CONFINEMENT and COLOR VORTICES in CHROMOSTATICS

dennis sivers U. Michigan & Portland Phys. Inst.

II. Lattice Studies of Color Vortices

Lattice QCD most important tool for understanding color confinement

Jeff Greensite An Intro. to the Confinement Problem

Detailed Calculations Hadron spectra & parton distributions

Engelhardt & Collaborators
Leinweber & Collaborators

Role of Color Vortices in the Confinement mechanism Emergent Structures

Lattice Gauge Field Theory and QCD

ab initio approach to solving no-Abelian field equations in 4-dim Euclidean space

Hadronic spectra: baryons, mesons, heavy-quark states and exotics

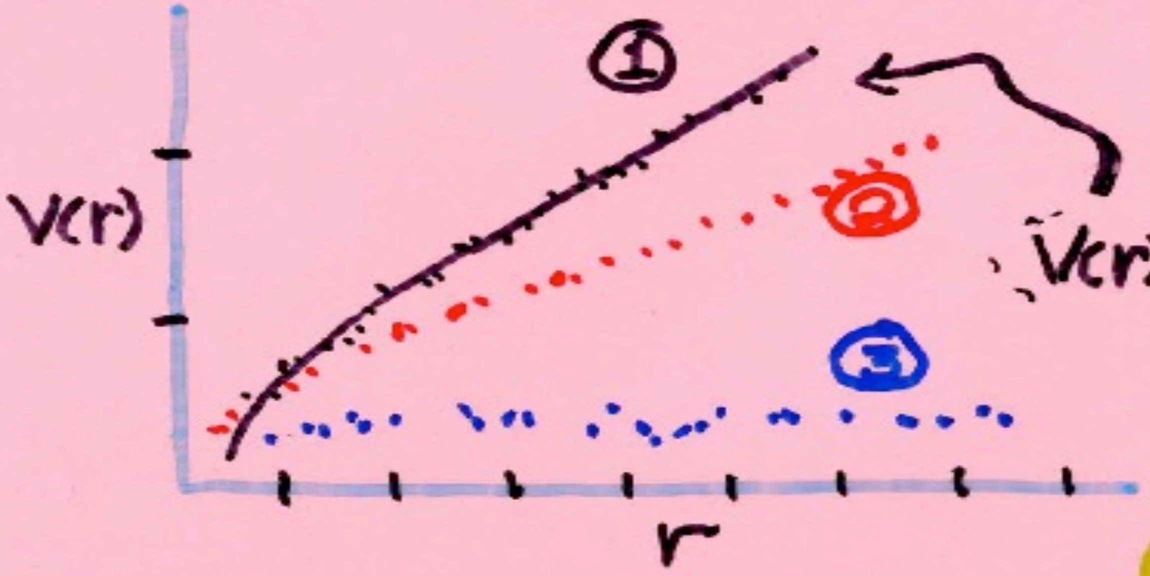
Area-law behavior for Wilson Loops

constituent distributions and non-perturbative structure

COLOR VORTICES and CONFINEMENT

Lattice Gauge field theory - Monte Carlo solutions to Yang-Mills Maxwell equations

It is possible to generate configurations with or without color vortices (center group)



1. regular
2. vortices only
3. vortices removed

$$V(r) = c + \frac{\alpha_s}{r} + \sigma r$$

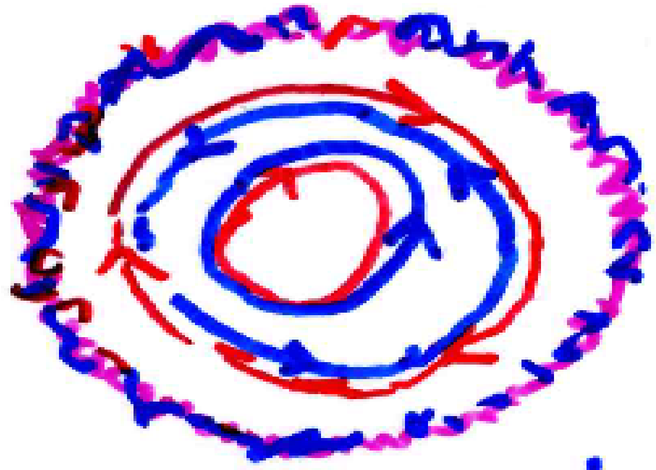
Cornell Potential ($Q\bar{Q}$)

Bowman et al
PR D84 034501 (2011)

Leinweber & Collaborators – dynamical fermions increase density of color vortices in vacuum

pure gauge	3277 ± 156
full QCD	5923 ± 259

Dynamical fermions improve fit for confining potential and for gluon Landau gauge propogator



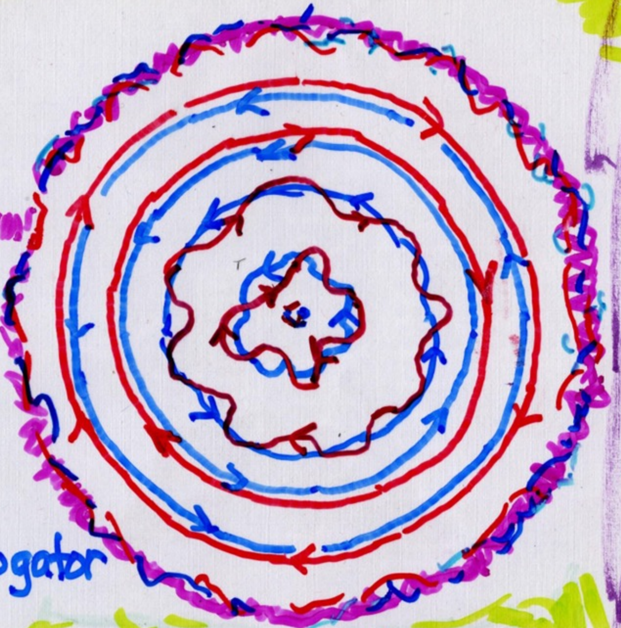
SU(2) Chromostatics
Interior of hadron
one chiral vortex

Leinweber & Collaborators

dynamical fermions increas density of color vortices in vacuum

pure gauge 3277± 156

full qcd 5923 ± 259



Improves fit for confining potential & Landau gauge propogator

$^3P_0$ $q\bar{q}$ pairs $J^{PC}=0^{++}$

Chromostatics

Interior Volume of hadron

one chiral adiabatic vortex $J^{PC}=0^{++}$

2019

The Strong Conjecture

The confinement mechanism for QCD involves a domain wall of topological (cp-odd) charge separating the interior volume of hadrons from an exterior volume

SU(2) Color

3 Pauli matrices: $\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\tau_a \tau_b = \delta_{ab} + i\,\varepsilon_{abc}\,\tau_c$ SU(2) generators $T_a = \frac{1}{2}\tau_a$

SU(2) color charge operator $Q = e\,T_3 = \frac{1}{2}\begin{pmatrix} e & 0 \\ 0 & -e \end{pmatrix}$

« bots » form a color doublet $|b\rangle = \begin{pmatrix} b_R \\ b_B \end{pmatrix}$ $Q|b\rangle = \frac{1}{2}e\begin{pmatrix} b_R \\ -b_B \end{pmatrix}$

The gauge connection carries adjoint charge

$A^\mu = A^\mu_a T_a$ $[Q, A_\mu] = e[T_3, A_\mu] = |C^+)E_{+1} + |C^-)E_{-1}$

'color'	b_R	b_B	A^μ_3	$\lvert C^+)$	$\lvert C^-)$
	$\frac{1}{2}e$	$-\frac{1}{2}e$	0	e	$-e$

vector in color space: $|C^+) = (b_R \bar{b}_B)$

$|C^0) = \frac{1}{2}(b_R \bar{b}_R + b_B \bar{b}_B)$

$|C^-) = (b_B \bar{b}_R)$

ADJOINT COLOR CHARGES SU(2) & SU(3)

charge operator SU(2)

$$Q = eT_3 = \frac{1}{2}\begin{pmatrix} e & 0 \\ 0 & -e \end{pmatrix}$$

$$[Q, A^{\mu}_{(2)}] = eC^{\mu}_{1}E_1 - eC^{\mu}_{-1}E_{-1}$$

charge operators SU(3)

$$Q_3 = gt_3 = \frac{1}{2}\begin{pmatrix} g & 0 & 0 \\ 0 & -g & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$Q_8 = gt_8 = \frac{1}{2\sqrt{3}}\begin{pmatrix} g & 0 & 0 \\ 0 & g & 0 \\ 0 & 0 & -2g \end{pmatrix}$$

$$[Q, A^{\mu}_{(3)}] = g\sum_{a=1}^{3} w_a(C^{\mu}_{a}E_a - C^{\mu}_{a}E_{-a})$$

spherical symmetry in the Bars-Witten gauge formalism aligns the radial directn in 3-space with the color-neutral diagonal direction or directions in group space. The transverse components of the gauge connection carry color charge

SU(N) {Gauge fields with Spherical Symmetry}

$$gA^a_0 = A_0(r,t)\,\hat r_a$$

$$gA^a_i = A_1(r,t)\,\rho_{ia} + \frac{a(r,t)}{r}\sin\omega(r,t)\,\delta^T_{ia} + \frac{a(r,t)\cos\omega(r,t)-1}{r}\,\varepsilon^T_{ia}$$

$$\rho_{ia} = \hat r_i \hat r_a \qquad \delta^T_{ia} = \hat\theta_i \hat\phi_a + \hat\phi_i \hat\theta_a \qquad \varepsilon^T_{ia} = \hat\theta_i \hat\theta_a - \hat\phi_i \hat\phi_a$$

$i = 1-3$
$a = 1-(N^2 \ldots$

for $N \geq 3$ there are $N-1$ axes for group space rotations

maximum abelian gauge

$$e^S_{ia}(\omega) = \delta^T_{ia}\cos(\omega(r,t)) - \varepsilon^T_{ia}\sin(\omega(r,t))$$

$$e^A_{ia}(\omega) = \delta^T_{ia}\sin(\omega(r,t)) + \varepsilon^T_{ia}\cos(\omega(r,t))$$

[gauge-dependent transverse tensors]

DEFINE ELECTRIC & MAGNETIC FIELDS

$$gE_i^a = E_L(r,t)\,\hat{r}_{ia} + E_s(r,t)\,\varepsilon^s_{ia}(\omega) + E_A(r,t)\,\varepsilon^A_{ia}(\omega)$$

$$gB_i^a = B_L(r,t)\,\hat{r}_{ia} + B_s(r,t)\,\varepsilon^s_{ia}(\omega) + B_A(r,t)\,\varepsilon^A_{ia}(\omega)$$

where $\omega = \omega(r,t)$

$$E_L(r,t) = -\frac{\partial}{\partial t}A_1 + \frac{\partial}{\partial r}A_0$$

$$E_s(r,t) = \frac{a}{r}\left[-\frac{\partial \omega}{\partial t} + A_0\right]$$

$$E_A(r,t) = -\frac{1}{r}\frac{\partial a}{\partial t}$$

$$B_L(r,t) = \frac{a^2-1}{r^2}$$

$$B_s(r,t) = -\frac{1}{r}\frac{\partial a}{\partial r}$$

$$B_A(r,t) = \frac{a}{r}\left[A_1 - \frac{\partial \omega}{\partial r}\right]$$

all nonlinearities associated with $B_L = \frac{a^2-1}{r^2}$ /

Yang-Mills Maxwell: $(D^{\mu} G_{\mu\nu})^a = J_\nu^a(r,t)$

$J_0^a(r,t) = \frac{1}{r^2} J_0(r,t)\, \hat{r}_a \quad J_i^a(r,t) = \frac{1}{r^2} J_1(r,t) P_{ia} + j_S(r,t)\, \epsilon^S_{ia}(\omega) + j_A(r,t)\, \epsilon_{ia}(\omega)$

classical adjoint currents

$$-\frac{\partial}{\partial r}(r^2 E_L) + 2 a r E_S = J_0(r,t)$$

$$-\frac{\partial}{\partial t}(r^2 E_L) + 2 a r B_A = J_1(r,t)$$

$$-\frac{\partial}{\partial t}(a r E_S) + \frac{\partial}{\partial r}(a r B_A) = a r j_S(r,t)$$

$$a\left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r^2}\right) a - r^2 (E_S^2 - B_A^2) - \frac{a^2(a^2-1)}{r^2} = a r j_A(r,t)$$

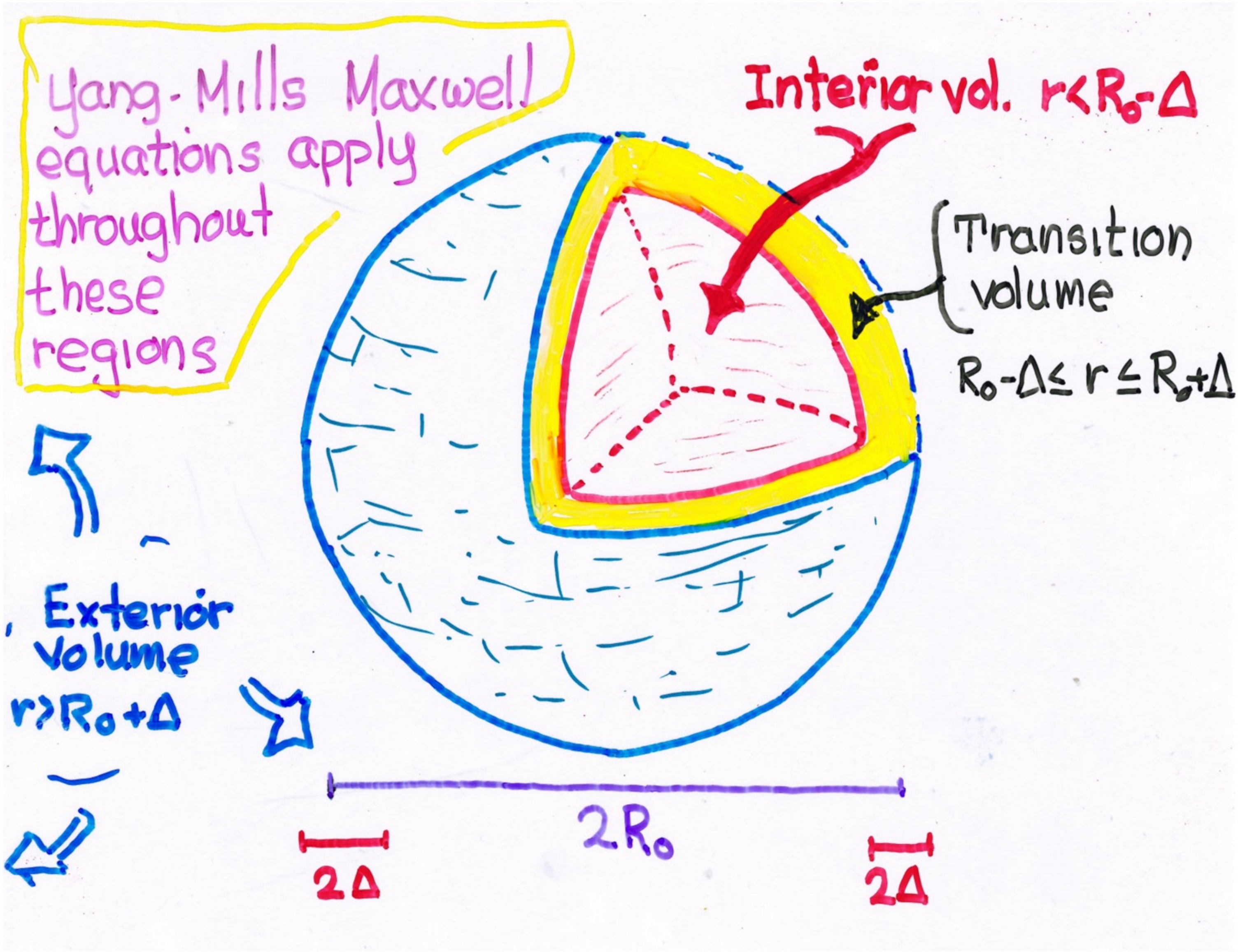
Bianchi constraints

$$\frac{\partial}{\partial r}(a r E_A) - \frac{\partial}{\partial t}(a r B_S) = 0$$

$$-E_L + \frac{\partial}{\partial r}(a r E_S) + \frac{\partial}{\partial t}(a r B_A) = g^2 r^2 E_i^a B_i^a$$

covariant current conservation

$$-\frac{\partial}{\partial t} J_0 + \frac{\partial}{\partial r} J_1 = 2 a r j_S = -2 \frac{\partial}{\partial t}(a r E_S) + 2 \frac{\partial}{\partial r}(a r B_A)$$



Yang-Mills Maxwell equations apply throughout these regions
Interior vol. r<R₀-Δ
Transition volume
R₀-Δ≤r≤R₀+Δ
Exterior Volume
r>R₀+Δ
2R₀
2Δ
2Δ

Spherical Symmetry as Dimensional Reduction

$$\int d^4x\, G^a_{\mu\nu} G^{a\mu\nu} = 4\pi \int dr\,dt\,(r^2 \mathcal{L}_g)$$

$$r^2 \mathcal{L}_g = r^2 F_{\ell m} F^{\ell m} + 2 D^\ell \phi D_\ell \phi^+ + \frac{1}{r^2}(|\phi|^2 - 1)$$

$\eta_{\ell m} = r^2 g_{\ell m}$

$\ell, m = 0, 1$

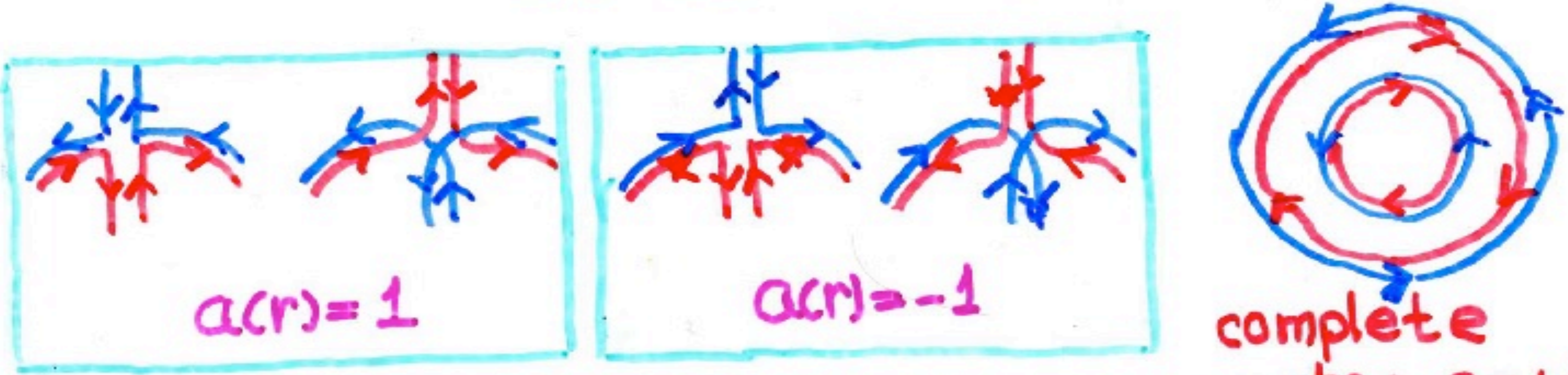
3+1 dim gauge theory becomes 1+1 dim Abelian Higgs theory with curved metric

$\phi = a e^{i\omega}$ transverse components of gauge connection act as colored pseudoscalars

The covariant derivative and the dynamical generation of color vortices

$$D_i^{ab}\hat{r}_b = \frac{a(r,t)}{r}\,\varepsilon^{S}_{ia}(\omega) \qquad [\hat{r}, D_i\hat{r}]^a = \frac{i\,a(r,t)}{r}\,\varepsilon^{A}_{ia}(\omega)$$

In the chromostatic limit interior of hadron is one color vortex



complete vortex $a=\pm 1$

TOPOLOGICAL CHARGE

- CP-odd condensate $E_i^a B_i^a(r,t)$

forms in regions with different values of constant $a(r,t)$ as a consequence of Yang-Mills Maxwell equations $-E_L + \frac{\partial}{\partial r}(arE_\theta) + \frac{\partial}{\partial t}(arB_\theta) = g^2 r^2 E_i^a B_i^a$

- not a conserved charge

Disappears after EW hadronic decays (ex. $\pi^0 \to \gamma\gamma$)

Instantons & Merons ..

II TOPOLOGICAL CHARGE
in Spherical Symmetric SU(2)

topological current $\partial^{l} K_{l}(r,t) = g^2 r^2 E_i^a B_i^a$

$$K_0(r,t) = (a^2 - 1)A_1 - a^2 \frac{\partial}{\partial r}\omega$$

$$K_1(r,t) = -(a^2 - 1)A_0 + a^2 \frac{\partial}{\partial t}\omega$$

when $a(r,t) = \pm 1$

$$K_0(r,t) = -\frac{\partial}{\partial r}\omega(r,t)$$

$$K_1(r,t) = +\frac{\partial}{\partial t}\omega(r,t)$$

$$\partial^{l} K_{l} = 0$$

Adler. Bell. Jackiw

$$J_5^\mu = i \sum \bar{\psi} \gamma^\mu \gamma_5 \psi$$

QFT requires

$$\partial_\mu J_5^\mu = \frac{g^2}{16\pi^2} {}^*G^a_{\mu\nu} G^{a\mu\nu}$$

$$= \frac{g^2}{8\pi^2} \eta_f \partial_\mu K^\mu$$

two related applications of topological (CP-odd) charge

Domain Wall

non-abelian field Eq'ns
spherical chromostatics

$$K_1' = r^2 E_i^a B_i^a$$

$$= [(a^2-1) A_0]'$$

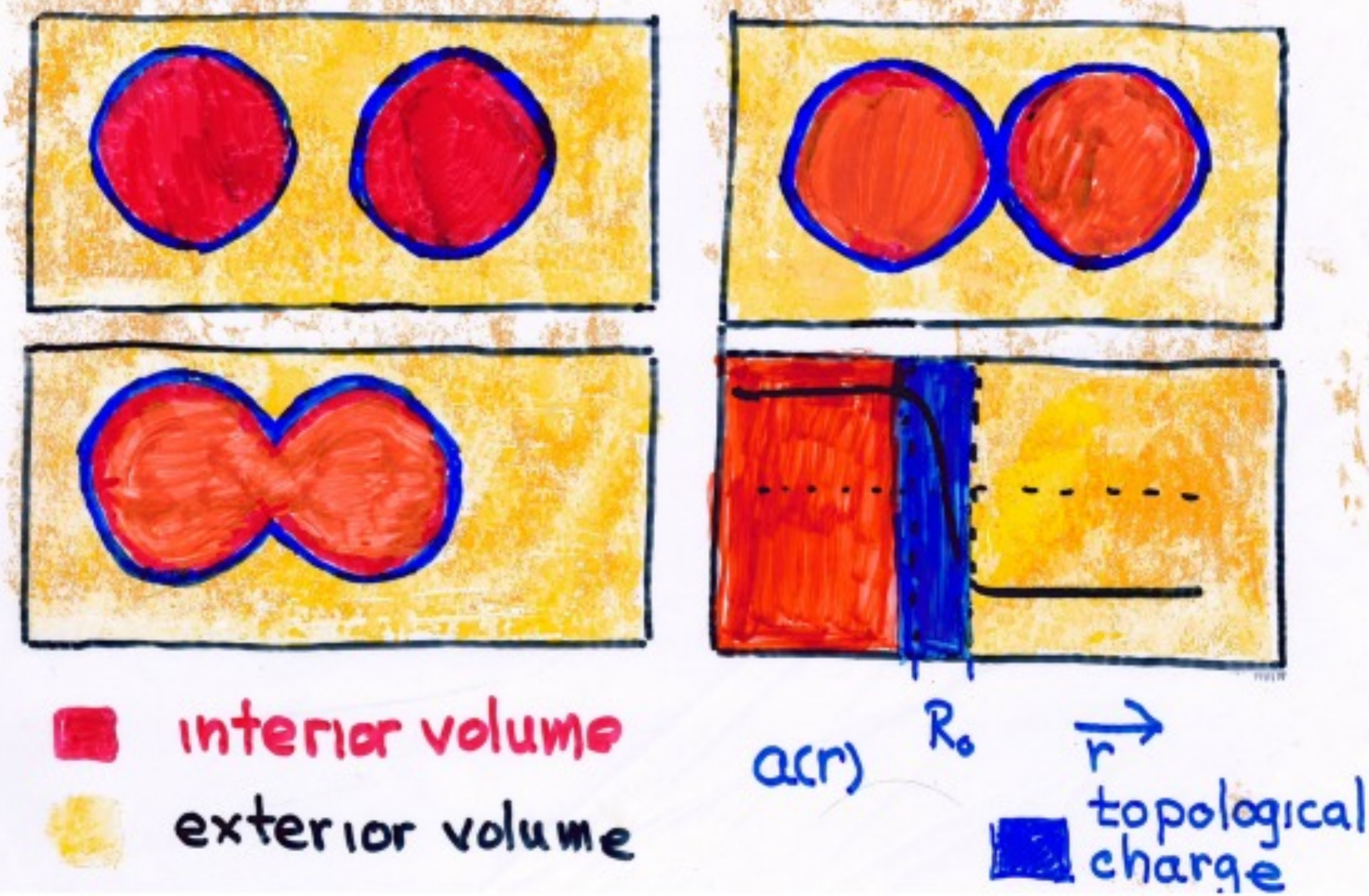
chiral transition soliton

$$a_{cl}(r) = -\tanh\left(\frac{\kappa}{\Delta}(r - R_0)\right) \quad \frac{\kappa}{\Delta} >> 1$$

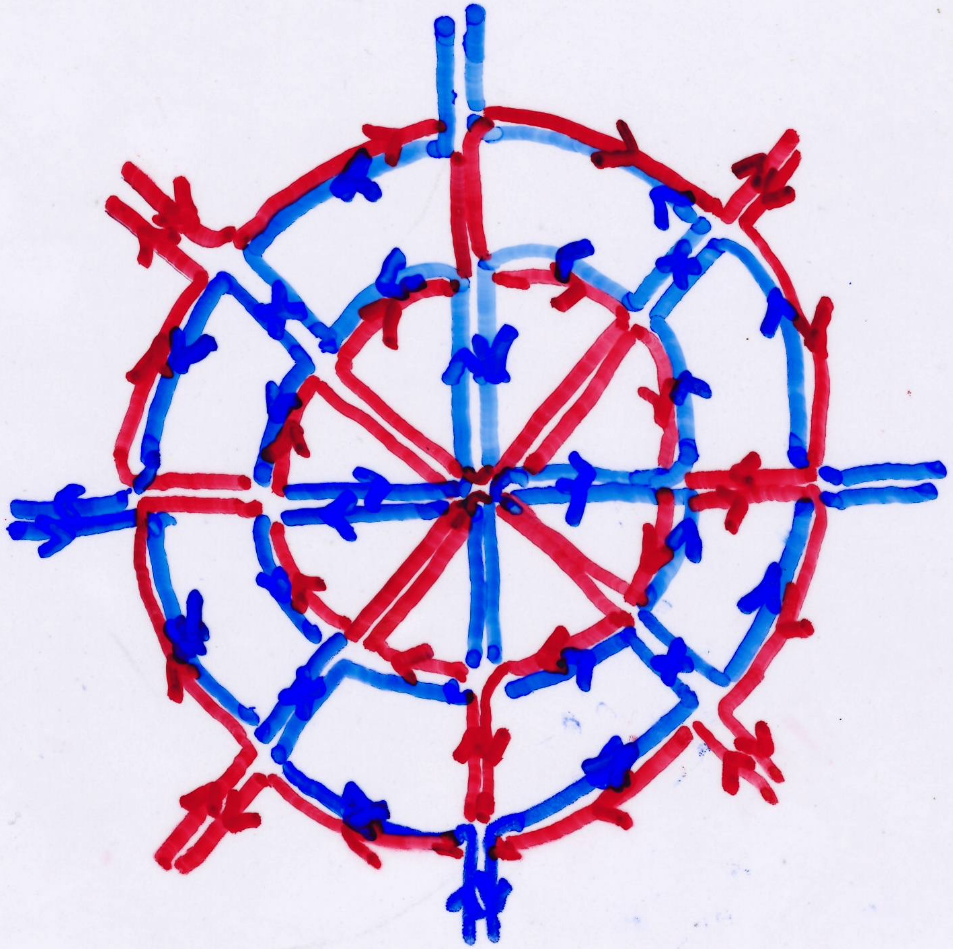
$$\omega(R_0 + \Delta) = \omega(R_0 - \Delta) \pm \pi.$$

$$A_0(R_0 + \Delta) = 0$$

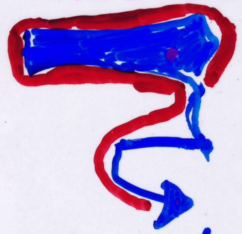
Domain Zones



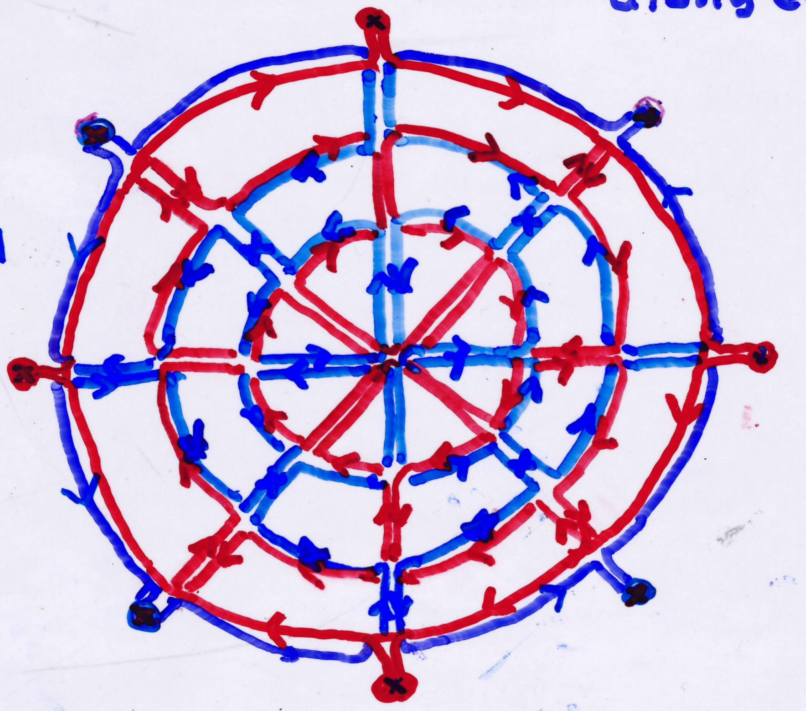
chiral nature covariant Derivatives generate



Here is another layer with $a(r)=1$ with ends tied off by a distributed SU(2) color source along each diagonal



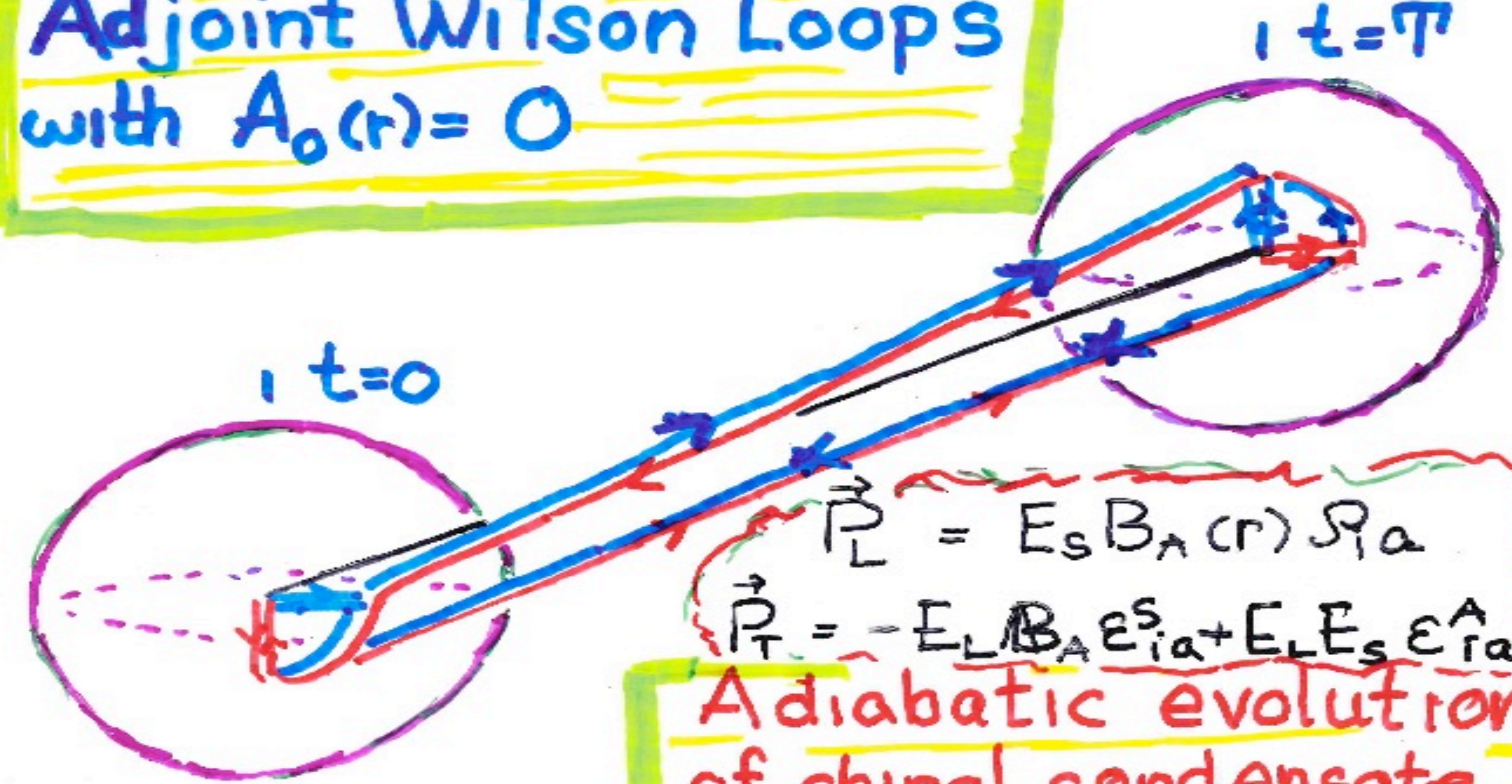
a standard nontopological sol'n to Yang-Mills Maxwell



$a=1$

the exterior volume is a sterile vacuum condensate with the same chirality $a=1$ as outer

Adjoint Wilson Loops with $A_0(r) = 0$



$$\vec{P}_L = E_S B_A(r)\,\Omega_a$$

$$\vec{P}_T = -E_L B_A \varepsilon^S_{ia} + E_L E_S \varepsilon^A_{ia}$$

Adiabatic evolution of chiral condensate

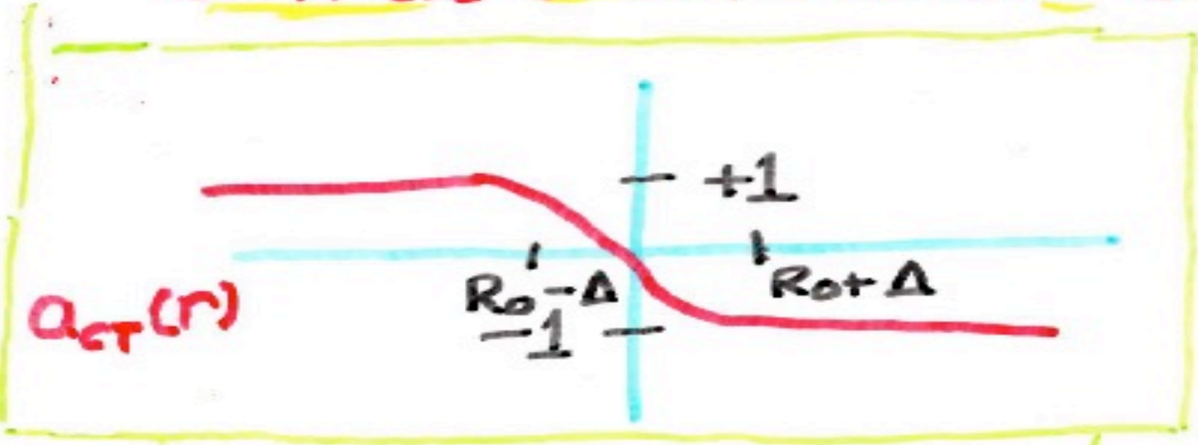
Classification of condensates in SU(2) chromostatics with spherical symmetry

$a(r) = \pm 1$	$E_i^a E_i^a \neq 0$	$B_i^a B_i^a = 0$		color electric
$a(r) = \pm 1$	$E_i^a E_i^a = 0$	$B_i^a B_i^a \neq 0$		color magnetic
$a(r) = \pm 1$	$E_i^a E_i^a \neq 0$	$B_i^a B_i^a \neq 0$		color glass
$a(r) = \pm 1$	$E_i^a E_i^a = 0$	$B_i^a B_i^a = 0$		sterile vacuum
$a(r) = 0$	$E_i^a E_i^a = 0$	$B_L B_L = (\pm)^2 \frac{1}{r^4}$		't Hooft Polyakov
$a(r) = C \neq \pm 1, 0$	$E_i^a E_i^a \neq 0$	$B_i^a B_i^a \neq 0$	$E_i^a B_i^a \neq 0$	topological or dyonic

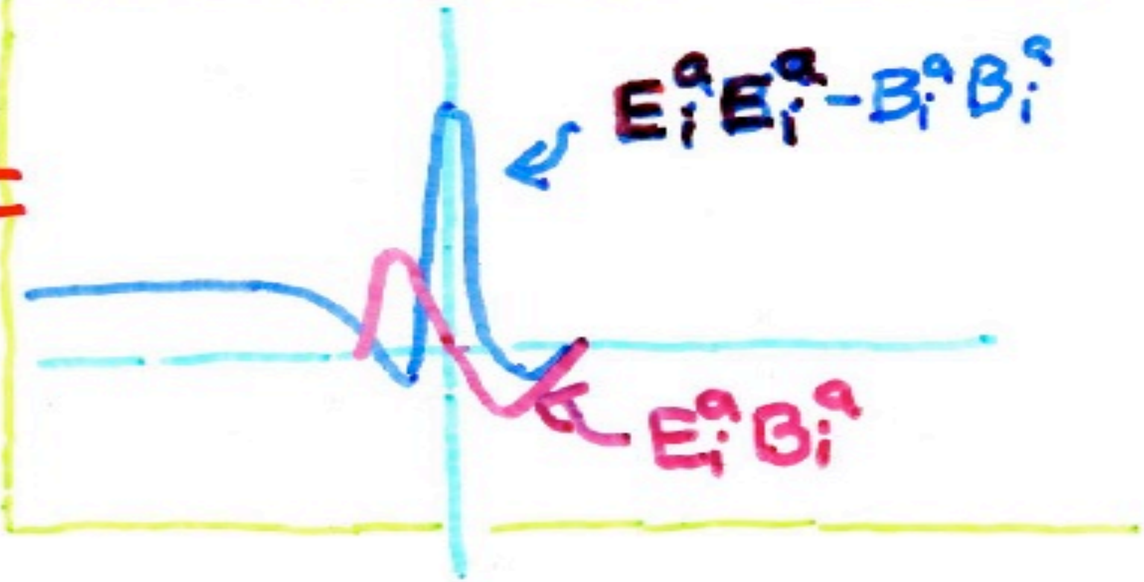
a domain wall is a region where $a'(r) \neq 0$ that separates other condensates and also carries topological charge

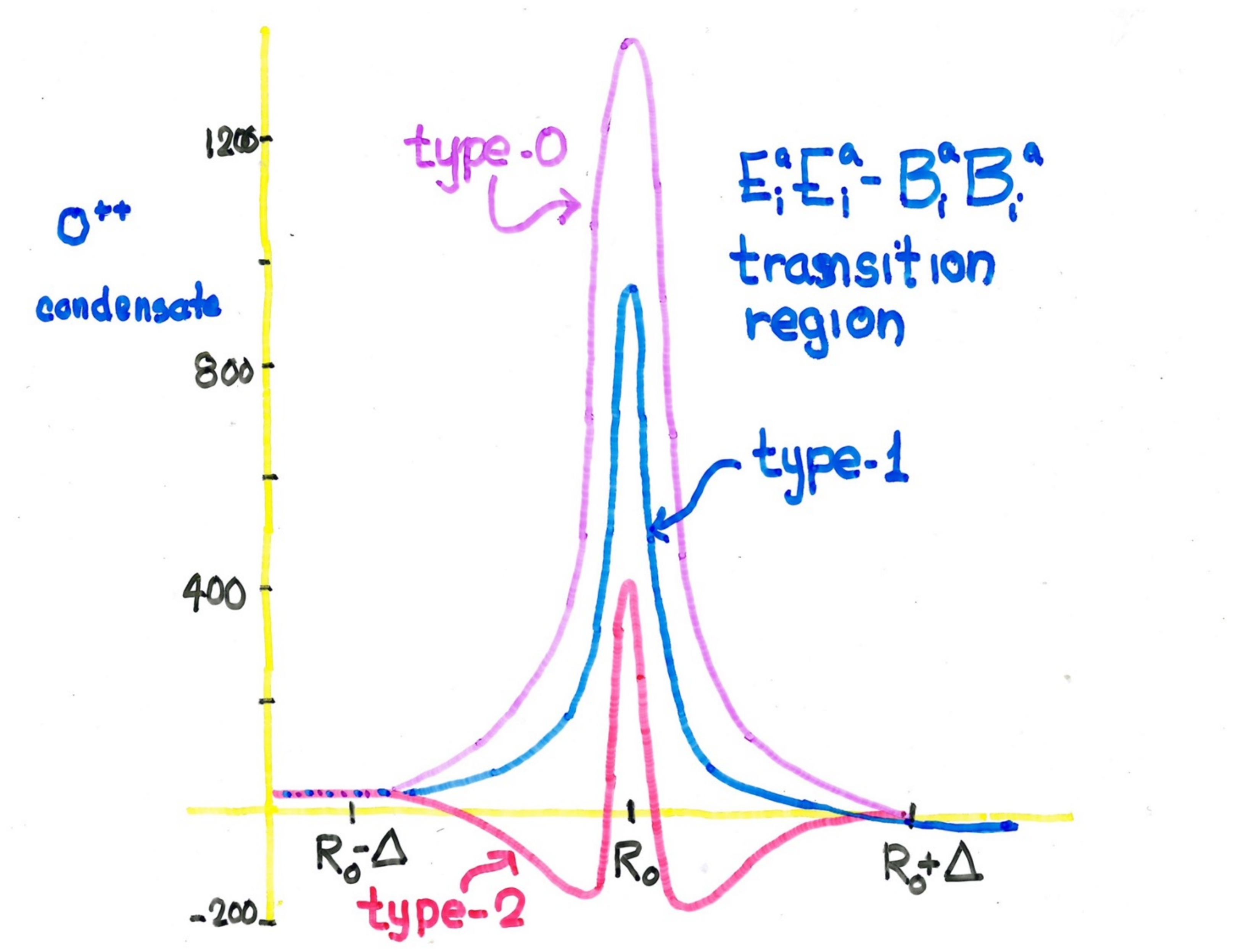
Chiral transition Soliton

DOMAIN
WALL
CONFINEMENT

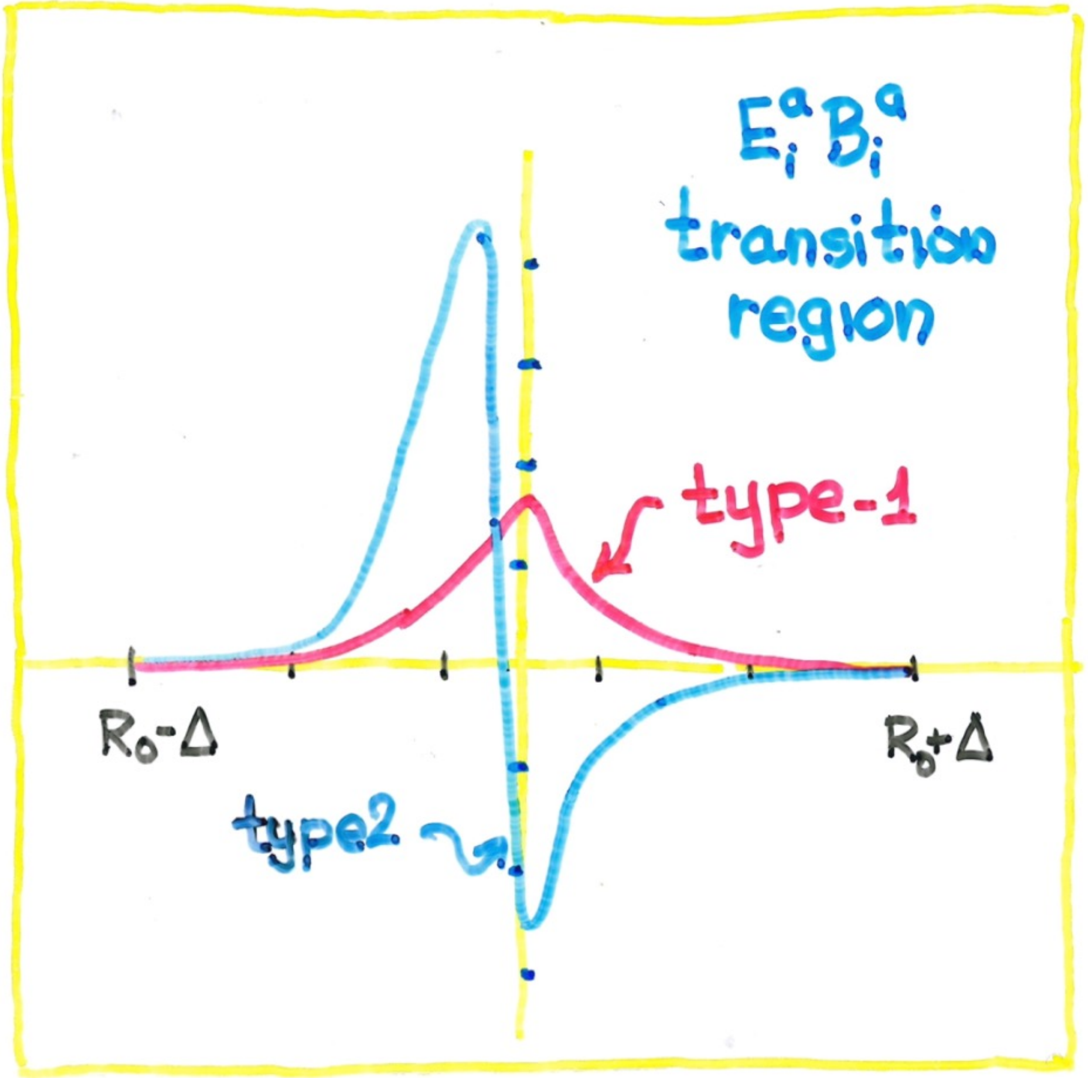


Lorentz-invariant
densities

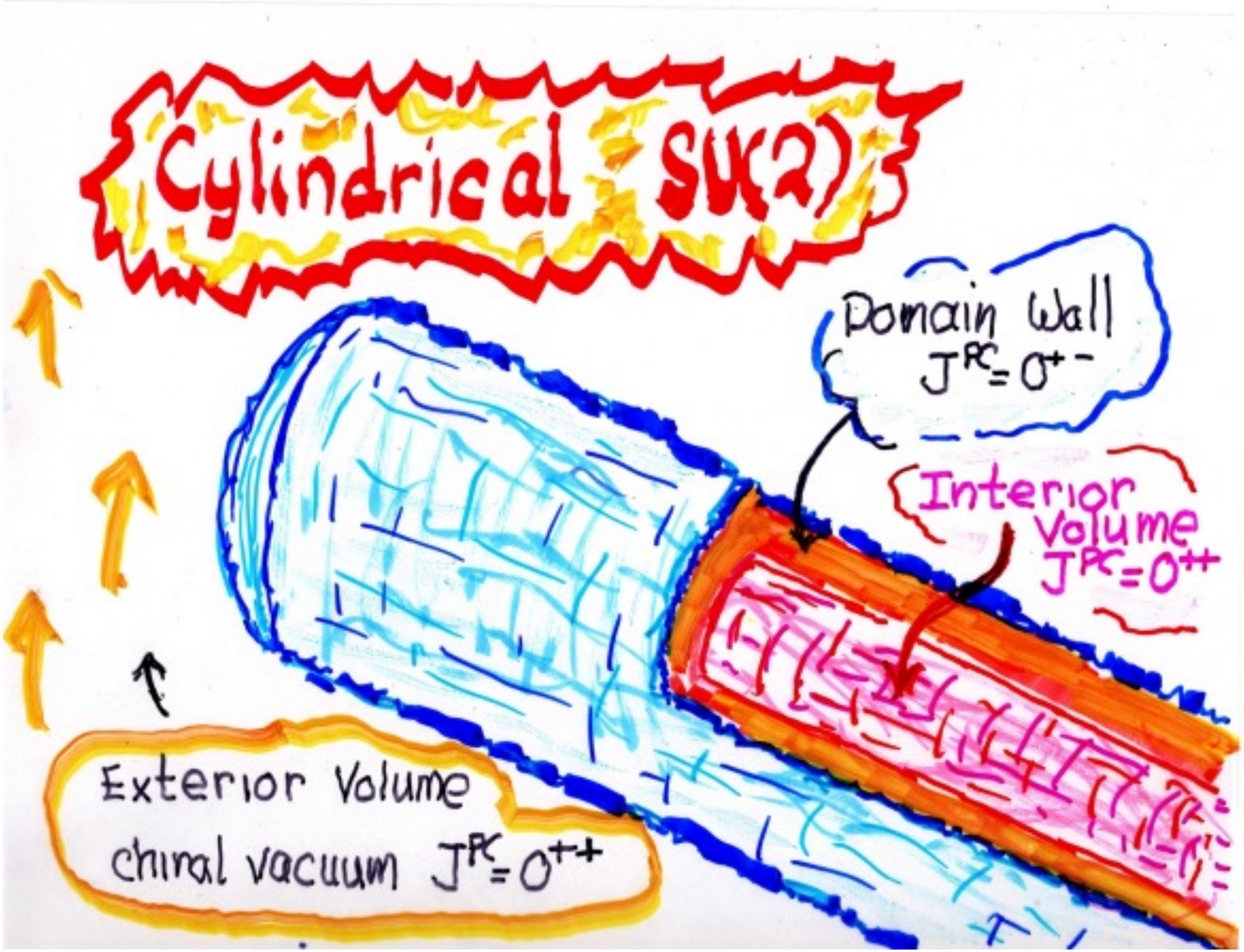




1200
800
400
-200
O^{++} condensate
type-0
$E_i^a E_i^a - B_i^a B_i^a$
transition region
type-1
type-2
$R_0 - \Delta$
R_0
$R_0 + \Delta$

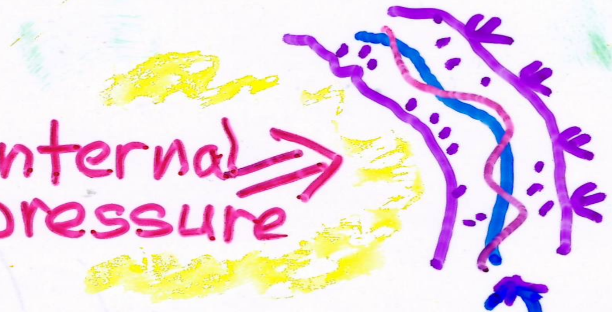


CP-odd condensate



Cylindrical SU(2)
Domain Wall
J^PC = 0^+-
Interior Volume
J^PC = 0^++
Exterior Volume
chiral vacuum J^PC = 0^++

The color-glass / sterile vacuum "kink" soliton with $\Delta a = \pm 2$ is also a topologically stable solution to SU(N) field equations

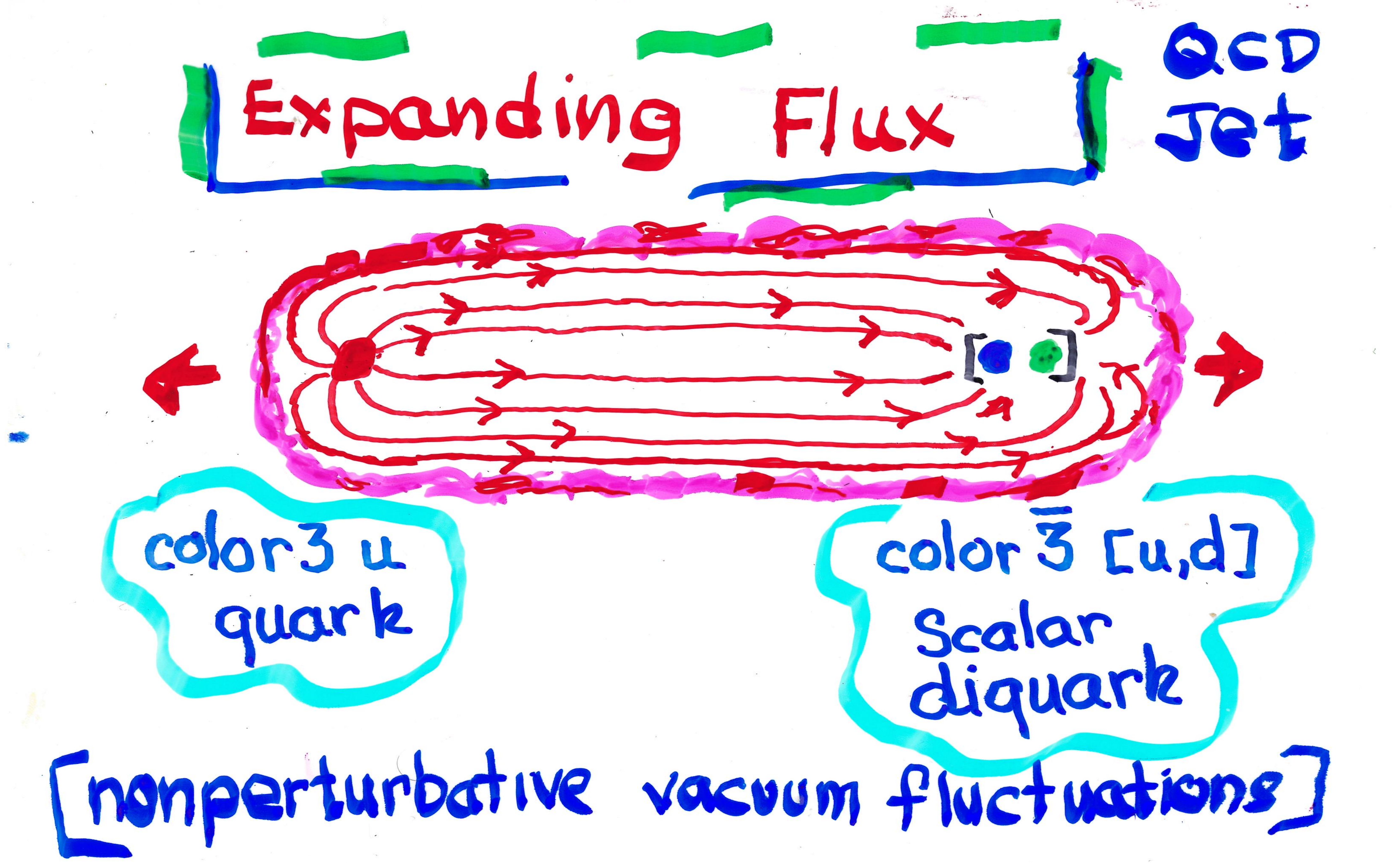


Confines Abelian Gluons - adjoint charge

replaces dual superconducting "QCD vacuum" condensate with a dual insulator

Translates to cylindrical symmetry - QCD jets & Collins functions

not yet studied extensively


Expanding Flux
QCD Jet
color 3 u quark
color $\bar{3}$ [u,d] Scalar diquark
[nonperturbative vacuum fluctuations]

Starting from the premise "Quantum fields are not completely understood either as laws of nature or from a mathematical point of view" Arthur Jaffe has defined a constructive field theory program whose "main goals" are to:

1. quantize classical field equations
2. prove the existence of solutions to equations in terms of quantum fields
3. establish the properties of these solutions

Streater-Wightman axioms define quantum fields as operators formed from tempered distributions with specialized renormalization procedures (Wilson)

Can cluster decomposition axiom can be reconciled with confinement in an SU(n) gauge theory?

The t'Hooft-Polyakov monopole is a topological stable solitonic solution to the SU(2) field eq'ns

Stability involves a pressure balance

external pressure from t'Hooft Polyakov condensate

Internal pressure (adiabatic + quantum fluctuations)

color glass

$a = \pm 1$

Domain wall "shrink wrap" pressure

$a = 0$

However, the CP-odd domain wall does not confine radial adjoint charge

Energy $\propto \langle A_0 A_1 \rangle \hat{r}_a e^{i(t-r)}$

"abelian" gluons can propogate though $a(r) = 0$ condensate

unless ... Dark matter.

P. Rossi Phys. Rep 86 (82)

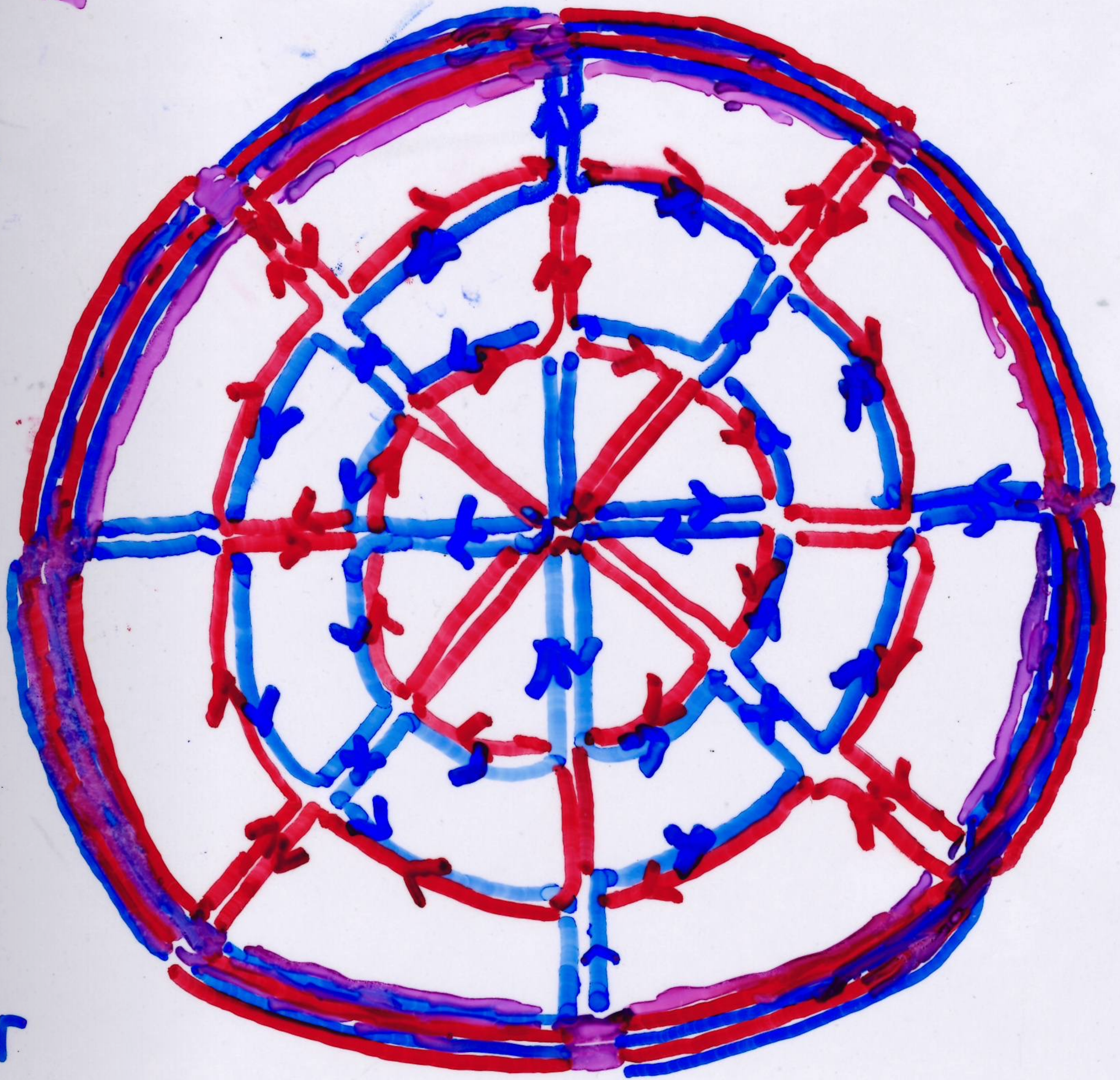
Here is a sketch showing what a type-1 sol'n could look like

topological $\hat{E}_i$ $\hat{B}_i$ shown in purple

Exterior region

't Hooft Polyakov condensate $B_L B_L = \frac{\pm 1}{r^4}$

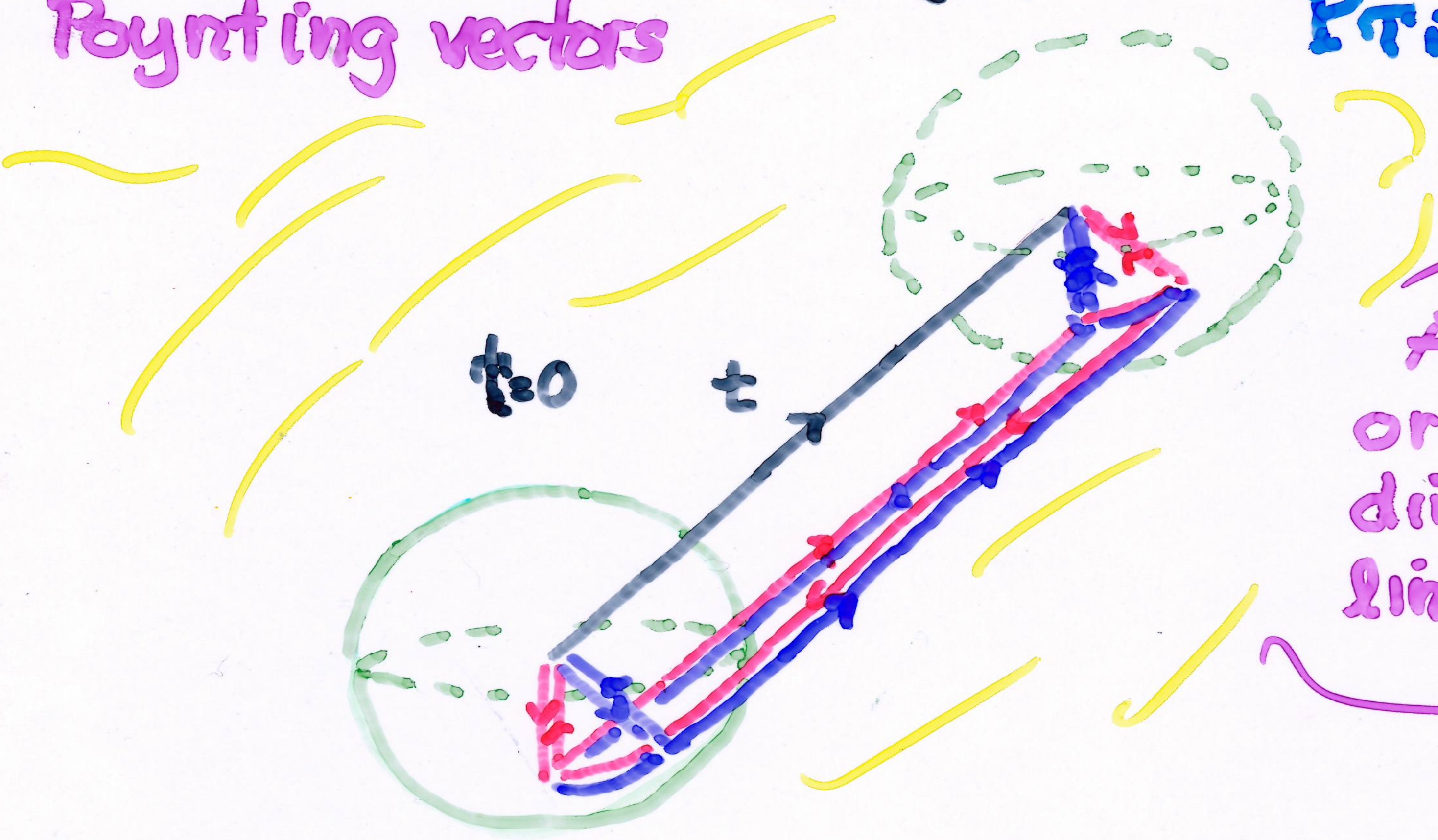
repels al color



Adiabatic Color-flow Poynting vectors

$$P_{L\,ia} = E_S B_A \rho_{ia}$$

$$P_{T\,ia} = -E_L B_A \varepsilon^{S}_{ia} + E_L E_S \varepsilon^{T}_{ia}$$



Away from the origin time-directed Wilson lines carry charge

Adjoint Wilson loops with $A_0(r) = 0$

Adiabatic Evolution Confined Condensate

Yang-Mills Maxwell : $\{ D^{\mu} G^{a}_{\mu\nu} = J^{a}_{\nu}(r,t) \}$

classical adjoint current : $J^{a}_{0}(r,t) = \frac{1}{r^2} J_0(r,t)\, \hat{r}_a$

$J^{a}_{i}(r,t) = \frac{1}{r^2} J_1(r,t)\, \rho_{ia} + j_S(r,t)\, \epsilon^{S}_{ia}(\omega) + j_A(r,t)\, \epsilon^{A}_{ia}(\omega)$

$$-\frac{\partial}{\partial r}(r^2 E_L) + 2arE_S = J_0(r,t)$$

$$-\frac{\partial}{\partial t}(r^2 E_L) + 2ar B_A = J_1(r,t)$$

$$-\frac{\partial}{\partial t}(arE_S) + \frac{\partial}{\partial t}(ar B_A) = ar j_S(r,t)$$

$$a\left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r^2}\right)a - r^2(E_S^2 - B_A^2) - \frac{a^2(a^2-1)}{r^2} = ar j_A(r,t)$$

Bianchi constraint: $\frac{\partial}{\partial r}(arE_A) - \frac{\partial}{\partial t}(ar B_S) = 0$

$$-E_L + \partial(arE_S) + \partial(arB_A) = r^2 E_i^a B_i^a$$