DEPARTMENT OF PHYSICS

LEADING POWER ACCURACY IN LATTICE QCD CALCULATION OF PARTON DISTRIBUTIONS

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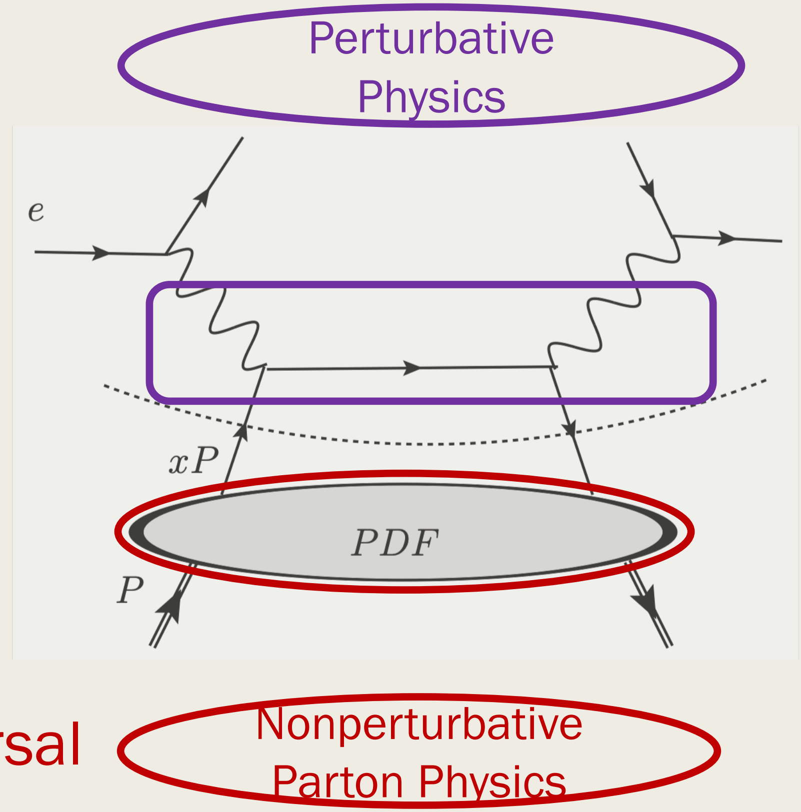
In collaboration with Xiangdong Ji (UMD), Yushan Su (UMD), and Jack Holligan (MSU)

Parton Distribution Functions



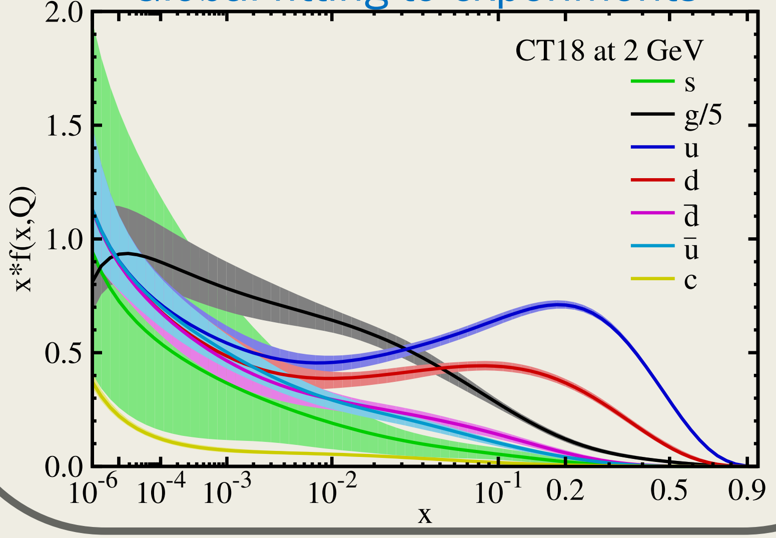
Universal

Parton Distribution Function

(Inclusive process)

probability density of finding a parton with momentum fraction x out of the hadron

Global fitting to experiments



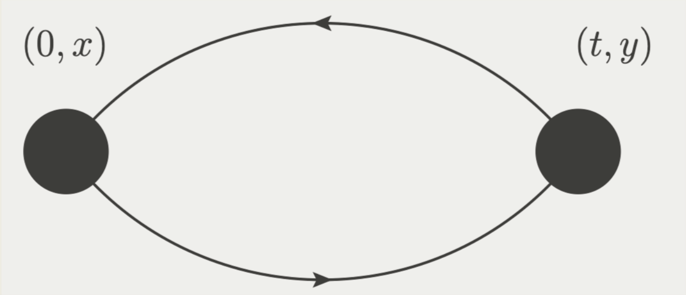
Important inputs to collider physics!

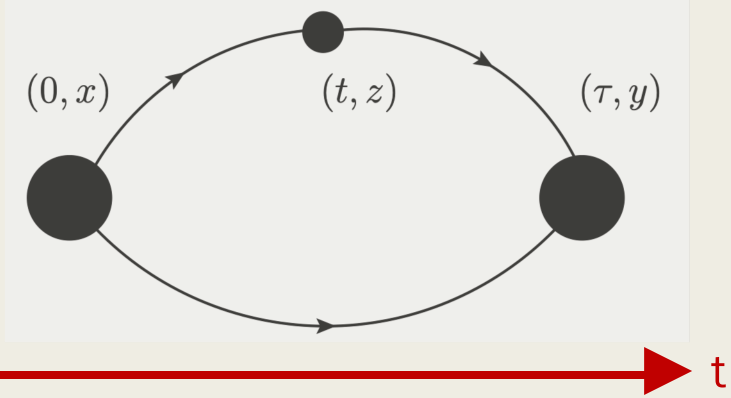
CTEQ PRD, (2019)

Lattice QCD

- Discretization of QCD action:
- Construction of correlators:

$C_2(t) = \langle \chi_{src}(0) | \chi_{snk}(t) \rangle$ $\quad$ $C_3(t) = \langle \chi_{src}(0) | O(t) | \chi_{snk}(\tau) \rangle$





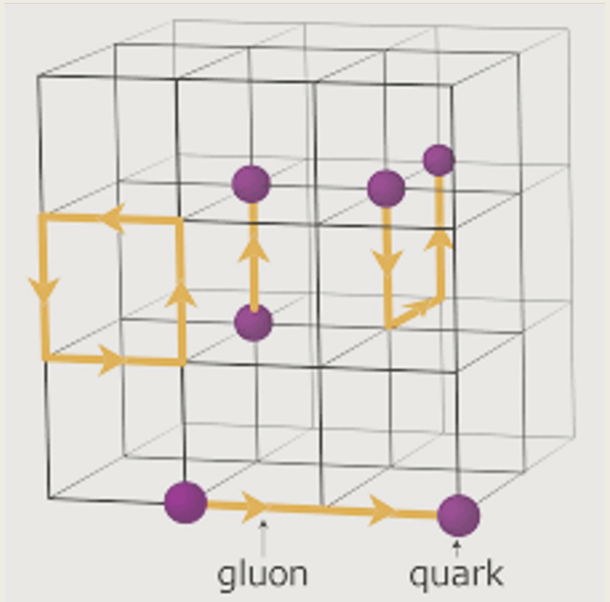
- Extraction of matrix elements:

$$C_2(t) = \sum |c_n|^2 e^{-E_n t}$$

$$C_3(t, \tau) = \sum c_m^* c_n \langle m | O | n \rangle e^{-E_m(\tau - t)} e^{-E_n t}$$

K.G. Wilson, Nobel Prize Winner (1982)

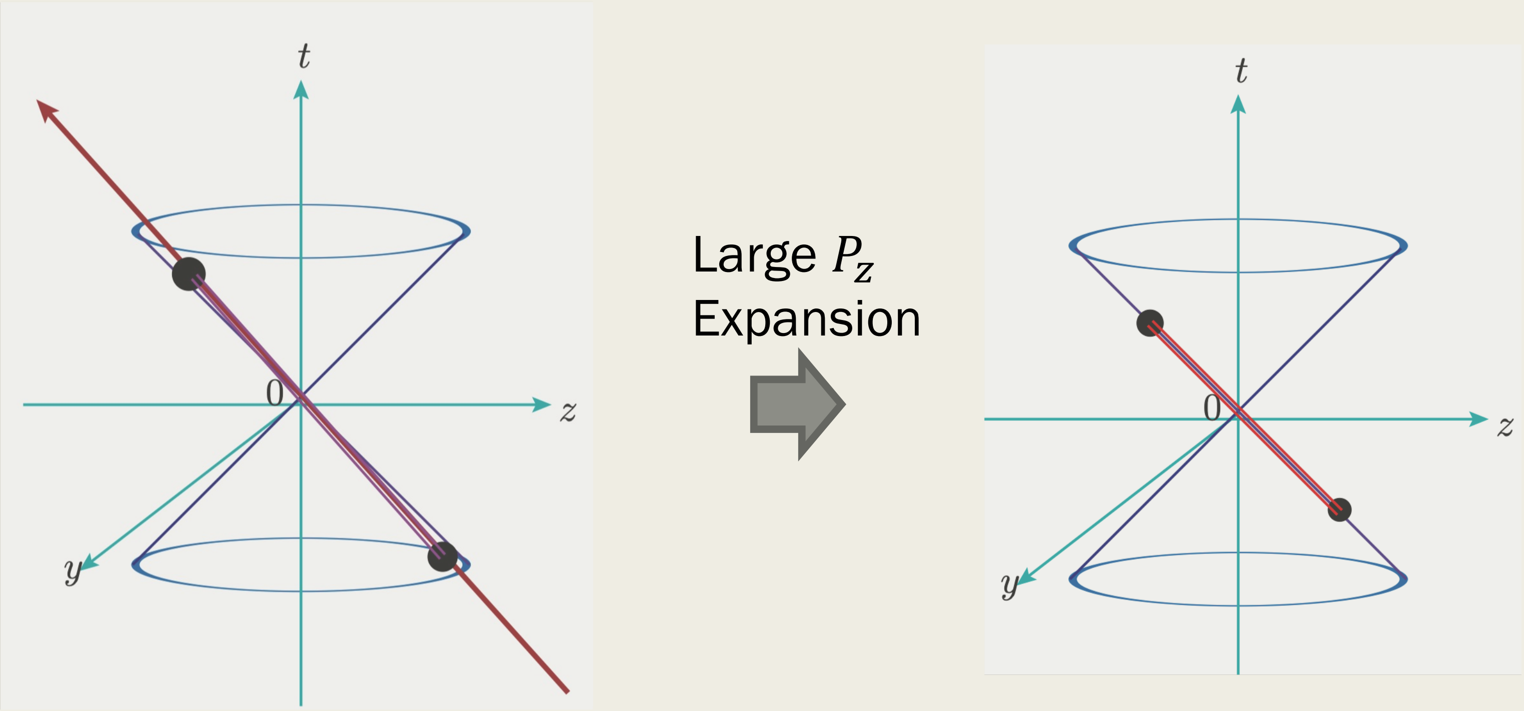
Euclidean 4D spacetime



Savage, NNPSS (2015)

Large Momentum Effective Theory (LaMET)

Ji, PRL (2013)
Ji, SCPMA(2014)



$+\mathcal{O}\left(\frac{1}{P_z^n}\right)$

Quasi-PDF: $\tilde{q}(x, P_z) = \int \frac{dzP_z}{2\pi} e^{ixzP_z}\langle P|\bar{q}(0)\gamma_t U(0,z)q(z)|P\rangle$

$C\ (x, y, \mu, P_z) \otimes q(y, \mu)$

$+\mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}\right)$

Recipe

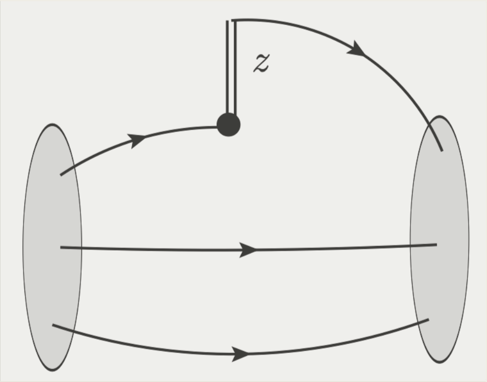
Lattice correlator

Fitting matrix elements

Renormalization

Extract x-dependence

Matching to parton physics

$C_{3pt} =$ 

$C_{3pt} = |c_0|^2 \langle 0|O|0\rangle e^{-E_0\tau} + \cdots$

$h^R(z) = \langle 0|O|0\rangle / Z^R(z,a)$

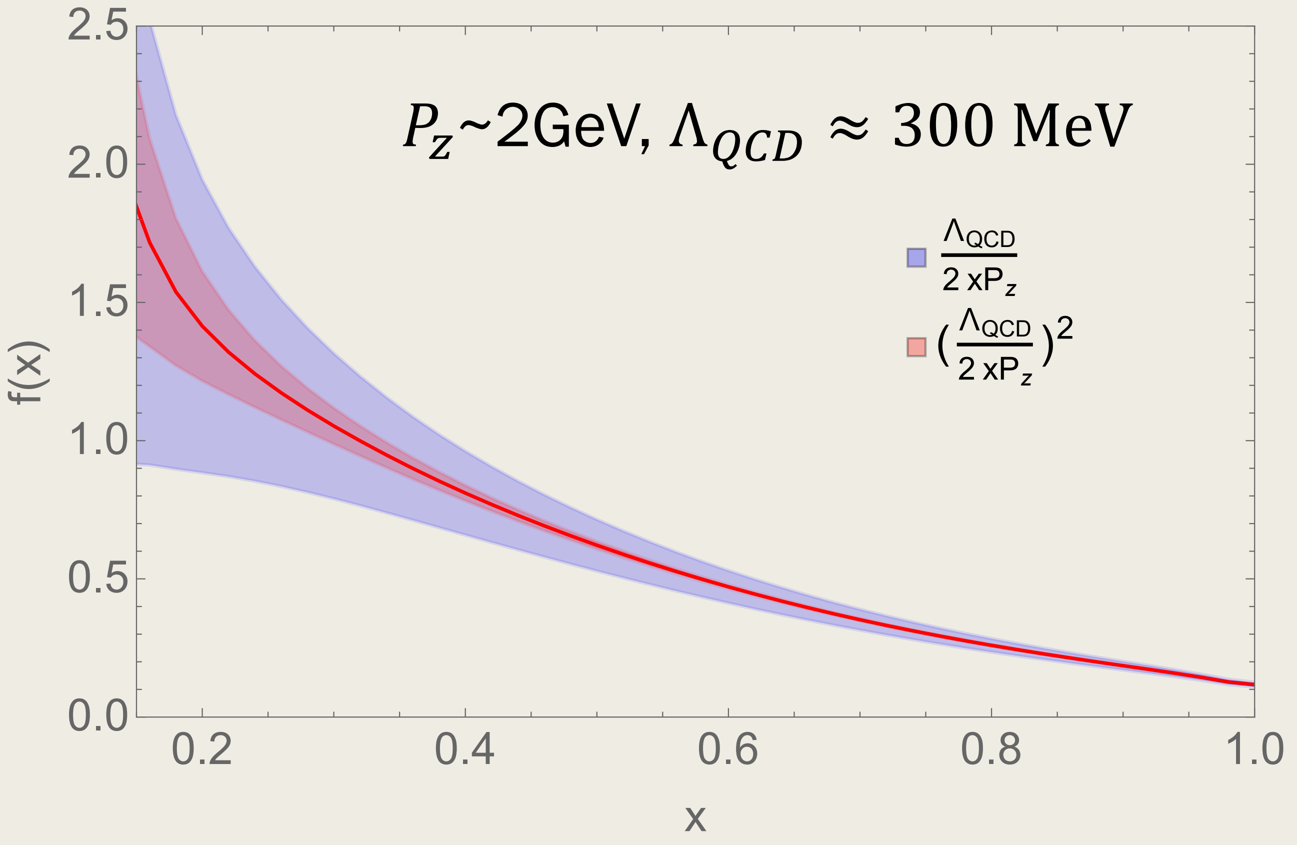
$\tilde{q}(x,P_z) = \int \frac{dzP_z}{2\pi} h^R(z) e^{ixzP_z}$

$q(x,\mu) = C^{-1}(x,y,\mu,P_z) \otimes \tilde{q}(y,P_z)$

Precision of Large Momentum Expansion

Ji, et.al, NPB (2021)

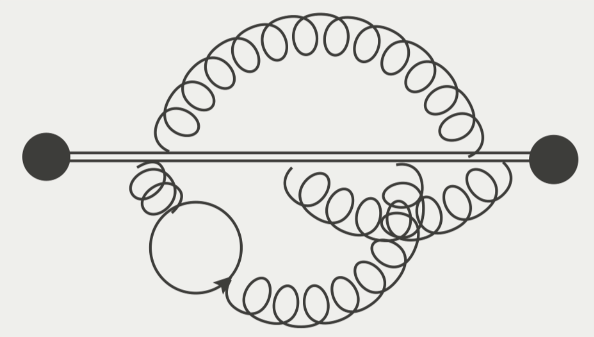
$$\tilde{q}(x,P_z) = C\,(x,y,\mu,P_z) \otimes q(y,\mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{x\,P_z}\right) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}\right)$$



Linear power correction must be eliminated!

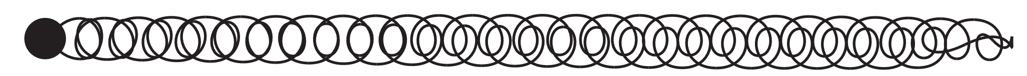
Not properly addressed in previous work

Why is there a linear correction in $1/P_z$?



Ji, et.al, PRL (2017)

- Non-local operator: $\bar{q}(0)\Gamma U(0,z)q(z)$
- Linearly divergent self-energy $\delta m(a) \sim \frac{1}{a}$
  - *A heavy quark propagating with "pole mass"* $\delta m(a)$
  - $h^B(\mathrm{z}) \sim e^{-\delta m(a)\cdot z}$
- What to subtract w/ linear divergence? Freedom to choose the scheme
- Pole mass of a "free" quark?



  - Long range interactions contributing $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ ambiguously

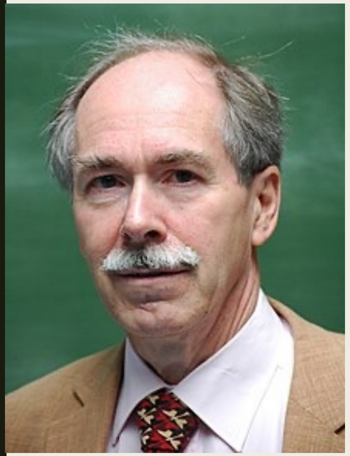
Beneke, PLB (1995)

- $h^R(\mathrm{z}) \sim h^B(\mathrm{z})e^{\delta m\cdot z}$ uncertain up to $e^{\mathcal{O}(\mathrm{z}\Lambda_{\mathrm{QCD}})} \to \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{\mathrm{x}\,P_z}\right)$ in $\tilde{q}$

Perturbative determination of $\delta m(a)$

- In perturbation theory, $\delta m = \frac{1}{a}\sum \alpha_s^{n+1}(a) r_n$
  - $r_n \sim n!$ ➡ Series is divergent for any α_s
- A lattice perturbative expansion of $\delta m(a)$ to 20th order

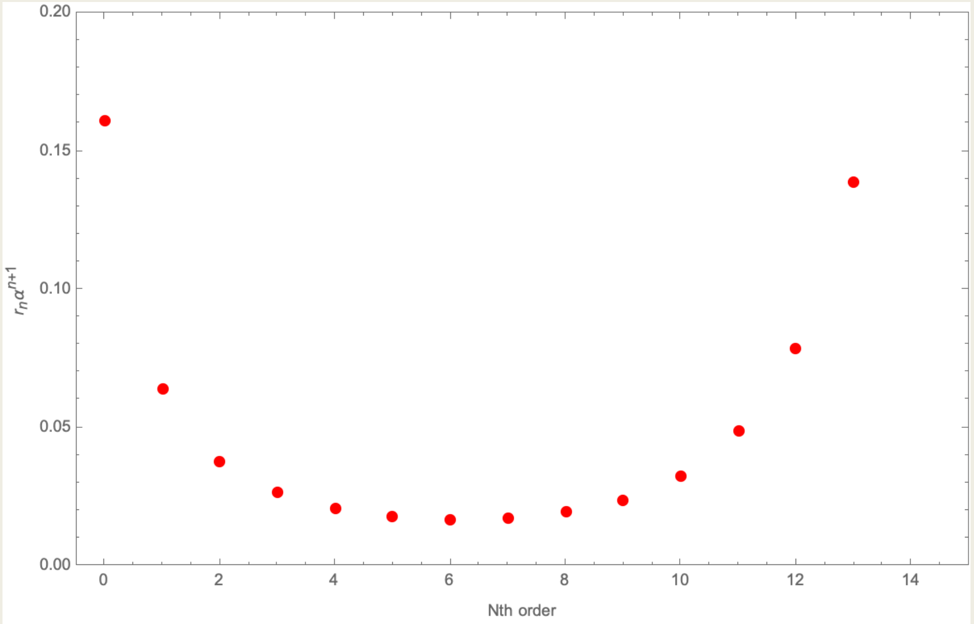
Bali, et al., PRD (2013)



Gerard 't Hooft
1999 Nobel Prize

Renormalon Divergence

Infrared renormalon is partly related to the strong coupling $\alpha_s(k)$ becoming non-perturbative in the region $k \sim \Lambda_{QCD}$.



Beneke, RMP (1998)

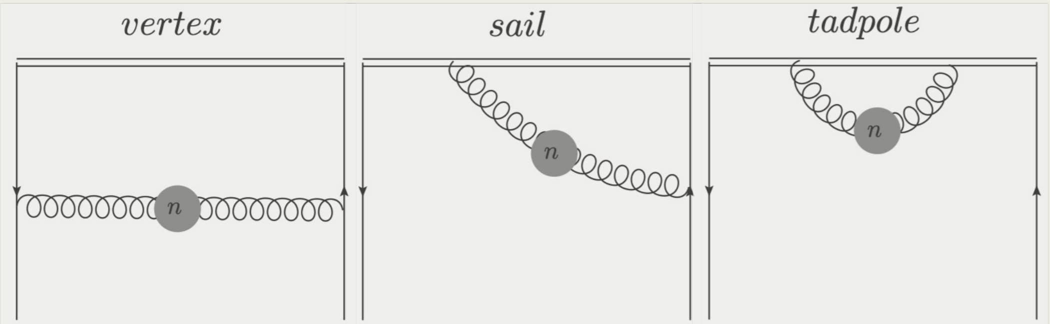
Linear correction in matching coefficients

$$\tilde{q}(x, P_z) = C(x, y, \mu, P_z) \otimes q(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{x\, P_z}\right) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}\right)$$

- $C(x, y, \mu, P_z)$ is obtained by perturbatively calculate the same operator, thus also has the same ambiguity:

$$C^{(n+1)} \sim n!$$

Braun, et.al., PRD(2018)



Achieve Power Accuracy

- Regularizing the infrared physics
  - *Explicit IR cut off:* $\int_0^{\Lambda_{\mathrm{UV}}} f(k)dk \to \int_{\Lambda_{\mathrm{IR}}}^{\Lambda_{\mathrm{UV}}} f(k)dk$

    Very difficult to calculate

  - *Resumming the series to all orders with some prescription:*

    Ayala, PRD (2019)
    Ayala, PRD (2020)

$$\sum_i^{\infty} \alpha_s^{i+1} r_i \to \int_C du\, e^{-u/\alpha_s} \sum_i^{\infty} \frac{r_i u^i}{i!}$$

    Seems impossible to know high order terms?

    But we know the divergent part

  - *Neutralize color charge of the heavy quark*
    - Non-perturbative determination of $\delta m(a)$
    - Depending on how to choose fitting parameters

    Applicable to lattice data

- Eliminate scheme dependence

Regularization scheme dependence:

- Introduce twist-three correction

Matching Coefficients

$$h^R(z, P_z, \mu, \tau) = \left(1 - m_0(\tau) z\right) \sum_{k=0}^{\infty} C_k\left(\alpha_s(\mu), \mu^2 z^2\right) \lambda^k a_{k+1}(\mu) + \mathcal{O}(z^2)$$

$\sim e^{\delta m(a)\cdot z} h^B(z)$

PDF moments

$$= \sum_{k=0}^{\infty} \left[C_k\left(\alpha_s(\mu), \mu^2 z^2\right) - z m_0(\tau)\right] \lambda^k a_{k+1}(\mu) + \mathcal{O}(z\alpha_s, z^2),$$

- Twist-3 ambiguities exist in both sides, h^R and C_k
- $m_0(\tau)$ matches schemes between renormalization of lattice data and regularization of the matching coefficients

Achieve Power Accuracy

$$\tilde{q}(x, P_z) = C(x, y, \mu, P_z) \otimes q(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{x P_z}\right) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{x^2 P_z^2}\right)$$

Renormalize with scheme-dependent non-perturbative parameter $m_0(\tau)$ $Z(a, \tau) \propto e^{(\delta m(a) + m_0(\tau)) \cdot z}$

+

Define τ scheme: Leading renormalon resummation (LRR)

Matching Condition

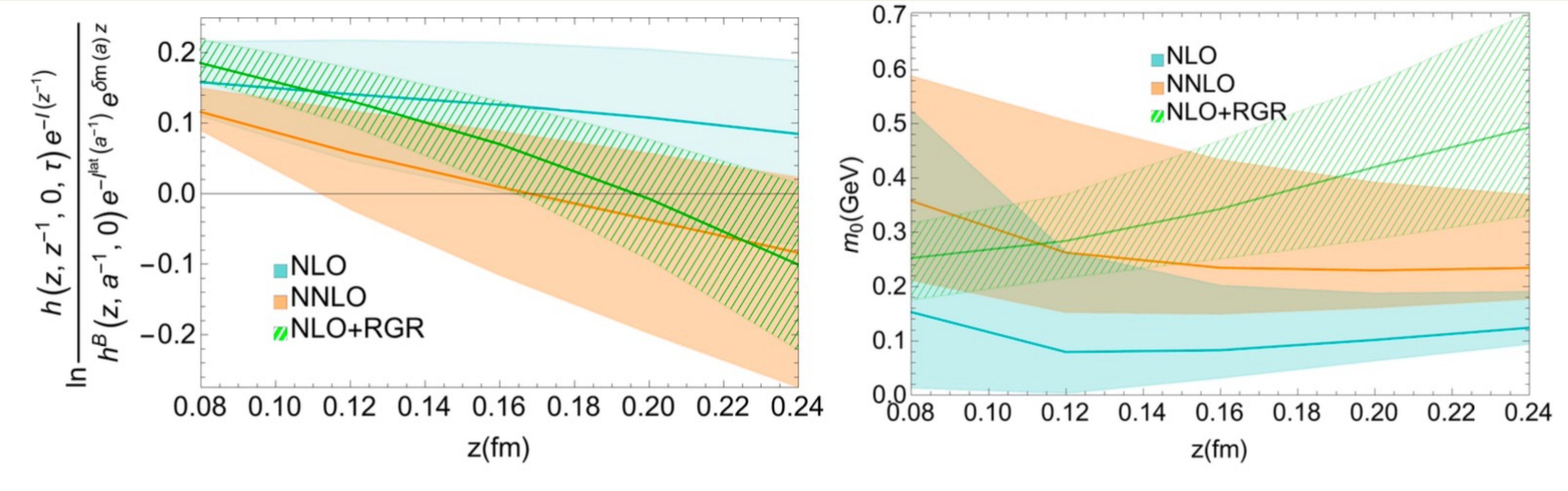
Extract $m_0(\tau)$: Fitting to $P_z = 0$ Lattce data

Data from Gao, et.al, PRL, (2022)

Extract $m_0(\tau)$ from fixed-order theory

$$\ln\left(\frac{h^R(z, P_z = 0, \mu)}{C_0(z, \mu^2 z^2)}\right) = c + m_0(\tau) z$$

Too large uncertainty!



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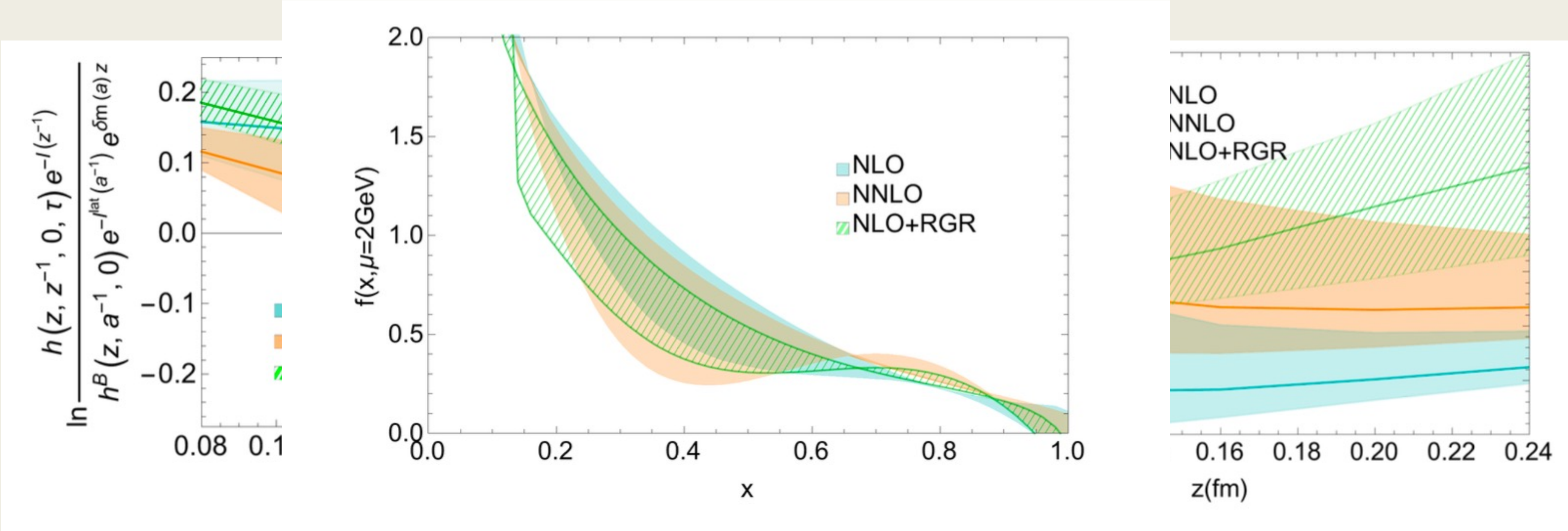
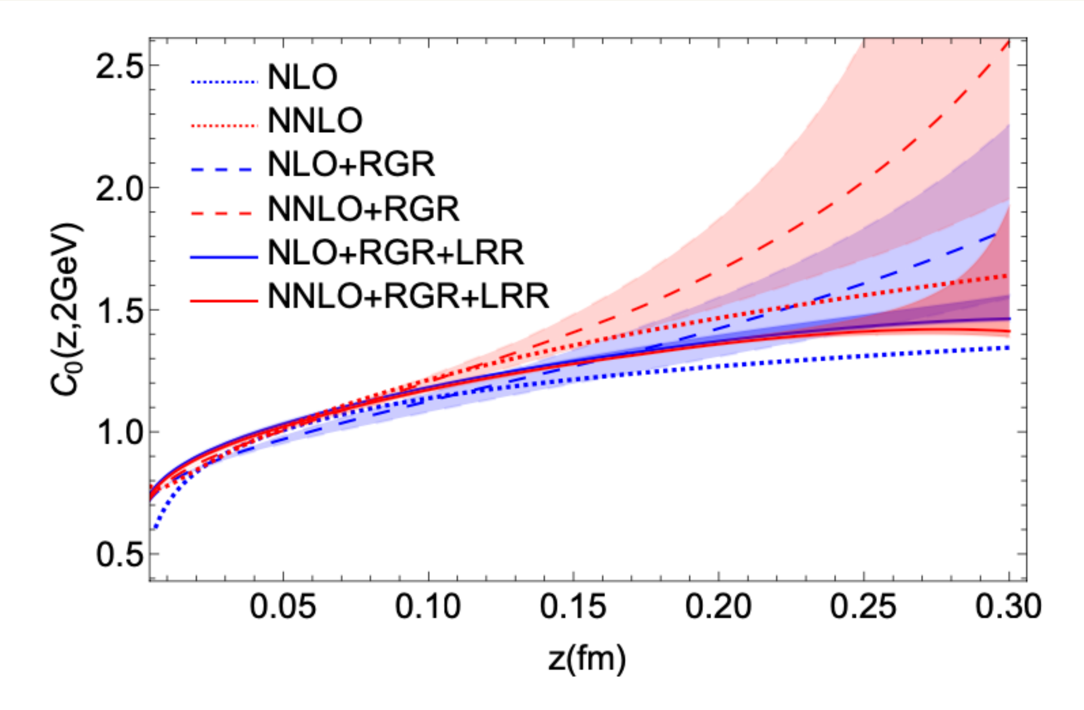
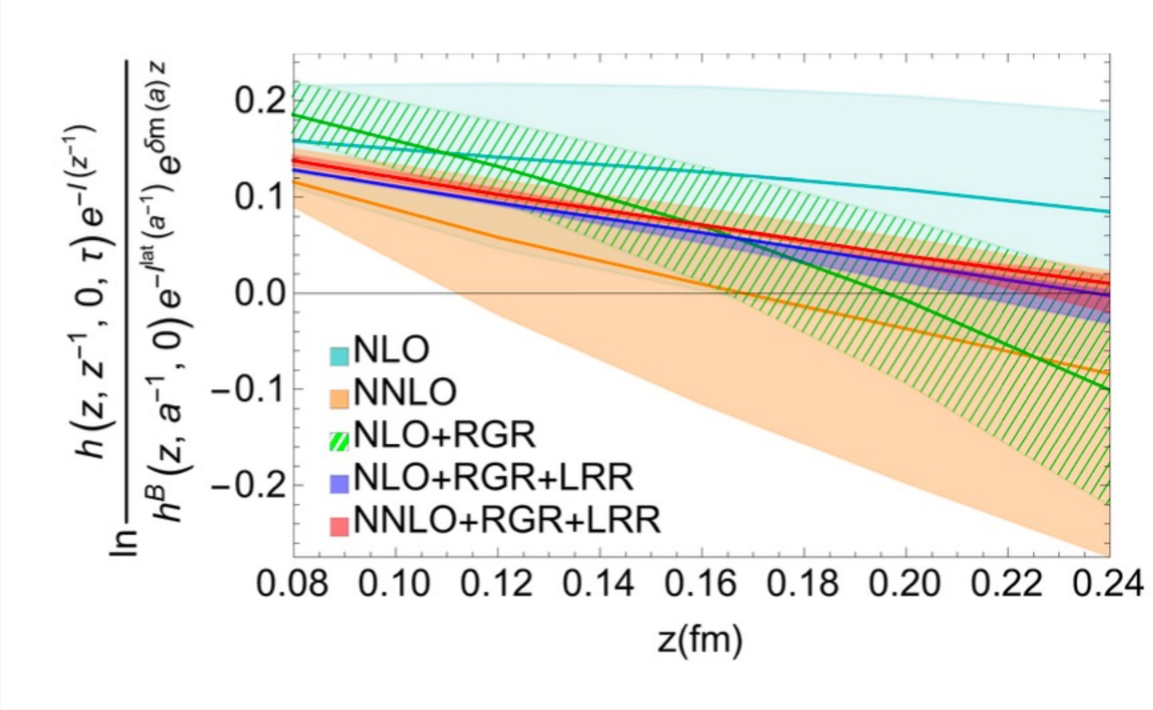


LRR improved perturbation theory

$C_0(z,\mu^2 z^2)$:



$\ln\left(\frac{h^R(z,P_z=0,\mu)}{C_0(z,\mu^2 z^2)}\right)$:



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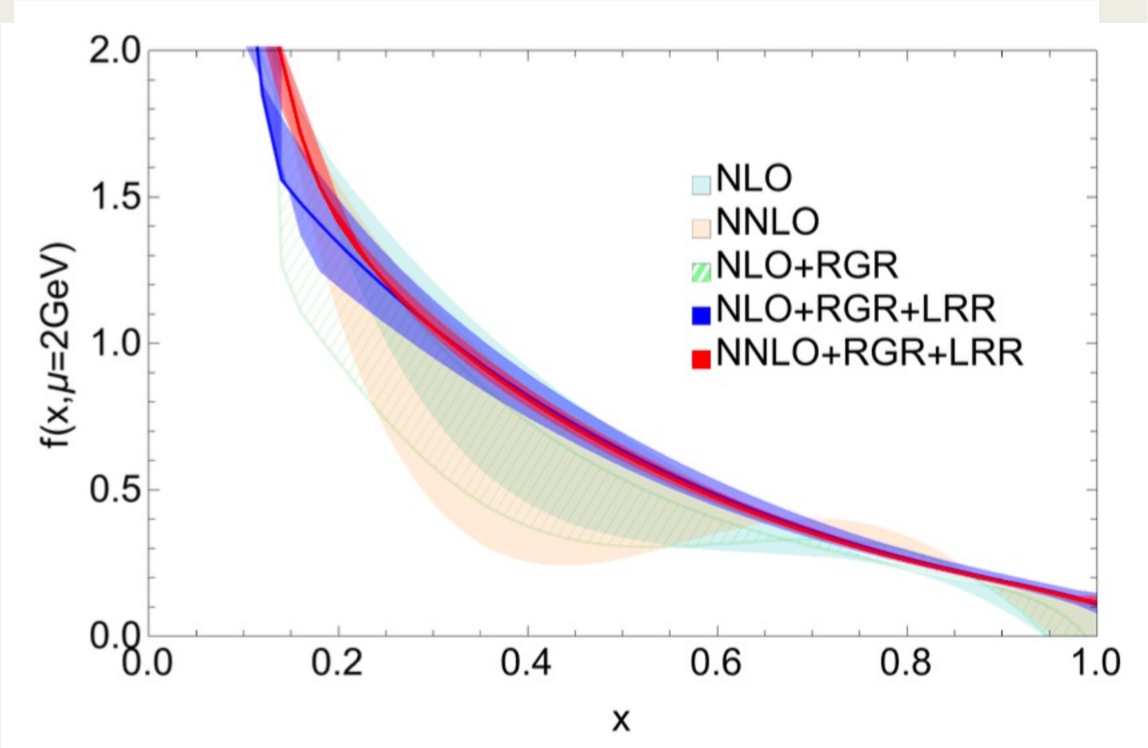
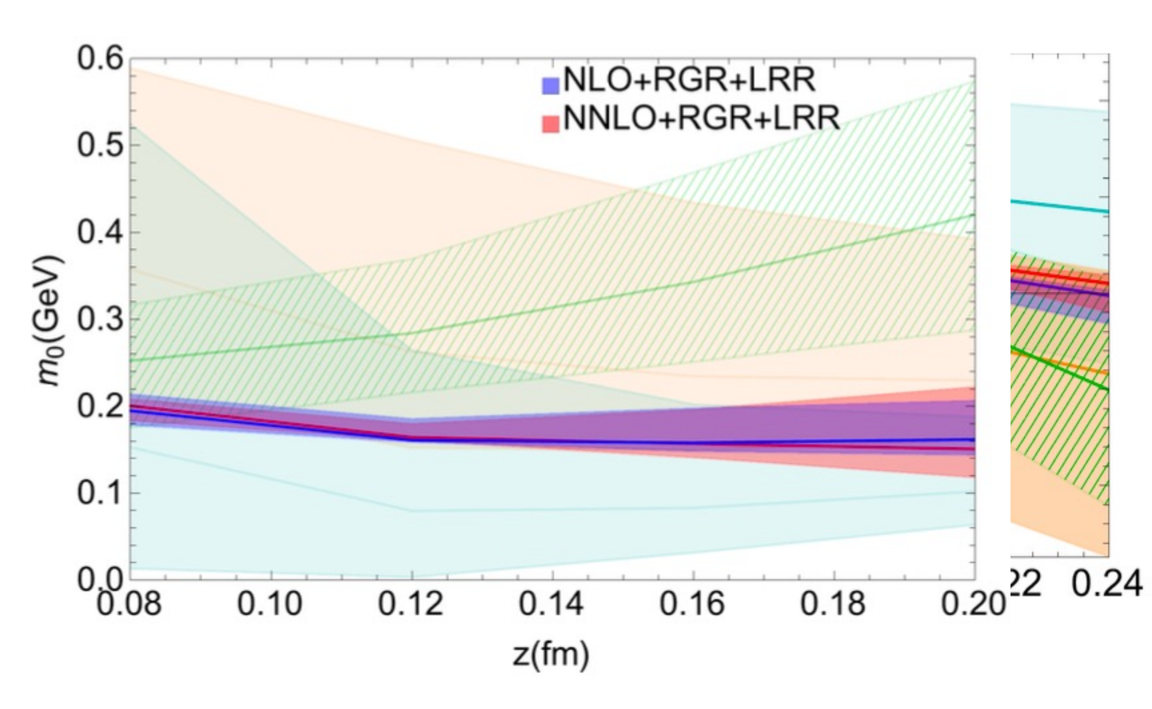


$\ln\left(\frac{h^R(z, P_z=0, \mu)}{C_0(z, \mu^2 z^2)}\right)$:



- Reduce the uncertainty 3~5 times from scale variation
- Improve the convergence when going to higher order

Summary

- Parton physics can be calculated from lattice QCD through large momentum expansion
- Power correction is an important source of systematic uncertainty
- We propose the first systematic approach to achieve $1/P_z$ accuracy

Outlook

- More solid determination of the renormalon contribution
- Generalization to more complicated parton observables