



Center for Theoretical  
Physics



Massachusetts  
Institute of  
Technology

# Polarized jet anisotropy at the future EIC

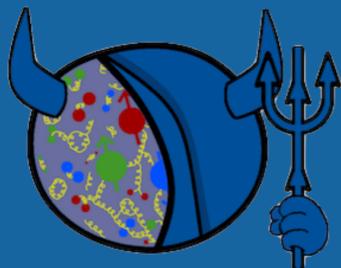
Fanyi Zhao

MIT CTP

R. Esha, Z.-B. Kang, K. Lee, D. Shao and [FZ](#), arXiv:2309.XXXX

Z.-B. Kang, H. Xing, Y. Zhou and [FZ](#), arXiv:2309.XXXX

September 26th, 2023

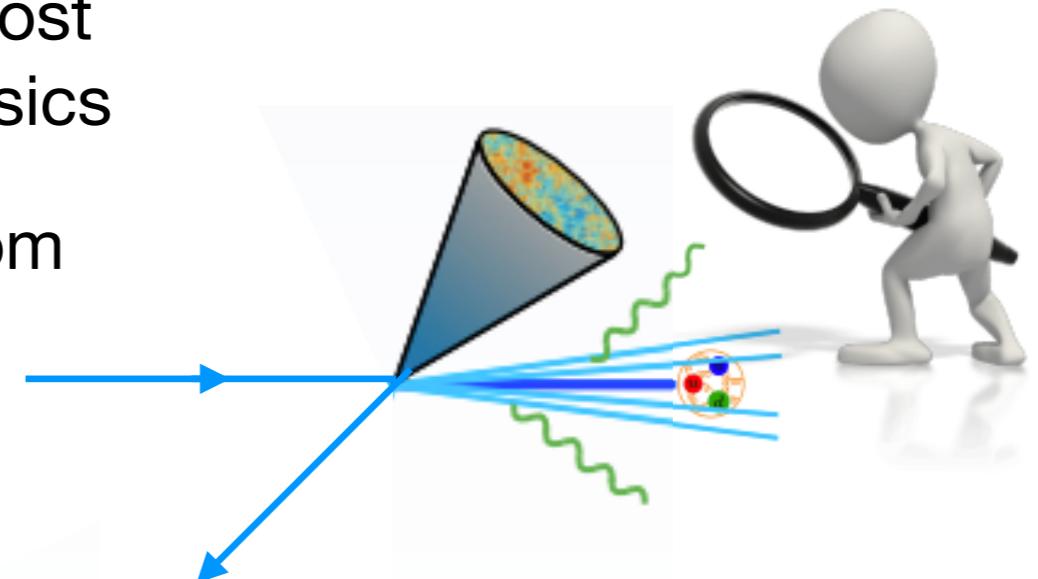


25th International Spin Symposium (SPIN 2023)

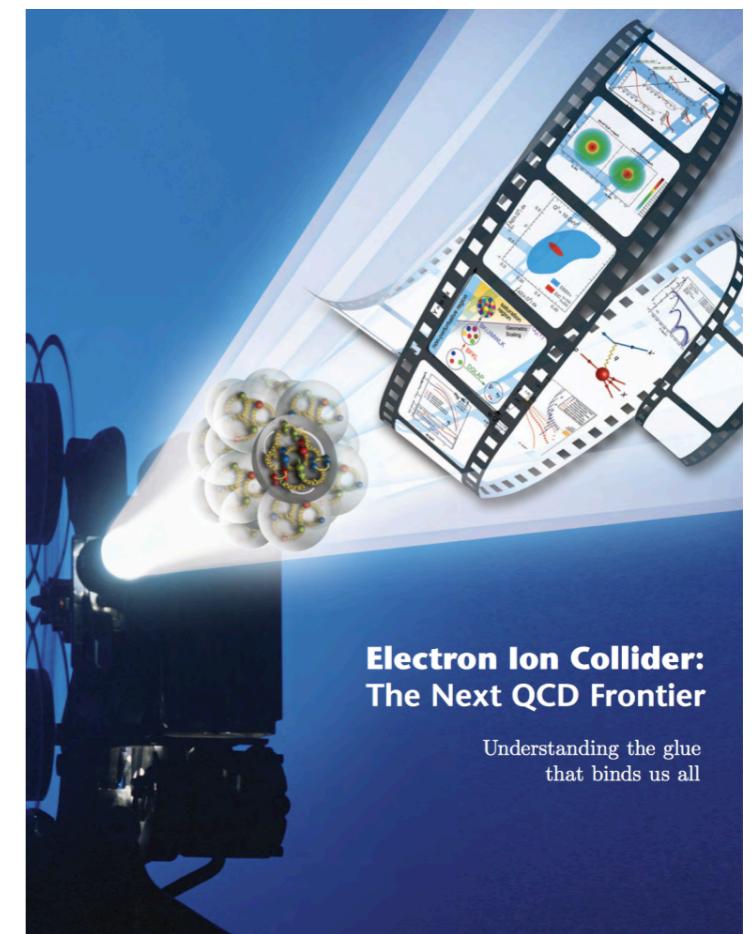
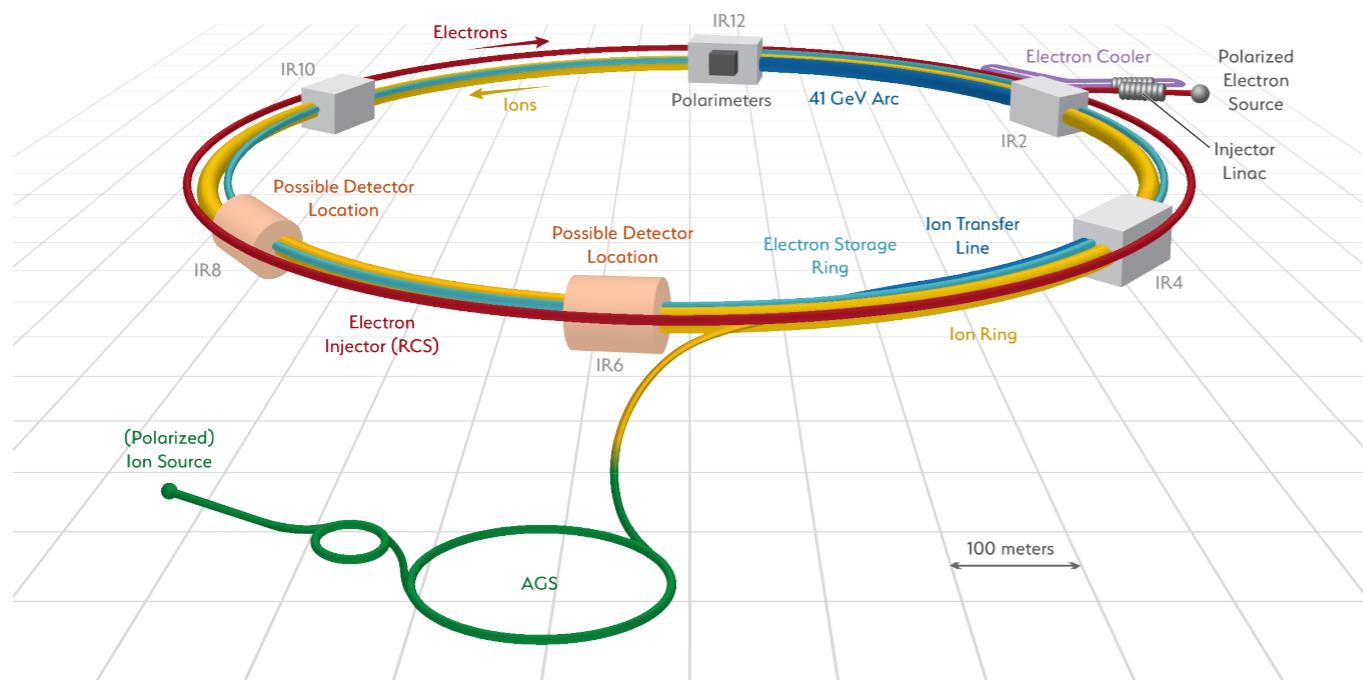
# Motivation

- 3D imaging of the nucleon: one of the most important research topics in hadron physics
- Back-to-back electron-jet production from  $ep$  collision  $\Rightarrow$  probe TMD PDFs

$$e(l) + p(P) \rightarrow e(l') + J_q(p_J) + X$$



Liu, Ringer, Vogelsang, Yuan PRL '18, PRD '20  
Arratia, Kang, Prokudin, Ringer PRD '20  
Kang, Lee, Shao, **Zhao**, JHEP '21



# Motivation

- Back-to-back electron-jet production from  $ep$  collision,

$$e(l) + p(P) \rightarrow e(l') + J_q(p_J) + X$$

$$q_T = |l'_T + p_T|,$$

$p_T$ : jet transverse momentum

Liu, Ringer, Vogelsang, Yuan PRL '18, PRD '20

Arratia, Kang, Prokudin, Ringer PRD '20

Kang, Lee, Shao, Zhao, JHEP '21

Unpolarized:

$$\frac{d\sigma^{ep \rightarrow e + \text{jet} + X}}{dp_T dq_T} = f_1(x, k_{c,\perp}) \otimes S_{\text{global}}(k_{\text{gs},\perp}) \otimes S_{sc}(k_{\text{sc},\perp}) \\ \times H(Q) J_c(p_T R)$$

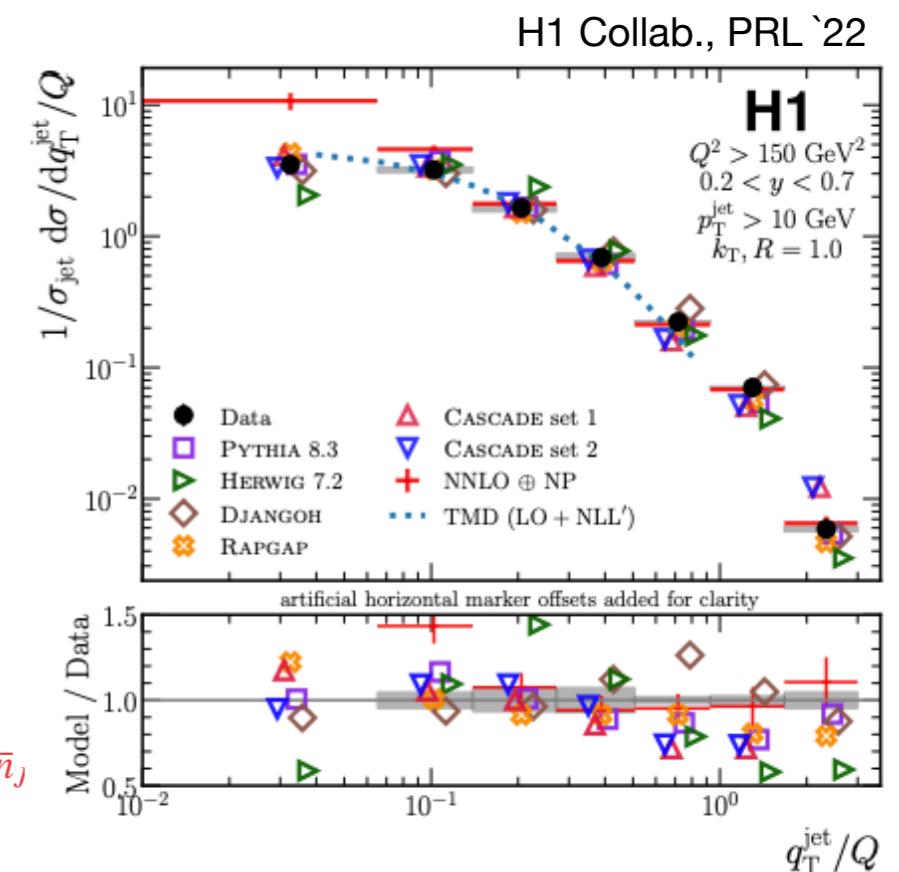
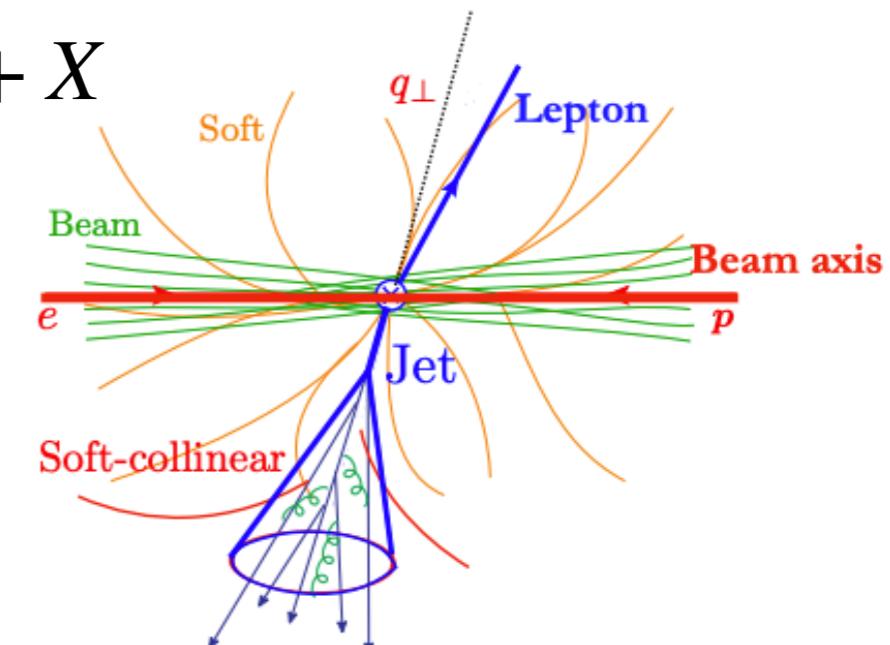
Relevant modes ( $\lambda = q_T/p_T$ ):

$$n\text{-collinear: } k_c \sim p_T(\lambda^2, 1, \lambda)_{n, \bar{n}}$$

$$\text{global soft: } k_{\text{gs}} \sim p_T(\lambda, \lambda, \lambda)$$

$$n_J\text{-collinear: } k_J \sim p_T(R^2, 1, R)_{n_J, \bar{n}_J}$$

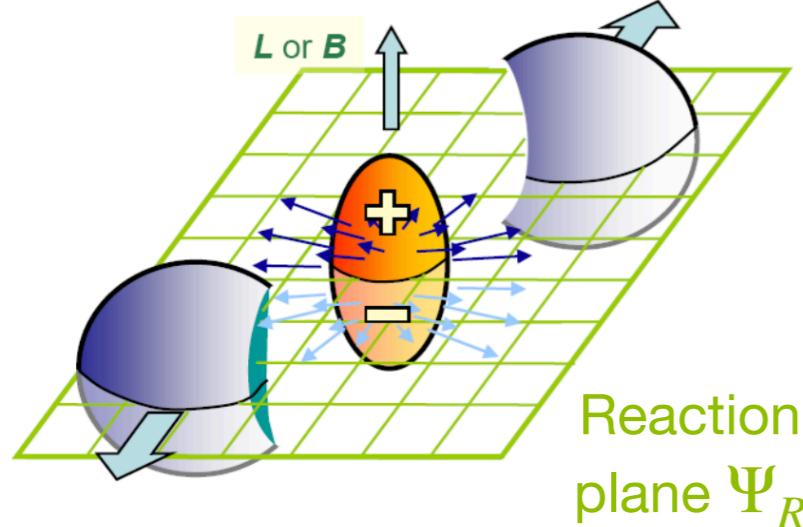
$$\text{Soft-collinear: } k_{\text{sc}} \sim p_T R(\lambda R, \lambda/R, \lambda)_{n_J, \bar{n}_J}$$



# Motivation

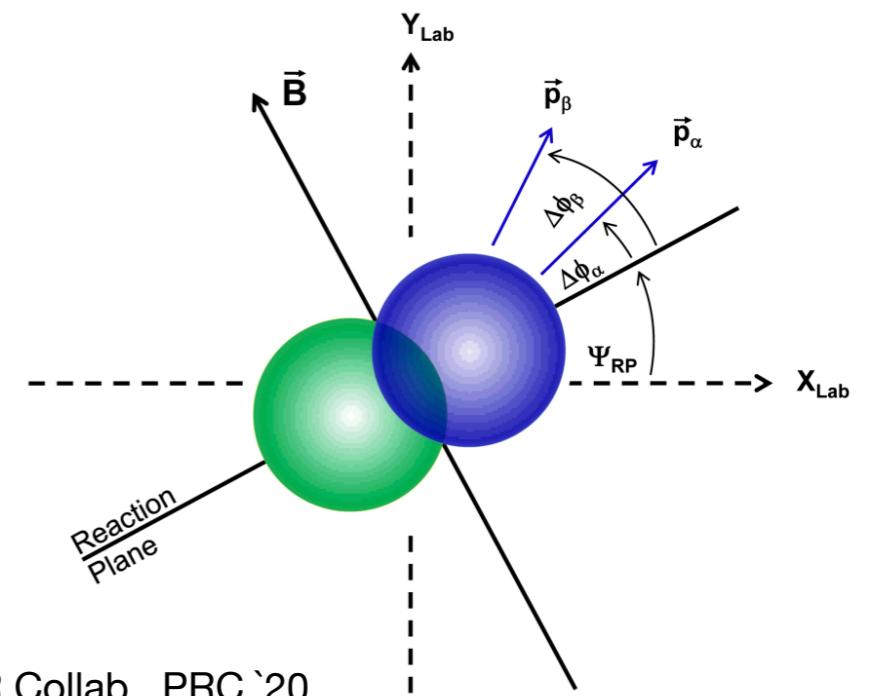
- Anisotropic flow expansions in heavy-ion collisions,

$$E \frac{d^3N}{d^3\mathbf{p}} = \frac{1}{2\pi} \frac{d^2N}{p_t dp_t dy} \left( 1 + 2 \sum_{n=1}^{\infty} v_n \cos [n(\varphi - \Psi_{RP})] \right), \quad v_n(p_t, y) = \langle \cos[n(\varphi - \Psi_{RP})] \rangle$$



STAR Collab., PRC '20

- $v_1$  : directed flow
- $v_2$  : elliptic flow
- $v_3$  : triangular flow
- $v_4$  : quadrupole flow ...

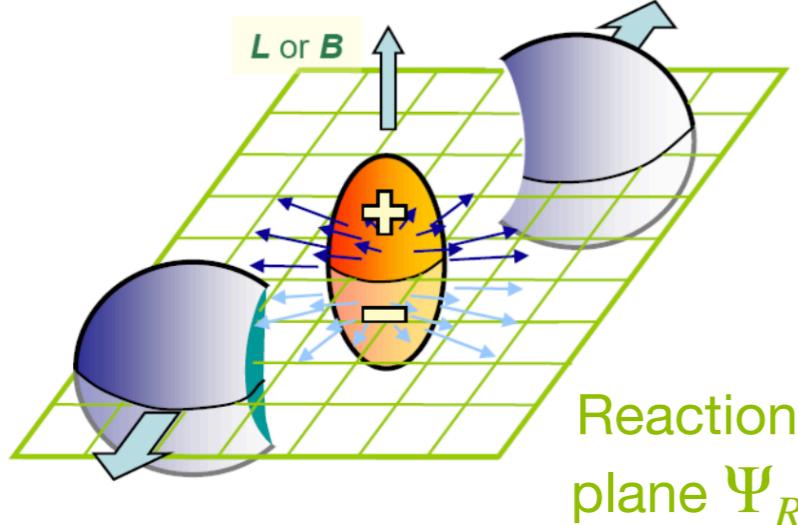


STAR Collab., PRC '20

# Motivation

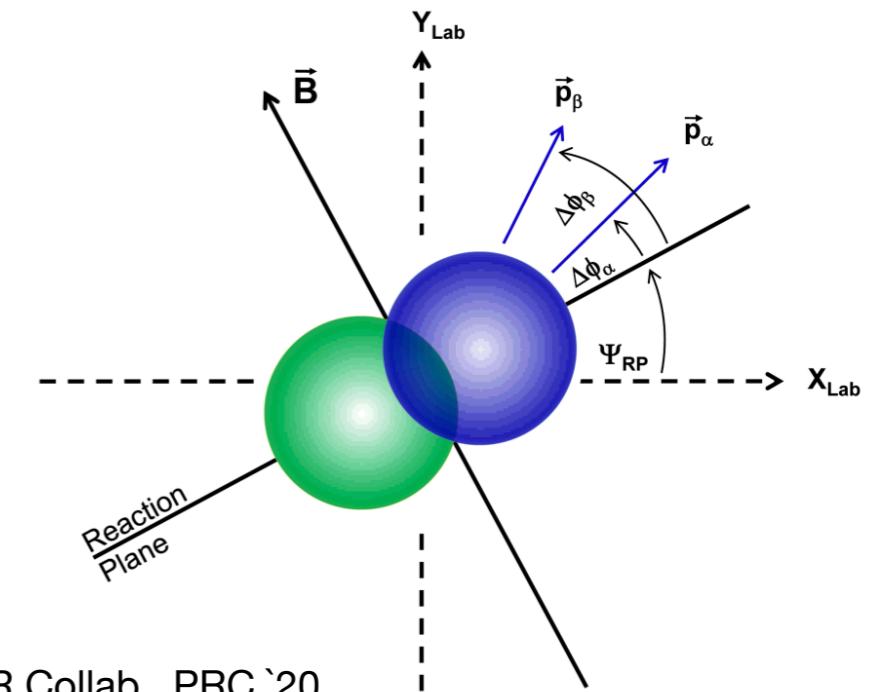
- Anisotropic flow expansions in heavy-ion collisions,

$$E \frac{d^3N}{d^3\mathbf{p}} = \frac{1}{2\pi} \frac{d^2N}{p_t dp_t dy} \left( 1 + 2 \sum_{n=1}^{\infty} v_n \cos [n(\varphi - \Psi_{RP})] \right), \quad v_n(p_t, y) = \langle \cos[n(\varphi - \Psi_{RP})] \rangle$$



STAR Collab., PRC '20

- $v_1$  : directed flow
- $v_2$  : elliptic flow
- $v_3$  : triangular flow
- $v_4$  : quadrupole flow ...



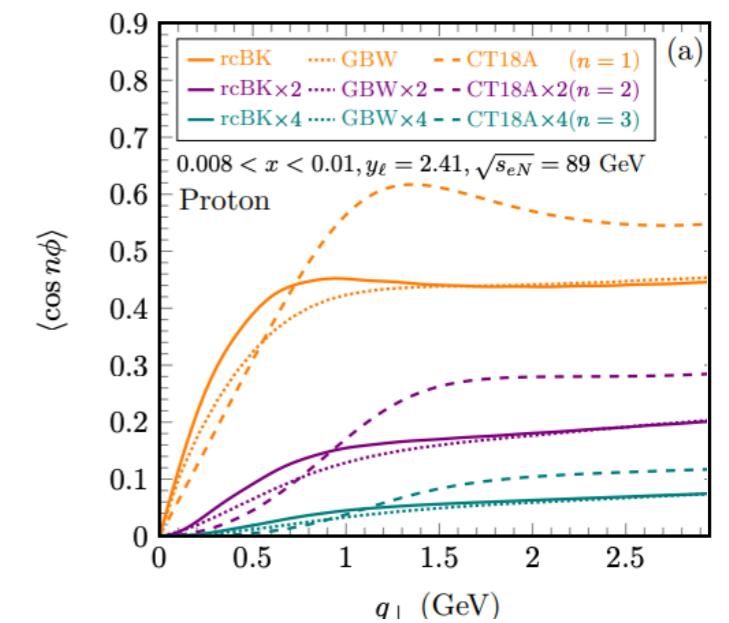
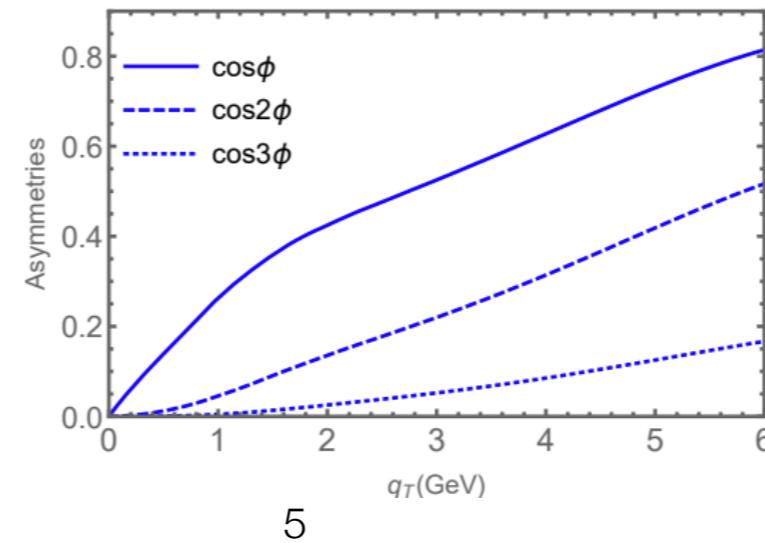
STAR Collab., PRC '20

- Azimuthal anisotropy in unpolarized  $ep$  collisions,

Hatta, Xiao, Yuan, Zhou PRL '21

Tong, Xiao, Zhang PRL '22

Esha, Kang, Lee, Shao, Zhao, '23



# Polarized jet anisotropy

- Back-to-back electron-jet production from  $ep$  collision,

$$e(l, \lambda_e) + p(P, S) \rightarrow e(l') + J_q(p_J) + X$$

Kang, Lee, Shao, Zhao, JHEP '21

$$\frac{d\sigma}{d^2\mathbf{p}_T dy_J d\phi_J d^2\mathbf{q}_T} = F_{UU} + \lambda_p \lambda_e F_{LL} + S_T \left[ \sin(\phi_q - \phi_S) F_{UT}^{\sin(\phi_q - \phi_S)} + \lambda_e \cos(\phi_q - \phi_S) F_{LT}^{\cos(\phi_q - \phi_S)} \right]$$

$\sim f_1$        $\sim g_1$        $\sim f_{1T}^\perp$        $\sim g_{1T}$

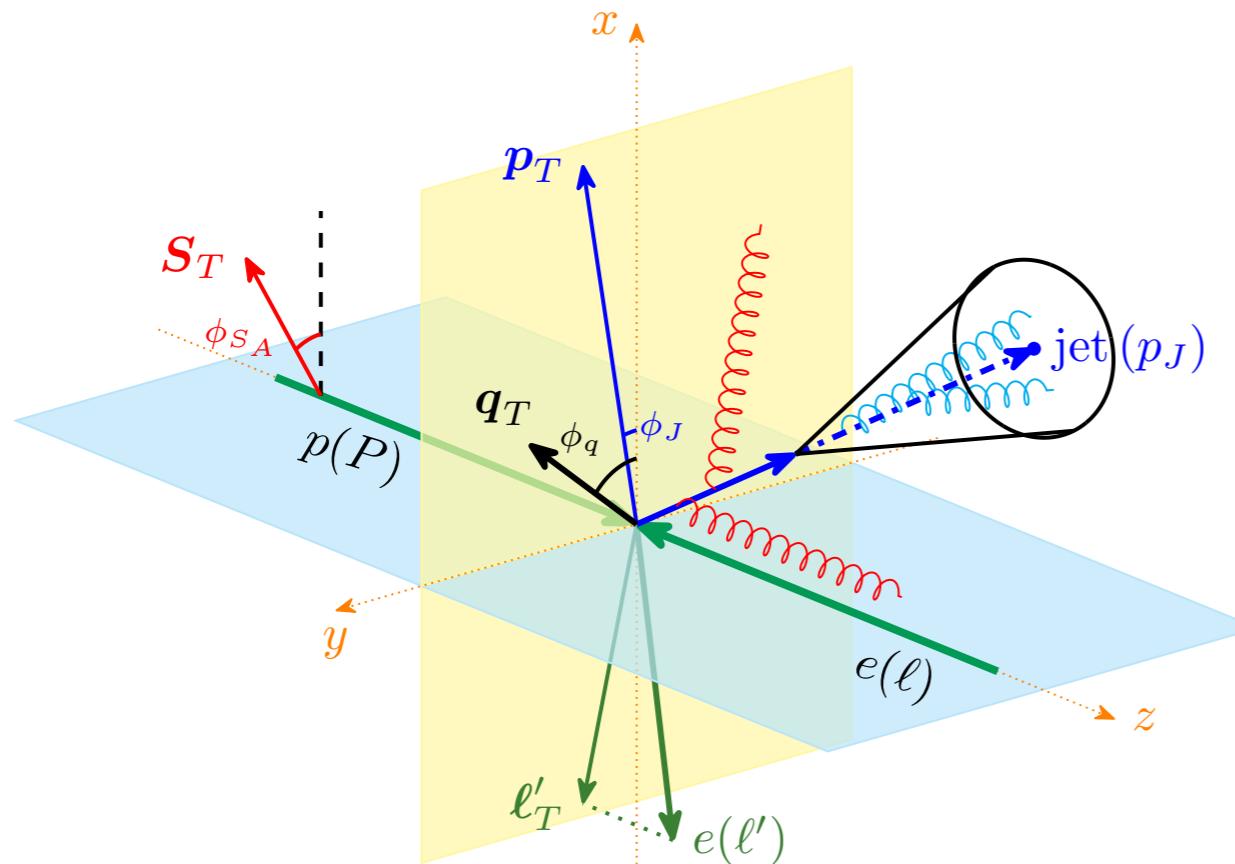
$$F_{AB} : AB = UU, LL, UT, LT$$

$A$  : electron polarization

$\lambda_e$

$B$  : proton polarization

$\lambda_p, S_T$



# Polarized jet anisotropy

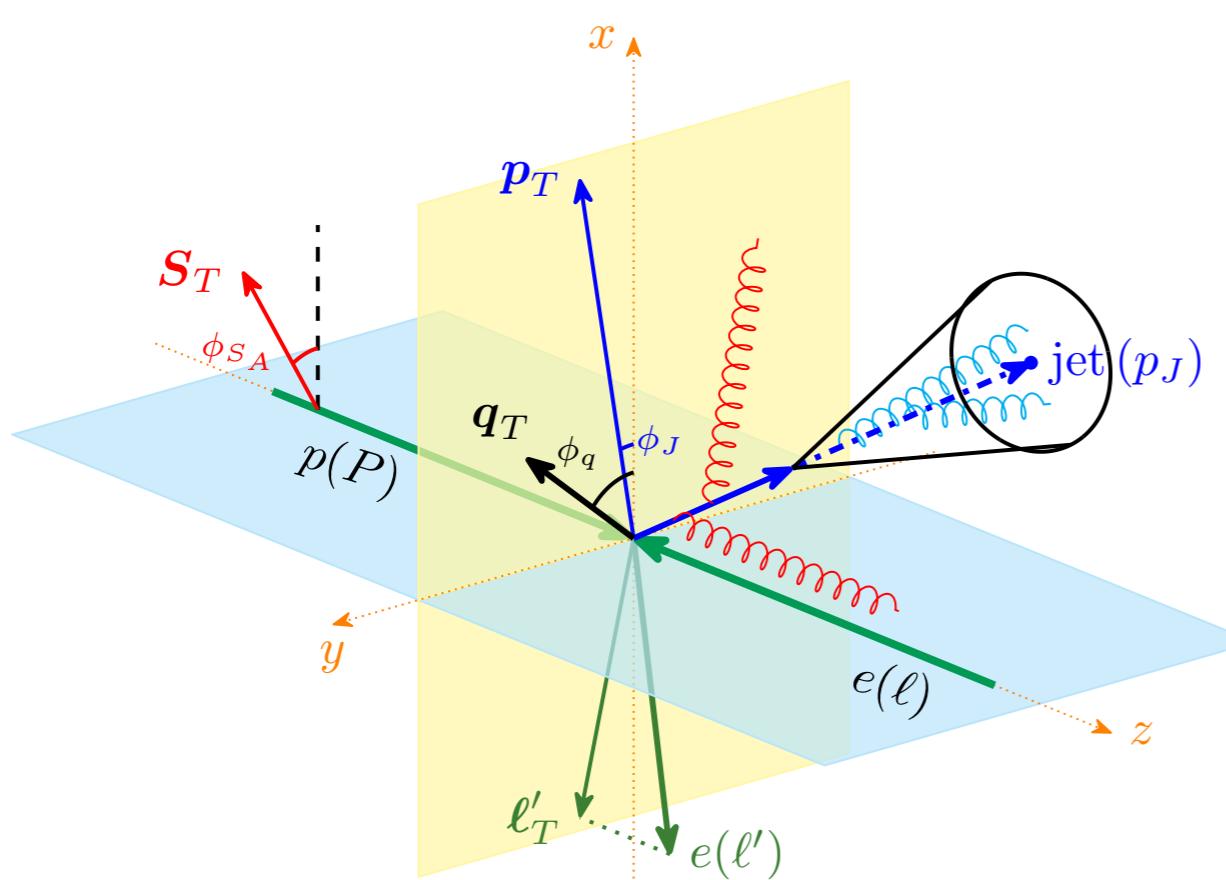
- Back-to-back electron-jet production from  $ep$  collision,

$$e(l, \lambda_e) + p(P, S) \rightarrow e(l') + J_q(p_J) + X$$

Kang, Lee, Shao, Zhao, JHEP '21

$$\frac{d\sigma}{d^2\mathbf{p}_T dy_J d\phi_J d^2\mathbf{q}_T} = F_{UU} + \lambda_p \lambda_e F_{LL} + S_T \left[ \sin(\phi_q - \phi_S) F_{UT}^{\sin(\phi_q - \phi_S)} + \lambda_e \cos(\phi_q - \phi_S) F_{LT}^{\cos(\phi_q - \phi_S)} \right]$$

$$\rightarrow = \frac{d\sigma}{2\pi d^2\mathbf{p}_T dy_J q_T dq_T} \left[ 1 + 2 \sum_{n=1}^{\infty} \sum_{AB} v_{AB}^n \cos(n(\phi_q - \phi_J)) \right]$$



$$v_{AB}^n(p_T, y_J, \lambda_e, S, \phi_S, \phi_q)$$

$$AB = UU, LL, UT, LT$$

$A$  : electron polarization

$B$  : proton polarization

$q_T$  : transverse momentum imbalance

$$\mathbf{q}_T = \mathbf{l}'_T + \mathbf{p}_T$$

$p_T$  : jet transverse momentum

$y_J$  : jet rapidity

# Polarized jet anisotropy

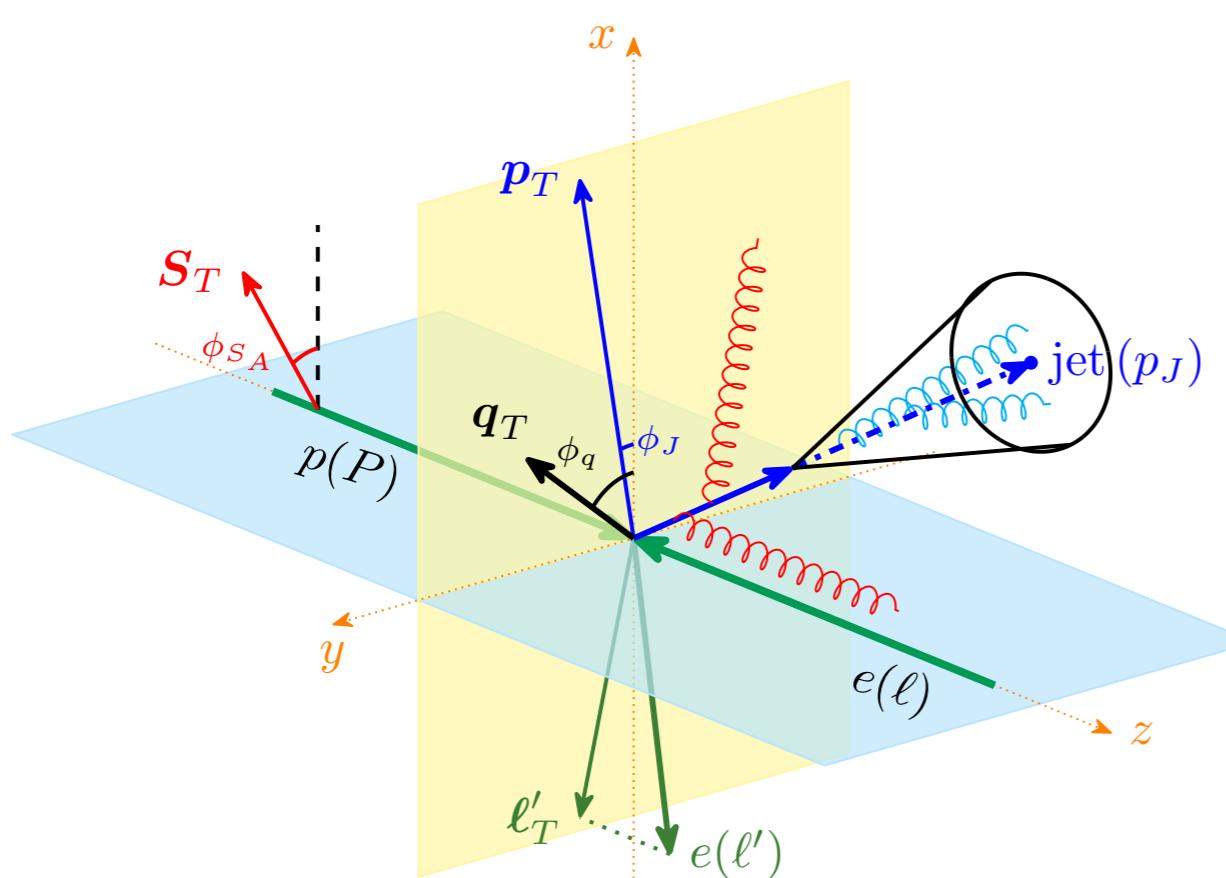
- Back-to-back electron-jet production from  $ep$  collision,

$$e(l, \lambda_e) + p(P, S) \rightarrow e(l') + J_q(p_J) + X$$

Kang, Lee, Shao, Zhao, JHEP '21

$$\frac{d\sigma}{d^2\mathbf{p}_T dy_J d\phi_J d^2\mathbf{q}_T} = F_{UU} + \lambda_p \lambda_e F_{LL} + S_T \left[ \sin(\phi_q - \phi_S) F_{UT}^{\sin(\phi_q - \phi_S)} + \lambda_e \cos(\phi_q - \phi_S) F_{LT}^{\cos(\phi_q - \phi_S)} \right]$$

$$\rightarrow = \frac{d\sigma}{2\pi d^2\mathbf{p}_T dy_J q_T dq_T} \left[ 1 + 2 \sum_{n=1}^{\infty} \sum_{AB} v_{AB}^n \cos(n(\phi_q - \phi_J)) \right]$$



$$v_{AB}^n(p_T, y_J, \lambda_e, S, \phi_S, \phi_q)$$

$\phi_q$  : azimuthal angle of transverse momentum imbalance

$\phi_J$  : azimuthal angle of jet transverse momentum

$\phi_S$  : azimuthal angle of initial proton transverse spin

$v_{AB}^n$  : anisotropic Fourier coefficients

# Polarized jet anisotropy

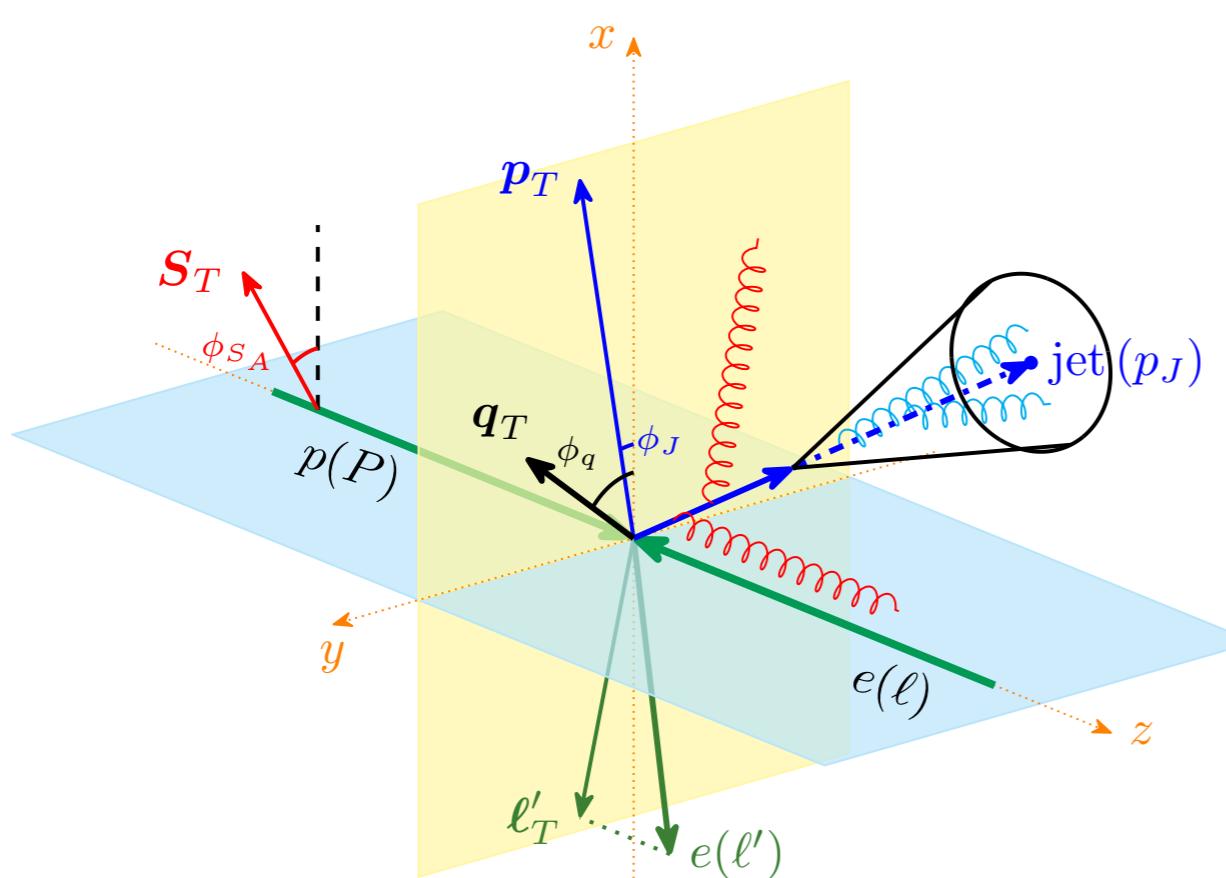
- Back-to-back electron-jet production from  $ep$  collision,

$$e(l, \lambda_e) + p(P, S) \rightarrow e(l') + J_q(p_J) + X$$

Kang, Lee, Shao, Zhao, JHEP '21

$$\frac{d\sigma}{d^2\mathbf{p}_T dy_J d\phi_J d^2\mathbf{q}_T} = F_{UU} + \lambda_p \lambda_e F_{LL} + S_T \left[ \sin(\phi_q - \phi_S) F_{UT}^{\sin(\phi_q - \phi_S)} + \lambda_e \cos(\phi_q - \phi_S) F_{LT}^{\cos(\phi_q - \phi_S)} \right]$$

$$\Rightarrow \frac{d\sigma}{2\pi d^2\mathbf{p}_T dy_J q_T dq_T} = \left[ 1 + 2 \sum_{n=1}^{\infty} \sum_{AB} v_{AB}^n \cos(n(\phi_q - \phi_J)) \right]$$



$$v_{AB}^n(p_T, y_J, \lambda_e, S, \phi_S, \phi_q)$$

$\phi_q$  : azimuthal angle of transverse momentum imbalance

$\phi_J$  : azimuthal angle of jet transverse momentum

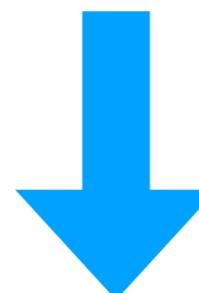
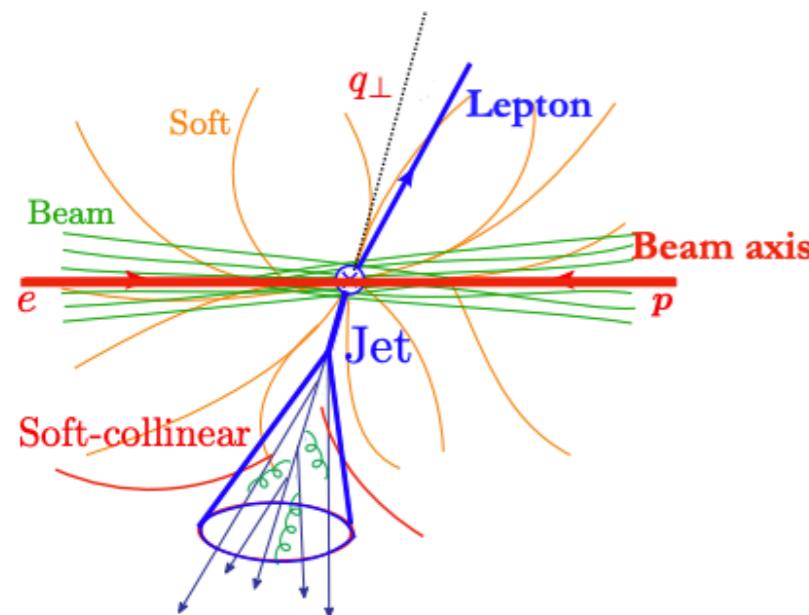
$\phi_S$  : azimuthal angle of initial proton transverse spin

$v_{AB}^n$  : anisotropic Fourier coefficients

# Polarized jet anisotropy

- E.g.  $F_{UT}^{\sin(\phi_q - \phi_S)} :$

$$F_{UT}^{\sin(\phi_q - \phi_S)} = \hat{\sigma}_0 H(Q, \mu) \sum_q e_q^2 \mathcal{J}_q(p_T R, \mu) \int \frac{b db}{2\pi} e^{-iq_T b} x \tilde{f}_{1T}^{\perp, q, (1)}(x, b^2, \mu, \zeta) \\ \times \int \frac{d\phi_{bq}}{2\pi} i b \cos(\phi_{bq}) M S_{\text{global}}(\mathbf{b}, \mu) S_{cs}(\mathbf{b}, R, \mu)$$



$$ib \cos(\phi_{bq}) e^{-ibq_T \cos(\phi_{bq})} \\ = -b \left( -J_1 + \sum_{n=1}^{\infty} -i^n (J_{n-1} - J_{n+1}) \cos(n\phi_{bq}) \right)$$

Jacobi-Anger identity

$$F_{UT}^{\sin(\phi_q - \phi_S)} = \sum_{n=0}^{\infty} A_n^{\text{Sivers}}$$

$$A_n^{\text{Sivers}} \equiv \hat{\sigma}_0 H(Q, \mu) \sum_q e_q^2 \mathcal{J}_q(p_T R, \mu) \int \frac{b^2 db}{2\pi} x M \tilde{f}_{1T}^{\perp, q, (1)}(x, b^2, \mu, \zeta) (-i)^n \left( \frac{1}{2} \right)^{\delta_{0n}} (J_{n+1}(q_T b) - J_{n-1}(q_T b))$$

$$\times \int \frac{d\phi_{bJ}}{2\pi} \cos(n(\phi_{bJ} - \phi_{qJ})) S_{\text{global}}(\mathbf{b}, \mu) S_{cs}(\mathbf{b}, R, \mu)$$

# Polarized jet anisotropy

- E.g.  $F_{UT}^{\sin(\phi_q - \phi_S)}$  :

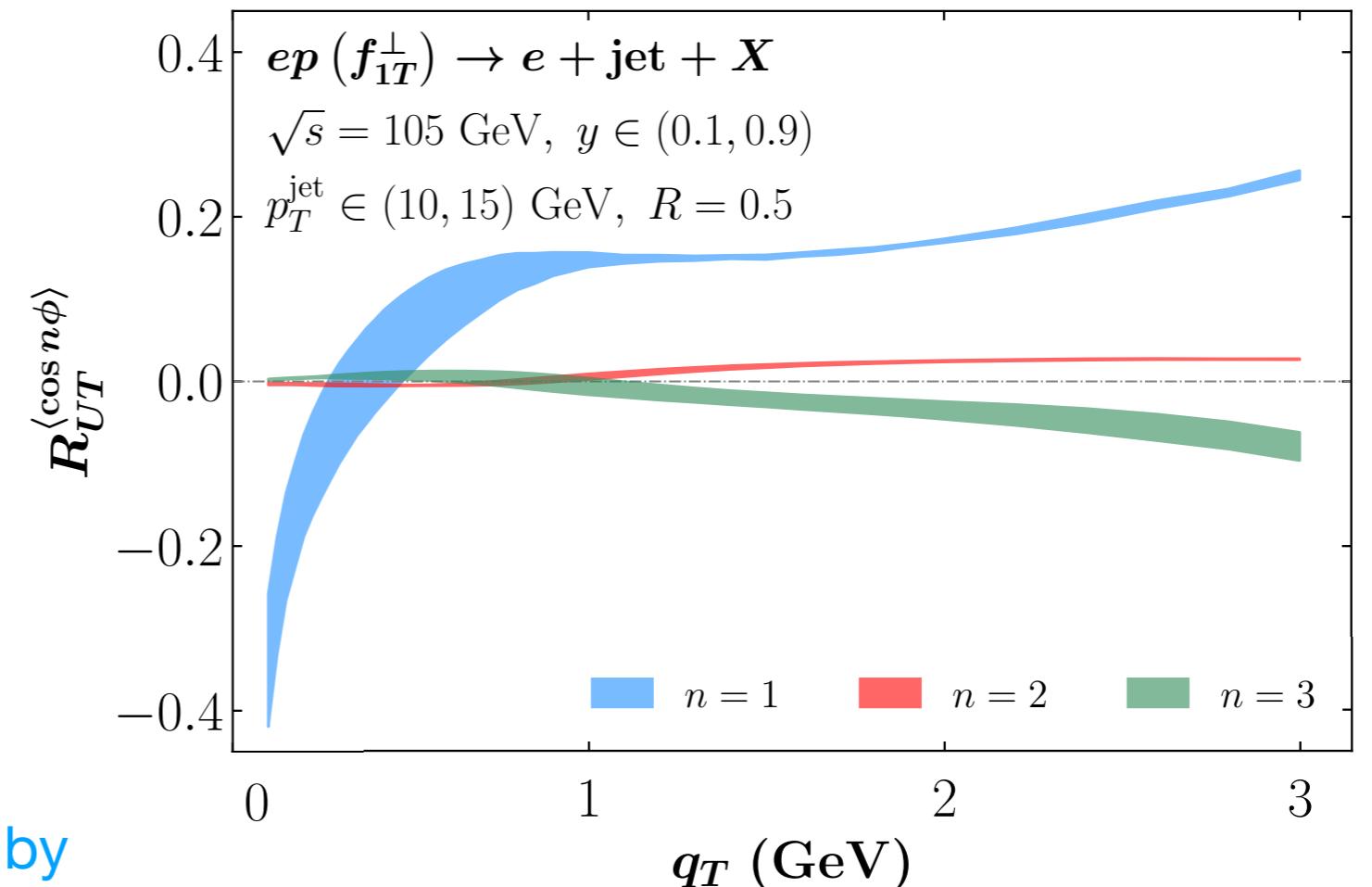
$$F_{UT}^{\sin(\phi_q - \phi_S)} = \sum_{n=0}^8 A_n^{\text{Sivers}}$$

$$R_{UT}^{\langle \cos(n\phi) \rangle} = \frac{\int A_n^{\text{Sivers}} \cos(n\phi) \frac{d\phi}{2\pi}}{\int A_0^{\text{Sivers}} \frac{d\phi}{2\pi}},$$

$$\phi = \phi_{qJ}$$

Anisotropic flows produced by  
global and collinear soft functions

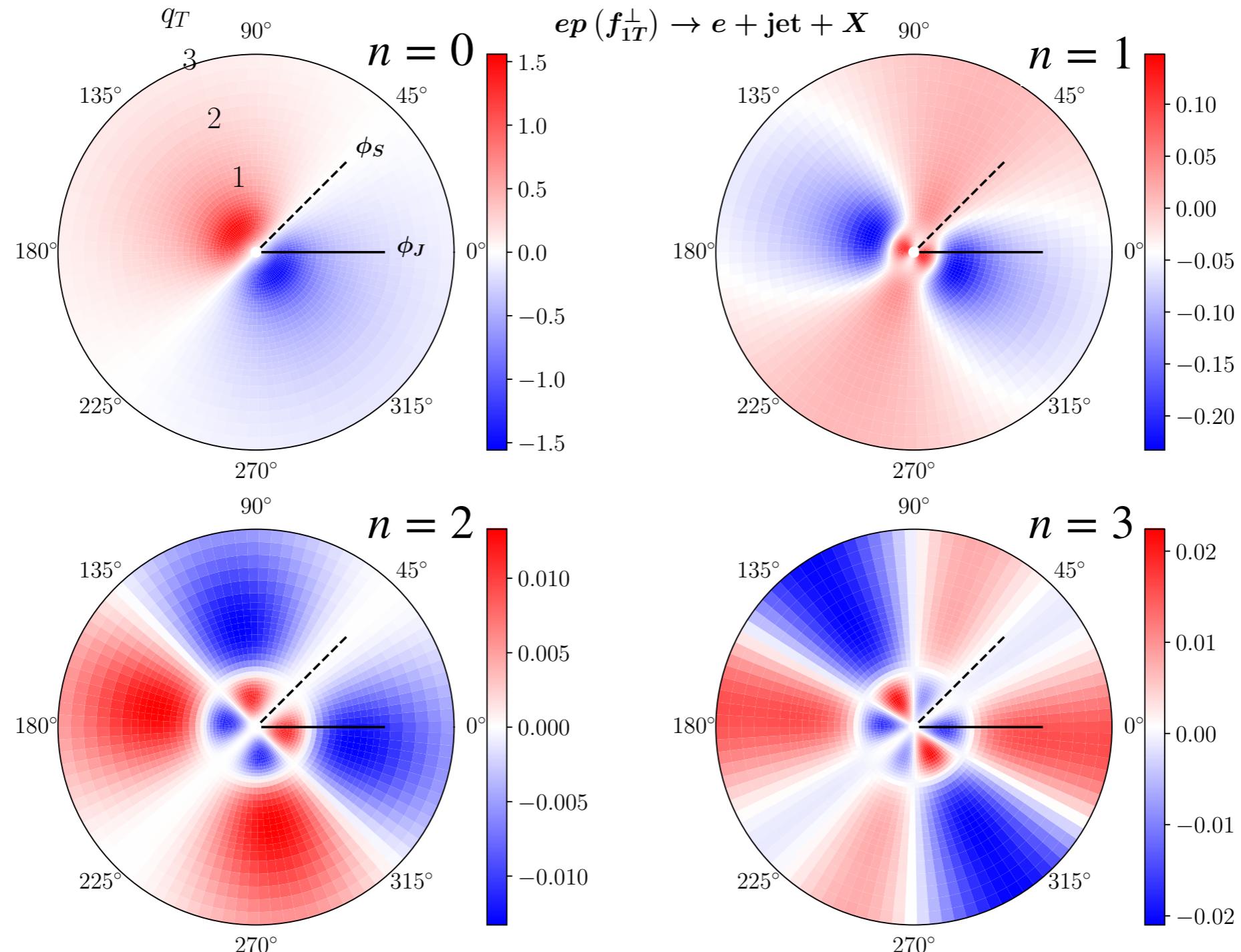
$$A_n^{\text{Sivers}} \equiv \hat{\sigma}_0 H(Q, \mu) \sum_q e_q^2 \mathcal{J}_q(p_T R, \mu) \int \frac{b^2 db}{2\pi} x M \tilde{f}_{1T}^{\perp q, (1)}(x, b^2, \mu, \zeta) (-i)^n \left(\frac{1}{2}\right)^{\delta_{0n}} (J_{n+1}(q_T b) - J_{n-1}(q_T b)) \\ \times \int \frac{d\phi_{bJ}}{2\pi} \cos(n(\phi_{bJ} - \phi_{qJ})) S_{\text{global}}(\mathbf{b}, \mu) S_{cs}(\mathbf{b}, R, \mu)$$



# Polarized jet anisotropy

12

- E.g.  $F_{UT}^{\sin(\phi_q - \phi_s)}$  :  $\phi_s = \frac{\pi}{4}$ ,  $\phi_J = 0$  **Rich spin structure!**



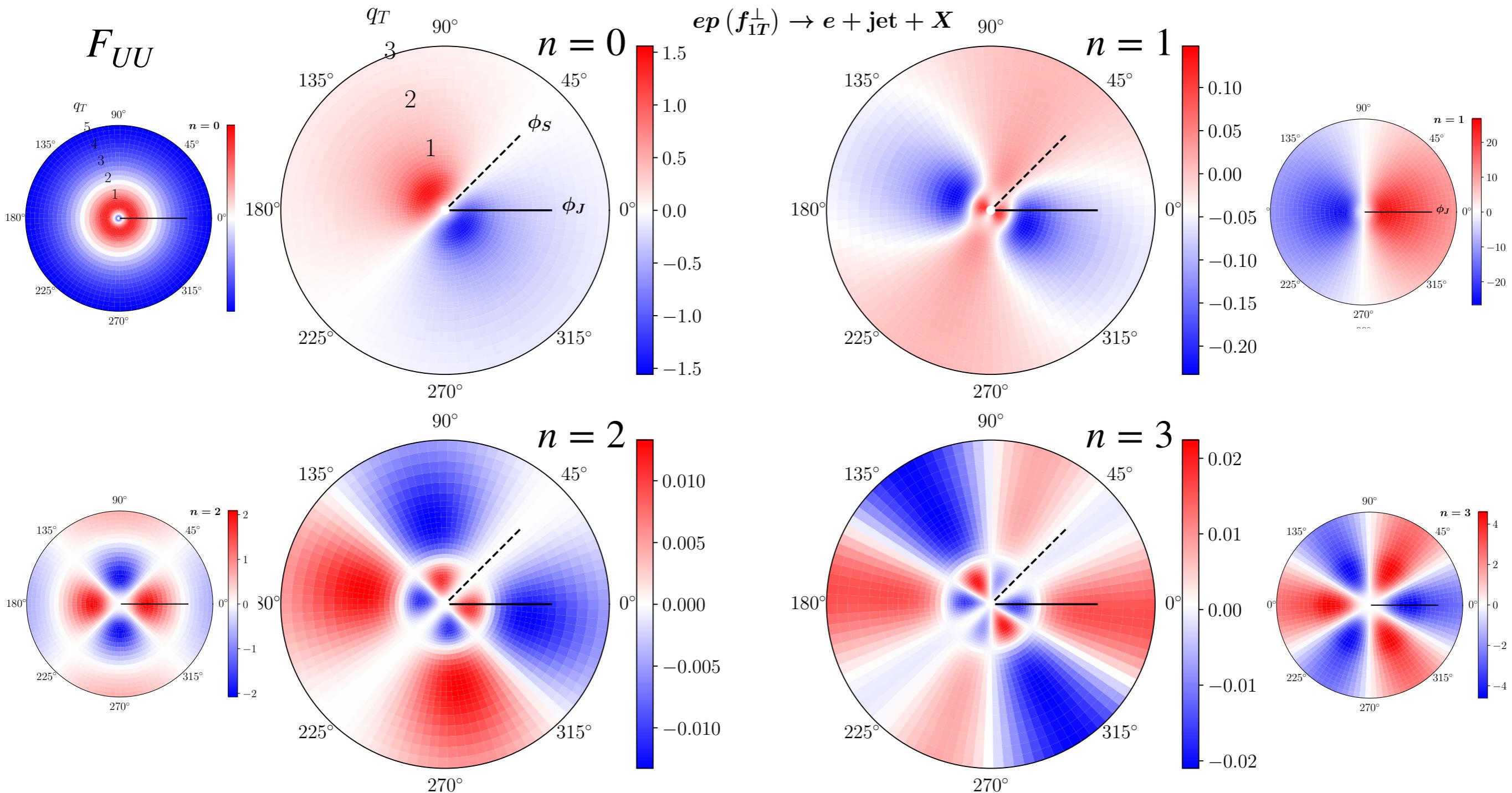
$$A_n^{\text{Sivers}} \equiv \hat{\sigma}_0 H(Q, \mu) \sum_q e_q^2 \mathcal{J}_q(p_T R, \mu) \int \frac{b^2 db}{2\pi} x M \tilde{f}_{1T}^{\perp q, (1)}(x, b^2, \mu, \zeta) (-i)^n \left(\frac{1}{2}\right)^{\delta_{0n}} (J_{n+1}(q_T b) - J_{n-1}(q_T b)) \int \frac{d\phi_{bJ}}{2\pi} \cos(n(\phi_{bJ} - \phi_{qJ})) S_{\text{global}}(\mathbf{b}, \mu) S_{cs}(\mathbf{b}, R, \mu)$$

# Polarized jet anisotropy

13

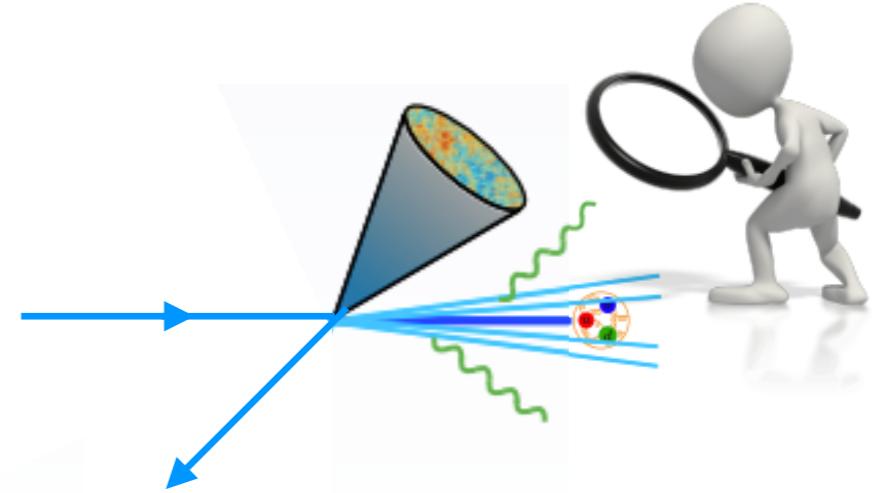
- E.g.  $F_{UT}^{\sin(\phi_q - \phi_s)}$  :  $\phi_s = \frac{\pi}{4}$ ,  $\phi_J = 0$

**Rich spin structure!**



$$A_n^{\text{Sivers}} \equiv \hat{\sigma}_0 H(Q, \mu) \sum_q e_q^2 \mathcal{J}_q(p_T R, \mu) \int \frac{b^2 db}{2\pi} x M \tilde{f}_{1T}^{\perp q, (1)}(x, b^2, \mu, \zeta) (-i)^n \left(\frac{1}{2}\right)^{\delta_{0n}} (J_{n+1}(q_T b) - J_{n-1}(q_T b)) \int \frac{d\phi_{bJ}}{2\pi} \cos(n(\phi_{bJ} - \phi_{qJ})) S_{\text{global}}(\mathbf{b}, \mu) S_{cs}(\mathbf{b}, R, \mu)$$

# Summary



- We study back-to-back polarized lepton-jet production in polarized  $ep$  collisions. In this work, we study the azimuthal anisotropy with azimuthal angle dependence on  $\phi_q$ ,  $\phi_J$  and  $\phi_S$
- Soft gluon radiation would lead to even more rich dynamics with nontrivial and novel features in polarized scattering: how soft gluon radiation would distort the azimuthal asymmetric pictures in the transverse plane related to the Sivers function
- The polarized jet anisotropy observables have measurable magnitudes and can be a valuable tool in exploring spin structures, polarized TMD PDFs in the lepton-jet correlations in future EIC.



# 25th International Spin Symposium (SPIN 2023)

## Backup

# Polarized jet anisotropy

- The global soft function: soft radiation that has no phase space restriction and does not resolve the jet cone.

$$S_{\text{global}}(\mathbf{b}, \mu, \nu) = 1 + \frac{\alpha_s}{2\pi} C_F \left\{ \left[ -\frac{2}{\eta} + \ln \frac{\mu^2}{\nu^2} + 2y_J + 2 \ln(-2i \cos(\phi_{bJ})) \right] \left( \frac{1}{\epsilon} + \ln \frac{\mu^2}{\mu_b^2} \right) + \frac{2}{\epsilon^2} + \frac{1}{\epsilon} \ln \frac{\mu^2}{\mu_b^2} - \frac{\pi^2}{6} \right\}$$

- The collinear-soft function: soft radiation which is only sensitive to the jet direction and resolves the jet cone

$$S_{cs}(\mathbf{b}, R, \mu) = 1 - \frac{\alpha_s}{2\pi} C_F \left[ \frac{1}{\epsilon^2} + \frac{2}{\epsilon} \ln \left( \frac{-2i \cos(\phi_{bJ}) \mu}{\mu_b R} \right) + 2 \ln^2 \left( \frac{-2i \cos(\phi_{bJ}) \mu}{\mu_b R} \right) + \frac{\pi^2}{4} \right]$$

$\mu_b = 2e^{-\gamma_E}/b$ ,  $\phi_{bJ} \equiv \phi_b - \phi_J$  with  $\phi_b$  and  $\phi_J$  are the azimuthal angles of the vector  $\mathbf{b}$  and jet transverse momentum  $\mathbf{p}_T$

# Polarized jet anisotropy

