Properties of gluon fields at early times in relativistic heavy ion collisions

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Introduction

goal: describe early time ($\tau \leq 1$ fm) dynamics of heavy-ion collisions
- evolution of system during this early stage not well understood
- importance: initial conditions for subsequent hydro evolution

more generally:
want to understand transition btwn the early-time dynamics and hydro phase
1. microscropic theory of non-abelian gauge fields (far from equilibrium)
$\rightarrow$
2. macroscopic effective theory based on universal conservation laws
- valid close to equilibrium

for more details on our work:
MEC, Czajka, Mrówczyński:
2001.05074, 2012.03042, 2105.05327, 2112.06812, 2202.00357
MEC, Cowie, Friesen, Mrówczyński, Pickering: 2304.03241.

method - Colour Glass Condensate (CGC) effective theory

CGC = high energy density largely gluonic matter
- associated with wavefunction of a high energy hadron
- initial state in high energy hadronic collisions

after collision CGC fields are transformed into glasma fields
- initially longitudinal color electric and magnetic fields

method is based on a separation of scales between
1. valence partons with large nucleon momentum fraction (x)
2. gluon fields with small x and large occupation numbers

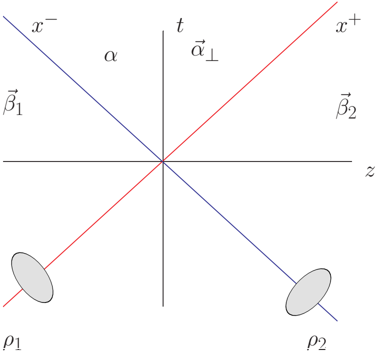
basic picture

dynamics of gluon fields determined from classical YM equation
$\rightarrow$ source provided by the valence partons

L. D. McLerran and R. Venugopalan, Phys. Rev. D, **49**, 2233 (1994); Phys. Rev. D, **49**, 3352 (1994); Phys. Rev. D, **50**, 2225 (1994).

theoretical framework

take the collision axis to be the z-axis
light-cone coordinates $x^\pm = (t \pm z)/\sqrt{2}$
Milne coordinates $\tau = \sqrt{2x^+x^-}$ and $\eta = \ln(x^+/x^-)/2$.



use YM equation in the pre-collision region

$\rho_1(x^-,\vec{x}_\perp) \to \beta_1^i(x^-,\vec{x}_\perp)$ and $\rho_2(x^+,\vec{x}_\perp) \to \beta_2^i(x^+,\vec{x}_\perp)$

boundary conditions: boost invariant initial glasma fields

$$\alpha_\perp^i(0,\vec{x}_\perp) = \alpha_\perp^{i(0)}(\vec{x}_\perp) = \lim_{w\to 0}\left(\beta_1^i(x^-,\vec{x}_\perp) + \beta_2^i(x^+,\vec{x}_\perp)\right)$$
$$\alpha(0,\vec{x}_\perp) = \alpha^{(0)}(\vec{x}_\perp) = -\frac{ig}{2}\lim_{w\to 0}\,[\beta_1^i(x^-,\vec{x}_\perp),\beta_2^i(x^+,\vec{x}_\perp)]$$

describe glasma fields (at early times) with proper time expansion ($\tau Q_s \ll 1$)

R. J. Fries, J. I. Kapusta and Y. Li, Nucl. Phys. A 774, 861 (2006).

$$\alpha(\tau,\vec{x}_\perp) \quad = \quad \alpha(0,\vec{x}_\perp) + \tau\alpha^{(1)}(\vec{x}_\perp) + \tau^2\alpha^{(2)}(\vec{x}_\perp) + \cdots$$

and similarly for $\alpha_\perp^i(\tau,\vec{x}_\perp)$ ...

coefs of expansion: require $\alpha(\tau,\vec{x}_\perp)$ and $\alpha_\perp^i(\tau,\vec{x}_\perp)$ satisfy sourceless YM eqn
$\to \alpha^{(n)}(\vec{x}_\perp)$ and $\vec{\alpha}_\perp^{(n)}(\vec{x}_\perp)$ in terms of $\alpha(0,\vec{x}_\perp)$ and $\vec{\alpha}_\perp(0,\vec{x}_\perp)$

$$\underbrace{\rho^n(x^\pm,\vec{x}_\perp)}_{\text{static valence parton sources}} \rightarrow \underbrace{\beta^n(x^\pm,\vec{x}_\perp)}_{\text{CGC pre-collision fields}} \rightarrow \underbrace{\alpha(0,\vec{x}_\perp)}_{\text{initial glasma fields (boost invariant)}} \rightarrow \underbrace{\alpha(\tau,\vec{x}_\perp)}_{\text{glasma fields}}$$

next: colour charge distributions are not known

- assume Gaussian distribution of colour charges in each nucleus

$$\langle\rho_1(x^-,\vec{x}_\perp)\rho_1(y^-,\vec{y}_\perp)\rangle \sim g^2\,\mu_1(\vec{x}_\perp)\delta(x^- - y^-)\delta^2(\vec{x}_\perp - \vec{y}_\perp)$$

$\mu(\vec{x}_\perp)$ is surface colour charge density

result for correlator of 2 potentials: ($\vec{R} = \frac{1}{2}(\vec{x}_\perp + \vec{y}_\perp)$, $\vec{r} = \vec{x}_\perp - \vec{y}_\perp$)

$$\delta_{ab}B^{ij}(\vec{x}_\perp,\vec{y}_\perp) \equiv \lim_{\mathrm{w}\to 0}\langle\beta^i_a(x^-,\vec{x}_\perp)\beta^j_b(y^-,\vec{y}_\perp)\rangle$$

$$\lim_{r\to 0} B^{ij}(\vec{x}_\perp,\vec{y}_\perp) = \delta^{ij}g^2\frac{\mu(\vec{R})}{8\pi}\left(\ln\left(\frac{Q_s^2}{m^2}+1\right) - \frac{Q_s^2}{Q_s^2+m^2}\right) + \cdots$$

infra-red regulator $m \sim \Lambda_{\mathrm{QCD}} \sim 0.2$ GeV

ultra-violet regulator = saturation scale = $Q_s = 2$ GeV

$\cdots$ *kept to 2nd order in grad expansion of* μ

J. Jalilian-Marian, A. Kovner, L. McLerran, H. Weigert, Phys. Rev. D 55, *5414 (1997); H. Fujii, K. Fukushima, Y. Hidaka, Phys. Rev. C* 79, *024909 (2009); G. Chen, R. Fries, J. Kapusta, Y. Li, Phys. Rev. C* 92, *064912 (2015).*

summary of method:

YM eqn with average over gaussian distributed valence sources
$\rightarrow$ correlators of pre-collision fields
$\rightarrow$ glasma field correlators (b. conds, sourceless YM eqn, τ exp)
$\rightarrow$ correlators of glasma chromodynamic $\vec{E}$ and $\vec{B}$ fields
$\Rightarrow$ observables

1. energy momentum tensor
   - isotropization of transverse/longitudinal pressures
   - azimuthal momentum distribution and spatial eccentricity
   - angular momentum
2. momentum broadening of hard probes

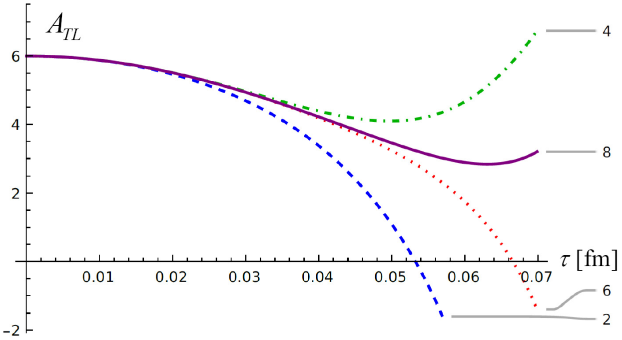
comment: many numerical approaches to study initial dynamics
our method is fully analytic
- allows control over different approximations and sources of errors
- can be systematically extended
- it has limitations (classical / no fluctuations of positions of nucleons)

compare longitudinal and transverse pressures

$$A_{TL} \equiv \frac{3(p_T - p_L)}{2p_T + p_L}$$

J. Jankowski, S. Kamata, M. Martinez and M. Spaliński, Phys. Rev. D **104**, 074012 (2021).

in equilibrium $(p_L = p_T = \mathcal{E}/3) \longrightarrow A_{TL} = 0$



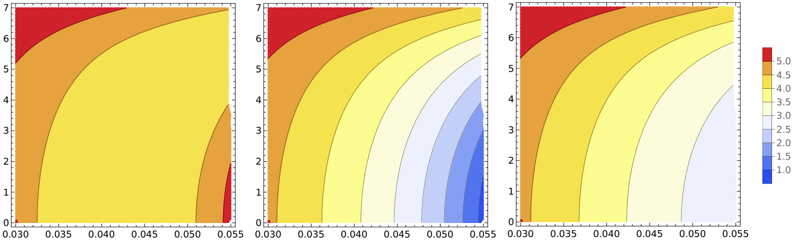


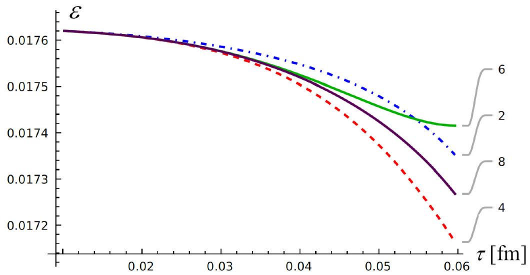
Figure: A_{TL} at fourth, sixth and eighth order. The vertical/horizontal axes are R and τ in fm.

Azimuthal asymmetry

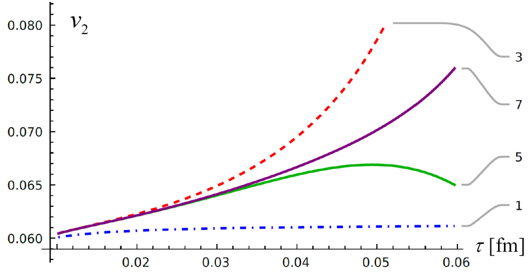
in a non-central collision - initial spatial asymmetry
relativistic collision $\to$ spatial asymmetries rapidly decrease
$\to$ anisotropic momentum flow can develop only in the first fm$/c$
• sensitive to system properties very early in its evolution
• provides direct information about the early stages of the system
spatial asymmetries from moments of the energy density
asymmetry in momentum from Fourier coefficients of the flow

$\cdots$ write in terms of components T^{00}, T^{0x} and T^{0y}

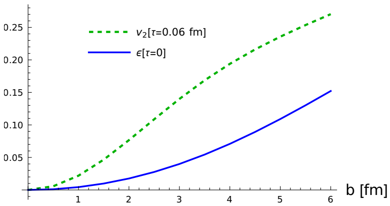
$$\varepsilon = -\frac{\int d^2R\, \frac{R_x^2-R_y^2}{\sqrt{R_x^2+R_y^2}}\, T^{00}}{\int d^2R\, \sqrt{R_x^2+R_y^2}\, T^{00}} \quad \text{and} \quad v_2 = \frac{\int d^2R\, \frac{T_{0x}^2-T_{0y}^2}{\sqrt{T_{0x}^2+T_{0y}^2}}}{\int d^2R\, \sqrt{T_{0x}^2+T_{0y}^2}}$$

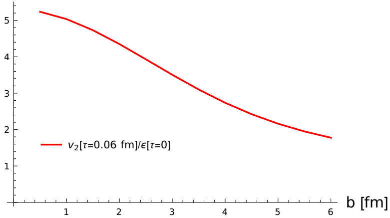


$\mathcal{E}$
0.0176
0.0175
0.0174
0.0173
0.0172
0.02
0.03
0.04
0.05
0.06
τ [fm]
6
2
8
4



v_2
0.080
0.075
0.070
0.065
0.060
0.02
0.03
0.04
0.05
0.06
τ [fm]
3
7
5
1





relative change in v_2 as b: $1 \to 6$ fm $\gg$ relative change in $v_2/\varepsilon(0)$

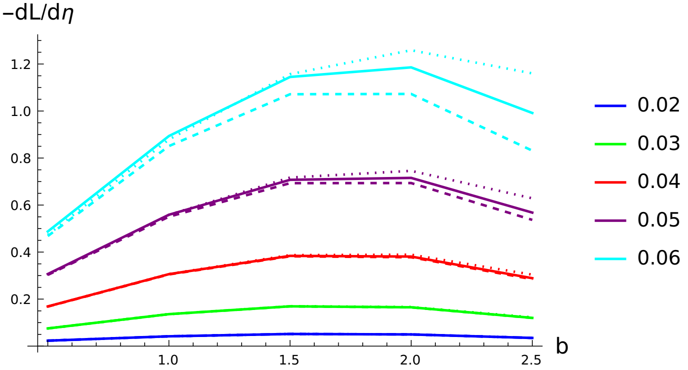
$\to$ correlation btwn spatial asymmetry from initial geometry and anisotropy of azimuthal momentum distribution

- mimics behaviour of hydrodynamics

result: angular momentum per unit rapidity

$$\frac{dL^y}{d\eta} = -\tau^2 \int d^2\vec{R}\, R^x\, T^{0z}$$

ions moving in $+/\text{-}z$ dirns displaced in $+/\text{-}$ x dirns $\rightarrow$ L_y is negative



dotted/dashed/solid lines are orders 4/6/8 comparison:

$L_y \sim 10^5$ at RHIC energies for initial system of colliding ions

- even larger at LHC energies

J. H. Gao, S. W. Chen, W. t. Deng, Z. T. Liang, Q. Wang and X. N. Wang, Phys. Rev. C 77, 044902 (2008);
F. Becattini, F. Piccinini and J. Rizzo, Phys. Rev. C 77, 024906 (2008).

idea: initial rapid rotation of glasma

$\rightarrow$ could be observed via polarization of final state hadrons

- large $\vec{L}$ & spin-orbit coupling $\rightarrow$ alignment of spins with $\vec{L}$

many experimental searches for this polarization

- effect of a few percent observed at RHIC
- at LHC result consistent with zero

- difficult to measure . . .
F. Becattini, M.A. Lisa, Ann. Rev. Nucl. Part. Sci. 70, 395 (2020).

these results supports our calculation

glasma carries only tiny imprint of the $\vec{L}$ of the intial state

$\rightarrow$ majority of the angular momentum is carried by valence quarks

hard probes - momentum broadening

hard probes produced via hard interactions at earliest phase of HIC

- propagate through the evolving medium

- suppression of high-p_T probes (jet quenching)

$\Rightarrow$ signal of formation of QGP

- deconfined state of matter = significant braking of hard partons

physics: frequent small $\vec{p}$ exchanges btwn probe and glasma fields
$\rightarrow$ transport equation in Fokker-Planck form

- describes interactions of hard probe interacting with glasma fields

$$\hat{q} = \frac{1}{v}\left(\delta^{\alpha\beta} - \frac{v^\alpha v^\beta}{v^2}\right)\frac{\langle \Delta p^\alpha \Delta p^\beta \rangle}{\Delta t}$$

$$= \frac{2}{v}\left(\delta^{\alpha\beta} - \frac{v^\alpha v^\beta}{v^2}\right) X^{\alpha\beta}(\vec{v})$$

$$X^{\alpha\beta}(\vec{v}) \equiv \frac{1}{2N_c}\int_0^t dt' \, \mathrm{Tr}\left[\left\langle \mathcal{F}^\alpha(t,\vec{x})\mathcal{F}^\beta\left(t-t',\vec{x}-\vec{v}t'\right)\right\rangle\right]$$

colour Lorentz force: $\vec{\mathcal{F}}(t,\mathrm{x}) \equiv g\left(\vec{E}(t,\vec{x}) + \vec{v}\times\vec{B}(t,\vec{x})\right)$

notation: $\alpha \in (1,2,3)$
$\vec{v}$ is the velocity of the probe

note: combination of two approaches

1. medium that the hard probe interacts with is a glasma
$\rightarrow$ described with CGC effective theory with proper time expansion
*** description is valid only at very early times*

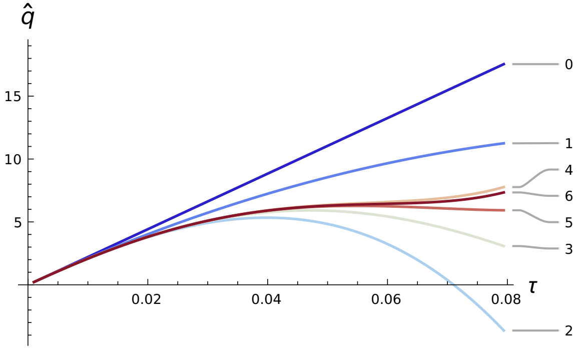
2. FP eqn describes interactions of hard probe with glasma fields
*** valid at times long enough that collision terms saturate*

$\Rightarrow$ conflict btwn assumptions that set these two time scales

also:
- FP description requires gradient expansion type approximations
- our CGC approach assumes boost invariance

** can all these conditions can be satisfied simultaneously?

result: $\hat{q}$ as a function of τ at different orders in the expansion



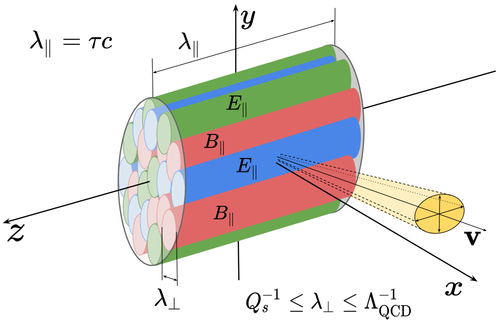
key: saturation regime appears before τ expansion breaks down

caution:

figure above obtained for $v_\perp = v$

method works less well if $v_\parallel$ large (experimentally less interesting)

reason: at very early times
glasma fields represented as longitudinal flux tubes



$\hat{q}$ built up during time probe is in domain of correlated fields
at zeroth order this time is determined by
- transverse correlation length (inferred from 2-point correlator)
- orientation and magnitude of the probe's velocity

$\rightarrow$ saturation is faster if $v_\parallel = 0$

note: probe's velocity also enters through the Lorentz force

impact of the glasma on jet quenching

radiative Eloss/length of probe traversing medium of length L
$\propto$ total accumulated transverse momentum broadening

$$\Delta p_T^2 = \int_0^L dt \hat{q}(t)$$

our calculation gives $\hat{q}_{\rm max} = 6\ \mathrm{GeV}^2/\mathrm{fm}$ at $t_{\rm max} = 0.06$ fm
typical equilibrium values are much smaller
- but the glasma exists for a very short time ...
$\rightarrow$ contro of pre-equilibrium phase to jet quenching usually ignored

an estimate:
$\hat{q}$ decreases from $\hat{q}_{\rm max}$ until hydrodynamic evolution takes over
A. Ipp, D. I. Müller and D. Schuh, Phys. Lett. B 810, 135810 (2020)
assume hydro evolution from $t_0 = 0.6$ fm, T_0=0.45 GeV and $\hat{q}_0 = 1.4\mathrm{GeV}^2/\mathrm{fm}$

$$\frac{\Delta p_T^2[\text{non-equib}]}{\Delta p_T^2[\text{equib}]} \approx 0.93$$

$\Rightarrow$ glasma plays an important role in jet quenching

conclusions

1. 8th order τ expansion can be trusted to $\tau \approx 0.07$ fm
2. glasma moves towards equilibrium
3. correlation btwn elliptic flow coef v_2 / spatial eccentricity
   - spatial asymmetry introduced by initial geometry is effectively transmitted to azimuthal distribution of gluon momentum field

   $\rightsquigarrow$ this behaviour mimics hydrodynamics
4. most of the angular momentum of the intial system not transmitted to glasma
   - contradicts picture of a rapidly rotating initial glasma state
5. glasma plays an important role in jet quenching