Weak decays of hadrons using high-precision lattice simulations

Davide Giusti

Universität Regensburg

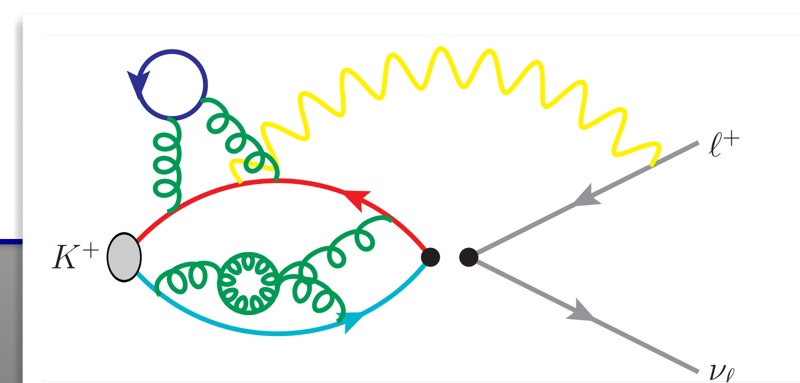
10th Biennial Workshop of the APS Topical Group on Hadronic Physics

Minneapolis

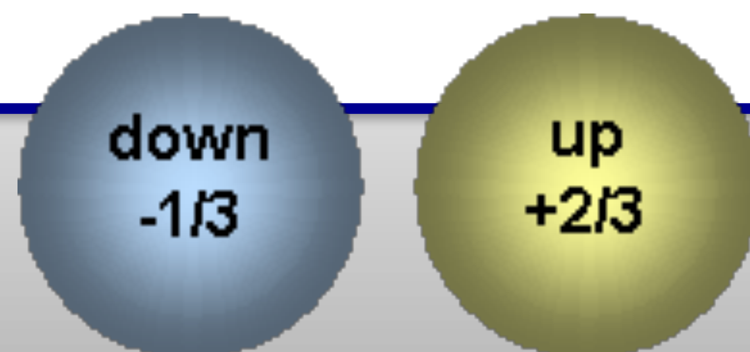
14th April 2023

OUTLINE

- Motivations
- Isospin-breaking effects on the lattice: the RM123 method
- Light meson leptonic decays



Phenomenological motivations



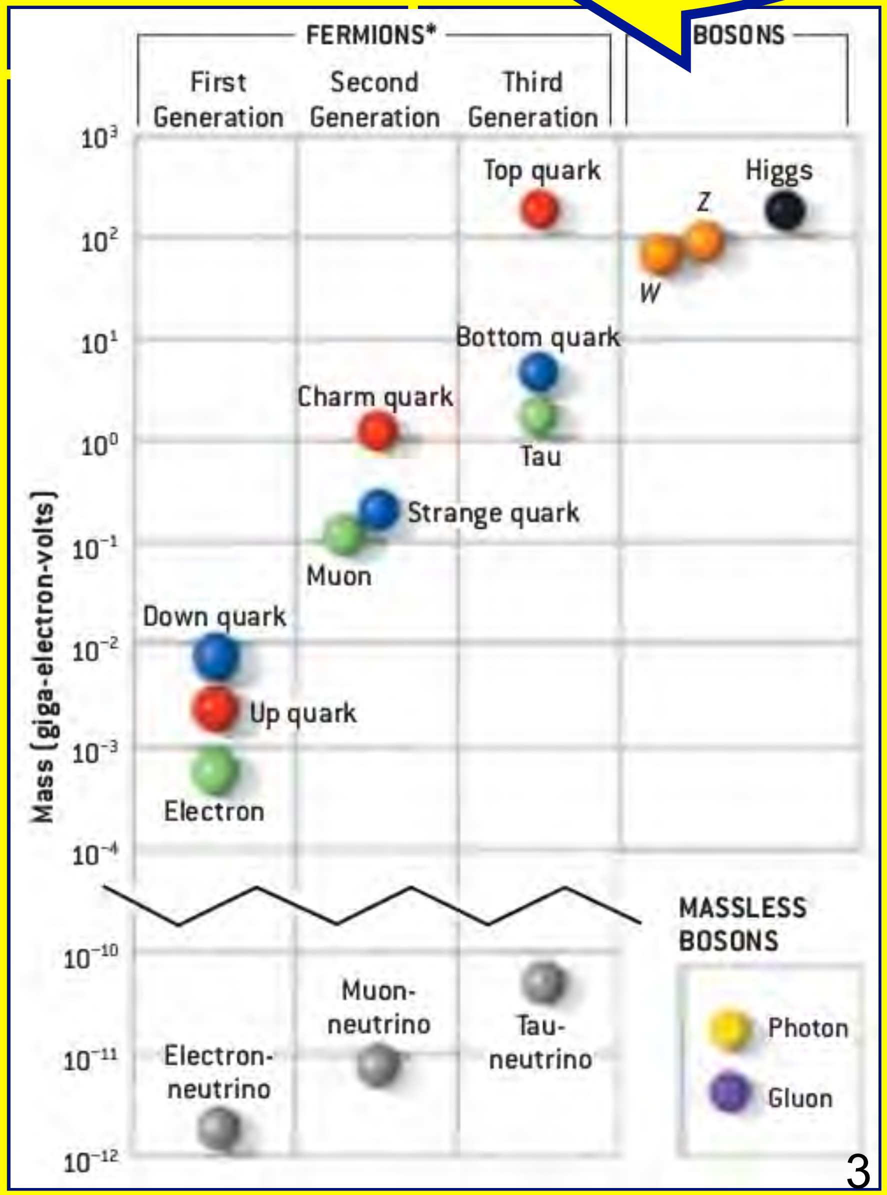
Flavor physics is (well) described but not explained in the Standard Model:

A large number of free parameters in the flavor sector (10 parameters in the quark sector only, 6 m_q + 4 CKM)

- Why 3 families?

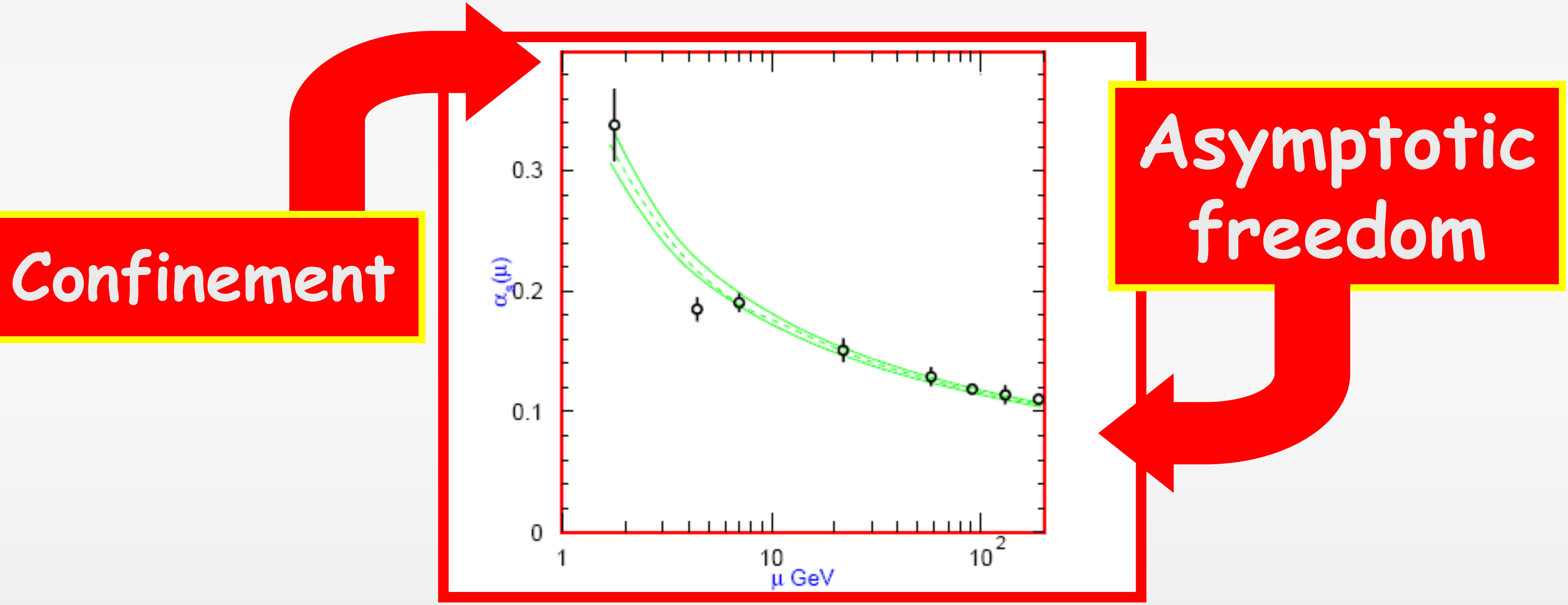
- Why the spectrum of quarks and leptons covers 5 orders of magnitude? ($m_q \sim v \sim G_F^{-1/2}$...)

- What give rise to the pattern of quark mixing and the magnitude of CP violation?



Lattice QCD

Strong interactions are non-perturbative at low energies



LQCD is a non-perturbative approach

The Functional Integral

The Green Functions can be written in terms of Functional Integrals over classical fields:

$$G(x_1, x_2, x_3, x_4) = \langle \phi(x_1)\, \phi(x_2)\, \phi(x_3)\, \phi(x_4) \rangle \equiv$$

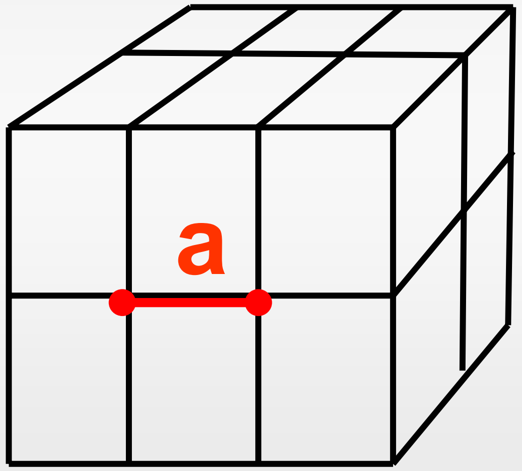
$$Z^{-1} \int [d\phi]\, \phi(x_1)\, \phi(x_2)\, \phi(x_3)\, \phi(x_4)\, e^{-S(\phi)}$$

The functional integral is defined by discretizing the space-time on a hypercubic 4-dimensional lattice

$$\phi(x) \to \phi(a\,n)$$

$$n = (n_x, n_y, n_z, n_t)$$

$$\partial_\mu \phi(x) \to \nabla_\mu \phi(x) = [\phi(x + a\,n_\mu) - \phi(x)]/a$$



The Lattice regularization

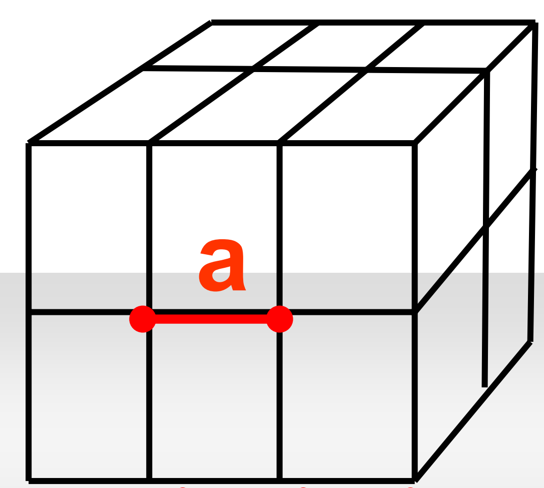
The functional integral is a formal definition because of the infrared and ultraviolet divergences. These are cured by introducing an infrared and an ultraviolet cutoff

1) The ultraviolet cutoff: $|p| \leq \pi/a$

The momentum p is cutoff at the first Brioullin zone

2) The infrared cutoff: $p_{min}\, a = 2\pi/L$

The lattice is defined in a finite volume

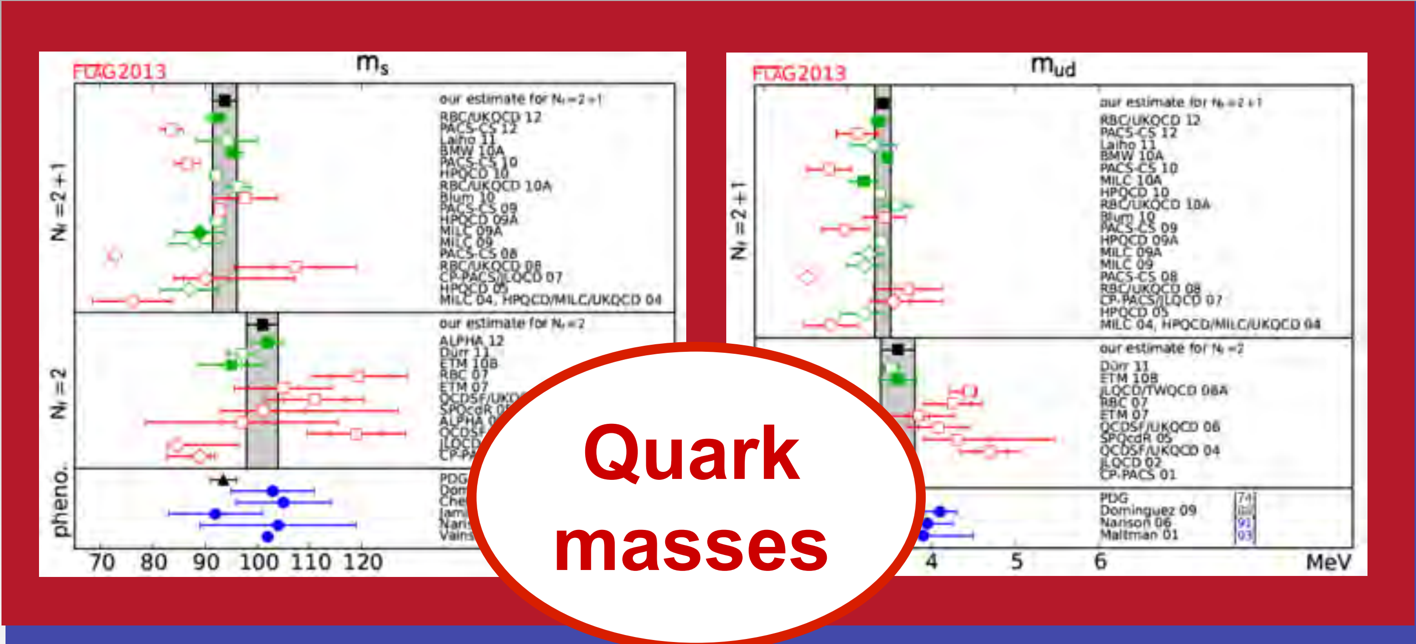


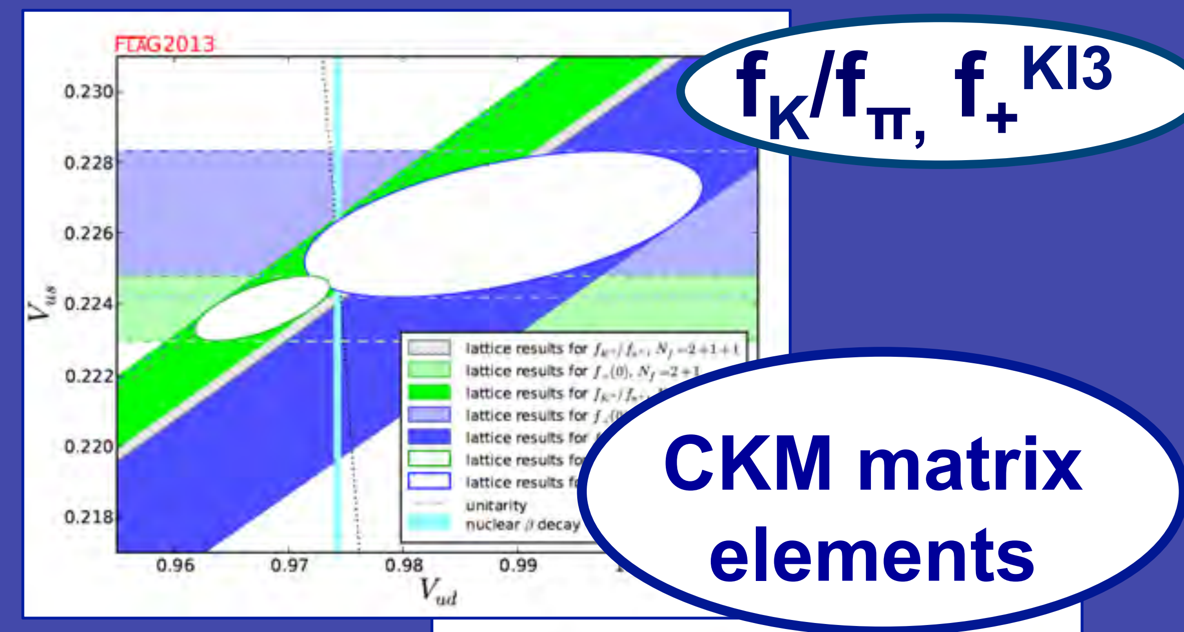
The physical theory is obtained in the limit

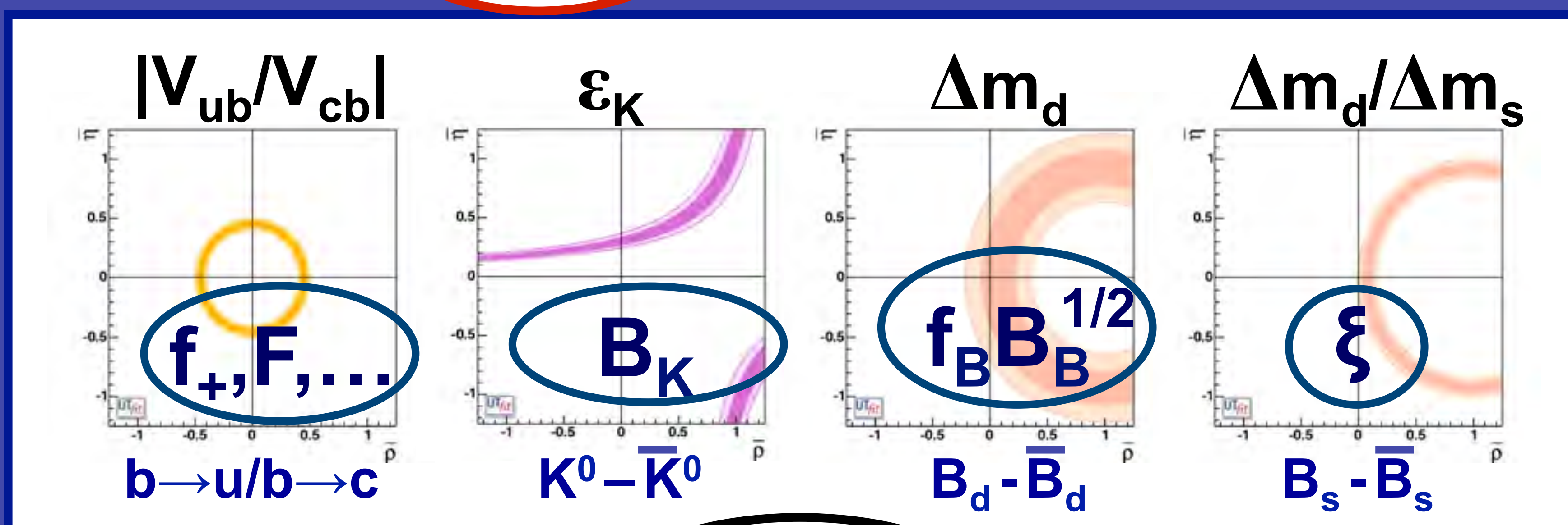
$a \rightarrow 0$ **Continuum limit** ; $L \rightarrow \infty$ **Thermodinamic limit**

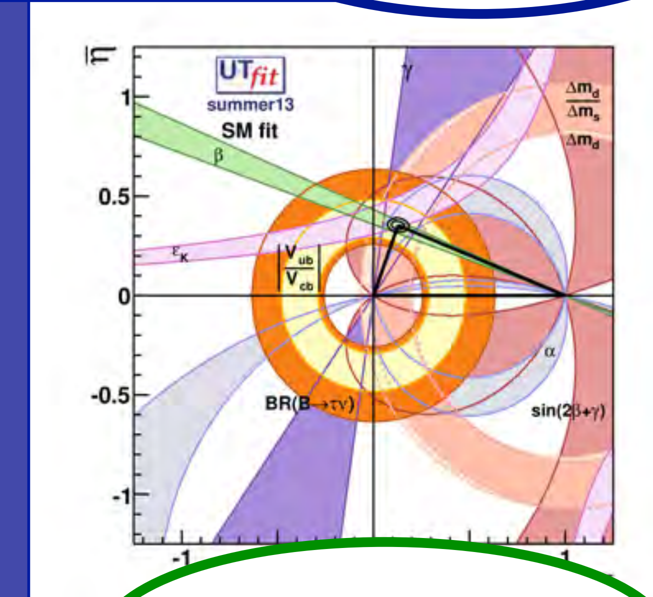
Use the most powerful supercomputers in the world

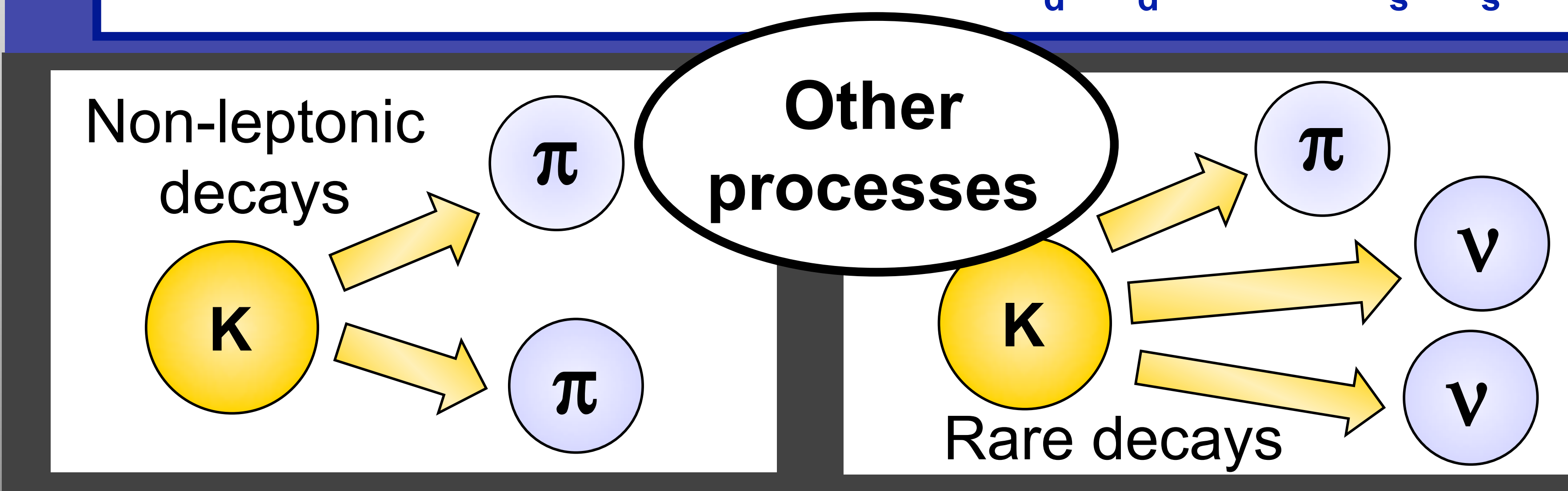
Lattice QCD and flavor physics

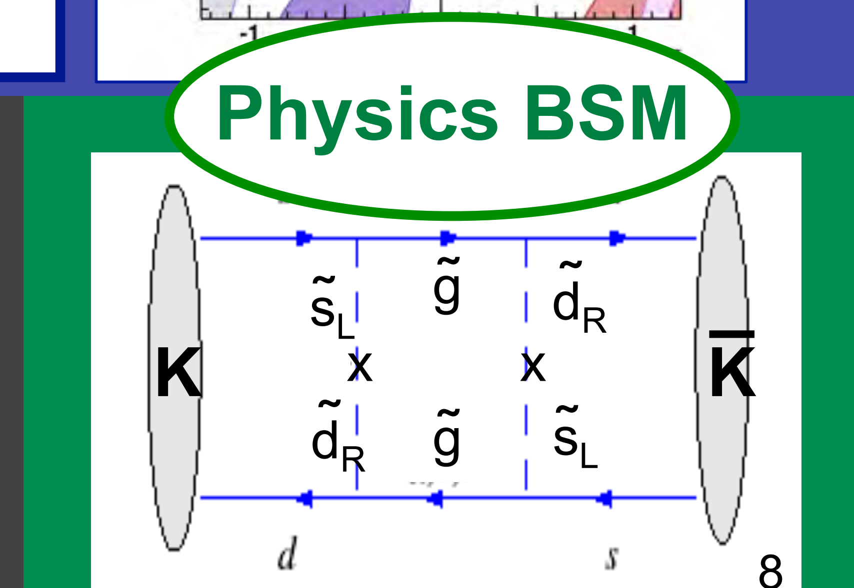




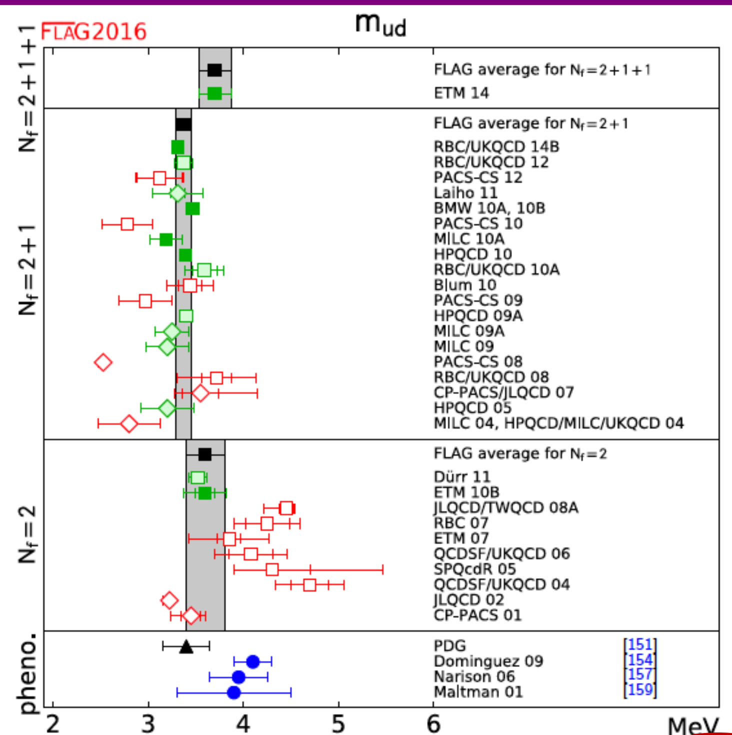




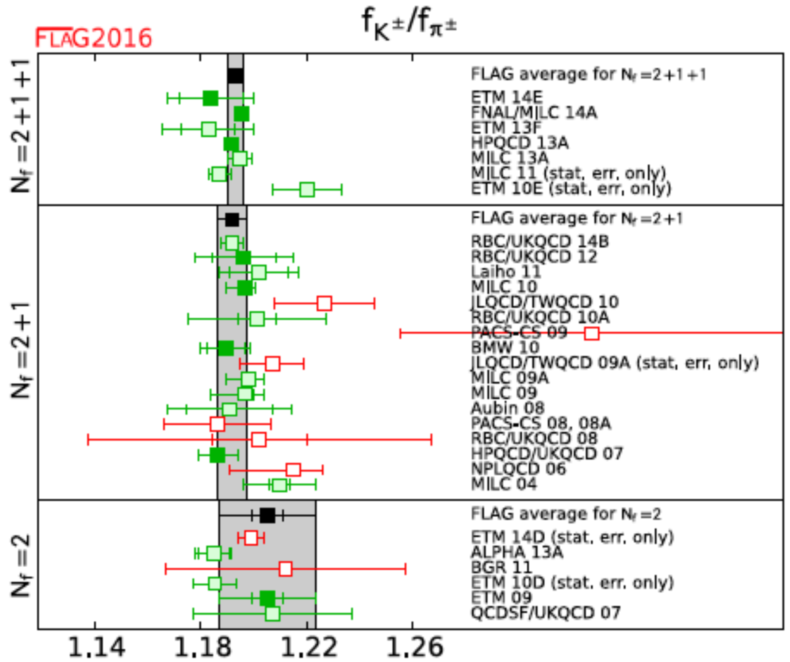




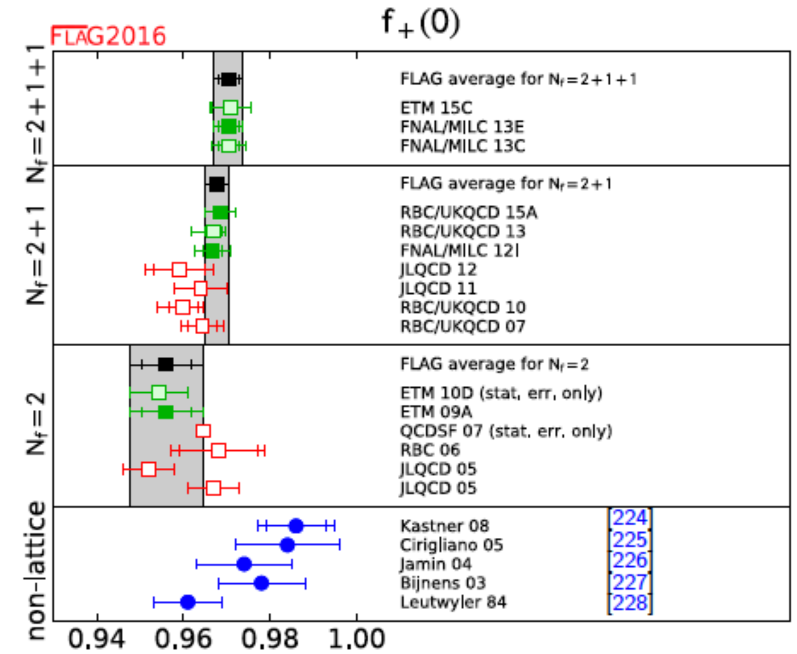
PRECISION PHYSICS from LATTICE QCD



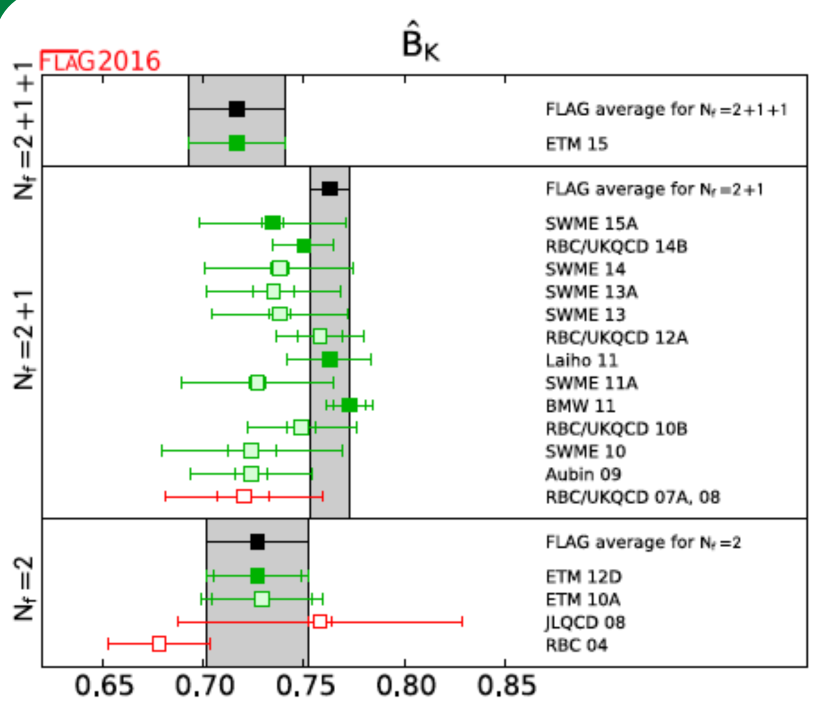
$\overline{m}_{ud}$= 3.37(8) MeV **2.3%**



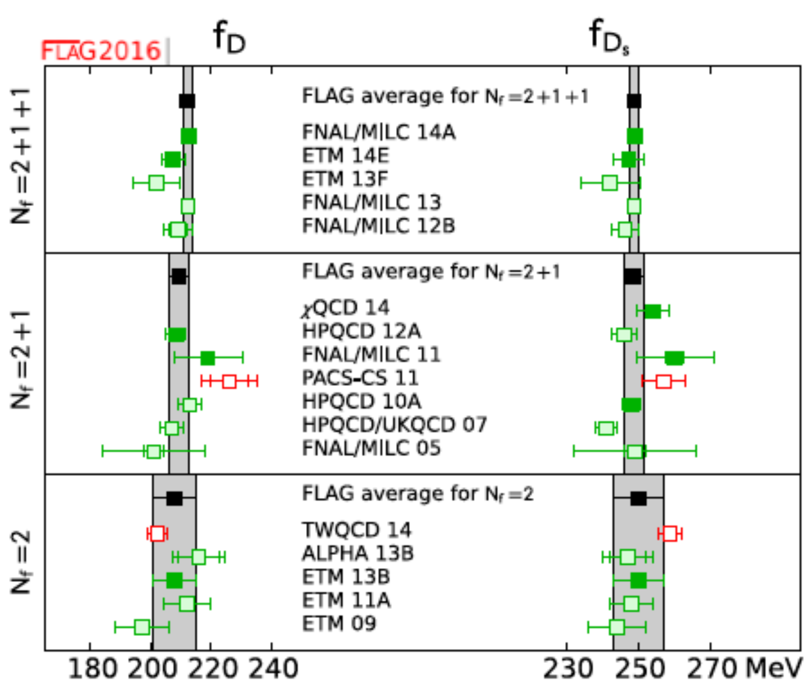
f_K / f_π = 1.193(3) **0.3%**



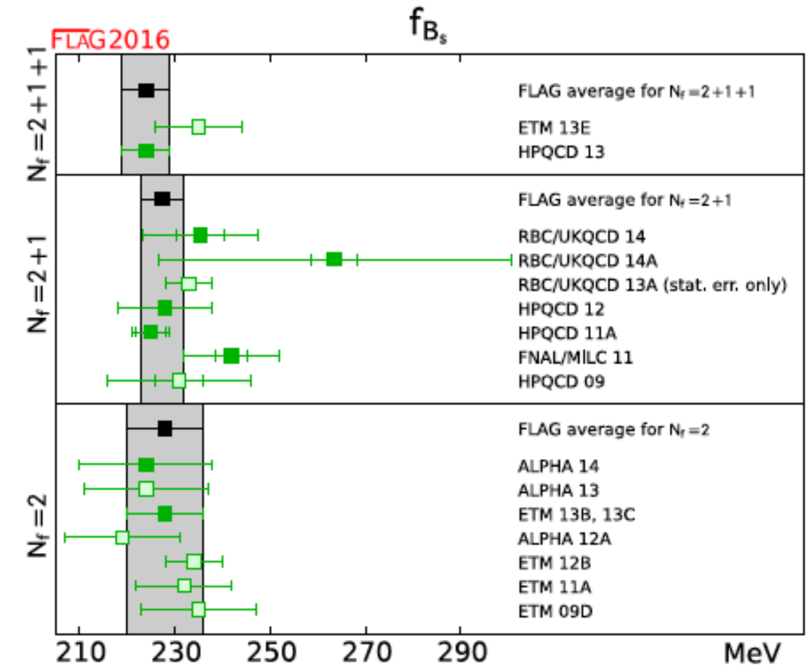
$f_+^{K\pi}(0)$ = 0.970(3) **0.3%**



$\hat{B}_K$ = 0.7625(97) **1.3%**



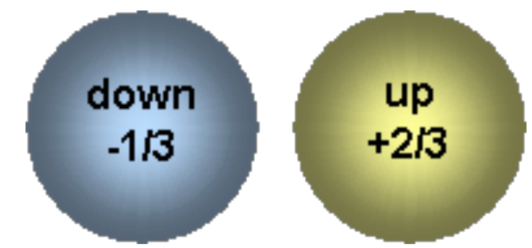
f_{Ds} = 249(1) MeV **0.4%**



f_{Bs} = 228(4) MeV **2.0%**

ISOSPIN-BREAKING EFFECTS

Isospin symmetry is an almost exact property of the strong interactions



Isospin-breaking effects are induced by:

$m_u \neq m_d$: $O[(m_d - m_u)/\Lambda_{QCD}] \approx 1/100$ "Strong"

$Q_u \neq Q_d$: $O(\alpha_{em}) \approx 1/100$ "Electromagnetic"

Since electromagnetic interactions renormalize quark masses the two corrections are intrinsically related

Though small, IB effects play often a very important role (quark masses, Mn - Mp, leptonic decay constants, vector form factor)

Isospin-breaking effects on the lattice

RM123 method

A strategy for Lattice QCD:

The isospin-breaking part of the Lagrangian is treated as a perturbation

Expand in: $m_d - m_u$ + α_{em}

JHEP

PUBLISHED FOR SISSA BY SPRINGER

RECEIVED: *November 7, 2011*

REVISED: *March 16, 2*

ACCEPTED: *April 2, 2*

PUBLISHED: *April 26, 2*

arXiv:1110.6294

Isospin breaking effects due to the up-down mass difference in lattice QCD

RM123 collaboration

G.M. de Divitiis,[a,b] P. Dimopoulos,[c,d] R. Frezzotti,[a,b] V. Lubicz,[e,f] G. Martinelli,[g,d] R. Petronzio,[a,b] G.C. Rossi,[a,b] F. Sanfilippo,[c,d] S. Simula,[f] N. Tantalo[a,b] and C. Tarantino[e,f]

PHYSICAL REVIEW D **87**, 114505 (2013)

Leading isospin breaking effects on the lattice

G. M. de Divitiis,[1,2] R. Frezzotti,[1,2] V. Lubicz,[3,4] G. Martinelli,[5,6] R. Petronzio,[1,2] G. C. Rossi,[1,2] F. Sanfilippo,[7] S. Simula,[4] and N. Tantalo[1,2]

(RM123 CollaboratioarXiv:1303.4896

[1]*Dipartimento di Fisica, Università di Roma "Tor Vergata", Via della Ricerca Scientifica 1, I-00133 Rome, Italy*

[2]*INFN, Sezione di Roma "Tor Vergata", Via della Ricerca Scientifica 1, I-00133 Rome, Italy*

[3]*Dipartimento di Matematica e Fisica, Università Roma Tre, Via della Vasca Navale 84, I-00146 Rome, Italy*

[4]*INFN, Sezione di Roma Tre, Via della Vasca Navale 84, I-00146 Rome, Italy*

[5]*SISSA, Via Bonomea 265, 34136 Trieste, Italy*

[6]*INFN, Sezione di Roma, Piazzale Aldo Moro 5, I-00185 Rome, Italy*

[7]*Laboratoire de Physique Théorique (Bâtiment 210), Université Paris Sud, F-91405 Orasay-Cedex, France*

(Received 3 April 2013; published 7 June 2013)

RM123 Collaboration

The (m_d-m_u) expansion

- Identify the isospin-breaking term in the QCD action

$$S_m = \sum_x \left[m_u \bar{u}u + m_d \bar{d}d \right] = \sum_x \left[\frac{1}{2}(m_u + m_d)(\bar{u}u + \bar{d}d) - \frac{1}{2}(m_d - m_u)(\bar{u}u - \bar{d}d) \right] =$$

$$= \sum_x \left[m_{ud} (\bar{u}u + \bar{d}d) - \Delta m (\bar{u}u - \bar{d}d) \right] = S_0 - \Delta m \hat{S} \longleftarrow \boxed{\hat{S} = \Sigma_x(\bar{u}u - \bar{d}d)}$$

- Expand the functional integral in powers of Δm

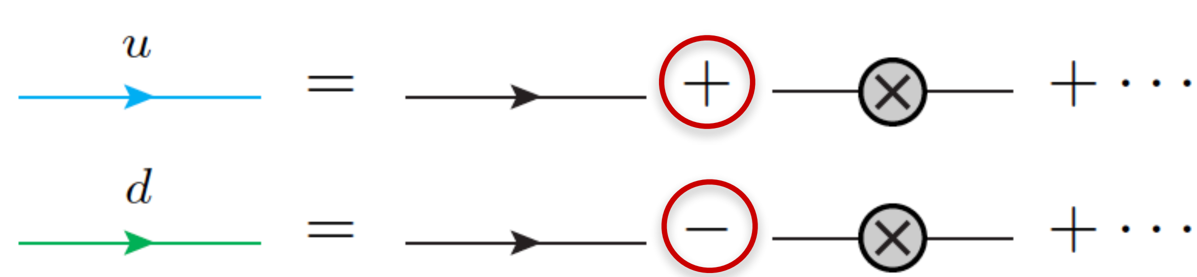
Advantage: factorised out

$$\langle O \rangle = \frac{\int D\phi\, O e^{-S_0 + \Delta m \hat{S}}}{\int D\phi\, e^{-S_0 + \Delta m \hat{S}}} \overset{1st}{\simeq} \frac{\int D\phi\, O e^{-S_0} \left(1 + \Delta m \hat{S}\right)}{\int D\phi\, e^{-S_0} \left(1 + \Delta m \hat{S}\right)} \simeq \frac{\langle O \rangle_0 + \Delta m \langle O \hat{S} \rangle_0}{1 + \cancel{\Delta m \langle \hat{S} \rangle_0}} = \langle O \rangle_0 + \Delta m \langle O \hat{S} \rangle_0$$

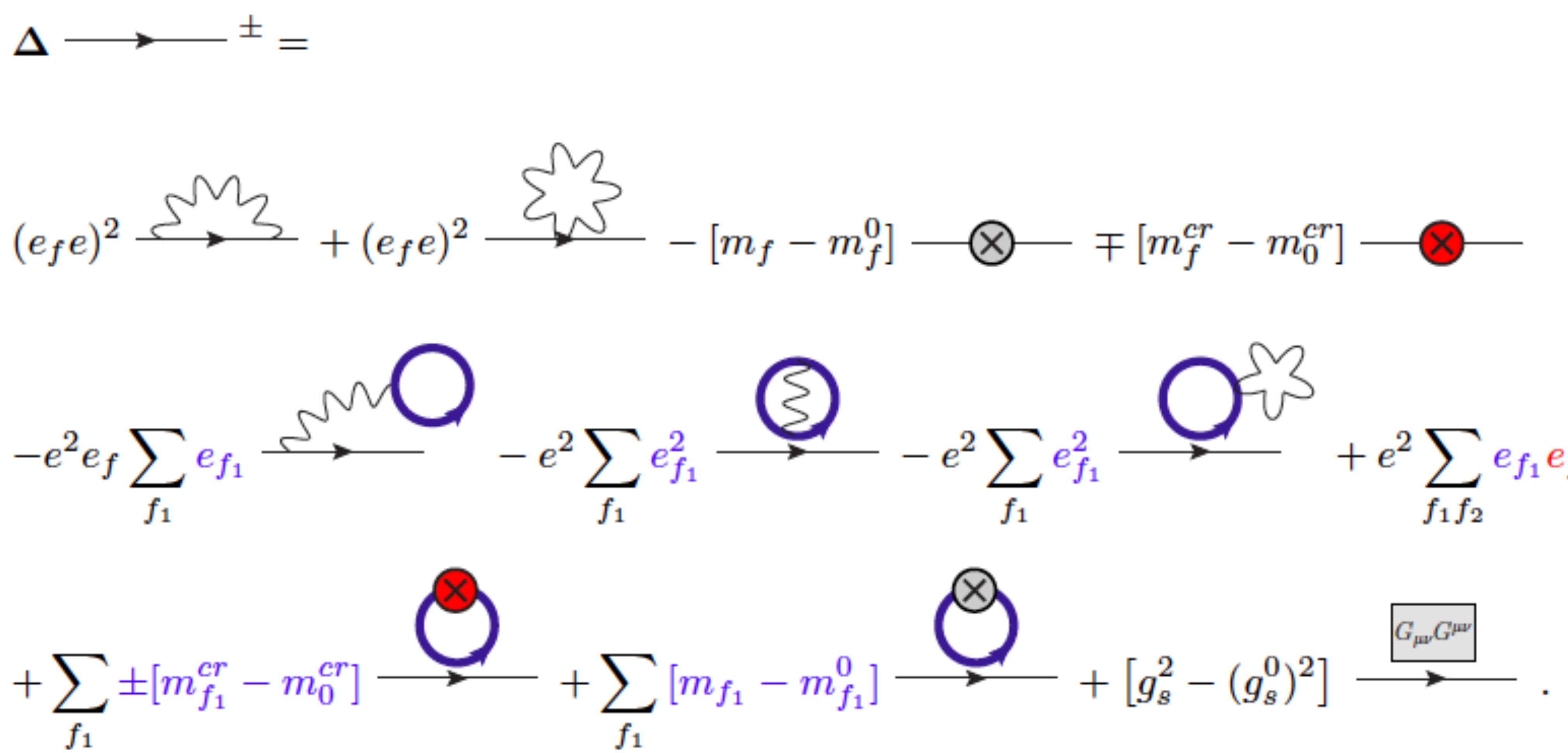
for isospin symmetry

- At leading order in Δm the corrections only appear in the valence-quark propagators:

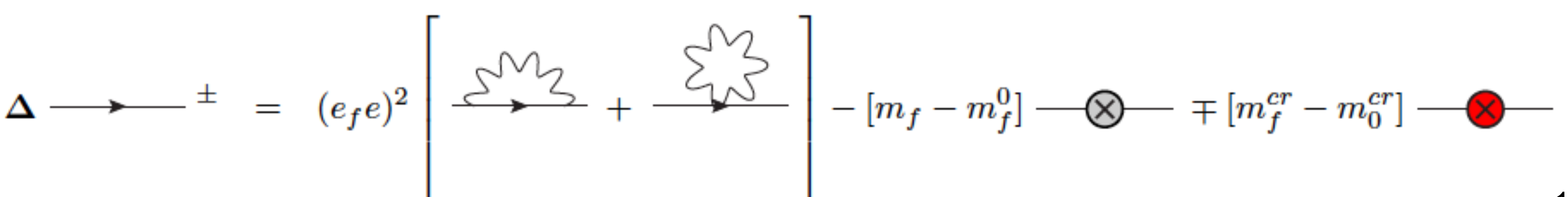
(disconnected contractions of $\bar{u}u$ and $\bar{d}d$ vanish due to isospin symmetry)



The QED expansion for the quark propagator



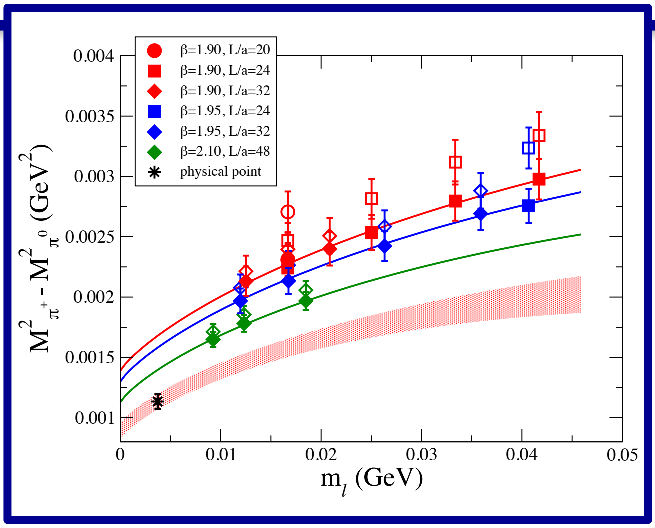
In the *electro-quenched* approximation:



Spheres of application

QED and isospin corrections

The RM123 method



- Quark and Hadron masses

PHYSICAL REVIEW D 87, 114505 (2013)

PHYSICAL REVIEW D 95, 114504 (2017)

- Decay rates of hadrons

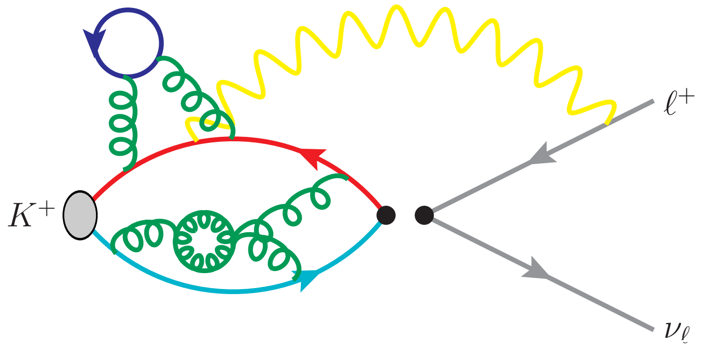
PHYSICAL REVIEW D 91, 074506 (2015)

PHYSICAL REVIEW D 95, 034504 (2017)

PHYSICAL REVIEW LETTERS 120, 072001 (2018)

PHYSICAL REVIEW D 100, 034514 (2019) [Editor's suggestion]

PHYSICAL REVIEW D 103, 014502 (2021)

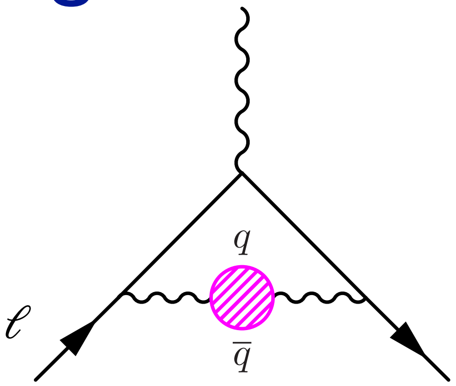


- Hadronic corrections to lepton anomalous magnetic moments

JOURNAL OF HIGH ENERGY PHYSICS 10 (2017) 157

PHYSICAL REVIEW D 99, 114502 (2019)

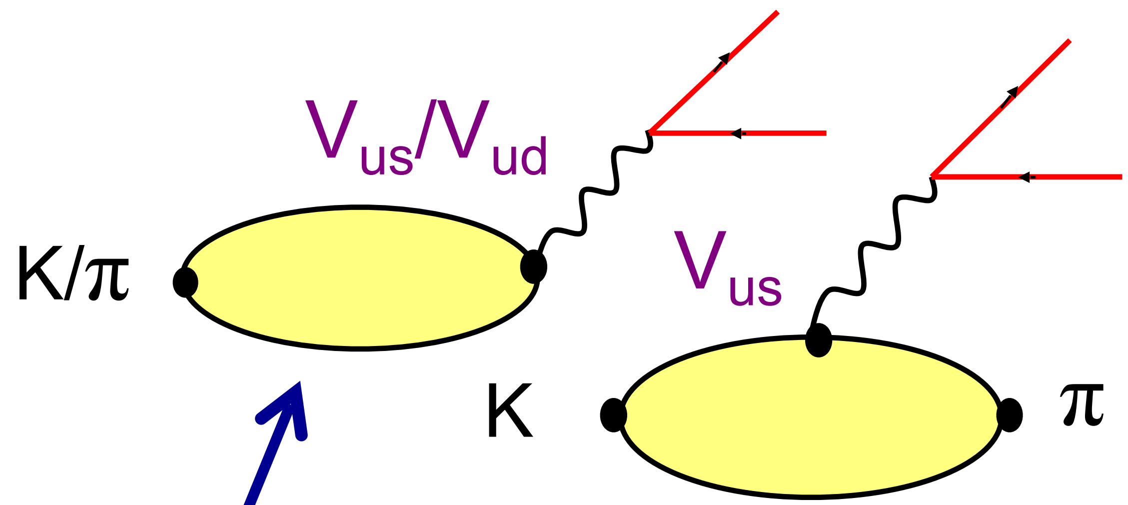
PHYSICAL REVIEW D 102, 054503 (2020)



QED corrections to hadronic decays

The determination of Vus and Vud

The relevant processes are leptonic and semileptonic K and π decays



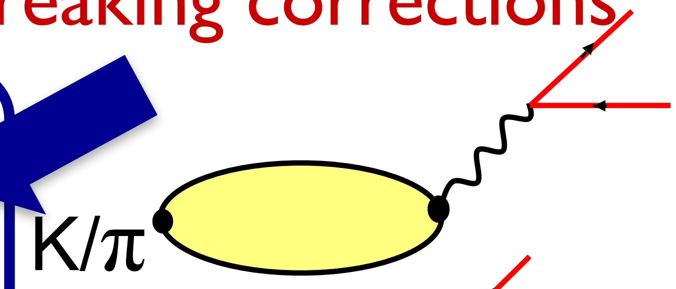
$$\frac{\Gamma\left(K^+ \to \ell^+ \nu_\ell(\gamma)\right)}{\Gamma\left(\pi^+ \to \ell^+ \nu_\ell(\gamma)\right)} = \left(\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi}\right)^2 \frac{M_{K^+}\left(1-m_\ell^2/M_{K^+}^2\right)^2}{M_{\pi^+}\left(1-m_\ell^2/M_{\pi^+}^2\right)^2}\left(1+\delta_{EM}+\delta_{SU(2)}\right)$$

$$\Gamma\left(K^{+,0} \to \pi^{0,-}\ell^+\nu_\ell(\gamma)\right) = \frac{G_F^2 M_{K^{+,0}}^5}{192\pi^3} C_{K^{+,0}}^2 \left|V_{us} f_+^{K^0\pi^-}(0)\right|^2 I_{K\ell}^{(0)} S_{EW}\left(1+\delta_{EM}^{K^{+,0}\ell}+\delta_{SU(2)}^{K^{+,0}\pi}\right)$$

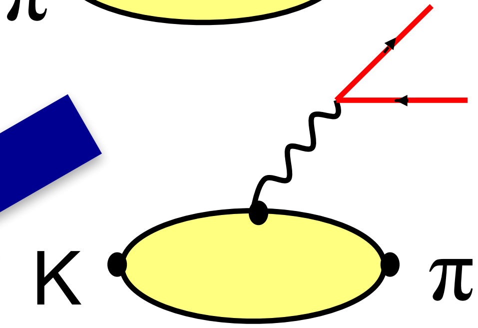
Electromagnetic and isospin-breaking effects

Given the present exper. and theor. (LQCD) accuracy, an important source of uncertainty are long distance electromagnetic and SU(2)-breaking corrections

$$\frac{\Gamma\left(K^+ \to \ell^+ \nu_\ell(\gamma)\right)}{\Gamma\left(\pi^+ \to \ell^+ \nu_\ell(\gamma)\right)} = \left(\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi}\right)^2 \frac{M_{K^+}\left(1-m_\ell^2/M_{K^+}^2\right)^2}{M_{\pi^+}\left(1-m_\ell^2/M_{\pi^+}^2\right)^2} \boxed{\left(1+\delta_{EM}+\delta_{SU(2)}\right)}$$



$$\Gamma\left(K^{+,0} \to \pi^{0,-} \ell^+ \nu_\ell(\gamma)\right) = \frac{G_F^2 M_{K^{+,0}}^5}{192\pi^3} C_{K^{+,0}}^2 \left|V_{us} f_+^{K^0\pi^-}(0)\right|^2 I_{K\ell}^{(0)} S_{EW} \boxed{\left(1+\delta_{EM}^{K^{+,0}\ell}+\delta_{SU(2)}^{K^{+,0}\pi}\right)}$$



For $\Gamma_{Kl2}/\Gamma_{\pi l2}$ At leading order in ChPT both δ_{EM} and $\delta_{SU(2)}$ can be expressed in terms of physical quantities (e.m. pion mass splitting, f_K/f_π, …)

- $\boxed{\delta_{EM} = -0.0069(17)}$ 25% of error due to higher orders ➡ 0.2% on $\Gamma_{Kl2}/\Gamma_{\pi l2}$

  M.Knecht *et al.*, 2000; V.Cirigliano and H.Neufeld, 2011

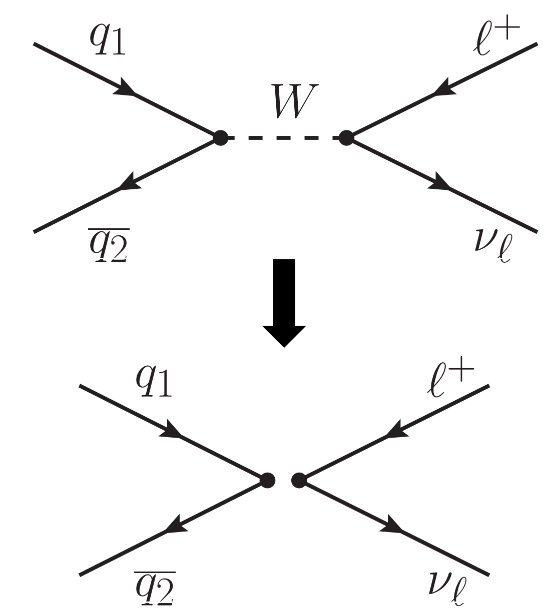
- $\boxed{\delta_{SU(2)} = \left(\frac{f_{K^+}/f_{\pi^+}}{f_K/f_\pi}\right)^2 - 1 = -0.0044(12)}$ 25% of error due to higher orders ➡ 0.1% on $\Gamma_{Kl2}/\Gamma_{\pi l2}$

  J.Gasser and H.Leutwyler, 1985; V.Cirigliano and H.Neufeld, 2011

ChPT is not applicable to D and B decays

Leptonic decays at tree level

Since the masses of the pion and kaon are much smaller than M_W we use the effective Hamiltonian



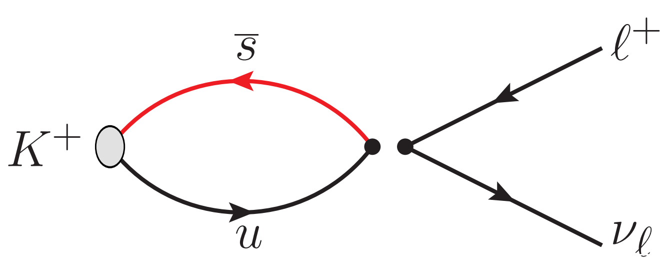
$$H_{eff} = \frac{G_F}{\sqrt{2}} V^*_{q_1 q_2} \left(\overline{q_2}\gamma^\mu (1-\gamma_5) q_1\right)\left(\overline{\nu_\ell}\gamma_\mu (1-\gamma_5)\ell\right)$$

This replacement is necessary in a lattice calculation, since $1/a \ll M_W$

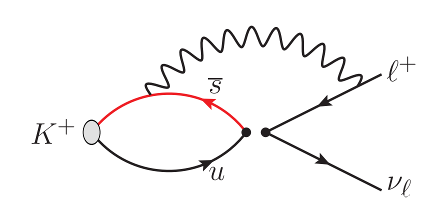
The rate is:

$$\Gamma^{(tree)}_{P^\pm}\left(P^\pm \to \ell^\pm \nu_\ell\right) = \frac{G_F^2}{8\pi}\left|V_{q_1 q_2}\right|^2 \left[f_P^{(0)}\right]^2 M_{P^\pm} m_\ell^2 \left(1 - \frac{m_\ell^2}{M_{P^\pm}^2}\right)^2$$

In the absence of electromagnetism, the non-perturbative QCD effects are contained in a single number, the pseudoscalar decay constant



$$A_P^{(0)} \equiv \langle 0 | \overline{q}_2 \gamma_4 \gamma_5 q_1 | P^{(0)} \rangle = f_P^{(0)} M_P^{(0)}$$

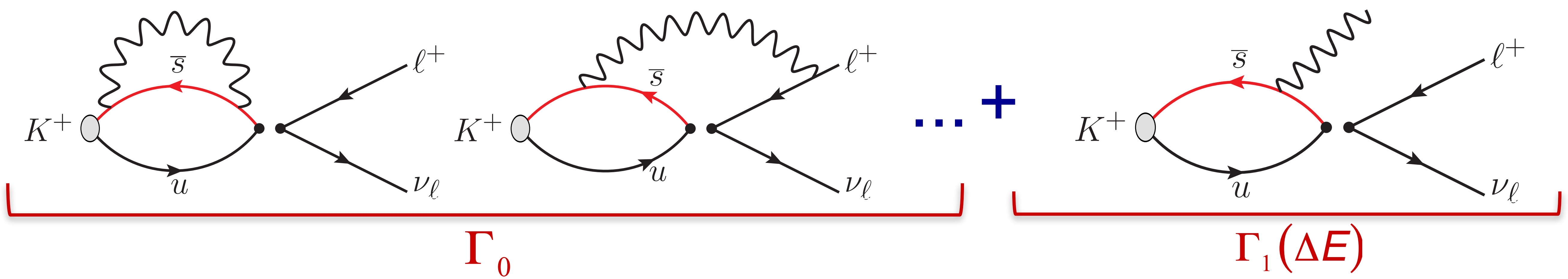


In the presence of electromagnetism it is not even possible to give a physical definition of f_P

J. Gasser and G.R.S. Zarnauskas, PLB 693 (2010) 122

Leptonic decays at O(α): the IR problem

At O(α), Γ_0 contains **infrared divergences**. One has to consider:



$$\boxed{\Gamma(P^\pm_{\ell 2}) = \Gamma(P^\pm \to \ell^\pm \nu_\ell) + \Gamma(P^\pm \to \ell^\pm \nu_\ell \gamma(\Delta E)) \equiv \Gamma_0 + \Gamma_1(\Delta E)}$$

with $0 \le E_\gamma \le \Delta E$. The sum is infrared finite

F. Bloch and A. Nordsieck, PR 52 (1937) 54

Both Γ_0 and $\Gamma_1(\Delta E)$ can be evaluated in a fully non-perturbative way in lattice simulations.

The strategy

In order to ensure the cancellation of IR divergences with good numerical precision, we rewrite:

$$\Gamma\left(P^{\pm}_{\ell 2}\right) = \left(\Gamma_0 - \Gamma_0^{pt}\right) + \left(\Gamma_0^{pt} + \Gamma_1^{pt}(\Delta E)\right) + \left(\Gamma_1(\Delta E) - \Gamma_1^{pt}(\Delta E)\right)$$

- Both the quantities Γ_0 and $\Gamma_1(\Delta E)$ are evaluated on the lattice

$$\frac{d^2\Gamma_1}{dx_\gamma dx_\ell} = \frac{\alpha_{em}\Gamma^{(tree)}}{4\pi}\left\{\frac{d^2\Gamma_{pt}}{dx_\gamma dx_\ell} + \frac{d^2\Gamma_{SD}}{dx_\gamma dx_\ell} + \frac{d^2\Gamma_{INT}}{dx_\gamma dx_\ell}\right\} \qquad x_\gamma = \frac{2p\cdot k}{m_P^2} \quad x_\ell = \frac{2p\cdot p_\ell - m_\ell^2}{m_P^2}$$

- The contribution $\Gamma_1 - \Gamma_1^{pt} = \Gamma_{SD} + \Gamma_{INT}$ can be computed in the infinite-volume limit requiring the knowledge of the structure dependent form factors $F_{A,V}(x_\gamma)$ and of f_P

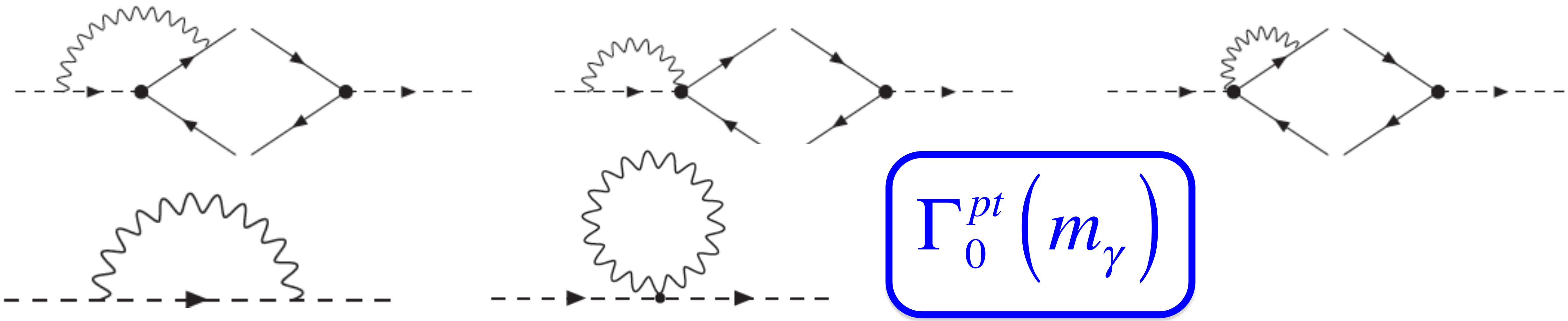
The strategy

$$\Gamma\left(P_{\ell 2}^{\pm}\right)=\left(\Gamma_0-\Gamma_0^{pt}\right)+\left(\Gamma_0^{pt}+\Gamma_1^{pt}(\Delta E)\right)$$
$$+\left(\Gamma_1(\Delta E)-\Gamma_1^{pt}(\Delta E)\right)$$

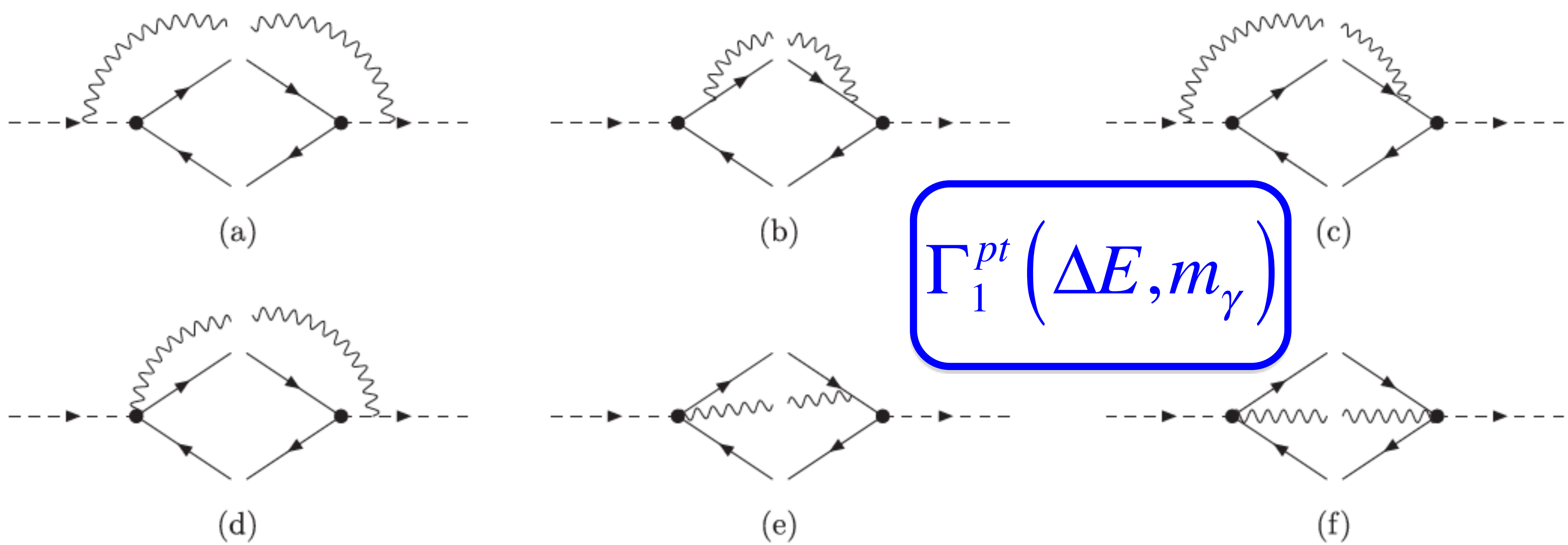
- The contributions from soft virtual photon to Γ_0 and Γ_0^{pt} in the first term are exactly the same and the IR divergence cancels in the difference $\Gamma_0-\Gamma_0^{\mathrm{pt}}$.
- The sum $\Gamma_0^{\mathrm{pt}}+\Gamma_1^{\mathrm{pt}}\left(\Delta E\right)$ in the second term is IR finite since it is a physically well defined quantity. This term can be thus calculated in perturbation theory with a different IR cutoff.
- The difference $\Gamma_1-\Gamma_1^{pt}$ in the third term is also IR finite.
- The three terms are also separately gauge invariant.

$$\Delta\Gamma_0(L)=\Gamma_0(L)-\Gamma_0^{pt}(L) \qquad \Gamma^{pt}(\Delta E)=\lim_{m_\gamma\to 0}\left[\Gamma_0^{pt}\left(m_\gamma\right)+\Gamma_1^{pt}\left(\Delta E,m_\gamma\right)\right]$$

$$\Gamma^{pt}(\Delta E) = \lim_{m_\gamma \to 0}\left[\Gamma_0^{pt}(m_\gamma) + \Gamma_1^{pt}(\Delta E, m_\gamma)\right]$$



$$\boxed{\Gamma_0^{pt}(m_\gamma)}$$



$$\boxed{\Gamma_1^{pt}(\Delta E, m_\gamma)}$$

$$\Gamma^{pt}(\Delta E) = \lim_{m_\gamma \to 0} \left[\Gamma_0^{pt}(m_\gamma) + \Gamma_1^{pt}(\Delta E, m_\gamma) \right]$$

The result is:

$$\Gamma(\Delta E) = \Gamma_0^{\text{tree}} \times \left(1 + \frac{\alpha}{4\pi} \left\{ 3\log\left(\frac{m_\pi^2}{M_W^2}\right) + \log\left(r_\ell^2\right) - 4\log(r_E^2) + \frac{2-10r_\ell^2}{1-r_\ell^2}\log(r_\ell^2) \right. \right.$$

$$-2\frac{1+r_\ell^2}{1-r_\ell^2}\log(r_E^2)\log(r_\ell^2) - 4\frac{1+r_\ell^2}{1-r_\ell^2}\,\mathrm{Li}_2(1-r_\ell^2) - 3$$

$$+\left[\frac{3+r_E^2-6r_\ell^2+4r_E(-1+r_\ell^2)}{(1-r_\ell^2)^2}\log(1-r_E) + \frac{r_E(4-r_E-4r_\ell^2)}{(1-r_\ell^2)^2}\log(r_\ell^2) \right.$$

$$\left.\left.\left. -\frac{r_E(-22+3r_E+28r_\ell^2)}{2(1-r_\ell^2)^2} - 4\frac{1+r_\ell^2}{1-r_\ell^2}\,\mathrm{Li}_2(r_E) \right] \right\} \right)$$

$$\boxed{r_E = 2\Delta E/m_\pi}$$

$$\boxed{r_\ell = m_\ell/m_\pi}$$

NEW

IMPORTANT CHECK: For $\Delta E = \Delta E_{MAX}$ the well known result for the total rate as in S. M. Berman, PRL 1 (1958) 468 and T. Kinoshita, PRL 2 (1959) 477 is reproduced

$$\Delta\Gamma_0(L) = \Gamma_0(L) - \Gamma_0^{pt}(L)$$

- $\Delta\Gamma_0(L)$ is the first term in the master formula

$$\Gamma(P_{\ell 2}^{\pm}) = \lim_{L\to\infty}\left[\Gamma_0(L) - \Gamma_0^{pt}(L)\right] + \left[\Gamma^{pt} + \Gamma_{SD} + \Gamma_{INT}\right](\Delta E)$$

Montecarlo simulation
Lattice QCD

1

PHYSICAL REVIEW LETTERS **120,** 072001 (2018)

arXiv:1711.06537

First Lattice Calculation of the QED Corrections to Leptonic Decay Rates

D. Giusti,[1] V. Lubicz,[1] G. Martinelli,[2] C.T. Sachrajda,[3]
F. Sanfilippo,[4] S. Simula,[4] N. Tantalo,[5] and C. Tarantino[1]

Light-meson leptonic decay rates in lattice QCD+QED

arXiv:1904.08731

M. Di Carlo,[1] D. Giusti,[2] V. Lubicz,[2] G. Martinelli,[1]
C.T. Sachrajda,[3] F. Sanfilippo,[4] S. Simula,[4] and N. Tantalo[5]

2

Perturbation theory
with pointlike pion
in finite volume

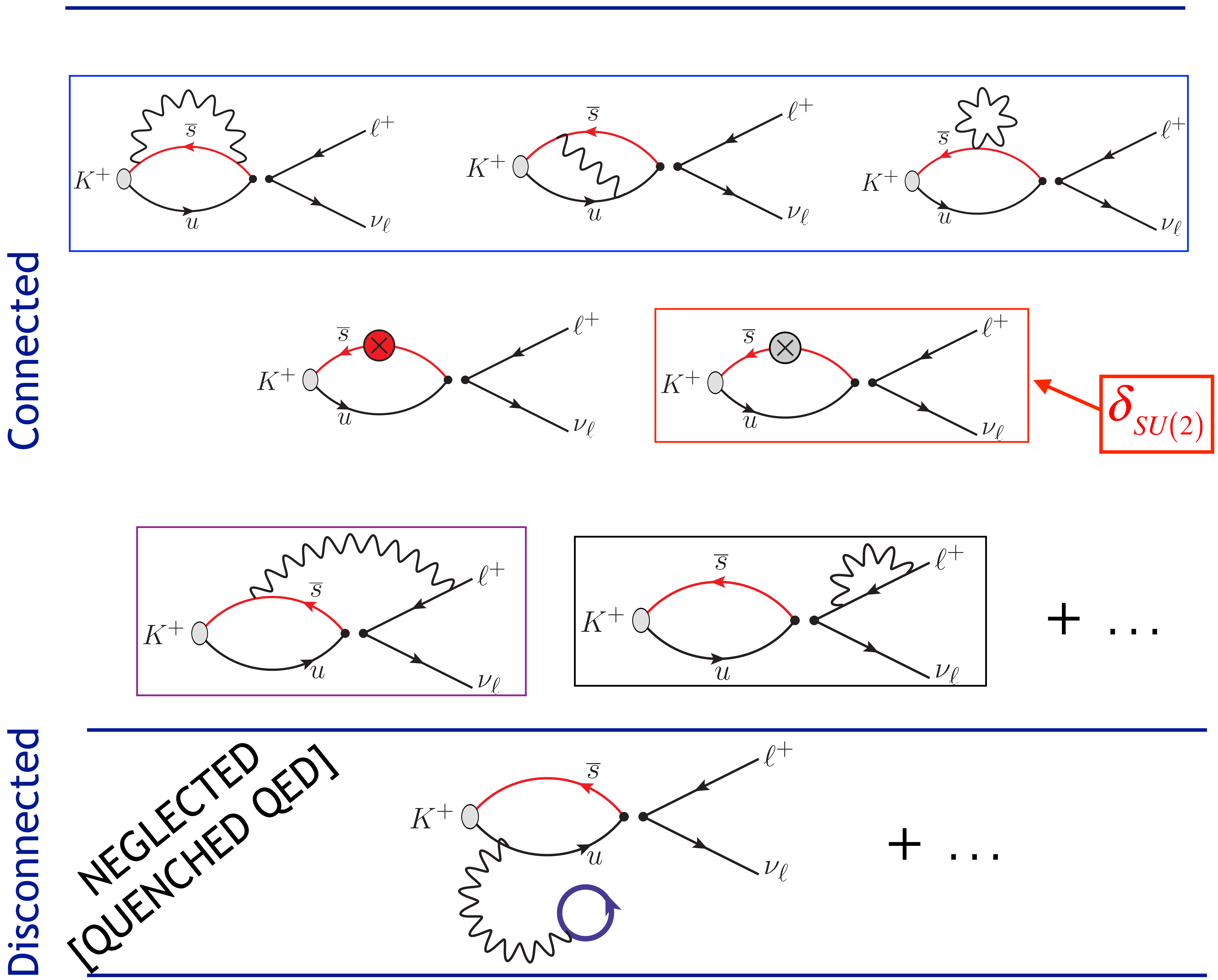
PHYSICAL REVIEW D **95,** 034504 (2017)

Finite-volume QED corrections to decay amplitudes in lattice QCD

V. Lubicz,[1] G. Martinelli,[2,3] C. T. Sachrajda,[4] F. Sanfilippo,[4] S. Simula,[5] and N. Tantalo[6]

Lattice calculation of $\Gamma_0(L)$ at $O(\alpha)$

The Feynman diagrams at O(α) can be divided in 3 classes



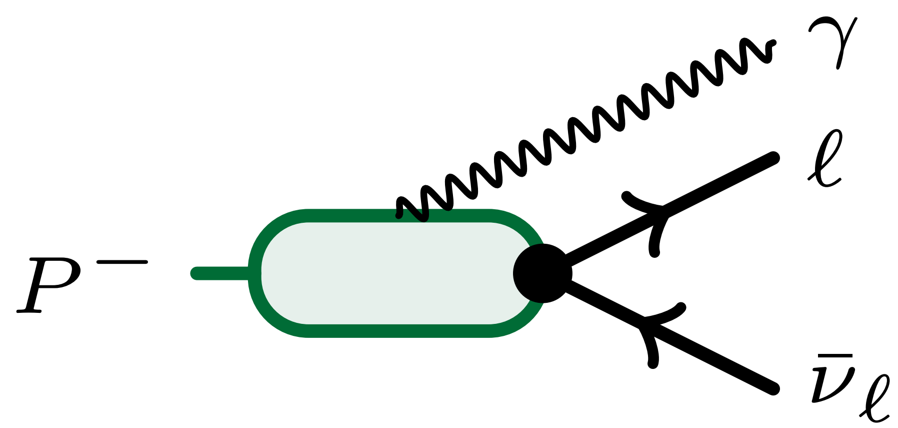
① The photon is attached to quark lines

② The photon connects one quark and one charged lepton line

③ Leptonic wave function renormalization. It cancels in

$$\Gamma_0(L) - \Gamma_0^{\mathrm{pt}}(L)$$

Real photon emission amplitude



$$\boxed{H_W^{\alpha r}(k,p) = \epsilon_\mu^r(k)\, H_W^{\alpha\mu}(k,p) = \epsilon_\mu^r(k) \int d^4y\, e^{ik\cdot y}\, \mathtt{T}\langle 0|j_W^\alpha(0) j_{em}^\mu(y)|P(\boldsymbol{p})\rangle}$$

By setting $k^2 = 0$, at fixed meson mass, the form factors depend on $p \cdot k$ only. Moreover, by choosing a *physical* basis for the polarization vectors, *i.e.* $\epsilon_r(\mathbf{k}) \cdot k = 0$, one has

$$H_W^{\alpha r}(k,p) = \epsilon_\mu^r(\boldsymbol{k}) \Bigg\{ -i\frac{F_V}{m_P}\varepsilon^{\mu\alpha\gamma\beta} k_\gamma p_\beta + \left[\frac{F_A}{m_P} + \frac{f_P}{p\cdot k}\right](p\cdot k\, g^{\mu\alpha} - p^\mu k^\alpha) + \frac{f_P}{p\cdot k} p^\mu p^\alpha \Bigg\}$$

Form factors: results

PHYSICAL REVIEW D **103,** 014502 (2021)

arXiv:2006.05358

First lattice calculation of radiative leptonic decay rates of pseudoscalar mesons

A. Desiderio,[1] R. Frezzotti,[1] M. Garofalo,[2] D. Giusti,[3,4] M. Hansen,[5] V. Lubicz,[2] G. Martinelli,[6] C. T. Sachrajda,[7] F. Sanfilippo,[4] S. Simula,[4] and N. Tantalo[1]

$$F^P_{A,V}(x_\gamma) = C^P_{A,V} + D^P_{A,V} x_\gamma$$

F_A

$$C_A^\pi = 0.010 \pm 0.003\,; \quad D_A^\pi = 0.0004 \pm 0.0006\,; \quad \rho_{C_A^\pi, D_A^\pi} = -0.419\,;$$

$$C_A^K = 0.037 \pm 0.009\,; \quad D_A^K = -0.001 \pm 0.007\,; \quad \rho_{C_A^K, D_A^K} = -0.673\,;$$

$$C_A^D = 0.109 \pm 0.009\,; \quad D_A^D = -0.10 \pm 0.03\,; \quad \rho_{C_A^D, D_A^D} = -0.557\,;$$

$$C_A^{D_s} = 0.092 \pm 0.006\,; \quad D_A^{D_s} = -0.07 \pm 0.01\,; \quad \rho_{C_A^{D_s}, D_A^{D_s}} = -0.745\,.$$

F_V

$$C_V^\pi = 0.023 \pm 0.002\,; \quad D_V^\pi = -0.0003 \pm 0.0003\,; \quad \rho_{C_V^\pi, D_V^\pi} = -0.570\,;$$

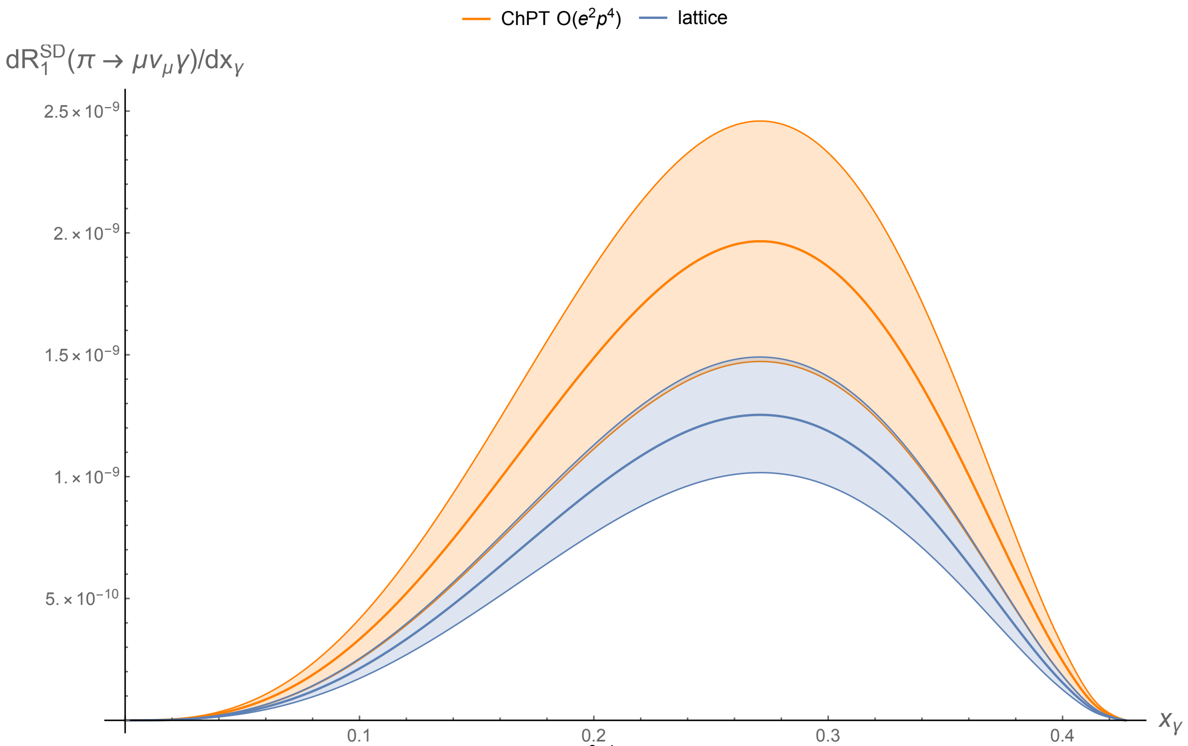
$$C_V^K = 0.12 \pm 0.01\,; \quad D_V^K = -0.02 \pm 0.01\,; \quad \rho_{C_V^K, D_V^K} = -0.714\,;$$

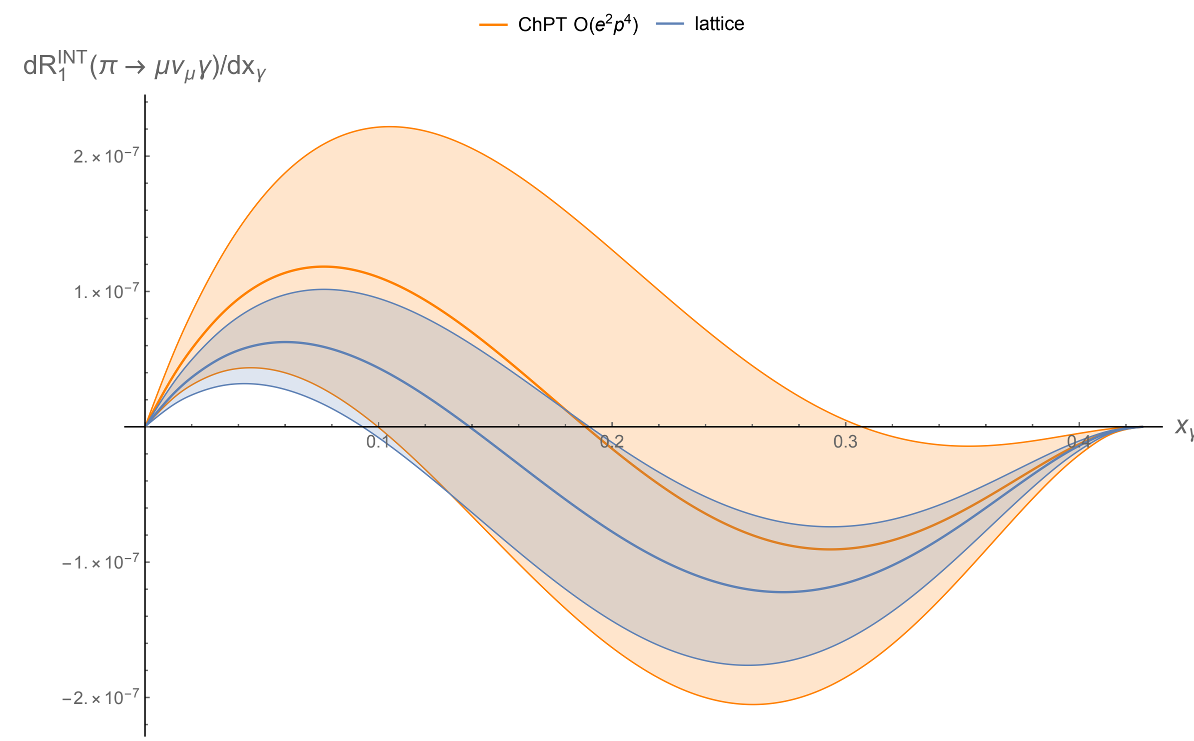
$$C_V^D = -0.15 \pm 0.02\,; \quad D_V^D = 0.12 \pm 0.04\,; \quad \rho_{C_V^D, D_V^D} = -0.580\,;$$

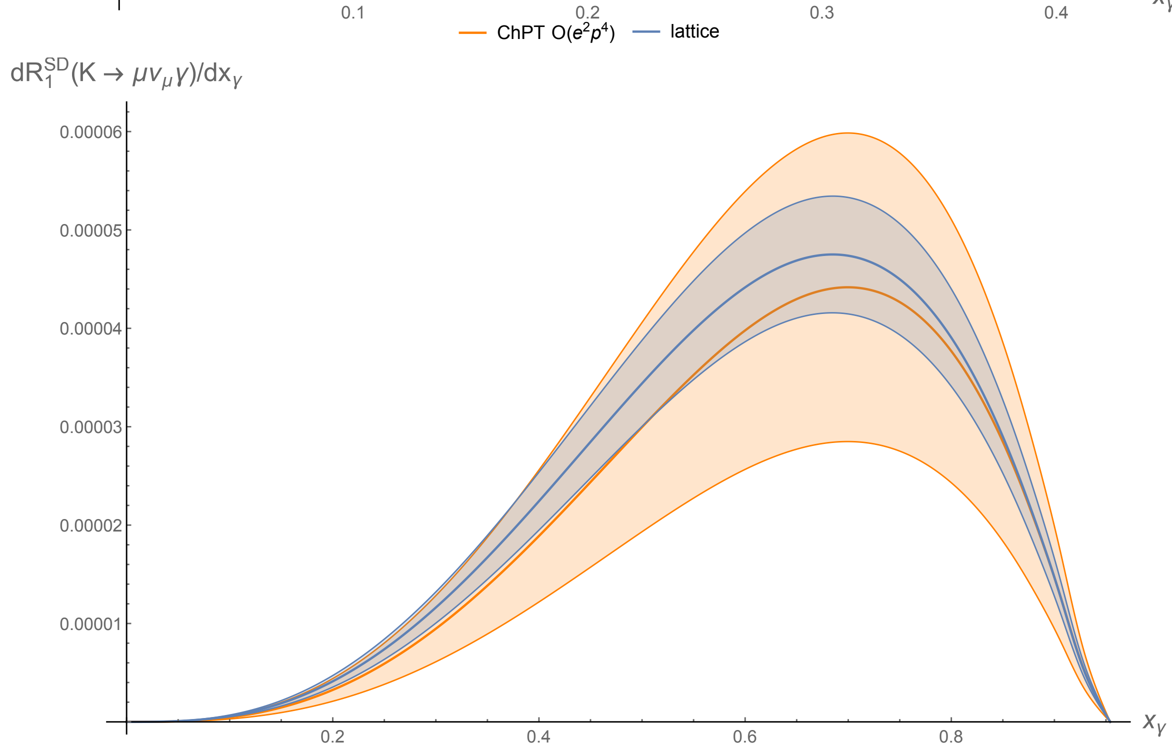
$$C_V^{D_s} = -0.12 \pm 0.02\,; \quad D_V^{D_s} = 0.16 \pm 0.03\,; \quad \rho_{C_V^{D_s}, D_V^{D_s}} = -0.900\,.$$

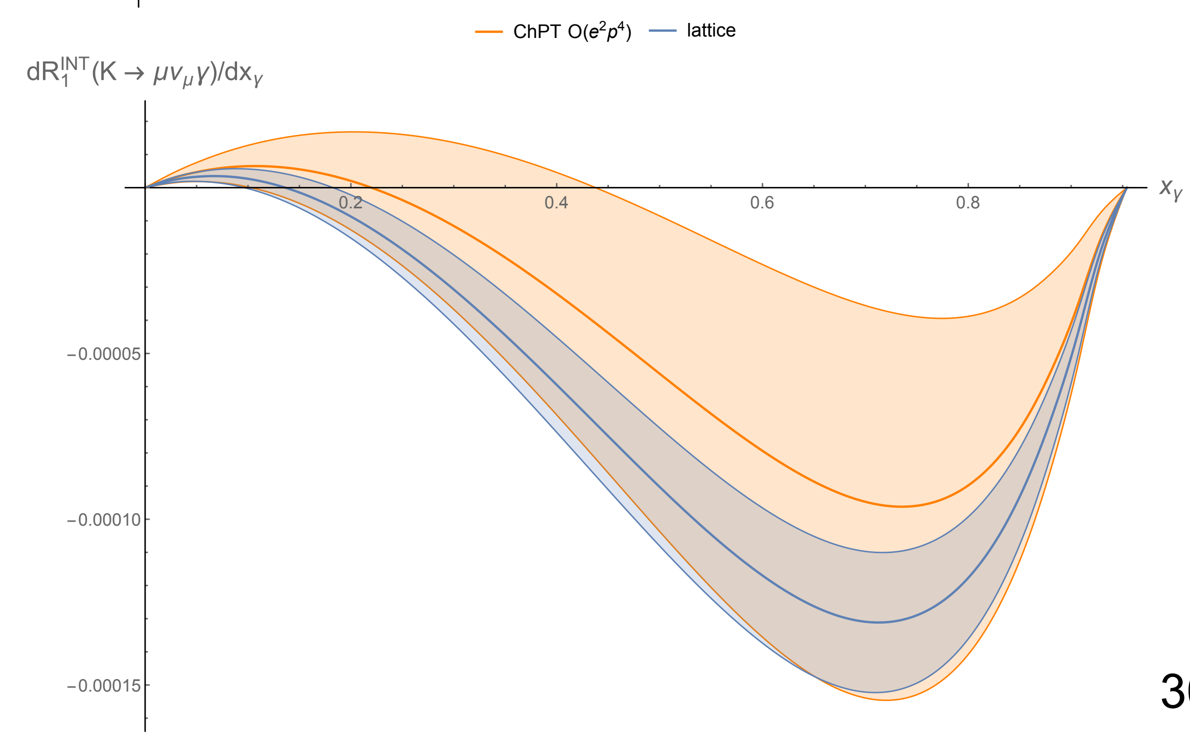
$$\frac{4\pi}{\alpha\,\Gamma_0^{\rm tree}}\,\frac{d\Gamma_1^{\rm SD}}{dx_\gamma} = \frac{m_P^2}{6 f_P^2\, r_\ell^2\,(1-r_\ell^2)^2}\left[F_V(x_\gamma)^2 + F_A(x_\gamma)^2\right] f^{\rm SD}(x_\gamma)$$

$$\frac{4\pi}{\alpha\,\Gamma_0^{\rm tree}}\,\frac{d\Gamma_1^{\rm INT}}{dx_\gamma} = -\frac{2\,m_P}{f_P\,(1-r_\ell^2)^2}\left[F_V(x_\gamma)\, f_V^{\rm INT}(x_\gamma) + F_A(x_\gamma)\, f_A^{\rm INT}(x_\gamma)\right]$$

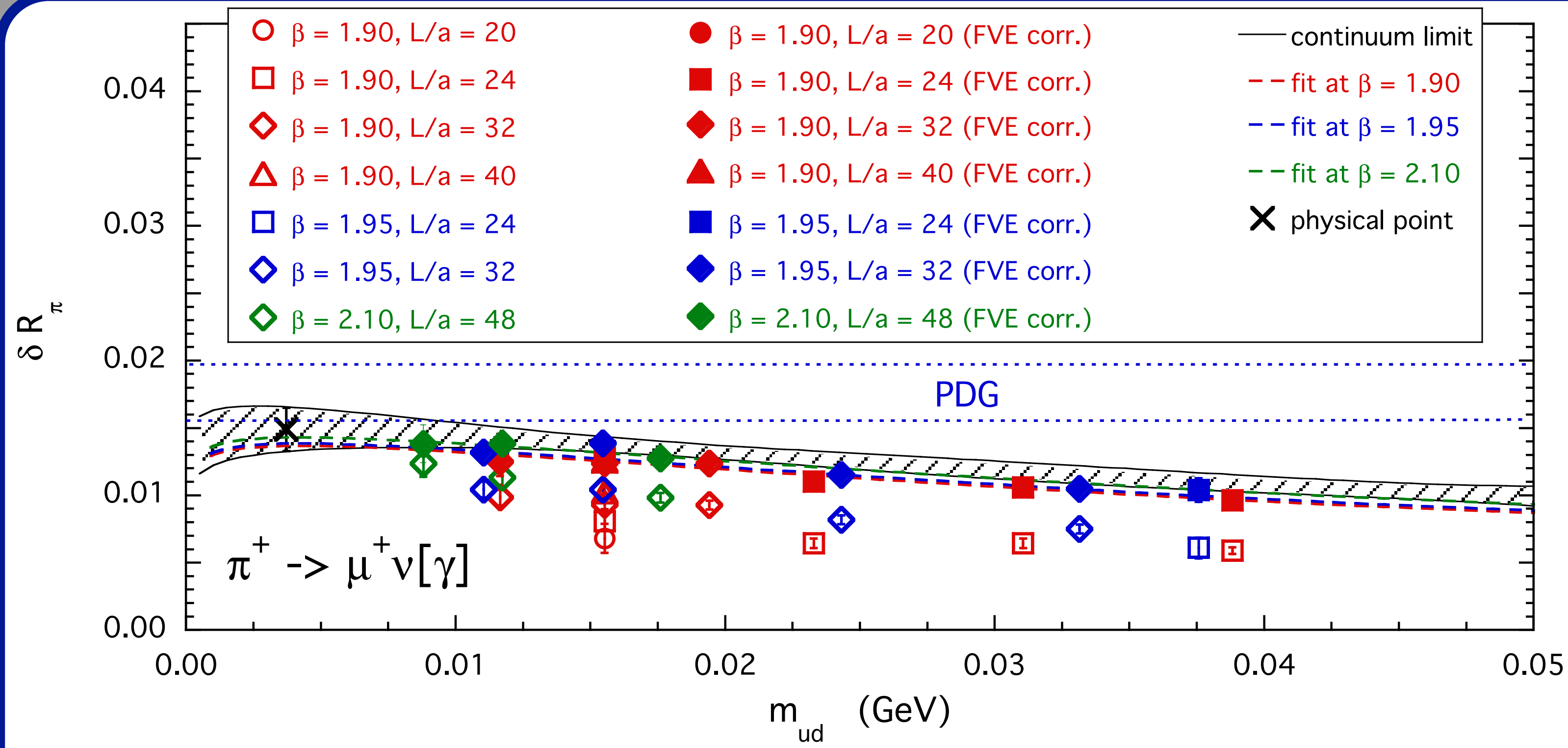








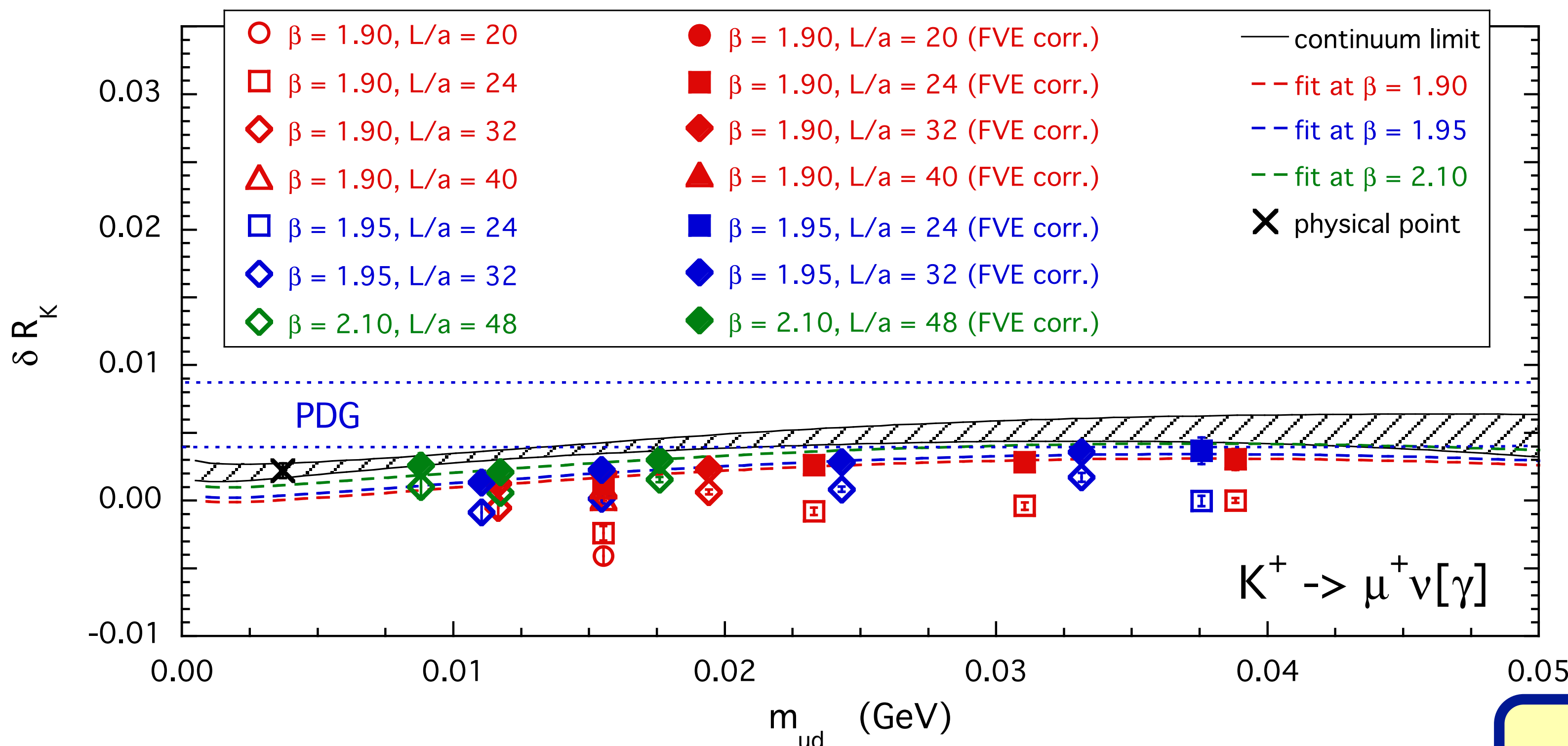
Leptonic decays at O(α): RESULTS



$$\Gamma\left(P^{\pm} \to \mu^{\pm}\nu_{\mu}[\gamma]\right) = \Gamma^{(tree)}\left[1 + \delta R_{P^{\pm}}\right]$$

$$\delta R_{\pi^{\pm}} = 0.0153(16)_{stat}(10)_{syst}$$
$$= 0.0153(19)$$

ChPT/PDG $\delta R_{\pi} = 0.0176(21)$



$$\delta R_{K^{\pm}} = 0.0024(6)_{stat}(8)_{syst}$$
$$= 0.0024(10)$$

ChPT/PDG $\delta R_{K} = 0.0064(24)$

V.Cirigliano, H.Neufeld; PLB 700 (2011) 7

$$f_K^{(0)}|V_{us}| = 35.23(4)_{\exp}(2)_{th} \text{ MeV}$$

$$f_{\pi}^{(0)}|V_{ud}| = 127.28(2)_{\exp}(12)_{th} \text{ MeV}$$

$$f_K^{(0)} = 156.11(21) \text{ MeV}$$

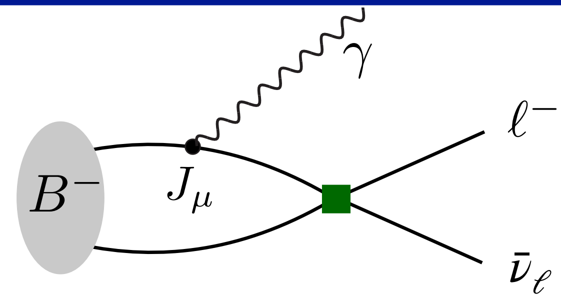
FLAG(2019) N_f=2+1+1

$$|V_{us}| = 0.22567(42)$$

$|V_{us}| = 0.2252(5)$ PDG

Radiative corrections to leptonic heavy-meson decays

$B^- \to \ell^- \bar{\nu}_\ell \gamma$



- The emission of a real hard photon removes the $(m_\ell/M_B)^2$ helicity suppression
- This is the simplest process that probes (for large E_γ) the first inverse moment of the B-meson LCDA

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega} \Phi_{B+}(\omega, \mu)$$

λ_B is an important input in QCD-factorization predictions for non-leptonic B decays but is poorly known

M. Beneke, V. M. Braun, Y. Ji, Y.-B. Wei, 2018

High-precision determination of radiative-leptonic-decay form factors using lattice QCD: a study of methods

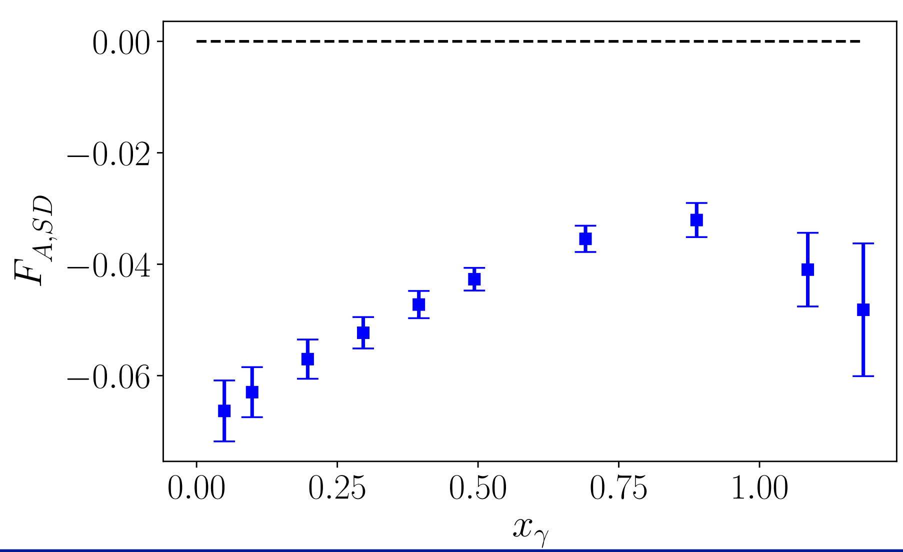
arXiv:2302.01298

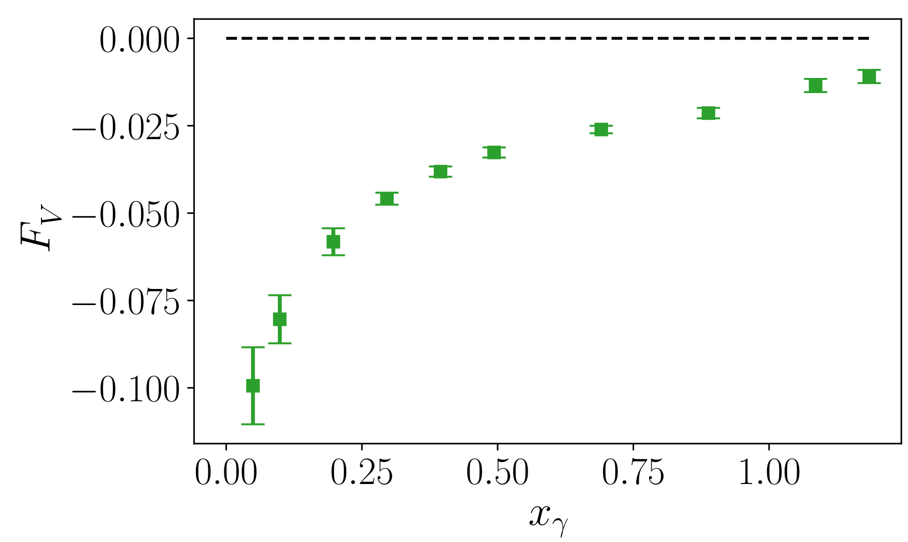
Davide Giusti,[1] Christopher F. Kane,[2] Christoph Lehner,[1] Stefan Meinel,[2] and Amarjit Soni[3]

[1] *Fakultät für Physik, Universität Regensburg, 93040, Regensburg, Germany*
[2] *Department of Physics, University of Arizona, Tucson, AZ 85721, USA*
[3] *Brookhaven National Laboratory, Upton, NY 11973, USA*

(Dated: February 2, 2023)

- $D_s \to \ell \nu_\ell \gamma$
- full kinematically allowed photon-energy range





Another application: the muon g-2

PHYSICAL REVIEW D **99,** 114502 (2019)

arXiv:1901.10462

Electromagnetic and strong isospin-breaking corrections to the muon $g-2$ from lattice QCD+QED

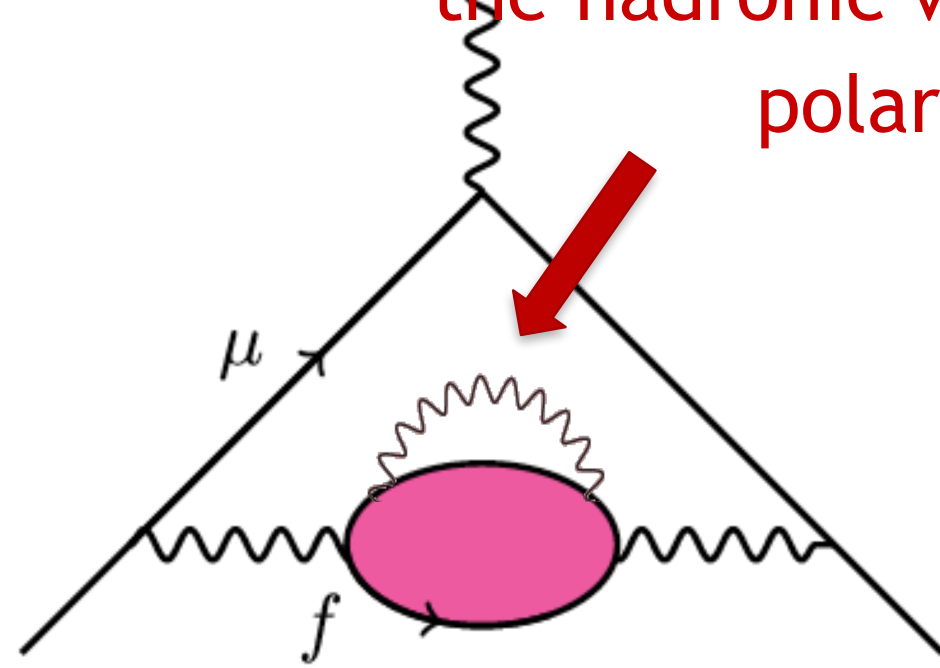
D. Giusti and V. Lubicz
Dipartimento di Matematica e Fisica, Università degli Studi Roma Tre and INFN, Sezione di Roma Tre, Via della Vasca Navale 84, I-00146 Rome, Italy

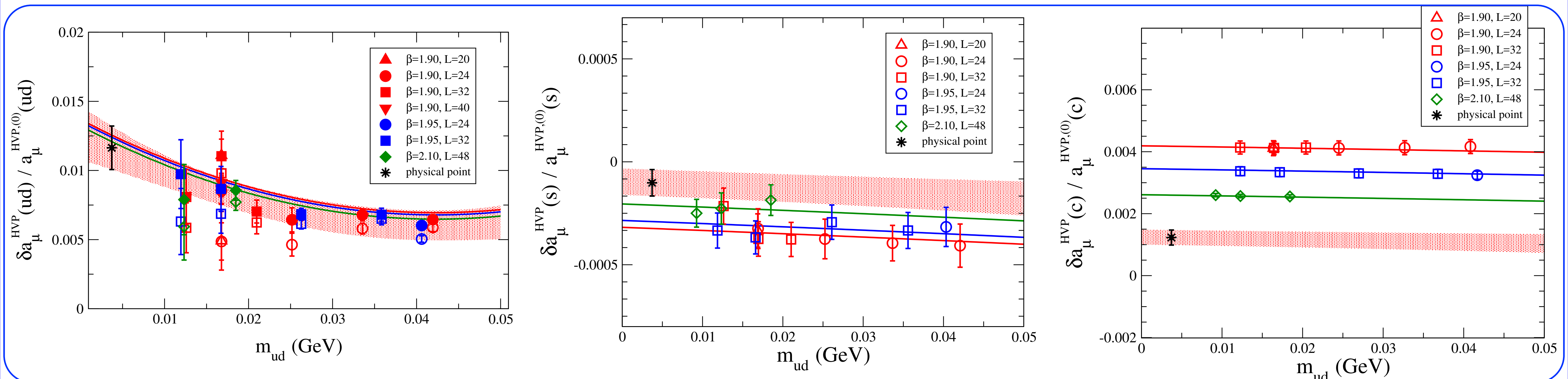
G. Martinelli
Dipartimento di Fisica, Università degli Studi di Roma "La Sapienza" and INFN, Sezione di Roma, Piazzale Aldo Moro 5, 00185 Roma, Italy

F. Sanfilippo and S. Simula
Istituto Nazionale di Fisica Nucleare, Sezione di Roma Tre, Via della Vasca Navale 84, I-00146 Rome, Italy

(for ETM Collaboration)

Electromagnetic correction to the hadronic vacuum polarization





Conclusions and future perspectives

- The experimental and theoretical accuracy reached in flavor physics for some hadronic observables implies that electromagnetic and strong isospin breaking effects cannot be neglected anymore

- We have developed a method to compute isospin breaking effects in hadronic processes with lattice QCD and presented the first calculation for light-meson leptonic decay rates

- For hadronic decays, the presence of IR divergences in the intermediate steps of the calculation requires a dedicated procedure

- Extension to leptonic heavy-light meson decays (arXiv:2111.15614, arXiv:2302.01298) and neutron beta decay is in progress

Supplementary slides

Details of the lattice simulation

We have used the gauge field configurations generated by ETMC, European Twisted Mass Collaboration, in the pure isosymmetric QCD theory with Nf=2+1+1 dynamical quarks

ensemble	β	V/a^4	$a\mu_{ud}$	$a\mu_\sigma$	$a\mu_\delta$	N_{cf}	$a\mu_s$	M_π (MeV)	M_K (MeV)
$A40.40$	1.90	$40^3 \cdot 80$	0.0040	0.15	0.19	100	0.02363	317(12)	576(22)
$A30.32$		$32^3 \cdot 64$	0.0030			150		275(10)	568(22)
$A40.32$			0.0040			100		316(12)	578(22)
$A50.32$			0.0050			150		350(13)	586(22)
$A40.24$		$24^3 \cdot 48$	0.0040			150		322(13)	582(23)
$A60.24$			0.0060			150		386(15)	599(23)
$A80.24$			0.0080			150		442(17)	618(14)
$A100.24$			0.0100			150		495(19)	639(24)
$A40.20$		$20^3 \cdot 48$	0.0040			150		330(13)	586(23)
$B25.32$	1.95	$32^3 \cdot 64$	0.0025	0.135	0.170	150	0.02094	259 (9)	546(19)
$B35.32$			0.0035			150		302(10)	555(19)
$B55.32$			0.0055			150		375(13)	578(20)
$B75.32$			0.0075			80		436(15)	599(21)
$B85.24$		$24^3 \cdot 48$	0.0085			150		468(16)	613(21)
$D15.48$	2.10	$48^3 \cdot 96$	0.0015	0.1200	0.1385	100	0.01612	223 (6)	529(14)
$D20.48$			0.0020			100		256 (7)	535(14)
$D30.48$			0.0030			100		312 (8)	550(14)

- Gluon action: Iwasaki
- Quark action: twisted mass at maximal twist (automatically O(a) improved)
  OS for *s* and *c* valence quarks

Pion masses in the range 220 - 490 MeV

4 volumes @ $M_\pi \simeq 320$ MeV and $a \simeq 0.09$ fm

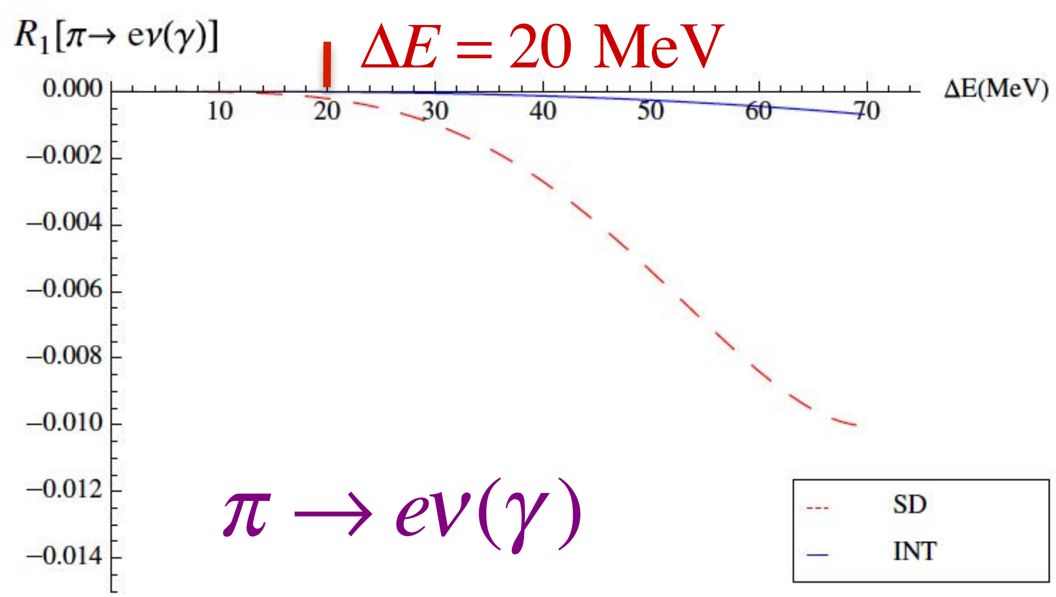
$M_\pi L \simeq 3.0 \div 5.8$

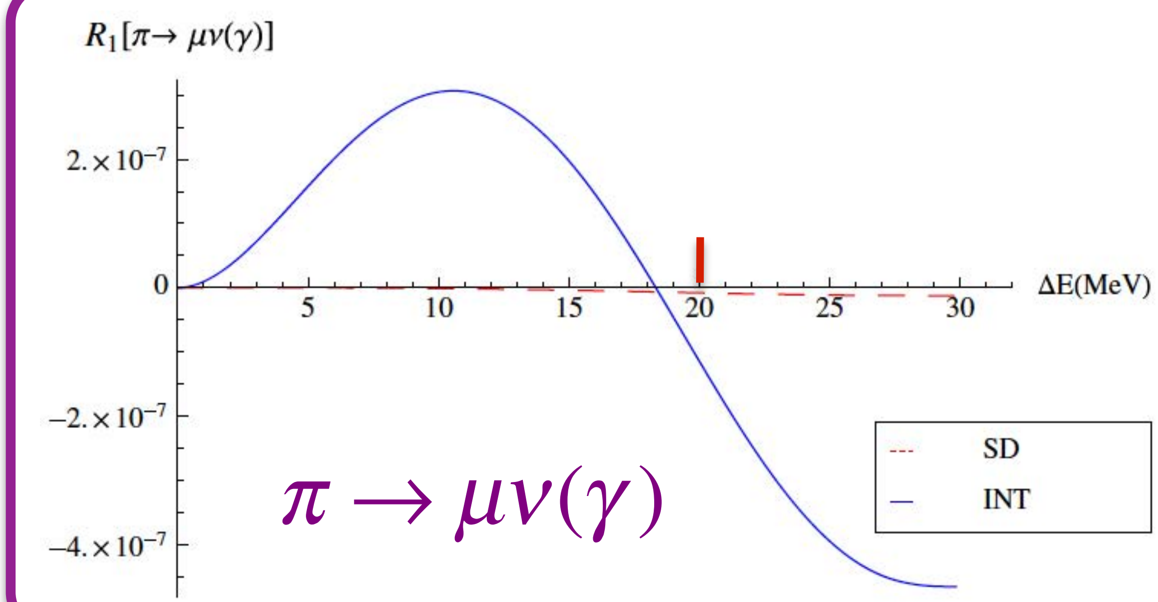
Extended Twisted Mass Collaboration

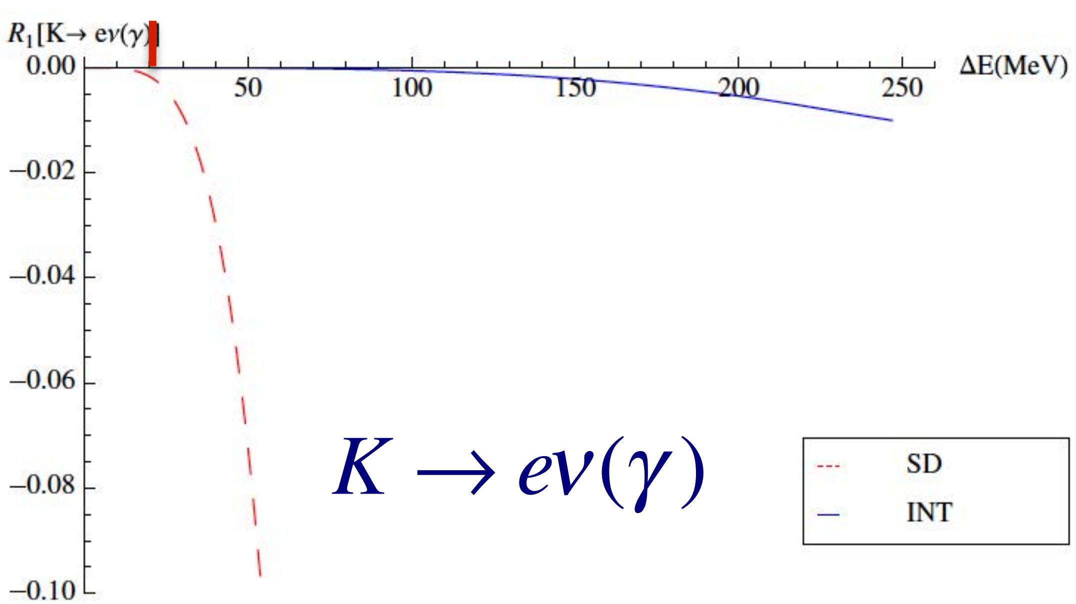
$$R_1^A(\Delta E) = \frac{\Gamma_1^A(\Delta E)}{\Gamma_0^{\alpha,\mathrm{pt}} + \Gamma_1^{\mathrm{pt}}(\Delta E)} \quad , \quad A = \{\mathrm{SD}, \mathrm{INT}\}$$

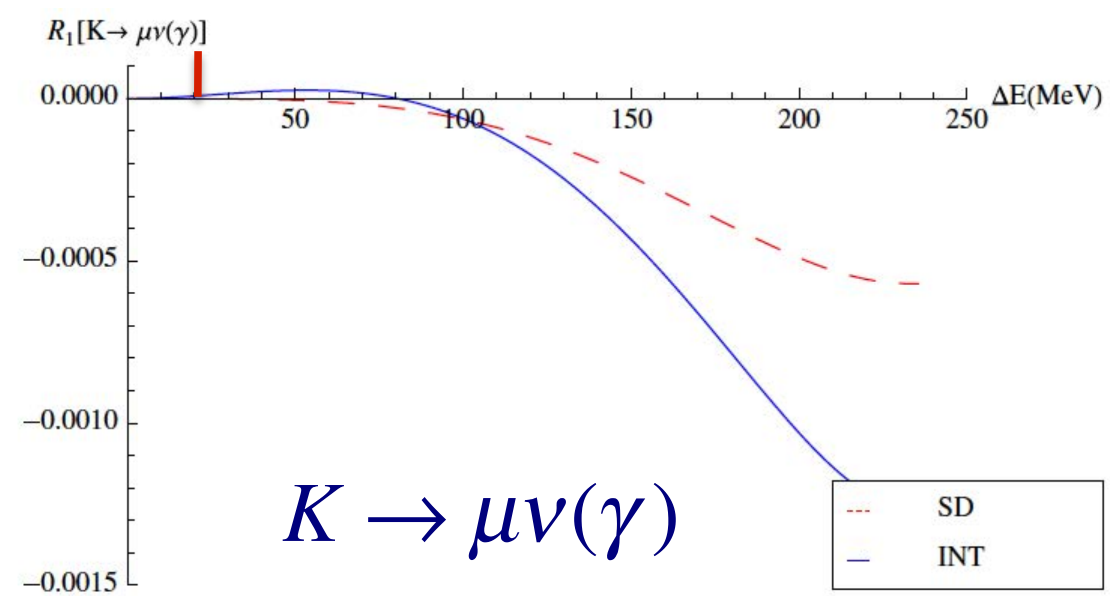
SD = structure dependent

INT = interference



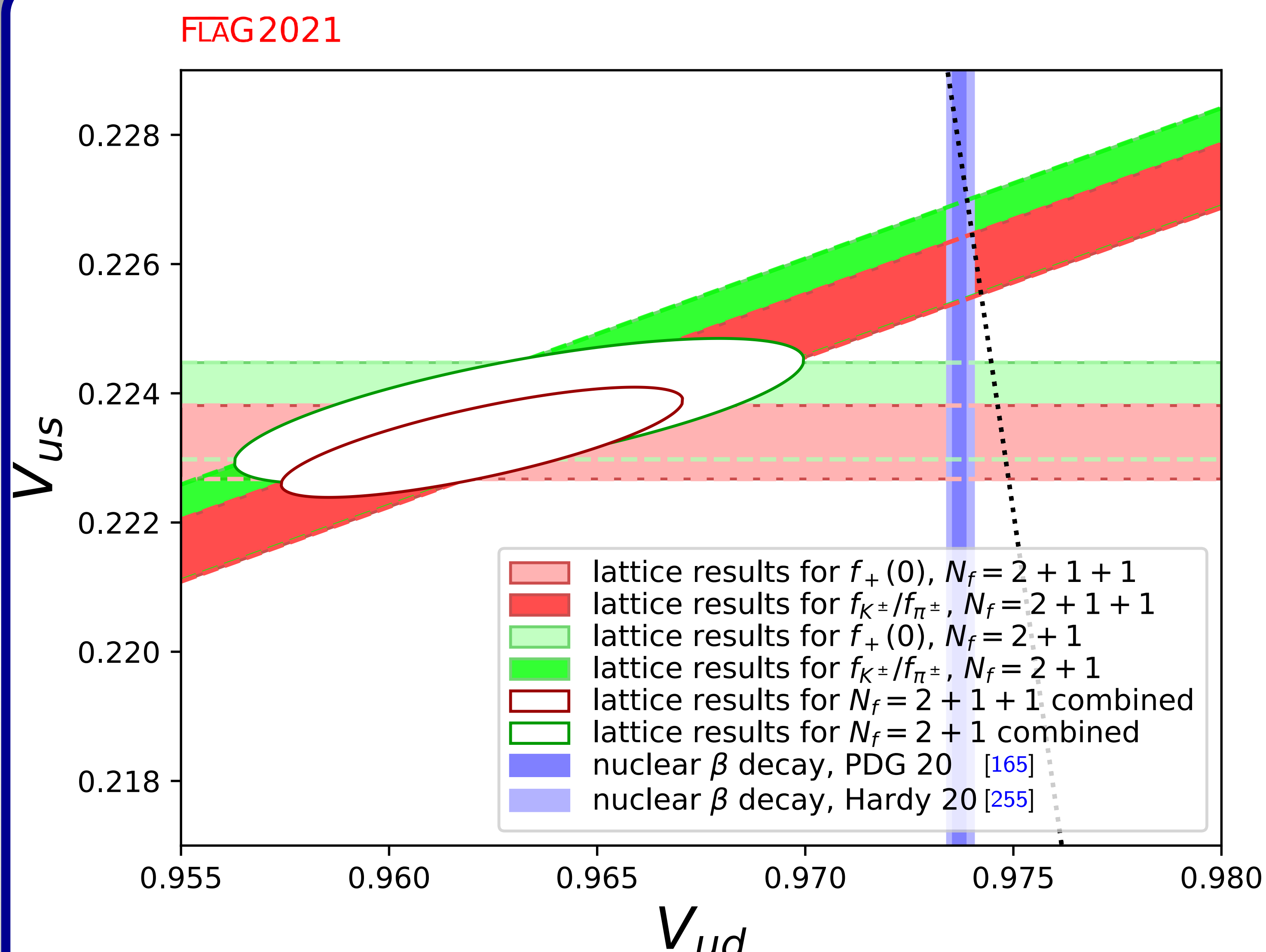






- Interference contributions are negligible in all the decays
- Structure-dependent contributions can be sizable for $K \to e\nu(\gamma)$ but they are negligible for $\Delta E < 20\ \mathrm{MeV}$ (which is experimentally accessible)

Unitarity of the CKM first-row



$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.27599(38)$$

$$|V_{us}|\, f_+(0) = 0.21654(41)$$

$$\boxed{|V_u|^2 \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2}$$

$\boxed{f_+(0)}$	$\boxed{f_{K^\pm}/f_{\pi^\pm}}$	$\lvert V_{ud}\rvert$ from
$\lvert V_u\rvert^2 = 0.99884(53) \approx 2.2\sigma$	$\lvert V_u\rvert^2 = 0.99988(46) \approx 0.4\sigma$	PDG 18
$\lvert V_u\rvert^2 = 0.99794(37) \approx 5.6\sigma$	$\lvert V_u\rvert^2 = 0.99885(34) \approx 3.4\sigma$	PDG 20