Numerical Study of Twist-3 Longitudinal-Transverse Double-Spin Asymmetries: a Probe of Quark-Gluon-Quark Correlations in Hadrons

Brandon Bauer
Lebanon Valley College, Annville, PA, USA

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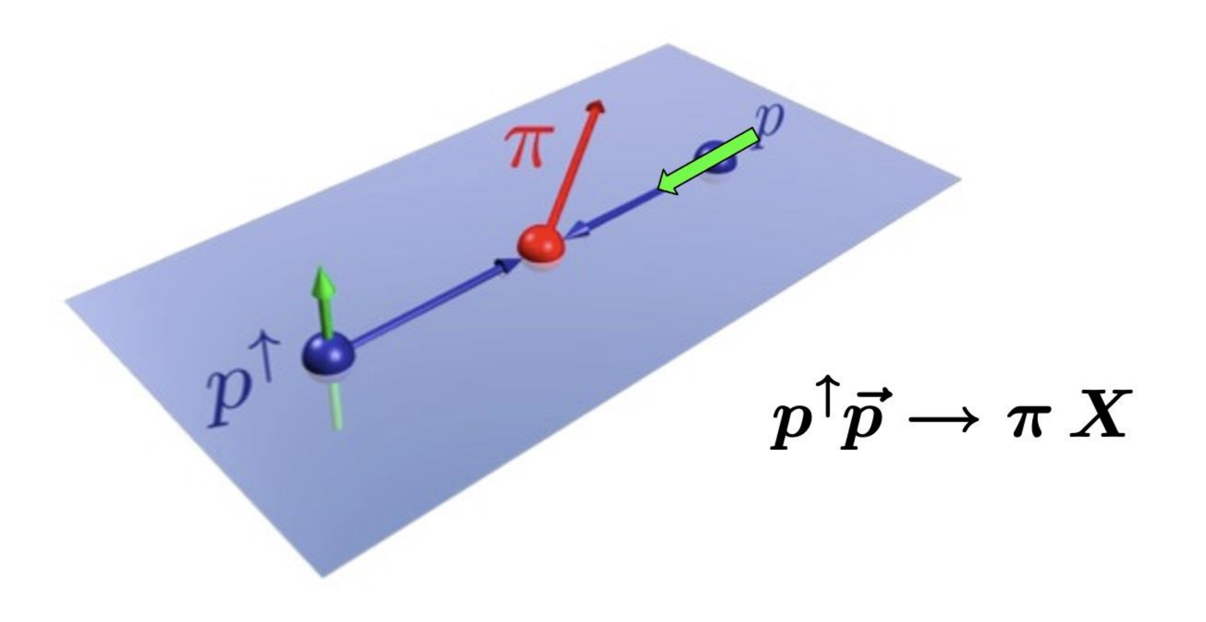
Motivation and Background

- A_{LT} - longitudinally polarized electron or proton colliding with a transversely polarized proton, with a single pion, photon, or jet detected in the final state

- Limited numerical work and only one measurement (from Jefferson Lab Hall A)

- These collisions give access to twist-3 parton distribution functions (PDFs) and fragmentation functions (FFs)

- By generating these predictions, we hope to motivate future experiments in order to gain more insight into the quark-gluon-quark interactions that occur inside of hadrons



$$A_{LT} \equiv \frac{\frac{1}{4}\left\{[d\sigma_{LT}(+,\uparrow) - d\sigma_{LT}(-,\uparrow)] - [d\sigma_{LT}(+,\downarrow) - d\sigma_{LT}(-,\downarrow)]\right\}}{d\sigma_{unp}}$$

- A_{LT} depends on the transverse momentum P_T and rapidity η of the final-state pion, photon, or jet, as well as the center-of-mass energy of the collision

Theoretical Input

- We need input for the PDFs and FFs that show up in the analytical calculation of A_{LT} :

$f_1(x)$ (unpolarized PDF) - Probability to find an unpolarized parton inside an unpolarized nucleon carrying a fraction x of the nucleon's momentum (use CT18)

$D_1(z)$ (unpolarized FF) - Probability for a parton to fragment into a hadron that carries a fraction z of the parton's momentum (use DSS14)

$h_1(x)$ (transversity PDF) - Probability to find a transversely polarized quark inside a transversely polarized nucleon carrying a fraction x of the nucleon's momentum (use JAM3D-22)

$g_1(x)$ (helicity PDF) - Probability to find a longitudinally polarized parton inside a longitudinally polarized nucleon carrying a fraction x of the nucleon's momentum (use NNPDFpol1.1)

Theoretical Input

$g_{1T}(x,k_T)$ ("worm gear" TMD PDF) - Probability to find a longitudinally polarized quark inside a transversely polarized nucleon carrying a fraction *x* of the nucleon's momentum and transverse momentum k_T (use Bhattacharya, et al. (2021) and also a Wandzura-Wilczek (WW) approx.)

$g_T(x)$ - This function does not have a simple probabilistic interpretation, but it is related to a quark-gluon-quark (qgq) correlation in a transversely polarized nucleon (use a WW approx. and qgq approx.)

$E(z)$ - This function does not have a simple probabilistic interpretation, but it is related to a quark-gluon-quark correlation in the fragmentation to an unpolarized hadron (use H~ from JAM3D-22)

Theoretical Input

More about E(z)...

$$E^{h/q}(z) = -2z\left(\int_z^\infty \frac{dz_1}{z_1^2}\frac{\hat{H}_{FU}^{\Re,h/q}(z,z_1)}{\frac{1}{z}-\frac{1}{z_1}} - \frac{m_q}{2M_h}D_1^{h/q}(z)\right)$$

$$\tilde{H}^{h/q}(z) = 2z\int_z^\infty \frac{dz_1}{z_1^2}\frac{\hat{H}_{FU}^{\Im,h/q}(z,z_1)}{\frac{1}{z}-\frac{1}{z_1}}$$

$$\sum_h \sum_{S_h} M_h \int_0^1 dz\, E^{h/q}(z) = M_j$$

$$\sum_h \sum_{S_h} M_h \int_0^1 dz\, \tilde{H}^{h/q}(z) = 0$$

Connection to dynamical quark mass generation in QCD – M_j is the mass of a "dressed" quark (Accardi, Signori (2019, 2020))

Look at 3 scenarios:
E = H~, E = 0, or E = -H~

Theoretical Input

$h_{1L}^{\perp}(x,k_T)$ ("worm gear" TMD PDF) - Probability to find a transversely polarized quark inside a longitudinally polarized nucleon carrying a fraction x of the nucleon's momentum and transverse momentum k_T (use WW approx. to write in terms of $h_1(x)$)

$h_L(x)$ - This function does not have a simple probabilistic interpretation, but it is related to a quark-gluon-quark correlation in a longitudinally polarized nucleon (use WW approx. to write in terms of $h_1(x)$)

$$h_{1L}^{\perp(1)a/N}(x) \overset{\text{WW}}{\approx} x^2 \int_x^1 dy\, \frac{h_1^{a/N}(y)}{y^2}$$

$$h_L^{a/N}(x) \overset{\text{WW}}{\approx} 2x \int_x^1 dy\, \frac{h_1^{a/N}(y)}{y^2}$$

Theoretical Input

Electron-nucleon collisions (Kanazawa, et al. (2015), Kanazawa, et al. (2016))

$$A_{LT}^{\vec{e}N^{\uparrow}\to\pi X} = \frac{\int_{z_{min}}^{1} \frac{dz}{z^3}\left(\frac{-4P_T}{S+T/z}\right)\frac{1}{x}\sum_a e_a^2\left[\frac{M}{\hat{u}}\,D_1^{\pi/a}(z)\,\mathcal{G}^{a/N}(x,\hat{s},\hat{t},\hat{u}) + \frac{M_\pi}{z\hat{t}}\,h_1^{a/N}(x)\,E^{\pi/a}(z)\left(-\frac{\hat{s}}{\hat{t}}\right)\right]}{\int_{z_{min}}^{1} \frac{dz}{z^2}\,\frac{1}{S+T/z}\,\frac{1}{x}\sum_a e_a^2\, f_1^{a/N}(x)\,D_1^{\pi/a}(z)\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right)}$$

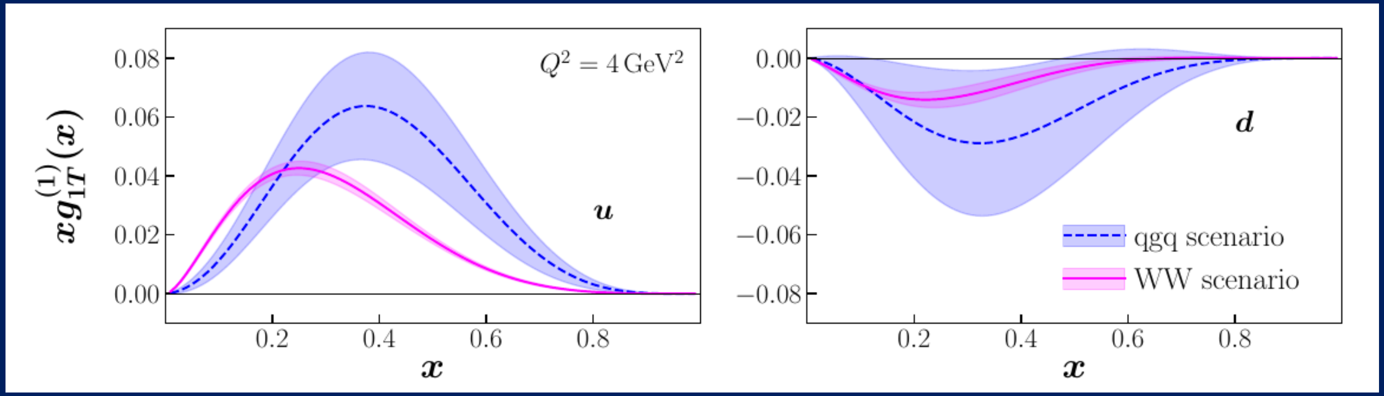
$$\mathcal{G}(x,\hat{s},\hat{t},\hat{u}) = \left(g_{1T}^{(1)}(x) - x\frac{dg_{1T}^{(1)}(x)}{dx}\right)\left(\frac{\hat{s}(\hat{s}-\hat{u})}{2\hat{t}^2}\right) + x\,g_T(x)\left(-\frac{\hat{s}\hat{u}}{\hat{t}^2}\right) + x\,g_1(x)\left(\frac{\hat{u}(\hat{s}-\hat{u})}{2\hat{t}^2}\right)$$

"hard factors" - encode the electron-quark scattering

Numerical Scenarios

Quark-Gluon-Quark (qgq) scenario:

-Use direct extraction of $g_{1T}(x,k_T)$ from Bhattacharya, et al. (2021) and the following for $g_T(x)$ (with $G_{FT} = 0$):

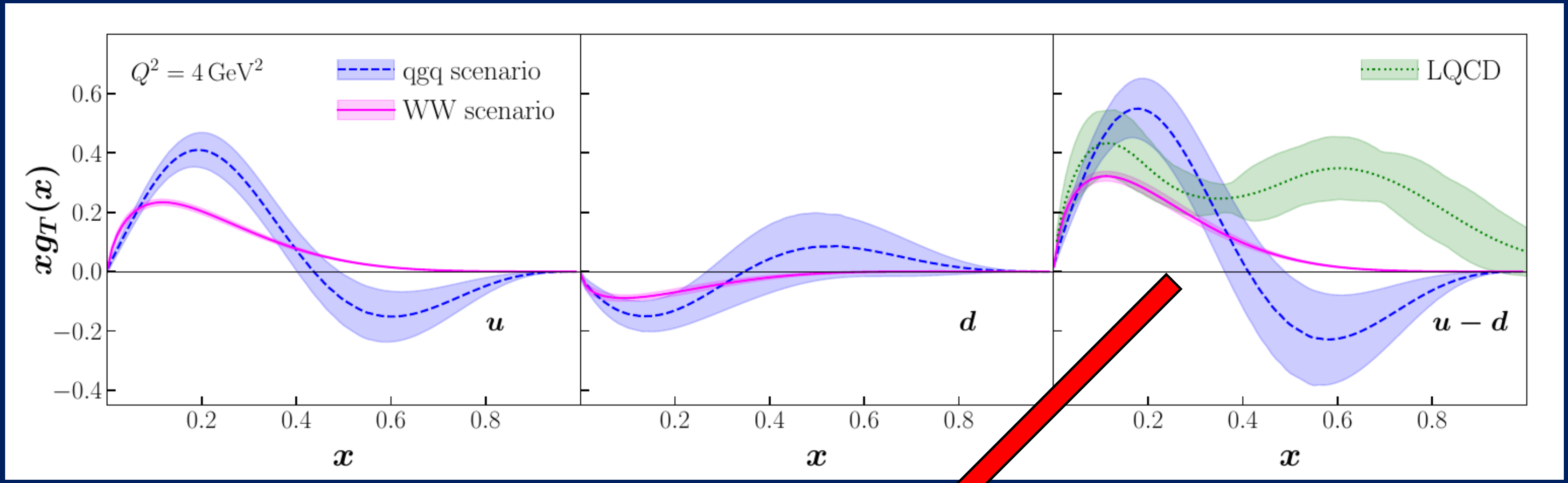


$$g_T^{q/N}(x) = g_1^{q/N}(x) + \frac{dg_{1T}^{(1)q/N}(x)}{dx} - 2\mathcal{P}\int_{-1}^{1} dy\, \frac{G_{FT}^{q/N}(x,y)}{(x-y)^2}$$

WW scenario:

$$g_{1T}^{(1)a/N}(x) \overset{\mathrm{WW}}{\approx} x\int_x^1 dy\, \frac{g_1^{a/N}(y)}{y}$$

$$g_T^{a/N}(x) \overset{\mathrm{WW}}{\approx} \int_x^1 dy\, \frac{g_1^{a/N}(y)}{y}$$



Also, E = H~, E = 0, or E = -H~ for each

Could possibly use lattice QCD data to extract information about the dynamical twist-3 function G_{FT}

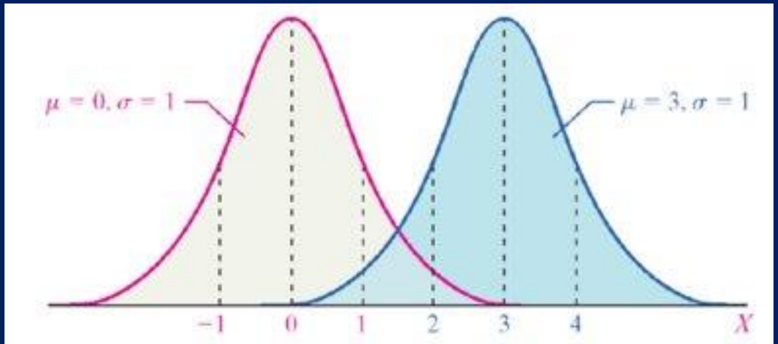
Quantifying Uncertainties via Bootstrapping

- Since A_{LT} involves several PDFs and FFs that have been extracted by different groups, it is not reasonable to calculate the full result using all replicas (e.g., 100 replicas for g_1 from NNPDF and 200 replicas for g_{1T} from Bhattacharya, et al. = 20,000 computations of A_{LT}) - need to bootstrap!

- Randomly sample a replica for each function (with replacement) and calculate A_{LT} vs. P_T. Repeat this *N* times and then *N'* times (with $N' > N$). This forms two distributions of A_{LT} values at each P_T value.

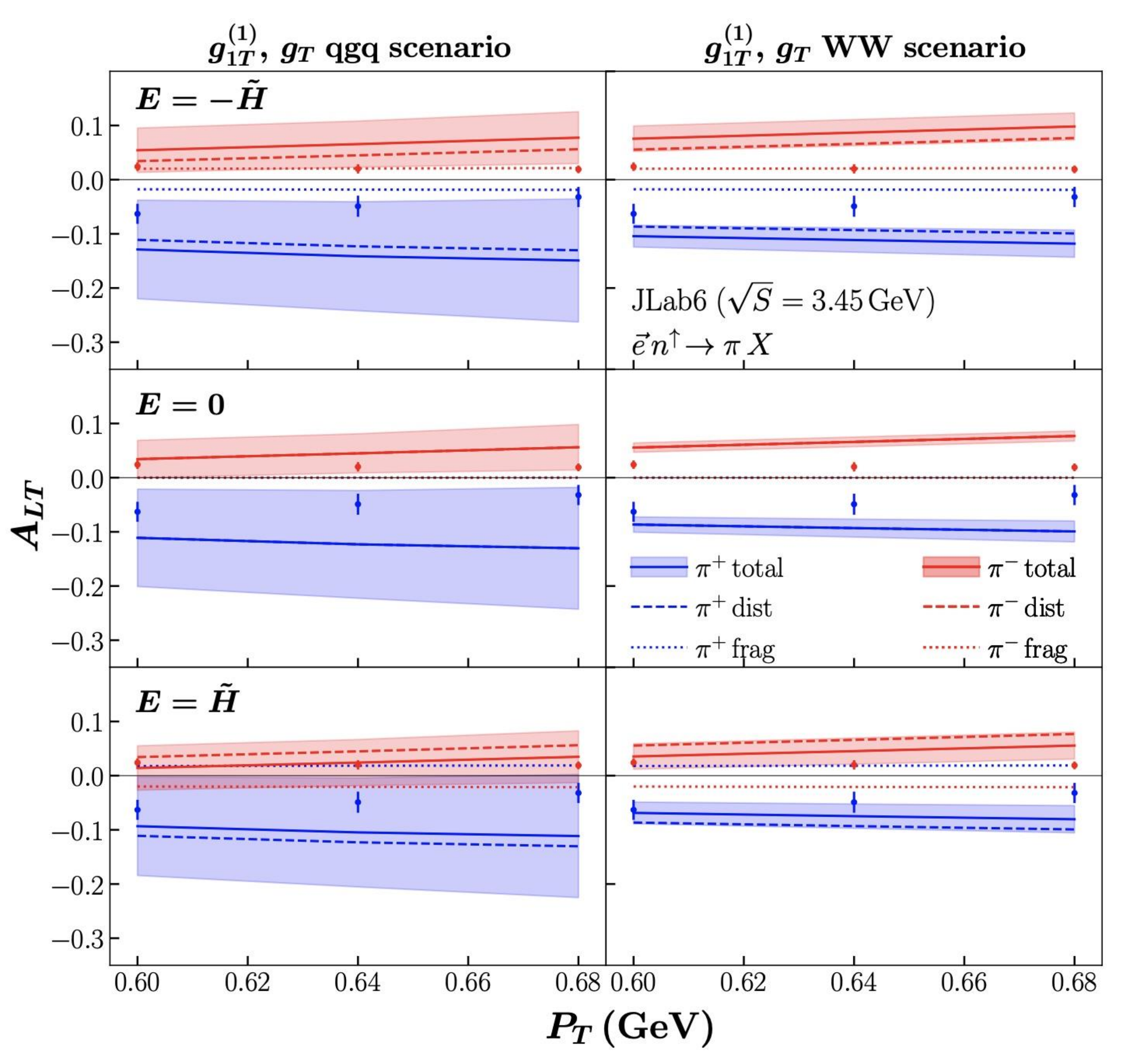
- Use Welch's t-statistic to determine when the two distributions are "equal" (p-value > 0.1) signifying convergence of the resampling

$$t = \frac{\mu_1 - \mu_2}{\sqrt{\sigma_1^2/N_1 + \sigma_2^2/N_2}}$$



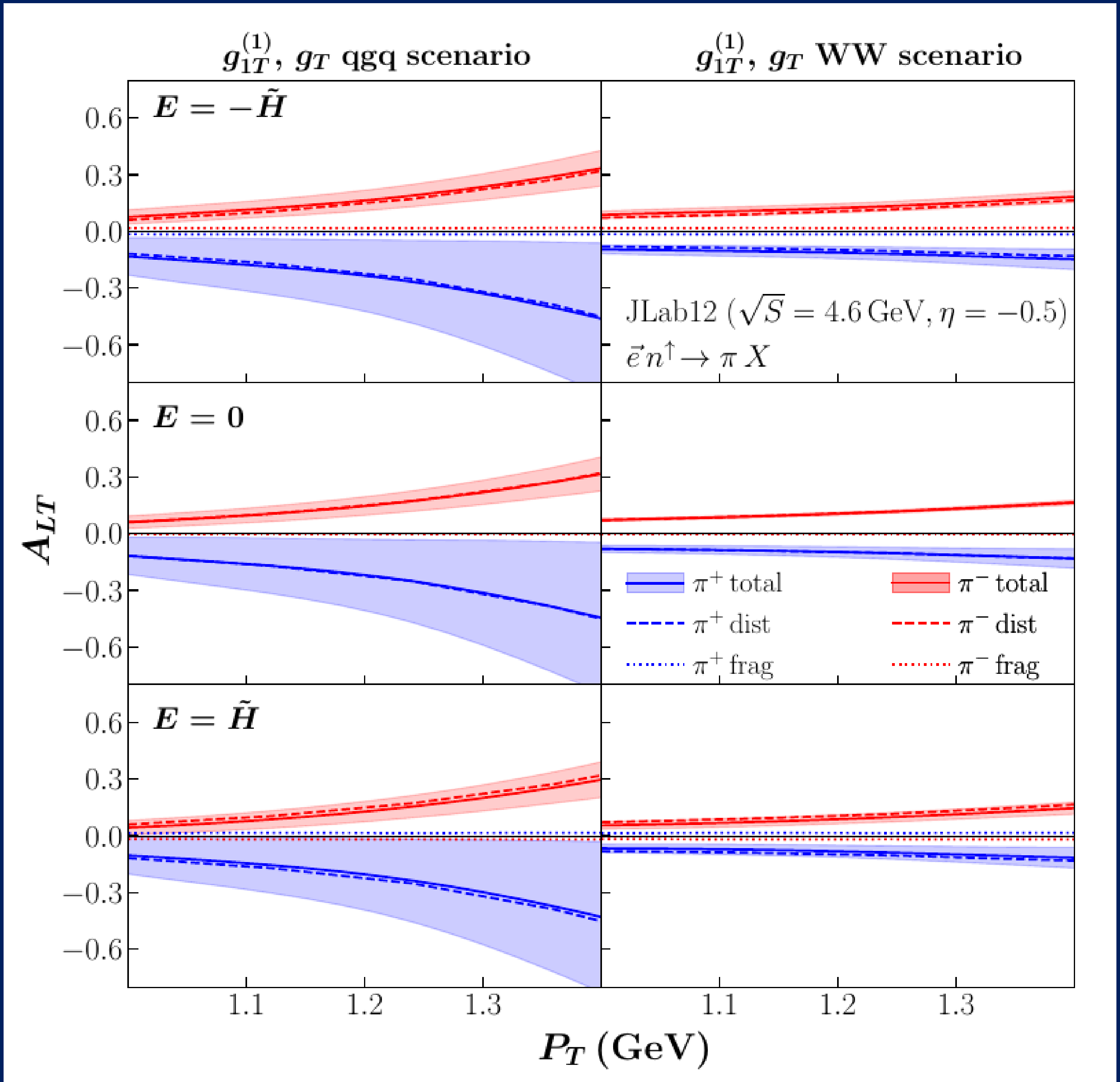
Comparison With JLab6

- Measurements of A_{LT} for the electron-neutron case from Jefferson Lab (JLab Hall A (2015))
- We are able to describe the data reasonably well with all scenarios
- Distribution term plays a dominant role over the fragmentation term



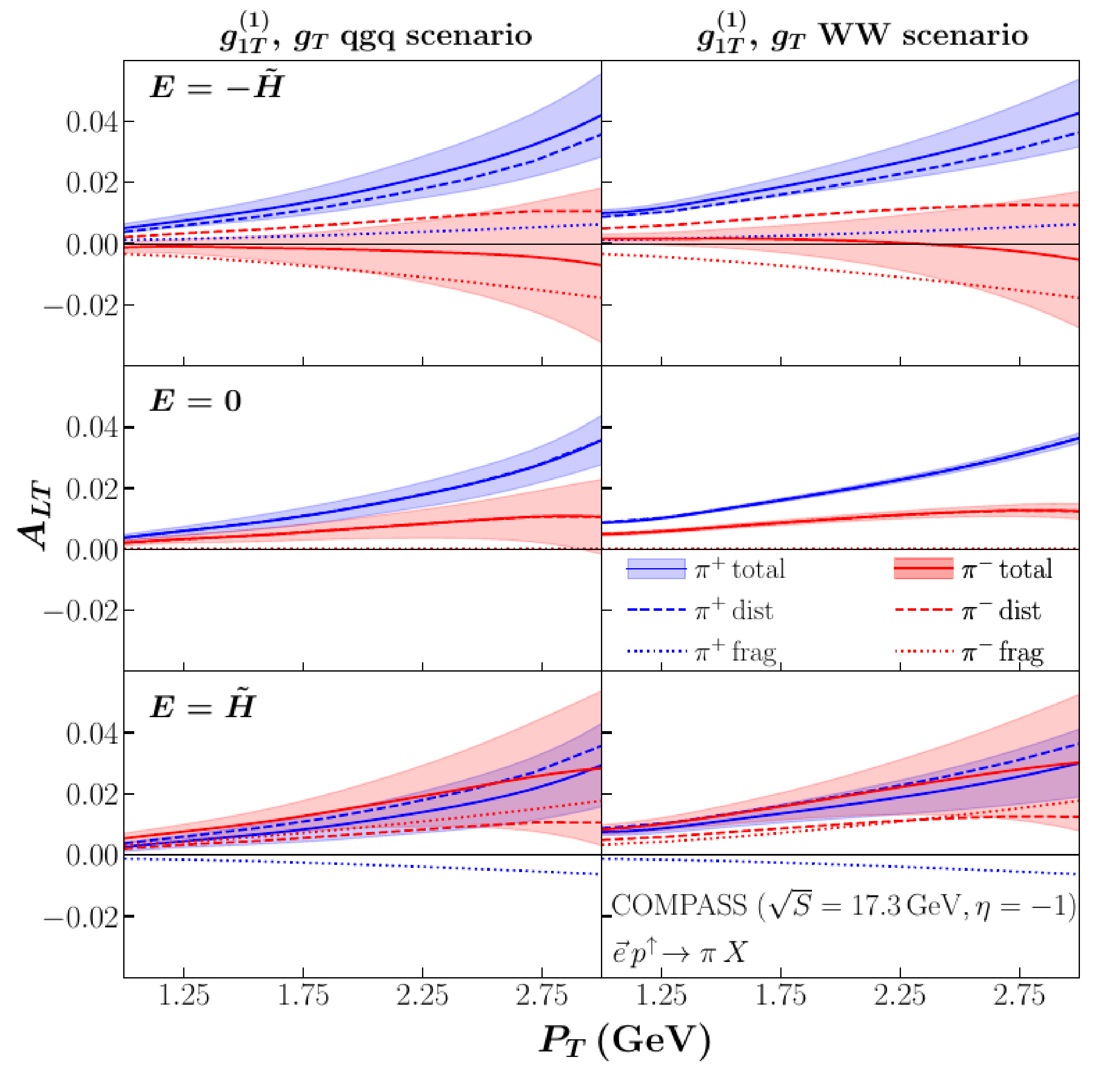
A_{LT} vs. P_T for JLab12

- Neutron target
- Asymmetries of 15-30% are predicted which grow more substantial with increasing P_T
- A_{LT} is dominated by the distribution term
- One may use JLab12 data to test the WW approximation and extract information about dynamical quark-gluon-quark correlations in the nucleon



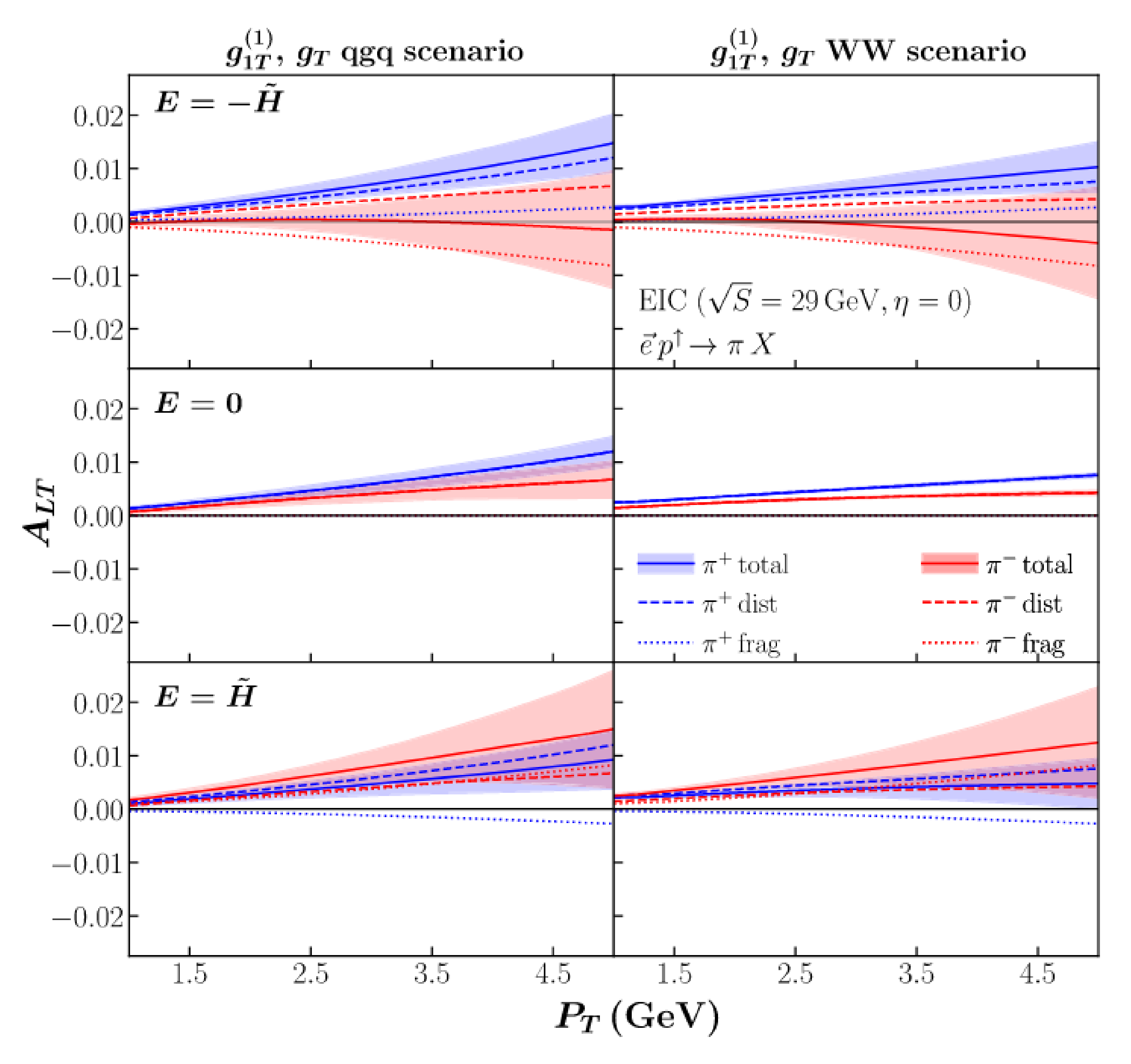
A_{LT} vs. P_T for COMPASS

- Proton target
- Compass results are measurable at ~2-4%
- A_{LT} fragmentation term can be comparable to the distribution term for π^- production
- E(z) = 0 case: A_{LT} for π^- is positive, so a measured negative asymmetry would be a likely indication of quark-gluon-quark fragmentation effects
- The qgq and WW scenarios may be difficult to distinguish at COMPASS since they give similarly-sized effects



A_{LT} vs. P_T for Low-Energy EIC

- Predictions at midrapidity show a decrease in the size of the asymmetry compared to JLab12 and COMPASS, with A_{LT} now ~0.5-1.5%
- A clearly negative signal for π^- production would be caused by E(z) (quark-gluon-quark fragmentation) - connected to dynamical quark mass generation



Theoretical Input

Proton-proton collisions

$$A_{LT}^{p^{\uparrow}\vec{p}\to\pi X} = \frac{d\sigma_{LT}^{\text{Tdist}} + d\sigma_{LT}^{\text{Ldist}} + d\sigma_{LT}^{\text{frag}}}{d\sigma_{unp}}$$

$$d\sigma_{unp} = \int_{z_{min}}^{1} dz \int_{x_{min}}^{1} \frac{dx}{x} \frac{1}{x' z^2 (xS + U/z)} \sum_{i} \sum_{a,b,c} f_1^{a/p}(x)\, f_1^{b/p}(x')\, D_1^{\pi/c}(z)\, H_U^i(\hat{s},\hat{t},\hat{u})$$

sum over all "channels" of how quarks and gluons in two protons can interact, e.g., qq --> qq, qg --> qg, etc.

"hard factors" - encode the interactions between quarks and gluons (too lengthy to explicitly write out)

Theoretical Input

Proton-proton collisions

$$A_{LT}^{p^\uparrow \vec{p} \to \pi X} = \frac{d\sigma_{LT}^{\text{Tdist}} + d\sigma_{LT}^{\text{Ldist}} + d\sigma_{LT}^{\text{frag}}}{d\sigma_{unp}}$$

Metz, et al. (2012)

$$d\sigma_{LT}^{\text{Tdist}} = -2MP_T \int_{z_{min}}^{1} dz \int_{x_{min}}^{1} \frac{dx}{x} \frac{1}{x' z^3 (xS + U/z)} \sum_i \sum_{a,b,c} \frac{1}{\hat{m}_i} \mathcal{G}_i^{a/p^\uparrow}(x, \hat{s}, \hat{t}, \hat{u})\, g_1^{b/\vec{p}}(x')\, D_1^{\pi/c}(z)$$

$$\mathcal{G}_i(x, \hat{s}, \hat{t}, \hat{u}) = \left(g_{1T}^{(1)}(x) - x \frac{d g_{1T}^{(1)}(x)}{dx} \right) H_{\tilde{g}}^i(\hat{s}, \hat{t}, \hat{u}) + x g_T(x)\, H_{1,G_{DT}}^i(\hat{s}, \hat{t}, \hat{u}) + \frac{x}{2} \left(g_1(x) - g_T(x) \right) H_{3,G_{DT}}^i(\hat{s}, \hat{t}, \hat{u})$$
$$+ \left[g_{1T}^{(1)}(x) + \mathcal{P} \int_{-1}^{1} \frac{dx_1}{x_1} \frac{x\, (F_{FT}(x, x_1) + G_{FT}(x, x_1))}{x - x_1} \right] H_{2,G_{DT}}^i(\hat{s}, \hat{t}, \hat{u})\,.$$

Neglect because no input is available

Theoretical Input

Proton-proton collisions

$$A_{LT}^{p^\uparrow \vec{p} \to \pi X} = \frac{d\sigma_{LT}^{\mathrm{Tdist}} + d\sigma_{LT}^{\mathrm{Ldist}} + d\sigma_{LT}^{\mathrm{frag}}}{d\sigma_{unp}}$$

Koike, Pitonyak, Yoshida (2016)

$$d\sigma_{LT}^{\mathrm{Ldist}} = -2MP_T \int_{z_{min}}^{1} dz \int_{x_{min}}^{1} \frac{dx}{x} \frac{1}{z^3(xS + U/z)} \sum_i \sum_{a,b,c} h_1^{a/p^\uparrow}(x)\, \mathcal{H}^{b/\vec{p}}(x', \hat{s}, \hat{t}, \hat{u})\, D_1^{\pi/c}(z)$$

$$\mathcal{H}(x', \hat{s}, \hat{t}, \hat{u}) = h_1(x')\, H_{1L}^{i}(\hat{s}, \hat{t}, \hat{u}) + h_L(x')\, H_{2L}^{i}(\hat{s}, \hat{t}, \hat{u}) + \frac{dh_{1L}^{\perp(1)}(x')}{dx'}\, H_{3L}^{i}(\hat{s}, \hat{t}, \hat{u})$$

Theoretical Input

Proton-proton collisions

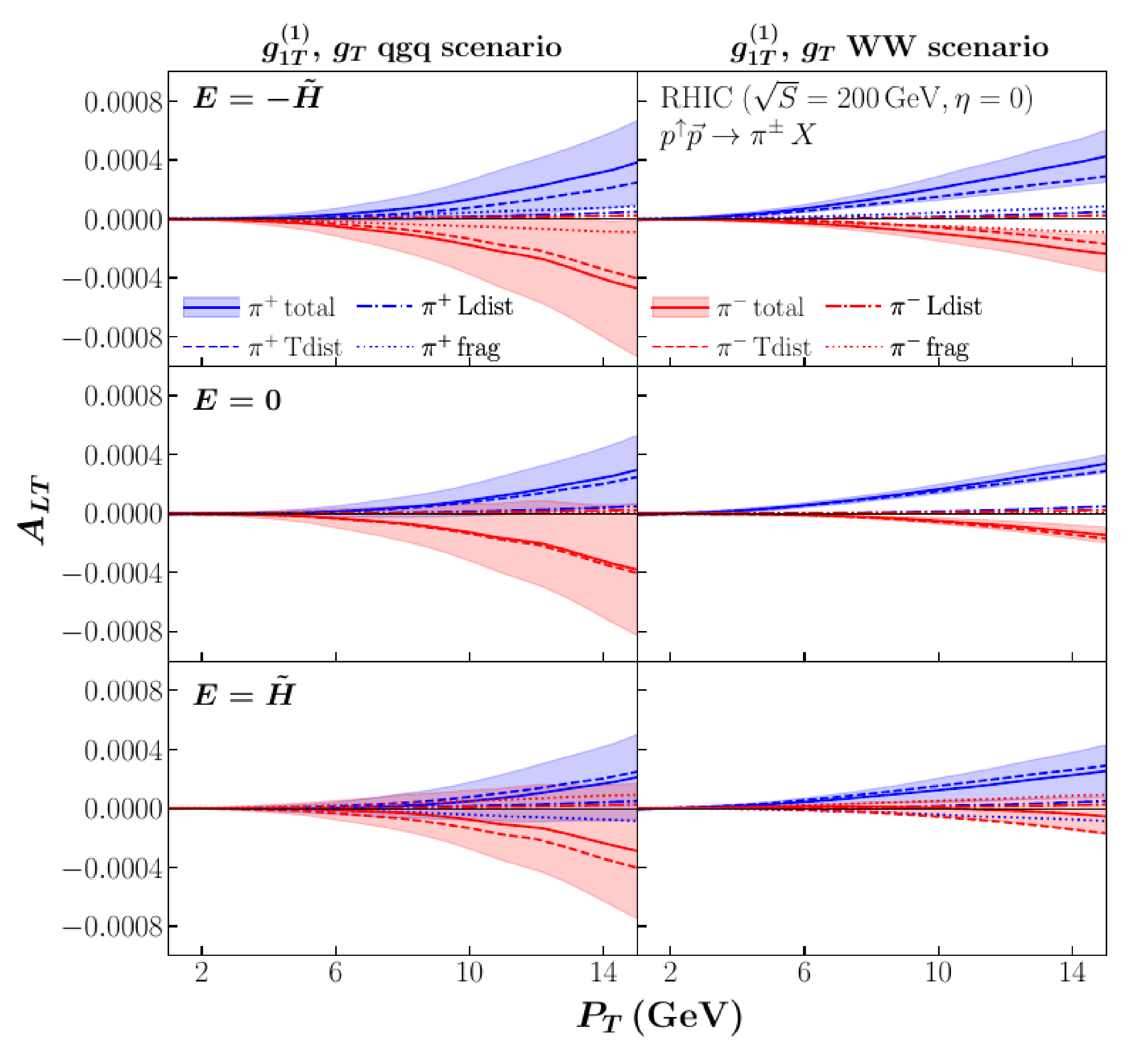
$$A_{LT}^{p^{\uparrow}\vec{p}\to\pi X} = \frac{d\sigma_{LT}^{\mathrm{Tdist}} + d\sigma_{LT}^{\mathrm{Ldist}} + d\sigma_{LT}^{\mathrm{frag}}}{d\sigma_{unp}}$$

Koike, et al. (2016)

$$d\sigma_{LT}^{\mathrm{frag}} = 2M_h P_T \int_{z_{min}}^{1} dz \int_{x_{min}}^{1} \frac{dx}{x} \frac{1}{x' z^4 (xS + U/z)} \sum_i \sum_{a,b,c} h_1^{a/p^{\uparrow}}(x)\, g_1^{b/\vec{p}}(x')\, E^{\pi/c}(z)\, H_f^i(\hat{s},\hat{t},\hat{u})$$

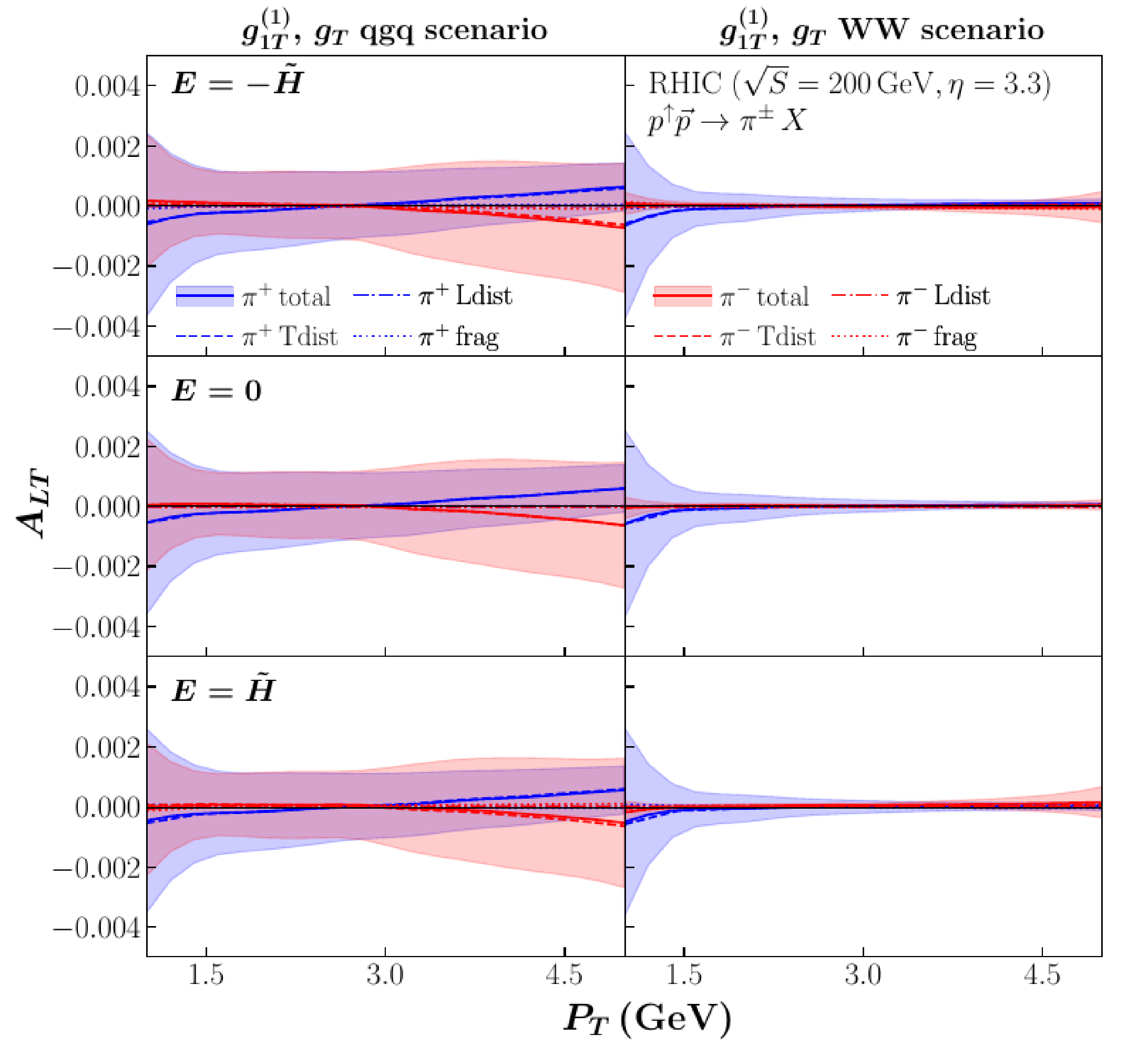
RHIC Proton-Proton A_{LT} vs. P_T

- Predictions for charged pion production at midrapidity (η= 0) reach ~0.02-0.05% for $\pi^{\pm}$ at the highest P_T
- The transverse distribution term gives the largest contribution to A_{LT}
- The fragmentation term plays a non-negligible role



RHIC Proton-Proton A_{LT} vs. P_T

- At forward rapidity (η= 3.3) the qgq scenario has larger error bands that are consistent with zero but range from ~−0.3% to +0.2%
- WW scenario uncertainties are smaller at larger P_T and again consistent with zero.
- In either case, the transverse distribution term gives the entirety of A_{LT} at forward rapidity



Conclusions and Outlook

- We found good agreement with JLab6 data, which is the only A_{LT} measurement available (for single-inclusive observables)
- Electron-nucleon collisions - the asymmetry decreases with increasing center-of-mass energy
- If significant deviations from the $E(z) = 0$ scenario are measured, it could provide direct information on $E(z)$, which is connected to dynamical quark mass generation
- One may also be able to test the validity of the Wandzura-Wilczek approximation for g_{1T}, g_T and probe dynamical twist-3 PDFs, especially with precision measurements at the EIC
- Proton-proton collisions – very small asymmetries; any measurable effect at RHIC would be direct evidence of dynamical quark-gluon-quark correlations
- We hope these predictions motivate future measurements