Baryon Number-Violating Amplitudes on a Lattice
with Physical Chirally-Symmetric Quarks

Sergey Syritsyn,
with Enrico Rinaldi, Michael Wagman, Jun-Sik Yoo,
Taku Izubuchi, Amarjit Soni, Peter Boyle, Yasumichi Aoki

25th International Symposium on Spin Physics
Duke University, Durham, NC, USA
Sep 26 2023

- Neutron-antineutron oscillation amplitudes
    - *Experimental lifetime limits & outlook*
    - *Nucleon model uncertainties*
    - *Hadron masses and energies*
    - *Extraction of matrix elements*
    - *Operator renormalization*

- Proton decay amplitudes
    - *Experimental lifetime limits & outlook*
    - *Effective operators for* $p \to \pi\ell$, $K\ell$ *decays*
    - *Momentum & continuum extrapolations*
    - *Proton decay constants (* $p \to 3\ell$ *)*

- Summary&Outlook

Baryogenesis and Broken Symmetries

Why More Matter > Antimatter? $\frac{n_B - n_{\bar{B}}}{n_\gamma} \approx 6 \cdot 10^{-10}$

- Three necessary components [A.Sakharov (1967)] :

proton decay, neutron oscillations

Baryon number-changing interactions

(alternatively, leptogenesis + sphalerons)

neutrinoless beta-decays

Violations of C- and CP-symmetries

(electric dipole moments of p, n, e^-, nuclei, atoms)

Interactions out of equlibrium

Baryon Number : accidental symmetry of SM, violated by sphalerons

- neutron-antineutron oscillations (ΔB=2)
- proton decay (ΔB=1)

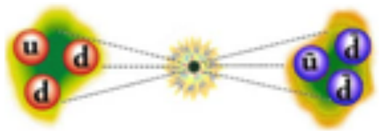
Missing piece of Grand-Unified Theories

Limit on nuclear matter stability?

ΔB=2 Number Violation : n–n̄ Oscillations

Baryon number not conserved ?⇒ (anti)neutrons are not energy eigenstates:

- (n,n̄) Hamiltonian with ΔB=2 $\mathcal{H} = \begin{pmatrix} n \\ \bar{n} \end{pmatrix}^\dagger \begin{pmatrix} M_n + \frac{1}{2}\Delta M & \delta m \\ \delta m & M_n + \frac{1}{2}\Delta M \end{pmatrix} \begin{pmatrix} n \\ \bar{n} \end{pmatrix}$
- n→n̄ transition probability $P_{n\to\bar{n}}(t) \approx \left[\frac{2\delta m}{\Delta M}\right]^2 \sin^2\left[\frac{1}{2}\Delta M\, t\right]$
- If $t \ll (\Delta M)^{-1}$: n→n̄ transition in $\tau_{n\bar{n}} = (2\delta m)^{-1}$
- current limit $\delta m \lesssim (10^8\,\mathrm{s})^{-1} \approx O(10^{-24})$ eV

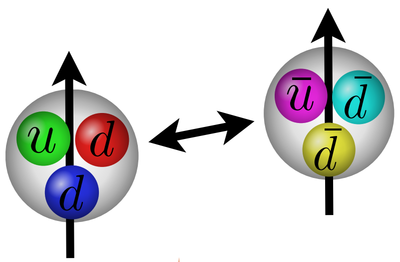


Medium effects dominate $\Delta M \gg \delta m$

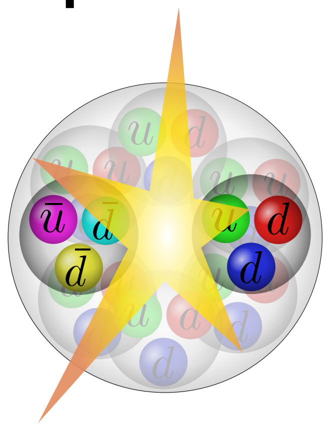
- In vacuum ("quasi-free" n–n̄) B~0.5 Gauss:

  $\Delta M = 2\mu_n B_\oplus \approx 6 \cdot 10^{-12}$ eV
- In nuclei :

  $\Delta M \sim O(100\,\mathrm{MeV})$



magnetic moment flip



interactions with nuclear medium

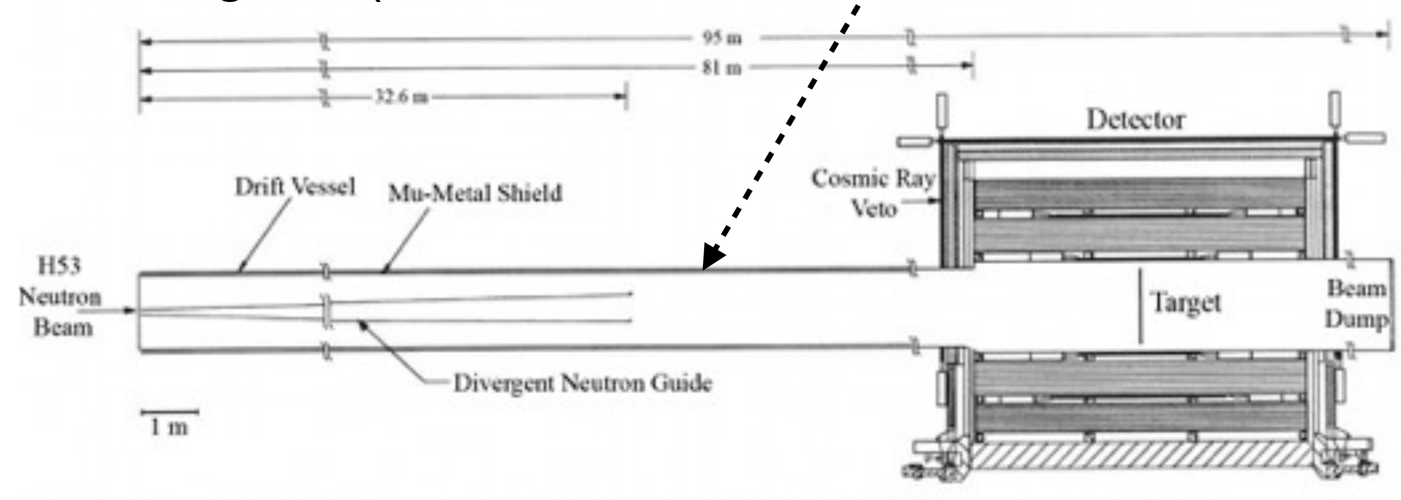
N–$\bar{\text{N}}$ Oscillations: Experimental Status

- "Quasi-free" reactor neutrons
  - ILL Grenoble high-flux reactor [M.Baldo-Ceolin et al, 1994)]

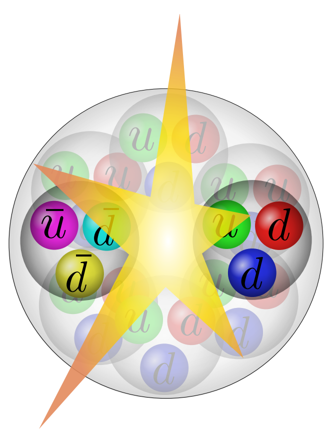
$$\tau_{n\bar{n}} \gtrsim 10^8\ s$$
$$\delta m \lesssim 6 \cdot 10^{-24}\ \text{eV}$$

Mag.field screening
for flight ~ $(\Delta M)^{-1} > 1\text{s}$

$$B < (2\mu_n t)^{-1} = 5\ \text{nT} = 10^{-4} B_\oplus$$



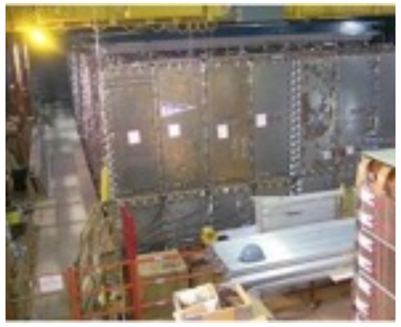
- In nuclei :
  - $\tau(^{56}Fe) \gtrsim 0.72{\cdot}10^{32}$ yr
    $\Longrightarrow \tau_{N\bar{N}} \gtrsim 1.4{\cdot}10^8$ s [Soudan]
  - $\tau(^{16}O) \gtrsim 1.77{\cdot}10^{32}$ yr
    $\Longrightarrow \tau_{N\bar{N}} \gtrsim 3.3{\cdot}10^8$ s [Super-K]
  - $\tau(^{2}H) \gtrsim 0.54{\cdot}10^{32}$ yr
    $\Longrightarrow \tau_{N\bar{N}} \gtrsim 1.96{\cdot}10^8$ s [SNO]



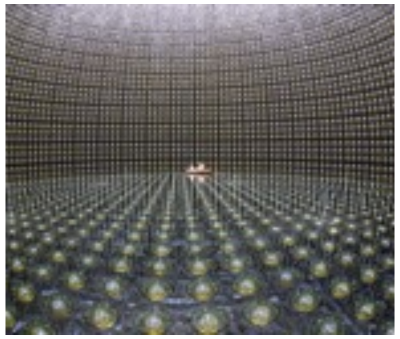
Nuclear decays from (ΔB=2) transitions: suppressed by nuclear medium:

$$T_d = R\tau_{n\bar{n}}^2$$
$$R \sim 10^{23}\ s^{-1}$$

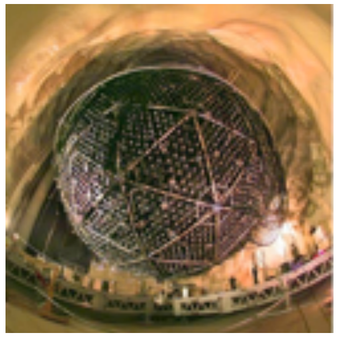
nuclear model uncertainty
~ 10-15% for ^{16}O
[E.Friedman, A.Gal (2008)]



Soudan



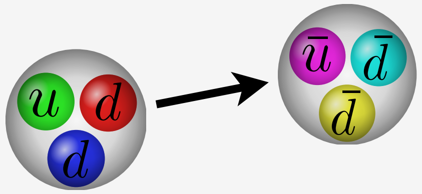
Super Kamiokande



SNO

Sensitivity is limited by atmospheric neutrinos

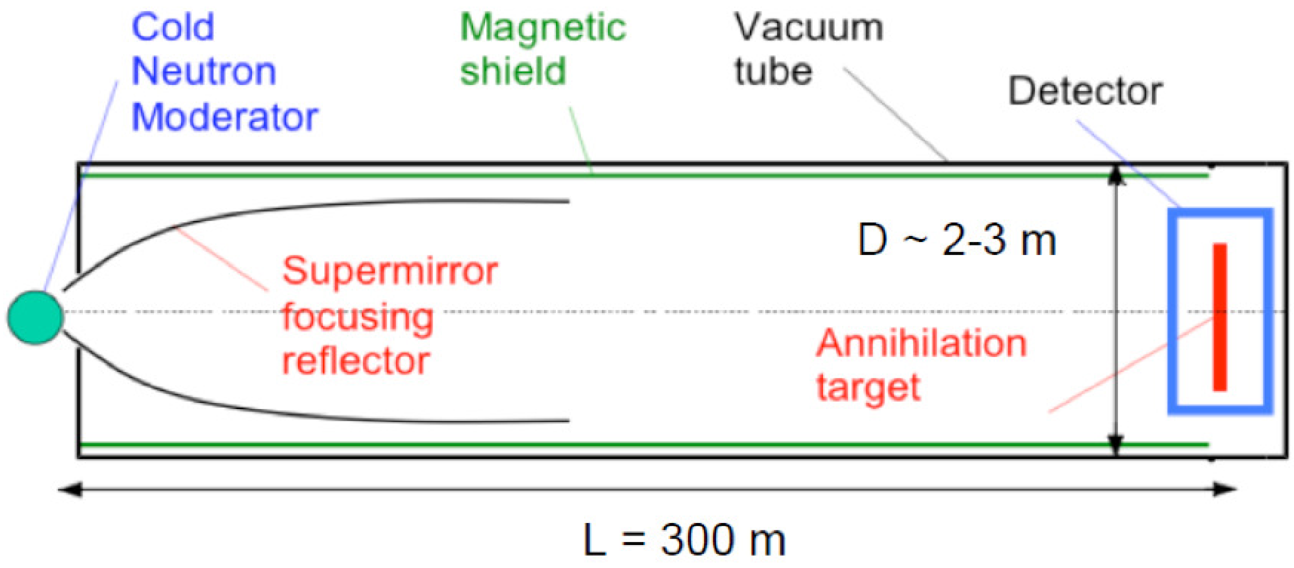
N–N̄: Experimental Outlook



Maximize *Probability of oscillation* ~ $N_n \ (T_{\mathrm{free}})^2$

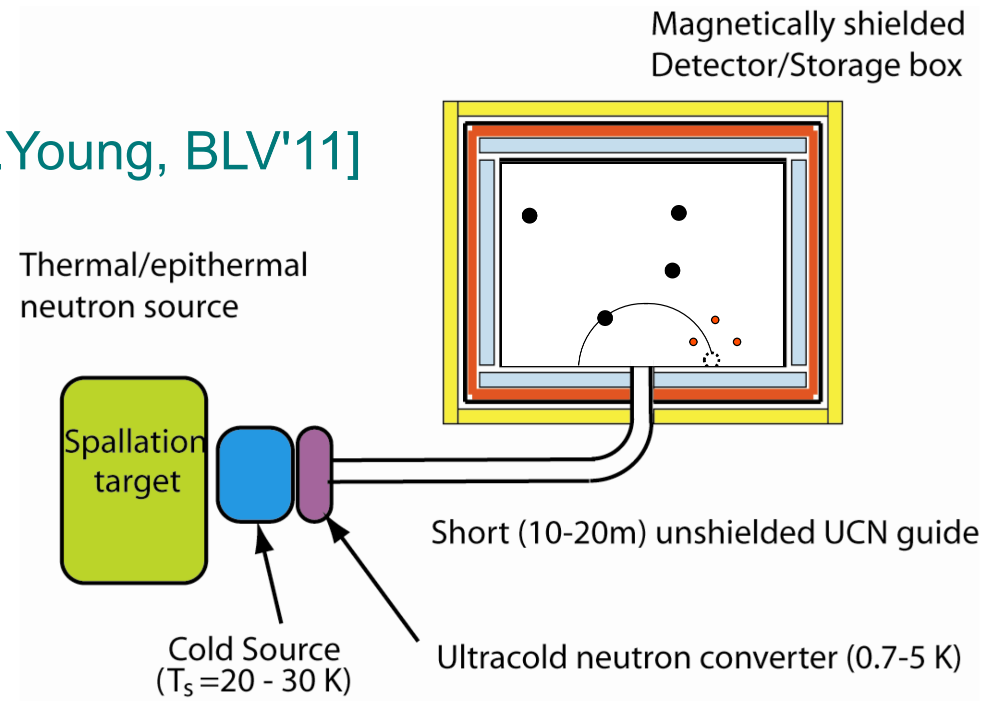
- Shielded beam (similar to ILL):
  Expected sensitivity x**10²-10³** ILL
  $\tau_{n\text{-}\bar{n}} \gtrsim 10^9\text{-}10^{10}$ s
  - ✦ Spallation sources: x12 flux @ESS
  - ✦ Elliptic focussing mirror
  - ✦ Better magnetic shielding (B < 1 nT)

[Phillips et al, arXiv:1410.1100]



- stored ultra-cold neutrons
  $\tau_{n\text{-}\bar{n}} \gtrsim 2.2 \cdot 10^8$ s

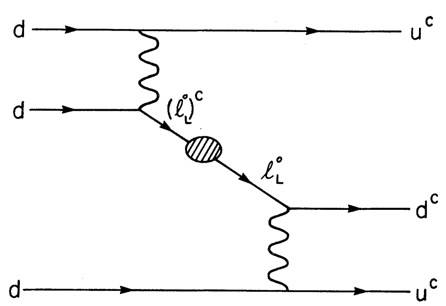
[A.Young, BLV'11]

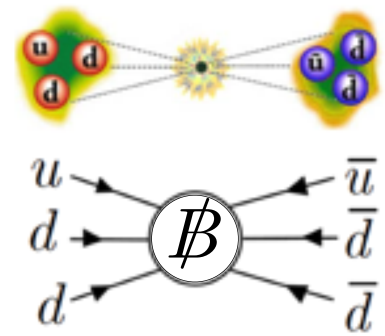


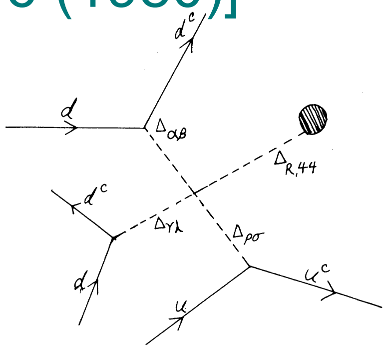
- Further improvements
  - ✦ Larger vessels
  - ✦ Better magnetic shielding (B < 1 nT)
  - ✦ Parabolic floor concentrators
  - ✦ Multiple coherent reflections

BSM Models and QCD Input

- GUT + massive Majorana lepton
  [T.K.Kuo, S.T.Love, PRL45:93 (1980)]
- partial unification and (B–L) viol.
  [R.N.Mohapatra, R.E.Marshak, PRL44:1316 (1980)]







Effective *ΔB=2* interaction

$$\mathcal{L}_{\rm eff} = \sum_i \left[c_i \mathcal{O}_i^{6q} + \text{h.c.} \right]$$

oscillation rate

$$(2\tau_{n\bar n})^{-1} = \delta m = -\langle \bar n | \int d^4x\, \mathcal{L}_{\rm eff}\, | n \rangle = -\sum_i \frac{c_i}{M_X^5} \langle \bar n | \mathcal{O}_i^{6q} | n \rangle$$

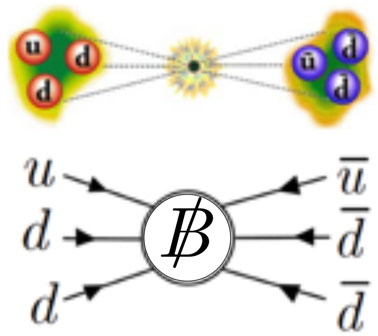
BSM scale suppression
M_X ≳ (200–300) TeV

n–n̄ amplitude from NP QCD $\sim \int d^3x\, (\rho_q)^3$

sensitive to spatial quark distribution

ΔB=2 Operators

Classification of all ΔI=1 6-quark operators



- Light-flavor $SU(2)_f$ multiplets
  [T.Kuo, S.Love, PRL45:93 (1980);
  S.Rao, R.Shrock, PLB116:238 (1982)]
- 2-loop perturbative running
  [Buchoff, Wagman, PRD93:016005(2015)]

$$(q_1 q_2) \doteq (q_1^T C q_2)$$
$$(q_1 q_2)^A \in (\bar{\mathbf{3}}_{\text{color}}, \mathbf{1}_{\text{flavor}})$$
$$(q_1 q_2)^S \in (\mathbf{6}_{\text{color}}, \mathbf{3}_{\text{flavor}})$$

$$(\mathbf{1}_L, \mathbf{3}_R) \qquad \begin{aligned} Q_1 &= -4\,(ud)_R^{A_1}\,(ud)_R^{A_2}\,(dd)_R^{S_3}\,T^{A_1 A_2 S_3} \\ Q_2 &= -4\,(ud)_L^{A_1}\,(ud)_R^{A_2}\,(dd)_R^{S_3}\,T^{A_1 A_2 S_3} \\ Q_3 &= -4\,(ud)_L^{A_1}\,(ud)_L^{A_2}\,(dd)_R^{S_3}\,T^{A_1 A_2 S_3} \end{aligned}$$

$$(\mathbf{1}_L, \mathbf{7}_R) \qquad Q_4 = -\frac{4}{5}\left[(uu)(dd) + 4(ud)_R(ud)\right]_{RR}^{S_1 S_2}\,(dd)_R^{S_3}\,T^{S_1 S_2 S_3}$$

$$(\mathbf{3}_L, \mathbf{5}_R) \qquad Q_5 = (uu)_R^{S_1}\,(dd)_L^{S_2}\,(dd)_L^{S_3}\,T^{S_1 S_2 S_3}$$ (not $SU(2)_L$-symmetric)

(and also $Q_{6,7}$ related by Wigner-Eckart thm)

Must have chiral symmetry to protect the operators from mixing

Fundamental Theory: QCD on a Lattice

Lattice Field Theory ⇔ Numerical evaluation of the Path Integral

$$\langle q_x \bar{q}_y \ldots \rangle = \int \mathcal{D}\big(Glue\big) \int \mathcal{D}\big(Quarks\big)\; e^{-S_{Glue} - \bar{q}\left(\not{D}+m\right) q} \; [q_x \bar{q}_y \ldots]$$

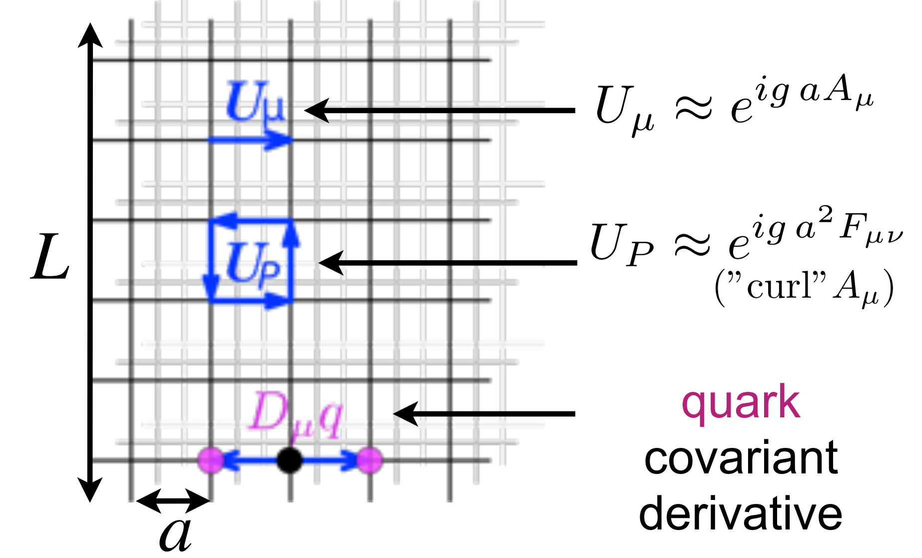
Grassmann integration

$$= \int \mathcal{D}\big(Glue\big)\; e^{-S_{Glue}}\; \mathrm{Det}\big(\not{D}+m\big) \; \left[\big(\not{D}+m\big)^{-1}_{x,y} \ldots\right]$$

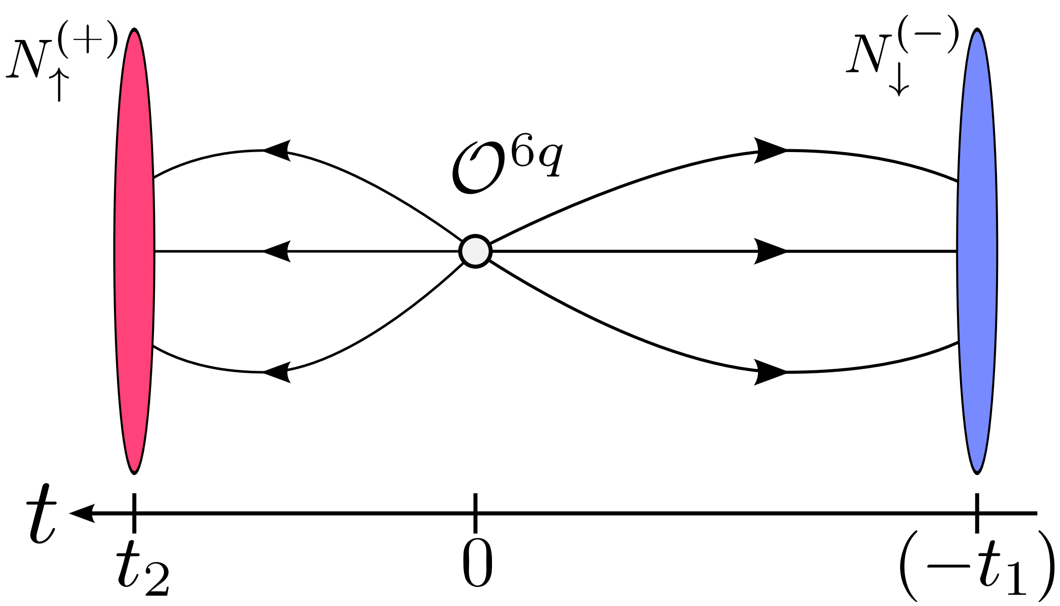
Hybrid Monte Carlo sampling of gluon background

Quark Eqns. of Motion

QCD on Euclidean 4D lattice



Lattice correlation fcn. for $\langle \bar{n} | Q_\alpha | n \rangle$ matrix elements
[M.Buchoff, C.Schroeder, J.Wasem PRD93:016005(2015)]



Systematic effects

- discretization errors
- finite volume
- ~~unphysical heavy pion(quark) mass~~
- ~~chiral symmetry breaking~~
- ~~excited states~~
- ~~renormalization / $\overline{\text{MS}}$ matching~~

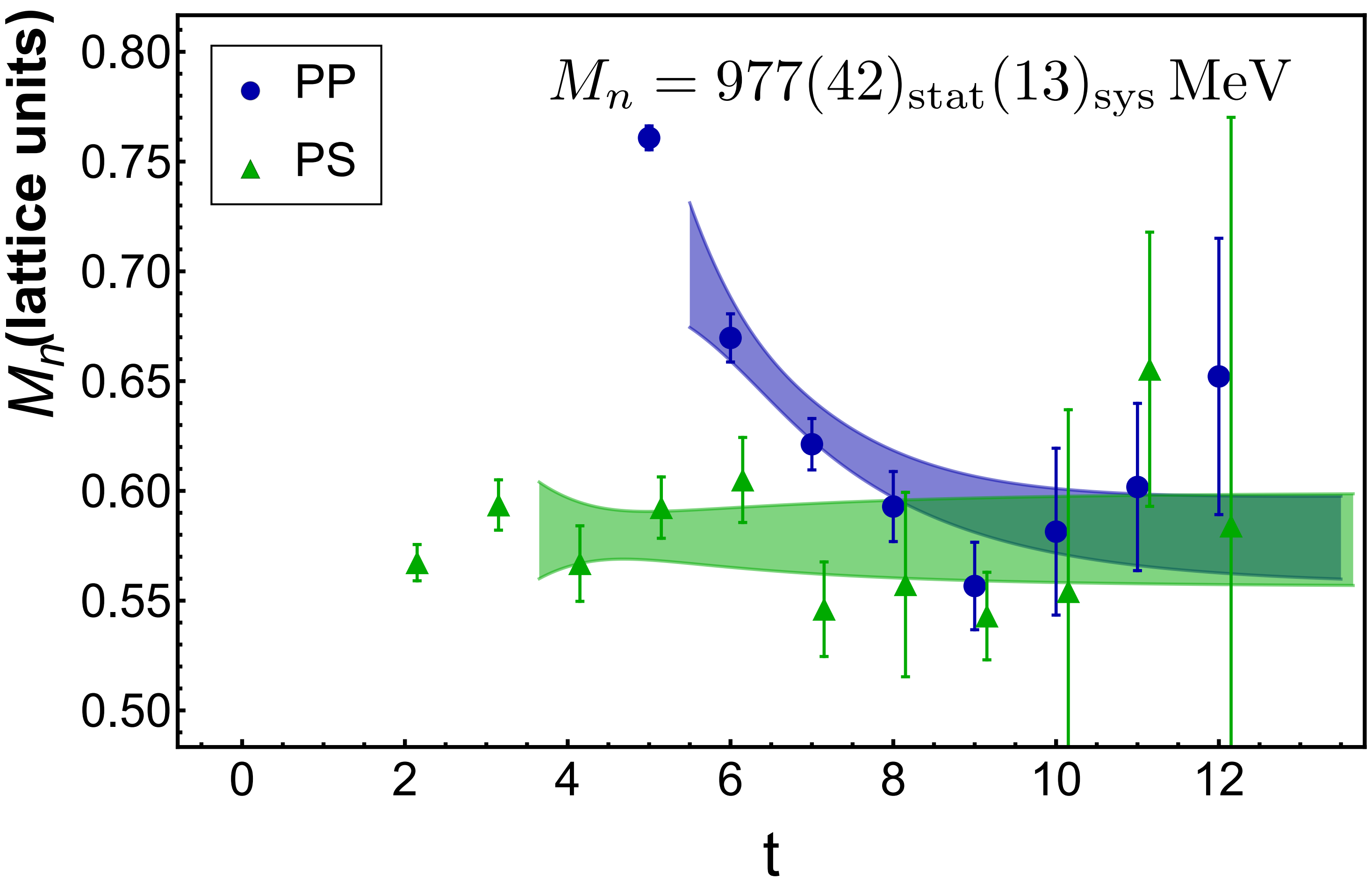
Lattice Details

- Physical quark masses, m_π = 140 MeV
  [T.Blum et al (RBC/UKQCD), PRD93:074505 arXiv:1411.7017]
  - ✦ lattice $48^3 \times 96 = 5.5^3 \times 10.9$ fm
  - ✦ spacing $a = 0.1392(4)$ fm, $\delta(a^{-6}) \approx$ **1.7%**
- chiral (Möbius Domain Wall Fermions)
- 2268 (28 x 81) MC samples

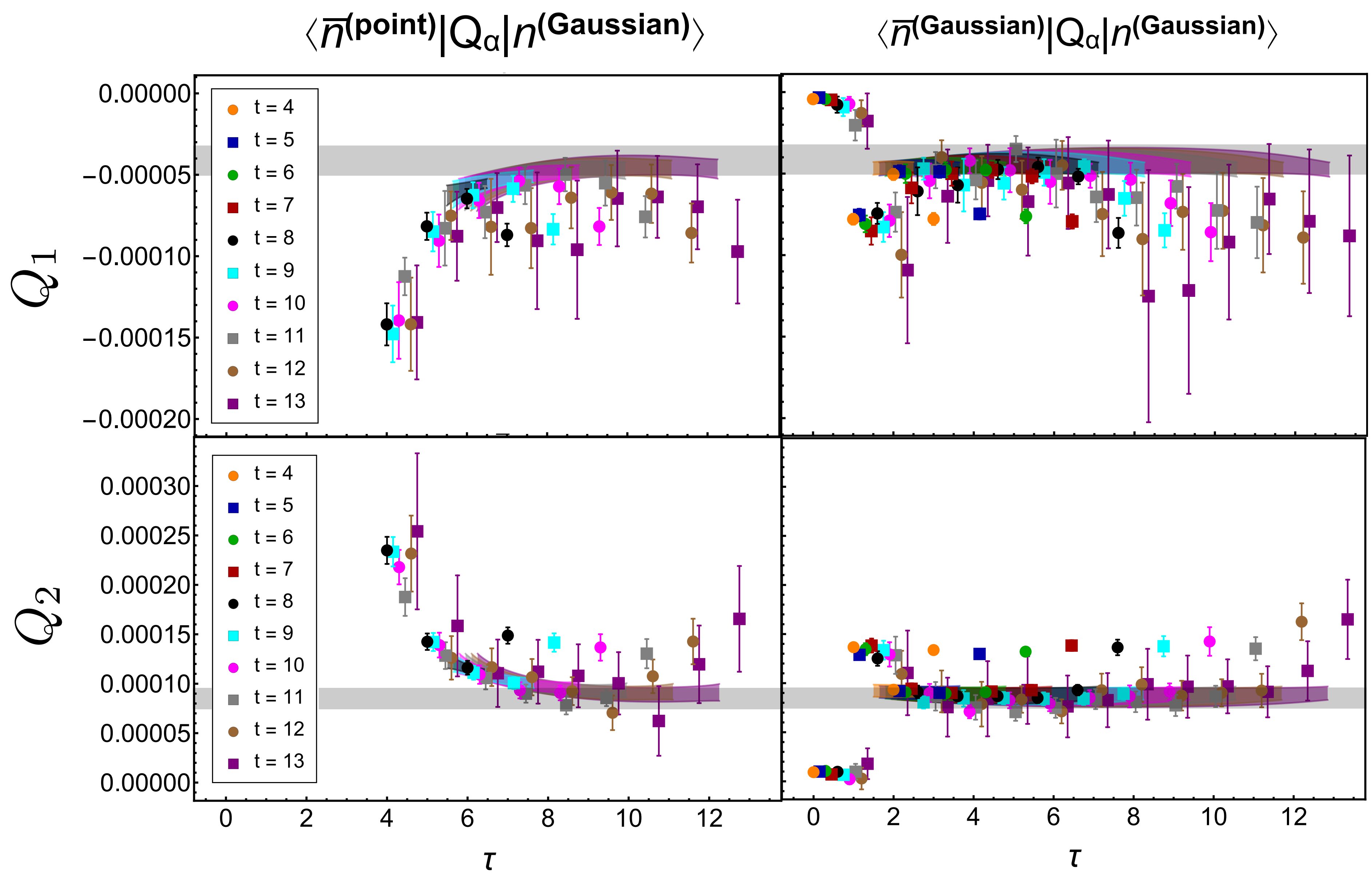
Nucleon effective mass

$$M_n^{\text{eff}}(t) = \frac{1}{a} \log \frac{C_{nn}(t)}{C_{nn}(t+a)}$$

- lattice data
- Variational analysis on quark w.f.'s: point-like vs. Gaussian(smeared)
- fits $C_{nn}(t) \sim C_0 e^{-E_0 t} + C_1 e^{-E_1 t}$
  sim. PP+PS with $t \approx 0.5 \ldots 1.5$ fm



n↔n̄ Amplitudes: Ground and Excited States

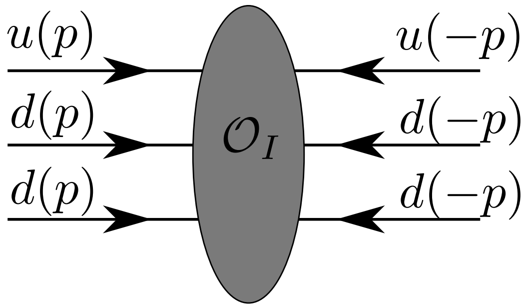


Excited state analysis:

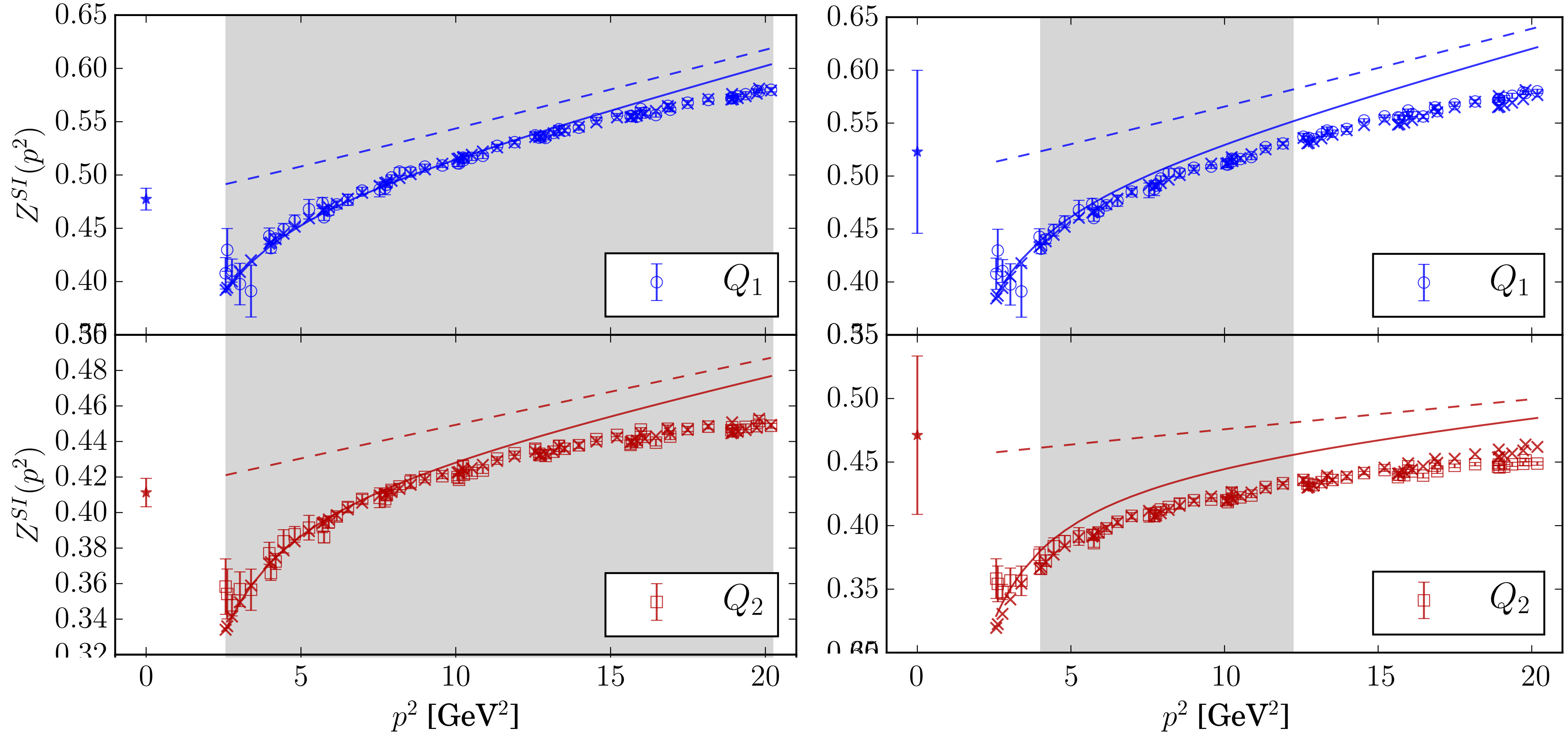
- Variational analysis: point-like vs. Gaussian-like quark w.f.'s in (anti)neutrons
- Data points: ratios of lattice correlators $C_{3pt}(T)/C_{2pt}(T) \to \langle N|Q|\bar{N}\rangle$
- Bands: 2-state fits of lattice data with Tsep ≈ 0.5 ... 1.5 fm
- Variance across fits → systematic uncertainty

Nonperturbative Operator Renormalization

- 6-Quark Green's functions on a lattice
  quark momentum scheme for 2-loop pQCD
  [Buchoff, Wagman PRD93:016005(2015);
  Rinaldi, SS, Wagman, et al PRD99:074510 (2019)]



+ permutations
to enforce ***SU*(2)*f***

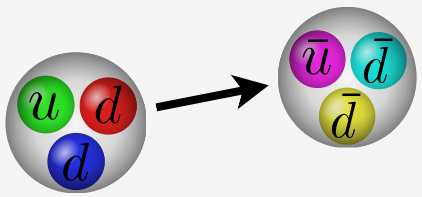


- p-dependence fit:
  (window $\Lambda_{\rm QCD} \ll p \ll \frac{\pi}{a}$)

$$Z^{SI}(p^2) = Z_{\rm final} + \underbrace{c_{NP}\frac{\Lambda^2_{QCD}}{p^2}}_{\rm nonpert.} + \underbrace{c_2(ap)^2 + c_{[4]}\frac{a^4 p^{[4]}}{(ap)^2}}_{\rm discretization}$$

- Variation with p² fits ranges, pQCD 1-,2-loop matching → systematic uncertainty

Lattice QCD Result: Enhanced N⇔$\bar{\text{N}}$



Lattice QCD with physical-mass, chiral-symmetric quarks:

x(5-10) larger N-Nbar oscillation vs. nucleon Bag model

[E.Rinaldi, S.S., M.Wagman, et al, PRD99:074510 (2019)]
[E.Rinaldi, S.S., M.Wagman, et al, PRL122:162(2018)]

	$\mathcal{O}^{\overline{MS}}(2\text{ GeV})$	Bag "A"	$\frac{\text{LQCD}}{\text{Bag "A"}}$	Bag "B"	$\frac{\text{LQCD}}{\text{Bag "B"}}$
$[(RRR)_\mathbf{3}]$	0	0	–	0	–
$[(RRR)_\mathbf{1}]$	45.4(5.6)	8.190	5.5	6.660	6.8
$[R_\mathbf{1}(LL)_\mathbf{0}]$	44.0(4.1)	7.230	6.1	6.090	7.2
$[(RR)_\mathbf{1}L_\mathbf{0}]$	-66.6(7.7)	-9.540	7.0	-8.160	8.1
$[(RR)_\mathbf{2}L_\mathbf{1}]^{(1)}$	-2.12(26)	1.260	-1.7	-0.666	3.2
$[(RR)_\mathbf{2}L_\mathbf{1}]^{(2)}$	0.531(64)	-0.314	-1.7	0.167	3.2
$[(RR)_\mathbf{2}L_\mathbf{1}]^{(3)}$	-1.06(13)	0.630	-1.7	-0.330	3.2
	$[10^{-5}\text{ GeV}^{-6}]$	$[10^{-5}\text{ GeV}^{-6}]$		$[10^{-5}\text{ GeV}^{-6}]$	

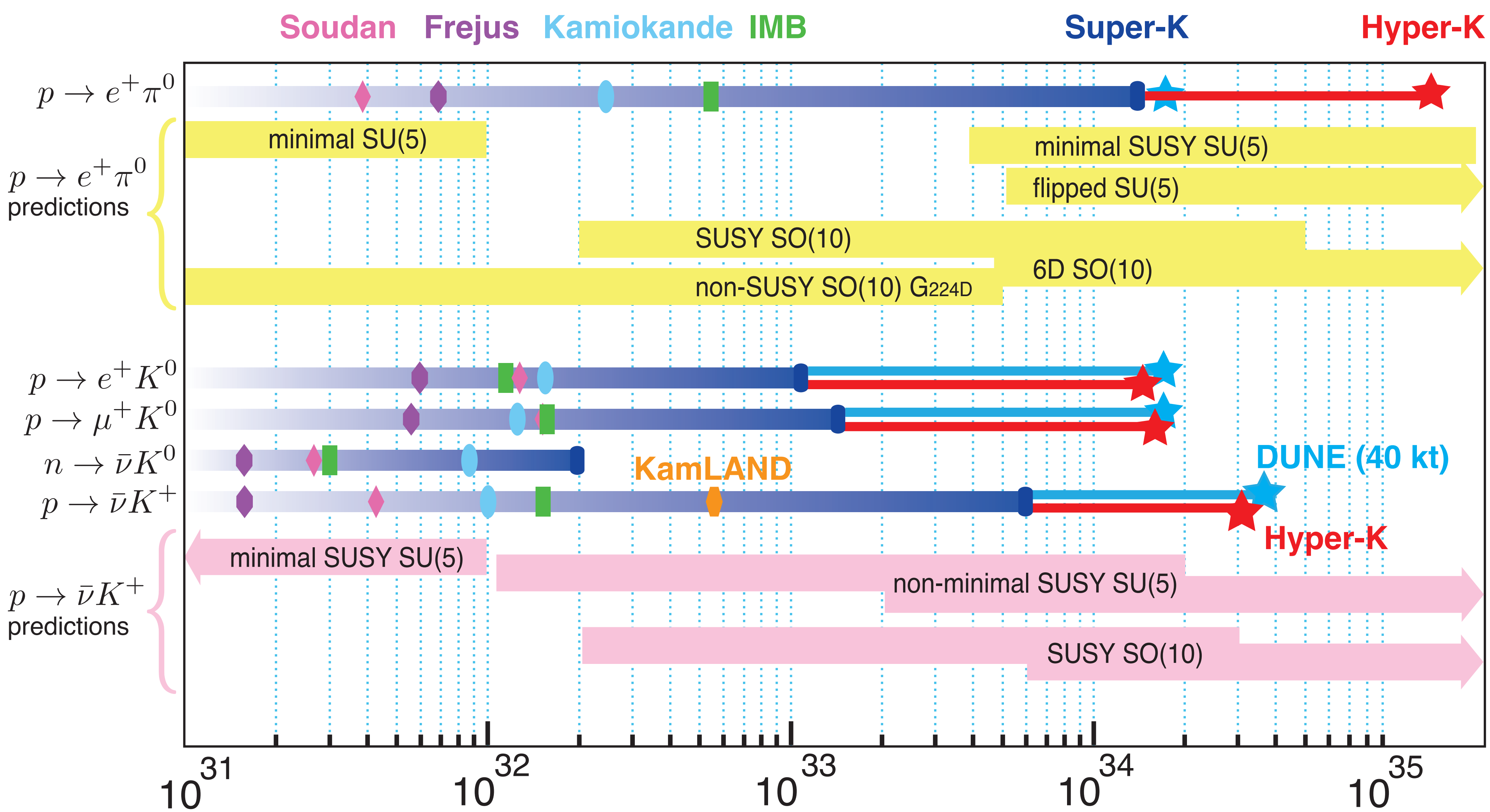
comparison to MIT Bag model [S.Rao, R.Shrock, PLB116:238 (1982)]

Next steps:

- Fully quantify systematic uncertainties : finite volume, continuum limit
- "Crossed" amplitudes : 2-neutron annihilation $\langle \text{vac}|O^{6q}|nn\rangle$
- Nuclear medium effects $\langle\, A{-}2 \,|\, O^{6q} \,|\, A \,\rangle$

Searches for Proton Decays

- Missing piece of Grand-Unified Theories
- Limits on stability of nuclear matter



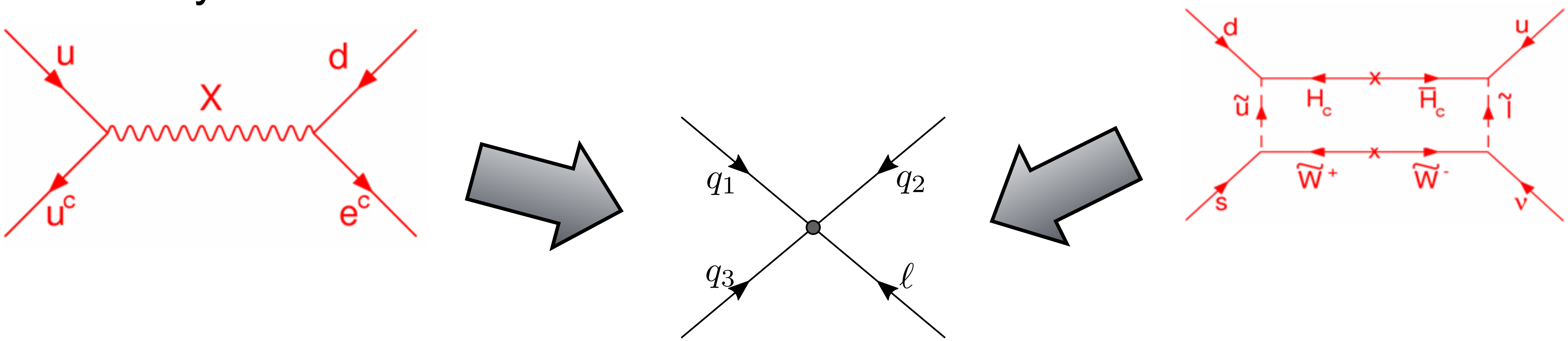
[LBNF and DUNE CDR, R.Acciarri et al (2015)]

- Expect x10 improvement on lifetime limit from Hyper-K and DUNE
- Better sensitivity to $p \to \bar{\nu}K^+$ that affects supersymmetric GUT models

Proton Decay Amplitudes and Rate

ordinary GUT

supersymmetric GUT



- Effective interaction

$$\mathcal{L}_{\rm eff} = \sum_I C_I \mathcal{O}_I \; + \; \text{h.c.}$$

$$\mathcal{O}_I = \epsilon^{abc}(\bar{q}_1^{aC} P_{\chi_I} q_2^b)(\bar{\ell}^C P_{\chi_I'} q_3^c) = \bar{\ell}^C_\alpha \, \mathcal{O}^{3q}_{I,\alpha}$$

$$q_{1,2,3} \in \{u, d, s\}\,, \quad P_{\chi_I^{(\prime)}} = \frac{1 \pm \gamma_5}{2}$$

- Decay width $p \to \Pi\bar{\ell}$ $\quad (\Pi = \pi, \mathrm{K}, \eta)$

$$\Gamma(p \to \Pi\bar{\ell}) = \frac{m_N}{32\pi}\left[1 - \left(\frac{m_\Pi}{m_N}\right)^2\right]^2 \Big|\sum_I C_I W^I_{\bar{\ell}}\Big|^2 \qquad \textit{where} \quad W_{\bar{\ell}} = \Big[W_0 + \underbrace{W_1 \cdot O(m_{\bar{\ell}}/m_N)}\Big]_{q^2 = m_{\bar{\ell}}^2}$$

negligible for e^+
≈10% for μ^+

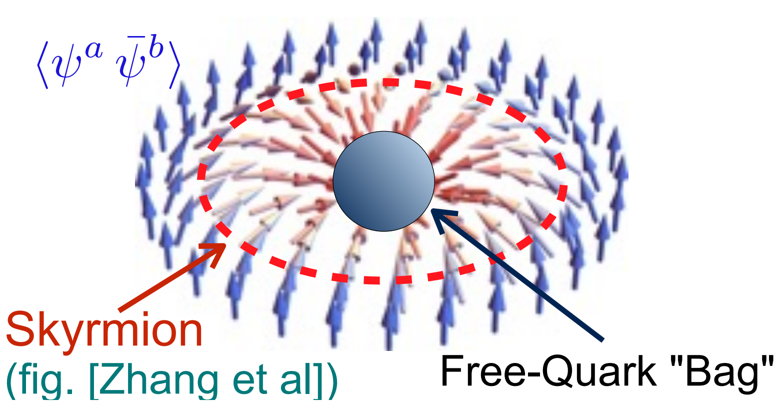
- Decay matrix elements $(W_{0,1})_I$ [S.Aoki et al, PRD62:014506 (2000)]

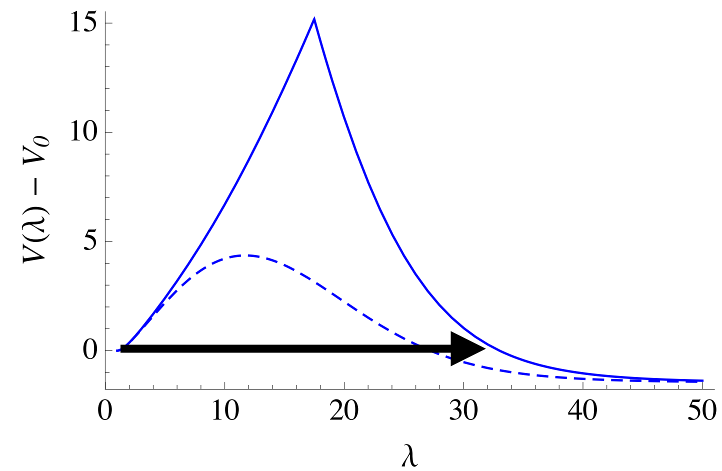
$$\langle \bar{\ell}(q)\Pi(p)|\mathcal{O}^{\chi'}|N(k)\rangle = \bar{v}^C_{\ell\alpha}(q)\, P_{\chi'} \left[W_0(-q^2) - \frac{i\not{q}}{m_N} W_1(-q^2)\right] u_N(k)$$

Proton Decay Matrix Elements

Is proton inherently stable?

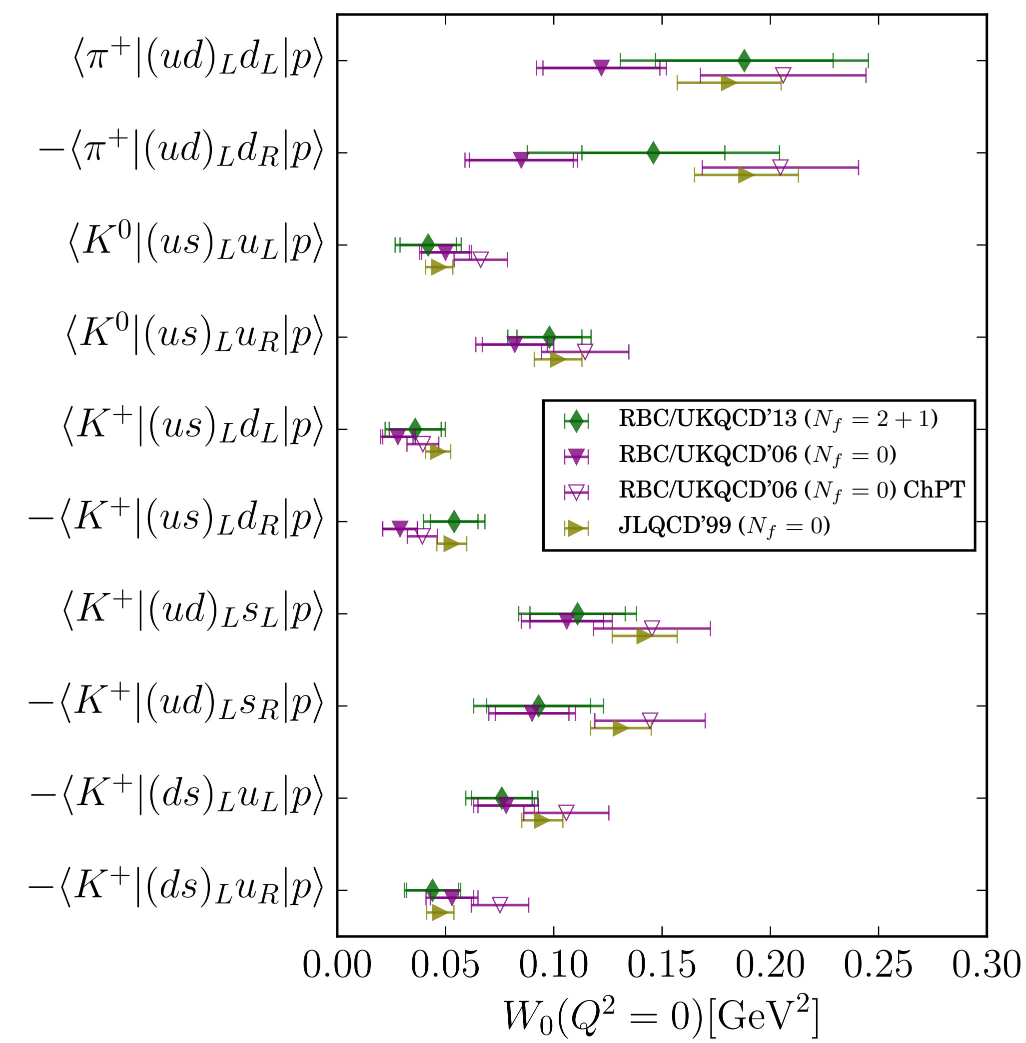
Conjecture [A.Martin, G.Stavenga '12]
Topological stability of "Chiral Bag" proton :





Nucleon-to-meson amplitudes
($p \to \pi\bar\ell, K\bar\ell,$ decays)

lattice calculations
with $m_\pi \gtrsim 330$ MeV



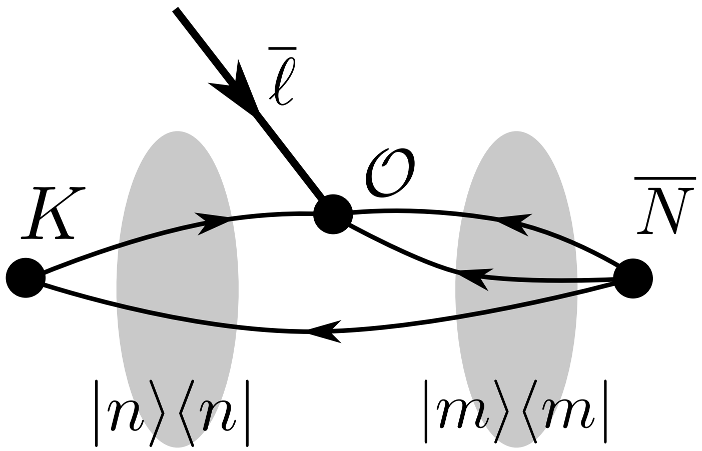
Lattice calculations:

- "direct" $p \to \pi\bar\ell, K\bar\ell$ decay matrix elements
  prior work at $m_\pi \gtrsim 300$ MeV:[S.Aoki et al (2000)]
  [Y.Aoki et al (2006), (2013), (2017)]
- "indirect" $p \to$ **vacuum** proton "decay constants"
  + LO-ChPT

Topological stability may strongly depend on
quark mass , chiral symmetry
$\Longrightarrow$ *Realistic physical-point calculation is necessary*

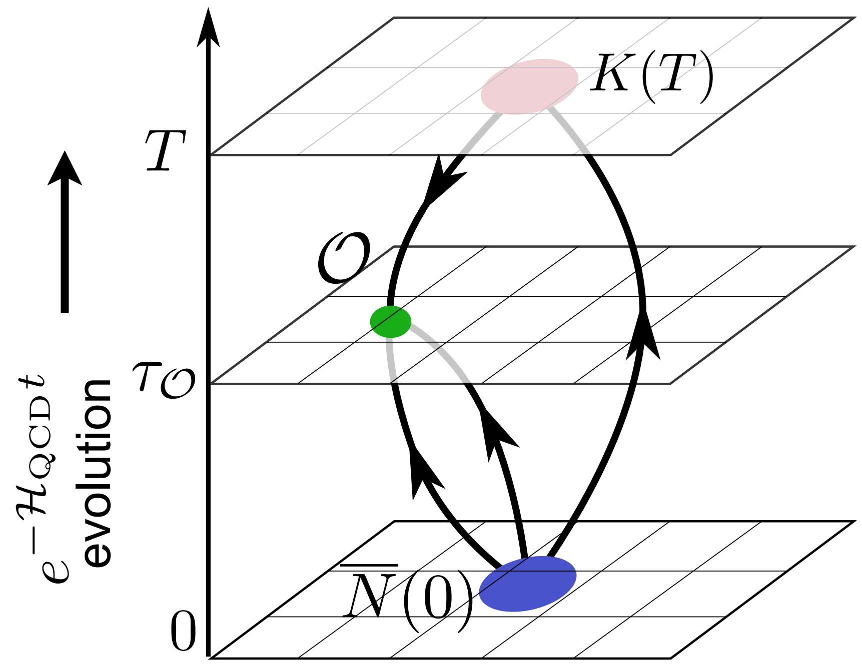
Proton→Meson Correlators in Lattice QCD

$$C_{3\mathrm{pt}}^{K\mathcal{O}N} = \langle K(T)\mathcal{O}(\tau)\overline{N}(0)\rangle =$$



$$= \sum_{m(K),n(N)} Z_m e^{-E_m(T-\tau)} \langle m(K)|\mathcal{O}|n(N)\rangle e^{-E_n\tau} Z_n^*$$

(Fit lattice data with $n,m = 0,1$)

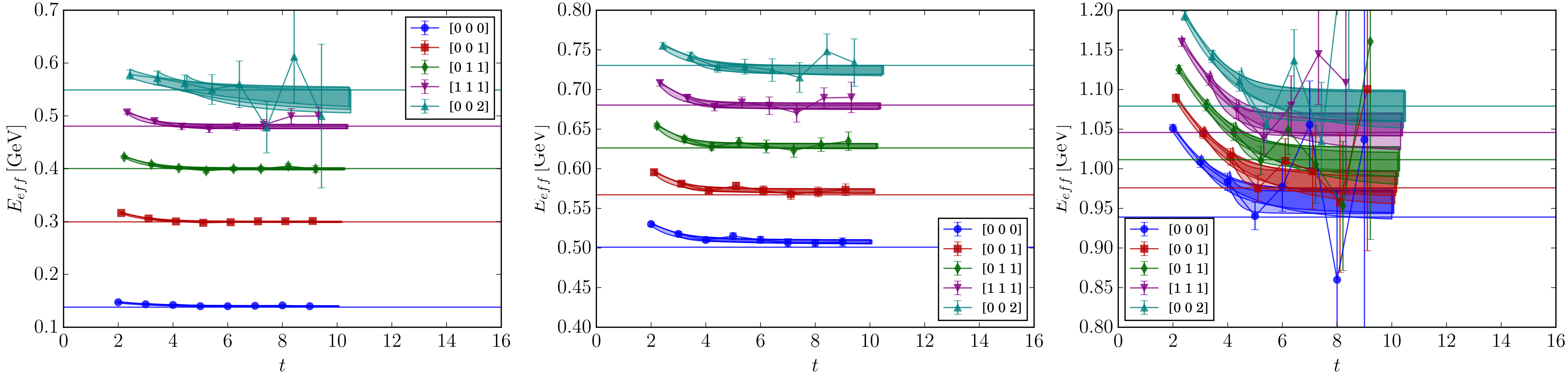


Quark lines = $(\not{D}+m)^{-1}\cdot\psi$

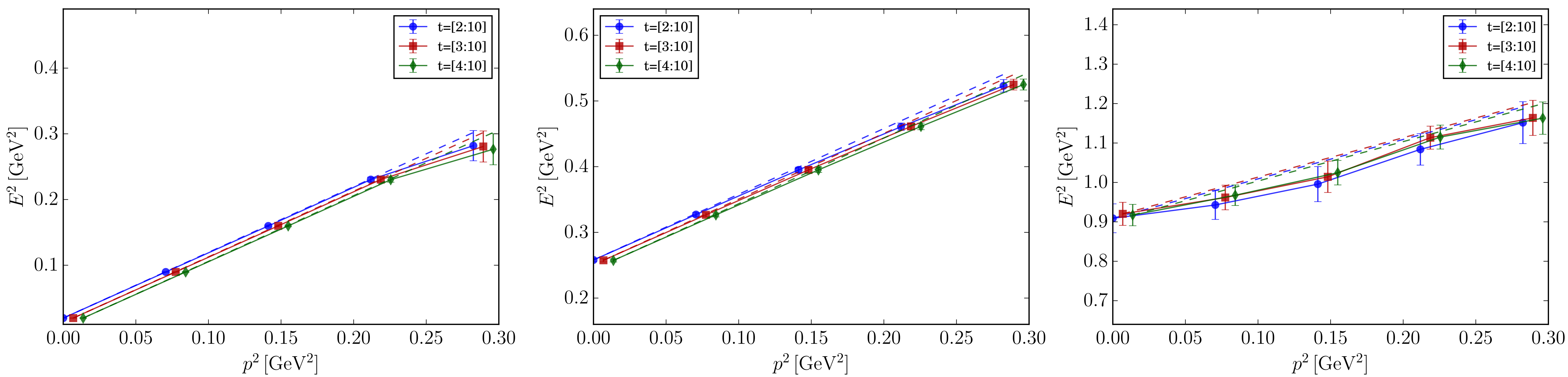
- Two lattice field ensembles:
  - $32^3\times 64$(a=0.14 fm) [32ID]
  - $24^3\times 64$(a=0.20 fm) [24ID]
- Chirally-symmetric (Mobius-)Domain Wall fermion action with physical light and strange quark masses
- Iwasaki gauge action+ Dislocation-supp. det.ratio (DSDR)

Proton and Meson Spectrum

- hadron effective energies



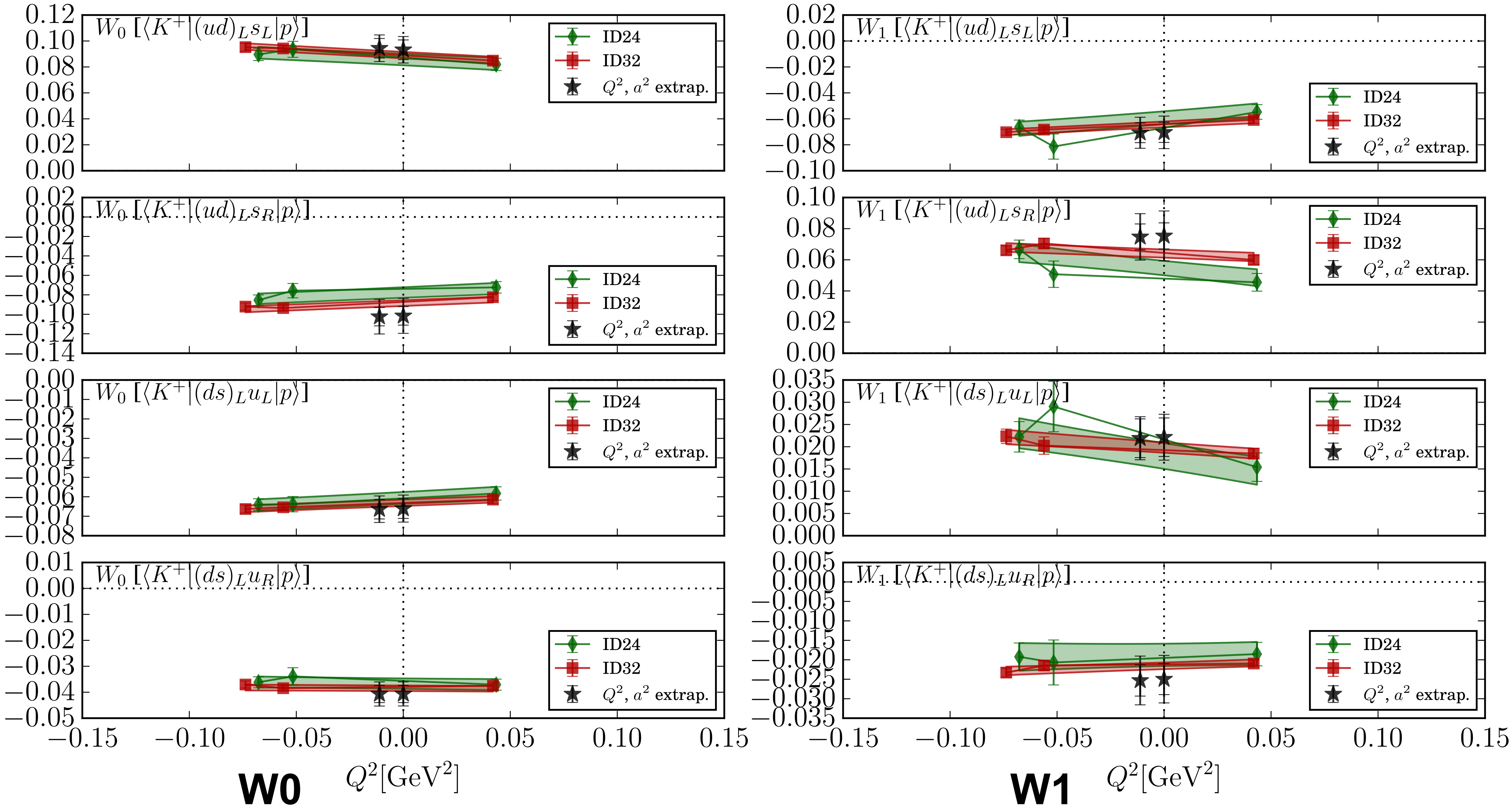
- hadron dispersion relations $E^2(p^2)$



pion **kaon** **nucleon**

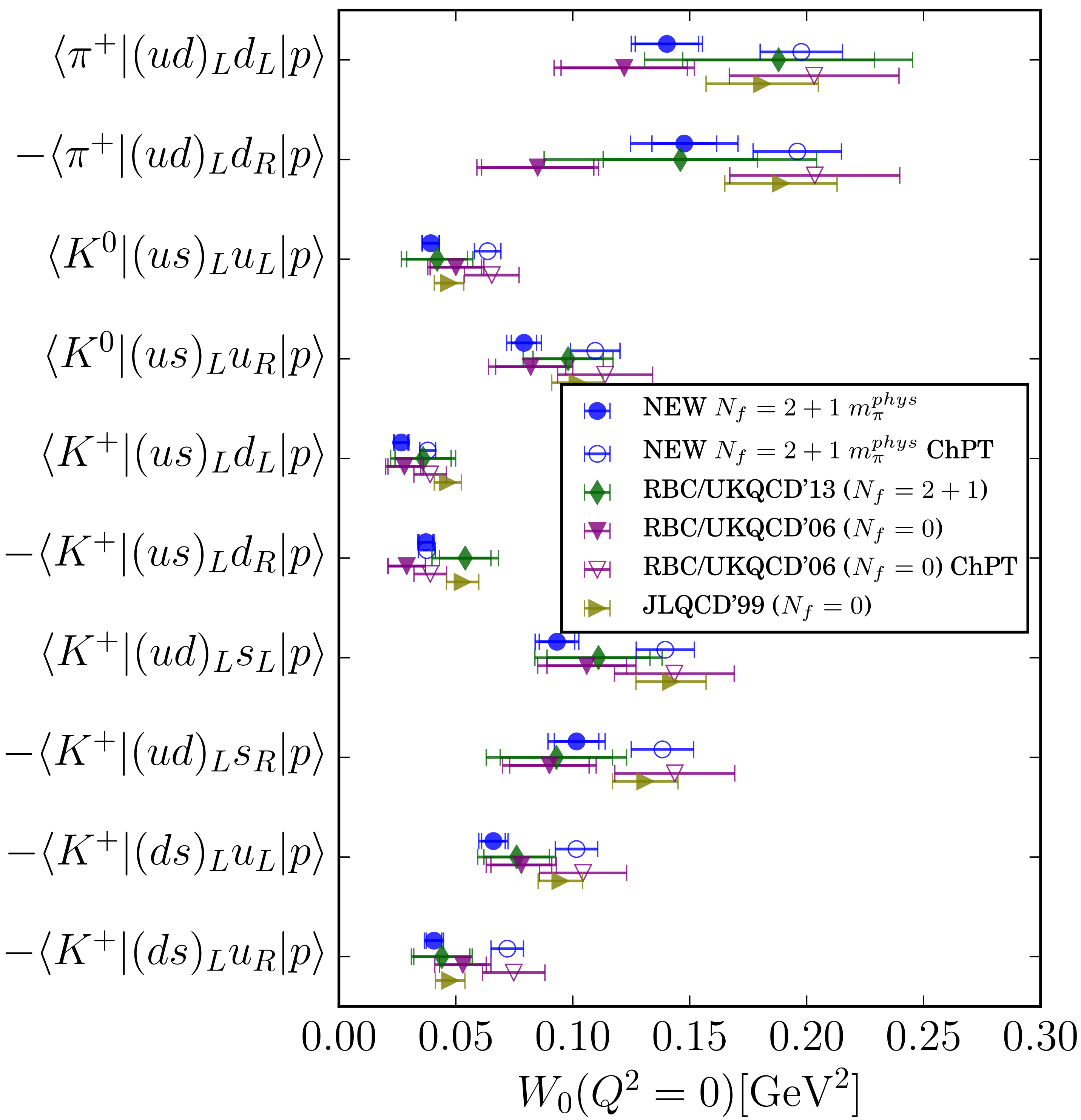
- 24ID ensemble (lattice spacing a=0.20 fm, $a^{-1} \approx$ 1 GeV)
- Two-state fits + priors from large-t_{min} one-state fits

Momentum and Continuum Extrapolation



- linear momentum extrapolation $Q^2 \rightarrow m_e^2, m_\mu^2$ to the decay kinematics
- Continuum extrapolation $A(a^2) \sim (A_0 + A_2\, a^2)$; sys.error = $|A_0 - A_{[a=0.14\text{fm}]}|$

Comparison to Previous Work



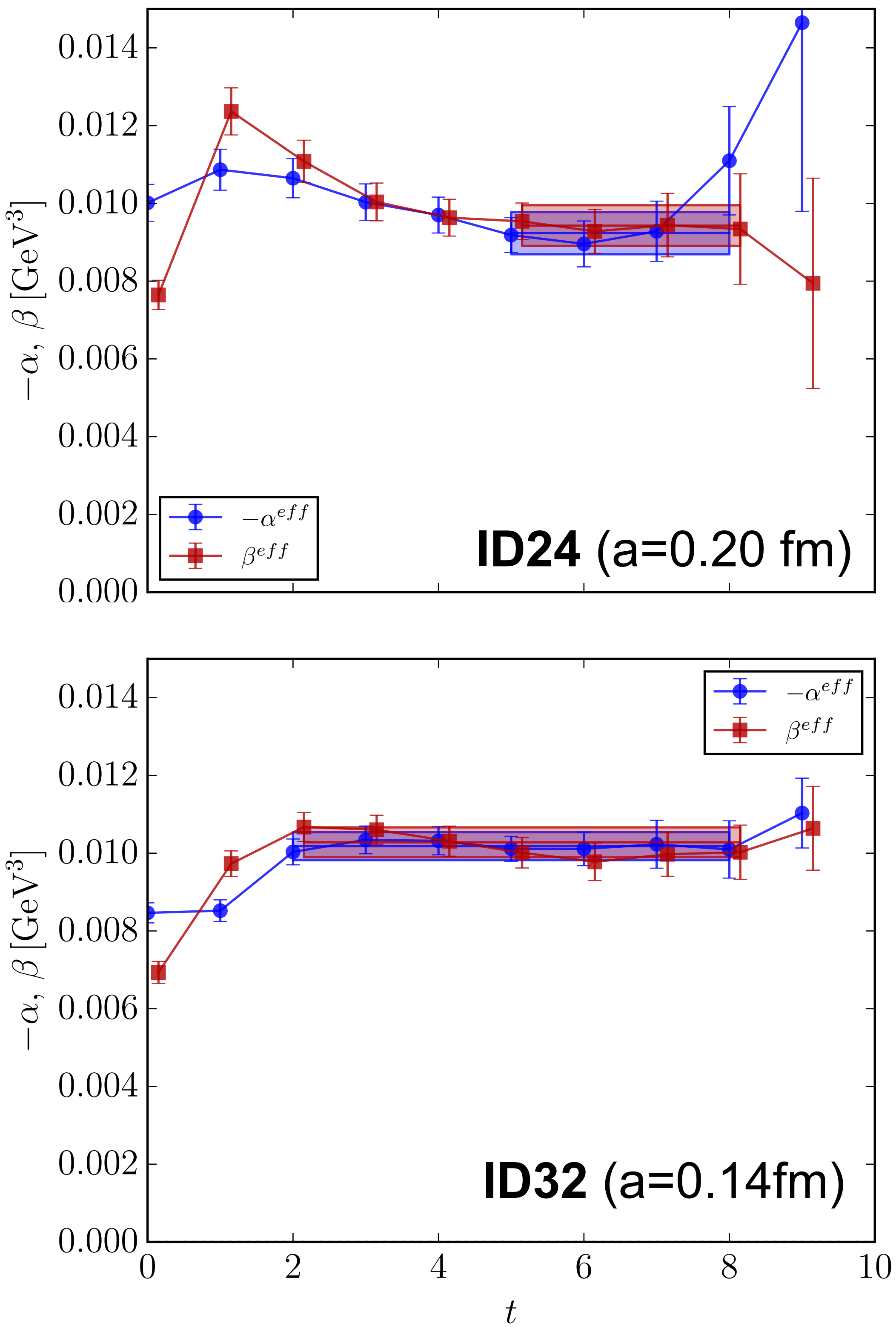
[J. Yoo, SS, et al, PRD105:074501 (2021)]

- New results:
  conservative sys. errors
  (stat+sys) precision ~ 10-20%
- No FVE study, mπ L~3.4
- physical-point results agree with prev. calculations at mπ ≳300 MeV

  [S.Aoki et al (2000)]
  [Y.Aoki et al (2006)]
  [Y.Aoki et al (2013)]

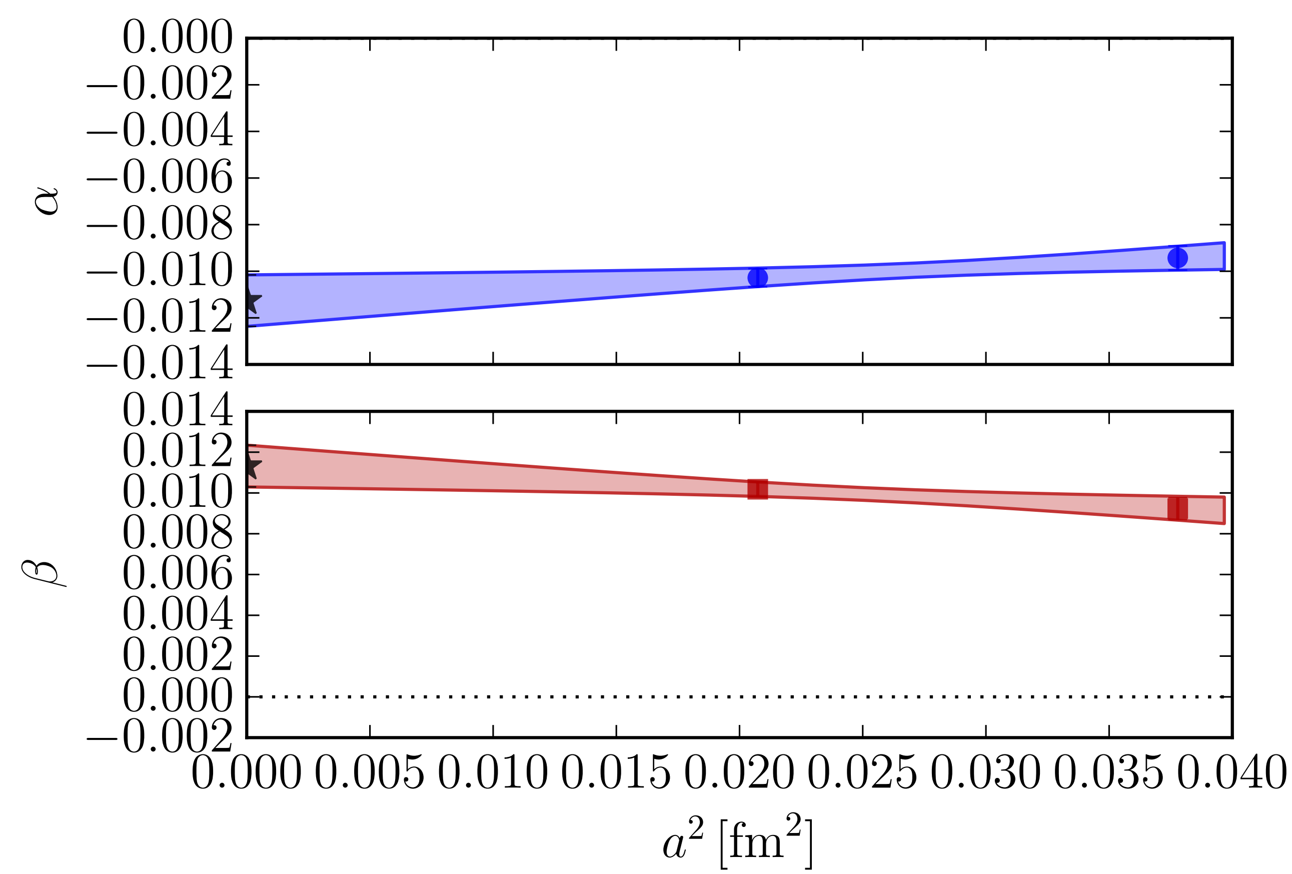
No suppression of nucleon decay due to chiral skyrmion topology

Proton Annihilation Amplitudes



$$\langle \mathrm{vac}|\epsilon^{abc}(\bar{u}^{aC} d^b)_R u^c_L|N\rangle = \alpha\, P_L U_N$$
$$\langle \mathrm{vac}|\epsilon^{abc}(\bar{u}^{aC} d^b)_L u^c_L|N\rangle = \beta\, P_L U_N$$

- $\langle \pi/\mathrm{K}|O^{3q}|N\rangle$ with soft-pion theorem
- nonpert.QCD component of $p \to 3\ell$



Summary & Conclusions

- Amplitudes of quark BNV operators computed in lattice QCD with realistic, chirally-symmetric quarks

- Neutron-antineutron oscillation

  Amplitudes × (6 ... 8) larger than from pheno.models

  Continuum limit study pending

  NEXT: nn→vacuum amplitudes, $n \to \bar{n}$ in nuclear medium

- Proton decays $p \to \pi/K$, p→leptons

  No topological suppression of nucleon decay found; confirm limits on GUTs

  Finer spacing, larger volume calculations desirable

  Need NLO ChPT for $p \to \pi/K$: cross-check vs. p→vacuum amplitude

  NEXT: $p \to \rho \to \pi\pi$, $p \to K^* \to \pi K$ amplitudes

BACKUP

This Work: Lattice Setup

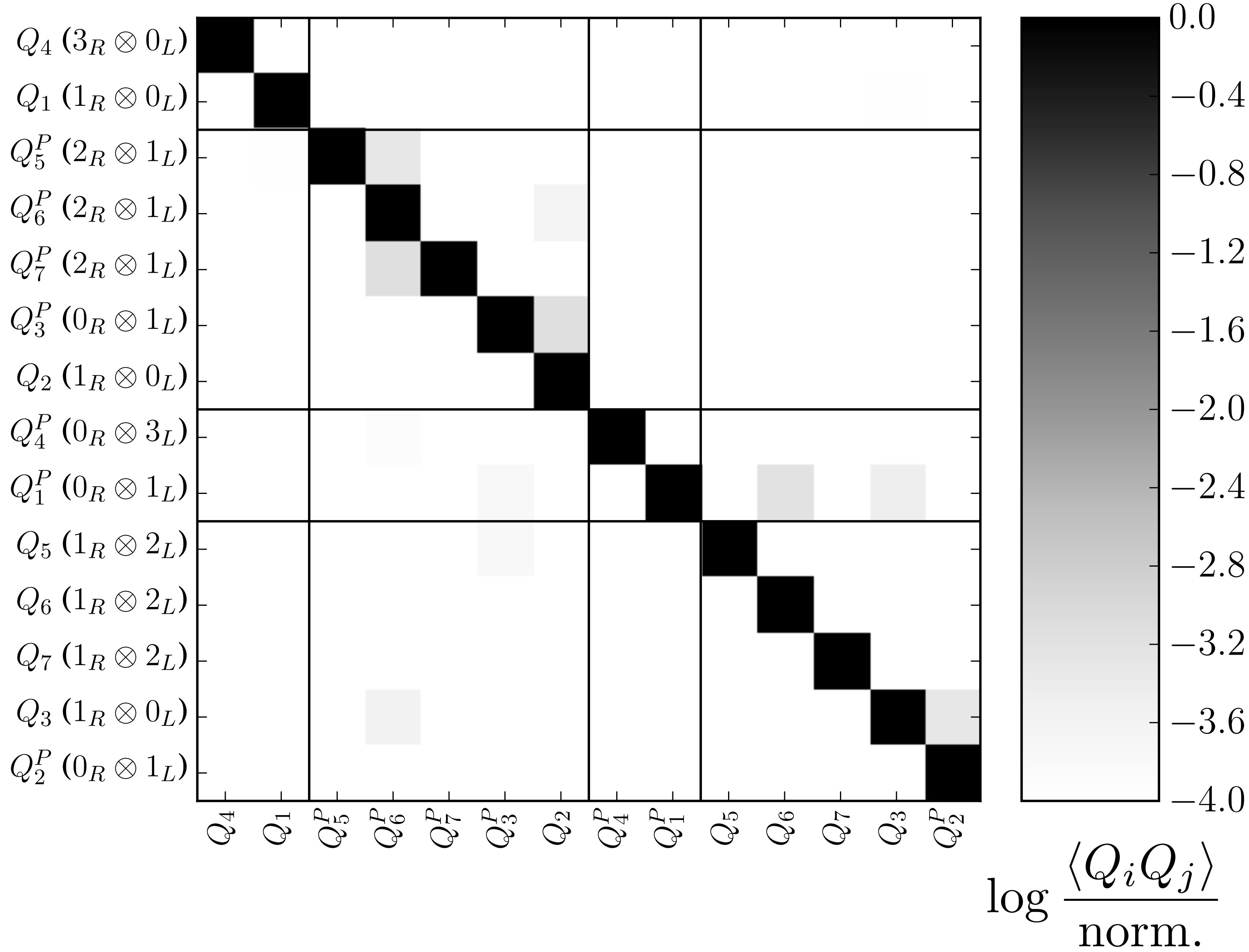
- Two ensembles: [32ID] $32^3{\times}64(a{=}0.14\ \mathrm{fm})$ and [24ID] $24^3{\times}64(a{=}0.20\ \mathrm{fm})$
- Iwasaki gauge action+ Dislocation-supp. det.ratio (DSDR)
- $N_f = 2{+}1$ Chirally-symmetric (Mobius-)Domain Wall fermion action with physical light and strange quark masses
- Multigrid deflation of z-Mobius operator + AMA
- "Direct" ($p \to \pi,K$ matrix elements) and "Indirect" ($p \to$vacuum + ChPT)
- Nonperturbative renormalization
- Two state-fit analysis of π,K,N spectrum and $p \to \pi,K$ matrix elements
- a^2 Continuum extrapolation

	24ID $24^3 \times 64$	32ID $32^3 \times 64$
β	1.633	1.75
a, fm	0.20	0.14
a^{-1}, GeV	1.02	1.37
$m_\pi L$	3.4	3.3
N_{conf}	134	94
N_{samp}	4288	3008

- three kinematic (Q^2) points to interpolate matrix elements to decay kinematic $Q^2 = -(m_{\bar{\ell}})^2$

Π	$\vec{n}_\Pi$	$\vec{n}_N$	Q^2(GeV^2) (24c)	Q^2(GeV^2) (32c)
π	[1 1 1]	[0 0 0]	0.010	−0.012
	[1 1 1]	[0 1 0]	0.113	0.095
	[0 0 2]	[0 0 0]	−0.116	−0.140
K	[0 1 1]	[0 0 0]	−0.034	−0.042
	[0 1 1]	[0 1 0]	0.058	0.056
	[0 0 1]	[0 0 0]	0.075	0.074

Nonperturbative Mixing of ⟨$\overline{n}$|Q|n⟩ Operators

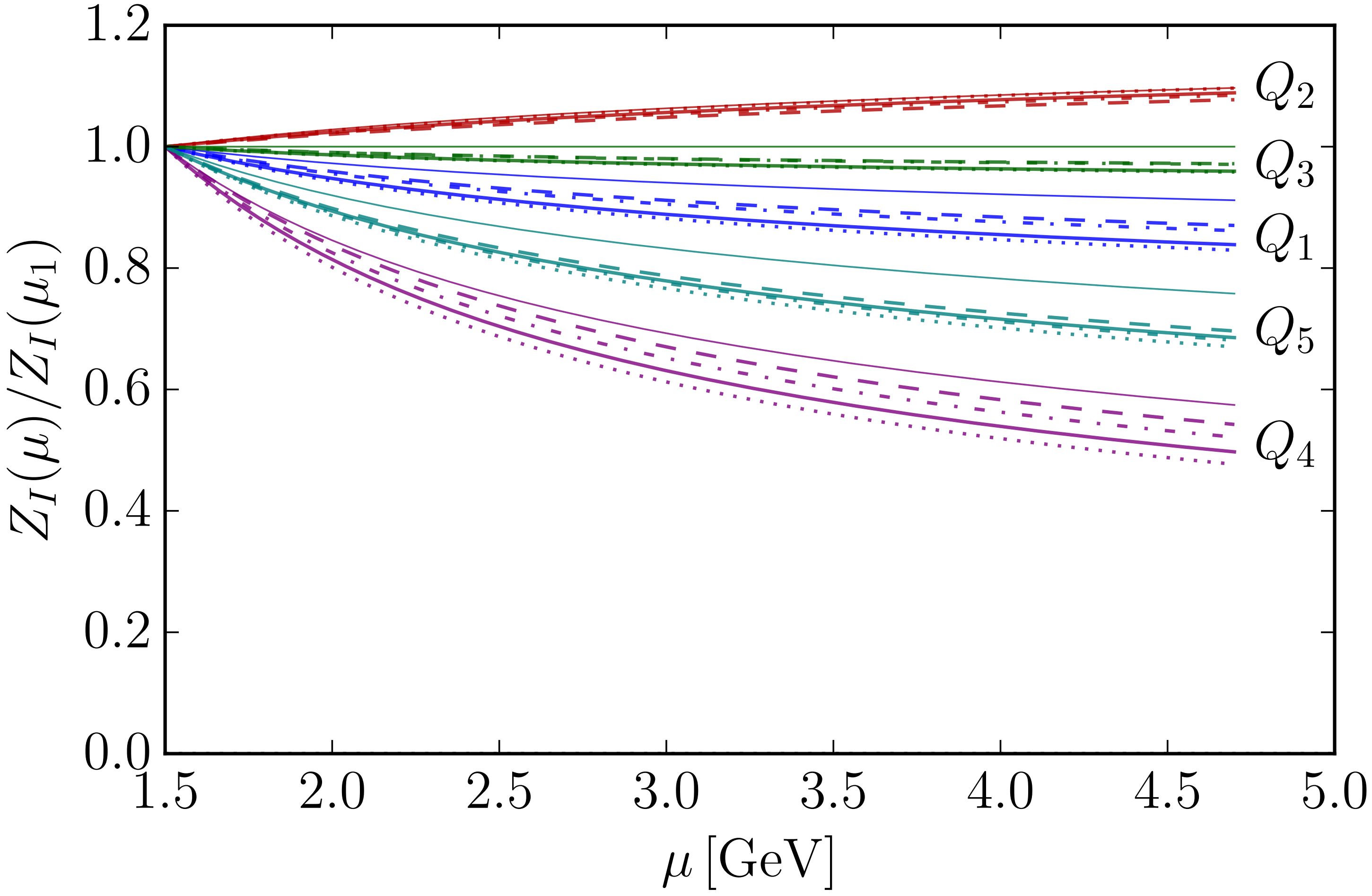


Nonperturbative mixing, normalized by diagonal ⟨Q_i Q_i⟩ correlators

- RI-MOM scheme: N_f=3 (solid) and N_f=4 (dotted)
- MSbar scheme: N_f=3 (dashed) and N_f=4 (dash-dotted)

Negligible mixing due to chiral symmetry of quark action

Perturbative Running of ⟨n̄|Q|n⟩ Operators



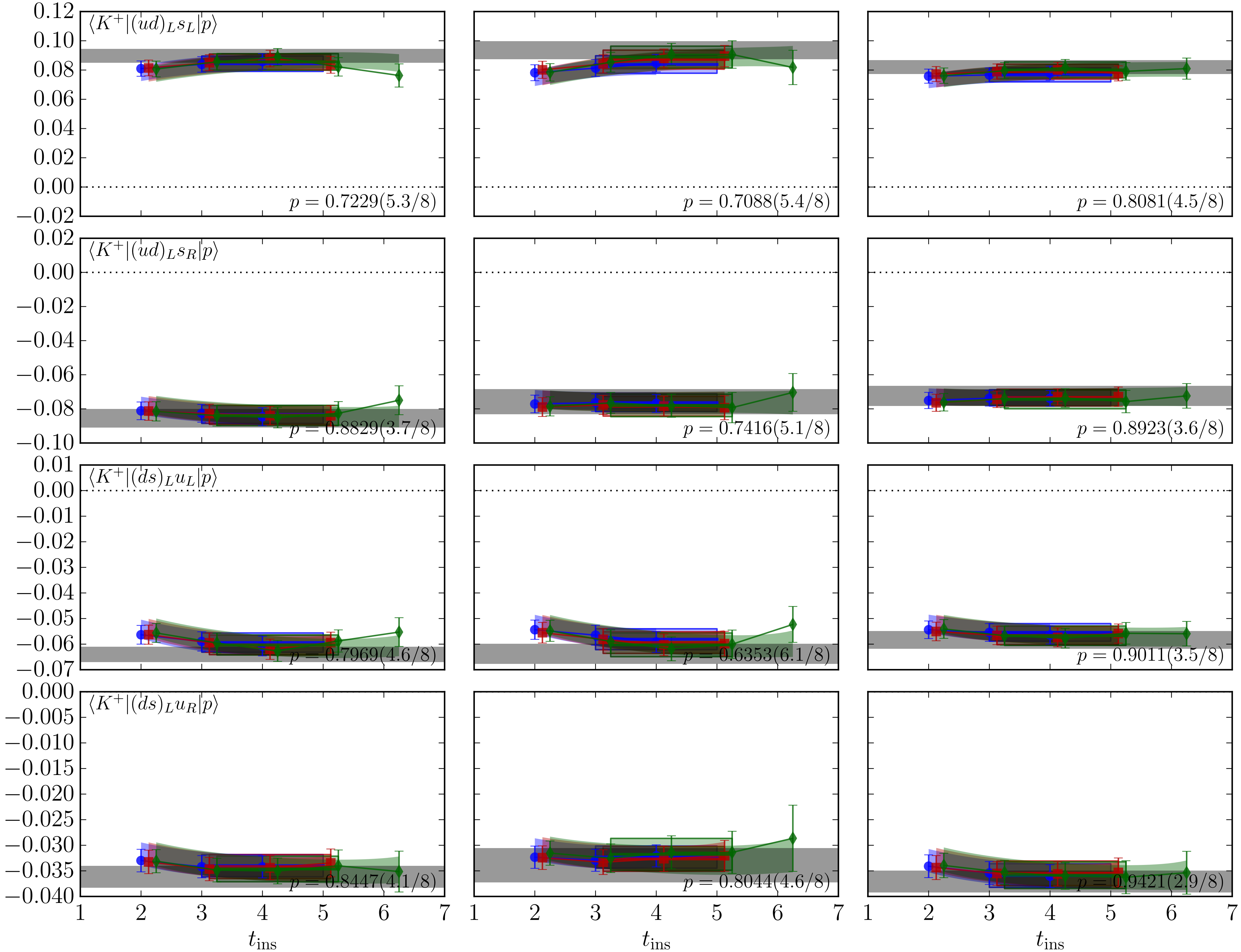
Perturbative running

- RI-MOM scheme: N_f=3 (solid) and N_f=4 (dotted)
- MSbar scheme: N_f=3 (dashed) and N_f=4 (dash-dotted)

Extraction of Matrix Elements

32ID

W₀



- Two-state fits with energies fixed from spectrum fits

Nonperturbative Renormalization

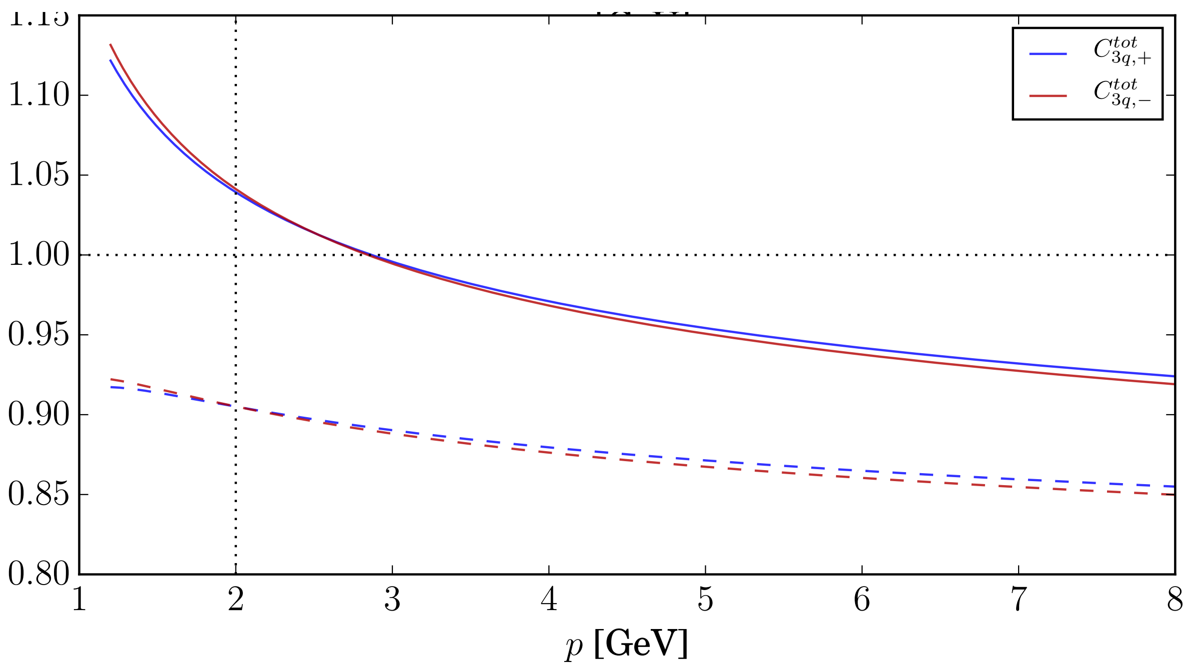
- symmetry-allowed mixing

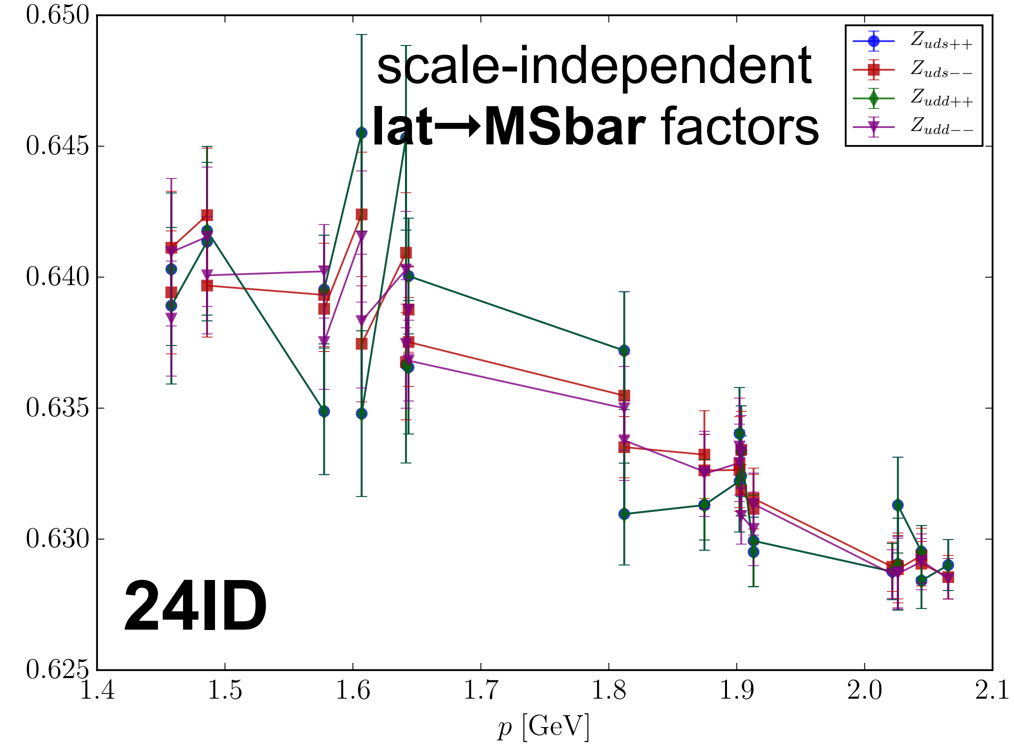
	$\mathcal{S}=-1$	$\mathcal{S}=+1$
$\mathcal{P}=-1$	SS, PP, AA	VV, TT
$\mathcal{P}=+1$	SP, PS, AV	VA, TQ

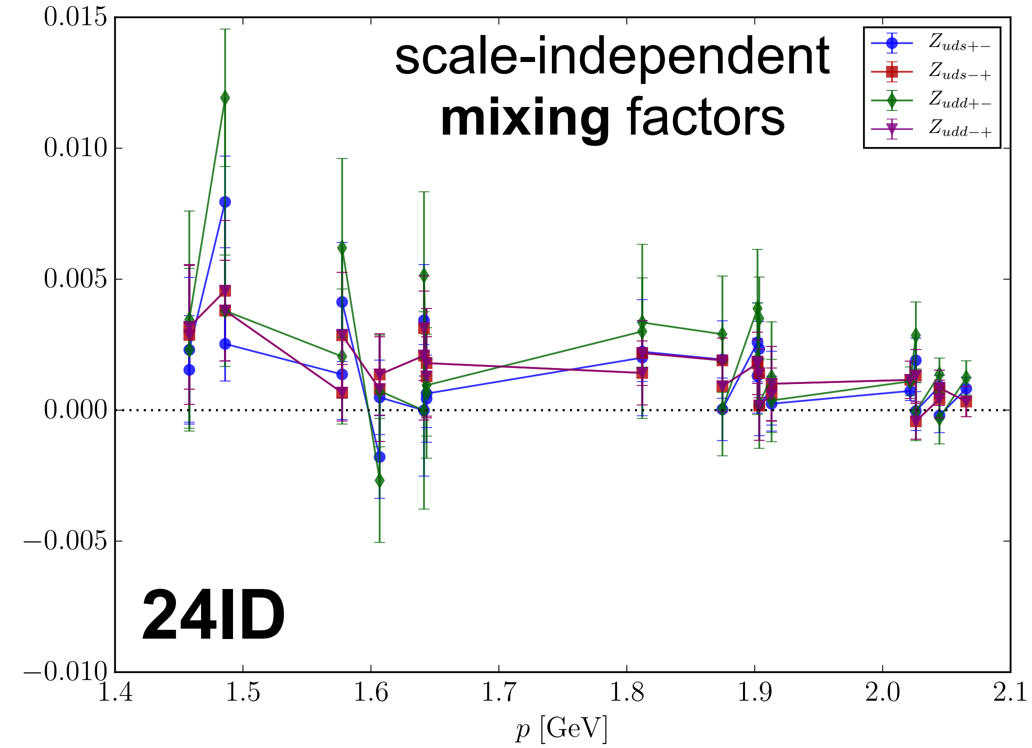
- *symmMOM* scheme : $p+q+r=0,\ p^2=q^2=r^2=\mu^2$

$$Z^{3q}_{IK}(\mu)\ \mathrm{Proj}_J\left[\langle\bar q_1(p)\bar q_2(q)\bar q_3(r)\,\mathcal{O}^{3q}_K\rangle_{\rm amp}\right]=\delta_{IJ}$$

- symmMOM(p)→MSbar(2 GeV) perturbative conversion at $O(\alpha^3)$ [J.Gracey, JHEP09:052 (2012)]







- chiral symmetry suppresses mixing of L⟺R fields & operators