

# Linking the $^3P_0$ decay model to Landau gauge QCD

R. Alkofer, F.J. Llanes-Estrada<sup>†</sup> & A. Salas-Bernárdez

Univ. Complutense de Madrid (FJLE, ASB) & Univ. of Graz (RA)

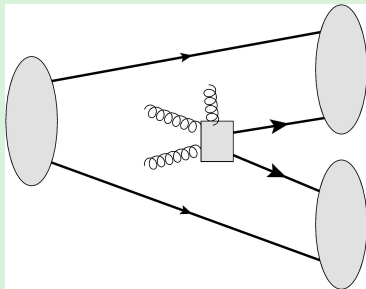
Presented at 10th Workshop of APS Hadron Physics group  
14/4/2023



# Thanking the organization for the **spirited** dicussions

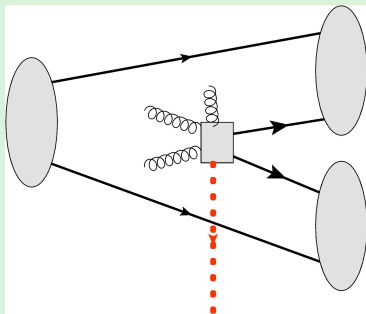


# $q\bar{q}$ OZI-allowed meson decays



L. Micu, NPB 10 (1969) 521-526

# $q\bar{q}$ OZI-allowed meson decays

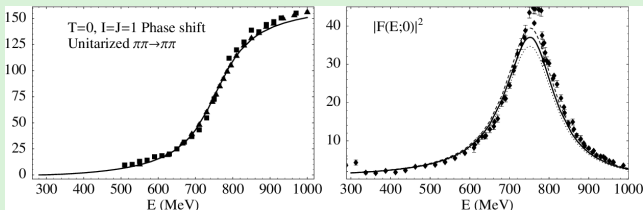


$${}^1S_0, {}^1P_1, {}^3S_1, {}^3P_0, {}^3P_1, {}^3P_2 \dots {}^{2S+1}L_J$$

Possible Q# of produced pair



# $q\bar{q}$ OZI-allowed meson decays

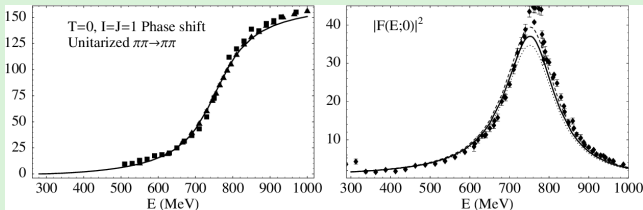


$$\cancel{1S_0}, \cancel{1P_1}, {}^3S_1, {}^3P_0, {}^3P_1, {}^3P_2 \dots$$

Think of  $\rho(\uparrow\uparrow) \rightarrow \pi(\uparrow\downarrow)\pi(\uparrow\downarrow)$

A. Gómez-Nicola *et al.* PLB **606** 351-360 (2005)

# $q\bar{q}$ OZI-allowed meson decays

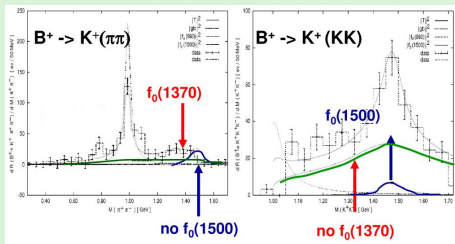


$${}^1S_0, {}^1P_1, \cancel{{}^3S_1}, {}^3P_0, {}^3P_1, {}^3P_2 \dots$$

$$\text{Think of } \rho(l=0) \rightarrow \underbrace{\pi(l=0)\pi(l=0)}_{L=1}$$

A. Gómez-Nicola *et al.* PLB **606** 351-360 (2005)

# $q\bar{q}$ OZI-allowed meson decays

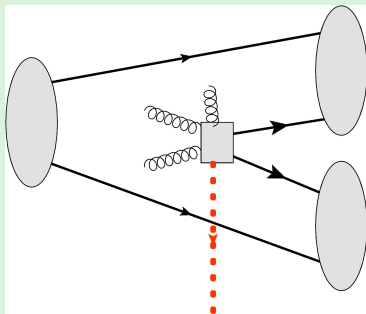


$${}^1S_0, {}^1P_1, {}^3S_1, {}^3P_0, \cancel{{}^3P_1}, \cancel{{}^3P_2} \dots$$

Move on to  $f_0 \rightarrow \underbrace{\pi\pi}_{J=0}$

E. Klempt <https://slideplayer.com/slide/14648261/>

# Lore: important $^3P_0$ pair production mechanism



$$^1S_0, ^1P_1, ^3S_1, ^3P_0, ^3P_1, ^3P_2 \dots ^{2S+1}L_J$$

Possible Q# of produced pair

# Not visible in QCD (or QED)

- $\int d^3x \bar{\psi} \boldsymbol{\gamma} \cdot \mathbf{A} \psi$  seems  ${}^3S_1$
- Chiral-symmetry respecting at all orders in perturbation theory
- But  ${}^3P_0$  breaks chiral symmetry

# Not very good rejection tests of $S = 1$ with light quarks

Famous selection rule:  $A(S = 0) \not\rightarrow B(S = 0) + C(S = 0)$

(tests the “3” part of  ${}^3P_0$ )

# Not very good rejection tests of $S = 1$ with light quarks

Famous selection rule:  $A(S = 0) \nrightarrow B(S = 0) + C(S = 0)$

(tests the “3” part of  ${}^3P_0$ )

List of  $S = 0$  quantum numbers:

$L$	$J^{P(C)}$	
0	$0^{-(+)}$	$\pi, \eta, K \dots$
1	$1^{+(-)}$	$h_1, b_1, \dots$
2	$2^{-(+)}$	$\pi_2, \eta_2 \dots$
3	$3^{+(-)}$	$h_3, b_3, \dots$
...		

# Not very good rejection tests with light quarks

- $0^{-+} \rightarrow 0^{-+} 0^{-+}$  forbidden by  $J^{PC}$  (can't test anything)



# Not very good rejection tests with light quarks

- $0^{-+} \rightarrow 0^{-+} 0^{-+}$  forbidden by  $J^{PC}$  (can't test anything)
- $1^{+-} \rightarrow 1^{+-} 0^{-+}$  no excited  $h_1, b_1$  mesons;  $K_1$  mixed  $\sim 20^\circ$

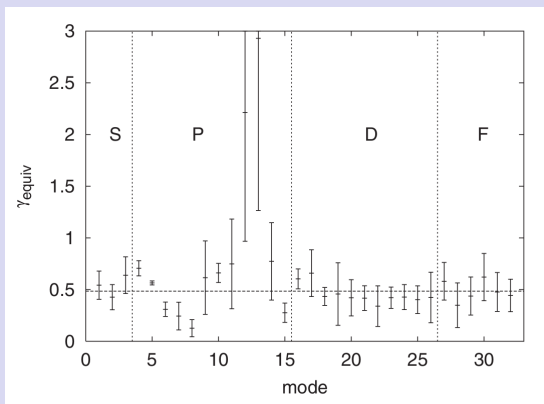
L. Abreu, A. Favero, FLE PRD 2018

# Not very good rejection tests with light quarks

- $0^{-+} \rightarrow 0^{-+} 0^{-+}$  forbidden by  $J^{PC}$  (can't test anything)
- $1^{+-} \rightarrow 1^{+-} 0^{-+}$  no excited  $h_1, b_1$  mesons;  $K_1$  mixed  $\sim 20^\circ$   
L. Abreu, A. Favero, FLE PRD 2018
- $2^{-+} \rightarrow 1^{+-} 1^{+-}$  phase-space closed...

(Plenty of room at Jlab to improve on the light meson spectrum)

# $D/D_s$ spectrum: 32 modes studied by Close and Swanson



F. Close and E.S. Swanson PRD72 094004 (2005)

# Effective Hamiltonian

$$H_{3P_0} = \sqrt{3}g_s \int d^3\mathbf{x} \bar{\psi}(\mathbf{x})\psi(\mathbf{x})$$

$$\gamma = \frac{g_s}{2m}$$

# Effective Hamiltonian

$$H_{3P_0} = \sqrt{3}g_s \int d^3\mathbf{x} \bar{\psi}(\mathbf{x})\psi(\mathbf{x})$$

$$\gamma = \frac{g_s}{2m}$$

Chiral-symmetry breaking:

$$[Q_5, H_{3P_0}] = \left[ \int d^3\mathbf{x} \psi^\dagger(\mathbf{x})\gamma_5\psi(\mathbf{x}), H_{3P_0} \right] \neq 0$$

# Effective Hamiltonian

$$H_{3P_0} = \sqrt{3}g_s \int d^3\mathbf{x} \bar{\psi}(\mathbf{x})\psi(\mathbf{x})$$

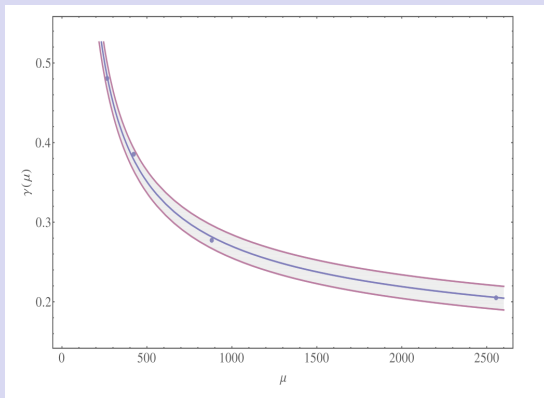
$$\gamma = \frac{g_s}{2m}$$

Chiral-symmetry breaking:

$$[Q_5, H_{3P_0}] = \left[ \int d^3\mathbf{x} \psi^\dagger(\mathbf{x})\gamma_5\psi(\mathbf{x}), H_{3P_0} \right] \neq 0$$

$$\begin{aligned} i(2\pi)^4 \delta^{(4)}(p+q) \mathcal{M}_{3P_0}^{ss'}(p, q) &= \\ \langle \mathbf{p}s, \mathbf{q}s' | iT_{3P_0} | 0 \rangle &= \\ (i(2\pi)^4 \delta^{(4)}(p+q)) (-\sqrt{3}g_s) \bar{u}^s(p) v^{s'}(q) & \end{aligned}$$

# Dependence on the quark mass by Salamanca group



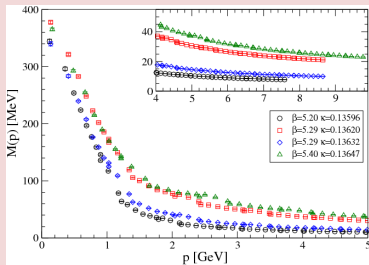
J. Segovia, D. R. Entem, F. Fernández Phys.Lett.B **715** (2012) 322-327

# Ongoing work: connect Quark-model pheno w. Landau gauge QCD

- $N$ -gluon to  $\bar{q}q$  kernel not known from first principles
- What to do with the information at hand?

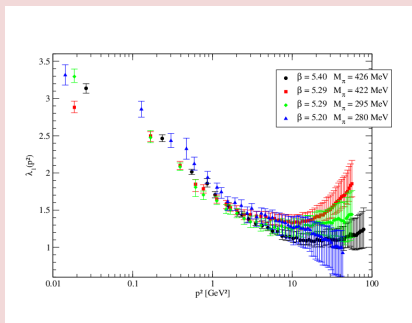
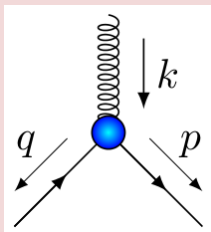


# Extensive lattice+DSE work on Landau gauge primitive Green's functions



Lattice data from O. Oliveira *et al.* Acta Phys.Polon.Supp. 9 (2016) 363-368

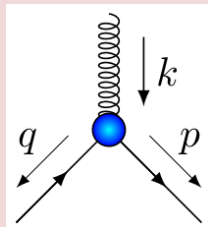
# Extensive lattice+DSE work on Landau gauge primitive Green's functions



And also the pure Yang-Mills primitive Green's functions...

Lattice data from O. Oliveira *et al.* Acta Phys.Polon.Supp. 9 (2016) 363-368

# Quark-Gluon vertex spin structure

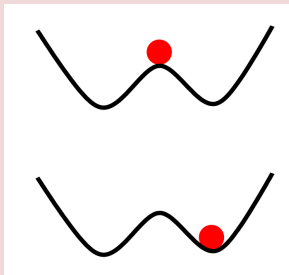


$$\Gamma_T^\mu(q_E, p_E; k_E) = \sum_{i=1}^8 g_i(\bar{p}_E^2) \rho_i^\mu(q_E, p_E)$$

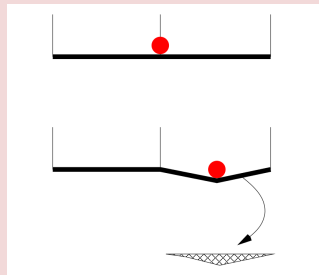
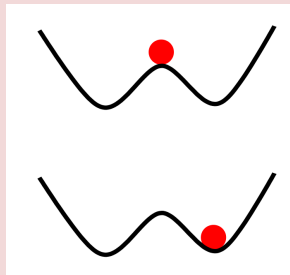
$$k^\mu = p^\mu + q^\mu$$

- It includes chiral-symmetry respecting and **breaking** pieces

# Chiral symmetry breaking

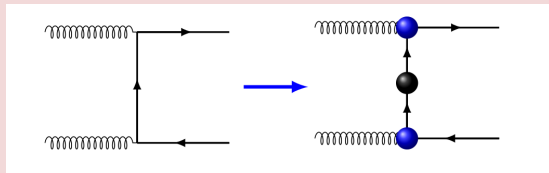


# Chiral symmetry breaking

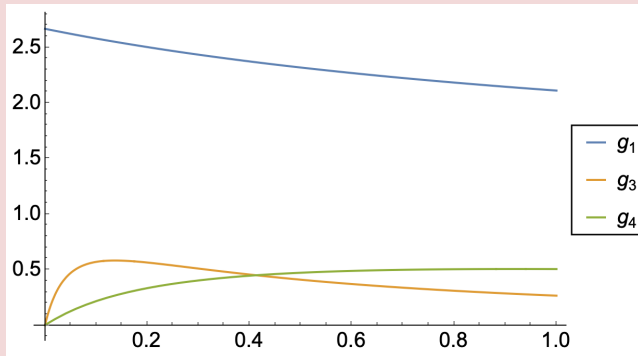


R. Alkofer *et al.* *Annals Phys.* **324** (2009) 106-172

# Ongoing work: connect Quark-model pheno w. Landau gauge QCD



# Euclidean $q_E^2$ functions (input from lattice, DSEs)



# Parametrization of transverse part of the vertex

- The tree-level vertex  $\rho_{1,E}^\mu = (\delta^{\mu\nu} - \hat{k}_E^\mu \hat{k}_E^\nu) \gamma_E^\mu \equiv \gamma_{T,E}^\mu$

with  $g_1(x) = 1 + \frac{1.67+0.204x}{1+0.683x+0.000851x^2}$



# Parametrization of transverse part of the vertex

- Chiral-symmetry breaking structures ( $s_E^\mu = (\delta^{\mu\nu} - \hat{k}_E^\mu \hat{k}_E^\nu) \bar{p}_E^\nu$ )  
 $\rho_{2,E}^\mu = i \hat{s}_E^\mu$  and  $\rho_{3,E}^\mu = i \hat{k}_E^\mu \gamma_{T,E}^\mu$

with  $g_3(x) = -1.45 g_2(x) = \frac{0.365x}{0.0187 + 0.353x + x^2}$  ;

# Parametrization of Landau-transverse part of the vertex

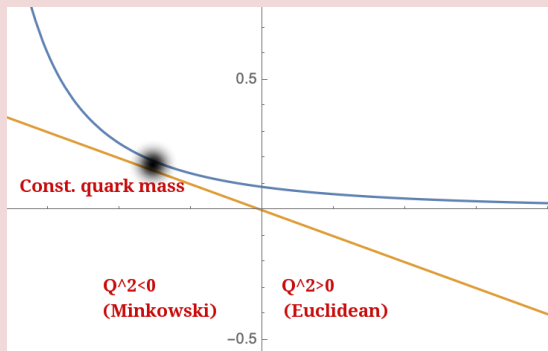
- The chirally symmetric structures

$$\rho_{4,E}^\mu = \hat{k}_E s_E^\mu \quad \text{and} \quad \rho_{7,E}^\mu = \hat{g}_E \hat{k}_E \gamma_{T,E}^\mu$$

$$\text{with } g_4(x) = g_7(x) = \frac{2.59x}{0.859+3.27x+x^2} .$$

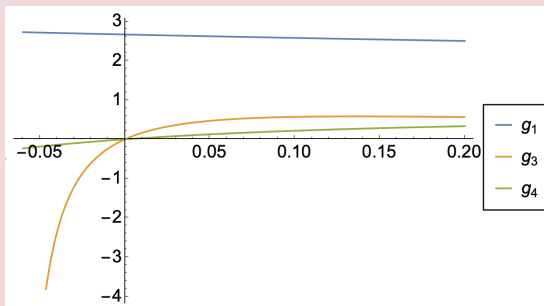
# Extension to physical Minkowski space

First, the propagator mass function:



# Extension to physical Minkowski space

Next, the vertex dressing form factors:



Note the  $Q^2 < 0$  enhancement of the chiral symmetry breaking piece!

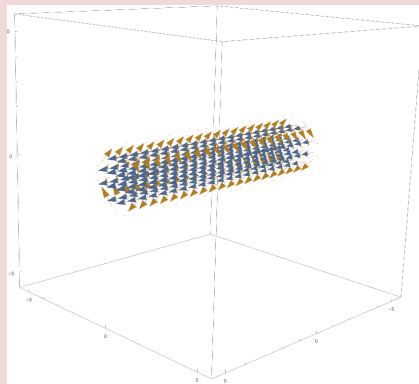
$$\begin{aligned}
& \langle \mathbf{p}s, \mathbf{q}s' | iT_{\text{singlet}} | 0 \rangle = \\
& \langle \mathbf{p}s, \mathbf{q}s' | -\frac{g^2}{2} \int d^4x \bar{\psi}_i(x) T_{ij}^a A_\mu^a(x) \Gamma^\mu \psi_j(x) \int d^4y \bar{\psi}_i(y) T_{ij}^a A_\nu^a(y) \Gamma^\nu \psi_j(y) | 0 \rangle = \\
& = -g^2 \int d^4x d^4y \int \frac{d^4t}{(2\pi)^4} \tilde{A}_0^a(p-t) \tilde{A}_0^a(q+t) \mathcal{K}_{ab}^{ss'}(p, q, t)
\end{aligned}$$

where

$$\mathcal{K}_{ab}^{ss'}(p, q, t) \equiv \left[ \bar{u}_i^s(p) T_{ij}^a \Gamma^0(p, -t) S(t) T_{jk}^b \Gamma^0(q, t) v_k^{s'}(q) \right] + \text{crossed amplitude}$$

and  $S(t)$  is the dressed fermion propagator.

# In a constant chromoelectric flux tube:



- Simplify to a constant chromo- $E$  (parallel-plate capacitor)  
Background Landau-gauge field  $(A_\rho, A_\theta, A_z, A_0) = (0, 0, 0, -Ez)$
- Think of the Schwinger pair-creation mechanism in QED

# A relation between the gluon to quark kernel $\mathcal{K}$ and the pair production amplitude

$$\langle \mathbf{p}S, \mathbf{q}S' | iT_{\text{singlet}} | 0 \rangle = -(2\pi)^4 \delta^{(4)}(\mathbf{p} + \mathbf{q}) (gE)^2 \left[ \frac{\partial}{\partial p^3} \frac{\partial}{\partial q^3} \mathcal{K}_{ab}^{ss'}(\mathbf{p}, \mathbf{q}, t) \right] \Big|_{t=q}$$

- With the primitive Green's functions construct this skeleton kernel ✓
- Project it over  ${}^{2S+1}L_J$  and numerically compare  
(But you can see in slides 25, 26 that the chiral symmetry breaking part will be important, perhaps even dominant)

- Historical  ${}^3P_0$  mechanism of strong decays needs QCD grounding
- Working on it from Landau gauge Green's functions (ideas welcome)
- It appears that the chiral symmetry breaking piece is indeed important



# Linking the $^3P_0$ decay model to Landau gauge QCD

R. Alkofer, F.J. Llanes-Estrada<sup>†</sup> & A. Salas-Bernárdez

Univ. Complutense de Madrid (FJLE, ASB) & Univ. of Graz (RA)

Presented at 10th Workshop of APS Hadron Physics group  
14/4/2023



# Acknowledgments

Work of FJLE done as part of the Exotic Hadrons (ExoHad) Topical Coll. This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 824093; grants MICINN: PID2019-108655GB-I00, PID2019-106080GB-C21 (Spain); UCM research group 910309 and the IPARCOS institute.



# Mixing of the $f_0$ s



EPJ.ST 230 1575-1592 (2021)