Light front time and rest frame densities of hadrons

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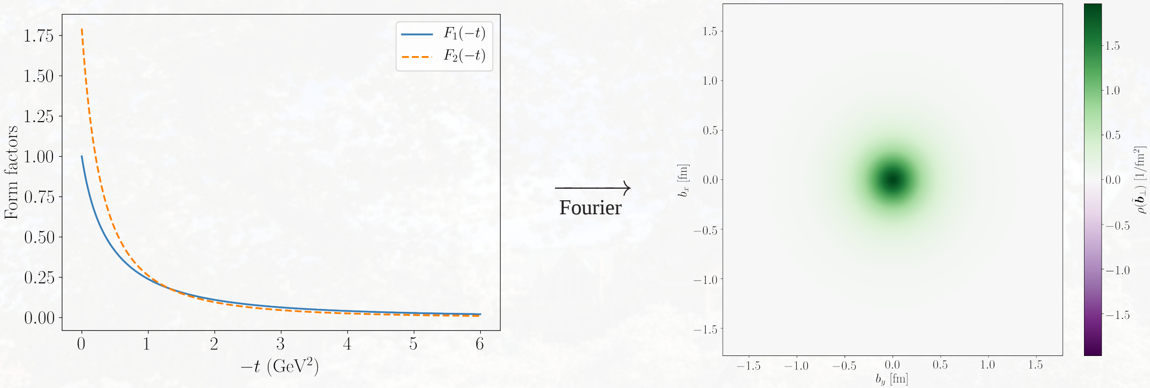
based on work in arXiv:2302.09171 (PRD, in press)

in collaboration with Jerry Miller

Introduction

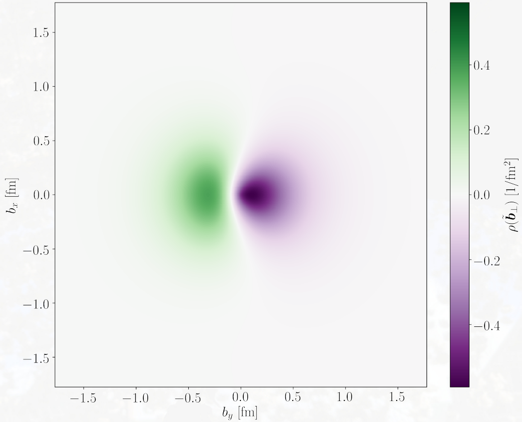
- Form factors are extracted from experimental measurements.
  - Electromagnetic, axial, gravitational, …
- 2D Fourier transforms provide 2D spatial densities at **fixed light front time**.
  - My talk is about what **fixed light front time** means.
  - And why these are rest frame (not infinite momentum frame) densities.
  - Building on ideas pioneered by Burkardt (2000, 2003), Diehl (2002), Miller (2007)



Phenomenology of spatial densities

- Densities for transverse polarization have azimuthal modulations.
  - cf. Burkardt, Int. J. Mod. Phys. A18 (2003)
- These are often attributed to kinematic effects from boosts to infinite momentum.
- I will argue this is **not** the case.
  - This is a rest state density.
  - Modulations are what we'd really see.
  - They arise from **synchronization** effects.

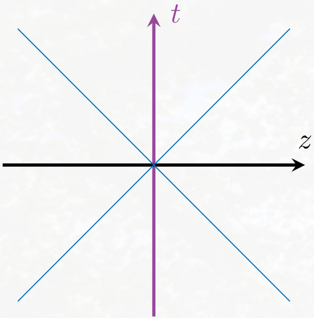
Transpol. neutron, charge density



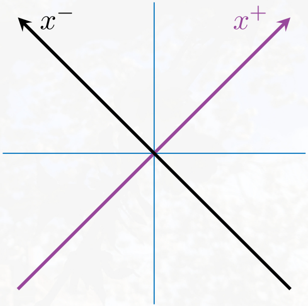
- This talk is about using light front for **phenomenology**.
  - Use form factors from experiment (Kelly 2004, Riordan 2010).
  - I'm not arguing for light front quantization.

Light front coordinates are a different foliation of spacetime.

$$x^{\pm} = t \pm z \qquad \mathbf{x}_{\perp} = (x, y) \qquad x^{+} = t + z = \text{time}$$



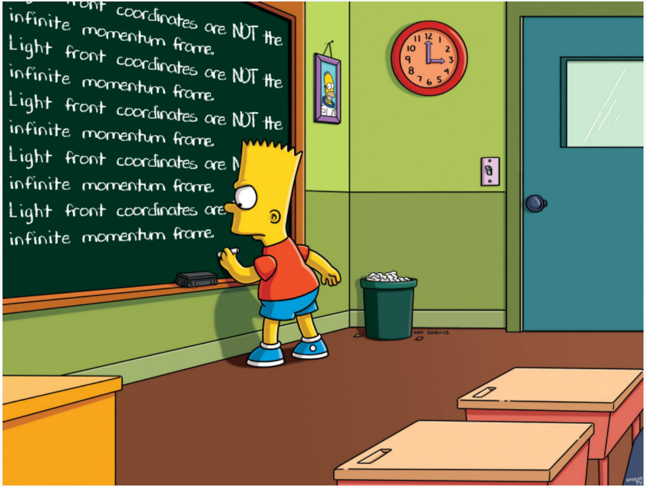
Minkowski coordinates



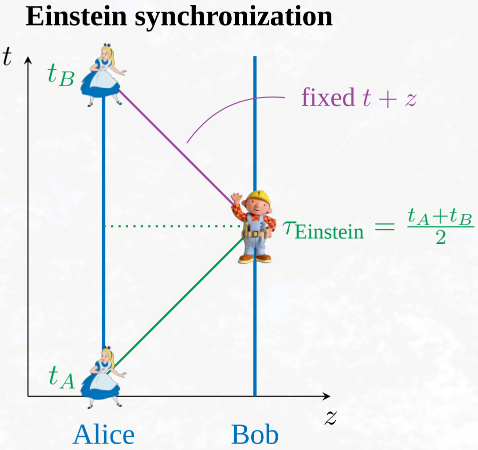
Light front coordinates

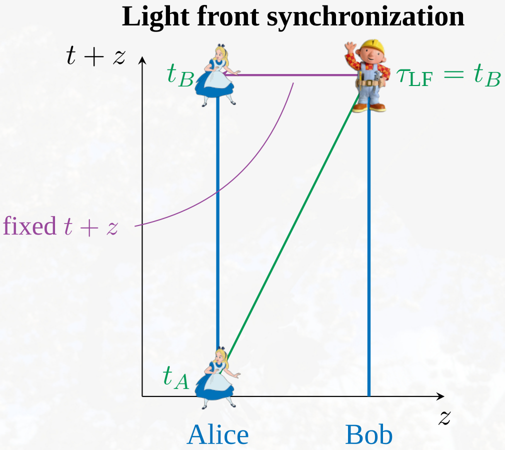
Not the IMF!

- Light front coordinates are valid in **any** frame.
    - They're not a reference frame.
- Light front coordinates are **not** the infinite-momentum frame.
    - A common misconception.
- Light front coordinates redefine **synchronization convention**.
    - What we mean by "simultaneous."



Synchronization conventions





- **Einstein synchronization** defined to be isotropic.
- **Light front synchronization** defines hyperplanes with fixed $t+z$ to be "simultaneous."
  - Light travels instantaneously in $-z$ direction *by definition.*
  - We take what we see as literally happening now.

Equal-“time” surfaces are just a convention

- Relativity requires *round-trip* speed of light to be invariant.
- Convention that one-way speed of light be c is a *definition*, not an empirical fact.
  - Pointed out in Einstein’s original paper.
- Redefining “time” coordinate means changing this definition.
  - Light front coordinates do exactly this!

894 *A. Einstein.*

B durch einen in *B* befindlichen Beobachter möglich. Es ist aber ohne weitere Festsetzung nicht möglich, ein Ereignis in *A* mit einem Ereignis in *B* zeitlich zu vergleichen; wir haben bisher nur eine „*A*-Zeit“ und eine „*B*-Zeit“, aber keine für *A* und *B* gemeinsame „Zeit“ definiert. Die letztere Zeit kann nun definiert werden, indem man *durch Definition* festsetzt, daß die „Zeit“, welche das Licht braucht, um von *A* nach *B* zu gelangen, gleich ist der „Zeit“, welche es braucht, um von *B* nach *A* zu gelangen. Es gehe nämlich ein Lichtstrahl zur „*A*-Zeit“ t_A von *A* nach *B* ab, werde zur „*B*-Zeit“ t_B in *B* gegen *A* zu reflektiert und gelange zur „*A*-Zeit“ t'_A nach *A* zurück. Die beiden Uhren laufen definitionsgemäß synchron, wenn

$$t_B - t_A = t'_A - t_B.$$

Einstein, Ann. Phys. 322 (1905) 891

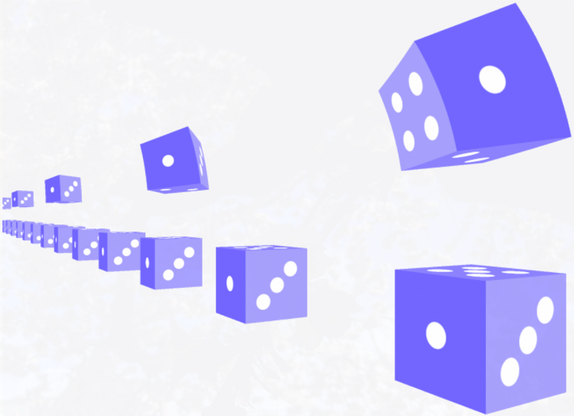
- **Technical review**: Anderson, Stedman & Vetharaniam, Phys. Rept. 295 (1998) 93
- **Didactic overview**: Veritasium, “Why No One Has Measured The Speed of Light” (YouTube)

- Lorentz-boosted objects *appear rotated*.
  - **Terrell rotation** (PR116, 1959)
  - Optical effect: contraction + delay

- **Light front transverse boost** *undoes* Terrell rotation:

$$B_x^{(\mathrm{LF})} = K_x - J_y$$

  - Standard boost + counter-rotation
  - Leaves x^+ (time) invariant
  - Part of the **Galilean subgroup**



Dice images by Ute Kraus,
`https://www.spacetimetravel.org/`

Galilean subgroup

- Poincaré group has a $(2+1)$D **Galilean subgroup**.
  - x^+ is time and $\boldsymbol{x}_\perp$ is space under this subgroup.
  - $P^+ = E_{\boldsymbol{p}} + p_z$ is the central charge.
  - x^+ and P^+ are invariant under this subgroup!
- Light front synchronization gives **fully relativistic** 2D picture that looks a lot like non-relativistic physics.
  - But with P^+ in place of m.

$$\frac{\mathrm{d}\boldsymbol{P}_\perp}{\mathrm{d}x^+} = P^+ \frac{\mathrm{d}^2\boldsymbol{x}_\perp}{\mathrm{d}x^{+2}}$$

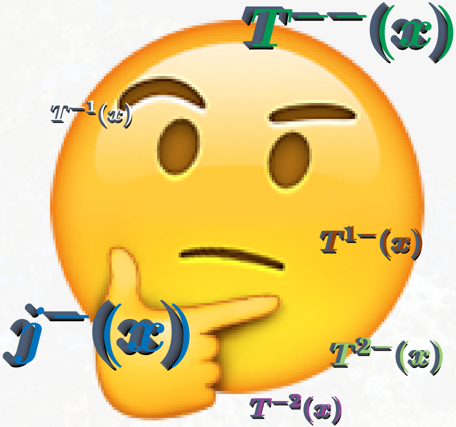
$$H = H_{\text{rest}} + \frac{\boldsymbol{P}_\perp^2}{2P^+}$$

$$\boldsymbol{v}_\perp = \frac{\boldsymbol{P}_\perp}{P^+}$$

etc.

What about x^-?

- x^- doesn't have a clear intuitive meaning.
- We actually integrate it out for densities.
- Current & energy-momentum tensor have components with unclear meanings.
  - $j^-(x)$
  - $T^{i-}(x), T^{-i}(x), T^{--}(x)$
- **Why not just use z?**



Tilted light front coordinates

Tilted coordinates

$$\tilde{\tau} = t + z$$
$$\tilde{x} = x$$
$$\tilde{y} = y$$
$$\tilde{z} = z$$

- Mind the strange metric…

$$\tilde{g}_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$$

- First defined by Blunden, Burkardt & Miller.
  - Phys. Rev. C61 (2000) 025206
- Use light front time.
  - Use light front synchronization!
  - Time invariant under **Galilean subgroup**.
- Use Cartesian spatial coordinates.
  - Ordinary intuition applies!

$$\mathrm{d}s^2 = \mathrm{d}\tilde{\tau}^2 - 2\,\mathrm{d}\tilde{\tau}\,\mathrm{d}\tilde{z} - \mathrm{d}\tilde{\boldsymbol{x}}_\perp^2$$
$$\partial^2 = -2\tilde{\partial}_z\tilde{\partial}_\tau - \tilde{\boldsymbol{\nabla}}^2$$

Momentum and velocity

- Energy & momentum are spacetime translation generators.

$$\mathrm{i}[\tilde{E}, \hat{M}] = \frac{\partial \hat{M}}{\partial \tilde{\tau}} \qquad -\mathrm{i}[\tilde{\boldsymbol{p}}, \hat{M}] = \tilde{\boldsymbol{\nabla}} \hat{M}$$

- On-shell dispersion relation:

$$\tilde{E} = \frac{m^2 + \tilde{\boldsymbol{p}}^2}{2\tilde{p}_z} = \frac{m^2 + \tilde{p}_z^2}{2\tilde{p}_z} + \frac{\tilde{\boldsymbol{p}}_\perp^2}{2\tilde{p}_z}$$

Energy-momentum

$$\tilde{E} = E$$
$$\tilde{p}_x = p_x$$
$$\tilde{p}_y = p_y$$
$$\tilde{p}_z = E + p_x = p^+$$

Velocity

$$\tilde{\boldsymbol{v}} = \boldsymbol{\nabla}_{\boldsymbol{p}} \tilde{E}$$
$$\tilde{v}_x = \tilde{p}_x / \tilde{p}_z$$
$$\tilde{v}_y = \tilde{p}_y / \tilde{p}_z$$
$$\tilde{v}_z = 1 - \tilde{E} / \tilde{p}_z$$

- **Rest** occurs when $\tilde{\boldsymbol{v}} = 0$.

Electromagnetic densities

- Physical four-current density:

$$\int \mathrm{d}\tilde{z} \langle \Psi | \hat{j}^\mu(x) | \Psi \rangle = \int \mathrm{d}^3 \tilde{\boldsymbol{R}}\, \mathscr{P}^\mu{}_\nu(\tilde{\boldsymbol{R}}, \tilde{\tau}, \Psi)\, \tilde{j}^\nu_{\text{internal}}(\boldsymbol{x}_\perp - \boldsymbol{R}_\perp)$$

Internal density

Smearing function

invariant under LF boosts

  - **Smearing function** contains all wave packet & velocity dependence.
  - Only **smearing function** modified by Lorentz boosts.
  - **Internal density** is boost-invariant. (due to Galilean subgroup)
  - **Internal density** is rest frame density!
- $\tilde{z}$ *still* must be integrated out for initial & final state to have same central charge.
  - That's why we're stuck with 2D densities.
  - But we made it clear we're dealing with ordinary space.

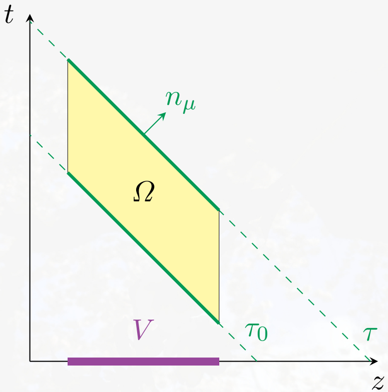
Charge density

- Charge density *at fixed* $\tilde{\tau} = t + z$.
  - Since we're using light front synchronization.
- Charge density given by:

$$\tilde{j}^0 = j^0 + j^3 = j^+$$

- Temporal part of continuity equation:

$$\tilde{\partial}_\mu \tilde{j}^\mu = \frac{\partial \tilde{j}^0}{\partial \tilde{\tau}} + \tilde{\boldsymbol{\nabla}} \cdot \tilde{\boldsymbol{j}} = 0$$

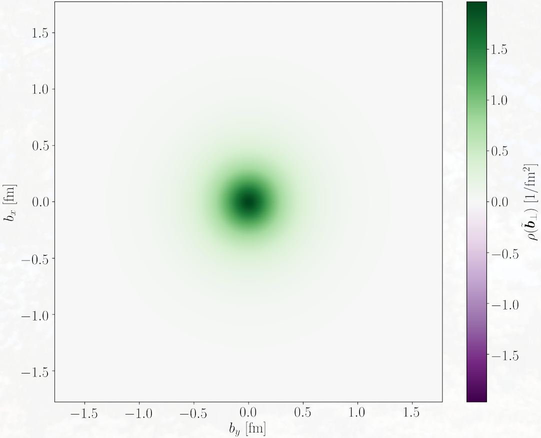


- Simple formula due to invariance under **Galilean subgroup**:

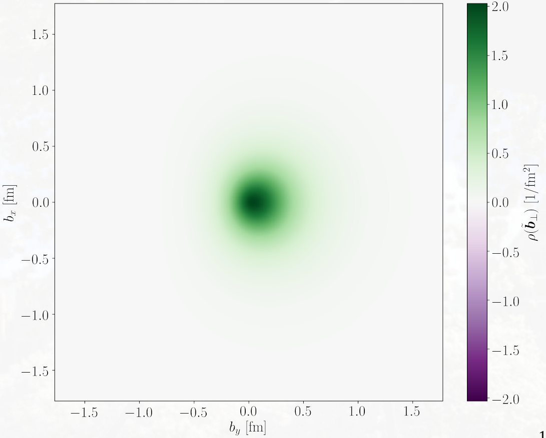
$$\tilde{j}^0_{\text{internal}}(\tilde{\boldsymbol{b}}_\perp, \hat{\boldsymbol{s}}) = \int \frac{\mathrm{d}^2 \tilde{\boldsymbol{\Delta}}_\perp}{(2\pi)^2} \frac{\langle p', \hat{\boldsymbol{s}} | \hat{j}^+(0) | p, \hat{\boldsymbol{s}} \rangle}{2p^+} \mathrm{e}^{-\mathrm{i}\tilde{\boldsymbol{\Delta}}_\perp \cdot \tilde{\boldsymbol{b}}_\perp}$$

$$\tilde{j}^0(\tilde{\boldsymbol{b}}_\perp, \hat{\boldsymbol{s}}) = \int \frac{\mathrm{d}^2 \tilde{\boldsymbol{\Delta}}_\perp}{(2\pi)^2} \left(F_1(-\tilde{\boldsymbol{\Delta}}_\perp^2) + \frac{(\hat{\boldsymbol{s}} \times \mathrm{i}\tilde{\boldsymbol{\Delta}}_\perp) \cdot \hat{z}}{2m} F_2(-\tilde{\boldsymbol{\Delta}}_\perp^2) \right) \mathrm{e}^{-\mathrm{i}\tilde{\boldsymbol{\Delta}}_\perp \cdot \tilde{\boldsymbol{b}}_\perp} \,,$$

Longitudinal polarization

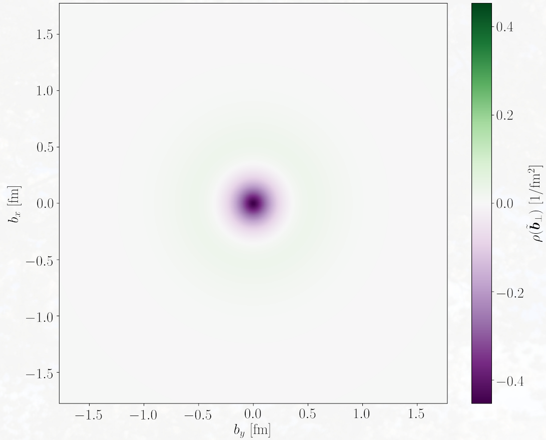


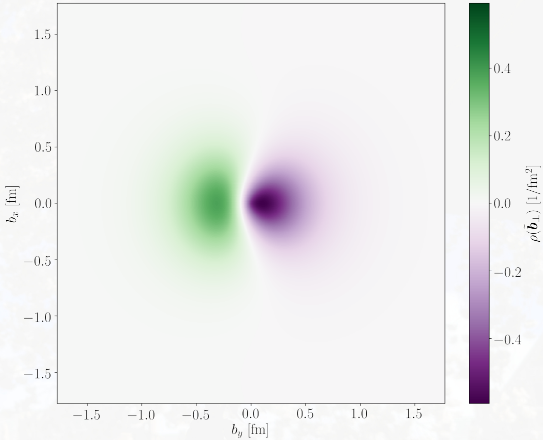
Transverse polarization



Neutron charge density

Longitudinal polarization | **Transverse polarization**

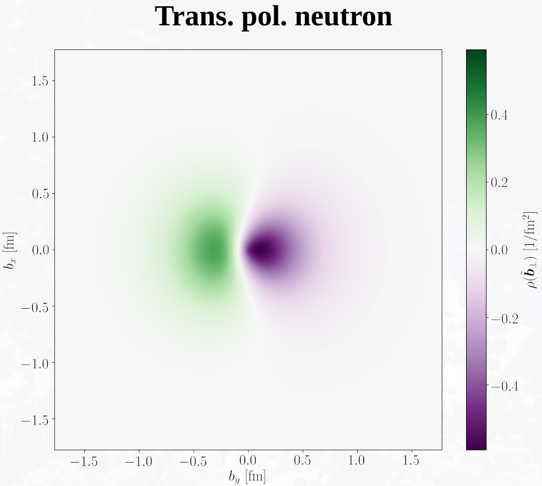




- Longitudinal polarization: negative core & diffuse positive cloud
  - Reproduces Miller, Phys. Rev. Lett. 99 (2007) 112001
- Transverse polarization: apparent electric dipole
  - Reproduces Carlson & Vanderhaegen, Phys. Rev. Lett. 100 (2008) 032004 (up to a sign)

So why modulations?

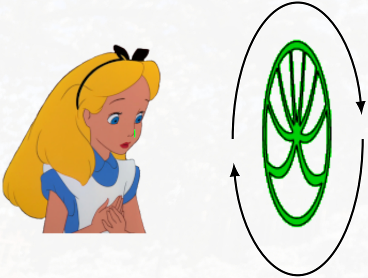
- Charge density of transpol. neutron.
  - Spin up ↑ along vertical axis.
- This is the charge density in every frame.
  - Includes the rest frame.
- Not an IMF artifact!
  - I never went to the IMF.
- Effect of **synchronization scheme**.
  - Effect of taking what we see literally.
  - This is a known effect; relativistic wheel.
  - Explained by George Gammow in 1938, *Mr Tompkins in Wonderland*



The relativistic wheel

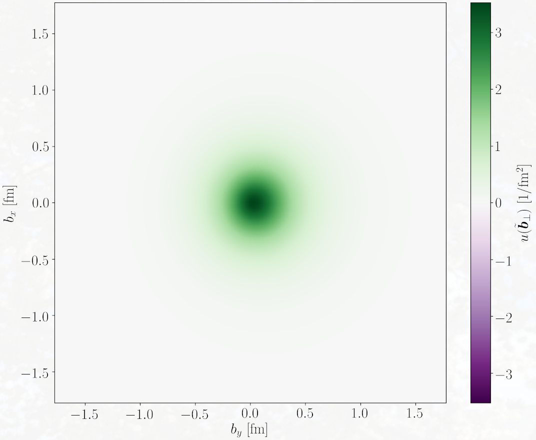
Static wheel

Spinning wheel

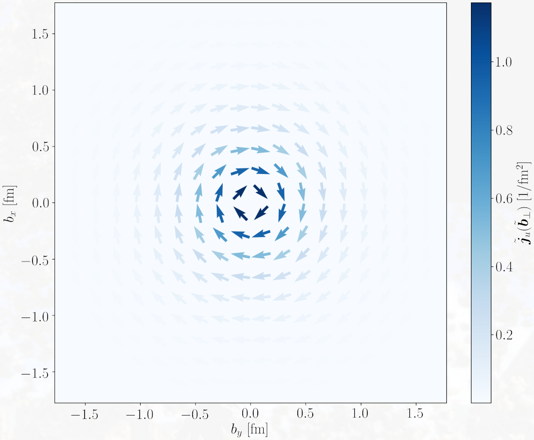


- **Static wheel** has regularly-placed spokes.
- Consider **spinning wheel**, axis oblique to observer.
  - *The wheel is considered at rest.*
- Spokes moving away are **redshifted**.
  - *Appear to* move slower.
  - Pile up; *appear to* become denser.
- Spokes moving towards are **blueshifted**.
  - *Appear to* move faster.
  - *Appear to* become rarer.
- These same distortions are present in nucleons!
- Also see videos at:
  `https://www.spacetimetravel.org/rad`
  (green wheel is relevant case)

Up quark density (from side)



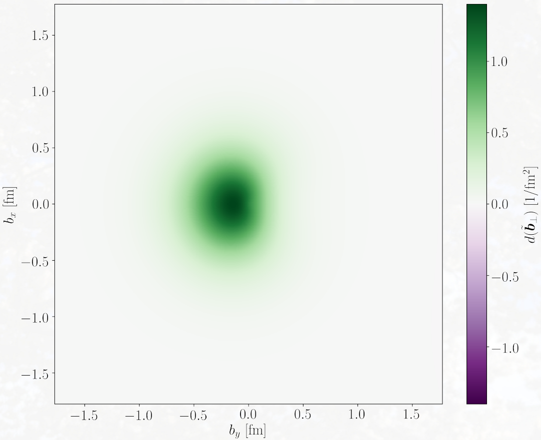
Up quark current (from below)



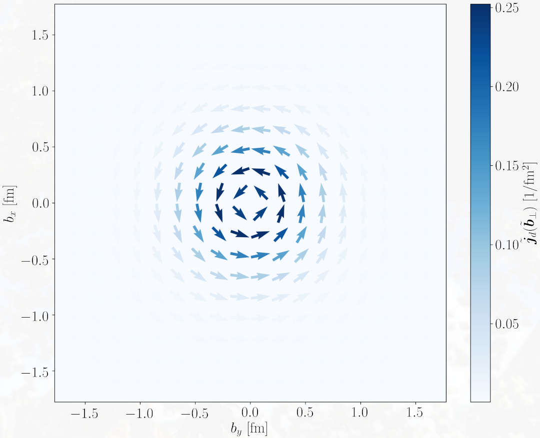
- Convert proton & neutron $\rightarrow$ up & down (**flavor separation**).
- Small distortion for up quarks, but consistent with wheel picture.

Down quark density & current in the proton

Down quark density (from side)



Down quark current (from below)



- Bigger distortion in down quarks!
- Orbit & bunching in opposite direction from up quark.

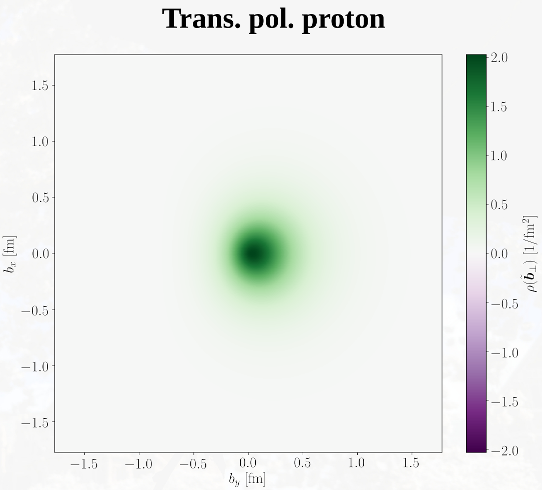
- Up quarks orbit along with proton spin.

$$\omega_u \approx \quad 0.417\ c/\mathrm{fm} = \quad 125\ \mathrm{ZHz}$$

- Down quarks orbit (much faster) against proton spin.

$$\omega_d \approx -0.922\ c/\mathrm{fm} = -276\ \mathrm{ZHz}$$

- Constructively contribute to *apparent* dipole moment.
  - In transversely polarized states.
- Would be what a viewer really sees!
  - Known effect: the relativistic wheel.



Energy density

Momentum densities

$$\tilde{T}^{\mu}{}_{\nu}(x) = \begin{bmatrix} \tilde{T}^{0}{}_{0}(x) & \tilde{T}^{0}{}_{1}(x) & \tilde{T}^{0}{}_{2}(x) & \tilde{T}^{0}{}_{3}(x) \\ \tilde{T}^{1}{}_{0}(x) & \tilde{T}^{1}{}_{1}(x) & \tilde{T}^{1}{}_{2}(x) & \tilde{T}^{1}{}_{3}(x) \\ \tilde{T}^{2}{}_{0}(x) & T^{2}{}_{1}(x) & T^{2}{}_{2}(x) & T^{2}{}_{3}(x) \\ \tilde{T}^{3}{}_{0}(x) & \tilde{T}^{3}{}_{1}(x) & \tilde{T}^{3}{}_{2}(x) & \tilde{T}^{3}{}_{3}(x) \end{bmatrix}$$

Energy fluxes

Stress tensor

- All 16 components of EMT have clear meaning in tilted coordinates.
- The **energy density** integrates to the usual "instant form" energy.

  $$\tilde{E} = E$$

  - *Relativistically exact* energy density.
  - Will give standard mass decomposition.
  - Can describe system at rest.
- Work in progress!

Acknowledgments

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APS Advancing Physics

Thank you for your time!