Selected results on the electric dipole moments from lattice QCD

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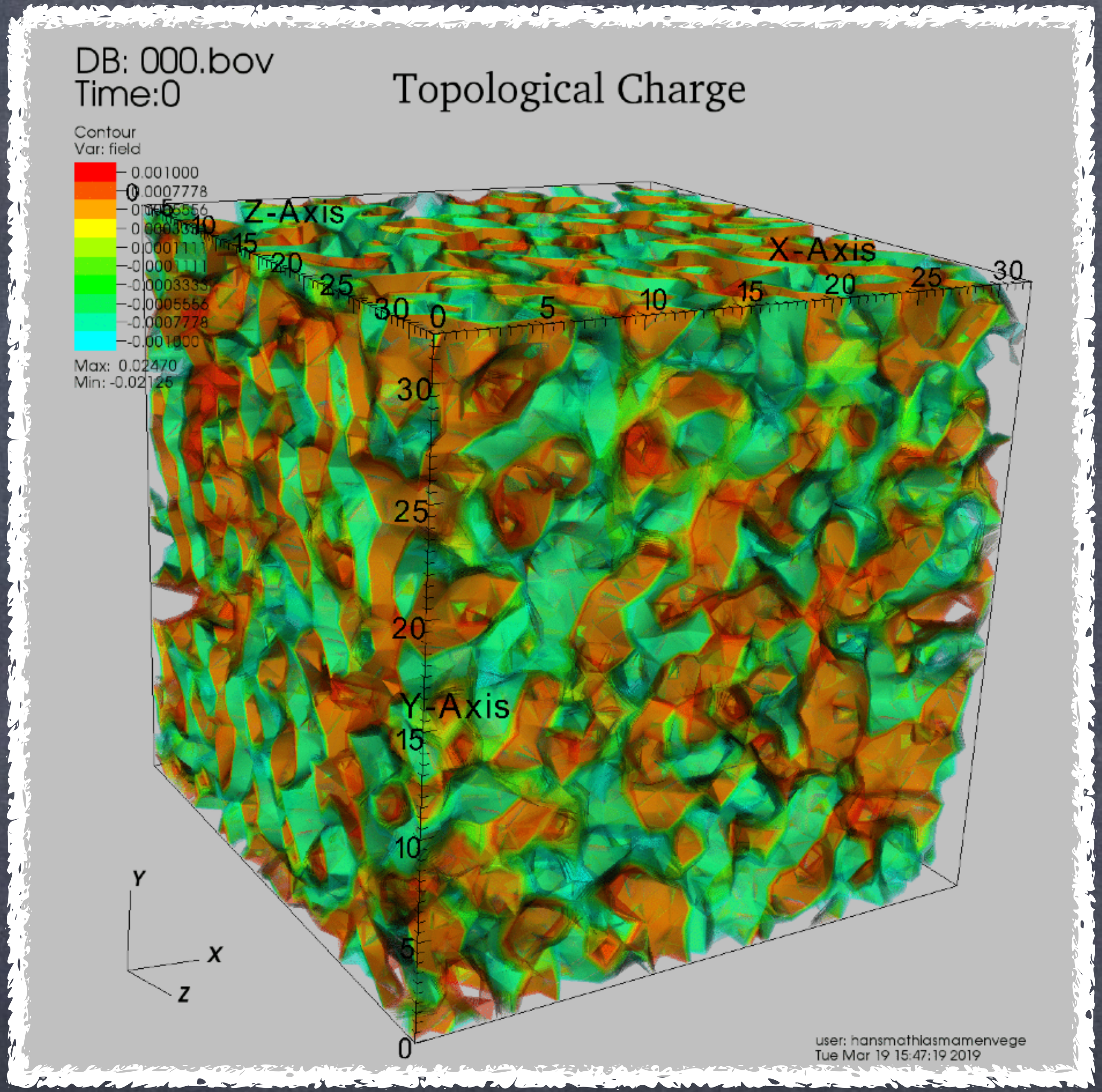

2308.16221
2304.03451
2302.02165
2212.09824
2111.11449
2106.07633
2005.04199
1902.03254
1810.10301
1810.05637
1809.03487
1711.04730
1507.02343
1409.2735

SYMLAT
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Berkeley
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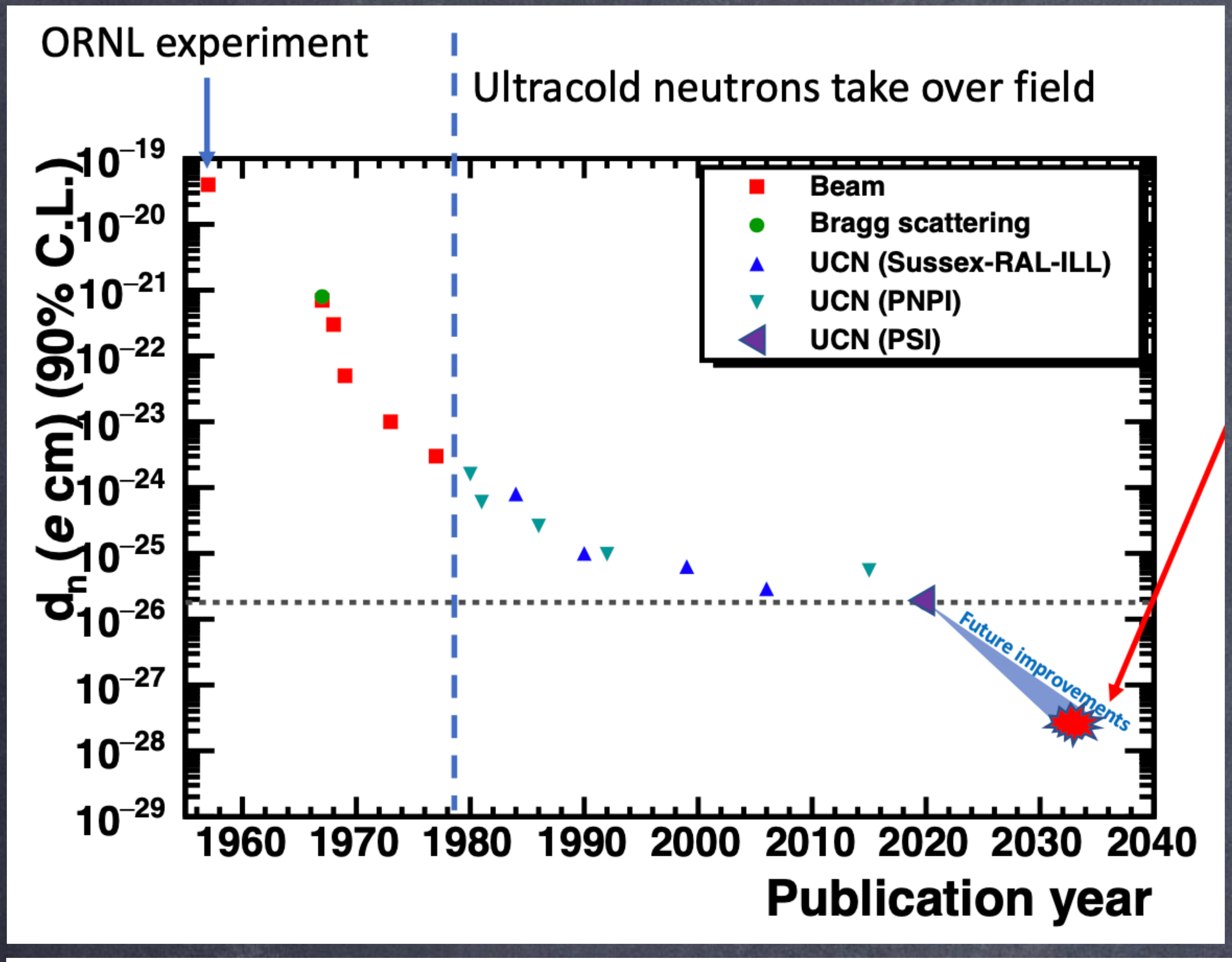
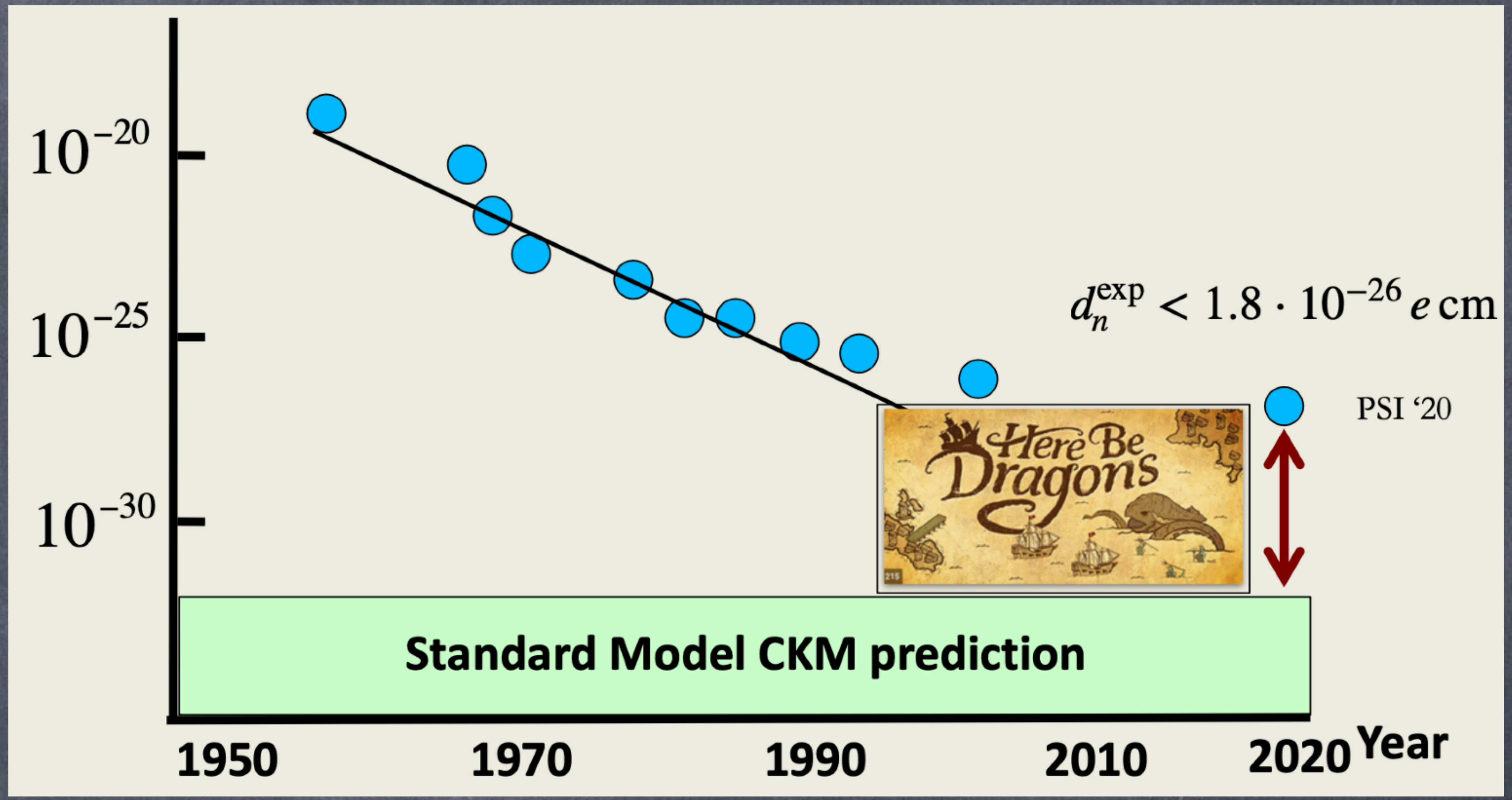
SPIN 2023 Durham Convention Center 25.09.2023

Neutron EDM



Abel et al.: 2020 (PSI)

$$|d_n| < 1.8 \times 10^{-26}\ e\ \text{cm (90\% C.L.)}$$

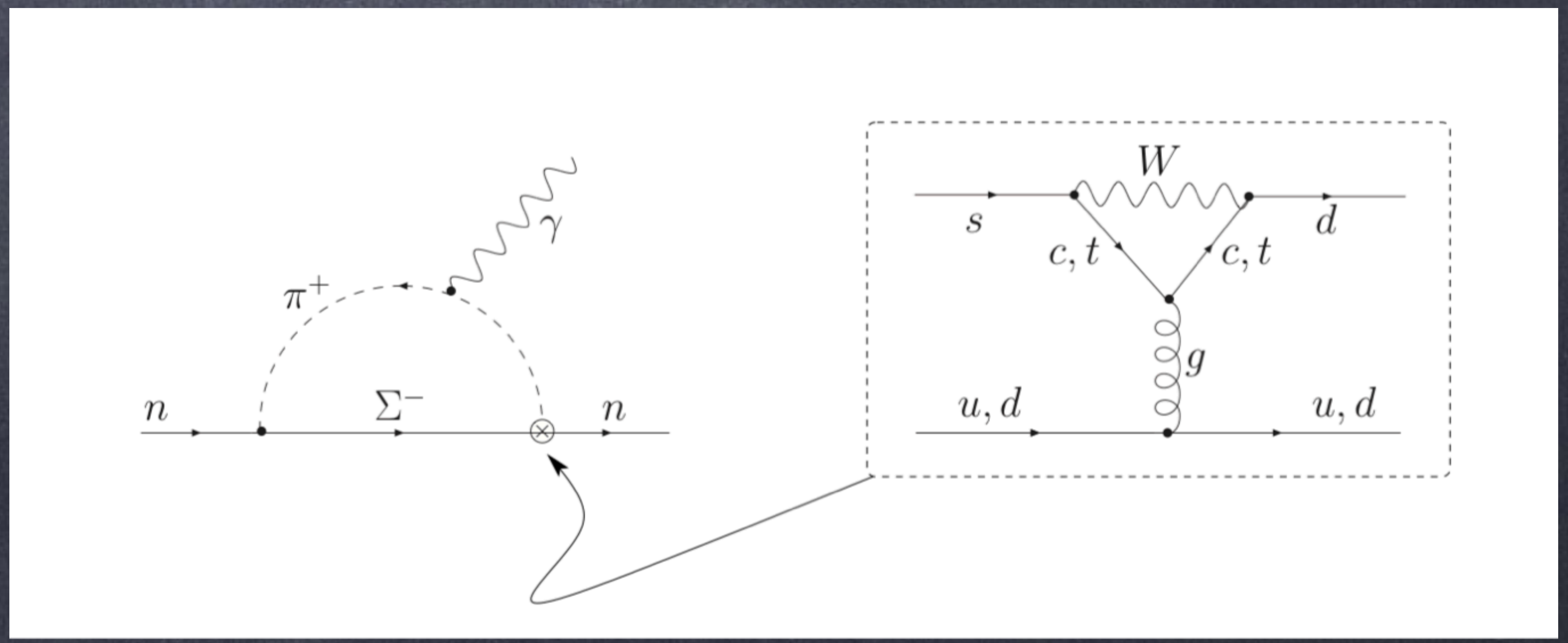


Alarcon et al.: 2022
Snowmass Summer Study Report

$$(d_n)_{\text{SM}} = (1-6) \times 10^{-32} e \quad \text{cm}$$

Shabalin: 1978-1980 Khriplovich, Zhitnitsky: 1982
Gavela et al. : 1982 Seng: 2015

Experiment: Facility	Neutron Source	Measurement Cell	Measurement Techniques	90% C.L. (10^{-28} e-cm) With 300 Live Days	Year 90% C.L. Data Acquired
Crystal: JPARC	Cold Neutron Beam	Solid	Crystal Diffraction (High Internal $\vec{E}$)	< 100	Development
Beam: ESS	Cold Neutron Beam	Vacuum	Pulsed Beam	< 50	~ 2030
PNPI: ILL	ILL Turbine (UCN) PNPI/LHe (UCN)	Vacuum	Ramsey Technique, $\vec{E} = 0$ Cell for Magnetometry	Phase 1 < 100 < 10	Development Development
n2EDM: PSI	Solid D_2 (UCN)	Vacuum	Ramsey Technique, External Cs Magnetometers, Hg Co-Magnetometer	< 15	~ 2026
PanEDM ILL/Munich	Superfluid ^{4}He (UCN), Solid D_2 (UCN)	Vacuum	Ramsey Technique, Hg Co-External ^{3}He and Cs Magnetometers	< 30	~ 2026
TUCAN: TRIUMF	Superfluid ^{4}He (UCN)	Vacuum	Ramsey Technique, Hg Co-Magnetometer, External Cs Magnetometers	< 20	~ 2027
nEDM: LANL	Solid D_2 (UCN)	Vacuum	Ramsey Technique, Hg Co-Magnetometer, Hg External Magnetometer, OPM	< 30	~ 2026
nEDM@SNS: ORNL	Superfluid ^{4}He (UCN)	^{4}He	Cryogenic High Voltage, ^{3}He Capture for ω, ^{3}He Co-Magnetometer with SQUIDs, Dressed Spins, Superconducting Magnetic Shield	< 20 < 3	~ 2029 ~ 2031

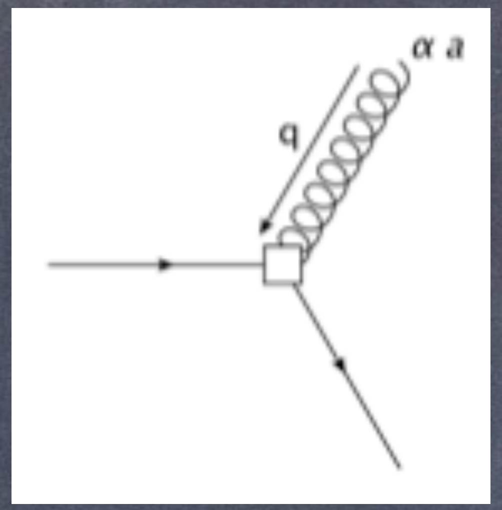


CP-violating sources

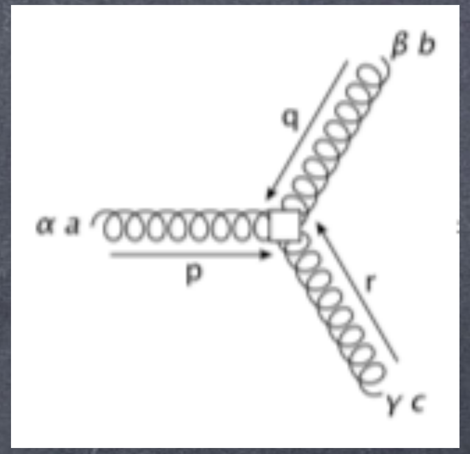
- Full list of dimension 5 and 6 operators is known

Buchmuller, Wyler: 1986
de Rujula et al.: 1991
Grzadkowski et al: 2010

$$\mathcal{O}_{\mathrm{CE}} = \sum_{f=u,d,s,\ldots} \overline{\psi}_f(x)\gamma_5\sigma_{\mu\nu}G_{\mu\nu}\psi_f(x)$$



$$\mathcal{O}_{\widetilde{G}}(x) = \frac{1}{g^2}\mathrm{Tr}\left[G_{\mu\rho}(x)G_{\rho\nu}(x)\widetilde{G}_{\nu\mu}(x)\right]$$



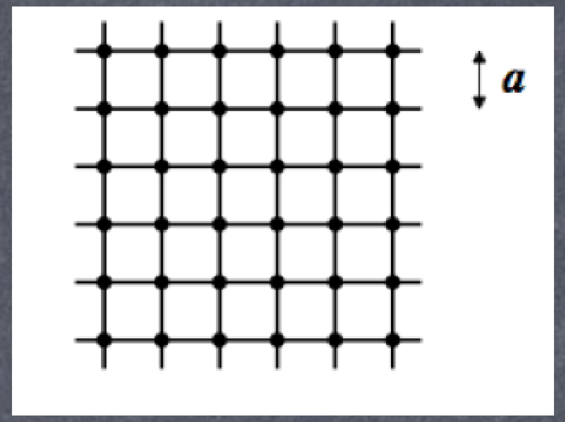
Weinberg: 1989

$$\mathcal{O}_{\mathrm{q}} = \sum_{f=u,d,s,\ldots} \overline{\psi}_f(x)\gamma_5\sigma_{\mu\nu}\psi_f(x)F_{\mu\nu}$$

$$\mathcal{L}_{\mathrm{QCD}+\theta} = \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \sum_{f=u,d,s,\ldots} \overline{\psi}_f(x)\left\{\gamma_\mu\left[\partial_\mu + gA^a_\mu T^a\right] + m_f\right\}\psi_f(x) - i\bar{\theta}q(x)$$

$$q(x) = \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{tr}\left\{G_{\mu\nu}(x)G_{\rho\sigma}(x)\right\}$$

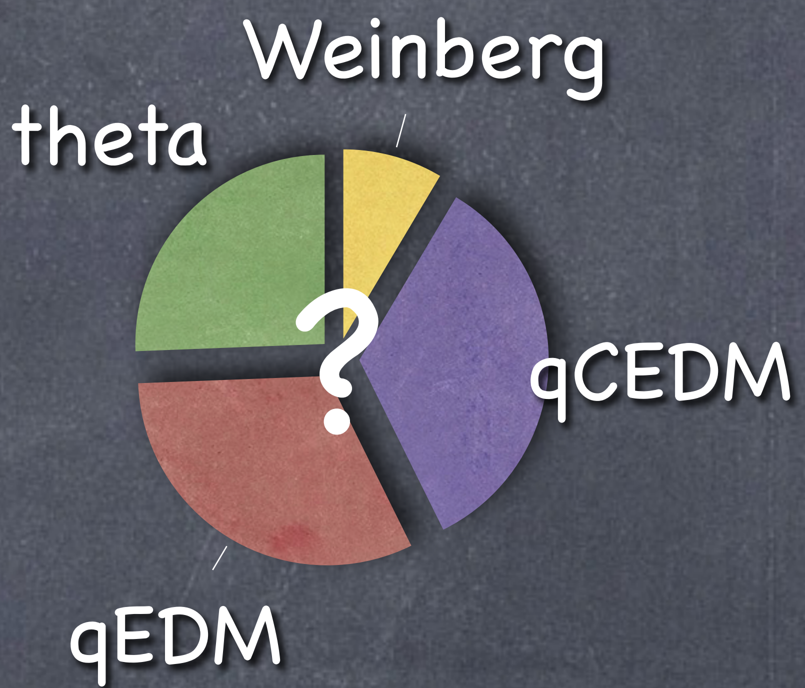
The role of lattice QCD



$$d_{\mathrm{N}} = M_{\mathrm{N}}^{\theta}\bar{\theta} + \left(\frac{v}{\Lambda}\right)^2 \sum_i M_{\mathrm{N}}^{(i)}\tilde{d}_i \qquad \langle N|J_\mu \mathcal{O}_{\cancel{CP}}|N\rangle \to d\ \underline{E}\cdot\underline{S}$$

M_{N}^{θ} ⟶ Hadronic matrix element topological charge

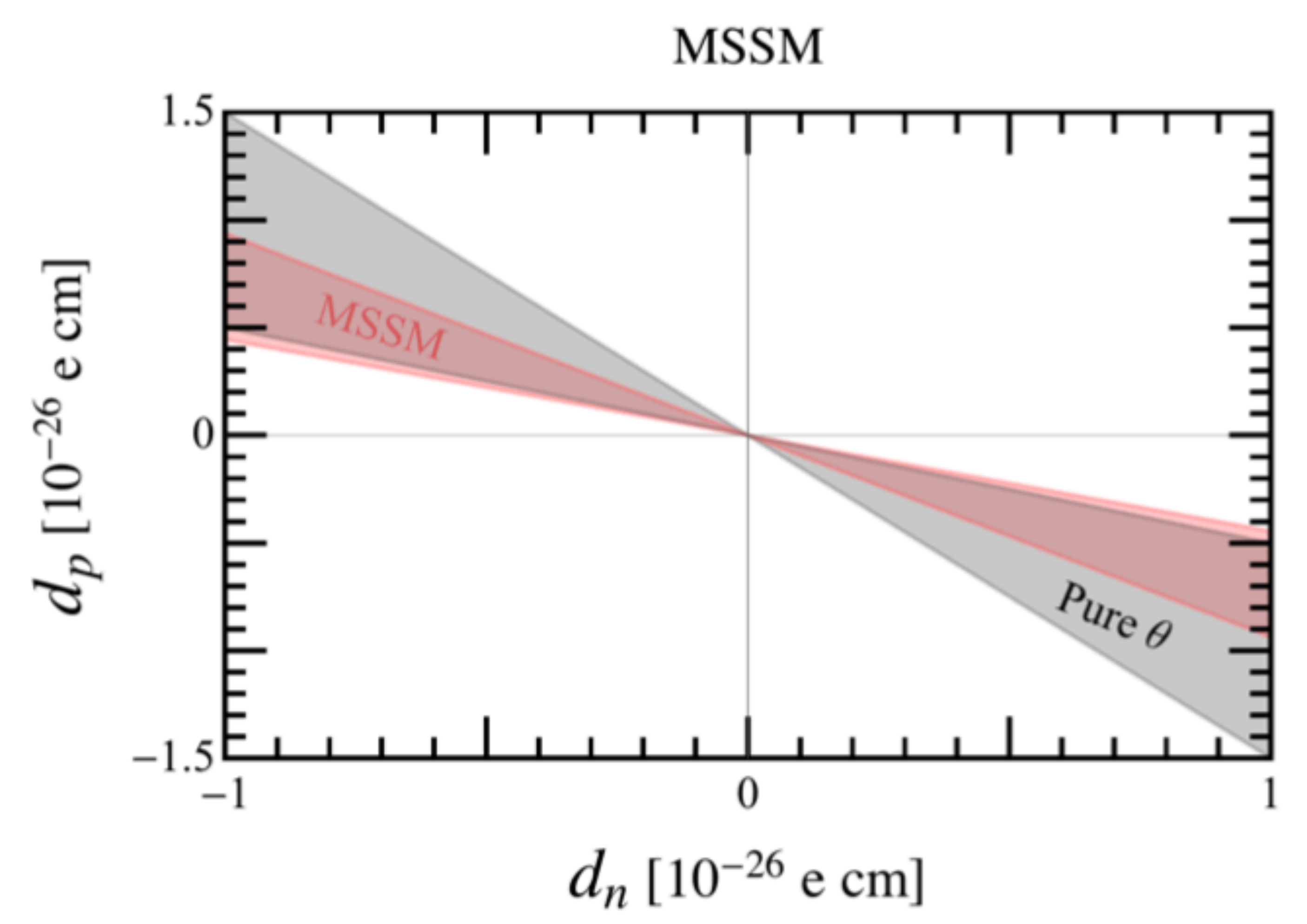
$M_{\mathrm{N}}^{(i)}$ ⟶ Hadronic matrix element CP odd operators

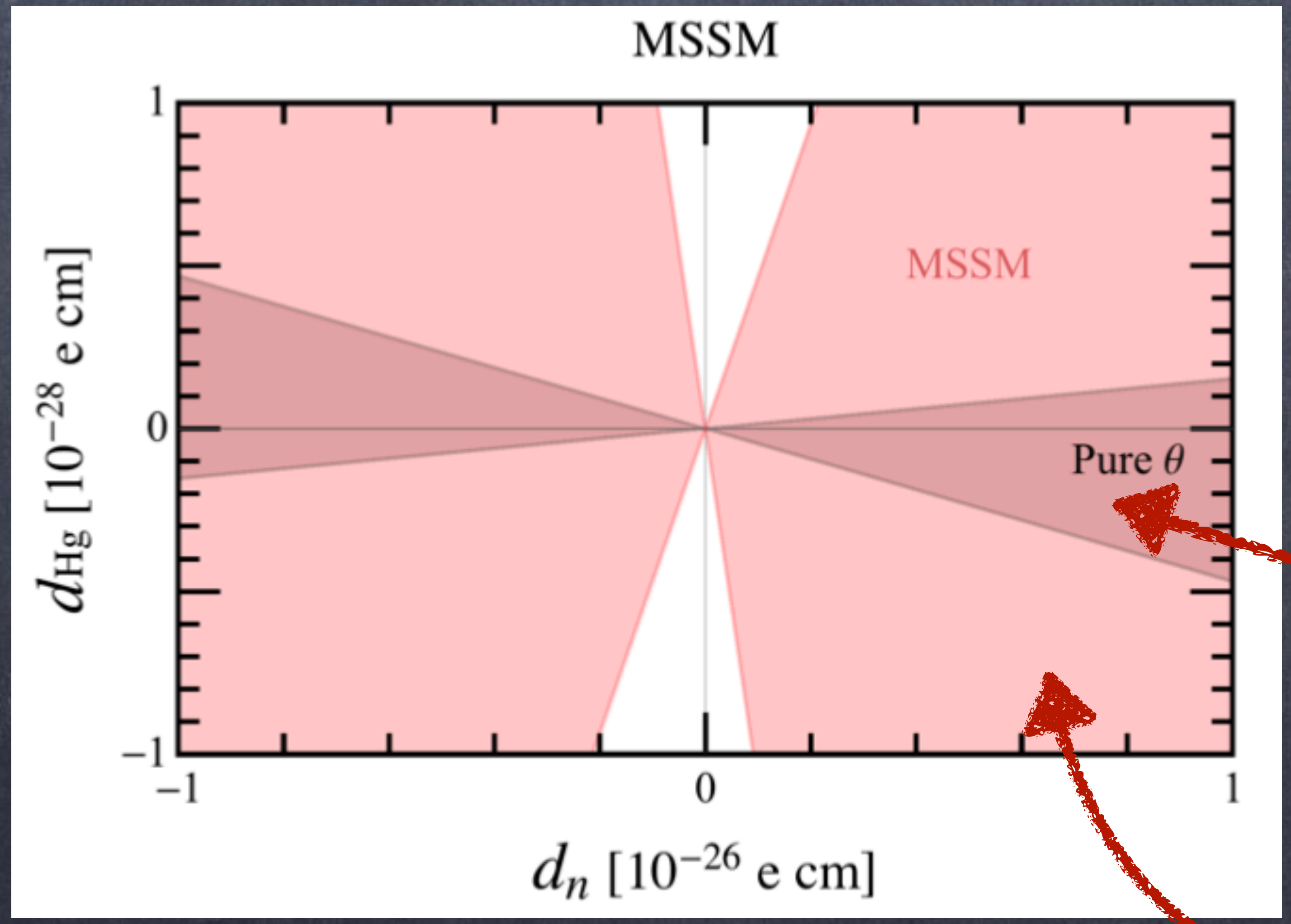


Alarcon et al.: 2022
Snowmass Summer Study Report

$$\begin{aligned} d_n = \quad & -\ (1.5\pm 0.7)\cdot 10^{-3}\bar{\theta}e\ \mathrm{fm} \\ & -\ (0.2\pm 0.01)d_u + (0.78\pm 0.03)d_d + (0.0027\pm 0.016)d_s \\ & -\ (0.55\pm 0.28)e\tilde{d}_u - (1.1\pm 0.55)e\tilde{d}_d + (50\pm 40)\mathrm{MeV}e\tilde{d}_G \end{aligned}$$

Shintani et al.: 2005
Berruto, Blum, Orginos, Soni 2006
A.S., Luu, de Vries: 2014-2015
Guo, Meißner, et al. : 2010-
Liang, Draper, Liu, Yang
Alexandrou et al. (ETMC): 2015-2020
Abramczyk et al. : 2017-
Dragos, Kim, Luu, Monahan, Rizik,
A.S., de Vries, Yousif: 2015-2021
Yoon, Bhattacharya, Cirigliano,
Gupta, Mereghetti: 2015-2021





$d_n\ \bar{g}_0\ \bar{g}_1$

$d_n\ d_p\ \bar{g}_1$ qCEDM

de Vries et al.: 2021

Theta-term

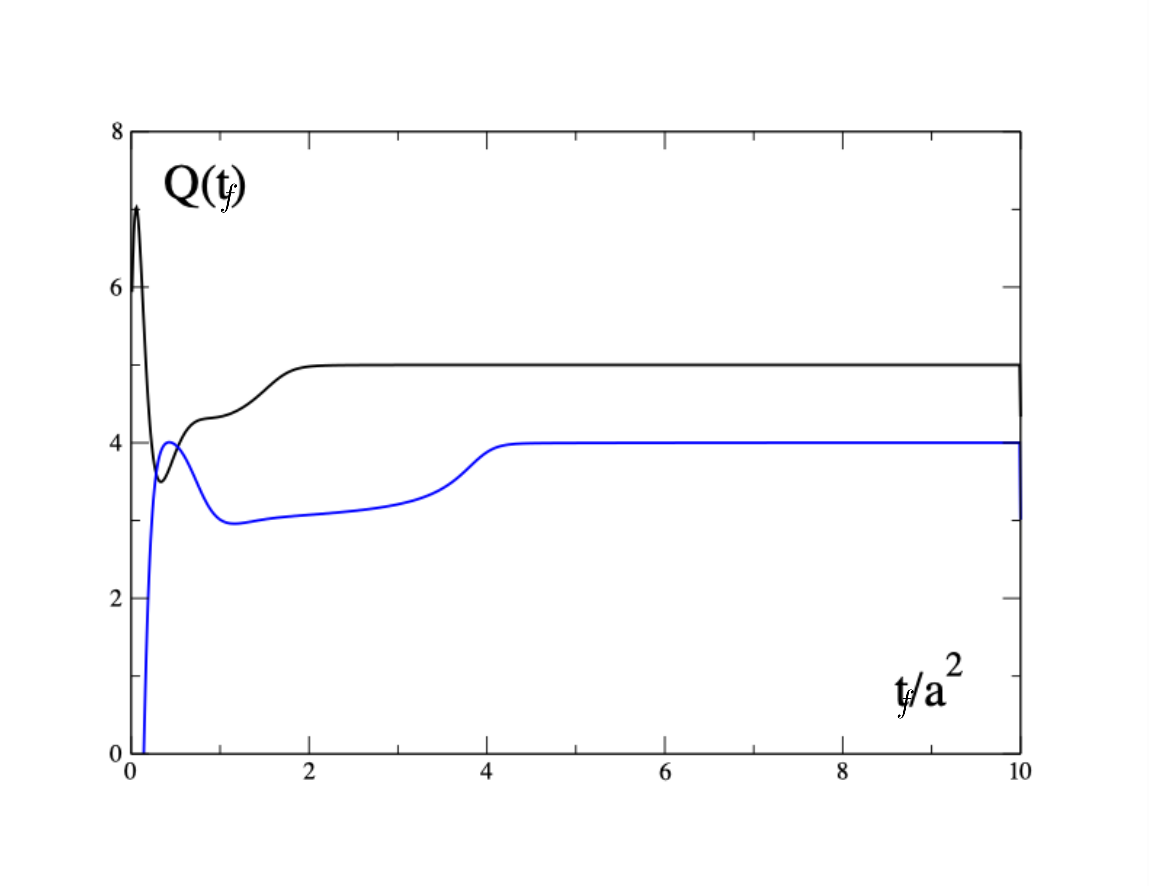
Gradient flow

Narayanan, Neuberger:2006

Lüscher 2010
Lüscher, Weisz 2011

$$x_\mu \quad t = \text{flow-time} \quad [t] = -2 \qquad A_\mu(x) = A^a_\mu(x) T^a \;\rightarrow\; \text{gluon fields}$$

$$\boxed{\begin{aligned} \partial_t B_\mu(x,t) &= D_\nu G_{\nu\mu}(x,t) \\ B_\mu(x,t)\big|_{t=0} &= A_\mu(x) \end{aligned}}$$



$$D_\nu = \partial_\nu + [B_\nu(x,t), \cdot] \qquad G_{\mu\nu}(x,t) = \partial_\mu B_\nu(x,t) - \partial_\nu B_\mu(x,t) + [B_\mu, B_\nu]$$

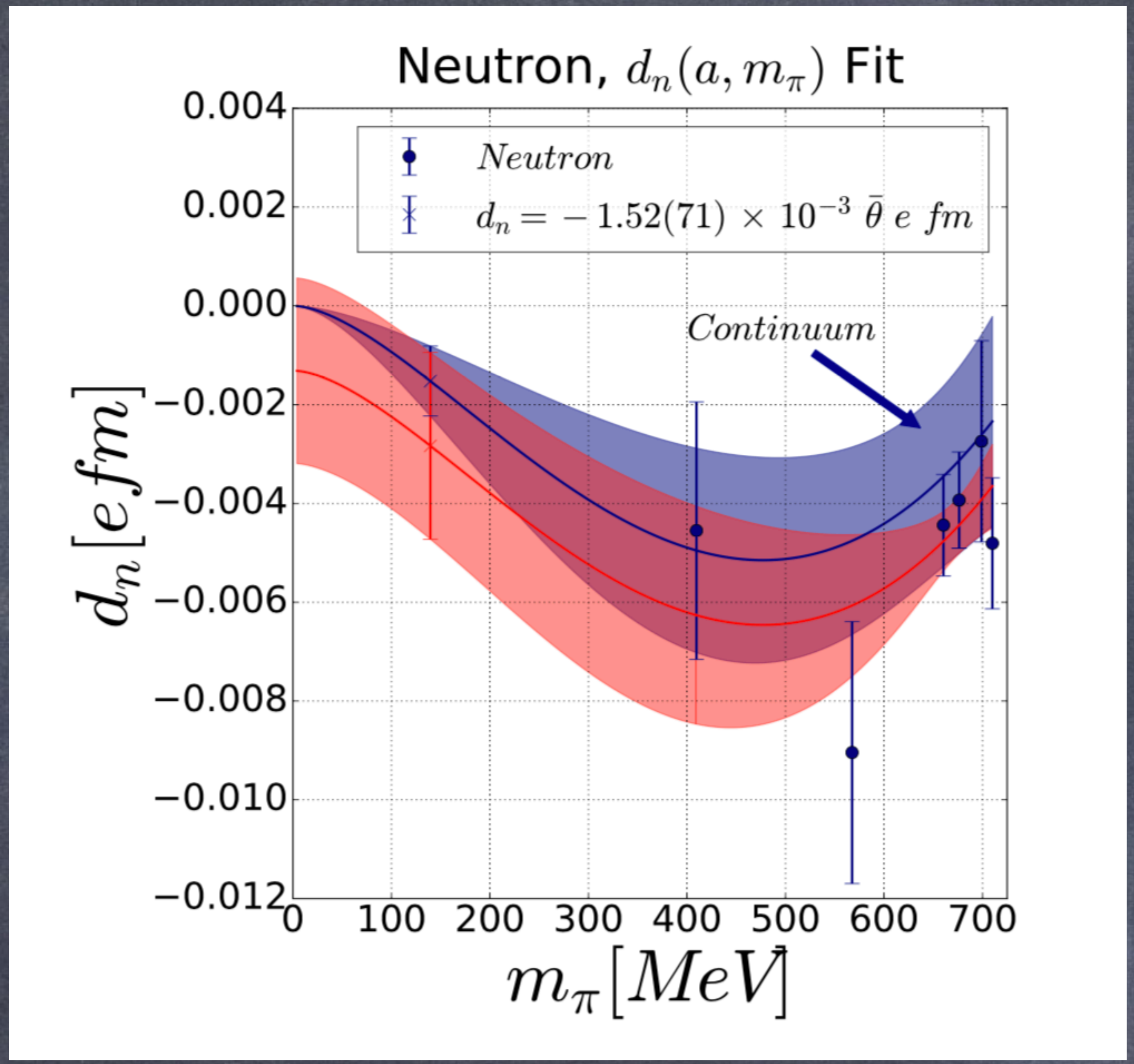
$$\partial_t Q(t) = 0 \qquad Q = \int d^4x \; q(x,t)$$

Equivalent to fermionic definition

Polyakov: 1987, Lüscher: 2010 Ce', Consonni, Engel, Giusti: 2015 Lüscher: 2021 Lüscher, Weisz: 2021

Low-energy Constant

Dragos, Luu, A.S., de Vries, Yousif: 2019



$$d_n(\bar\theta) = \bar d_n - \frac{e g_A \bar g_0^{\bar\theta}}{8\pi^2 F_\pi} \ln \frac{m_\pi^2}{M_N^2}$$

Ottnad et al.: 2010
Mereghetti et al.: 2011

$$d_{n/p}(a, m_\pi) = C_1^{n/p} m_\pi^2 + C_2^{n/p} m_\pi^2 \ln \frac{m_\pi^2}{M_N^2} + C_3^{n/p} a^2$$

	$C_1\ [\bar\theta\, e\ \mathrm{fm}^3]$	$C_2\ [\bar\theta\, e\ \mathrm{fm}^3]$	$C_3\ \left[\frac{\bar\theta\, e\ \mathrm{fm}}{\mathrm{fm}^2}\right]$	χ^2_{PDF}	$\bar g_0^{\bar\theta}\ [\bar\theta]$
proton	$-3.6(5.3) \times 10^{-4}$	$-6.8(6.6) \times 10^{-4}$	0.20(31)	2.0(1.4)	$-9.9(9.6) \times 10^{-3}$
neutron	$3.1(3.2) \times 10^{-4}$	$8.8(4.4) \times 10^{-4}$	−0.16(23)	1.8(1.5)	$-12.8(6.4) \times 10^{-3}$

$$\boxed{d_n^{\mathrm{phys}} = -0.00152(71)\ \bar\theta\ e\ \mathrm{fm}}$$

$$|\bar\theta| < 1.98 \times 10^{-10} (90\%\mathrm{CL})$$

$$\bar g_0^{\bar\theta} = -1.28(64) \cdot 10^{-2} \bar\theta$$

Ab-initio determination of $\bar g_0^{\theta}$

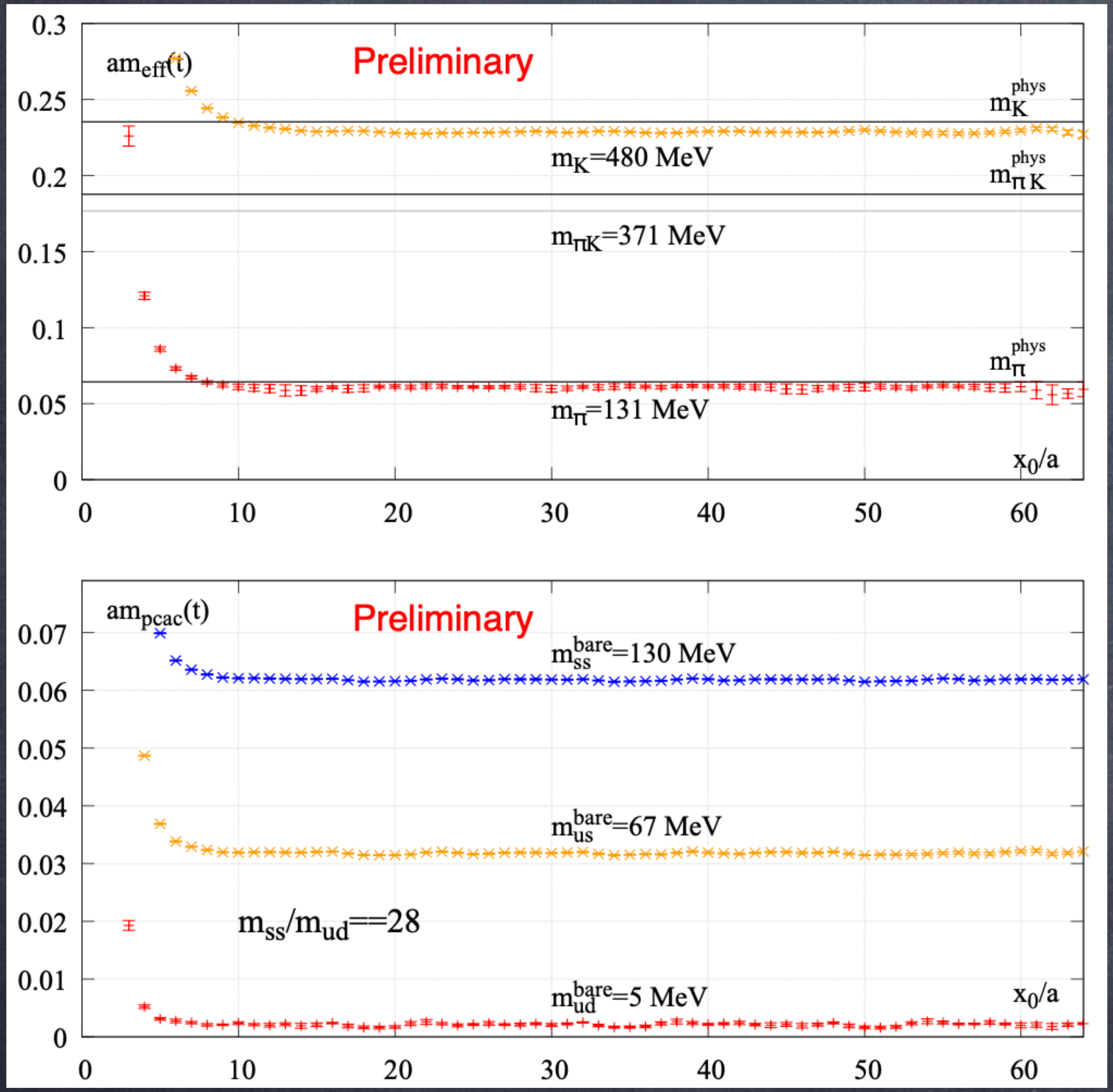
$$\bar g_0^{\bar\theta} = -1.47(23) \cdot 10^{-2} \bar\theta$$

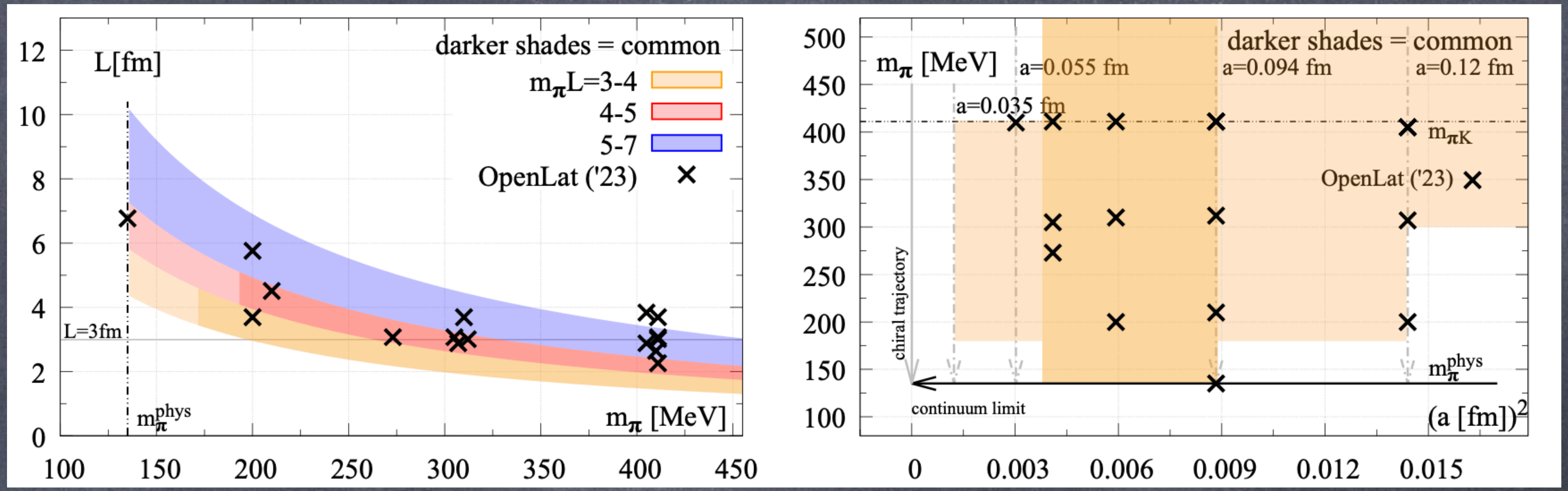
Crewther et al.: 1980
de Vries et al.: 2015

Future with Open Science

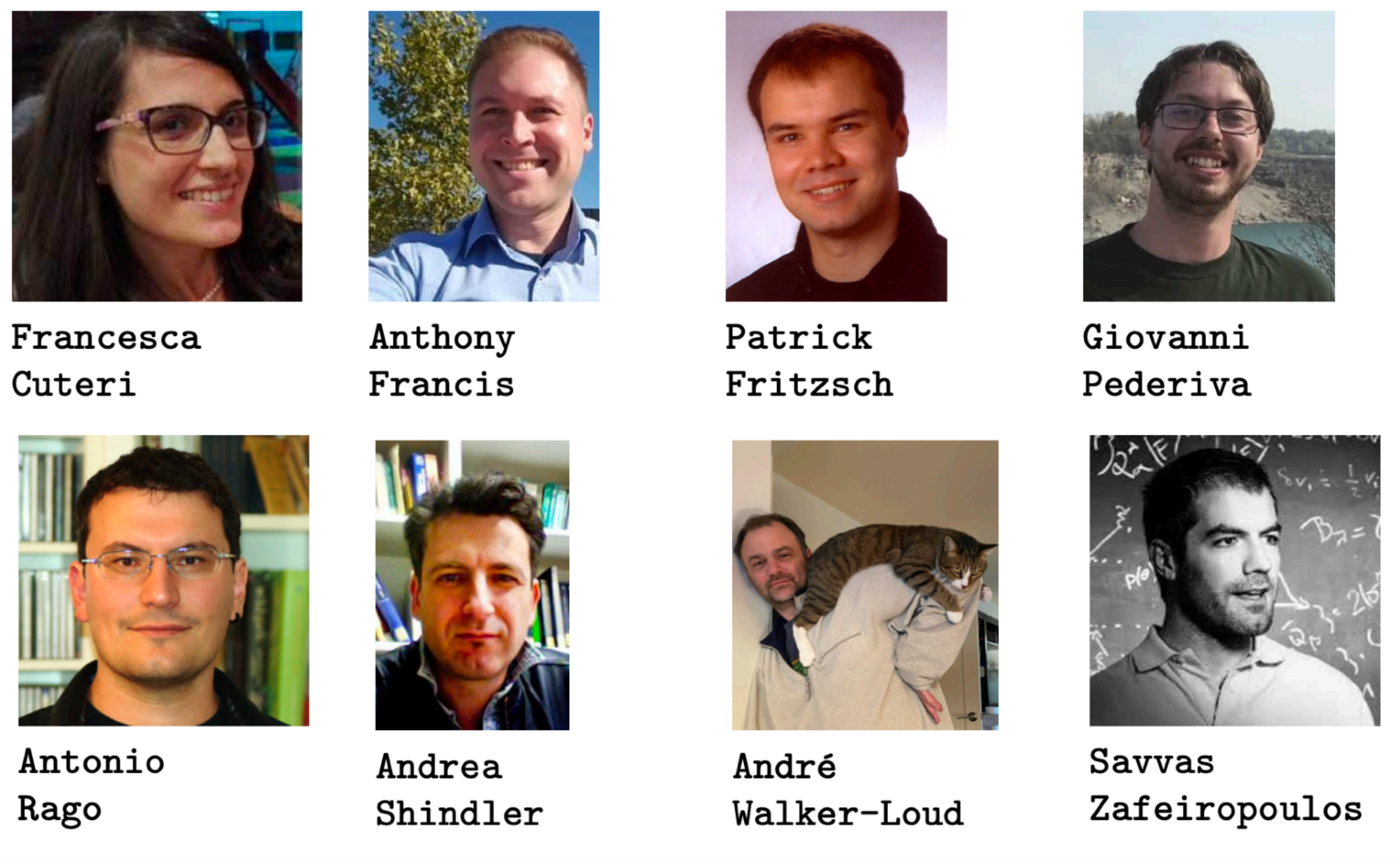
OPEN LATtice initiative

- OpenLat: open science initiative. Gauges with SWF open to the whole community





Ensemble	N_{conf}
a12m412	1200
a12m300	→ 700
a12m200*	→ 20*
a094m412	1500
a094m300**	→ 250**
a094m200	50
a094m135	→ 40
a077m412	→ 1000
a077m300	→ 100
a077m200	→ 50
a064m412	→ 1100
a064m300	→ 700
a055m412	→ 100



https://openlat1.gitlab.io

OpenLat

Neutron EDM & Schiff moment

m_π = 400 MeV

t/t_0 = 1.9

a = 0.065-0.12 fm

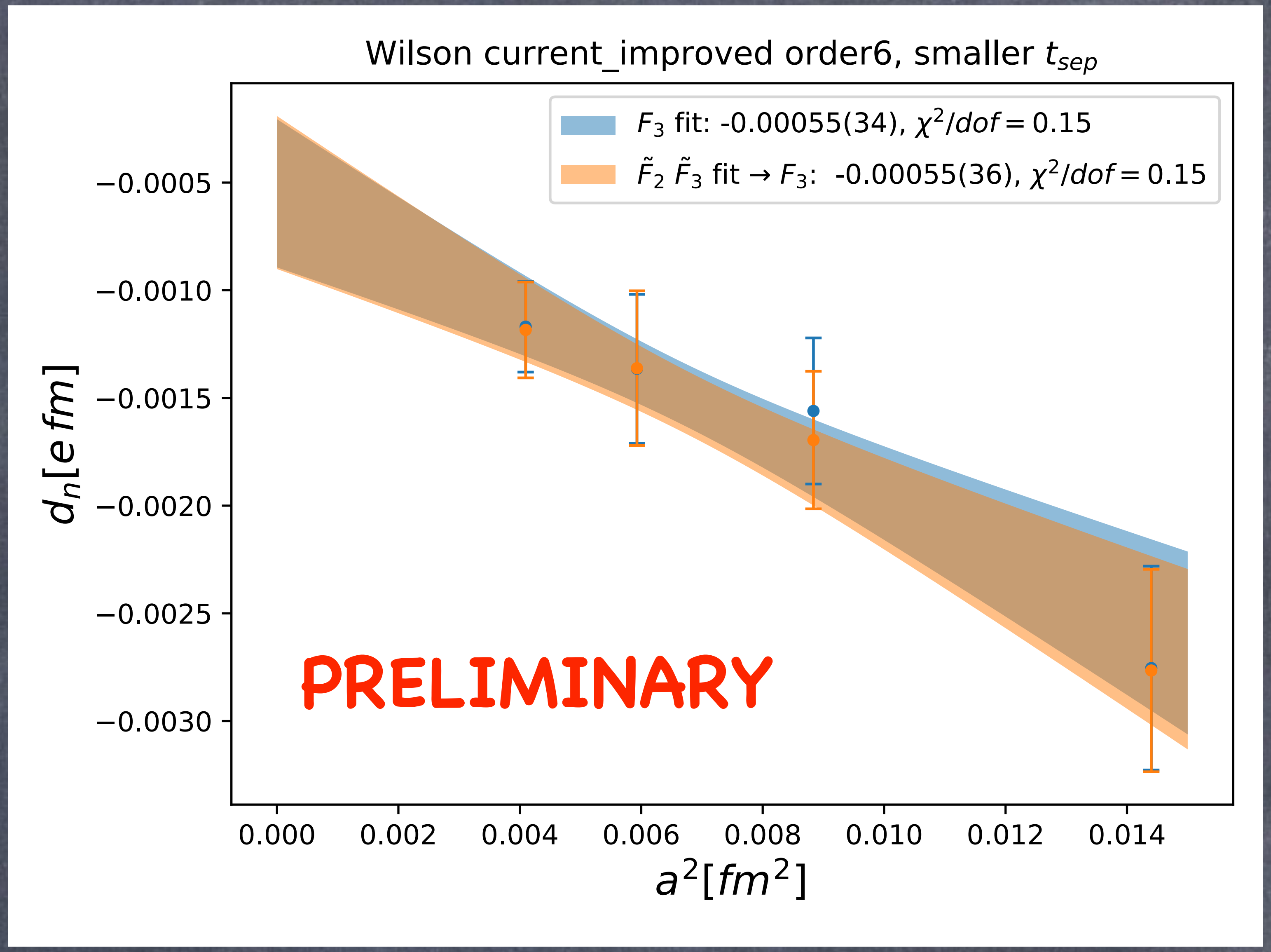
c_V still missing

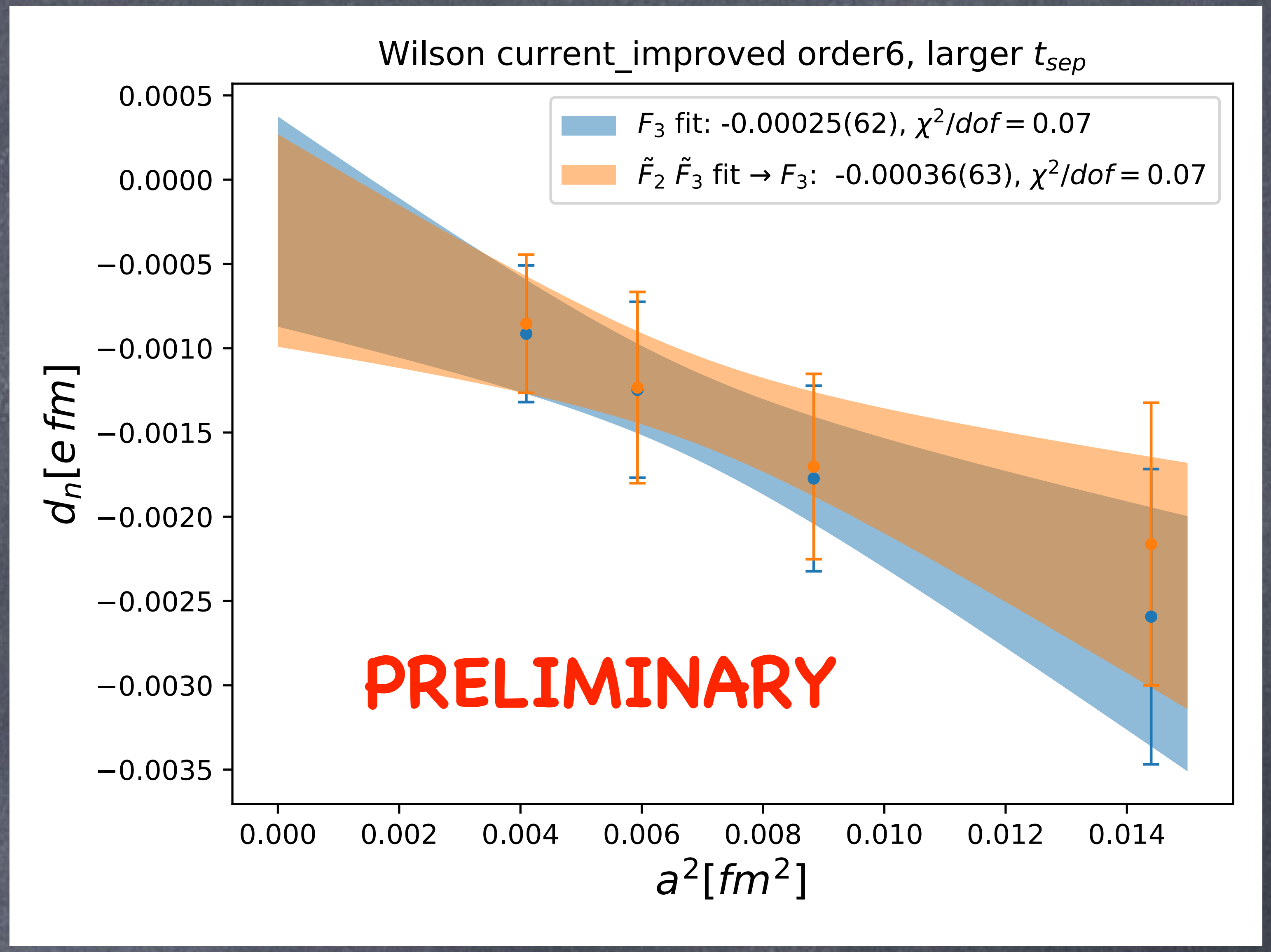
Systematics

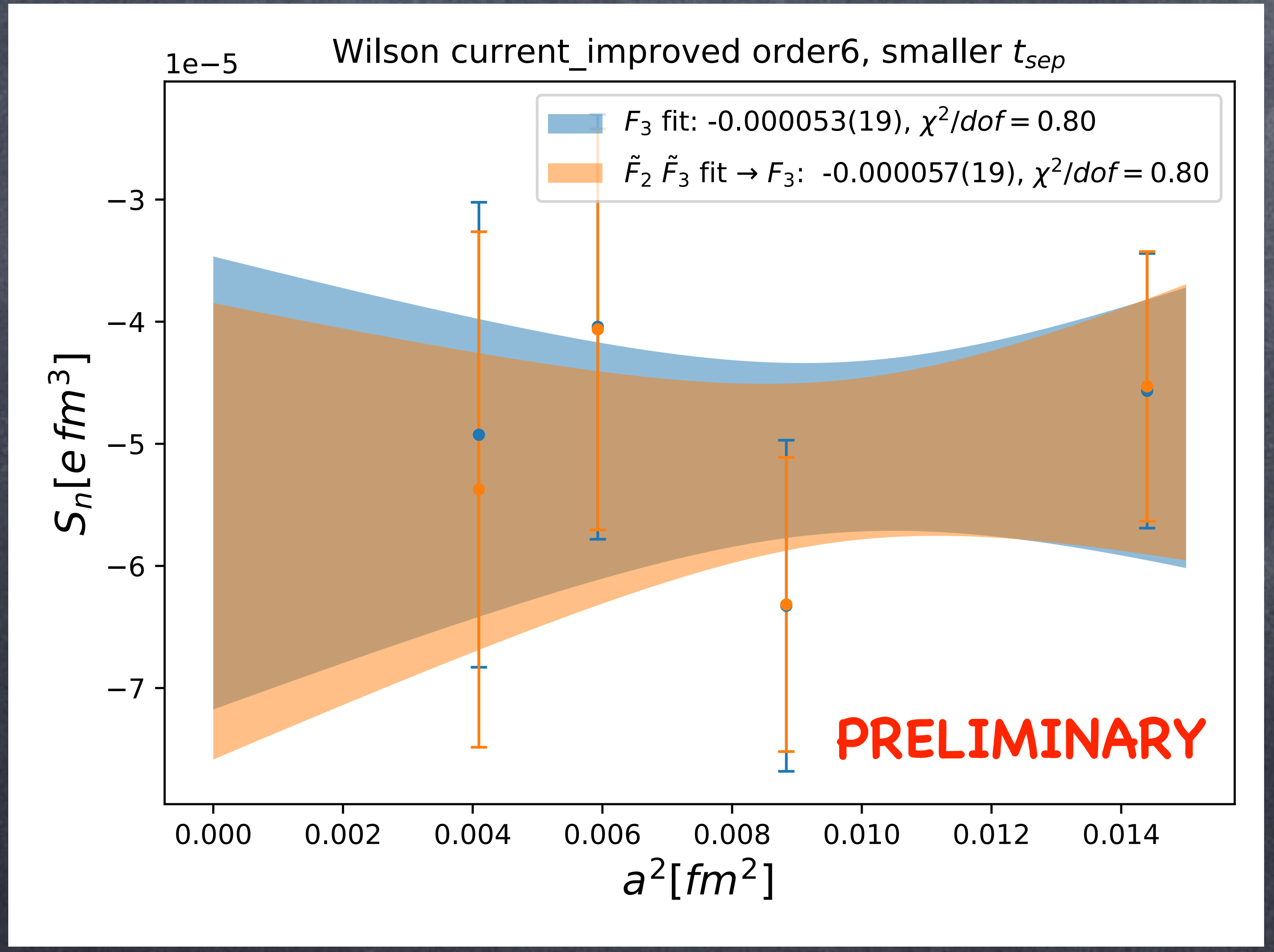
- Excited state contamination
- Finite volume effects
- Pion mass dependence
- Different definition of charge

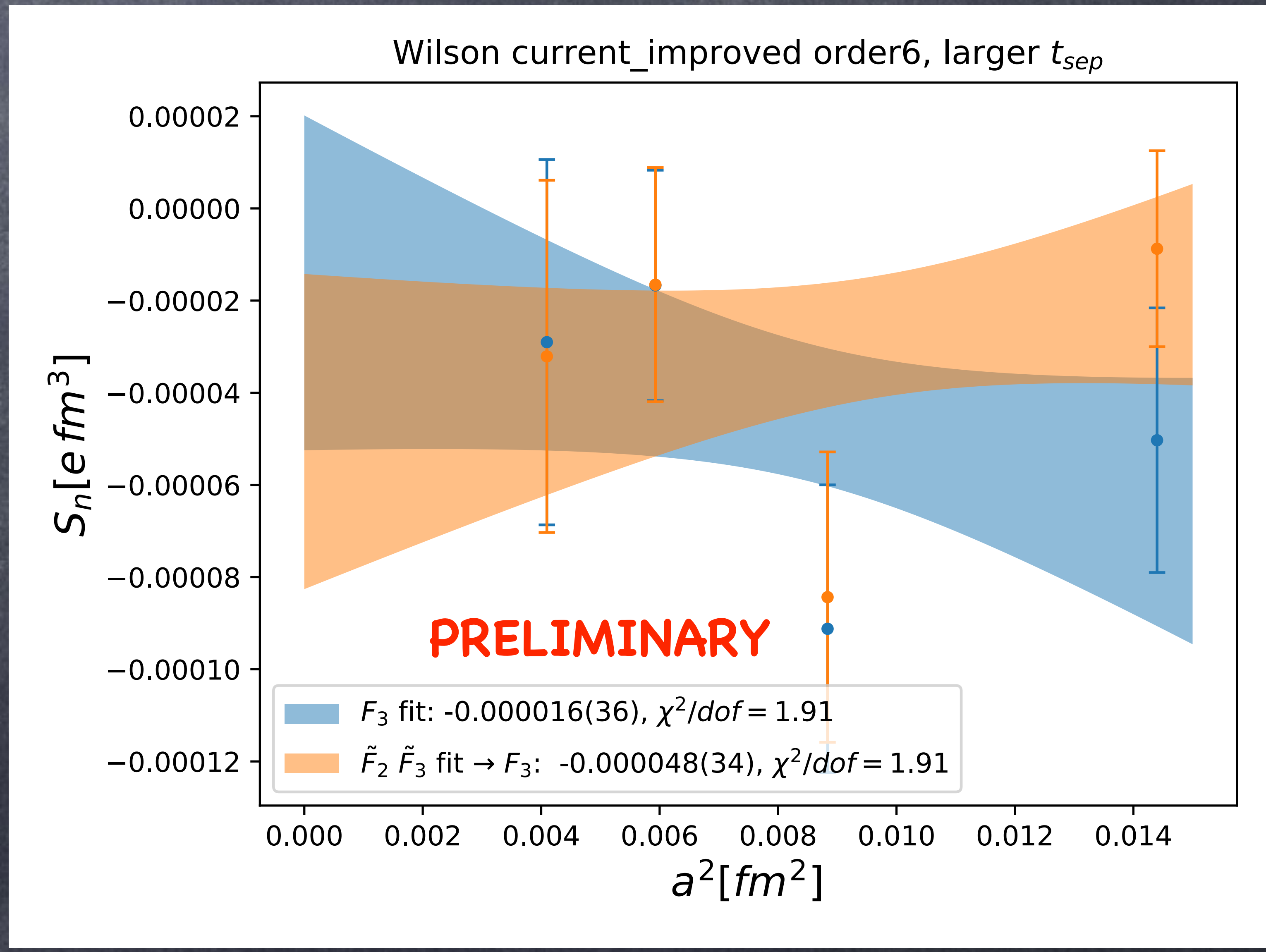
$$|S_N| = 1.7(3)\times 10^{-4}\bar{\theta}\ e\ \mathrm{fm}^3$$

Mereghetti et al.: 2011







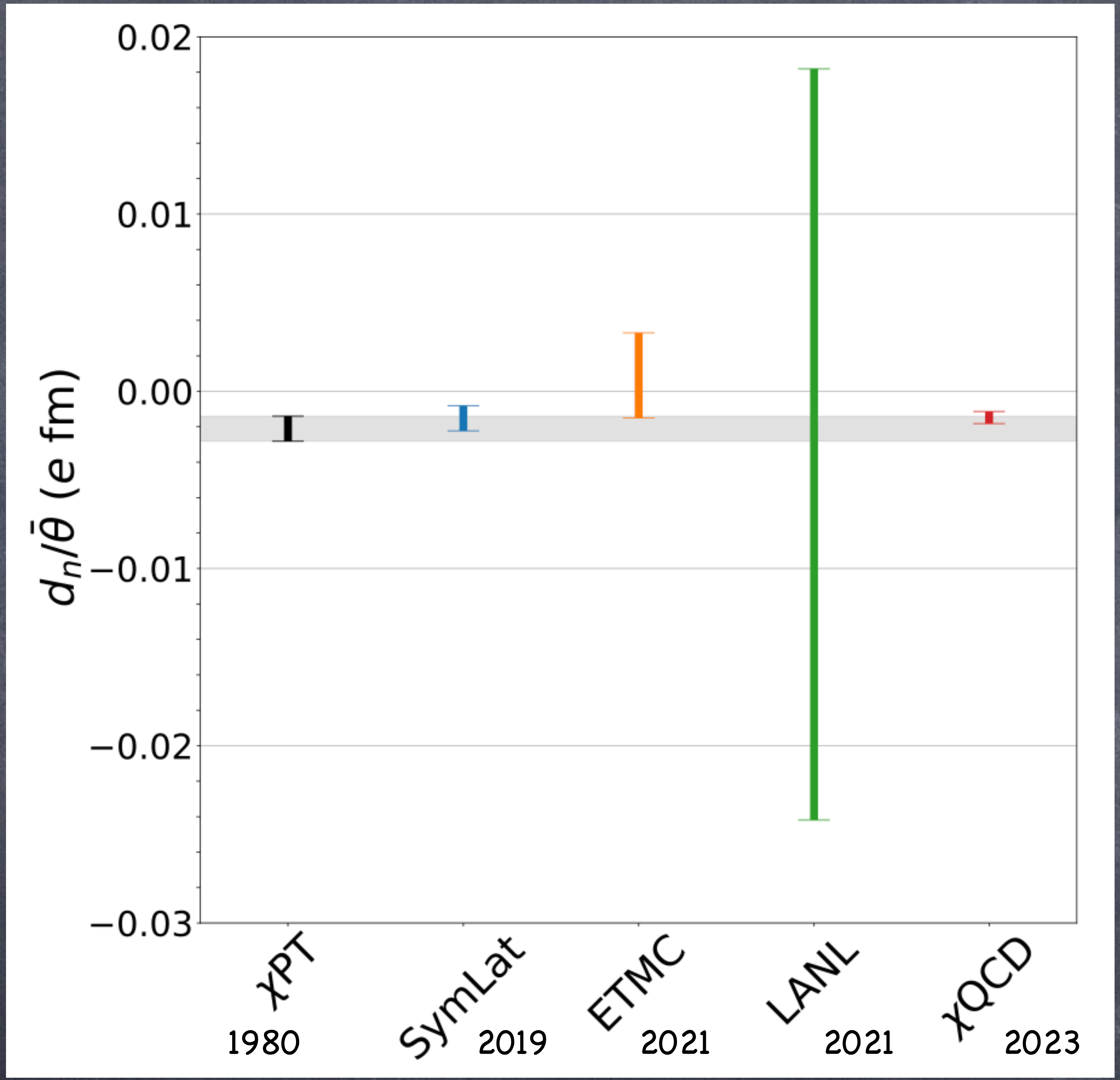


Theta-term Neutron EDM - status

χPT
Crewther, Di Vecchia, Veneziano, Witten: 1980

SymLat
Dragos, Luu, A.S., de Vries, Yousif: 2019

ETMC
Alexandrou, Athenodorou, Hadjiyiannakou, Todaro: 2021



LANL
Bhattacharya, Cirigliano, Gupta, Mereghetti: 2021

χQCD
Liang, Alexandru, Draper, Liu, B. G. Wang, Wang, Yang: 2023

Acharya et al.: 2023
(White Paper for 2023 NSAC LRP)

Quark-Chromo EDM

Renormalization

Bhattacharya, Cirigliano, Gupta, Mereghetti, Yoon: 2015

$$\mathcal{O}_{\mathrm{CE}}(x) = \overline{\psi}(x)\gamma_5\sigma_{\mu\nu}G_{\mu\nu}t^a\psi(x)$$

$$[\mathcal{O}_{\mathrm{CE}}]_R = Z_{\mathrm{qcEDM}}\left[\mathcal{O}_{\mathrm{CE}} - \frac{C}{a^2}P\right] + \cdots$$

$$P(x) = \overline{\psi}(x)\gamma_5 t^a\psi(x)$$

RI-MOM Off-shell

$\frac{1}{a}$ d=4 —> 2 operators + 3 O(m)

$\log a$ d=5 —> 3 operators + (7 + 5) O(m,m^2) + 4 “nuisance”

Power divergences need to be subtracted non-perturbatively

Maiani, Martinelli, Sachrajda: 1992

Gradient flow

Lüscher: 2013

$$\boxed{\begin{array}{l} \partial_t \chi(x,t) = \Delta \chi(x,t) \qquad \partial_t \bar{\chi}(x,t) = \bar{\chi}(x,t)\overleftarrow{\Delta} \\ \chi(x,t=0) = \psi(x) \\ \bar{\chi}(x,t=0) = \bar{\psi}(x) \end{array}}$$

$x_\mu = (x_0, \mathbf{x}) \qquad t = \mathrm{flow-time} \qquad [t] = -2$

$$\Delta = D_{\mu,t} D_{\mu,t} \qquad D_{\mu,t} = \partial_\mu + B_{t,\mu}$$

$$\chi_R(x,t) = Z_\chi^{1/2} \chi(x,t) \qquad \mathcal{O}(x,t) = \overline{\chi}(x,t)\Gamma(x,t)\chi(x,t) \qquad \mathcal{O}_R = Z_\chi \mathcal{O}$$

$$\Sigma_t = \langle \overline{\chi}(x,t)\chi(x,t) \rangle \qquad \Sigma_{t,R} = Z_\chi \Sigma_t$$

Lüscher: 2010, 2013
Lüscher, Weisz: 2011

No additive divergences

All fermion operators renormalize multiplicatively with same factor

Strategy - Short flow-time expansion

A.S., Luu, de Vries: 2014-2015
Dragos, Luu, A.S. de Vries: 2018-2019
Rizik, Monahan, A.S.: 2018-2020
A.S.: 2020
Kim, Luu, Rizik, A.S.: 2020
Mereghetti, Monahan, Rizik, A.S., Stoffer: 2021
Crosas, Monahan, Rizik, A.S., Stoffer: 2023

$$[\mathcal{O}_i(t)]_{\mathrm{R}} = \sum_i c_{ij}(t,\mu)\,[\mathcal{O}_i(t=0,\mu)]_{\mathrm{R}} + O(t)$$

LQCD PT - LQCD

$$c_{ij}(t,\mu) = \delta_{ij} + \frac{\alpha_s(\mu)}{4\pi} c_{ij}^{(1)}(t,\mu) + O(\alpha_s^2)$$

- Calculation of matrix elements with flowed fields
  - Multiplicative renormalization (no power divergences and no mixing)
- Calculation of Wilson coefficients
  - Insert OPE in off-shell amputated 1PI Green's functions
- Power divergences subtracted non-perturbatively (LQCD)
- Determination of the physical renormalized matrix element at zero flow-time

Quark-Chromo EDM

Rizik, Monahan, A.S.: 2020
Mereghetti, Monahan, Rizik, A.S., Stoffer : 2021

$$[\mathcal{O}_i(t)]_{\mathrm{R}} = \sum_i c_{ij}(t,\mu)\,[\mathcal{O}_i(t=0,\mu)]_{\mathrm{R}} + O(t)$$

$$\mathcal{O}_{CE}(x,t) = \bar{\chi}(x,t)\tilde{\sigma}_{\mu\nu}G_{\mu\nu}(x,t)\chi(x,t)$$

$$\tilde{\sigma}^{\mathrm{HV}}_{\mu\nu} = -\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\sigma_{\alpha\beta} \qquad \tilde{\sigma}^{\mathrm{NDR}}_{\mu\nu} = \sigma_{\mu\nu}\gamma_5$$

$$\mathcal{O}_P(x) = \bar{\psi}(x)\gamma_5\psi(x)$$

$$\mathcal{O}_{m^2P}(x) = m^2\bar{\psi}(x)\gamma_5\psi(x)$$

$$\mathcal{O}_{m\theta}(x) = m\mathrm{tr}[G_{\mu\nu}\widetilde{G}_{\mu\nu}]$$

$$\mathcal{O}_E(x) = \bar{\psi}(x)\tilde{\sigma}_{\mu\nu}F_{\mu\nu}(x)\psi(x)$$

$$\begin{aligned}\mathcal{O}^R_{CE}(x;t) = {} & c_P(t,\mu)\mathcal{O}^{\mathrm{MS}}_P(x;\mu) + c_{m\theta}(t,\mu)\mathcal{O}^{\mathrm{MS}}_{m\theta}(x;\mu) + c_E(t,\mu)\mathcal{O}^{\mathrm{MS}}_E(x;\mu)\\ & + c_{CE}(t,\mu)\mathcal{O}^{\mathrm{MS}}_{CE}(x;\mu) + c_{mP^2}(t,\mu)\mathcal{O}^{\mathrm{MS}}_{m^2P}(x;\mu) + \cdots\end{aligned}$$

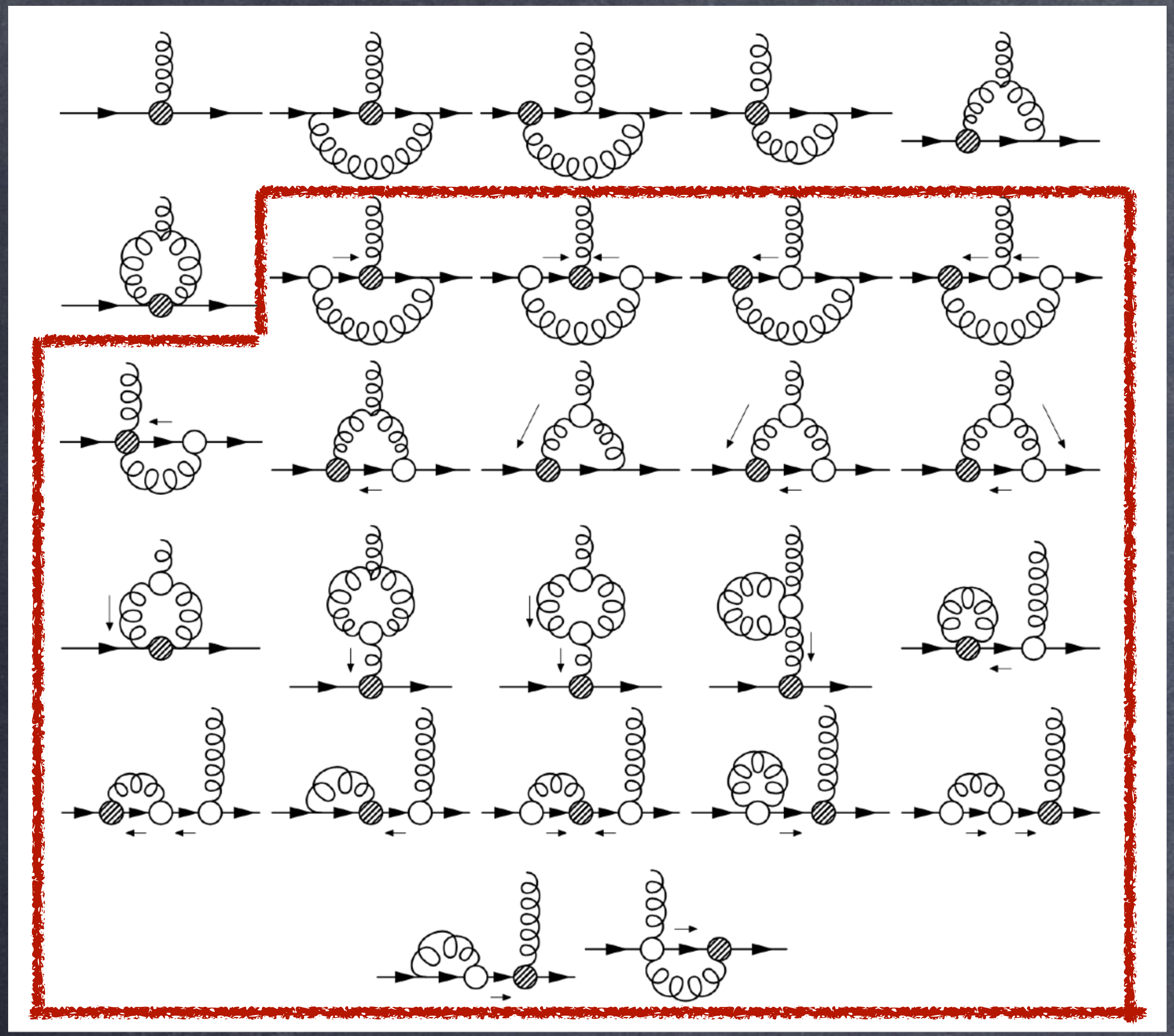
$$Z_\chi^{-n/2}\left\langle(\psi)^{n_\psi}\,(\bar{\psi})^{n_{\bar{\psi}}}\,(A_\mu)^{n_A}\,\mathcal{O}_i(t)\right\rangle^{\mathrm{amp}} = c_{ij}(t)\,\left(Z^{\mathrm{MS}}_{jk}\right)^{-1}\left\langle(\psi)^{n_\psi}\,(\bar{\psi})^{n_{\bar{\psi}}}\,(A_\mu)^{n_A}\,\mathcal{O}_k\right\rangle^{\mathrm{amp}}$$

$$c_{ij}(t,\mu) = \delta_{ij} + \frac{\alpha_s(\mu)}{4\pi}c^{(1)}_{ij}(t,\mu) + O(\alpha_s^2)$$

Quark-Chromo EDM

Rizik, Monahan, A.S.: 2020
Mereghetti, Monahan, Rizik, A.S., Stoffer : 2021

$c_{CE}(t,\mu)$



- Expand integrands of loop integrals in all scales excluding t
  - Analytic structure altered –> distortion of IR structure
  - in matching equation the IR modification drops out in the difference
  - Expanding loop integrals in the RHS vanish in DR –> UV and IR are identical
  - The LHS is UV-finite, beside the renormalization of the bare parameters and flowed fermion fields
  - The IR singularities on the LHS exactly match the UV MS counterterms

$$c_{CE}(t,\mu) = \zeta_\chi^{-1} + \frac{\alpha_s}{4\pi}\left[2(C_F - C_A)\log(8\pi\mu^2 t) - \frac{1}{2}\Big((4+5\delta_{\rm HV})C_A + (3-4\delta_{\rm HV})C_F\Big)\right]$$

$$= 1 + \frac{\alpha_s}{4\pi}\bigg[(5C_F - 2C_A)\log(8\pi\mu^2 t) - \frac{1}{2}\Big((4+5\delta_{\rm HV})C_A + (3-4\delta_{\rm HV})C_F\Big) - \log(432)C_F\bigg]$$

3-gluon CP-odd operator

3 gluon CP-odd

Rizik, Monahan, A.S.: 2020
Crosas, Monahan, Rizik, A.S., Stoffer : 2023

$$[\mathcal{O}_i(t)]_{\mathrm{R}} = \sum_i c_{ij}(t,\mu)\,[\mathcal{O}_i(t=0,\mu)]_{\mathrm{R}} + O(t)$$

$$\mathcal{O}_{\widetilde{G}}(x,t) = \frac{1}{g^2}\mathrm{Tr}[G_{\mu\nu}G_{\nu\lambda}\widetilde{G}_{\lambda\mu}]$$

$$\mathcal{O}_{\widetilde{G}}(x,t) = \sum_i C_i(t,\mu) O_i^{\mathrm{MS}}(x,\mu) + \ldots$$

$$\mathcal{O}_\theta = \frac{1}{g_0^2}\mathrm{Tr}[G_{\mu\nu}\widetilde{G}_{\mu\nu}]$$

$$\mathcal{O}_{\widetilde{G}} = \frac{1}{g_0^2}\mathrm{Tr}[G_{\mu\nu}G_{\nu\lambda}\widetilde{G}_{\lambda\mu}]$$

$$\mathcal{O}_{CE} = \overline{\chi}\sigma_{\mu\nu}G_{\mu\nu}\mathcal{M}\chi$$

$$\mathcal{O}_{\partial G} = \frac{1}{g_0^2}\partial_\nu \mathrm{Tr}[(D_\mu G_{\mu\lambda})\widetilde{G}_{\nu\lambda}]$$

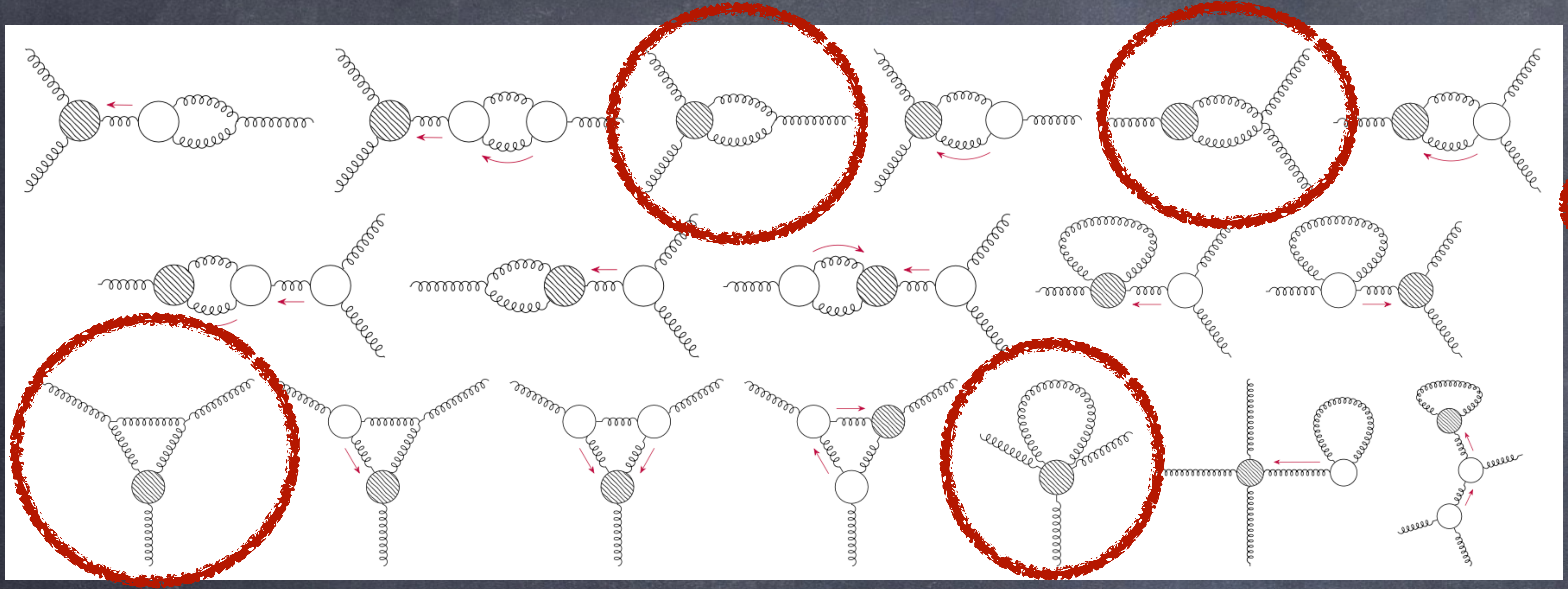
$$\mathcal{O}_{\Box\theta} = \frac{1}{g_0^2}\Box\mathrm{Tr}[G_{\mu\nu}\widetilde{G}_{\mu\nu}]$$

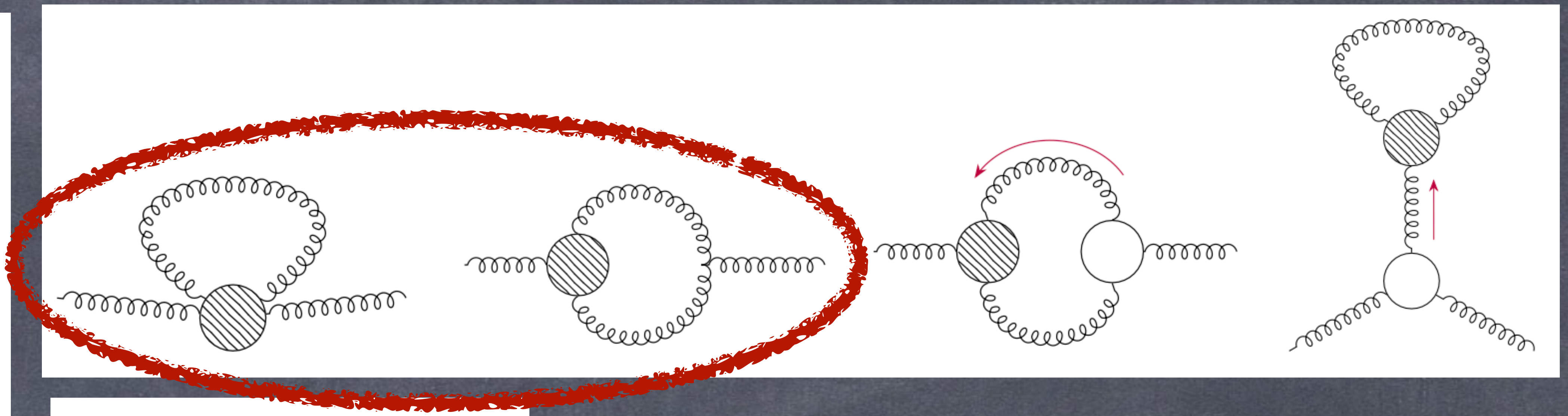
$$Z_\chi^{-n/2}\left\langle(\psi)^{n_\psi}\,(\bar{\psi})^{n_{\bar{\psi}}}\,(A_\mu)^{n_A}\,\mathcal{O}_i(t)\right\rangle^{\mathrm{amp}} = c_{ij}(t)\,\left(Z_{jk}^{\mathrm{MS}}\right)^{-1}\left\langle(\psi)^{n_\psi}\,(\bar{\psi})^{n_{\bar{\psi}}}\,(A_\mu)^{n_A}\,\mathcal{O}_k\right\rangle^{\mathrm{amp}}$$

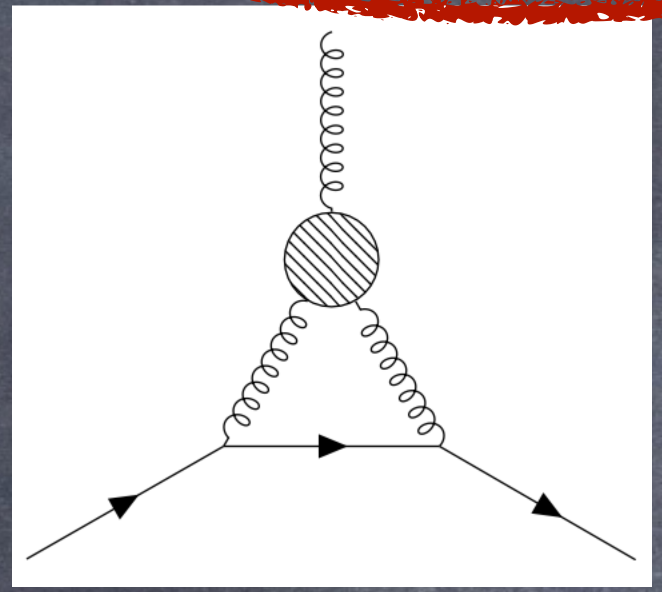
$$c_{ij}(t,\mu) = \delta_{ij} + \frac{\alpha_s(\mu)}{4\pi}c_{ij}^{(1)}(t,\mu) + O(\alpha_s^2)$$

3 gluon CP-odd

Rizik, Monahan, A.S.: 2020
Crosas, Monahan, Rizik, A.S., Stoffer : 2023







$$C_\theta = -\frac{9\alpha_s C_A}{16\pi}\frac{1}{t}$$

$$\mathcal{O}_{\widetilde{G}}(x,t) = \sum_i C_i(t,\mu) O_i^{\mathrm{MS}}(x,\mu) + \ldots$$

$$C_{\widetilde{G}} = 1 + \frac{\alpha_s C_A}{12\pi} + \frac{3\alpha_s C_A \log(8\pi\mu^2 t)}{2\pi}$$

$$C_{CE} = \frac{31i\,\alpha_s C_A}{192\pi} + \frac{3i\,\alpha_s C_A \log(8\pi\mu^2 t)}{32\pi}$$

$$C_{\partial G} = -\frac{179\alpha_s C_A}{96\pi}$$

$$C_{\Box\theta} = 0$$

SYMLAT Status

- Theta-term nucleon EDM —> first results 1409.2735 1507.02343 1809.03487 1902.03254
  - Renormalization, S/N
- Quark-chromo EDM —> renormalization
  - Power divergences —> PT 1810.05637 2005.04199 2111.1149
    Non-perturbative 1810.10301 2106.07633
  - Logs/mixing —> 2111.1149 2212.09824
- 3 gluon operator —> PT power divergences 2005.04199
  Preliminary studies for renormalization (power divergences) 1711.04730 1810.05637
  —> Logs/mixing 2308.16221

Neutron EDM from Lattice QCD

Quark EDM —>
simplest calculation with Lattice QCD. Precision 3%-5%. No Disc.

Theta-term nucleon EDM —> few calculations: 2 σ effect

—> new result have stronger signal

3 gluon operator —> No Lattice QCD calculation, 1-loop matching

4-fermion operators —> No Lattice QCD calculation, 1-loop matching

Quark-chromo EDM —>
First result with LO renormalization
New promising approach based on gradient flow —>
1-loop matching, NP power divergence,
2-loop in progress

	Renormalization	Continuum limit	Chiral extrapolation	Finite Volume	Excited States
θ - term	🟢	🟢	🟡	🟡	🟡
quark EDM	🟢	🟢	🟢	🟢	🟢
quark-chromo	🟢	🔴	🟡	🔴	🔴
3-gluon	🟡	🔴	🟡	🔴	🔴
4-fermion	🟡	🔴	🔴	🔴	🔴

Work in progress

- Improve determination of nEDM from theta-term — OpenLat
- Matching coefficients of qCEDM at 2-loops — Borgulat, Harlander, Rizik, A.S.
- Non-perturbative determination of power divergences with SWF
- Calculation of the qCEDM in a nucleon — Kim, Luu, Pederiva, Rizik, A.S.
- OpenLat: open science initiative. Gauges with SWF open to the whole community — Cuteri, Francis, Fritzsch, Pederiva, Rago, A.S., Walker-Loud, Zafeiropoulos

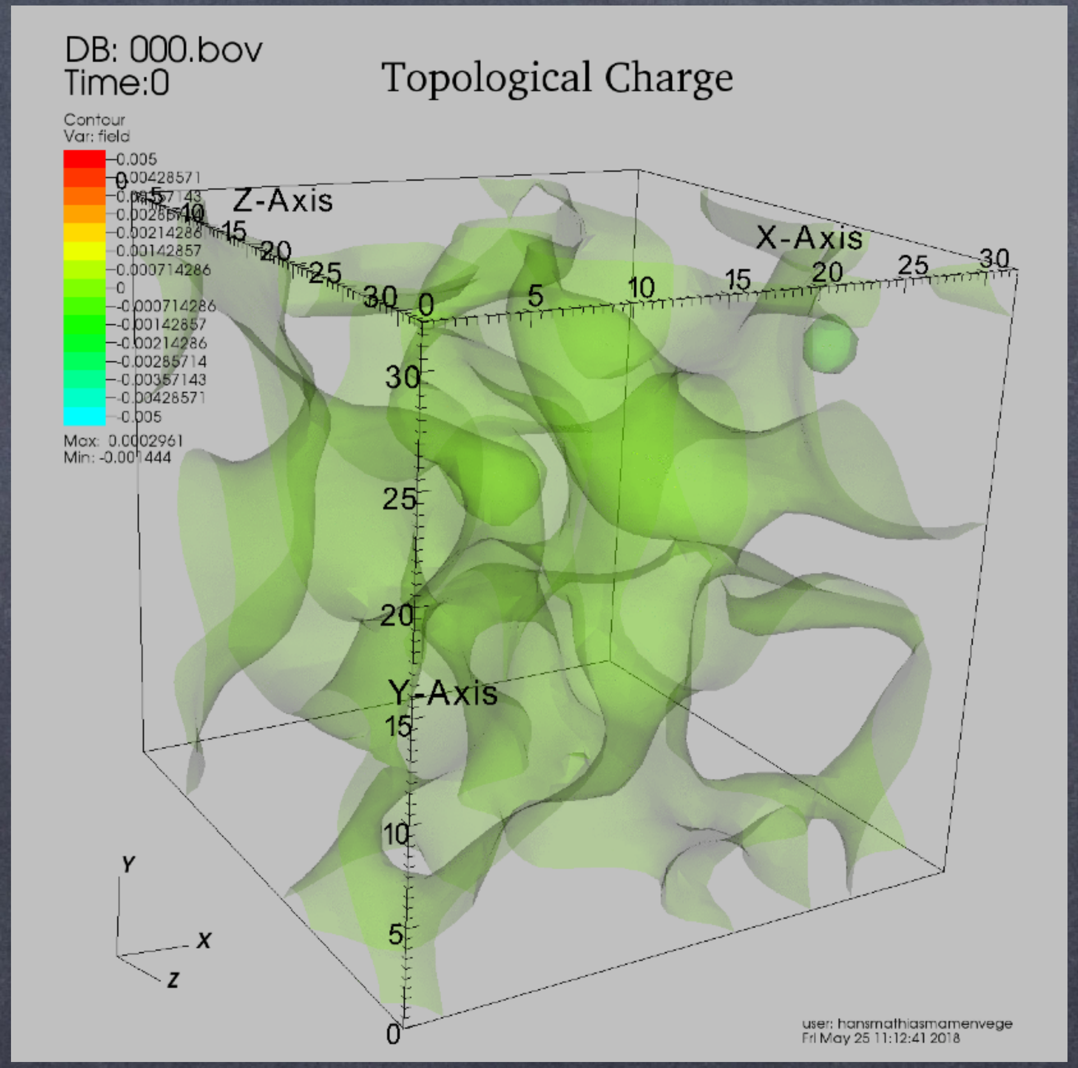
Near term goals

- Several calculations of the theta EDM. ChiPT is consistent with our results
- Calculate theta-term contribution to the nucleon EDM with GF and improved S-to-N ratios. O(20%) for theta-term in the next 2-3 years
- Extension to 2-loops the calculation of the matching factors
- Non-perturbative renormalization and first calculations of qCEDM and 3 gluon operator matrix elements. Use of the gradient flow is critical.

Lattice QCD is moving towards a determination of nucleon EDM

Stay tuned

Thank you!



Backup Slides

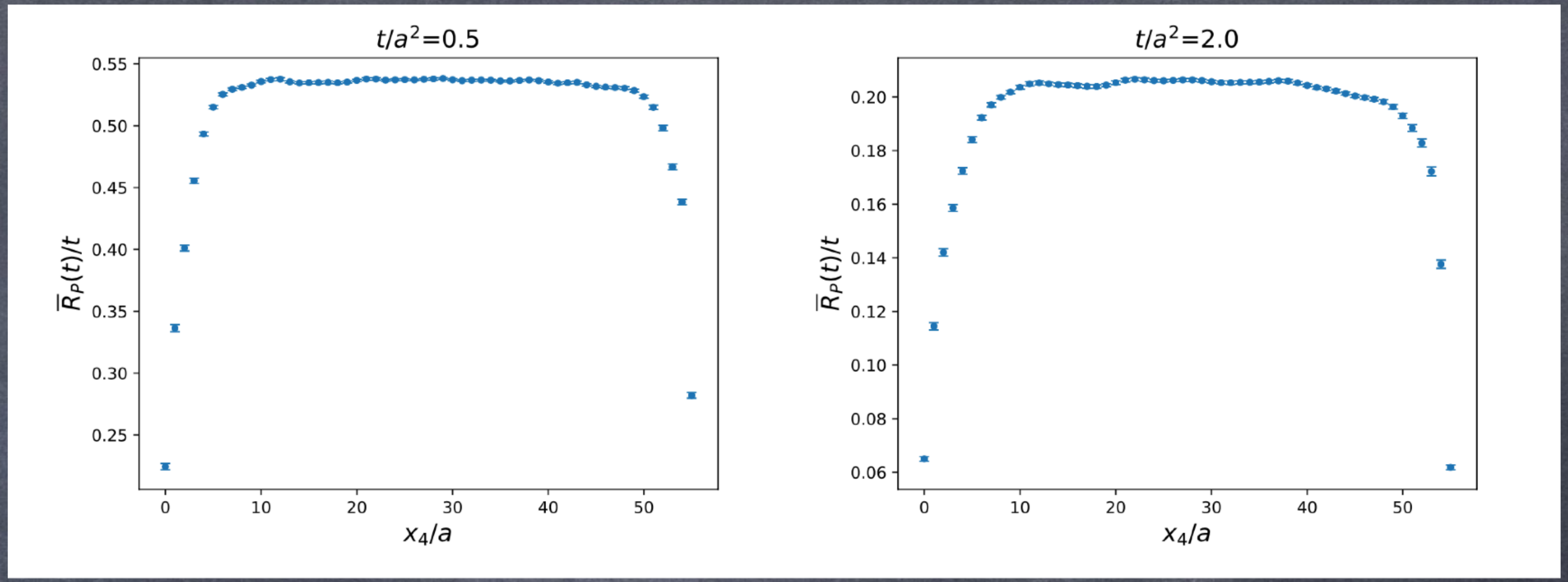
Quark-Chromo EDM: power divergences

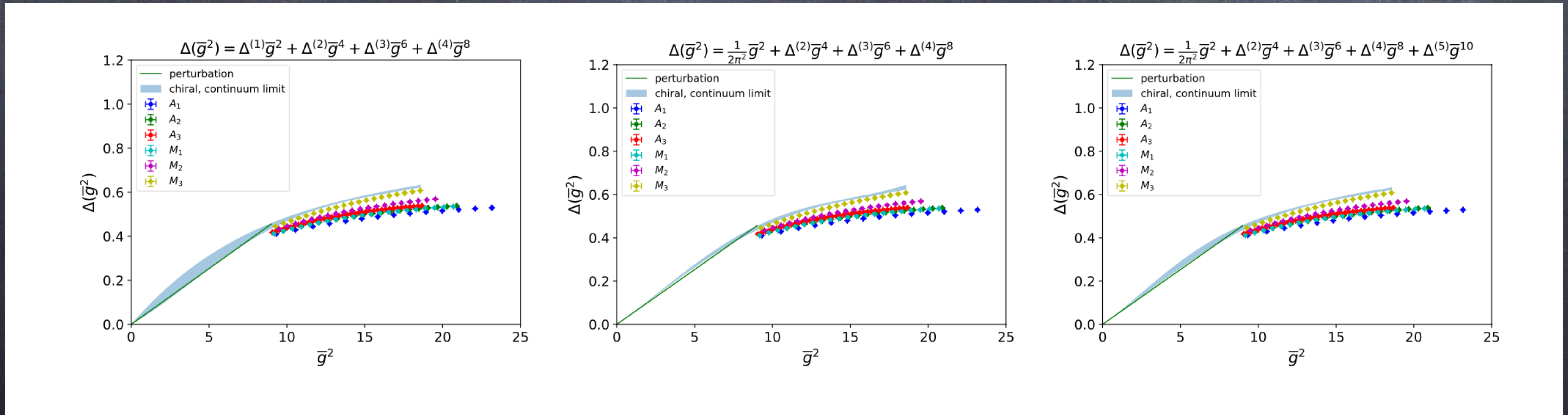
Kim, Luu, Rizik, A.S.:2020

$$\Gamma_{CP}(x_4;t) = a^3 \sum_{\mathbf{x}} \left\langle \mathcal{O}^{ij}_{CE}(x_4,\mathbf{x};t) P^{ji}(0,\mathbf{0};0) \right\rangle$$

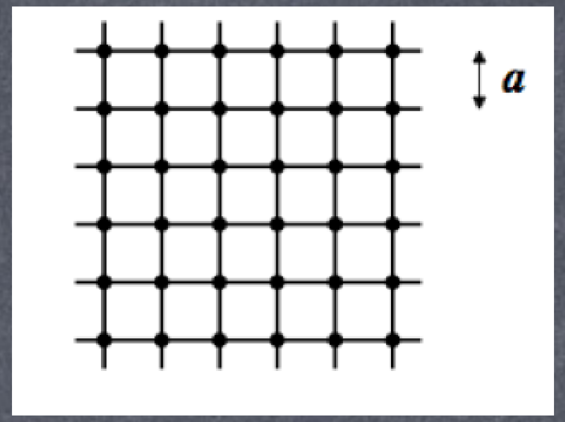
$$\Gamma_{PP}(x_4;t) = a^3 \sum_{\mathbf{x}} \langle P^{ij}(x_4,\mathbf{x};t) P^{ji}(0,\mathbf{0}) \rangle$$

$$\overline{R}_{\mathrm{P}}(x_4;t) = t\frac{\Gamma_{\mathrm{CP}}(x_4;t)}{\Gamma_{\mathrm{PP}}(x_4,t)}$$





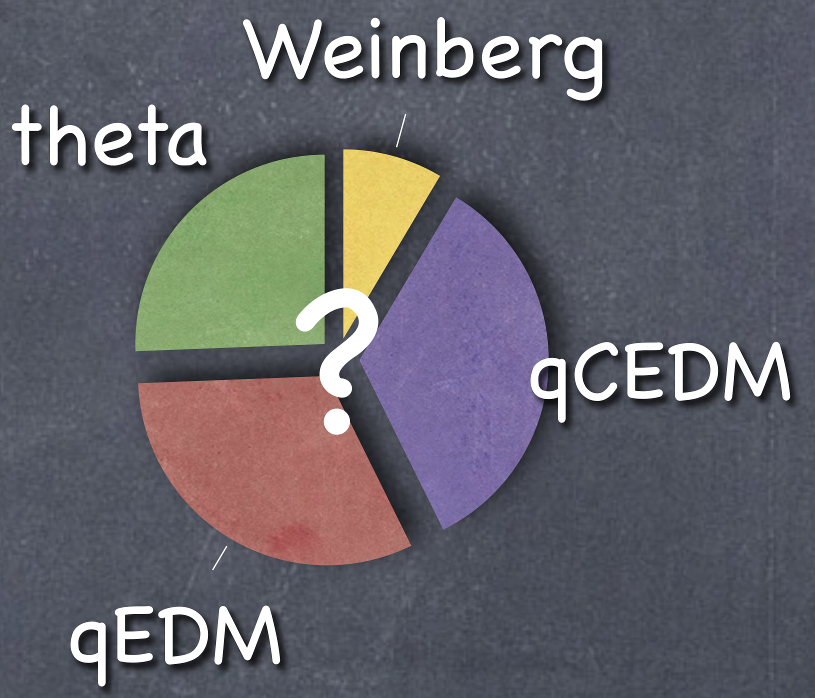
The role of lattice QCD



$$d_{\rm N} = M_{\rm N}^{\theta}\bar{\theta} + \left(\frac{v}{\Lambda}\right)^2 \sum_i M_{\rm N}^{(i)}\tilde{d}_i \qquad \langle N|J_\mu \mathcal{O}_{\cancel{CP}}|N\rangle \to d\;\underline{E}\cdot\underline{S}$$

$M_{\rm N}^{\theta}$ ⟶ Hadronic matrix element topological charge

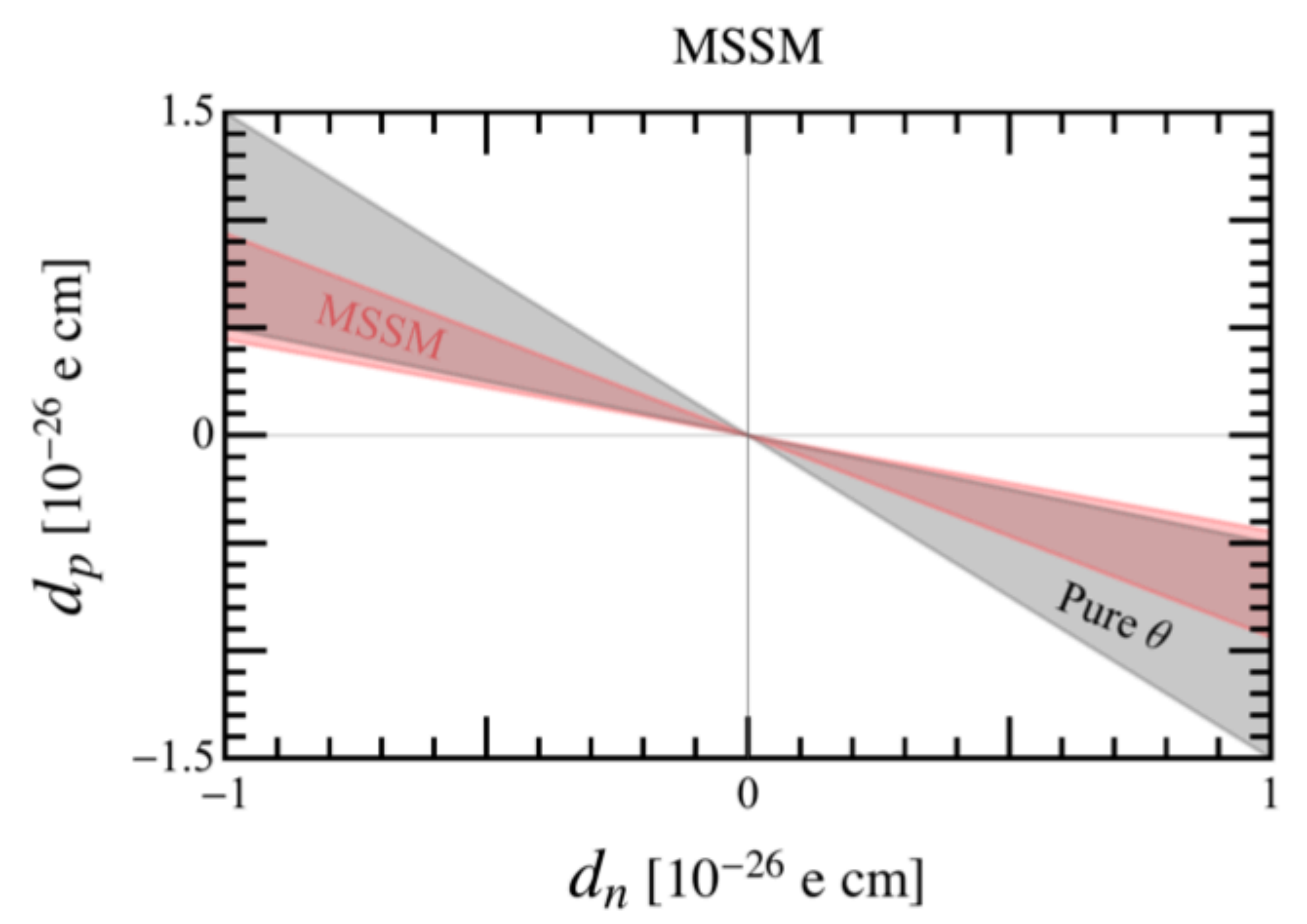
$M_{\rm N}^{(i)}$ ⟶ Hadronic matrix element CP odd operators

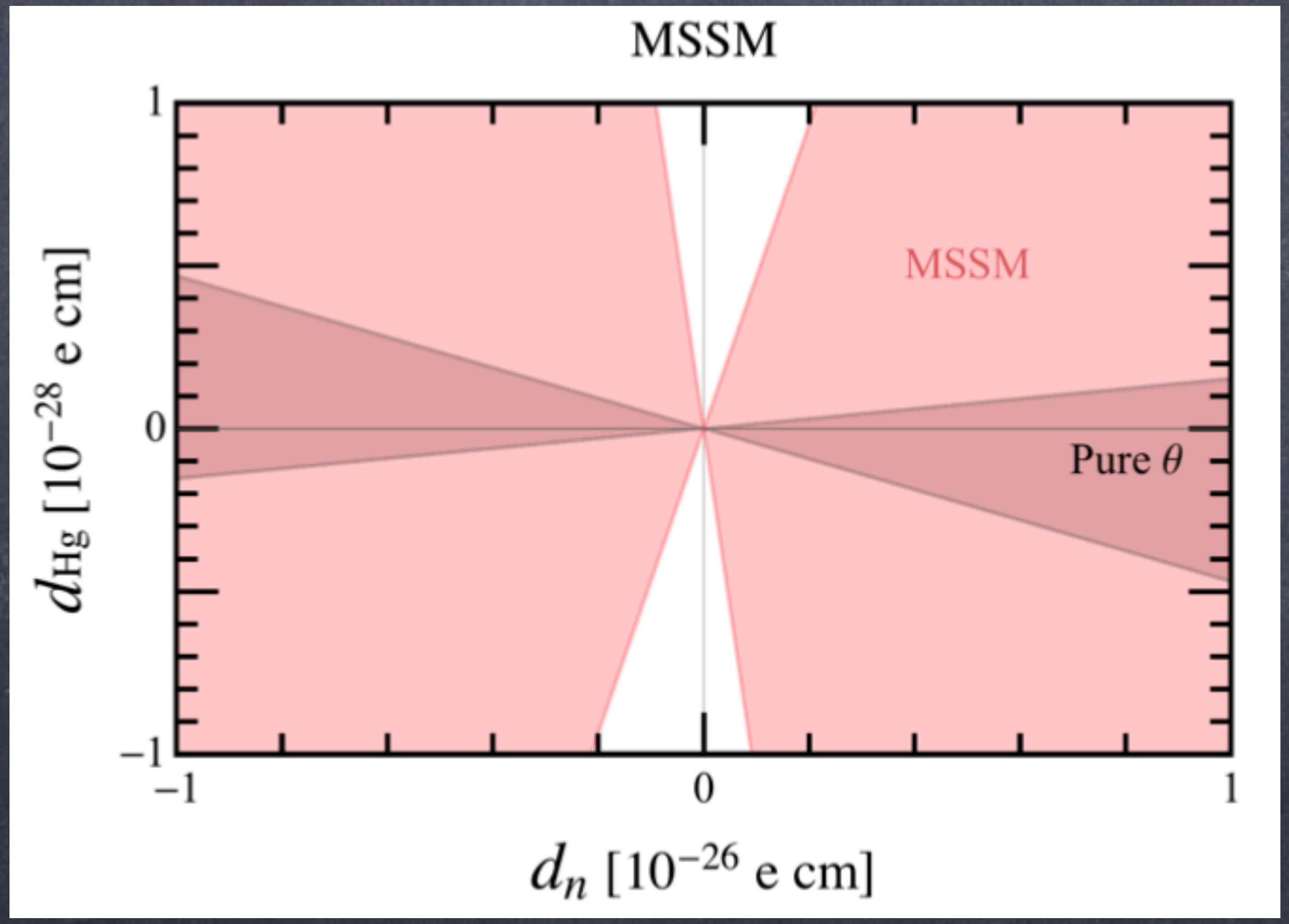


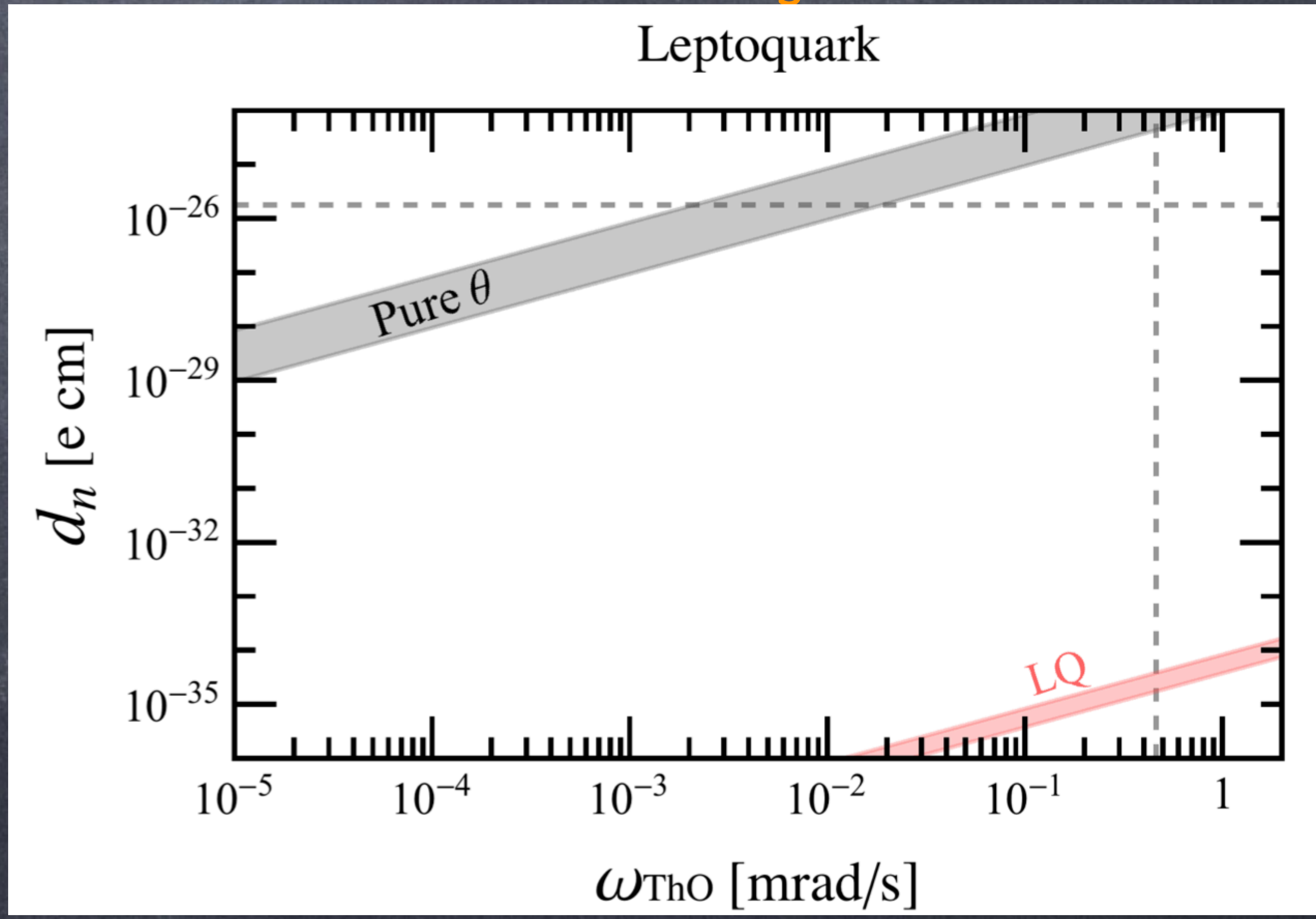
Alarcon et al.: 2022
Snowmass Summer Study Report

$$\begin{aligned} d_n = \; & - \quad (1.5 \pm 0.7)\cdot 10^{-3}\bar{\theta} e \text{ fm} \\ & - \quad (0.2 \pm 0.01) d_u + (0.78 \pm 0.03) d_d + (0.0027 \pm 0.016) d_s \\ & - \quad (0.55 \pm 0.28) e\tilde{d}_u - (1.1 \pm 0.55) e\tilde{d}_d + (50 \pm 40)\text{MeV} e\tilde{d}_G \end{aligned}$$

Shintani et al.: 2005
Berruto, Blum, Orginos, Soni 2006







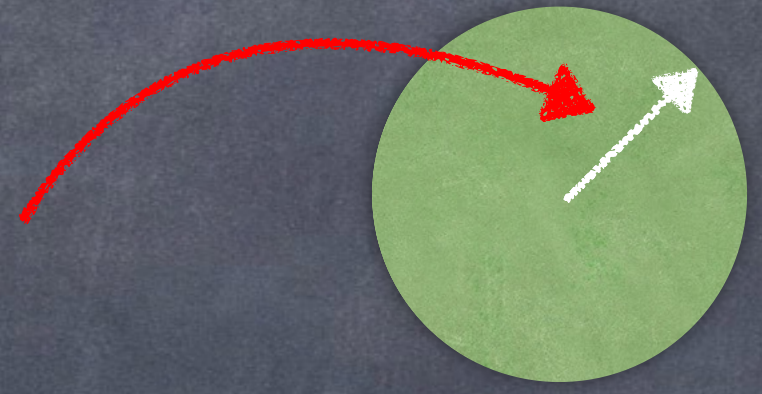
de Vries et al.: 2021

Gradient flow

Lüscher: 2013

$$\chi(x,t) = \int d^4y K(x-y,t)\psi(y) \quad K(x,t) = \frac{1}{4\pi t^2} \mathrm{e}^{-\frac{x^2}{4t}}$$

- Smoothing over a range $\sqrt{8t}$



- Gaussian damping at large momenta

$$\chi_R(x,t) = Z_\chi^{1/2}\chi(x,t) \qquad \mathcal{O}(x,t) = \overline{\chi}(x,t)\Gamma(x,t)\chi(x,t) \qquad \mathcal{O}_R = Z_\chi \mathcal{O}$$

$$\Sigma_t = \langle \overline{\chi}(x,t)\chi(x,t)\rangle \qquad \Sigma_{t,R} = Z_\chi \Sigma_t$$

Lüscher: 2010, 2013
Lüscher, Weisz: 2011

No additive divergences

All fermion operators renormalize multiplicatively with same factor

New physics scale

$$d_f \sim e q_f \sin(\delta_{\mathrm{CPV}}) \left(\frac{g^2}{16\pi^2}\right)^l \xi_{\mathrm{FV}} \frac{m_f}{M_{\mathrm{NP}}^2}$$

$$d_e \leq d_e^{\max}$$

$$M_{\mathrm{NP}} \gtrsim \sqrt{\frac{10^{29} e \text{ cm}}{d_e^{\max}}} \times \begin{cases} 50 \text{ TeV}, & \text{if } l = 1 \\ 2 \text{ TeV}, & \text{if } l = 2 \end{cases}$$

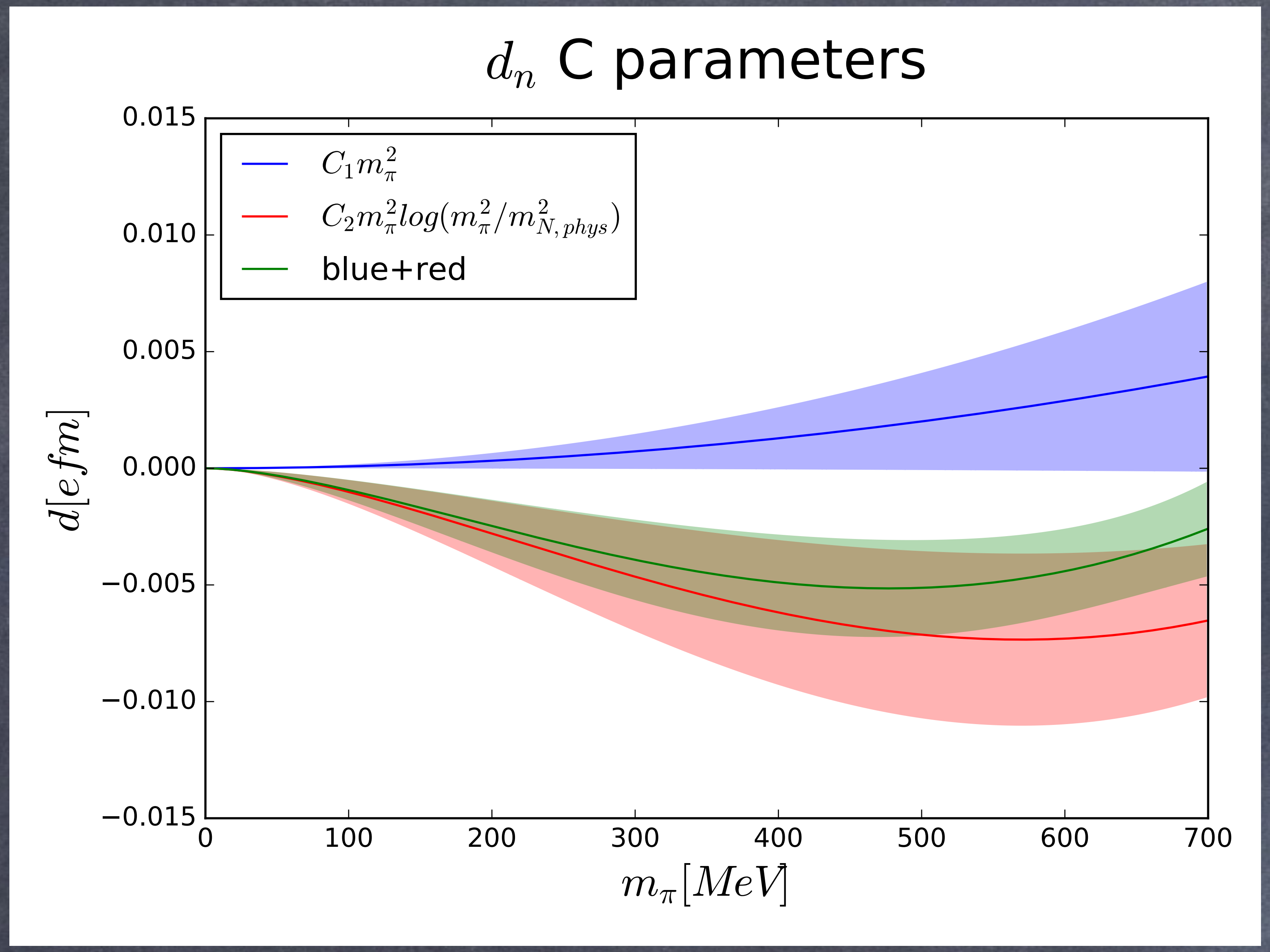
- The discovery potential in EDM searches can be roughly quantified by the reach in mass scale, assuming maximal CP violation.
- In all cases we see that the mass reach is very high - EDMs are exploring uncharted territory.
- if we insist that the scale of new physics is close to the electroweak scale, EDMs probe very small CP-violating couplings, still providing invaluable information for model building and understanding the nature of CP symmetry and its breaking.

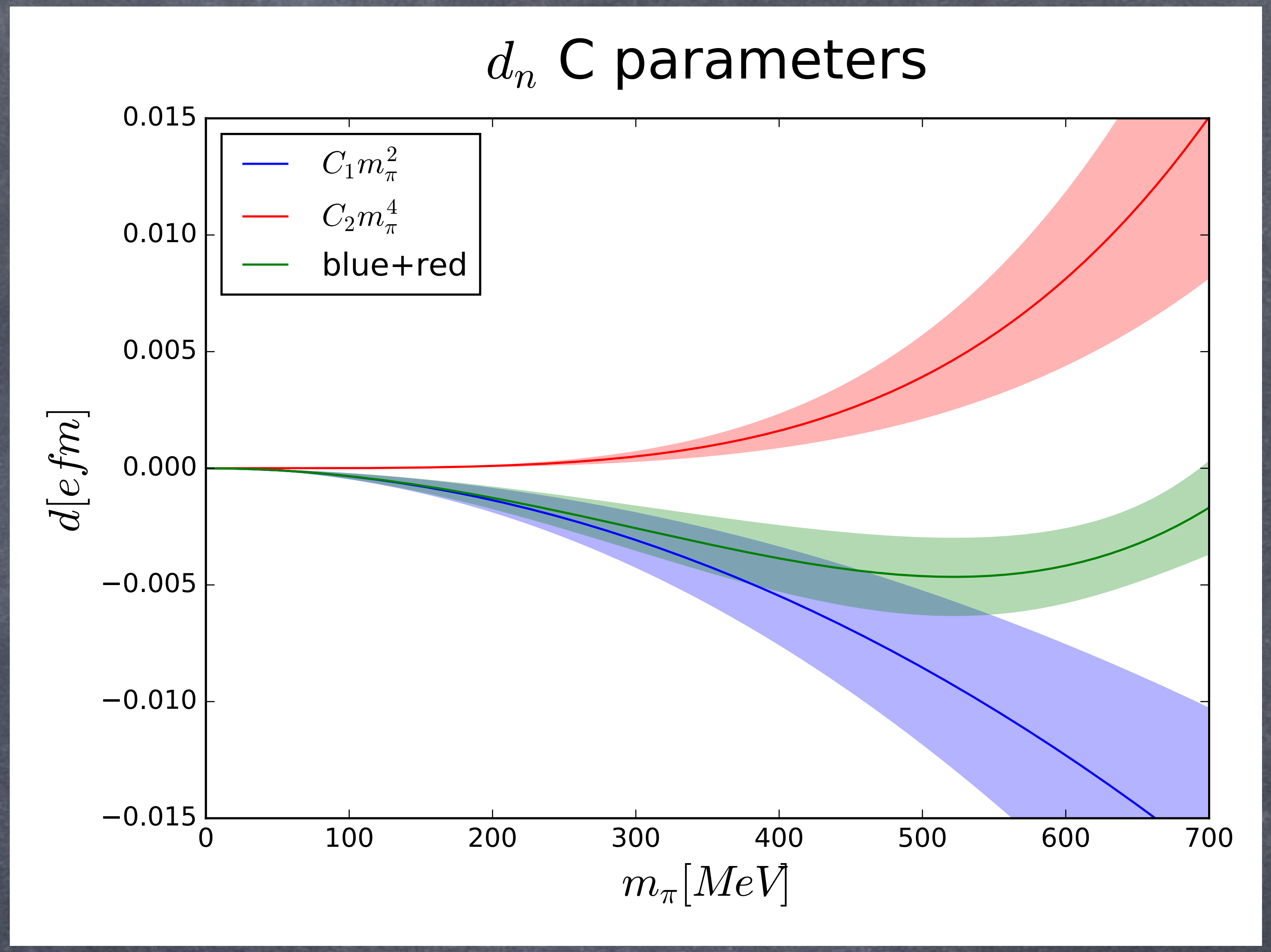
Operator	Loop order	Mass reach
Electron EDM	1	$48\,\mathrm{TeV}\sqrt{10^{-29}\,e\,\mathrm{cm}/d_e^{\max}}$
	2	$2\,\mathrm{TeV}\sqrt{10^{-29}\,e\,\mathrm{cm}/d_e^{\max}}$
Up/down quark EDM	1	$130\,\mathrm{TeV}\sqrt{10^{-29}\,e\,\mathrm{cm}/d_q^{\max}}$
	2	$13\,\mathrm{TeV}\sqrt{10^{-29}\,e\,\mathrm{cm}/d_q^{\max}}$
Up-quark CEDM	1	$210\,\mathrm{TeV}\sqrt{10^{-29}\,\mathrm{cm}/\tilde{d}_u^{\max}}$
	2	$20\,\mathrm{TeV}\sqrt{10^{-29}\,\mathrm{cm}/\tilde{d}_u^{\max}}$
Down-quark CEDM	1	$290\,\mathrm{TeV}\sqrt{10^{-29}\,\mathrm{cm}/\tilde{d}_d^{\max}}$
	2	$28\,\mathrm{TeV}\sqrt{10^{-29}\,\mathrm{cm}/\tilde{d}_d^{\max}}$
Gluon CEDM	2 ($\propto m_t$)	$22\,\mathrm{TeV}\sqrt[3]{10^{-29}\,\mathrm{cm}/(100\,\mathrm{MeV})/\tilde{d}_G^{\max}}$
	2	$260\,\mathrm{TeV}\sqrt{10^{-29}\,\mathrm{cm}/(100\,\mathrm{MeV})/\tilde{d}_G^{\max}}$

Alarcon et al.: 2022
Snowmass Summer Study Report

ChPT-inspired fit

Dragos, Luu, A.S., de Vries, Yousif: 2019





$$d_{n/p}(m_\pi) = C_1^{n/p} m_\pi^2 + C_2^{n/p} m_\pi^2 \ln \frac{m_\pi^2}{M_N^2}$$

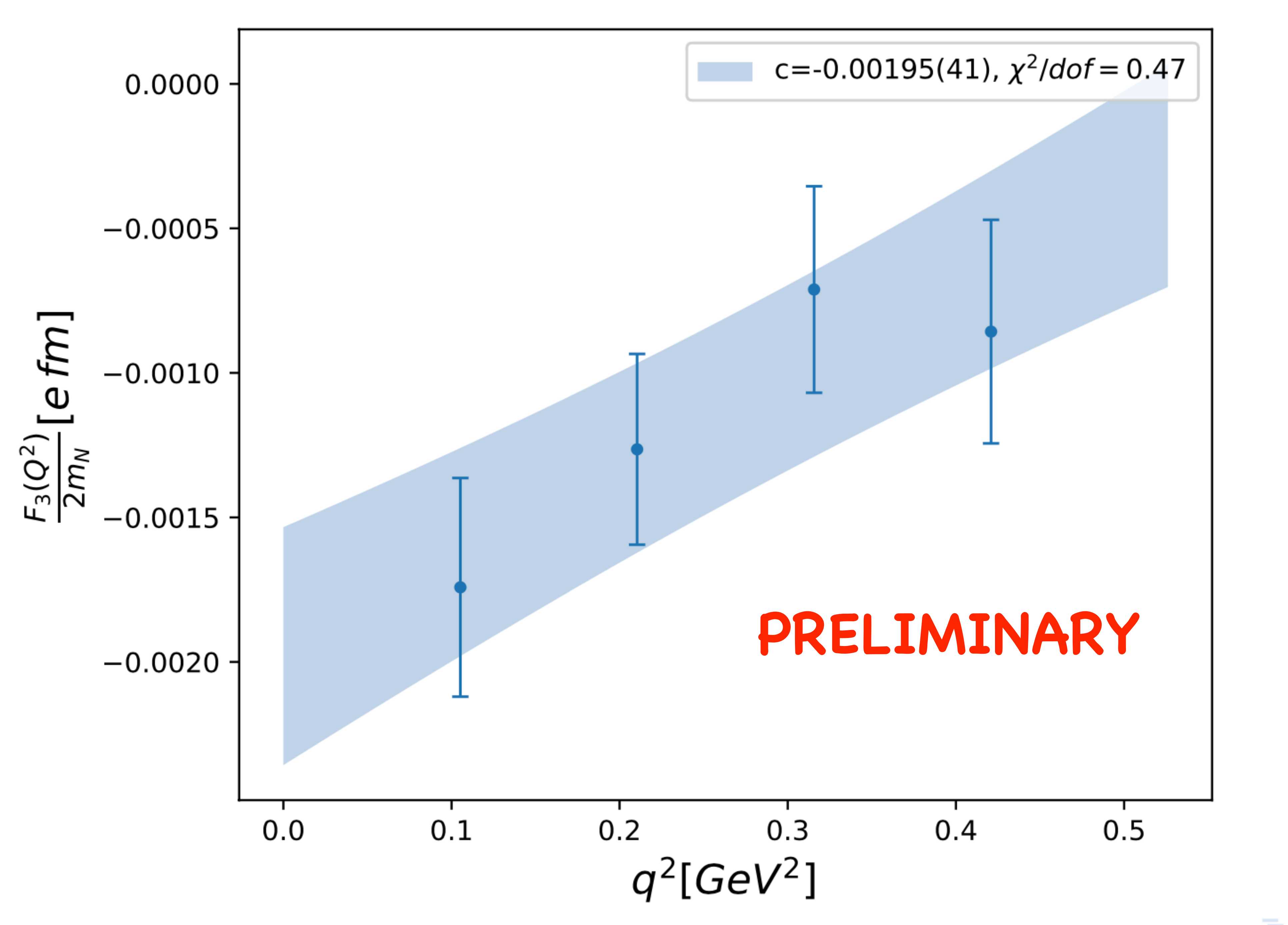
$$d_{n/p}(m_\pi) = C_1^{n/p} m_\pi^2 + C_2^{n/p} m_\pi^4$$

Data naturally favor the ChiPT-inspired pion mass dependence ==> log dominance

CP-odd form factor

OpenLat

m_π = 400 MeV
t/t_0 = 1.9

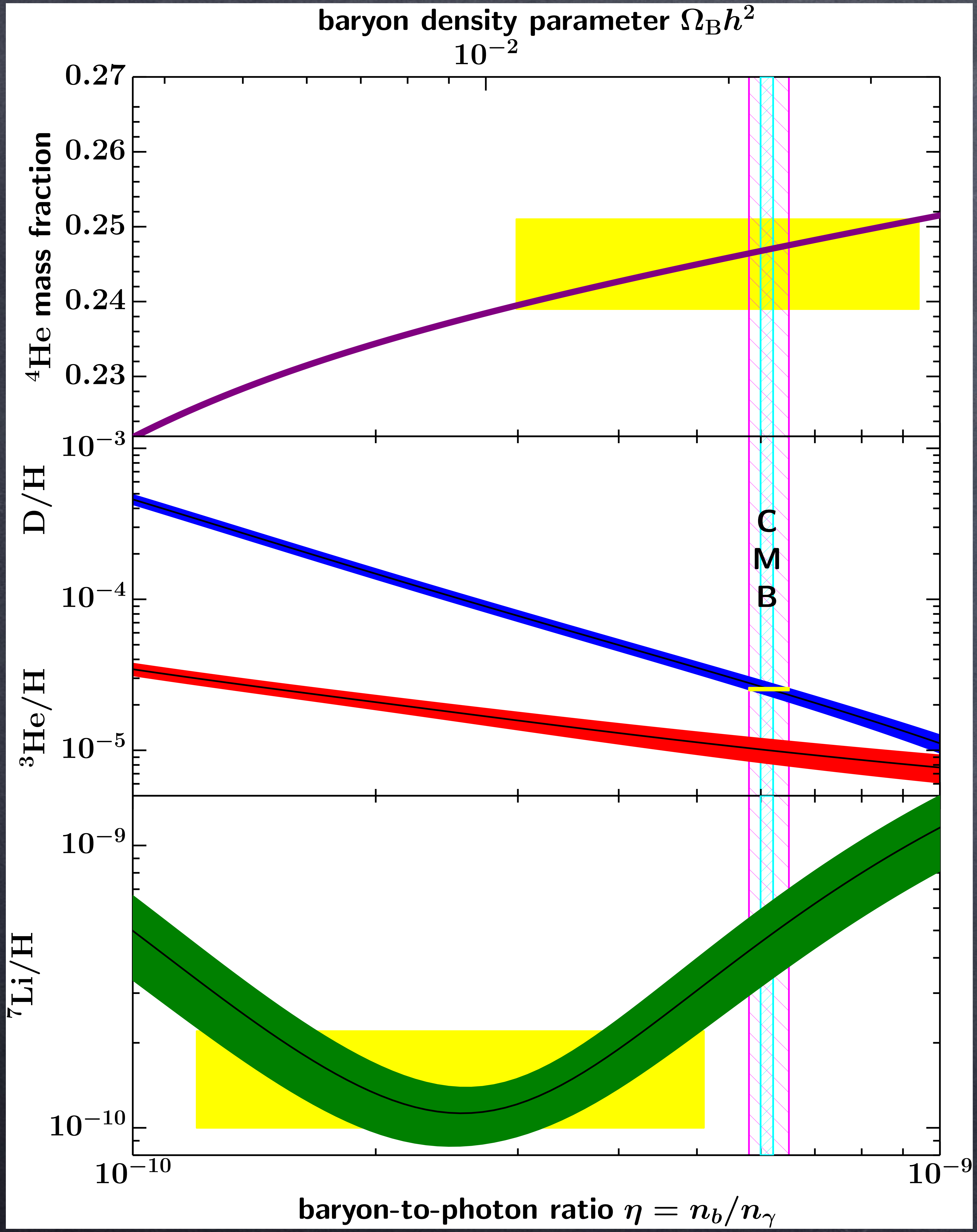


$$\frac{F_3^{P/N}(Q^2)}{2M_N} = d_{P/N} - S_{P/N}Q^2 + H_{P/N}(Q^2)$$

$$\frac{d_P}{d_N} < 0 \qquad \frac{S_P}{S_N} < 0$$

Mereghetti et al.: 2011

Matter antimatter asymmetry



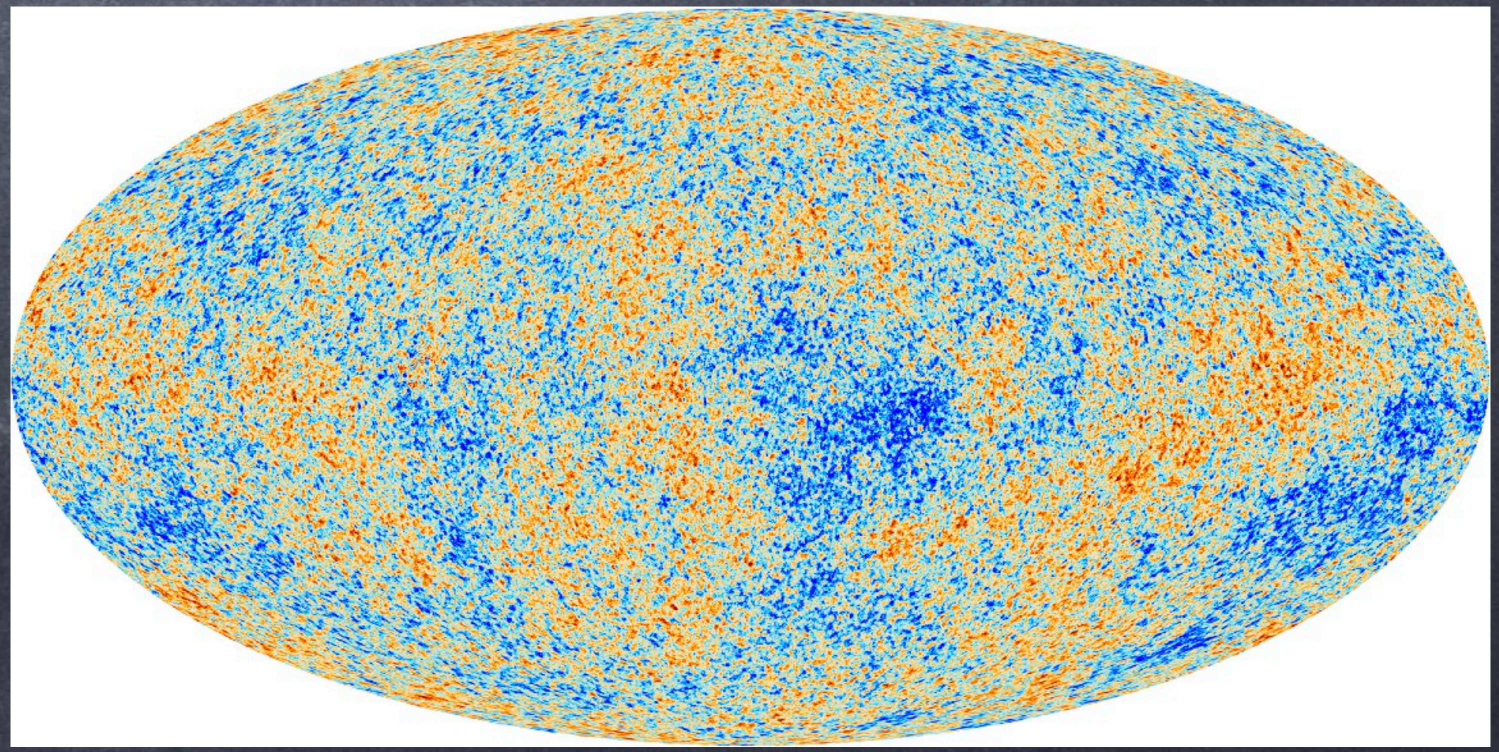
PDG 2021

$$\eta = \frac{n_B}{n_\gamma} \qquad n_B = n_b - n_{\bar{b}} \qquad \eta = \frac{\text{(matter) - (antimatter)}}{\text{relic photons}}$$

$$\eta = (6.143 \pm 0.190) \times 10^{-10}$$ Concordance range

$$\Omega_b = \frac{\rho_b}{\rho_{\text{crit}}} \simeq \eta h^{-2}/274 \times 10^{10} = 0.02244 \pm 0.00069\ h^{-2}$$

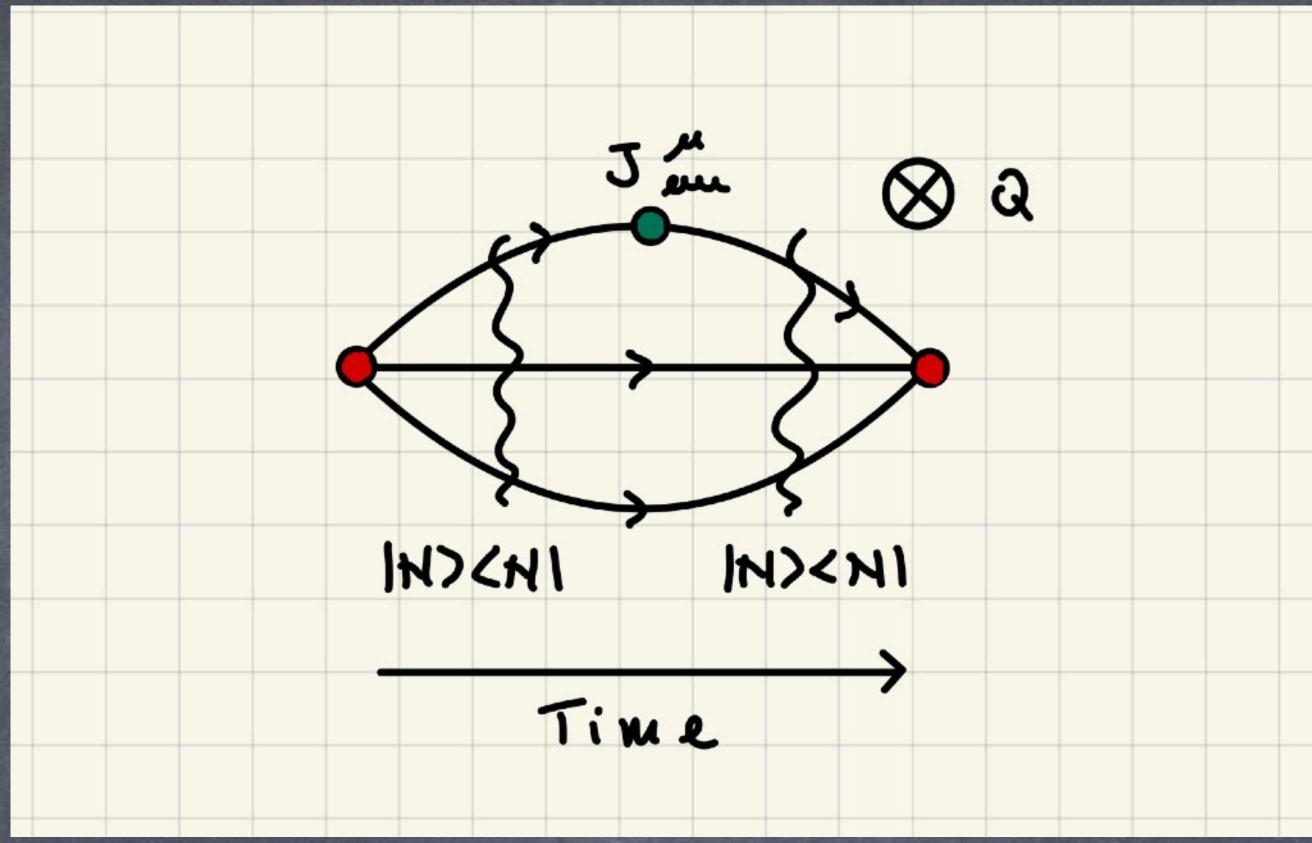
$$\Omega_b h^2 = 0.02230 \pm 0.00021 \Rightarrow \eta = (6.104 \pm 0.058) \times 10^{-10}$$



PLANCK

Fields, Olive, Yeh, Young: 2020

EDM from θ-term



$$G^{\theta}_{N J_\mu N} = \langle N(y_0, \underline{p}_2) J^{\mu}_{\text{em}}(x_0, \underline{q}) N^{\dagger}(0, \underline{p}_1) \rangle_{\theta}$$

$$\left\langle N^{\theta}(\underline{p}', s') | J^{\text{em}}_{\mu} | N^{\theta}(\underline{p}, s) \right\rangle = \overline{u}_N(\underline{p}', s') \Gamma^{\bar{\theta}}_{\mu}(q^2) u_N(\underline{p}, s)$$

$$\mathrm{e}^{-S} \simeq \mathrm{e}^{-S_{\text{QCD}}} \left[1 + i\theta Q\right]$$

$$\langle \mathcal{O} \rangle_{\bar{\theta}} \simeq \langle \mathcal{O} \rangle_{\bar{\theta}=0} + i\bar{\theta} \, \langle \mathcal{O} \; Q \rangle_{\bar{\theta}=0} + O(\bar{\theta}^2) \qquad Q = \int d^4x \; q(x)$$

Problem: definition of Q on the lattice

Shintani et al.: 2005
Berruto, Blum, Orginos, Soni: 2005

Topological charge on the lattice

Discretize $\longrightarrow$ $$q(x) = \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{tr}\left\{G_{\mu\nu}(x)G_{\rho\sigma}(x)\right\}$$

Continuity in space is lost
On the lattice it has no topological significance

Geometrical definition:

extend the lattice gauge field to a continuous one
Field between lattice points -> judicious interpolation
Smooth gauge fields (bound on field tensor)

Lüscher: 1982
Phillips, Stones: 1986

Fermionic definition:

Anomalous Ward Identity

Smit: 1980

$$\partial_\mu A_\mu = 2mP + \text{extra terms}$$

$$\partial_\mu A_\mu = 2mP + 2iN_f q_L$$

$$Q_L \propto m \sum_x \mathrm{Tr}\left(\gamma_5 S\right)$$

$$Q = n_+ - n_-$$

Atiyah, Singer: 1971

P. Hasenfratz: 1998
P. Hasenfratz, Laliena, Niedermeyer: 1998

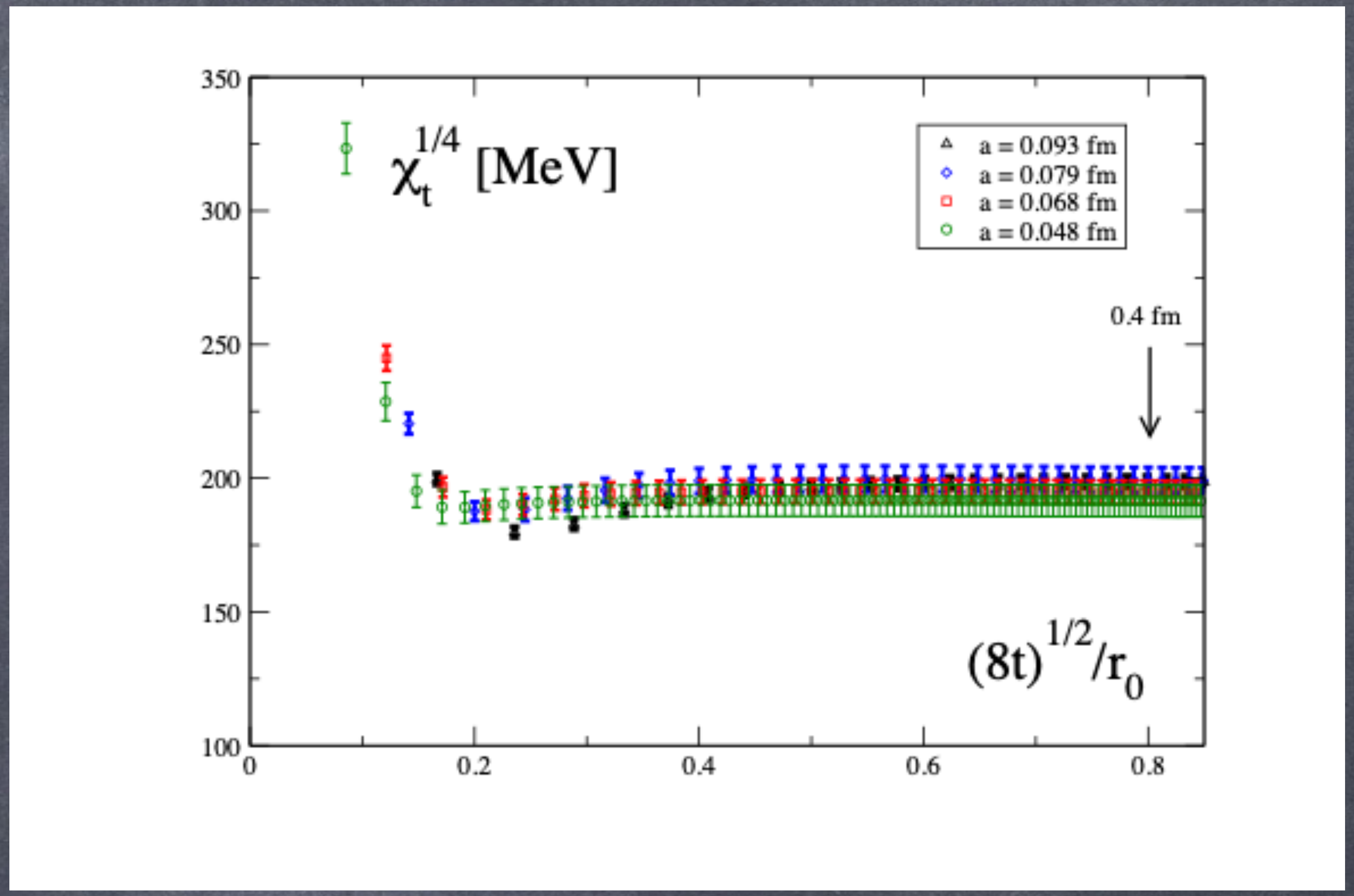
Topological charge

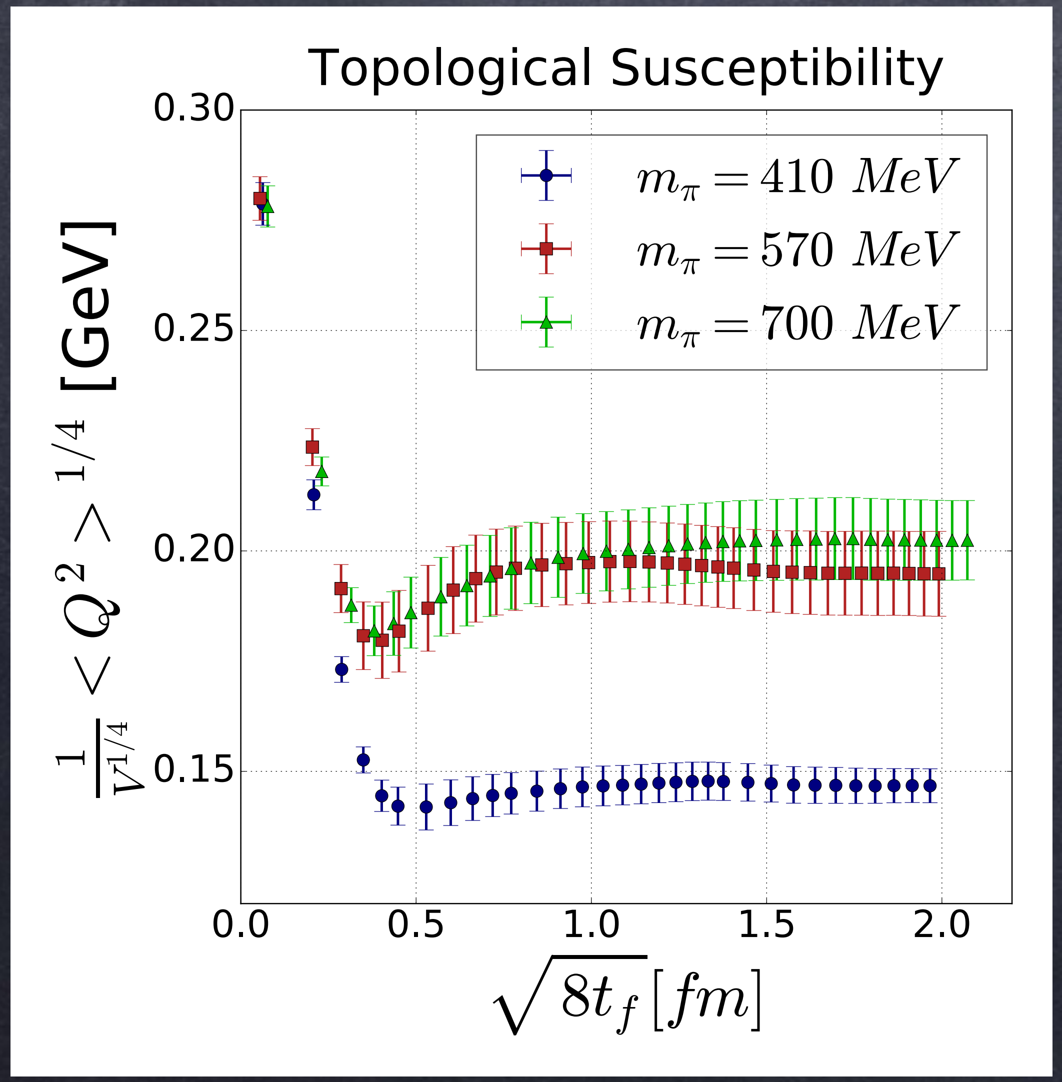
$$q(x,t_f) = \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{tr}\left\{G_{\mu\nu}(x,t_f)G_{\rho\sigma}(x,t_f)\right\}$$

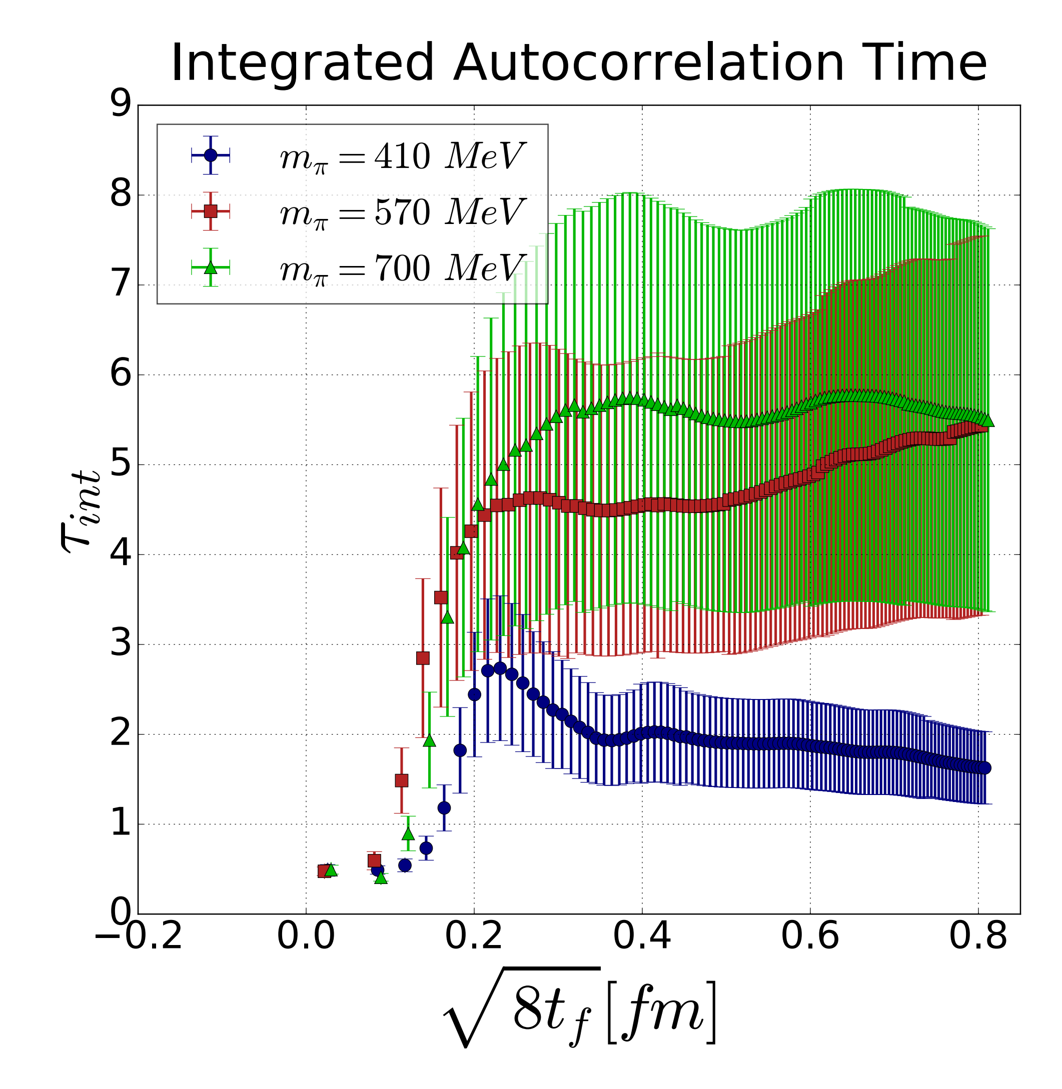
$$Q(t_f) = \int d^4x\; q(x,t_f)$$

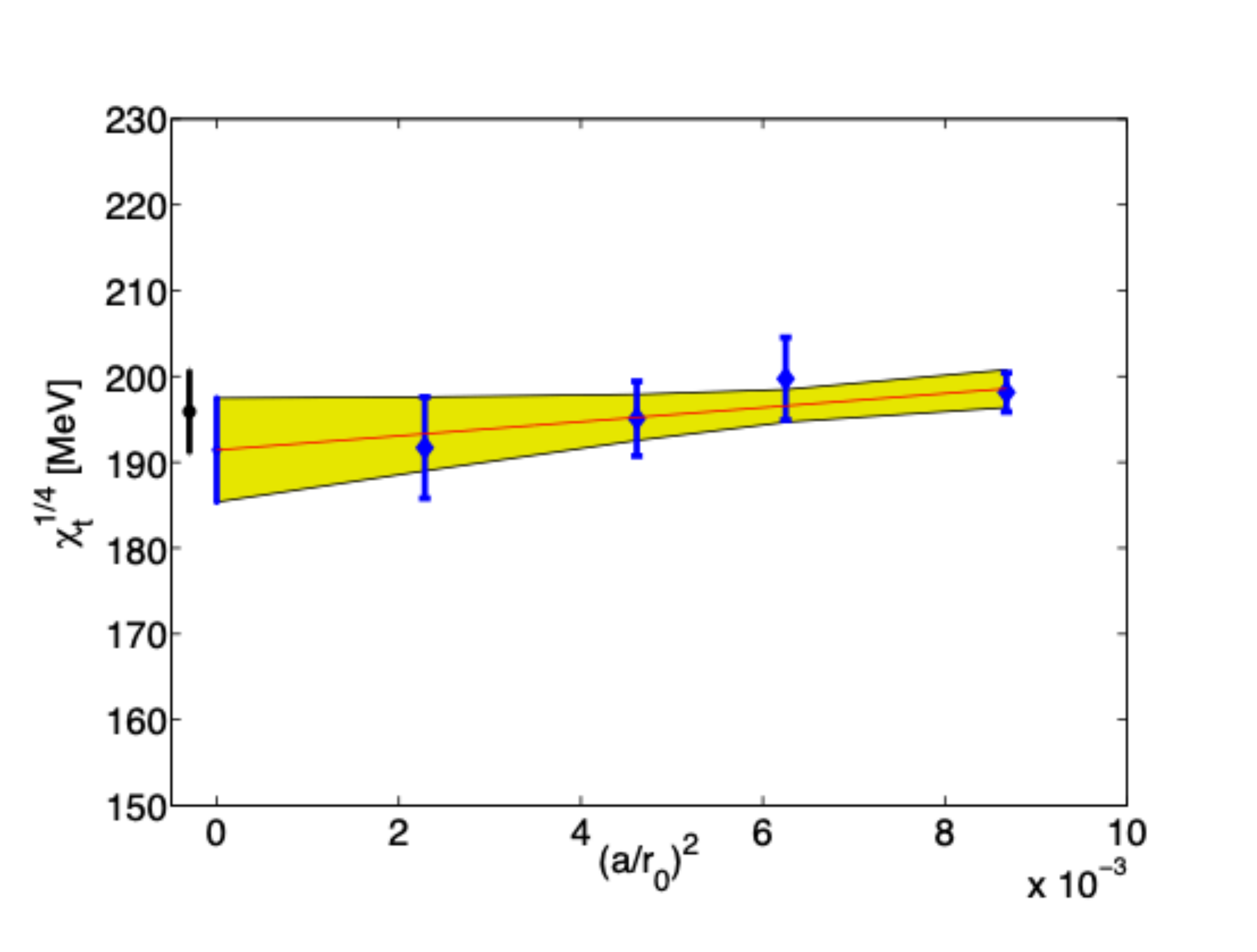
$$\chi_t = \frac{1}{V}\int d^4x\; d^4y\;\; \langle q(x,t_f)q(y,t_f)\rangle$$

$$\chi_t^{1/4} = 191(7)\ \mathrm{MeV}$$









A.S., de Vries, Luu: 2014, 2015

Perturbation theory with flowed fields

Lüscher, Weisz:2010,2011
Lüscher: 2013

Rizik, Monhahan, A.S.:
2018, 2020

$$B_\mu(x;t) = \int \mathrm{d}^d y \Big[K_{\mu\nu}(x-y;t)A_\nu(y) + \int_0^t \mathrm{d}s K_{\mu\nu}(x-y;t-s)R_\nu(y;s) \Big],$$

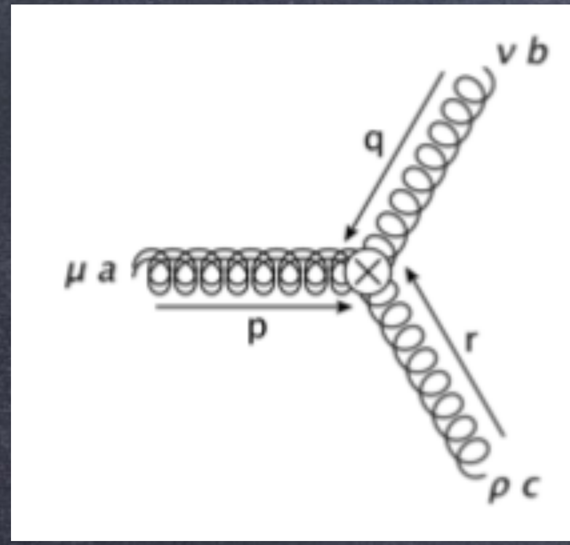
$$\chi(x,t) = \int \mathrm{d}^d y \left[J(x-y;t)\psi(y) + \int_0^t \mathrm{d}s\, J(x-y;t-s)\Delta'\chi(y;s) \right],$$

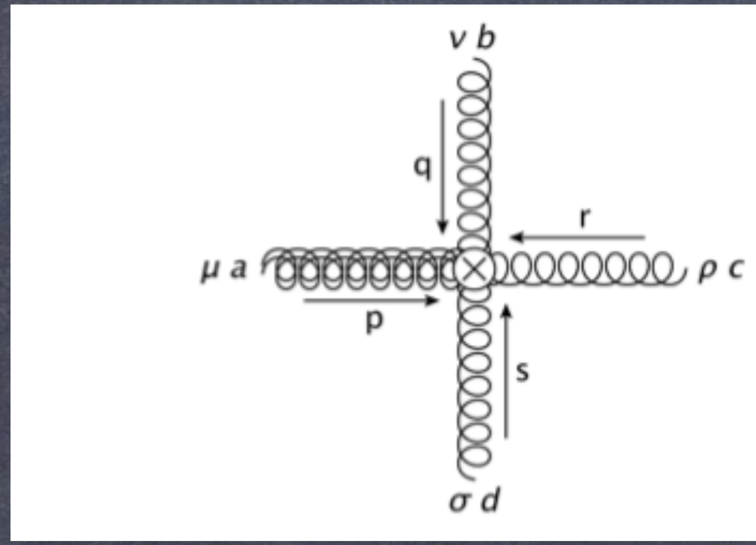
$$\overline{\chi}(x,t) = \int \mathrm{d}^d y \left[\overline{\psi}(y)\bar{J}(x-y;t) + \int_0^t \mathrm{d}s\, \overline{\chi}(y;s)\overleftarrow{\Delta}'\bar{J}(x-y;t-s) \right].$$

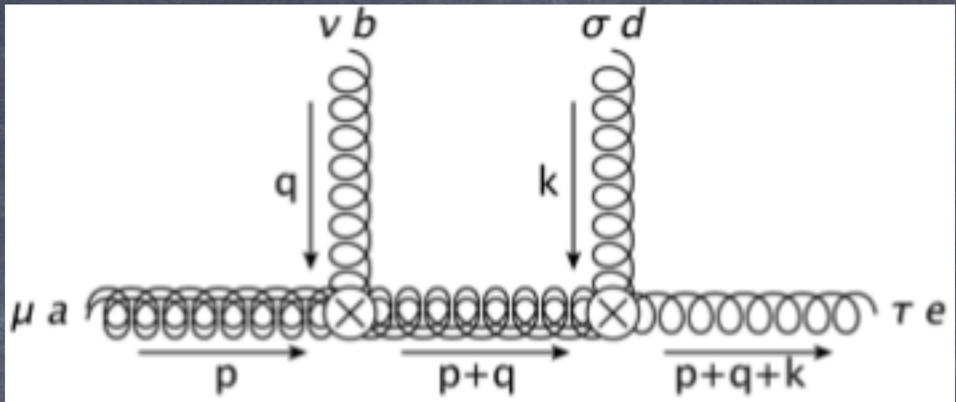
$$\partial_t \chi_t = \Delta \chi_t \qquad \partial_t \overline{\chi_t} = \overline{\chi_t}\overleftarrow{\Delta}$$

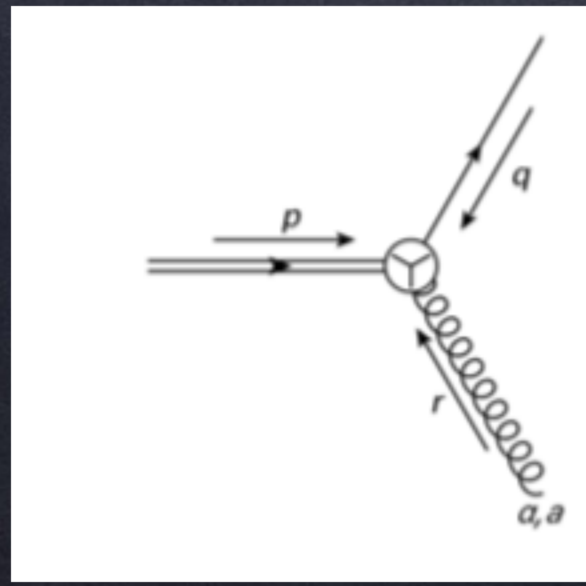
$$\chi_t(x)|_{t=0} = \psi(x)$$

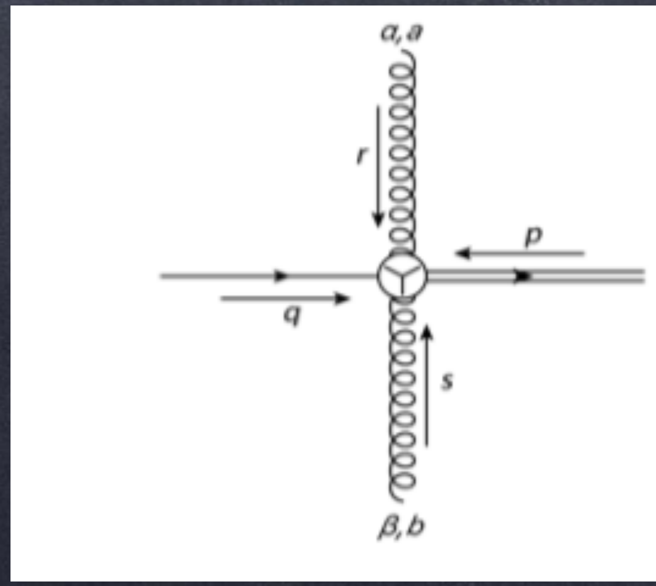
$$\overline{\chi}_t(x)|_{t=0} = \overline{\psi}(x)$$

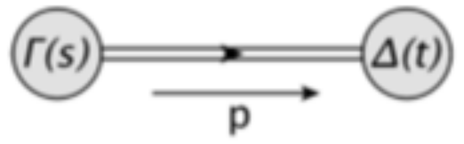




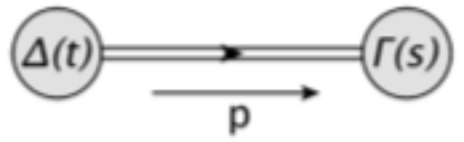








$$= \int_0^\infty ds\ \theta(t-s)\Delta(t)\tilde{J}_{t-s}(p)\Gamma(s)\,,$$



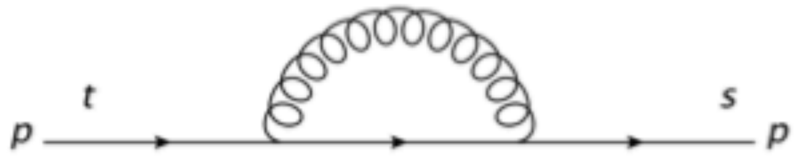
$$= \int_0^\infty ds\ \theta(t-s)\Gamma(s)\tilde{\bar{J}}_{t-s}(p)\Delta(t)\,,$$

Sample calculation: quark propagator

Lüscher: 2013

Rizik, Monhahan, A.S.: 2018, 2020

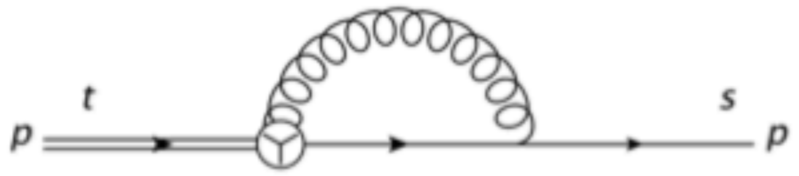
$$\Sigma_1^{(2)}(p) =$$



$$\text{(C5a)}$$

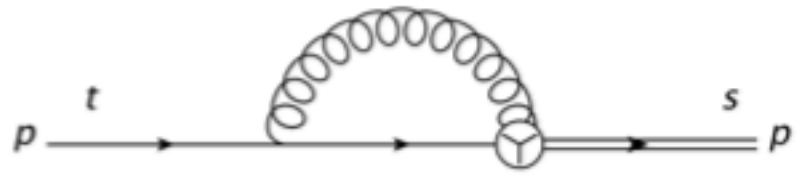
$$= -g_0^2 \frac{C_2(F)}{(4\pi)^2} \left\{ \left[\frac{1}{\epsilon} + \log\left(\frac{4\pi\mu^2}{p^2}\right) - \gamma_E + 1 \right] i\not{p} + 4 \left[\frac{1}{\epsilon} + \log\left(\frac{4\pi\mu^2}{p^2}\right) - \gamma_E + \frac{3}{2} \right] m_0 + R\left(\frac{m_0^2}{p^2}\right) \right\} + \mathcal{O}(\epsilon),$$

$$\Gamma_{2,a}^{(2)}(p;t) =$$



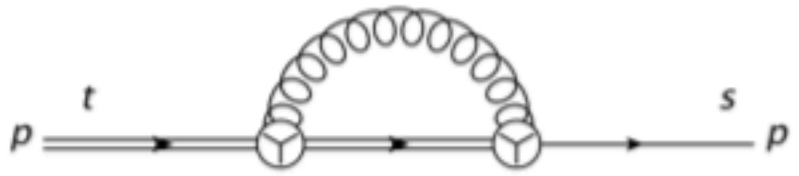
$$= g_0^2 \frac{C_2(F)}{(4\pi)^2} \left[\frac{1}{\epsilon} + \log\left(8\pi\mu^2 t\right) + 1 \right] + \mathcal{O}(\epsilon, t), \qquad \text{(C5b)}$$

$$\Gamma_{2,b}^{(2)}(p;s) =$$



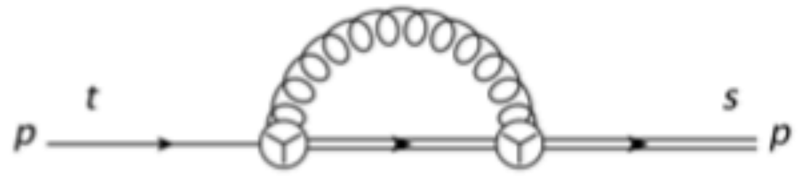
$$= g_0^2 \frac{C_2(F)}{(4\pi)^2} \left[\frac{1}{\epsilon} + \log\left(8\pi\mu^2 s\right) + 1 \right] + \mathcal{O}(\epsilon, s), \qquad \text{(C5c)}$$

$$\Gamma_{3,a}^{(2)}(p;t) =$$



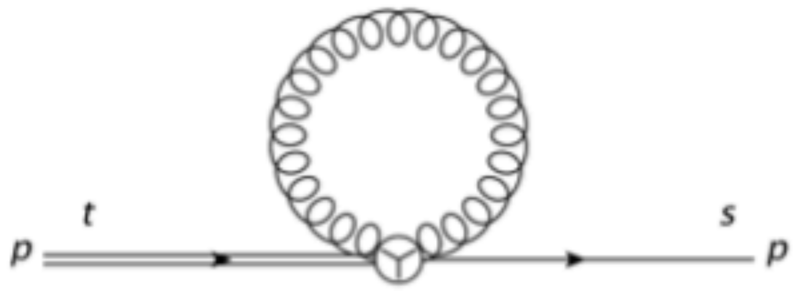
$$= 0 + \mathcal{O}(t), \qquad \text{(C5d)}$$

$$\Gamma_{3,b}^{(2)}(p;s) =$$



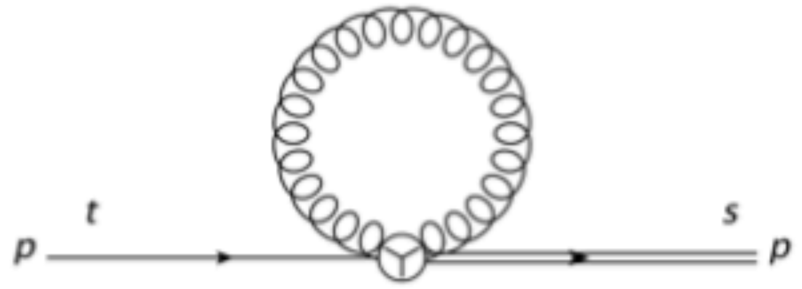
$$= 0 + \mathcal{O}(s), \qquad \text{(C5e)}$$

$$\Gamma_{4,a}^{(2)}(p;t) =$$



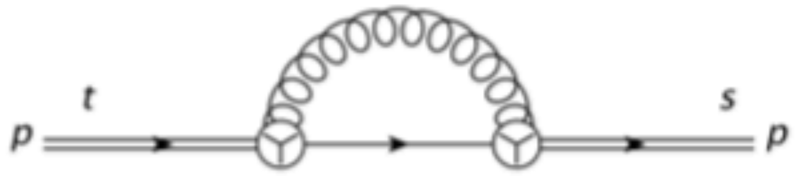
$$= -2g_0^2 \frac{C_2(F)}{(4\pi)^2} \left[\frac{1}{\epsilon} + \log\left(8\pi\mu^2 t\right) + \frac{1}{2} \right] + \mathcal{O}(\epsilon, t), \qquad \text{(C5f)}$$

$$\Gamma_{4,b}^{(2)}(p;s) =$$



$$= -2g_0^2 \frac{C_2(F)}{(4\pi)^2} \left[\frac{1}{\epsilon} + \log\left(8\pi\mu^2 s\right) + \frac{1}{2} \right] + \mathcal{O}(\epsilon, s), \qquad \text{(C5g)}$$

$$\Gamma_5^{(2)}(p;t,s) =$$



$$= 0 + \mathcal{O}(s, t), \qquad \text{(C5h)}$$

$$Z_\chi = 1 + g_0^2 \frac{C_2(F)}{(4\pi)^2} \left\{ \frac{3}{\epsilon} + \log(4\pi) - \gamma_E + 1 \right\}$$

Numerical details

Dragos, Luu, A.S., de Vries, Yousif: 2019

NP improved Wilson + Iwasaki gauge

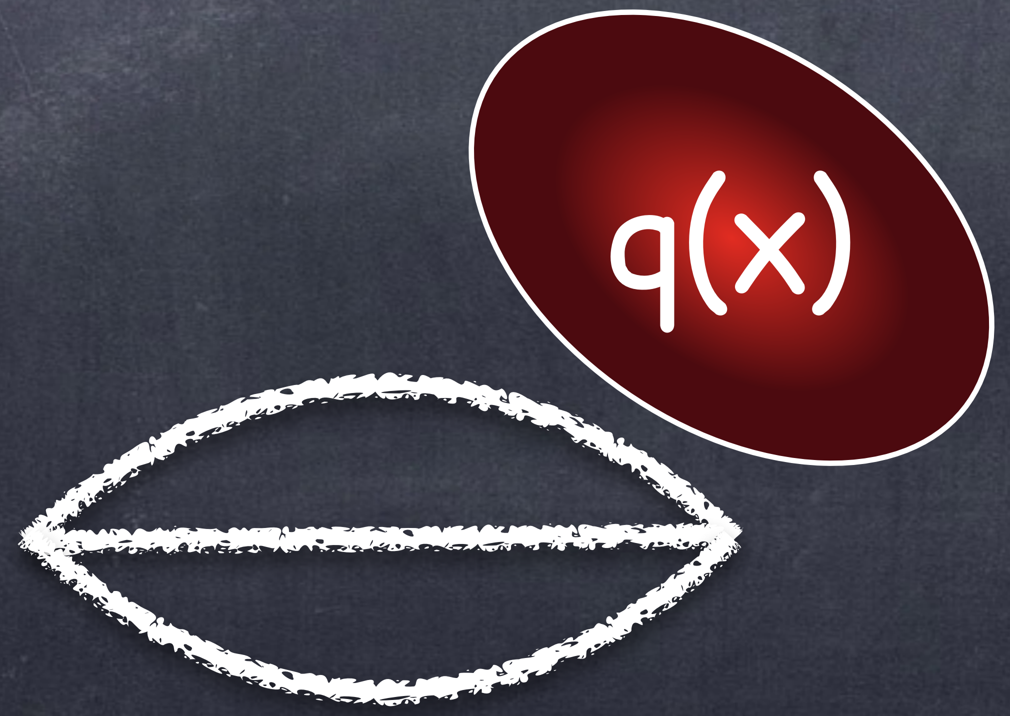
a=0.1-0.068 fm
mpi=400-700 MeV

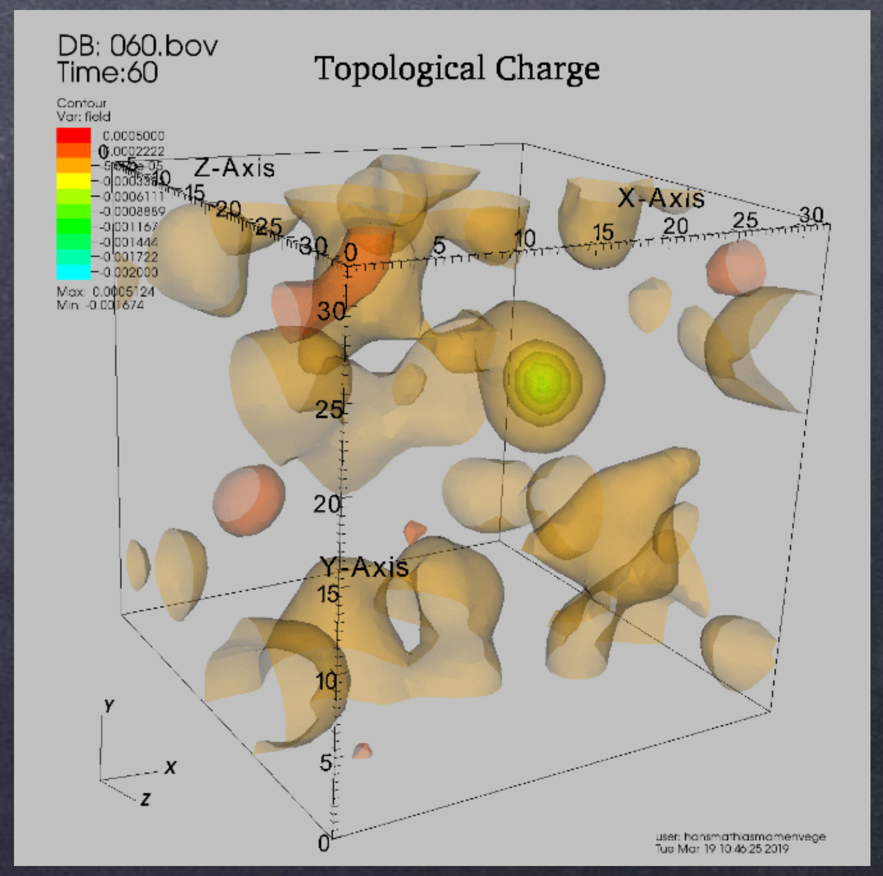
O(L/2a) Stochastic source locations

3 Gaussian smearings

	β	κ_l	κ_s	L/a	T/a	$c_{\rm sw}$	N_G	$N_{\rm corr}$
M_1	1.90	0.13700	0.1364	32	64	1.715	322	30094
M_2	1.90	0.13727	0.1364	32	64	1.715	400	20000
M_3	1.90	0.13754	0.1364	32	64	1.715	444	17834
A_1	1.83	0.13825	0.1371	16	32	1.761	800	15220
A_2	1.90	0.13700	0.1364	20	40	1.715	789	15407
A_3	2.05	0.13560	0.1351	28	56	1.628	650	12867

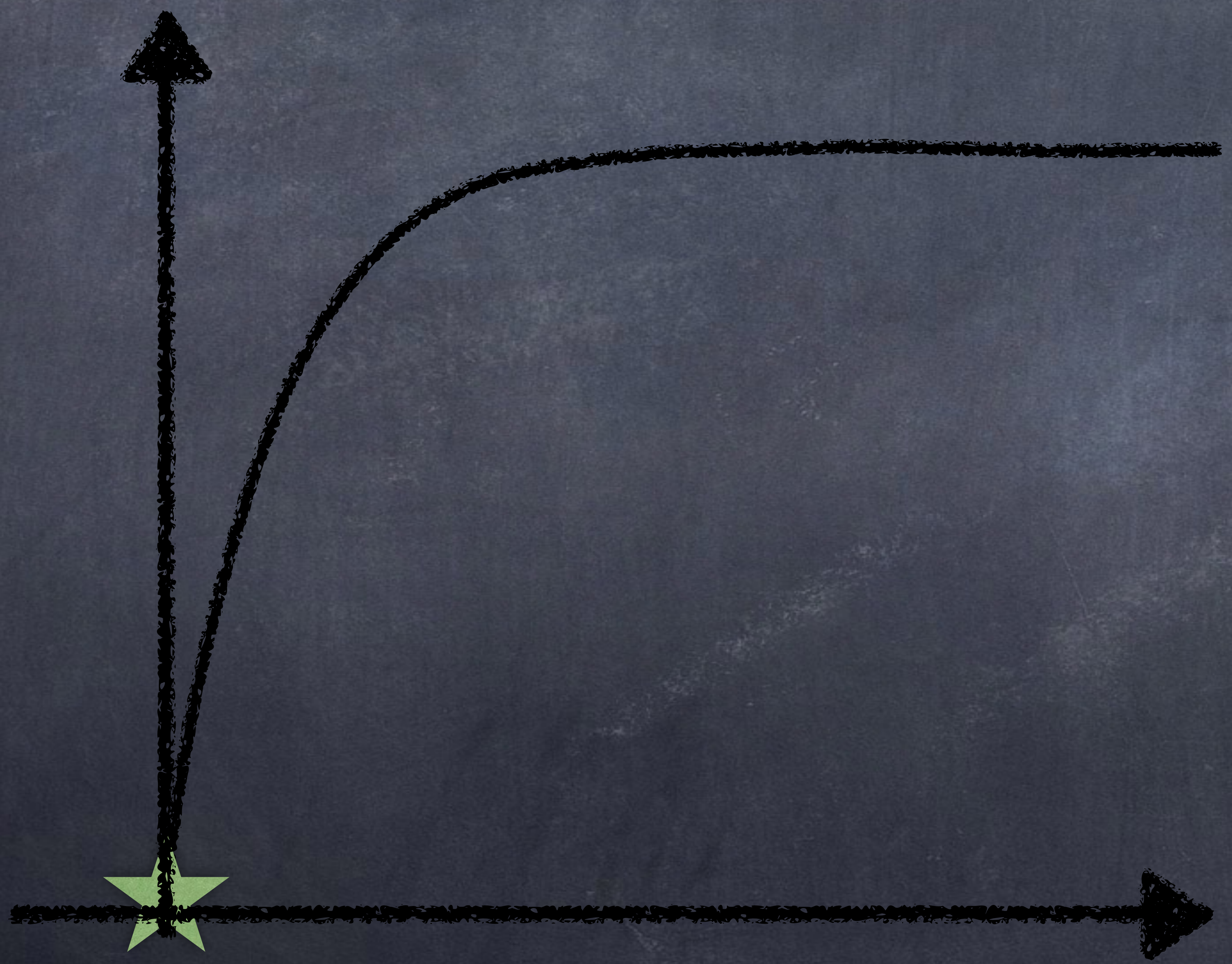
PACS-CS: 2009



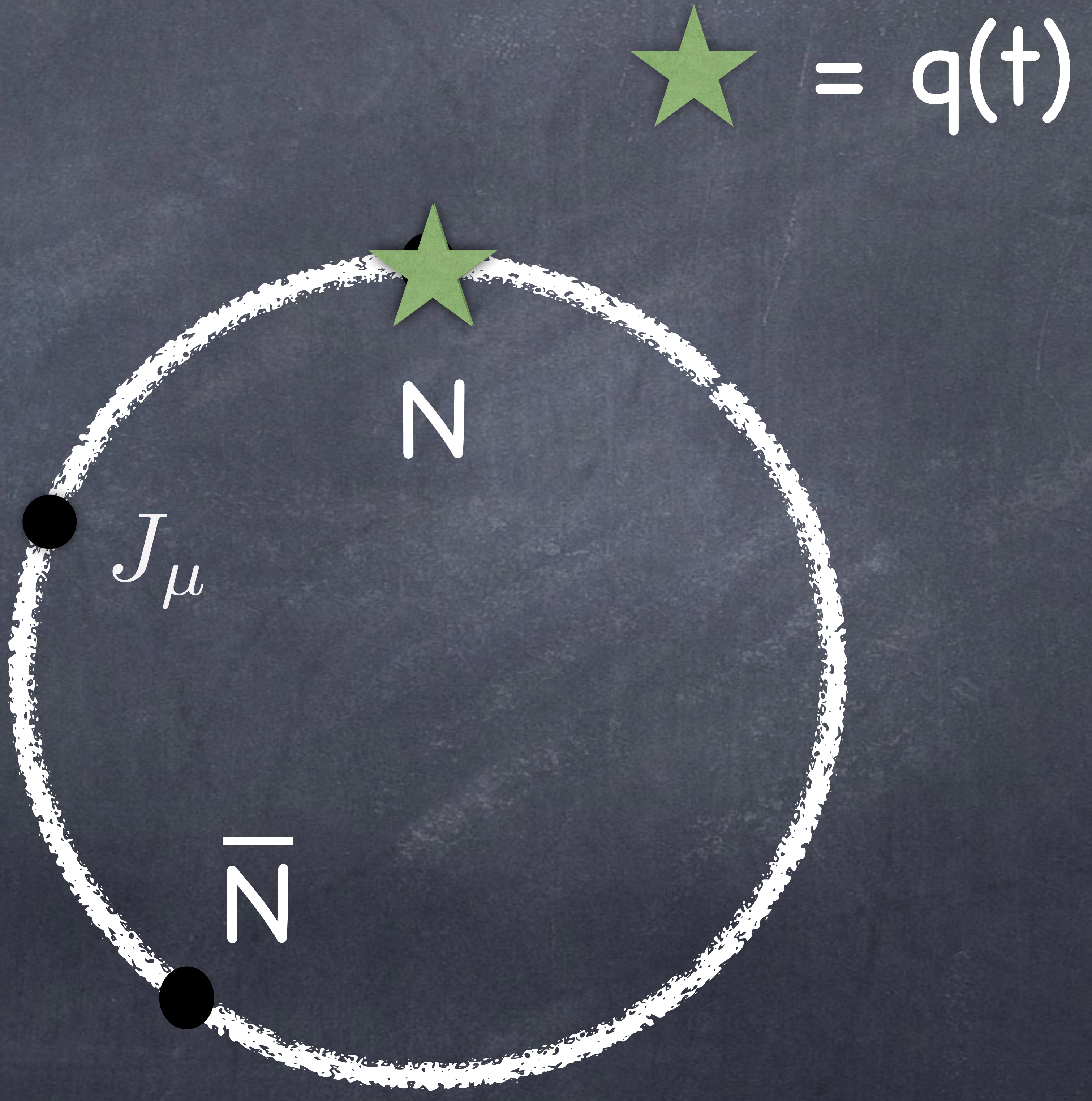


Signal-to-noise improvement

Is there a space-time region dominated by noise that can be neglected in the 4-d integration?



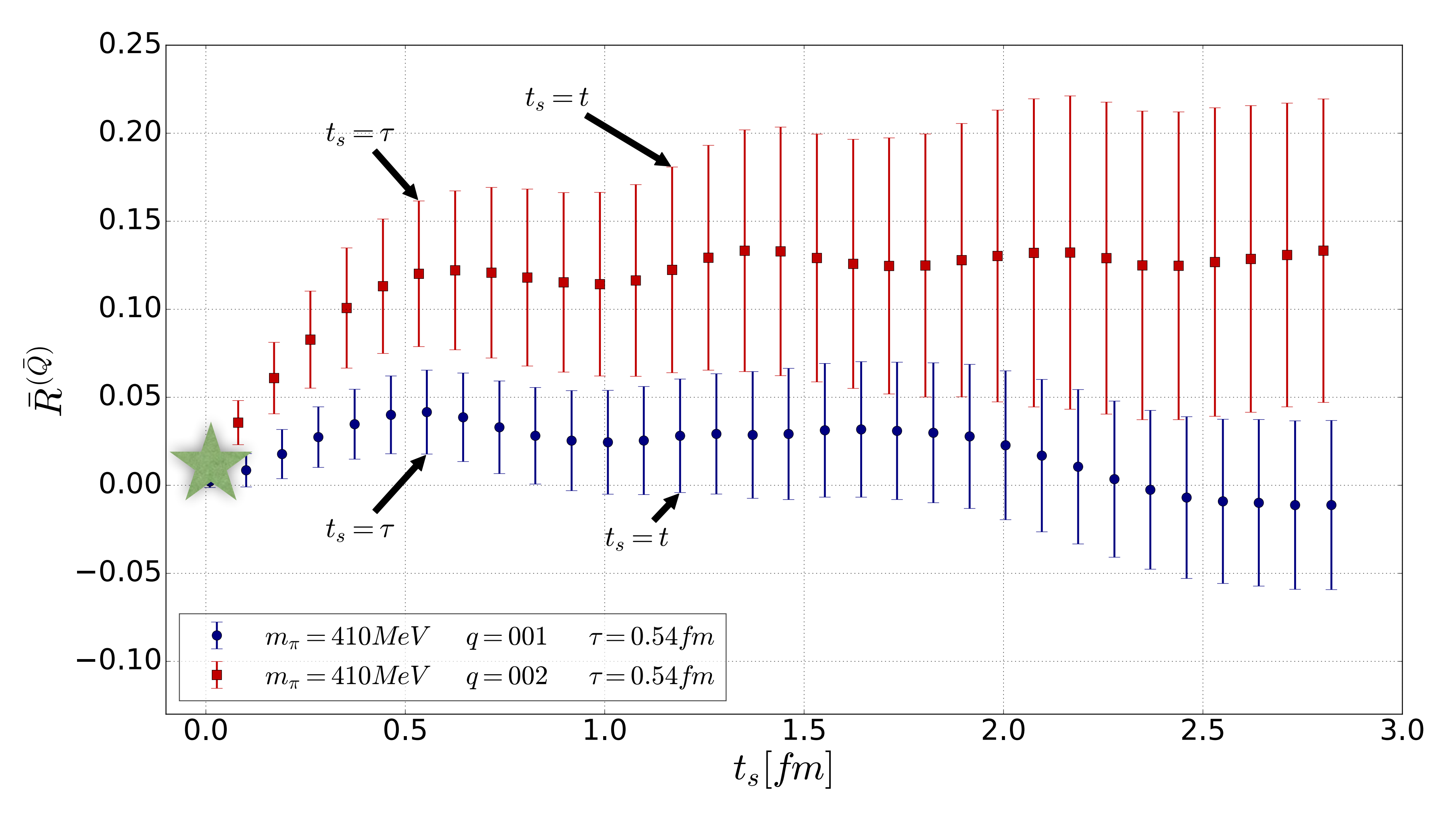
Dragos, Luu, A.S., de Vries, Yousif: 2019

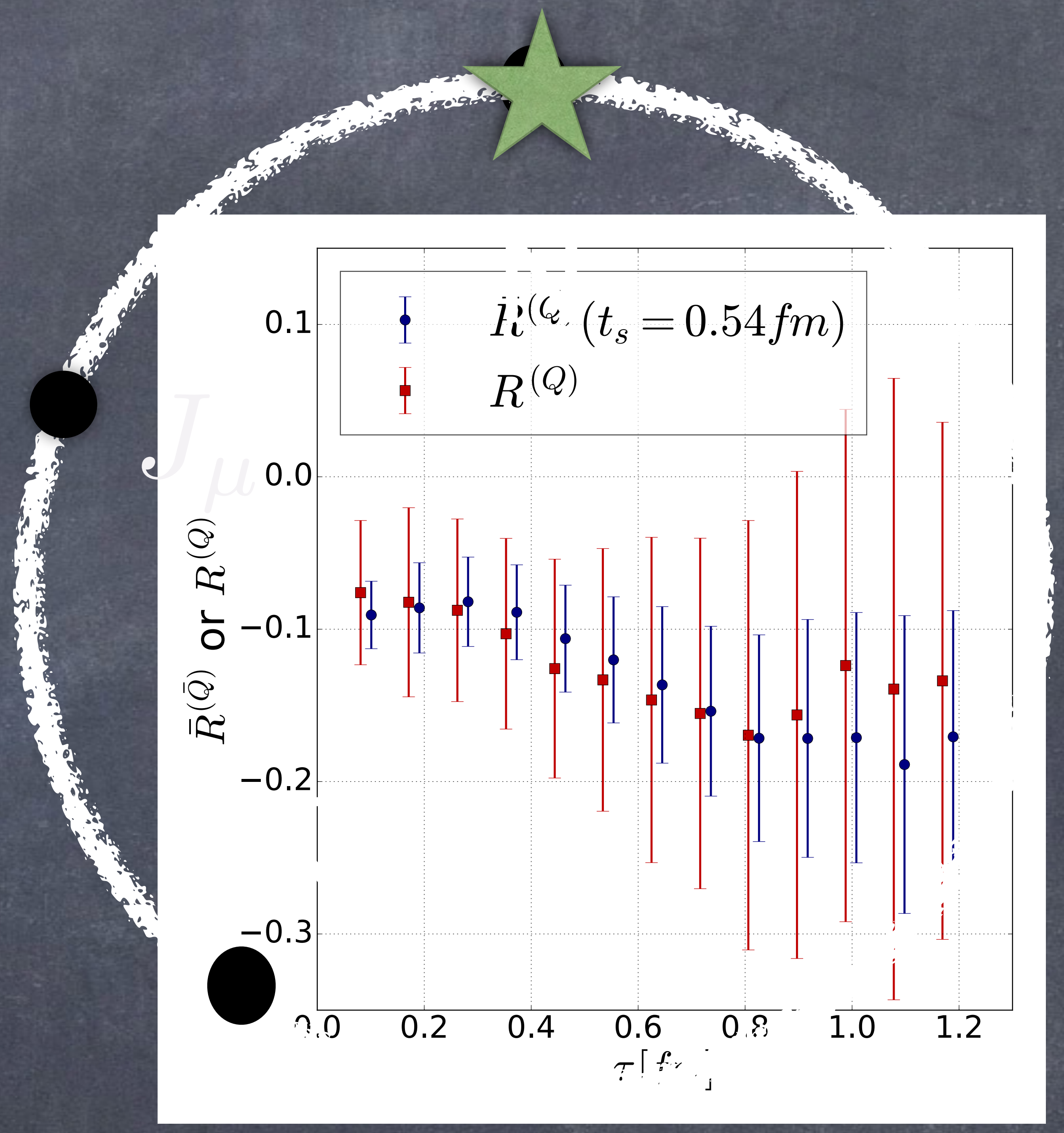


$$\langle N | J_\mu \mathcal{O}_{\cancel{CP}} | N \rangle \rightarrow d\ \underline{E} \cdot \underline{S}$$

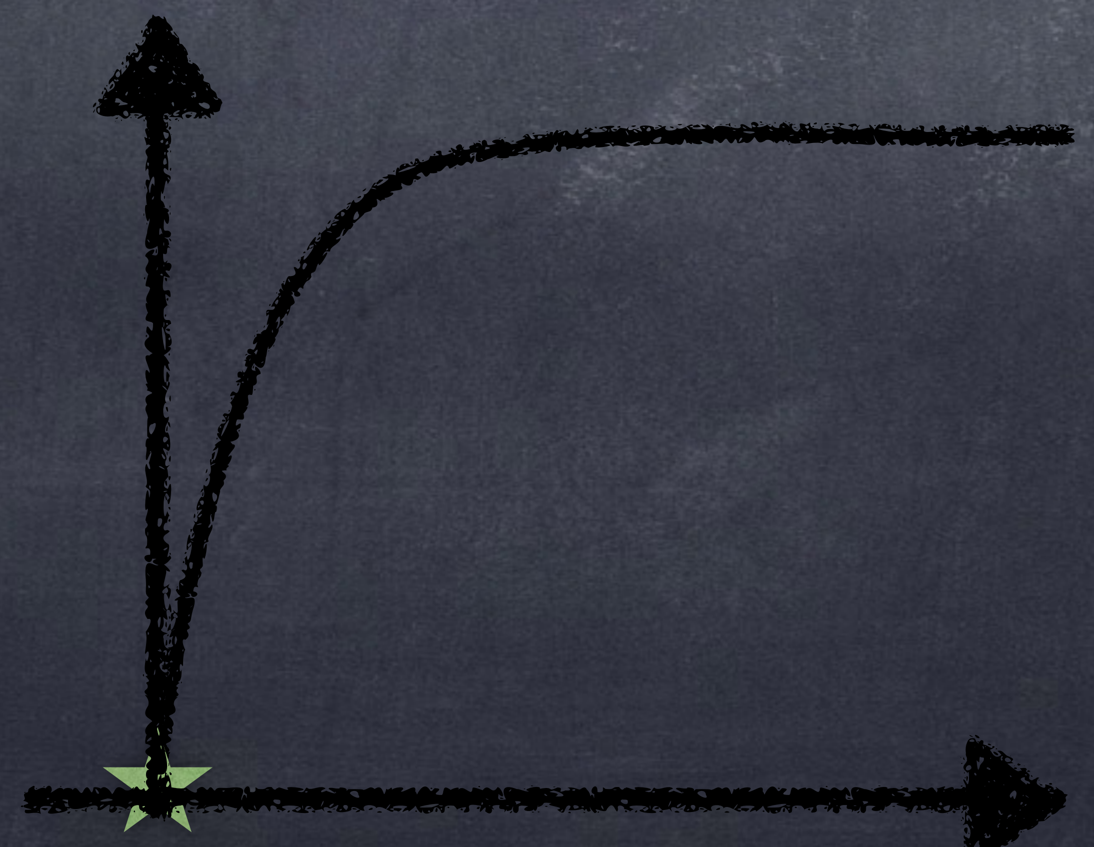
Signal-to-noise improvement

 = q(t)



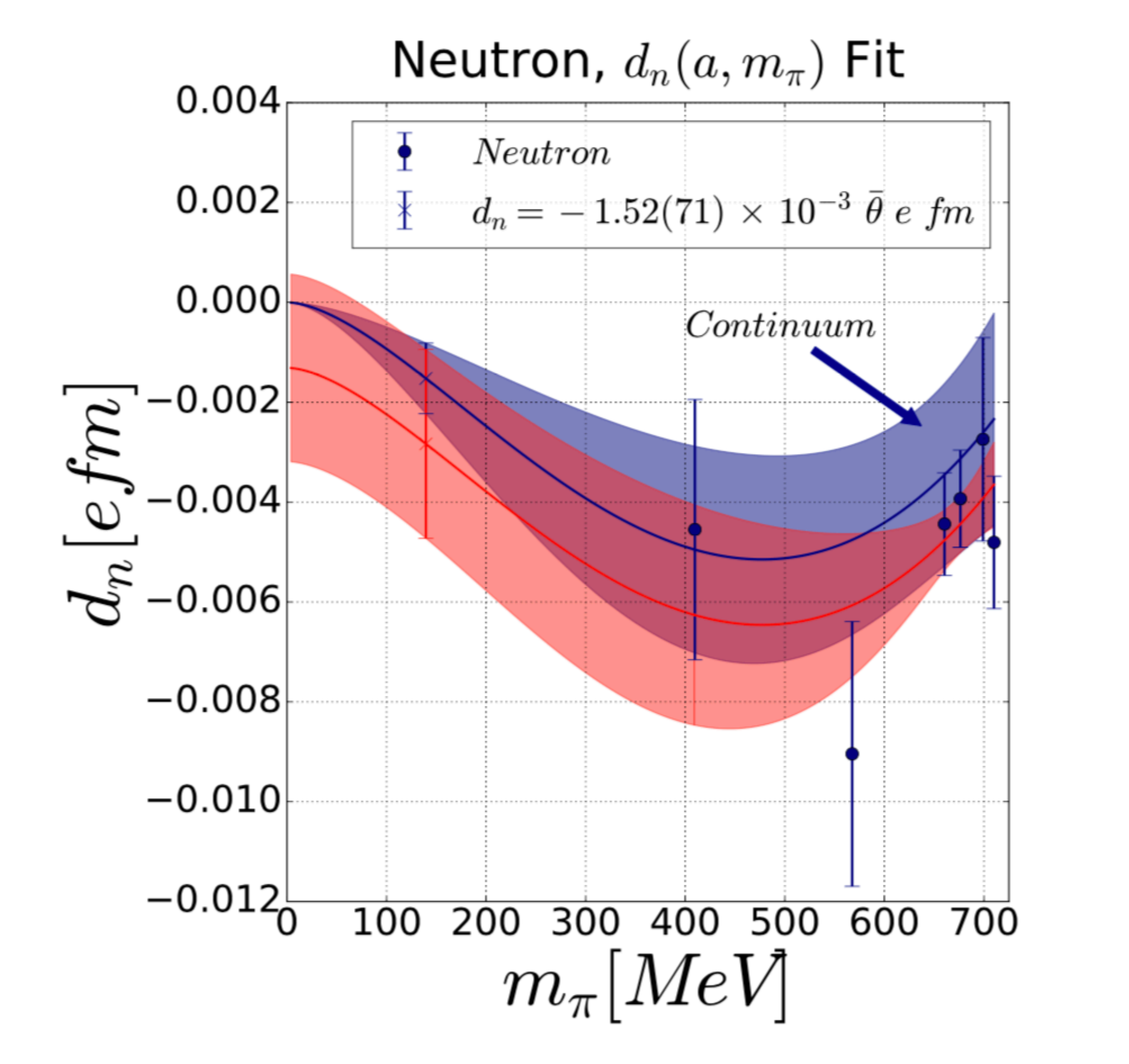


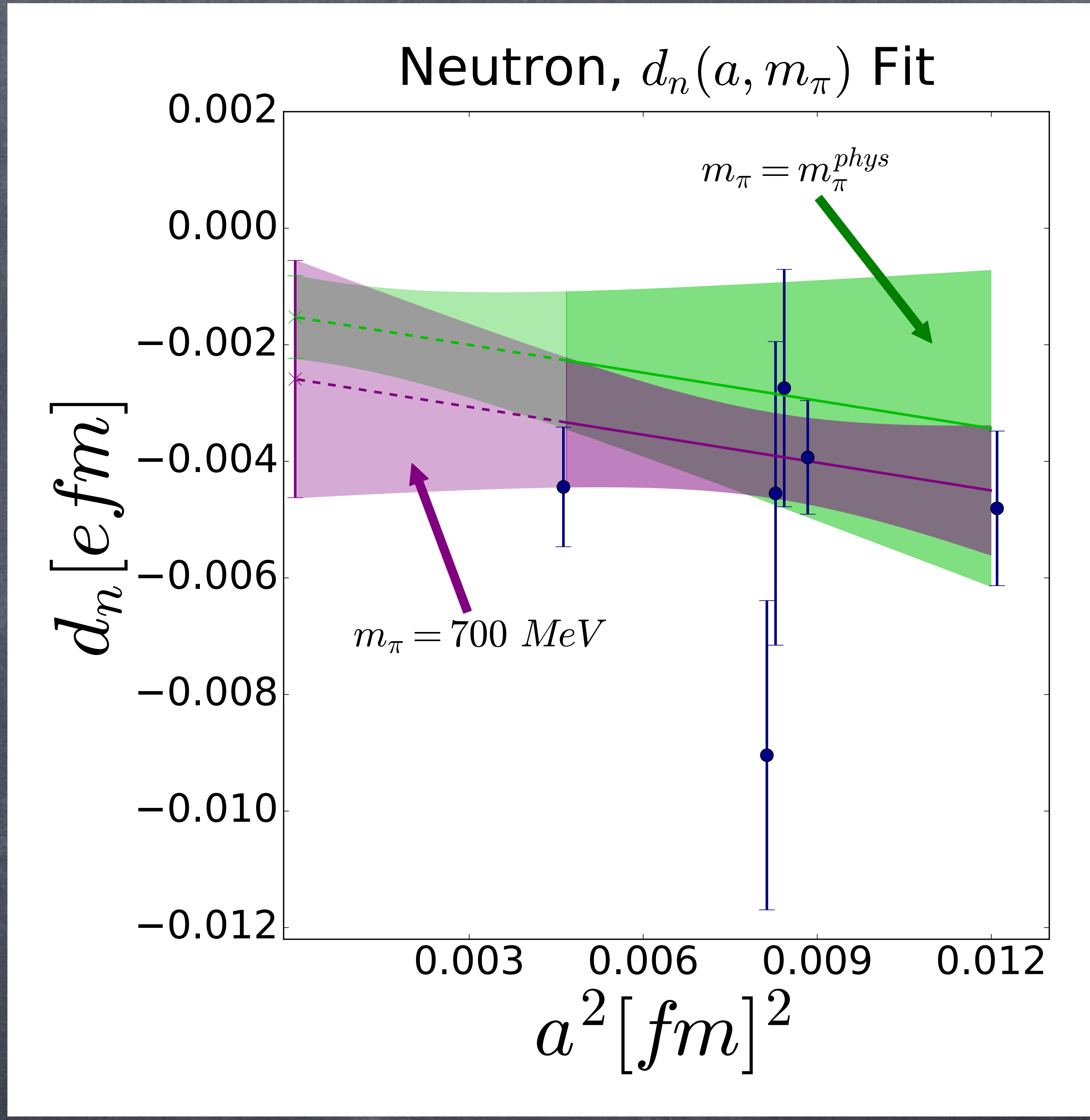
$$R^Q = \frac{G_3^Q(t,\tau,t_f;\underline{p}',\underline{q})}{G_2(\underline{p}',t)} \cdot K(t,\tau;\underline{p}',\underline{q})$$



Dragos, Luu, A.S.,
de Vries, Yousif: 2019

Chiral interpolation





$$d_{n/p}(a, m_\pi) = C_1^{n/p} m_\pi^2 + C_2^{n/p} m_\pi^2 \ln \frac{m_\pi^2}{M_N^2} + C_3^{n/p} a^2$$

$$d_n(\bar{\theta}) = \bar{d}_n - \frac{e g_A \bar{g}_0^{\bar{\theta}}}{8\pi^2 F_\pi} \ln \frac{m_\pi^2}{M_N^2}$$

Dragos, Luu, A.S., de Vries, Yousif: 2019

Ottnad et al.: 2010
Mereghetti et al.: 2011

4+1 Local field theory

Lüscher
2010-2013

$$S = S_{\mathrm{G}} + S_{\mathrm{G,fl}} + S_{\mathrm{F,QCD}} + S_{\mathrm{F,fl}}$$

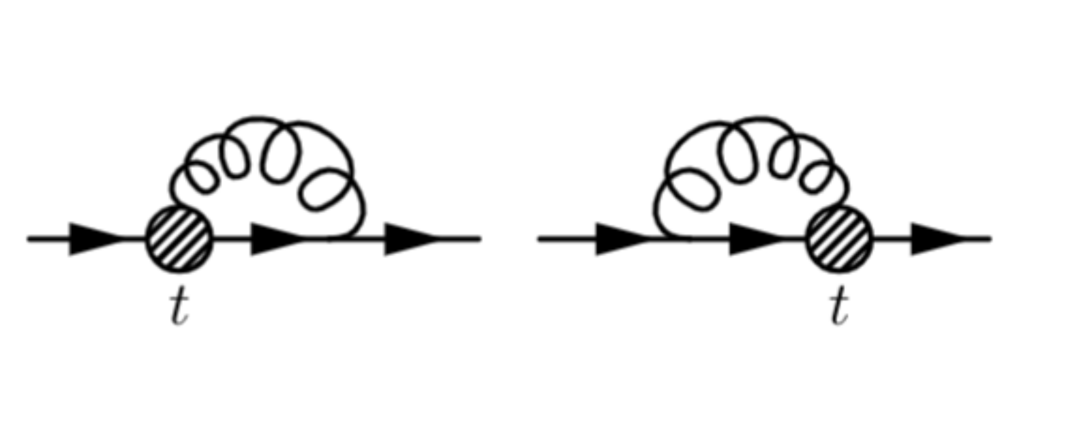
$$S_{\mathrm{F,fl}} = \int_0^\infty dt \int d^4x \left[\overline{\lambda}(t,x)\left(\partial_t - \Delta\right)\chi(t,x) + \overline{\chi}(t,x)\left(\overleftarrow{\partial}_t - \overleftarrow{\Delta}\right)\lambda(t,x)\right]$$

- Wick contractions
- Renormalization. All order proof for gauge sector — Lüscher, Weisz: 2011
- Chiral symmetry and Ward identities — Lüscher: 2013, A.S.:2013
- Wilson twisted mass — A.S.: 2013

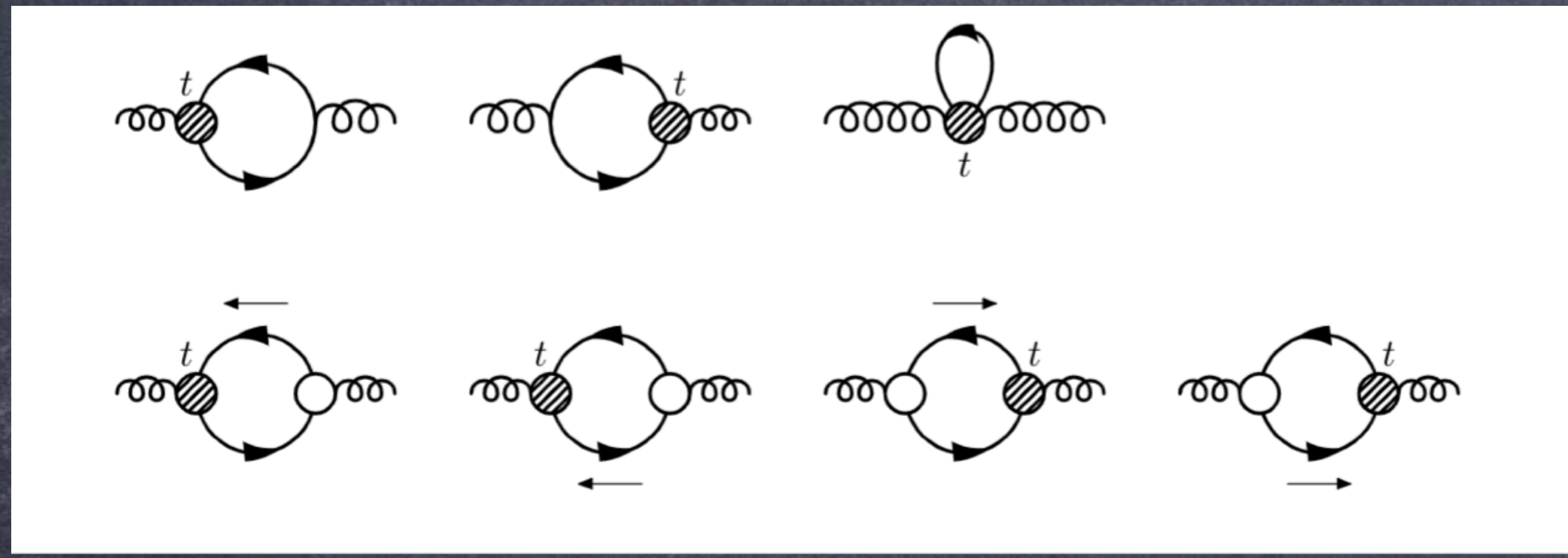
Quark-Chromo EDM

Rizik, Monahan, A.S.: 2020
Mereghetti, Monahan, Rizik, A.S., Stoffer : 2021

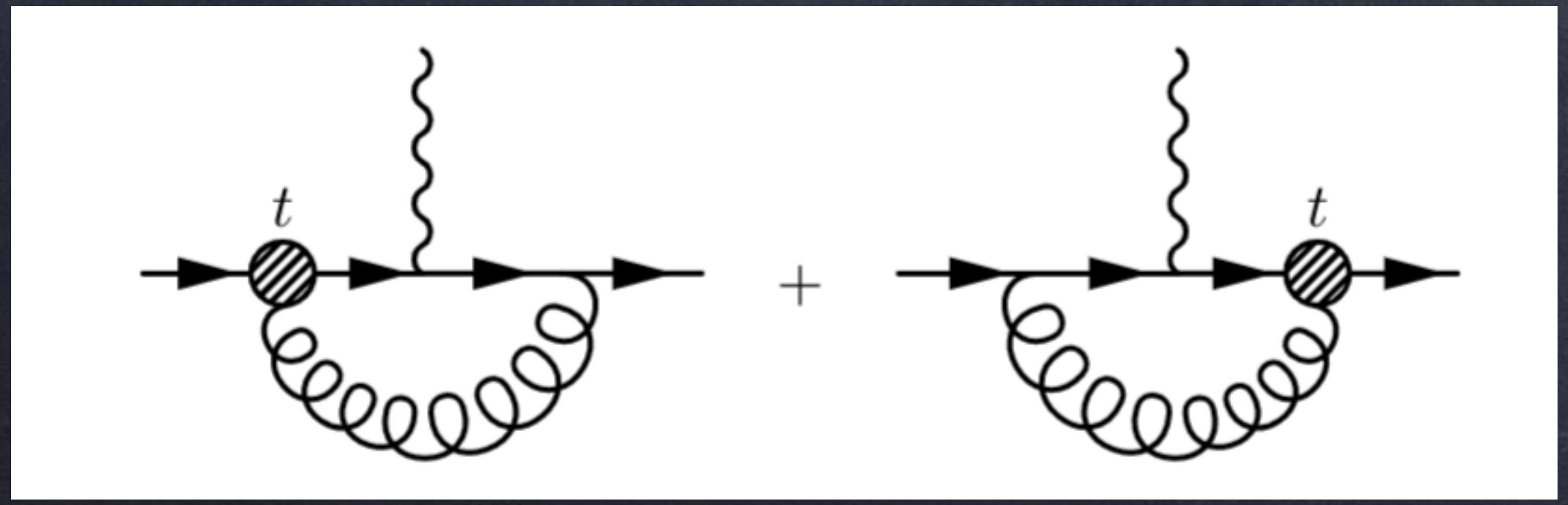
$c_P(t,\mu)$ $c_{m^2P}(t,\mu)$

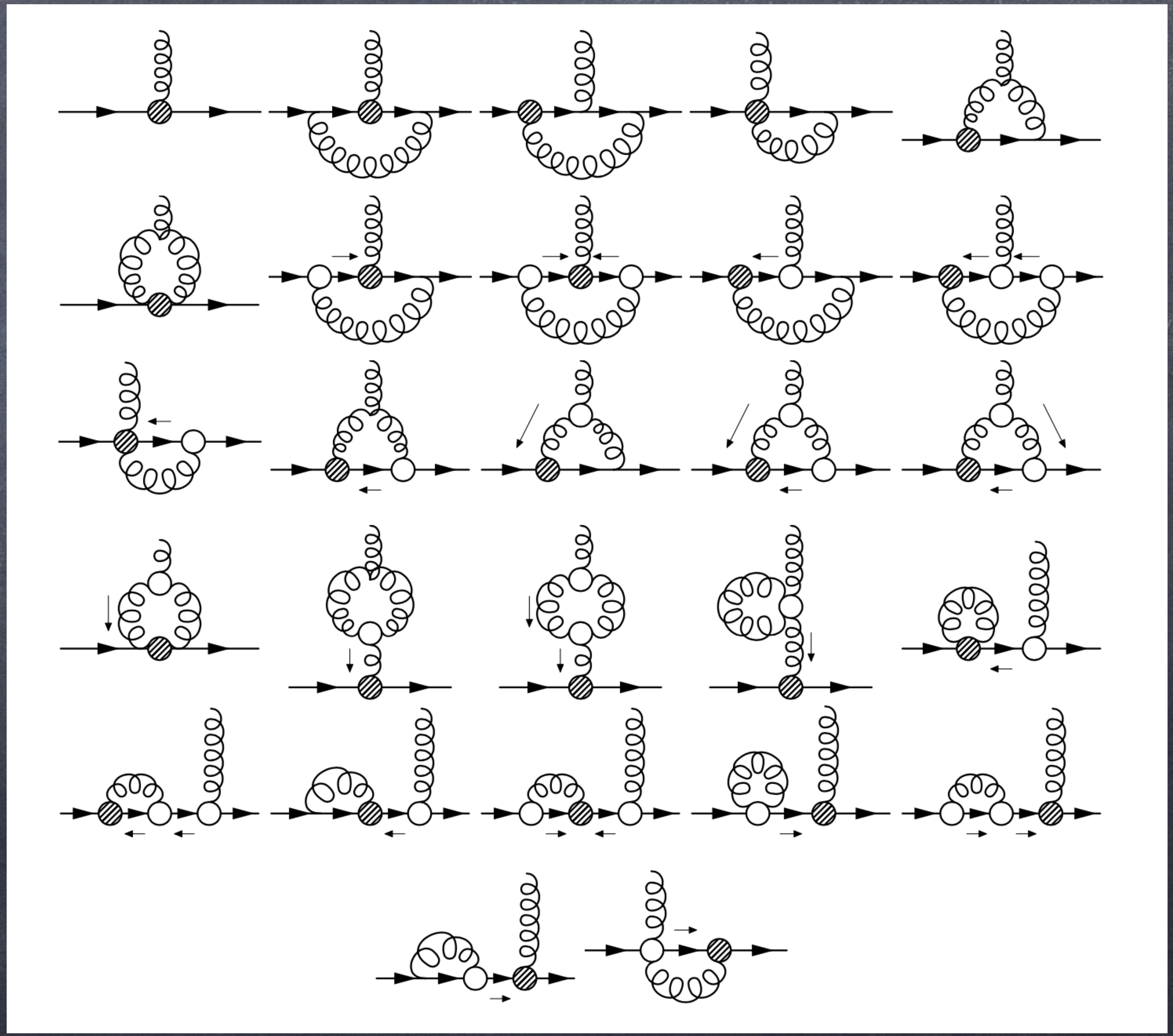


$c_{m\theta}(t,\mu)$



$c_E(t,\mu)$

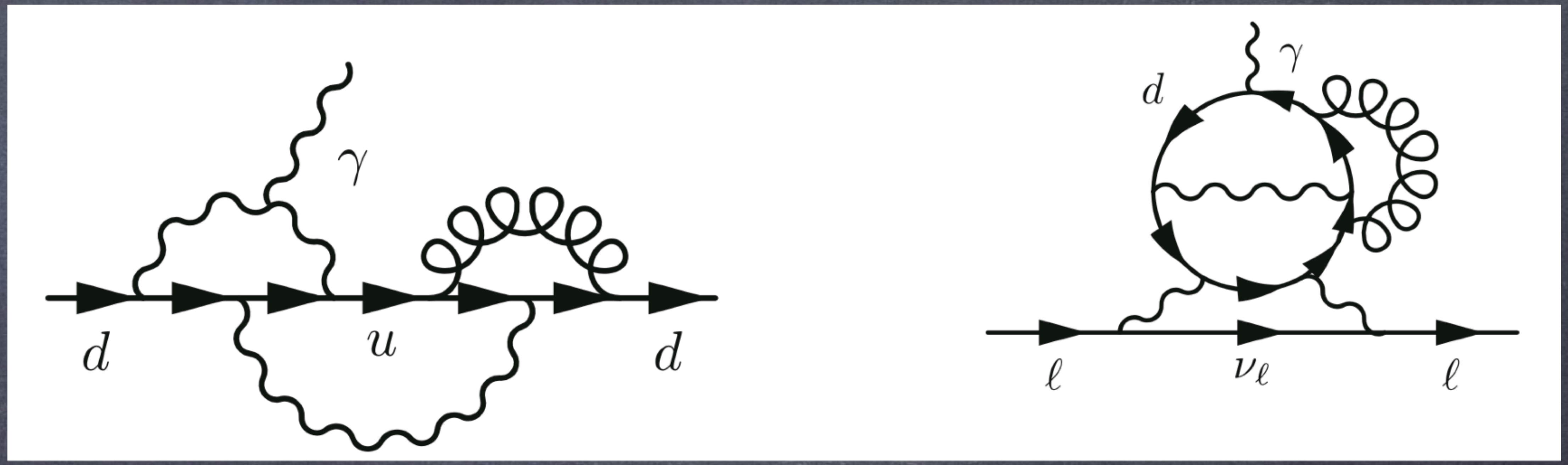




$c_{CE}(t,\mu)$

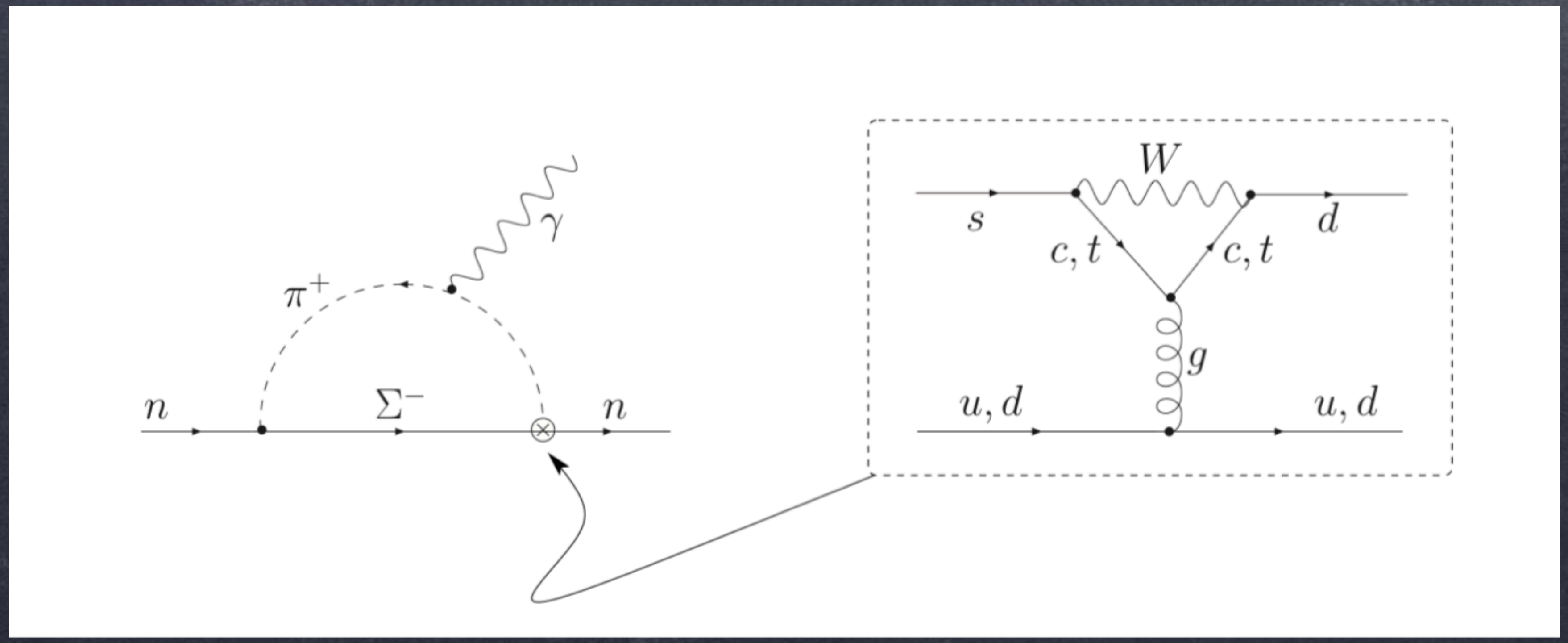
EDM from the Standard Model

- No EDMs at 1-loop
- At 2-loops individual diagrams are non-zero, but the sum vanishes
- Quark EDM are induced at 3-loops
- Electron EDM are induced at 4-loops



Electron EDM can be larger due to hadronic loops
EDM of not elementary particles can be larger

$$(d_n)_{\mathrm{SM}=(1-6)\times 10^{-32}e \quad \mathrm{cm}}$$



Shabalin: 1978-1980 Khriplovich, Zhitnitsky: 1982
Gavela et al. : 1982 Seng: 2015