# CGC for ultra-peripheral $\mathrm{Pb}+\mathrm{Pb}$ collisions at the Large Hadron Collider 

Haowu Duan<br>North Carolina State University<br>Based on JHEP 12 (2022) 077, with Alex Kovner and Vladi Skokov

10th GHP Workshop, 2023

This work is supported by DOE
Special thanks to The Gordon and Betty Moore Foundation and APS

## NC STATE <br> UNIVERSITY

## What's new?

Two particle angular correlation observed in UPC measurement at LHC

(a) PHYSICAL REVIEW C 104, 014903 (2021), ATLAS

(b) Backgrounds \& signals

## Ridge correlation with different system size



Figure: Fig from Schlichting, Tribedy (2016)

## NC STATE UNIVERSITY

## Elliptic flow


(a) Peripheral collision for AA

(b) $v_{2} \rightarrow$ viscosity

Small viscosity $\eta / s$ leads to higher $v_{2}$. ( Figures from Raimond Snellings (2011) )

$$
\frac{d N}{d q_{1}^{2} d q_{2}^{2}} \propto 1+\sum_{n} 2 v_{n}^{2} \cos (n \Delta \theta)
$$

## Ridge correlation in small systems ?

- If ridge correlation indicates fluid behavior, what is the smallest collision system to create QGP?
- High multiplicity p+p (2010), p+Pb (2012) at LHC
- $p+A u, d+A u,{ }^{3} \mathrm{He}+\mathrm{Au}$ at RHIC (2013-2020)
- Is there additional origin of the angular correlation?
- Opportunities to probe novel effects
- The smallest projectile is DIS photon!


## Ultra-peripheral collisions



- $b>R_{A}+R_{B}$
- Equivalent photon approximation
- Weizsäcker-Williams field
- $Q^{2} \lesssim(60 M e v)^{2}$ for $\mathrm{A}=16$



## NC STATE <br> UNIVERSITY

## Origins of the angular correlation in UPC

- Hydrodynamic

Collectivity in Ultra-Peripheral $\mathrm{Pb}+\mathrm{Pb}$ Collisions at the Large Hadron Collider

Wenbin Zhao, ${ }^{1}$ Chun Shen, ${ }^{1,2}$ and Björn Schenke ${ }^{3}$<br>${ }^{1}$ Department of Physics and Astronomy, Wayne State University, Detroit, Michigan 48201, USA<br>${ }^{2}$ RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA<br>${ }^{3}$ Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA

- Color domain effect in the target

Exploring the Collective Phenomenon at the Electron-Ion Collider
Yu Shi, ${ }^{1}$ Lei Wang, ${ }^{1}$ Shu-Yi Wei, ${ }^{2, *}$ Bo-Wen Xiao, ${ }^{3,}{ }^{\dagger}$ and Liang Zheng ${ }^{4, \ddagger}$
${ }^{1}$ Key Laboratory of Quark and Lepton Physics (MOE) and Institute of Particle Physics, Central China Normal University, Wuhan 430079, China
${ }^{2}$ European Centre for Theoretical Studies in Nuclear Physics and Related Areas $\left(E C T^{*}\right)$ and Fondazione Bruno Kessler, Strada delle Tabarelle 286, I-38123 Villazzano (TN), Italy
${ }^{3}$ School of Science and Engineering, The Chinese University of Hong Kong, Shenzhen 518172, China
${ }^{4}$ School of Mathematics and Physics, China University of Geosciences (Wuhan), Wuhan 430074, China

- Quantum correlations (explored in our work)
- Bose-Einstein correlation
- HBT(Hanbury Brown and Twiss) effect
- Dominated by the correlations in projectile


## Bose enhancement

Two particle correlator in a free boson gas,

$$
D(\boldsymbol{x}, \boldsymbol{y})=\int_{\boldsymbol{p}, \boldsymbol{p}^{\prime}, \boldsymbol{q}, \boldsymbol{q}^{\prime}} e^{-i \boldsymbol{x} \cdot\left(\boldsymbol{p}^{\prime}-\boldsymbol{p}\right)} e^{-i \boldsymbol{y} \cdot\left(\boldsymbol{q}^{\prime}-\boldsymbol{q}\right)}\left\langle\hat{\alpha}_{a}^{\dagger}(\boldsymbol{p}) \hat{a}_{b}^{\dagger}(\boldsymbol{q}) \hat{a}_{a}\left(\boldsymbol{p}^{\prime}\right) \hat{a}_{b}\left(\boldsymbol{q}^{\prime}\right)\right\rangle
$$

There are three different scenarios

- $\boldsymbol{p}=\boldsymbol{p}^{\prime}, \boldsymbol{q}=\boldsymbol{q}^{\prime}:\left\langle\hat{a}_{a}^{\dagger}(\boldsymbol{p}) \hat{a}_{b}^{\dagger}(\boldsymbol{q}) \hat{a}_{a}\left(\boldsymbol{p}^{\prime}\right) \hat{a}_{b}\left(\boldsymbol{q}^{\prime}\right)\right\rangle$, uncorrelated, $\mathcal{O}(1)$
- $\boldsymbol{p}=\boldsymbol{q}^{\prime}, \boldsymbol{q}=\boldsymbol{p}^{\prime}:\left\langle\hat{a}_{a}^{\dagger}(\boldsymbol{p}) \hat{a}_{b}^{\dagger}(\boldsymbol{q}) \hat{a}_{a}\left(\boldsymbol{p}^{\prime}\right) \hat{a}_{b}\left(\boldsymbol{q}^{\prime}\right)\right\rangle, \mathcal{O}\left(\frac{1}{N_{c}^{2}}\right)$
- $\boldsymbol{p}=\boldsymbol{q}^{\prime}=\boldsymbol{q}=\boldsymbol{p}^{\prime}$, suppressed by $\frac{1}{N_{c}^{2}}$ and $\frac{1}{V}$


## NC STATE <br> UNIVERSITY

## HBT

$$
\begin{aligned}
D_{\mathrm{HBT}}\left(\boldsymbol{k}_{1}, \boldsymbol{k}_{2}\right)= & \sum_{a, b} \int_{\boldsymbol{x}_{0}, \boldsymbol{x}_{0}^{\prime}, \boldsymbol{y}_{0}, \boldsymbol{y}_{0}^{\prime}} \int_{\boldsymbol{x}_{1}, \boldsymbol{x}_{2}^{\prime}, \boldsymbol{y}_{3}, \boldsymbol{y}_{4}^{\prime}} e^{i \boldsymbol{k}_{1} \cdot\left(\boldsymbol{x}_{0}^{\prime}-\boldsymbol{x}_{0}\right)} e^{i \boldsymbol{k}_{2} \cdot\left(\boldsymbol{y}_{0}^{\prime}-\boldsymbol{y}_{0}\right)} \\
& \times\left\langle\hat{a}_{a}^{\dagger}\left(\boldsymbol{x}_{0}\right) \hat{a}_{b}^{\dagger}\left(\boldsymbol{y}_{0}\right) \hat{a}_{a}\left(\boldsymbol{x}_{0}^{\prime}\right) \hat{a}_{b}\left(\boldsymbol{y}_{0}^{\prime}\right)\right\rangle \\
& \times G\left(\boldsymbol{x}_{0}-\boldsymbol{x}_{1}\right) G\left(\boldsymbol{y}_{0}-\boldsymbol{y}_{1}\right) G\left(\boldsymbol{x}_{0}^{\prime}-\boldsymbol{x}_{1}^{\prime}\right) G\left(\boldsymbol{y}_{0}^{\prime}-\boldsymbol{y}_{1}^{\prime}\right) \\
& \times\left\langle J_{a}\left(\boldsymbol{x}_{1}\right) J_{b}\left(\boldsymbol{y}_{1}\right) J_{a}\left(\boldsymbol{x}_{1}^{\prime}\right) J_{b}\left(\boldsymbol{y}_{1}^{\prime}\right)\right\rangle
\end{aligned}
$$

- The "wrong" contraction is enforced by the ensemble average of the source correlator

$$
\left\langle J_{a}\left(\boldsymbol{x}_{1}\right) J_{b}\left(\boldsymbol{y}_{1}\right) J_{a}\left(\boldsymbol{x}_{1}^{\prime}\right) J_{b}\left(\boldsymbol{y}_{1}^{\prime}\right)\right\rangle
$$

## NC STATE <br> UNIVERSITY

## Dipole model $\left(|Q|<\Lambda_{Q C D}\right)$



- Dipole model to approximate the photon Small $Q^{2}$ suppresses the longitudinal polarization

$$
\Psi_{\lambda}^{T}\left(z, \boldsymbol{r}, s_{1}\right)=-i \frac{2 e e_{f}}{2 \pi} \delta_{s_{1},-s_{2}}\left(2 z-1+2 \lambda s_{1}\right) \sqrt{z(1-z)} \frac{\boldsymbol{r} \cdot \boldsymbol{\epsilon}_{\boldsymbol{\lambda}}}{|r|} \varepsilon_{f} K_{1}\left(\varepsilon_{f}|r|\right)
$$

Note: UPC photon is actually linearly polarized (Small correction to the correlation).

## NC STATE <br> UNIVERSITY

## MV model



- Inspired by Vector Meson Dominance Model
- Due to the existence of the high energy fixed point, $\rho$-meson at asymptotically high energy $\equiv$ nucleus
- Valence degrees of freedom $\rho_{a}(\boldsymbol{x})$ follow the distribution defined by McLerran-Venugopalan (MV) model

$$
W\left(\rho_{a}\right)=\exp \left\{-\int_{\boldsymbol{x}} \frac{\rho_{a}(\boldsymbol{x}) \rho_{a}(\boldsymbol{x})}{2 \mu^{2}}\right\}
$$

## Organize the cross section

Organize the cross section $\Sigma$ according to the order of $\rho$

$$
\Sigma=\Sigma_{2}+\Sigma_{3}+\Sigma_{4}
$$


(a) $\Sigma_{2}\left(\rho^{2}\right)$

(b) $\Sigma_{3}\left(\rho^{3}\right)$

(c) $\Sigma_{4}\left(\rho^{4}\right)$

## NC STATE

UNIVERSITY

## How cross section calculated

Use $\Sigma_{2}$ as example, in coordinate space,

$$
\begin{aligned}
\Sigma_{2} & =4 \int d^{2} \boldsymbol{x} \int d^{2} \overline{\boldsymbol{x}} f^{i}\left(\bar{u}_{1}-\boldsymbol{x}\right) f^{i}\left(u_{1}-\overline{\boldsymbol{x}}\right) f^{j}\left(\bar{u}_{2}-\bar{u}_{1}\right) f^{j}\left(u_{2}-u_{1}\right)\left\langle\rho_{d^{\prime}}(\overline{\boldsymbol{x}}) \rho_{d}(\boldsymbol{x})\right\rangle_{P} \\
& \left\langle\left[\left[U^{\dagger}\left(u_{1}\right) T^{a} U\left(u_{1}\right)\right]\left[U^{\dagger}\left(u_{2}\right)-U^{\dagger}\left(u_{1}\right)\right]\left[U\left(\bar{u}_{2}\right)-U\left(\bar{u}_{1}\right)\right]\left[U^{\dagger}\left(\bar{u}_{1}\right) T^{a} U\left(\bar{u}_{1}\right)\right]\right]_{d^{\prime} d}\right\rangle_{T}
\end{aligned}
$$

where $f^{i}(\boldsymbol{x})=\frac{g}{(2 \pi)^{2}} \frac{x_{i}}{x^{2}}$.

- Kinematic factors (Eikonal emission vertices)
- Projectile (photon)
- Target (nucleus)


## NC STATE <br> UNIVERSITY

## Expectation values for projectile and target

## Symmetrization(isolating the signal)

- Symmetrization of $\hat{\rho} s$ (MV model)

$$
\begin{aligned}
\hat{\rho}_{a}(\boldsymbol{x}) \hat{\rho}_{b}(\boldsymbol{y}) & =\frac{1}{2}\left\{\hat{\rho}_{a}(\boldsymbol{x}), \hat{\rho}_{b}(\boldsymbol{y})\right\}+\frac{1}{2}\left[\hat{\rho}_{a}(\boldsymbol{x}), \hat{\rho}_{b}(\boldsymbol{y})\right] \\
& =\rho_{a}(\boldsymbol{x}) \rho_{b}(\boldsymbol{y})-\frac{1}{2} \delta^{(2)}(x-y) T_{a b}^{c} \rho_{c}(\boldsymbol{x})
\end{aligned}
$$

- Symmetrization of color factors (Dipole model)

$$
t^{a} t^{b}=\frac{1}{2}\left\{t^{a}, t^{b}\right\}+\frac{1}{2} i f_{a b}^{c} t^{c}
$$



NC STATE Figure: The color structure for the correction term UNIVERSITY

## Target average

- Factorized Dipole Approximation

Phys. Rev. D 96, 074018, Kovner, Rezaeian

- Dense target $\rightarrow$ Saturated
$\gg \frac{1}{Q_{s}} \quad \frac{1}{Q_{s}}$ serves the role of correlation length in transverse plane
- For the example configuration

$$
\begin{aligned}
& \operatorname{Tr}\left[U\left(x_{1}\right) U^{\dagger}\left(x_{2}\right) U\left(x_{3}\right) U^{\dagger}\left(x_{4}\right)\right] \\
& \approx \\
& \frac{1}{N_{c}^{2}-1} \operatorname{Tr}\left[U\left(x_{1}\right) U^{\dagger}\left(x_{2}\right)\right] \operatorname{Tr}\left[U\left(x_{3}\right) U^{\dagger}\left(x_{4}\right)\right]+
\end{aligned}
$$

## NC STATE <br> UNIVERSITY

## Angular correlation from the cross section

From the cross section of the two gluon production

$$
\Sigma=\frac{d \mathcal{N}}{d \eta d q_{1}^{2} d \xi d q_{2}^{2}}
$$

one can extract the angular correlation function

$$
C(q, \theta)=\frac{\Sigma(q, \theta)}{\frac{1}{2 \pi} \int_{0}^{2 \pi} \Sigma(q, \theta) d \theta}
$$

set $\left|q_{1}\right|=\left|q_{2}\right|=q$, and $\theta$ is the angle between the two particles

## NC STATE <br> UNIVERSITY

## $\Sigma_{2}, q=Q_{s}$



(a) Dipole

(b) MV

As expected, a strong back-to-back correlation.

## $\Sigma_{3}, q=Q_{s}$



(a) Dipole

(b) MV
$\Sigma_{4}^{\text {nsym }}$, non-symmetric part, $q=Q_{s}$

(a) Dipole

(b) MV

Also gives us back-to-back correlation. large error bar comes from the fact that monstrous dipole $\Sigma_{4}^{n s y m}$ is not Monte Carlo friendly.

## NC STATE

UNIVERSITY
$\sum_{4}^{s y m}$, symmetric part, $q=Q_{s}$


As what was done in experimental analysis, we subtract backgrounds and normalize the signal. The results show similar correlations in CGC calculation.

## NC STATE

UNIVERSITY
$v_{2}$ and $v_{2}^{2}$
Recall,

$$
\frac{d N}{d \boldsymbol{q}_{1}^{2} d \boldsymbol{q}_{2}^{2}} \propto 1+\sum_{n} 2 v_{n}^{2} \cos (n \Delta \theta)
$$

One first define,

$$
V_{n}\left(q_{1}\right)=\int d \theta_{1} \int_{0}^{p_{\perp}^{\max }} d^{2} \boldsymbol{q}_{2} \exp (i n \Delta \theta) \frac{d N}{d \boldsymbol{q}_{1}^{2} d \boldsymbol{q}_{2}^{2} d \eta d \xi}
$$

by definition,

$$
v_{2}^{(2)}\left(p_{\perp}\right)=\sqrt{\frac{V_{2}\left(p_{\perp}\right)}{V_{0}\left(p_{\perp}\right)}}
$$

assuming factorization,

$$
v_{2}\left(p_{\perp}\right)=\frac{V_{2}\left(p_{\perp}\right) / V_{0}\left(p_{\perp}\right)}{\sqrt{V_{2} / V_{0}}}
$$

## $v_{2}$ results




- Different behavior above 2 Gev due to the lack of HBT contribution on the left.
- In the ATLAS analysis, $P_{\text {Max }}=2 G e v$


## NC STATE <br> UNIVERSITY

## Factorization test

## Theoretical calculation



Figure: The elliptic flow $v_{2}$ for three different kinematic ranges of the trigger particle. Here as in the previous figure, $Q_{s}=2 \mathrm{GeV}$. The size of the projectile is set by $R=1 / \mathrm{GeV}$.

## NC STATE UNIVERSITY

## Average in momentum bins



Figure: Parameters are the same as previous slides but binned with the same bin choice as the ATLAS analysis.

Binning the particles decreases the differences between the models.

## NC STATE UNIVERSITY

## Summary and outlook

- We analytically derived inclusive two gluon production in UPC at mid-rapidity.
- To estimate systematic uncertainty originated from the poor knowledge of the real photon wave function, we studied two limiting cases.
- Both models result in qualitatively similar correlation. Quantitatively, the amplitude of azimuthal anisotropy for MV model is about two times the dipole model.
- Our results show similar correlation as experimental data.
- Further developments
- Phenomenology
- To extend to EIC physics (large $Q^{2}$, work in progress)
- To incorporate rapidity dependence


## Backup slides

## NC STATE UNIVERSITY

# Gluon production 

## Create gluons within initial states

One account for the emission of the gluons using coherent operators

$$
C=\mathcal{P} e^{i \sqrt{2} \int d^{2} x d \xi \hat{b}_{a}^{i}(\xi, \boldsymbol{x})\left[a_{i, a}^{\dagger}(\xi, \boldsymbol{x})+a_{i, a}(\xi, \boldsymbol{x})\right]}
$$

with the background field

$$
\hat{b}_{a}^{i}(\xi, \boldsymbol{x})=\frac{g}{2 \pi} \int d^{2} y \frac{(\boldsymbol{x}-\boldsymbol{y})^{i}}{|\boldsymbol{x}-\boldsymbol{y}|^{2}} \hat{\rho}_{\mathrm{P}}^{a}(\xi, \boldsymbol{y})
$$

- MV model classical source $\rho_{a}$
- $\hat{\rho}_{D}^{a}(\boldsymbol{x})=b_{\alpha \sigma}^{\dagger}\left(\boldsymbol{x}_{\mathbf{1}}\right) t_{\alpha \beta}^{a} b_{\beta \sigma}\left(\boldsymbol{x}_{\mathbf{1}}\right) \delta^{(2)}\left(\boldsymbol{x}-\boldsymbol{x}_{\mathbf{1}}\right)-d_{\alpha \sigma}^{\dagger}\left(\boldsymbol{x}_{\mathbf{2}}\right) t_{\beta \alpha}^{a} d_{\beta \sigma}\left(\boldsymbol{x}_{\mathbf{2}}\right) \delta^{(2)}\left(\boldsymbol{x}-\boldsymbol{x}_{\mathbf{2}}\right)$
- $\hat{\rho}_{g}^{a}(\zeta, \boldsymbol{x})=a_{b}^{i \dagger}(\eta, \boldsymbol{x}) T_{b c}^{a} a_{c}(\eta, \boldsymbol{x})$



## Eikonal scattering through the shock wave



$$
U(\boldsymbol{x})=\mathcal{P} \exp \left\{i g \int_{-\infty}^{\infty} d x^{+} T^{a} A_{a}^{-}\left(x^{+}, \boldsymbol{x}\right)\right\}
$$

The strong gluon field $A_{a}^{-}\left(x^{+}, \boldsymbol{x}\right)$ is a functional of the valance source in the target.


## The cross section

$$
\frac{d \mathcal{N}}{d \eta d q_{1}^{2} d \xi d q_{2}^{2}}=\frac{1}{(2 \pi)^{4}} \int d^{2} u_{1} d^{2} u_{2} d^{2} \bar{u}_{1} d^{2} \bar{u}_{2} e^{-i \boldsymbol{q}_{1}\left(\boldsymbol{u}_{1}-\overline{\boldsymbol{u}}_{1}\right)} e^{-i \boldsymbol{q}_{2}\left(\boldsymbol{u}_{2}-\overline{\boldsymbol{u}}_{2}\right)} \Sigma
$$

and

$$
\Sigma=\left\langle\gamma^{*}\right| C^{\dagger} \hat{S}^{\dagger} C a_{i, a}^{\dagger}\left(\eta, \boldsymbol{u}_{1}\right) a_{j, b}^{\dagger}\left(\xi, \boldsymbol{u}_{2}\right) a_{i, a}\left(\eta, \overline{\boldsymbol{u}}_{1}\right) a_{j, b}\left(\xi, \overline{\boldsymbol{u}}_{2}\right) C^{\dagger} \hat{S} C\left|\gamma^{*}\right\rangle
$$

where $C=C_{\xi} C_{\eta}$, and $\eta \gg \xi$,

$$
\begin{aligned}
C_{\eta} & \simeq 1+i \sqrt{2} \int d^{2} v_{1} \hat{b}_{D a}^{i}\left(\boldsymbol{v}_{1}\right)\left[a_{a}^{i \dagger}\left(\eta, \boldsymbol{v}_{1}\right)+a_{a}^{i}\left(\eta, \boldsymbol{v}_{1}\right)\right] \\
C_{\xi} & \simeq 1+i \sqrt{2} \int d^{2} v_{2}\left(\hat{b}_{D b}^{j}\left(\boldsymbol{v}_{2}\right)+\delta \hat{b}_{b}^{j}\left(\eta, \boldsymbol{v}_{2}\right)\right)\left[a_{b}^{j \dagger}\left(\xi, \boldsymbol{v}_{2}\right)+a_{b}^{j}\left(\xi, \boldsymbol{v}_{2}\right)\right]
\end{aligned}
$$

- $C\left|\gamma^{*}\right\rangle$ Initial state
- $\hat{S}$ S-matrix
- $C a_{j, b}\left(\xi, \bar{u}_{2}\right) C^{\dagger}$ dressed gluons in the final state


## NC STATE

UNIVERSITY

## Dipole expectation values

- Expectation values for $q \bar{q}$

$$
\begin{aligned}
& \langle q \bar{q}| \hat{\rho}_{d^{\prime}}(\overline{\boldsymbol{x}}) \hat{\rho}_{d}(\boldsymbol{x})|q \bar{q}\rangle=\frac{\delta^{d d^{\prime}}}{2}\left(\delta^{2}\left(\overline{\boldsymbol{x}}-\boldsymbol{z}_{1}\right)-\delta^{2}\left(\overline{\boldsymbol{x}}-\boldsymbol{z}_{2}\right)\right)\left(\delta^{2}\left(\boldsymbol{x}-\boldsymbol{z}_{1}\right)-\delta^{2}\left(\boldsymbol{x}-\boldsymbol{z}_{2}\right)\right) \\
& \langle q \bar{q}| \hat{\rho}^{a}\left(\boldsymbol{x}_{\mathbf{1}}\right) \hat{\rho}^{b}\left(\boldsymbol{x}_{\mathbf{2}}\right) \hat{\rho}^{c}\left(\boldsymbol{x}_{\mathbf{3}}\right)|q \bar{q}\rangle \\
& =\frac{i f_{a b c}}{4}\left(\delta^{(2)}\left(\boldsymbol{x}_{\mathbf{2}}-\boldsymbol{z}_{\mathbf{1}}\right)+\delta^{(2)}\left(\boldsymbol{x}_{\mathbf{2}}-\boldsymbol{z}_{\mathbf{2}}\right)\right) \prod_{i=1,3}\left(\delta^{(2)}\left(\boldsymbol{x}_{\boldsymbol{i}}-\boldsymbol{z}_{\mathbf{1}}\right)-\delta^{(2)}\left(\boldsymbol{x}_{\boldsymbol{i}}-\boldsymbol{z}_{\mathbf{2}}\right)\right)
\end{aligned}
$$

$\boldsymbol{z}_{1}, \boldsymbol{z}_{2}$ are the transverse coordinates of quark and anti-quark.

- Average over different dipole size $\boldsymbol{r}=\boldsymbol{z}_{1}-\boldsymbol{z}_{2}$

$$
\left\langle\rho_{d^{\prime}}(\overline{\boldsymbol{x}}) \rho_{d}(\boldsymbol{x})\right\rangle_{P} \approx \sum_{s_{1}} \int_{z} \int d^{2} \boldsymbol{r} \Psi_{\lambda}^{T *}\left(z, r, s_{1}\right) \Psi_{\lambda}^{T}\left(z, r, s_{1}\right)\langle q \bar{q}| \rho_{d^{\prime}}(\overline{\boldsymbol{x}}) \rho_{d}(\boldsymbol{x})|q \bar{q}\rangle
$$

## NC STATE <br> UNIVERSITY

## MV model projectile average

- MV model describes the distribution of classical color source not quantum operators.

$$
W\left(\rho_{a}\right)=\exp \left\{-\int_{\boldsymbol{x}} \frac{\rho_{a}(\boldsymbol{x}) \rho_{a}(\boldsymbol{x})}{2 \mu^{2}}\right\}
$$

$$
\mu^{2}(\boldsymbol{x})=\mathcal{N} \exp \left\{-\frac{\boldsymbol{x}^{2}}{R^{2}}\right\}
$$

- Two and three point correlators

$$
\begin{aligned}
\left\langle\hat{\rho}_{a}(\boldsymbol{x}) \hat{\rho}_{b}(\boldsymbol{y})\right\rangle_{\mathrm{MV}} & =\left\langle\rho_{a}(\boldsymbol{x}) \rho_{b}(\boldsymbol{y})\right\rangle_{\mathrm{MV}}=\mu^{2} \delta^{(2)}(\boldsymbol{x}-\boldsymbol{y}) \delta_{a b} \\
\left\langle\hat{\rho}_{a}(\boldsymbol{x}) \hat{\rho}_{b}(\boldsymbol{y}) \hat{\rho}_{c}(\boldsymbol{z})\right\rangle_{\mathrm{MV}} & =-\frac{1}{2} \delta^{(2)}(\boldsymbol{x}-\boldsymbol{y}) \delta^{(2)}(\boldsymbol{y}-\boldsymbol{z}) T_{b c}^{a} \mu^{2}
\end{aligned}
$$

- Symmetrization of $\hat{\rho} s$

$$
\begin{aligned}
\hat{\rho}_{a}(x) \hat{\rho}_{b}(y) & =\frac{1}{2}\left\{\hat{\rho}_{a}(\boldsymbol{x}), \hat{\rho}_{b}(\boldsymbol{y})\right\}+\frac{1}{2}\left[\hat{\rho}_{a}(\boldsymbol{x}), \hat{\rho}_{b}(\boldsymbol{y})\right] \\
& =\rho_{a}(\boldsymbol{x}) \rho_{b}(\boldsymbol{y})-\frac{1}{2} \delta^{(2)}(x-y) T_{a b}^{c} \rho_{c}(\boldsymbol{x})
\end{aligned}
$$

