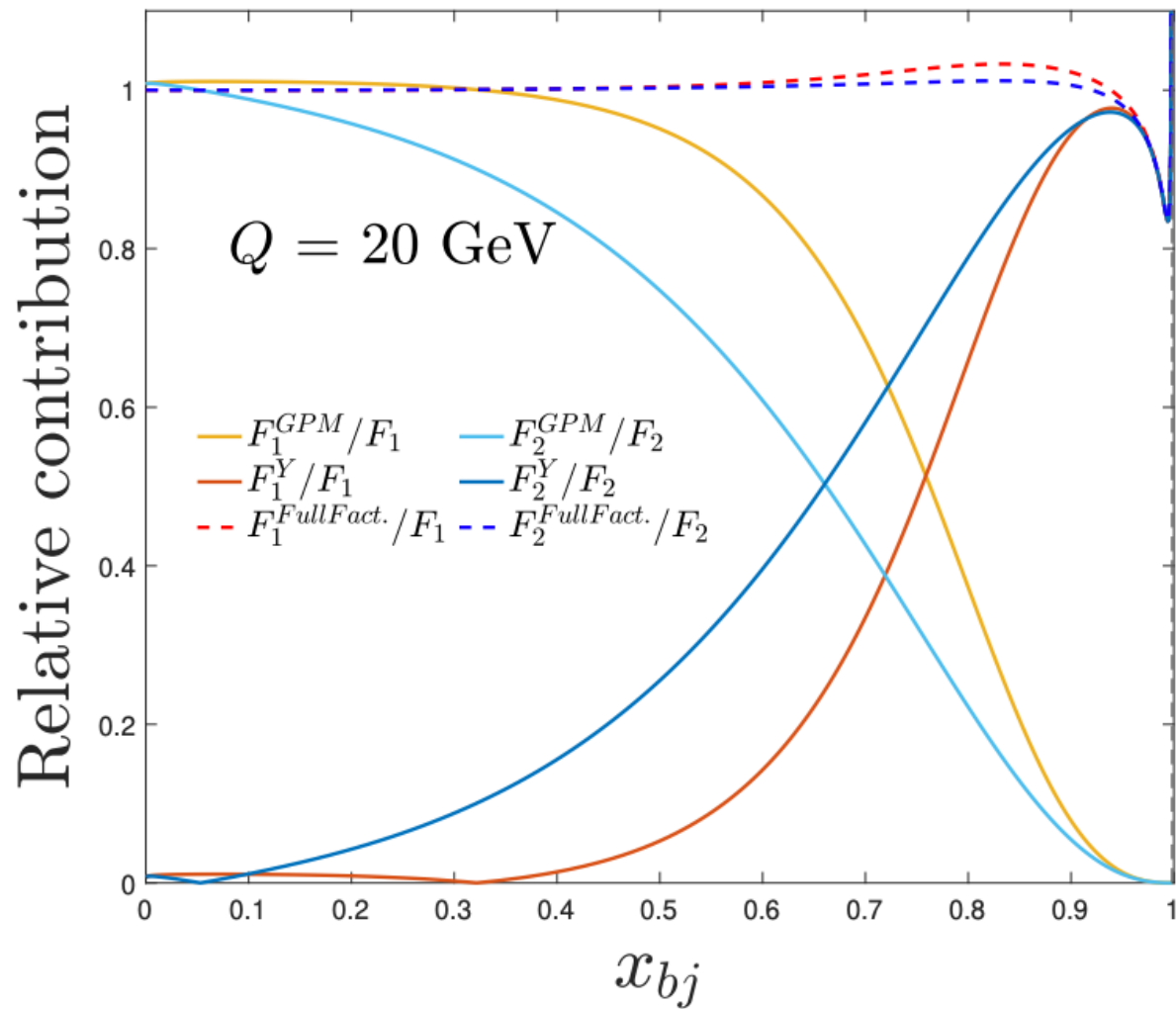
EXACT VS FACTORIZED RESULTS



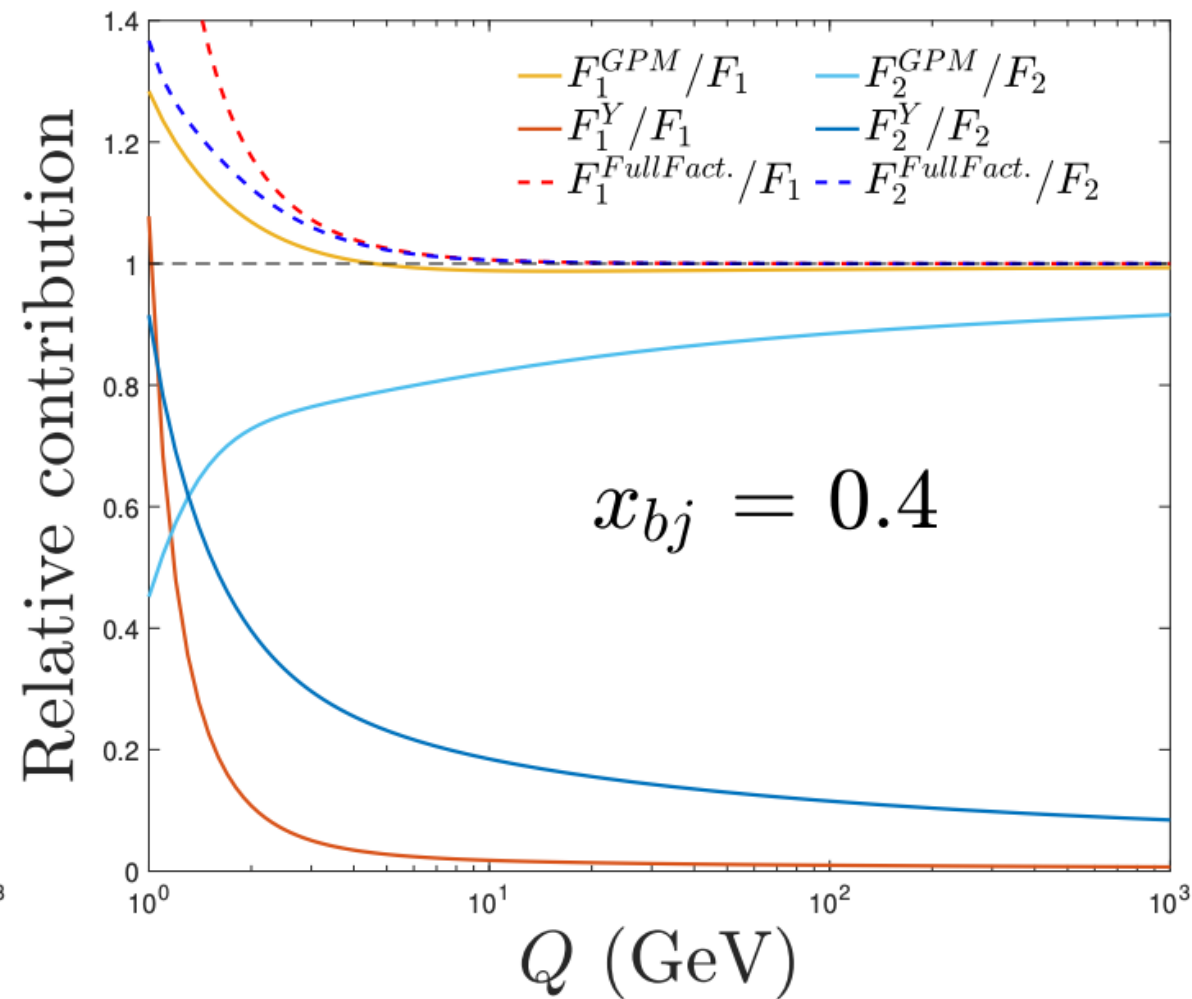
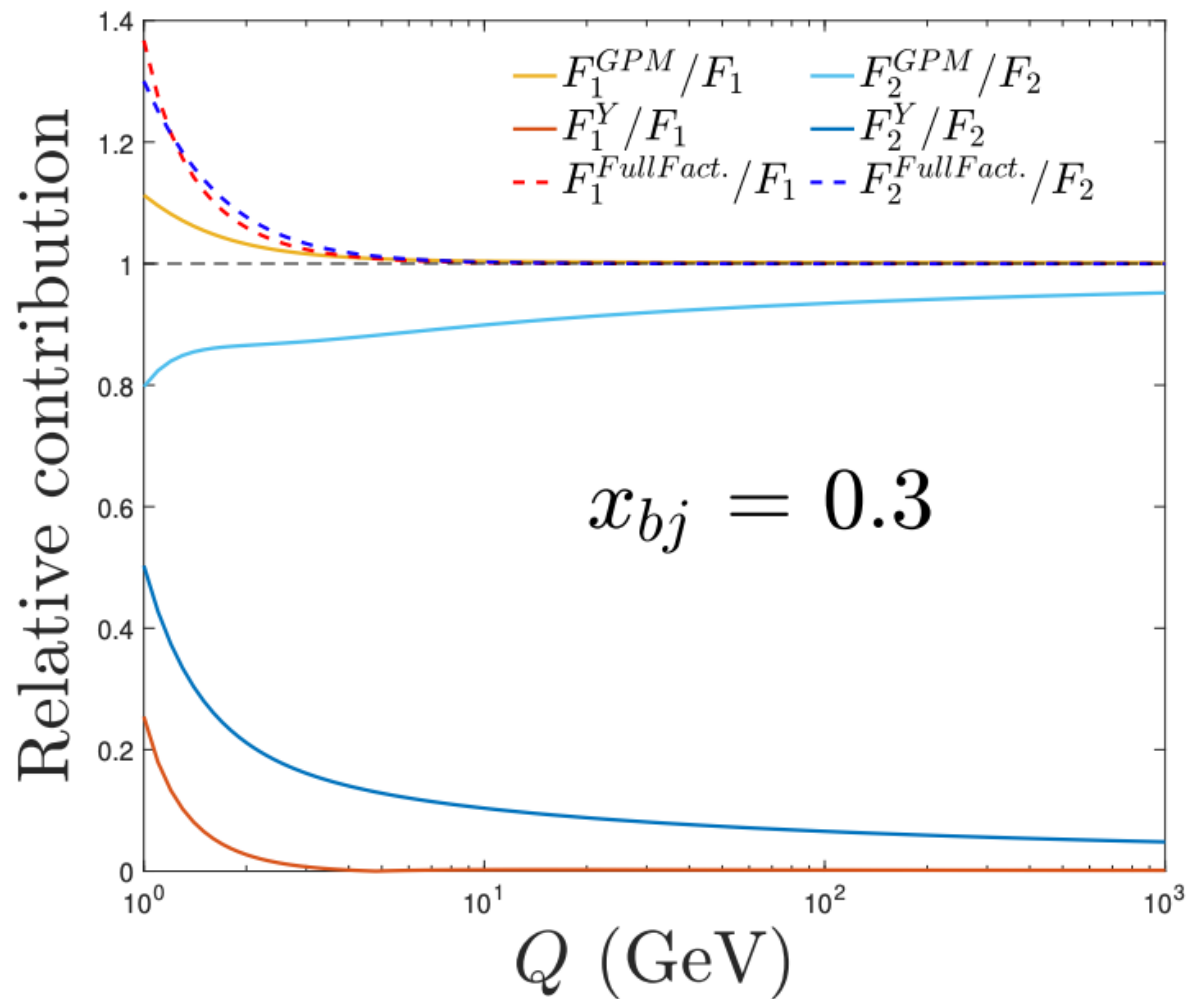
EXACT VS FACTORIZED RESULTS



EXACT VS FACTORIZED RESULTS



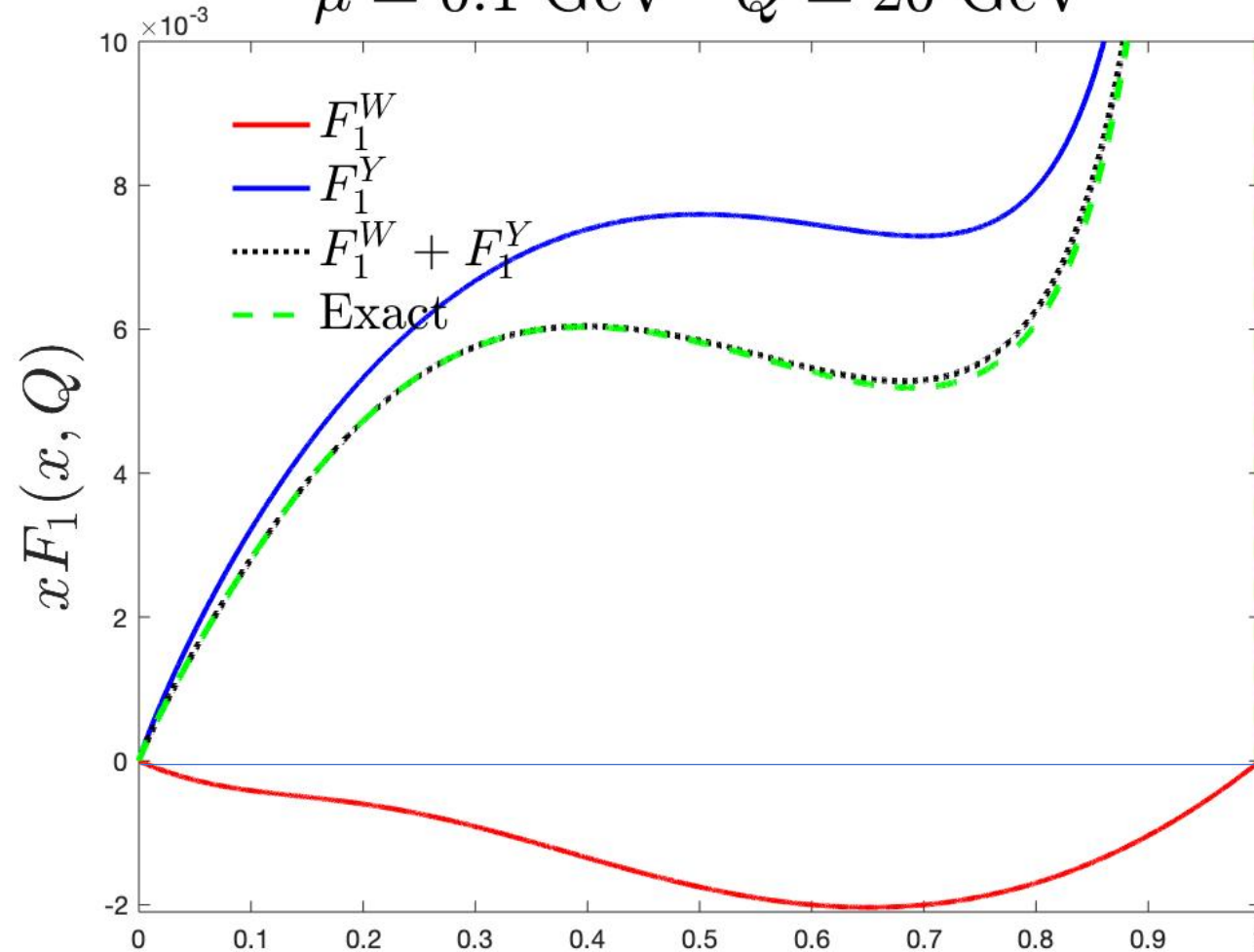
EXACT VS FACTORIZED RESULTS



EXACT VS FACTORIZED RESULTS

$$F_{1,2}(x_{bj}, Q; \mu) = F_{1,2}^W(x_{bj}, Q; \mu) + F_{1,2}^Y(x_{bj}, Q; \mu) + \mathcal{O}\left(\frac{m^2}{Q^2}\right).$$

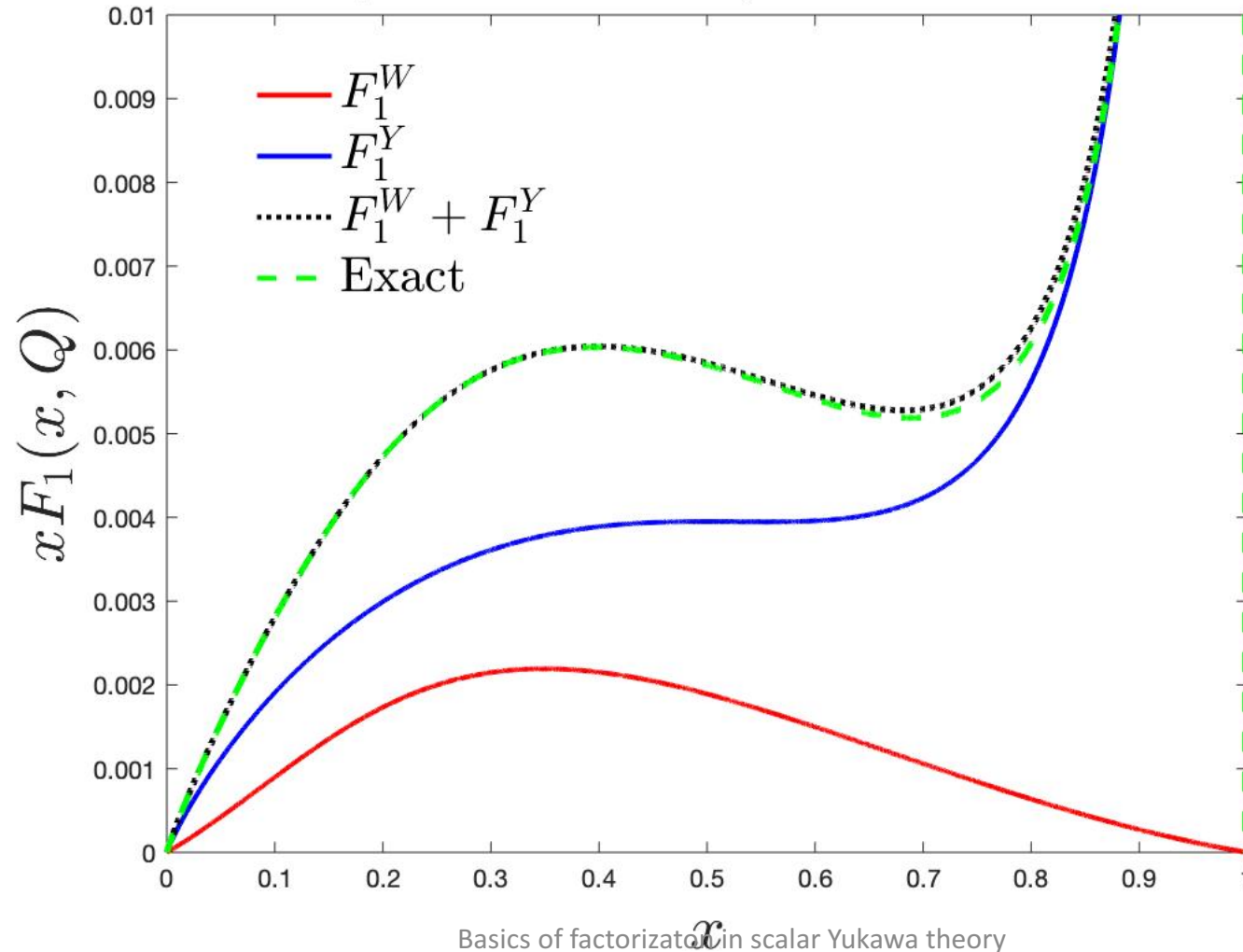
$$\mu = 0.1 \text{ GeV} \quad Q = 20 \text{ GeV}$$



EXACT VS FACTORIZED RESULTS

$$F_{1,2}(x_{bj}, Q; \mu) = F_{1,2}^W(x_{bj}, Q; \mu) + F_{1,2}^Y(x_{bj}, Q; \mu) + \mathcal{O}\left(\frac{m^2}{Q^2}\right).$$

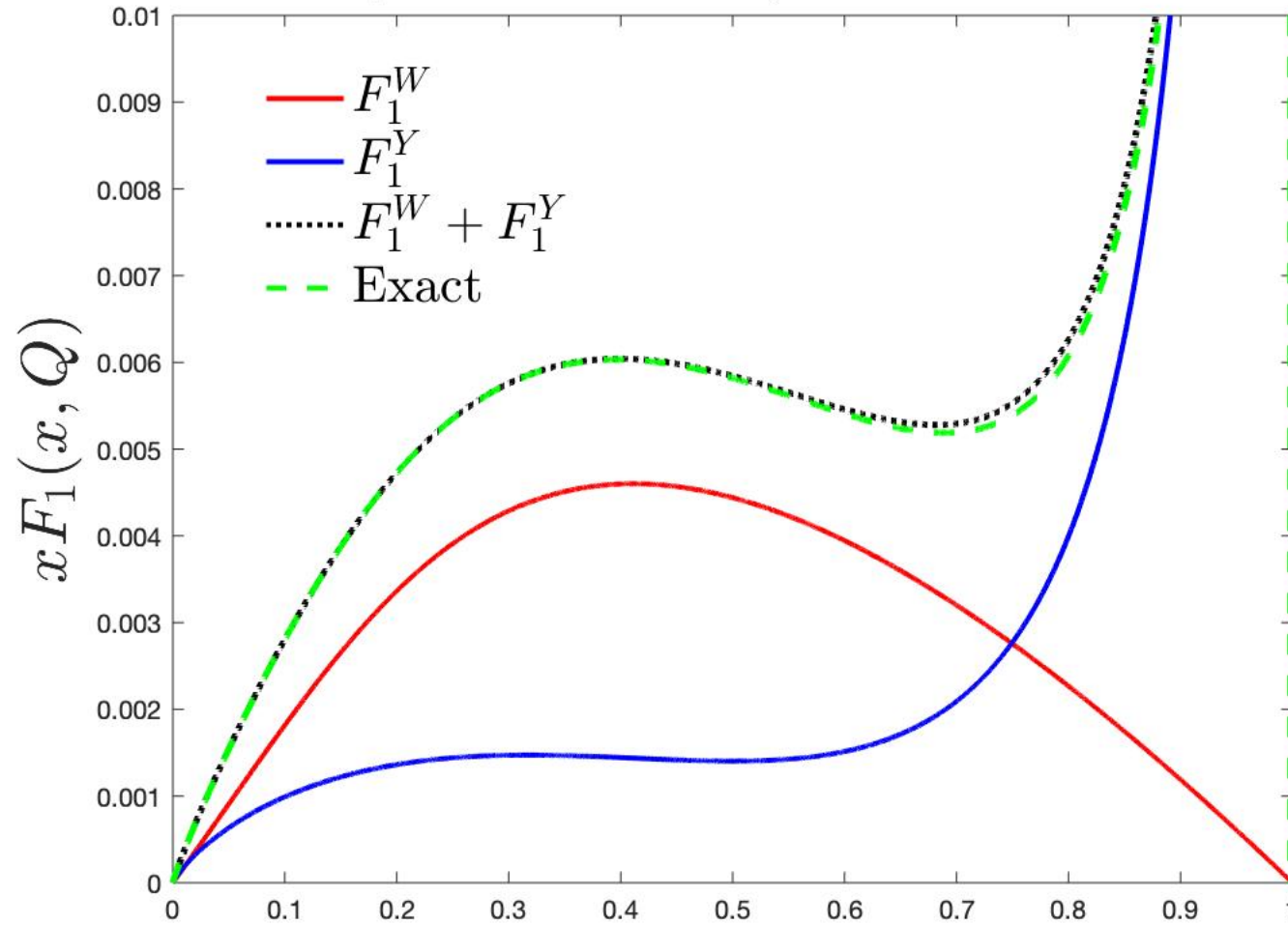
$$\mu = 1 \text{ GeV} \quad Q = 20 \text{ GeV}$$



EXACT VS FACTORIZED RESULTS

$$F_{1,2}(x_{bj}, Q; \mu) = F_{1,2}^W(x_{bj}, Q; \mu) + F_{1,2}^Y(x_{bj}, Q; \mu) + \mathcal{O}\left(\frac{m^2}{Q^2}\right).$$

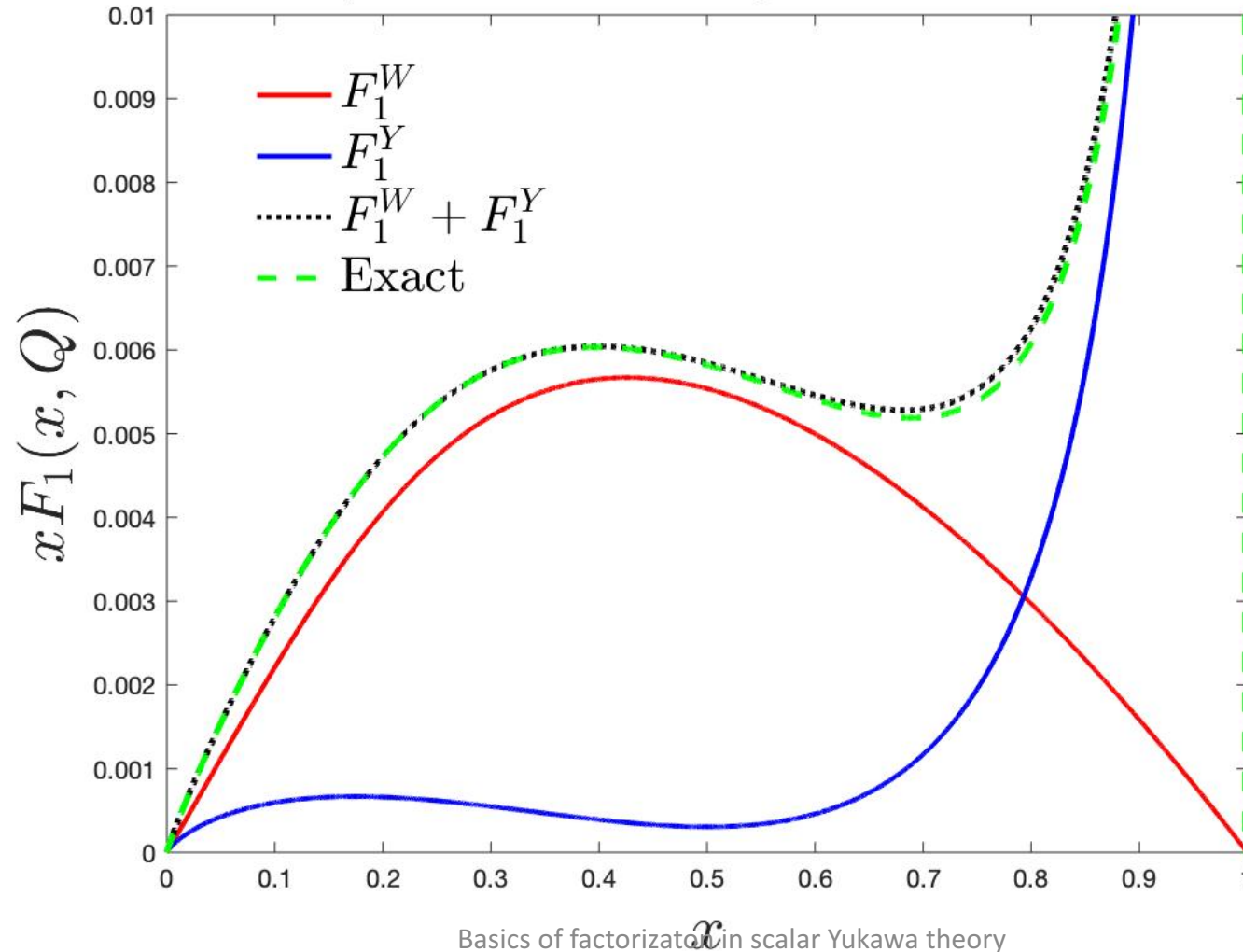
$$\mu = 5 \text{ GeV} \quad Q = 20 \text{ GeV}$$



EXACT VS FACTORIZED RESULTS

$$F_{1,2}(x_{bj}, Q; \mu) = F_{1,2}^W(x_{bj}, Q; \mu) + F_{1,2}^Y(x_{bj}, Q; \mu) + \mathcal{O}\left(\frac{m^2}{Q^2}\right).$$

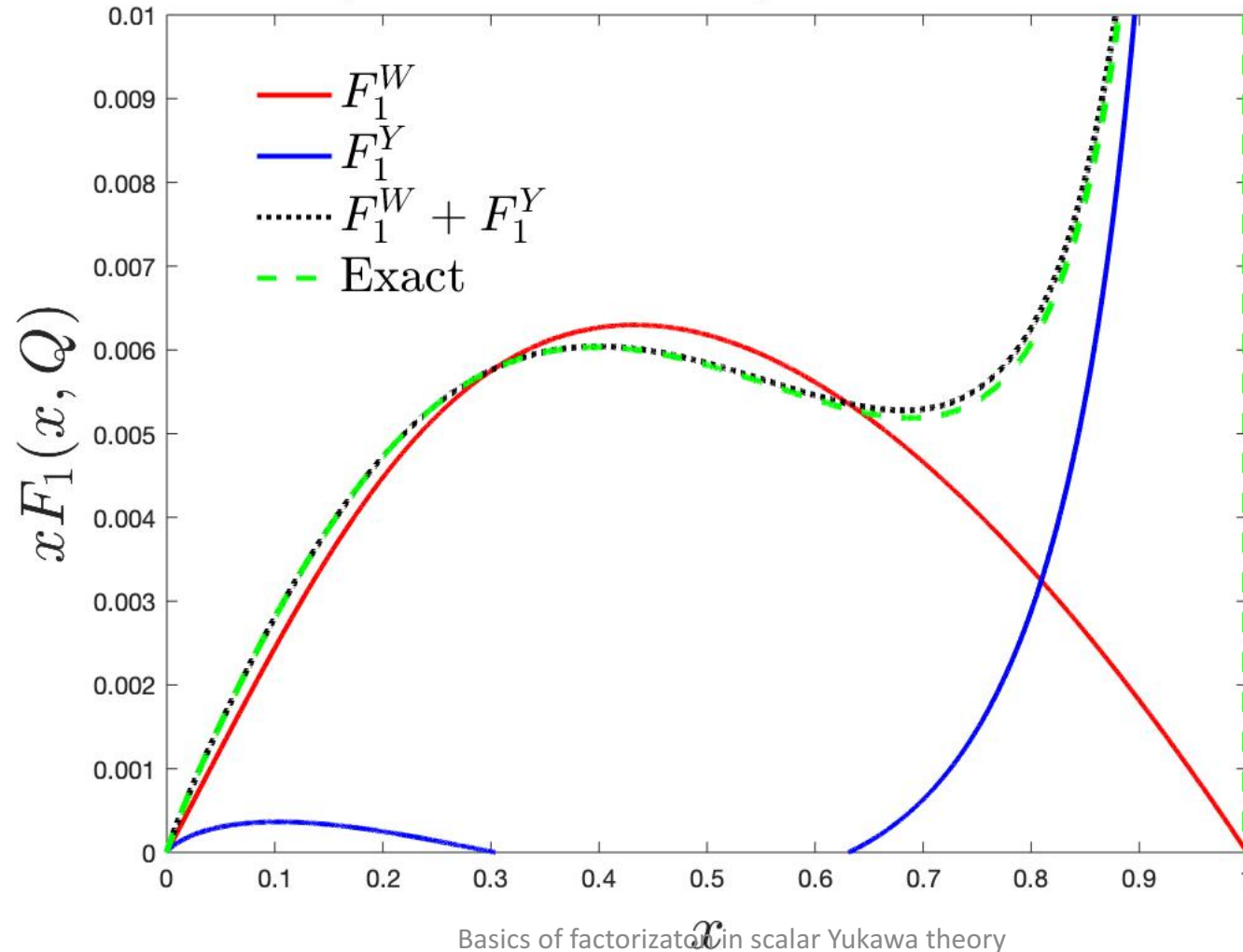
$$\mu = 10 \text{ GeV} \quad Q = 20 \text{ GeV}$$



EXACT VS FACTORIZED RESULTS

$$F_{1,2}(x_{bj}, Q; \mu) = F_{1,2}^W(x_{bj}, Q; \mu) + F_{1,2}^Y(x_{bj}, Q; \mu) + \mathcal{O}\left(\frac{m^2}{Q^2}\right).$$

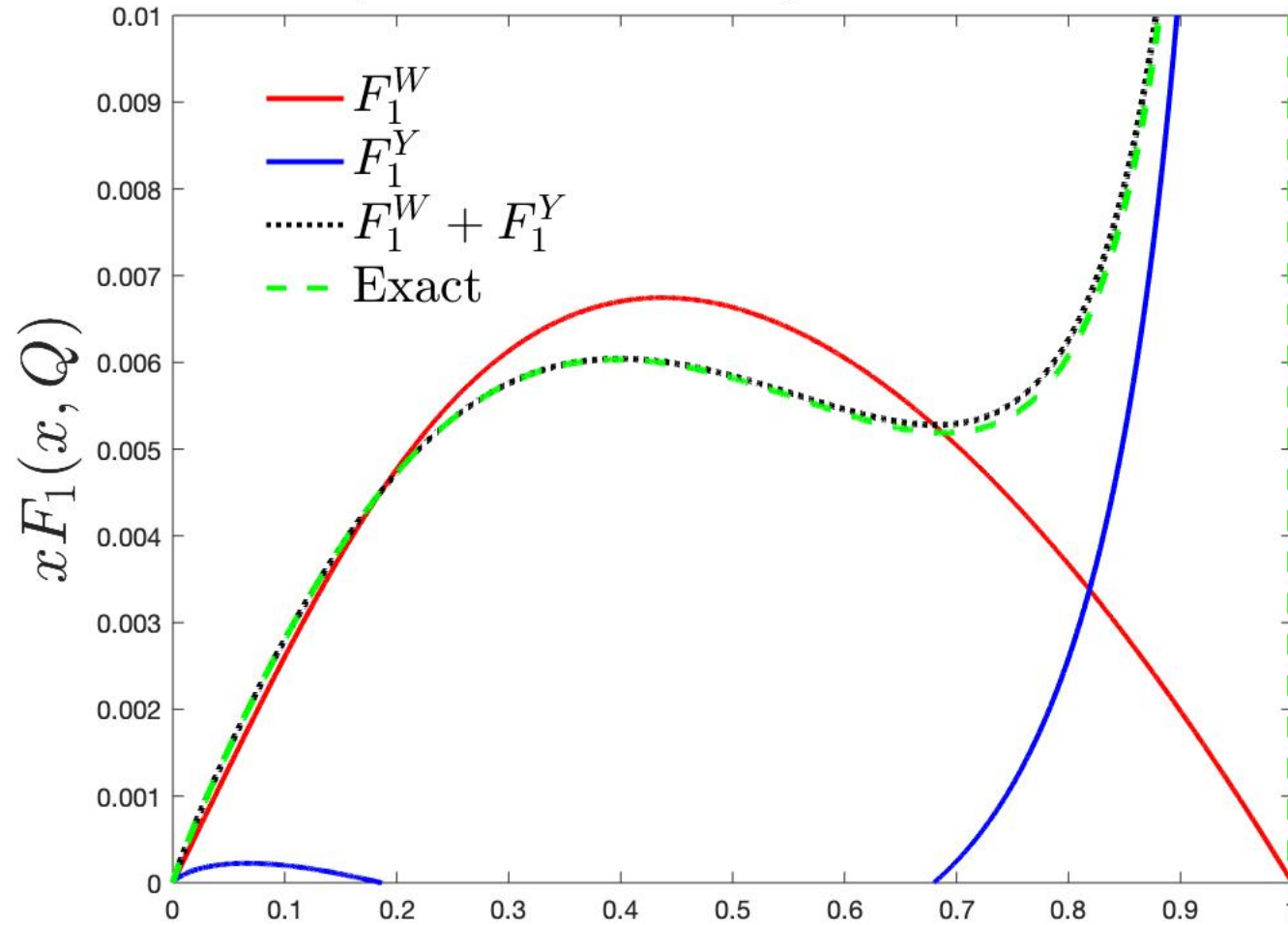
$$\mu = 15 \text{ GeV} \quad Q = 20 \text{ GeV}$$



EXACT VS FACTORIZED RESULTS

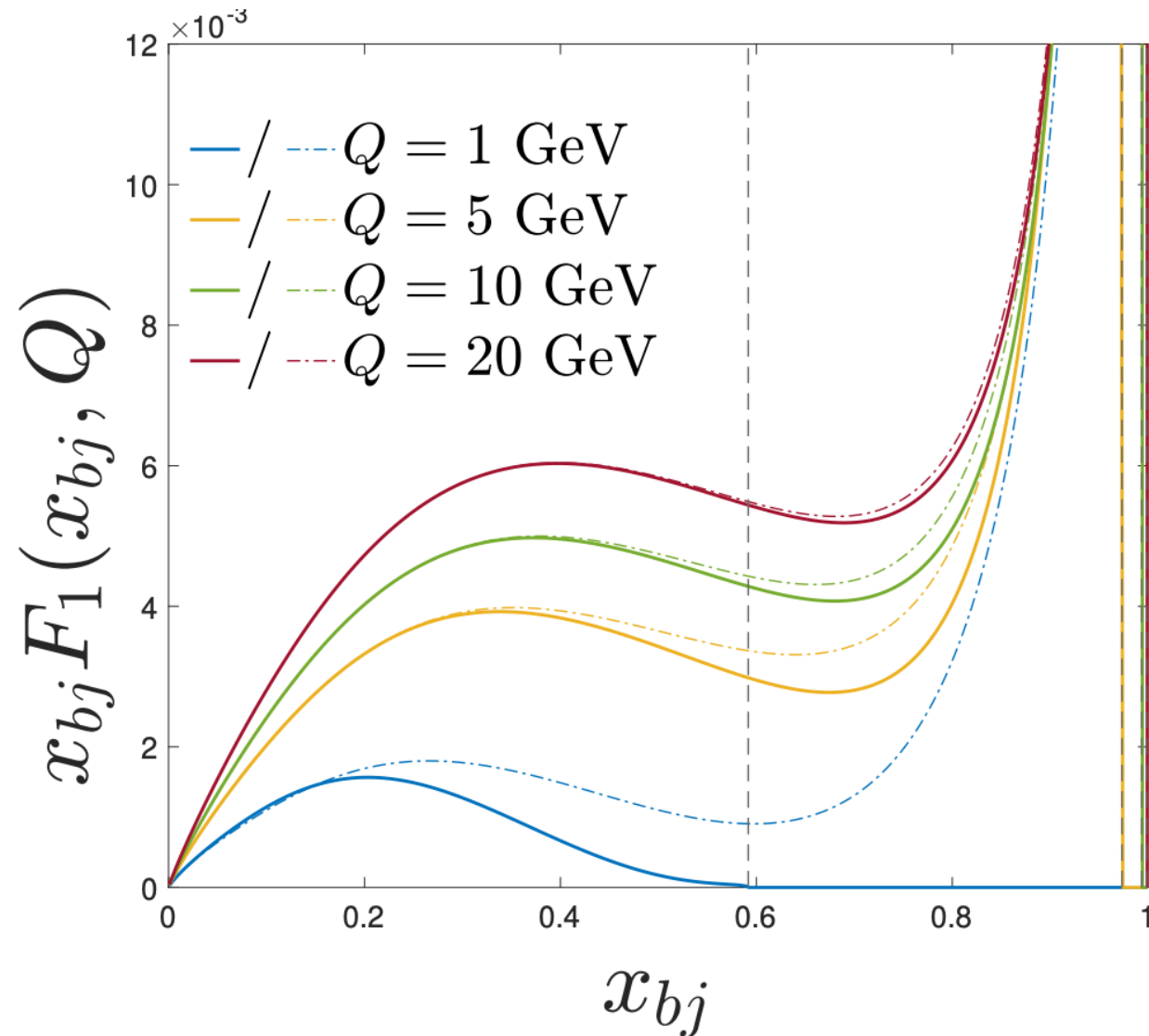
$$F_{1,2}(x_{bj}, Q; \mu) = F_{1,2}^W(x_{bj}, Q; \mu) + F_{1,2}^Y(x_{bj}, Q; \mu) + \mathcal{O}\left(\frac{m^2}{Q^2}\right).$$

$$\mu = 20 \text{ GeV} \quad Q = 20 \text{ GeV}$$



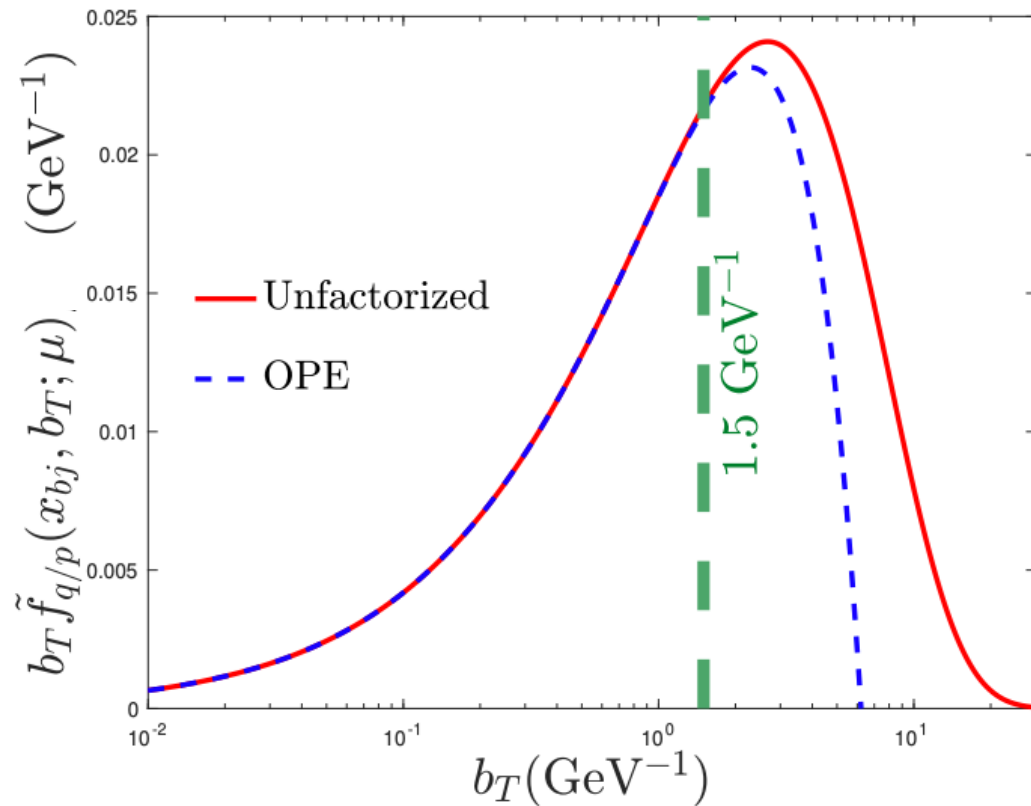
THE INPUT SCALE (Q_0)

$$F_1(x_B; Q^2) \rightarrow \int \frac{d\xi}{\xi} \hat{F}_1[x_B/\xi, \mu^2/Q^2; g(\mu)] f[\xi; \mu/\Lambda; g(\mu)] + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

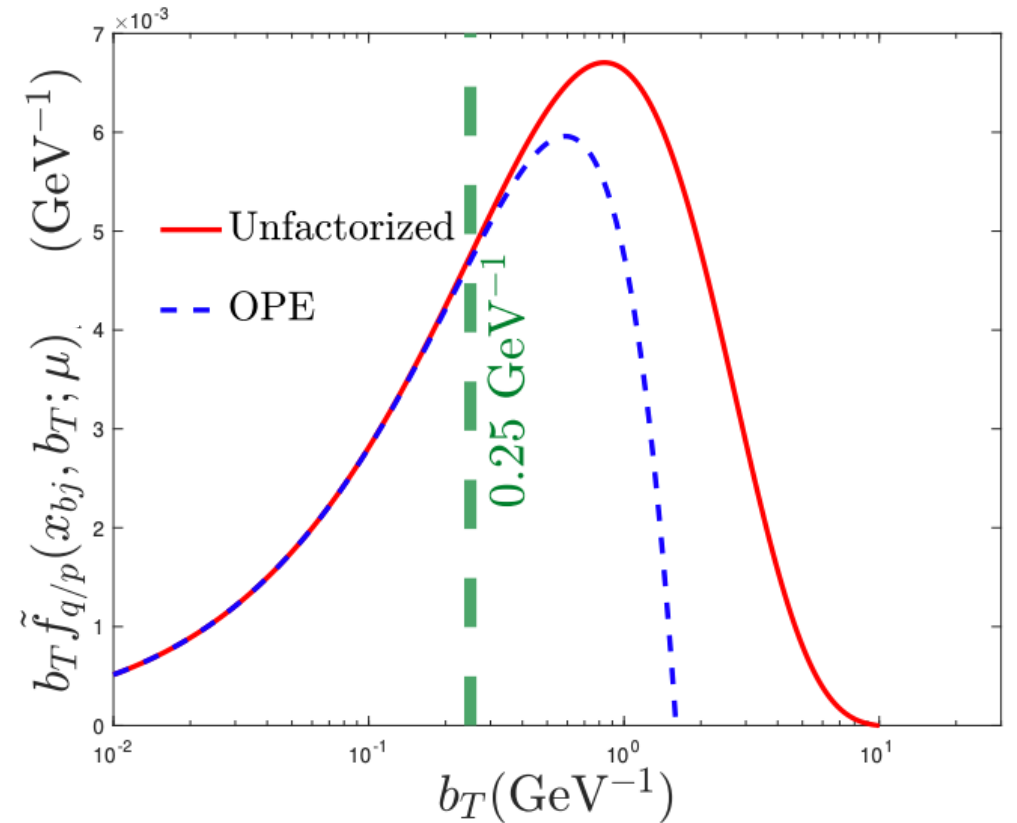


TRANSVERSE COORDINATE (b_T) SPACE

$$f(x_{bj}, b_T) = \begin{cases} f^{\text{OPE}}(x_{bj}, b_*) & b_T \ll b_{max} \\ f^{\text{OPE}}(x_{bj}, b_*) e^{-g(x_{bj}, b_T)} & b_T > b_{max} \end{cases}$$



$m_q = 0.3 \text{ GeV}, m_p = 1.0 \text{ GeV}, m_s = 1.0 \text{ GeV}$



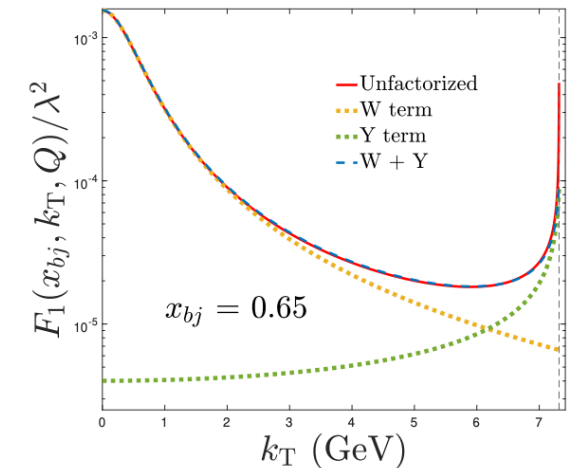
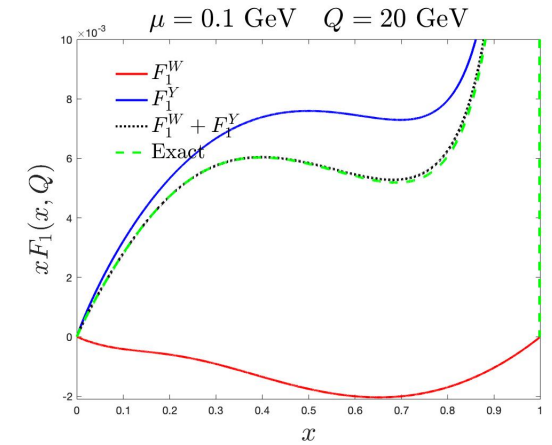
$m_q = 0.3 \text{ GeV}, m_p = 1.0 \text{ GeV}, m_s = 1.5 \text{ GeV}$

CONCLUSIONS

- The naïve expectations from a parton model framework may not hold for the theories that require renormalization
- The Y-term contribution is necessary to maintain a good agreement with the exact results

OUTLOOK

- Repeating the calculation for the polarized case



THANK YOU FOR YOUR ATTENTION