

# GLUON TMD STUDIES WITH HEAVY QUARKONIUM STATES

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In collaboration with: A. Bacchetta, M. G. Echevarria, J.-P. Lansberg, M. Ozcelik, M. Radici, A. Signori

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**HAS QCD**  
HADRONIC STRUCTURE AND  
QUANTUM CHROMODYNAMICS



Trento Institute for  
Fundamental Physics  
and Applications



**talento**  
cm Programa de atracción  
de talento investigador  
Comunidad de Madrid

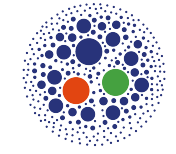


Madrid  
**UAH**



# 1. Introductory remarks

# Gluon TMD PDFs: a largely unexplored territory



Theory: different gauge-link structures...



...more diversified kind of modified universality!



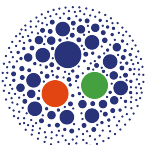

Pheno: golden channels for extraction

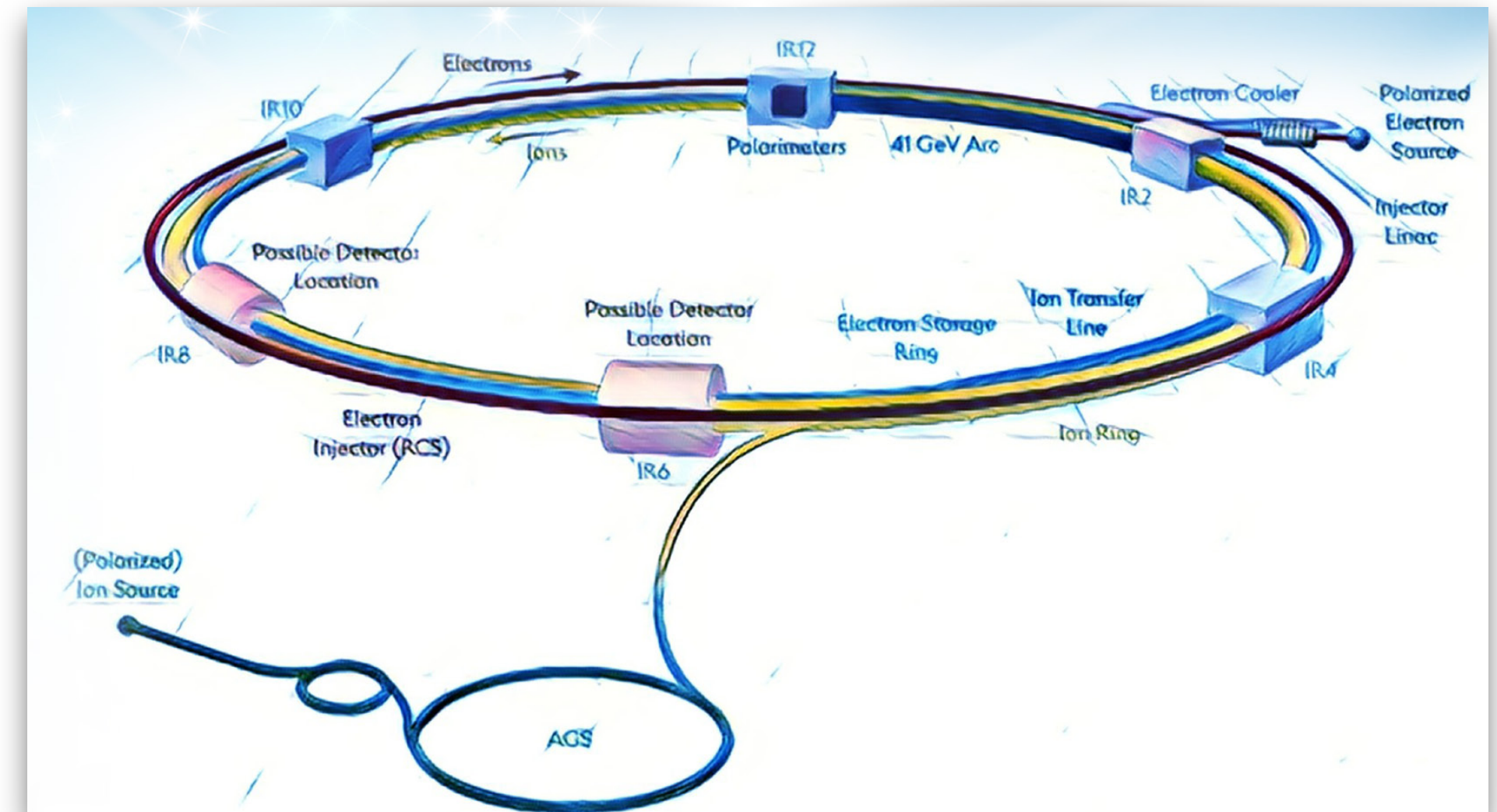
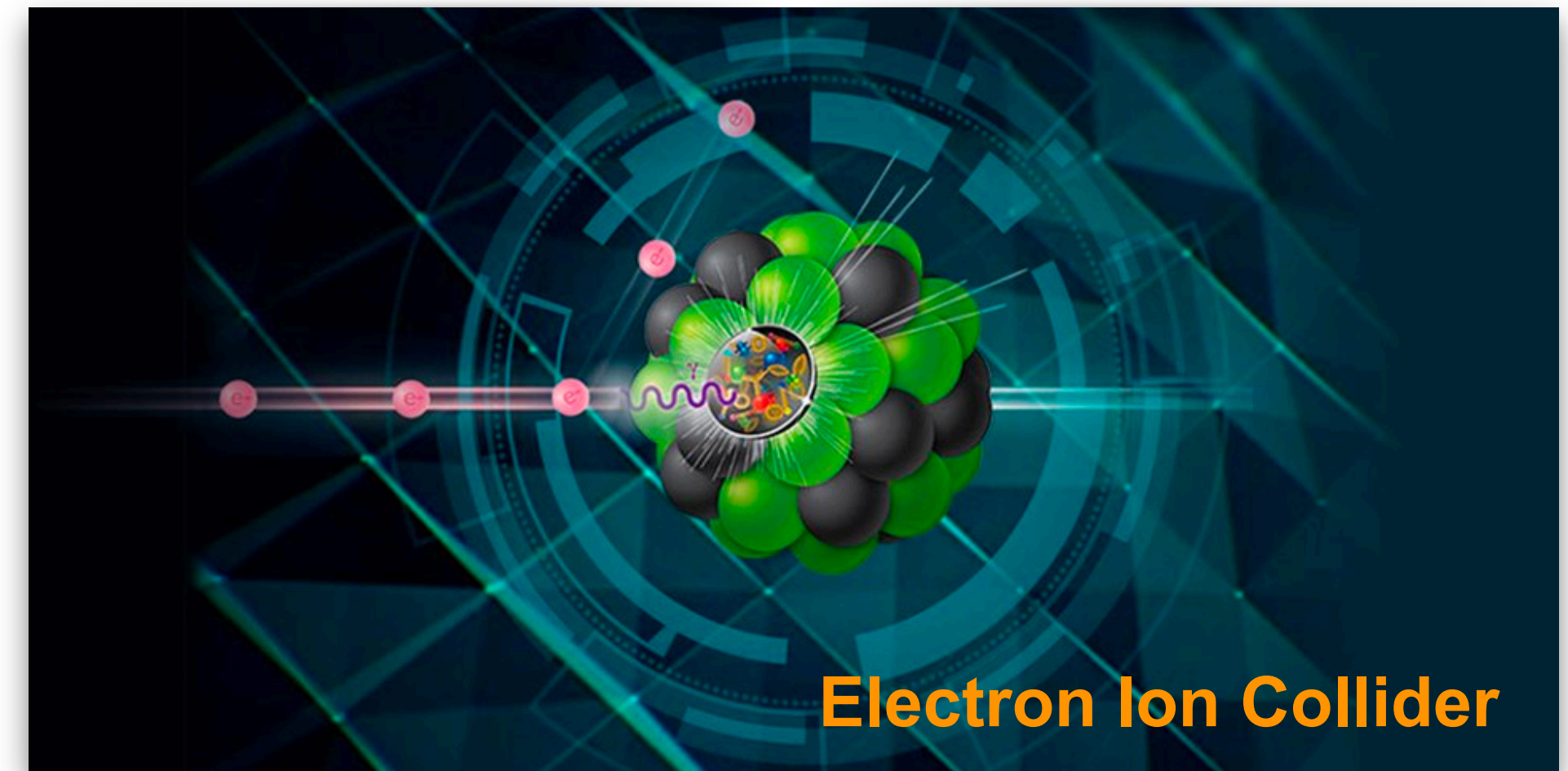
of quark TMDs are subleading for gluon TMDs

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## 3D proton imaging





-  Glueon TMD PDFs  $\Rightarrow$  core sector of EIC studies
-  Need for a flexible model, suited to pheno

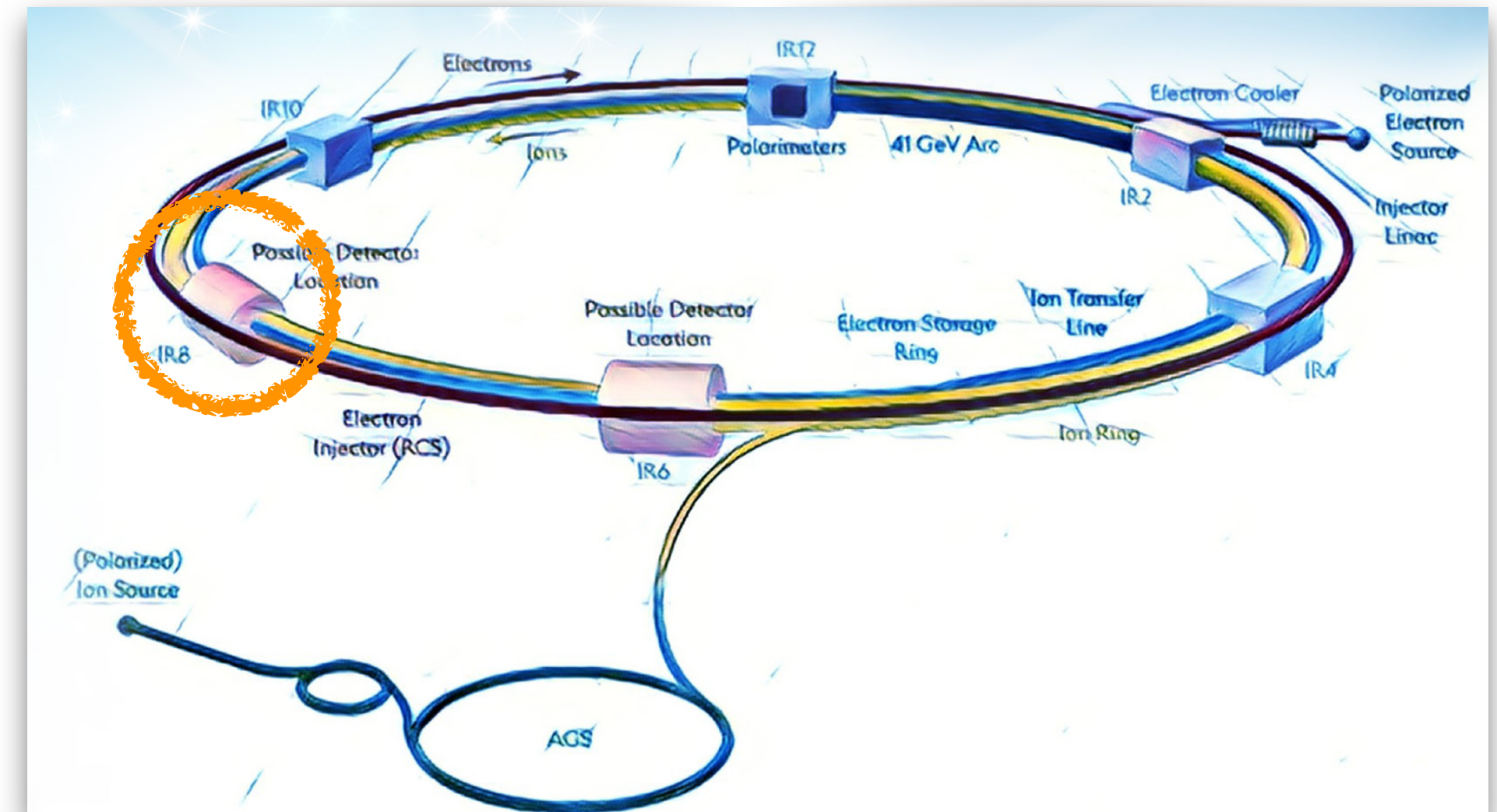
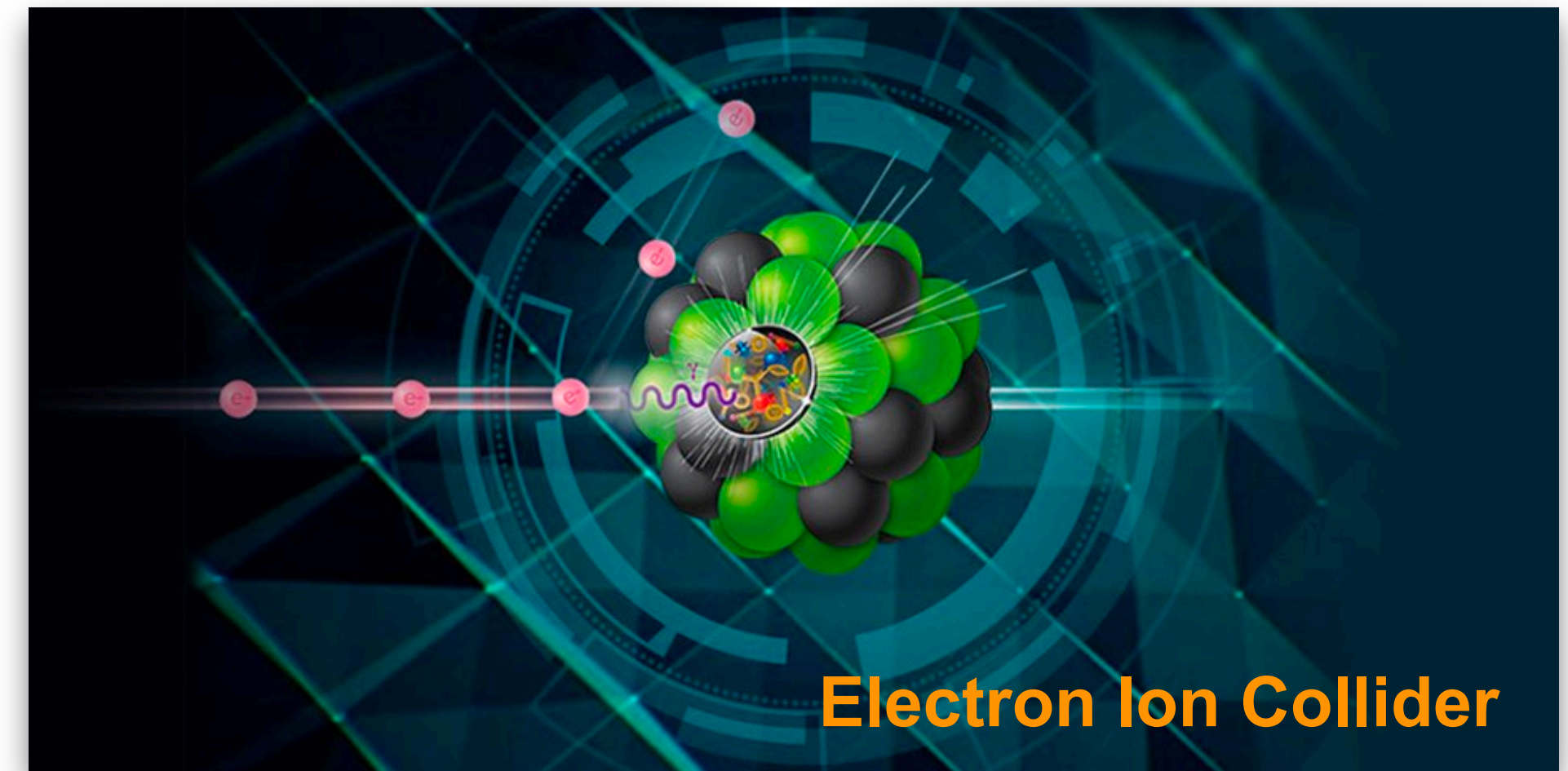


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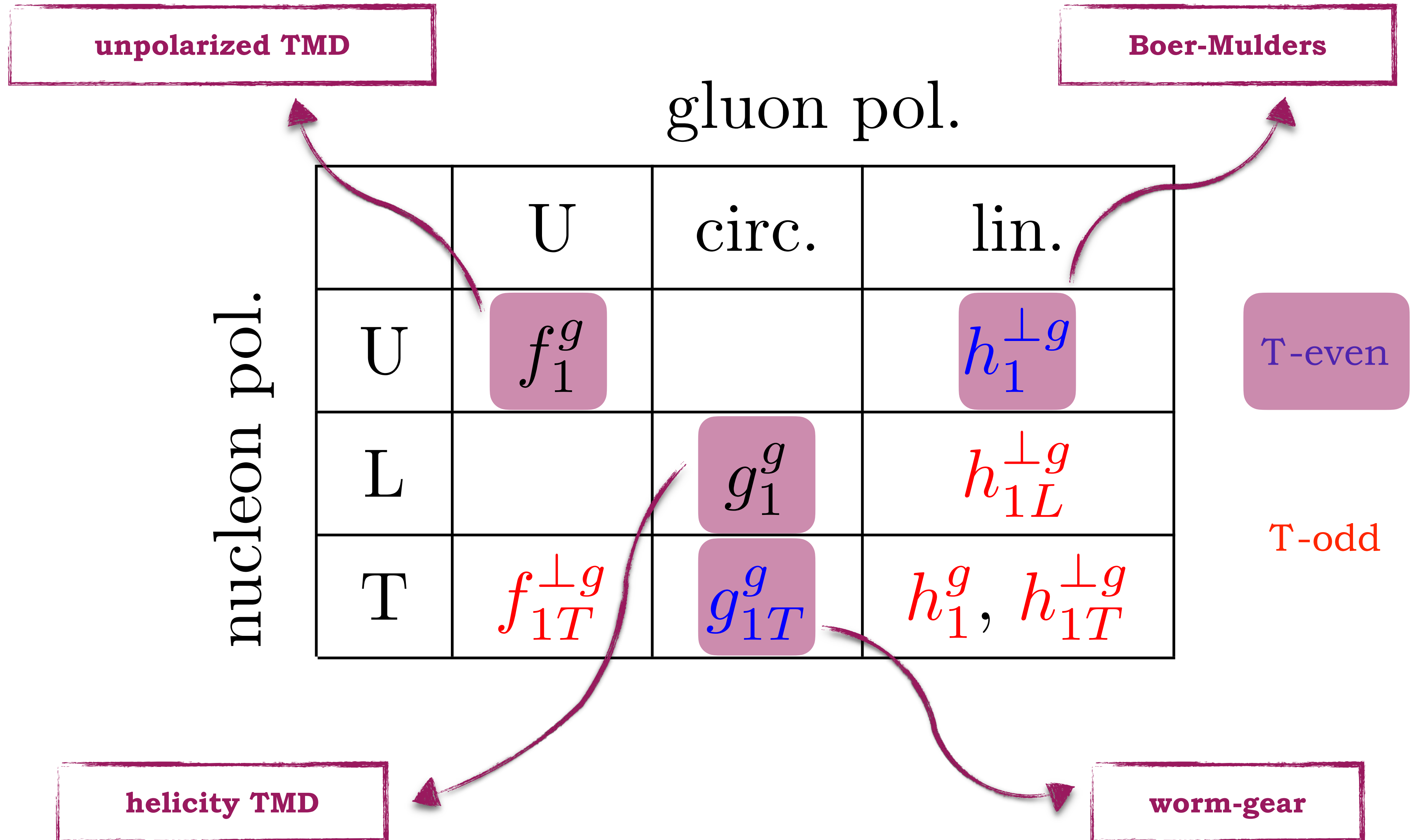
-  Gluon TMD PDFs  $\Rightarrow$  core sector of EIC studies
-  Need for a flexible model, suited to pheno
-  Gluon and nucleon polarization at twist-2
-  Window of opportunities also at a 2<sup>nd</sup> detector



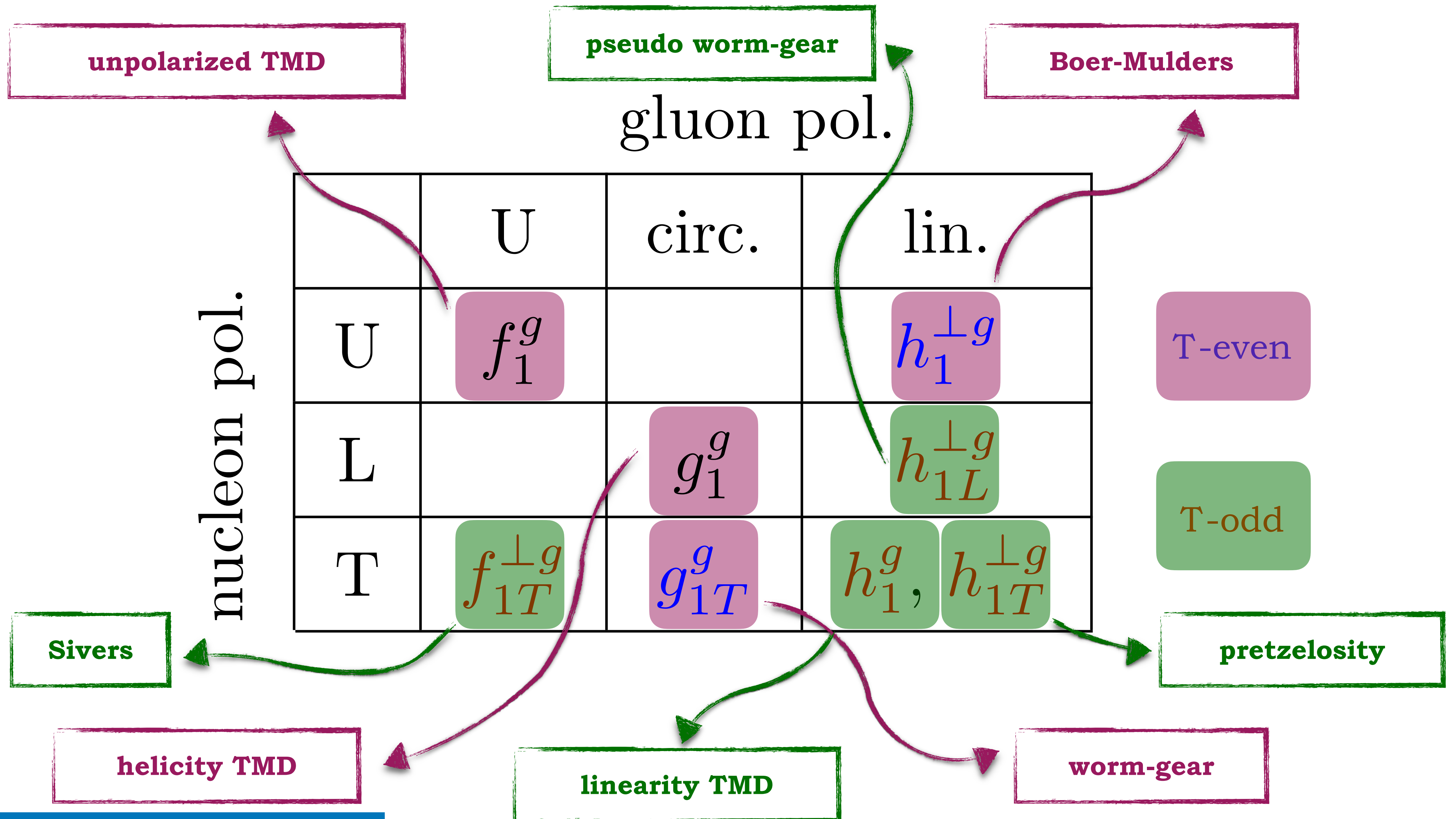
# Gluon TMD PDFs at leading twist

		gluon pol.			
		U	circ.	lin.	
nucleon pol.	U	$f_1^g$		$h_1^{\perp g}$	T-even
	L		$g_1^g$	$h_{1L}^{\perp g}$	T-odd
	T	$f_{1T}^{\perp g}$	$g_{1T}^g$	$h_1^g, h_{1T}^{\perp g}$	

# Gluon TMD PDFs at leading twist



# Gluon TMD PDFs at leading twist



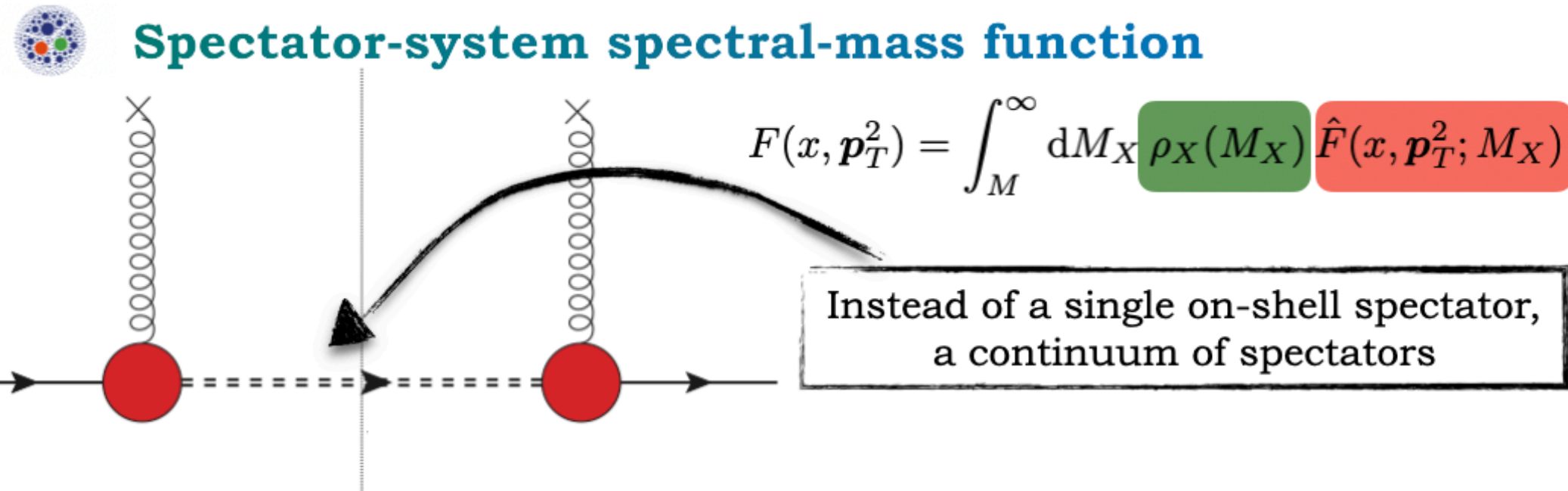


The background features a stylized, semi-transparent illustration of a nucleon (proton or neutron) with a complex internal structure. It shows various colored spheres (red, blue, green) representing quarks and yellow wavy lines representing gluons, all contained within a light blue, textured volume. The overall aesthetic is scientific and modern.

## 2. Spectator-model gluon TMDs

# Spectator-model gluon TMDs

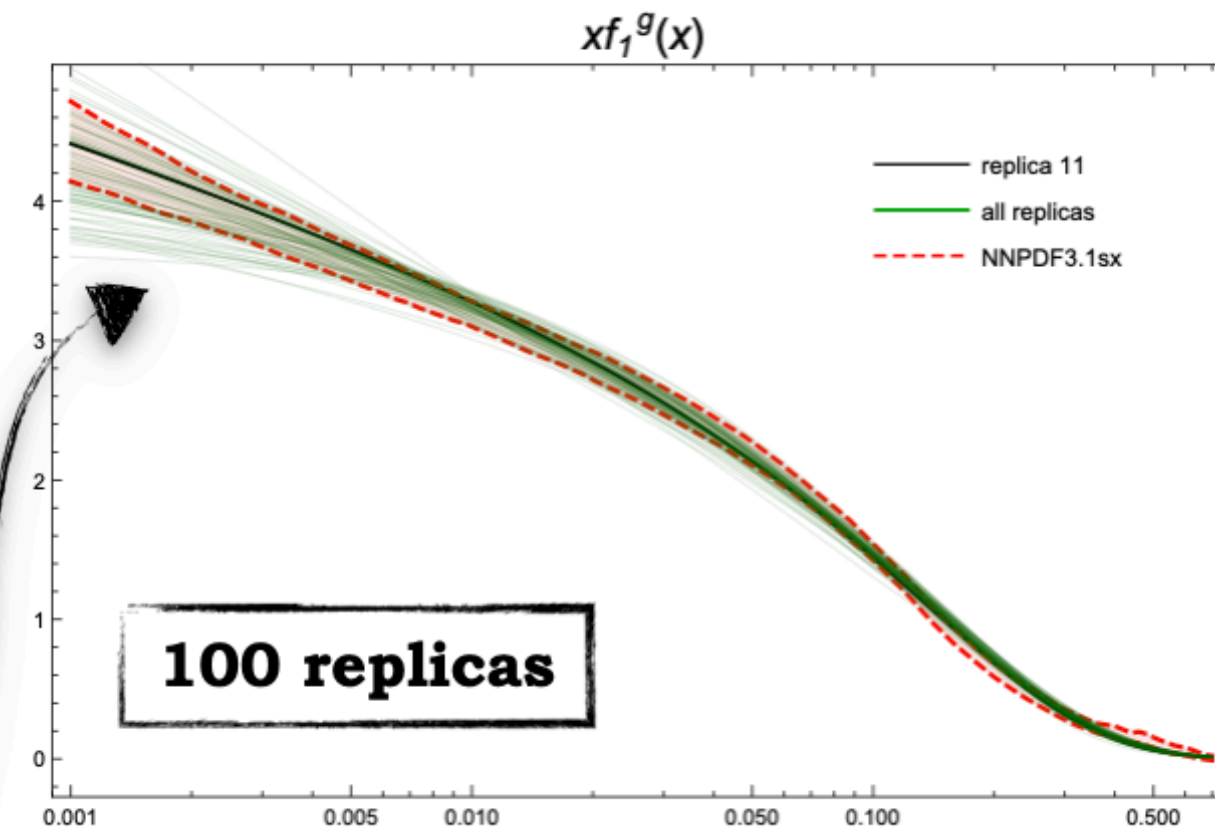
## Our model at a glance



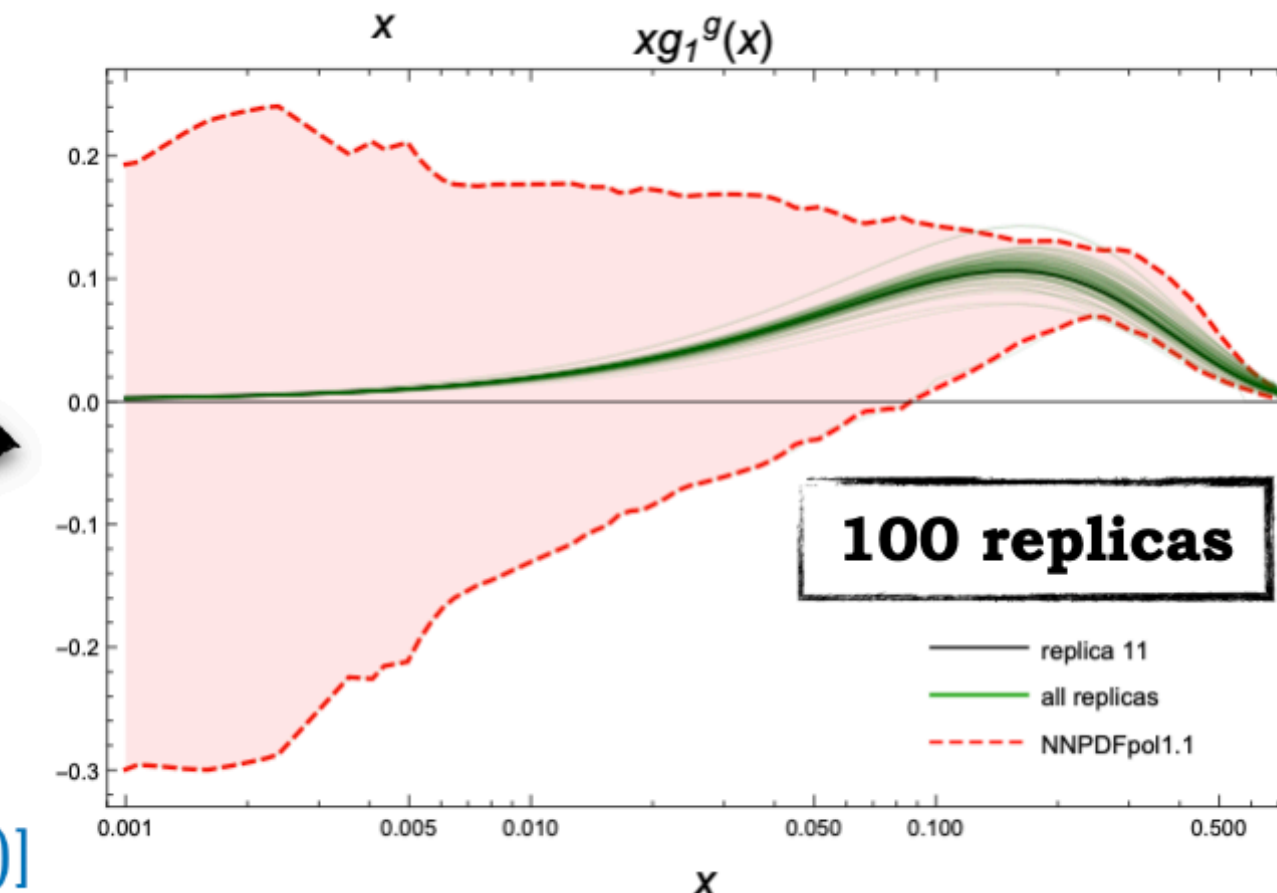
**Link with collinear factorization**

$p_T$ -integrated TMDs **have to** reproduce PDFs at the lowest scale ( $Q_0$ ) *before* evolution

Spectral function **learns** small- and moderate- $x$  info encoded in **NNPDF** collinear parametrizations (NNPDF3.1sx + NNPDFpol1.1)





- ✓ Simultaneous fit of  $f_1$  and  $g_1$  PDFs
- ✓ Inclusion of small- $x$  resummation effects (**BFKL**)
- ✓ Calculation of all leading-twist T-even gluon TMDs



**2.3 Modeling gluon TMDs** [A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]

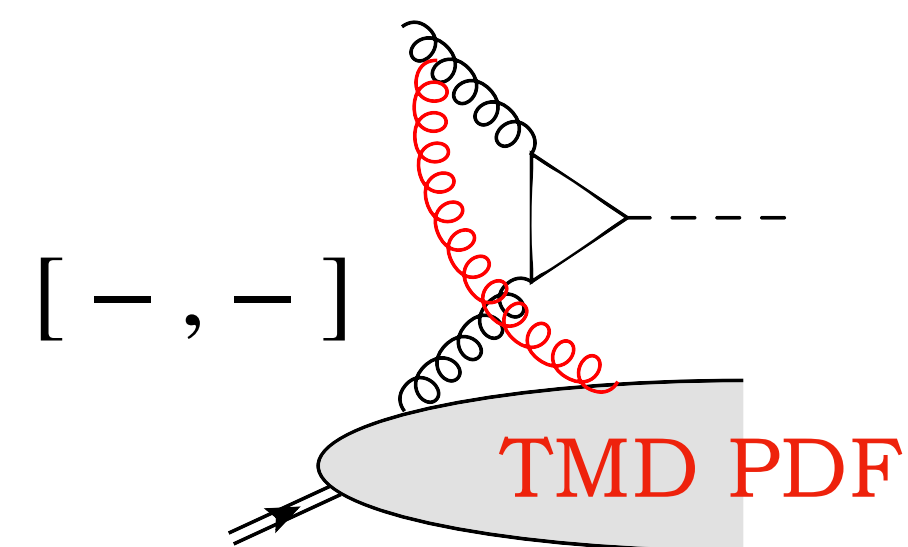
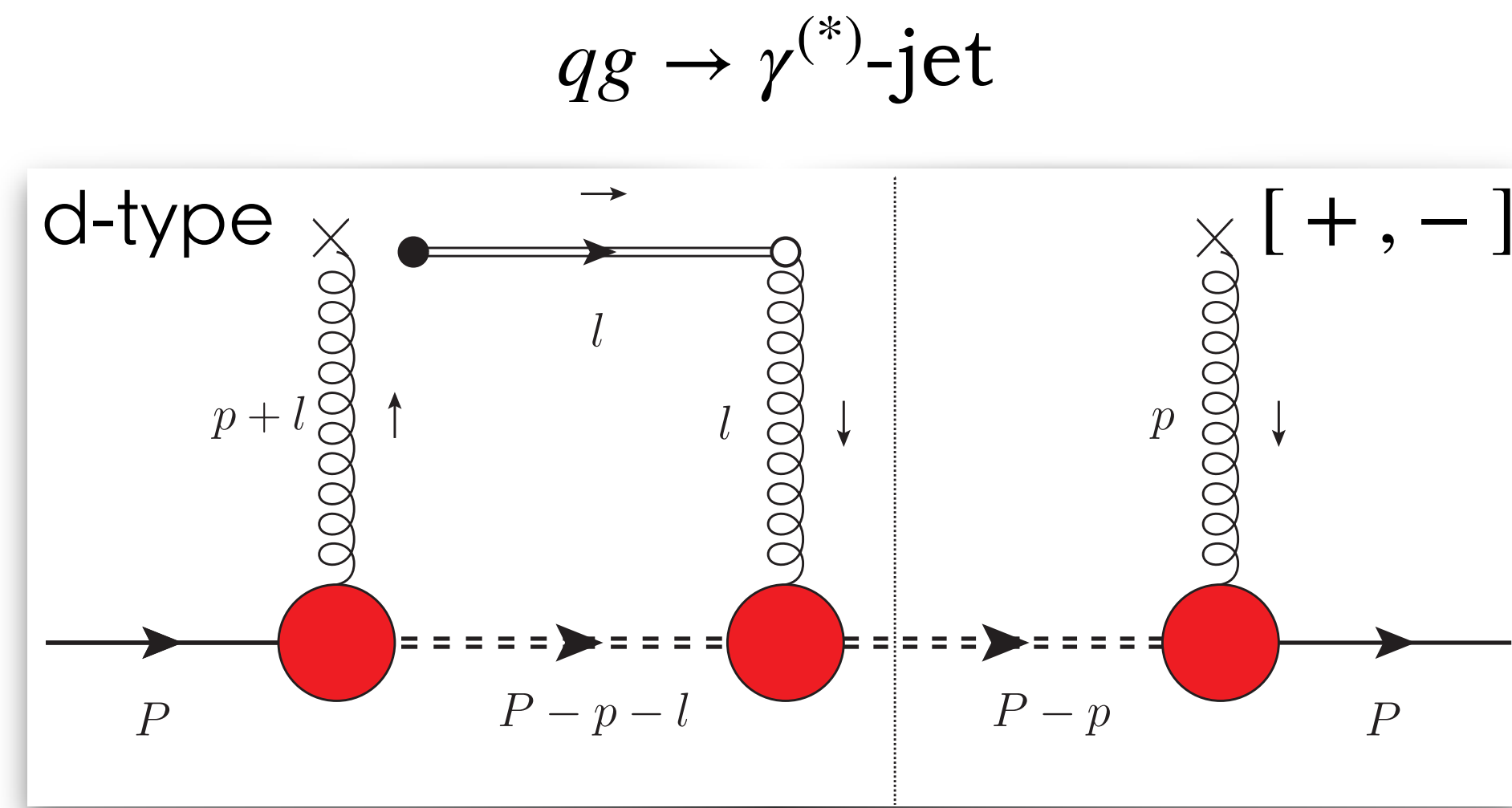
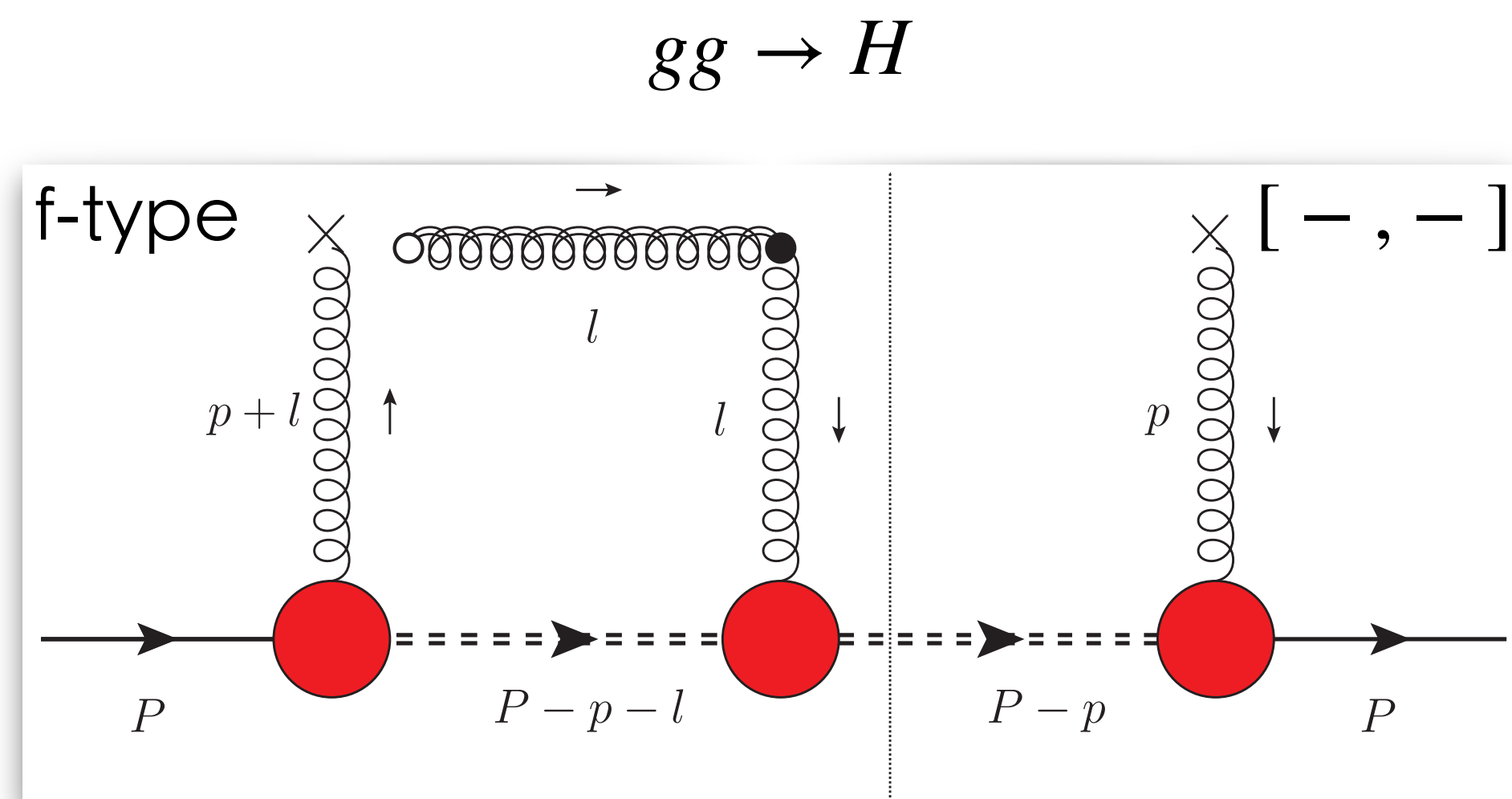
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# T-odd gluon TMDs in a spectator model

-  No residual gluon-spectator interaction at tree level
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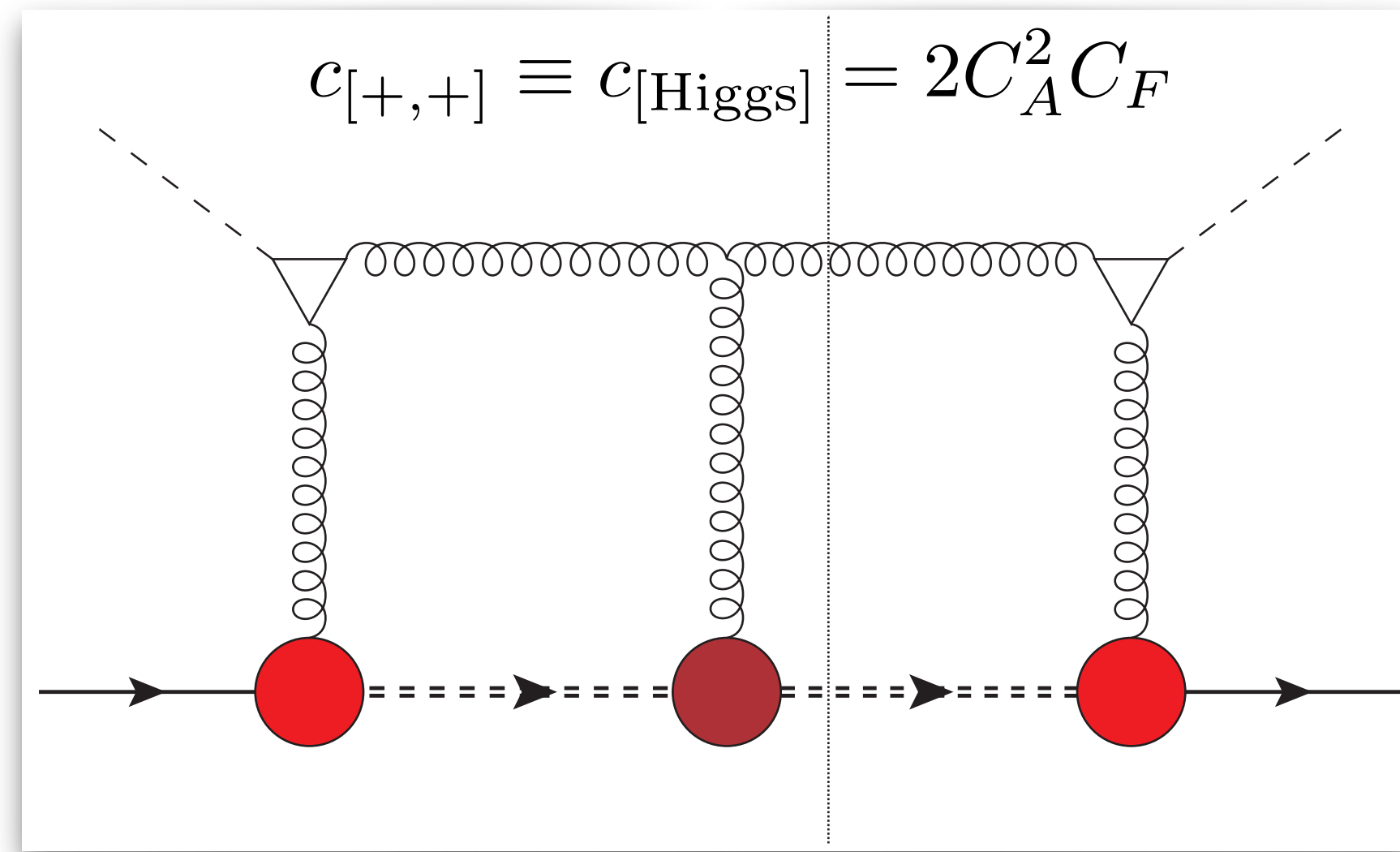
f-type (WW) structure



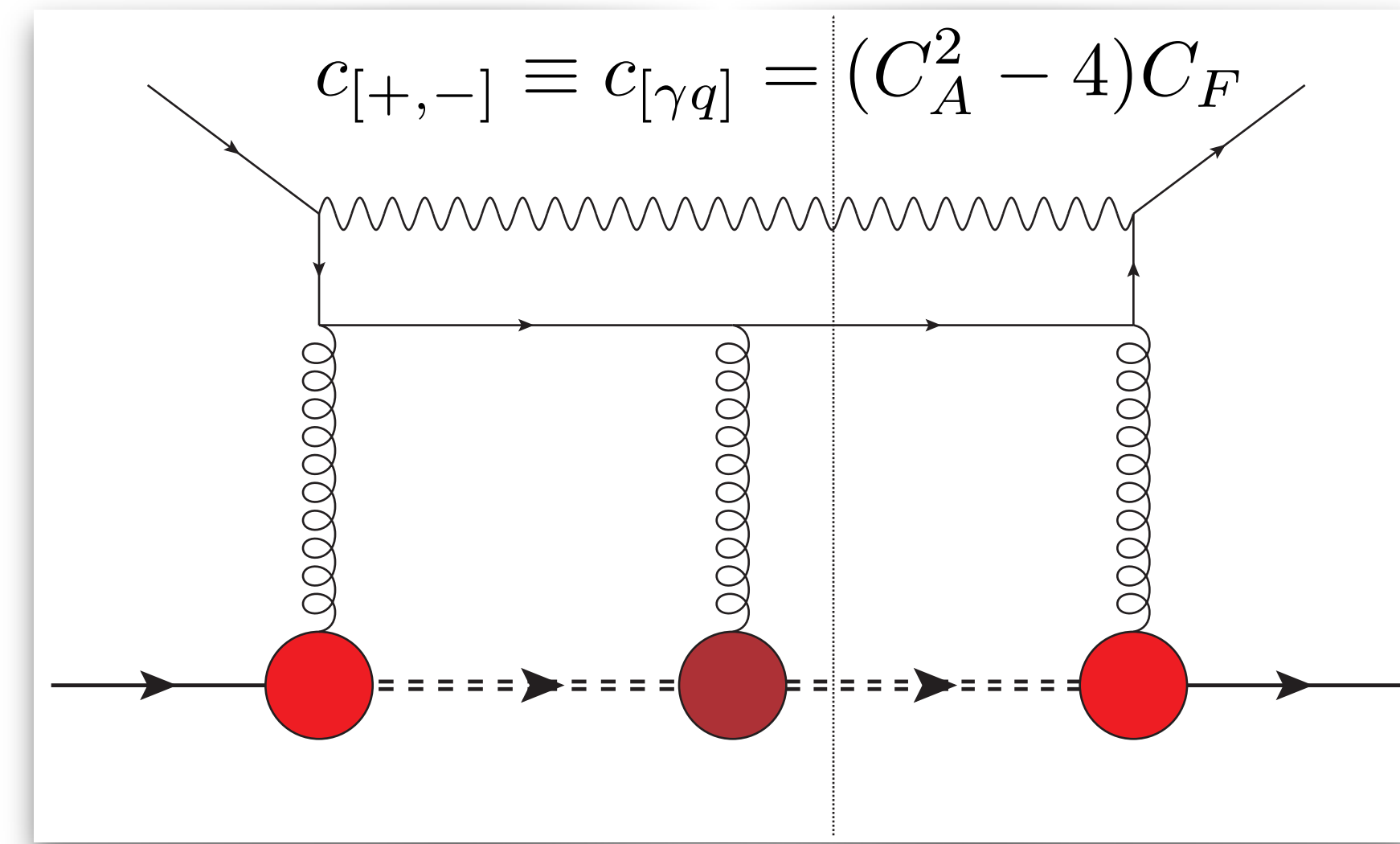
d-type (dipole) structure

# T-odd gluon TMDs in a spectator model

Higgs-gluon fusion  $\Rightarrow$  f-type  $[+, +]$



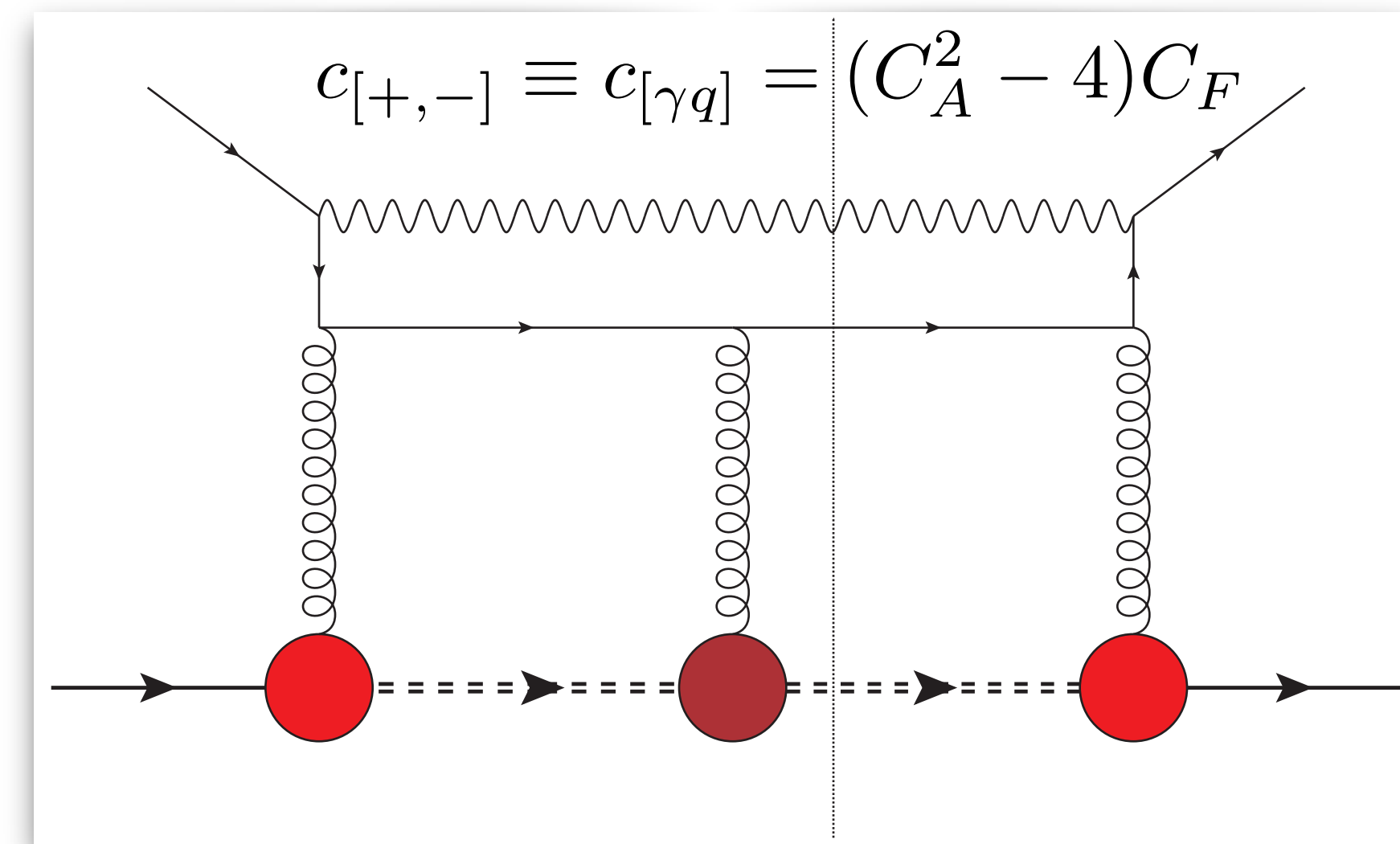
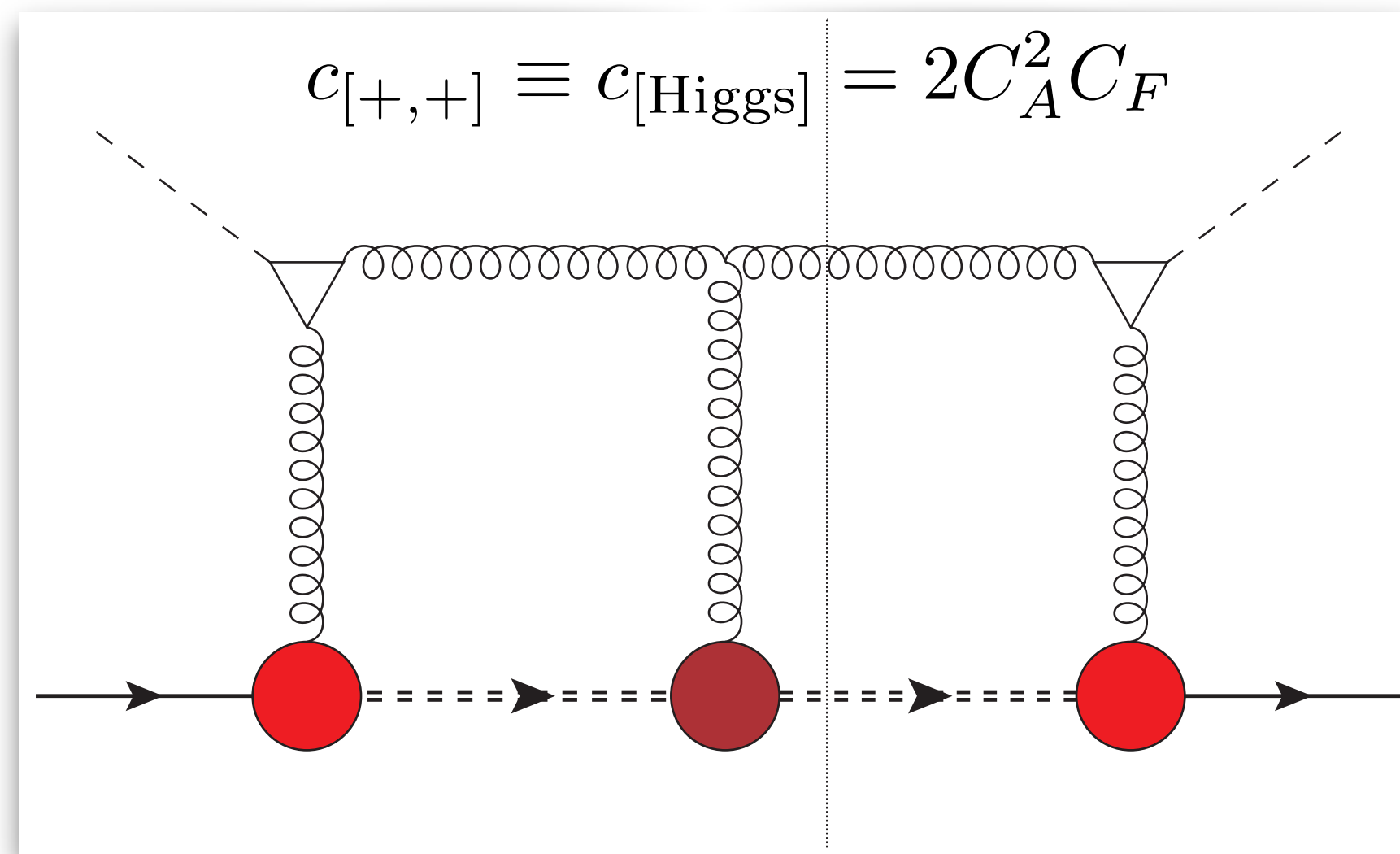
Photon-jet emission  $\Rightarrow$  d-type  $[+, -]$



# T-odd gluon TMDs in a spectator model

Higgs-gluon fusion  $\Rightarrow$  f-type [ + , + ]

Photon-jet emission  $\Rightarrow$  d-type [ + , - ]



 nucleon-gluon-spectator

 spectator-gluon-spectator

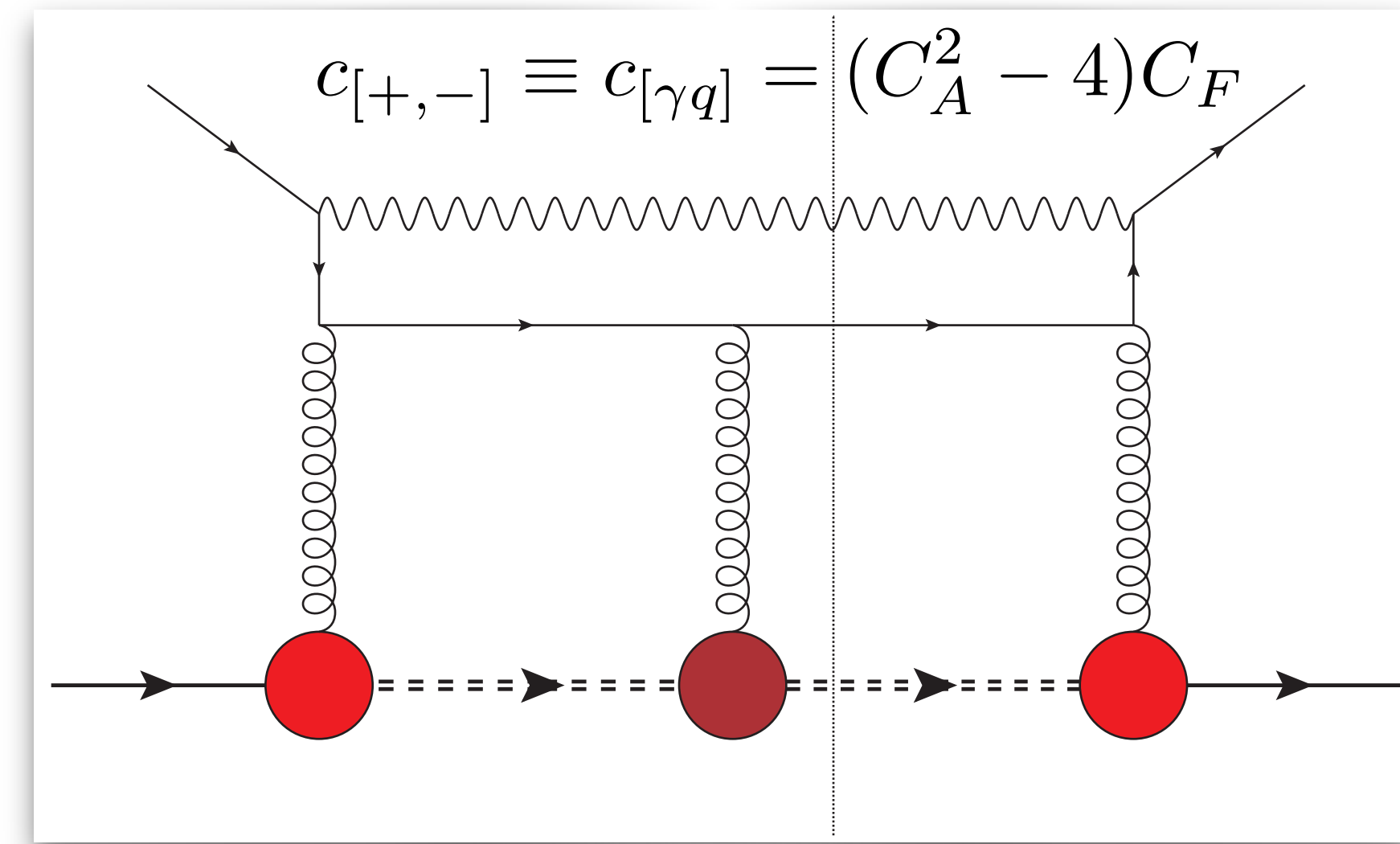
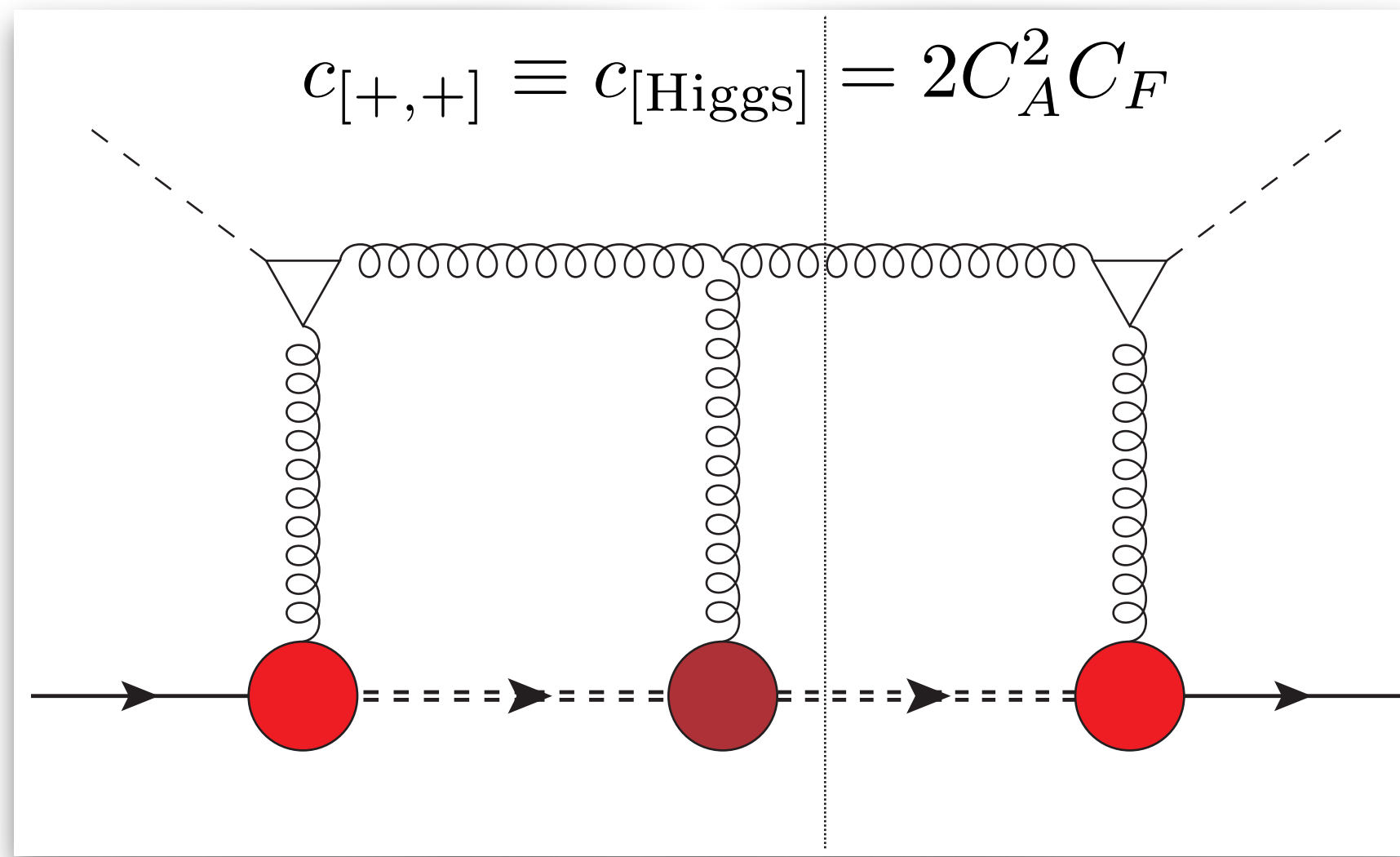
$$\mathcal{Y}_{bc}^{\mu}(p^2) = \delta_{bc} \left[ g_1(p^2) \gamma_{\mu} + g_2(p^2) \frac{i}{2M} \sigma^{\mu\nu} p_{\nu} \right]$$

$$\mathcal{X}_{abc}^{\mu}(p^2) = f^{abc} \left[ g_1^f(p^2) \gamma^{\mu} + g_2^f(p^2) \frac{i}{2M} \sigma^{\mu\nu} p_{\nu} \right] - i d^{abc} \left[ g_1^d(p^2) \gamma^{\mu} + g_2^d(p^2) \frac{i}{2M} \sigma^{\mu\nu} p_{\nu} \right]$$

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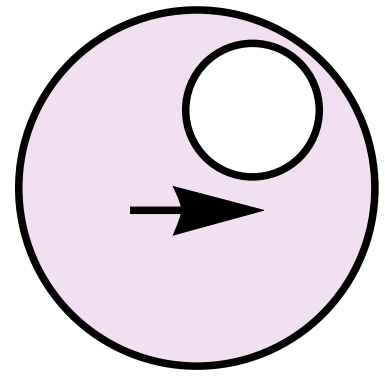
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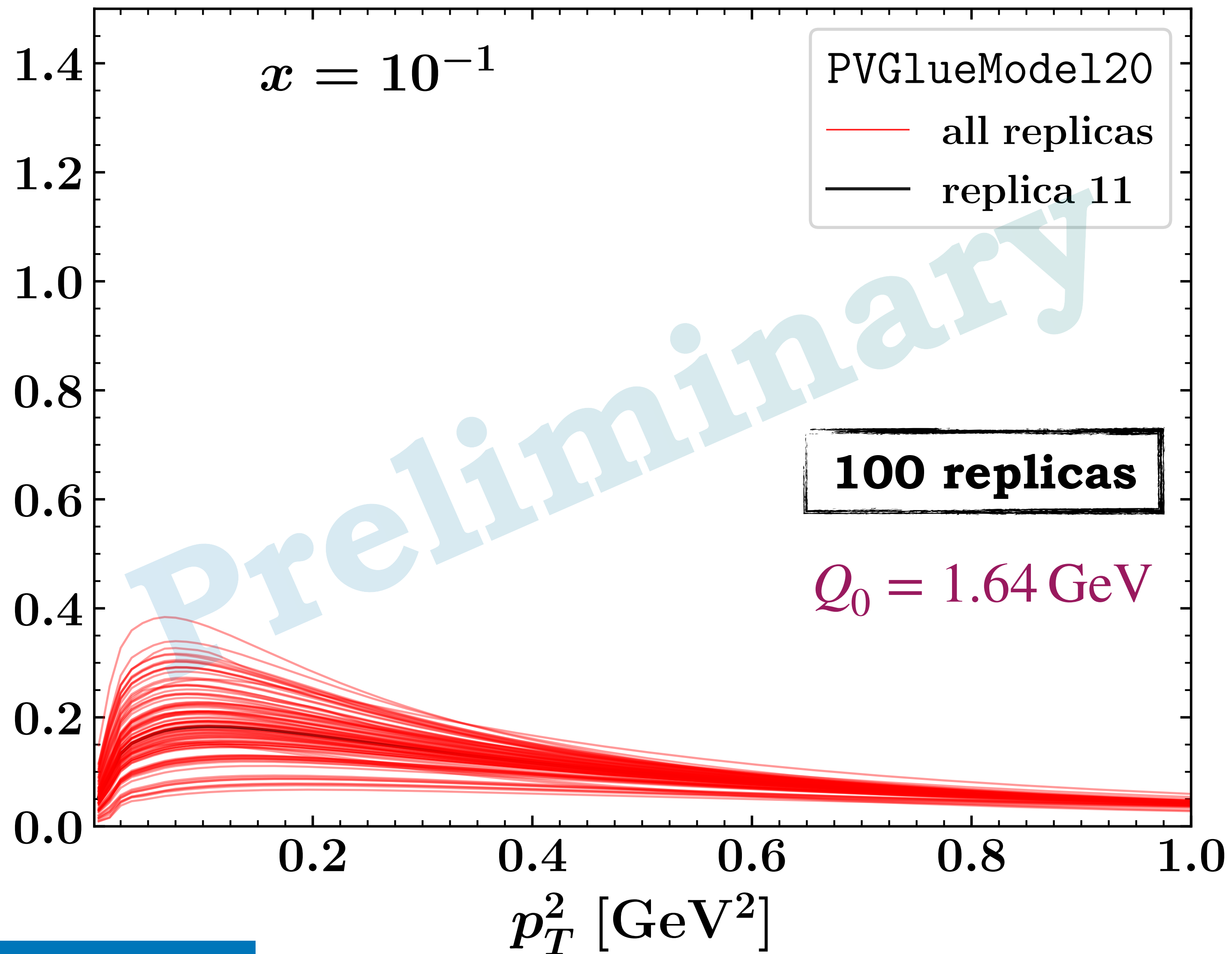
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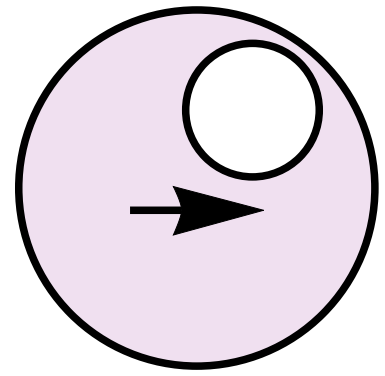
Assumption:  $g_{1,2}^d(p^2) = g_{1,2}^f(p^2) \equiv g_{1,2}(p^2) \quad \Leftrightarrow \quad f_{1T}^{\perp[+,-]} = \frac{c_{[+,-]}}{c_{[+,+]}} f_{1T}^{\perp[+,+]} \equiv \frac{10}{18} f_{1T}^{\perp[+,+]}$



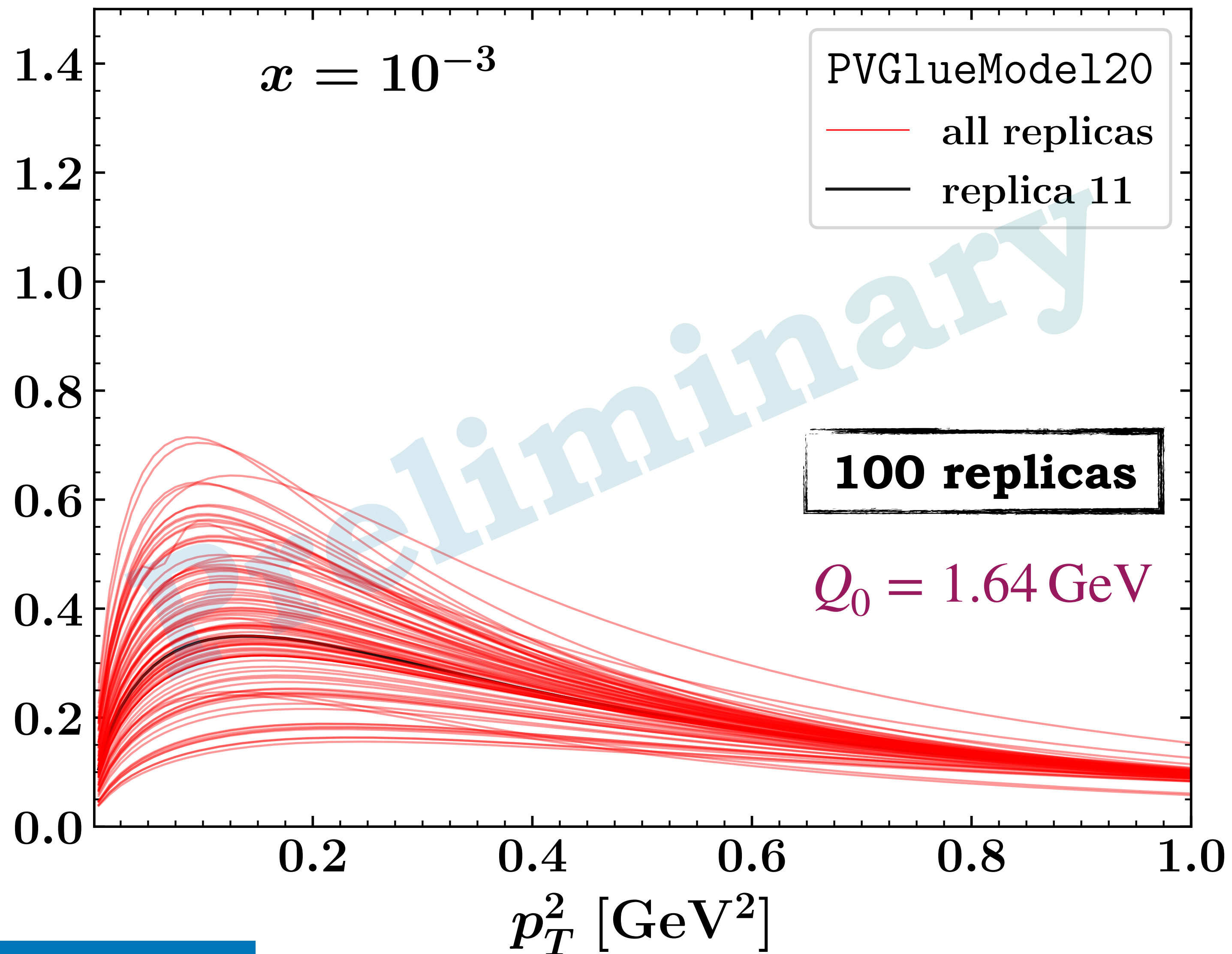
$$x \frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)$$







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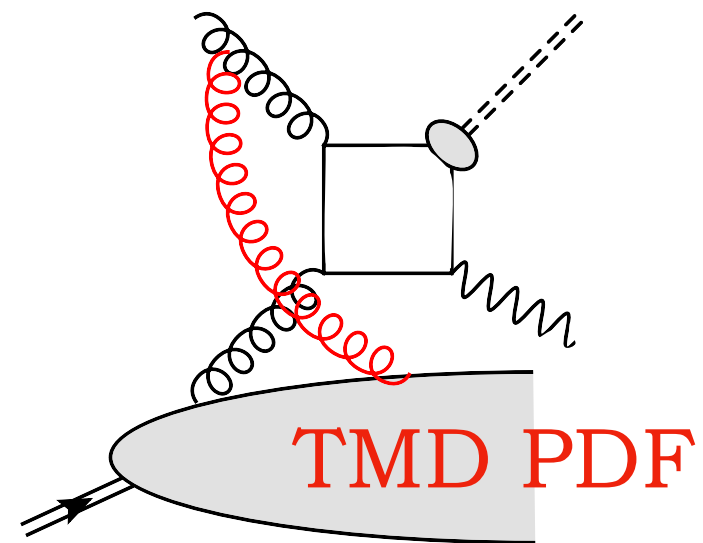
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# Quarkonia: assets & challenges

## Assets

 Onia  $\Rightarrow$  clean channels of f-type gluon TMDs



Initial-state color flow  $\Rightarrow$   $[-, -]$  gauge link

(overview)  [D. Boer (2017)]

Sivers	$ep^\dagger \rightarrow e' Q \bar{Q} X$ $ep^\dagger \rightarrow e' j_1 j_2 X$
$f_{1T}^{\perp g[-,-]}$	✓
$f_{1T}^{\perp g[+,-]}$	×

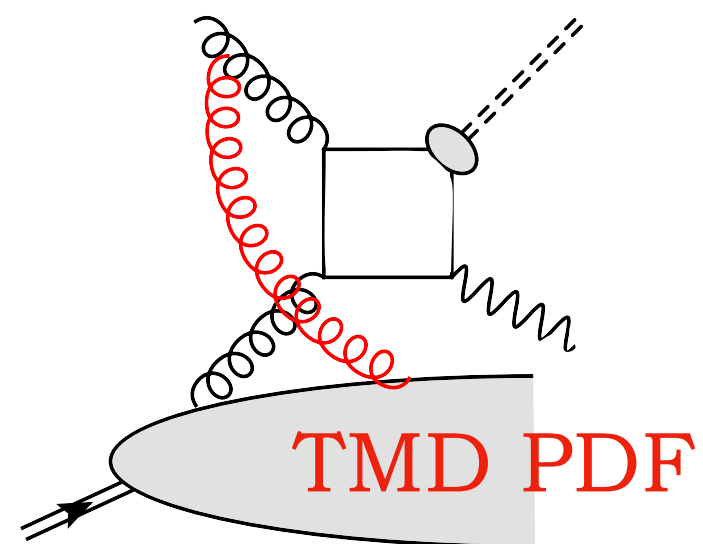
Boer-Mulders	$ep \rightarrow e' Q \bar{Q} X$ $ep \rightarrow e' j_1 j_2 X$
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  $\eta_{c,b}$   $\Rightarrow$  LHC complementarity, TMD factorization

$$\frac{d\sigma}{dq_T} \sim$$

at low transverse momentum  
for [pseudo]scalar state

$$\sim \mathcal{C} [ f_1^{g/A} f_1^{g/B} ] \pm \mathcal{C} [ h_1^{\perp g/A} h_1^{\perp g/B} ]$$

unpolarized gluons      lin. polarized gluons

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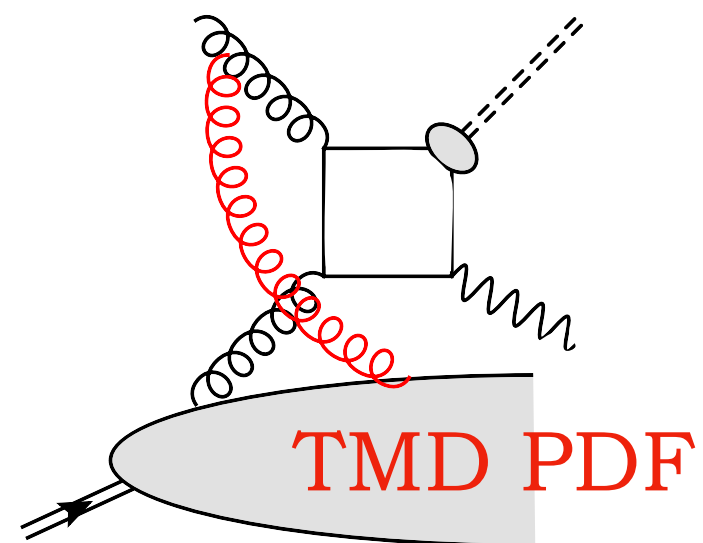
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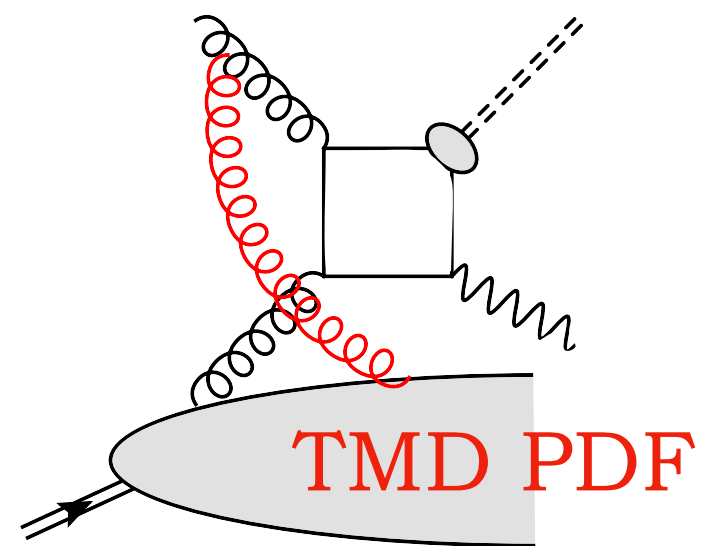
Precision TMD  $\Leftrightarrow$  production mechanism(s)

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Color Evaporation Model

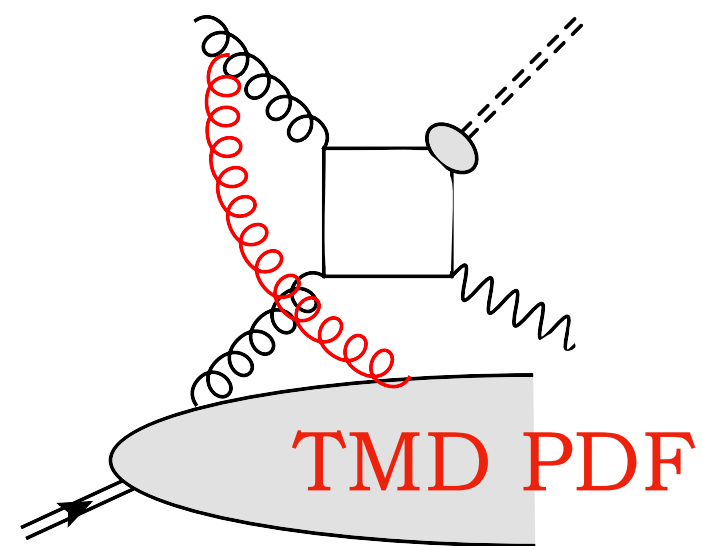
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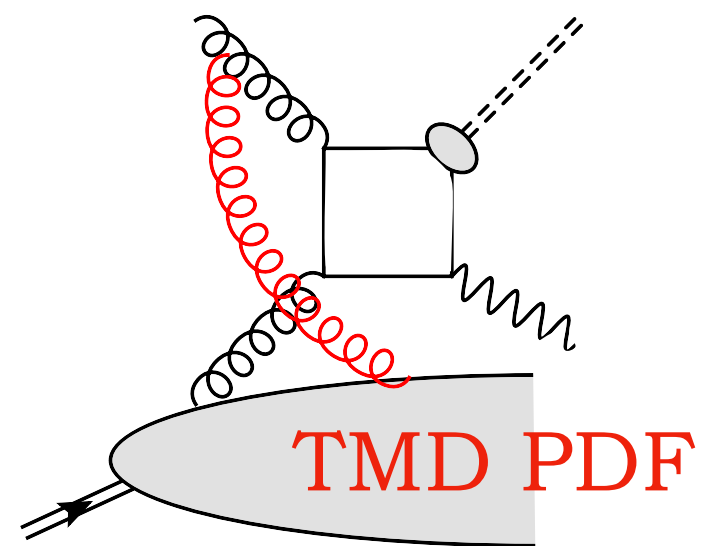
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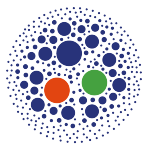
### NRQCD and Color Octet

Higher Fock states, soft gluon emissions  
Problems at low  $p_T$ , fails on polarization



## TMD & shape functions

 NRQCD  $\Rightarrow$  double expansion:  $\alpha_s \oplus v$

 NRQCD  $\Rightarrow$   $d\sigma(|Q\rangle) \propto \mathcal{H} \otimes \text{LDME}$


$$|Q\rangle = \mathcal{O}(1) |Q\bar{Q}[^3S_1^{(1)}]\rangle + \mathcal{O}(v) |Q\bar{Q}[^3P_J^{(8)}g]\rangle + \mathcal{O}(v^2) |Q\bar{Q}[^1S_0^{(8)}g]\rangle \\ + \mathcal{O}(v^2) |Q\bar{Q}[^3S_1^{(1,8)}gg]\rangle + \mathcal{O}(v^2) |Q\bar{Q}[^3D_J^{(1,8)}gg]\rangle + \dots$$

S-wave quarkonium wave function

# Quarkonia: a path toward precision

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S-wave quarkonium wave function

 TMD  $\Rightarrow$  from LDMEs to shape functions (ShFs)

 2 mechanisms: bound state + soft-gluon

(factorization)  [M. Garcia Echevarria (2019)]

(SCET)  [S. Fleming, Y. Makris, T. Mehen (2020)]

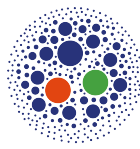
(unpol.  $J/\psi$ )  [D. Boer, U. D'Alesio, F. Murgia, C. Pisano, P. Taelis (2020)]

(pol.  $J/\psi$ )  [D. Boer, U. D'Alesio, L. Maxia, F. Murgia, C. Pisano, R. Sangem (2022)]

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 NRQCD  $\Rightarrow$  double expansion:  $\alpha_s \oplus v$

 NRQCD  $\Rightarrow$   $d\sigma(|Q\rangle) \propto \mathcal{H} \otimes \text{LDME}$

$$|\mathcal{Q}\rangle = \mathcal{O}(1) |Q\bar{Q}[^3S_1^{(1)}]\rangle + \mathcal{O}(v) |Q\bar{Q}[^3P_J^{(8)}g]\rangle + \mathcal{O}(v^2) |Q\bar{Q}[^1S_0^{(8)}g]\rangle + \mathcal{O}(v^2) |Q\bar{Q}[^3S_1^{(1,8)}gg]\rangle + \mathcal{O}(v^2) |Q\bar{Q}[^3D_J^{(1,8)}gg]\rangle + \dots$$

S-wave quarkonium wave function

 TMD  $\Rightarrow$  from LDMEs to shape functions (ShFs)

 2 mechanisms: bound state + soft-gluon

(factorization)  [M. Garcia Echevarria (2019)]

(SCET)  [S. Fleming, Y. Makris, T. Mehen (2020)]

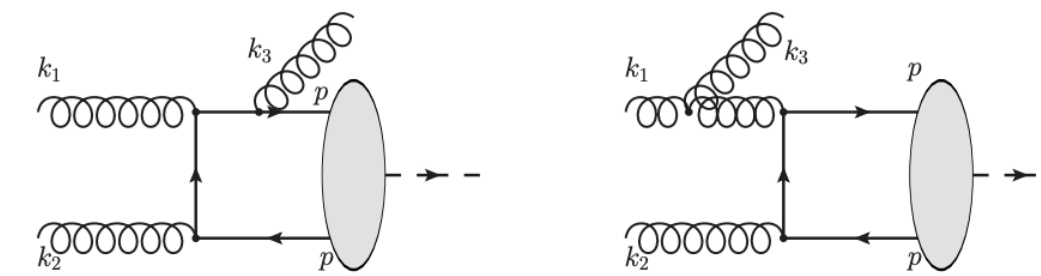
(unpol.  $J/\psi$ )  [D. Boer, U. D'Alesio, F. Murgia, C. Pisano, P. Taelis (2020)]

(pol.  $J/\psi$ )  [D. Boer, U. D'Alesio, L. Maxia, F. Murgia, C. Pisano, R. Sangem (2022)]

## NLO collinear negative distributions

### Recap of NLO calculation & origin of negative numbers

$\hat{s}$ -dependence only present in real corrections ( $g(k_1) + g(k_2) \rightarrow \eta_Q(P) + g(k_3)$ )



- **Real-emission corrections** are perfect square ( $|\mathcal{M}^{(\text{Real})}|^2$ ) and thus **positive**
- **IR singularities** in the real emissions only reveal themselves after taking the **phase-space integration**:  $\bar{\sigma}_{gg}^{\text{NLO},z\neq 1}(z) = \int d\hat{t} \frac{\bar{\sigma}_{gg}^{\text{NLO},z\neq 1}}{d\hat{t}}$

$$\bar{\sigma}_{gg}^{\text{NLO},z\neq 1}(z) = -\frac{1}{\epsilon_{\text{IR}}} \frac{\alpha_s}{\pi} \left( \frac{4\pi\mu_R^2}{M_Q^2} \right)^\epsilon \Gamma(1+\epsilon) \hat{\sigma}_0^{\text{LO}} z P_{gg}(z) + 2C_A \frac{\alpha_s}{\pi} \hat{\sigma}_0^{\text{LO}} \bar{A}_{gg}(z)$$

- For  $\epsilon_{\text{IR}} \rightarrow 0^-$ ,  $\bar{\sigma}_{gg}^{\text{NLO},z\neq 1} \geq 0$  for all  $0 \leq z < 1$  as expected
- Initial-state collinear **divergences are absorbed/subtracted into PDF** via *process-independent* Altarelli-Parisi counterterm in  $\overline{\text{MS}}$ -scheme

$$\bar{\sigma}_{gg}^{\text{AP-CT}}(z) = \frac{1}{\epsilon_{\text{IR}}} \frac{\alpha_s}{\pi} \left( \frac{4\pi\mu_R^2}{\mu_F^2} \right)^\epsilon \Gamma(1+\epsilon) \hat{\sigma}_0^{\text{LO}} z P_{gg}(z)$$

J.P. Lansberg (IJCLab)

Quarkonium production

September 28, 2022 5 / 20

(this slide)  [Diffraction 2022's talk by Jean-Philippe Lansberg]

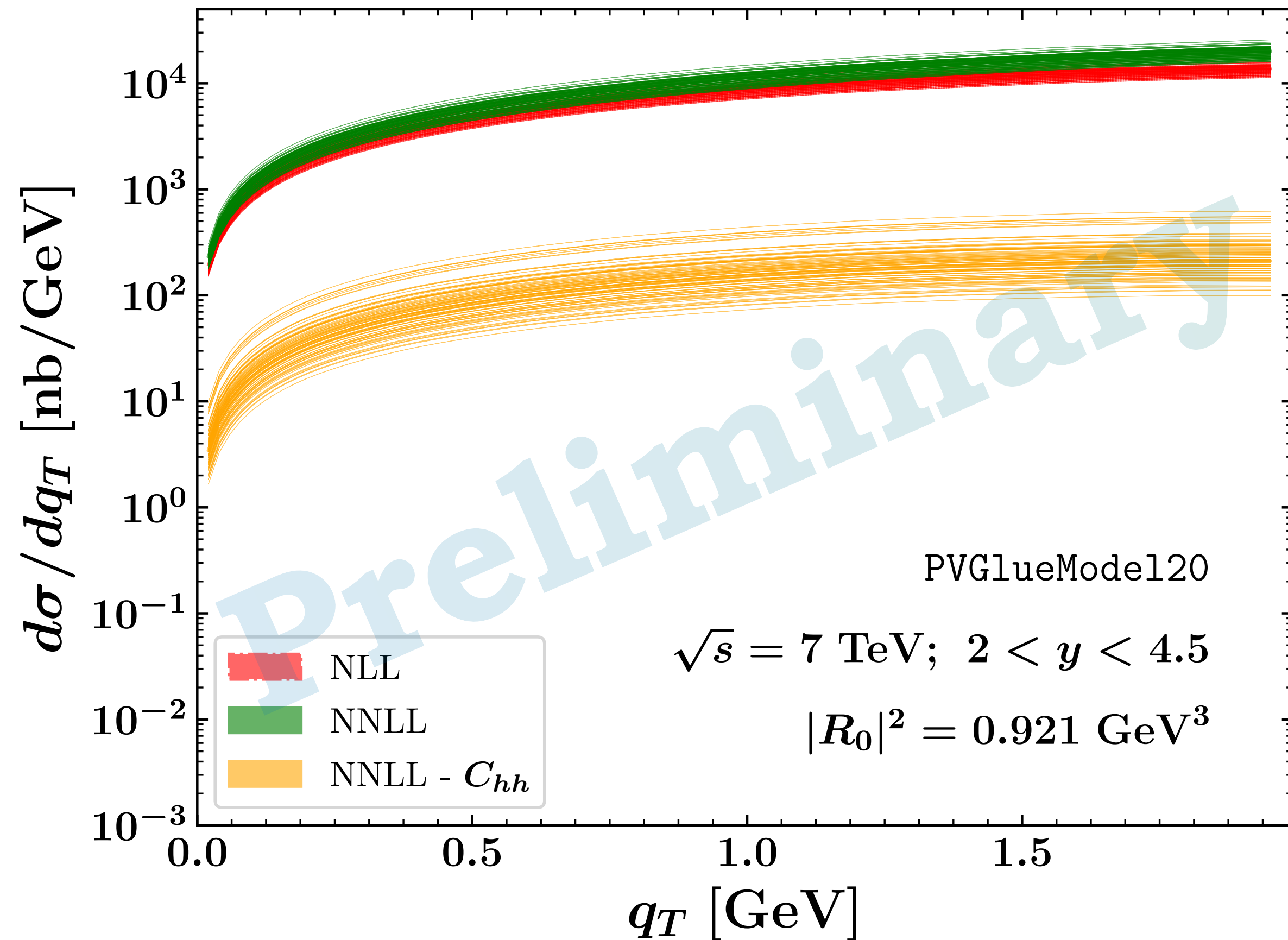
(collinear/HEF matching)  [J.-P. Lansberg, M. Nefedov, M.A. Ozelik (2022)]

(scale fixing)  [A. Colpani Serri et al. (2021)]

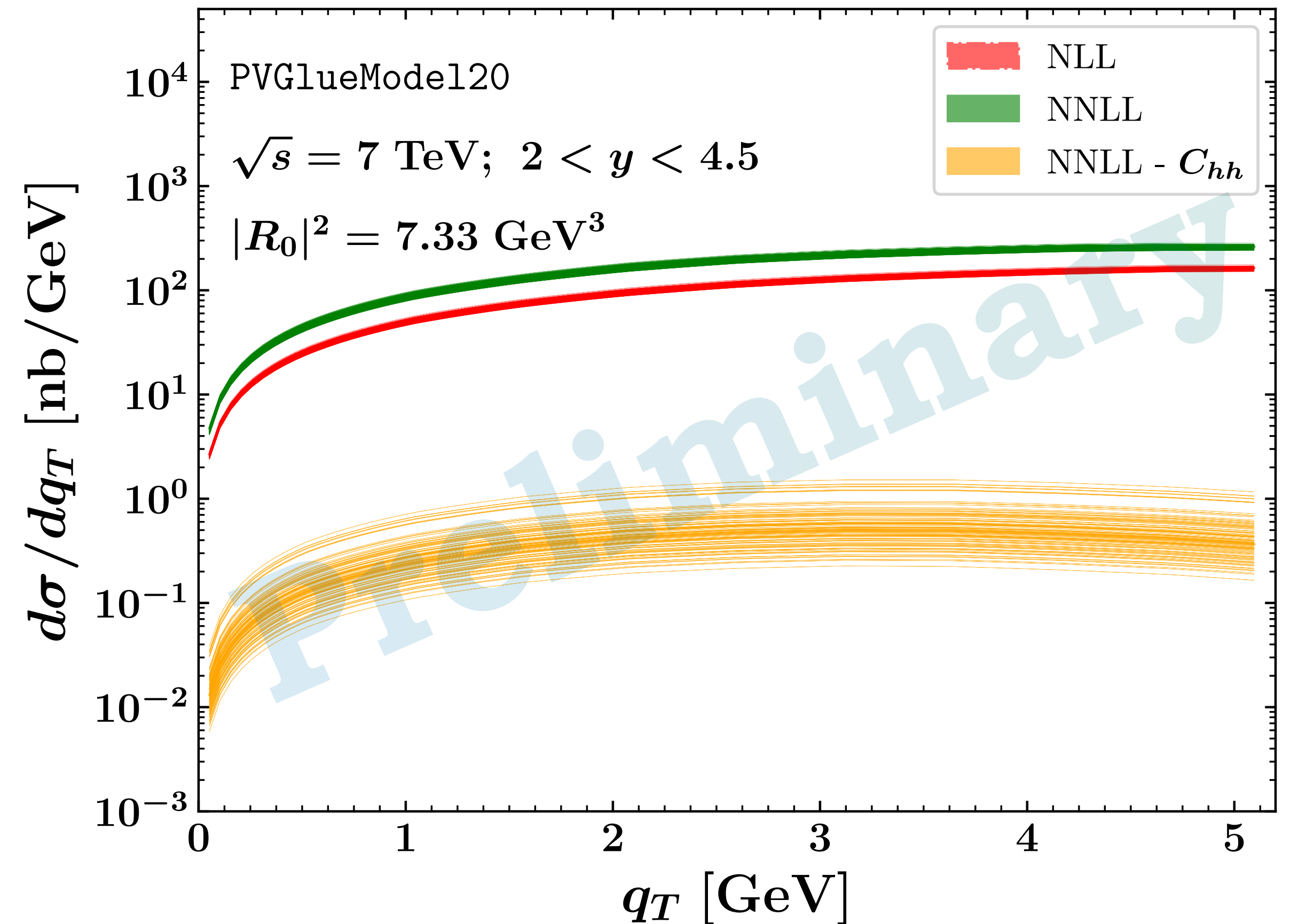
# $\eta_{c,b}$ production @ 7TeV LHCb


 Perturbative scales fixed, NP-evolution parameters fixed, TMD 100-replica analysis, [NRQCD](#) w/o [ShFs](#)

$$p(P_1) + p(P_2) \rightarrow \eta_c(q_T)$$

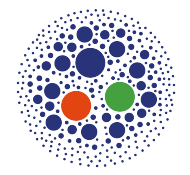


$$p(P_1) + p(P_2) \rightarrow \eta_b(q_T)$$

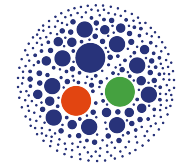


[A. Bacchetta, F.G.C., M.G. Echevarria, J.-P. Lansberg, M. Ozelik, M. Radici, A. Signori (in preparation)]

# Quarkonia & gluon TMDs: a win-win strategy

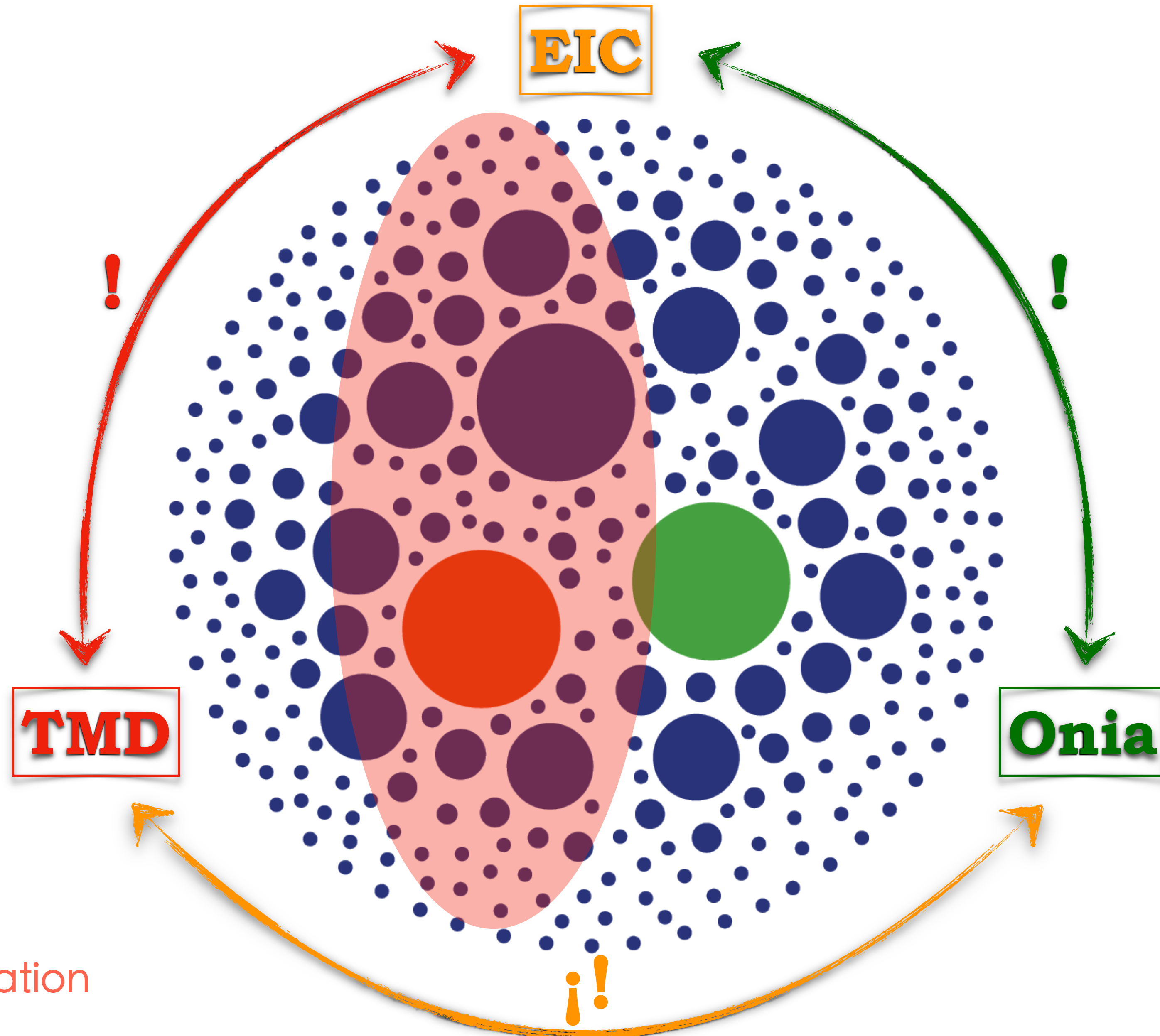


Onia as Tools  $\Rightarrow$  3D proton imaging via gluon TMDs



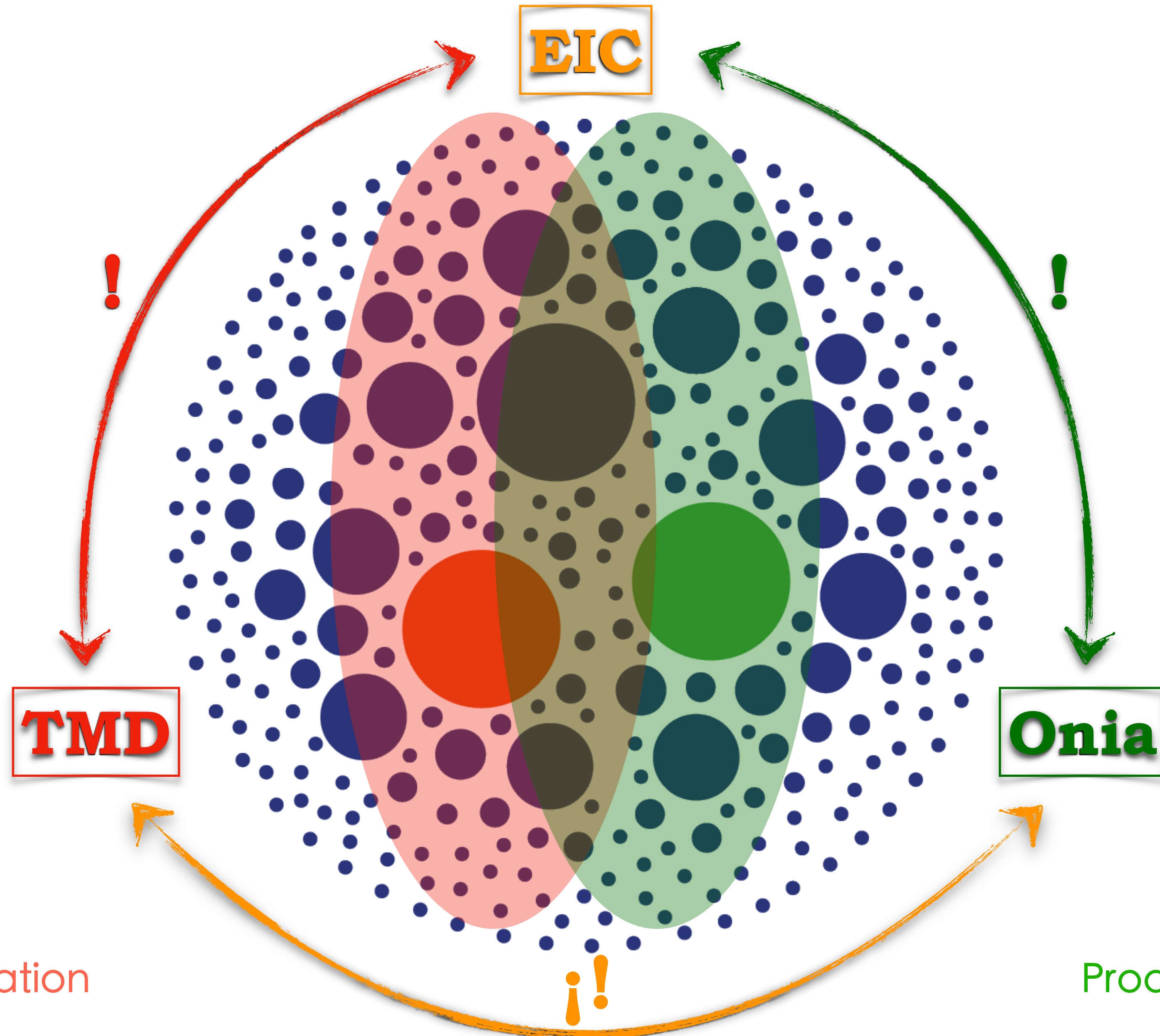
Gluon TMDs as Tools  $\Rightarrow$  unveil onium production mechanisms

# Quarkonia & gluon TMDs: a win-win strategy



Models & polarization

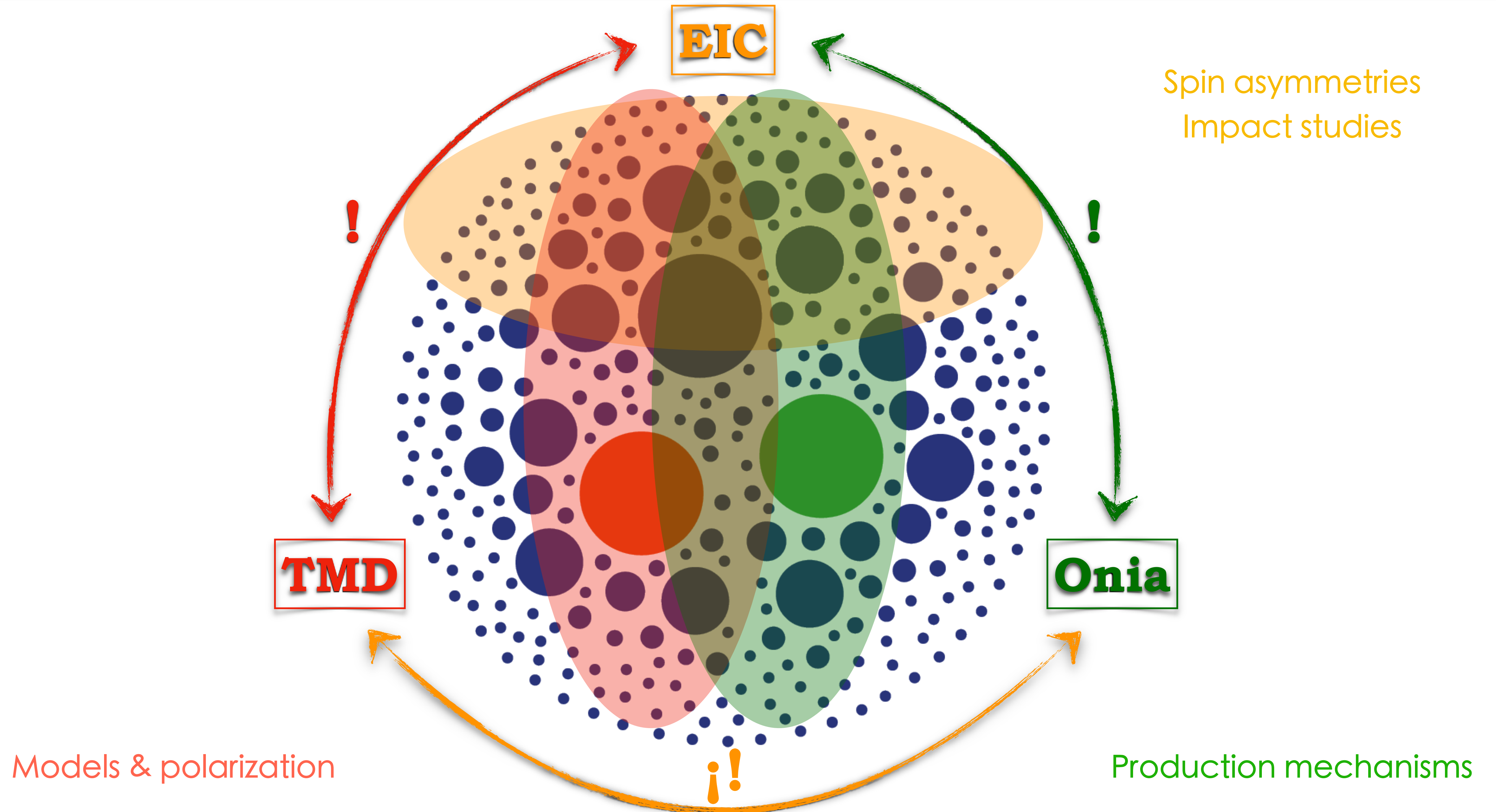
# Quarkonia & gluon TMDs: a win-win strategy



Models & polarization

Production mechanisms

# Quarkonia & gluon TMDs: a win-win strategy



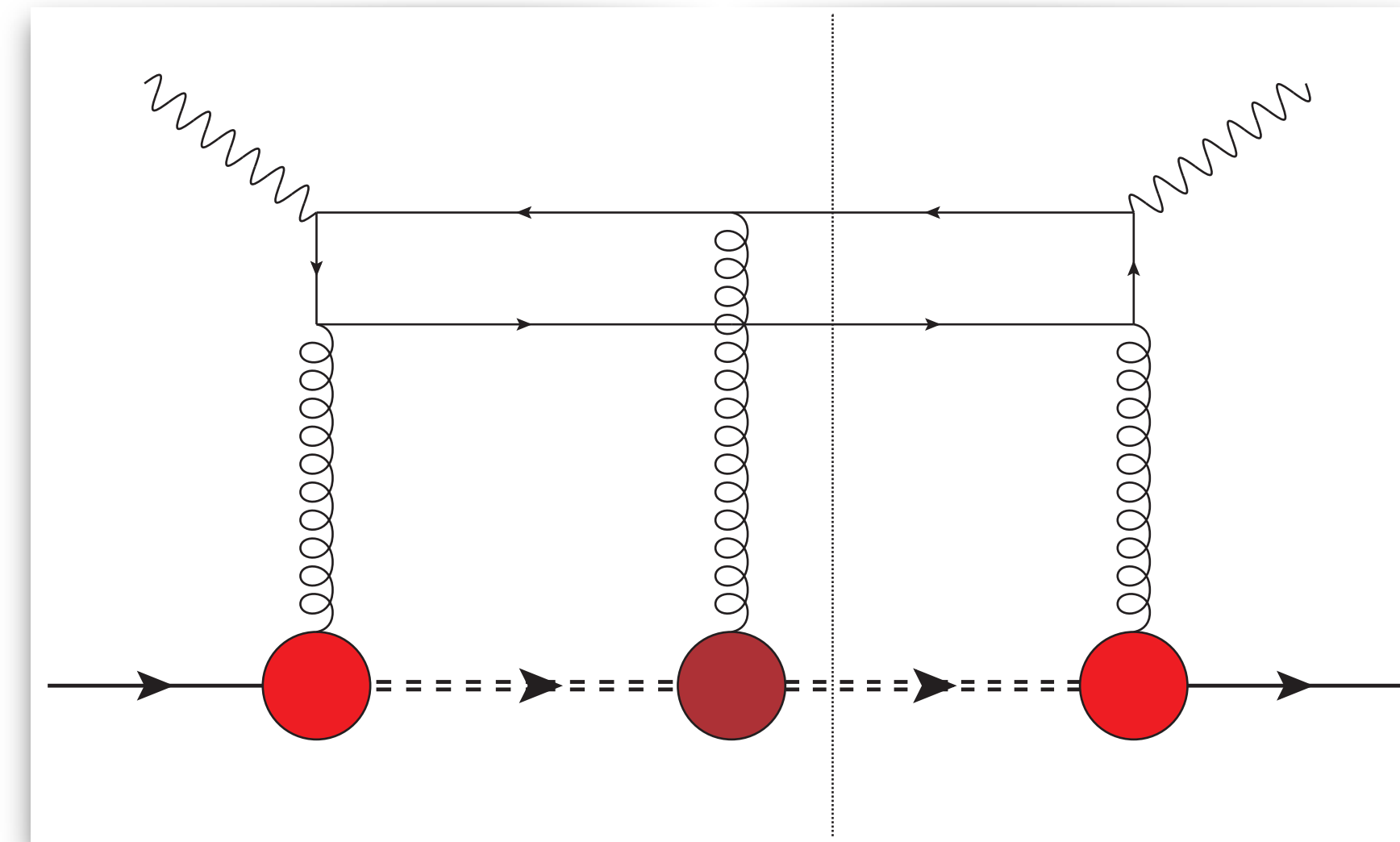
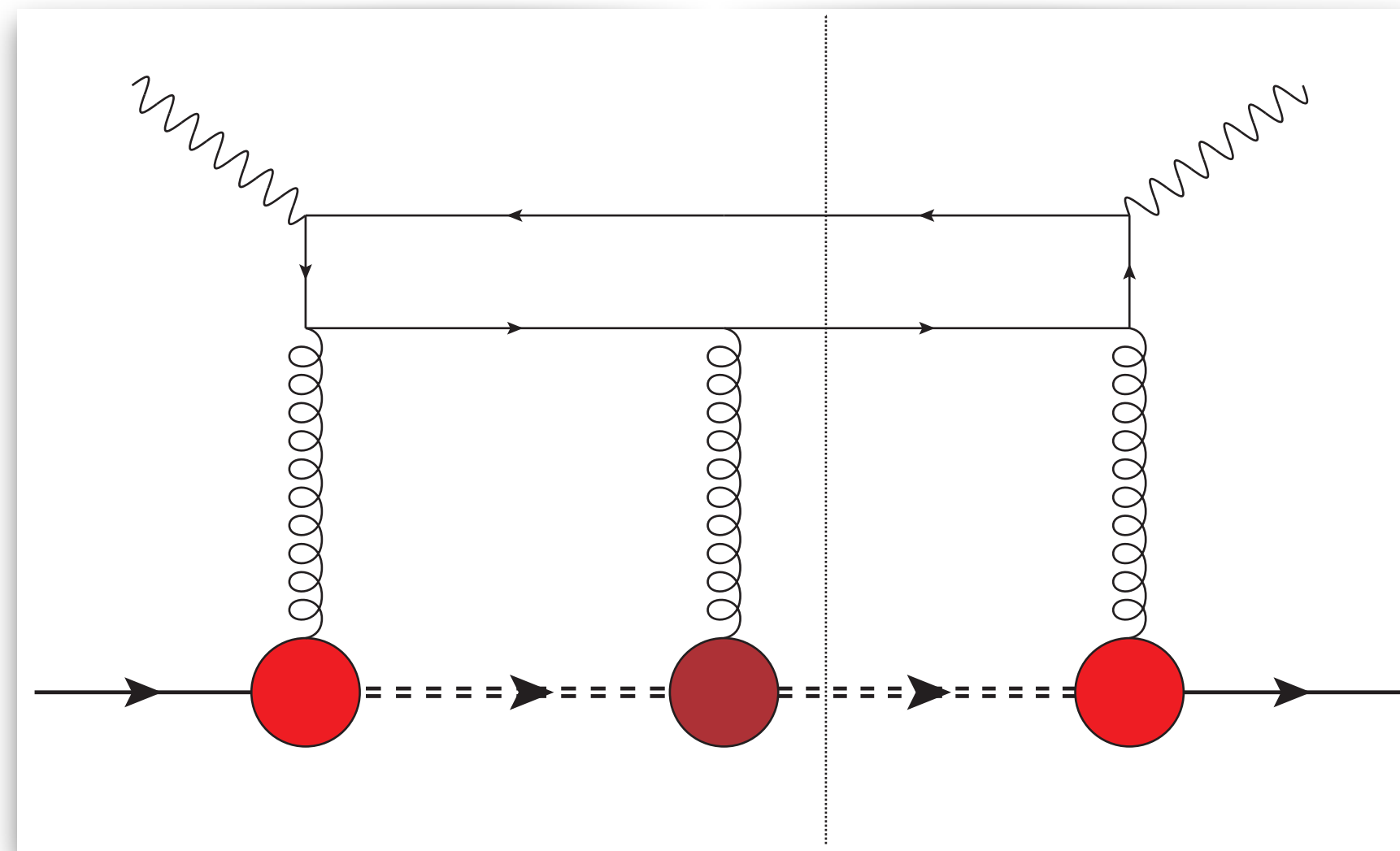




# Extras

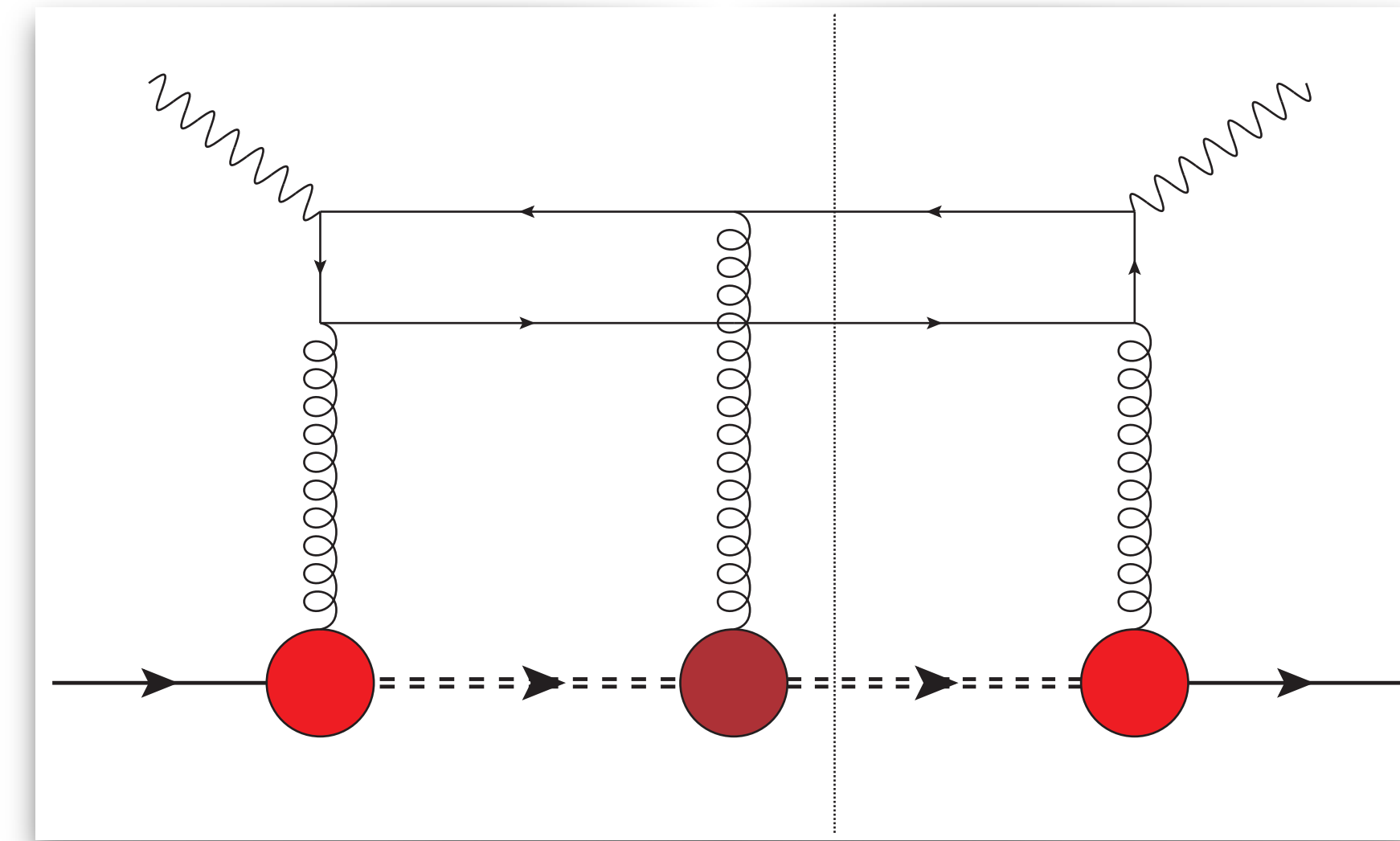
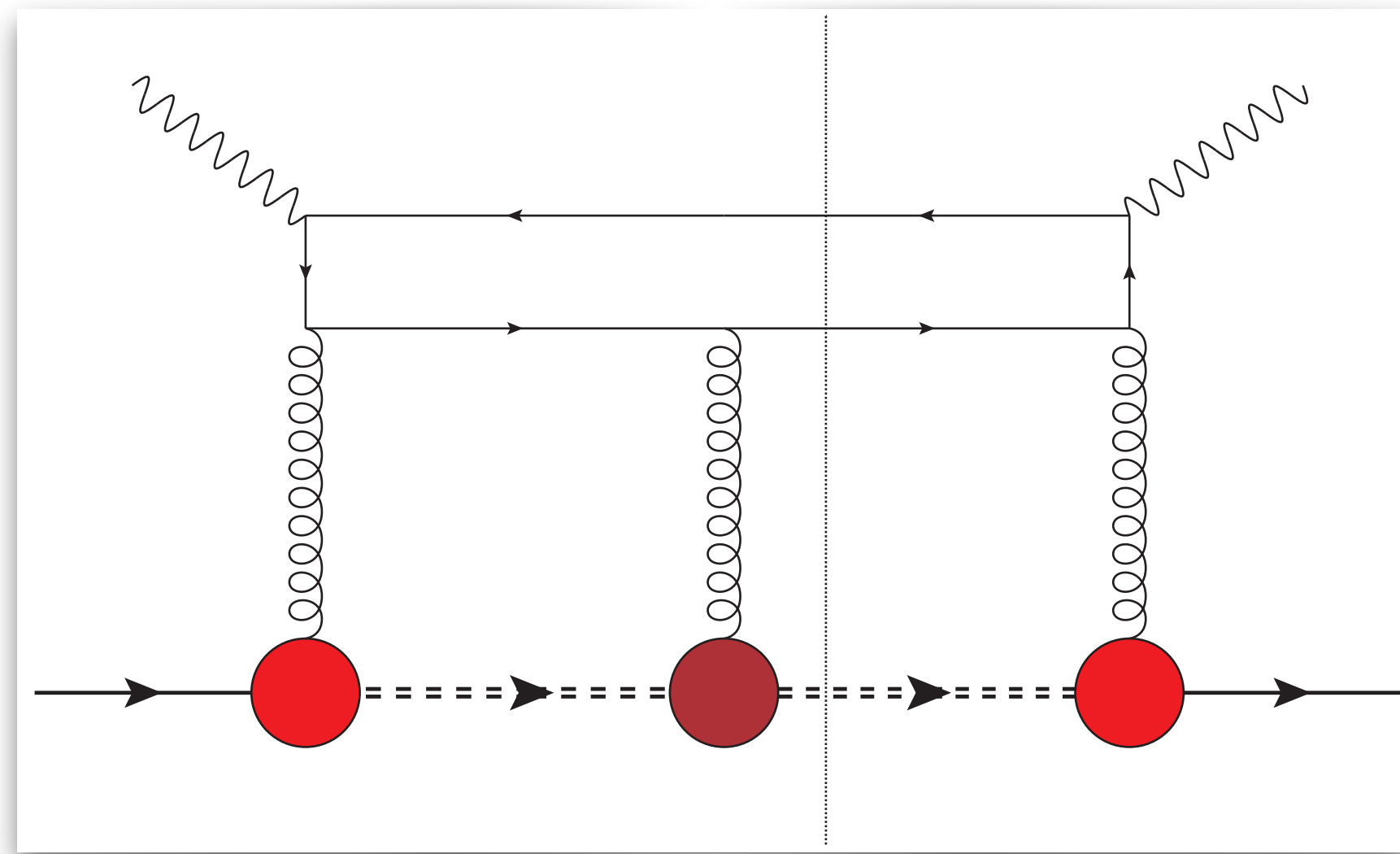
# Analytic structure of T-odd gluon TMDs

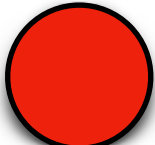
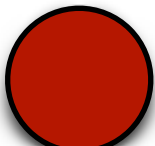
Two-jet SIDIS  $\Rightarrow f$ -type  $[+, +]$



# Analytic structure of T-odd gluon TMDs

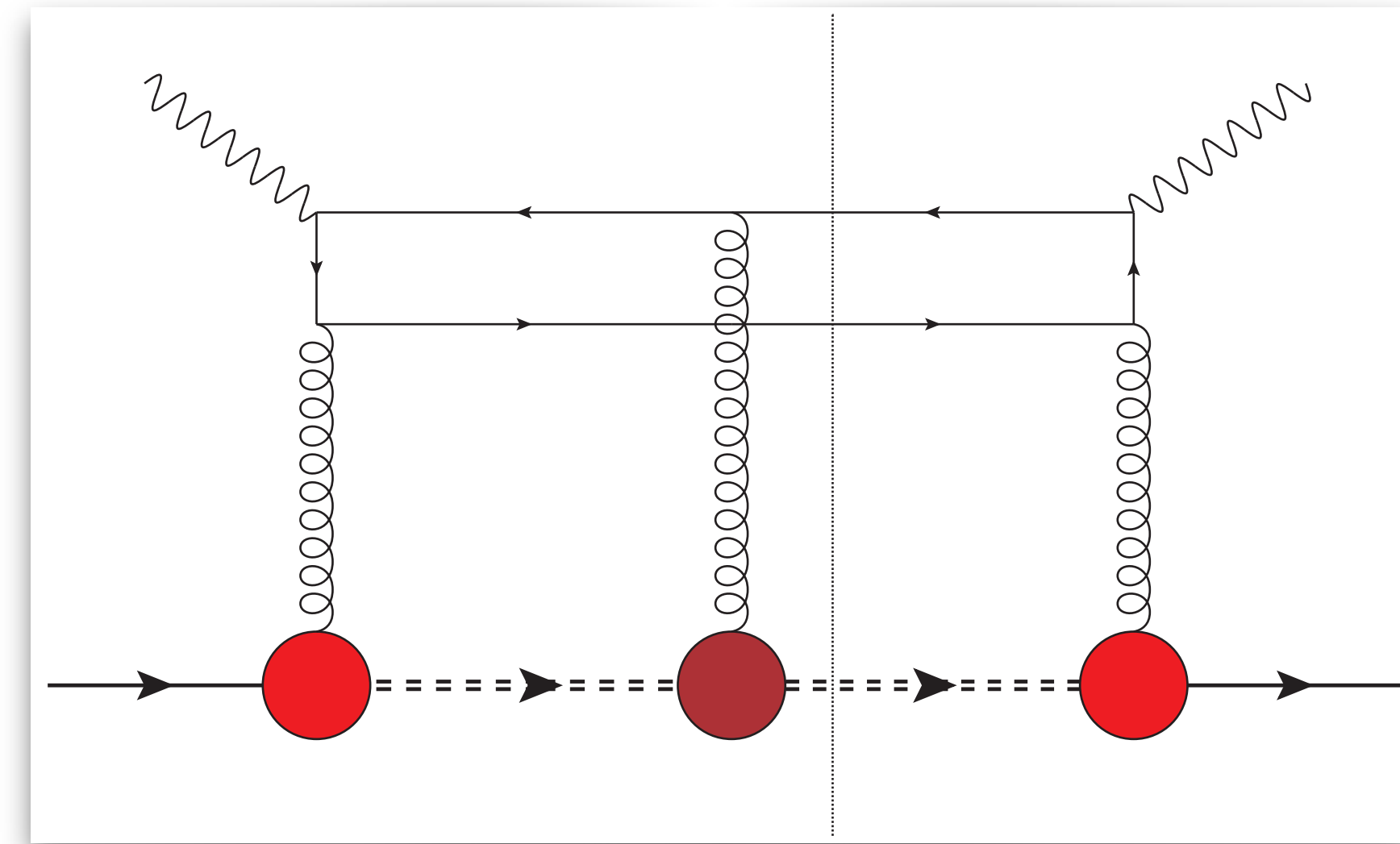
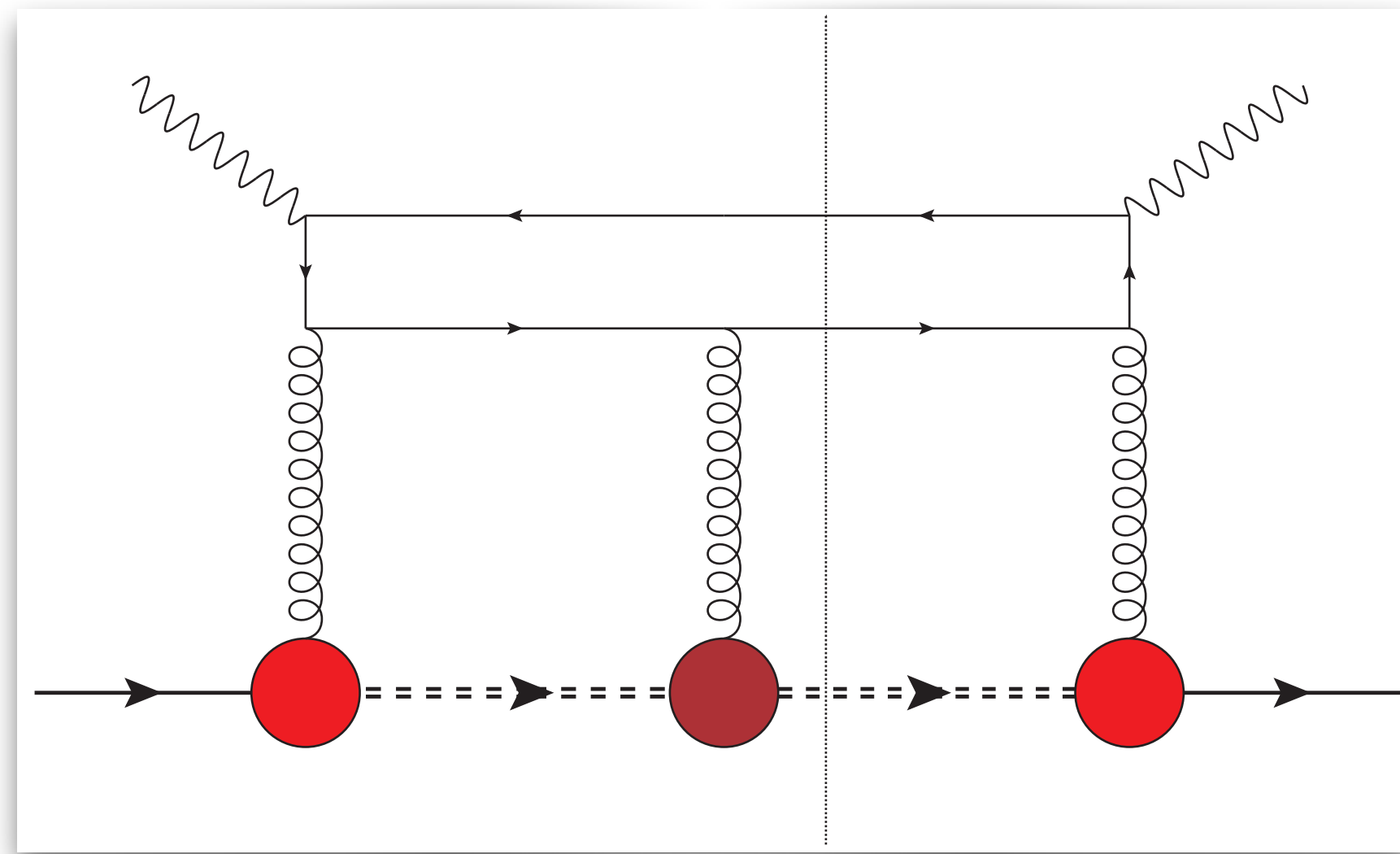
Two-jet SIDIS  $\Rightarrow f$ -type [ + , + ]

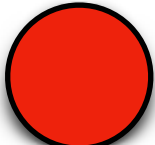
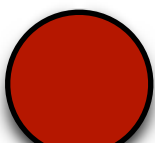


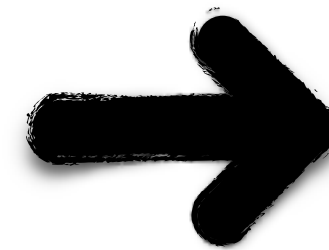
-  nucleon-gluon-spectator
-  spectator-gluon-spectator

# Analytic structure of T-odd gluon TMDs

Two-jet SIDIS  $\Rightarrow f$ -type  $[+, +]$



-  nucleon-gluon-spectator
-  spectator-gluon-spectator



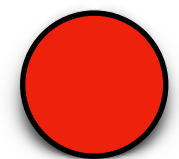
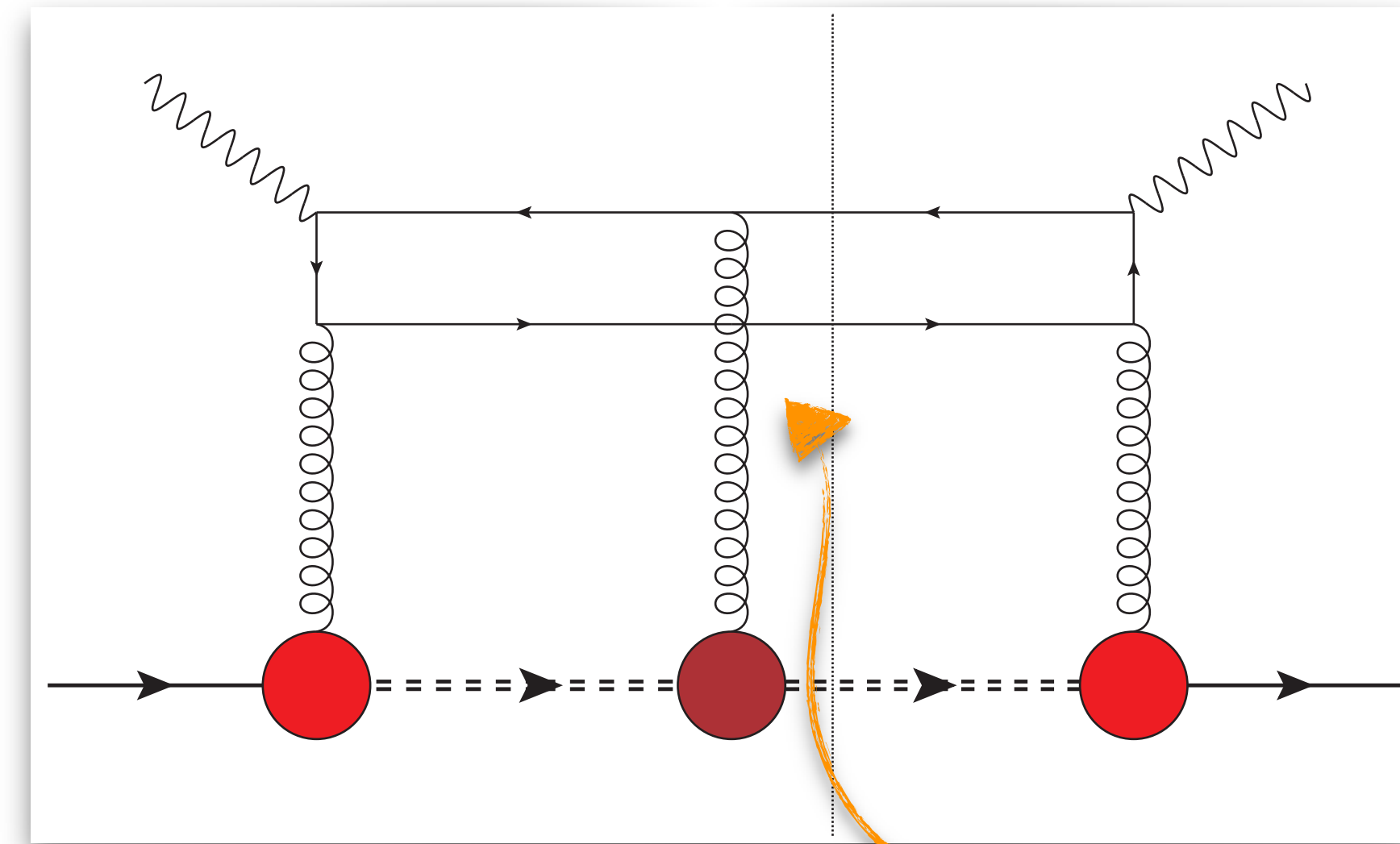
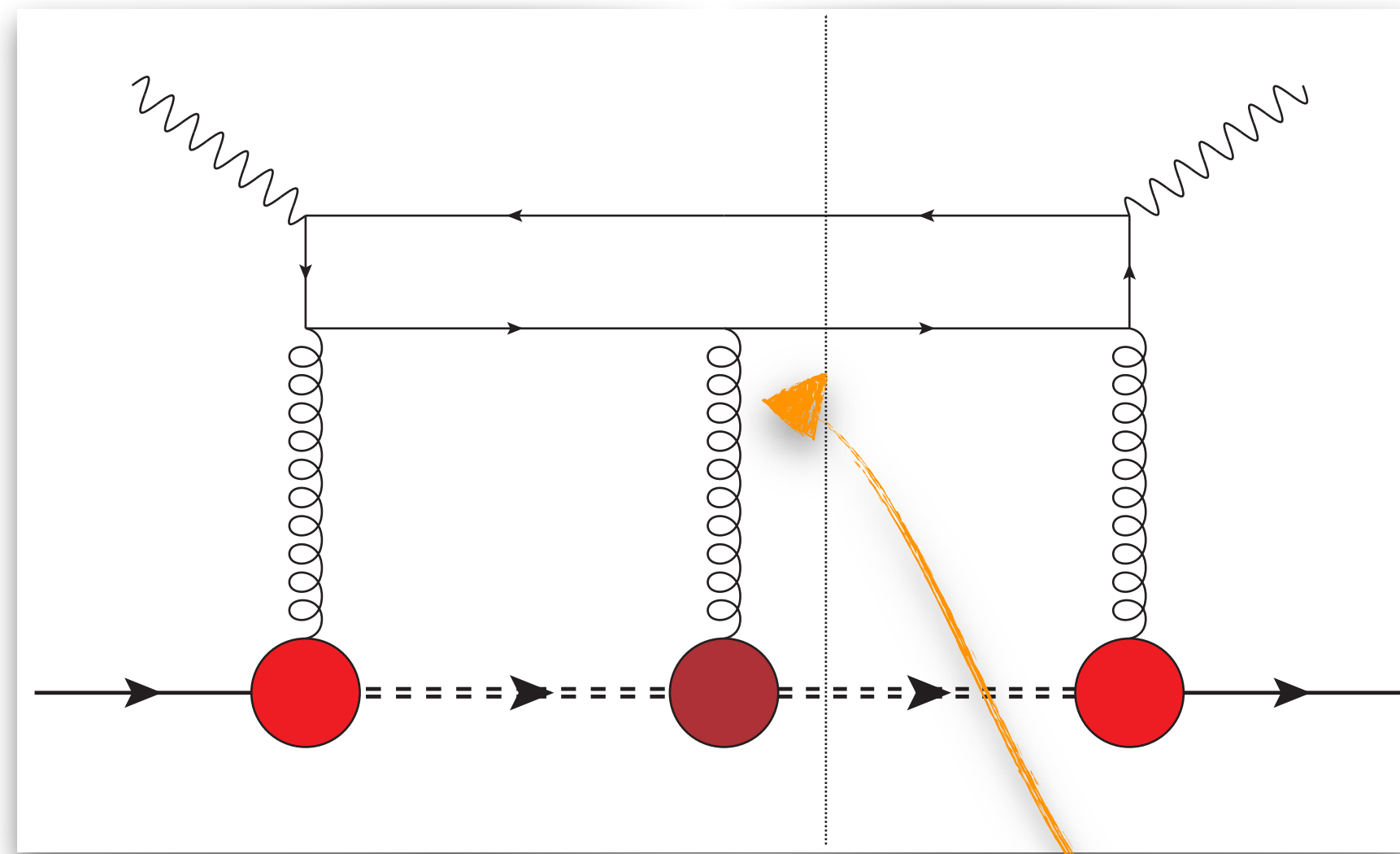
$8 \times 7 \times 4$

$$F(x, \mathbf{p}_T^2) = \sum_{i,j,k}^{1,2} C_{ijk}^{(F)}(x, \mathbf{p}_T^2) g_i(\mathbf{p}_T^2) g_j(\mathbf{p}_T^2) g_k(\mathbf{p}_T^2)$$

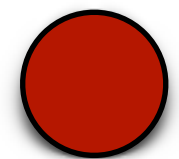
$$C_{ijk}^{(F)}(x, \mathbf{p}_T^2) = \sum_{l=1}^7 C_{ijk}^{(F),l}(x, \mathbf{p}_T^2) \mathcal{D}_l(x, \mathbf{p}_T^2)$$

# Analytic structure of T-odd gluon TMDs

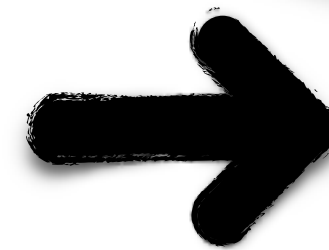
Two-jet SIDIS  $\Rightarrow f$ -type [ + , + ]



nucleon-gluon-spectator



spectator-gluon-spectator



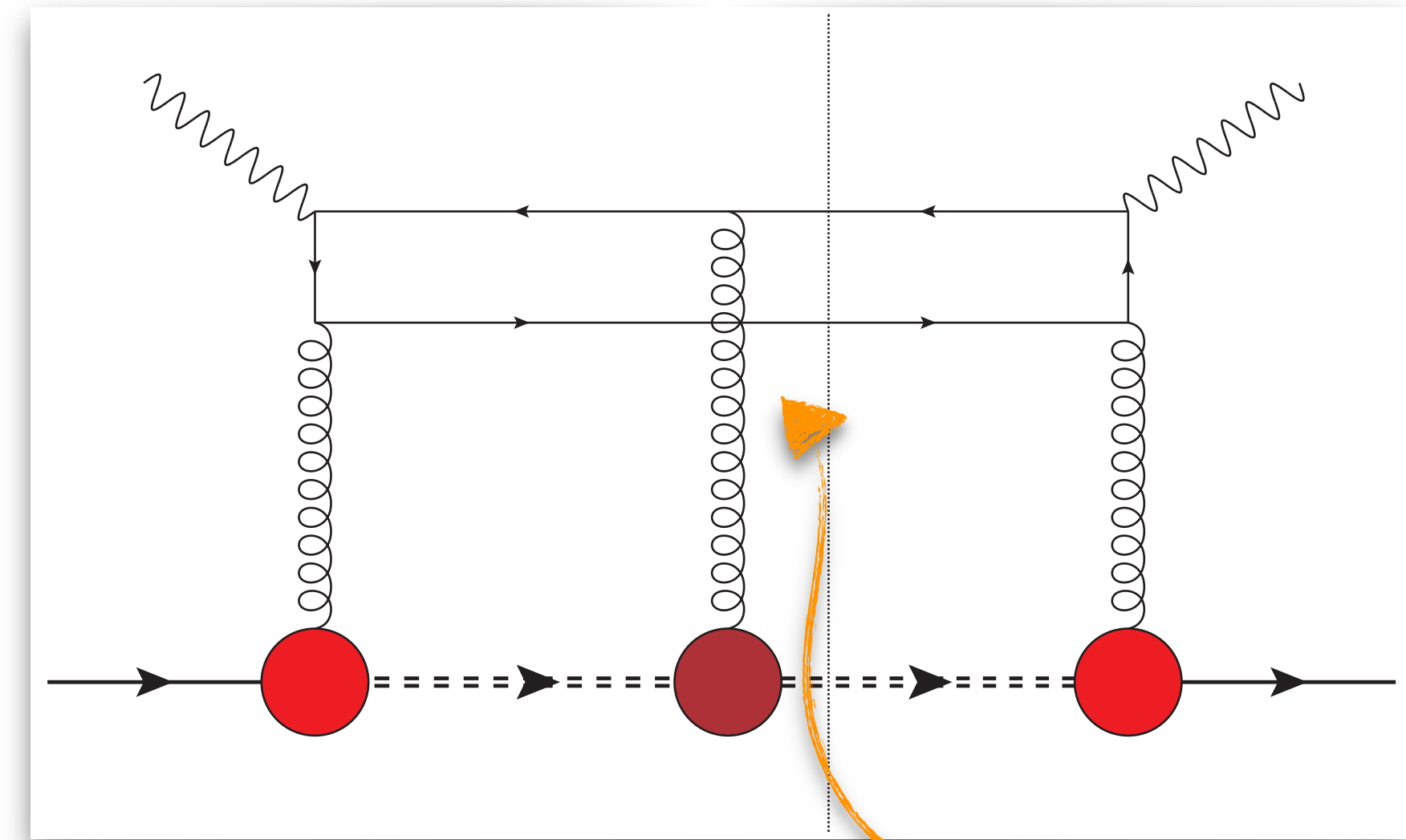
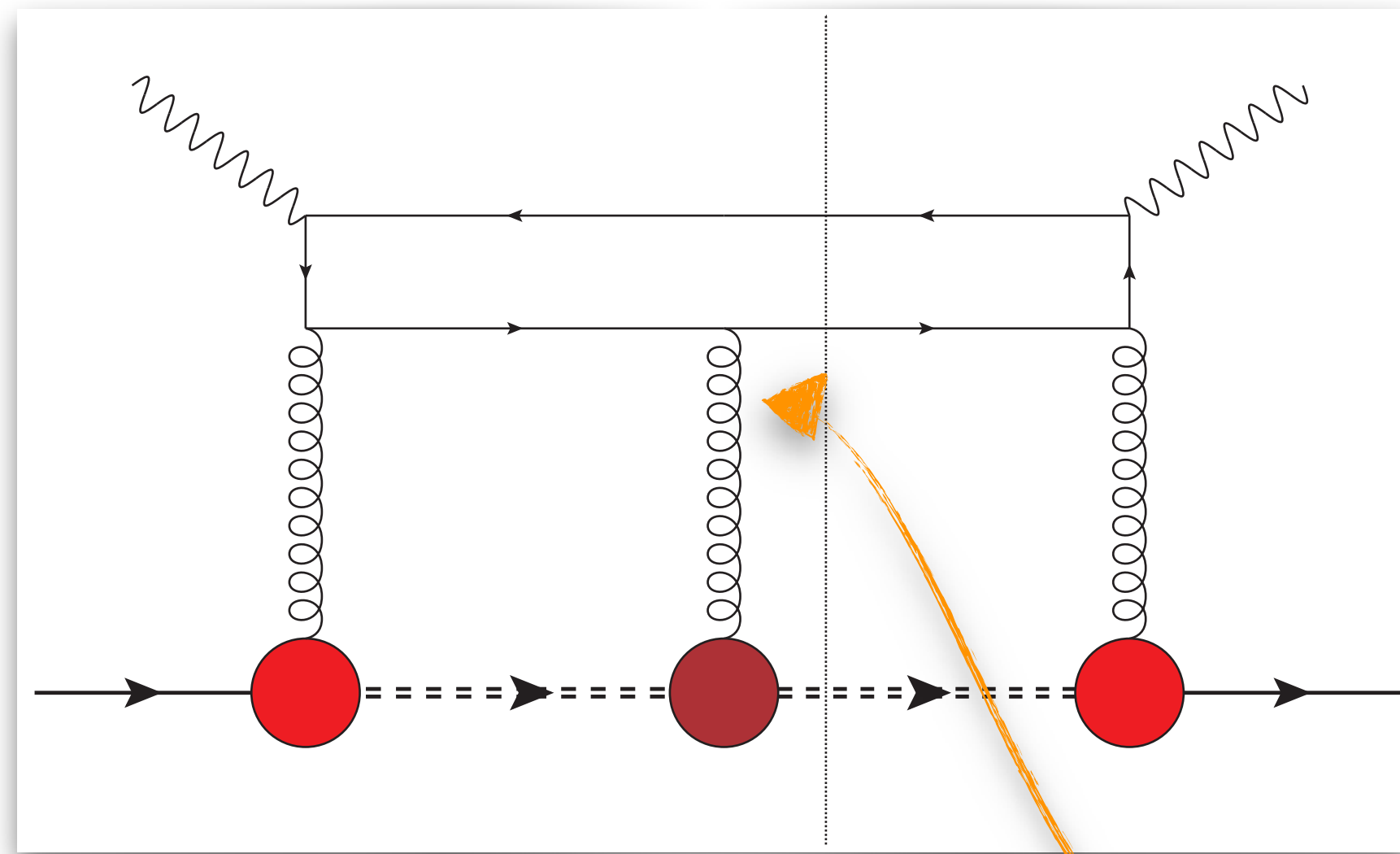
$$8 \times 7 \times 4$$

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# Analytic structure of T-odd gluon TMDs

Two-jet SIDIS  $\Rightarrow f$ -type  $[+, +]$



- nucleon-gluon-spectator
- spectator-gluon-spectator

➔

8 × 7 × 4

$$F(x, \mathbf{p}_T^2) = \sum_{i,j,k}^{1,2} C_{ijk}^{(F)}(x, \mathbf{p}_T^2) g_i(\mathbf{p}_T^2) g_j(\mathbf{p}_T^2) g_k(\mathbf{p}_T^2)$$

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# Anatomy of gluon TMDs

$$F(x, \mathbf{b}; \mu, \zeta) = \sum_j \left( C_j^{(F)} \otimes F^j \right) (x, b_*; \mu_b) e^{S(b_*; \mu_b, \mu, \zeta)} e^{S_{\text{NP}}(b)} F_{\text{NP}}(x, b)$$

matching coefficients collinear PDF nonperturbative Sudakov nonperturbative TMD function

**perturbative** expansion in  $\alpha_s(\mu)$

perturbative Sudakov

resummation of

$$L = \ln \frac{Q^2}{\mu_b^2}$$

define **logarithmic ordering**

slide adapted from C. Bissolotti

# Anatomy of gluon TMDs

$$F(x, \mathbf{b}; \mu, \zeta) = \sum_j \left( C_j^{(F)} \otimes F^j \right) (x, b_*; \mu_b) e^{S(b_*; \mu_b, \mu, \zeta)} e^{S_{\text{NP}}(b)} F_{\text{NP}}(x, b)$$

matching coefficients  
 collinear PDF  
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 nonperturbative TMD function

**perturbative** expansion in  $\alpha_s(\mu)$

perturbative Sudakov  
 resummation of

$$L = \ln \frac{Q^2}{\mu_b^2}$$

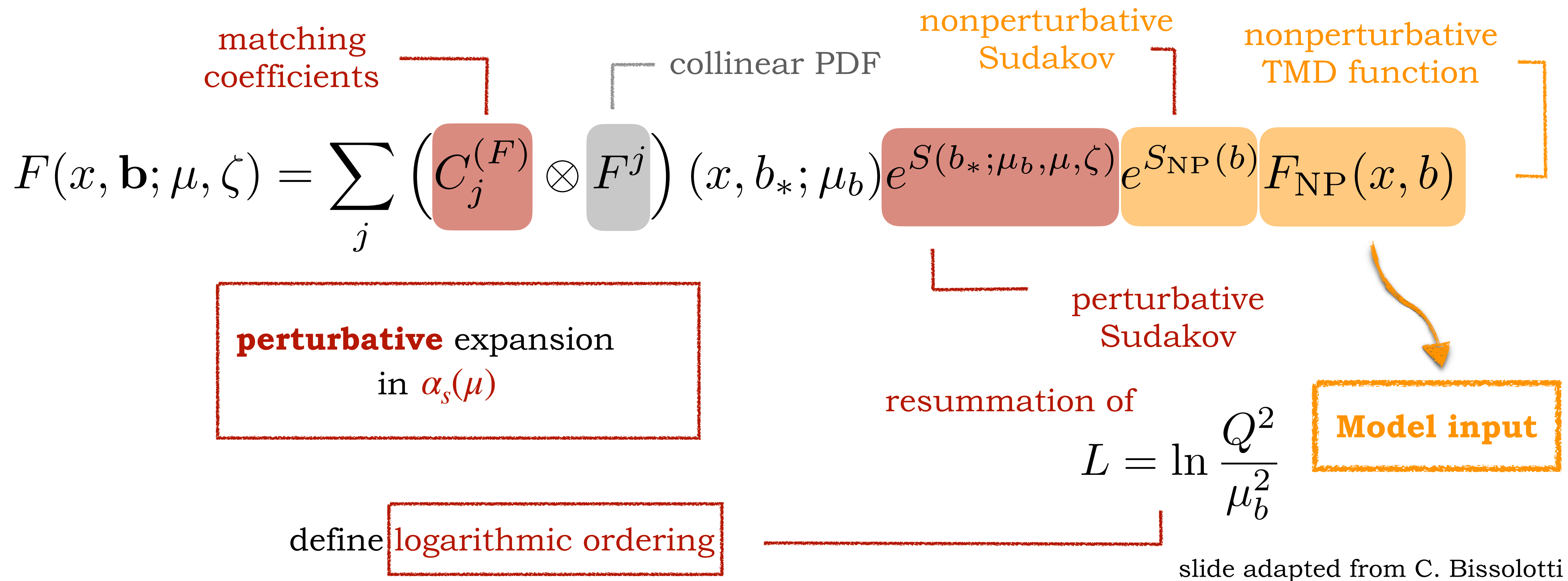
Model input

define logarithmic ordering

slide adapted from C. Bissolotti

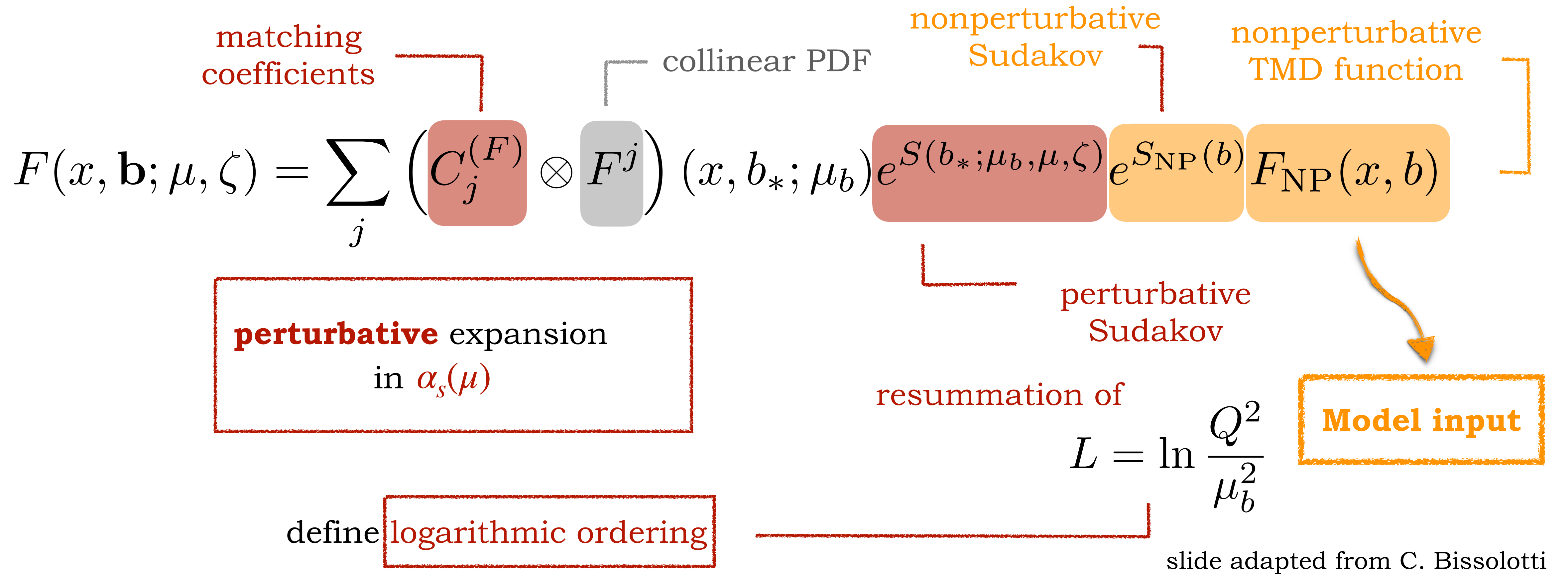


# Anatomy of gluon TMDs



$$f_1(x, \mathbf{b}, \mu, \zeta) \rightarrow C_j^{(f_1)} \otimes f_1^j = \left[ 1 + \mathcal{O}(\alpha_s) \right]_j \otimes f_1^j$$

# Anatomy of gluon TMDs



$$f_1(x, \mathbf{b}, \mu, \zeta) \rightarrow C_j^{(f_1)} \otimes f_1^j = [1 + \mathcal{O}(\alpha_s)]_j \otimes f_1^j$$

$$h_1^\perp(x, \mathbf{b}, \mu, \zeta) \rightarrow C_j^{(h_1^\perp)} \otimes f_1^j = [\mathcal{O}(\alpha_s)]_j \otimes f_1^j$$

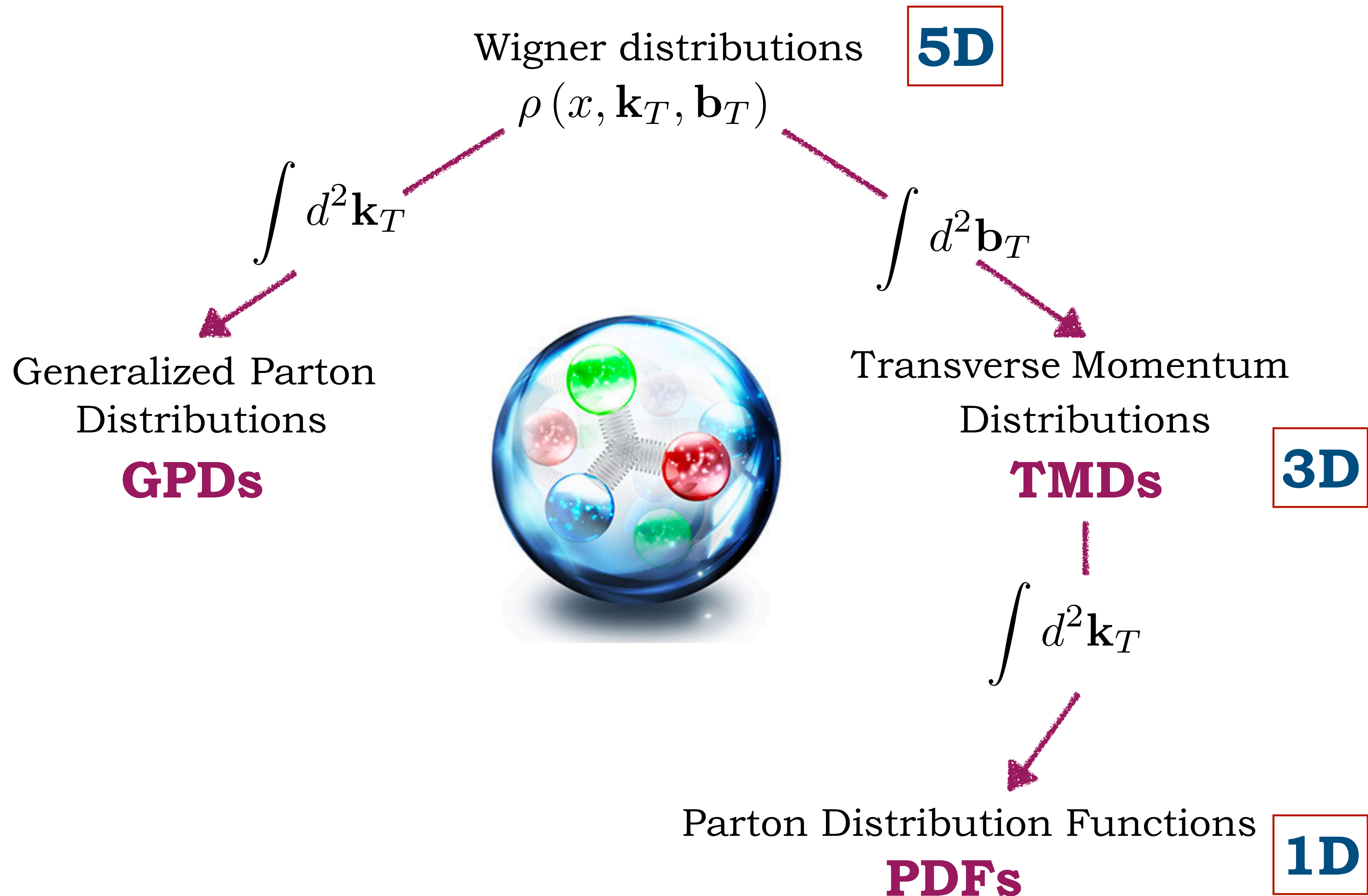
Backup

¿ Suppression of genuine NP effects ? ←



# TMD factorization

# Parton densities: an incomplete family tree

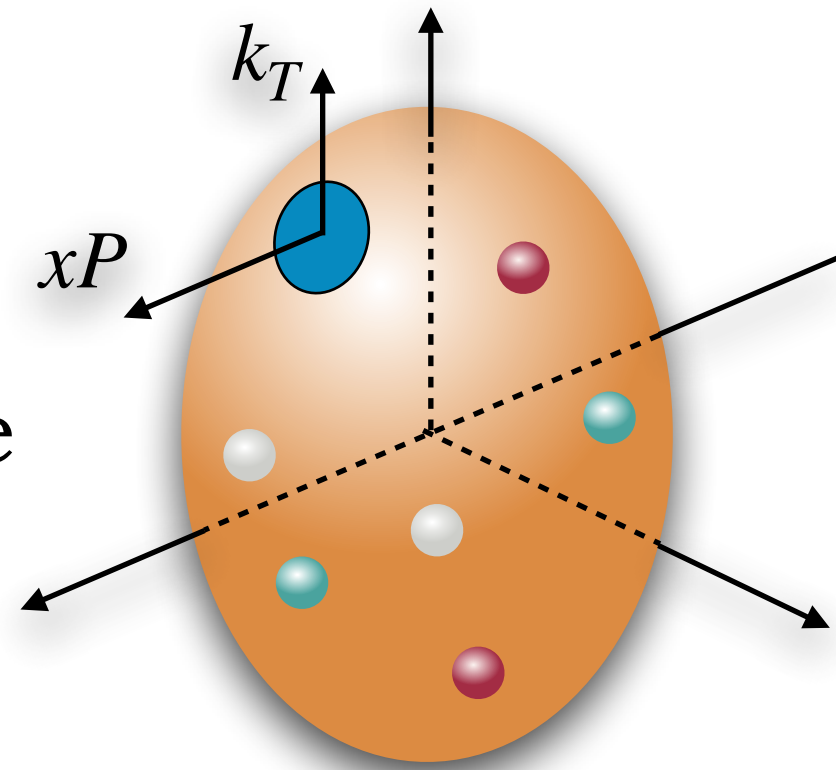


slide adapted from C. Bissolotti

# Parton densities: an incomplete family tree

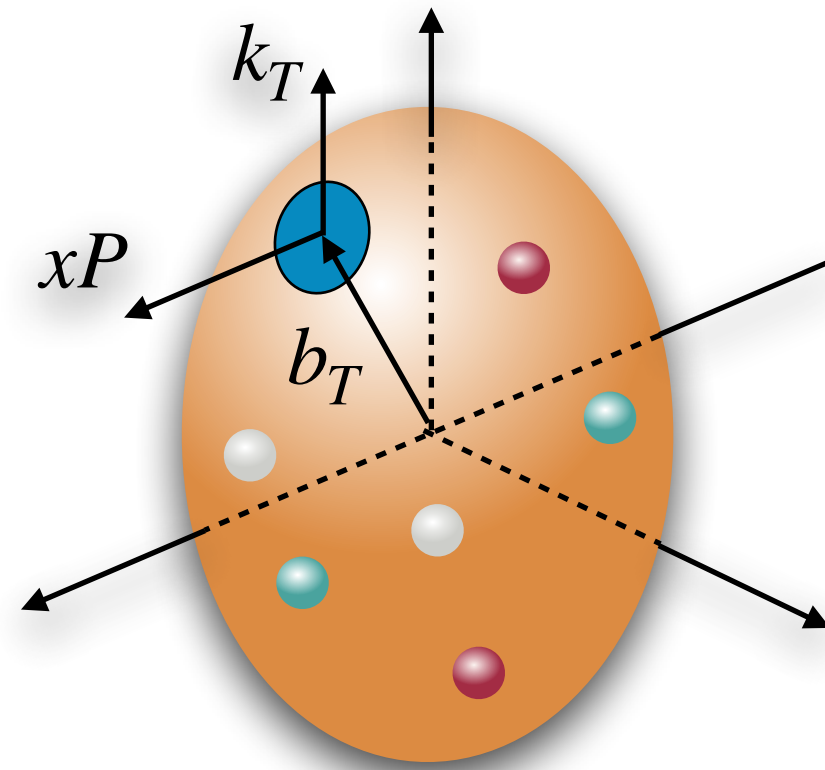
**3D**

TMDs  
(semi-)inclusive



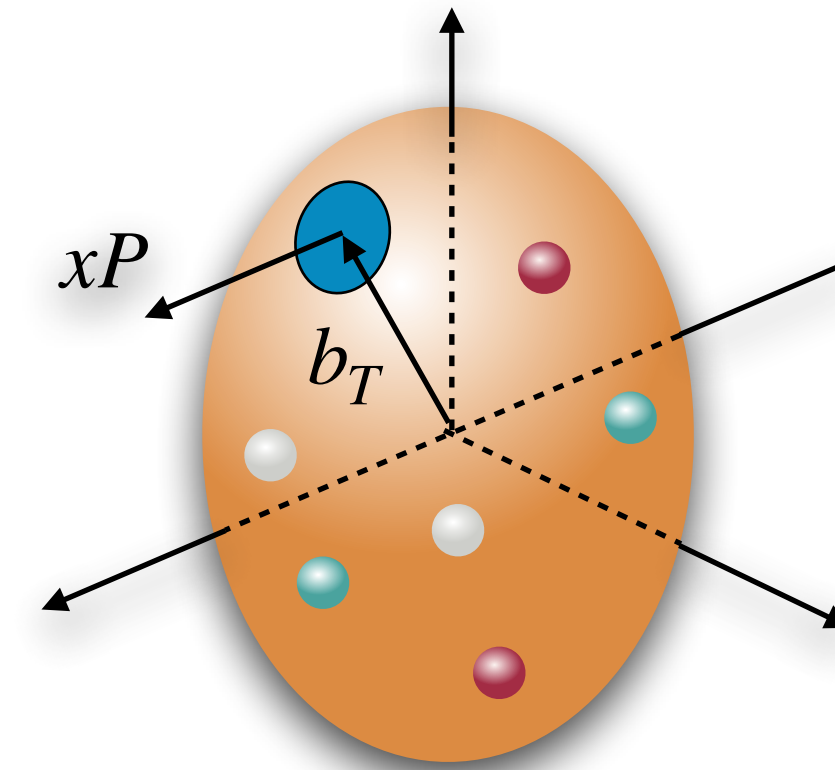
**5D**

Wigner distributions



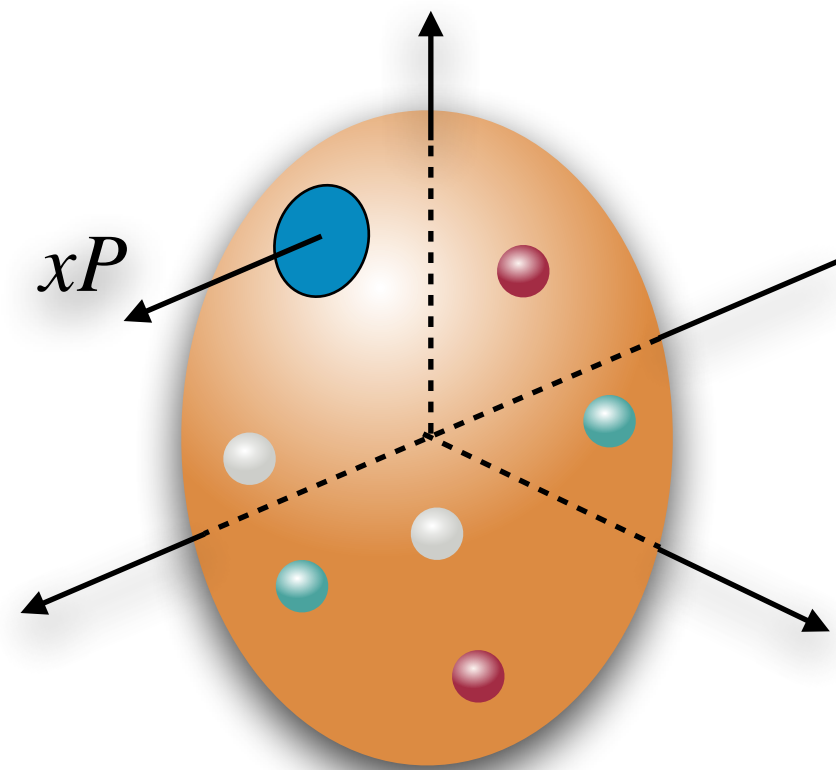
**3D**

FT of GPDs  
exclusive

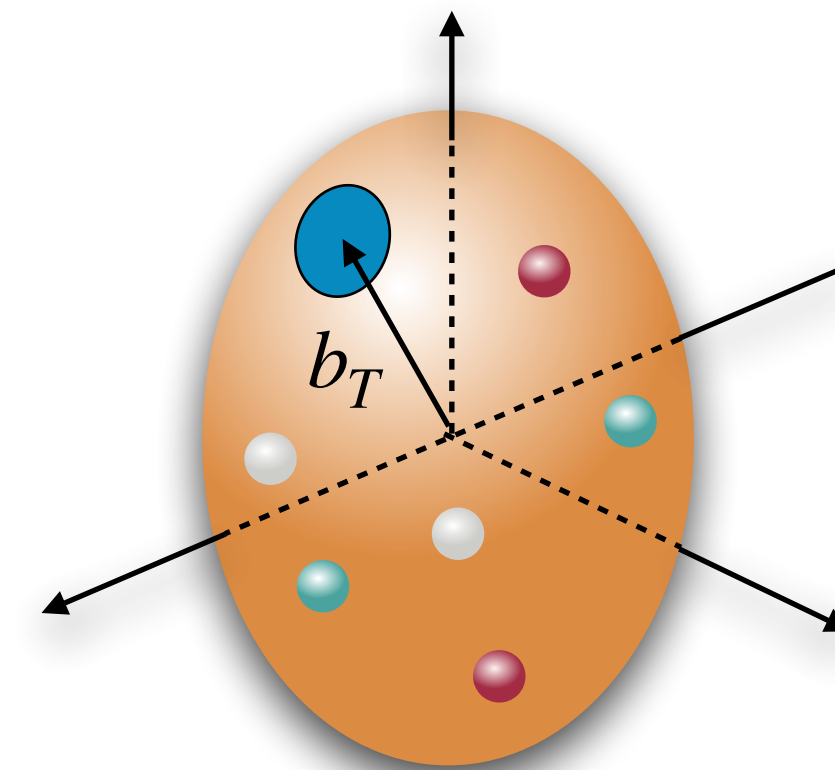


PDFs

(semi-)inclusive



FT of Form Factors



**1D**

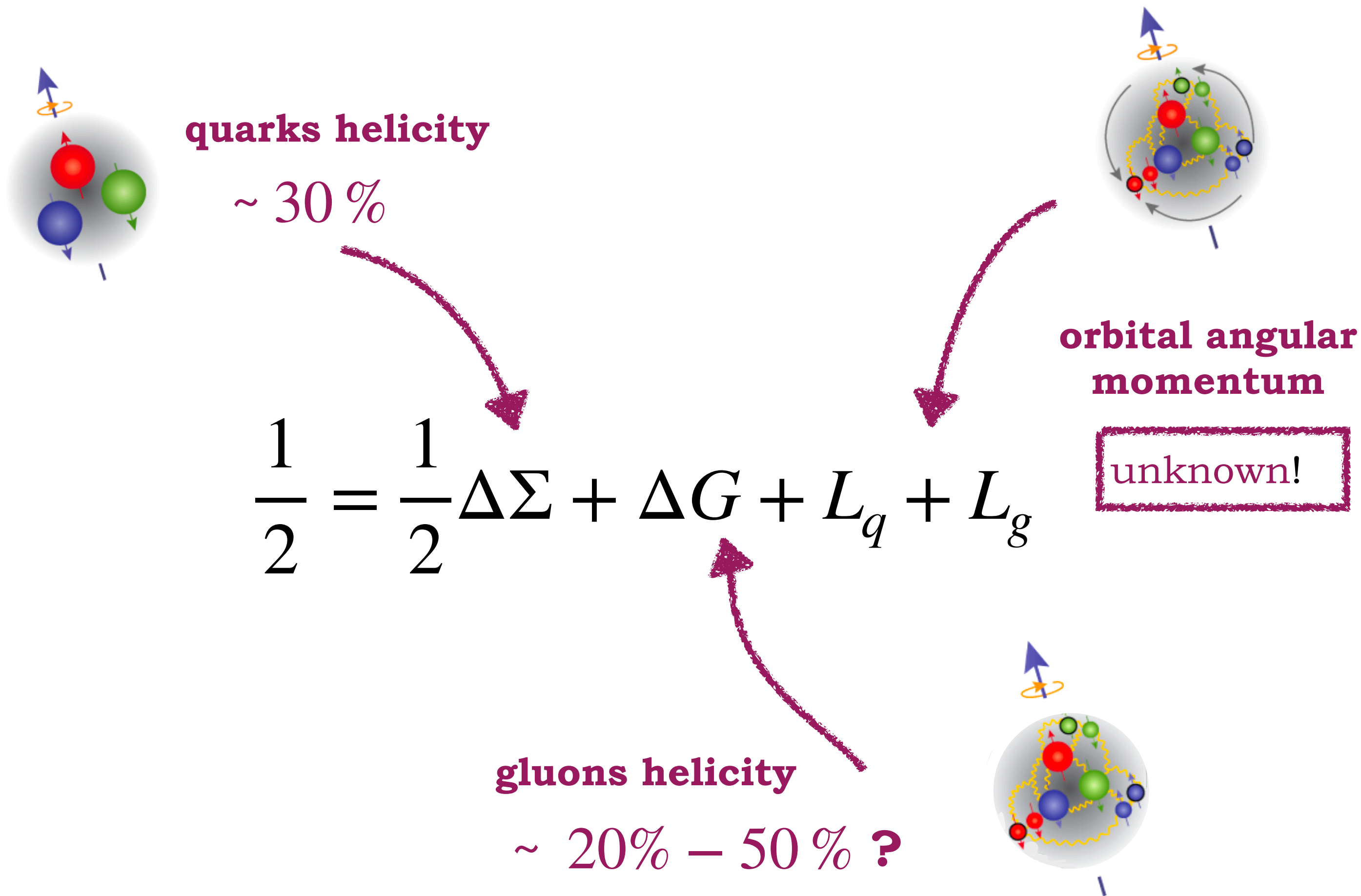
**2D**

→  $\vec{b}_\perp$  dependence  
 .....→  $\vec{k}_\perp$  dependence



these two variables are NOT Fourier conjugate

# The proton spin crisis



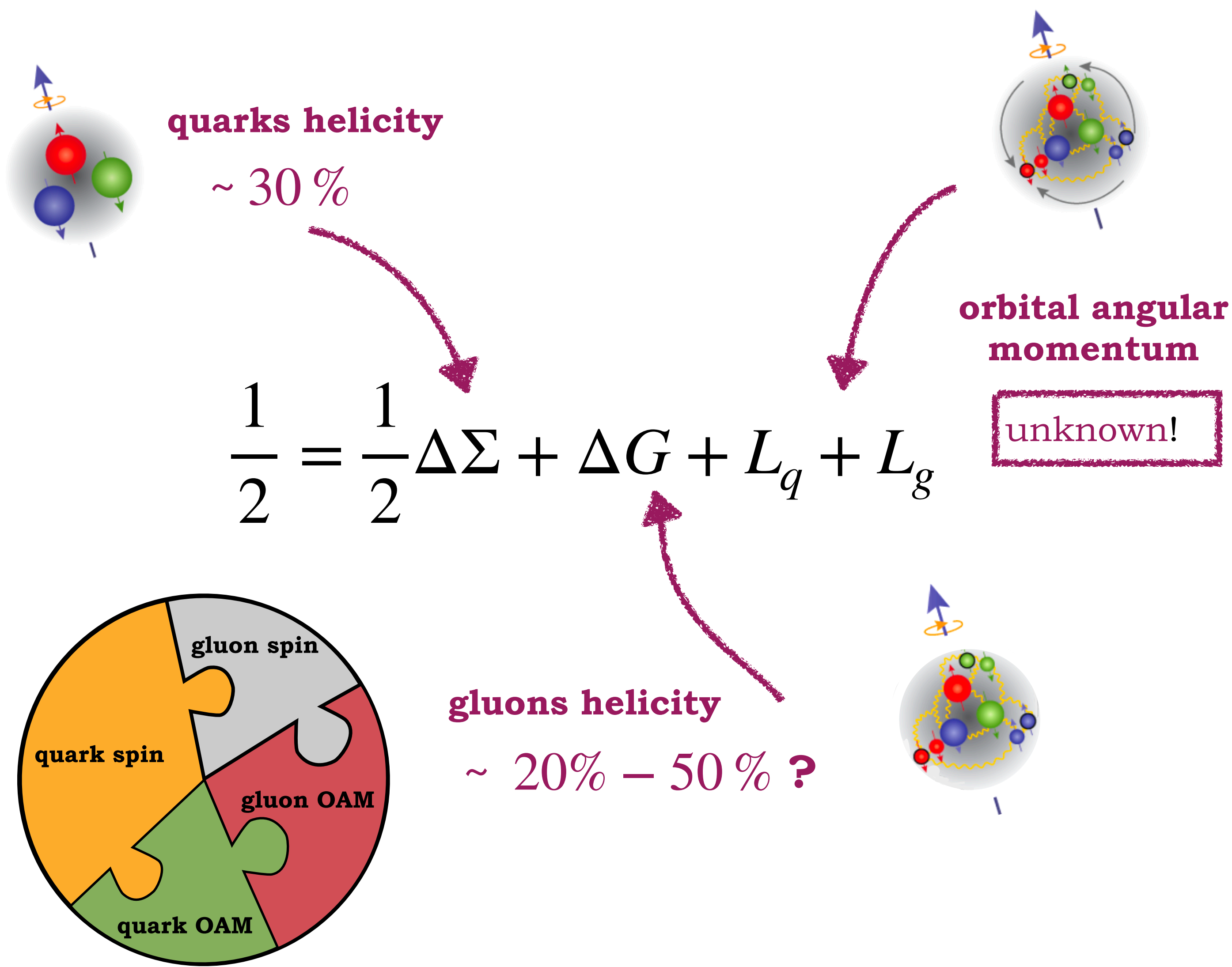
**Total spin carried by quarks and gluons does not amount to 1/2, one needs orbital angular momentum, then a 3D description...**

(proton spin crisis) [EMC Collaboration, CERN (1987)]

slide adapted from C. Bissolotti

**Backup**

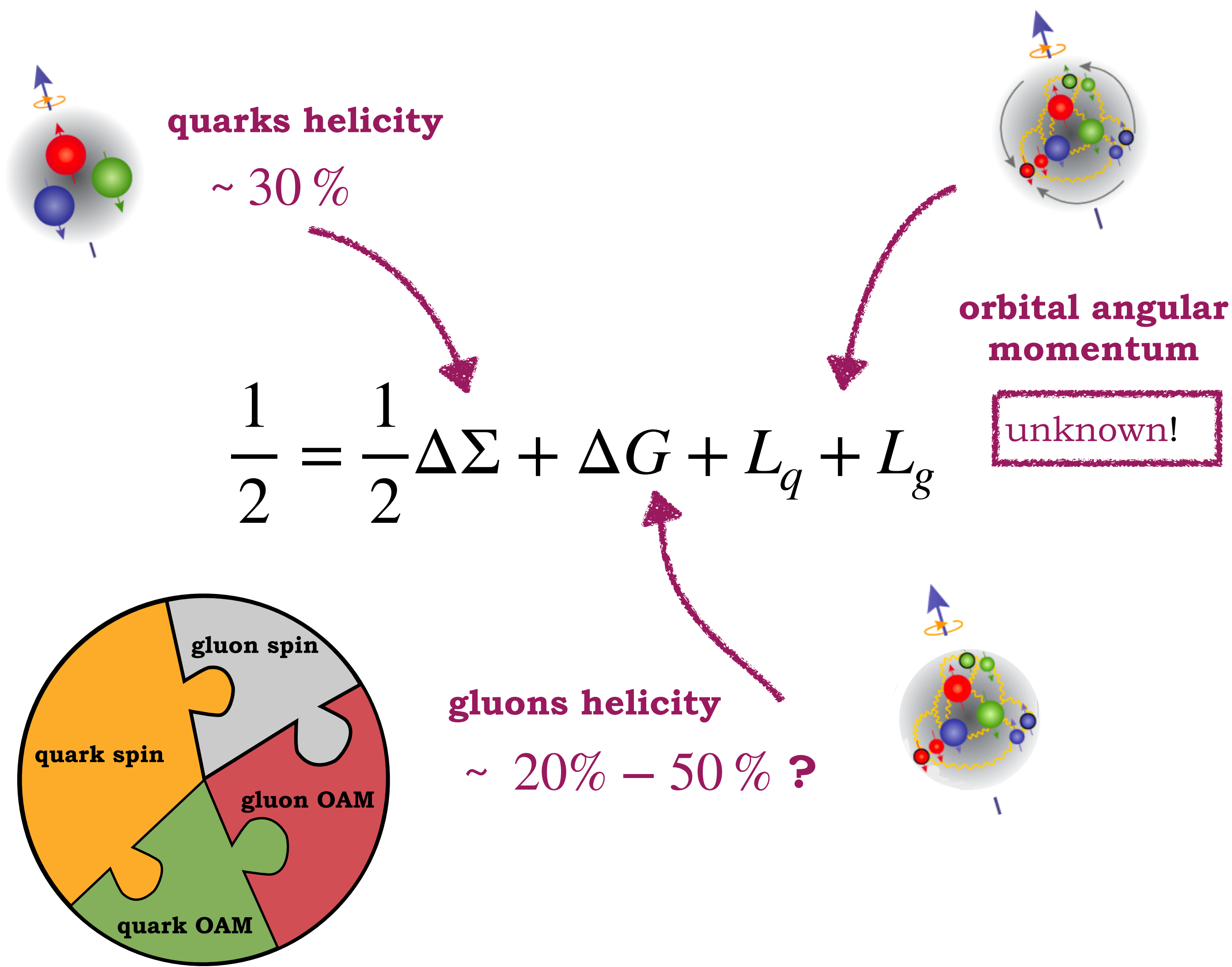
# The proton spin crisis



**Total spin carried by quarks and gluons does not amount to 1/2, one needs orbital angular momentum, then a 3D description...**

(proton spin crisis) [EMC Collaboration, CERN (1987)]

# The proton spin crisis



...many other effects in hadronic interactions cannot be understood in the purely collinear approach

Total spin carried by quarks and gluons does not amount to 1/2, one needs orbital angular momentum, then a 3D description...

(proton spin crisis) [EMC Collaboration, CERN (1987)]



# Gluon TMDs: gauge links and modified universality

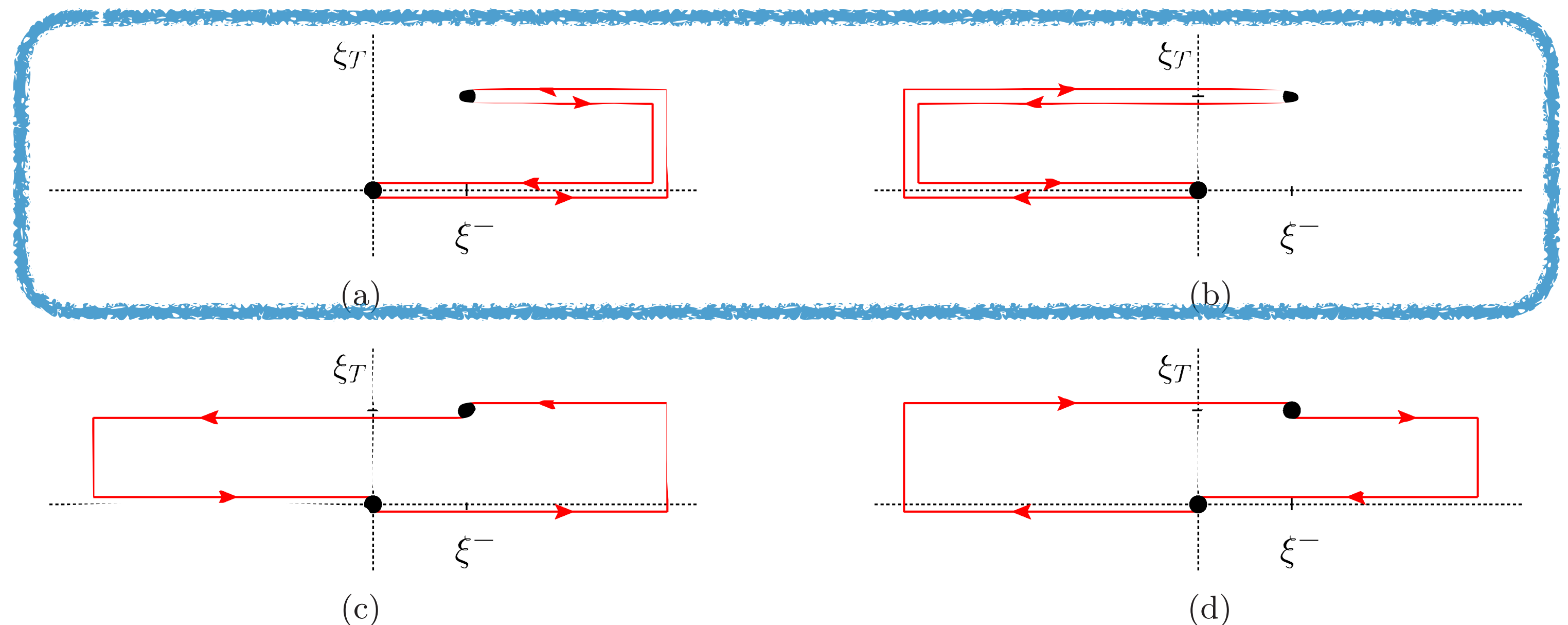
- \* Single-spin asymmetries  $\rightarrow$  process dependence of TMDs via gauge links
- \* Color flow  $\rightarrow$  integration paths of gauge links calculable
- \* Gluon TMDs  $\rightarrow$  more complicated structure with respect to quark staple links
- \* Factorization-preserving processes  $\rightarrow$  two main kinds of modified universality
- \* Different classes of processes  $\rightarrow$  distinct gluon TMDs, not related to each other

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## *f*-type (WW)

(a) [ + , + ] or (b) [ - , - ]

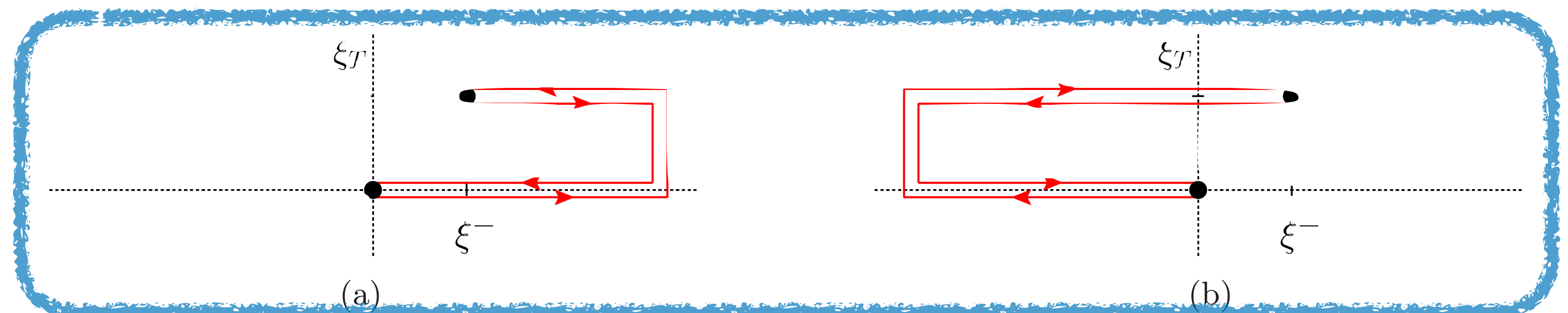


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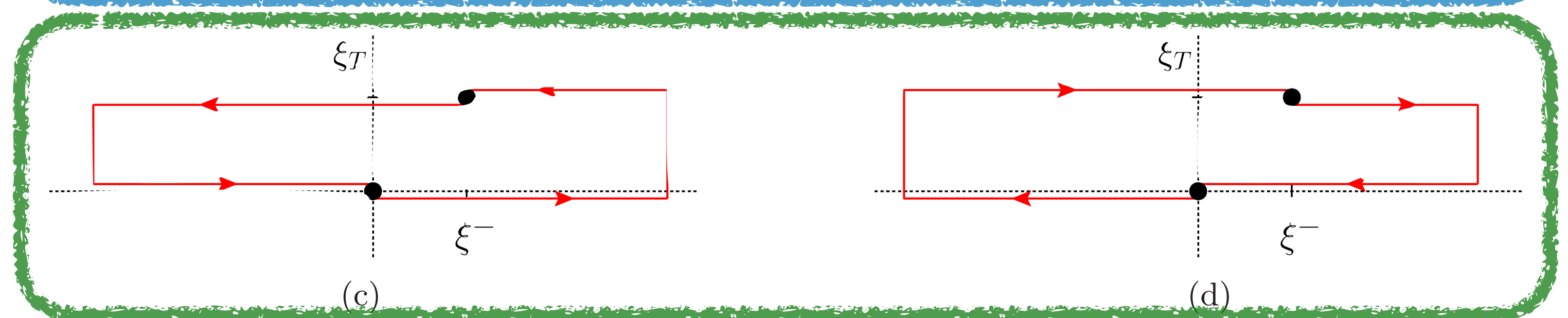
## ***f*-type (WW)**

(a)  $[+, +]$  or (b)  $[-, -]$



## ***d*-type (dipole)**

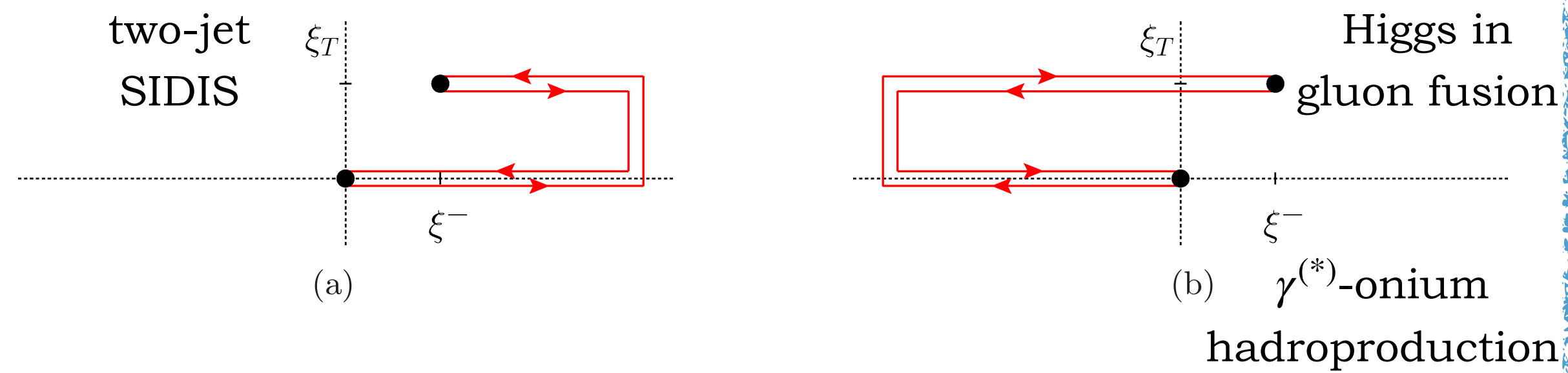
(c)  $[+, -]$  or (d)  $[-, +]$



# Accessing f-type and d-type gluon TMDs

## *f*-type (WW)

(a) [ + , + ] or (b) [ - , - ]

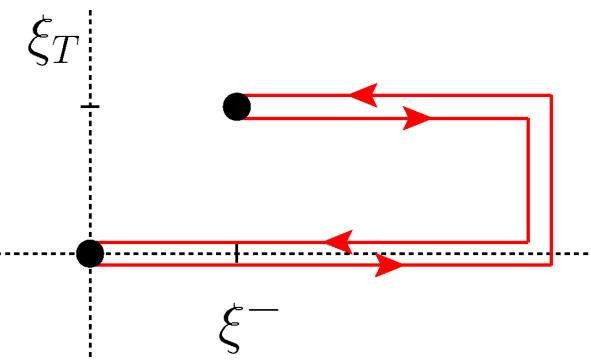


# Accessing f-type and d-type gluon TMDs

## *f*-type (WW)

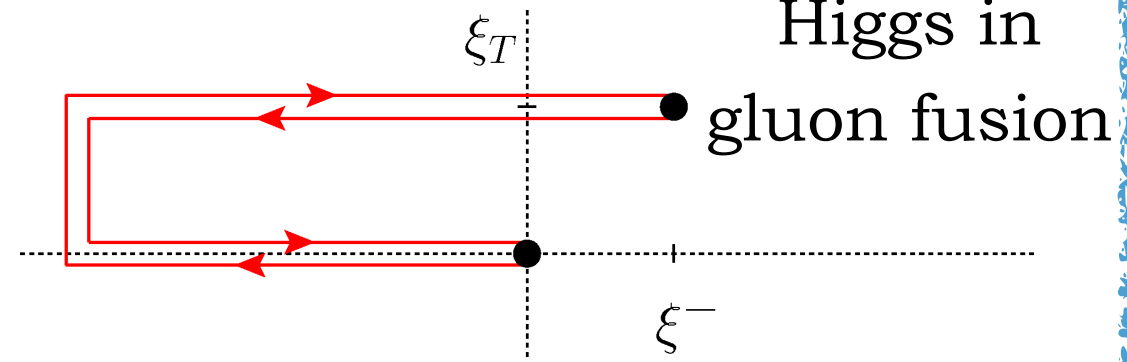
(a) [ + , + ] or (b) [ - , - ]

two-jet  
SIDIS



(a)

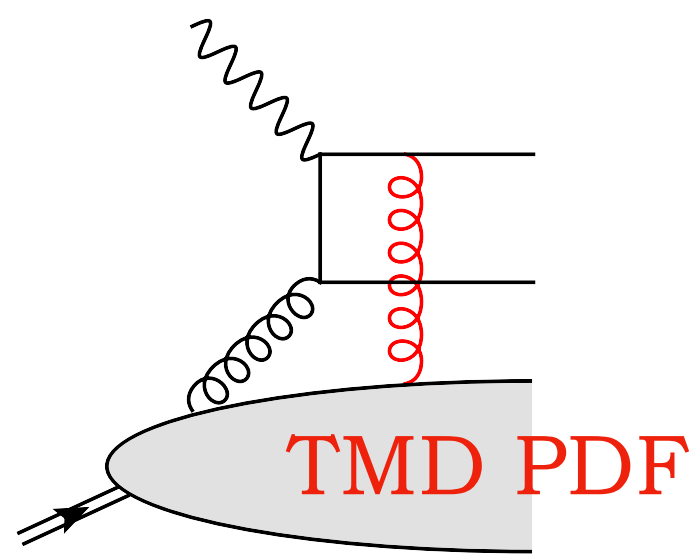
Higgs in  
gluon fusion



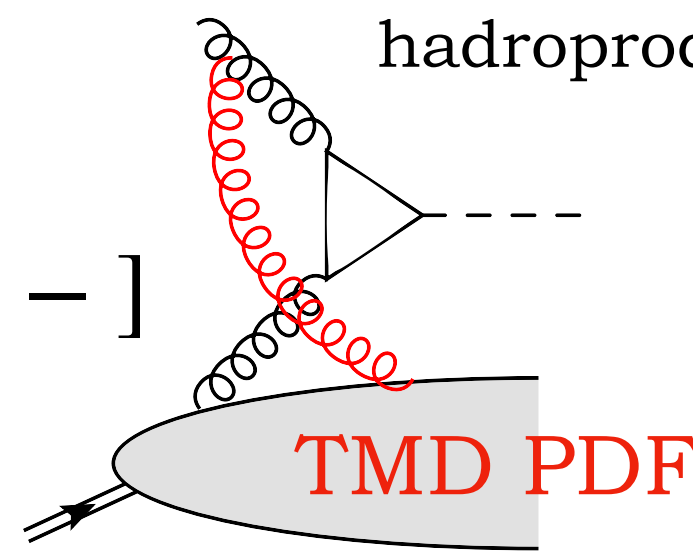
(b)

$\gamma^{(*)}$ -onium  
hadroproduction

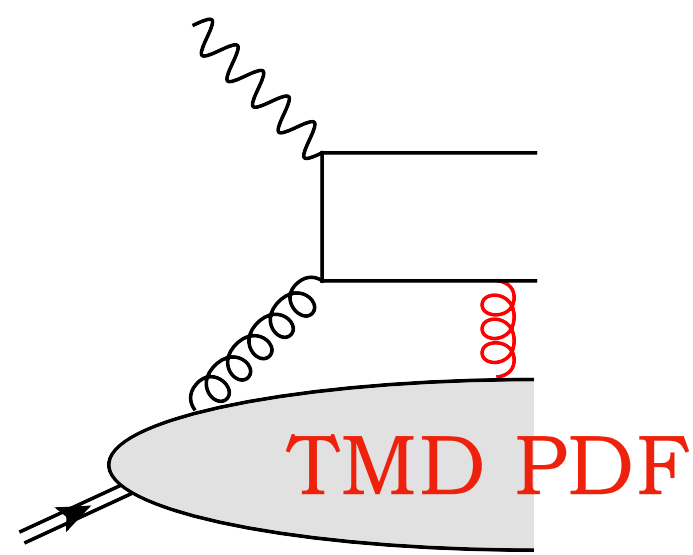
[ +



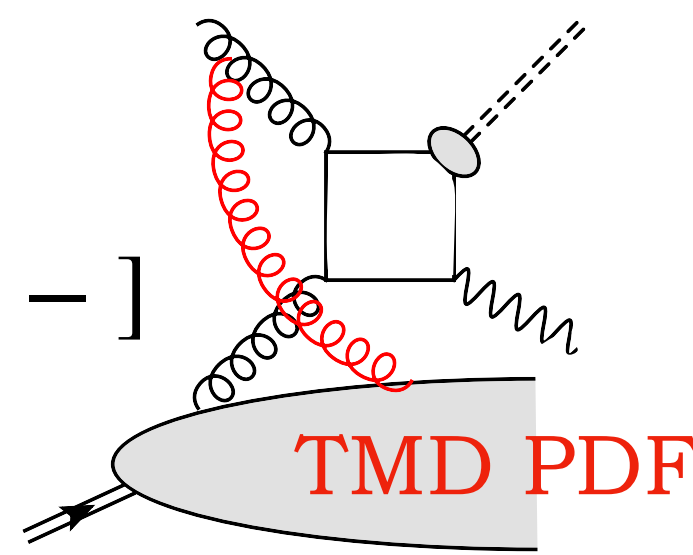
[ - , - ]



+ ]



[ - , - ]

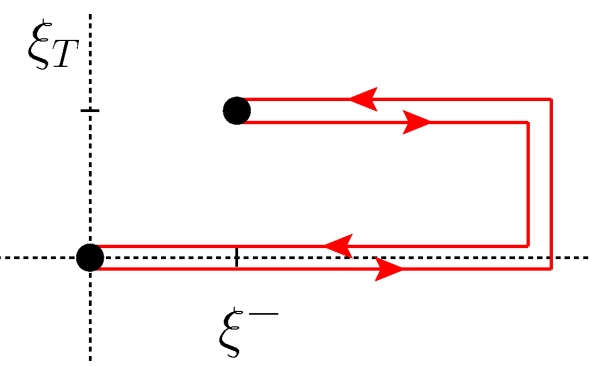


# Accessing f-type and d-type gluon TMDs

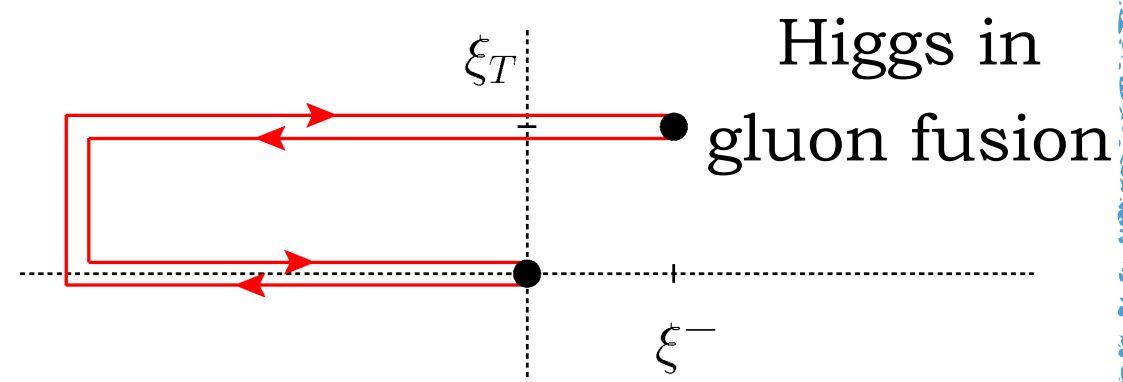
## *f*-type (WW)

(a)  $[+, +]$  or (b)  $[-, -]$

two-jet  
SIDIS



(a)

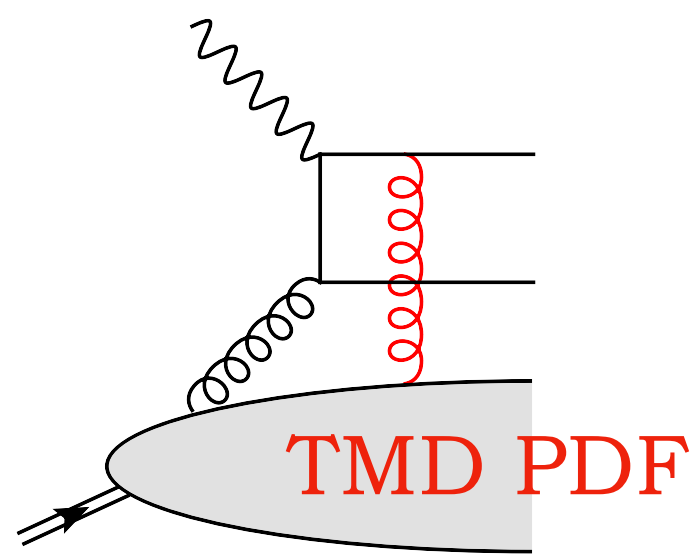


(b)

Higgs in  
gluon fusion

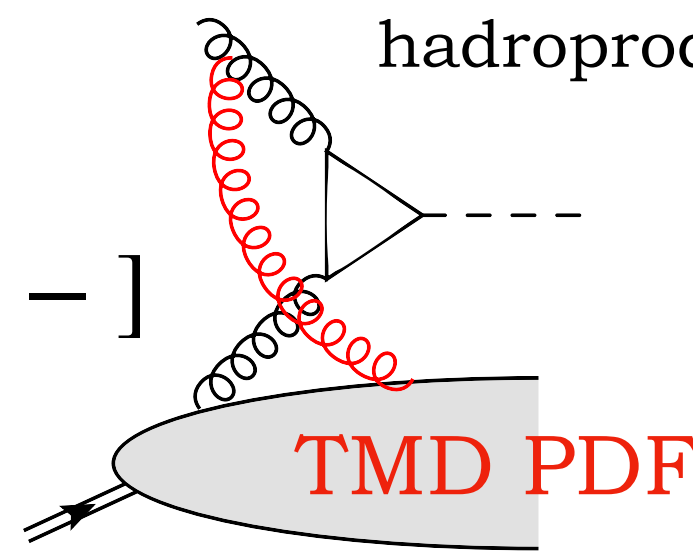
$\gamma^{(*)}$ -onium  
hadroproduction

[+]



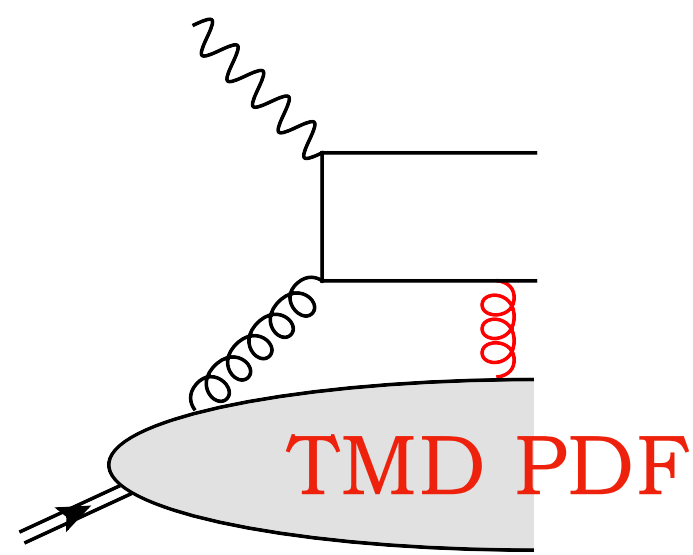
TMD PDF

[-, -]



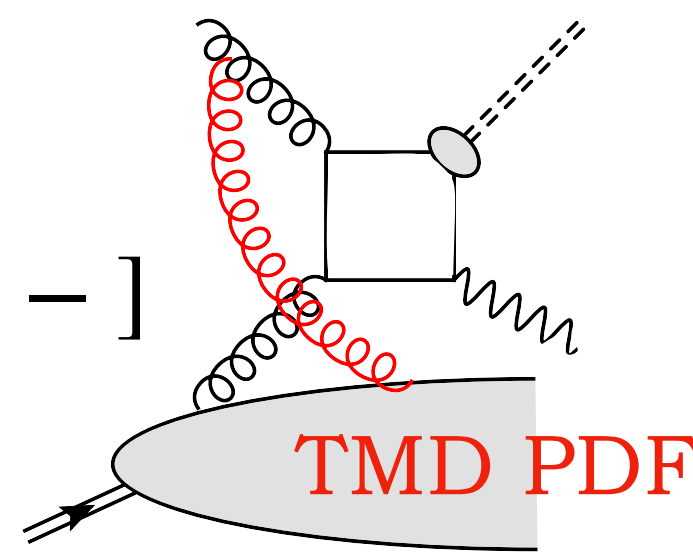
TMD PDF

[+]



TMD PDF

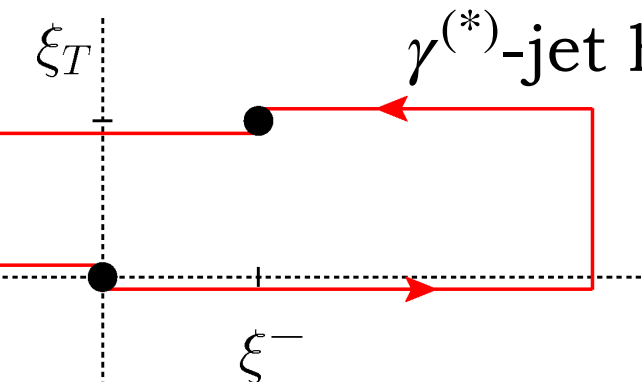
[-, -]



TMD PDF

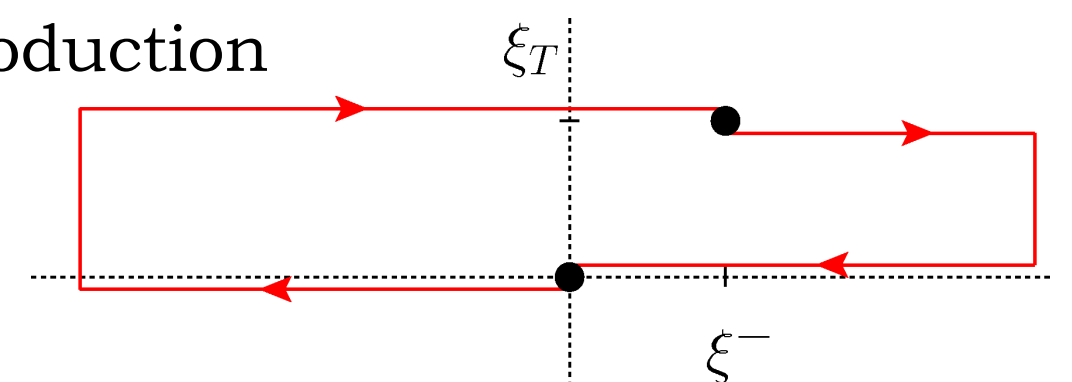
## *d*-type (DP)

(c)  $[+, -]$  or (d)  $[-, +]$



(c)

$\gamma^{(*)}$ -jet hadroproduction



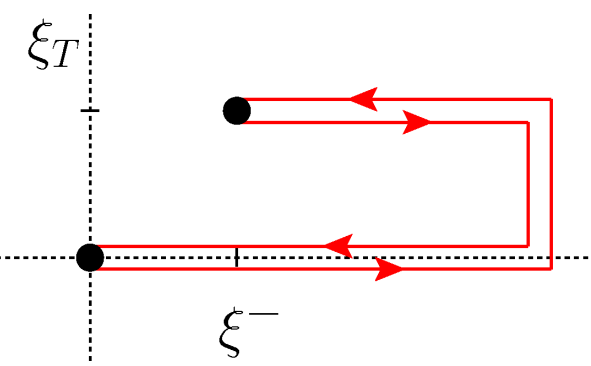
(d)

# Accessing f-type and d-type gluon TMDs

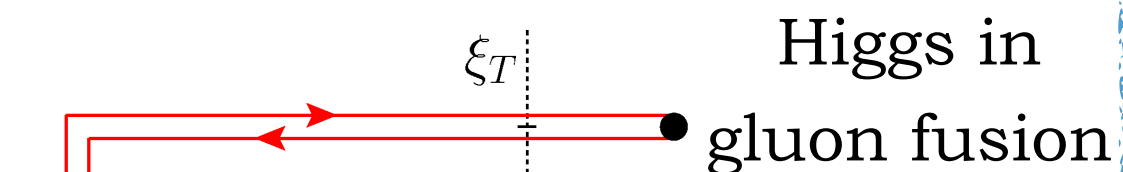
## *f*-type (WW)

(a)  $[+, +]$  or (b)  $[-, -]$

two-jet  
SIDIS



(a)

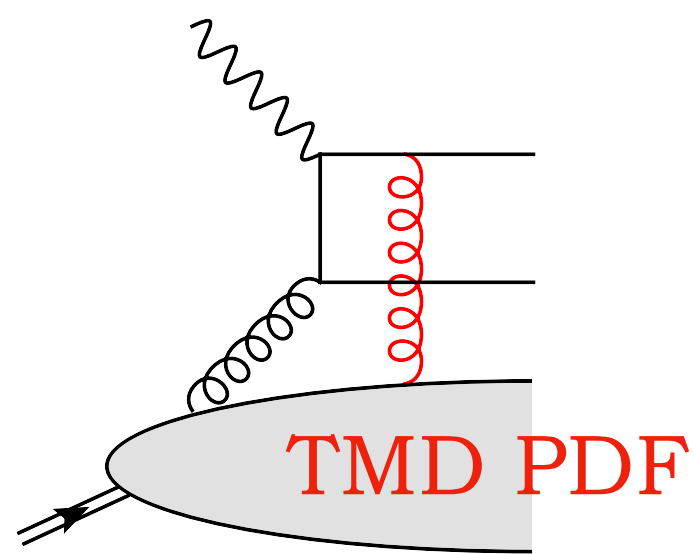


(b)

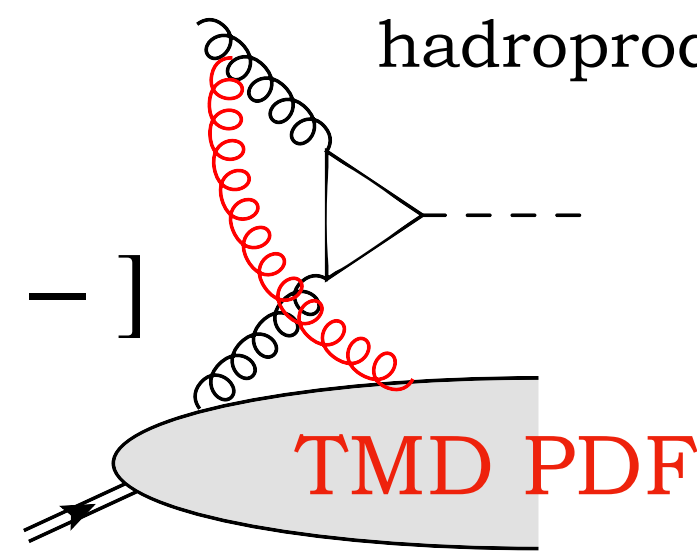
Higgs in  
gluon fusion

$\gamma^{(*)}$ -onium  
hadroproduction

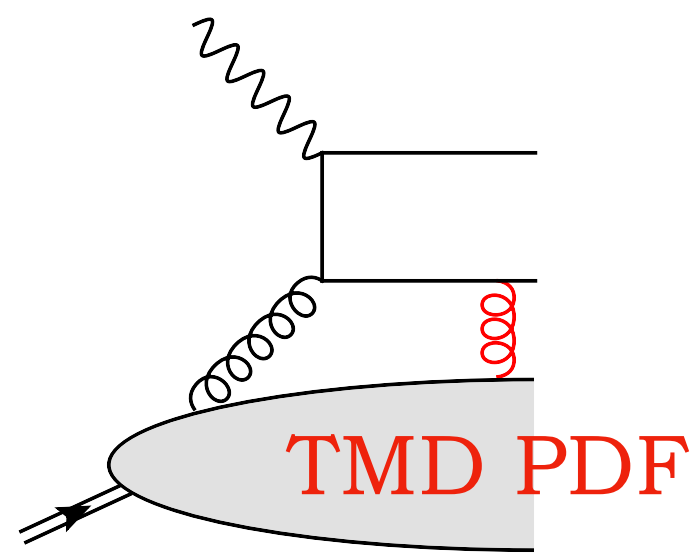
[+]



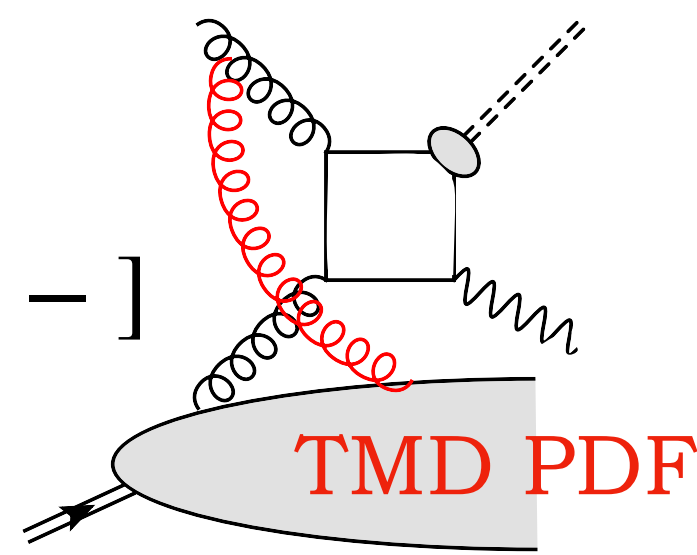
[-, -]



[+]



[-, -]



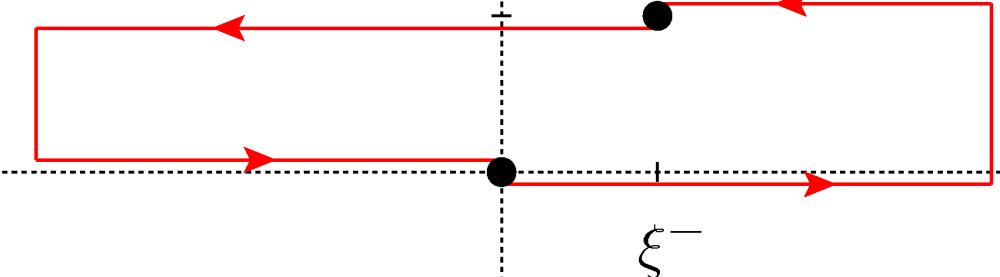
## *d*-type (DP)

(c)  $[+, -]$  or (d)  $[-, +]$

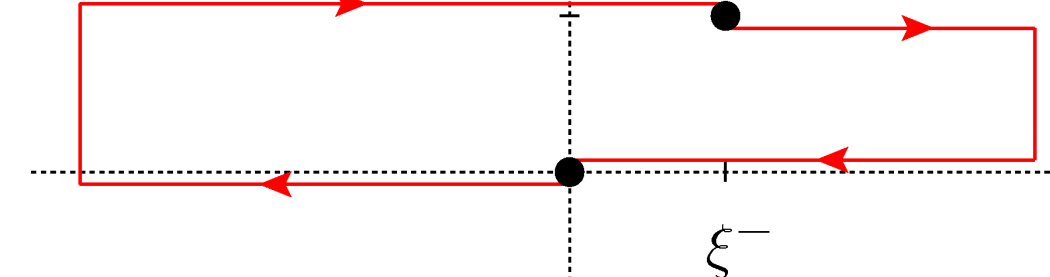
$\xi_T$

$\gamma^{(*)}$ -jet hadroproduction

$\xi_T$

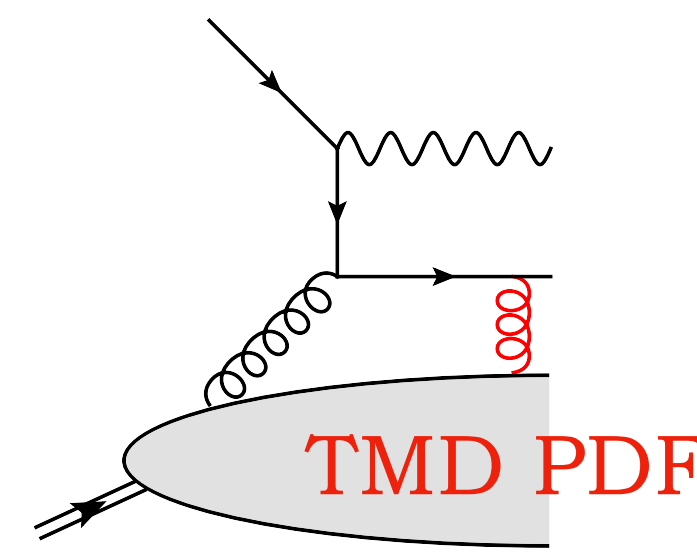


(c)

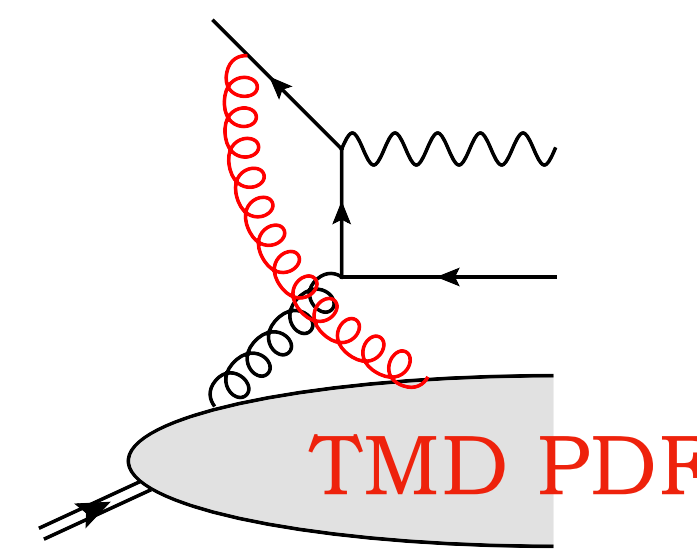


(d)

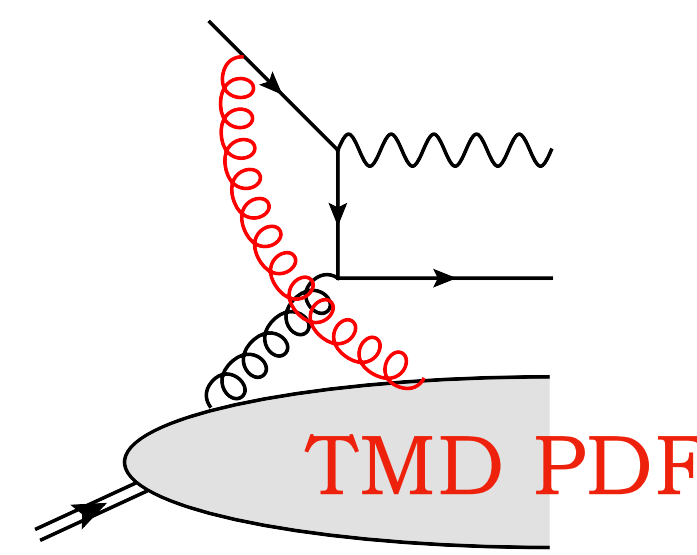
[+]



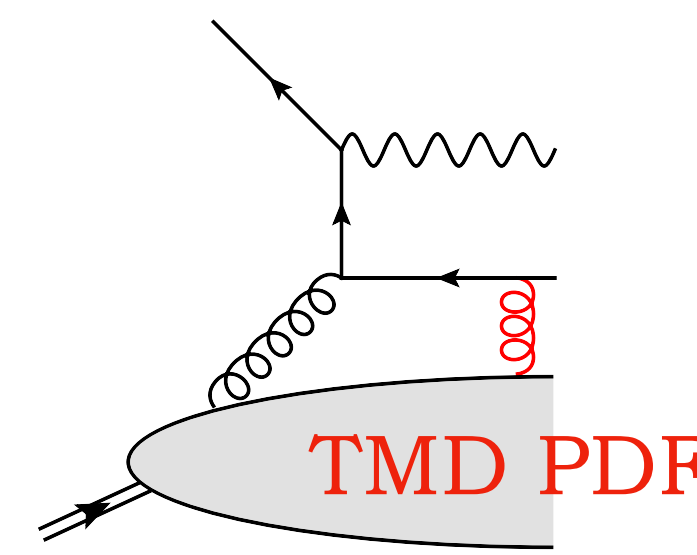
[-]



-]



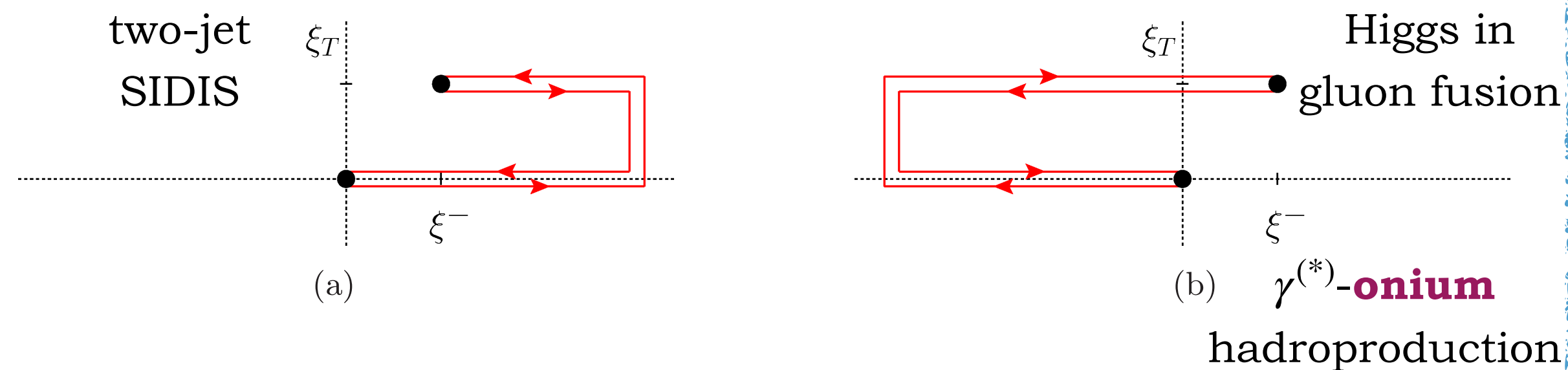
[+]



# Accessing f-type and d-type gluon TMDs

## *f*-type (WW)

(a) [ + , + ] or (b) [ - , - ]



\* Color flow annihilated within final/initial state

\* *f*-type gluon TMDs  $\rightarrow f^{abc}$  color structure

\* Modified universality:

$$f_1^{[+,+]} = f_1^{[-,-]},$$

$$f_{1T}^\perp[+,+] = -f_{1T}^\perp[-,-]$$

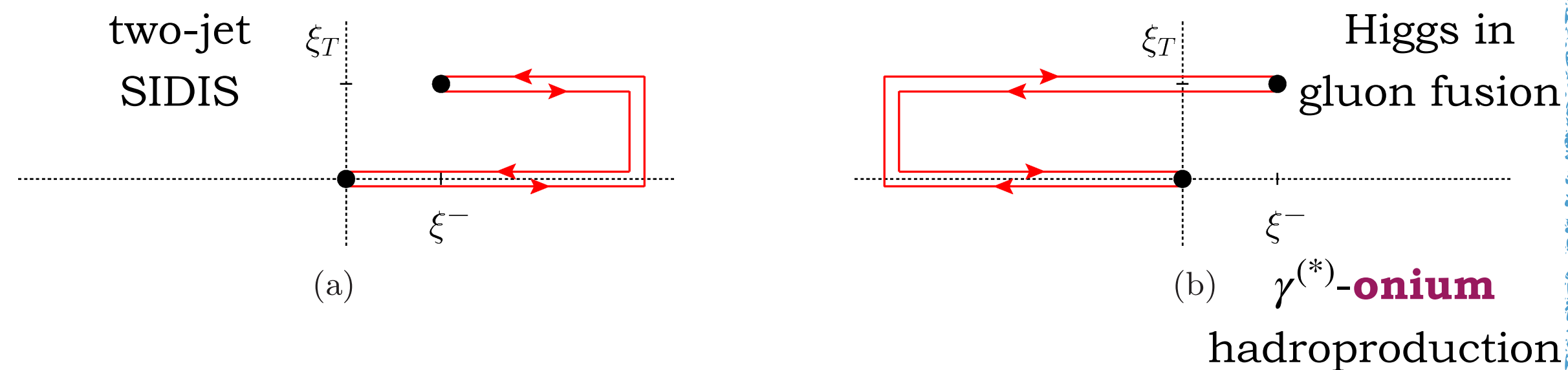
\* Phenomenology: Higgs, **quarkonia** or  $\gamma\gamma$  in  $pp$ , two-jet SIDIS, heavy-quark pair SIDIS



# Accessing f-type and d-type gluon TMDs

## f-type (WW)

(a) [ + , + ] or (b) [ - , - ]



- \* Color flow annihilated within final/initial state

- \* f-type gluon TMDs  $\rightarrow f^{abc}$  color structure

- \* Modified universality:

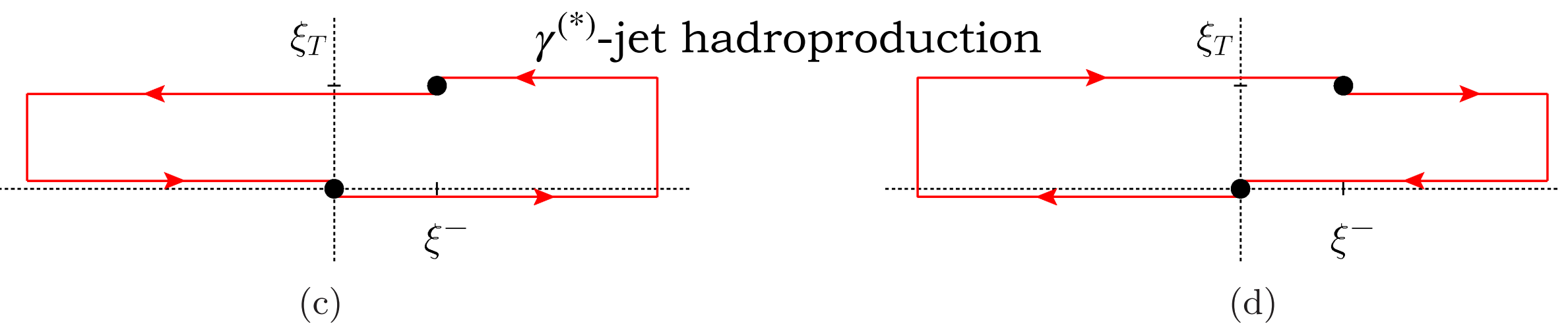
$$f_1^{[+,+]} = f_1^{[-,-]},$$

$$f_{1T}^\perp[+,+] = -f_{1T}^\perp[-,-]$$

- \* Phenomenology: Higgs, **quarkonia** or  $\gamma\gamma$  in  $pp$ , two-jet SIDIS, heavy-quark pair SIDIS

## d-type (DP)

(c) [ + , - ] or (d) [ - , + ]



- \* Color flow involving both initial and final states

- \* d-type gluon TMDs  $\rightarrow d^{abc}$  color structure

- \* Modified universality:

$$f_1^{[+,-]} = f_1^{[-,+]},$$

$$f_{1T}^\perp[+,-] = -f_{1T}^\perp[-,+]$$

- \* Phenomenology: single hadron or  $\gamma^{(*)}$ -jet hadroproduction, SIDIS or Drell-Yan (subleading)

Gauge link  $\rightarrow$  two main independent sets of TMDs, **not related** to each other

# Dihadron hadroproduction and factorization breaking

\* Proof of factorization violation [\[T. J. Rogers, P. J. Mulders \(2010\)\]](#)

\* Assumed factorization in SCET and CGC

\* Significance of low- $x$  studies

\* Size of factorization-breaking effects small?

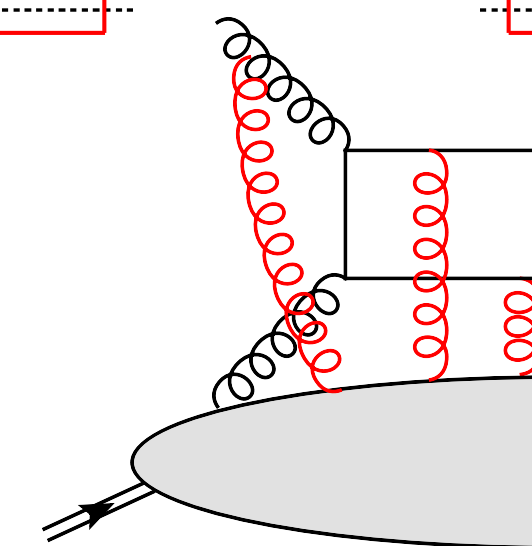
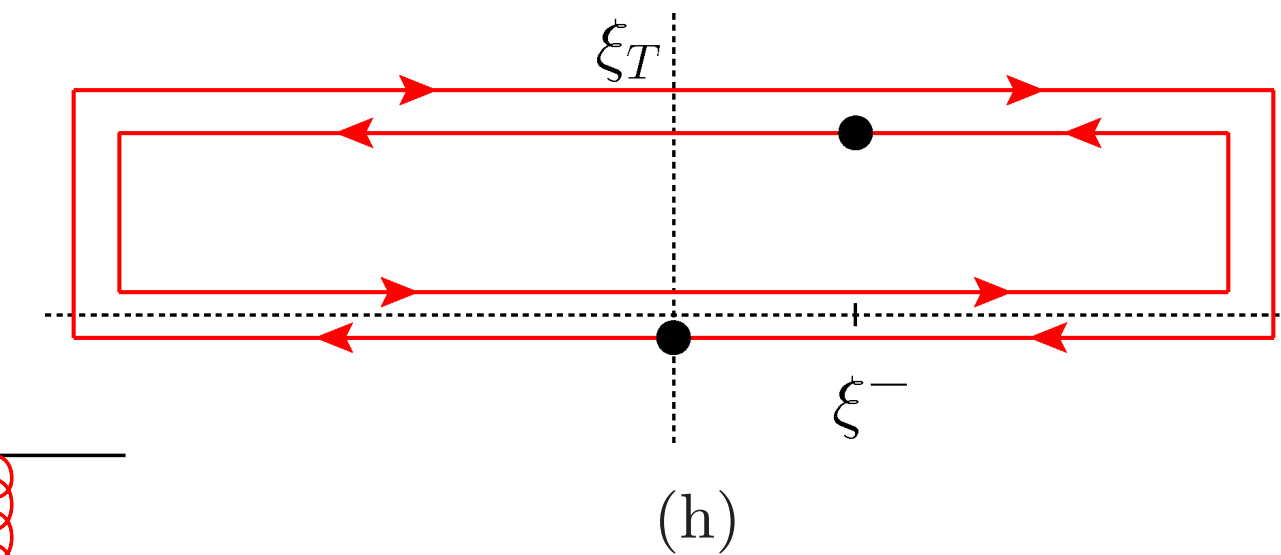
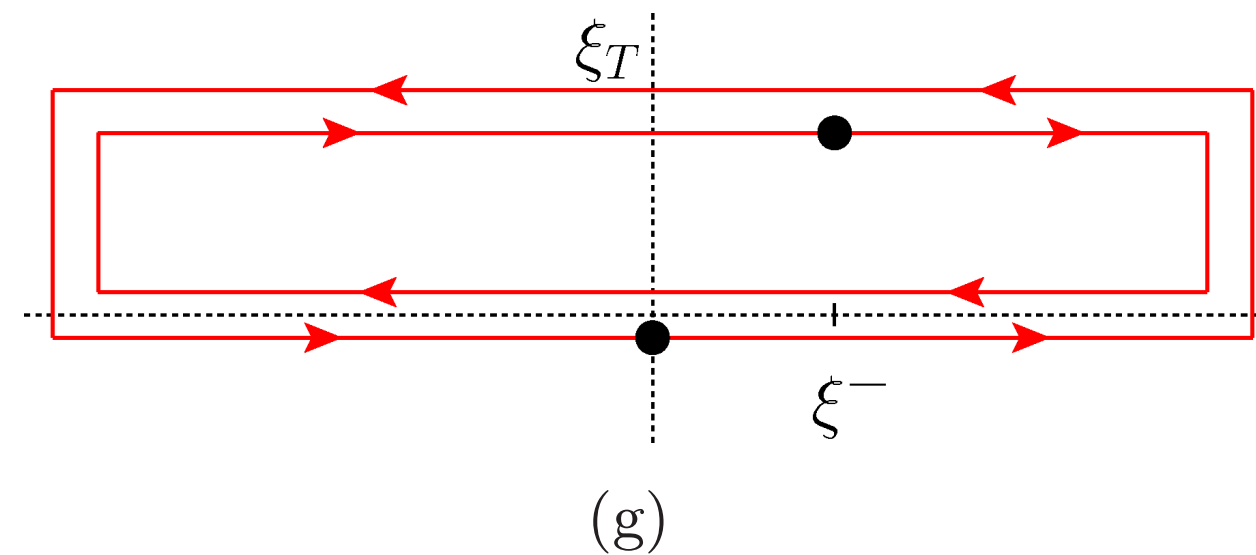
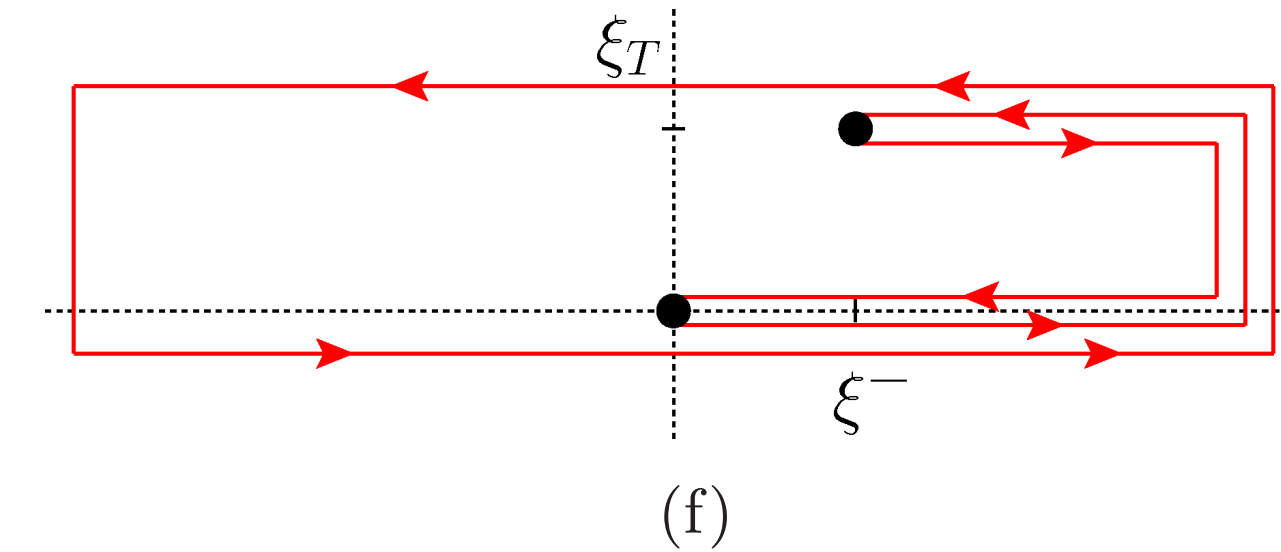
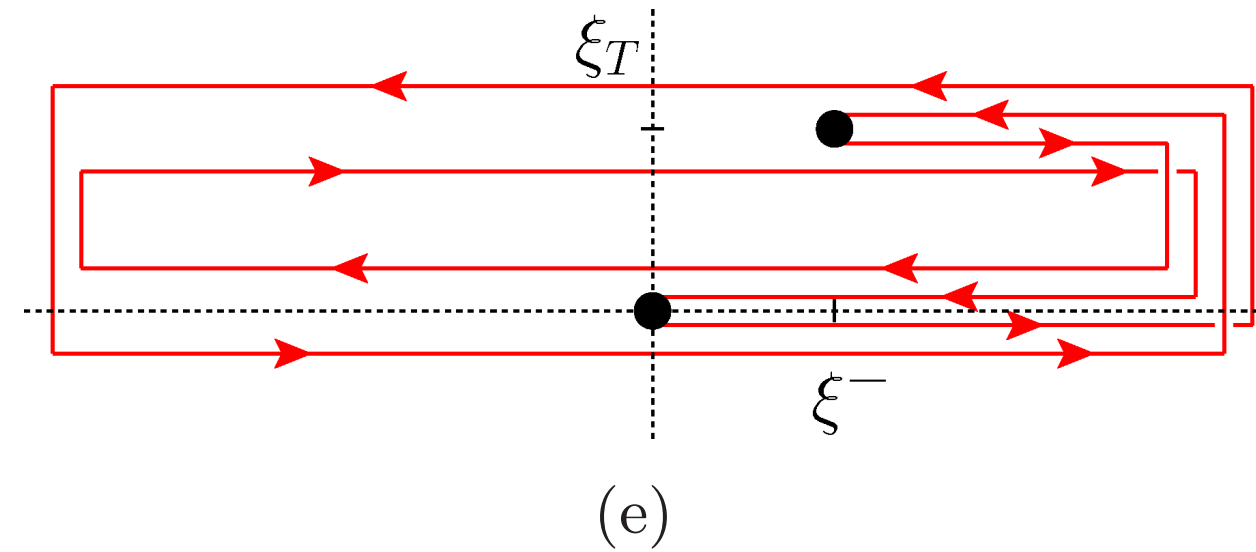
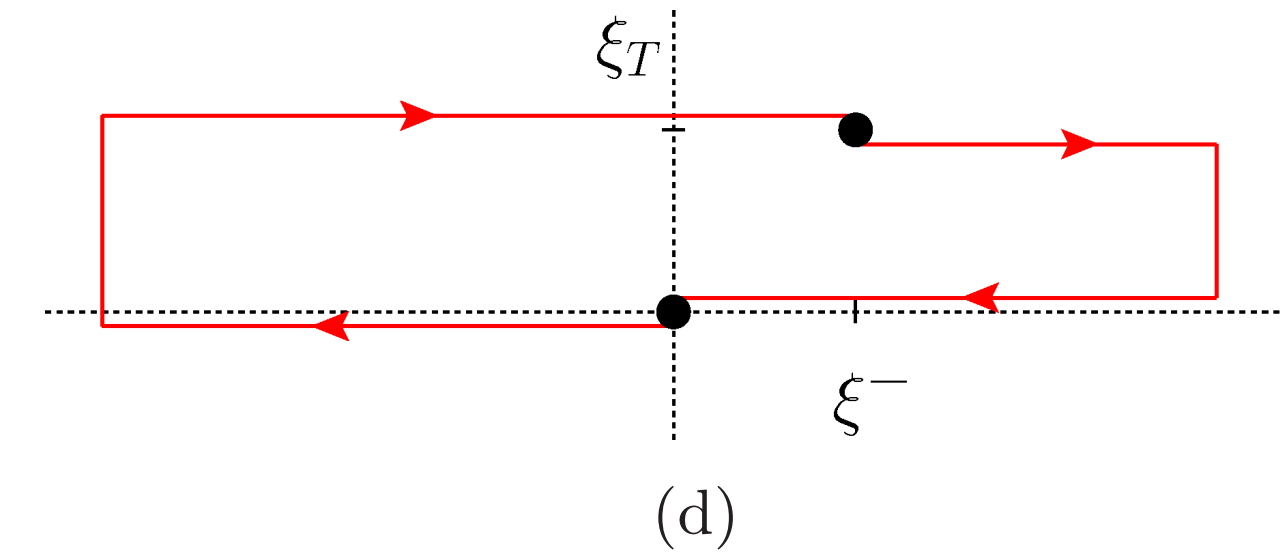
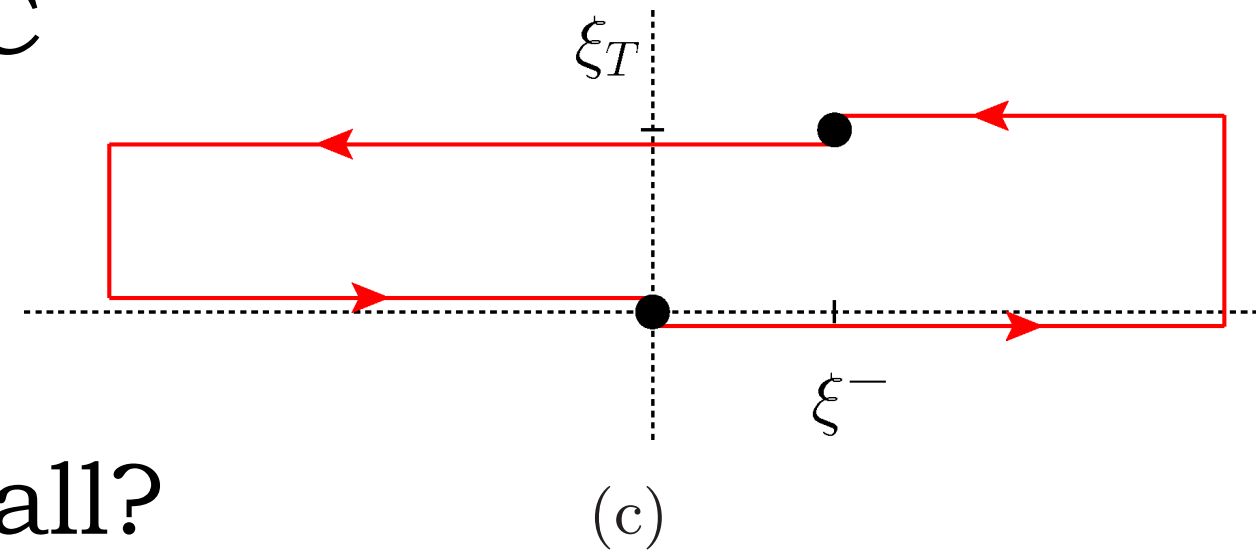
\* DP TMDs:

(c)  $[+, -]$  and (d)  $[-, +]$

\* Appearance of new gauge **loop links**:

(e)  $[+\square, +\square]$ , (f)  $[+, +\square]$ ,

(g)  $[\square, \square]$ , and (h)  $[\square, \square]$





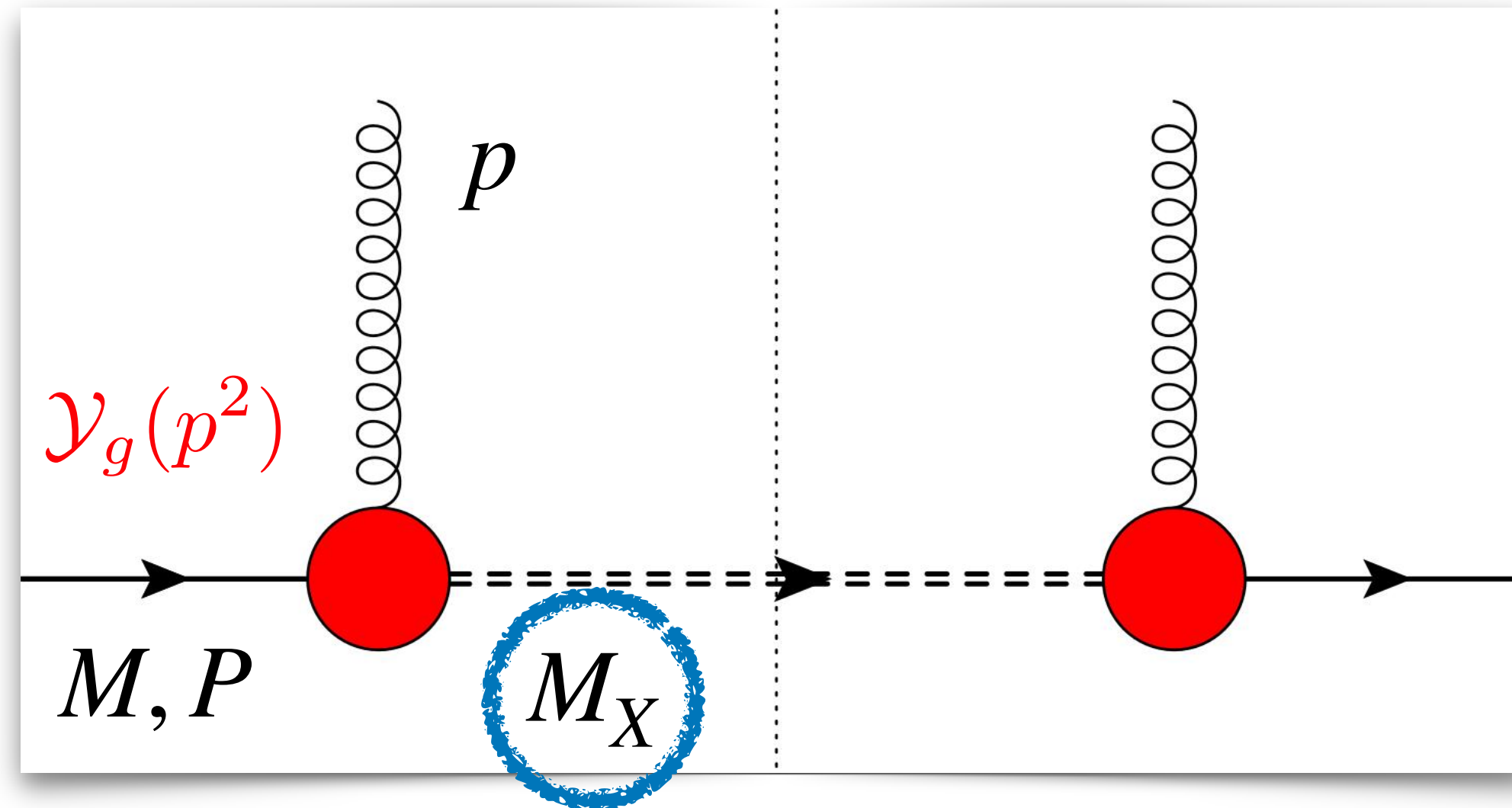
# Small-x improved gluon TMDs

# Assumptions of the model



## Spin-1/2 spectator

Lowest Fock state:  
**tri-quark** spectator  
on-shell and  
with mass  $M_X$

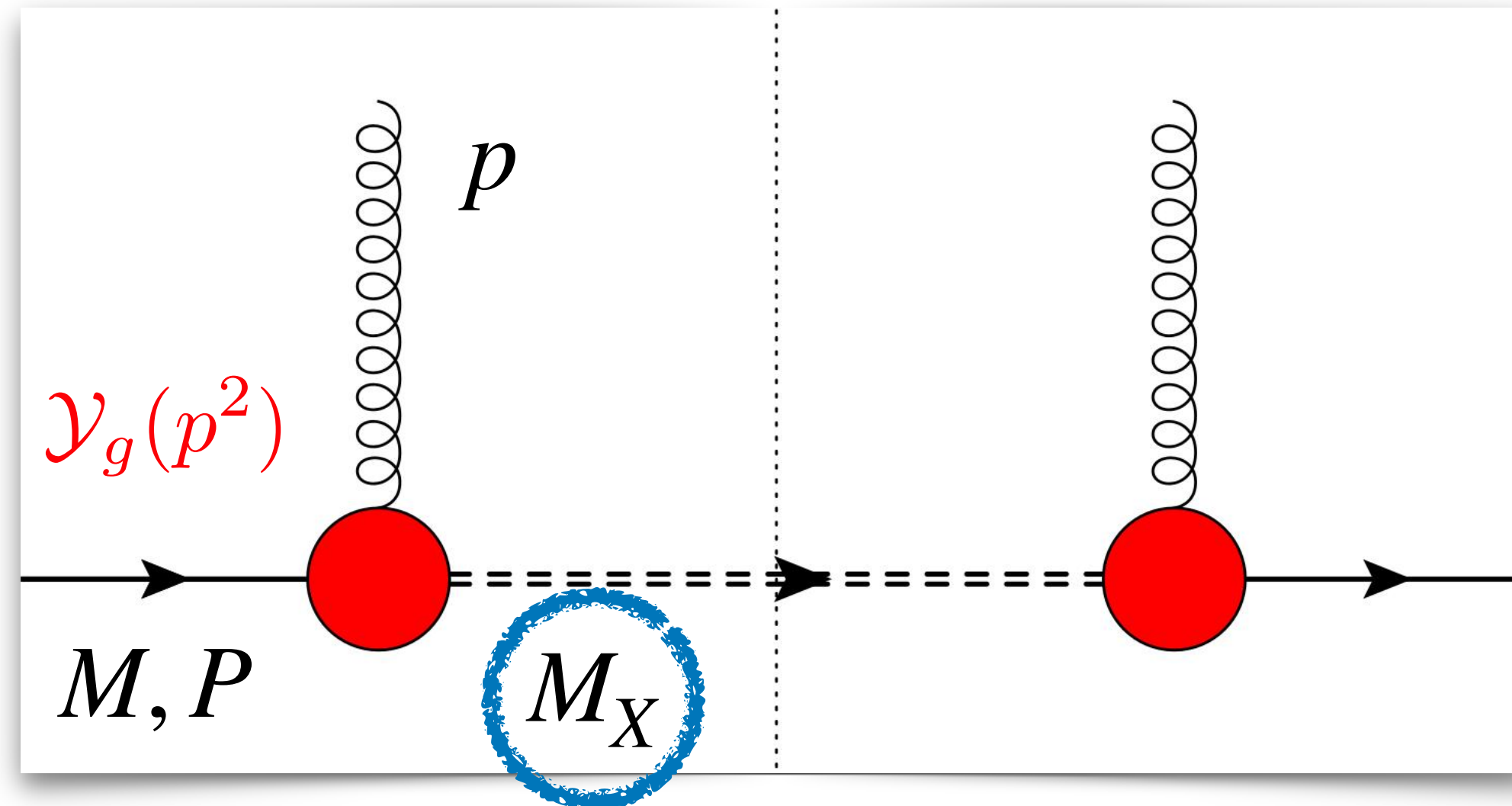


# Assumptions of the model



## Spin-1/2 spectator

Lowest Fock state:  
**tri-quark** spectator  
 on-shell and  
 with mass  $M_X$



## Nucleon-gluon-spectator vertex

$$\Phi_g = \frac{1}{2(2\pi)^3(1-x)P^+} \text{Tr} \left[ (\not{P} + M) \frac{1 + \gamma^5 \not{\xi}}{2} G_{\mu\rho}^*(p) G^{\nu\sigma}(p) \mathcal{Y}_g^{\rho*} \mathcal{Y}_{g\sigma} (\not{P} - \not{p} + M) \right]$$

$$\mathcal{Y}_g^\mu = g_1(p^2) \gamma^\mu + i \frac{g_2(p^2)}{2M} \sigma^{\mu\nu} p_\nu$$



mimics proton form factors  
 (conserved EM current  
 of a free nucleon)



## Link with collinear factorization

1.  $p_T$ -integrated TMDs **have to** reproduce PDFs at the lowest scale ( $Q_0$ ) *before* evolution
2. TMDs and PDFs *decouple* due to evolution

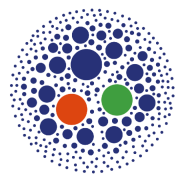
# Assumptions of the model



## Link with collinear factorization

1.  $p_T$ -integrated TMDs **have to** reproduce PDFs at the lowest scale ( $Q_0$ ) *before* evolution
2. TMDs and PDFs *decouple* due to evolution

$$g_{1,2}(p^2) = \kappa_{1,2} \frac{p^2}{|p^2 - \Lambda_X^2|^2}$$



## Dipolar form factor(s)

1. Cancels singularity of gluon propagator
2. Suppresses effects of high  $p_T$
3. Compensates log divergences arising from  $p_T$ -integration
4. Adds three more parameters:  $\kappa_{1,2}$  and  $\Lambda_X$

# Assumptions of the model



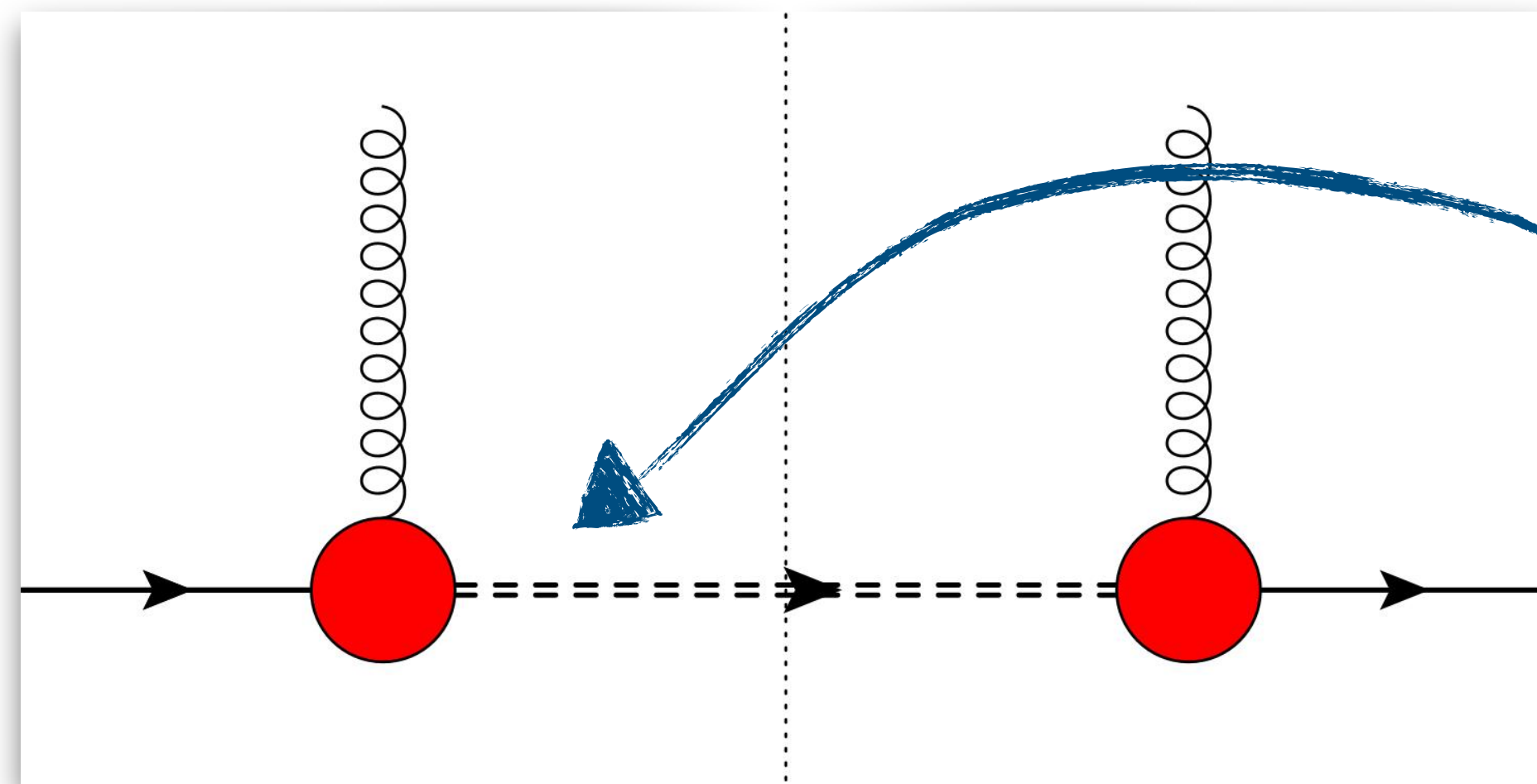
## Spectator-system spectral-mass function

spectral-mass function

$$F(x, \mathbf{p}_T^2) = \int_M^\infty dM_X \rho_X(M_X) \hat{F}(x, \mathbf{p}_T^2; M_X)$$

spectator-model TMD

[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]



Instead of a single on-shell spectator, a continuum of spectators

$\mathcal{V}_g(p^2)$



# Assumptions of the model



## Spectator-system spectral-mass function

**spectral-mass function**

$$F(x, \mathbf{p}_T^2) = \int_M^\infty dM_X \rho_X(M_X) \hat{F}(x, \mathbf{p}_T^2; M_X)$$

**spectator-model TMD**

[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]

$$\rho_X \left( M_X; \{X^{(\text{pars})}\} \equiv \{A, B, a, b, C, D, \sigma\} \right) = \mu^{2a} \left[ \frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}} \right]$$

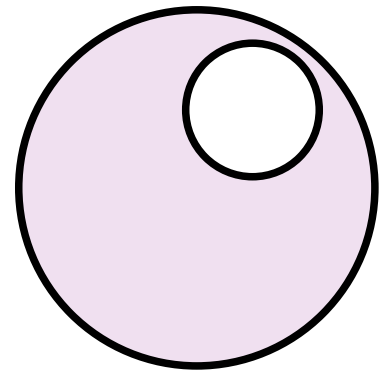
**low- $x$  (high- $\mu^2$ ) tail**  $\propto (a - b)$

$q\bar{q}$  contributions energetically available at large  $M_X$

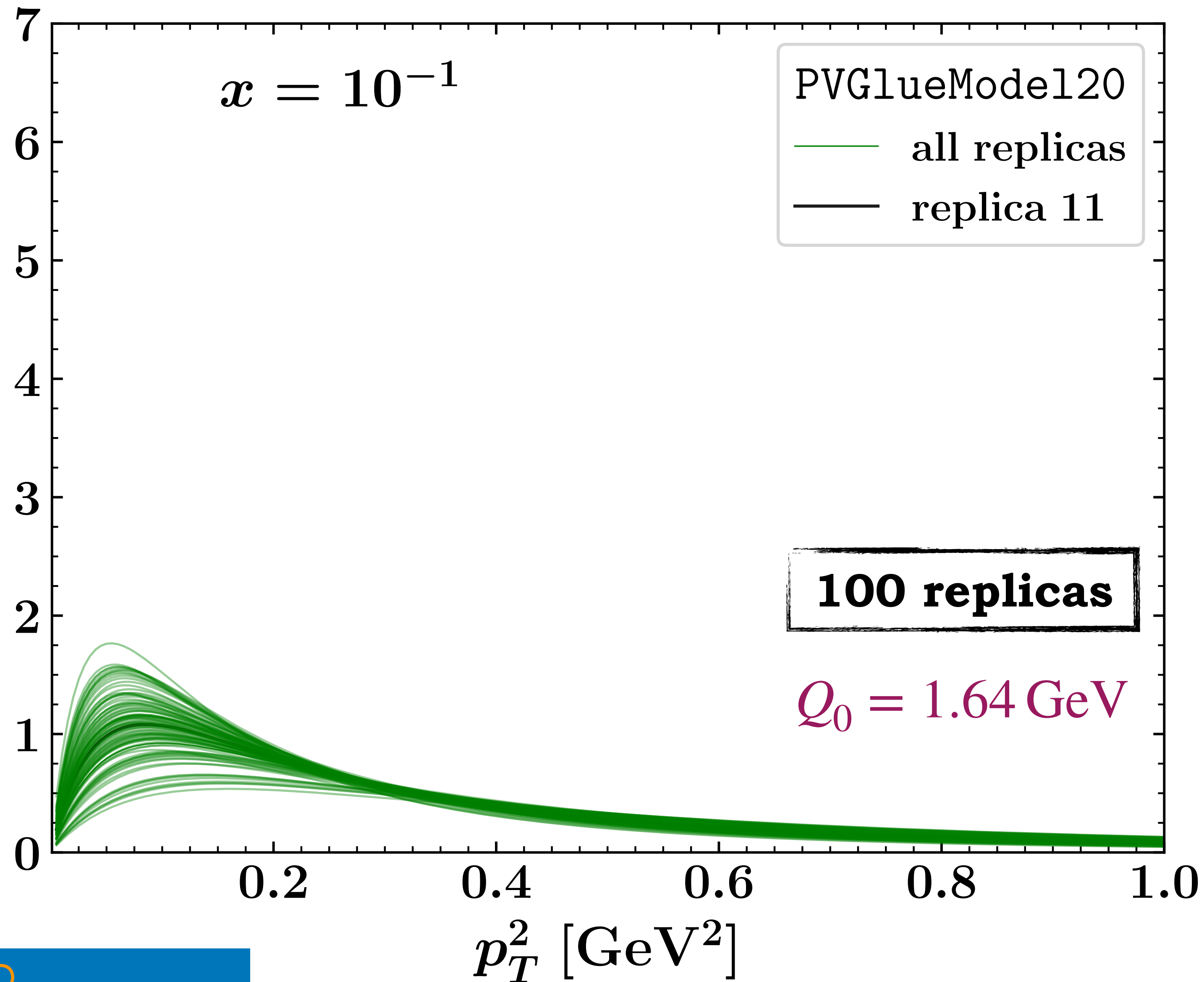
$$\mu^2 = M_X^2 - M^2$$

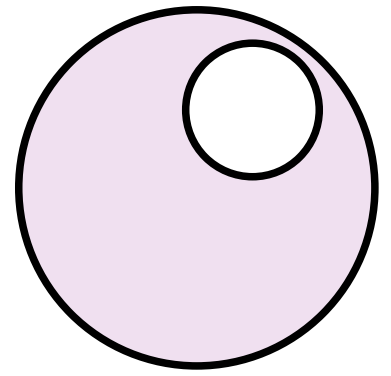
**moderate- $x$  trend**

pure tri-quark contribution at low  $M_X$

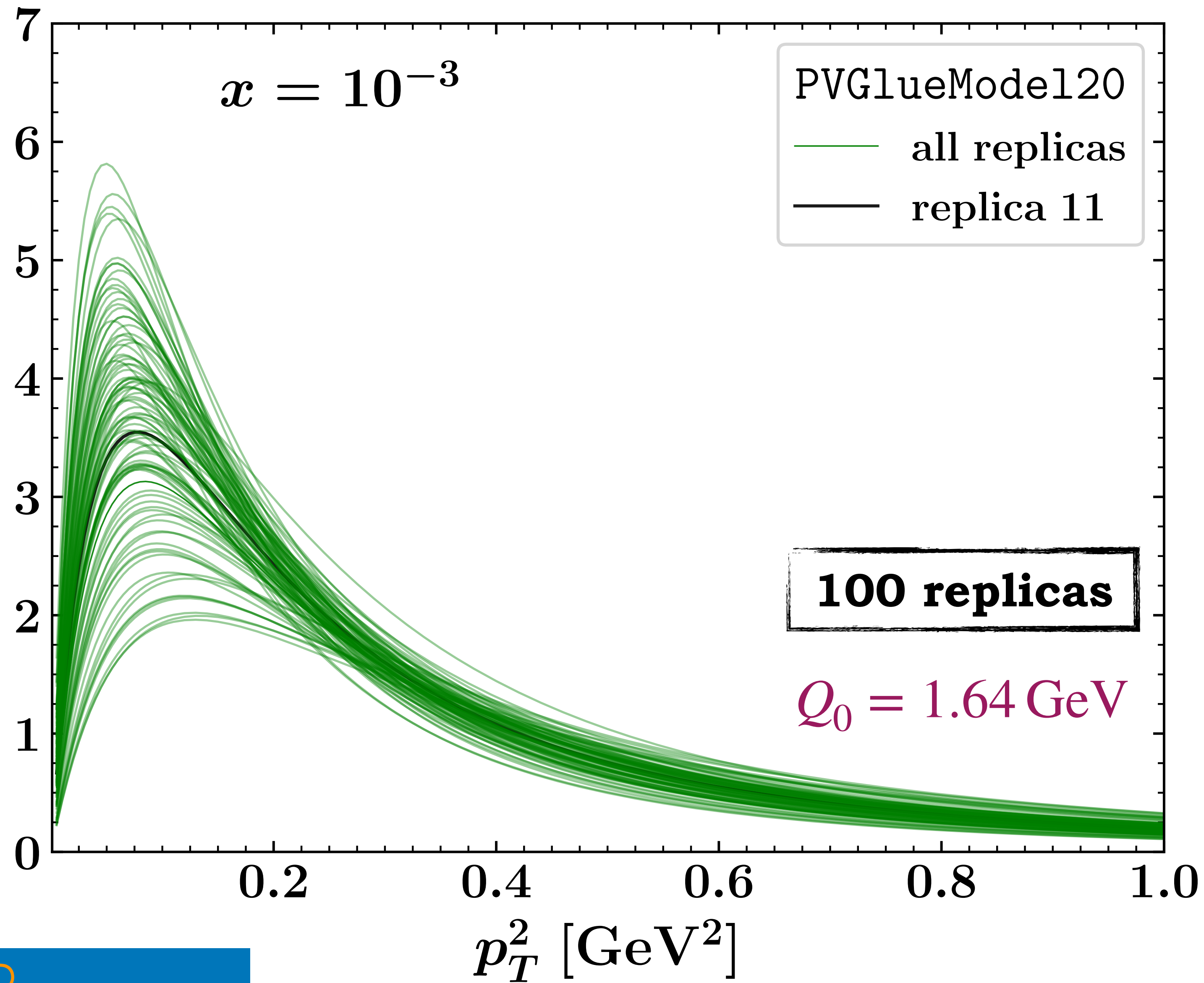


$$x f_1(x, p_T^2)$$



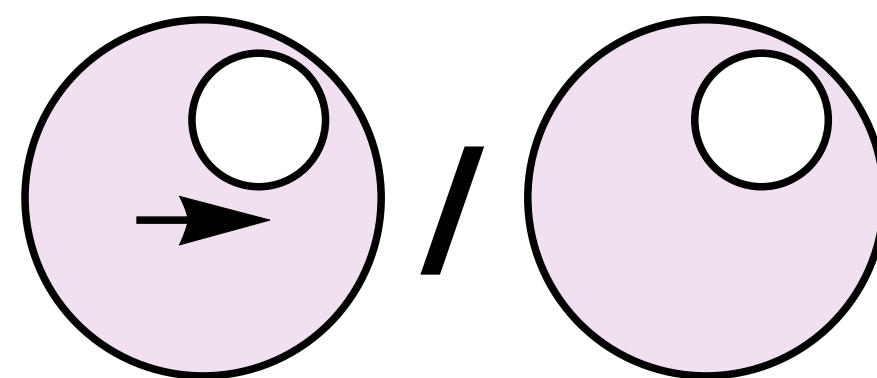


$$x f_1(x, p_T^2)$$



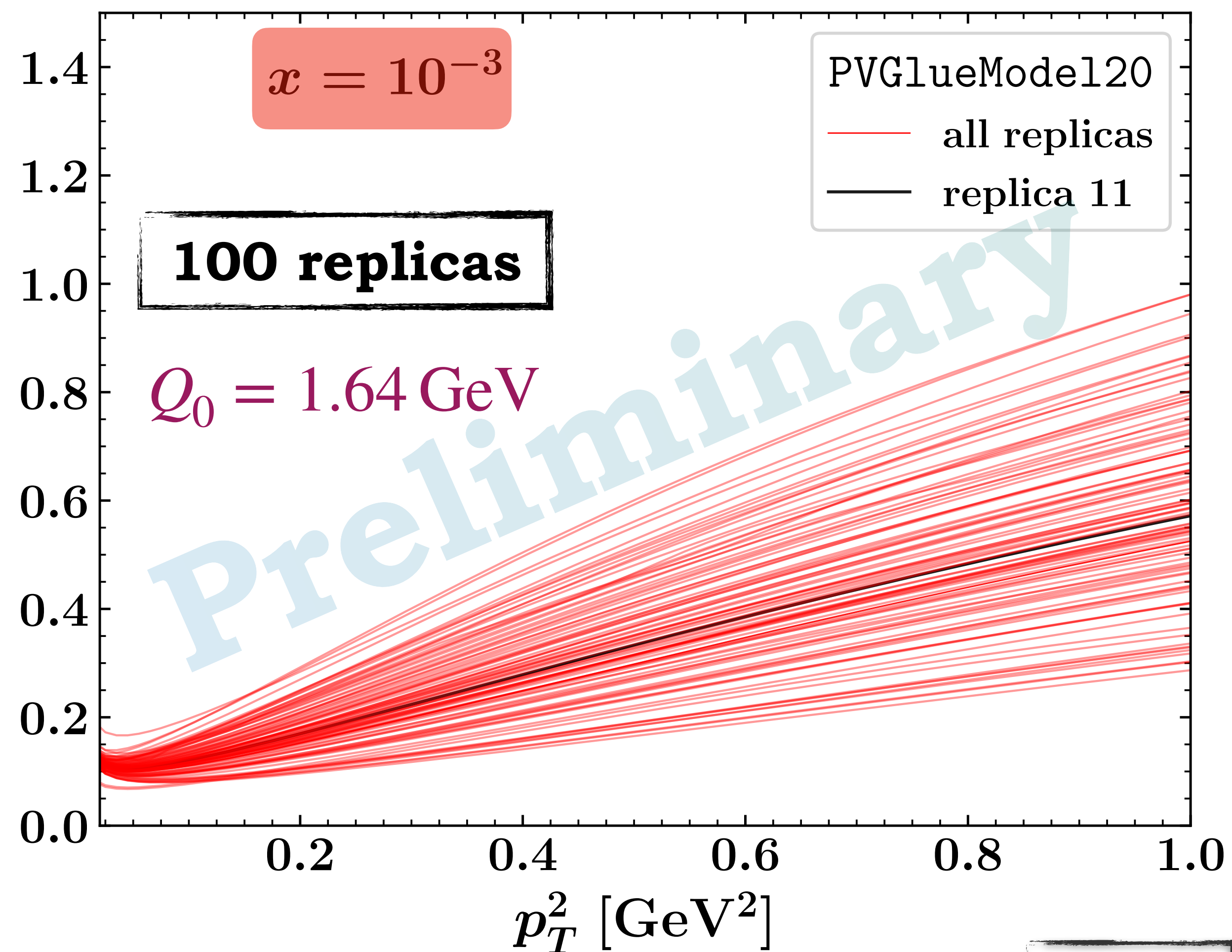
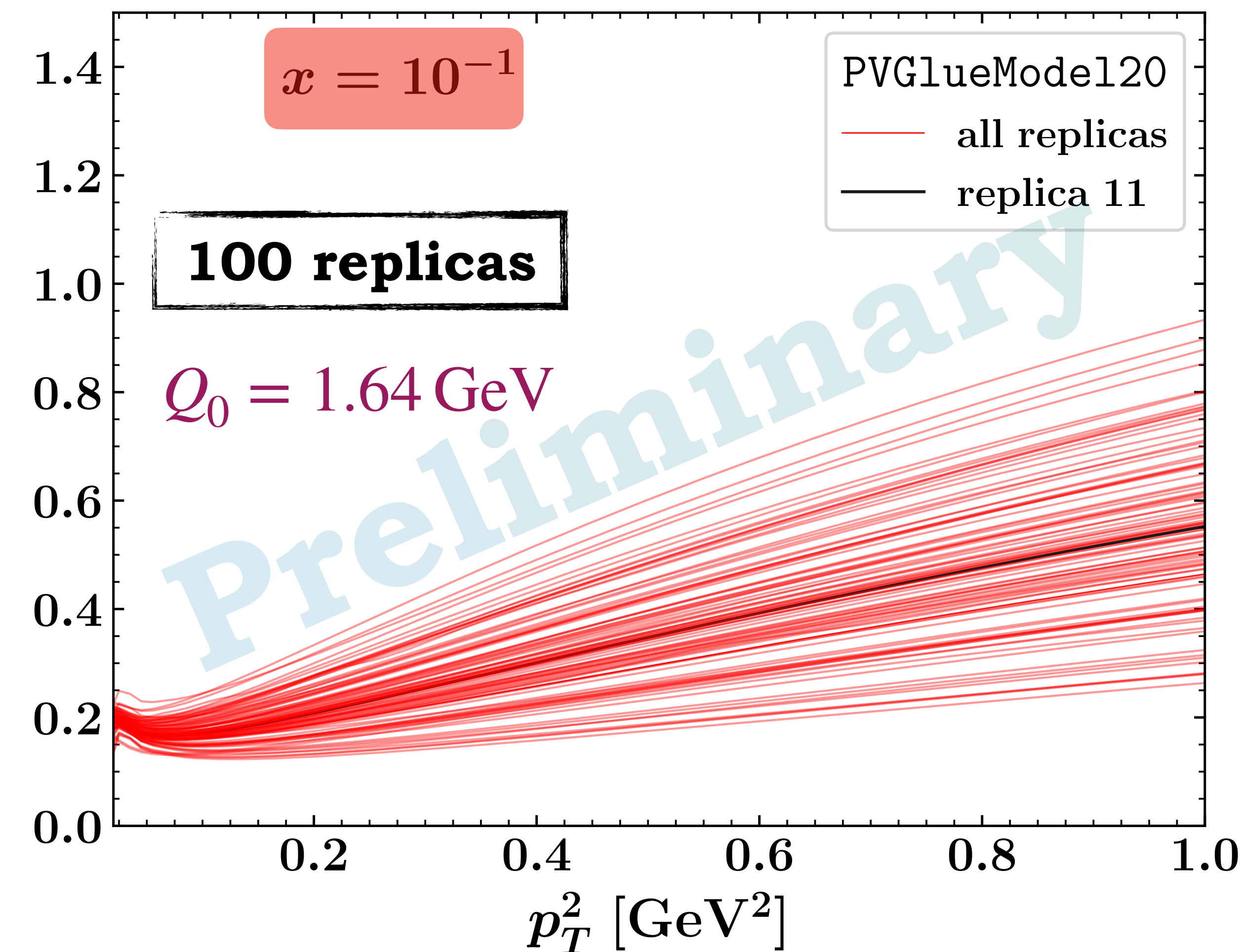
# $f$ -type Sivers/unpol.

A. Bacchetta, F.G. C., M. Radici (in preparation)



$$\frac{p_T}{M} f_{1T}^{\perp[+,+]} / f_1$$

$$\frac{p_T}{M} f_{1T}^{\perp[+,+]} / f_1$$



**Backup**

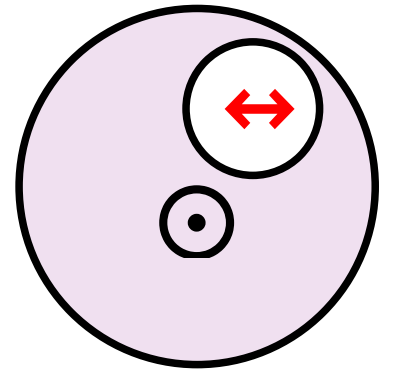
# Glueon TMD correlator and T-odd glueon densities

$$\Gamma_U^{ij}(x, \mathbf{k}) = x \left[ \delta_T^{ij} f_1(x, \mathbf{k}^2) + \frac{k_T^{ij}}{M^2} h_1^\perp(x, \mathbf{k}^2) \right]$$

$$\Gamma_L^{ij}(x, \mathbf{k}) = x \left[ i\epsilon_T^{ij} S_L g_1(x, \mathbf{k}^2) + \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_L}{2M^2} h_{1L}^\perp(x, \mathbf{k}^2) \right]$$

$$\Gamma_T^{ij}(x, \mathbf{k}) = x \left[ \frac{\delta_T^{ij} \epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, \mathbf{k}^2) + \frac{i\epsilon_T^{ij} \mathbf{k} \cdot \mathbf{S}_T}{M} g_{1T}(x, \mathbf{k}^2) \right. \\ \left. - \frac{\epsilon_T^{k_T \{i} S_T^{j\}} + \epsilon_T^{S_T \{i} k_T^{j\}}}{4M} h_1(x, \mathbf{k}^2) - \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_T}{2M^3} h_{1T}^\perp(x, \mathbf{k}^2) \right]$$

# Glueon TMD correlator and T-odd gluon densities

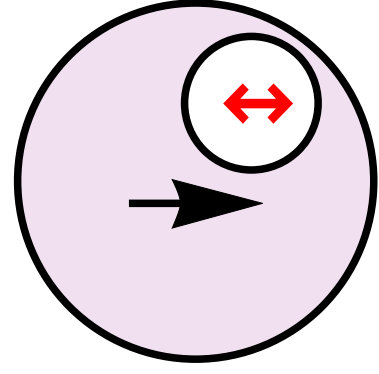


$$\Gamma_U^{ij}(x, \mathbf{k}) = x \left[ \delta_T^{ij} f_1(x, \mathbf{k}^2) + \frac{k_T^{ij}}{M^2} h_1^\perp(x, \mathbf{k}^2) \right]$$

$$\Gamma_L^{ij}(x, \mathbf{k}) = x \left[ i\epsilon_T^{ij} S_L g_1(x, \mathbf{k}^2) + \frac{\epsilon_T^{\{i} k_T^{j\} \alpha} S_L}{2M^2} h_{1L}^\perp(x, \mathbf{k}^2) \right]$$

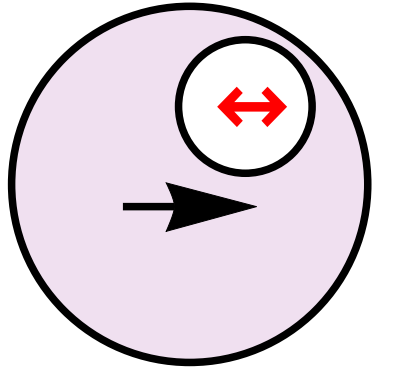
$$\Gamma_T^{ij}(x, \mathbf{k}) = x \left[ \frac{\delta_T^{ij} \epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, \mathbf{k}^2) + \frac{i\epsilon_T^{ij} \mathbf{k} \cdot \mathbf{S}_T}{M} g_{1T}(x, \mathbf{k}^2) \right. \\ \left. - \frac{\epsilon_T^{k_T \{i} S_T^{j\}} + \epsilon_T^{S_T \{i} k_T^{j\}}}{4M} h_1(x, \mathbf{k}^2) - \frac{\epsilon_T^{\{i} k_T^{j\} \alpha} S_T}{2M^3} h_{1T}^\perp(x, \mathbf{k}^2) \right]$$

**pseudo worm-gear**

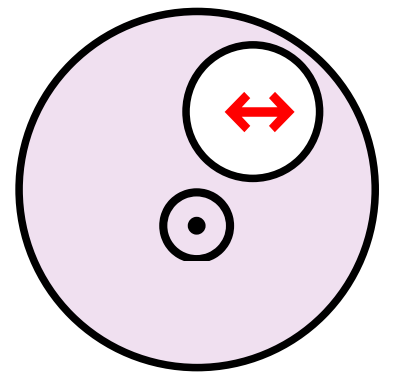


**linearity TMD**

**pretzelosity**



# Gluon TMD correlator and T-odd gluon densities



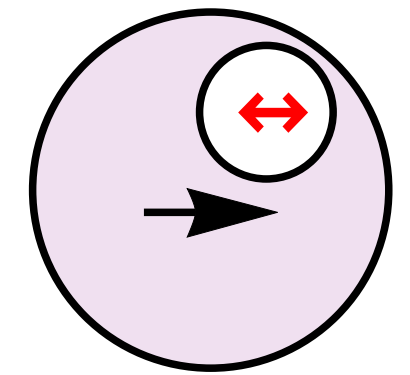
**pseudo worm-gear**

$$\Gamma_U^{ij}(x, k) = x \left[ \delta_T^{ij} f_1(x, k^2) + \frac{k_T^{ij}}{M^2} h_1^\perp(x, k^2) \right]$$

$$\Gamma_L^{ij}(x, k) = x \left[ i\epsilon_T^{ij} S_L g_1(x, k^2) + \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_L}{2M^2} h_{1L}^\perp(x, k^2) \right]$$

$$\Gamma_T^{ij}(x, k) = x \left[ \frac{\delta_T^{ij} \epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, k^2) + \frac{i\epsilon_T^{ij} \mathbf{k} \cdot \mathbf{S}_T}{M} g_{1T}(x, k^2) \right.$$

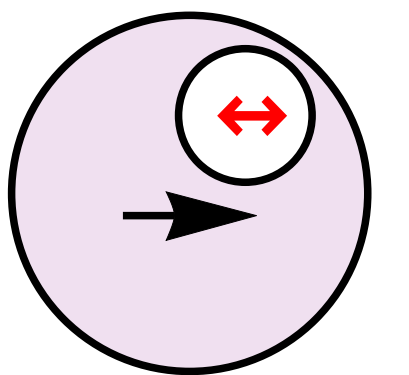
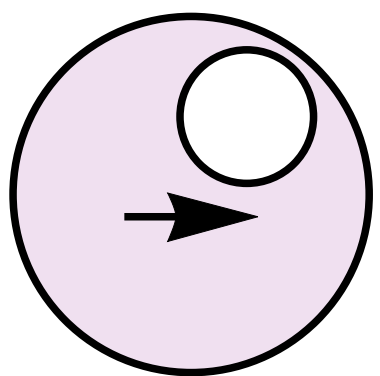
$$\left. - \frac{\epsilon_T^{k_T \{i} S_T^{j\}} + \epsilon_T^{S_T \{i} k_T^{j\}}}{4M} h_1(x, k^2) - \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_T}{2M^3} h_{1T}^\perp(x, k^2) \right]$$



**linearity TMD**

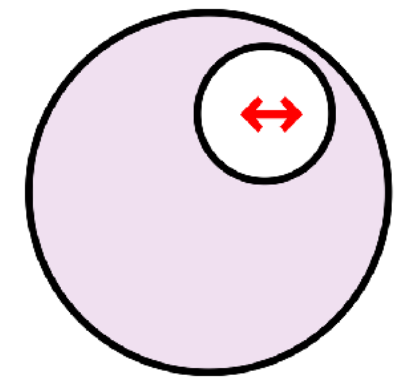
**pretzelosity**

**Sivers**



$$\frac{\epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, k^2) = \frac{1}{2} \delta_{Tij} \Gamma_T^{ij}(x, k)$$

# Boer-Mulders effect in unpolarized pp collisions

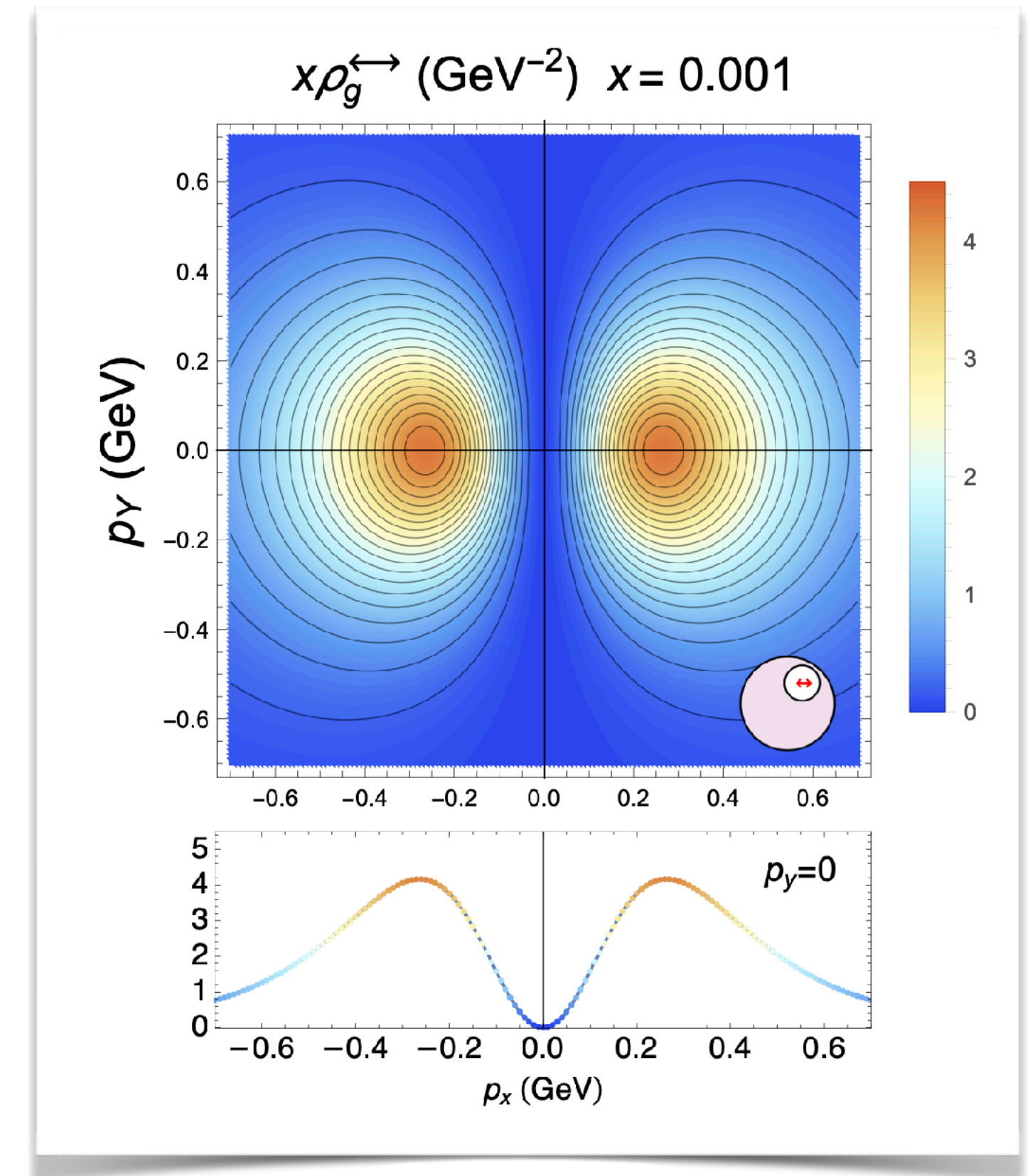


$[\leftrightarrow / \mathbf{u}]$

$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

unpol.

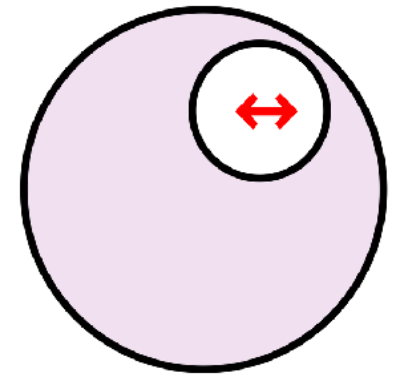
Boer-Mulders



[A. Bacchetta, F.G.C., M. Radici, P. Taelis (2020)]



# Boer-Mulders effect in unpolarized pp collisions



$[\leftrightarrow / \mathbf{u}]$

$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

unpol.

Boer-Mulders



(Pseudo)scalar  $p_T$ -distributions: Higgs,  $\eta_{c,b}$

$$\frac{d\sigma}{dq_T} \sim \Phi_A^U \Phi_B^U |\mathcal{M}|^2$$

at low transverse momentum  
for (pseudo)scalar state

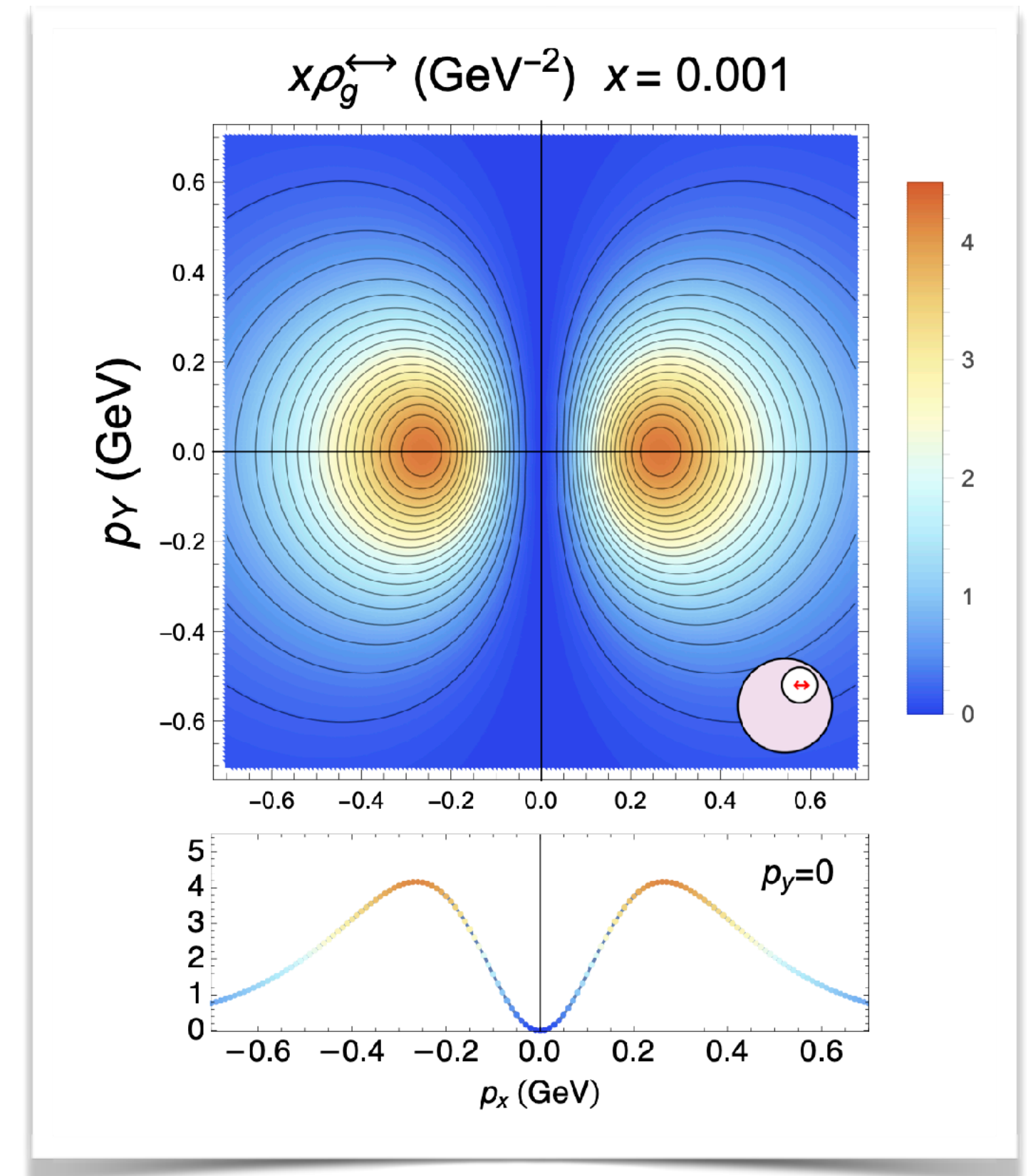
$$\sim \mathcal{C} \left[ \begin{array}{cc} f_1^{g/A} & f_1^{g/B} \end{array} \right] \pm \mathcal{C} \left[ \begin{array}{cc} h_1^{\perp g/A} & h_1^{\perp g/B} \end{array} \right]$$

unpolarized gluons

lin. polarized gluons

NRQCD

$$\frac{CS}{CO} \sim \frac{1}{v^4}$$

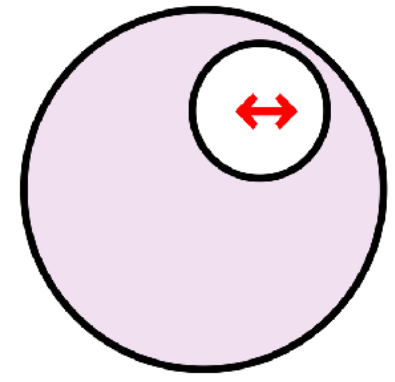


[D. Boer, W.J. den Dunnen, C. Pisano, M. Schlegel, W. Vogelsang (2012)]  
(Higgs+jet angular distributions)

[D. Boer, C. Pisano (2015)]

[A. Bacchetta, F.G.C., M. Radici, P. Taelis (2020)]

# Boer-Mulders effect in unpolarized pp collisions



$[\leftrightarrow / \mathbf{u}]$

$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

unpol.

Boer-Mulders



(Pseudo)scalar  $p_T$ -distributions: Higgs,  $\eta_{c,b}$

$$\frac{d\sigma}{dq_T} \sim \Phi_A^U \Phi_B^U |\mathcal{M}|^2$$

at low transverse momentum  
for (pseudo)scalar state

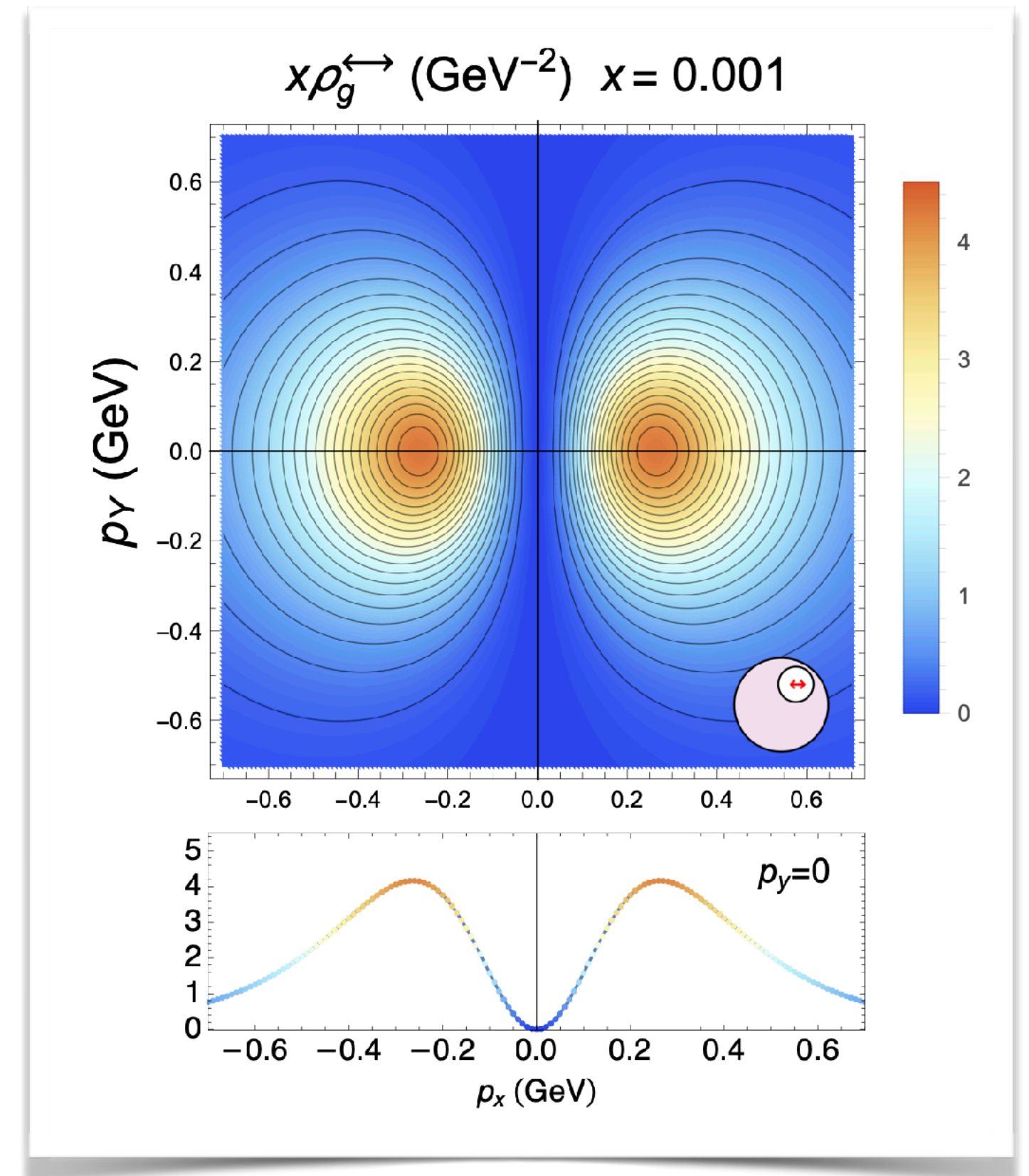
$$\sim \mathcal{C} \left[ \begin{array}{cc} f_1^{g/A} & f_1^{g/B} \end{array} \right] \pm \mathcal{C} \left[ \begin{array}{cc} h_1^{\perp g/A} & h_1^{\perp g/B} \end{array} \right]$$

unpolarized gluons

lin. polarized gluons

NRQCD

$$\frac{CS}{CO} \sim \frac{1}{v^4}$$



[D. Boer, W.J. den Dunnen, C. Pisano, M. Schlegel, W. Vogelsang (2012)]  
(Higgs+jet angular distributions)

[D. Boer, C. Pisano (2015)]

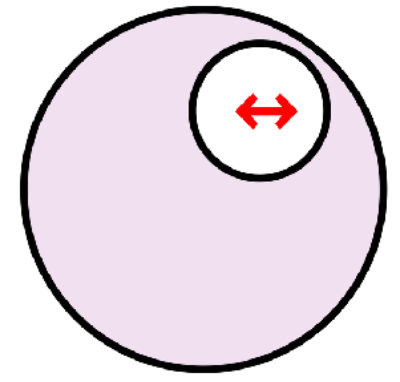
[A. Bacchetta, F.G.C., M. Radici, P. Taelis (2020)]



Model prediction at low-x

$$\frac{f_1^g(x, p_T^2)}{h_1^{\perp g}(x, p_T^2)} \underset{x \rightarrow 0^+}{\sim} \text{constant}$$

# Boer-Mulders effect in unpolarized pp collisions



$[\leftrightarrow / \mathbf{u}]$

$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

unpol.

Boer-Mulders



(Pseudo)scalar  $p_T$ -distributions: Higgs,  $\eta_{c,b}$

$$\frac{d\sigma}{dq_T} \sim \Phi_A^U \Phi_B^U |\mathcal{M}|^2$$

at low transverse momentum  
for (pseudo)scalar state

$$\sim \mathcal{C} [ f_1^{g/A} f_1^{g/B} ]$$

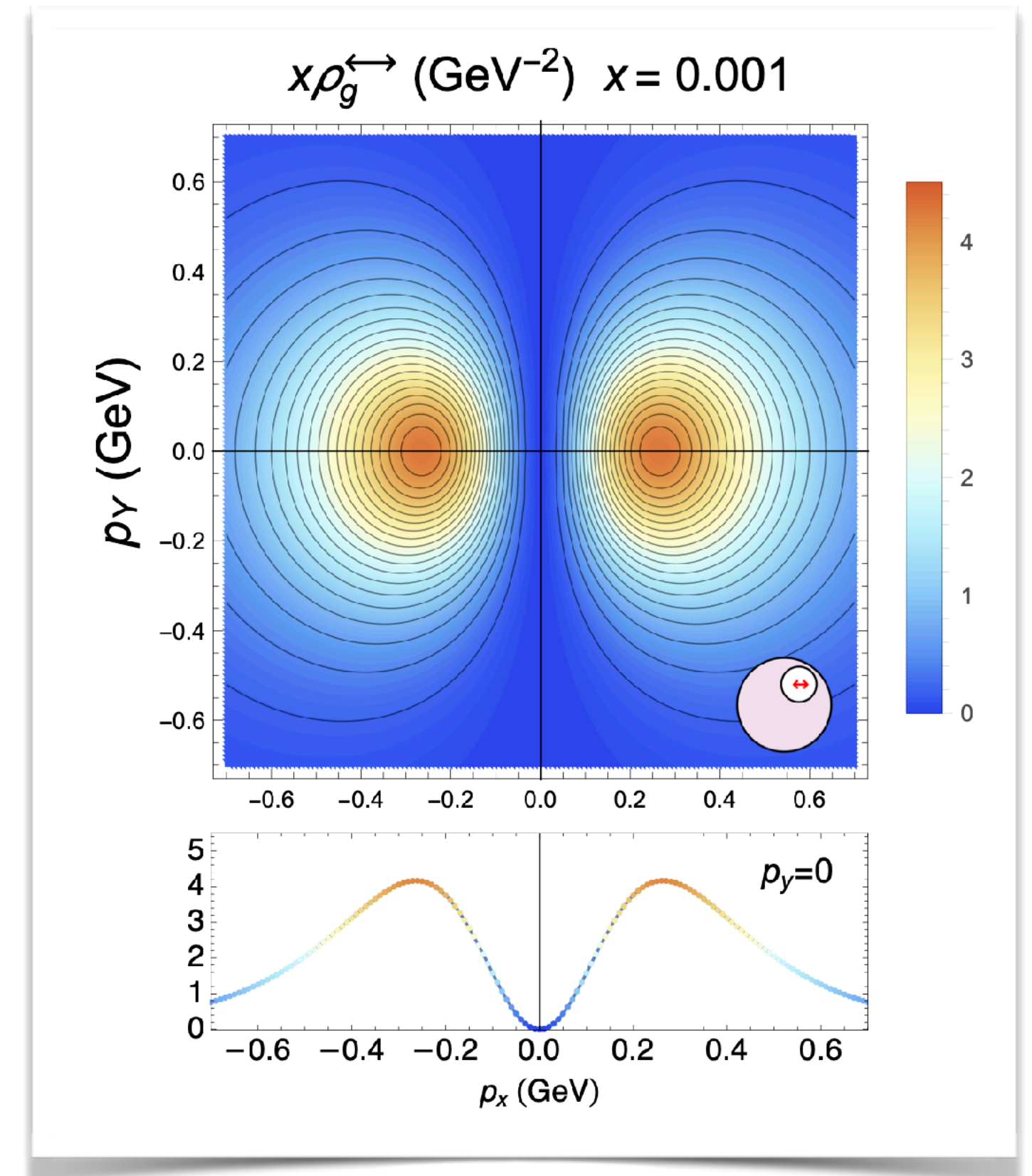
unpolarized gluons

$$\pm \mathcal{C} [ h_1^{\perp g/A} h_1^{\perp g/B} ]$$

lin. polarized gluons

NRQCD

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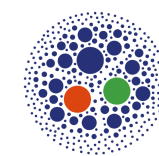
[D. Boer, C. Pisano (2015)]

[A. Bacchetta, F.G.C., M. Radici, P. Taelis (2020)]



Model prediction at low-x

$$\frac{f_1^g(x, p_T^2)}{h_1^{\perp g}(x, p_T^2)} \underset{x \rightarrow 0^+}{\sim} \text{constant}$$



BFKL regime (linear low-x evolution)



$$f_1^g(x, p_T^2) = h_1^{\perp g}(x, p_T^2) + \text{higher twist}$$

# Anatomy of gluon TMDs

$$F(x, \mathbf{b}; \mu, \zeta) = \sum_j \left( C_j^{(F)} \otimes F^j \right) (x, b_*; \mu_b) e^{S(b_*; \mu_b, \mu, \zeta)} e^{S_{\text{NP}}(b)} F_{\text{NP}}(x, b)$$

matching coefficients  
 collinear PDF  
 nonperturbative Sudakov  
 nonperturbative TMD function  
 perturbative Sudakov  
 resummation of  
 $L = \ln \frac{Q^2}{\mu_b^2}$   
 define logarithmic ordering  
 slide adapted from C. Bissolotti

# Anatomy of gluon TMDs

$$F(x, \mathbf{b}; \mu, \zeta) = \sum_j \left( C_j^{(F)} \otimes F^j \right) (x, b_*; \mu_b) e^{S(b_*; \mu_b, \mu, \zeta)} e^{S_{\text{NP}}(b)} F_{\text{NP}}(x, b)$$

matching coefficients  
 collinear PDF  
 nonperturbative Sudakov  
 nonperturbative TMD function

**perturbative** expansion in  $\alpha_s(\mu)$

perturbative Sudakov  
 resummation of

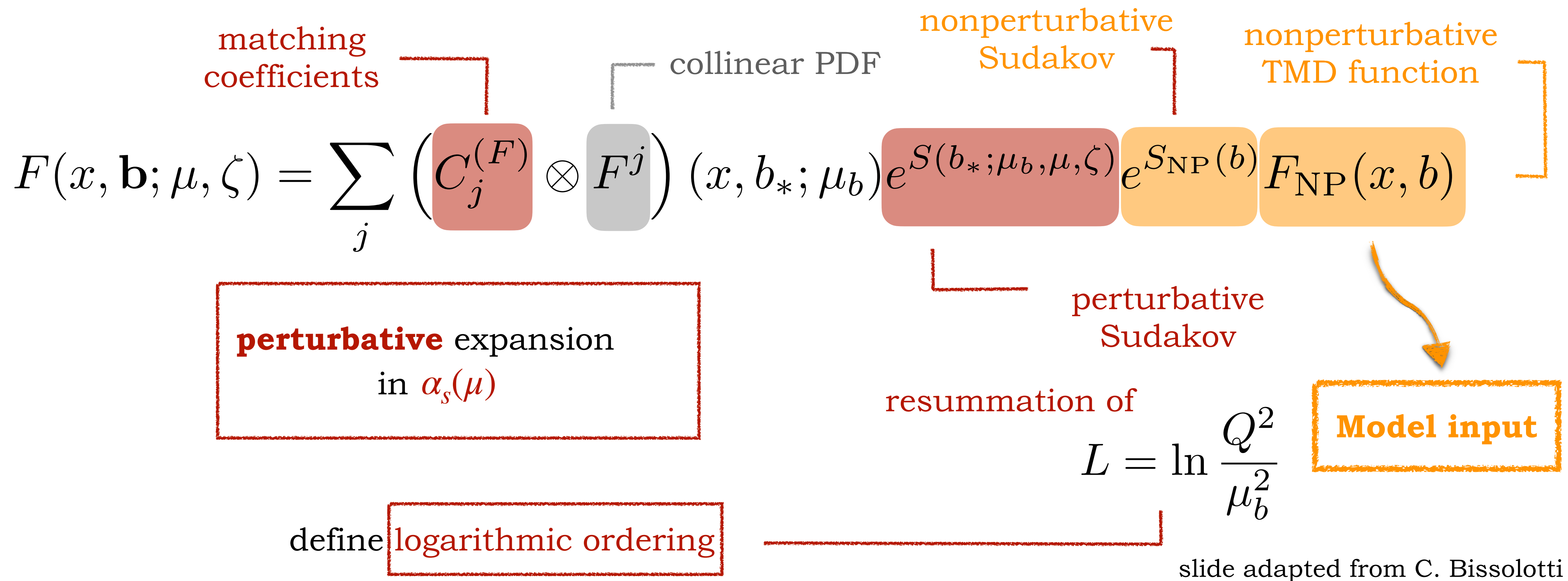
$$L = \ln \frac{Q^2}{\mu_b^2}$$

Model input

define logarithmic ordering

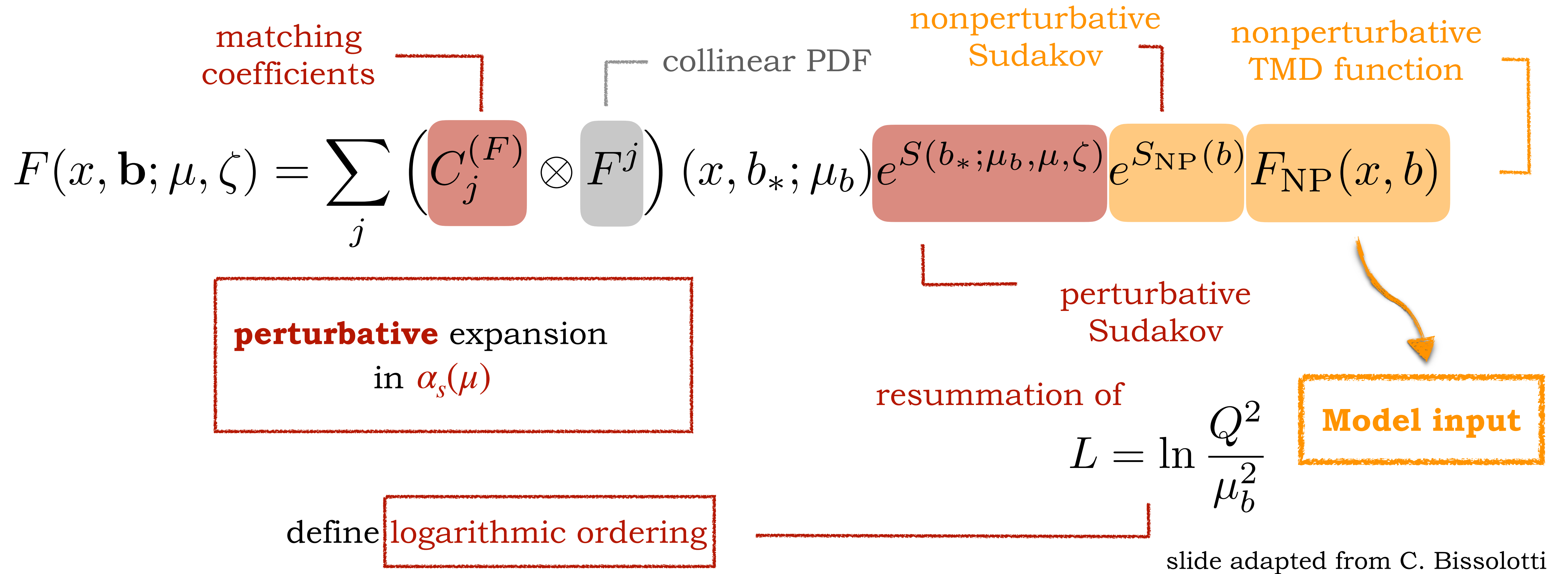
slide adapted from C. Bissolotti

# Anatomy of gluon TMDs



$$f_1(x, \mathbf{b}, \mu, \zeta) \rightarrow C_j^{(f_1)} \otimes f_1^j = [1 + \mathcal{O}(\alpha_s)]_j \otimes f_1^j$$

# Anatomy of gluon TMDs



$$f_1(x, \mathbf{b}, \mu, \zeta) \rightarrow C_j^{(f_1)} \otimes f_1^j = [1 + \mathcal{O}(\alpha_s)]_j \otimes f_1^j$$

$$h_1^\perp(x, \mathbf{b}, \mu, \zeta) \rightarrow C_j^{(h_1^\perp)} \otimes f_1^j = [\mathcal{O}(\alpha_s)]_j \otimes f_1^j$$

Suppression of genuine NP effects! ←

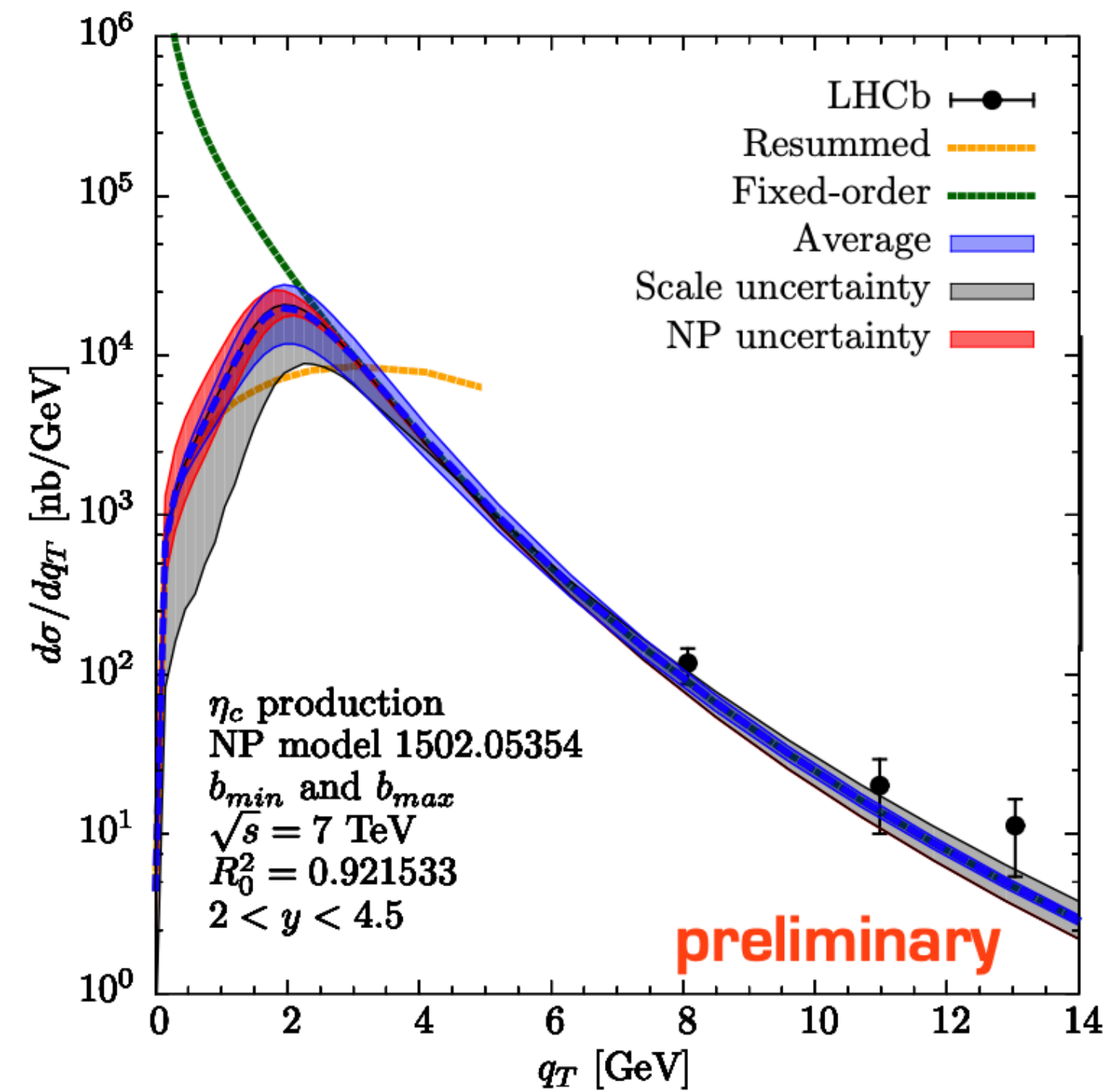
# Gluon TMD phenomenology

The background features a complex, multi-layered illustration of gluon transverse momentum distributions (TMDs). It consists of several overlapping, semi-transparent circular regions. Each region contains a network of yellow wavy lines representing gluons, with various colored spheres (red, blue, green) and arrows indicating interactions and spin. The overall aesthetic is scientific and abstract, with a light blue and white color palette and a subtle grid pattern.



## $\eta_c$ production at LHC

full transverse momentum spectrum:  
low  $q_T$  matched with high  $q_T$  region



**blue band:** uncertainty from matching

**grey band:** scale uncertainty

**red band:** nonpert. uncertainty

$$S_{NP}(\bar{b}_T) = - \left[ \frac{a_1}{2} + \frac{a_2}{2} \ln Q^2 \right] \bar{b}_T^2$$

$a_i = 0.5 \text{ GeV}^2$ , var. 50%, envelope

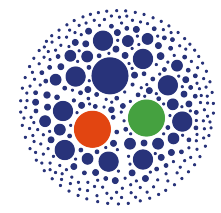
both for unpolarized and  
linearly polarized distributions

**the formalism is in good shape!**  
we need the data at low  $q_T$



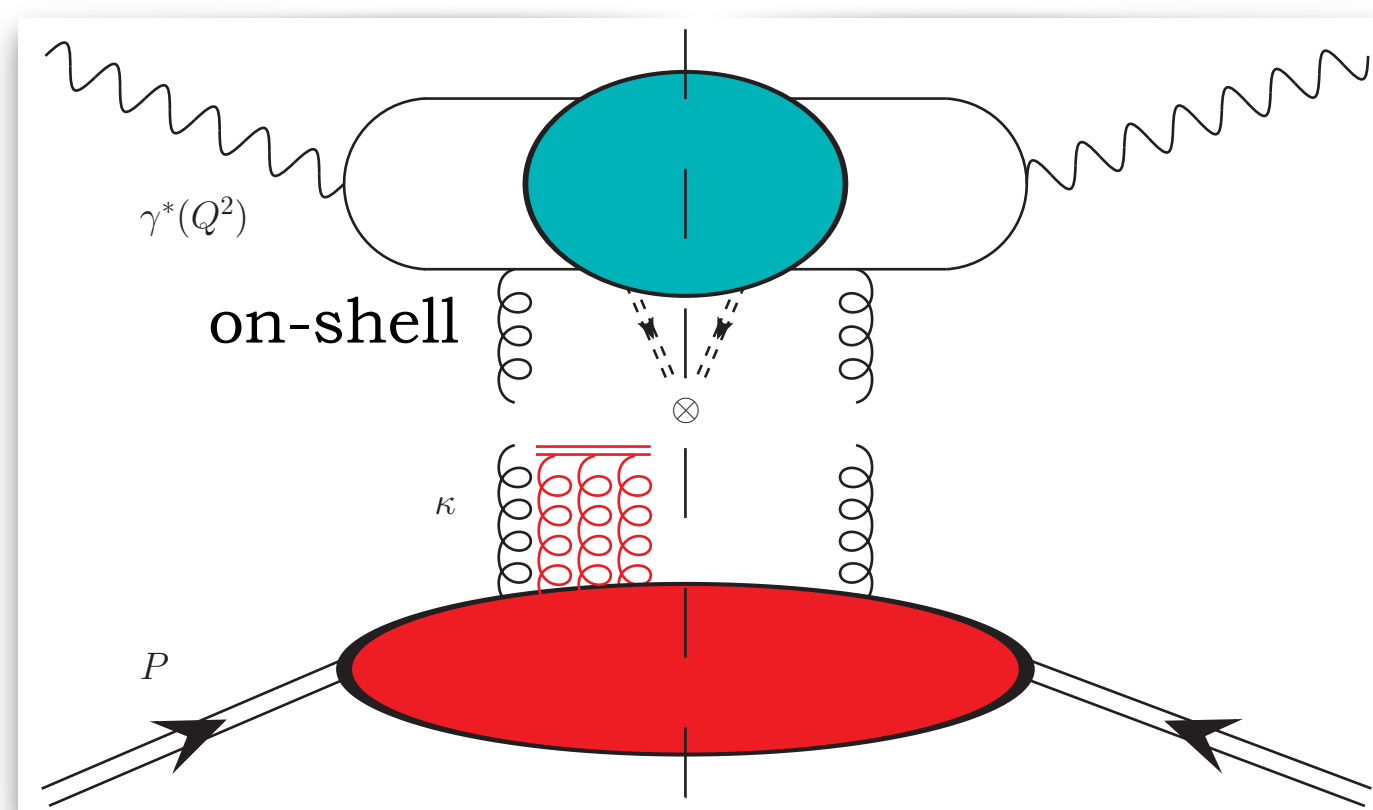
# HEF and the UGD

# TMD versus high-energy factorization



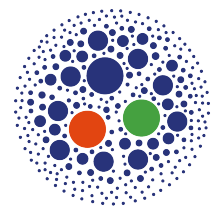
## TMD

- \* Semi-inclusive processes
- \*  $\kappa_T \ll$  hardest scale
- \* Language of **parton correlators**
- \* Diagram: SIDIS onium



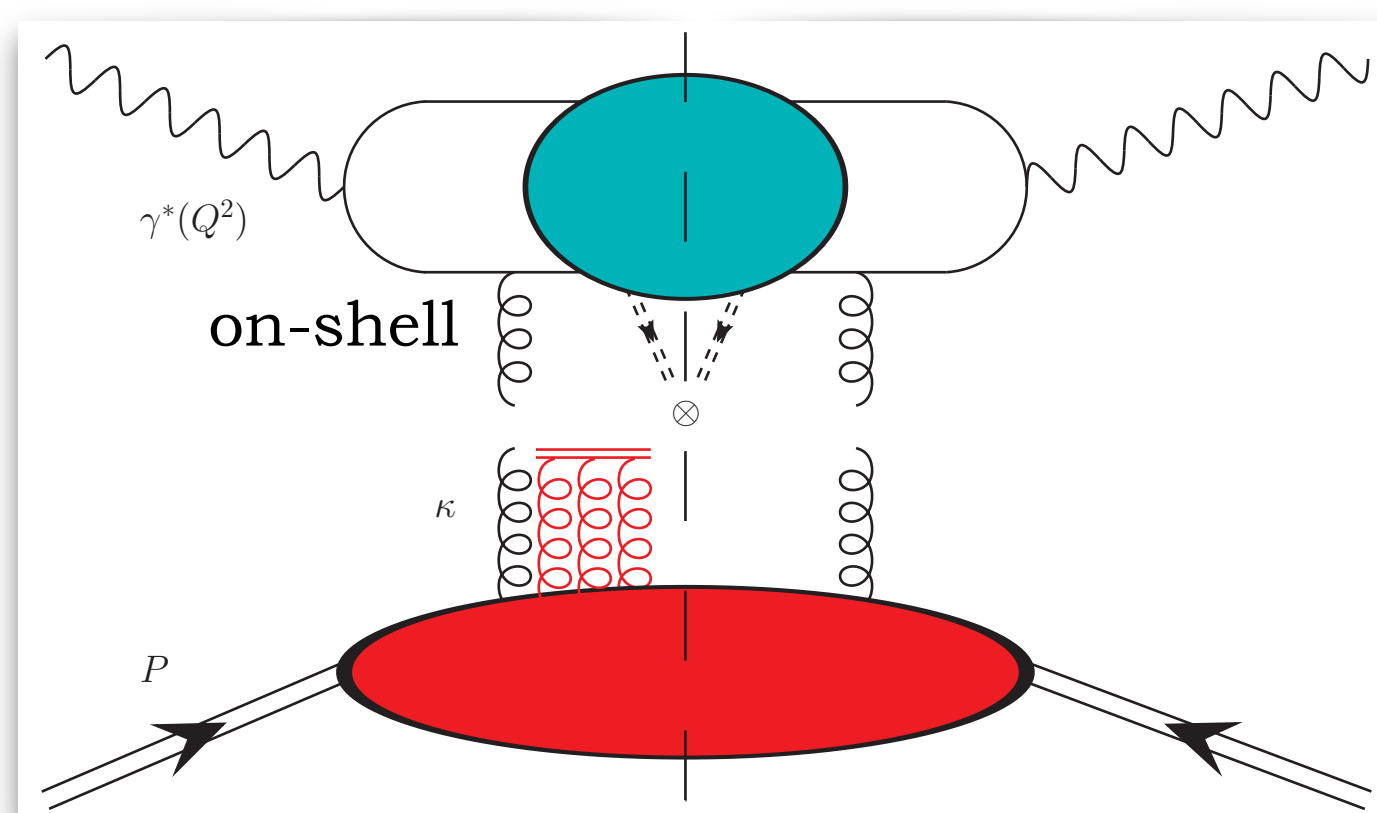
TMD  
PDF

# TMD versus high-energy factorization

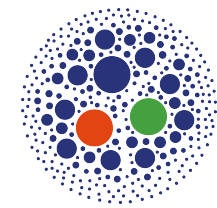


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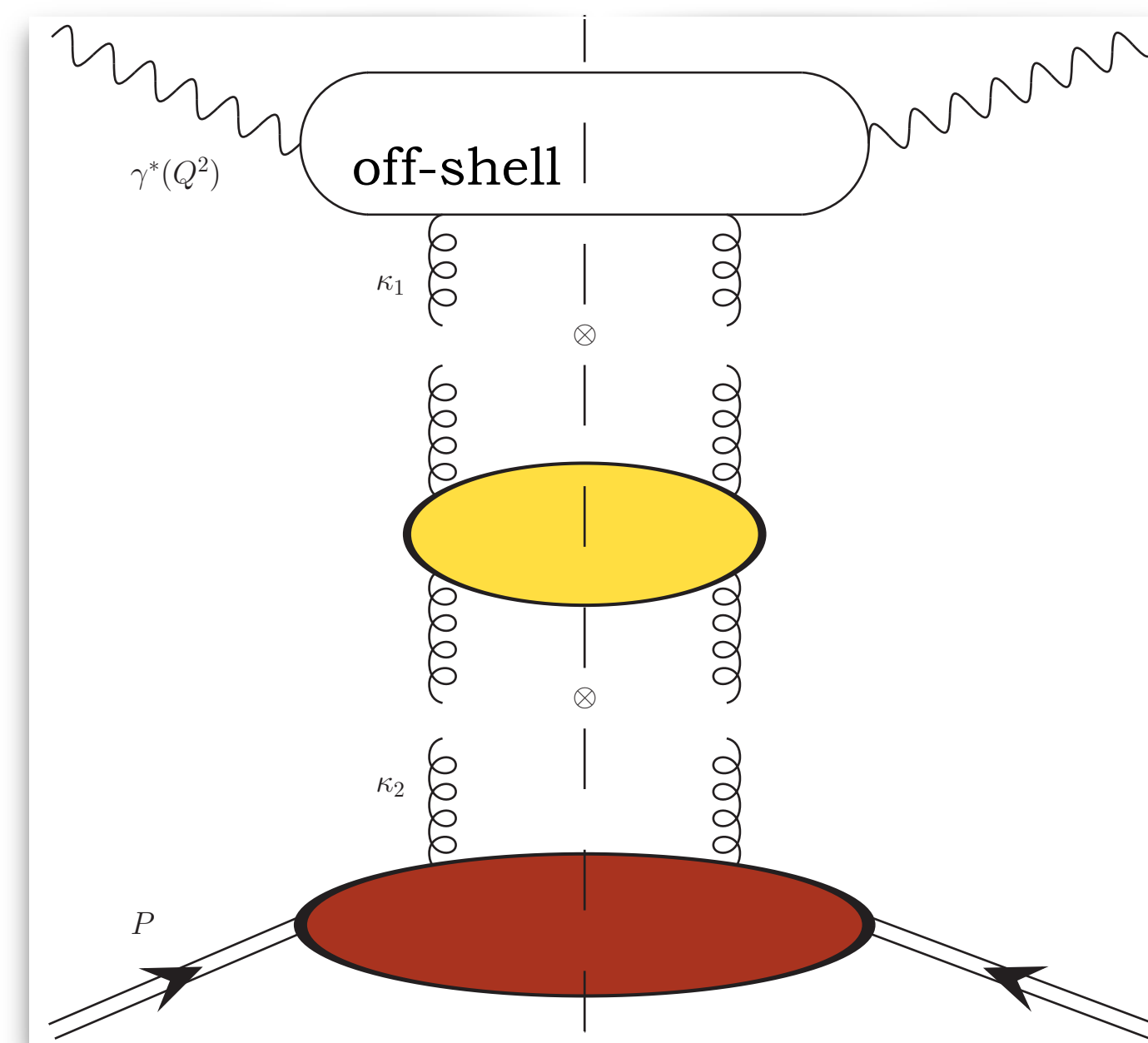


TMD  
PDF



## HEF

- \* Inclusive or exclusive processes (!)
- \* Small  $x$ , large  $\kappa_T$
- \* Language of **Reggeized gluons**
- \* Diagram: DIS



$\Phi \gamma^* \rightarrow \gamma^*$



$\mathcal{G}_{\text{BFKL}}$



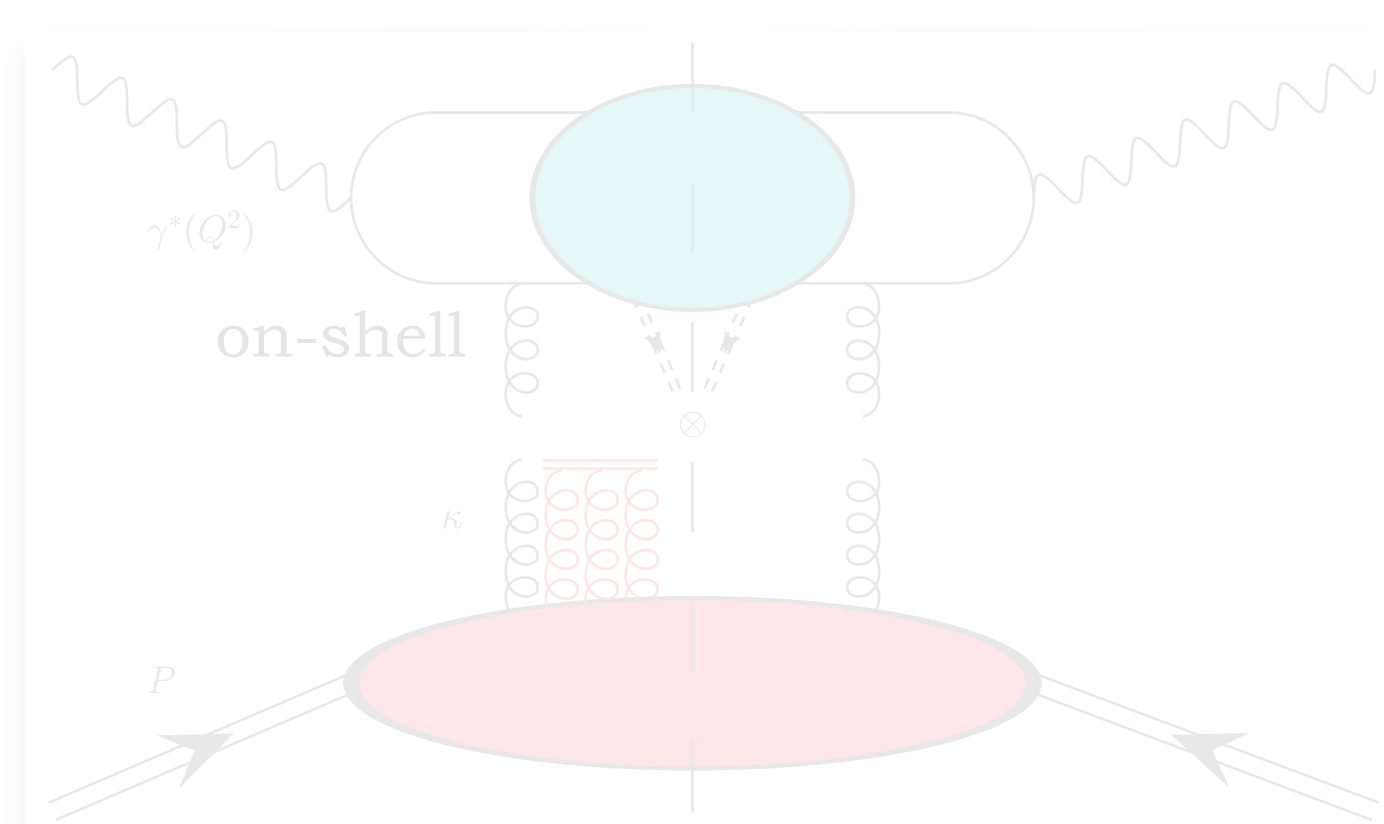
$\Phi^P_{[\text{NP}]}$

# TMD versus high-energy factorization

**TMD**

IR-safe colorless  $\{\Phi^{i \rightarrow 0}\}$   
 (Fadin-Martin theorem)  
 [V.S. Fadin, A.D. Martin (1999)]

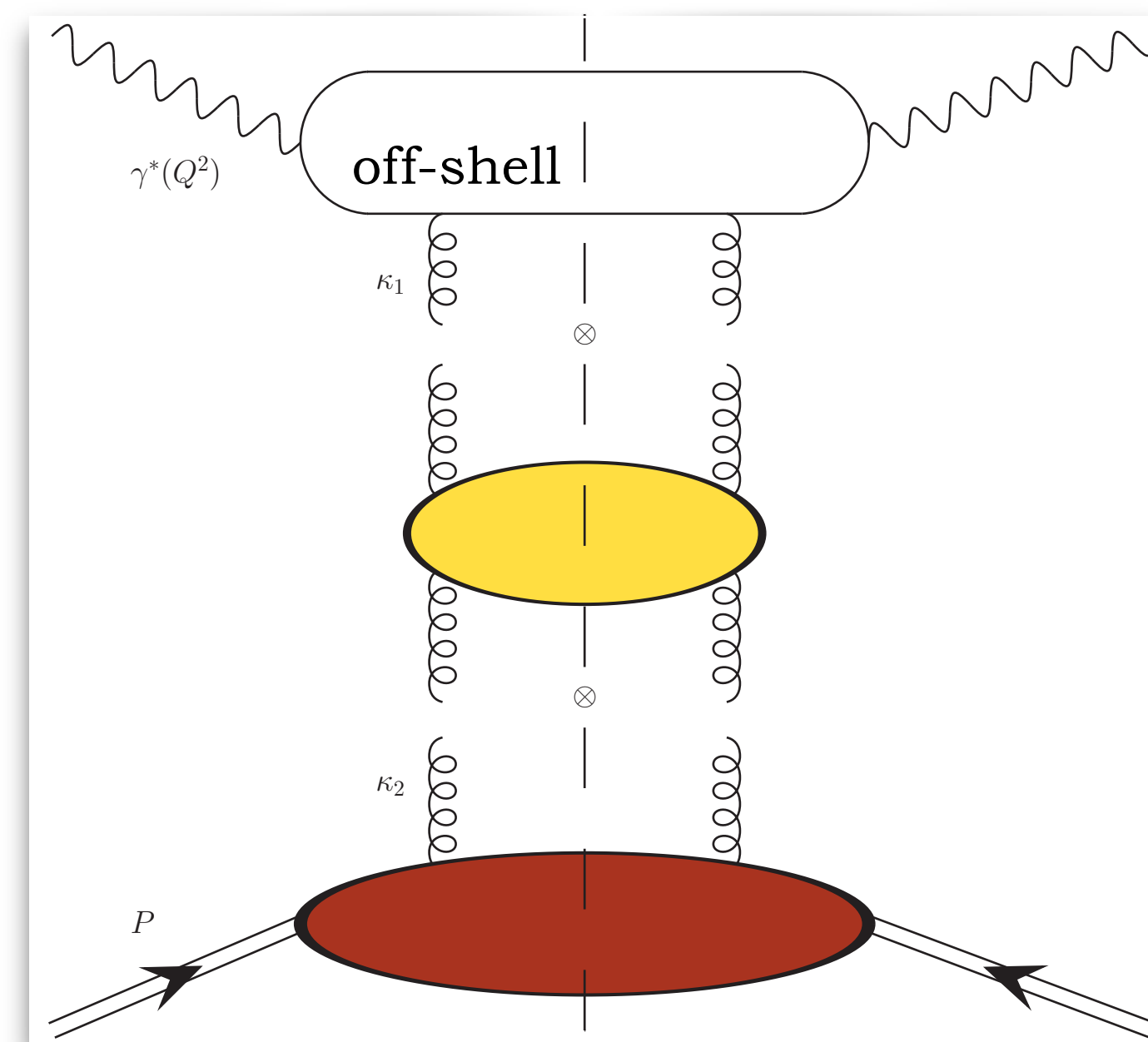
- \* Semi-inclusive processes
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TMD  
PDF

**HEF**

- \* Inclusive or exclusive processes (!)
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$\Phi^{\gamma^* \rightarrow \gamma^*}$

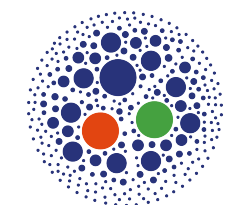
$\otimes$

$\mathcal{G}_{\text{BFKL}}$

$\otimes$

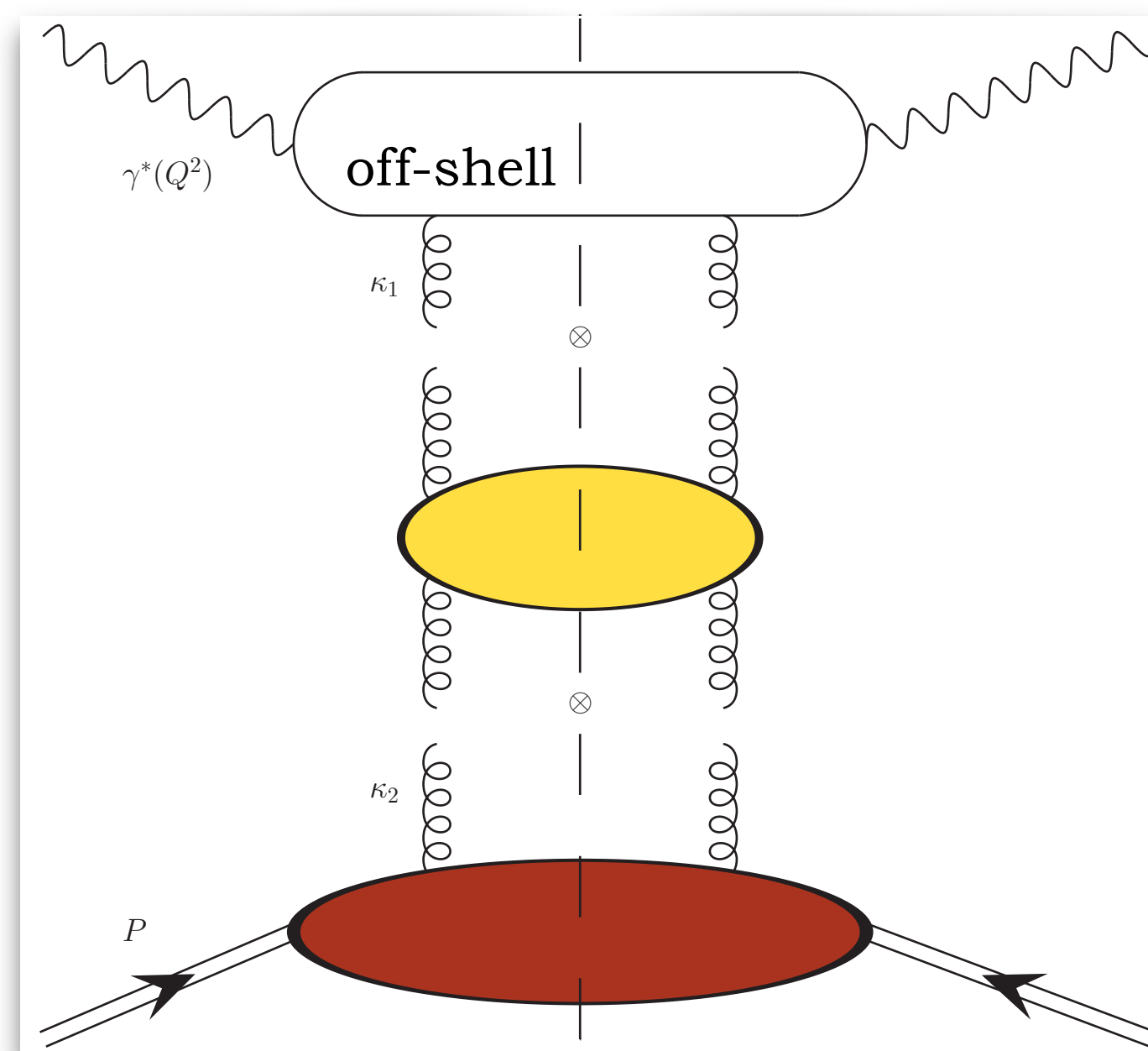
$\Phi^P_{[\text{NP}]}$

# TMD versus high-energy factorization



**HEF**

- \* Inclusive or exclusive processes (!)
- \* Small  $x$ , large  $\kappa_T$
- \* Language of **Reggeized gluons**
- \* Diagram: DIS



$\Phi^{\gamma^* \rightarrow \gamma^*}$

$\otimes$

$\mathcal{G}_{BFKL}$

$\otimes$

$\Phi^P_{[NP]}$

**IR-safe colorless  $\{\Phi^{i \rightarrow 0}\}$**

(Fadin-Martin theorem)

- \* Semi-inclusive processes  
[\[V.S. Fadin, A.D. Martin \(1999\)\]](#)

- \*  $\kappa_T \ll$  hardest scale

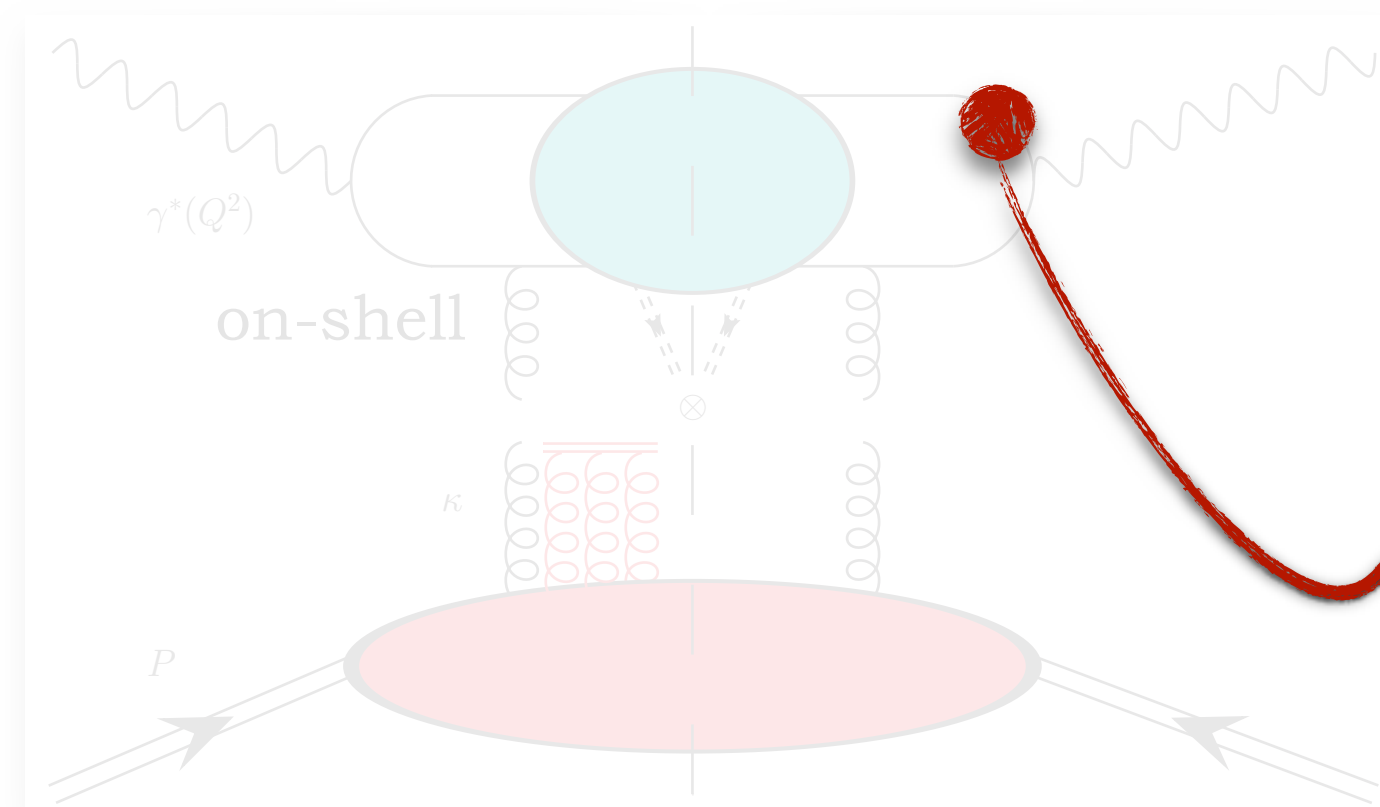
- \* Language of **parton correlators**

- \* Diagram: SIDISonium

**IR diffusion pattern**

(Bartels' cigar)

- \* [\[J. Bartels, H. Lotter \(1993\)\]](#)



$Q^2$

$\updownarrow$

$Q_0^2$

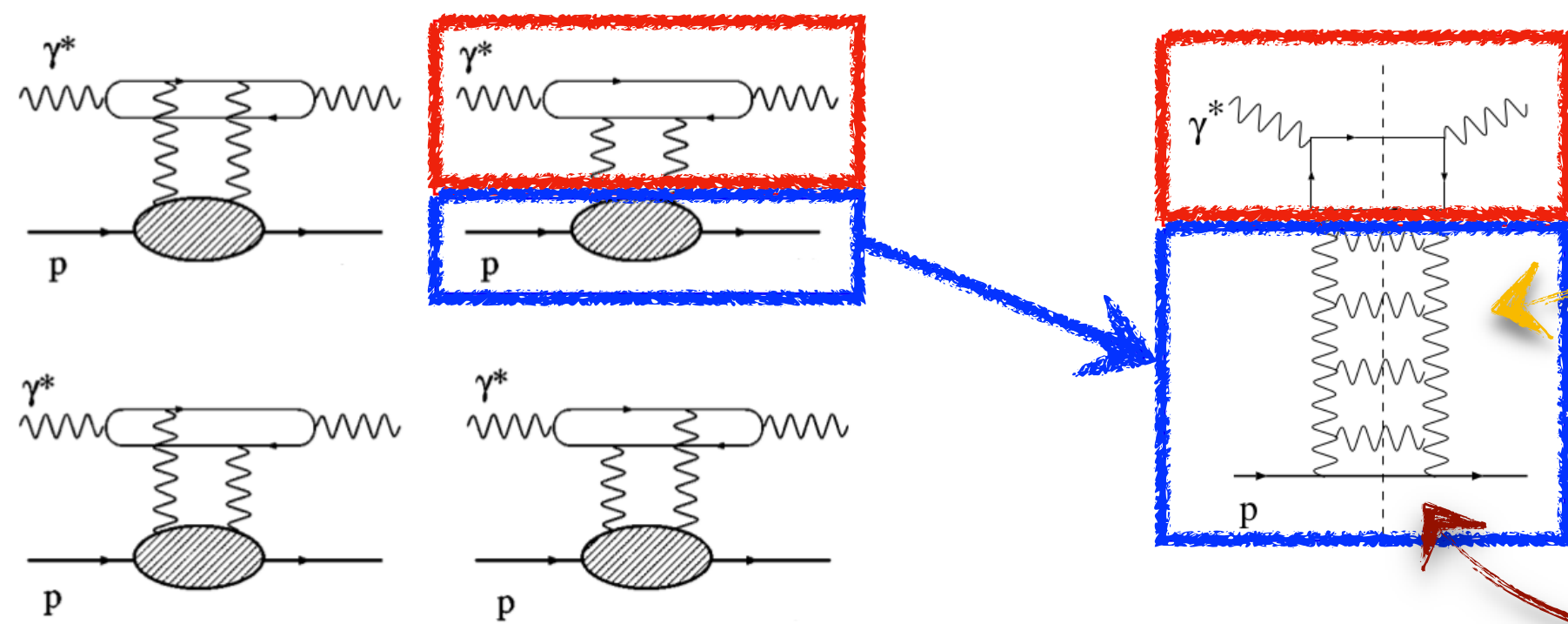
TMD  
PDF

# High-energy factorization and the UGD

- example: **virtual photoabsorption** in **high-energy factorization**

$$\sigma_{\text{tot}}(\gamma^* p \rightarrow X) \propto \text{Im}_s \{ \mathcal{A}(\gamma^* p \rightarrow \gamma^* p) \} \equiv \Phi_{\gamma^* \rightarrow \gamma^*} \circledast \mathcal{F}(x, \kappa^2)$$

- ◇  $\mathcal{F}(x, \kappa^2)$  is the **unintegrated gluon distribution (UGD)** in the proton

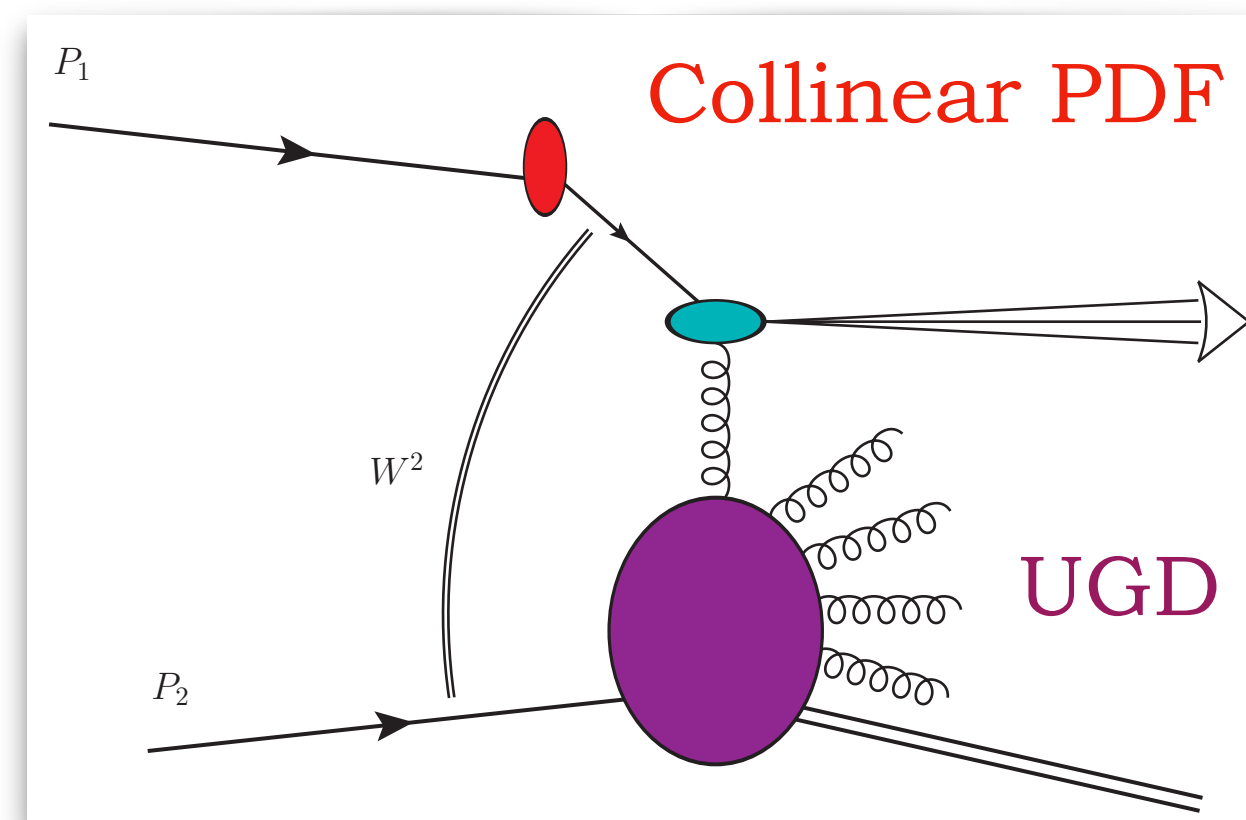


- ▶ Small- $x$  limit: **UGD** = [ **BFKL gluon ladder** ]  $\circledast$  [ **proton impact factor** ]
  - ◇ Takes into account the **resummation** of **high-energy logs**
  - ◇ Describes the **coupling** of the gluon Green's function to the **proton**
- ▶ Proton impact factor is non-perturbative  $\implies$  UGD needs to be modeled!

# Hybrid or pure factorization?

## Forward emissions

- \* *Asymmetric* config.  $\leftrightarrow$  fast parton + small- $x$  gluon
- \* Hybrid **high-energy** / **collinear** factorization



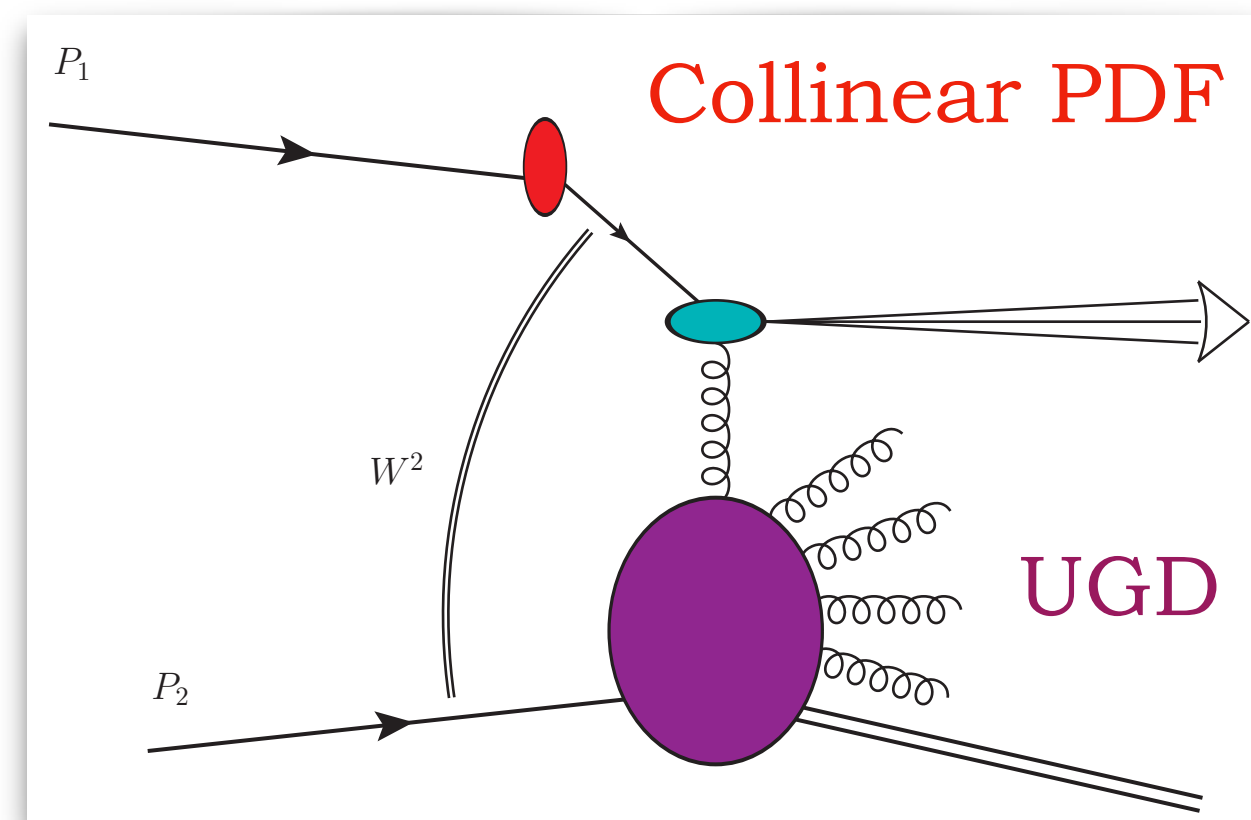
- \* *Distinctive signals* of small- $x$  dynamics **expected**
- \* Phenomenology:  
*forward jet, Drell-Yan, Higgs or vector meson*



# Hybrid or pure factorization?

## Forward emissions

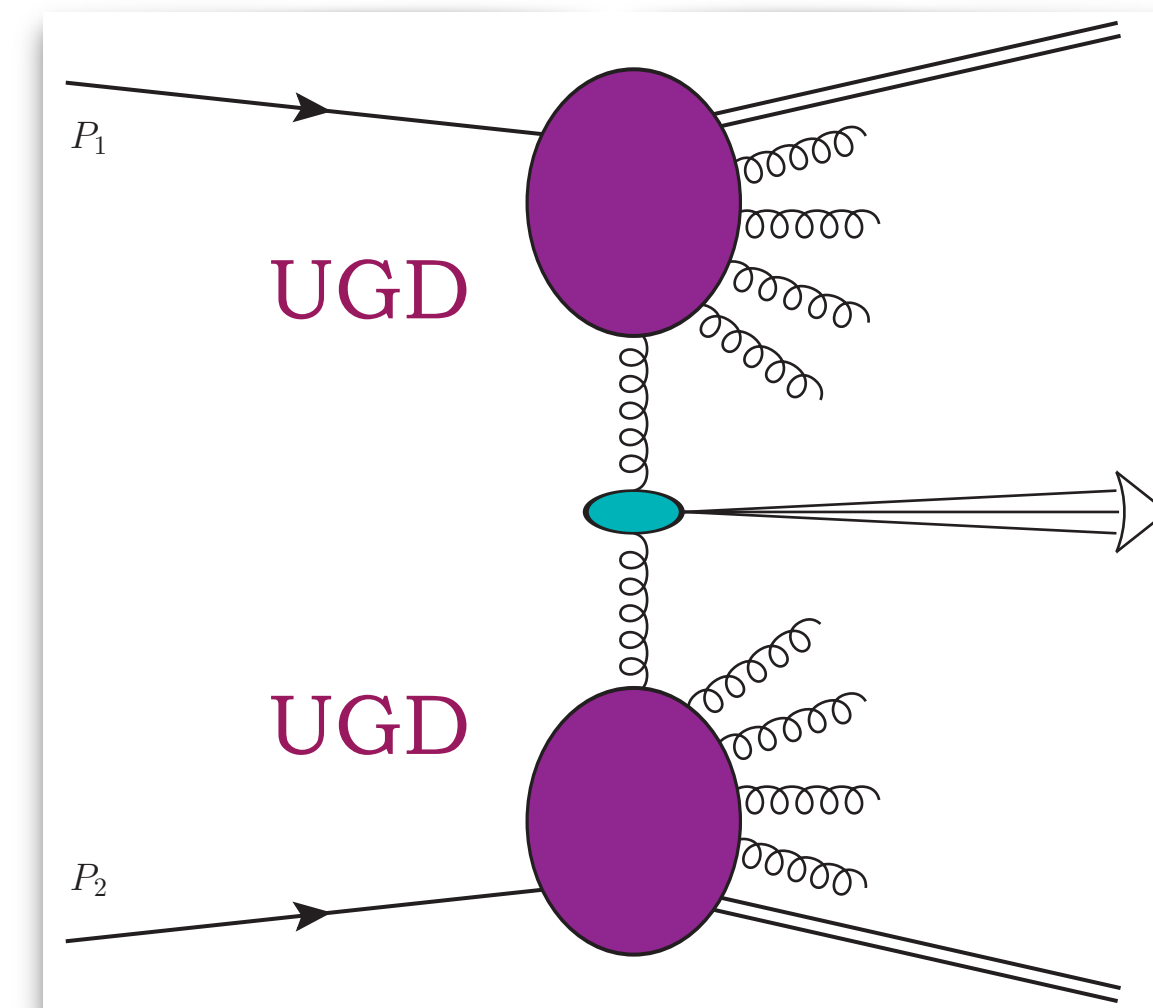
- \* *Asymmetric* config.  $\leftrightarrow$  fast parton + small- $x$  gluon
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- \* *Distinctive signals* of small- $x$  dynamics **expected**
- \* Phenomenology:  
*forward jet, Drell-Yan, Higgs or vector meson*

## Central emissions

- \* *Gluon induced*  $\leftrightarrow$  small- $x$  gluons
- \* Pure **high-energy** factorization

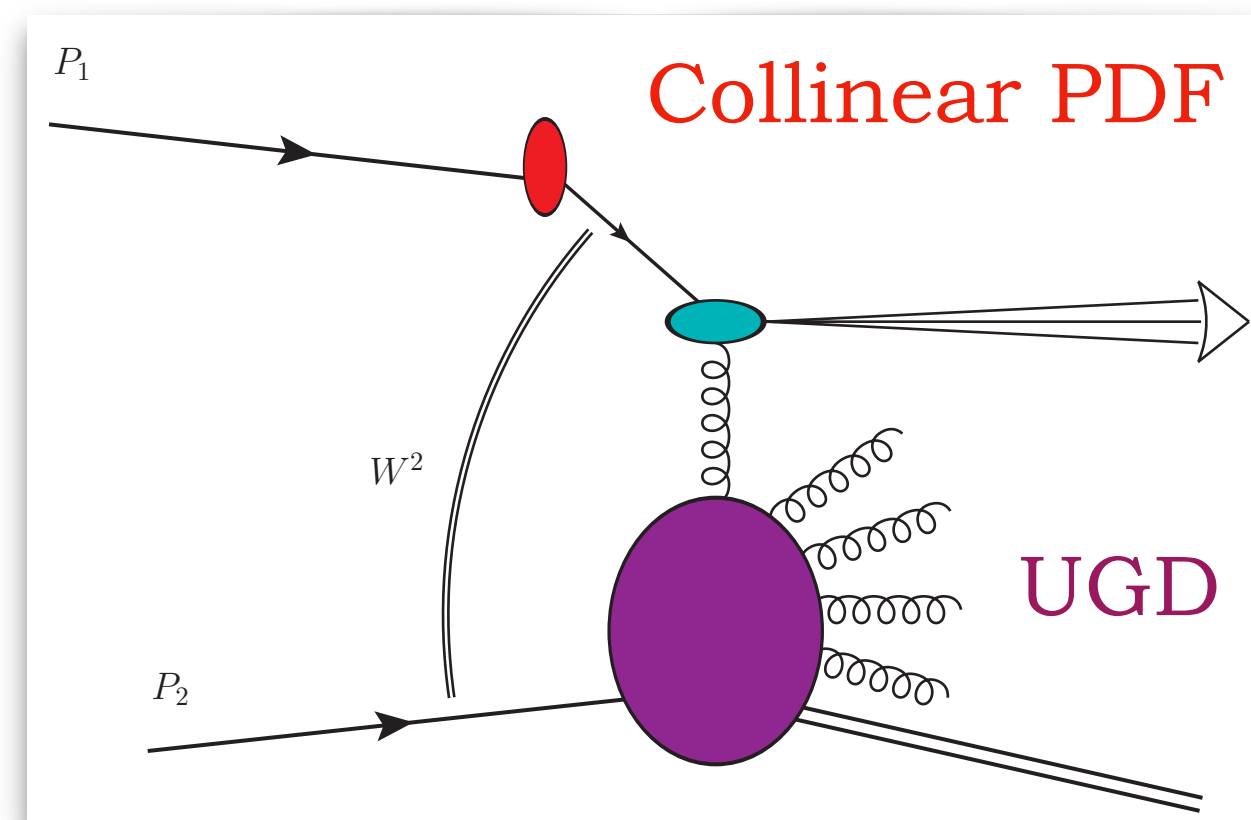


- \* Small- $x$  dynamics to **enhance** f.o. description
- \* Phenomenology:  
*central jet, Higgs or vector meson*

# Hybrid or pure factorization?

## Forward emissions

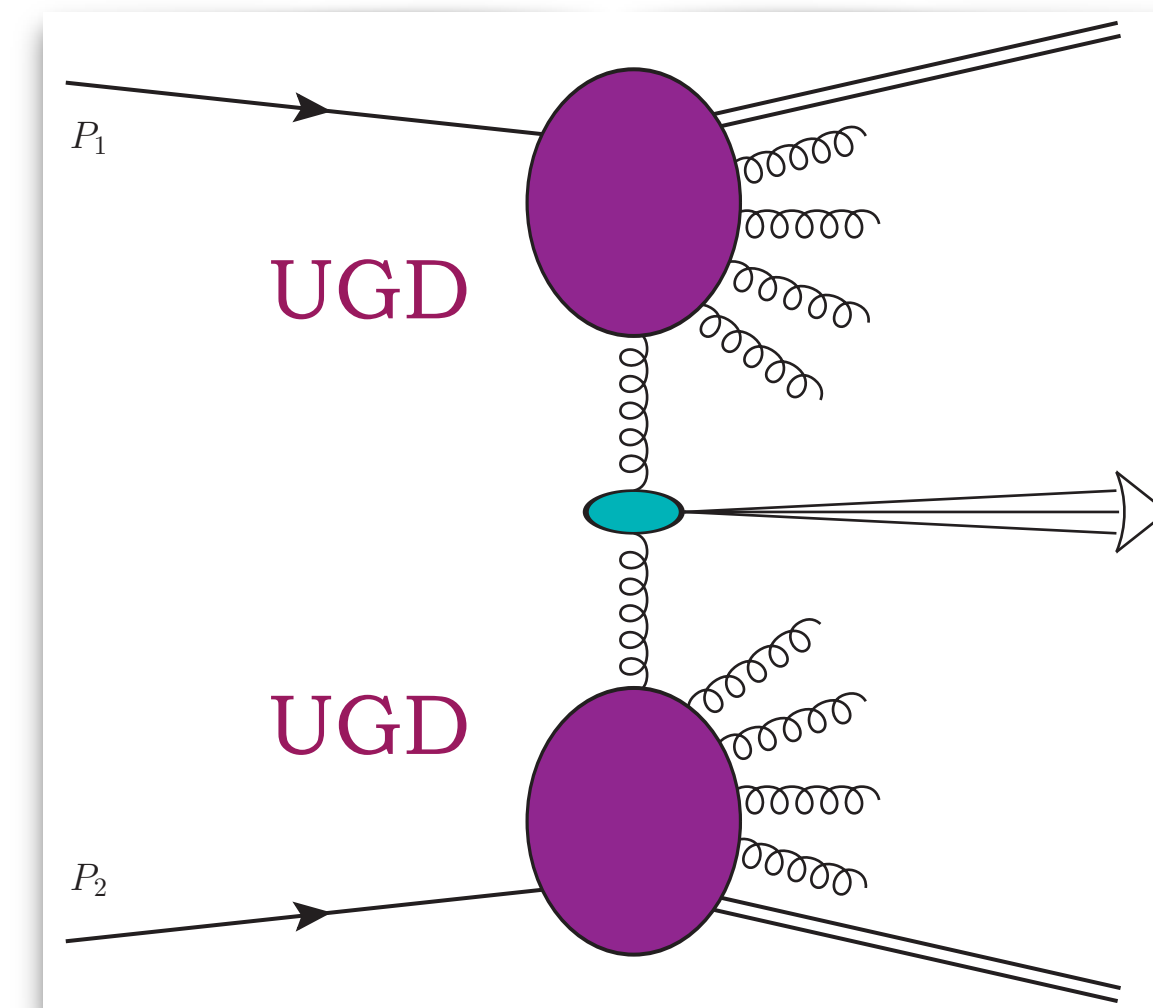
- \* *Asymmetric* config.  $\leftrightarrow$  fast parton + small- $x$  gluon
- \* Hybrid **high-energy** / **collinear** factorization



- \* *Distinctive signals* of small- $x$  dynamics **expected**
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## Central emissions

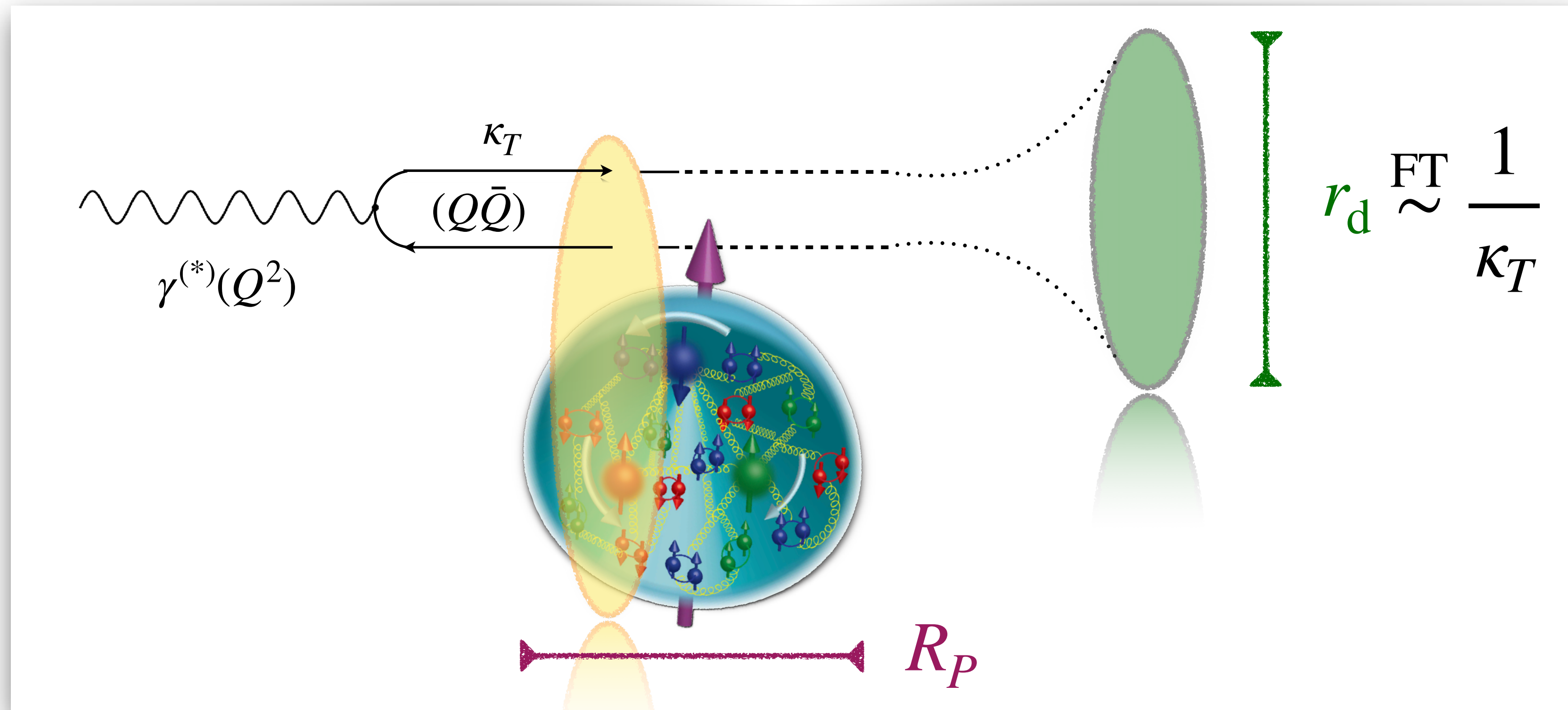
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- \* Pure **high-energy** factorization



- \* Small- $x$  dynamics to **enhance** f.o. description
- \* Phenomenology:  
*central* jet, Higgs or vector meson

Table complemented by *exclusive* counterparts and *lepto-hadronic* channels

# Diffractive $\gamma^*P$ scatterings and color dipoles



$$W_{\mu\nu} \propto \text{Im} \left\{ i \int d^4x e^{iq \cdot x} \langle P | T [J_\mu(x) J_\nu(0)] | P \rangle \right\}$$

- \* Small- $x \Rightarrow$  Ioffe time  $\gg R_P$
- \* At least one  $J_\mu$  outside proton...
- \* ...color dipole picture!

The background of the slide is a light blue and green gradient with several overlapping, semi-transparent Feynman diagrams. These diagrams illustrate particle interactions, featuring various particles represented by colored spheres (red, blue, green) and wavy lines (yellow, blue). Some diagrams include arrows indicating the direction of particle flow. The overall aesthetic is scientific and modern.

# Exclusive forward $\rho$ meson leptonproduction

## Exclusive light VM: $\rho^0, \omega, \phi$

\* *Small-size* dipoles  $\Rightarrow$  large  $\kappa_T$

\* **Collinear** description: twist-2/-3 LVM NP **DAs**

$$\Phi^{\gamma^* \rightarrow \rho} \propto \int_0^1 dz T_H^{\gamma^* \rightarrow \rho}(z, \kappa_T, Q, \mu_R, \mu_F) \phi^{\lambda_\rho}(z, \mu_F)$$

\* Significance of small  $\kappa_T$  under investigation...

\* HERA indication: no large- $r_d$  dynamics

\* **LVMs as tools**: discrimination among UGD models

\* **LVMs as tools**: UGD extraction  $\Leftarrow$  HERA + EIC fits

# Single forward emissions

## Exclusive light VM: $\rho^0, \omega, \phi$

- \* *Small-size* dipoles  $\Rightarrow$  large  $\kappa_T$
- \* **Collinear** description: twist-2/-3 LVM NP **DAs**
- $$\Phi^{\gamma^* \rightarrow \rho} \propto \int_0^1 dz T_H^{\gamma^* \rightarrow \rho}(z, \kappa_T, Q, \mu_R, \mu_F) \phi^{\lambda_\rho}(z, \mu_F)$$
- \* Significance of small  $\kappa_T$  under investigation...
- \* HERA indication: no large- $r_d$  dynamics
- \* **LVMs as tools**: discrimination among UGD models
- \* **LVMs as tools**: UGD extraction  $\Leftarrow$  HERA + EIC fits

## Quarkonia

- \* Size of dipoles  $\Rightarrow$  wide range of  $\kappa_T$
- \* Description: **NRQCD** (combined with LFWFs)
- $$[\text{LFWF} \otimes \mathcal{A}_{\text{dip.}}] \xleftrightarrow{\text{dilute}} [\Phi^{\gamma^* \rightarrow J/\Psi} \otimes \text{UGD}]$$
- \* Validity of *small-size* dipoles questionable...
- \* NRQCD: large- $r_d$  dynamics for  $\Psi(2s)$  ( $\Upsilon(2s)$  ?)
- [\[K. Suzuki et al. \(2000\)\]](#); [\[J. Cepila et al. \(2019\)\]](#); [\[M. Hentschinski et al. \(2020\)\]](#)
- \* **Onia as tools**: scan of TMD/HEF intersection range