Moments of nucleon GPDs from the leading-twist expansion of the quasi-GPD matrix element

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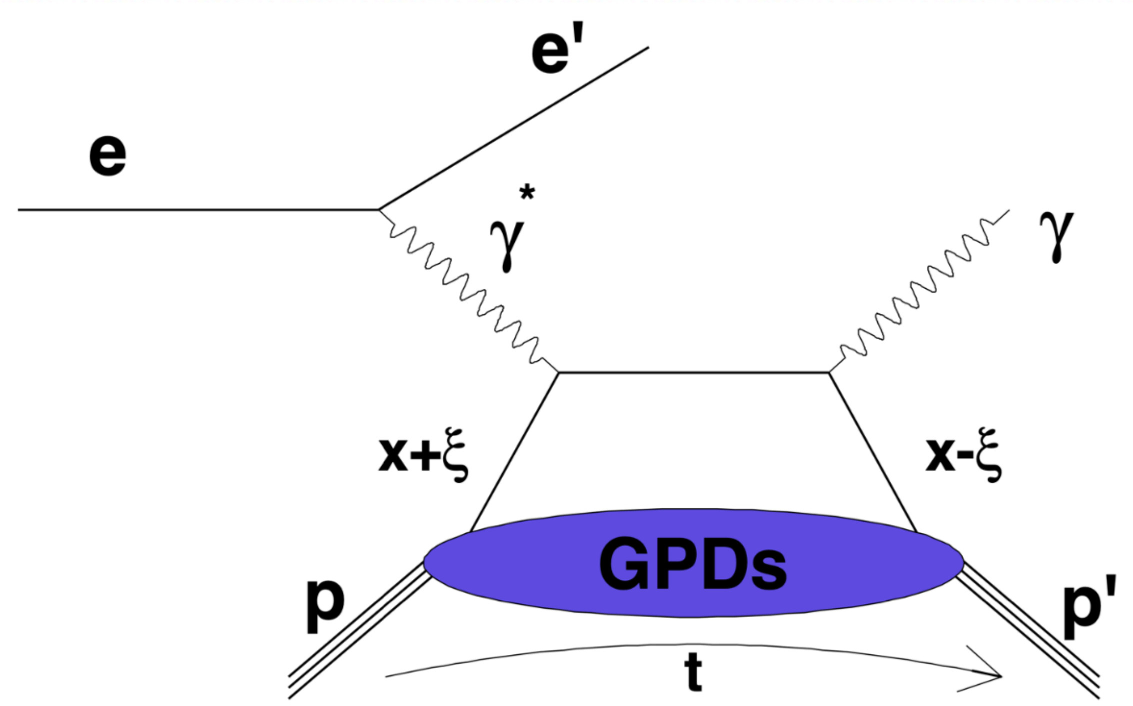
GHP2023, Apr 12 – 14

Generalized parton distributions

The gauge-invariant off-forward matrix elements,



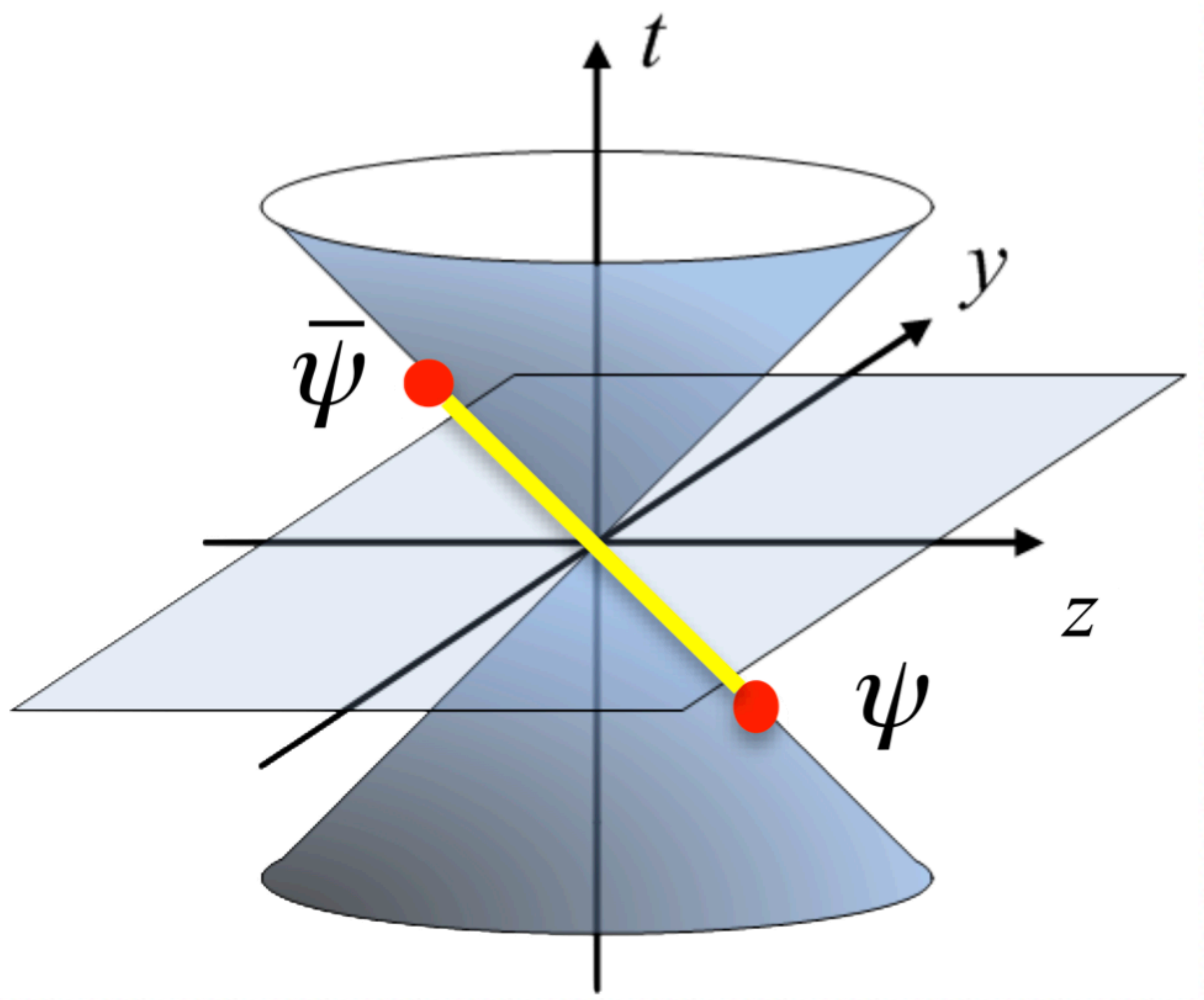
$$F^{\mu}(z,P,\Delta) = \langle p_f|\bar{q}(-\tfrac{z}{2})\gamma^{\mu}\mathscr{W}(-\tfrac{z}{2},\tfrac{z}{2})q(\tfrac{z}{2})\,|p_i\rangle$$

Light-cone GPDs,

$$F(x,\xi,\Delta,\mu) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} F^{\mu}(z,P,\Delta)$$

- $\gamma^{\mu} = \gamma^{+}$
- $z = ln_{-}$, $z^2 = 0$
- $\mathscr{W}(-\frac{z}{2},\frac{z}{2}) = \mathscr{P}\exp(i\int_{-ln\_/2}^{ln\_/2} dl' A^{+})$

$z + ct = 0, \quad z - ct \neq 0$



Generalized parton distributions

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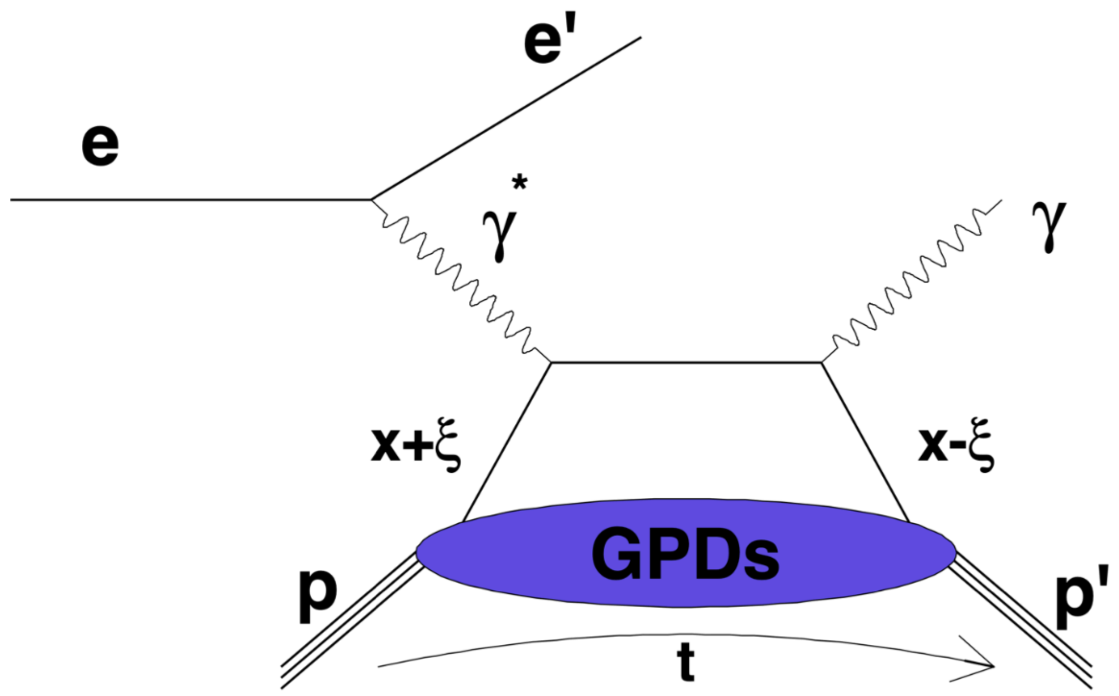

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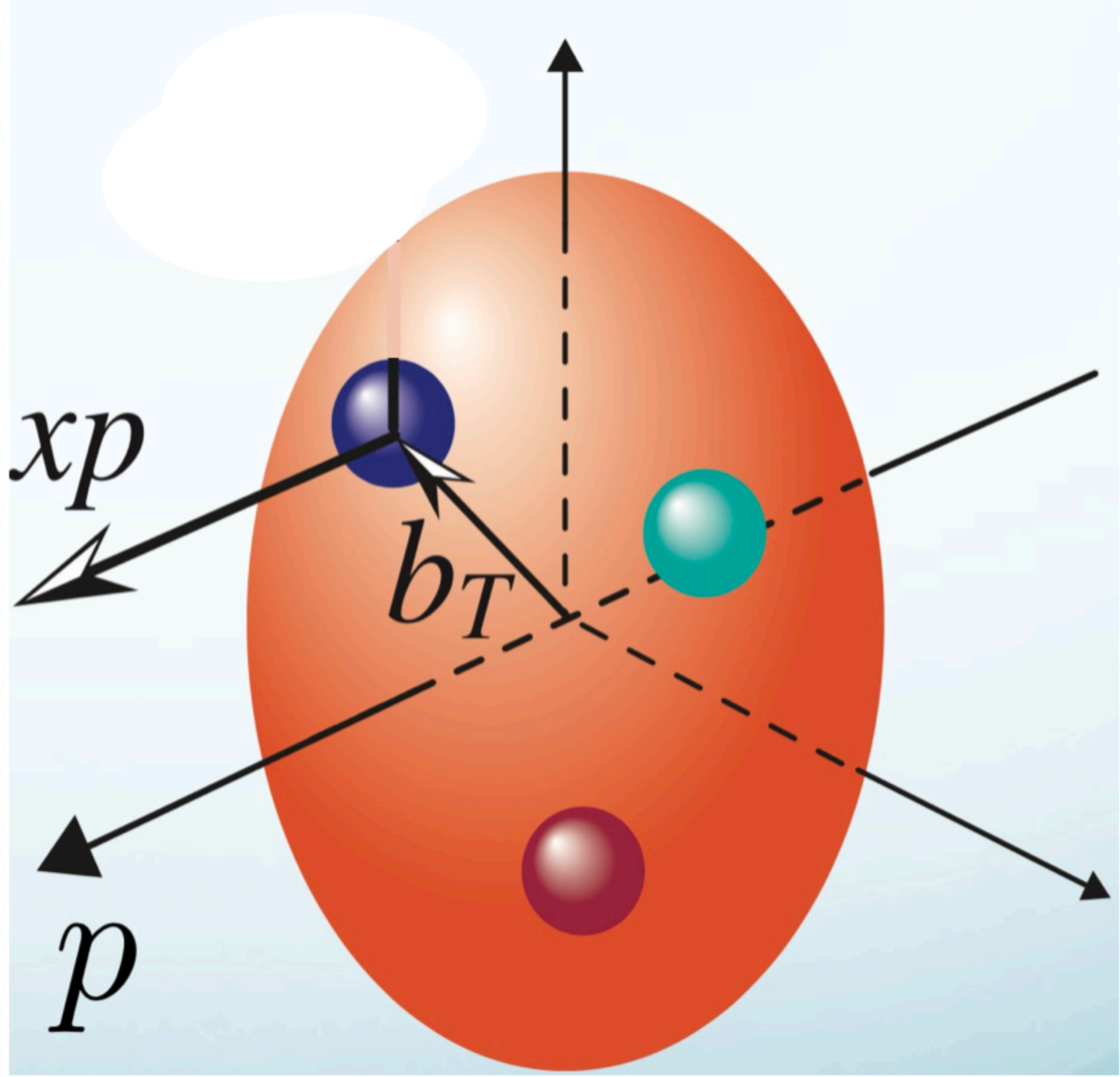
- $\mathcal{W}(-\frac{z}{2},\frac{z}{2}) = \mathcal{P}\exp(i\int_{-ln_\_/2}^{ln_\_/2} dl' A^{+})$

Depends on x and Lorentz invariant products of the vectors p_f, p_i (or $P = (p_f + p_i)/2$, $\Delta = p_f - p_i$) and $n_\_$, conventionally to be

- skewness $\xi = -(\Delta \cdot n_\_)/(2P \cdot n_\_)$
- momentum transfer $t = \Delta^2$

Generalized parton distributions



GPDs goes far beyond the 1D PDFs and the transverse structure encoded in the form factors,

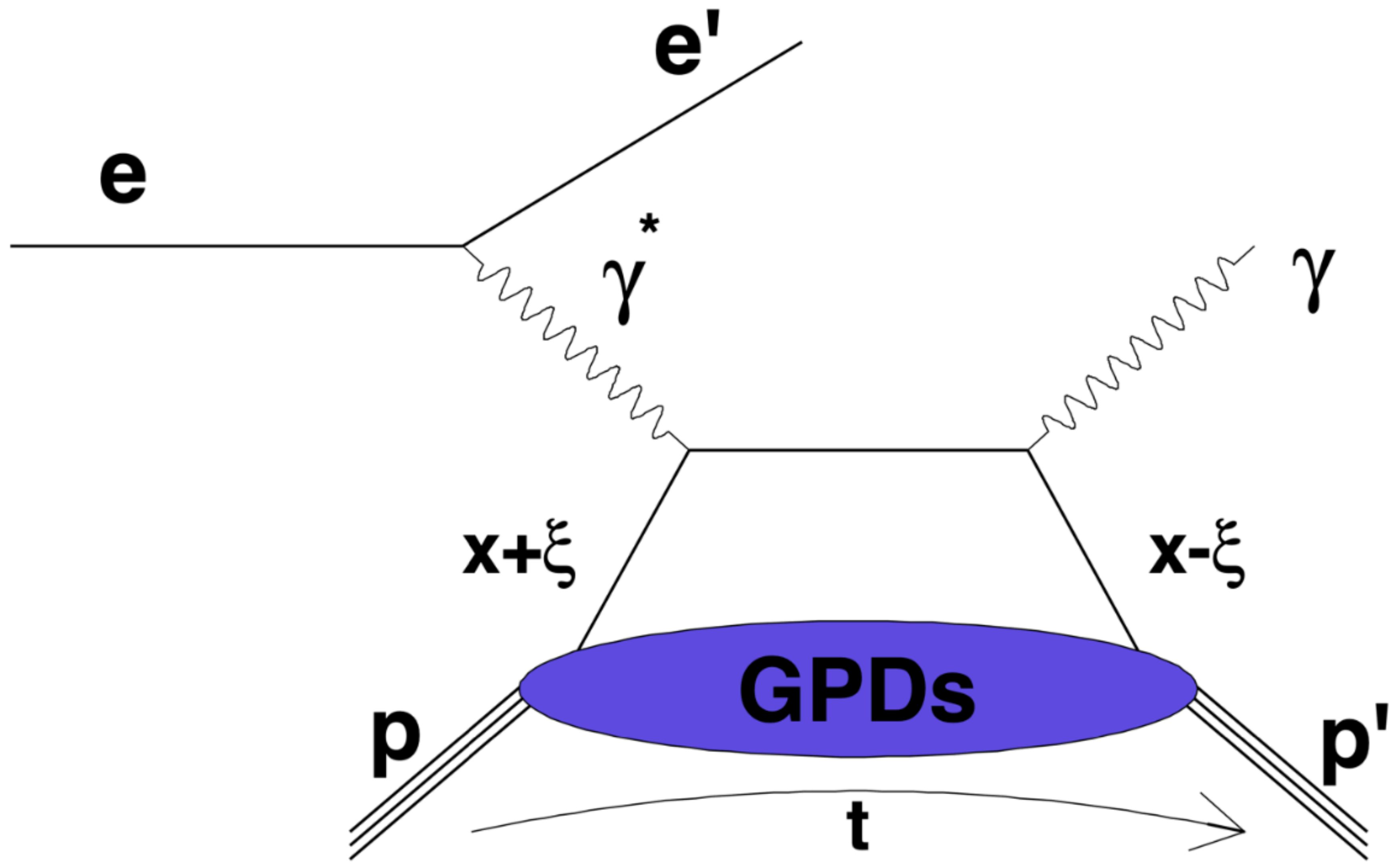
- Offer insights into the 3D image of hadrons.
- Give access to the orbital motion and spin of partons.
- Have a relation to pressure and shear forces inside hadrons.

$F(x, \Delta, \mu)$ $\xi = 0$ — F.T. ($\Delta \leftrightarrow b_T$) — $F(x, b_T, \mu)$

Impact parameter distribution

Generalized parton distributions

DVCS



The golden process to study the quark GPDs is DVCS

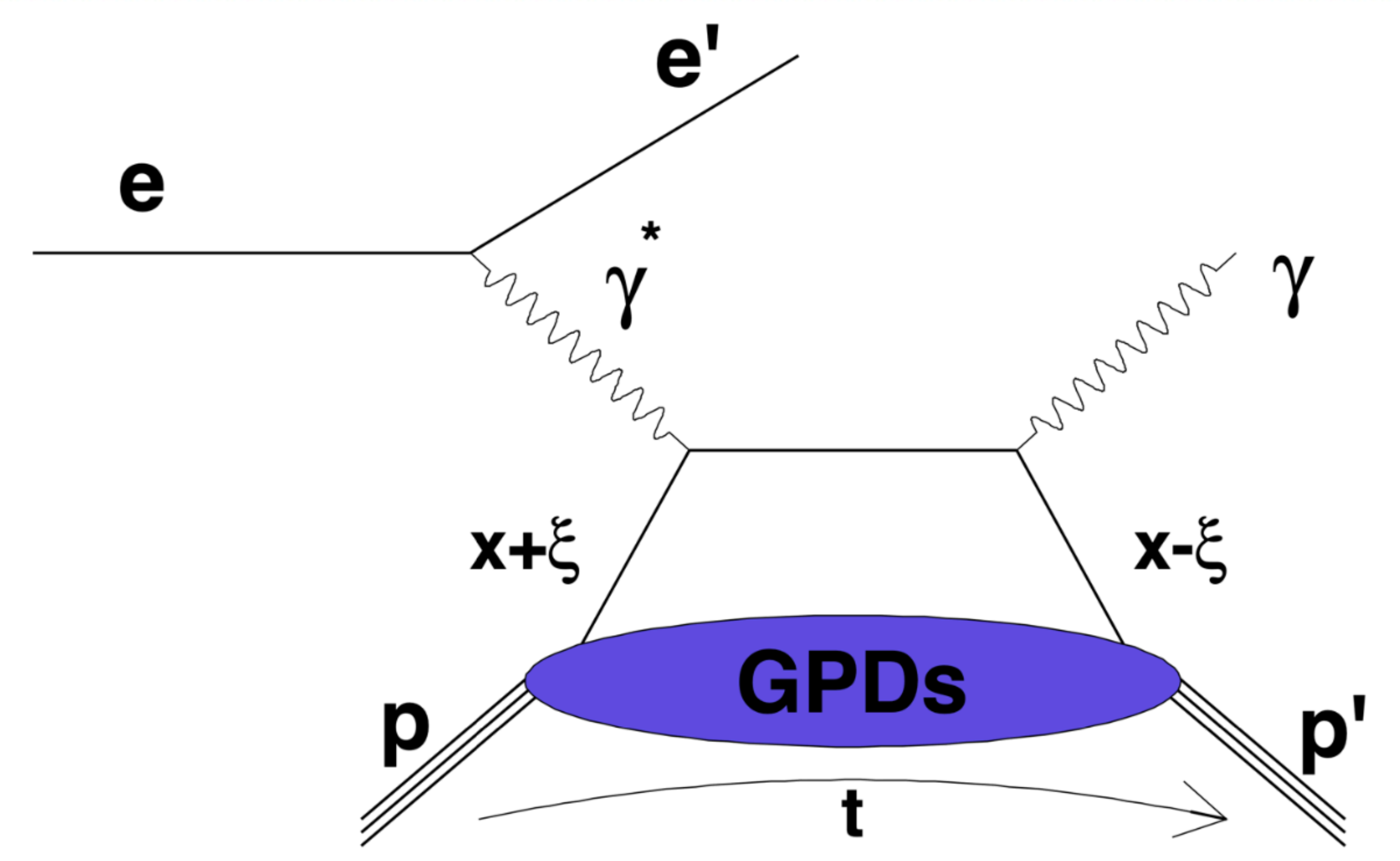
Challenging:

- observables appear at the amplitude level
- multi-dimensionality (x, ξ, t)
- the momentum fraction x is integrated over (Compton Form Factors)

$$\mathcal{F}(\xi, t; Q^2) = \int_{-1}^{1} \mathrm{d}x \left[\frac{1}{\xi - x - i\epsilon} \pm \frac{1}{\xi + x - i\epsilon} \right] F(x, \xi, t; Q^2)$$

Generalized parton distributions

DVCS



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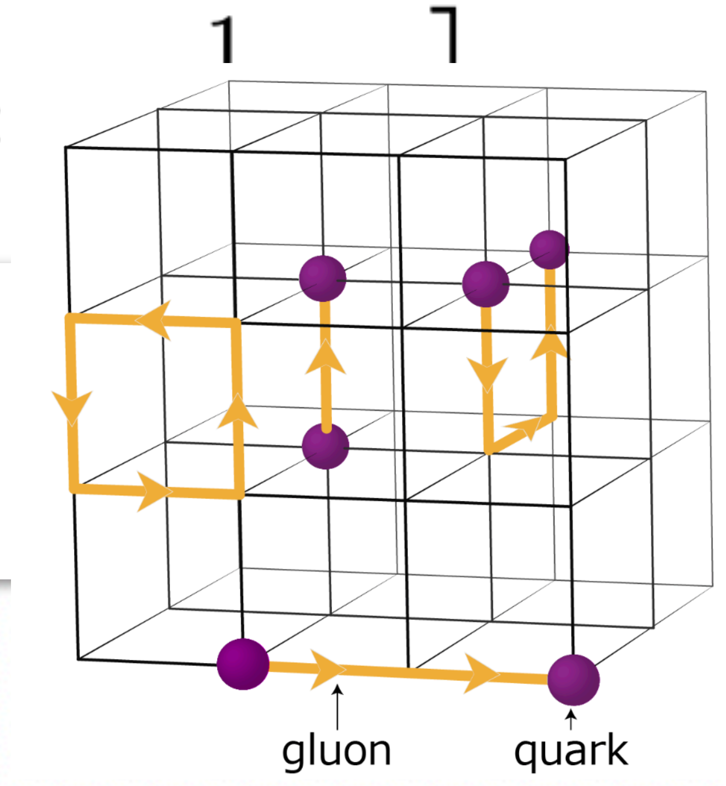
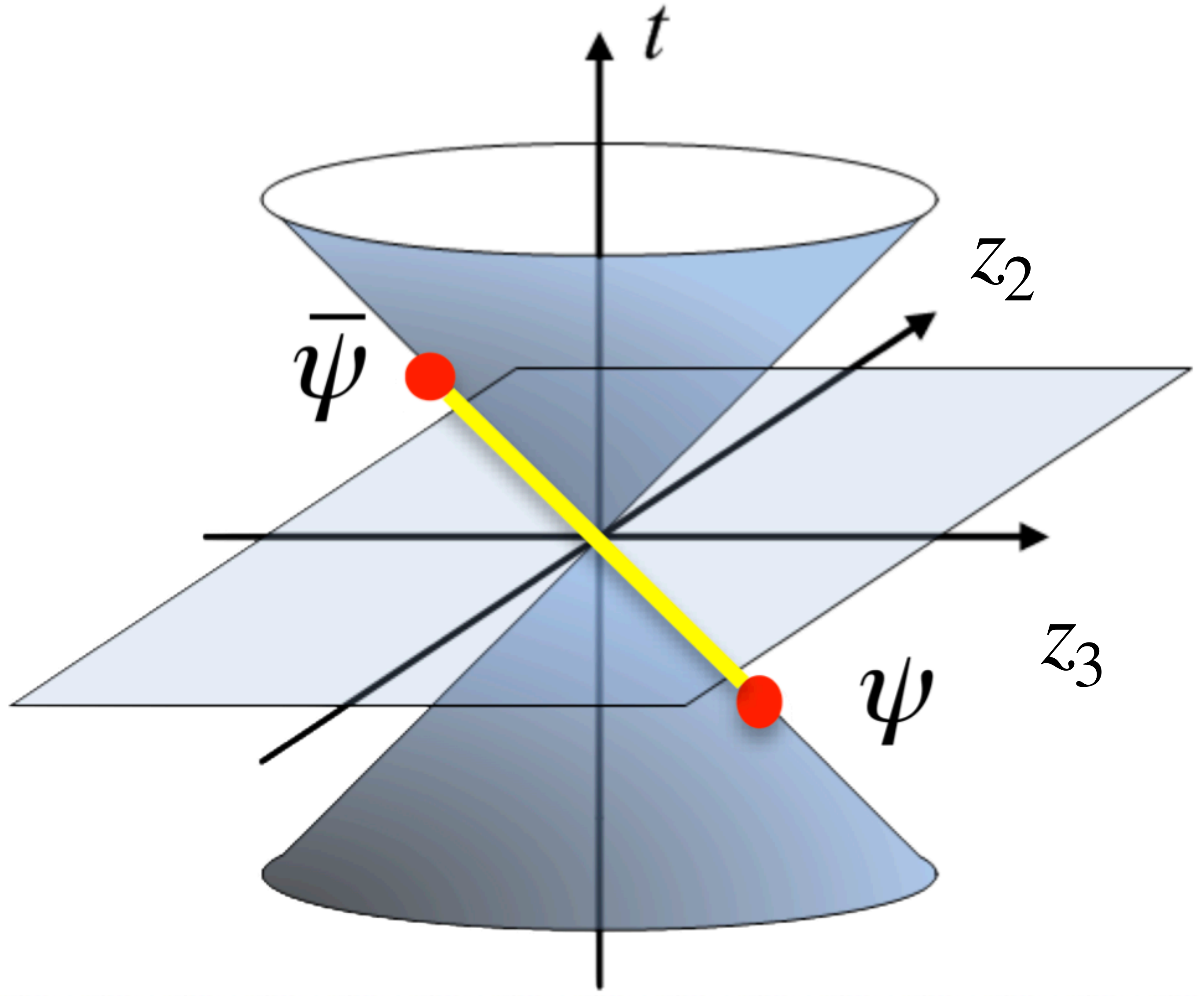
Complementary knowledge from lattice QCD is essential.



Generalized parton distributions

$$z_3 + ct = 0, \quad z_3 - ct \neq 0$$



Light-cone correlation: Cannot be calculated on the lattice

- Mellin or Gegenbauer Moments from leading-twist **local operators**.

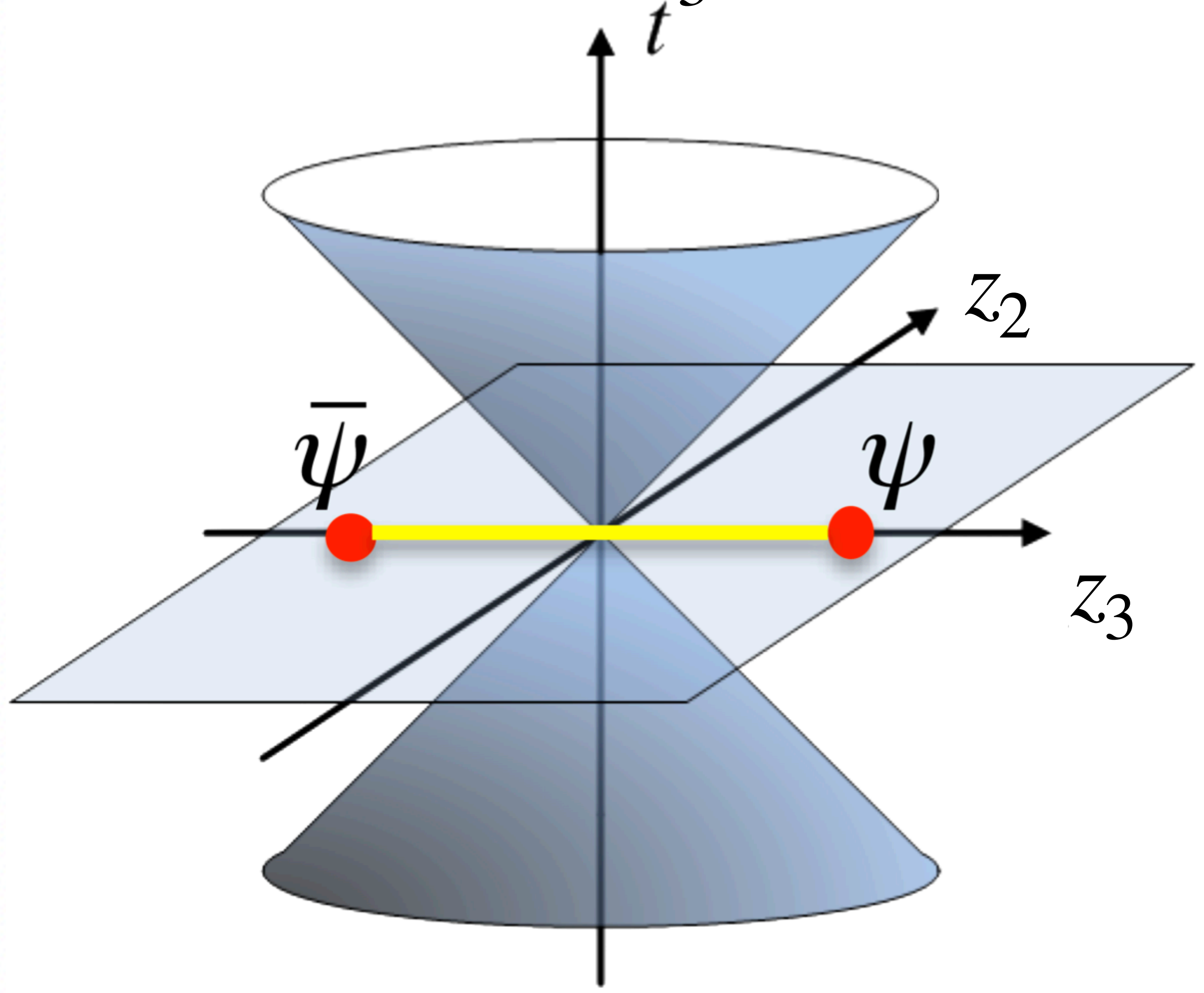
$$\bar{q}\gamma^{\sigma}\overleftrightarrow{D}^{\alpha_1}\ldots\overleftrightarrow{D}^{\alpha_n}q$$

ETMC, PRD 101 (2022)
ETMC, PRD 83 (2011)

Focusing on lowest moments, while high moments from high-dimensional operators are impossible from discretized lattice.

Generalized parton distributions

$t = 0, \quad z_3 \neq 0$



$$\mathcal{F}^{\mu}(z, P, \Delta) = \langle p_f | \bar{q}(-\tfrac{z}{2})\gamma^{\mu} \mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2}) q(\tfrac{z}{2}) | p_i \rangle$$

$$z = (0,0,0,z_3), \; z^2 = z_3^2$$

- Mellin or Gegenbauer Moments from leading-twist **local operators**.

$$\bar{q}\gamma^{\sigma} \overleftrightarrow{D}^{\alpha_1} \ldots \overleftrightarrow{D}^{\alpha_n} q$$

ETMC, PRD 101 (2022)
ETMC, PRD 83 (2011)

- **Large-momentum effective theory**: x -space matching of quasi-PDF.

X. Ji, PRL 2013
X. Ji, et al, RevModPhys 2021

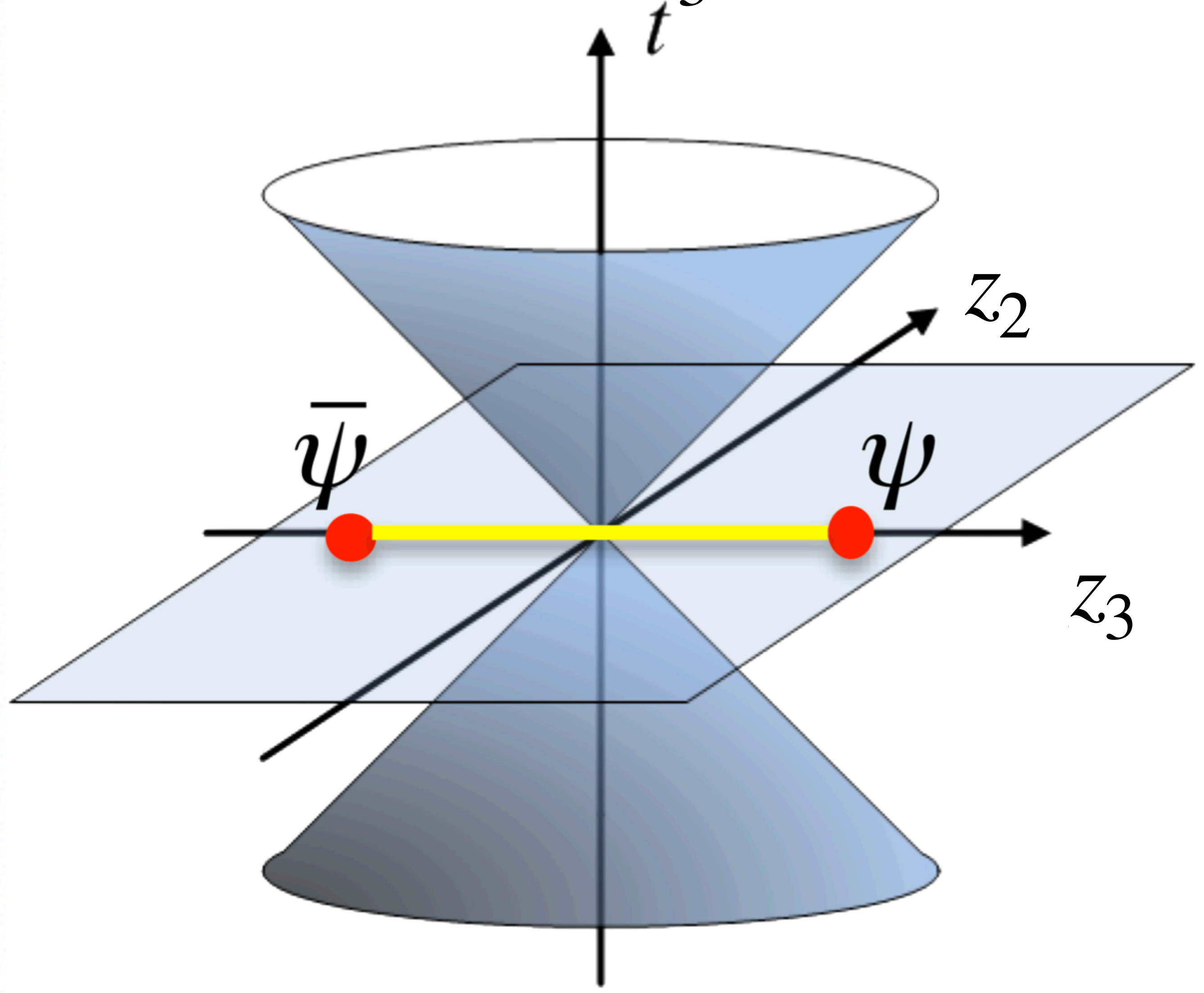
- **Short distance factorization** of the quasi-PDF matrix elements in position space or the pseudo-PDF approach.

  - A. Radyushkin, PRD 100 (2019)
  - A. Radyushkin, Int.J.Mod.Phys.A 2020

- …

Short distance factorization

$t = 0, \quad z_3 \neq 0$



$$\mathscr{F}^\mu(z, P, \Delta) = \langle p_f | \bar{q}(-\tfrac{z}{2})\gamma^\mu \mathscr{W}(-\tfrac{z}{2}, \tfrac{z}{2}) q(\tfrac{z}{2}) | p_i \rangle$$

$z = (0,0,0,z_3),\ z^2 = z_3^2$

SDF of the zero skewness GPD matrix elements:

- V. Braun et al., EPJC 55 (2008)
- A. V. Radyushkin et al., PRD 96 (2017)
- Y. Ma et al., PRL 120 (2018)
- T. Izubuchi et al., PRD 98 (2018)

$$\mathscr{F}^R(z, P, \Delta) = \int_{-1}^{1} d\alpha \mathscr{C}(\alpha, \mu^2 z^2) \int_{-1}^{1} dy e^{-iy\alpha\lambda} F(x, \xi, \Delta, \mu) + \mathscr{O}(z^2 \Lambda_{QCD}^2)$$

$$= \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} C_n(z^2\mu^2) \langle x^n \rangle (t;\mu) + \mathscr{O}(z^2 \Lambda_{\mathrm{QCD}}^2)$$

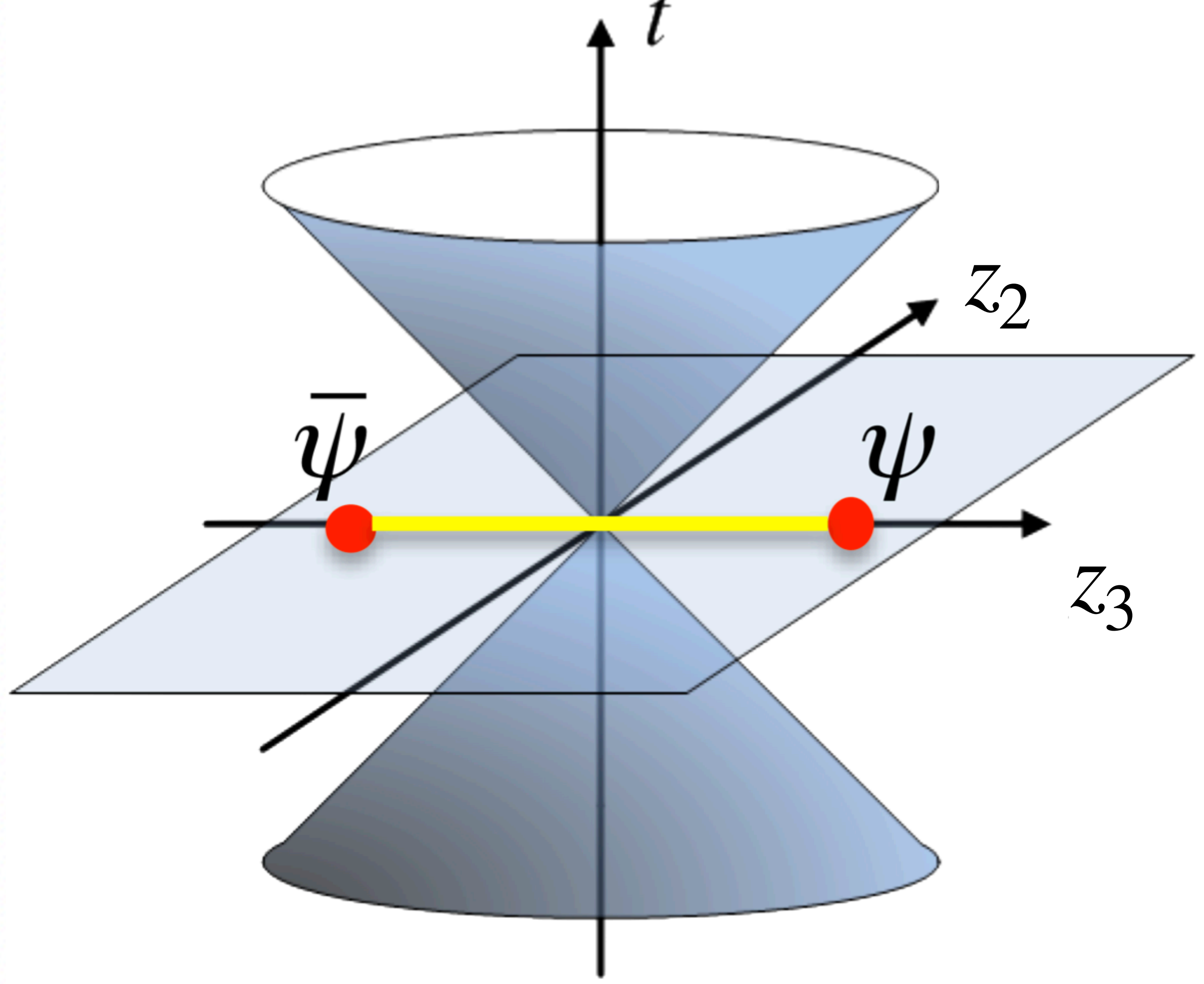
Perturbative matching

$\lambda = zP$

$$\int_{-1}^{1} dx x^n H^q(x, \xi = 0, t) = A^q_{n+1,0}(t)$$

$$\int_{-1}^{1} dx x^n E^q(x, \xi = 0, t) = B^q_{n+1,0}(t)$$

Short distance factorization



$$\mathcal{F}^{\mu}(z, P, \Delta)$$
$$= \langle p_f | \bar{q}(-\tfrac{z}{2})\gamma^{\mu}\mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2})q(\tfrac{z}{2}) | p_i \rangle$$

$z = (0,0,0,z_3), \ z^2 = z_3^2$

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$$= \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} C_n(z^2\mu^2) \langle x^n \rangle (t; \mu) + \mathcal{O}(z^2 \Lambda_{\rm QCD}^2)$$

Perturbative matching

$$\lambda = zP$$

- The perturbative matching is valid in short range of z_3.
- The information that lattice data contains is limited by the range of finite $\lambda = zP$.

quasi-GPD matrix elements

The matrix elements can be parametrized in terms of linearly-independent Dirac structures:

S. Bhattacharya, et al., PRD 106 (2022)

$$F^\mu(z,P,\Delta) = \bar{u}(p_f,\lambda')\left[\frac{P^\mu}{m}A_1 + mz^\mu A_2 + \frac{\Delta^\mu}{m}A_3 + im\sigma^{\mu z}A_4 + \frac{i\sigma^{\mu\Delta}}{m}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{m}A_6 + mz^\mu i\sigma^{z\Delta}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{m}A_8\right]u(p_i,\lambda)$$

$$A_i(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$$

light-cone GPDs H and E

$$F^+(z,P,\Delta) = \bar{u}(p_f,\lambda')\left[\gamma^+ H(z,P,\Delta) + \frac{i\sigma^{+\mu}\Delta_\mu}{2m}E(z,P,\Delta)\right]u(p_i,\lambda)$$

$$H(z,P,\Delta) = A_1 + \frac{\Delta^+}{P^+}A_3$$

$$E(z,P,\Delta) = -A_1 - \frac{\Delta^+}{P^+}A_3 + 2A_5 + 2P^+z^-A_6 + 2\Delta^+z^-A_8$$

- Lorentz invariant, frame independent
- $\dfrac{\Delta^+}{P^+} = \dfrac{\Delta\cdot z}{P\cdot z},\ z^2 = 0$

Commonly used quasi-GPD matrix elements

$$\mathscr{F}^0(z,P,\Delta) = \bar{u}(p_f,\lambda')\left[\gamma^0\mathscr{H}_0(z,P,\Delta) + \frac{i\sigma^{0\mu}\Delta_\mu}{2m}\mathscr{E}_0(z,P,\Delta)\right]u(p_i,\lambda)$$

$$\mathscr{H}_0^s(z,P^s,\Delta^s) = A_1 + \frac{\Delta^{0,s}}{P^{0,s}}A_3 - \frac{m^2\Delta^{0,s}z^3}{2P^{0,s}P^{3,s}}A_4 + \left[\frac{(\Delta^{0,s})^2z^3}{2P^{3,s}} - \frac{\Delta^{0,s}\Delta^{3,s}z^3P^{0,s}}{2(P^{3,s})^2} - \frac{z^3(\Delta_\perp^s)^2}{2P^{3,s}}\right]A_6 + \left[\frac{(\Delta^{0,s})^3z^3}{2P^{0,s}P^{3,s}} - \frac{(\Delta^{0,s})^2\Delta^{3,s}z^3}{2(P^{3,s})^2} - \frac{\Delta^{0,s}z^3(\Delta_\perp^s)^2}{2P^{0,s}P^{3,s}}\right]A_8$$

$\ldots$

- Frame dependent
- Computational expensive for multiple Q^2
- Encouraging results were reported

quasi-GPD matrix elements

light-cone GPDs H and E

$$H(z, P, \Delta) = A_1 + \frac{\Delta^+}{P^+} A_3$$

$$E(z, P, \Delta) = -A_1 - \frac{\Delta^+}{P^+} A_3 + 2A_5 + 2P^+ z^- A_6 + 2\Delta^+ z^- A_8$$

- Lorentz invariant, frame independent
- $\frac{\Delta^+}{P^+} = \frac{\Delta \cdot z}{P \cdot z}, \; A_i(z \cdot P, z \cdot \Delta, \Delta^2, z^2), \; z^2 = 0$

Lorentz invariant quasi-GPD matrix elements

$$\mathscr{H}(z, P, \Delta) = A_1 + \frac{\Delta \cdot z}{P \cdot z} A_3$$

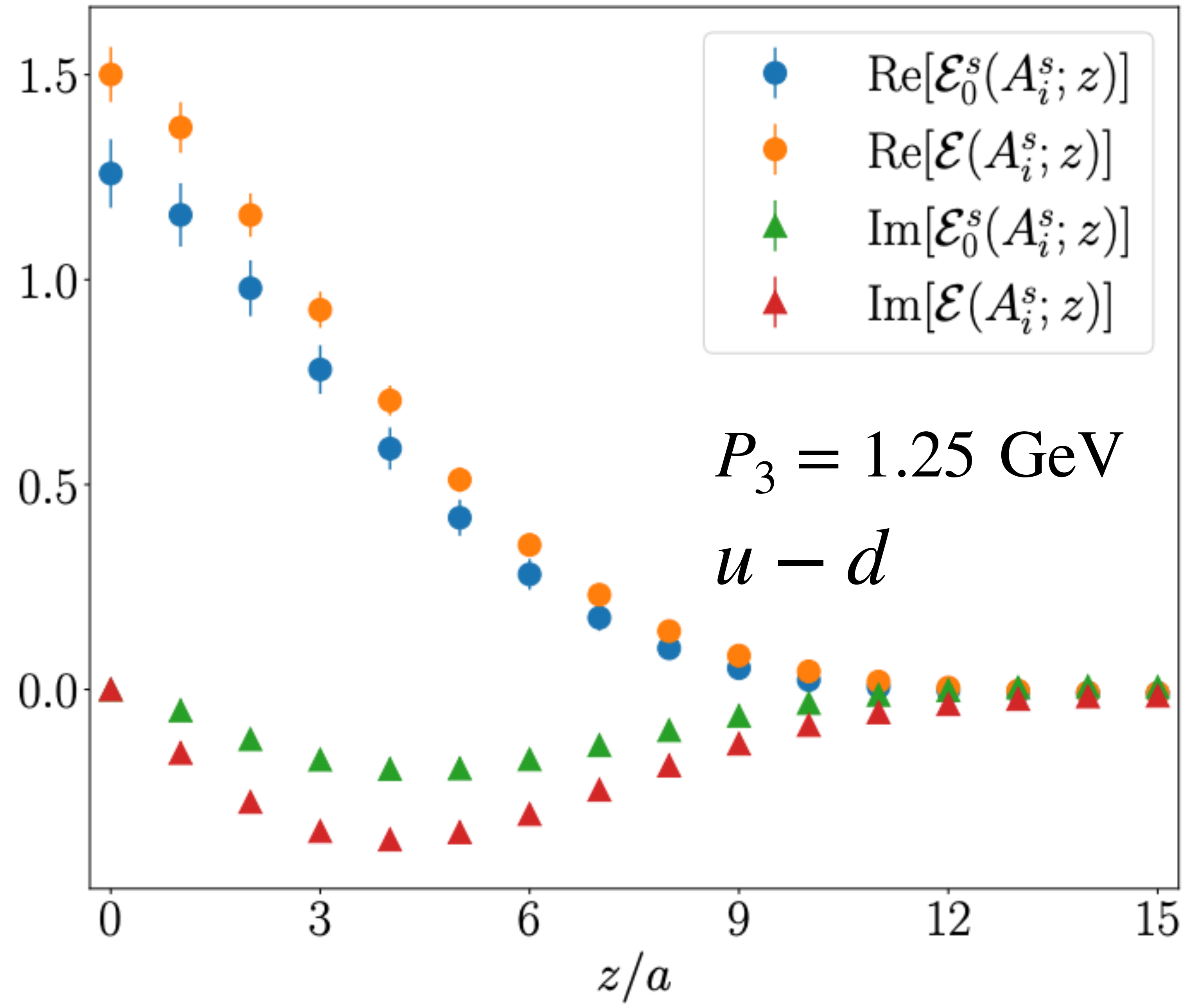
$$\mathscr{E}(z, P, \Delta) = -A_1 - \frac{\Delta \cdot z}{P \cdot z} A_3 + 2A_5 + 2P \cdot z A_6 + 2\Delta \cdot z A_8$$

- Lorentz invariant, frame independent
- $A_i(z \cdot P, z \cdot \Delta, \Delta^2, z^2), \; z^2 \neq 0$
- A_i can be solved from matrix elements of $\mathscr{F}^0, \mathscr{F}^1, \mathscr{F}^2$

S. Bhattacharya, et al., PRD 106 (2022)

Bare matrix elements and renormalization

Bare matrix elements of quasi-GPD E



- Nf=2+1+1 twisted mass (TM) fermions & clover improvement.
- $m_\pi = 260$ MeV, $a = 0.093$ fm, $32^3 \times 64$
- iso-vector (u-d) and iso-scalar (u+d), connected diagrams only.

The operator can be multiplicatively renormalized

- X. Ji, J. H. Zhang and Y. Zhao, PRL120.112001
- J. Green, K. Jansen and F. Steffens, PRL.121.022004

$$[\bar{q}(-\tfrac{z}{2})\gamma^\mu \mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})q(\tfrac{z}{2})]_B$$

$$\delta m = m_{-1}/a + m_0$$

$$= e^{-\delta m(a)|z|} Z(a)[\bar{q}(-\tfrac{z}{2})\gamma^\mu \mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})q(\tfrac{z}{2})]_R$$

- **Ratio scheme renormalization**

  - A. V. Radyushkin et al., PRD 96 (2017)
  - BNL, PRD 102 (2020)

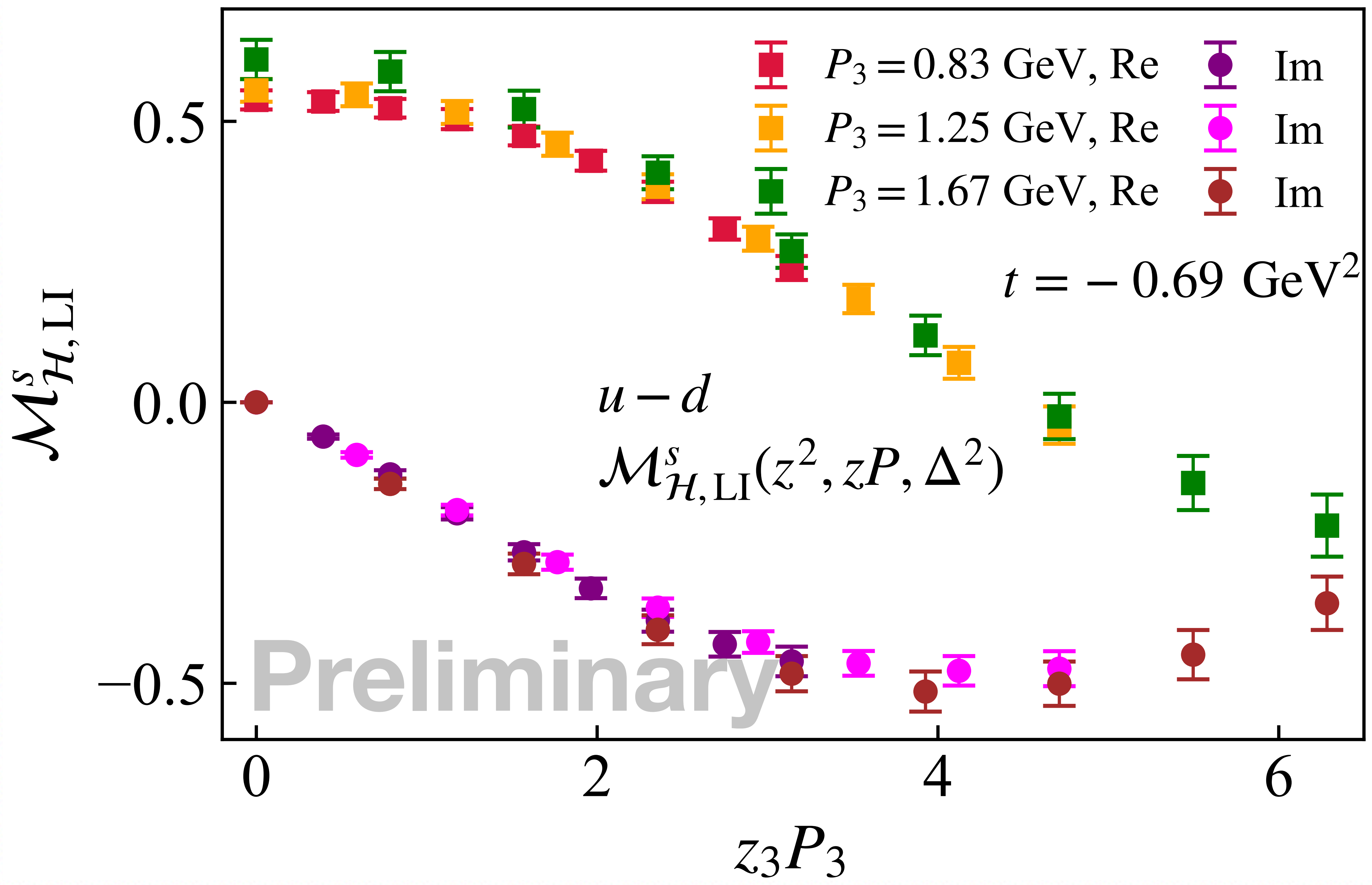
$$\mathcal{M}(z^2, zP, \Delta^2) = \frac{F^R(z,P,\Delta;\mu)}{F^R(z,P=0,\Delta=0;\mu)} = \frac{F^B(z,P,\Delta;a)}{F^B(z,P=0,\Delta=0;a)}$$

$$= \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} \frac{C_n(z^2\mu^2)}{C_n(z^2\mu^2)} \langle x^n \rangle(\Delta^2;\mu) + \mathcal{O}(z^2\Lambda^2_{\mathrm{QCD}})$$

Reduce to the standard Ioffe-time pseudo-distribution when $\Delta = 0$

Ratio scheme renormalization

Ratio-scheme matrix elements for H

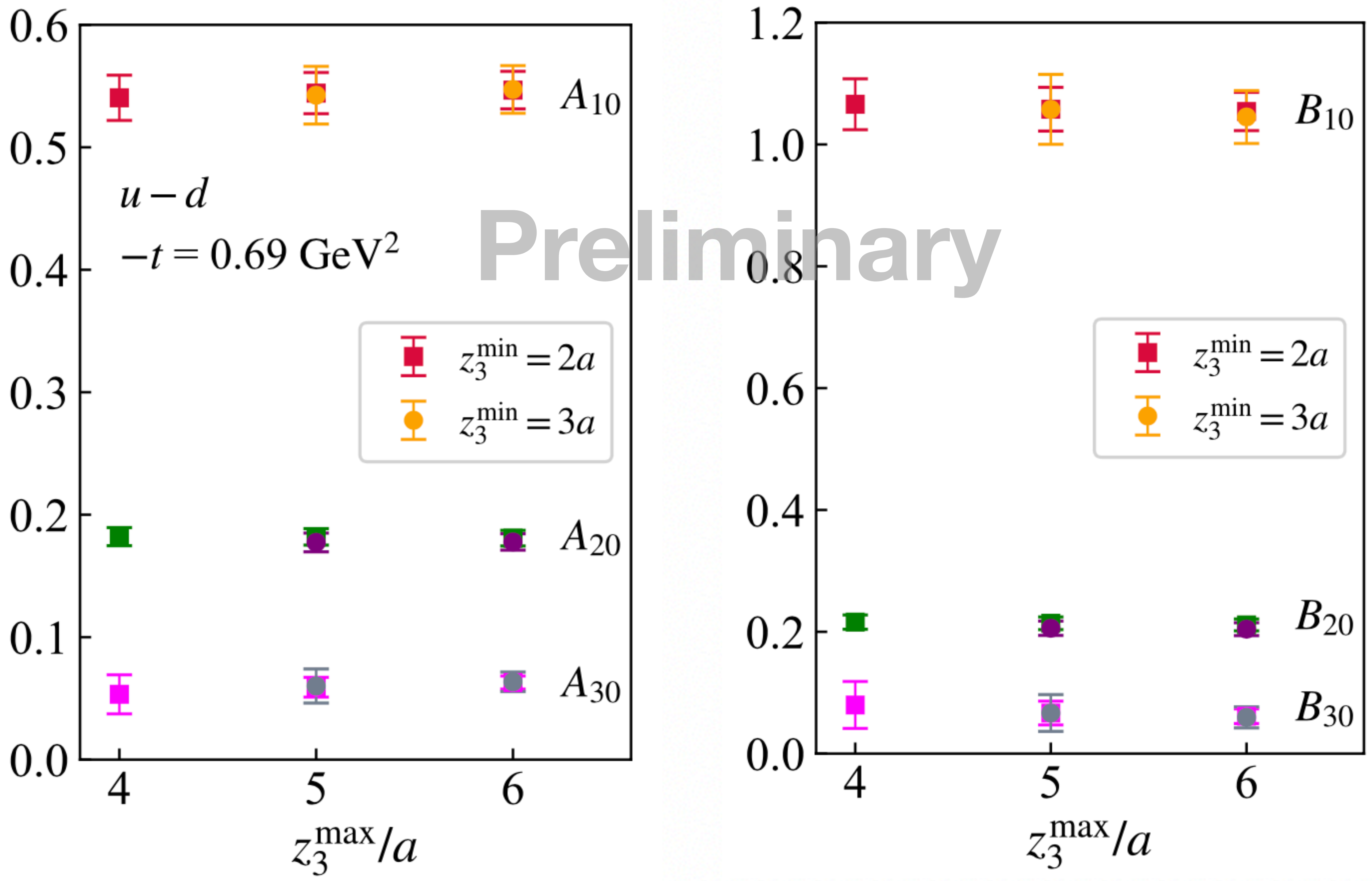


- At tree level ($\alpha_s = 0$, $C_n(\mu^2 z^2) = 1$) approximation, simply a polynomial function of zP.
- Beyond LO, the perturbative kernels $C_n(z^2\mu^2)/C_0(z^2\mu^2)$ are supposed to compensate the z-dependent evolution.
- Wilson-coefficients available up to NNLO for iso-vector case, while NLO for iso-scalar case.

$$\mathscr{M}(z^2, zP, \Delta^2) = \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} \frac{C_n(z^2\mu^2)}{C_n(z^2\mu^2)} \langle x^n \rangle (t; \mu) + \mathscr{O}(z^2 \Lambda^2_{\mathrm{QCD}})$$

Mellin moments of GPDs

$$\mathcal{M}(z^2, zP, \Delta^2) = \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} \frac{C_n(z^2\mu^2)}{C_n(z^2\mu^2)} \langle x^n \rangle + \mathcal{O}(z^2 \Lambda_{QCD}^2)$$



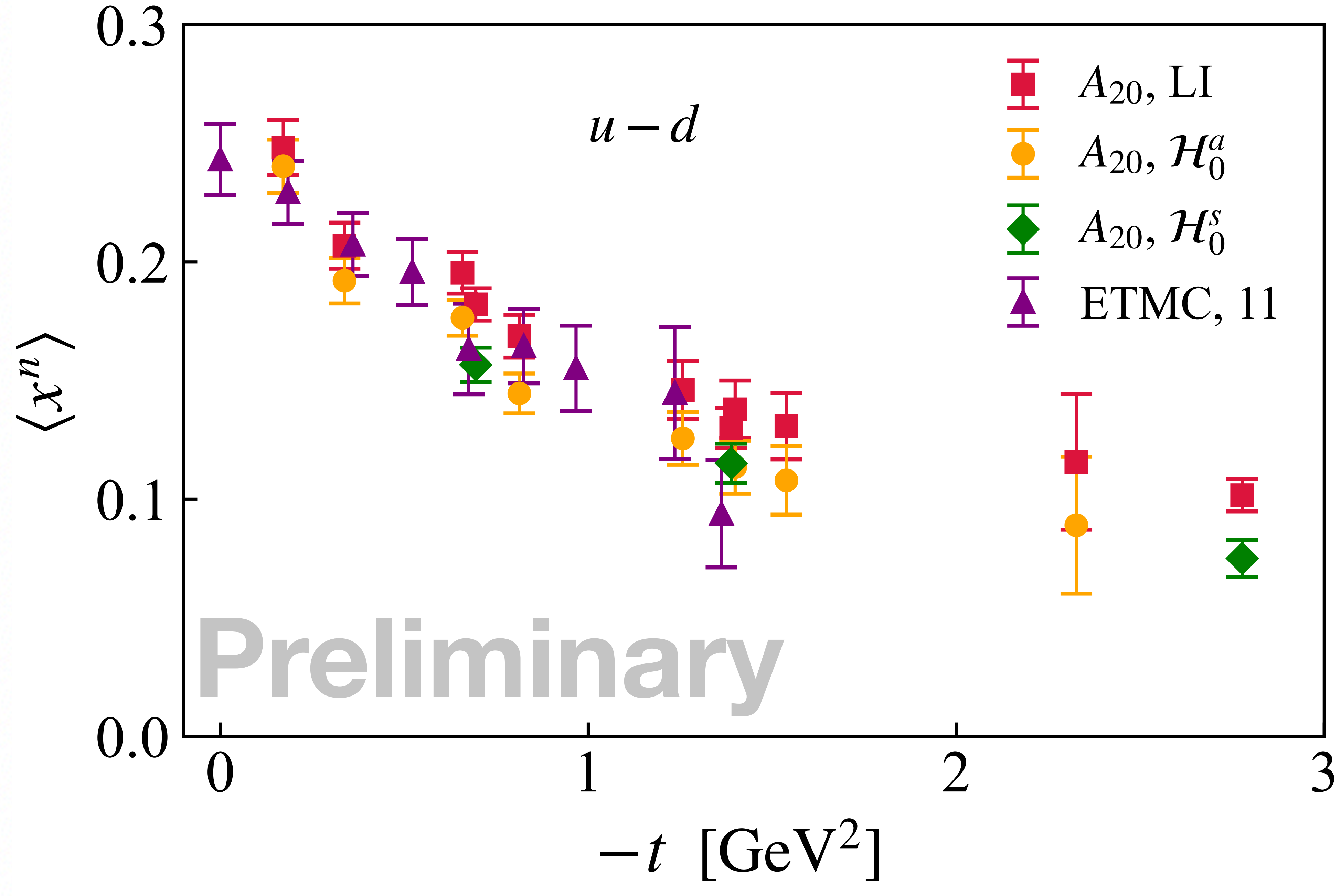
$z \in [z_{min}, z_{max}]$

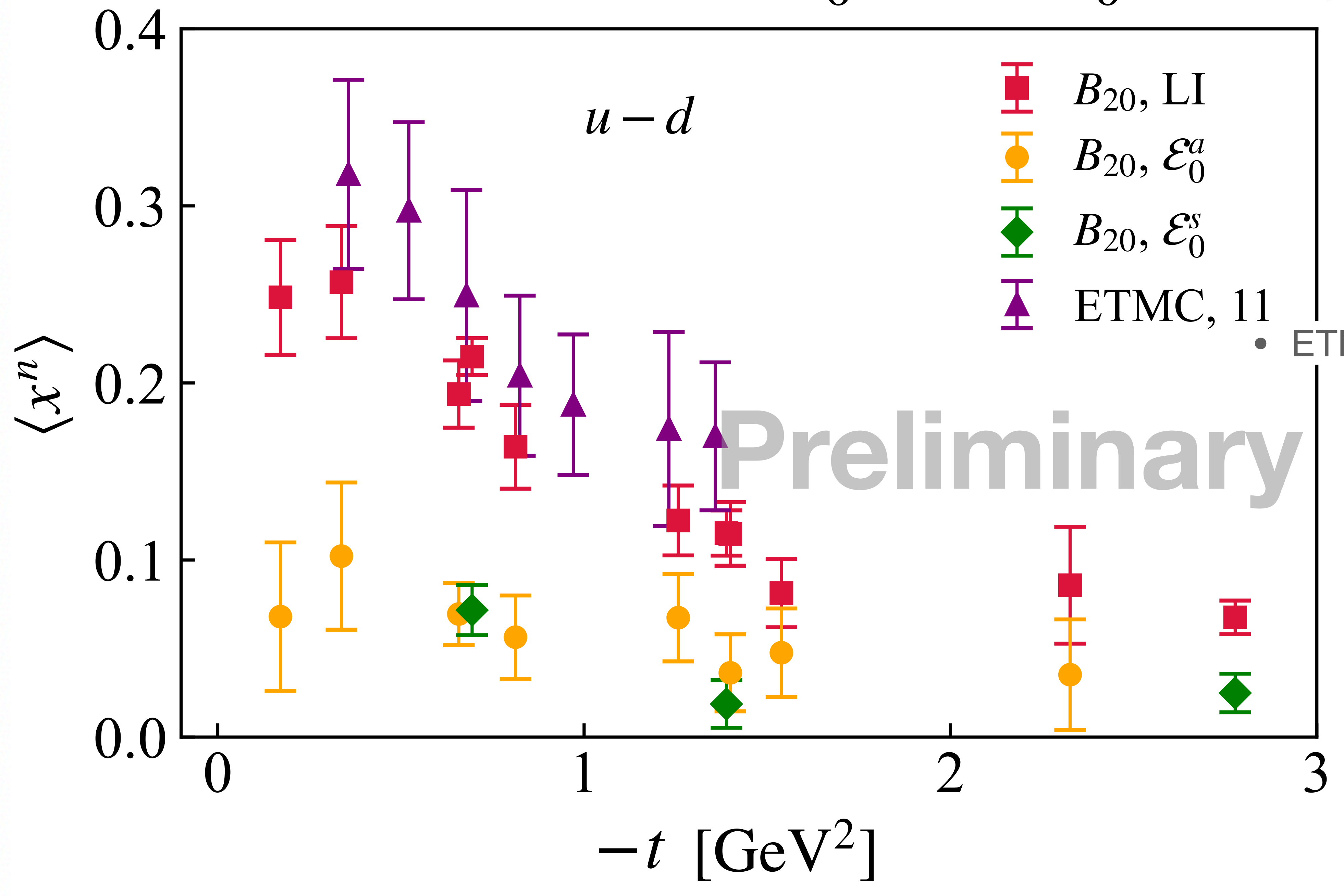
Latt. artifact? Higher twist?

- We vary z_{min} and z_{max} to estimate the systematic errors.
- Reasonable signal up to $\langle x^2 \rangle$ (A_{30} and B_{30}) is observed, higher momentum and statistics are needed to constrain higher moments.

Mellin moments of GPDs

- LI: Lorentz invariant definition.
- $\mathscr{H}_0^{s/a}$ and $\mathscr{E}_0^{s/a}$ are γ_0 definition.

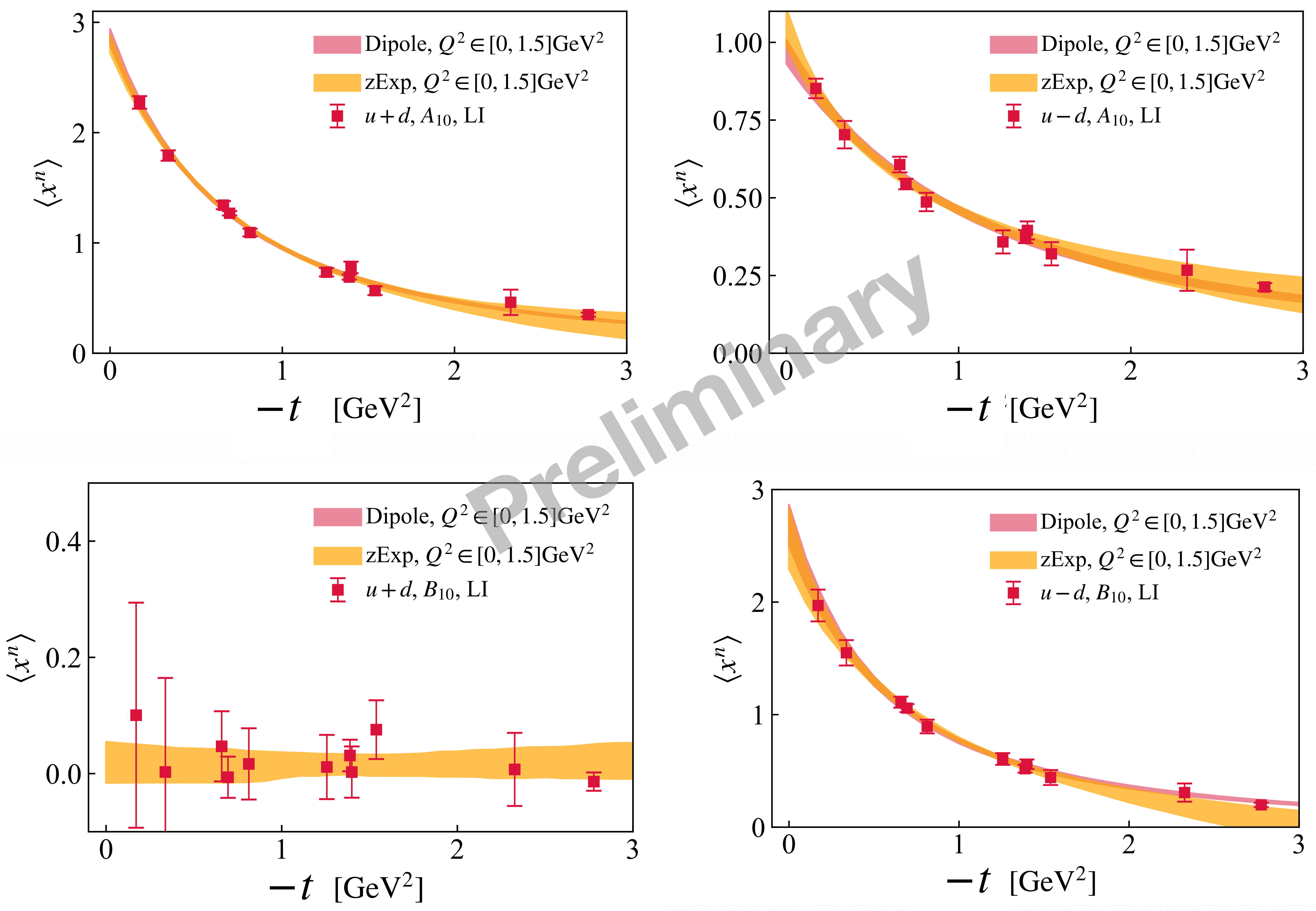




- ETMC, PRD 83 (2011)

- Results from different definitions are inconsistent especially for B_{20}.
- The Lorentz invariant (LI) definition have better agreement with the traditional moments calculation.

t-dependence of moments: A_{10} & B_{10}



Di-pole fit: $$\langle x^n \rangle(Q^2) = \frac{\langle x^n \rangle(0)}{(1+\frac{Q^2}{M^2})^2}$$

z-expension fit:

$$\langle x^n \rangle(Q^2) = \sum_{k=0}^{k_{\max}} a_k z(Q^2)^k$$

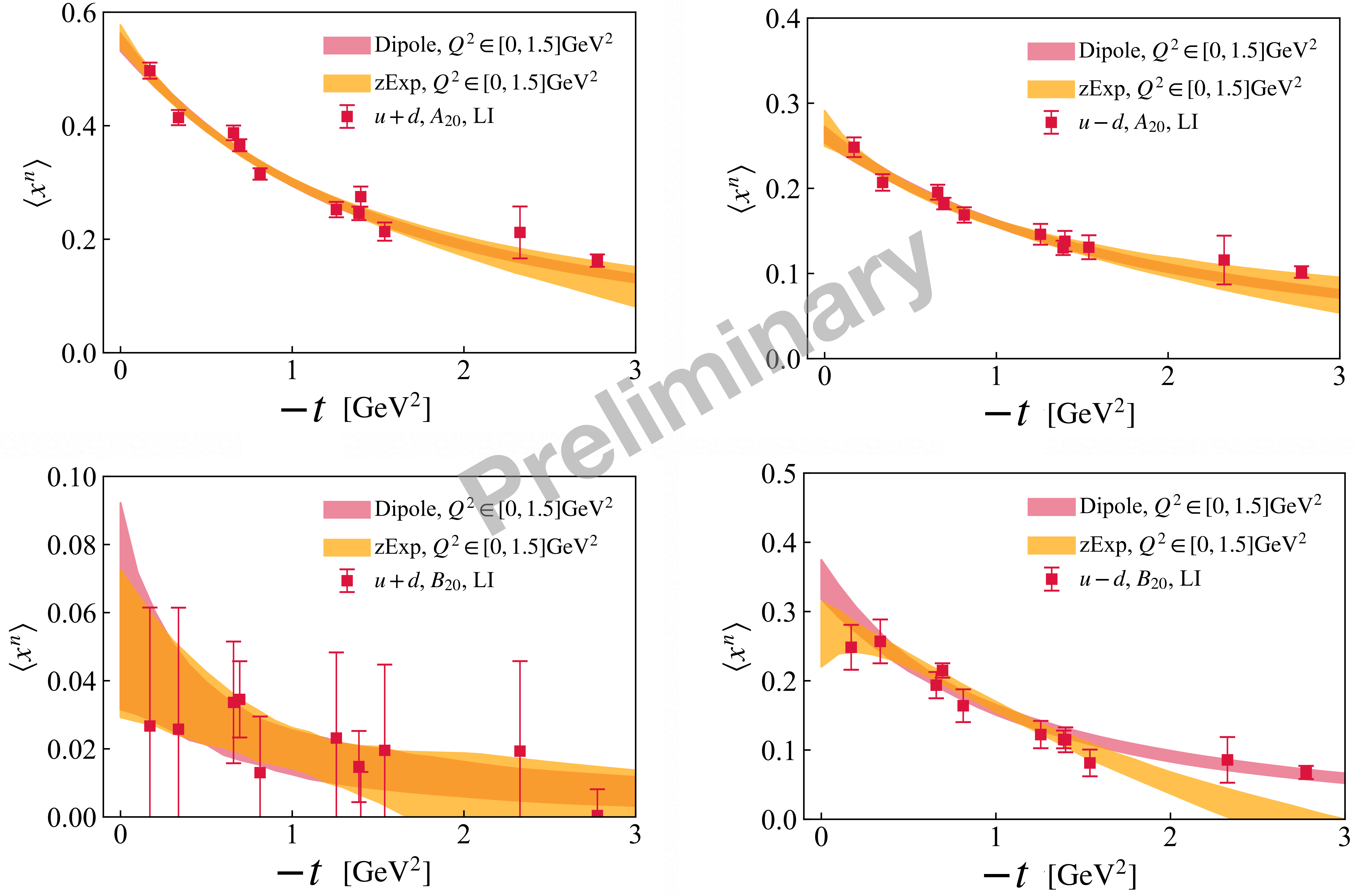
$$z(Q^2) = \frac{\sqrt{t_{\rm cut}+Q^2} - \sqrt{t_{\rm cut}-t_0}}{\sqrt{t_{\rm cut}+Q^2} + \sqrt{t_{\rm cut}-t_0}}$$

$$-t = Q^2$$

$-t \to 0$	Di-pole	z-expension	ETMC'11
A_{10}^{u-d}	0.97(04)	1.05(06)	1
A_{10}^{u+d}	2.87(07)	2.80(08)	–
B_{10}^{u-d}	2.68(18)	2.56(27)	2.61(23)
B_{10}^{u+d}	0.38(38)	0.19(36)	–

- The A_{10} and B_{10} are Dirac and Pauli form factors.

t-dependence of moments: A_{20} & B_{20}



$-t \to 0$	Di-pole	z-expension	ETMC'11
A_{20}^{u-d}	0.263(10)	0.270(21)	0.264(13)
A_{20}^{u+d}	0.547(16)	0.557(21)	0.613(14)
B_{20}^{u-d}	0.346(28)	0.267(47)	0.301(47)
B_{20}^{u+d}	0.065(29)	0.050(22)	-0.046(43)

Quark total angular momentum:

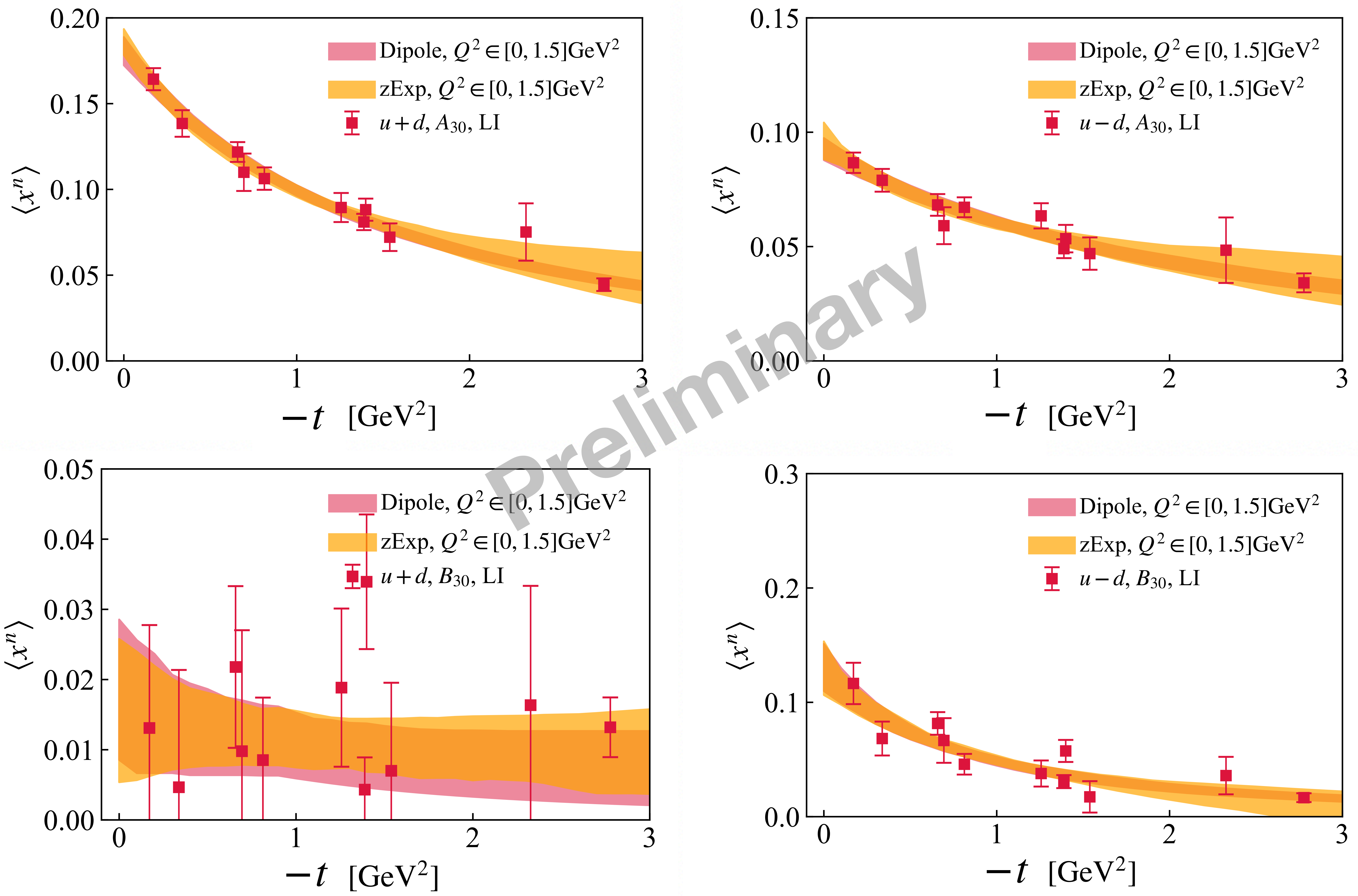
$$J^q = \frac{1}{2}(A_{20}^q(0) + B_{20}^q(0))$$

$$J^{u-d} = 0.267(27)(39)$$

$$J^{u+d} = 0.301(14)(02)$$

- The A_{20} and B_{20} are gravitational form factors.

t-dependence of moments: A_{30} & B_{30}



- A_{30} and B_{30} show reasonable signal and smooth t dependence.

$-t \to 0$	Di-pole	z-expension
A_{30}^{u-d}	0.093(05)	0.096(08)
A_{30}^{u+d}	0.181(08)	0.186(07)
B_{30}^{u-d}	0.130(21)	0.130(23)
B_{30}^{u+d}	0.018(10)	0.015(10)

Summary and outlook

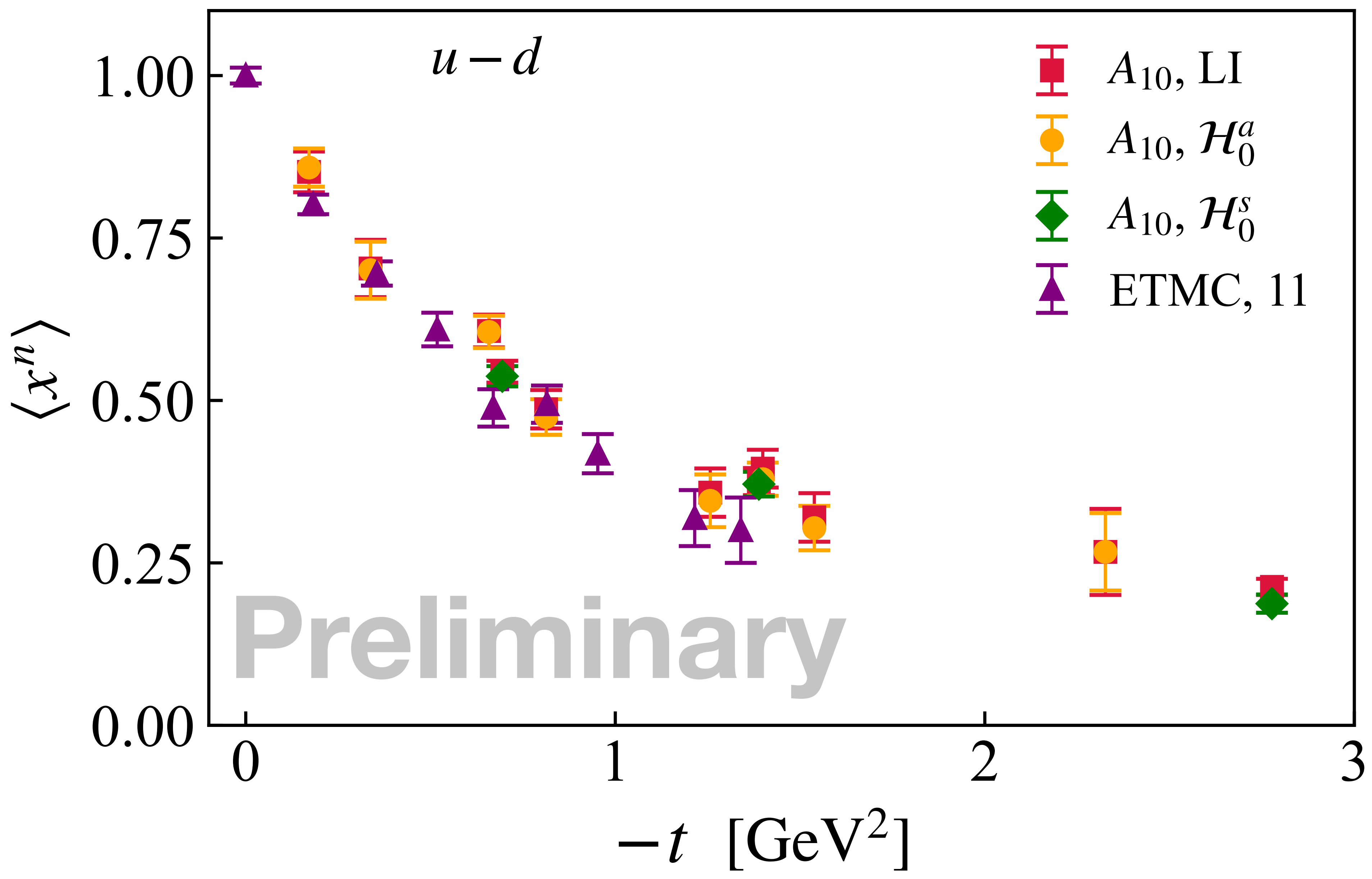
- We carried out lattice calculation of the quasi-GPD matrix elements of proton using the Lorentz invariant definition.
- The matrix elements are renormalized in ratio scheme and the first few Mellin moments up to A_{30} and B_{30} were extracted using the leading-twist short distance factorization frame work.
- Higher moments can be constrained with higher momentum and statistics, and the methods can be extended to non-zero skewness GPDs.
- Calculations with physical quark masses and smaller lattice spacings are needed to address the lattice artifacts.

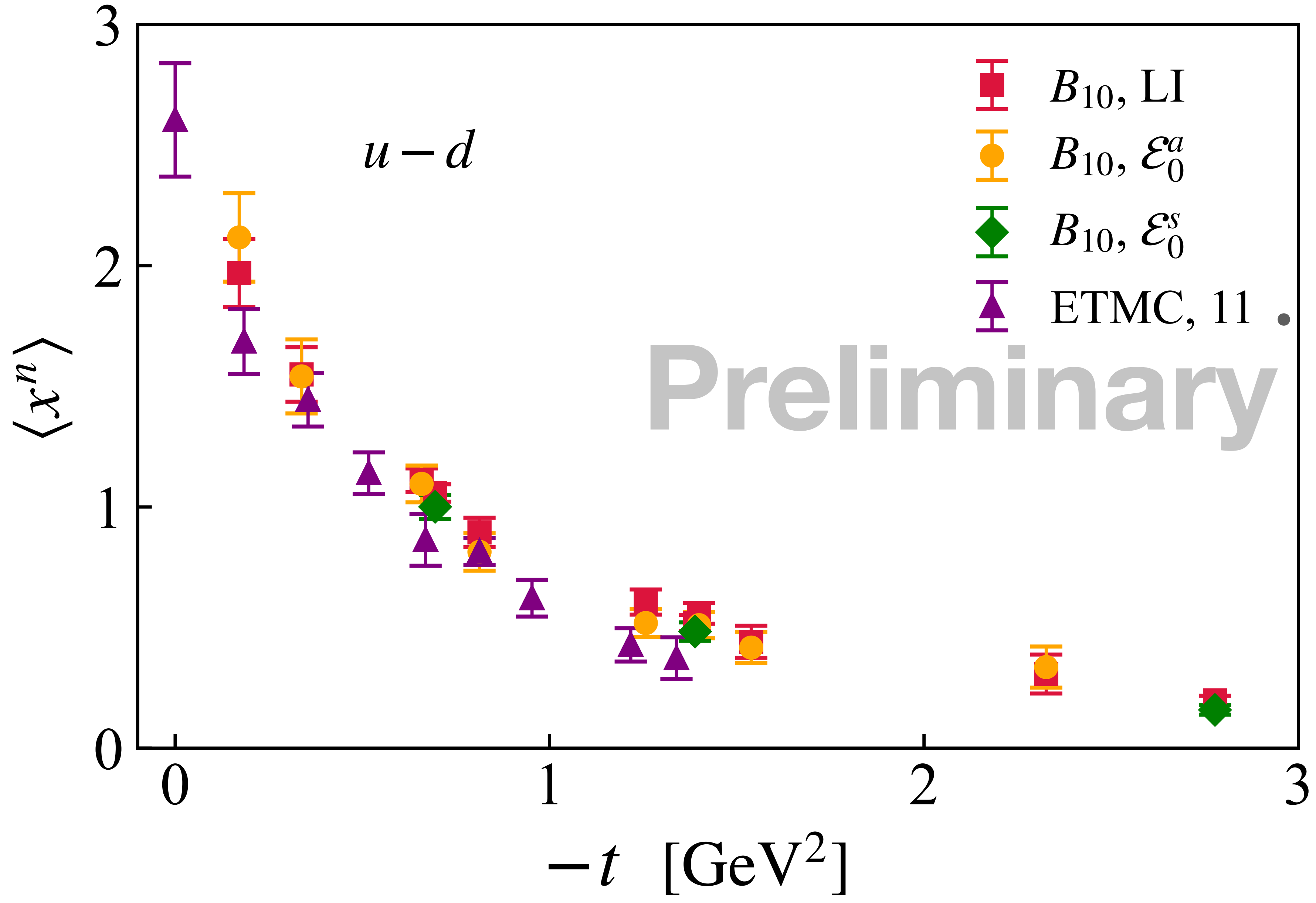
I acknowledge financial support from The Gordon and Betty Moore Foundation and the American Physical Society to present this work at the GHP 2023 workshop.

Thanks for your attention

Back up

t-dependence of moments



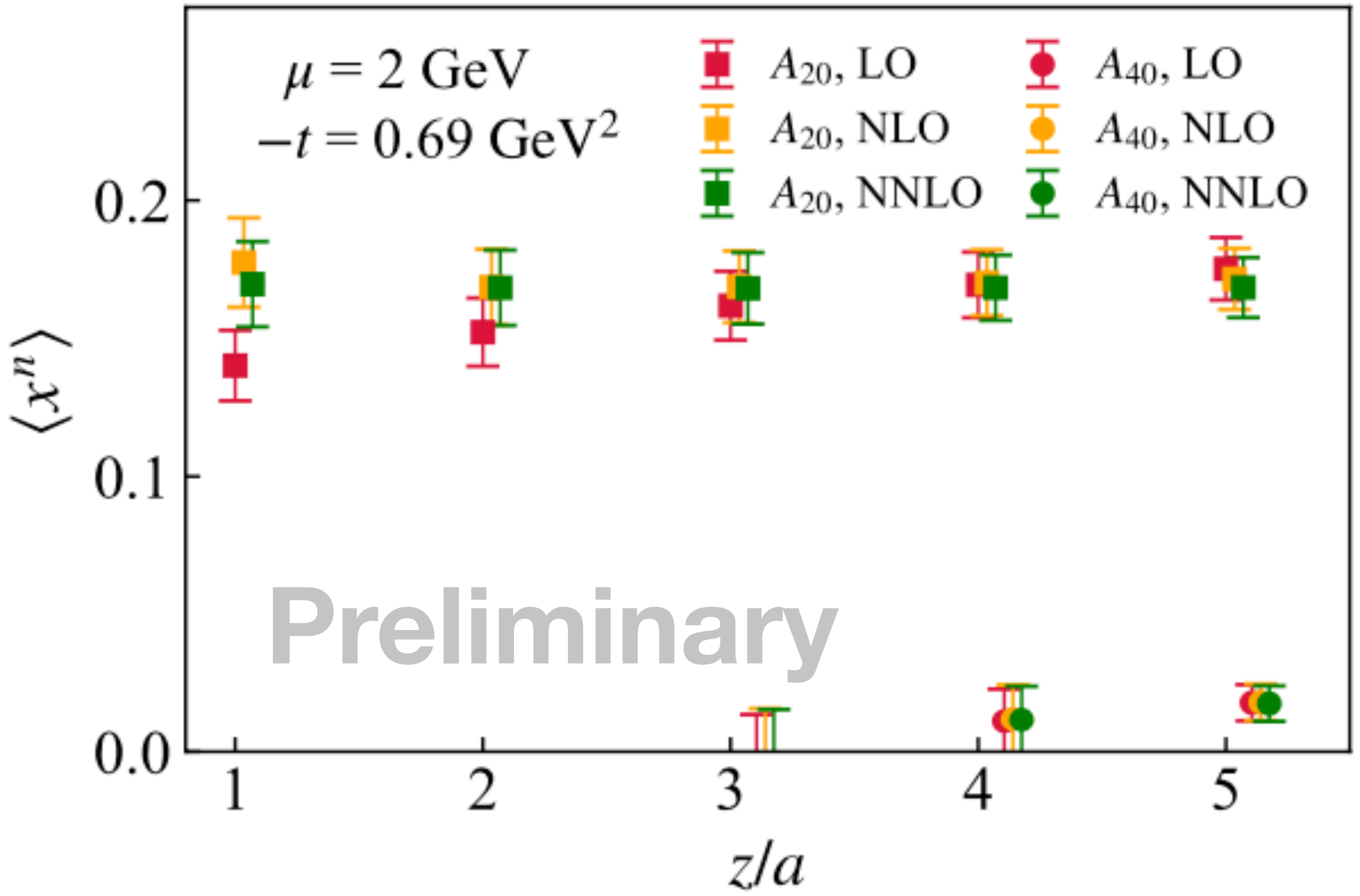


- ETMC, PRD 83 (2011)

- The A_{10} and B_{10} are Dirac and Pauli form factors.
- No difference between different definition.
- Good agreement with literatures using similar lattice setup.

Mellin moments of GPDs

$$\mathcal{M}(z^2, zP, \Delta^2) = \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} \frac{C_n(z^2\mu^2)}{C_n(z^2\mu^2)} \langle x^n \rangle + \mathcal{O}(z^2\Lambda^2_{\rm QCD})$$



Odd moments $\langle x \rangle$ and $\langle x^3 \rangle$ (or A_{20} and A_{40}) extracted from imaginary part of matrix element at each z by fitting P dependence.

- The tree-level ($\alpha_s = 0$) result show mild z dependence.
- Beyond LO, the z dependence is compensated by the Wilson coefficients and produce the z-independent plateau.
- NNLO produce similar results with NLO within current statistical errors.
- Signal for higher moments is weak: requiring higher momentum and statistics.