TMD Fragmentations from thrust dependent observables

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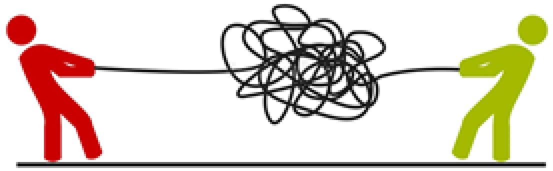
10th workshop of the APS Topical Group on Hadronic Physics

Extraction of TMD Fragmentation Functions

Access to the 3D-dynamics of confinement

Standard TMD factorization

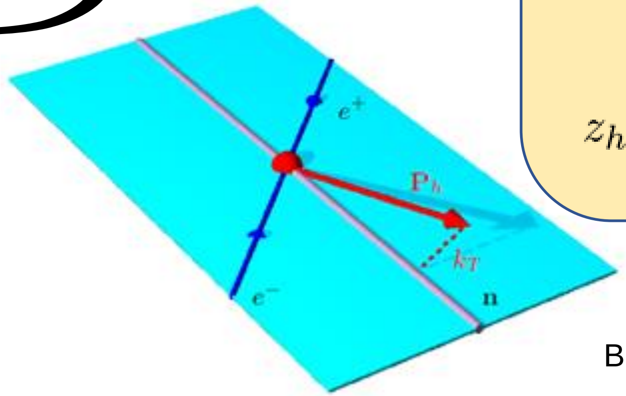
- SIDIS $d\sigma \sim H_{\text{SIDIS}}\, F\, D$
- DIA $d\sigma \sim H_{\text{DIA}}\, D_1\, D_2$



Always two TMDs that have to be extracted *simultaneously*

A process with a single hadron may offer a cleaner access to TMD FFs

$$d\sigma \overset{??}{\propto} D$$



Single-Inclusive Hadroproduction (SIA)

$$e^+e^- \rightarrow h\,X$$

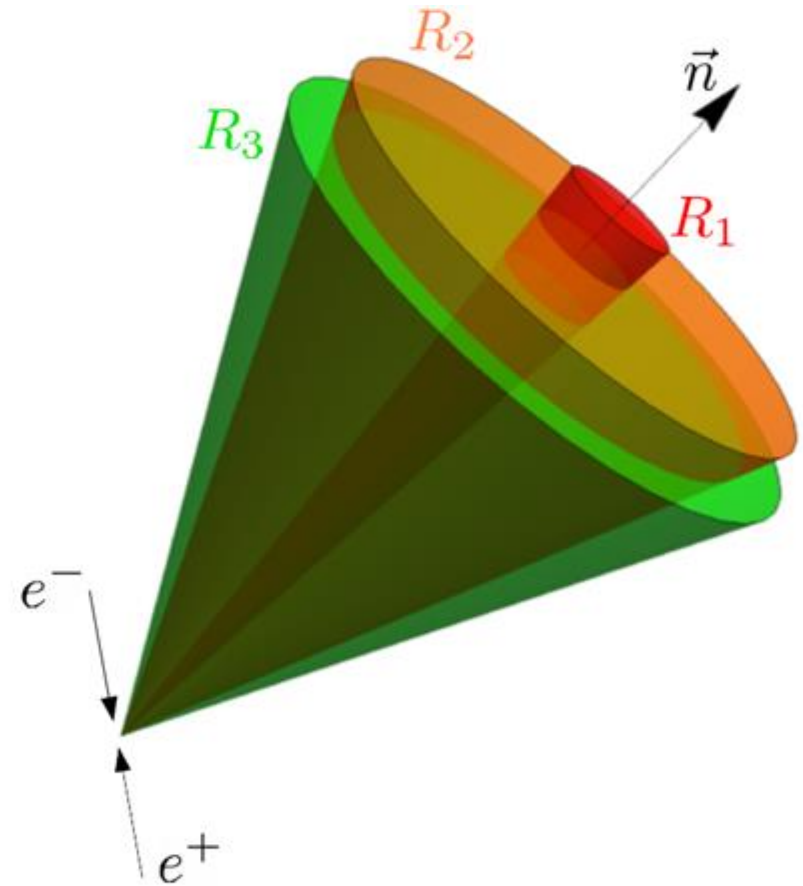
The transverse momentum of the detected hadron is measured w.r.t. the thrust axis

$$z_h = \frac{E}{Q/2}, \quad T = \frac{\sum_i |\vec{P}_{(\text{c.m.}),\,i} \cdot \hat{n}|}{\sum_i |\vec{P}_{(\text{c.m.}),\,i}|}, \quad P_T \text{ w.r.t } \vec{n}$$

BELLE collab., *Phys.Rev.D 99 (2019)*

Three kinematic regions

Depending on where the hadron is located within the jet the underlying kinematics can be remarkably different, resulting in different factorization theorems



The hadron is detected very close to the **axis** of the jet.

- ❑ Extremely small P_T
- ❑ Soft radiation affects significantly the transverse deflection of the hadron from the thrust axis

The hadron is detected in the **central region** of the jet.

- ❑ Most common scenario
- ❑ Majority of experimental data expected to fall into this case

The hadron is detected near the **boundary** of the jet.

- ❑ Moderately small
- ❑ The hadron transverse momentum affects the topology of the final state directly

The three regions are uniquely determined by the specific role of **soft** and **soft-collinear** radiation:

$u \to 1-T$

$b_T \to P_T/z$

	soft	soft-collinear	collinear
R_1	TMD-relevant	TMD-relevant	TMD-relevant
R_2	TMD-irrelevant	TMD-relevant	TMD-relevant
R_3	TMD-irrelevant	TMD-irrelevant	TMD-relevant

$$d\sigma \sim H\, J(u)\, \Sigma(u, b_T)\, D(z, b_T)$$

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TMD FF + NP soft

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TMD FF

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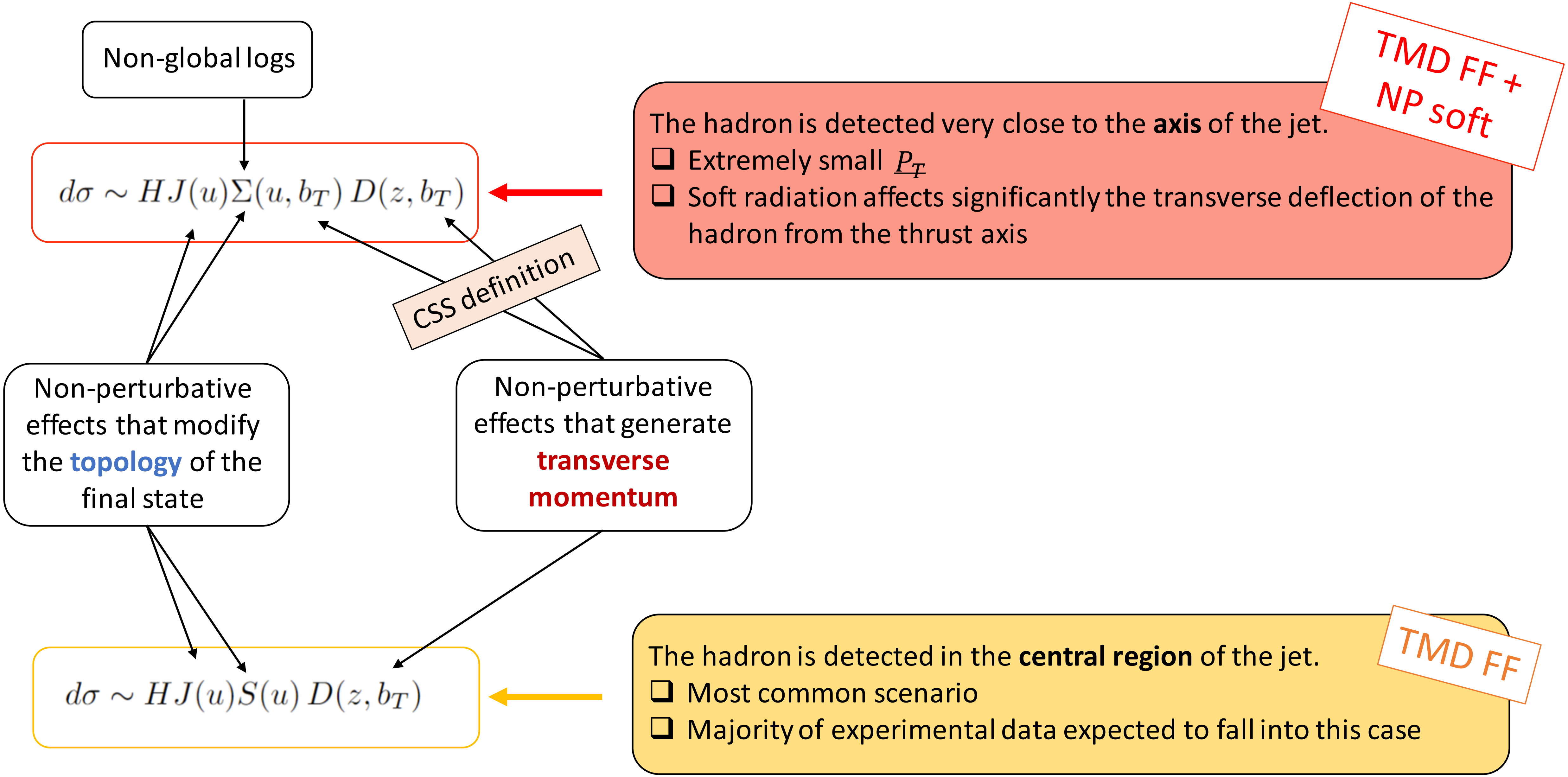
Generalized FJF


Non-global logs
$d\sigma \sim H J(u) \Sigma(u, b_T)\, D(z, b_T)$
CSS definition
Non-perturbative effects that modify the **topology** of the final state
Non-perturbative effects that generate **transverse momentum**
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TMD FF

Non-global logs

$$d\sigma \sim H J(u)\Sigma'(u, b_T)\, D^{\mathrm{CSS}}(z, b_T)$$

TMD FF + NP soft

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- ❑ Extremely small P_T
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Non-perturbative effects that modify the **topology** of the final state

Non-perturbative effects that generate **transverse momentum**

NOT THE SAME TMD FRAGMENTATION FUNCTION!

Is **universality** in danger?

$$d\sigma \sim H J(u) S(u)\, D(z, b_T)$$

TMD FF

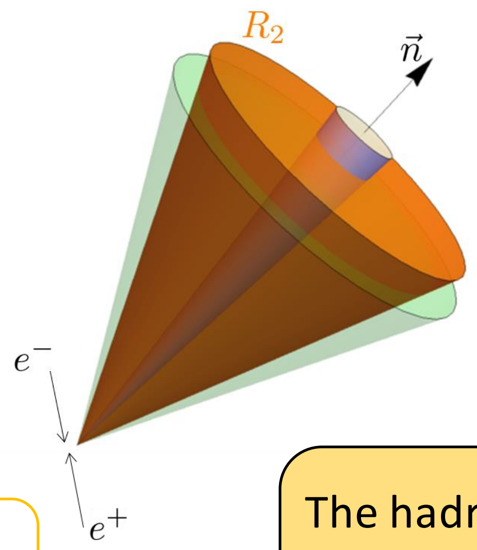
The hadron is detected in the **central region** of the jet.

- ❑ Most common scenario
- ❑ Majority of experimental data expected to fall into this case

Standard TMD factorization can be extended beyond the standard processes (DY, SIDIS, DIA) at the cost of including a new, independent, non-perturbative function (the **soft model**).

$$D^{\mathrm{CSS}}(z, b_T) = D(z, b_T)\,\sqrt{M_S(b_T)}$$

→ Universality is saved!



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TMD FF

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Rapidity divergences in the central region

$$\frac{\partial}{\partial y_1} \ldots \mathcal{S}(\tau, y_1, \ldots)\, D(z, b_T, y_1) \neq 0$$

SIA^{thr} has a **double nature**:
- Thrust dependent observable
- TMD observable

The thrust τ *naturally* regularizes the rapidity divergences.
The 2-jet limit $\tau \to 0$ corresponds to removing the regulator and to exposing the rapidity divergences in fixed order calculations.

So the final result depends on a regulator?
Yes, but...

1) The thrust is *measured*.
2) When the regulator is removed the (factorized) cross section vanishes, as showed by resummation.

The rapidity cut-offs $y_{1,2}$ *artificially* regularize the rapidity divergences.
The limits $y_{1,2} \to \pm\infty$ correspond to removing the regulator and to exposing the rapidity divergences in fixed order calculations.

So the final result depends on a regulator?
Yes (in principle), but…

1) The rapidity cut-offs are just mathematical tools.
2) In standard TMD factorization they cancel among themselves before the limit $y_{1,2} \to \pm\infty$ is taken and the final cross section is rapidity cut-offs independent.

Both kind of regularization coexists in SIA^{thr}.
Therefore, it should not be surprising that the two mechanisms intertwine themselves and hence that thrust and rapidity regulators are strictly related.

This signals a *redundancy* of regulators: one can be expressed in terms of the other.
In particular, the rapidity cut-off y_1 should be a function of thrust, such that when it is removed, also τ is removed. In other words:

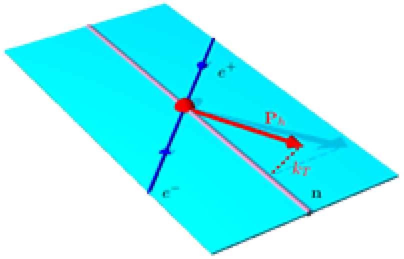
$$\tau \to 0 \Longleftrightarrow y_1 \to +\infty$$

Peculiar and very unique feature of the central region!

Naively from kinematics…

$$y_h \geq -\log\sqrt{\tau} \Rightarrow y_1 \propto -\log\sqrt{\tau}$$

Rapidity of detected hadron



…but also formally:
The double counting due to the overlap between soft and collinear (forward) radiation is cancelled only if the rapidity cut-off is fixed to a function of thrust and transverse momentum

SOFT-COLLINEAR

COLLINEAR → $k_T \lesssim c_1/b_T$

SOFT → $k_T \lesssim Q\, e^{y_1}/u_E$

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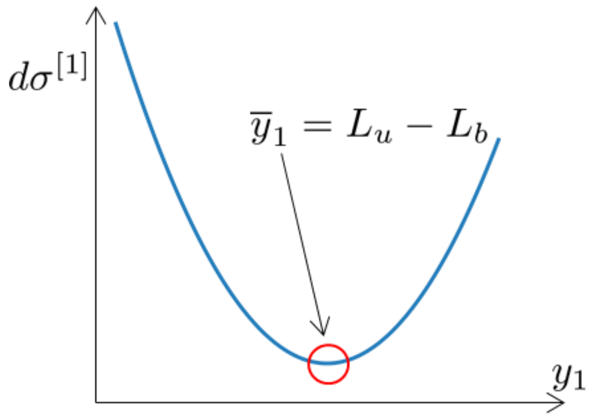
$$u_E = u\, e^{\gamma_E};\ c_1 = 2e^{-\gamma_E}$$
$$L_u = \log u_E$$
$$L_b = \log\left(b_T Q/c_1\right)$$



$$y_1 = L_u - L_b$$

This is also the **minimum** of the factorized cross section as a function of y_1 ↔ It is the (unique) solution of the Collins-Soper evolution equation!

$$\frac{\partial}{\partial y_1} d\sigma_{R_2} = 0$$

$$\overline{y}_1 = L_u - L_b^\star \left(1 + \frac{1 - e^{-\frac{2\beta_0}{\gamma_K^{[1]}}\left(g_K - \widetilde{K}^\star\right)}}{\lambda_b^\star}\right)$$

- ❑ Large and positive
- ❑ Consistent with pert. solution: $\overline{y}_1 = L_u - L_b$ as $b_T \to 0$
- ❑ Consistent with kinematics: $\widehat{y}_1 = -\log\sqrt{\tau} + b_T\text{-logs}$

Factorization theorem in the central region

$$d\sigma_{R_2} \sim H\ J(u)\,\frac{\mathcal{S}(u,\overline{y}_1,y_2)}{\mathcal{Y}_L(u,y_2)}\,\widetilde{D}_{h/j}(z,b_T,\overline{y}_1)$$

$$= H\,J\,\frac{\mathcal{S}}{\mathcal{Y}_L}\bigg|_{\text{ref. scale}} \exp\left\{\int_{\mu_J}^{Q}\frac{d\mu'}{\mu'}\gamma_J + \frac{1}{2}\int_{\mu_S}^{Q}\frac{d\mu'}{\mu'}\gamma_S\right\} \times \widetilde{D}_{h/j}(z,b_T)\Big|_{y_1=0}$$

$$\times \exp\left\{\frac{1}{2}\int_{\mu_S}^{\mu_S e^{\overline{y}_1}}\frac{d\mu'}{\mu'}\left[\widehat{g}-\gamma_K\,\log\left(\frac{\mu'}{\mu_S}\right)\right] - \overline{y}_1\ \widetilde{K}\Big|_{\mu_S}\right\}$$

Genuinely thrust. Exponent is *half* of standard thrust distributionin e+e- annihilation

Genuinely TMD. Reference scales as* in standard TMD factorization

Correlation part. It encodes the correlations between the measured variables

The function g_K does not only appear into the TMD FF!

$$\frac{d\sigma_{R_2}}{dz\,dT\,dP_T} \overset{\text{LL}}{=} -\frac{\sigma_B}{1-T}N_C\int\frac{d^2\vec{b}_T}{(2\pi)^2}\ \widetilde{D}^{\text{LL}}_{h/j}(z,b_T)\Big|_{y_1=0}$$

$$\times\exp\left\{-\log(1-T)f_1(\bullet)\right\}\ \gamma(\bullet)$$

$$\bullet = \{-a_S\,\beta_0\,\log(1-T),\,2\,a_S\,\beta_0\,L_b^\star,\,g_K(b_T)\}$$

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$$\times\exp\left\{-\log\left(1-T\right)f_1(\bullet)+f_2(\bullet)-\frac{1}{\log\left(1-T\right)}f_3(\bullet)\right\}\left(\gamma(\bullet)-\frac{1}{\log\left(1-T\right)}\rho(\bullet)\right)$$

$$\bullet=\{-a_S\,\beta_0\ \log\left(1-T\right),2\,a_S\,\beta_0\,L_b^{\star},g_K(b_T)\}$$

Phenomenology $e^+e^- \to \pi^\pm \; X$

BELLE collaboration
Phys.Rev.D 99 (2019) 11, 112006

BELLE data for charged **pions**

- $T = 0.825$: $0.275 \leq z \leq 0.675$, $P_T/z \leq 0.16Q$
- $T = 0.875$: $0.325 \leq z \leq 0.725$, $P_T/z \leq 0.15Q$
- $T = 0.925$: $0.375 \leq z \leq 0.775$, $P_T/z \leq 0.14Q$
- $T = 0.975$: $0.425 \leq z \leq 0.775$, $P_T/z \leq 0.13Q$

In total we select 230 data points.

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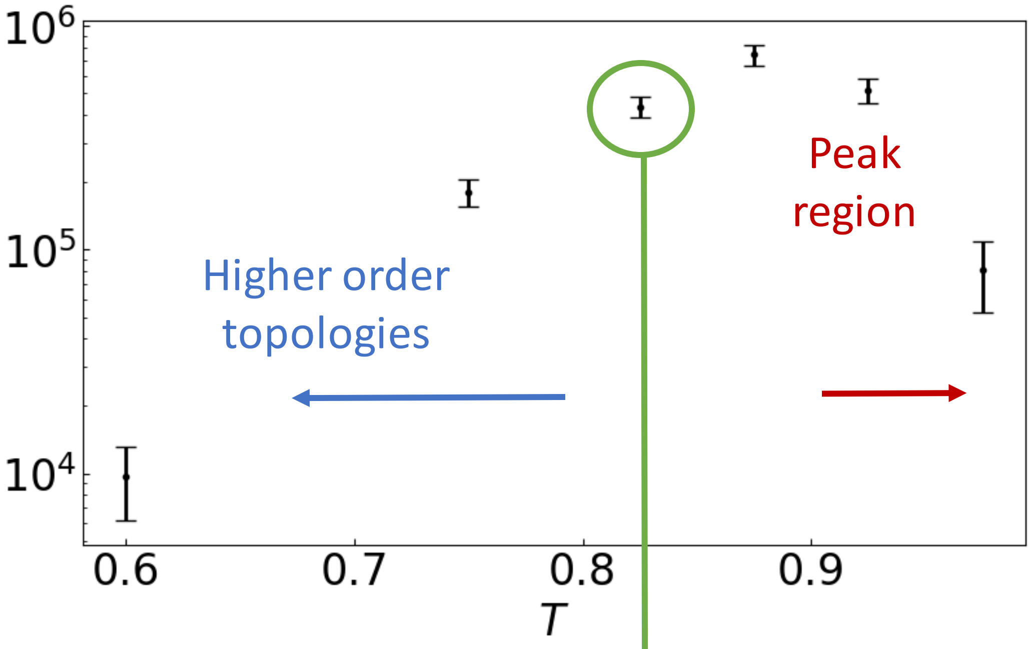


A preliminary fit in the subsample fixes the functional form of the non-perturbative content of the TMD FF

Subsample of 57 data points

"Pure" TMD extraction

Non-perturbative content of the TMD FF

1. g_K function, describing the long-distance behavior of the Collins-Soper kernel.

- Even function of b_T
- Parabolic behavior at small b_T $\quad g_K \sim g_2 b_T^2 + \ldots$ for $b_T \to 0$.
- Constant behavior at large b_T $\quad g_K \to g_0$ for $b_T \to \infty$.

$$g_k = g_0 \tanh\left(\beta^2 \frac{b_T^2}{b_{MAX}^2}\right)$$

2. M_D model for the (unpolarized) TMD FF, describing its characteristic long-distance behavior.

- Even function of b_T
- Gaussian behavior at small b_T $\quad M_D \sim e^{-c b_T^2} \times \ldots$ for $b_T \to 0$.
- Exponential decay at large b_T $\quad M_D \sim e^{-d b_T} \times \ldots$ for $b_T \to \infty$.

$$M_D\left(z_h, P_T, M, p\right) = \frac{\Gamma(p)}{\pi \Gamma(p-1)} M^{2(p-1)} \left(M^2 + \frac{P_T^2}{z_h^2}\right)^{-p}$$

Where $p = p(R, W)$ and $M = M(R, W)$

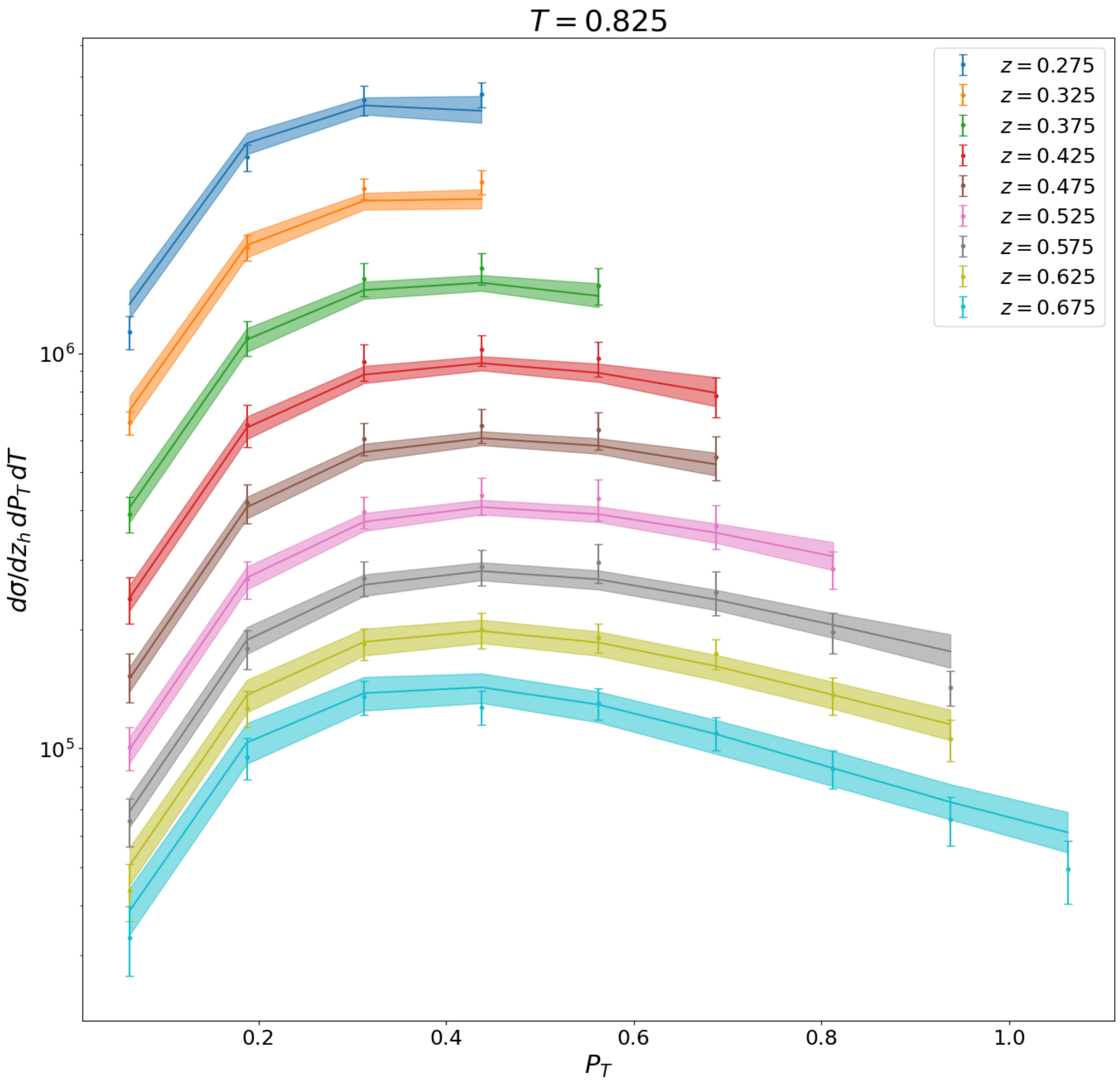
$$R(z) = 1 - \alpha \frac{f(z)}{f(z_0)} \text{ with } f(z) = z\,(1-z)^{\frac{1-z_0}{z_0}},$$

$$W(z) = \frac{m_\pi}{R(z)^2}.$$

Preliminary step

- 57 data points
- 4 free parameters

$\chi^2/\text{d.o.f.}$	0.618	
z_0	$0.5521^{+0.0414}_{-0.0397}$	M_D
α	$0.3644^{+0.0250}_{-0.0282}$	
g_0	$0.2943^{+0.0328}_{-0.0261}$	g_K
β	$4.7100^{+1.985}_{-1.985}$	



Non-perturbative effects of thrust

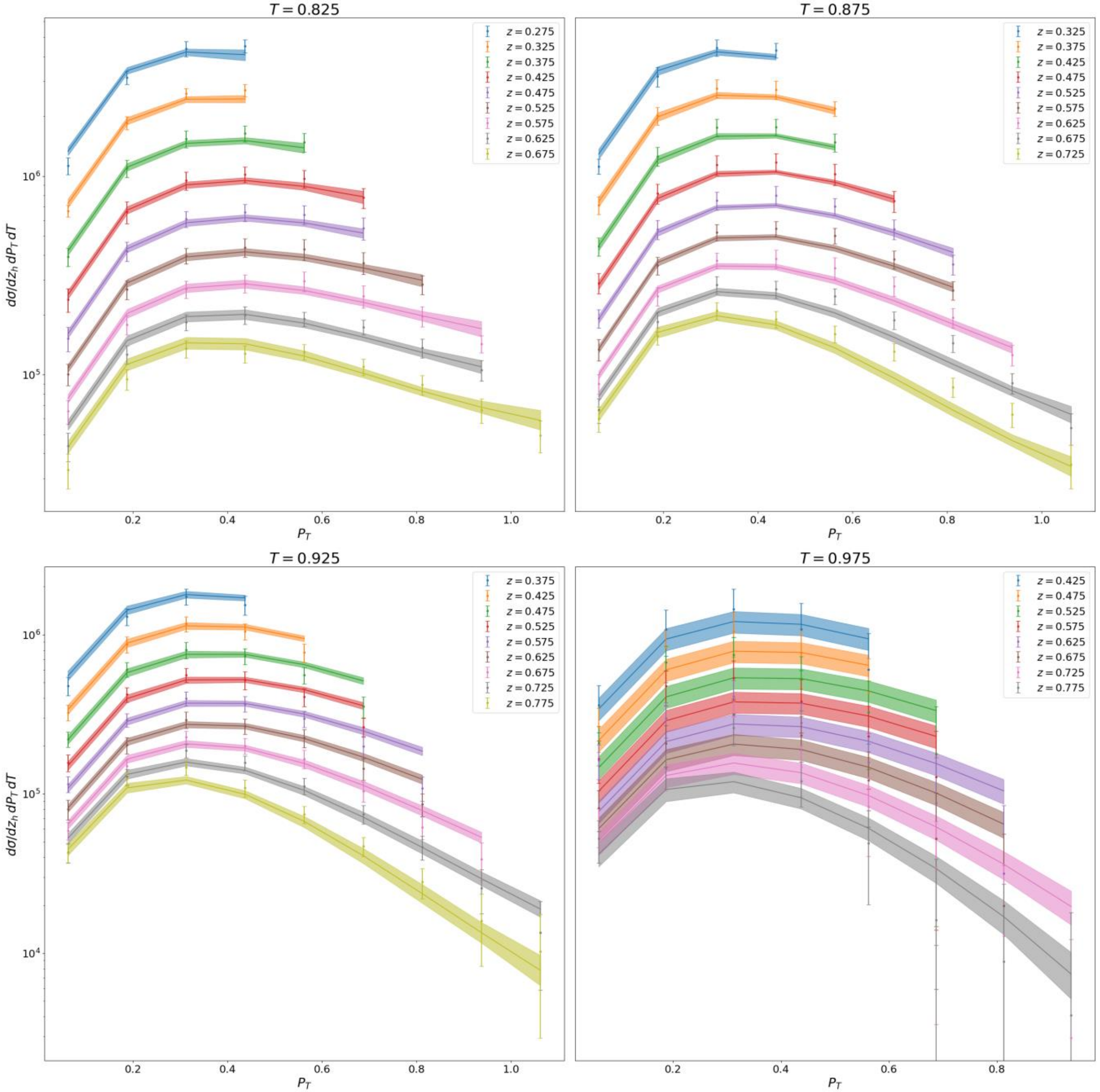
- 230 data points
- 6 free parameters

Minimal approach:

$$\frac{d\sigma}{dz\,dT\,d^2\vec{P}_T} = \frac{d\sigma^{\text{pert.}}}{dz\,dT\,d^2\vec{P}_T}\bigg|_{T-T_0} f_{\text{NP}}(T)$$

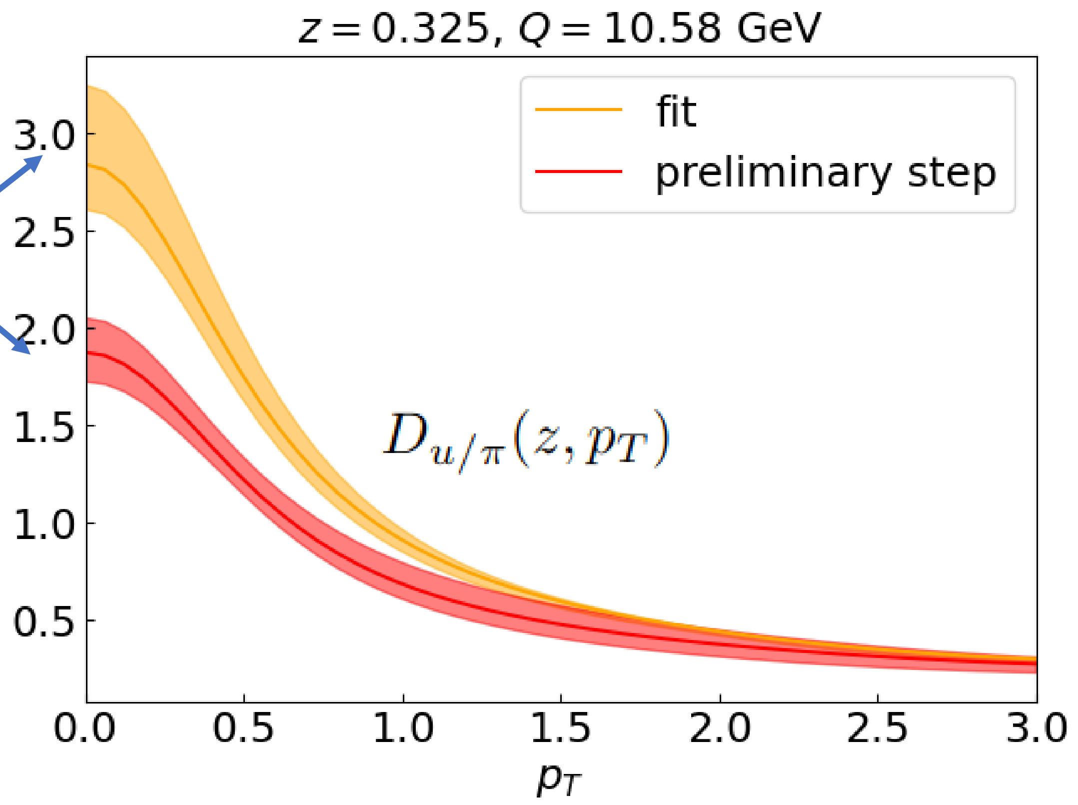
With: $f_{\text{NP}} = \tanh\left(\rho^2\,(1-T)^2\right)$

χ^2/d.o.f.	1.074
z_0	$0.5335^{+0.01938}_{-0.01802}$
α	$0.34029^{+0.01137}_{-0.0122}$
g_0	$0.1043^{+0.0445}_{-0.07419}$
β	$1.6765^{+0.8150}_{-0.8150}$
T_0	$0.0617^{+0.0294}_{-0.0133}$
ρ	$7.7204^{+0.2833}_{-0.2098}$



TMD Fragmentation Function

Correlation between thrust and g_K



The intertwine between the Collins-Soper kernel and the thrust dependence is a unique and special feature of SIA

Beyond the sole extraction of TMD FFs (soft physics, QCD vacuum etc...)

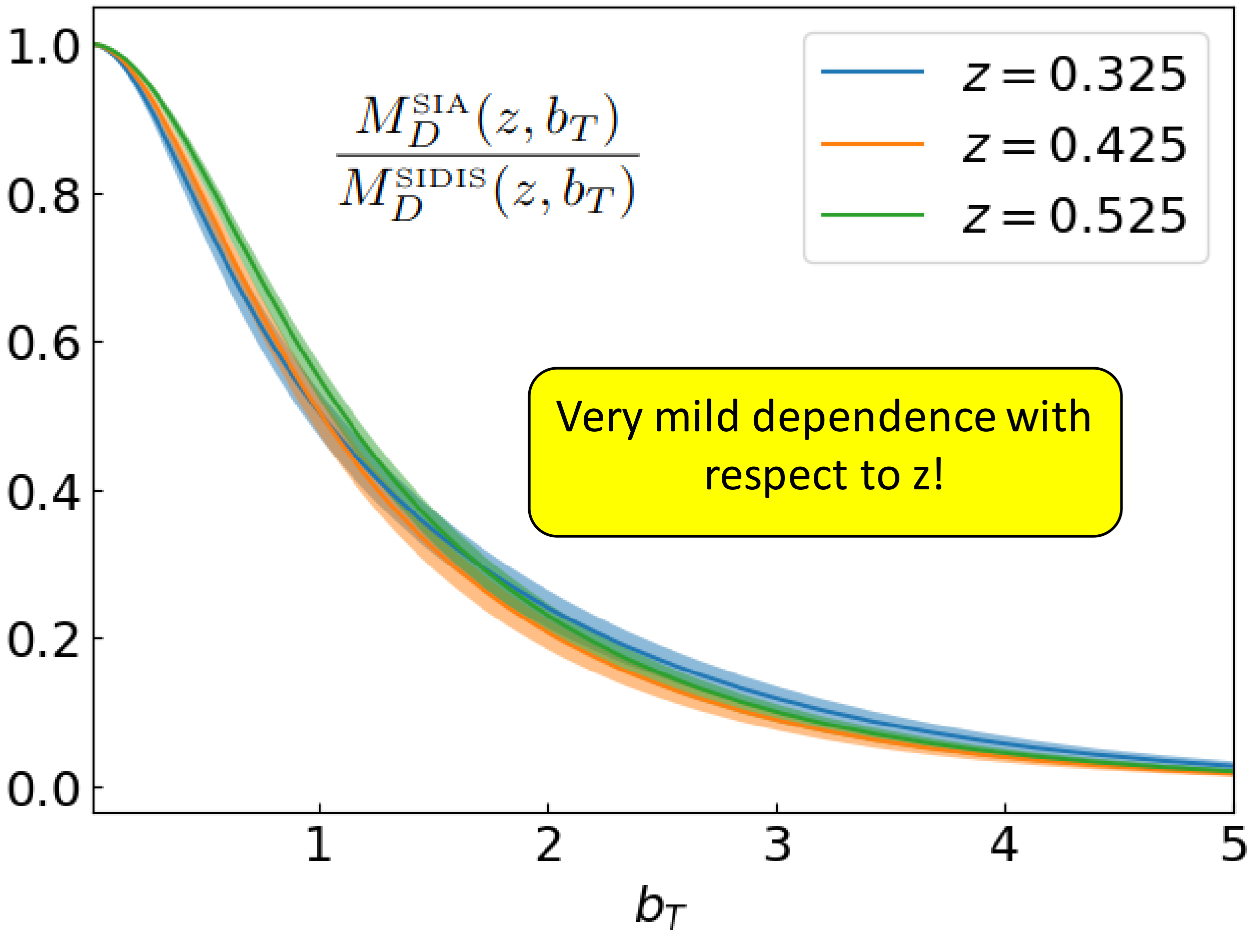
Universality and comparison with SIDIS

The comparison is relevant for the (indirect) extraction of the **soft model**

$$D^{\rm CSS}(z,b_T) = D(z,b_T)\,\sqrt{M_S(b_T)}$$

$\Updownarrow$

$$M_D^{\rm CSS}(z,b_T) = M_D(z,b_T)\,\sqrt{M_S(b_T)}$$



The model extracted from SIDIS is taken from SV19

$$D_{NP}(x,b) = \exp\left(-\frac{\eta_1 z + \eta_2(1-z)}{\sqrt{1+\eta_3(\boldsymbol{b}/z)^2}}\frac{\boldsymbol{b}^2}{z^2}\right)\left(1+\eta_4\frac{\boldsymbol{b}^2}{z^2}\right),$$

Non-perturbative structure of semi-inclusive deep-inelastic and Drell-Yan scattering at small transverse momentum
I.Scimemi, A.Vladimirov, *JHEP* 06 (2020) 137

In the future...

- ❑ Addressing more precisely the non-perturbative effects associated with thrust
- ❑ Comparison with standard TMD factorization (especially DIA!)
- ❑ Extension to other processes (EIC)
- ❑ Direct access to the soft sector

And much more!