# Muon g-2 in the SM: status and prospects

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University of Illinois

25TH INTERNATIONAL SPIN PHYSICS SYMPOSIUM

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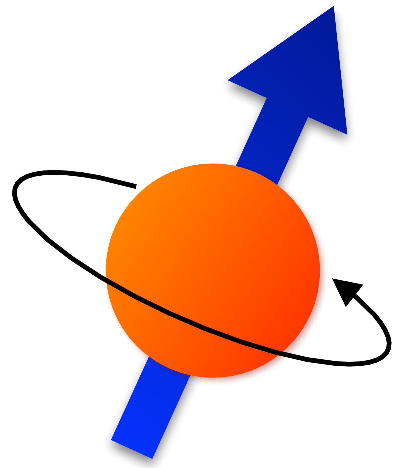
*Durham, NC, USA*

# Outline



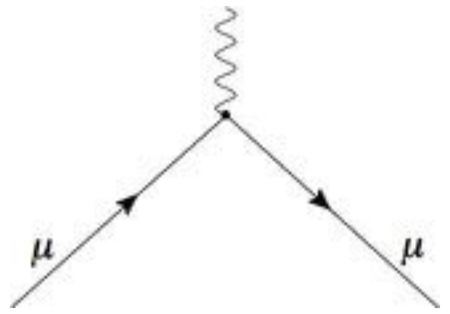
- Introduction
- Muon g-2 Theory Initiative
- Hadronic corrections in comparison
- HVP
  - puzzles
- HLbL
- Summary and Outlook
- Appendix

- ``The anomalous magnetic moment of the muon in the SM":
  1st White Paper published in 2020 (132 authors, 82 institutions) [T. Aoyama et al, arXiv:2006.04822, Phys. Repts. 887 (2020) 1-166.]
- ``Prospects for precise predictions of $a_\mu$ in the SM": 2022 Snowmass Summer Study, arXiv:2203.15810
- Summary statement on the status of Muon g-2 Theory in the SM: https://muon-gm2-theory.illinois.edu

# Anomalous magnetic moment

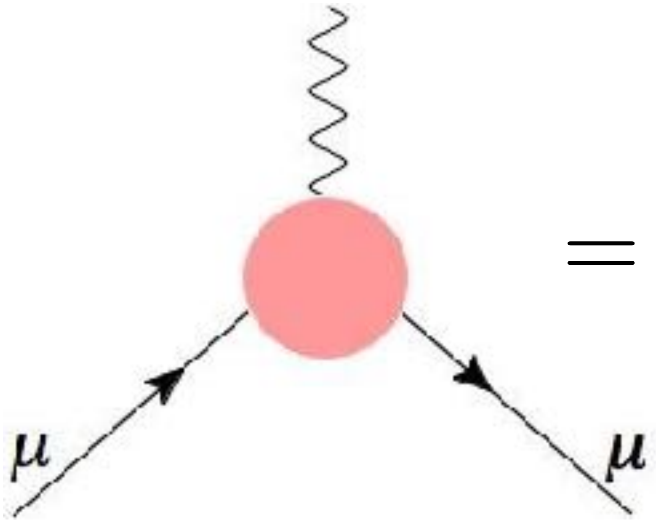
The magnetic moment of charged leptons $(e, \mu, \tau)$: $\vec{\mu} = g\dfrac{e}{2m}\vec{S}$

Dirac (leading order): $g = 2$




$$= (-ie)\,\bar{u}(p')\gamma^{\mu}u(p)$$

Quantum effects (loops):




$$= (-i\,e)\,\bar{u}(p')\left[\gamma^{\mu}F_1(q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2m}F_2(q^2)\right]u(p)$$

Note: $F_1(0) = 1$ and $g = 2 + 2\,F_2(0)$

Anomalous magnetic moment: $a \equiv \dfrac{g-2}{2} = F_2(0)$

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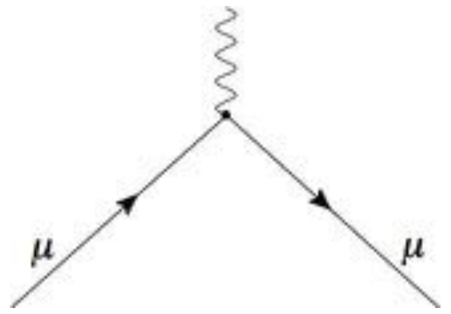


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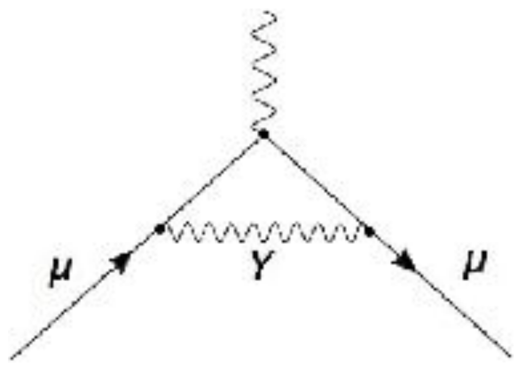
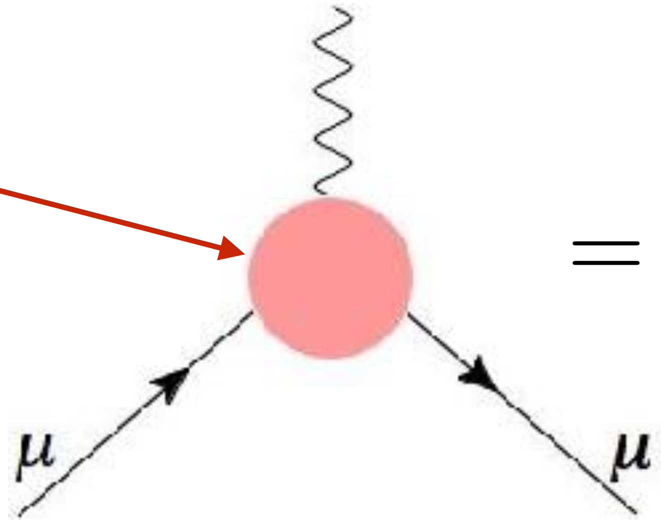



$$g = 2\left(1 + \frac{\alpha}{2\pi}\right)$$

All SM particles contribute

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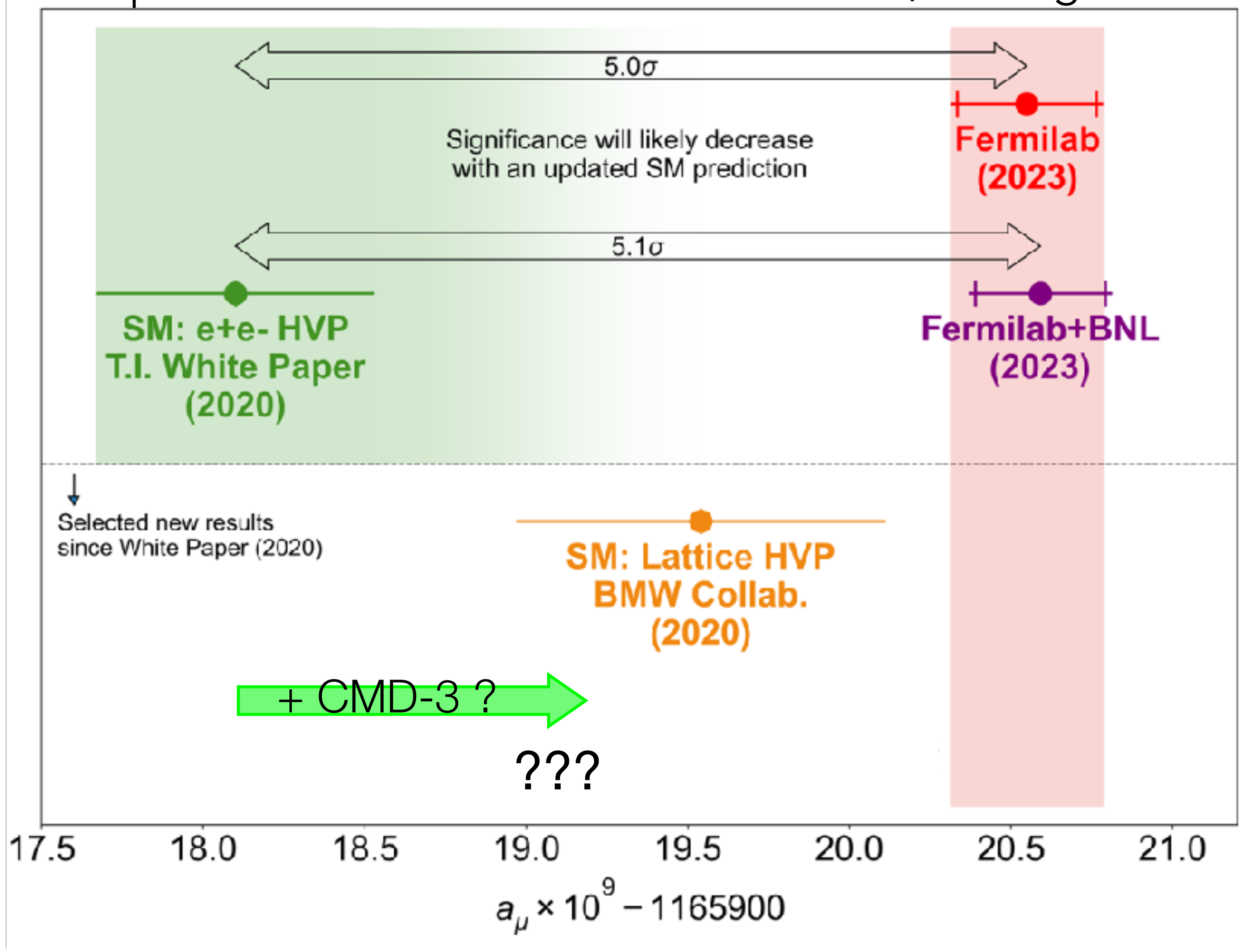
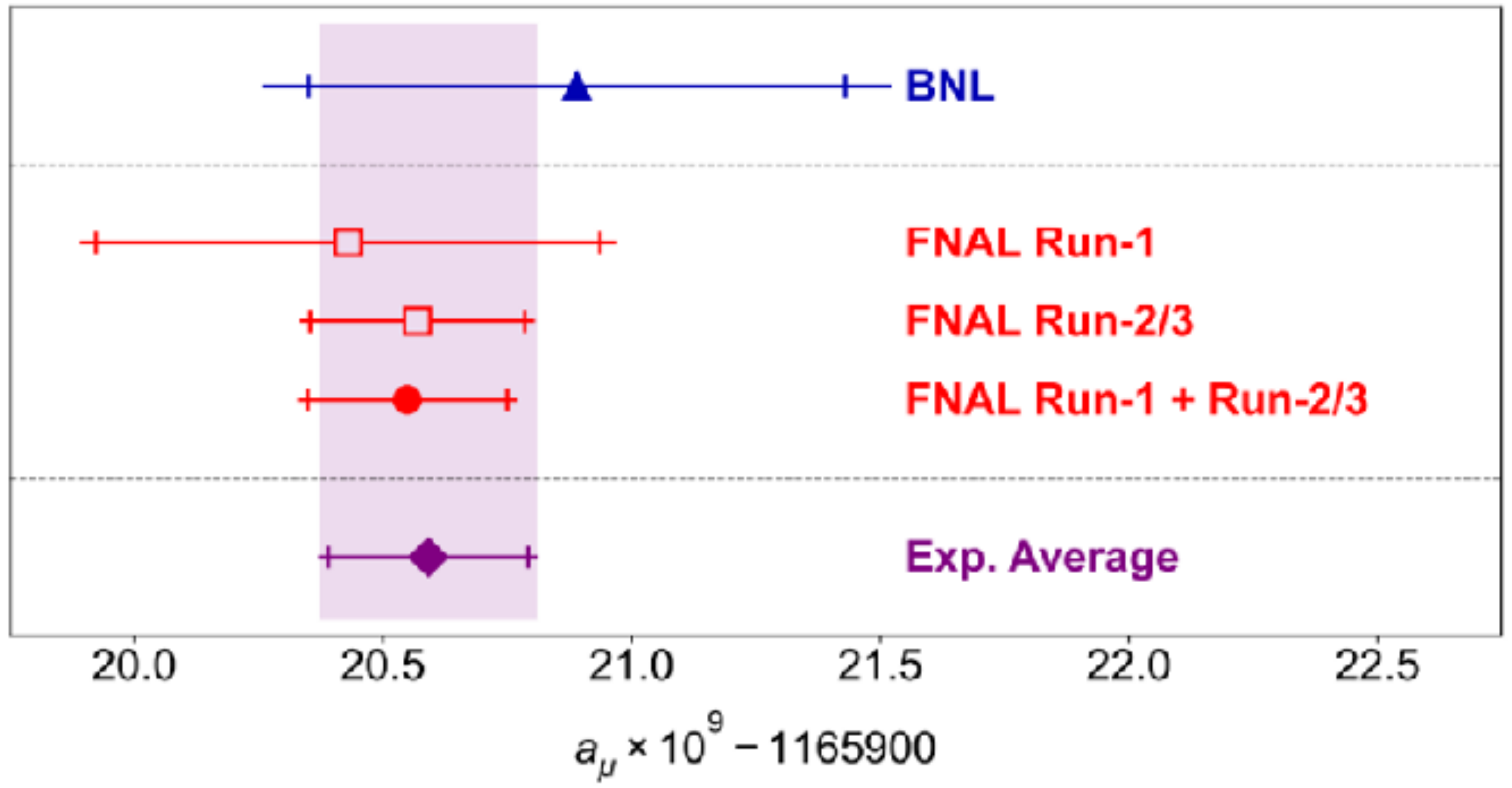
Anomalous magnetic moment: $a \equiv \frac{g-2}{2} = F_2(0)$ $= \frac{\alpha}{2\pi} + O(\alpha^2) + \ldots = 0.00116\ldots$

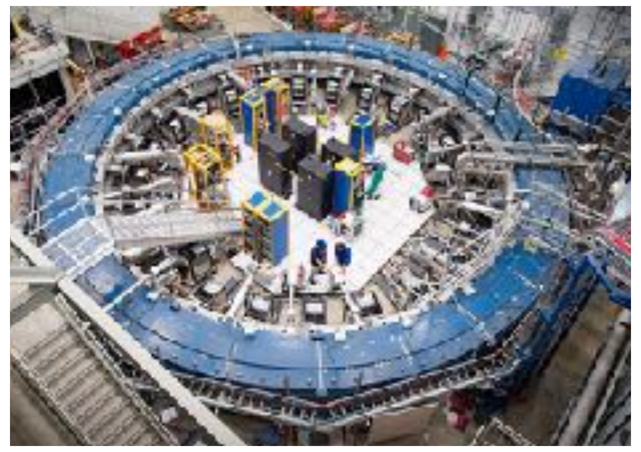
# Fermilab muon g-2 experiment

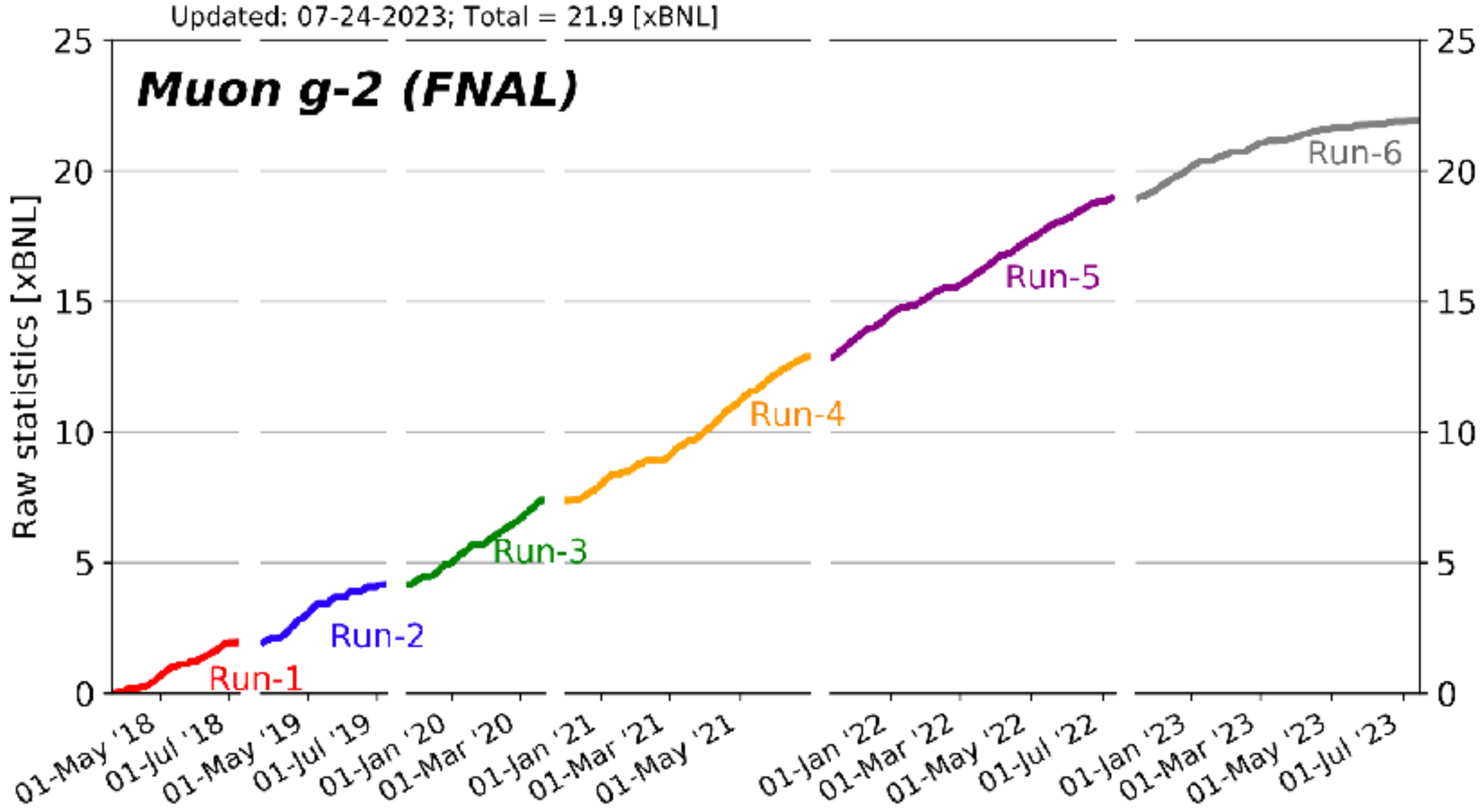
- The Fermilab experiment released the measurement result from their run 2&3 data on 10 Aug 2023. [D. Aguillard et al, 2308.06230]
- Run 6 completed summer 2023.

adapted from J. Mott @ Scientific Seminar, 10 Aug 2023





# Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

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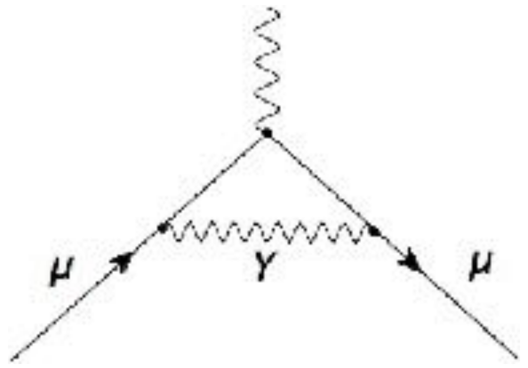
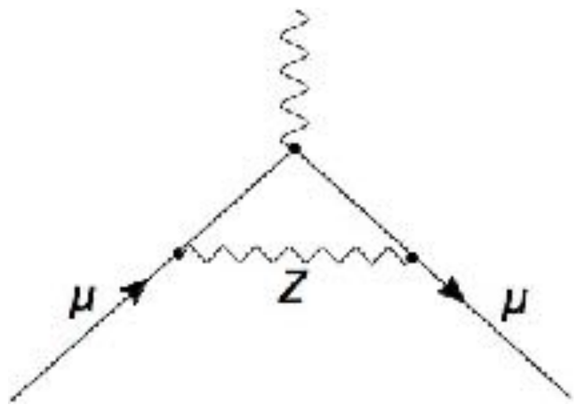
QED




$+\ldots$ (5 loops) $116\,584\,718.9\,(1) \times 10^{-11}$ $0.001$ ppm

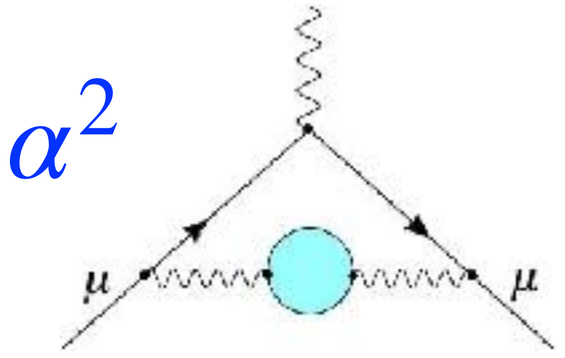
EW




$+\ldots$ (2 loops) $153.6\,(1.0) \times 10^{-11}$ $0.01$ ppm

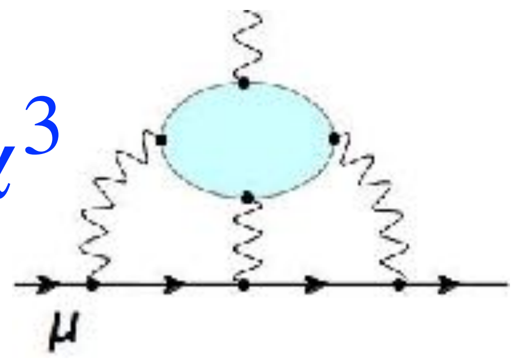
Hadronic corrections

HVP $\alpha^2$




$+\ldots$ (NNLO) $6845\,(40) \times 10^{-11}$ [0.6%] $0.34$ ppm

HLbL $\alpha^3$



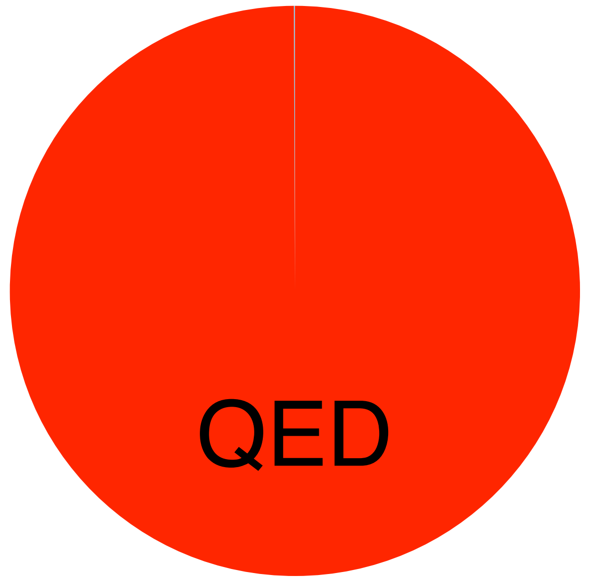

$+\ldots$ (NLO) $92\,(18) \times 10^{-11}$ [20%] $0.15$ ppm

# Muon g-2: SM contributions

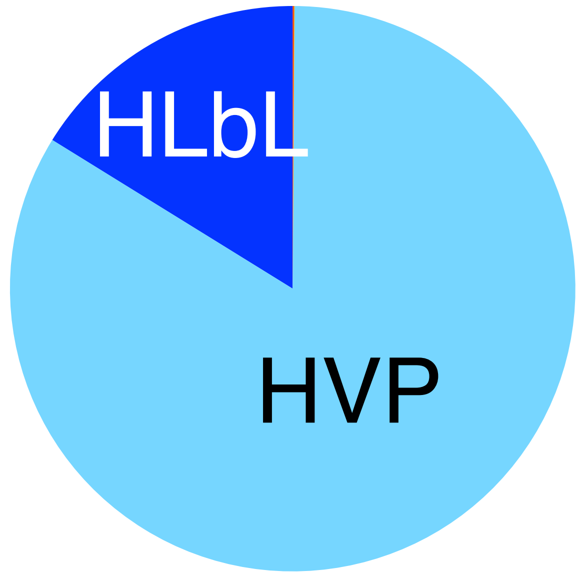
$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

contribution

- QED
- EW
- HVP
- HLbL




error$^2$

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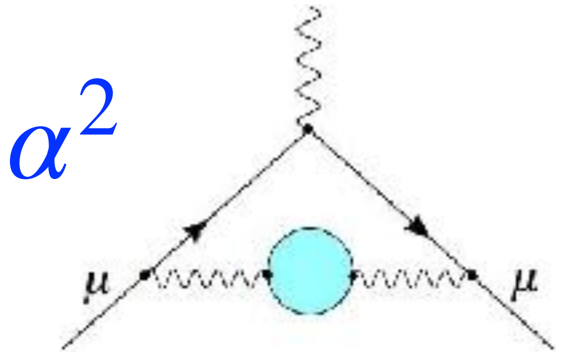


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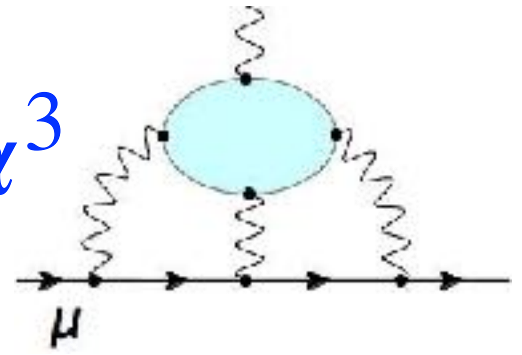


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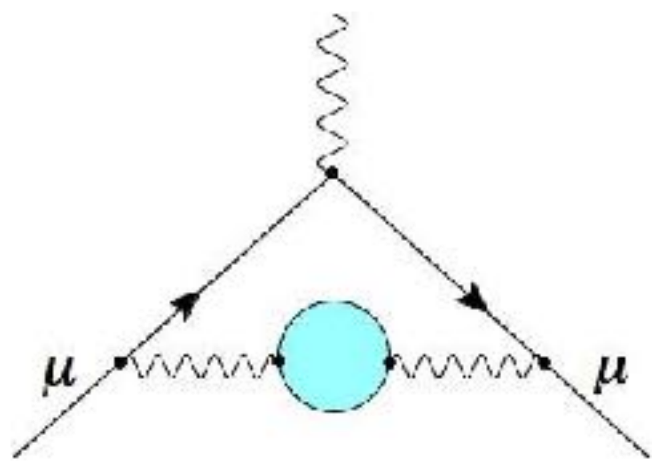
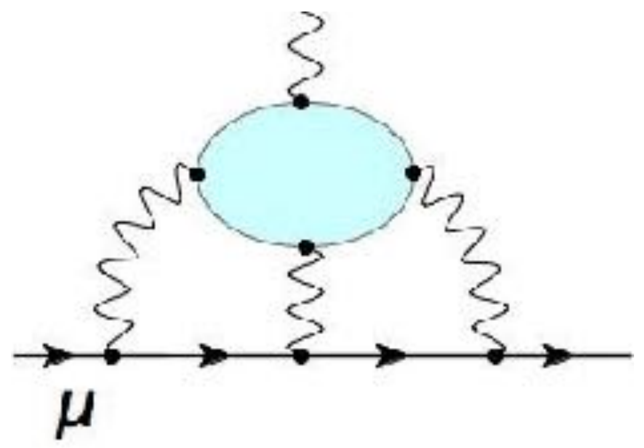
Hadronic corrections

# Muon g-2: hadronic corrections



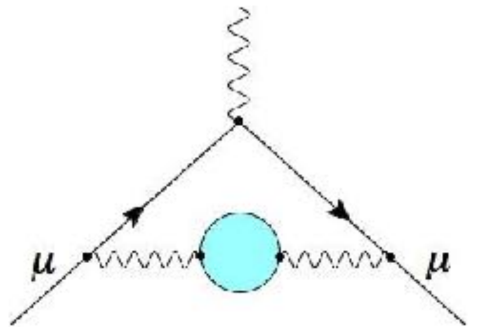




- Hadronic contributions are obtained by integrating over all possible virtual photon momenta, integral is weighted towards low $q^2$.
- Cannot use perturbation theory to reliably compute the hadronic bubbles
- Two-point & four-point functions:

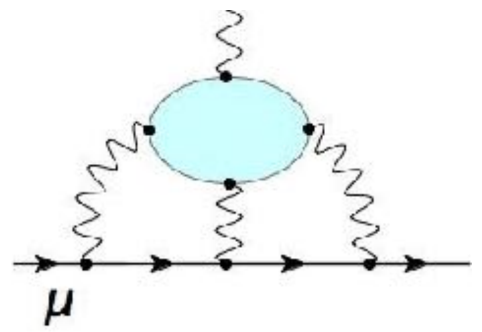
HVP: $\langle 0 | T\{j_\mu j_\nu\} | 0 \rangle$ HLbL: $\langle 0 | T\{j_\mu j_\nu j_\rho j_\sigma\} | 0 \rangle$

Two independent approaches

1. Dispersive, data-driven
2. Lattice QCD



# Hadronic Corrections

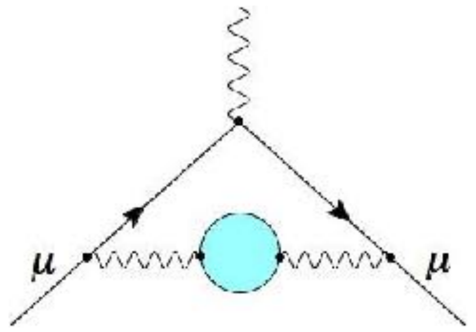



Two independent approaches:

- **For HVP:** use dispersion relations to rewrite integral in terms of hadronic cross section:


Many experiments (over 20+ years) have measured the $e^+e^-$cross sections for the different channels over the needed energy range with increasing precision.

For HLbL: new dispersive approach



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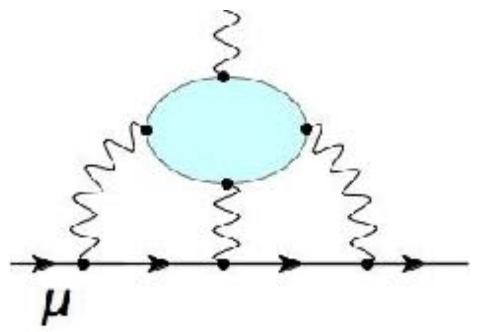

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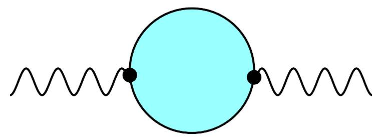
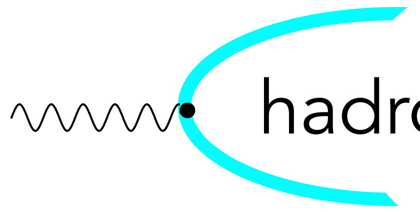
*Im*[  ] ~ |   $|^2$ ➠

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\,\hat{\Pi}(q^2) \;=\; \frac{m_\mu^2}{12\pi^3}\int ds\,\frac{\hat{K}(s)}{s}\,\sigma_{\mathrm{exp}}(s)$$

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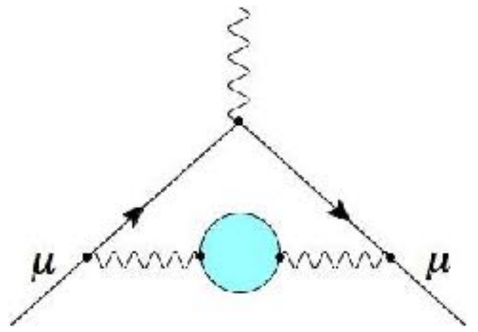

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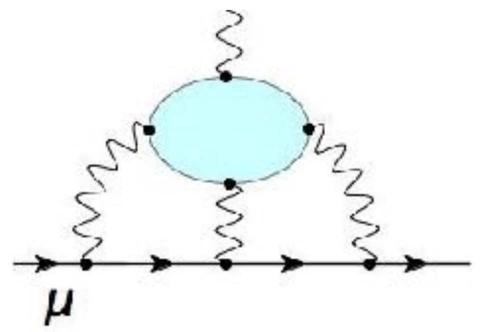


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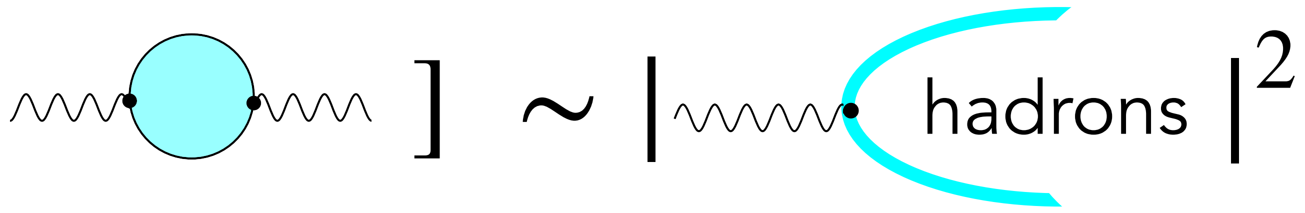


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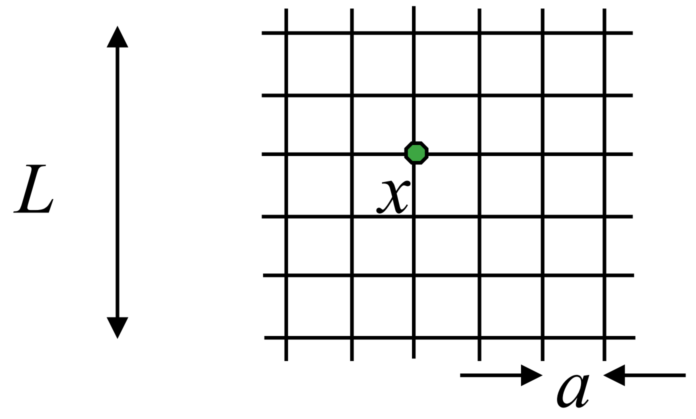
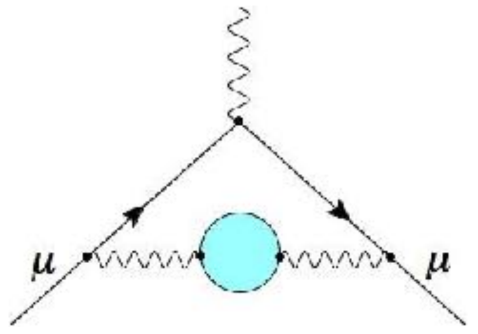



Approximations:
discrete space-time (spacing $a$)
finite spatial volume ($L$), and time extent ($T$)
…

- *ab-initio* method to quantify QCD effects
- already used for simple hadronic quantities with high precision
- requires large-scale computational resources
- allows for entirely SM theory based evaluations

Integrals are evaluated numerically using Monte Carlo methods.

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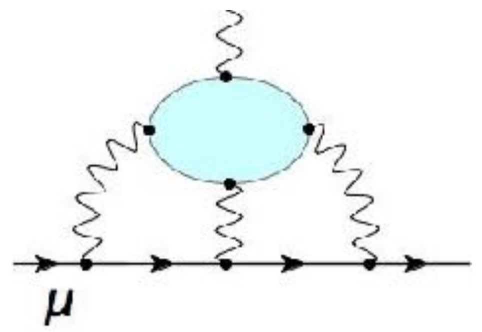


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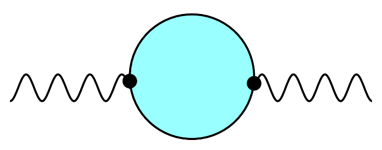
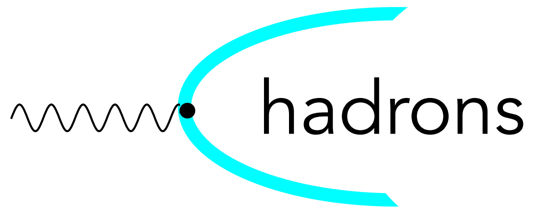
*Im*[  ] ~ |  |$^2$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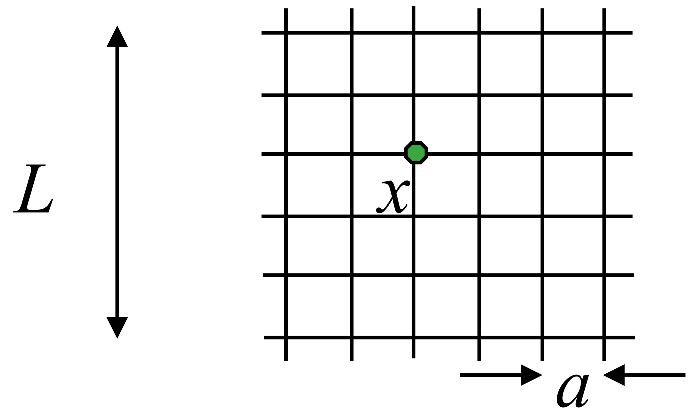


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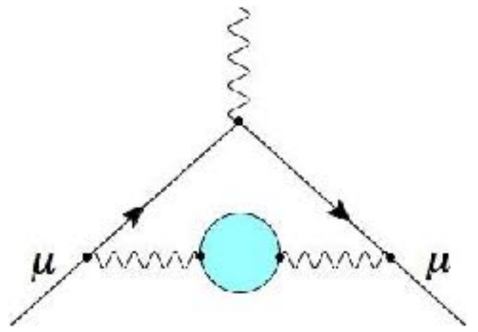


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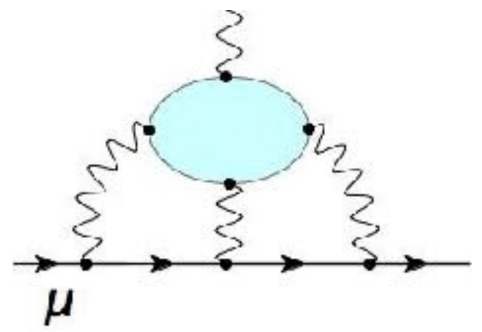


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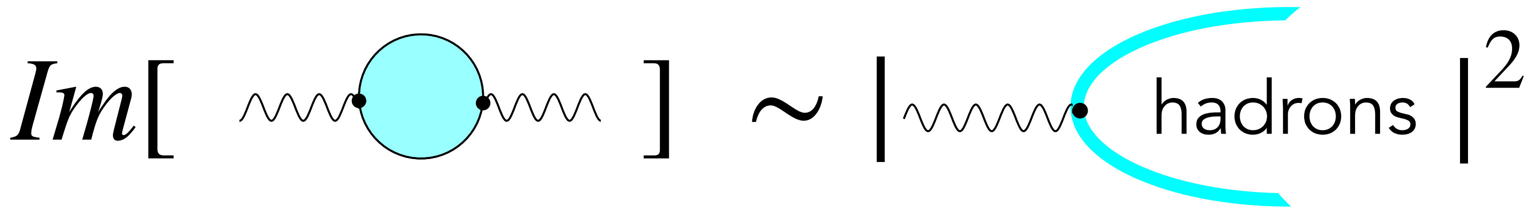


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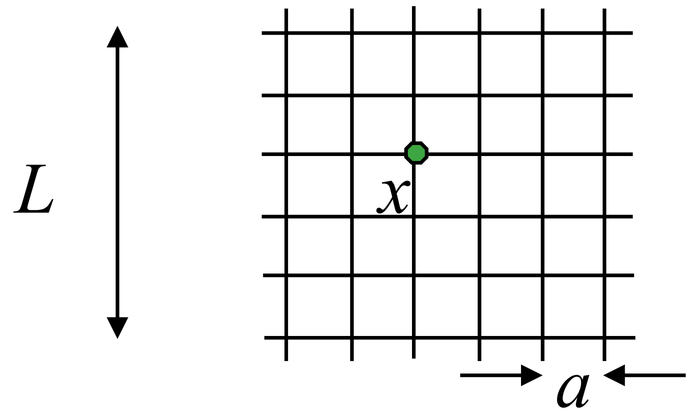


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# Muon g-2 Theory Initiative

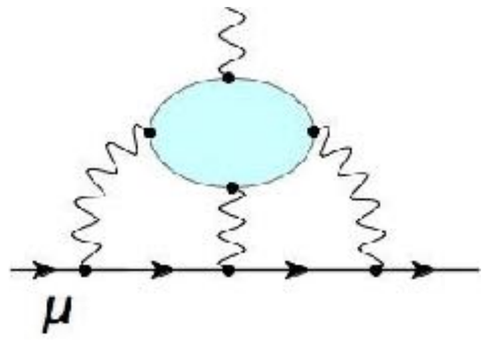
## Steering Committee

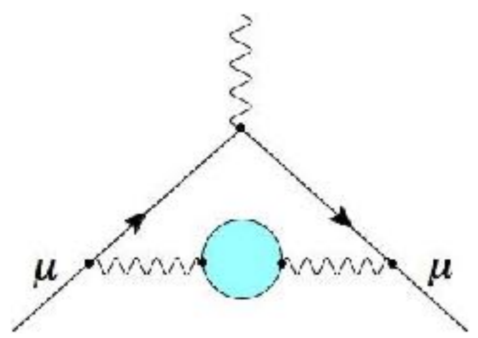
- Gilberto Colangelo (Bern)
- Michel Davier (Orsay) co-chair
- Aida El-Khadra (UIUC & Fermilab) chair
- Martin Hoferichter (Bern)
- Christoph Lehner (Regensburg University) co-chair
- Laurent Lellouch (Marseille)
- Tsutomu Mibe (KEK)
  J-PARC Muon g-2/EDM experiment
- Lee Roberts (Boston)
  Fermilab Muon g-2 experiment
- Thomas Teubner (Liverpool)
- Hartmut Wittig (Mainz)

https://muon-gm2-theory.illinois.edu

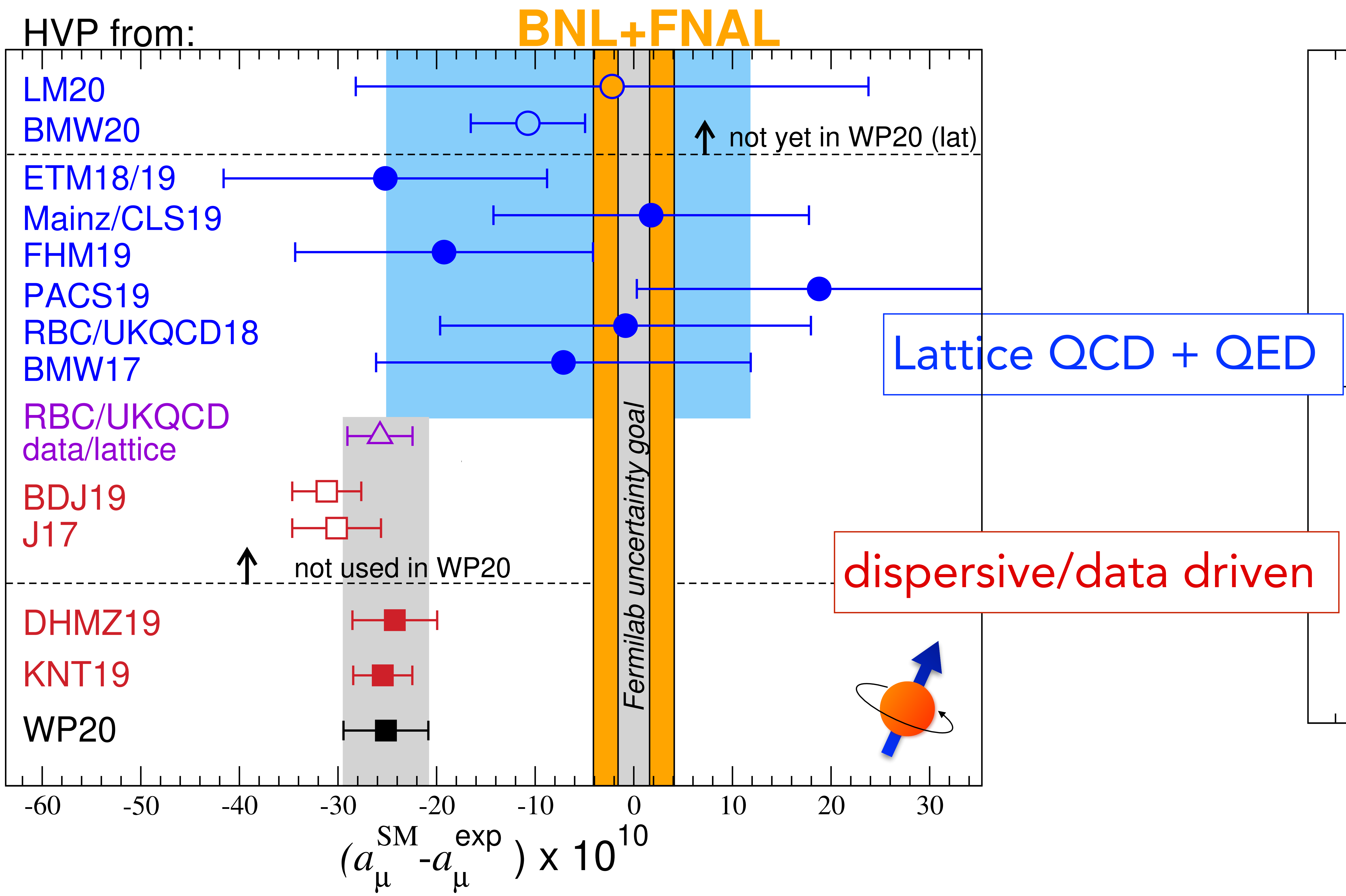
- Maximize the impact of the Fermilab and J-PARC experiments
  ➠ quantify and reduce the theoretical uncertainties on the hadronic corrections
- summarize the theory status and assess reliability of uncertainty estimates
- organize workshops to bring the different communities together:
  First plenary workshop @ Fermilab: 3-6 June 2017
  HVP workshop @ KEK: 12-14 February 2018
  HLbL workshop @ U Connecticut: 12-14 March 2018
  Second plenary workshop @ HIM (Mainz): 18-22 June 2018
  Third plenary workshop @ INT (Seattle): 9-13 September 2019
  Lattice HVP at high precision workshop (virtual): 16-20 November 2020
  Fourth plenary workshop @ KEK (virtual): 28 June - 02 July 2021
  Fifth plenary workshop @ Higgs Centre (Edinburgh): 5-9 September 2022
  Sixth plenary workshop @ University of Bern: 4-8 September 2023
  Seventh plenary workshop @ KEK (Japan): June 2024
  Eight plenary workshop: 2025 in the US — seeking proposals

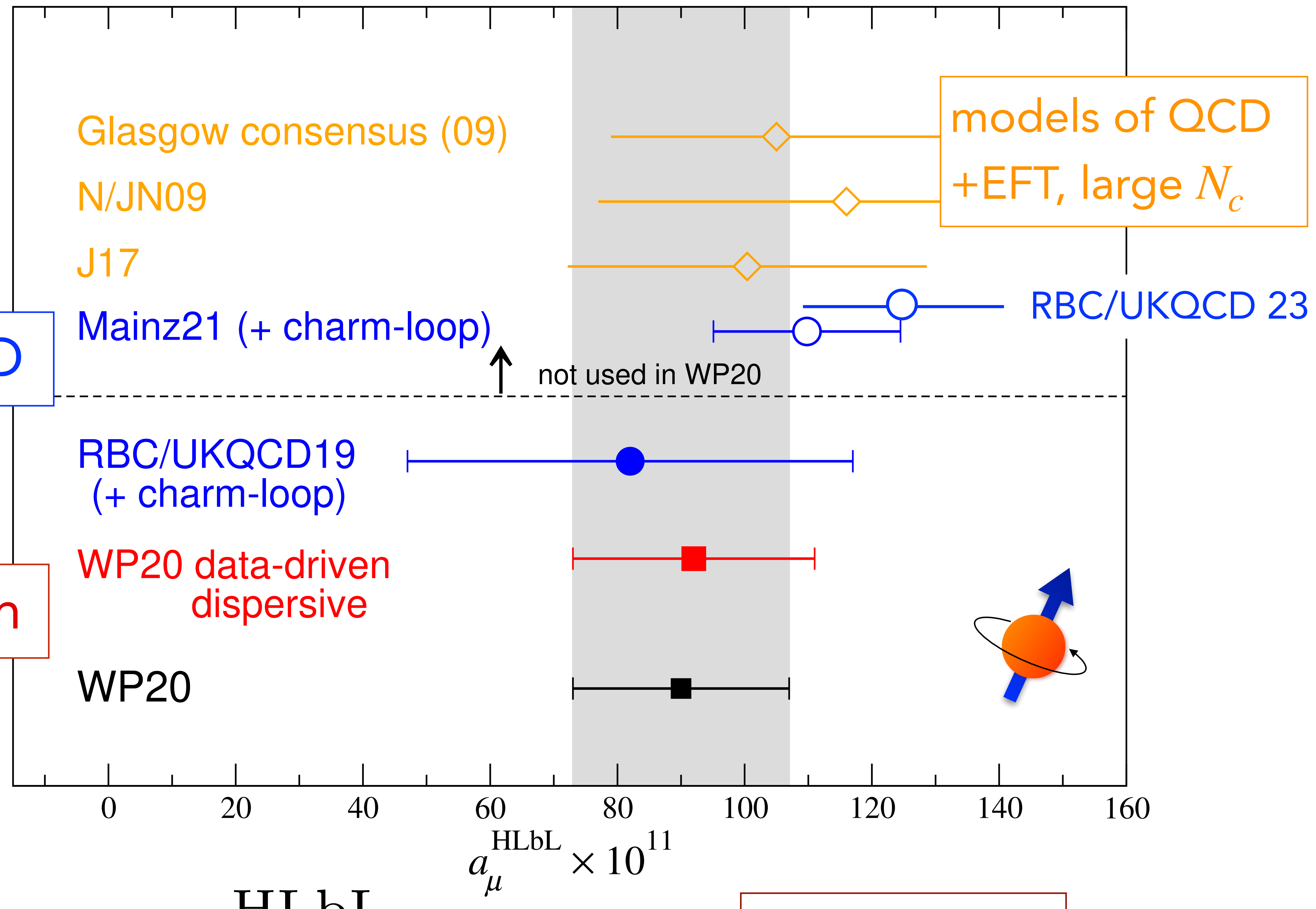
# Hadronic Corrections: Comparisons




## Before February 2023

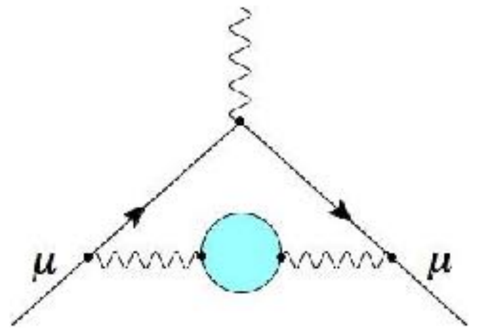



$a_\mu^{HLbL}$

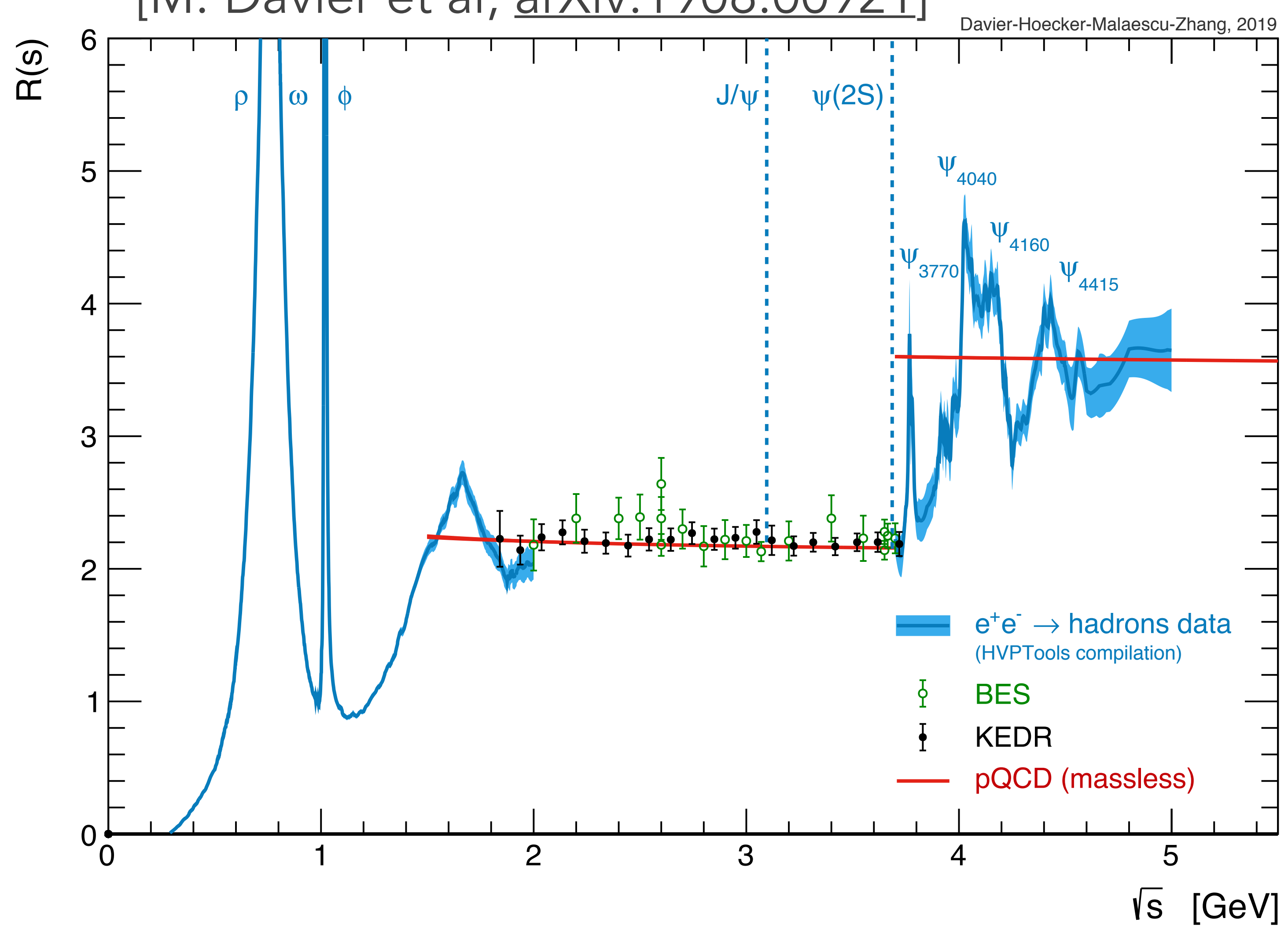
➠ appendix

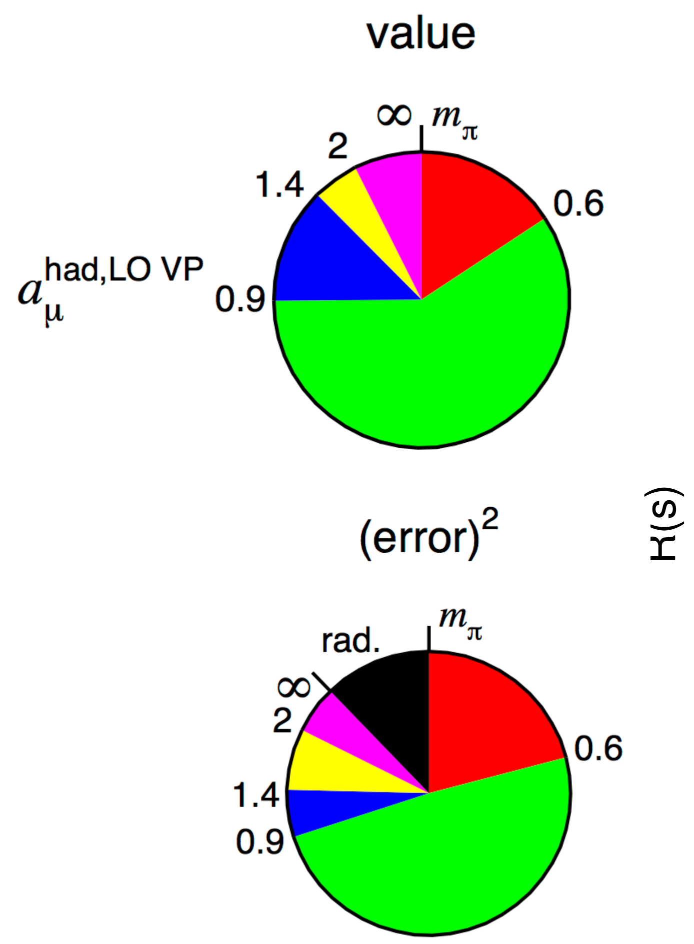
$$a_\mu^{SM} \quad a_\mu^{HVP} + \left[a_\mu^{QED} + a_\mu^{Weak} + a_\mu^{HLbL}\right]$$



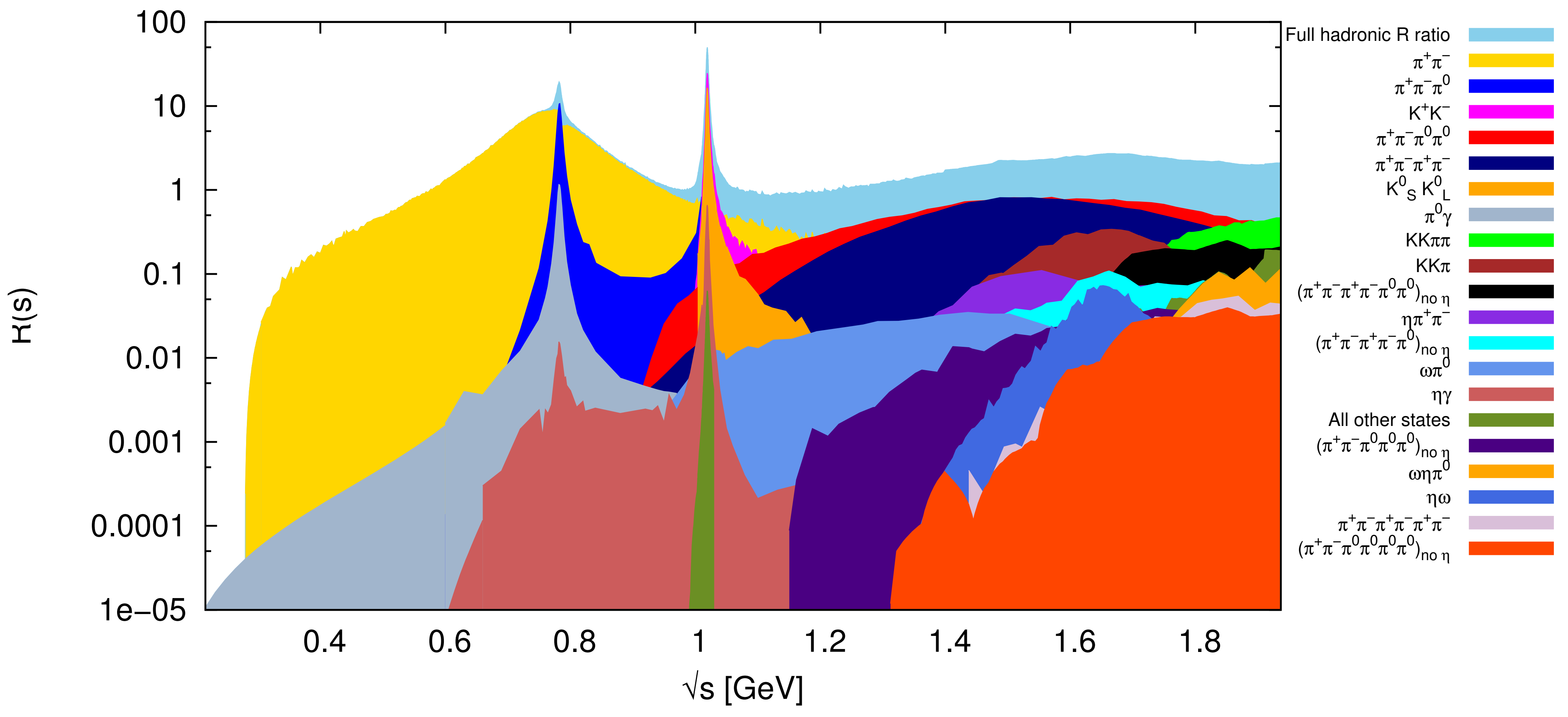
# HVP: data-driven

[M. Davier et al, arXiv:1908.00921]



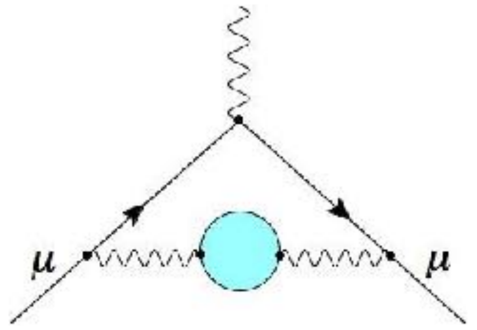

[A. Keshavarzi et al, arXiv:1802.02995]




- $\sigma_{\rm had}(s)$ defined to include real & virtual photons
- direct integration method: no modelling of $\sigma_{\rm had}(s)$, summing up contributions from all hadronic channels
- total hadronic cross section $\sigma_{\rm had}(s)$ from > 100 data sets in more than 35 channels summed up to $\sqrt{s} \sim 2\,{\rm GeV}$
- $\sqrt{s} > 2$ GeV: inclusive data + pQCD + narrow resonances
- two independent compilations (DHMZ, KNT) using the direct integration method

Tensions between BaBar and KLOE data sets:

- Cross checks using analyticity and unitarity relating pion form factor to $\pi\pi$ scattering
- Combinations of data sets affected by tensions
  ➠ conservative merging procedure




# HVP: data-driven

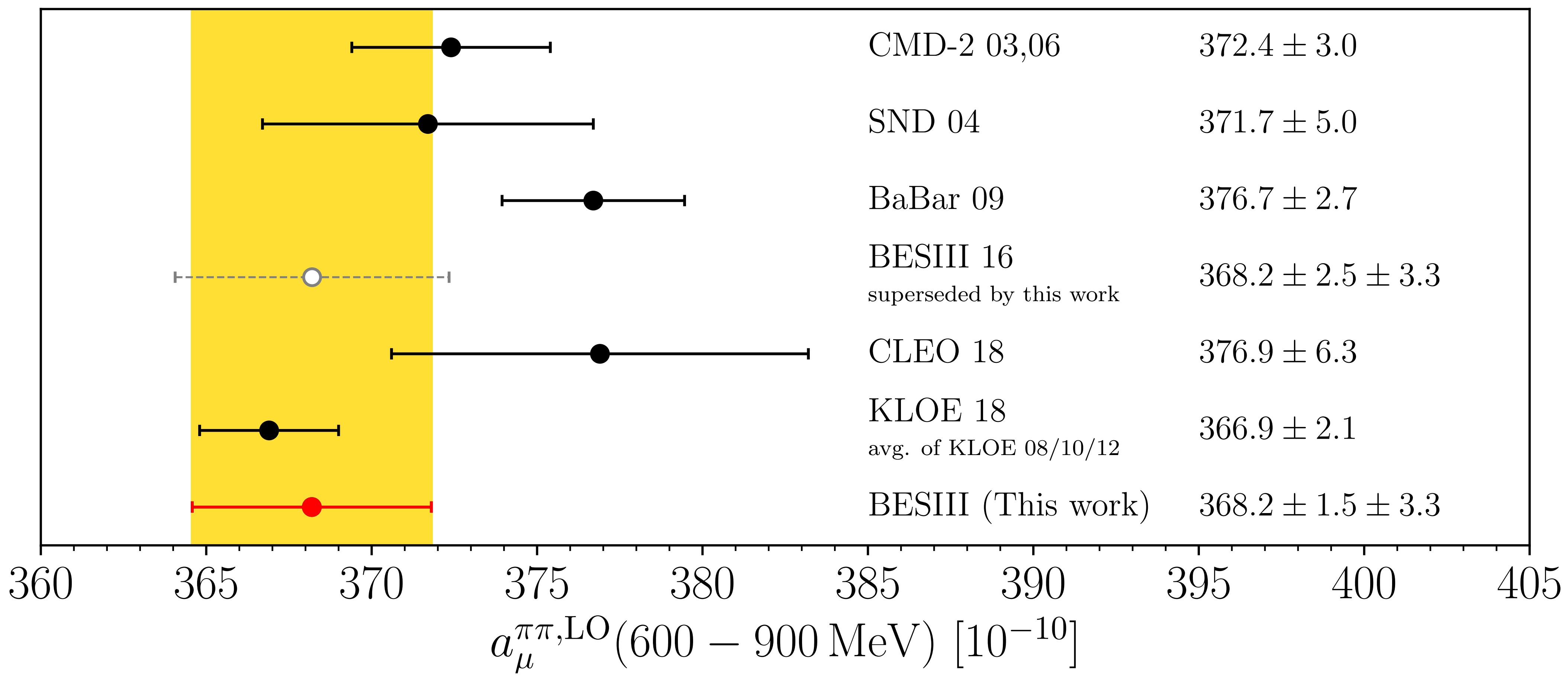
2020 White Paper [T. Aoyama et al, arXiv:2006.04822, Phys. Repts. 887 (2020) 1-166.]

Conservative merging procedure to obtain a realistic assessment of the underlying uncertainties:

- account for tensions between data sets
- account for differences in methodologies for compilation of experimental inputs and treatment of correlations between systematic errors
- include results using constraints from unitarity & analyticity in $\pi\pi$ and $\pi\pi\pi$ channels
  [Colangelo et al, 2018; Anantharayan et al, 2018; Davier et al, 2019; Hoferichter et al, 2019]
- Full NLO radiative corrections [Campanario et al, 2019]

$$a_\mu^{\mathrm{HVP,LO}} = 693.1\,(2.8)_{\mathrm{exp}}\,(0.7)_{\mathrm{DV+pQCD}}\,(2.8)_{\mathrm{BaBar-KLOE}} \times 10^{-10}$$
$$= 693.1\,(4.0) \times 10^{-10}$$

[M. Ablikim et al (BES III), arXiv:2009.05011]



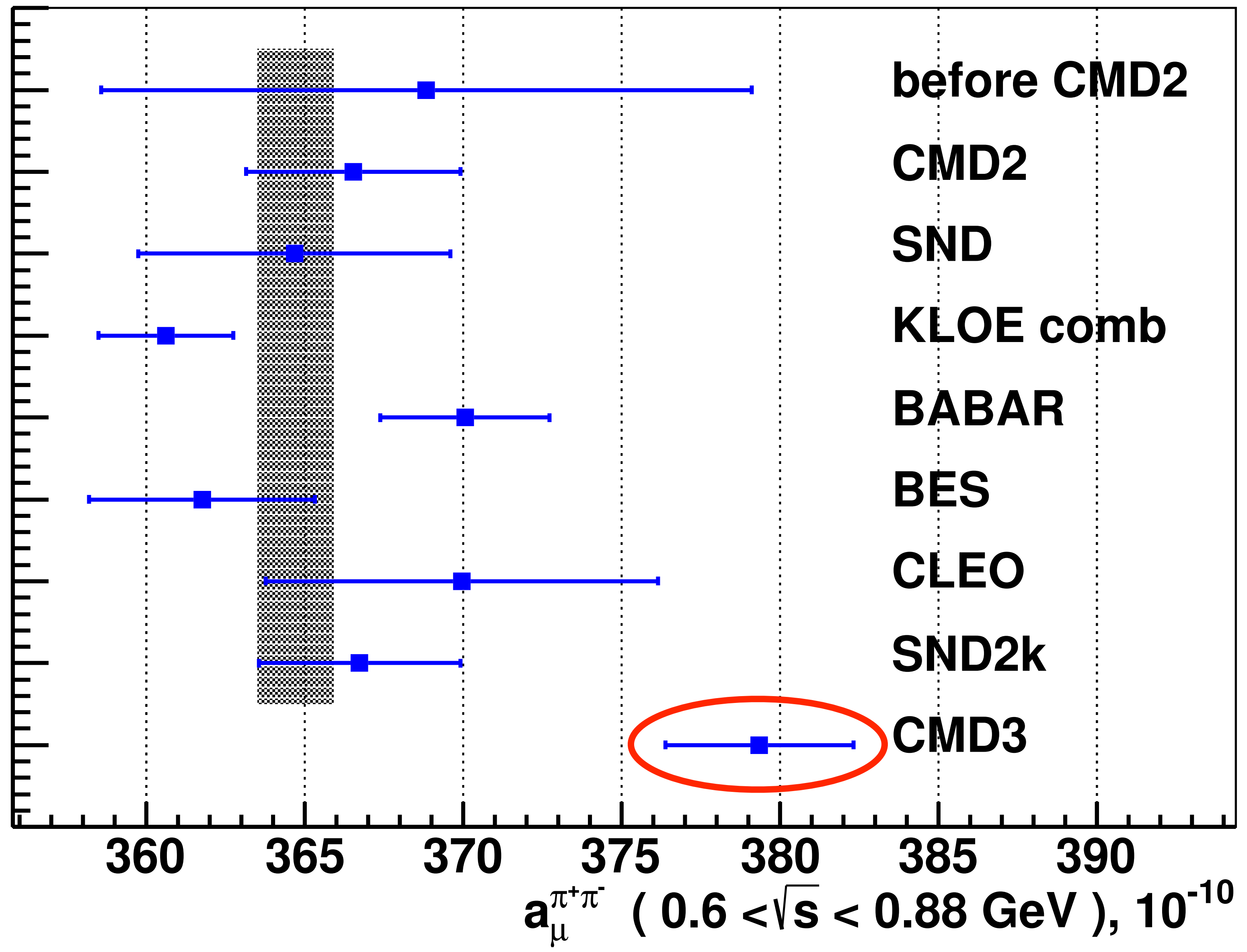
# HVP: data-driven

T. Teubner @ Zurich workshop

## New results for $\sigma_{\mathrm{had}}(s)$:

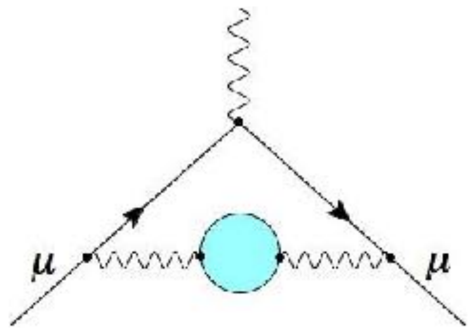
- **pi+pi-pi0**, BESIII (2019), arXiv:1912.11208
- **pi+pi-** [covariance matrix erratum], BESIII (2020), Phys.Lett.B 812 (2021)
- **K+K-pi0**, SND (2020), Eur.Phys.J.C 80 (2020) 12, 1139
- **etapi0gamma** (res. only), SND (2020), Eur.Phys.J.C 80 (2020) 11, 1008
- **pi+pi-**, SND (2020), JHEP 01 (2021) 113
- **etaomega** → pi0gamma, SND (2020), Eur.Phys.J.C 80 (2020) 11, 1008
- **pi+pi-pi0**, SND (2020), Eur.Phys.J.C 80 (2020) 10, 993
- **pi+pi-pi0**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112003
- **pi+pi-2pi0omega**, BaBar (2021), Phys. Rev. D 103, 092001
- **etaetagamma**, SND (2021), Eur.Phys.J.C 82 (2022) 2, 168
- **etaomega**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- **pi+pi-pi0eta**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- **omegaetapi0**, BaBar (2021), Phys. Rev. D 103, 092001
- **pi+pi-4pi0**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- **pi+pi-pi0pi0eta**, BaBar (2021), Phys.Rev.D 103 (2021) 9, 092001
- **pi+pi-3pi0eta**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- **2pi+2pi-3pi0**, BaBar (2021), Phys. Rev. D 103, 092001
- **omega3pi0**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- **pi+pi-pi+pi-eta**, BaBar (2021), Phys. Rev. D 103, 092001
- **inclusive**, BESIII (2021), Phys.Rev.Lett. 128 (2022) 6, 062004
- ...

**New:** from CMD-3 [F. Ignatov et al, arXiv:2302.08834]




### A new puzzle!

- discrepancies between experiments now $\gtrsim (3-5)\,\sigma$ need to be understood/resolved
- (virtual) scientific seminar + discussion panel on CMD-3 measurement March 27 (8:00 –11:00 am US CDT) Discussions are continuing….
- 6th Muon g-2 Theory Initiative workshop (4-8 Sep 2023, Bern)



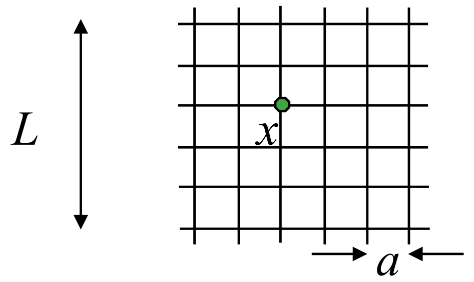

# HVP: data-driven

Ongoing work on experimental inputs:

- BaBar: new analysis of large data set in $\pi\pi$ channel, also $\pi\pi\pi$, other channels
- KLOE: new analysis of large data set in $\pi\pi$ channel, other channels
- SND: new results for $\pi\pi$ channel, other channels in progress
- BESIII: new results in 2021 for $\pi\pi$ channel, continued analysis also for $\pi\pi\pi$, other channels
- Belle II: arXiv:2207.06307 (Snowmass WP)
  Better ultimate statistics than BaBar or KLOE; similar or better systematics for low-energy cross sections

Ongoing work on theoretical aspects:

- better treatment of structure dependent radiative corrections (NLO) in $\pi\pi$ and $\pi\pi\pi$ channels
  so far: FsQED (scalar QED + pion form factor)
  tests of radiative corrections using exp. measurement of charge asymmetry [Ignatov + Lee, arXiv:2204.12235]
  new dispersive treatment [Colangelo at al, arXiv2207.03495]
- Developing NNLO Monte Carlo generators (STRONG 2020 workshop https://agenda.infn.it/event/28089/)
- including $\tau$ decay data: requires nonperturbative evaluation of IB correction [M. Bruno et al, arXiv:1811.00508]




# Lattice QCD Introduction

## The State of the Art

Lattice QCD calculations of simple quantities (with at most one stable meson in initial/final state) that **quantitatively account for all systematic effects** (discretization, finite volume, renormalization,…) in some cases with

- sub percent precision.
- total errors that are commensurate (or smaller) than corresponding experimental uncertainties.

Progress due to a virtuous cycle of theoretical developments, improved algorithms/methods and increases in computational resources (``Moore's law")

Scope of LQCD calculations is increasing due to continual development of new methods:

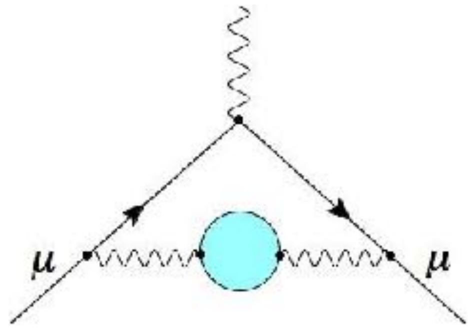
- nucleon matrix elements
- nonleptonic kaon decays ($K \to \pi\pi, \epsilon', \ldots$)
- resonances, scattering ($\pi\pi \to \rho, \ldots$)
- long-distance effects ($\Delta M_K, \ldots$)
- QED corrections
- radiative decay rates
- structure: PDFs, GPDs, TMDs, …
- inclusive decay rates ($B \to X_c \ell \nu, \ldots$)
- …

# 2013 ➠ present

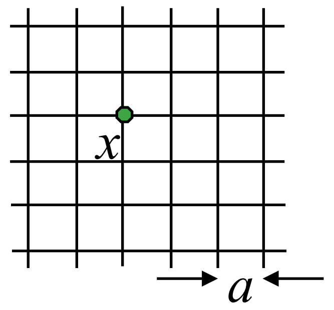
https://www.usqcd.org/documents/13flavor.pdf and [J. Butler et al, arXiv:1311.1076]

| Quantity | CKM element | 2013 expt. error | 2007 forecast lattice error | 2013 lattice error | forecast lattice error | 2021 FLAG Average | |
|---|---|---|---|---|---|---|---|
| $f_K/f_\pi$ | $\lvert V_{us}\rvert$ | 0.2% | 0.5% | 0.4% | 0.15% | 0.18 % | |
| $f_+^{K\pi}(0)$ | $\lvert V_{us}\rvert$ | 0.2% | – | 0.4% | 0.2% | 0.18 % | |
| $f_D$ | $\lvert V_{cd}\rvert$ | 4.3% | 5% | 2% | $< 1\%$ | 0.3 % | |
| $f_{D_s}$ | $\lvert V_{cs}\rvert$ | 2.1% | 5% | 2% | $< 1\%$ | 0.2 % | |
| $D \to \pi\ell\nu$ | $\lvert V_{cd}\rvert$ | 2.6% | – | 4.4% | 2% | 0.7 % | [from 2212.12648] |
| $D \to K\ell\nu$ | $\lvert V_{cs}\rvert$ | 1.1% | – | 2.5% | 1% | 0.6 % | |
| $B \to D^*\ell\nu$ | $\lvert V_{cb}\rvert$ | 1.3% | – | 1.8% | $< 1\%$ | ~1.5 % | [from 2105.14019, 2304.03137] |
| $B \to \pi\ell\nu$ | $\lvert V_{ub}\rvert$ | 4.1% | – | 8.7% | 2% | ~3 % | |
| $f_B$ | $\lvert V_{ub}\rvert$ | 9% | – | 2.5% | $< 1\%$ | 0.7 % (0.6 % for $f_{B_s}$) | |
| $\xi$ | $\lvert V_{ts}/V_{td}\rvert$ | 0.4% | 2–4% | 4% | $< 1\%$ | 1.3 % | |
| $\Delta m_s$ | $\lvert V_{ts}V_{tb}\rvert^2$ | 0.24% | 7–12% | 11% | 5% | 4.5 % | |
| $B_K$ | $\mathrm{Im}(V_{td}^2)$ | 0.5% | 3.5–6% | 1.3% | $< 1\%$ | 1.3 % | |

QED threshold:
QED corrections important/ dominant source of theory error in SM predictions

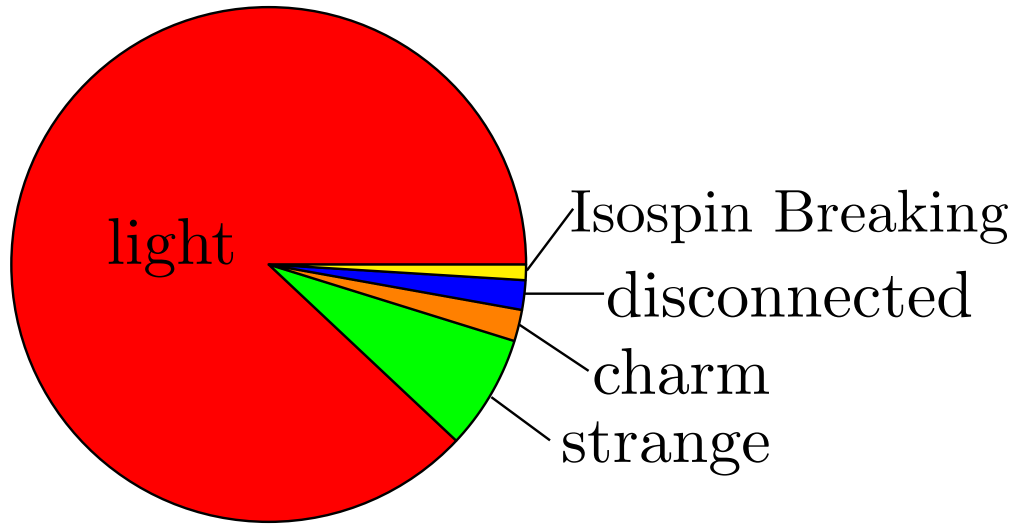



# Lattice HVP: Introduction




Calculate $a_\mu^{\rm HVP}$ in Lattice QCD: $$a_\mu^{\rm HVP,LO} = \sum_f a_{\mu,f}^{\rm HVP,LO} + a_{\mu,{\rm disc}}^{\rm HVP,LO}$$

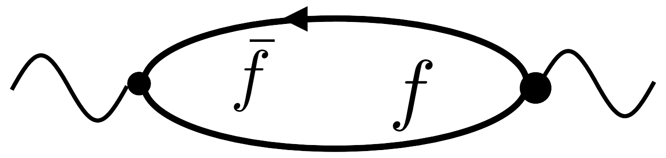
**$a_\mu$**




- Separate into connected for each quark flavor + disconnected contributions (gluon and sea-quark background not shown in diagrams)

  Note: almost always $m_u = m_d$

$f = ud, s, c, b$ $\sum_f$

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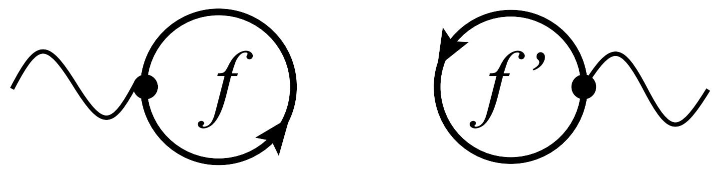
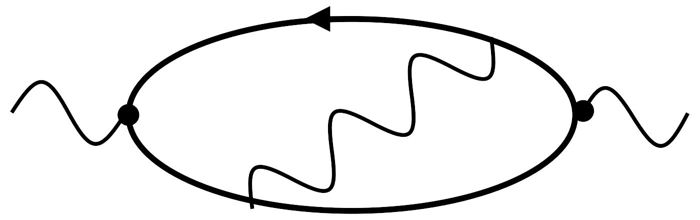



- light-quark connected contribution: $a_\mu^{\rm HVP,LO}(ud)$ ~90% of total
- $s,c,b$-quark contributions $a_\mu^{\rm HVP,LO}(s,c,b)$ ~8%, 2%, 0.05% of total
- disconnected contribution: $a_{\mu,{\rm disc}}^{\rm HVP,LO}$ ~2% of total
- Isospinbreaking (QED + $m_u \neq m_d$) corrections: $\delta a_\mu^{\rm HVP,LO}$ ~1% of total

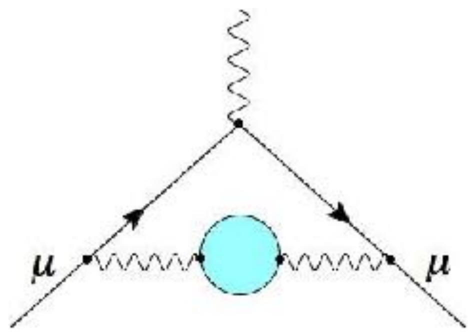
- need to add QED and strong isospin breaking ($\sim m_u - m_d$) corrections:

$+$

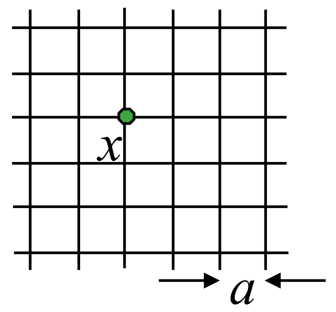


$+ \ldots$

$$a_\mu^{\rm HVP,LO} = a_\mu^{\rm HVP,LO}(ud) + a_\mu^{\rm HVP,LO}(s) + a_\mu^{\rm HVP,LO}(c) + a_{\mu,{\rm disc}}^{\rm HVP,LO} + \delta a_\mu^{\rm HVP,LO}$$

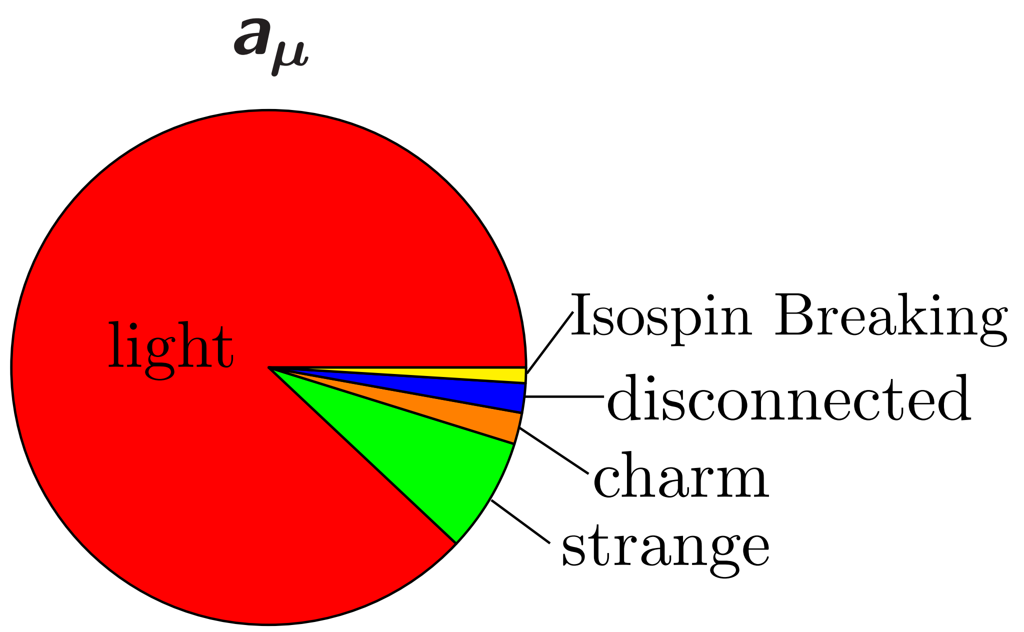



# Lattice HVP: challenges



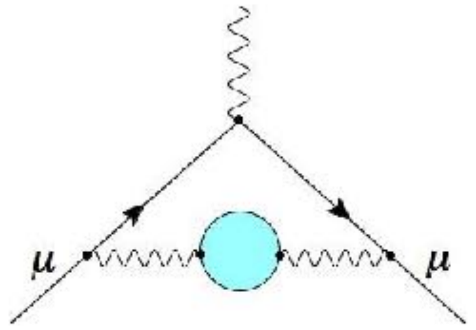

Calculate $a_\mu^{\rm HVP}$ in Lattice QCD: $$a_\mu^{\rm HVP,LO} = \sum_f a_{\mu,f}^{\rm HVP,LO} + a_{\mu,\rm disc}^{\rm HVP,LO}$$

$$a_\mu^{\rm HVP,LO} = a_\mu^{\rm HVP,LO}(ud) + a_\mu^{\rm HVP,LO}(s) + a_\mu^{\rm HVP,LO}(c) + a_{\mu,\rm disc}^{\rm HVP,LO} + \delta a_\mu^{\rm HVP,LO}$$

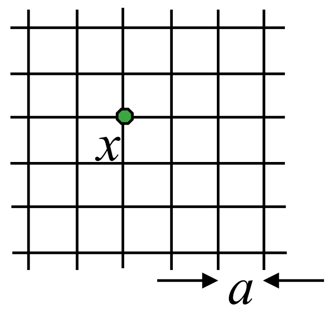



- $a_\mu^{\rm HVP,LO}$ needed with $< 0.5\,\%$ precision
- subpercent statistical precision:
  exponentially growing noise-to-signal in $C(t)$ as $t \to \infty$
  affects light-quark contributions
- sizable finite volume effects
- sensitivity to scale setting uncertainty
- control discretization effects
- quark-disconnected diagrams: control noise
- include isospin-breaking effects
  Separation of $a_\mu^{\rm HVP,LO}$ into $a_\mu^{\rm HVP,LO}(ud)$ and $\delta a_\mu^{\rm HVP,LO}$ is scheme dependent.

- light-quark connected contribution:
  $a_\mu^{\rm HVP,LO}(ud)$ ~90% of total
- *s*,*c*,*b*-quark contributions
  $a_\mu^{\rm HVP,LO}(s,c,b)$ ~8%, 2%, 0.05% of total
- disconnected contribution:
  $a_{\mu,\rm disc}^{\rm HVP,LO}$ ~2% of total
- Isospinbreaking (QED + $m_u \neq m_d$) corrections:
  $\delta a_\mu^{\rm HVP,LO}$ ~1% of total




# Windows in Euclidean time



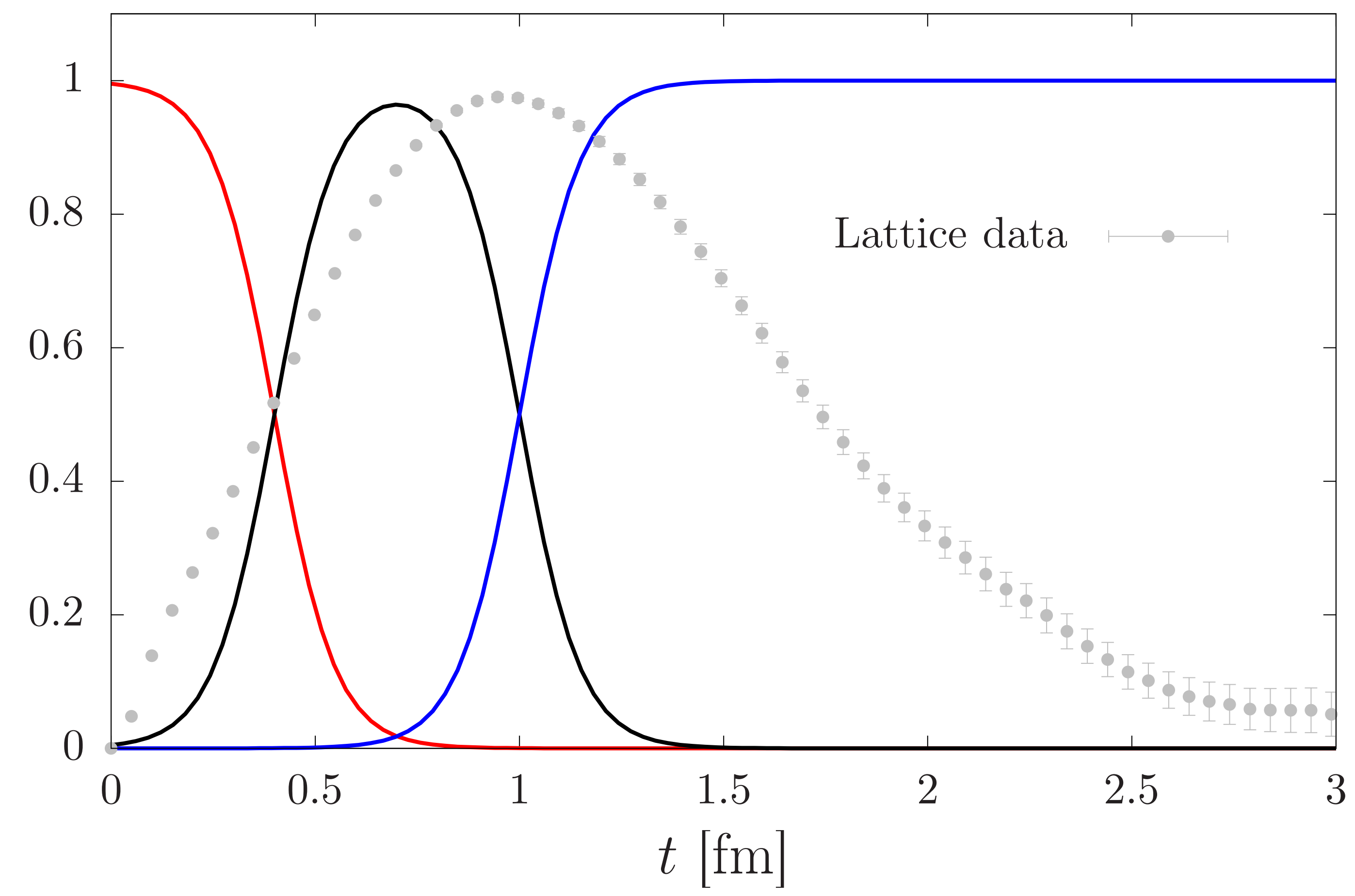

$$a_\mu^{\rm HVP,LO} = 4\,\alpha^2 \int_0^\infty dt\, C(t)\, \tilde{w}(t)$$

- Use windows in Euclidean time to consider the different time regions separately
  [T. Blum et al, arXiv:1801.07224, 2018 PRL]

Short Distance (SD) $t : 0 \to t_0$
Intermediate (W) $t : t_0 \to t_1$
Long Distance (LD) $t : t_1 \to \infty$

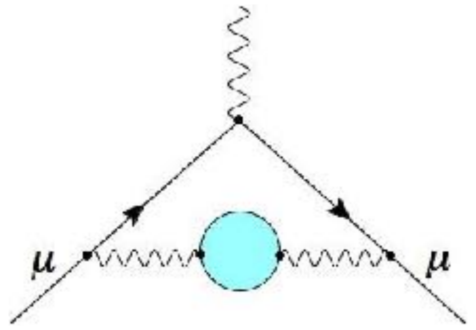
$t_0 = 0.4\,\text{fm}, t_1 = 1.0\,\text{fm}$

H. Wittig @ Lattice 2021

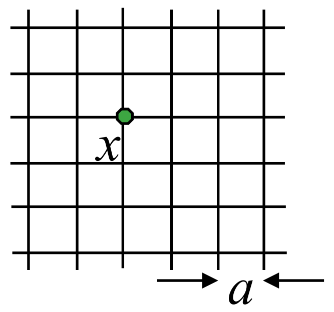



- disentangle systematics/statistics from long distance/FV and discretization effects
- intermediate window: easy to compute in lattice QCD; compare to disperse approach
- Internal cross check: compute each window separately (in continuum, infinite volume limits,…) and combine:

$$a_\mu = a_\mu^{\rm SD} + a_\mu^{\rm W} + a_\mu^{\rm LD}$$




# Lattice HVP: results



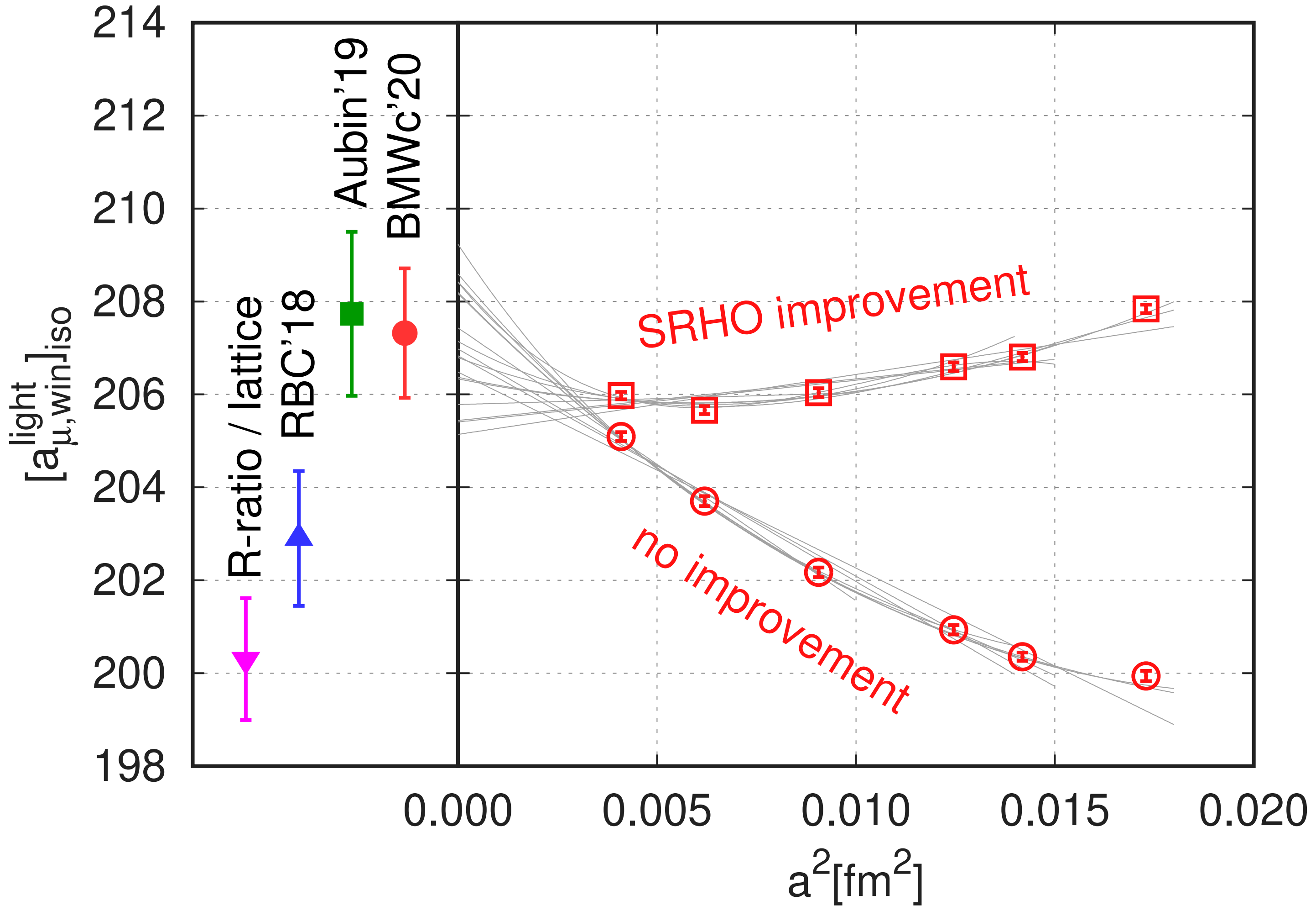

$$a_\mu^{\mathrm{HVP,LO}} = 4\,\alpha^2 \int_0^\infty dt\, C(t)\, \tilde{w}(t)$$

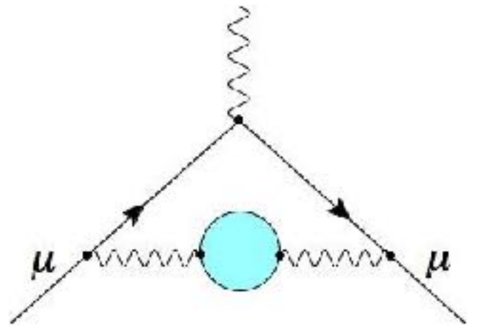
## In 2020 WP:

- Lattice HVP average at $2.6\,\%$ total uncertainty: $a_\mu^{\mathrm{HVP,LO}} = 711.6\,(18.4) \times 10^{10}$
- BMW 20 [Sz. Borsanyi et al, arXiv:2002.12347, 2021 Nature] first LQCD calculation with sub-percent ($0.8\,\%$) error in tension with data-driven HVP ($2.1\sigma$)
- Further tensions for intermediate window:

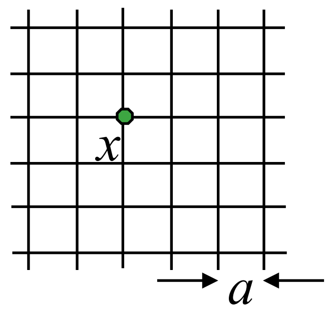
  -$3.7\sigma$ tension with data-driven evaluation

  -$2.2\sigma$ tension with RBC/UKQCD18

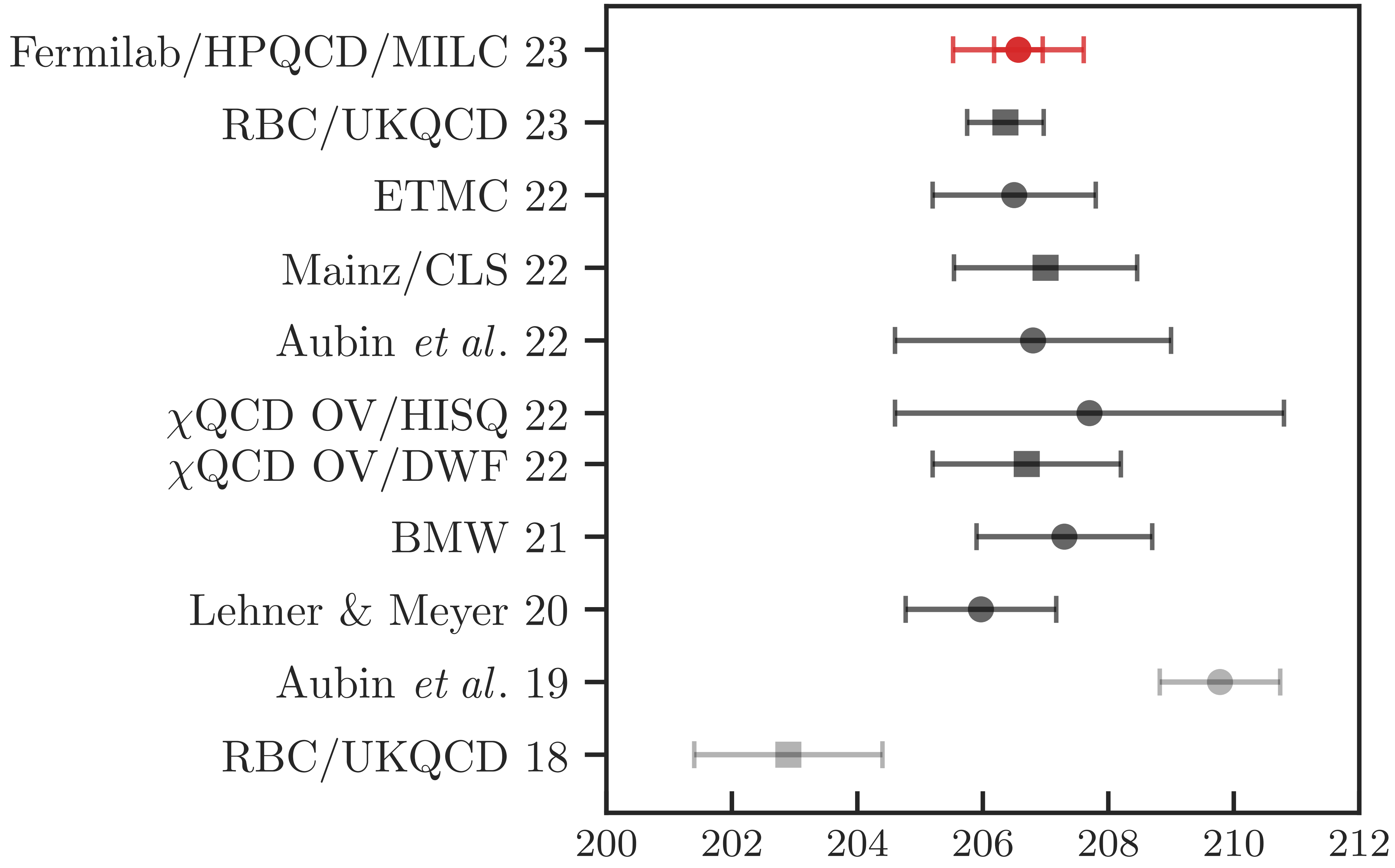




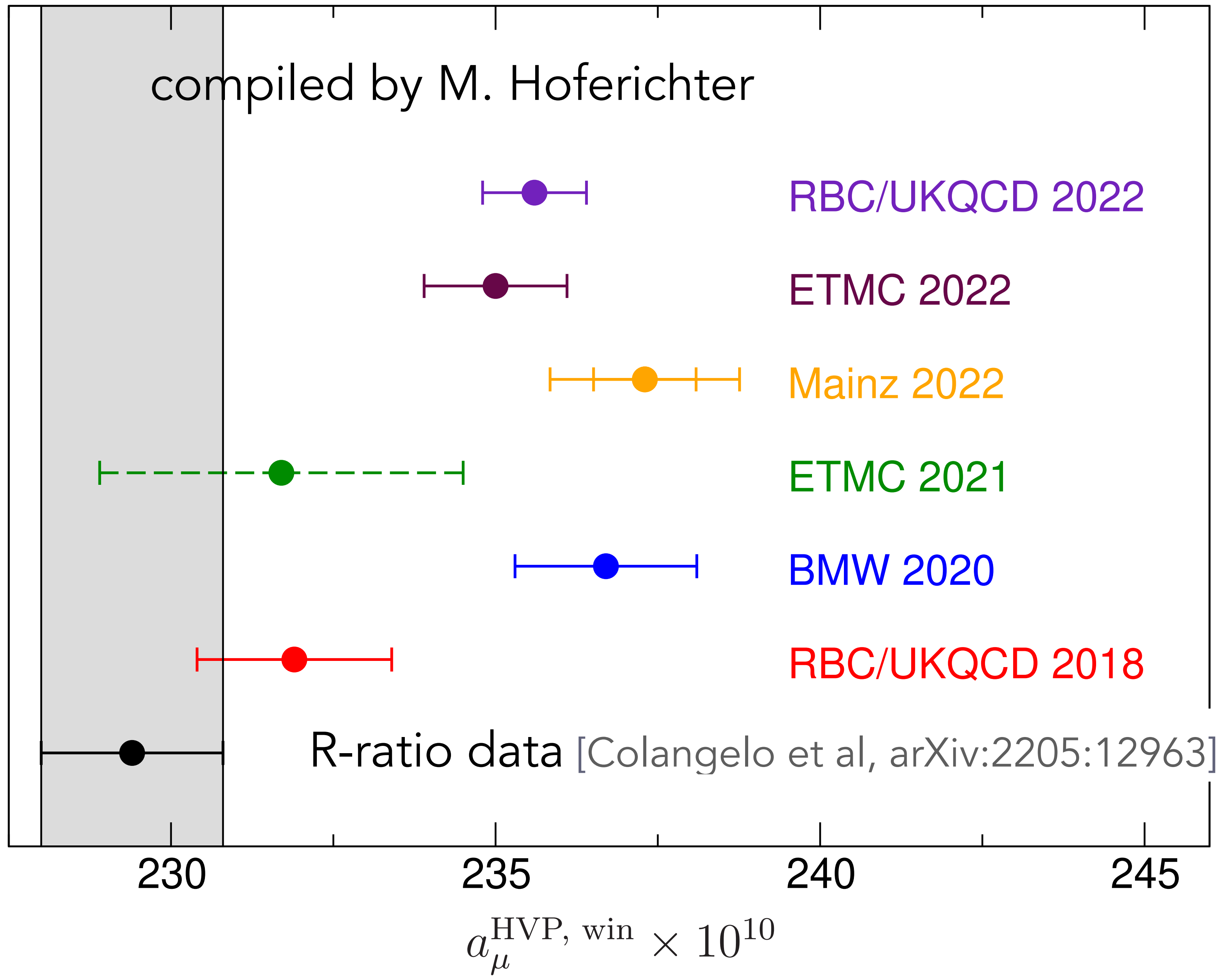
# Lattice HVP: results



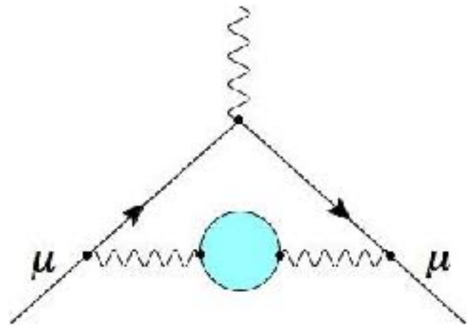

- new results in 2022/2023 for intermediate window, $a_\mu^W$ from six different lattice groups.
- blind analyses: Fermilab/HPQCD/MILC + RBC/UKQCD
- lattice-only comparison of light-quark connected contribution to intermediate window:



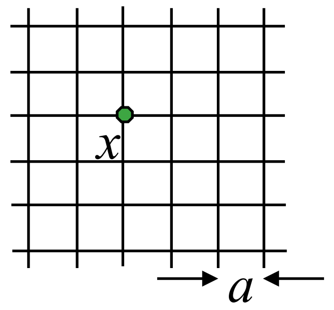

- LQCD results including all contributions




$\sim (3.5 - 4)\sigma$ tensions between LQCD and (pre-2023) data-driven evaluations




# Lattice HVP: results

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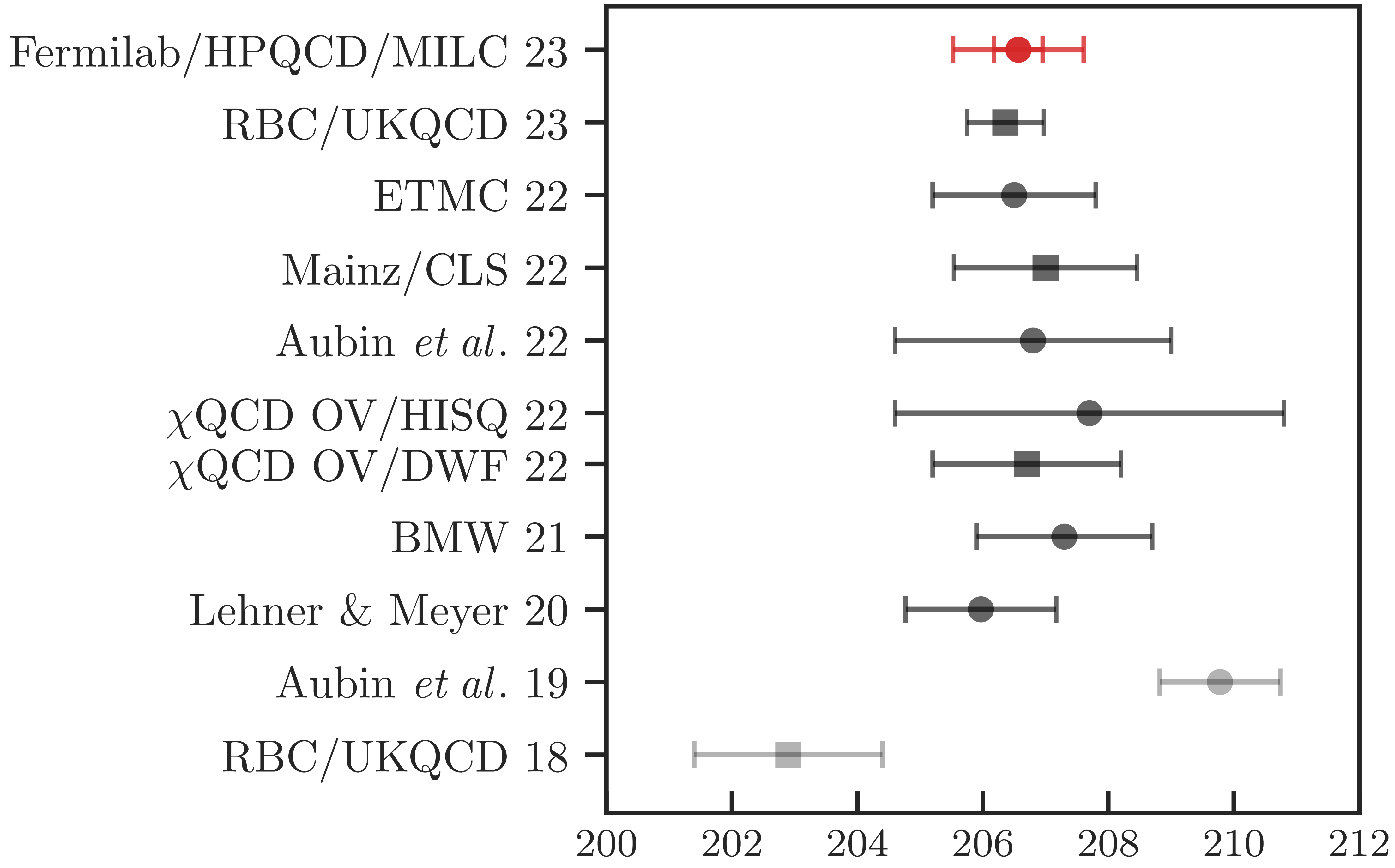
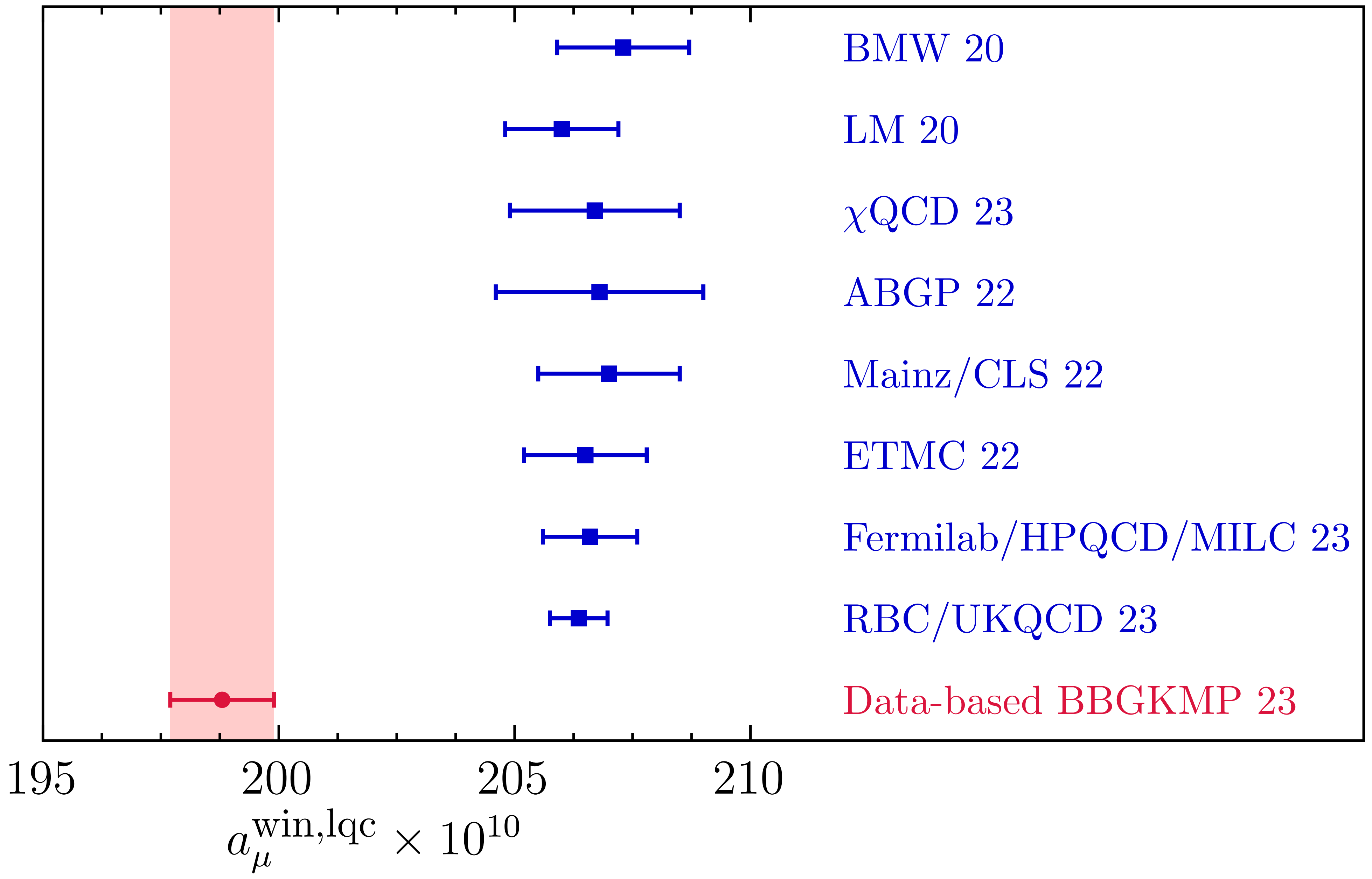
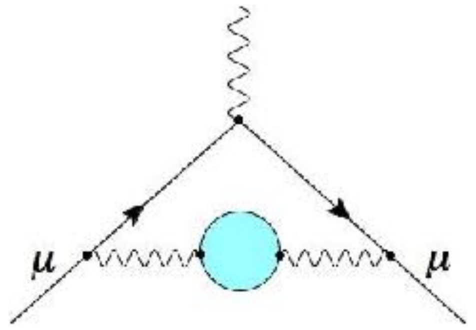



- dispersive evaluation of light-quark connected contribution

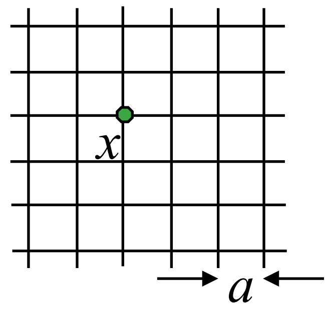
[G. Benton, et al, arXiv:2306.16808]




➠ primary source of tension




# Lattice HVP: outlook




## Ongoing work:

Evaluations of short-distance windows [ETMC, RBC/UKQCD]

Proposals for computing more windows:

- Use linear combinations of finer windows to locate the tension (if it persists) in $\sqrt{s}$ [Colangelo et al, arXiv:12963]
- Use larger windows, excluding the long-distance region $t \gtrsim 2\,\mathrm{fm}$ to maximize the significance of any tension [Davies at at, arXiv:2207.04765]

## For total HVP:

- independent lattice results at sub-percent precision: coming soon!
- Including $\pi\pi$ states for refined long-distance computation (Mainz, RBC/UKQCD, FNAL/MILC)
- include smaller lattice spacings to test continuum extrapolations (needs adequate computational resources)

g-2 sessions @ Lattice 2023 conference

# Summary

- consistent results from independent, precise LQCD calculations for light-quark connected contribution to **intermediate window** $a_\mu^{\mathrm{W}}$ (~ 1/3 of $a_\mu^{\mathrm{HVP,LO}}$) ➟ $3-4\,\sigma$ tension with data-driven results?
- still need independent LQCD results for long-distance contribution, total HVP: coming soon
  ➟ develop method average for lattice HVP results, **assess tensions (if any)** with data-driven average
- Programs and plans in place to improve by 2025:
  - data-driven HVP: **if differences are resolved/understood,** $\sim 0.3\,\%$
    new measurements from BaBar, KLOE, SND, Belle II,…. will shed light on current discrepancies (blind analyses are paramount!)
    improved treatment of structure dependent radiative corrections (NLO) in $\pi\pi$ and $\pi\pi\pi$ channels
  - lattice HVP: **if no tensions** between independent lattice results, $\sim 0.5\,\%$
  - dispersive HLbL and lattice HLbL: no puzzles, steady progress, $\sim 10\,\%$
- **IF tensions/differences between data-driven HVP and lattice HVP are resolved,** SM prediction will likely match precision goal of the Fermilab experiment.
- IF NOT, will need detailed comparisons, explore connections between HVP, $\sigma(e^+e^-)$, $\Delta\alpha$, global EW fits.

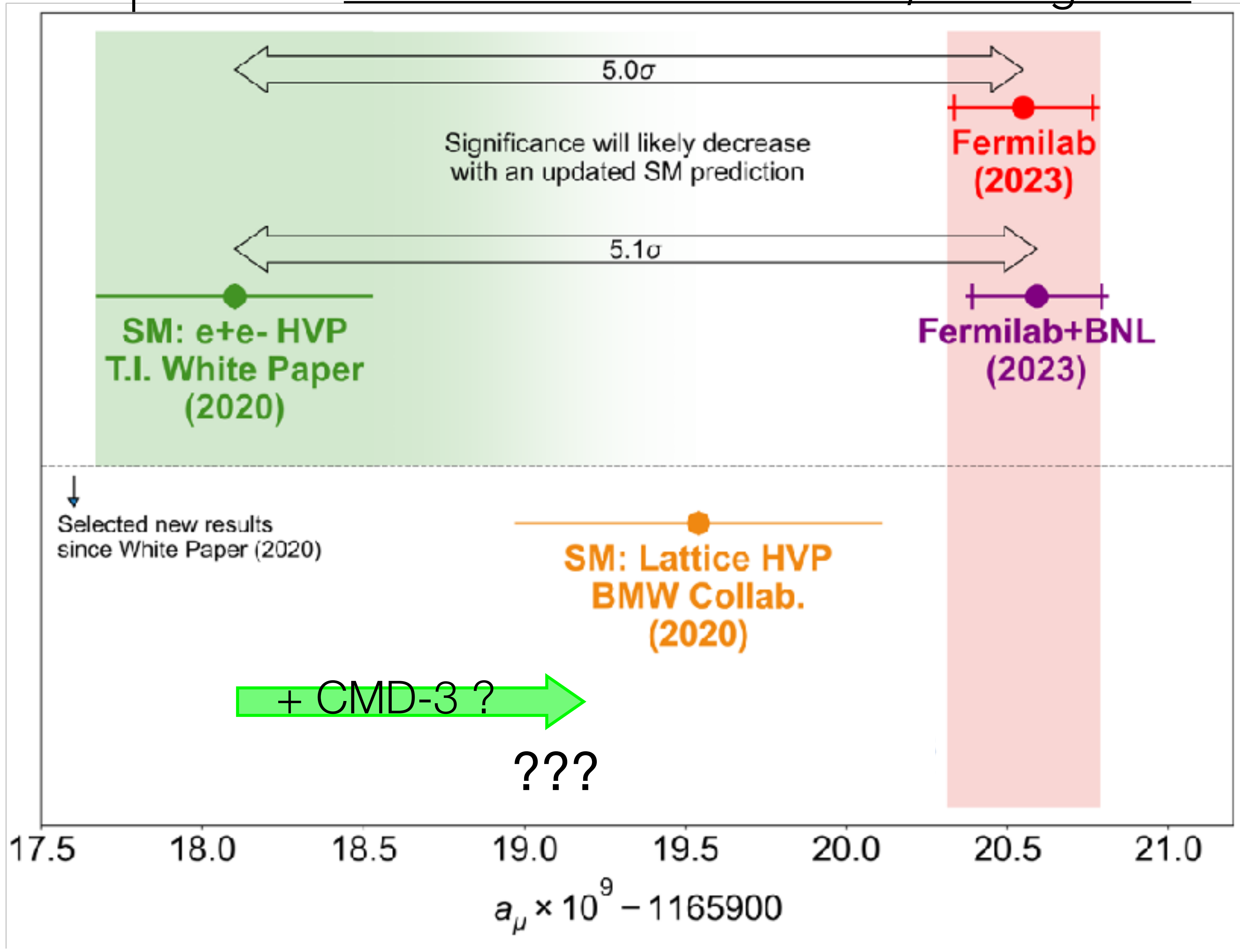
- BSM implications ➟ appendix

➟ **continued coordination by Theory Initiative: workshops, WPs, …**

# muon g-2: SM theory vs experiment

- The Fermilab experiment released the measurement result from their run 2&3 data on 10 Aug 2023. [D. Aguillard et al, 2308.06230]
- Run 6 completed summer 2023, final measurement result expected in 2025

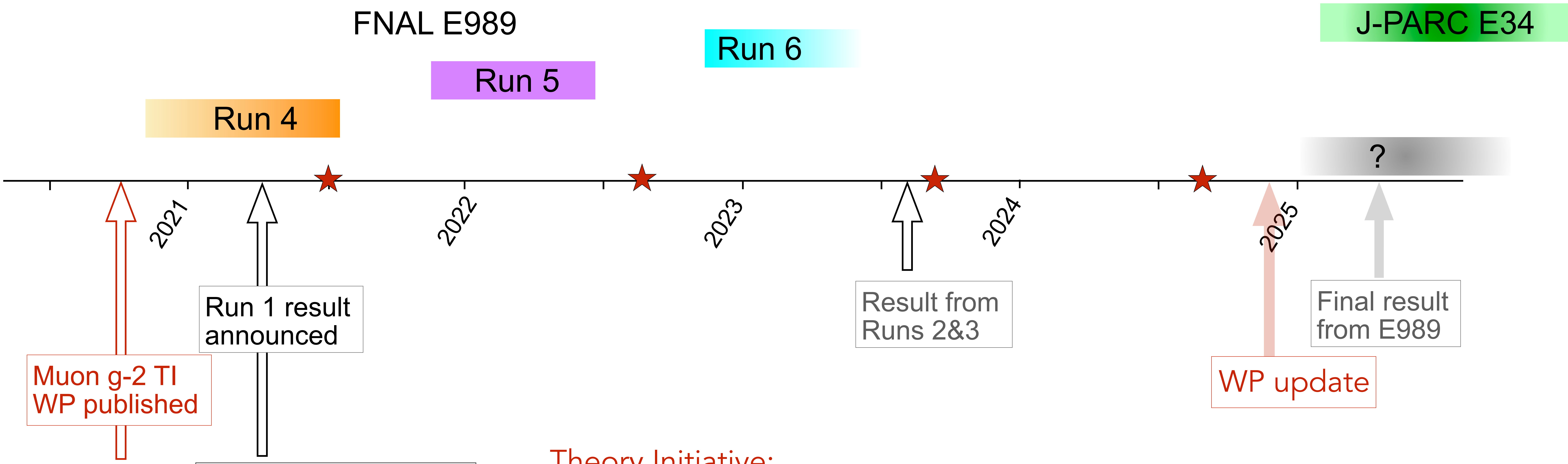
adapted from J. Mott @ Scientific Seminar, 10 Aug 2023



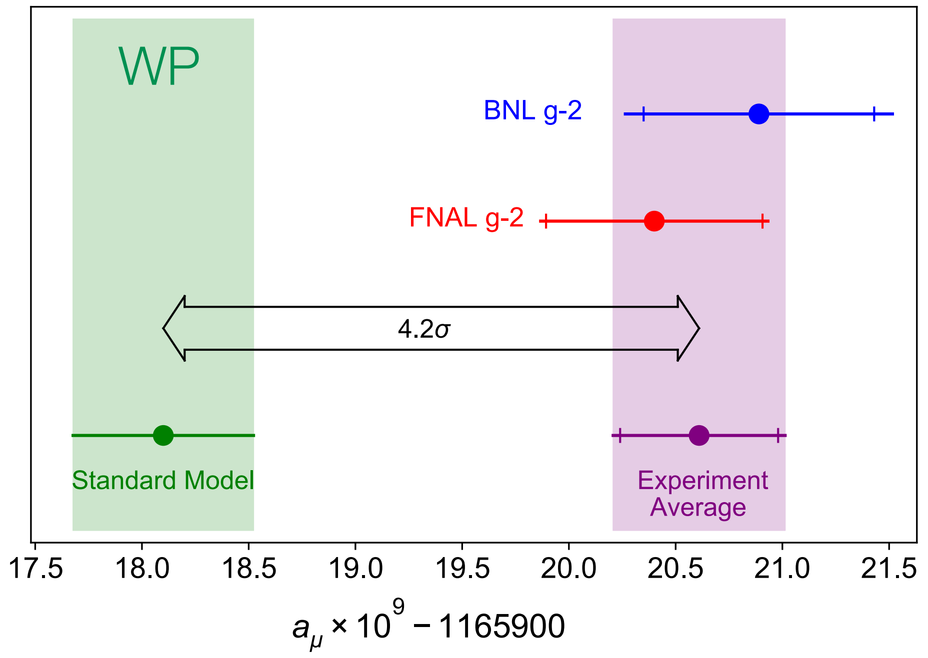

consolidated LQCD prediction (coming soon)

new measurements by BaBar, KLOE, SND,….

# Near-term Timeline








Theory Initiative:

- CMD-3 seminar (virtual): 27 March 2023 at 8:00am US CDT
- 2nd CMD-3 discussion meeting
- 8/9/2023: Status of Muon g-2 Theory in SM

★ TI workshops:
Jun 2021 @ KEK (virtual)
Sep 2022 @ Higgscentre
Sep 2023 @ Bern
Sep 2024 @ KEK or KMI

# Outlook

- Experimental program beyond 2025:
  - J-PARC: Muon g-2/EDM
  - Fermilab: future muon campus experiments?
  - Belle II, BESIII, Novosibirsk,…
  - Chiral Belle (?)
- Data-driven/dispersive program beyond 2025:
  - development of NNLO MC generators
  - for HLbL, improved experimental/lattice inputs together with further development of dispersive approach
- MUonE will provide a space-like measurement of HVP
- Lattice QCD beyond 2025:
  - access to future computational resources (coming Exascale) will enable improvements of all errors (statistical and systematic)
  - concurrent development of better methods and algorithms (gauge-field sampling, noise reduction) will accelerate progress
  - beyond g-2: a rich program relevant for all areas of HEP

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# Thank you!

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SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

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STRONG 2020

IP3

UCONN DEPARTMENT OF PHYSICS

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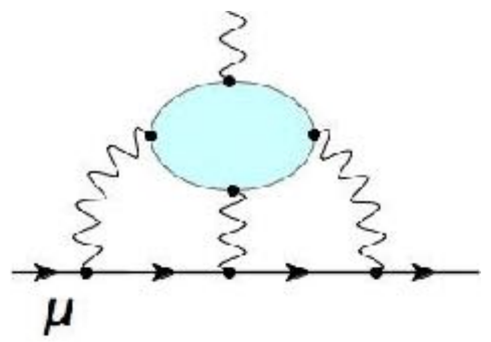
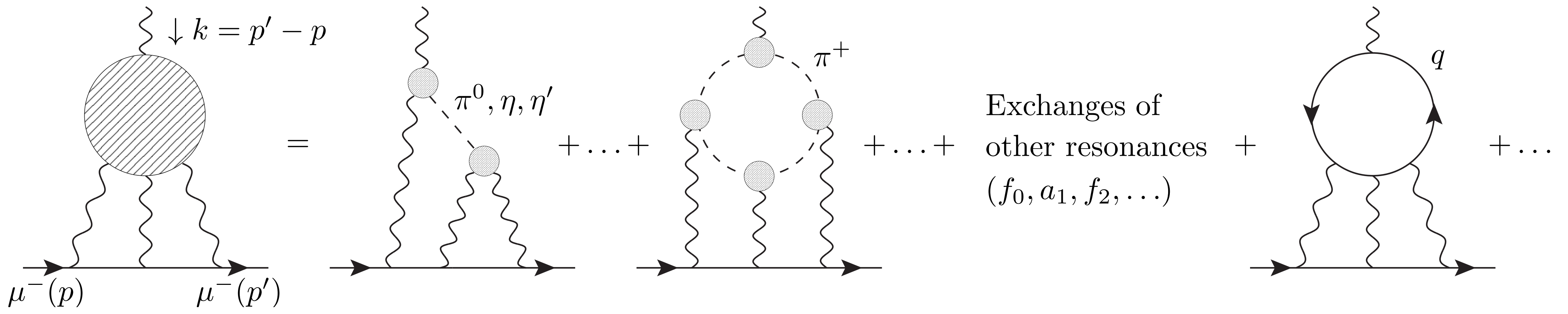
# Appendix




# Hadronic Light-by-light

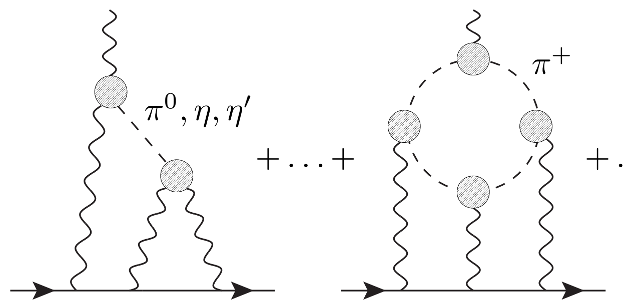



## Dispersive approach:

[Colangelo at al, 2014; Pauk & Vanderhaegen 2014; …]

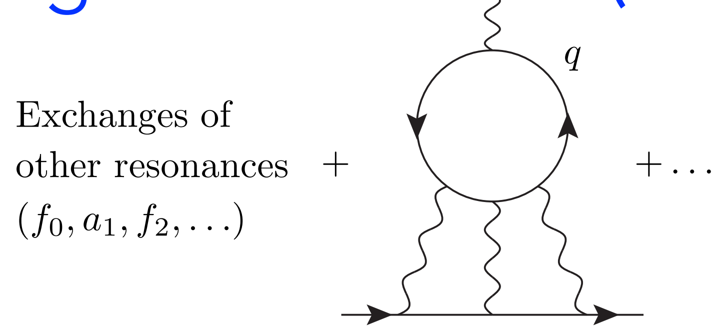
- ✦ model independent
- ✦ significantly more complicated than for HVP
- ✦ provides a framework for data-driven evaluations
- ✦ can also use lattice results as inputs

## Dominant contributions ( $\approx 75\,\%$ of total):



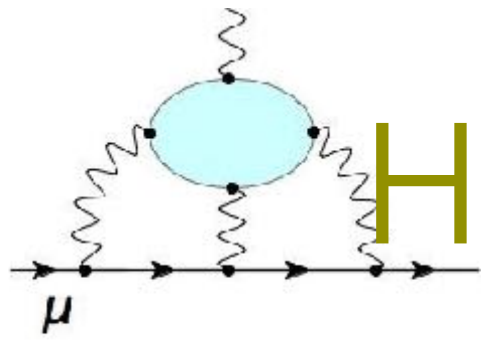

- ✦ Well quantified with $\approx 6\,\%$ uncertainty
- ✦ $\eta, \eta'$ pole contributions: Canterbury approximants only
- ✦ Ongoing work: consolidation of $\eta, \eta'$ pole contributions using disp. relations and LQCD

## Subleading contributions ( $\approx 25\,\%$ of total):




- ✦ Not yet well known
  ➠ dominant contribution to total uncertainty
- ✦ Ongoing work:
  - Implementation of short-distance constraints (now at 2-loop)
  - new dispersive formalism for higher spin intermediate states [Luedtke, Procura, Stoffer, 2023, in progress]
  - Mainz and BESIII ramping up $\gamma^{(*)}\gamma^*$ programs [A. Denig and C. Redmer @ Higgscentre workshop]

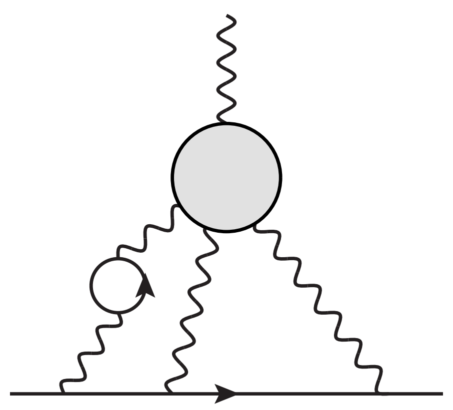
Dispersive, data-driven evaluation of HLbL with $\leq 10\,\%$ total uncertainty feasible by ~2025.




# HLbL: dispersive

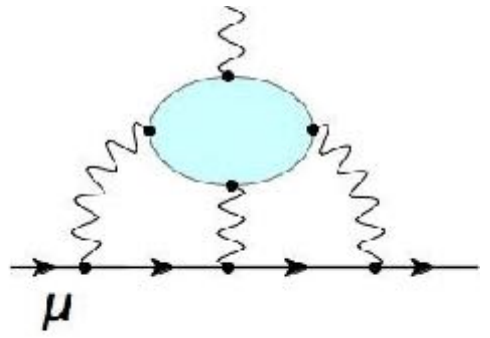
## Comparison:

| Contribution | PdRV(09) [471] | N/JN(09) [472, 573] | J(17) [27] | Our estimate |
|---|---|---|---|---|
| $\pi^0, \eta, \eta'$-poles | 114(13) | 99(16) | 95.45(12.40) | 93.8(4.0) |
| $\pi, K$-loops/boxes | −19(19) | −19(13) | −20(5) | −16.4(2) |
| $S$-wave $\pi\pi$ rescattering | −7(7) | −7(2) | −5.98(1.20) | −8(1) |
| subtotal | 88(24) | 73(21) | 69.5(13.4) | 69.4(4.1) |
| scalars | – | – | – | } − 1(3) |
| tensors | – | – | 1.1(1) | } |
| axial vectors | 15(10) | 22(5) | 7.55(2.71) | 6(6) |
| $u, d, s$-loops / short-distance | – | 21(3) | 20(4) | 15(10) |
| $c$-loop | 2.3 | – | 2.3(2) | 3(1) |
| total | 105(26) | 116(39) | 100.4(28.2) | 92(19) |

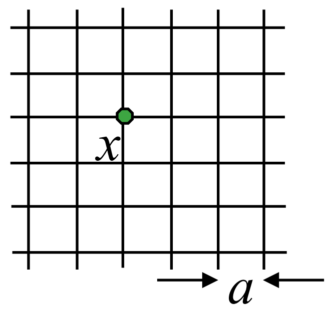


## NLO HLbL contribution:

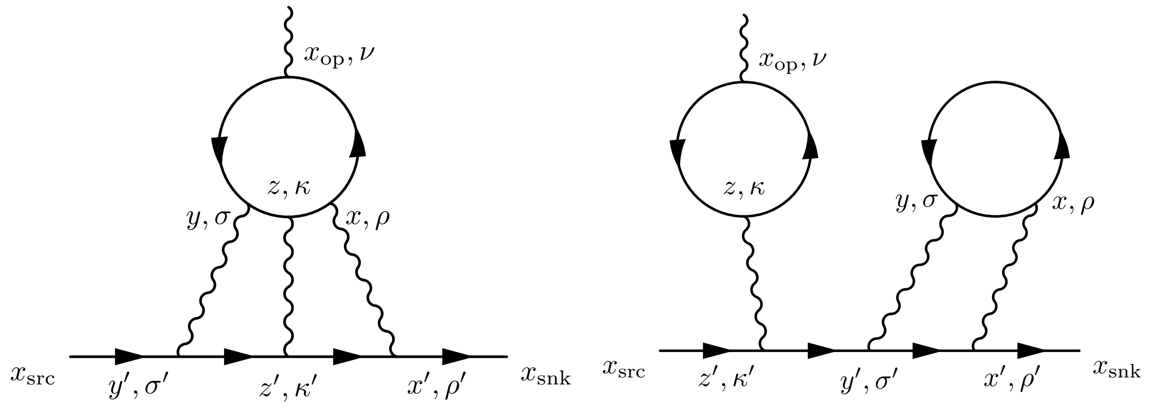
$$a_\mu^{\mathrm{HLbL,NLO}} = 2\,(1) \times 10^{-11}$$

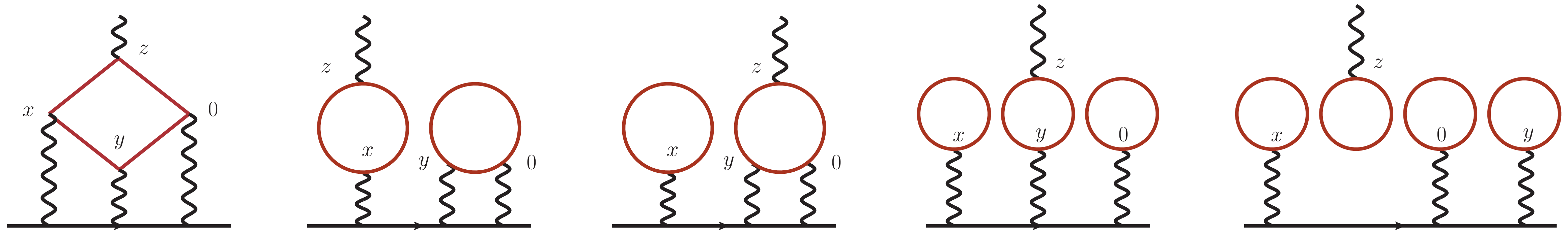



# Hadronic Light-by-light: lattice




Lattice QCD+QED: Two independent and complete direct calculations of $a_\mu^{\rm HLbL}$




- ✦ RBC/UKQCD
  [T. Blum et al, arXiv:1610.04603, 2016 PRL; arXiv:1911.08123, 2020 PRL]
- ✦ QCD + $\text{QED}_\text{L}$ (finite volume) stochastic

  DWF ensembles at/near phys mass,
  $a \approx 0.08 - 0.2\,\text{fm},\ L \sim 4.5 - 9.3\,\text{fm}$

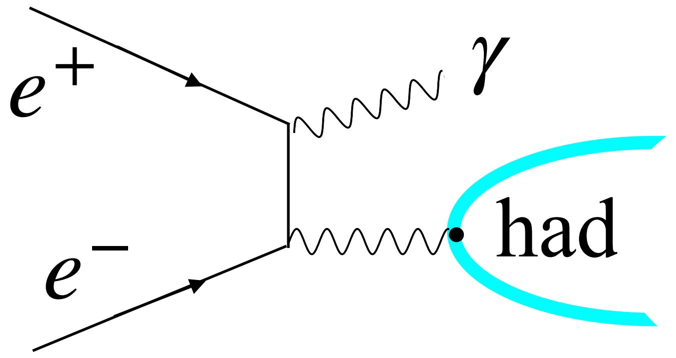
- ✦ Mainz group
  [E. Chao et al, arXiv:2104.02632]
- ✦ QCD + QED (infinite volume & continuum) analytic

  CLS (2+1 Wilson-clover) ensembles
  $m_\pi \sim 200 - 430\,\text{MeV},\ a \approx 0.05 - 0.1\,\text{fm},\ m_\pi L > 4$

- ✦ Cross checks between RBC/UKQCD & Mainz approaches in White Paper at unphysical pion mass
- ✦ Both groups are continuing to improve their calculations, adding more statistics, lattice spacings, physical mass ensemble (Mainz)
- ✦ new result from RBC/UKQCD [T. Blum et al, arXiv:2304.04423] using QCD + QED (inf.): $a_\mu^{\rm HLbL} = 124.7\,(11.5)\,(9.9) \times 10^{-11}$ consistent with previous calculations
- ✦ ongoing LQCD calculations of $\pi$, $\eta$, $\eta'$ transition form factors to determine pseudo scalar pole contributions [Mainz, ETMC, BMW]

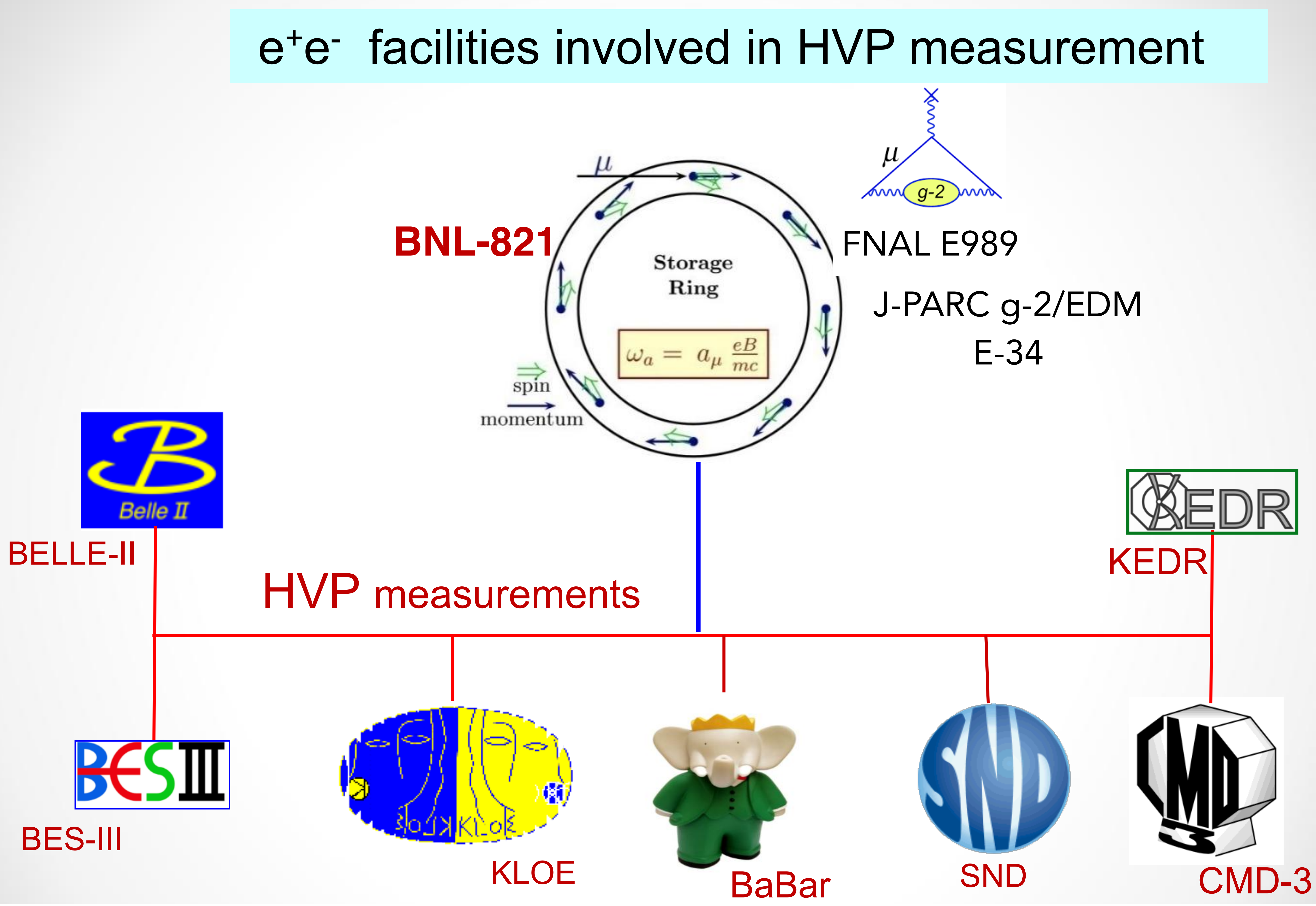
# Experimental Inputs to HVP

- two exp. approaches
  - ``Direct scan": change CM energy of $e^+e^-$ beams
  - ``Radiative Return":
    with fixed $e^+e^-$ CM energy, select events with initial state radiation (ISR)




- complemented by:
  - MC generators for $\sigma_{\text{had}}(s)$ (e.g. PHOKARA)
  - detailed studies of radiative corrections (now known through NLO)

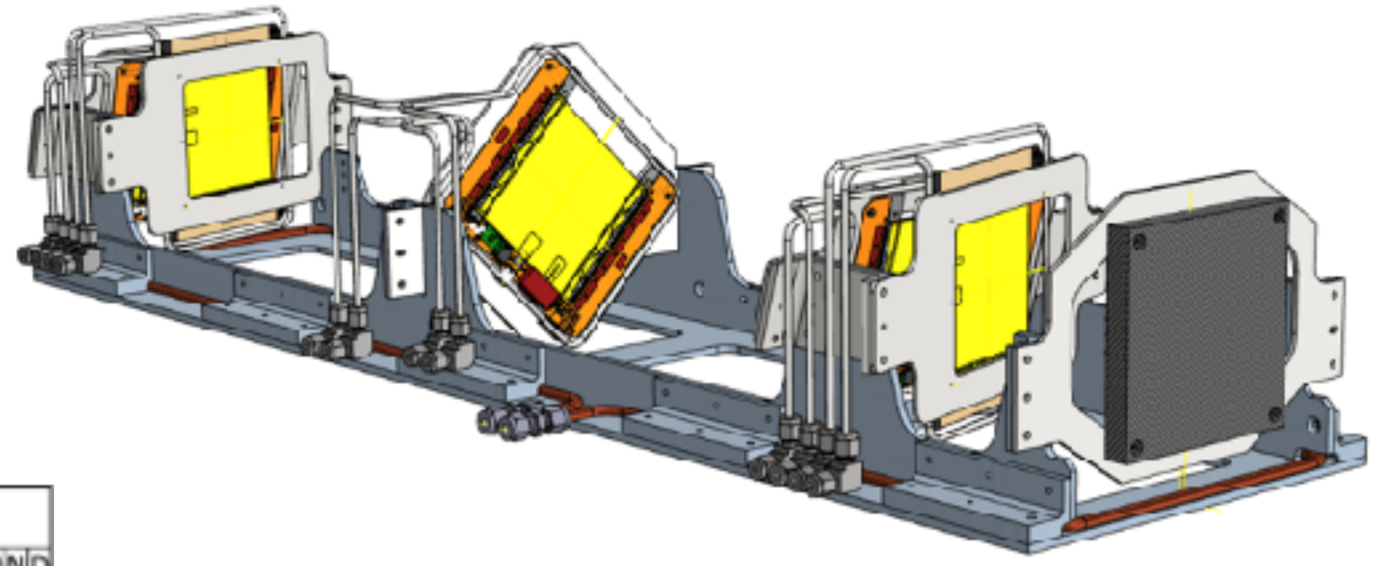
S. Serednyakov (for SND) @ HVP KEK workshop



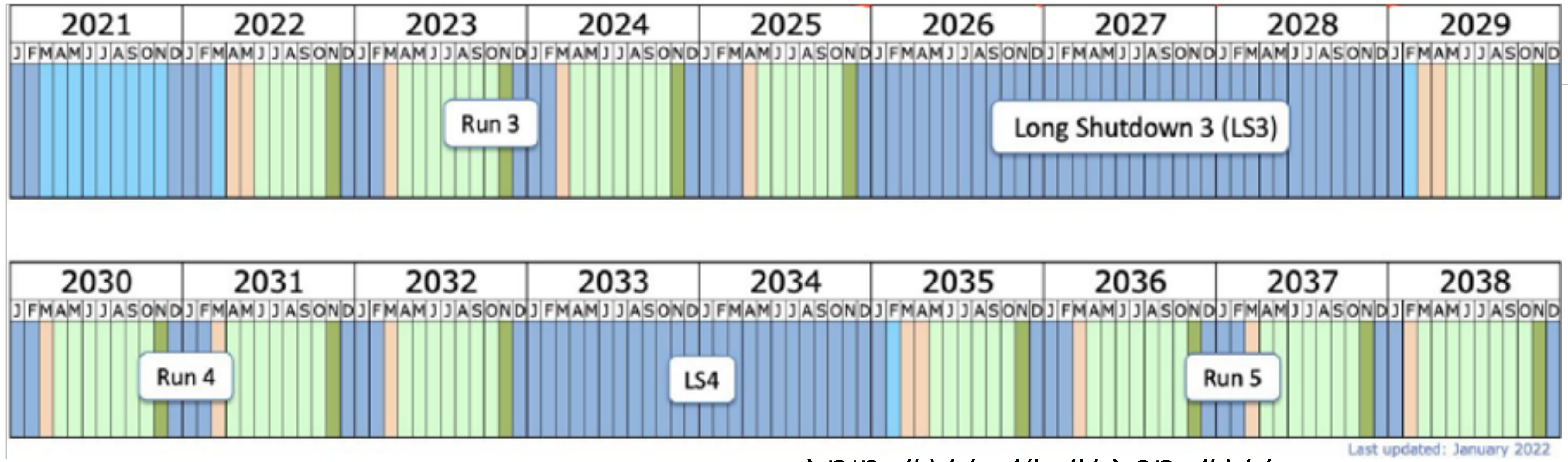
# Alternative measurement of HVP for $a_\mu^{\mathrm{HVP}}$ : MUonE at CERN


- Space-like determination of $a_\mu^{\mathrm{HVP}}$ at <0.5% through the scattering of 160 GeV muons on electron target
- Much progress in the last years, inc. detector optimization and development of $\mu - e$ (N)NLO MC
- Staged approach towards the full experiment: one station (2022), two stations (2023); possible 10 stations before LS3 (2026) (2% accuracy)
- Technical proposal towards full experiment in preparation
- Growing interest from both experimental and theory community

*-C. M. Carloni Calame et al PLB 746 (2015) 325*
*-G. Abbiendi et al Eur.Phys.J.C* 77 (2017) 3, 139
-LoI https://cds.cern.ch/record/2677471/files/SPSC-I-252.pdf



LHC schedule



# Efforts on Radiative Corrections for low energy

**Workstop+Conference in Zurich 5-9 June 2023** (LOC: A. Signer, G. Stagnitto, Y. Ulrich)

## Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in $e^+e^-$ collisions

5–9 Jun 2023
University of Zurich
Europe/Zurich timezone

Enter your search term

- Overview
- Timetable
- Contribution List
- Code of Conduct

Contact

yannick.ulrich@durham....

In this workstop, we will discuss radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in $e^+e^-$ collisions. This is to be seen as part of the Strong 2020 effort. We will cover

- leptonic processes at NNLO and beyond
- processes with hadrons
- parton shower
- experimental input

Each area will be given at least half a day, starting with an open 1h seminar followed by a lengthy discussion.

Just like previous workstops, we try to gather a small number of theorists who actively work on this topic to make very concrete progress. It should not just be about giving talks, but to actually learn from each other and put together the jigsaw pieces.

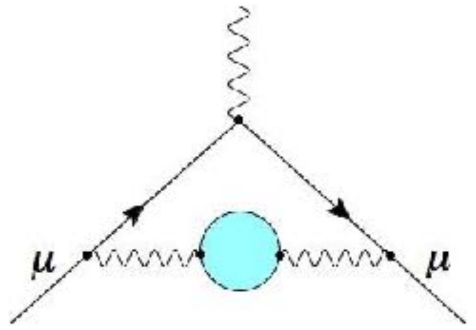
Additionally to the workstop that is only by-invite only, there is a broader conference directly following the workstop.

Three-day in-person (Workstop) + a three half day hybrid conference

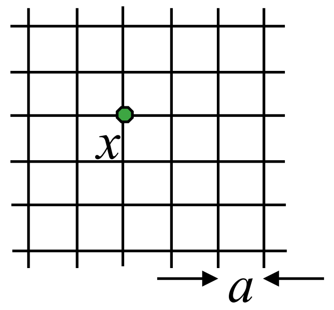
5 Working Groups:

- WP1: Leptonic processes at NNLO
- WP2: Form factor contributions at N3LO
- WP3: Processes with hadrons
- WP4: Parton showers
- WP5: Experimental input

**Final goal: full NNLO MC. Aim to write a report by Autumn 2023**




# Lattice HVP: outlook




## g-2 sessions @ Lattice 2023 conference

Long-distance window/total HVP:

- Fermilab/HPQCD/MILC: Preliminary & blinded (!) results for light-quark connected, total HVP
- RBC/UKQCD: progress on ongoing analysis of long-distance window, $\pi\pi$ analysis
- BMW: progress on generating lattice data @ 0.048 fm
  first look at $\pi\pi$ data
  comparisons between BMW and DHMZ observables with correlations

- M. Golterman: data-driven determination of light-quark connected windowed HVP.

Isospin-breaking corrections:

- Mainz: progress implementing new coordinate-space method

Plenary g-2 talks @ Lattice 2023 on Friday (special JTEP seminar)

# Connections

$$\sigma(e^+e^- \to \text{hadrons}) \quad \Leftrightarrow \quad a_\mu^{\text{HVP}} \quad \Leftrightarrow \quad \Delta\alpha_{\text{had}}(M_Z^2)$$

- $\Delta\alpha_{\text{had}}(M_Z^2)$ also depends on the hadronic vacuum polarization function, and can be written as an integral over $\sigma(e^+e^- \to \text{hadrons})$, but weighted towards higher energies.
- a shift in $a_\mu^{\text{HVP}}$ also changes $\Delta\alpha_{\text{had}}(M_Z^2)$: ➠ EW fits

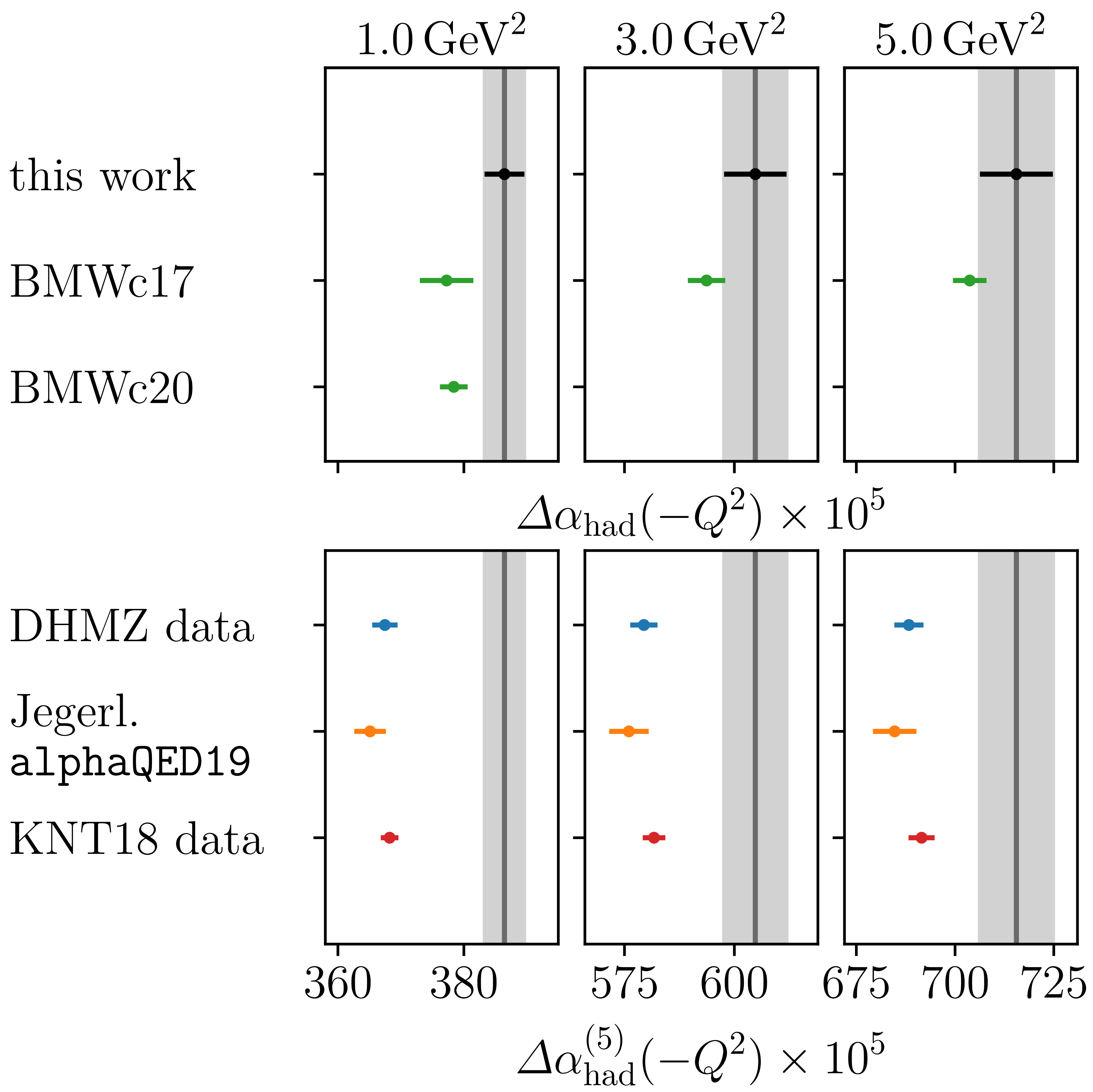
  [Passera, et al, 2008, Crivellin et al 2020, Keshavarsi et al 2020, Malaescu & Scott 2020]

  If the shift in $a_\mu^{\text{HVP}}$ is in the low-energy region ( $\lesssim 1\,\text{GeV}$), the impact on $\Delta\alpha_{\text{had}}(M_Z^2)$ and EW fits is small.

# Connections

$$\sigma(e^+e^- \to \text{hadrons}) \quad \Leftrightarrow \quad a_\mu^{\text{HVP}} \quad \Leftrightarrow \quad \Delta\alpha_{\text{had}}(M_Z^2)$$

H. Wittig @ Higgscentre workshop




Dispersion integral: $\Delta\alpha^{(5)}_{\text{had}}(q^2) = -\frac{\alpha\, q^2}{3\pi} \oint_{m^2_{\pi^0}}^{\infty} ds\, \frac{R(s)}{s(s-q^2)}$

Lattice QCD:

$$\Delta\alpha_{\text{had}}(-Q^2) = \frac{\alpha}{\pi}\frac{1}{Q^2}\int_0^\infty dt\, G(t)\left[Q^2t^2 - 4\sin^2\left(\tfrac{1}{2}Q^2t^2\right)\right]$$

- Direct lattice calculation of $\Delta\alpha(-Q^2)$ on the same gauge ensembles used in Mainz/CLS 22
  *[Cè et al., JHEP 08 (2022) 220, arXiv:2203.08676]*
- Tension of $\sim 3\sigma$ observed with data-driven evaluation of $\Delta\alpha_{\text{had}}(-Q^2)$ for $Q^2 \gtrsim 3\,\text{GeV}^2$
  → consistent with tension for window observable

# Connections

$$\sigma(e^+e^- \to \text{hadrons}) \quad \Leftrightarrow \quad a_\mu^{\text{HVP}} \quad \Leftrightarrow \quad \Delta\alpha_{\text{had}}(M_Z^2)$$

H. Wittig @ Higgscentre workshop

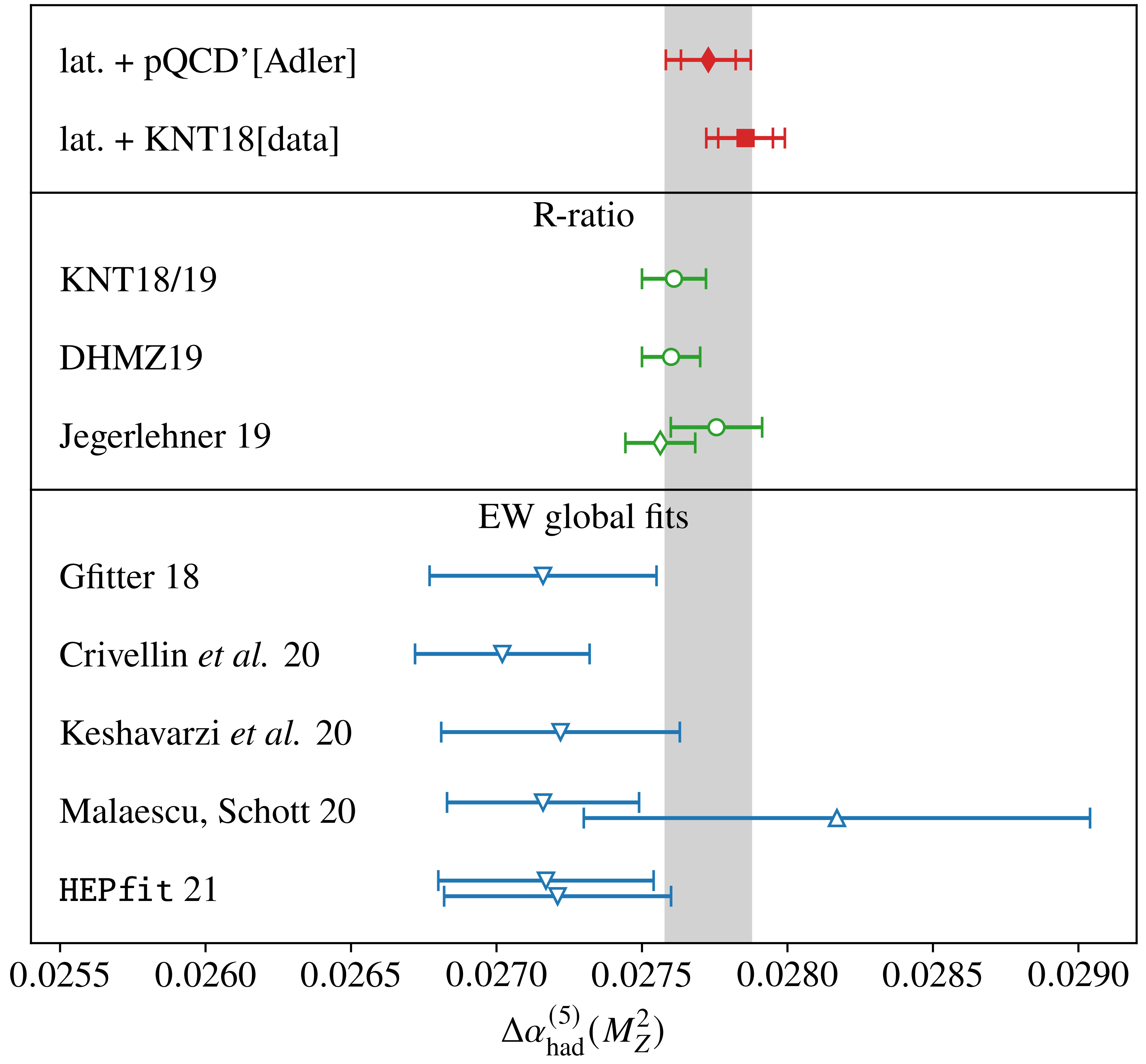
Adler function approach, aka. "Euclidean split technique"

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \Delta\alpha^{(5)}_{\text{had}}(-Q_0^2)$$
$$+[\Delta\alpha^{(5)}_{\text{had}}(-M_Z^2) - \Delta\alpha^{(5)}_{\text{had}}(-Q_0^2)]$$
$$+[\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) - \Delta\alpha^{(5)}_{\text{had}}(-M_Z^2)]$$

$$\Rightarrow \quad \Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = 0.027\,73(9)_{\text{lat}}(2)_{\text{btm}}(12)_{\text{pQCD}}$$

*[Cè et al., JHEP 08 (2022) 220, arXiv:2203.08676]*

- Agreement between lattice QCD and evaluations based on the $R$-ratio



# Connections

$$\sigma(e^+e^- \to \text{hadrons}) \quad \Leftrightarrow \quad a_\mu^{\text{HVP}} \quad \Leftrightarrow \quad \Delta\alpha_{\text{had}}(M_Z^2)$$

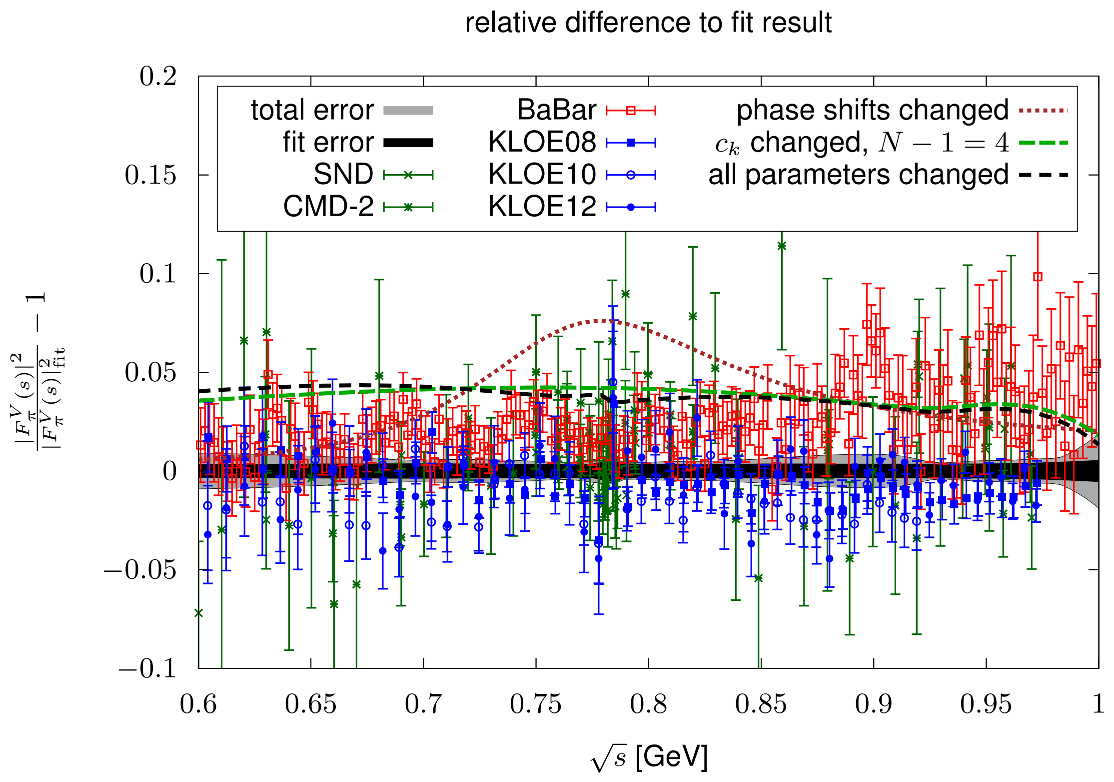
- $\Delta\alpha_{\text{had}}(M_Z^2)$ also depends on the hadronic vacuum polarization function, and can be written as an integral over $\sigma(e^+e^- \to \text{hadrons})$, but weighted towards higher energies.
- a shift in $a_\mu^{\text{HVP}}$ also changes $\Delta\alpha_{\text{had}}(M_Z^2)$: ➠ EW fits
  [Passera, et al, 2008, Crivellin et al 2020, Keshavarsi et al 2020, Malaescu & Scott 2020]
  If the shift in $a_\mu^{\text{HVP}}$ is in the low-energy region ( $\lesssim 1\,\text{GeV}$), the impact on $\Delta\alpha_{\text{had}}(M_Z^2)$ and EW fits is small.
- A shift in $a_\mu^{\text{HVP}}$ from low ( $\lesssim 2\,\text{GeV}$) energies
  ➠ $\sigma(e^+e^- \to \pi\pi)$
  must satisfy unitarity & analyticity constraints ➠ $F_\pi^V(s)$
  can be tested with lattice calculations
  [Colangelo, Hoferichter, Stoffer, arXiv:2010.07943]

Peter Stoffer @ Lattice HVP workshop

Constraints on the two-pion contribution to HVP — arXiv:2010.07943 [hep-ph]

Modifying $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$

- "low-energy" scenario: local changes in cross section of $\sim 8\%$ **around $\rho$**
- "high-energy" scenario: impact on **pion charge radius** and space-like VFF $\Rightarrow$ chance for **independent lattice-QCD checks**
- requires **factor $\sim 3$ improvement** over $\chi$QCD result:
  $\langle r_\pi^2 \rangle = 0.433(9)(13)\,\text{fm}^2$
  $\to$ arXiv:2006.05431 [hep-ph]



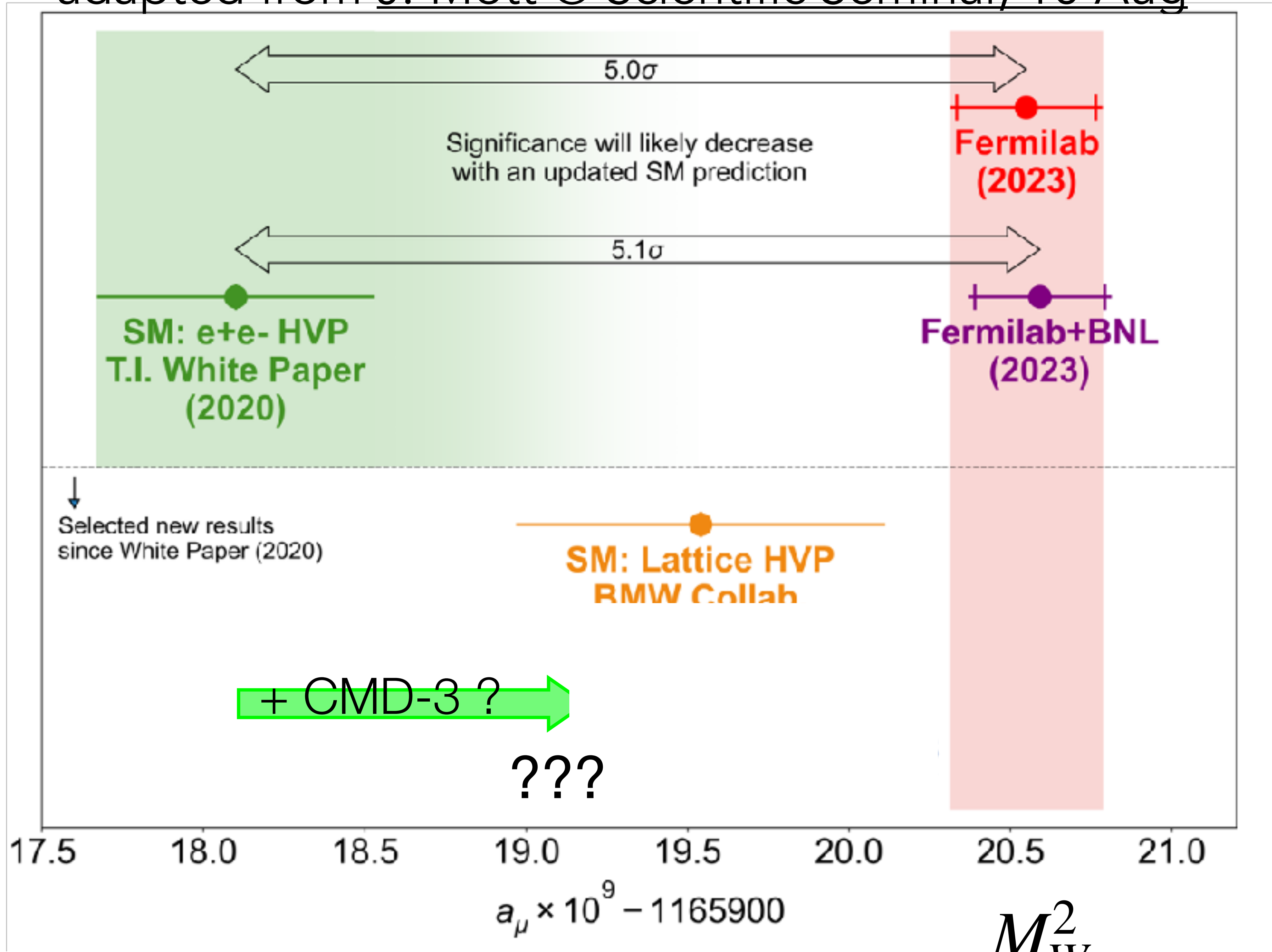
# Beyond the SM possibilities

$a_\mu$ is loop-induced, conserves CP & flavor, flips chirality.

The difference between Exp-WP2020 is large:

$$\Delta a_\mu = 249\,(48) \times 10^{-11} \;>\; a_\mu(\text{EW})$$

adapted from J. Mott @ Scientific Seminar, 10 Aug



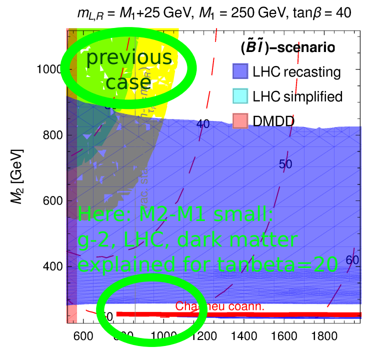

Generically expect: $a_\mu^{\text{NP}} \sim a_\mu^{\text{EW}} \times \dfrac{M_W^2}{\Lambda^2} \times \text{couplings}$

Can be accommodated by many BSM theories (800+ papers)

D. Stöckinger @ g-2 Days (http://pheno.csic.es/g-2Days21/)

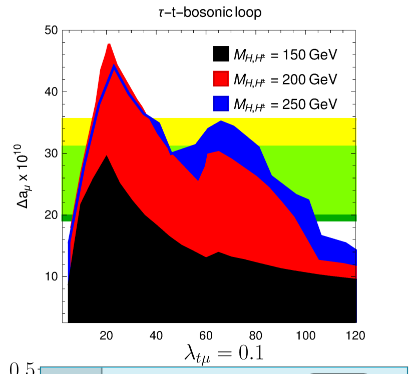
SUSY: MSSM, MRSSM

- MSugra. . . many other generic scenarios
- Bino-dark matter+some coannihil.+mass splittings
- Wino-LSP+specific mass patterns



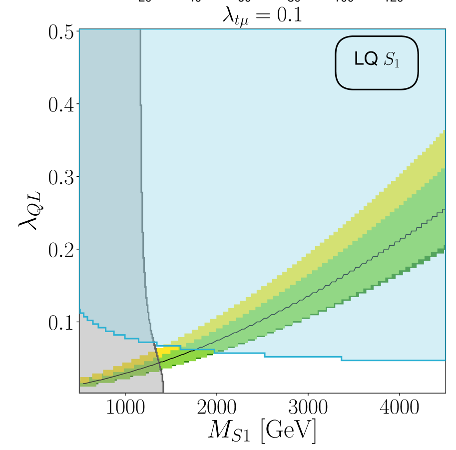

Two-Higgs doublet model

- Type I, II, Y, Type X(lepton-specific), flavour-aligned



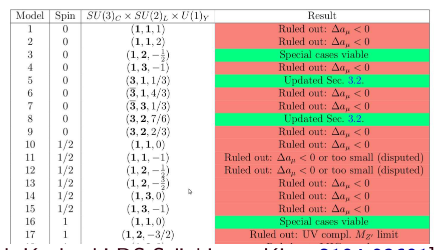
Lepto-quarks, vector-like leptons

- scenarios with muon-specific couplings to $\mu_L$ and $\mu_R$




Simple models (one or two new fields)

- Mostly excluded
- light N.P. (ALPs, Dark Photon, Light $L_\mu - L_\tau$)



[Athron,Balazs,Jacob,Kotlarski,DS,Stöckinger-Kim, 2104.03691]

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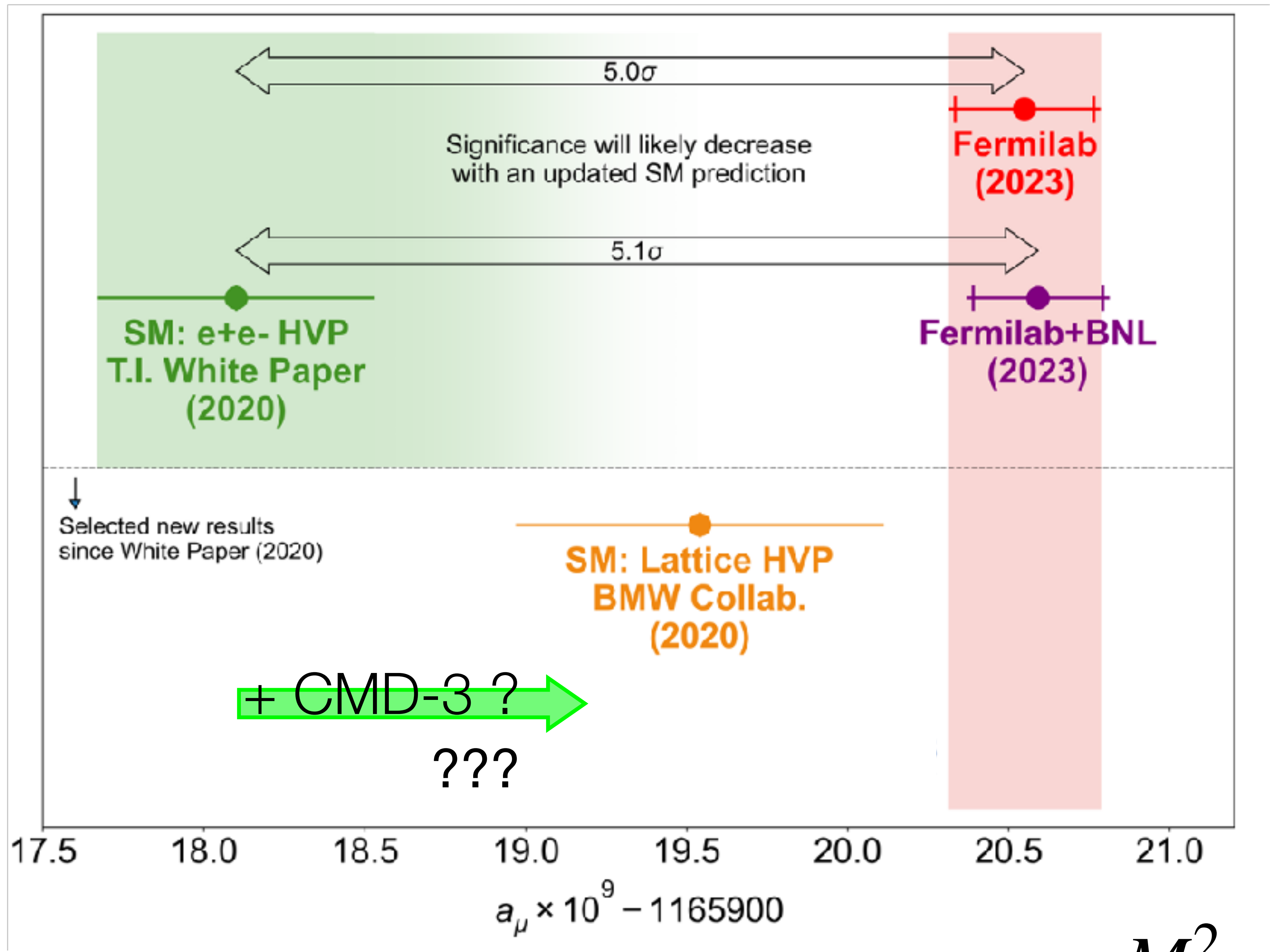
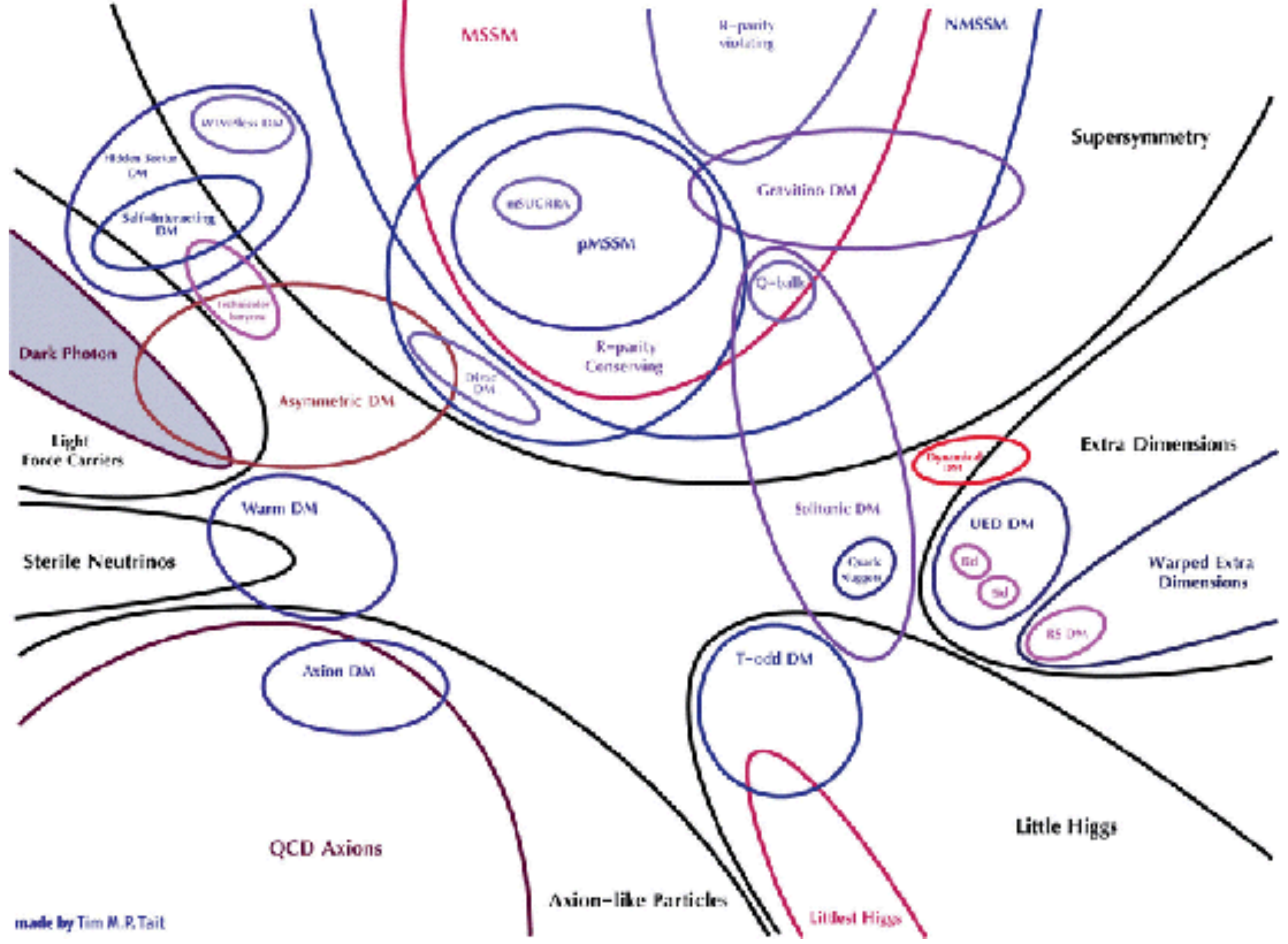


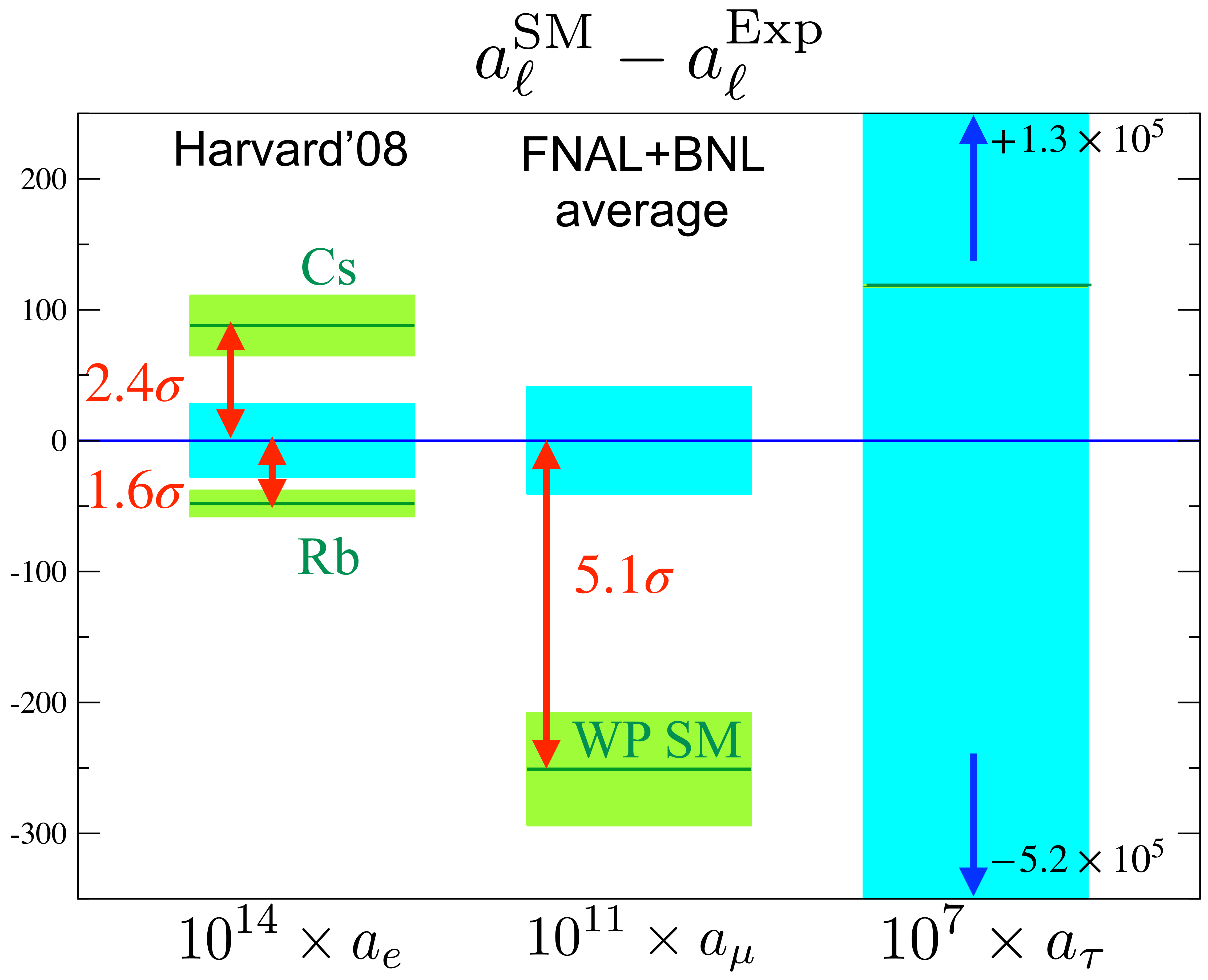

Generically expect: $a_\mu^{\mathrm{NP}} \sim a_\mu^{\mathrm{EW}} \times \frac{M_W^2}{\Lambda^2} \times$ c

Can be accommodated by many BSM theories (1k+ papers)




- Can new physics hide in the low-energy $\sigma(e^+e^- \to \pi\pi)$ cross section?
  ➠ No [Luzio, et al, arXiv:2112.08312]
- New boson at ~ 1GeV decays into $\mu^+\mu^-, e^+e^-$, affects $\sigma(e^+e^- \to \pi\pi)$ indirectly [L. Darmé et al, arXiv:2112.09139]
- Neutral, long-lived hadrons, heretofore undetected? [Farrar, arXiv:2206.13460]
- Z′ at < 1 GeV, coupling to 1st gen matter particles [Coyle, Wagner, arXiv:2305.02354]

# Lepton moments summary



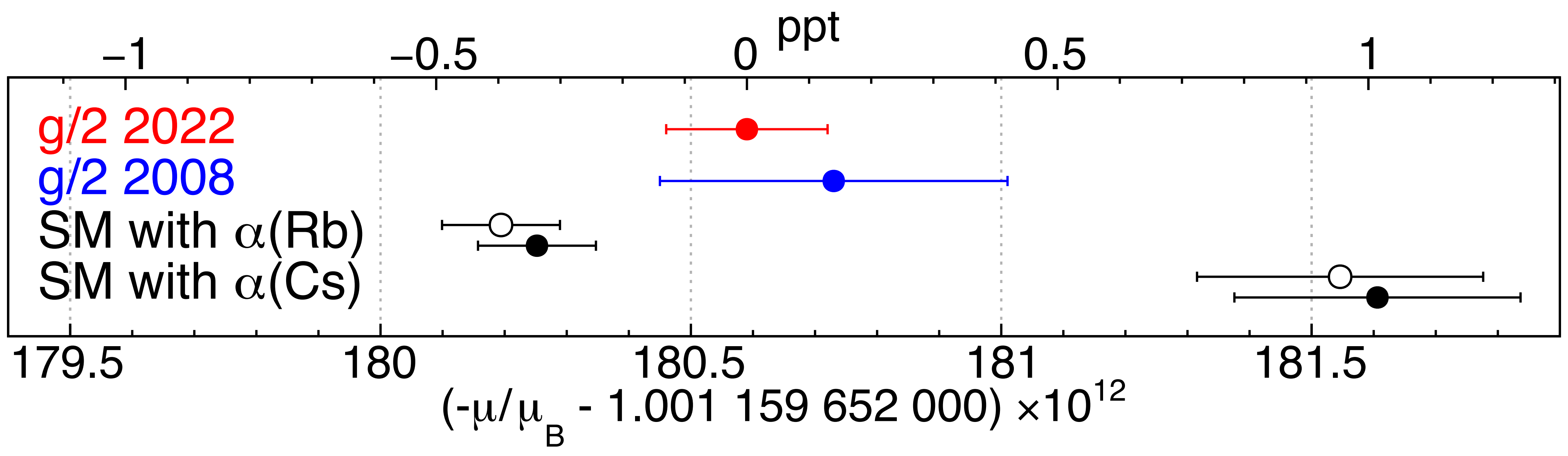

Cs: $\alpha$ from Berkeley group [Parker et al, Science 360, 6385 (2018)]

Rb: $\alpha$ from Paris group [Morel et al, Nature 588, 61–65(2020)]

Sensitivity to heavy new physics: $a_\ell^{\rm NP} \sim \frac{m_\ell^2}{\Lambda^2}$

$(m_\mu/m_e)^2 \sim 4 \times 10^4$

[X. Fan et al (Gabrielse group), arXiv:2209.13084]




Prospects for tau moment measurement:

Chiral Belle arXiv:2205.12847

- use polarized $e^-$ beam
- with $40ab^{-1}$ measurement of $a_\tau$ at $10^{-5}$ feasible
- with more statistics measurement at $10^{-6}$ possible