Simultaneous Global Analysis of Di-Hadron Fragmentation Functions and Transversity PDFs

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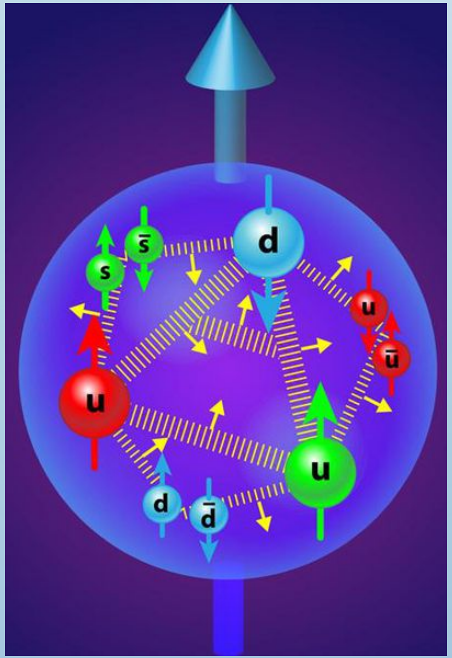
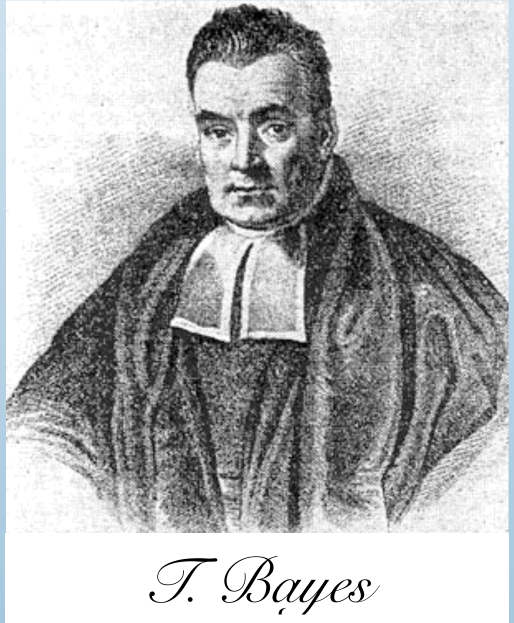
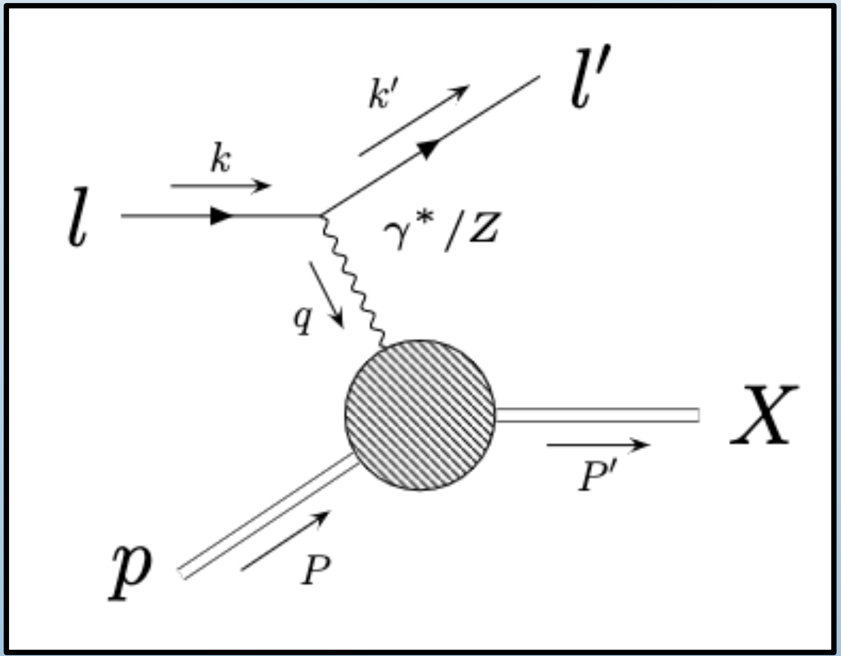
April 12, 2023

1. Introduction
2. Extraction of DiFFs
3. Extraction of Transversity PDFs
4. Extraction of Tensor Charges
5. Conclusions and Outlook



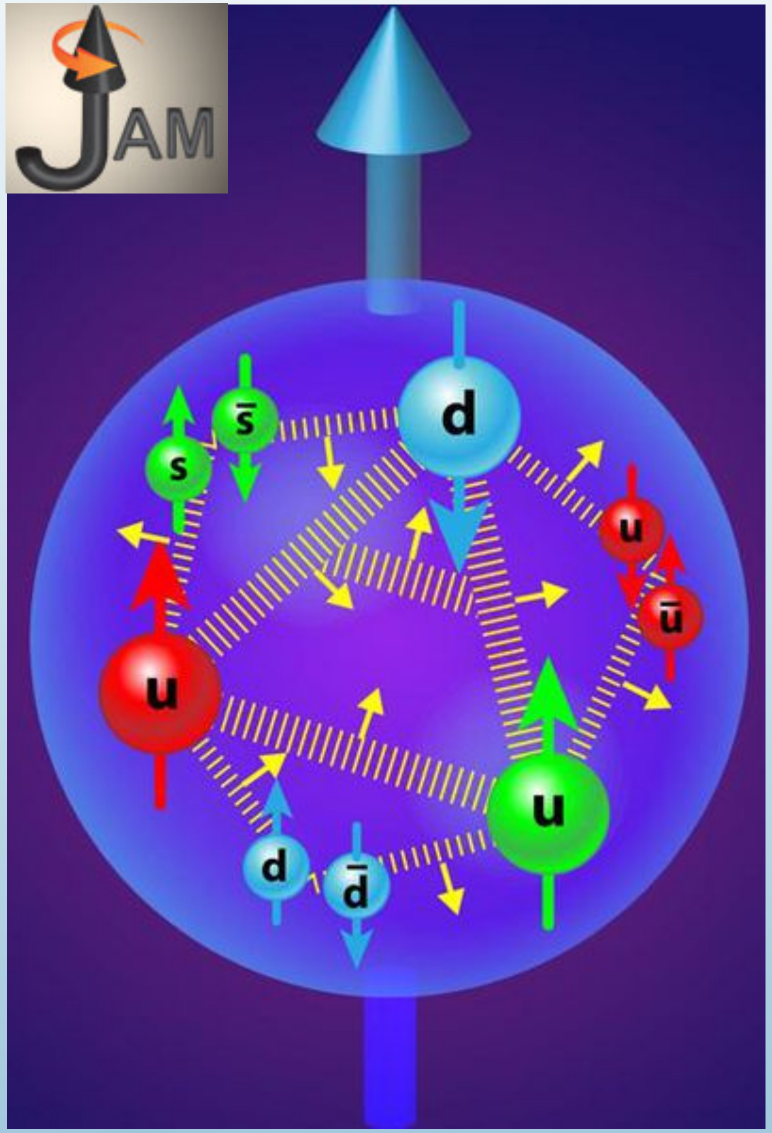




JAM Collaboration

3-dimensional structure of nucleons:
- <u>Parton distribution functions (PDFs)</u>
- <u>Fragmentation functions (FFs)</u>
- Transverse momentum dependent distributions (TMDs)
- Generalized parton distributions (GPDs)

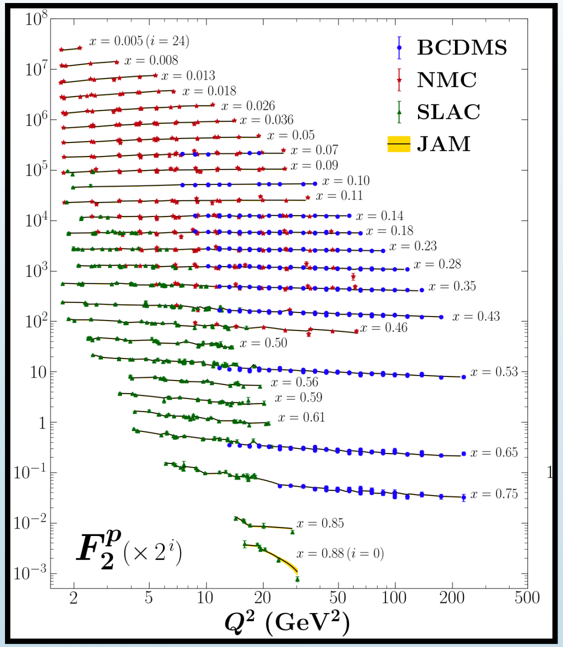
- Collinear factorization in perturbative QCD
- Simultaneous determinations of PDFs, FFs, etc.
- Monte Carlo methods for Bayesian inference



Introduction

$$\chi^2(\boldsymbol{a}) = \sum_{i,e} \left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}} \right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1 - N_e}{\delta N_e} \right)^2$$

χ^2 Minimization



Hadron Structure

$$\frac{d}{d\ \ln(\mu^2)} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z,\mu) f_j(\frac{x}{z},\mu)$$

Param. + Evolve + Factorization

$$\sigma = \sum_{i,j} H_{ij} \otimes f_i \otimes f_j$$

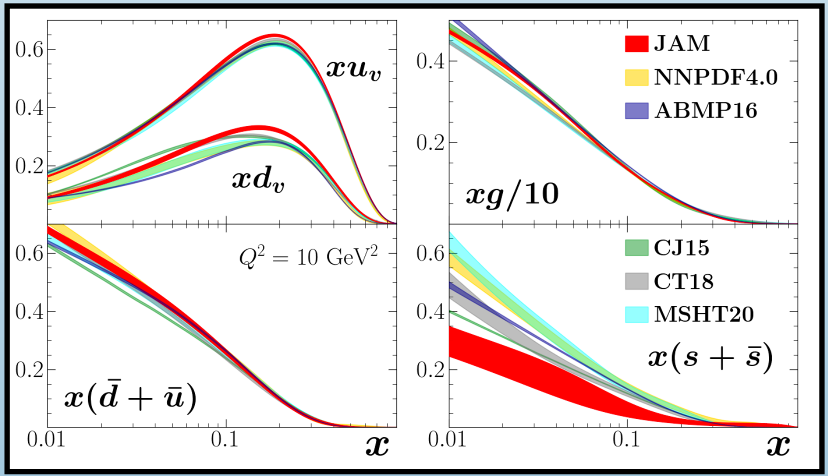
Global QCD Analysis

$$\mathcal{L}(\boldsymbol{a}, \text{data}) = \exp\left(-\frac{1}{2}\chi^2(\boldsymbol{a}, \text{data})\right)$$

$$\mathcal{P}(\boldsymbol{a}|\text{data}) \sim \mathcal{L}(\boldsymbol{a}, \text{data})\, \pi(\boldsymbol{a})$$

USQCD

Extended Twisted Mass Collaboration



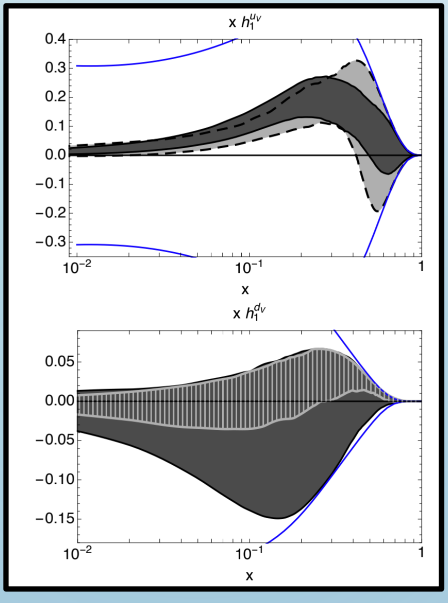
Data Resampling

$$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$

Approaches to Extract Transversity

Di-Hadron Frag.

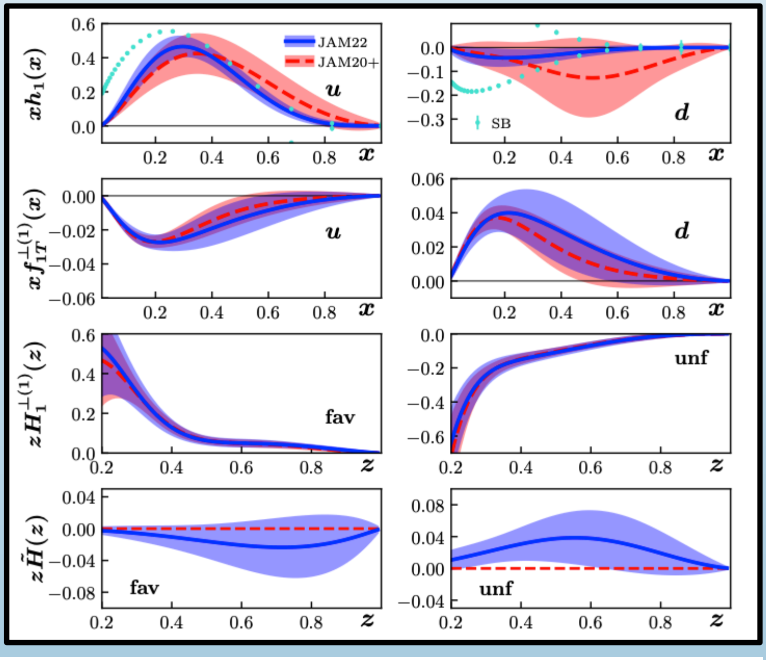
- Radici + Bacchetta (RB18)
- Benel + Courtoy + Ferro-Hernandez (2020)



M. Radici and A. Bacchetta,
Phys. Rev. Lett. **120**, no. 19, 192001 (2018)

TMD + Collinear Twist-3

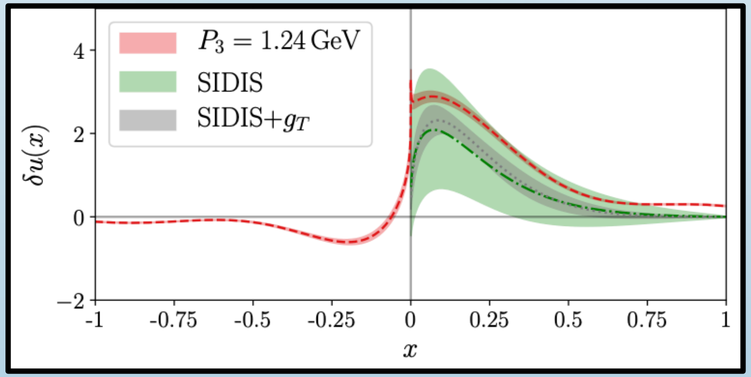
- JAM3D



L. Gamberg *et al.*, Phys. Rev. D **106**, no. 3, 034014 (2022)

Lattice QCD

- ETMC Collaboration
- PNDME Collaboration
- Hasan *et al.*

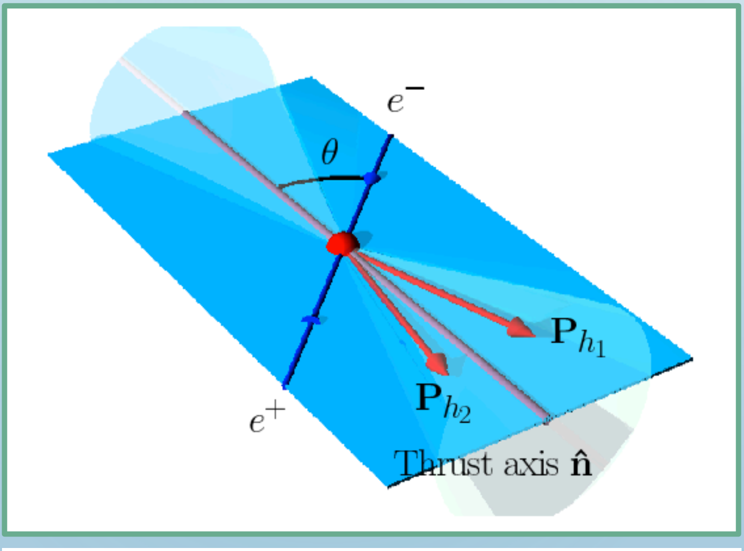


C. Alexandrou *et al.*, Phys. Rev. D **104**, no. 5, 054503 (2021)

JAM Global Analysis in the collinear DiFF Approach

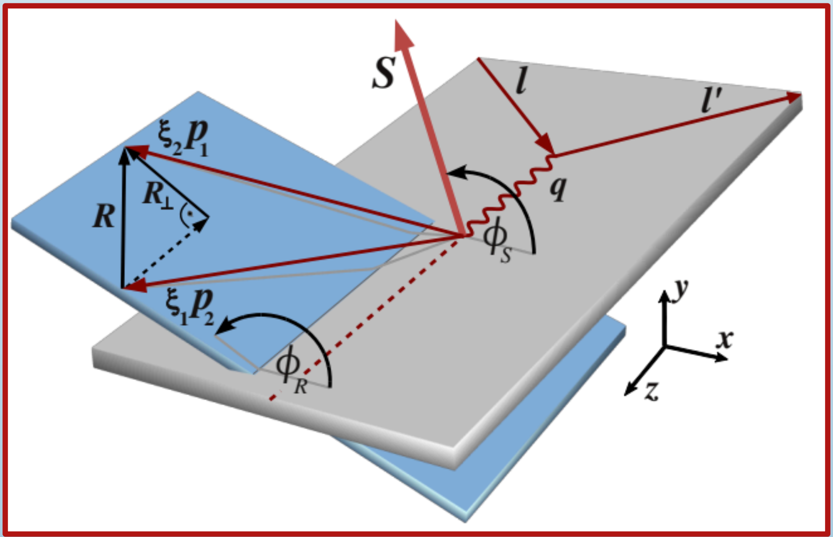
First *simultaneous* extraction of $\pi^+\pi^-$ DiFFs (D_1^q), IFFs ($H_1^{\sphericalangle,q}$), and transversity PDFs (h_1^q) at LO

Semi-Inclusive Annihilation



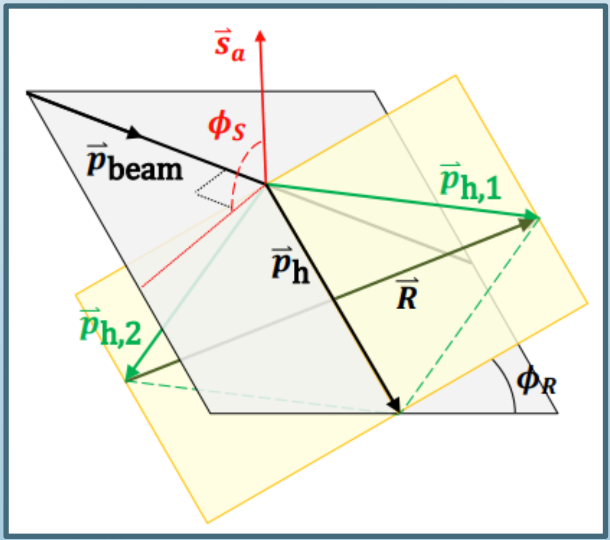
R. Seidl *et al.*, Phys. Rev. D **96**, no. 3, 032005 (2017)

Semi-Inclusive Deep Inelastic Scattering



C. Adolph *et al.*, Phys. Lett. B **713**, 10-16 (2012)

Proton-Proton Collisions



L. Adamczyk *et al.*, Phys. Rev. Lett. **115**, 242501 (2015)

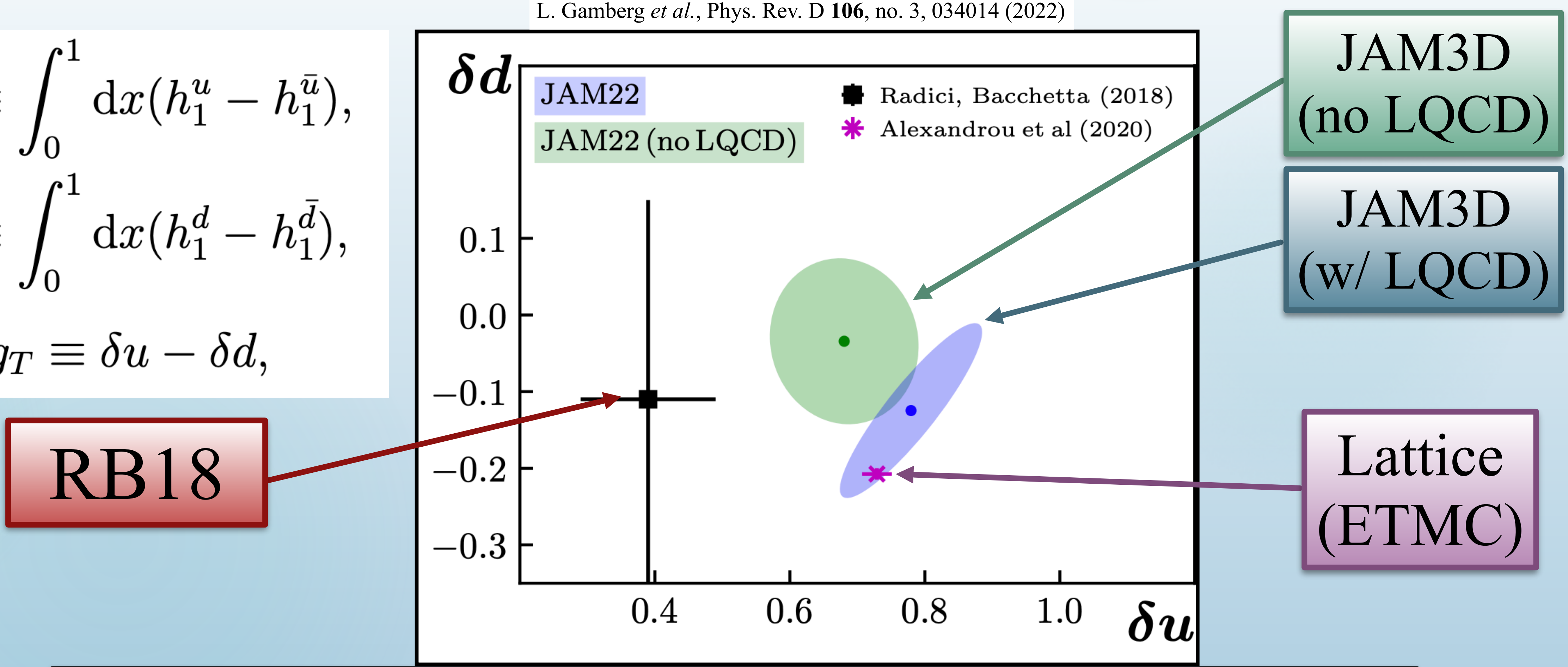
The Transverse Spin Puzzle?

$$\delta u \equiv \int_0^1 \mathrm{d}x (h_1^u - h_1^{\bar{u}}),$$

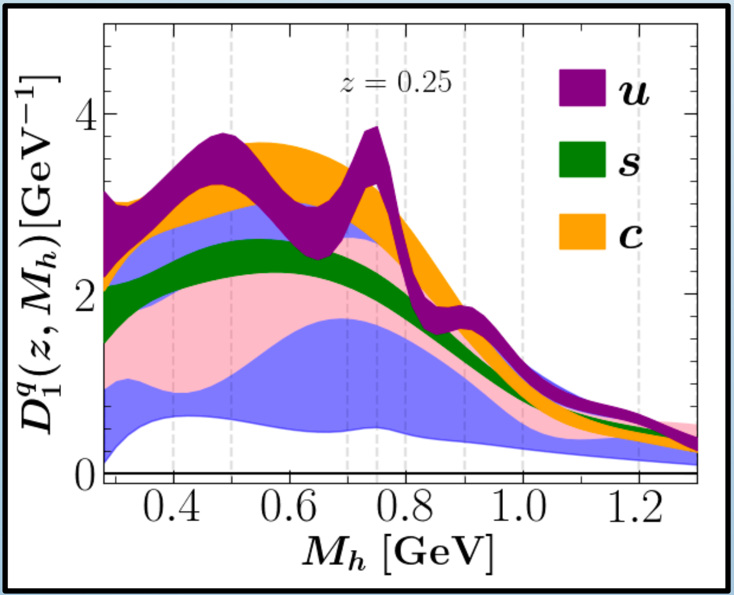
$$\delta d \equiv \int_0^1 \mathrm{d}x (h_1^d - h_1^{\bar{d}}),$$

$$g_T \equiv \delta u - \delta d,$$

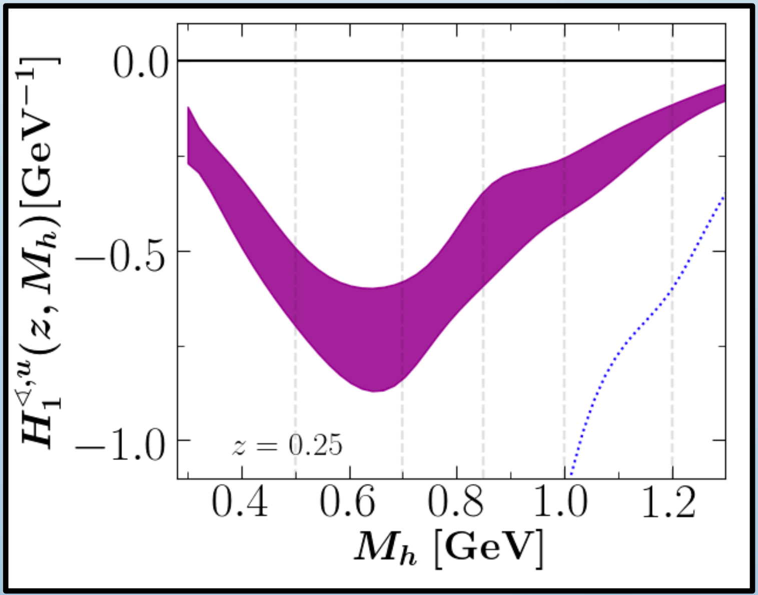
L. Gamberg *et al.*, Phys. Rev. D **106**, no. 3, 034014 (2022)



Large disagreements between three approaches…
Can this be solved?



$z = 0.25$
u
s
c
$D_1^q(z, M_h)[\mathrm{GeV}^{-1}]$
0
2
4
0.4
0.6
0.8
1.0
1.2
M_h [GeV]



$H_1^{\sphericalangle,u}(z, M_h)[\mathrm{GeV}^{-1}]$
0.0
−0.5
−1.0
$z = 0.25$
0.4
0.6
0.8
1.0
1.2
M_h [GeV]

Kinematics and Definitions

$$q(k) \to h_1(P_1) + h_2(P_2) + X \qquad z_{1,2} = P^-_{1,2}/k^-$$

$$M_h^2 \equiv P_h^2 \equiv (P_1 + P_2)^2 \qquad R \equiv \frac{1}{2}(P_1 - P_2) \qquad z \equiv z_1 + z_2 \qquad \zeta = \frac{z_1 - z_2}{z}$$

$$D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp}) \equiv \frac{1}{64\pi^3 z_1 z_2} \int \frac{\mathrm{d}\xi^+ \mathrm{d}^2 \vec{\xi}_T}{(2\pi)^3} e^{ik\cdot\xi} \, \mathrm{Tr}\left[\langle 0 | \psi_q(\xi) | h_1, h_2, X\rangle \langle h_1, h_2, X | \bar{\psi}_q(0) | 0\rangle \gamma^- \right]_{\xi^-=0}$$

Needed for number density interpretation

A. Majumder and X. N. Wang, J. Phys. G **31**, S533-S540 (2005)

$$D_1^{h_1 h_2/q}(z_1, z_2) = \int \mathrm{d}^2 \vec{P}_{1\perp} \mathrm{d}^2 \vec{P}_{2\perp} D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp} \vec{P}_{2\perp})$$

$$D_1^{h_1 h_2/q}(z, M_h) = \int \mathrm{d}\zeta \; D_1^{h_1 h_2/q}(z, \zeta, M_h) \quad \text{(extDiFFs)}$$

Checks of Definition

Number density

$$\sum_{h_1 h_2} \int \mathrm{d}z_1 \mathrm{d}z_2 D_1^{h_1 h_2/q}(z_1, z_2) = N^q(N^q - 1)$$

Momentum sum rule

$$\sum_{h_1} \int_0^{1-z_2} \mathrm{d}z_1 \int \mathrm{d}^2 \vec{P}_{1\perp} z_1 D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp}) = (1 - z_2) D_1^{h_2/q}(z_2, \vec{P}_{2\perp})$$

LO cross section for $e^- e^+ \to (h_1 h_2) X$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z_1 \mathrm{d}z_2} = \sum_{q\bar{q}} \hat{\sigma}^q D_1^{h_1 h_2/q}(z_1, z_2)$$

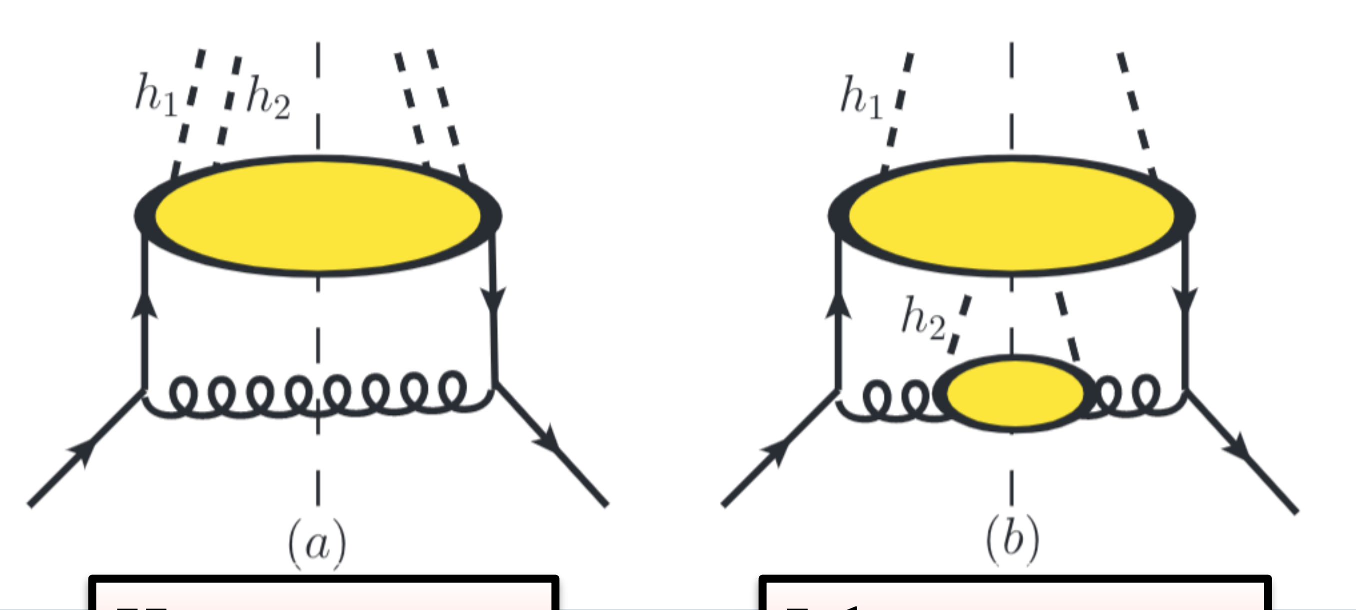
$$\frac{\mathrm{d}\sigma}{\mathrm{d}z \mathrm{d}M_h} = \sum_{q\bar{q}} \hat{\sigma}^q D_1^{h_1 h_2/q}(z, M_h)$$

$$\hat{\sigma}^q = \frac{4\pi e_q^2 \alpha_{\mathrm{em}}^2 N_c}{3Q^2}$$

Evolution

Evolution for extDiFFs (quark non-singlet)

$$\frac{\partial}{\partial \ln \mu^2} D_1^{h_1 h_2/q}(z, \zeta, \vec{R}_T^2; \mu) = \int_z^1 \frac{dw}{w} D_1^{h_1 h_2/q}(\frac{z}{w}, \zeta, \vec{R}_T^2; \mu) P_{q\to q}(w)$$



Homogeneous

Inhomogeneous

Homogeneous term only for extended DiFFs

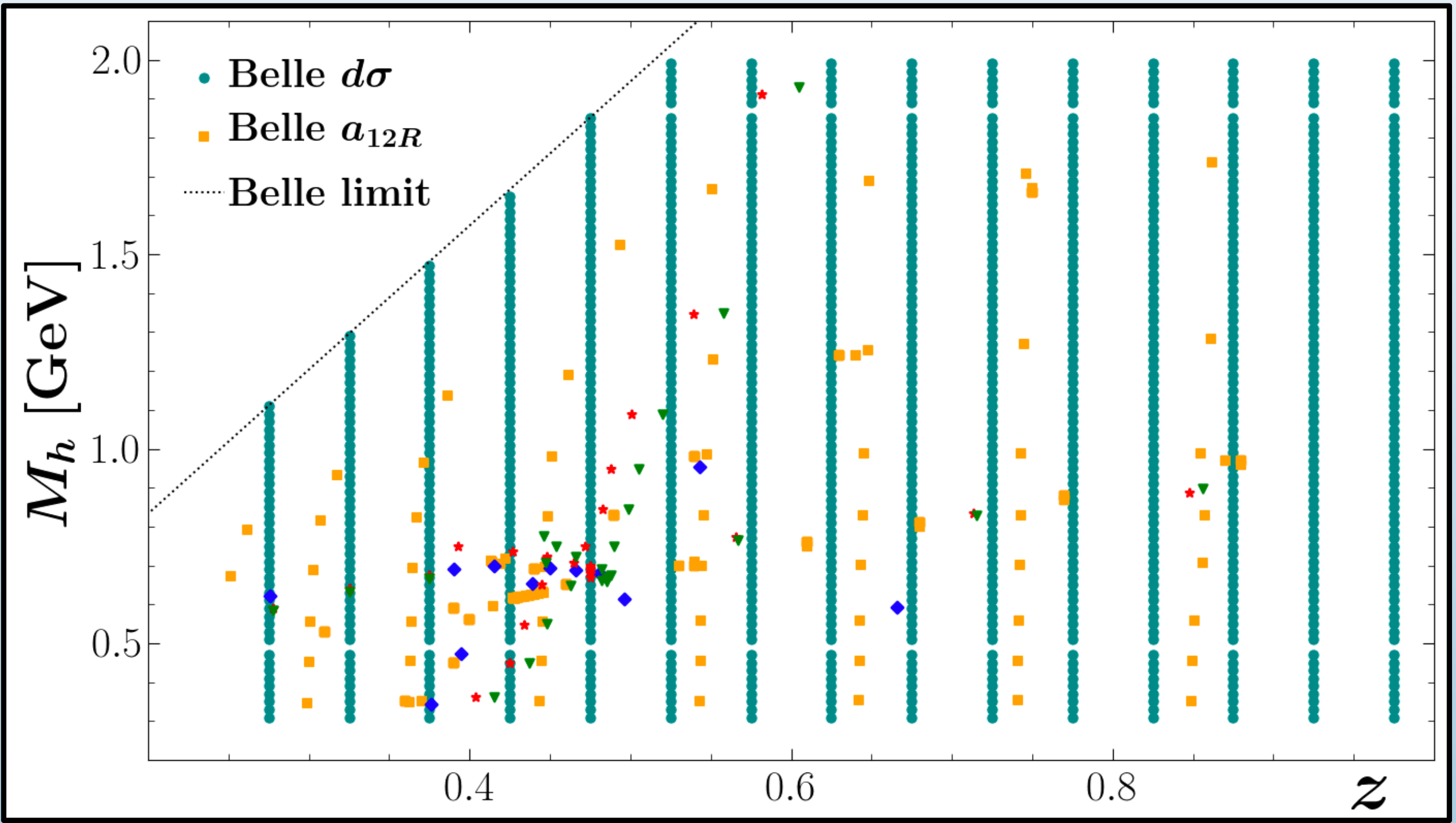
F. A. Ceccopieri, M. Radici, and A. Bacchetta, Phys. Lett. B **650**, 81 (2007)

Inhomogeneous term exists for $D_1^{h_1 h_2}(z_1, z_2)$

Analogous derivations done for $D_1^{h_1 h_2/g}$ and $H_1^{\sphericalangle, h_1 h_2/q}$

Data for DiFFs

SIA cross section	Belle	1121 points
SIA Artru-Collins	Belle	183 points



$\pi^+\pi^-$ DiFFs

$$D_1^u = D_1^d = D_1^{\bar{u}} = D_1^{\bar{d}},$$

$$D_1^s = D_1^{\bar{s}}, \quad D_1^c = D_1^{\bar{c}}, \quad D_1^b = D_1^{\bar{b}},$$

5 independent functions (w/ D_1^g)

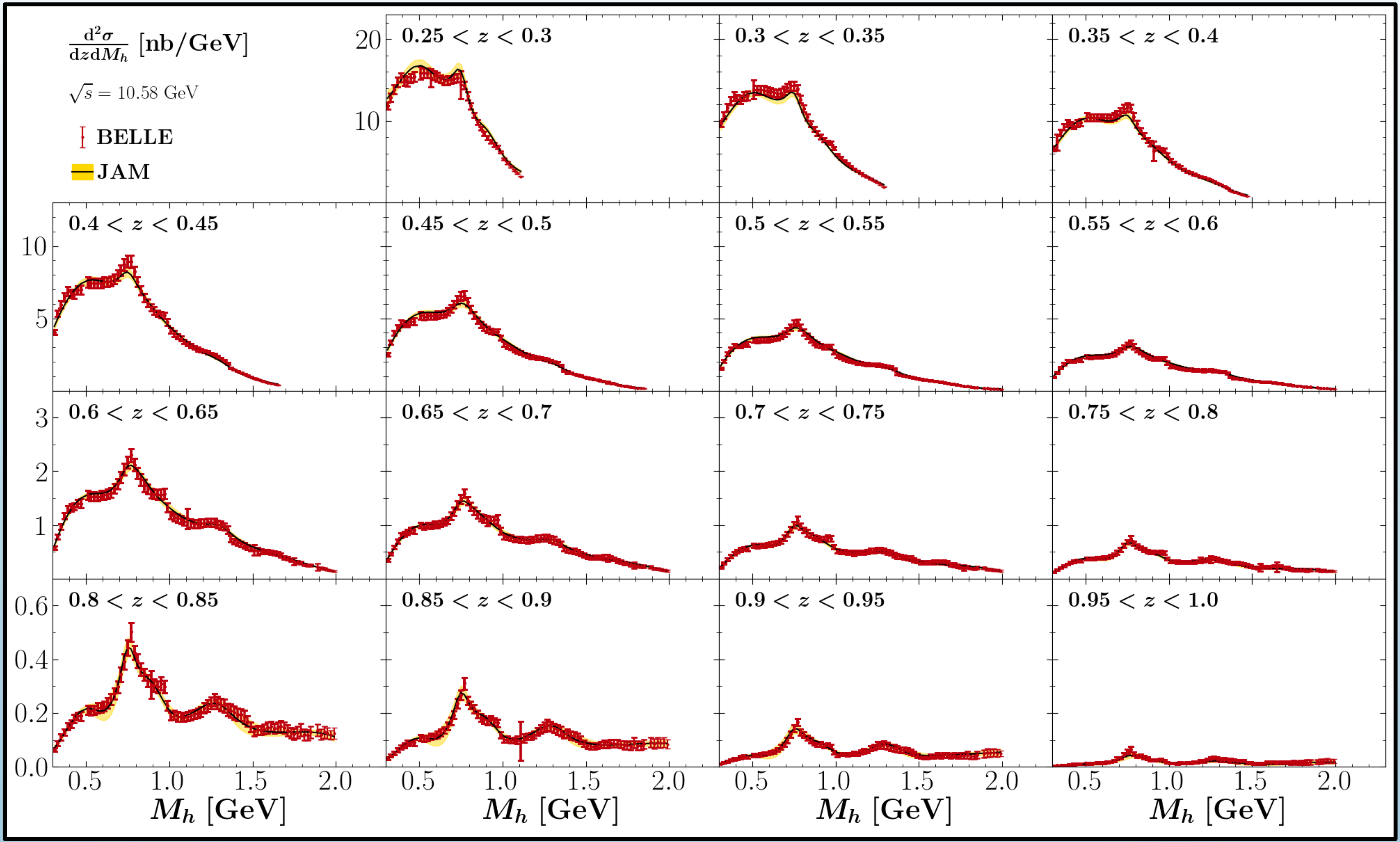
$$H_1^{\sphericalangle,u} = -H_1^{\sphericalangle,d} = -H_1^{\sphericalangle,\bar{u}} = H_1^{\sphericalangle,\bar{d}},$$

$$H_1^{\sphericalangle,s} = -H_1^{\sphericalangle,\bar{s}} = H_1^{\sphericalangle,c} = -H_1^{\sphericalangle,\bar{c}} = 0,$$

1 independent function

A. Courtoy et al., Phys. Rev. D **85**, 114023 (2012)

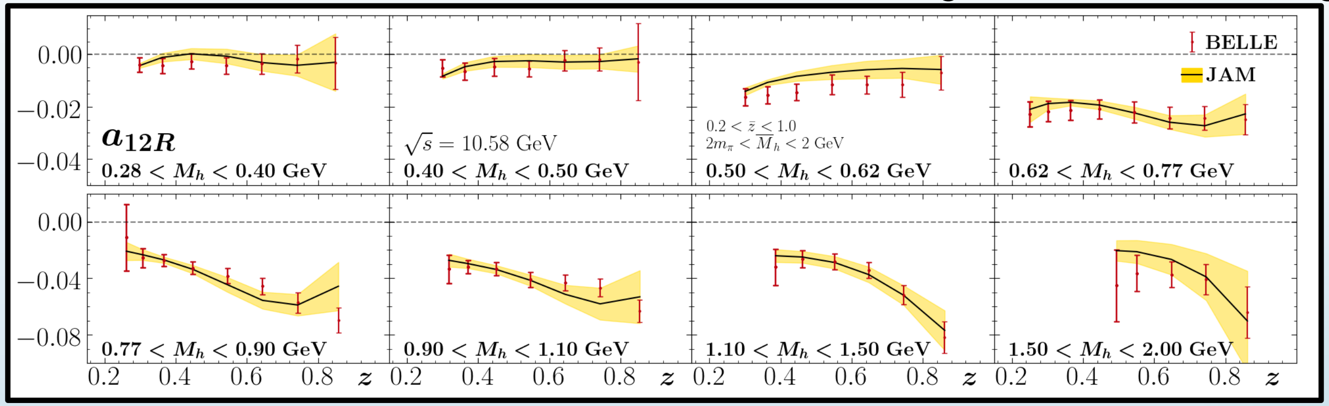
Quality of Fit (Unpolarized Cross Section)



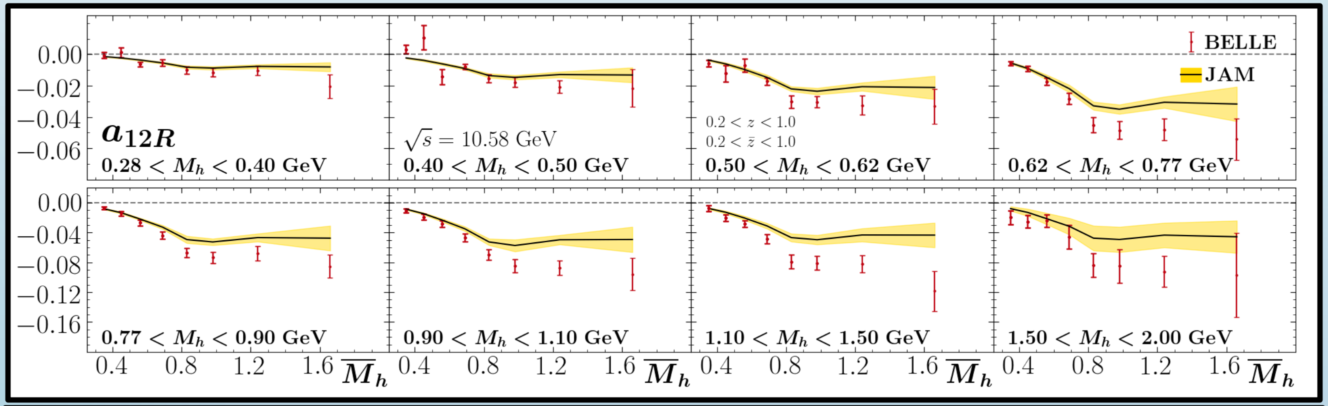
R. Seidl *et al.*, Phys. Rev. D **96**, 032005 (2017)

Quality of Fit (Artru-Collins Asymmetry)

(z, M_h) binning

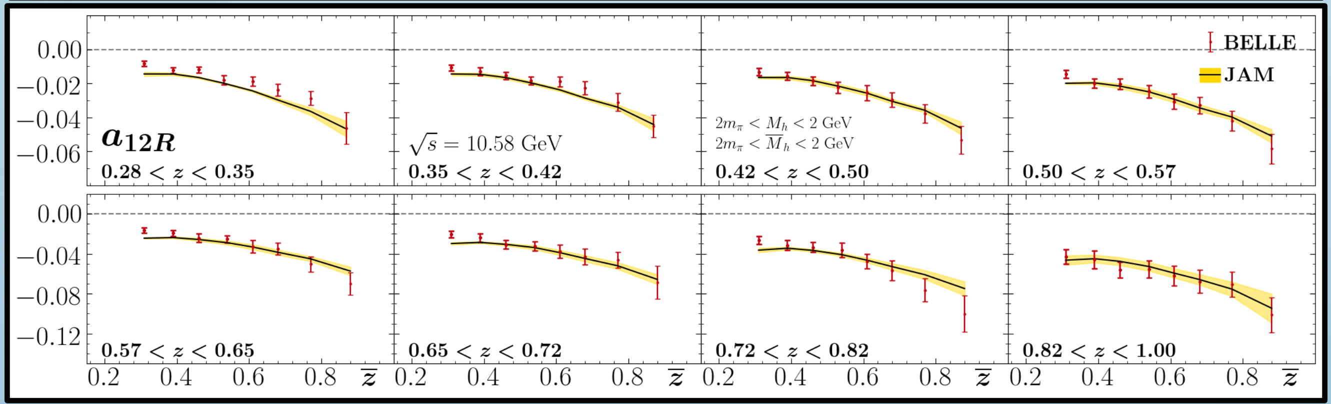


$(M_h, \overline{M}_h)$ binning

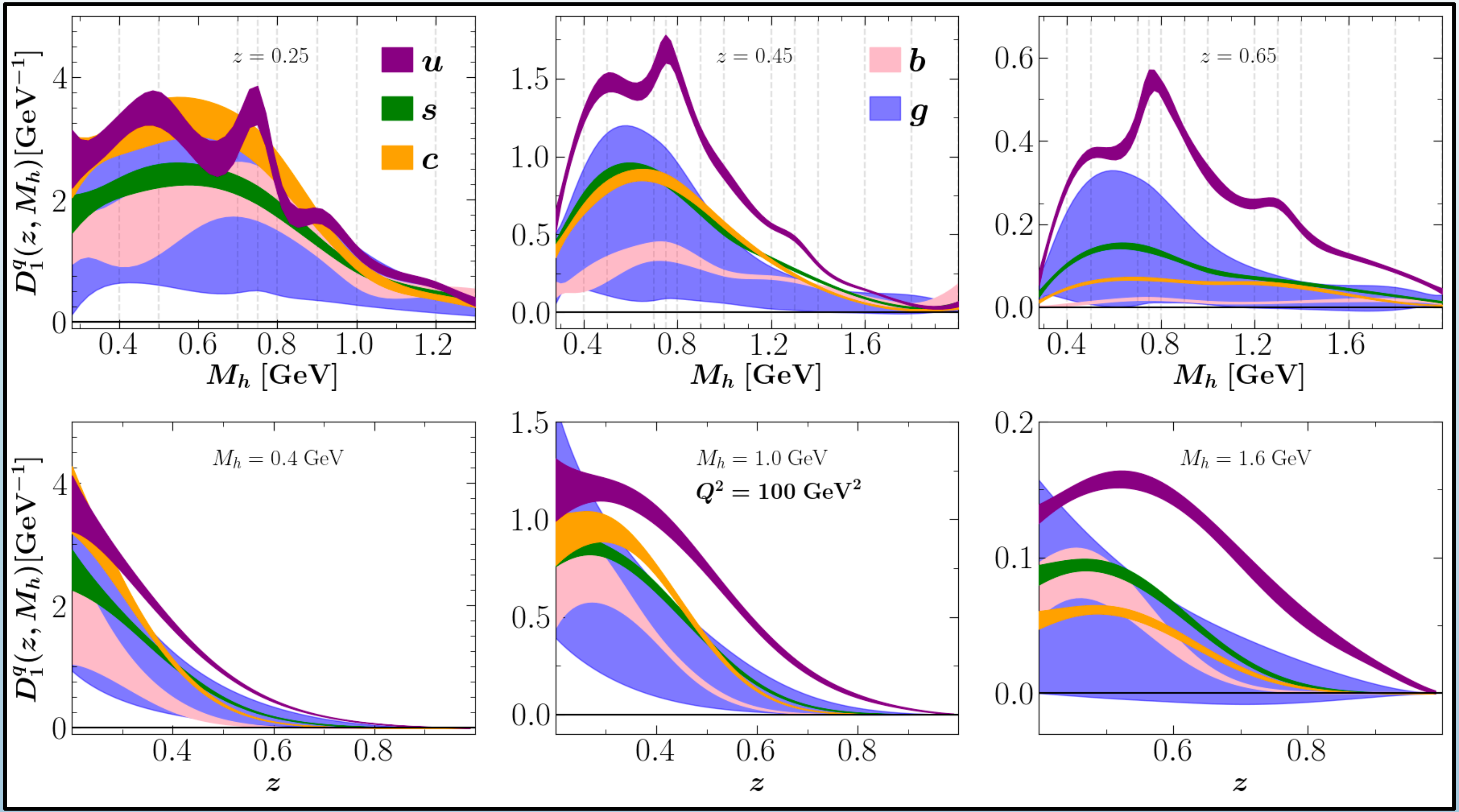


A. Vossen *et al.*,
Phys. Rev. Lett. **107**, 072004 (2011)

$(z, \bar{z})$ binning



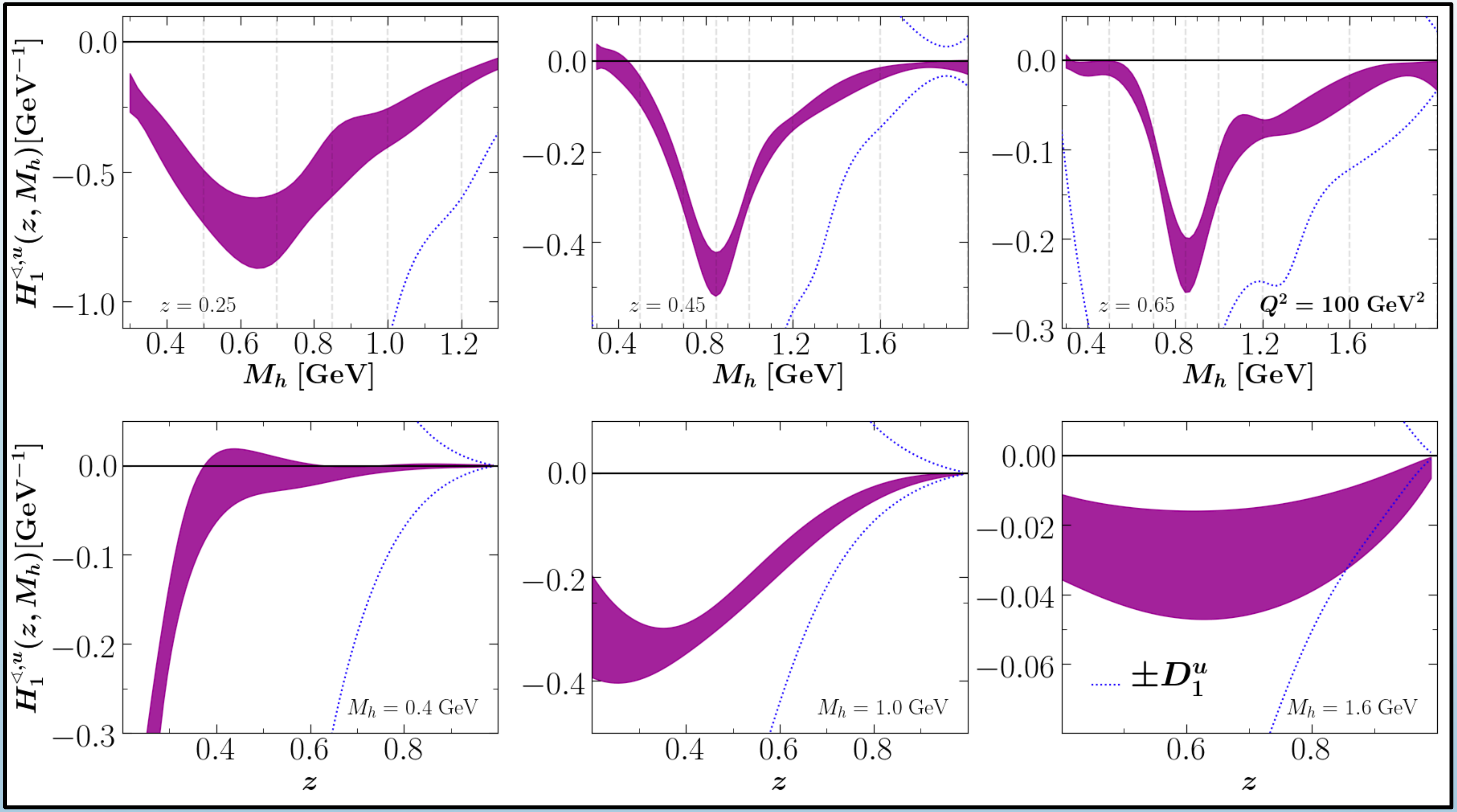
Extracted DiFFs



Bound: $D_1^q > 0$

A. Bacchetta and M. Radici, Phys. Rev. D **67**, 094002 (2003)

Extracted IFFs



Bound:

$$|H_1^{\sphericalangle,q}| < D_1^q$$

A. Bacchetta and M. Radici, Phys. Rev. D **67**, 094002 (2003)

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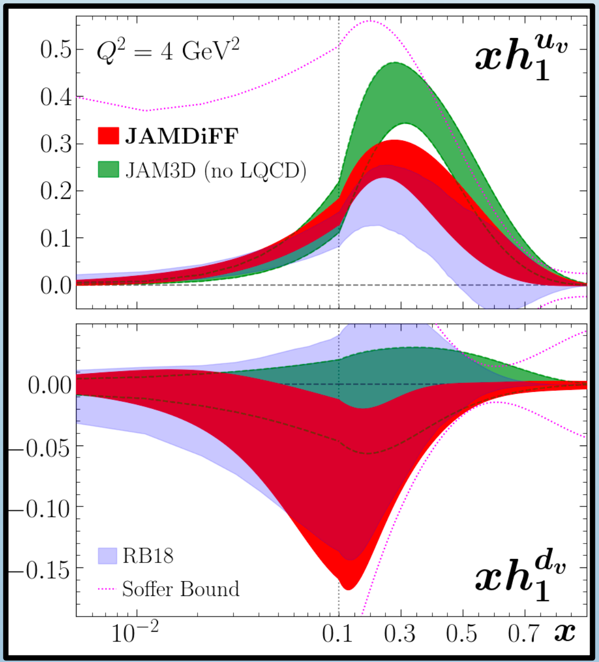
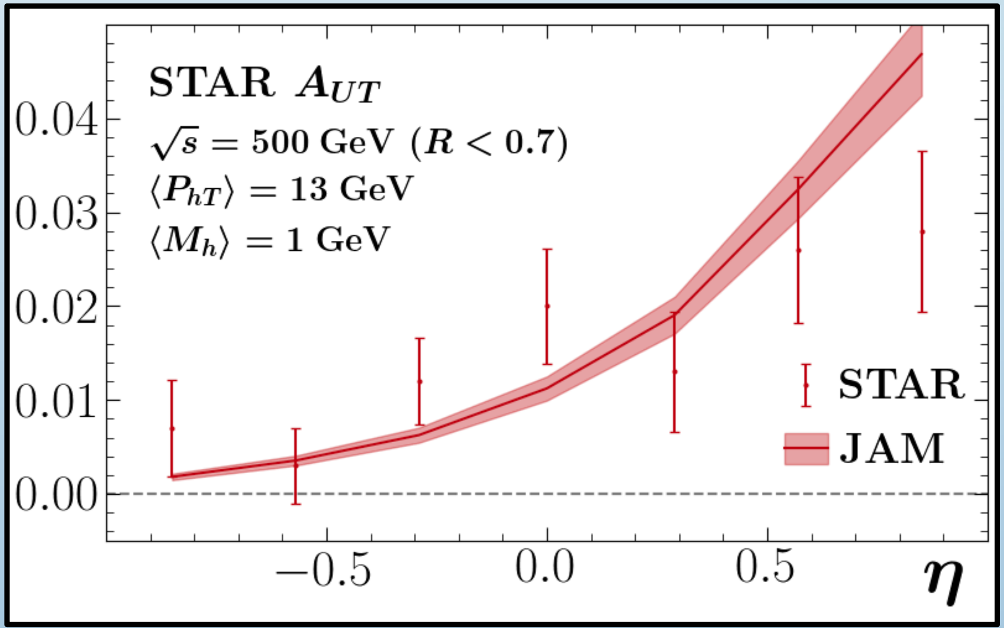
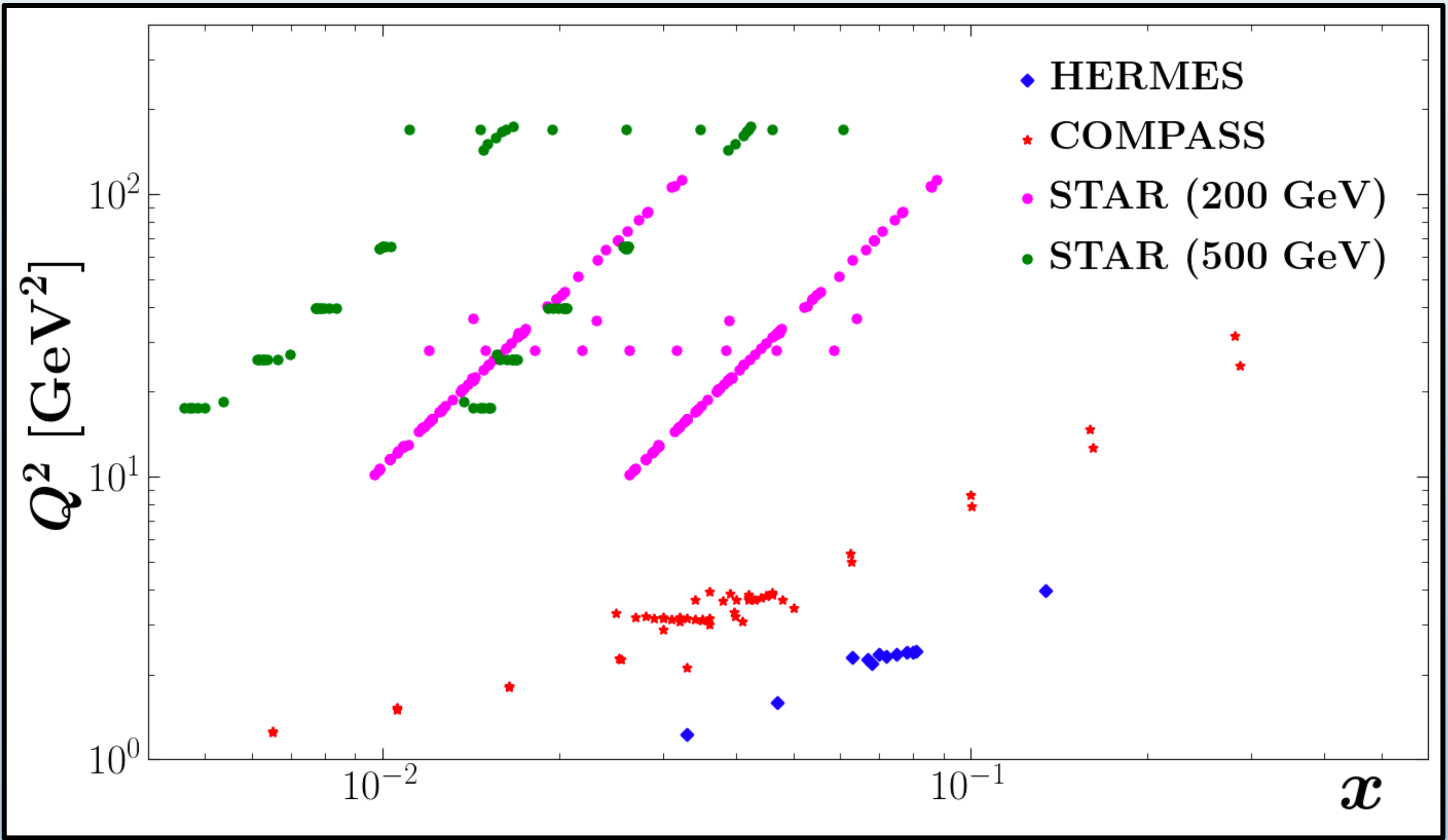




Data for PDFs

SIDIS (p, D)	COMPASS, HERMES	64 points
Proton-Proton	STAR	269 points



Parameterization Choices

3 independent observables
3 independent functions

$$h_1^{u_v}$$
$$h_1^{d_v}$$
$$h_1^{\bar{u}} = -h_1^{\bar{d}}$$

Prediction from large-N_c limit

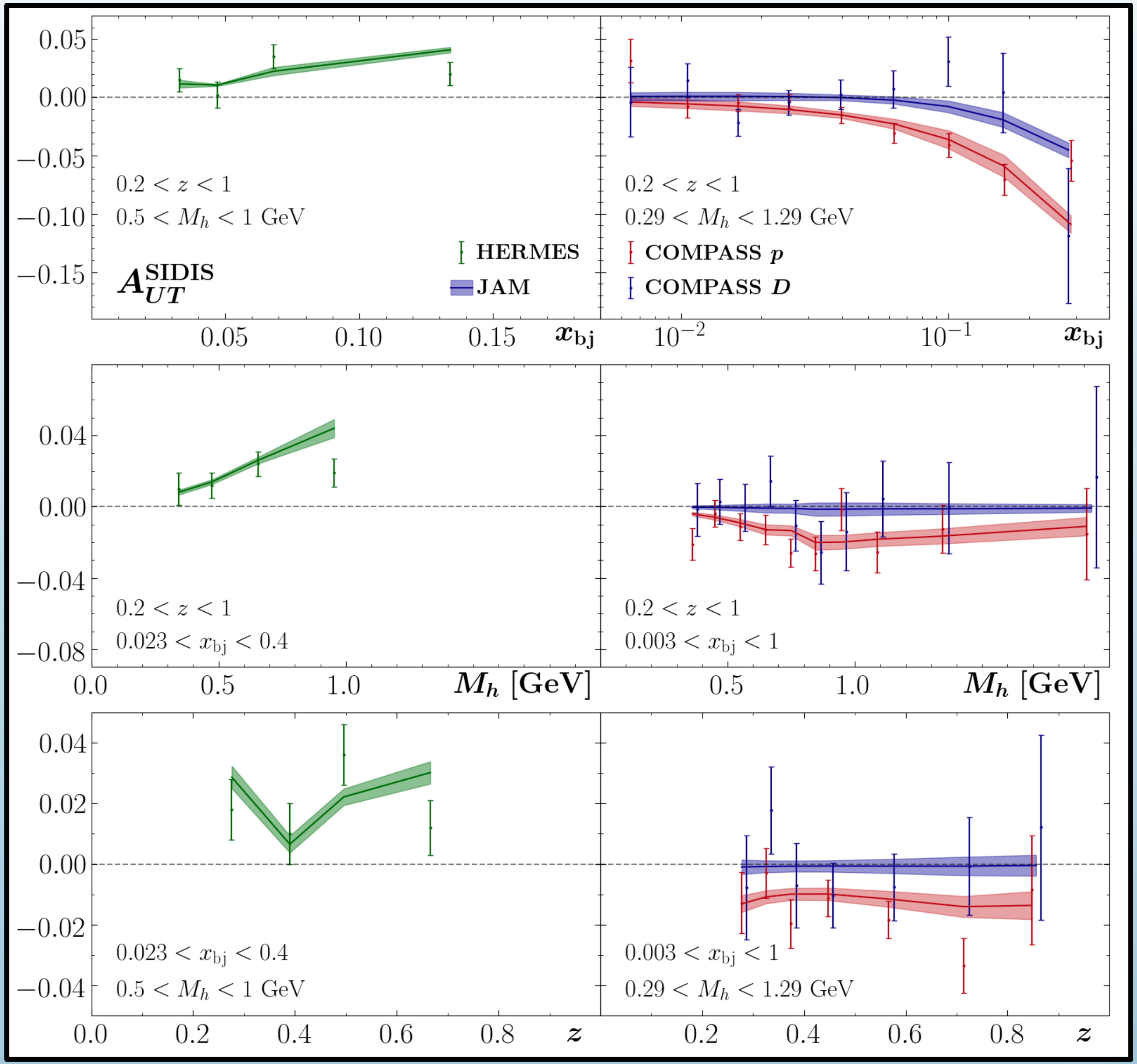
P. V. Pobylitsa, arXiv:hep-ph/0301236 (2003)

Quality of Fit (SIDIS)

x binning

M_h binning

z binning



A. Airapetian *et al.*, JHEP **06**, 017 (2008)

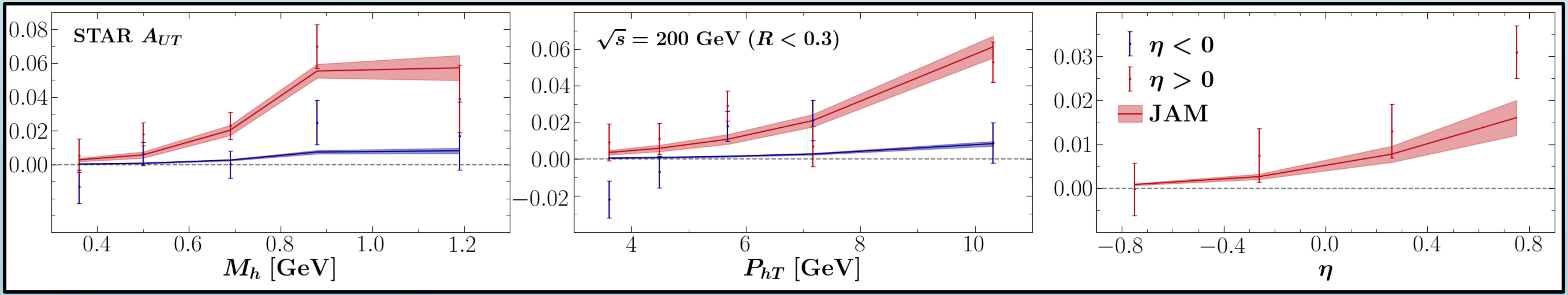
COMPASS, arXiv:hep-ph/2301.02013 (2023)

Quality of Fit (STAR $\sqrt{s} = 200$ GeV)

M_h binning

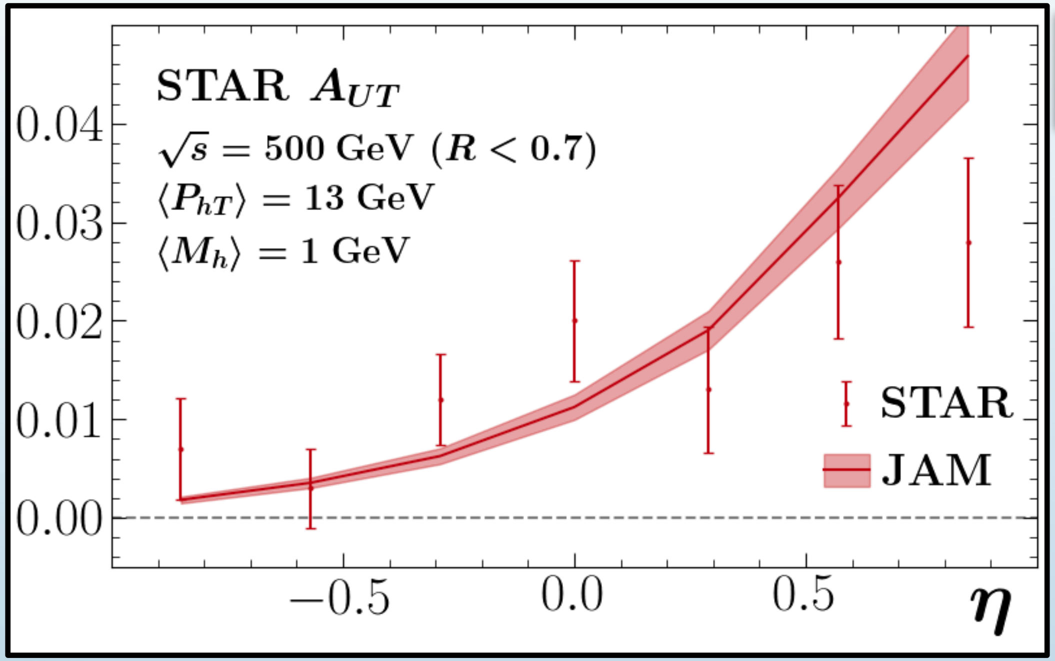
P_{hT} binning

η binning



L. Adamczyk *et al.*, Phys. Rev. Lett. **115**, 24501 (2015)

Quality of Fit (STAR $\sqrt{s} = 500$ GeV)

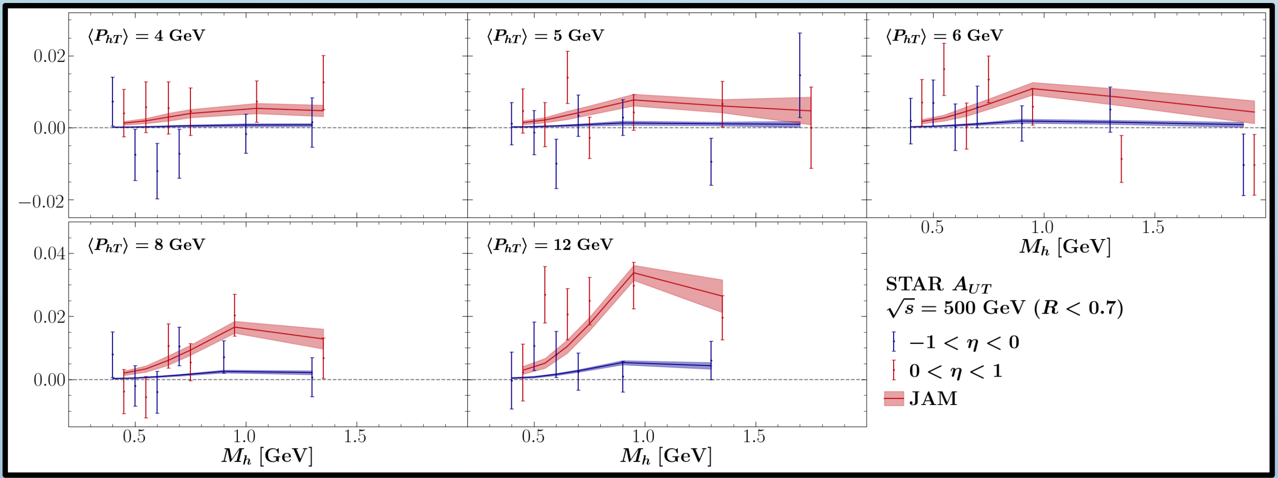


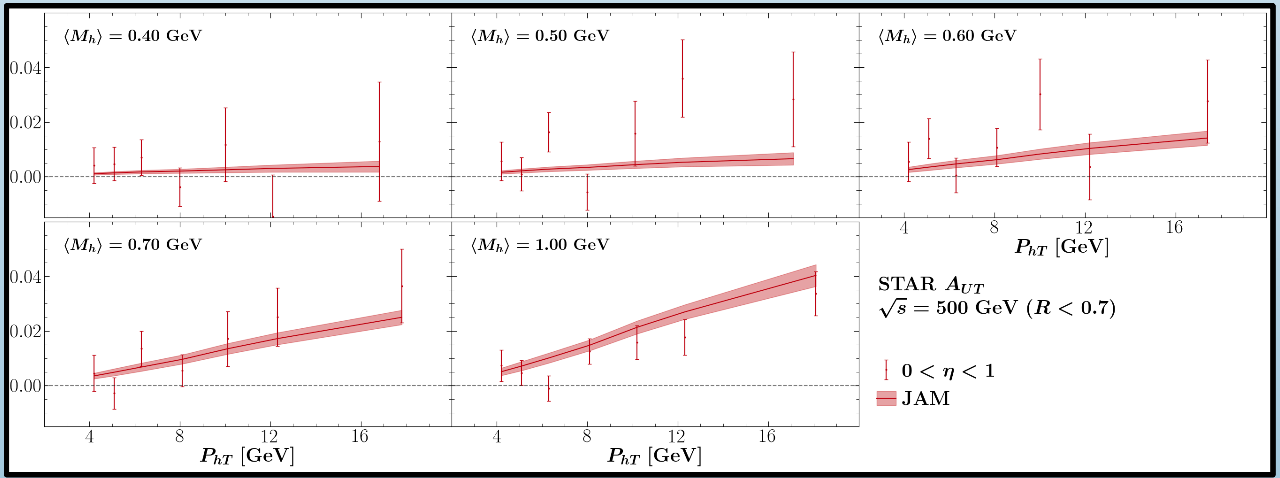
η binning

L. Adamczyk *et al.*, Phys. Rev. B **780**, 332-339 (2018)

M_h binning

P_{hT} binning





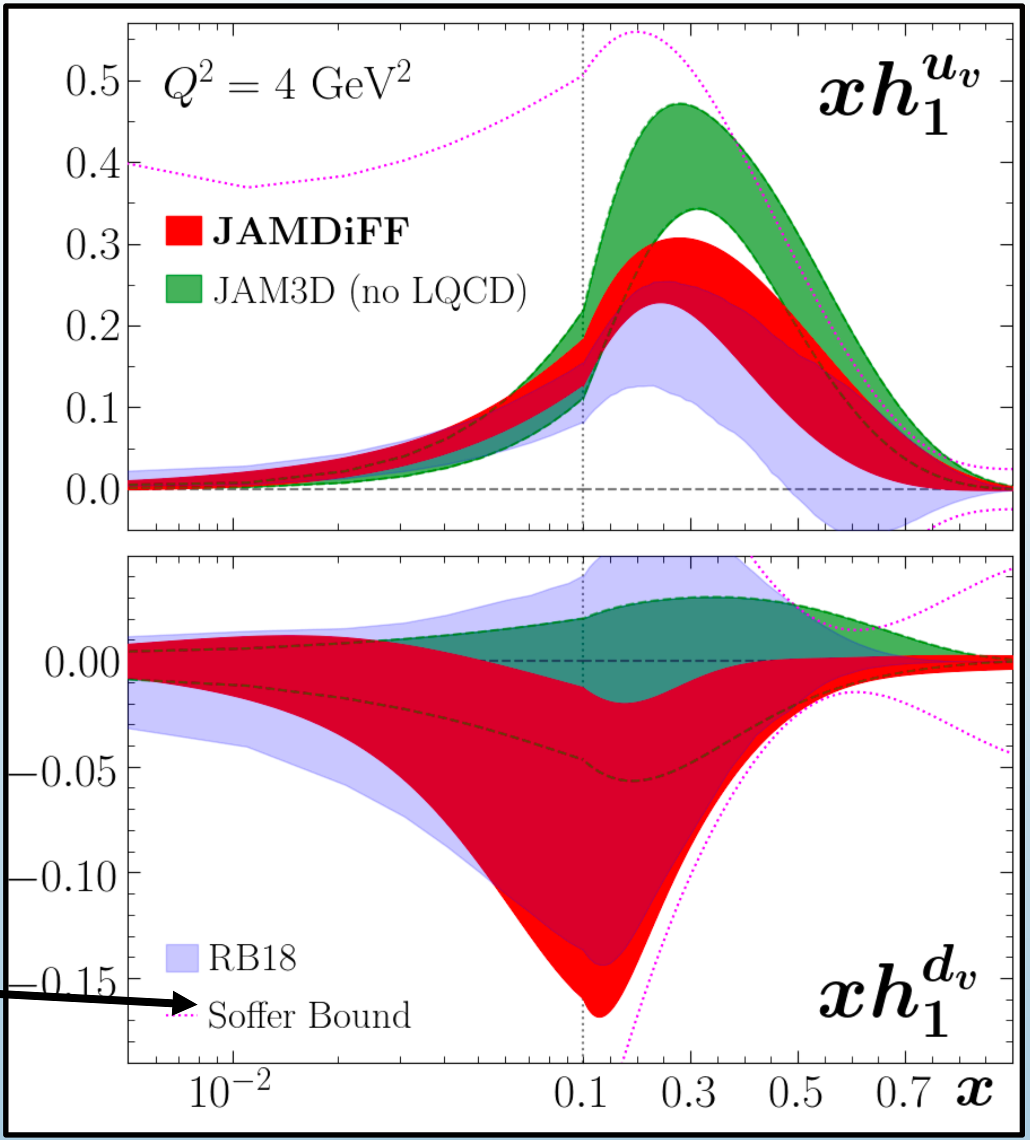
Transversity PDFs

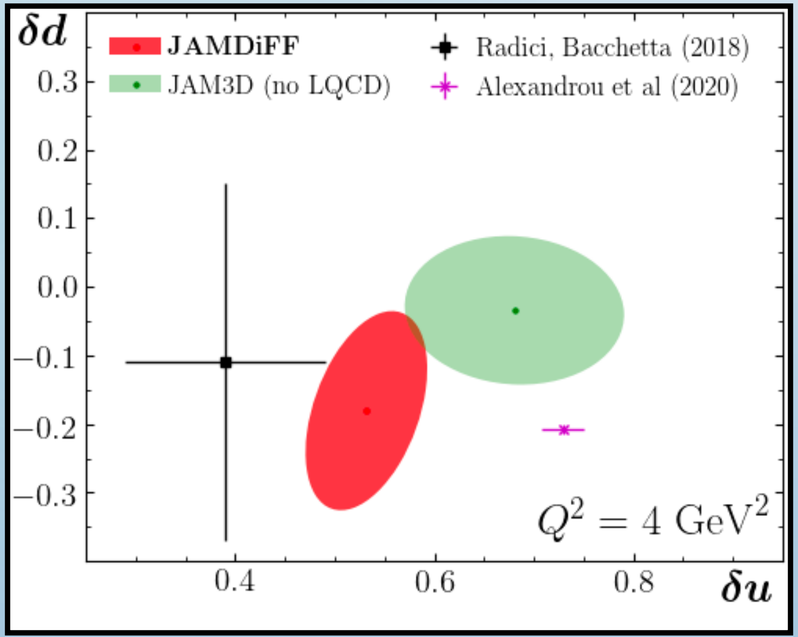
Agreement with RB18

Result for u_v smaller than result from JAM3D (no LQCD)

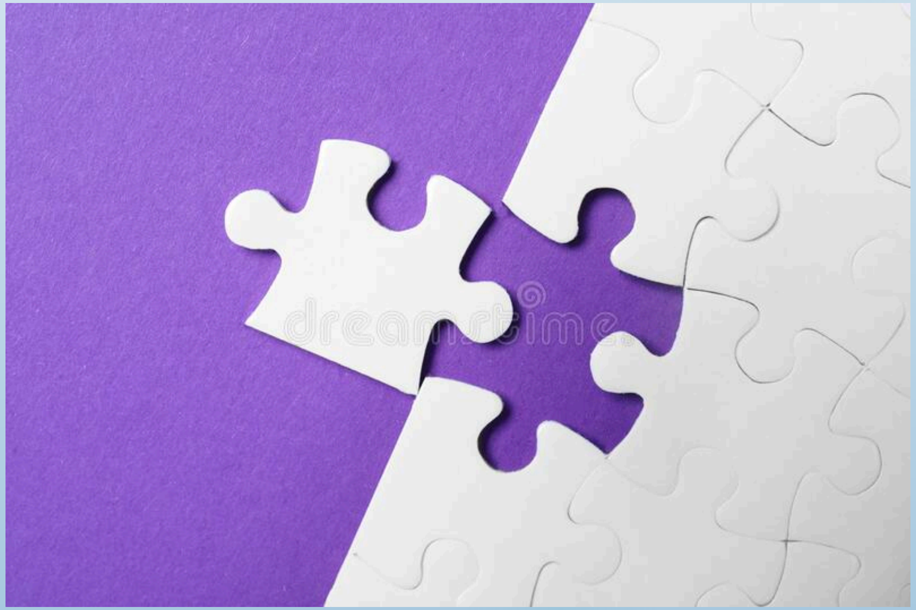
Soffer Bound: $|h_1^q| < \frac{1}{2}[f_1^q + g_1^q]$

J. Soffer, Phys. Rev. Lett. **74**, 1292-1294 (1995)

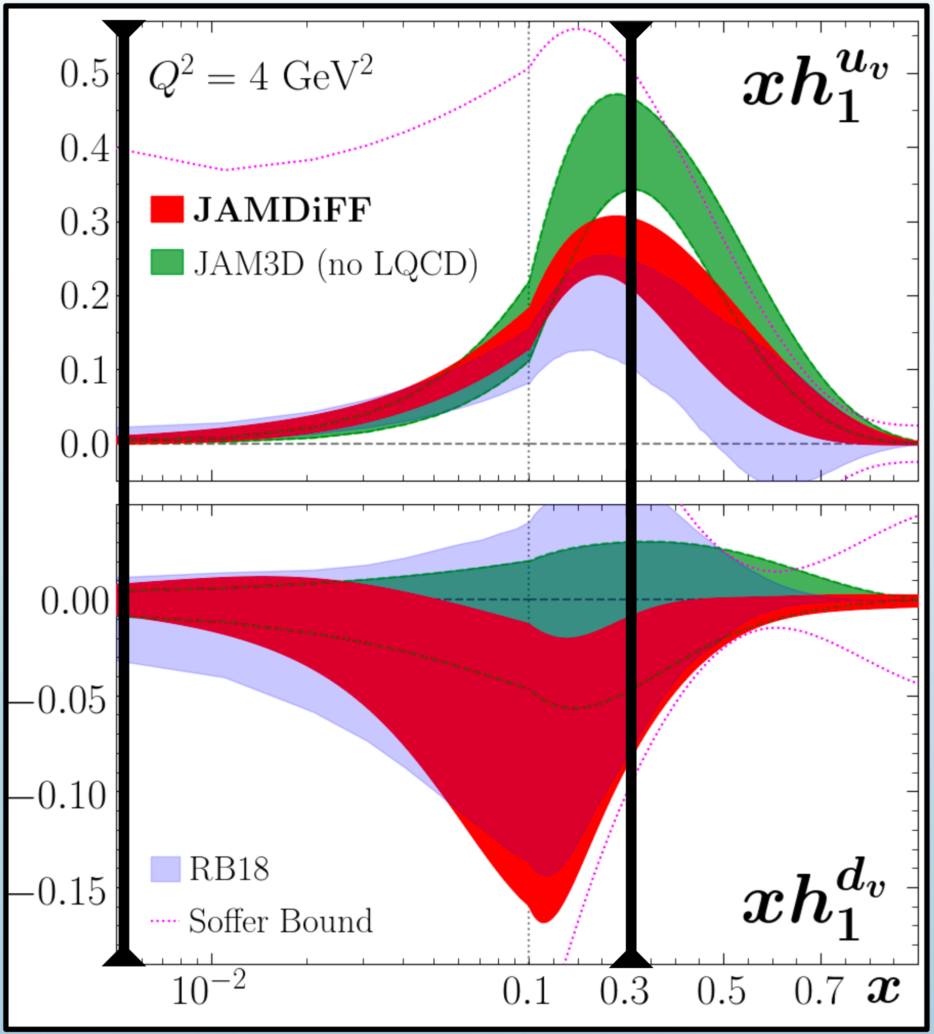




δd
JAMDiFF
JAM3D (no LQCD)
Radici, Bacchetta (2018)
Alexandrou et al (2020)
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
0.4
0.6
0.8
δu
$Q^2 = 4$ GeV^2



Controlling Extrapolation



Measured Region

$$\delta u \equiv \int_0^1 \mathrm{d}x (h_1^u - h_1^{\bar{u}}),$$

$$\delta d \equiv \int_0^1 \mathrm{d}x (h_1^d - h_1^{\bar{d}}),$$

$$g_T \equiv \delta u - \delta d,$$

Large $x \gtrsim 0.3$

Soffer Bound: $|h_1^q| < \frac{1}{2}\left[f_1^q + g_1^q\right]$

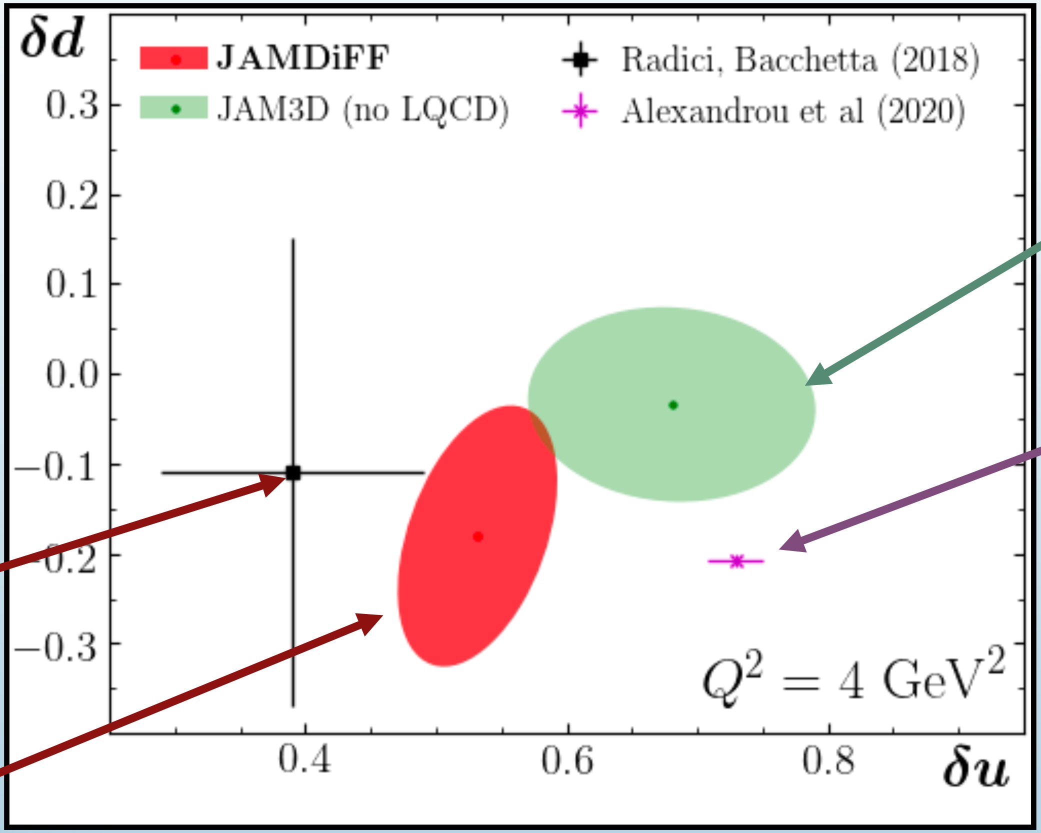
J. Soffer, Phys. Rev. Lett. **74**, 1292-1294 (1995)

Small $x \lesssim 0.005$

$$h_1^q \xrightarrow[x \to 0]{} x^{\alpha_q} \qquad \alpha_q = 1 - 2\sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 0.17 \pm 50\,\%$$

Y. V. Kovchegov and M. D. Sievert, Phys. Rev. D **99**, 054033 (2019)

Tensor Charges (no lattice in fit)



JAM3D (no LQCD)

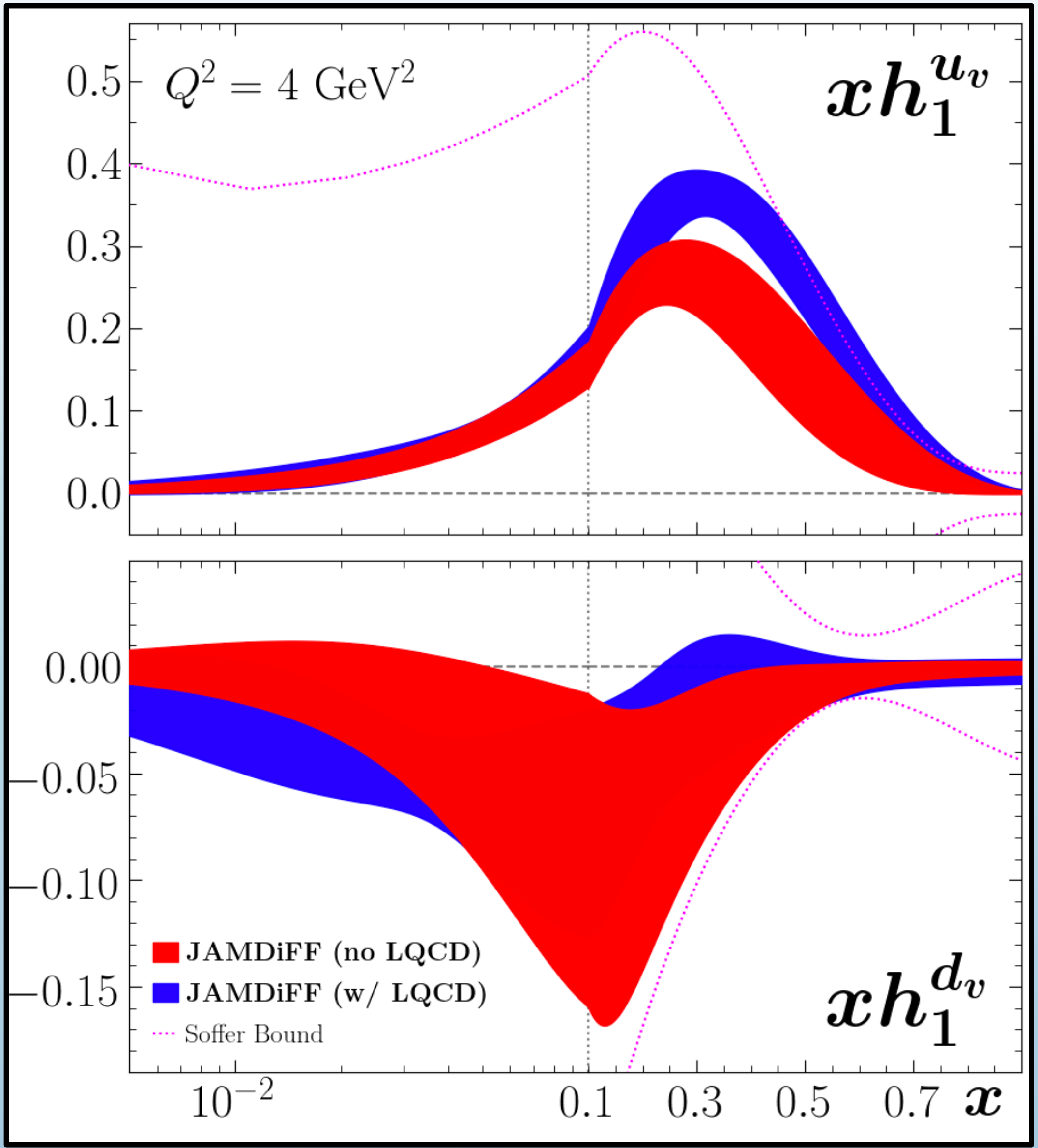
Lattice (ETMC)

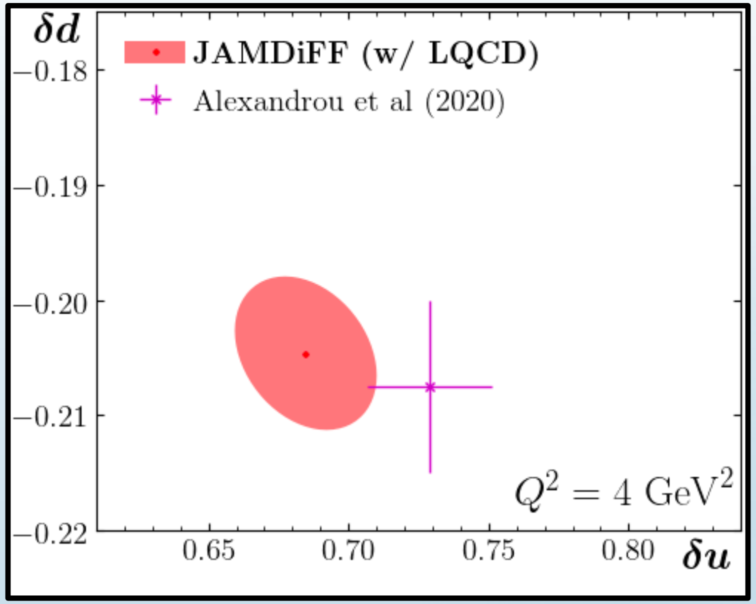
RB18

This analysis

Consistent with RB18 and JAM3D (no LQCD). What happens if lattice is included in our fit?

Tensor Charges (with lattice in fit)

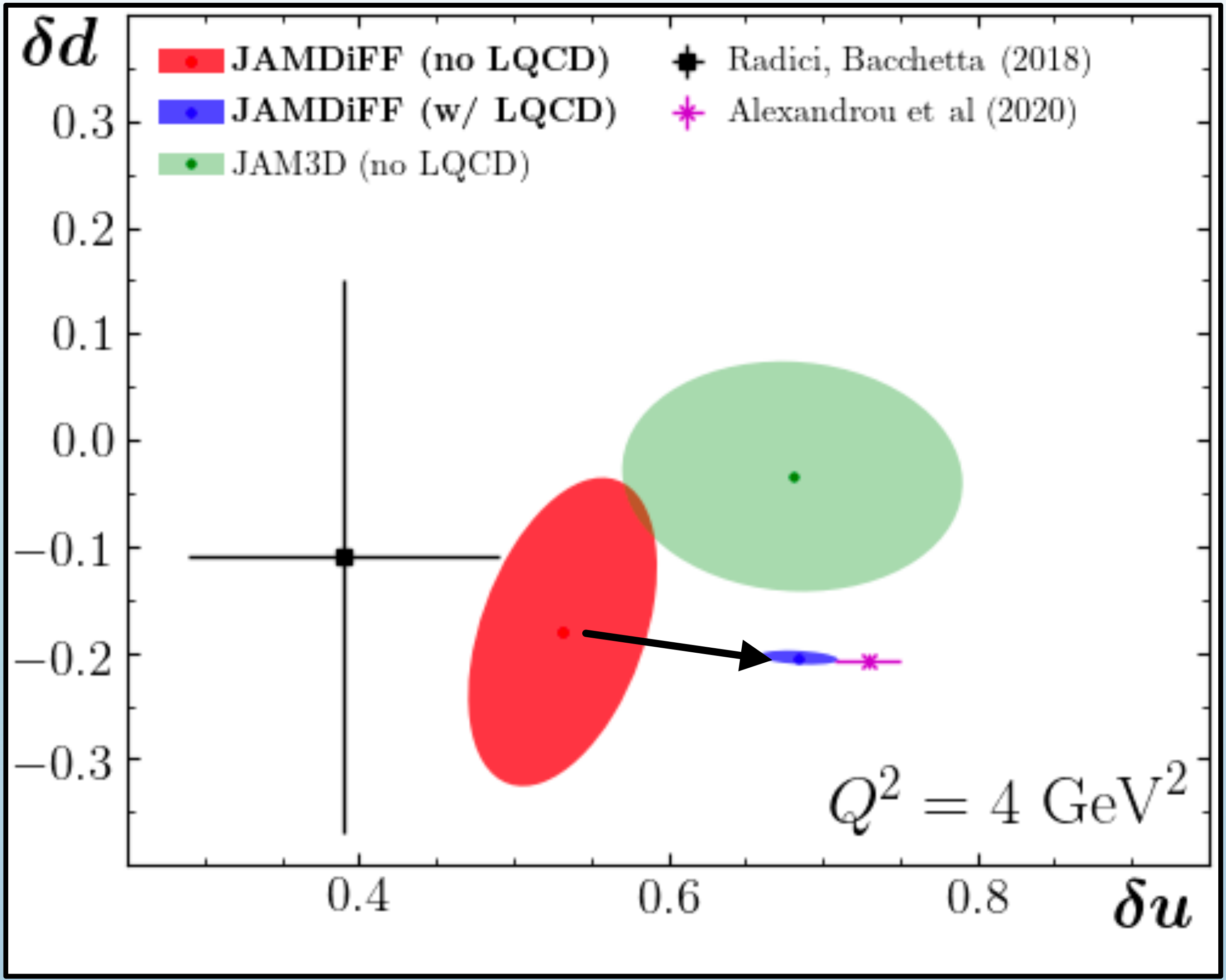




Fit is able to accommodate lattice data quite well!

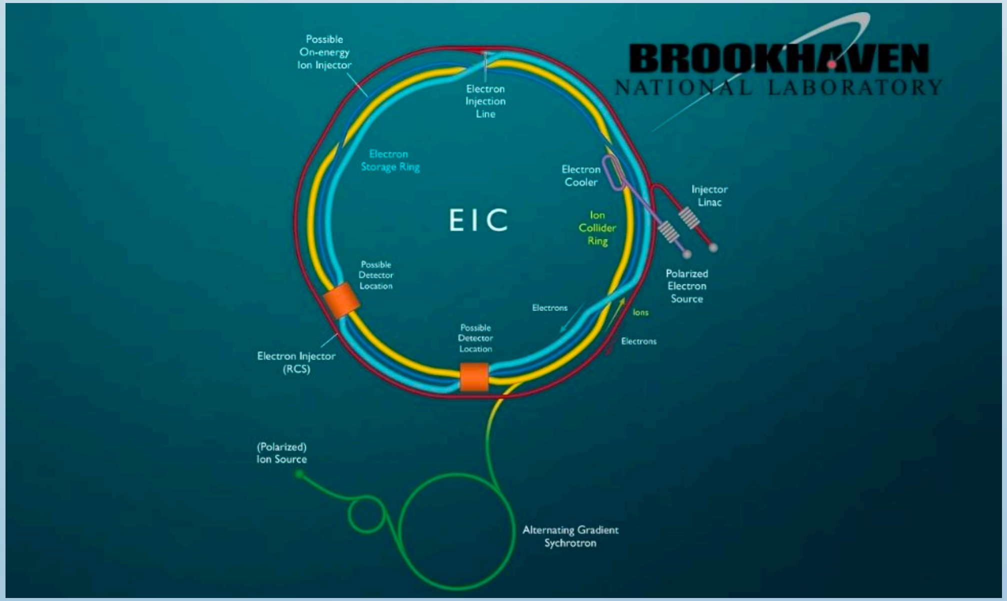
Global χ^2 without lattice: 1.11
Global χ^2 with lattice: 1.15

Tensor Charges (before and after lattice)

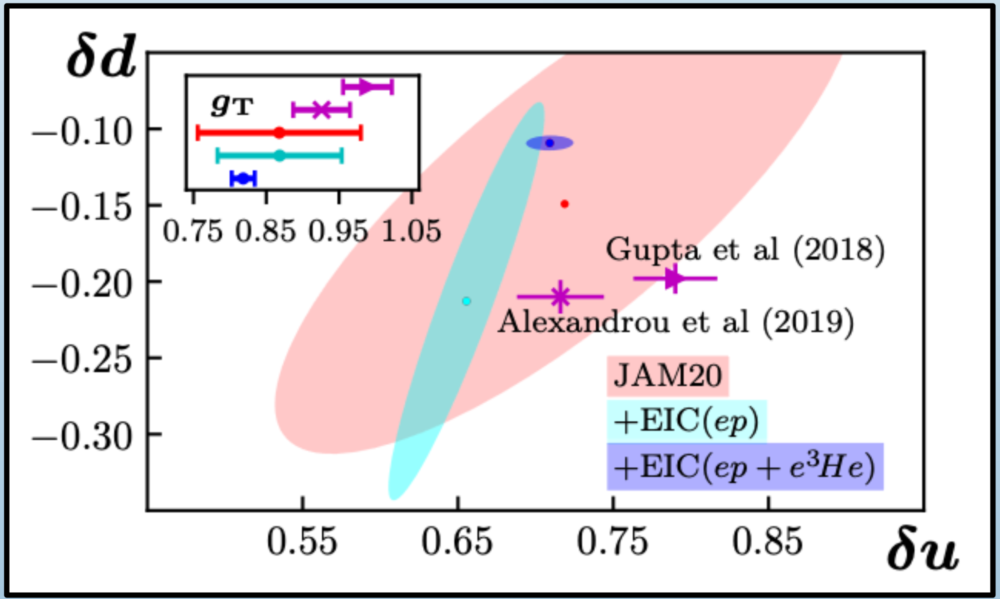


Noticeable shift from including lattice data

Experimental data has only weak preference, as it is not directly sensitive to the full moment



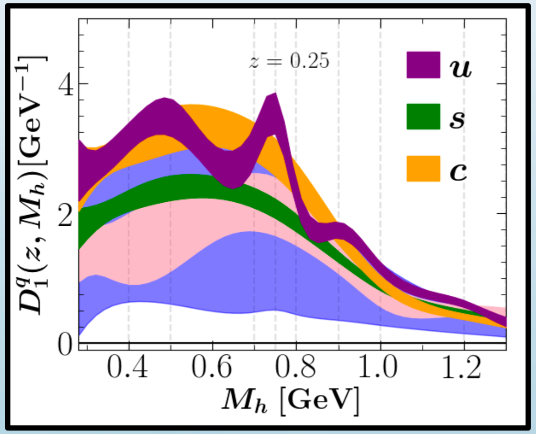
BROOKHAVEN
NATIONAL LABORATORY
EIC
Possible On-energy Ion Injector
Electron Injection Line
Electron Storage Ring
Electron Cooler
Injector Linac
Ion Collider Ring
Possible Detector Location
Polarized Electron Source
Electrons
Ions
Electron Injector (RCS)
(Polarized) Ion Source
Alternating Gradient Sychrotron

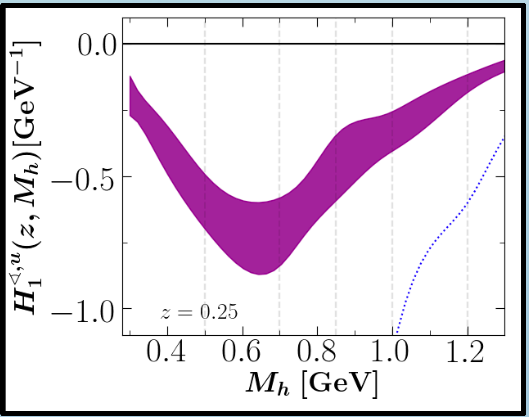


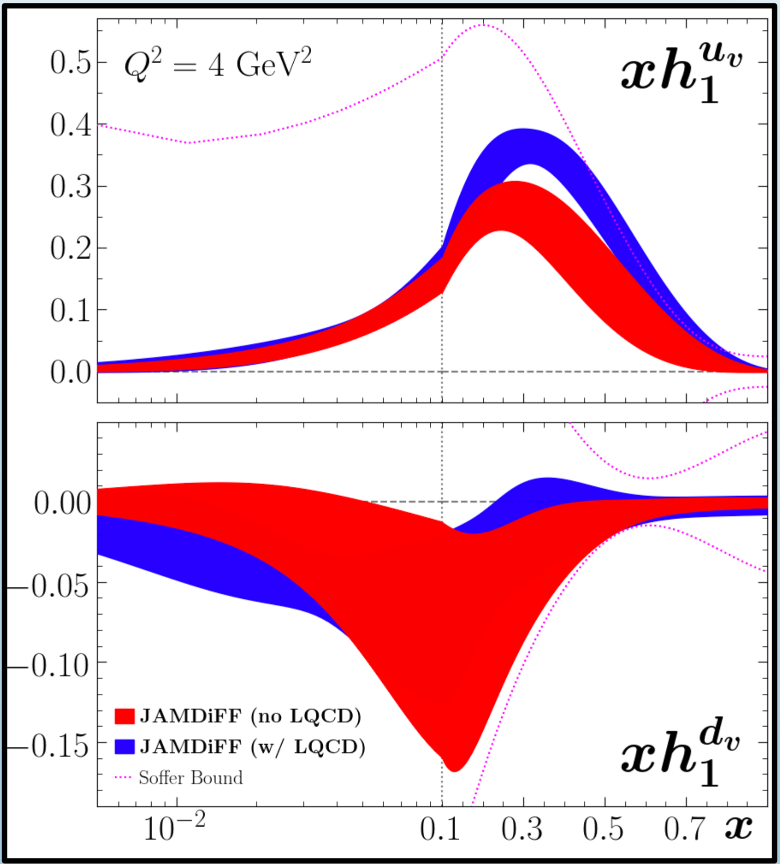
δd
-0.10
-0.15
-0.20
-0.25
-0.30
g_T
0.75 0.85 0.95 1.05
Gupta et al (2018)
Alexandrou et al (2019)
JAM20
+EIC(ep)
+EIC($ep + e^3He$)
0.55
0.65
0.75
0.85
δu

Conclusions

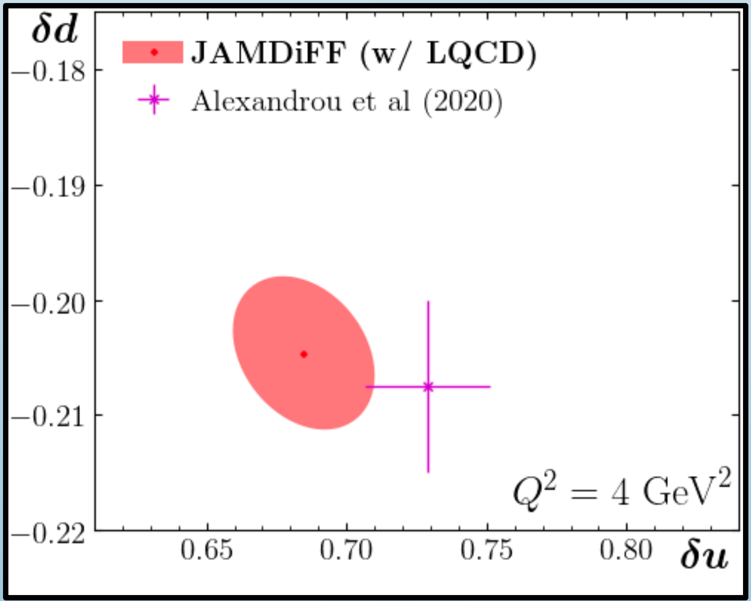
Simultaneous extraction of DiFFs and transversity PDFs





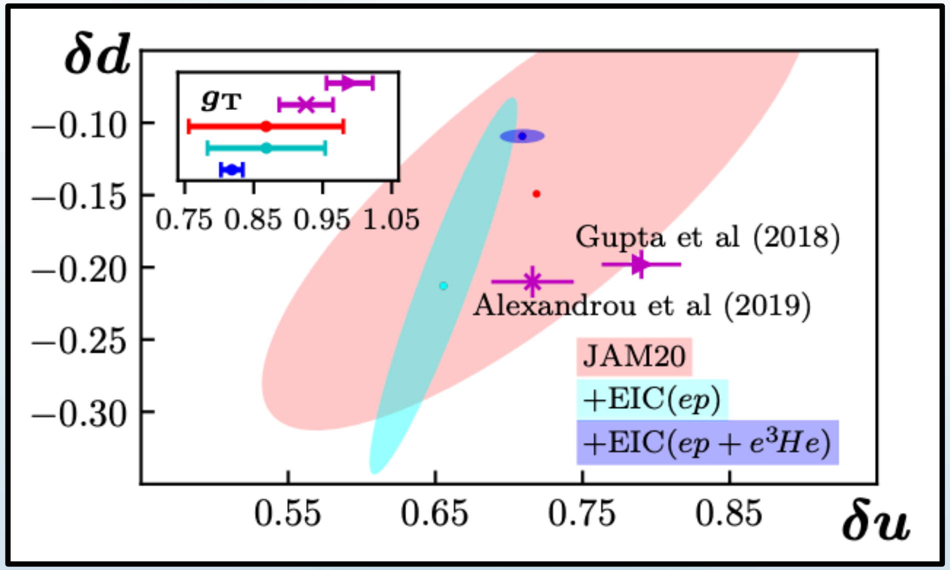


Transverse spin puzzle?



Outlook

L. Gamberg *et al.*, Phys. Lett. B **816**, 136255 (2021)



More data from RHIC
Proton-proton cross section

SIDIS multiplicities from COMPASS

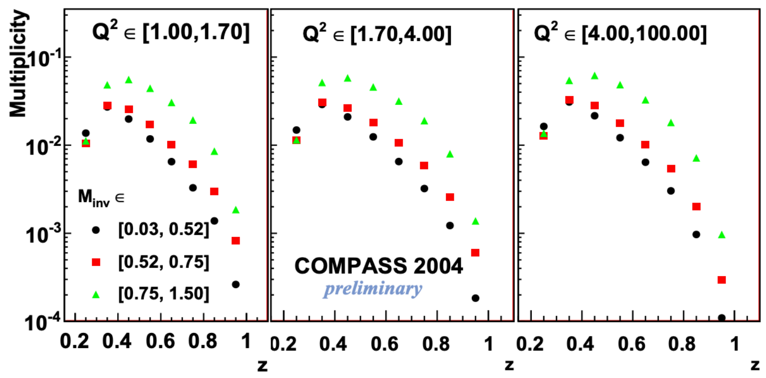


Figure 4: h^+h^- multiplicities measured versus z, M_{inv} and Q^2.

N. Makke, Phys. Part. Nucl. **45**, 138-140 (2014)

EIC can provide new information

Simultaneous fit of DiFF channel + TMD channel + Lattice QCD

Collaboration

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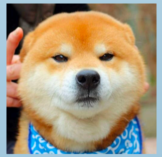
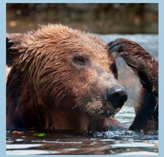
Nobuo Sato

Daniel Pitonyak

Thank you to Yiyu Zhou and Patrick Barry for helpful discussions





Extra Slides

Parameterize PDFs at input scale $Q_0^2 = m_c^2$

$$f_i(x) = Nx^{\alpha}(1-x)^{\beta}(1+\gamma\sqrt{x}+\eta x)$$

Evolve PDFs using DGLAP

$$\frac{\mathrm{d}}{\mathrm{d}\ \ln(\mu^2)} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z,\mu) f_j(\frac{x}{z},\mu)$$

Calculate Observables

$$d\sigma^{pp} = \sum_{ij} H_{ij}^{pp} \otimes f_i \otimes f_j$$

Mellin Space Techniques

$$\mathrm{d}\sigma^{pp} = \sum_{ijkl} \frac{1}{(2\pi i)^2} \int \mathrm{d}N \int \mathrm{d}M \tilde{f}_j(N,\mu_0)\tilde{f}_l(M,\mu_0) \otimes \left[x_1^{-N} x_2^{-M} \tilde{\mathcal{H}}_{ik}^{pp}(N,M,\mu) U_{ij}^S(N,\mu,\mu_0) U_{kl}^S(M,\mu,\mu_0)\right]$$

Experimentally measured cross-section

"Soft part" (process independent)
Describes internal structure

$$\sigma = \sum_{ij} H_{ij} \otimes f_i \otimes f_j + \mathcal{O}(1/Q)$$

"Hard part" (process dependent)
Cross-section at parton level
Calculated in perturbative QCD

Now that the observables have been calculated…

Data

Theory

Normalization

$$\chi^2(\boldsymbol{a}) = \sum_{i,e}\left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}}\right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1-N_e}{\delta N_e}\right)^2$$

Uncorrelated Uncertainties

Correlated Uncertainties

Normalization Uncertainty

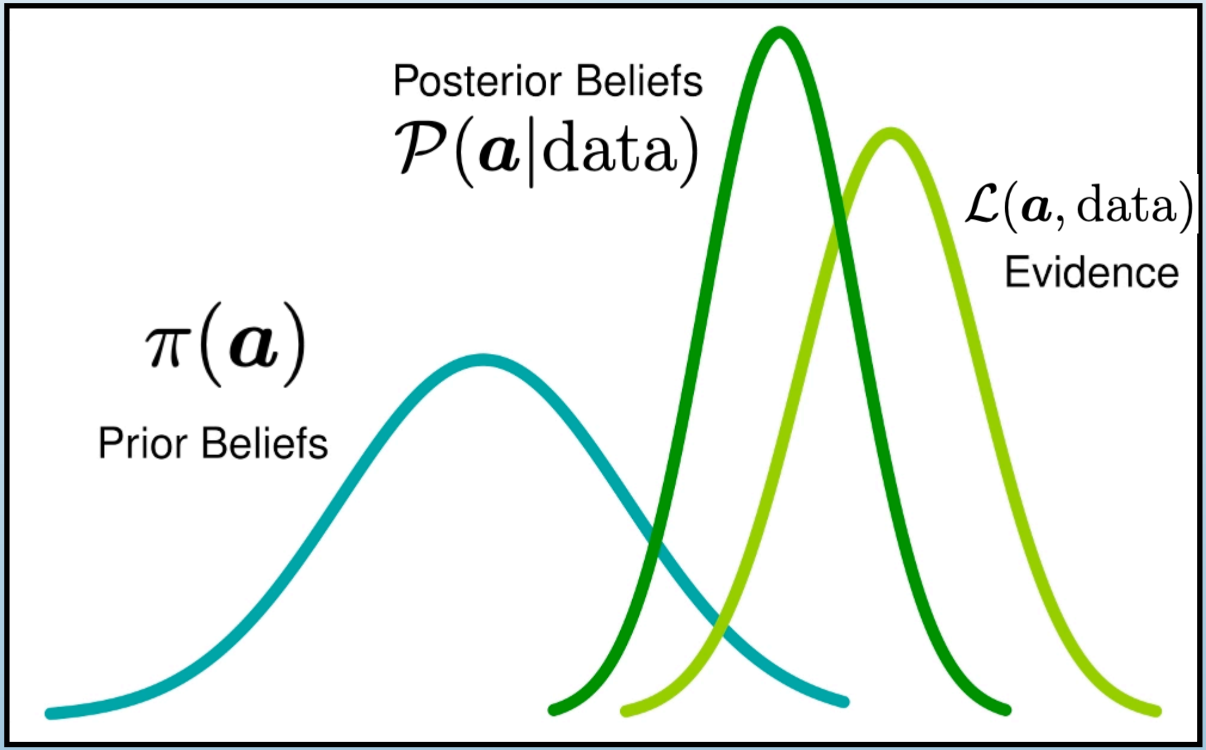
Now that we have calculated $\chi^2(\boldsymbol{a}, \text{data})$…

Likelihood Function

$$\mathcal{L}(\boldsymbol{a}, \text{data}) = \exp\left(-\frac{1}{2}\chi^2(\boldsymbol{a}, \text{data})\right)$$

Bayes' Theorem

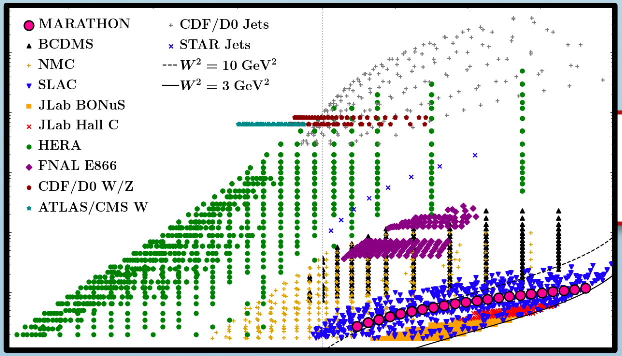
$$\mathcal{P}(\boldsymbol{a}|\text{data}) \sim \mathcal{L}(\boldsymbol{a}, \text{data})\,\pi(\boldsymbol{a})$$



Pseudo-Data → $$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$ ← Uncorrelated Uncertainties

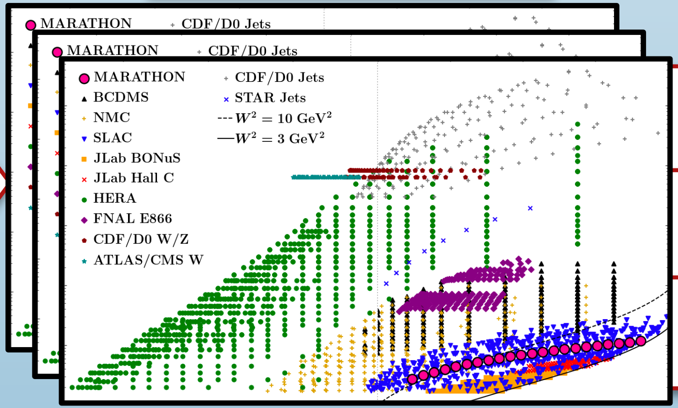
↑ Data

Original Data



DR

Replica Data

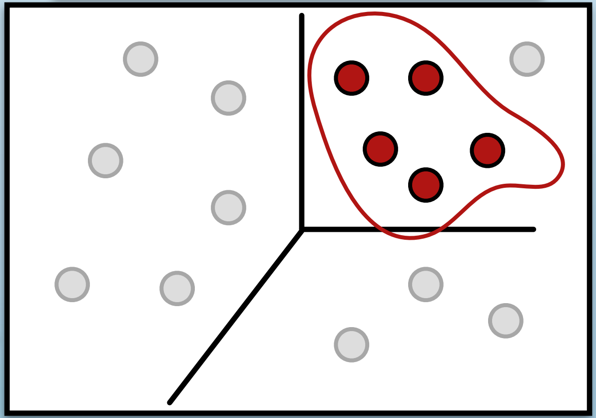


Maximum Likelihood

Maximum Likelihood

Maximum Likelihood

Parameter Space



For a quantity $O(\boldsymbol{a})$: (for example, a PDF at a given value of (x, Q^2))

$$E[O] = \int d^n a \; \rho(\boldsymbol{a}\,|\,data) \; O(\boldsymbol{a})$$

$$V[O] = \int d^n a \; \rho(\boldsymbol{a}\,|\,data) \; \left[O(\boldsymbol{a}) - E[O]\right]^2$$

Exact, but

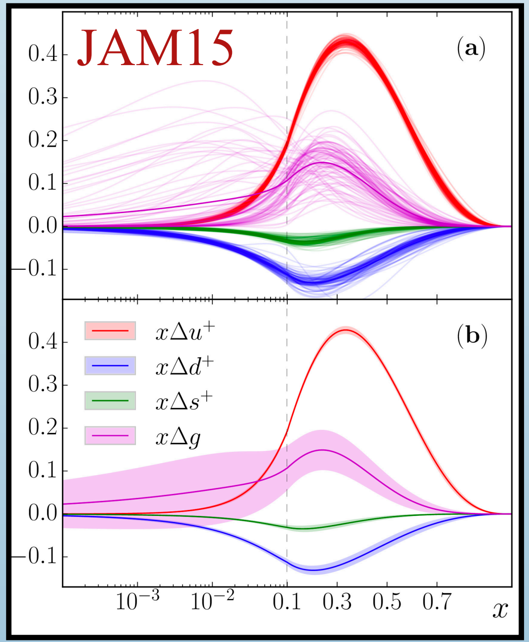
$n = \mathcal{O}(100)!$

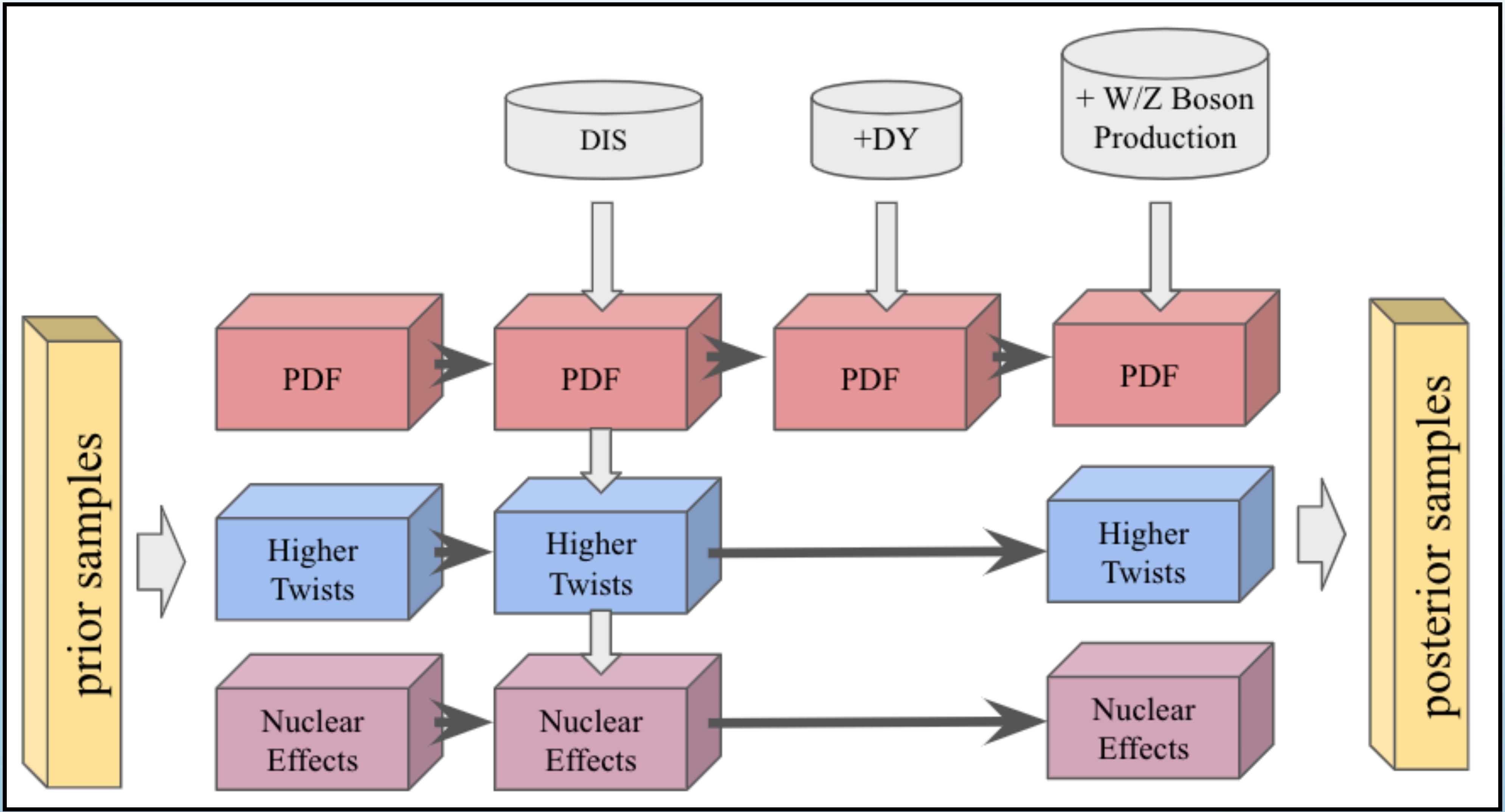
Build an MC ensemble

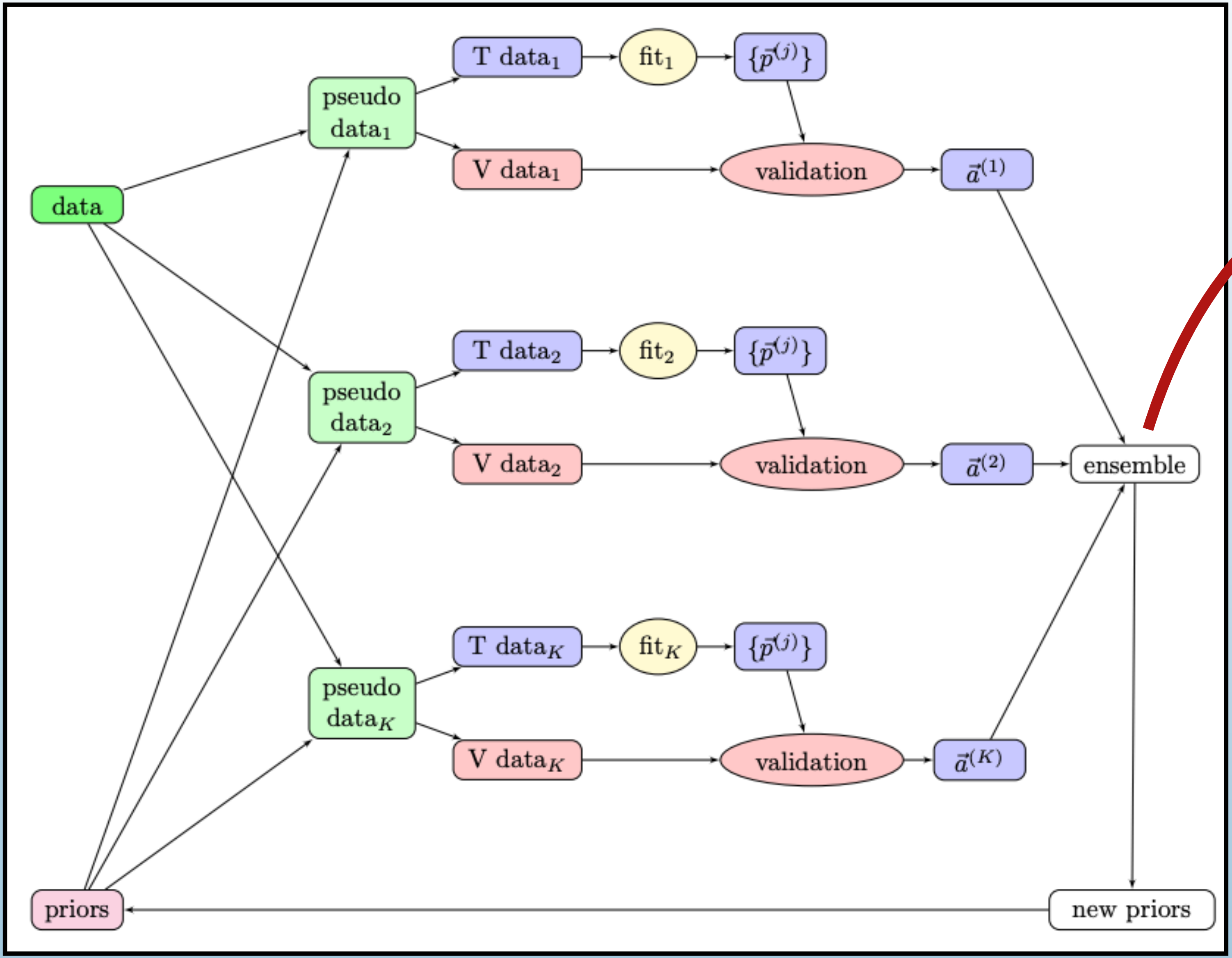
$$E[O] \approx \frac{1}{N} \sum_k O(\boldsymbol{a}_k)$$

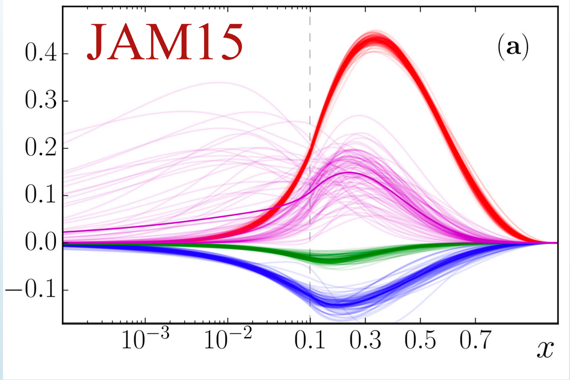
$$V[O] \approx \frac{1}{N} \sum_k \left[O(\boldsymbol{a}_k) - E[O]\right]^2$$

Average over k sets of the parameters (replicas)



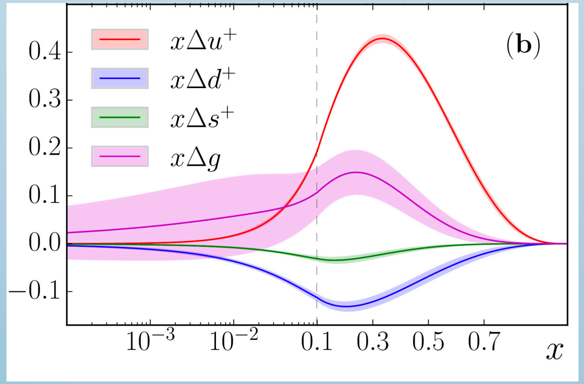

DIS
+DY
+ W/Z Boson
Production
prior samples
PDF
PDF
PDF
PDF
Higher
Twists
Higher
Twists
Higher
Twists
Nuclear
Effects
Nuclear
Effects
Nuclear
Effects
posterior samples



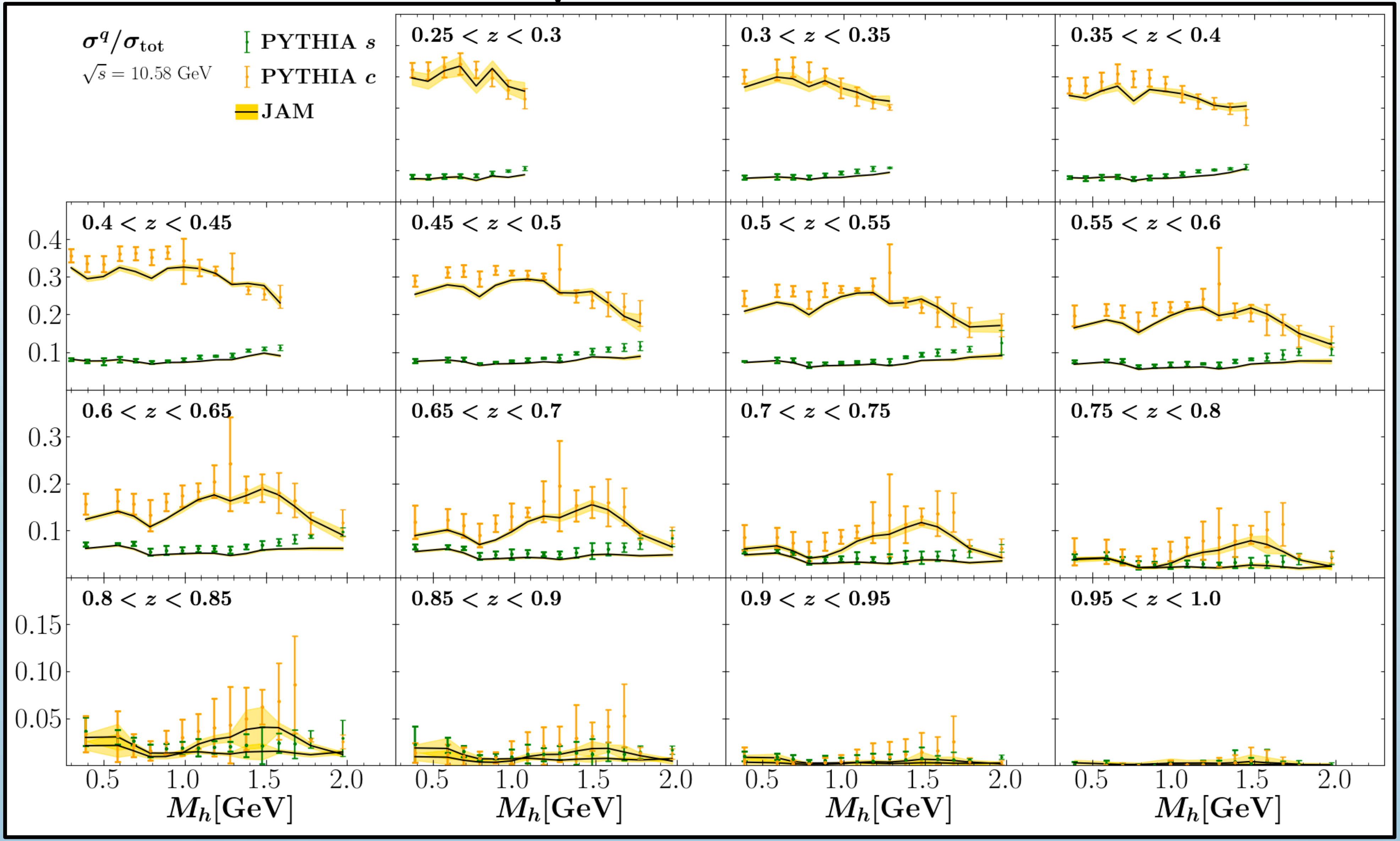


$$E[O] \approx \frac{1}{N} \sum_k O(\boldsymbol{a}_k)$$

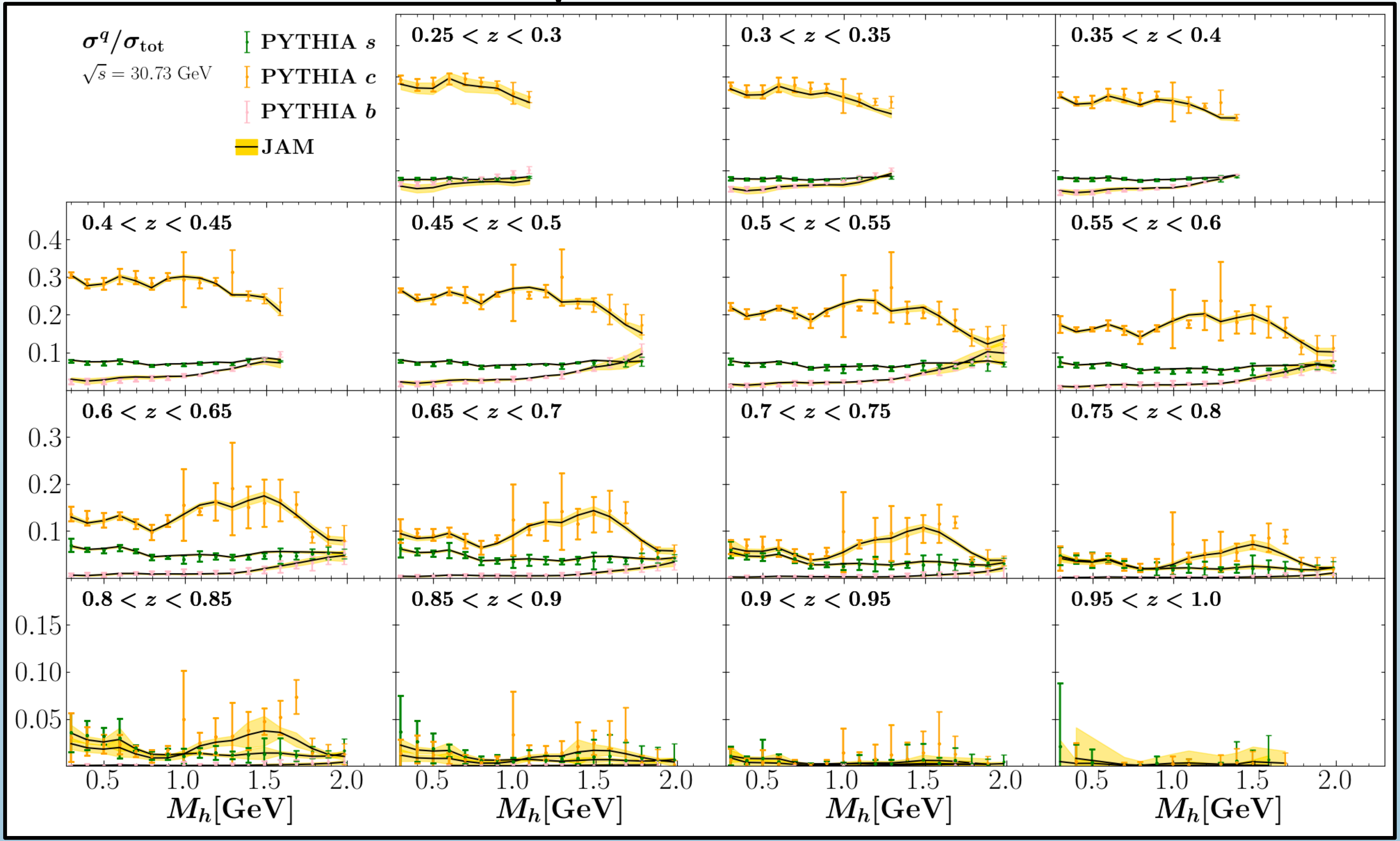
$$V[O] \approx \frac{1}{N} \sum_k \left[O(\boldsymbol{a}_k) - E[O]\right]^2$$



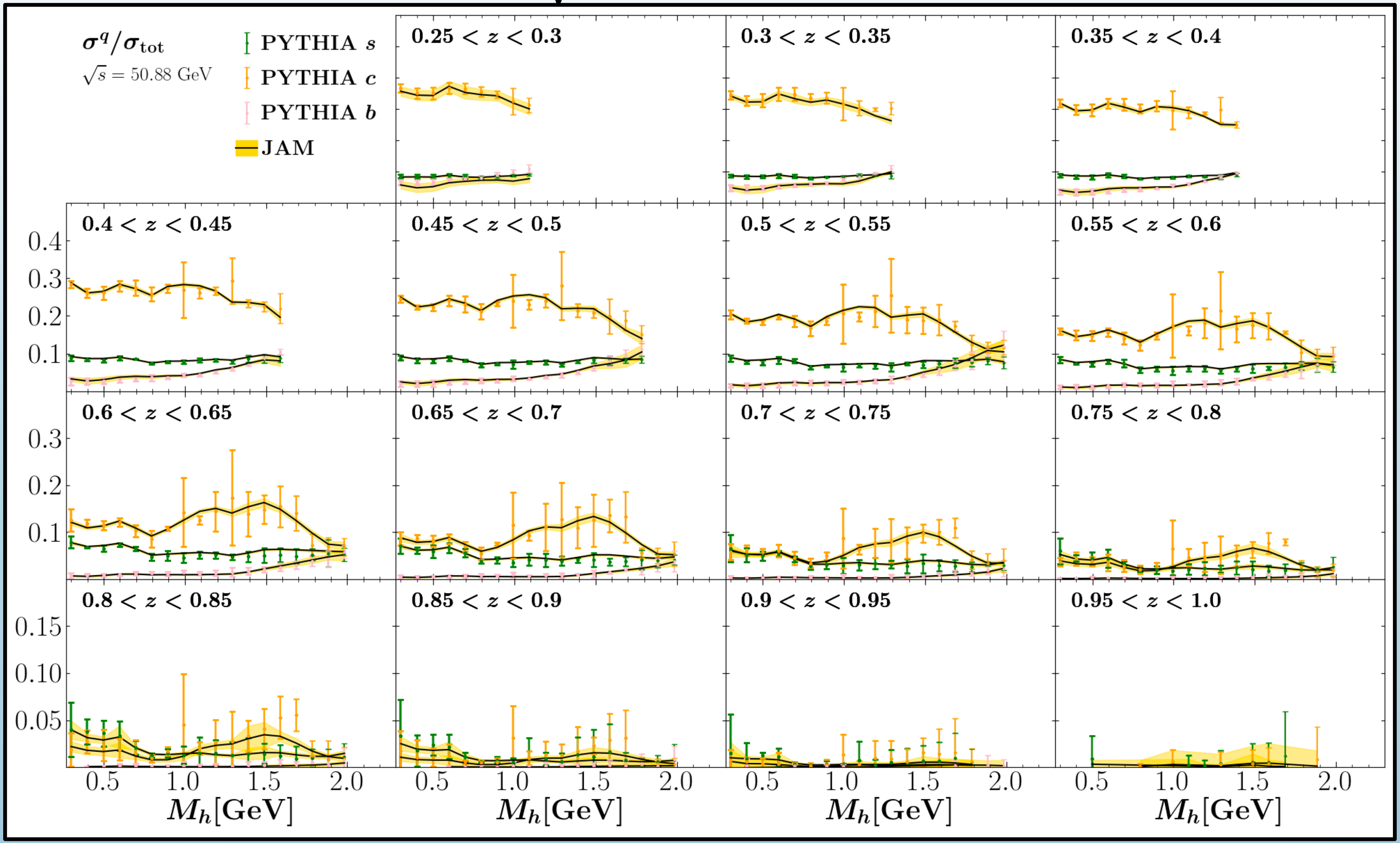
PYTHIA data ($\sqrt{s} = 10.58$ GeV)



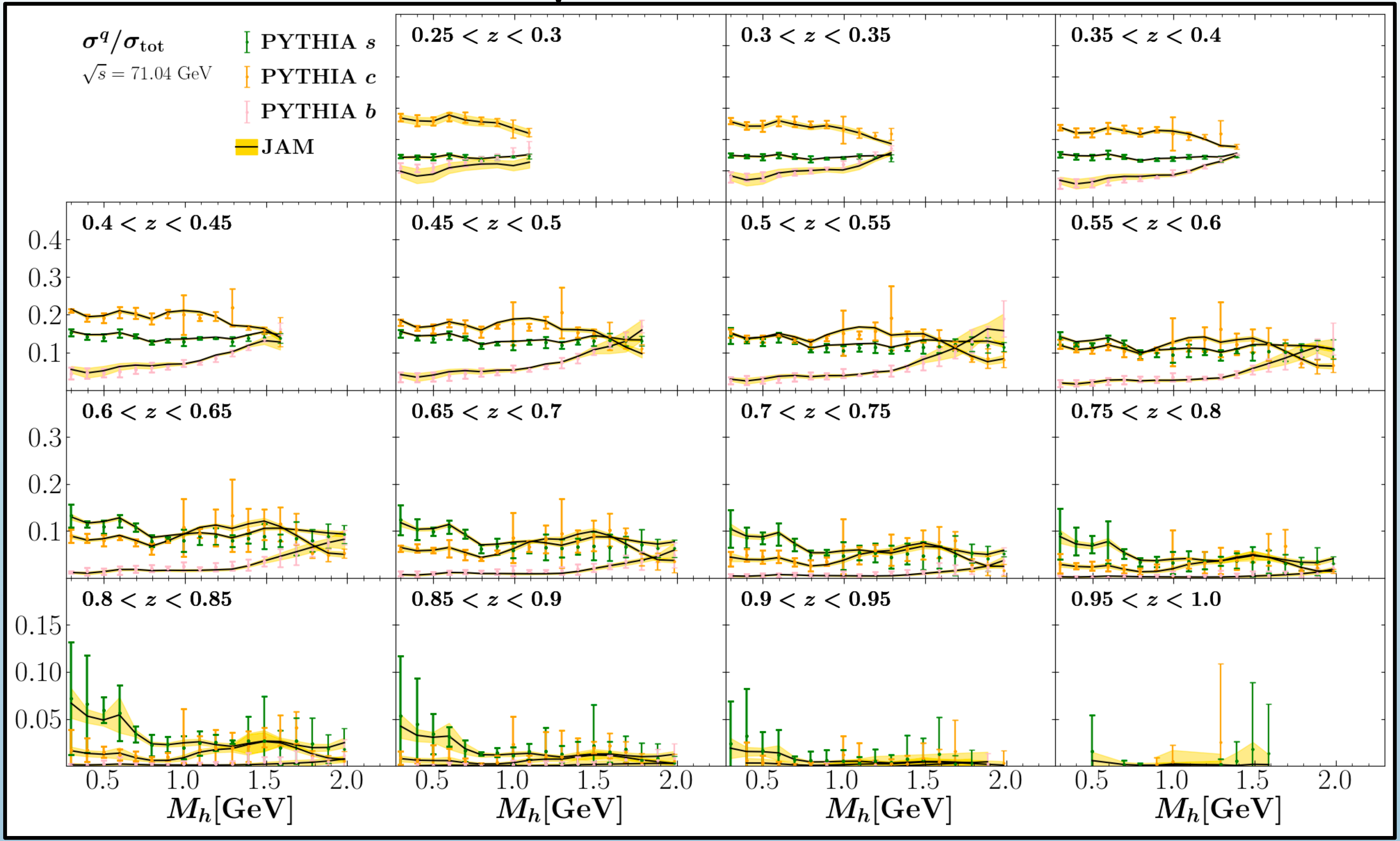
PYTHIA data ($\sqrt{s} = 30.73$ GeV)



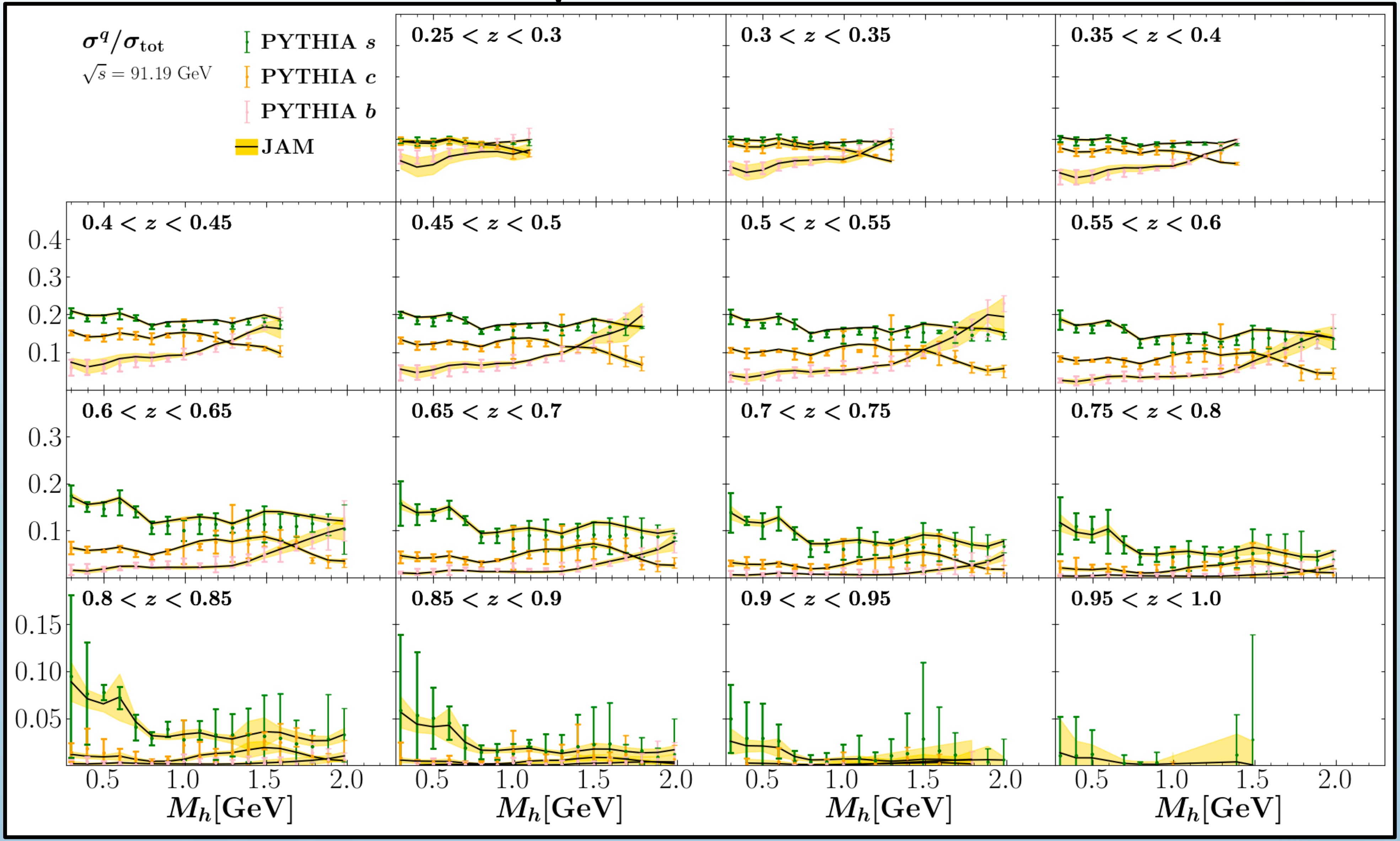
PYTHIA data ($\sqrt{s} = 50.88$ GeV)



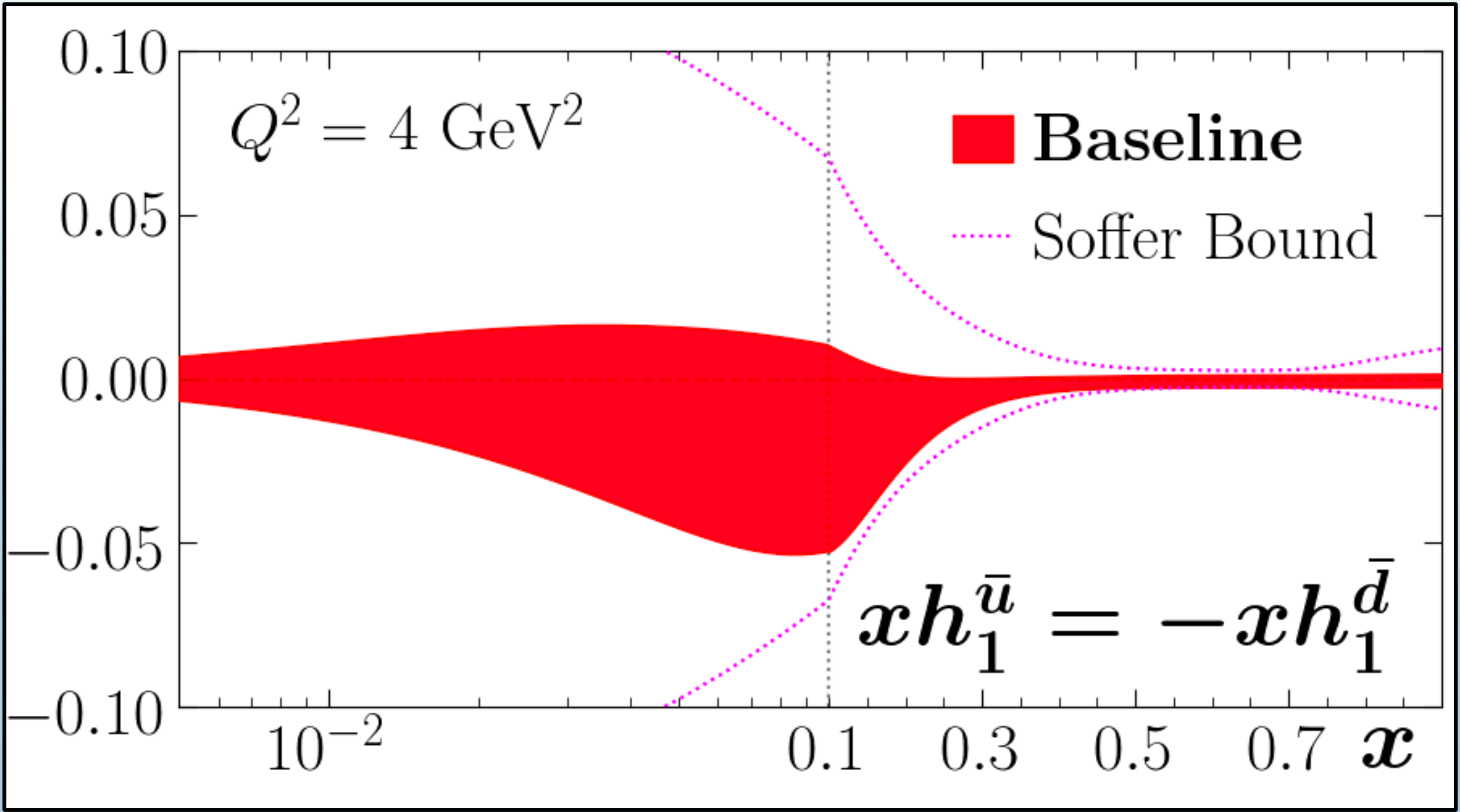
PYTHIA data ($\sqrt{s} = 71.04$ GeV)



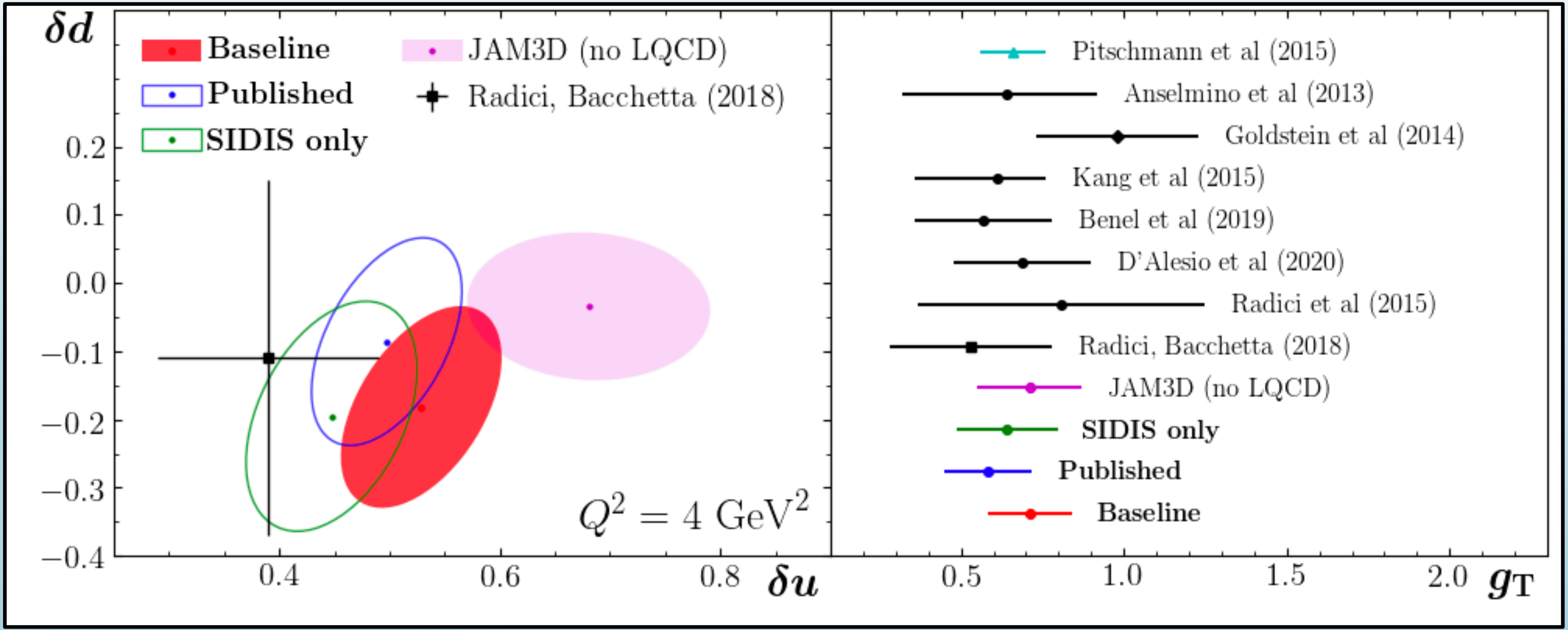
PYTHIA data ($\sqrt{s} = 91.19$ GeV)



Transversity PDFs (antiquarks)



Tensor Charges (Different Datasets)



DiFF Parameterization

$$\mathbf{M}_h^u = [2m_\pi, 0.40, 0.50, 0.70, 0.75, 0.80, 0.90, 1.00, 1.20, 1.30, 1.40, 1.60, 1.80, 2.00]\ \mathrm{GeV}.$$

$$D_1^q(z, \mathbf{M}_h^{q,i}) = \sum_{j=1,2,3} \frac{N_{ij}^q}{\mathcal{M}_{ij}^q} z^{\alpha_{ij}^q} (1-z)^{\beta_{ij}^q},$$

204 parameters for D_1

48 parameters for $H_1^{\sphericalangle}$

PDF Parameterization

$$h_1^{u_v}$$
$$h_1^{d_v}$$
$$h_1^{\bar{u}} = -h_1^{\bar{d}}$$

$$f(x, \mu_0^2) = \frac{N}{\mathcal{M}} x^{\alpha} (1-x)^{\beta} (1 + \gamma\sqrt{x} + \eta x),$$

15 parameters for h_1

χ^2 Tables

experiment	observable	binning	N_{dat}	χ^2_{red}	fitted norm.
Belle [2]	$\frac{d^2\sigma}{dz dM_h}$	z, M_h	1121	1.24	0.992(20)
Belle [3]	a_{12R}	z, M_h	55	0.53	—
		$M_h, \overline{M}_h$	64	3.43	
		$z, \bar{z}$	64	1.54	
HERMES [5]	A_{UT}^{HERMES}	x_{bj}	4	1.84	1.101(43)
		M_h	4	1.27	
		z	4	1.74	
COMPASS (p) [53]	A_{UT}^{COMPASS}	x_{bj}	9	0.88	0.994(4)
		M_h	10	1.12	
		z	7	1.58	
COMPASS (D) [53]	A_{UT}^{COMPASS}	x_{bj}	9	1.20	1.002(5)
		M_h	10	0.39	
		z	7	0.47	
STAR [6] $\sqrt{s} = 200$ GeV $R < 0.3$	A_{UT}^{pp}	$M_h, \eta < 0$	5	2.54	0.982(17)
		$M_h, \eta > 0$	5	1.52	
		$P_{hT}, \eta < 0$	5	0.92	
		$P_{hT}, \eta > 0$	5	1.05	
		η	4	1.72	
STAR [25] $\sqrt{s} = 500$ GeV $R < 0.7$	A_{UT}^{pp}	$M_h, \eta < 0$	32	0.78	1.078(27)
		$M_h, \eta > 0$	32	1.16	
		$P_{hT}, \eta > 0$	35	1.09	
		η	7	1.57	
STAR [76] $\sqrt{s} = 200$ GeV $R < 0.3$ PRELIMINARY	A_{UT}^{pp}	$M_h, \eta < 0$	31	0.94	0.955(16)
		$M_h, \eta > 0$	31	1.25	
		$P_{hT}, \eta < 0$	29	0.85	
		$P_{hT}, \eta > 0$	29	1.05	
		η	9	2.06	
Total			1627	**1.29**	

experiment	N_{dat}	Lattice	Baseline
HERMES [5]	12	1.92	1.62
COMPASS (p) [53]	26	1.28	1.16
COMPASS (D) [53]	26	0.71	0.69
STAR (2015) [6]	24	1.62	1.54
STAR (2018) [25]	106	1.09	1.05
STAR (PRELIM) [76]	129	1.09	1.10
ETMC δu [46]	1	4.04	—
ETMC δd [46]	1	0.15	—
Total	325	**1.15**	**1.11**