What are GPDs & how to access them in Lattice QCD?

Shohini Bhattacharya
RIKEN BNL
29th September 2023

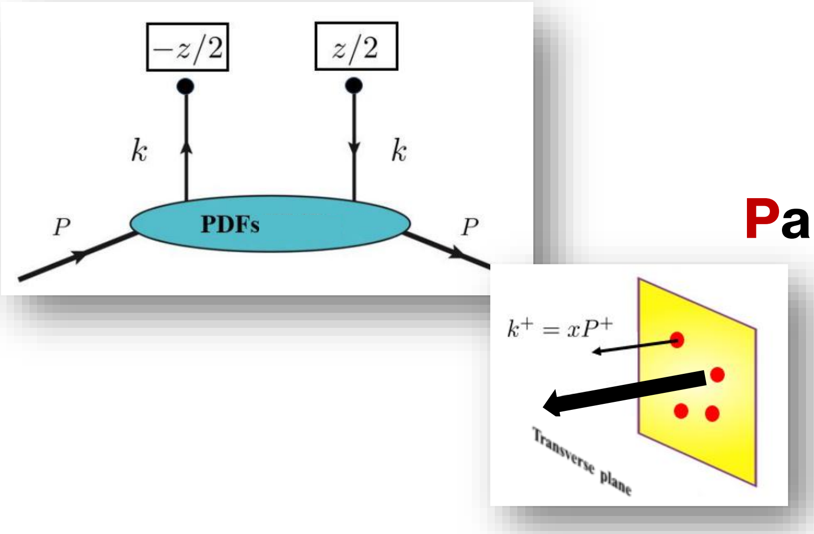
Duke University

Non-perturbative functions in QCD

← **Parton motion**

⬅ **Nucleon motion**

Snapshots of the nucleons



Parton **D**istribution **F**unctions

PDFs (x)

Non-perturbative functions in QCD

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⬅ **Nucleon motion**

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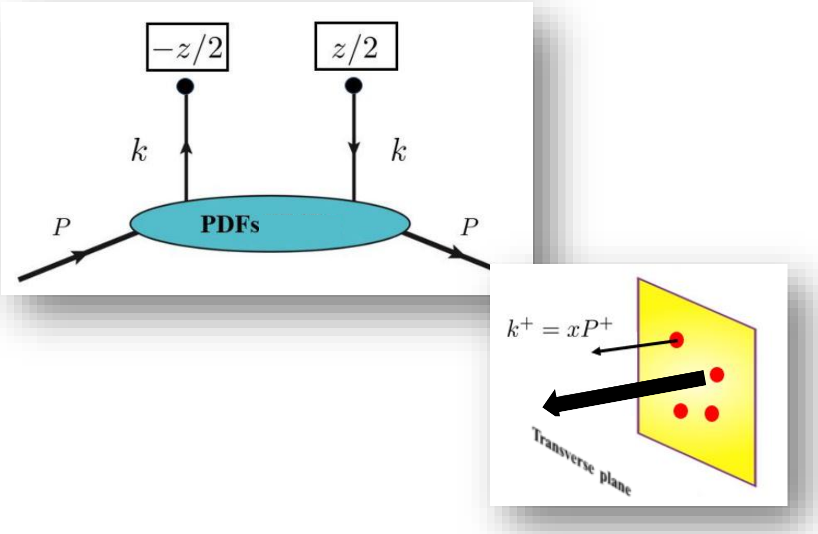
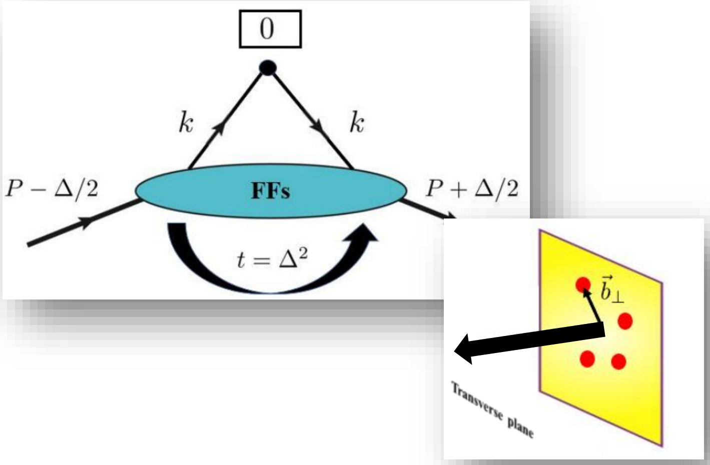


Form Factors

PDFs (x)

FFs (Δ)



Non-perturbative functions in QCD

Parton motion

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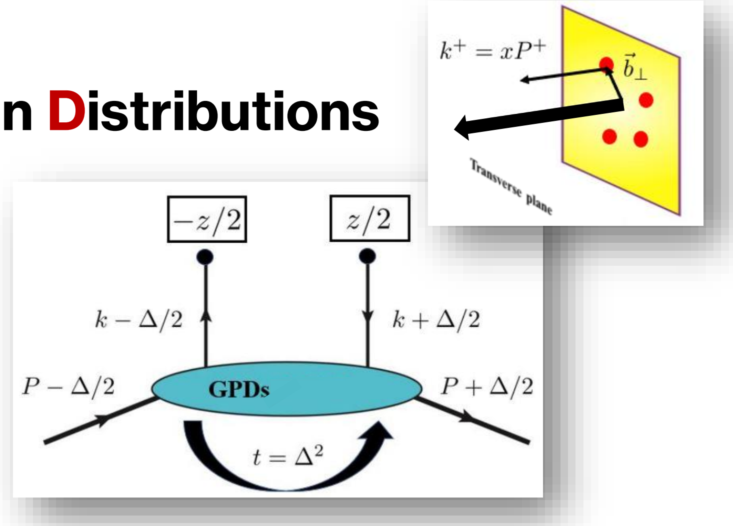
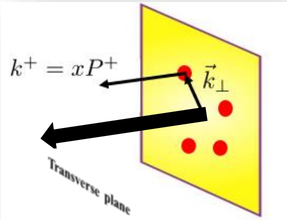
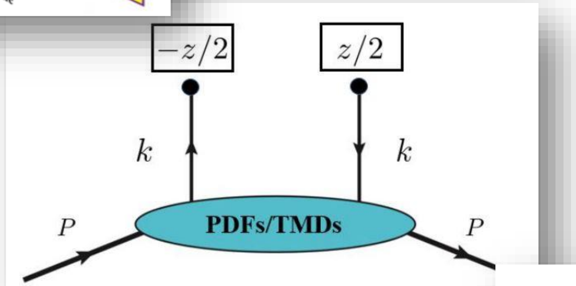
Snapshots of the nucleons

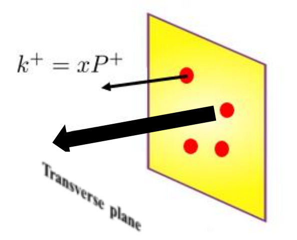
Generalized Parton Distributions

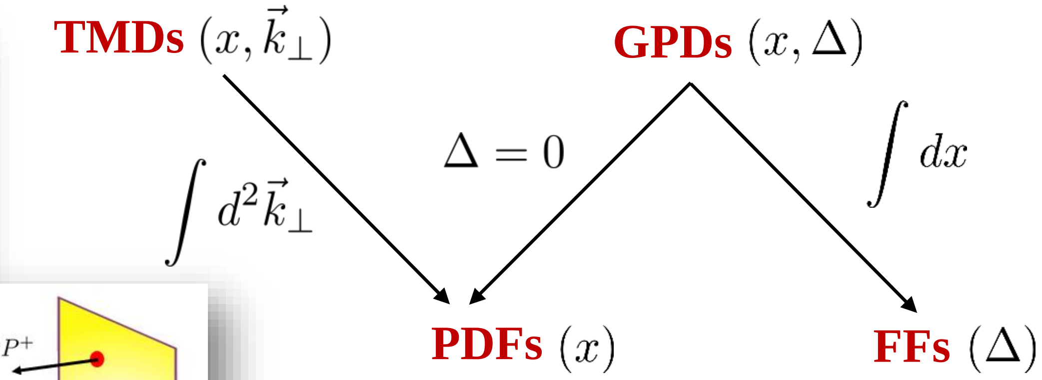


Transverse Momentum-dependent Distributions









TMDs $(x, \vec{k}_\perp)$

GPDs (x, Δ)

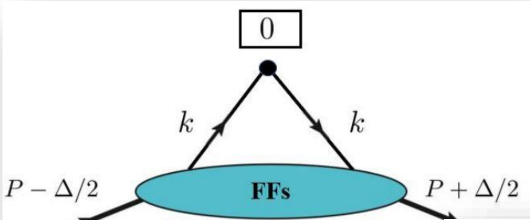
$\int d^2\vec{k}_\perp$

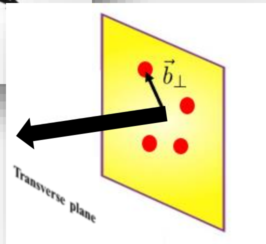
$\Delta = 0$

$\int dx$

PDFs (x)

FFs (Δ)





Non-perturbative functions in QCD

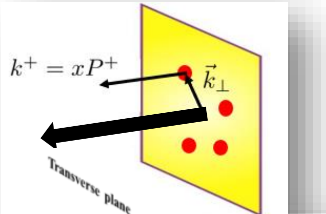
Parton motion

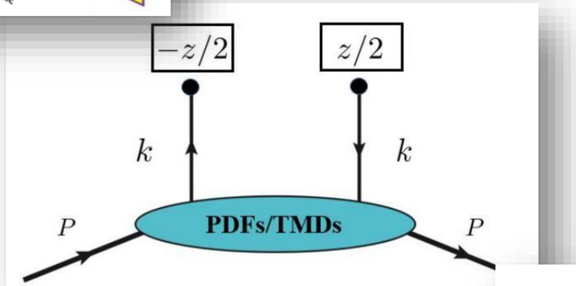
Nucleon motion

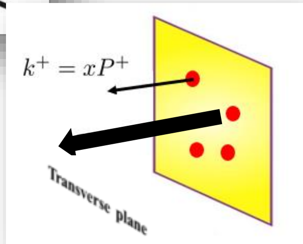
Snapshots of the nucleons

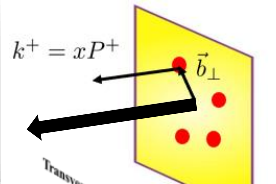
Generalized Transverse Momentum-dependent Distributions

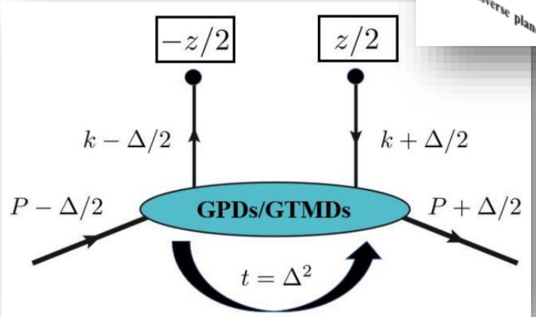
$$\begin{tikzcd}
 & \textbf{GTMDs}\ (x,\vec{k}_\perp,\Delta) \arrow[dl, "\Delta=0"'] \arrow[dr, "\int d^2\vec{k}_\perp"] & & \\
\textbf{TMDs}\ (x,\vec{k}_\perp) \arrow[dr, "\int d^2\vec{k}_\perp"'] & & \textbf{GPDs}\ (x,\Delta) \arrow[dl, "\Delta=0"'] \arrow[dr, "\int dx"] & \\
 & \textbf{PDFs}\ (x) & & \textbf{FFs}\ (\Delta)
\end{tikzcd}$$

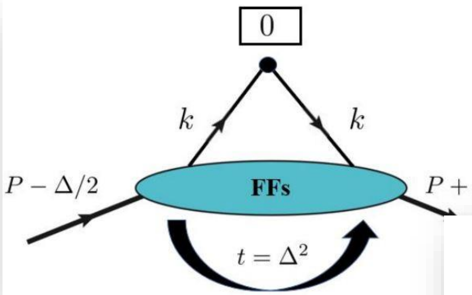


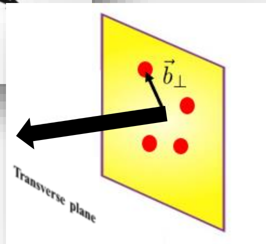






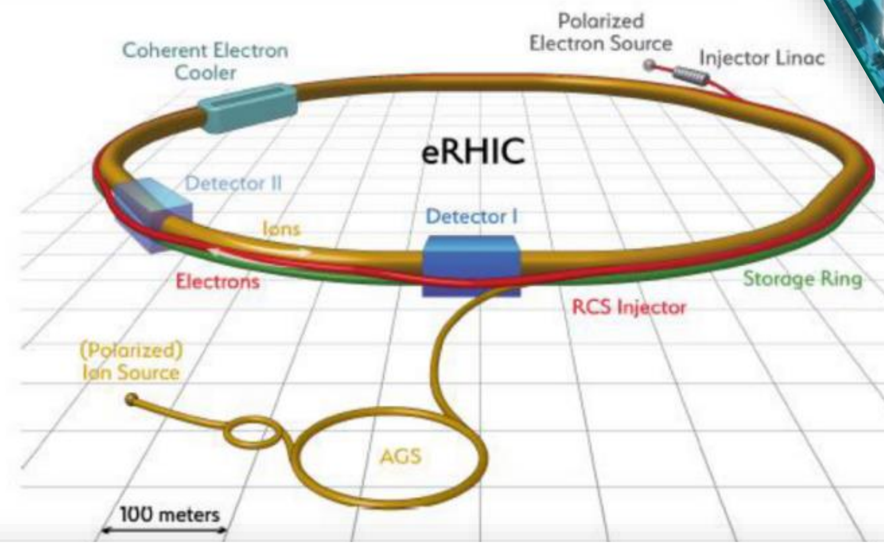






Efforts detailed in a decade worth of reports:

Electron-Ion Collider (EIC)



Electron-Ion Collider: The Next QCD Frontier (PDF)

The Electron-Ion Collider: Assessing the Energy Dependence of Key Measurements (PDF)

An Assessment of U.S.-Based Electron-Ion Collider Science

Conceptual Design Report Electron-Ion Collider

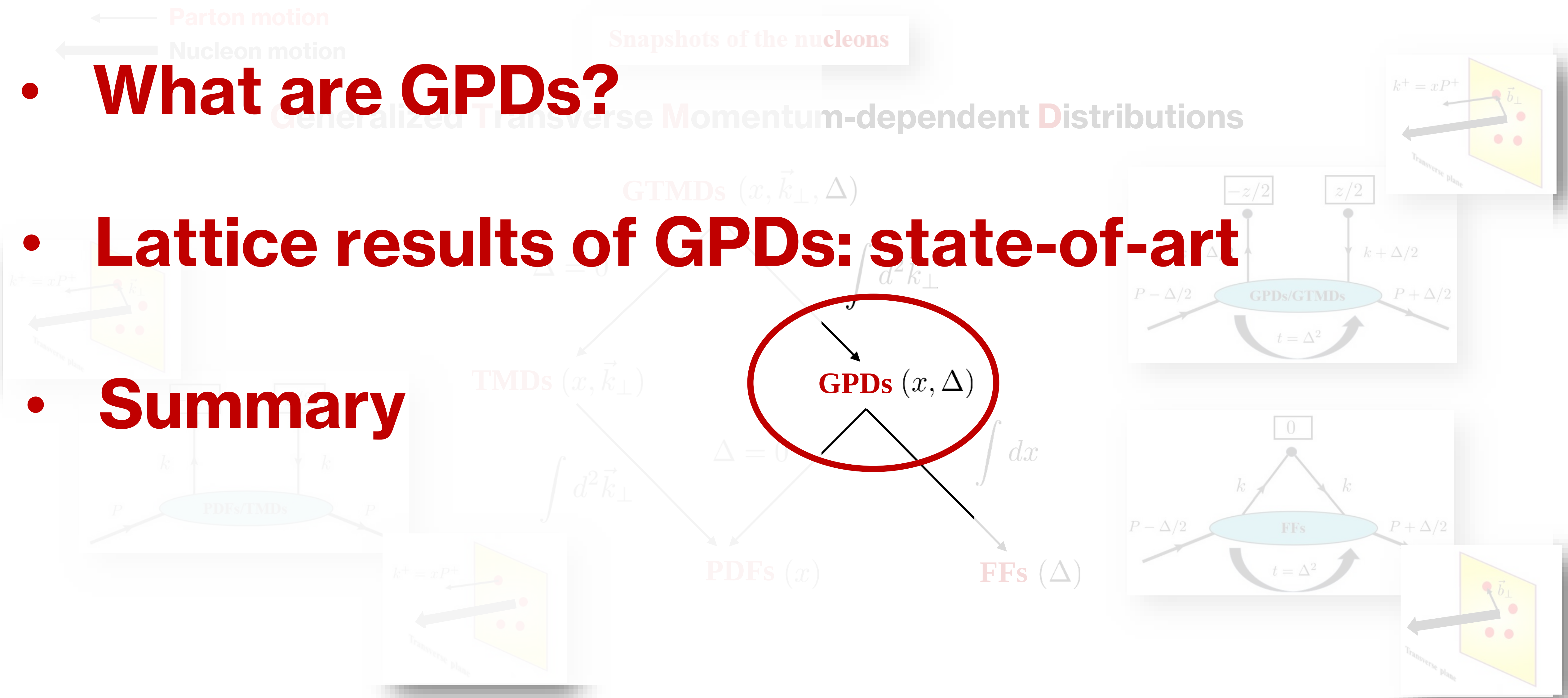
Science Requirements and Detector

Nucleon tomography (mapping partonic distributions) is one of the major goals of the EIC

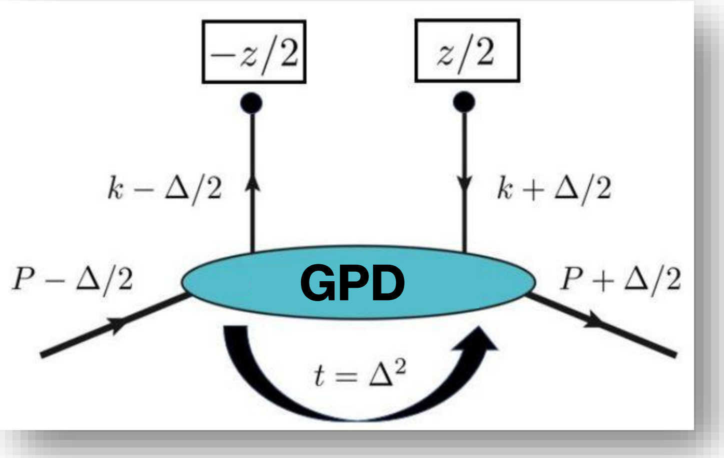
Lattice calculations can serve as a valuable complement to the ongoing efforts at the EIC

Outline

- **What are GPDs?**
- **Lattice results of GPDs: state-of-art**
- **Summary**



What are Generalized Parton Distributions?

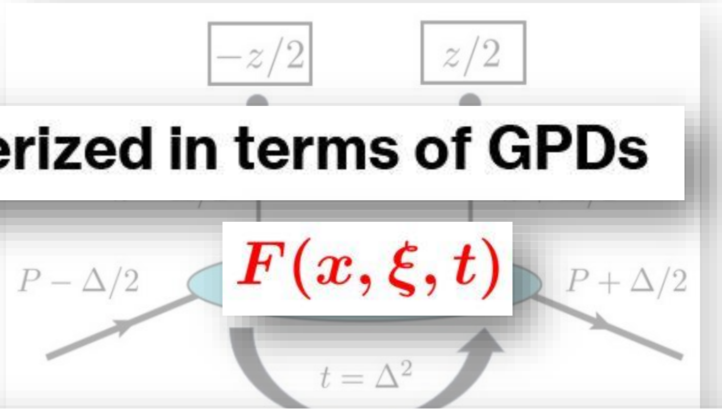


GPD correlator for quarks: Graphical representation

Definition of GPD correlator for quarks:

$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int\frac{dz^-}{2\pi}e^{ik\cdot z}\langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

What are Generalized Parton Distributions?



Correlator parameterized in terms of GPDs

$F(x,\xi,t)$

x : “average” longitudinal momentum fraction carried by parton

ξ : skewness parameter; longitudinal momentum transfer to nucleon

t : momentum transfer squared

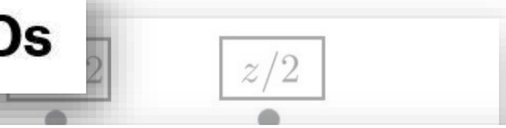
Definition of

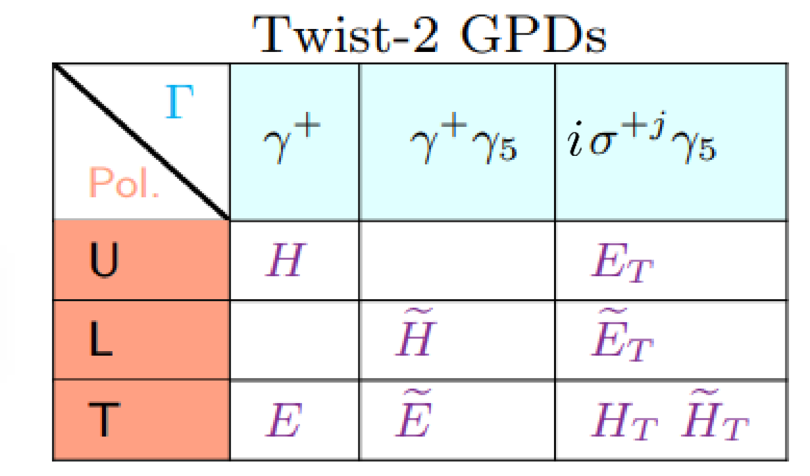
$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z}\langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

What are Generalized Parton Distributions?

Example:

At twist 2 there are 8 GPDs





Twist-2 GPDs

Pol. \ Γ	γ^+	$\gamma^+\gamma_5$	$i\sigma^{+j}\gamma_5$
U	H		E_T
L		$\widetilde{H}$	$\widetilde{E}_T$
T	E	$\widetilde{E}$	H_T $\widetilde{H}_T$

$F(x, \xi, t)$

GPD corr… …entation

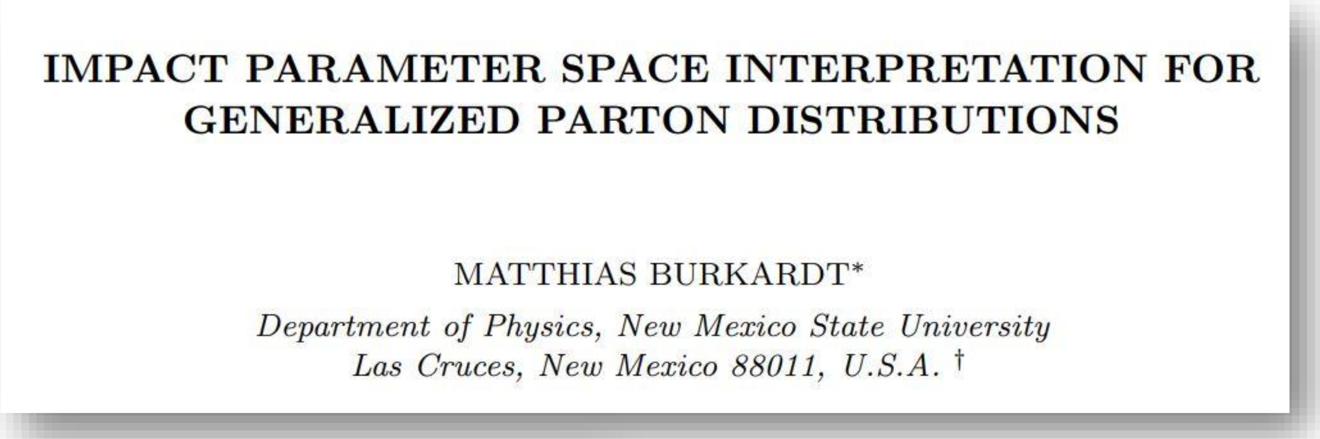
Definition of GPD correlator for quarks:

$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z}\langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

Motivation for studying GPDs

1)

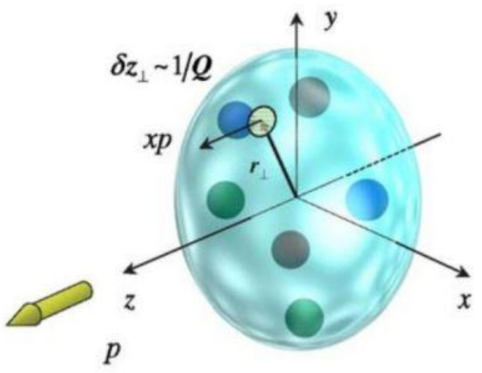
3D imaging (Burkardt, 0005108 ...)



IMPACT PARAMETER SPACE INTERPRETATION FOR GENERALIZED PARTON DISTRIBUTIONS

MATTHIAS BURKARDT*

Department of Physics, New Mexico State University
Las Cruces, New Mexico 88011, U.S.A. †



3D quark/gluon dist.

$$\boxed{F(x,\xi=0,\Delta_\perp) \xrightarrow{\mathcal{FT}} f(x,r_\perp)}$$

Motivation for studying GPDs

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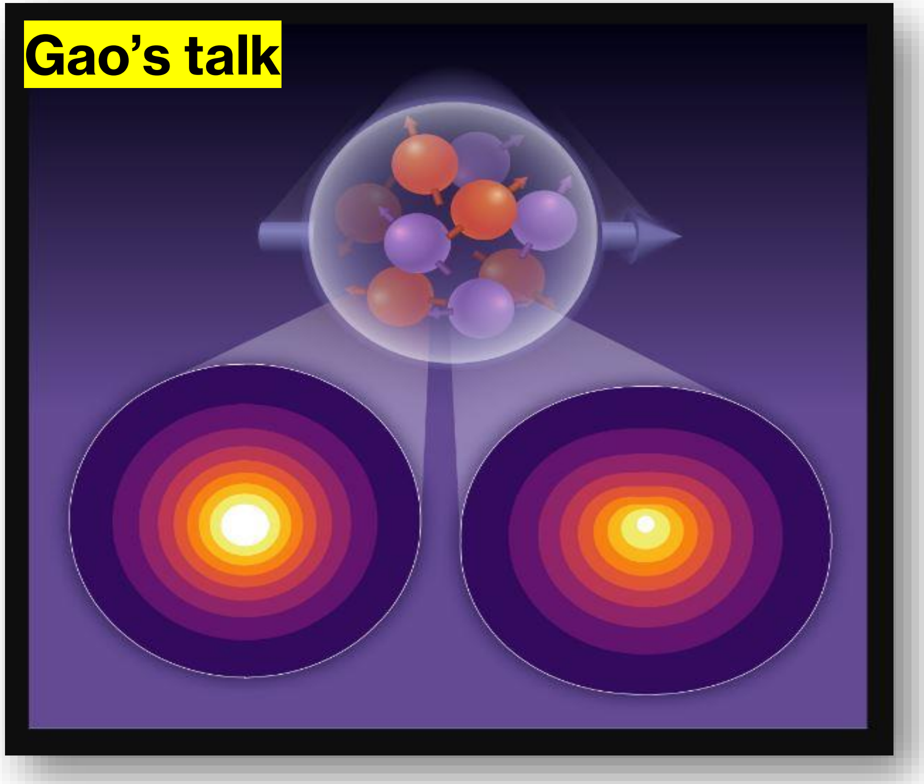
3D imaging (Burkardt, 0005108 ...)

Lattice QCD results of impact-parameter distributions:

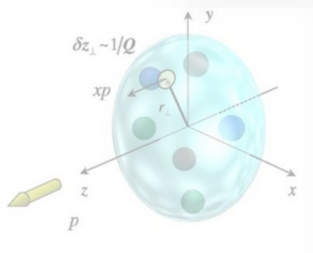


Gao's talk

Motivation for studying GPDs

3D imagin

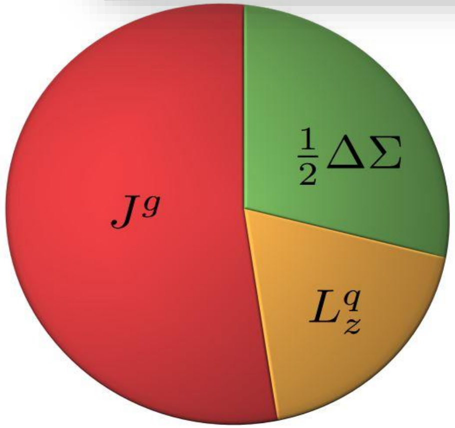
2) Spin sum rule & orbital angular momentum (Ji, 9603249)



3D quark/gluon dist.

GAUGE-INVARIANT DECOMPOSITION OF NUCLEON SPIN AND ITS SPIN-OFF *

Xiangdong Ji



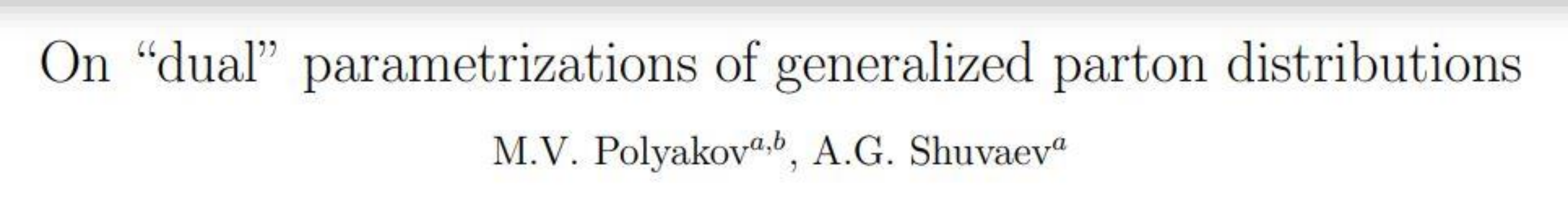
$$\frac{1}{2} = \underbrace{\frac{1}{2}\Delta\Sigma(\mu) + L_z^q(\mu)}_{J^q} + J^g(\mu)$$

Example:

$$J^q = \int_{-1}^{1} dx\, x\left(H^q + E^q\right)\Big|_{t=0}$$

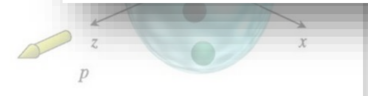
Motivation for studying GPDs

3) **Mechanical properties (pressure/shear) inside nucleon** (Polyakov, Shuvaev, 0207153 ...)



On "dual" parametrizations of generalized parton distributions

M.V. Polyakov[a,b], A.G. Shuvaev[a]



Lorcé, Meziani's talk

Exploits relations between GPDs & Gravitational Form Factors:

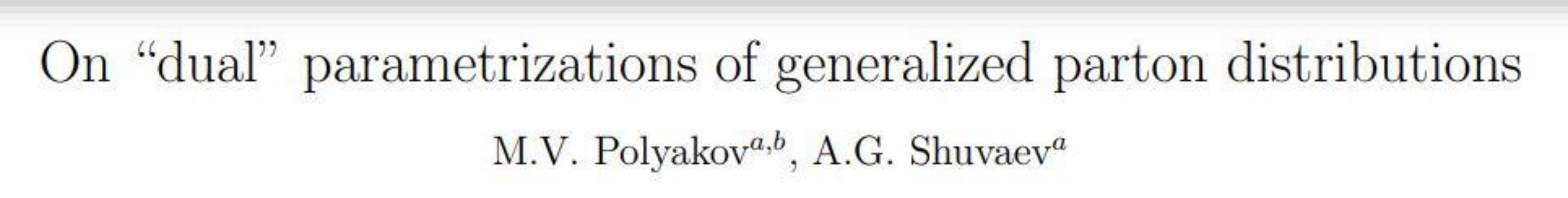
Gravitational Form Factors:

$$\langle P_2|\Theta_f^{\mu\nu}|P_1\rangle = \frac{1}{M}\bar{u}(P_2)\left[P^\mu P^\nu A_f+(A_f+B_f)\frac{P^{(\mu}i\sigma^{\nu)\rho}l_\rho}{2}+\frac{D_f}{4}(l^\mu l^\nu-g^{\mu\nu}l^2)+M^2\bar{C}_f g^{\mu\nu}\right]u(P_1)$$

$$A+\xi^2 D=\int_{-1}^{1} dx\, xH \qquad B-\xi^2 D=\int_{-1}^{1} dx\, xE$$

Motivation for studying GPDs

3) **Mechanical properties (pressure/shear) inside nucleon** (Polyakov, Shuvaev, 0207153 ...)



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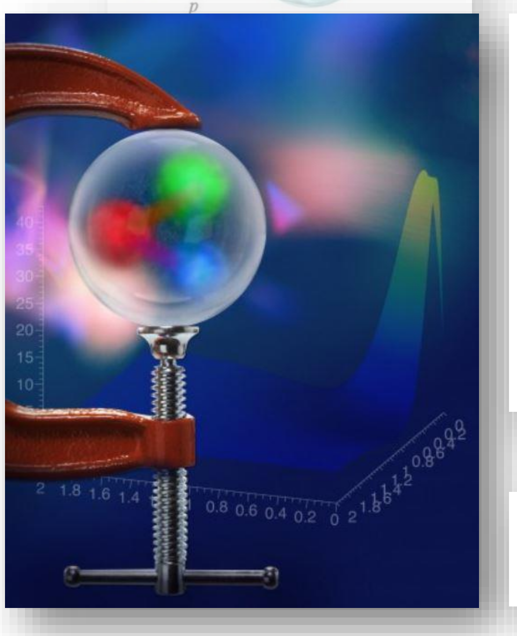


Courtesy: JLab media

LETTER

https://doi.org/10.1038/s41586-018-0060-z

The pressure distribution inside the proton

V. D. Burkert[1]*, L. Elouadrhiri[1] & F. X. Girod[1]

QUARKS FEEL THE PRESSURE IN THE PROTON

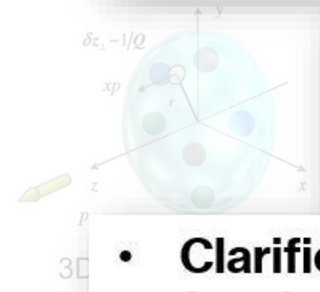
Motivation for studying GPDs

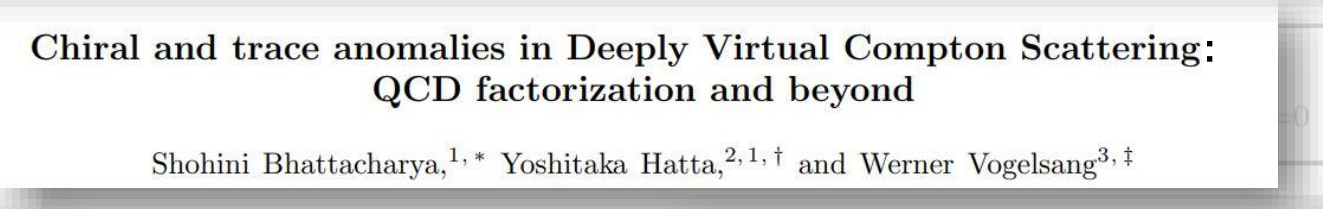
3D imagin

(Ji, 9603249)

4) **Mass generations & chiral symmetry breaking**

(SB, Hatta, Vogelsang, 2210.13419, 2305.09431)





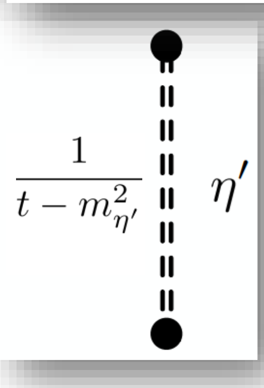
Chiral and trace anomalies in Deeply Virtual Compton Scattering: QCD factorization and beyond

Shohini Bhattacharya,[1,*] Yoshitaka Hatta,[2,1,†] and Werner Vogelsang[3,‡]

- **Clarified QCD factorization for the first time within $\Lambda_{\rm QCD}^2 \ll t \ll Q^2$ regime: Crucial topic for ongoing & future experiments including at EIC**

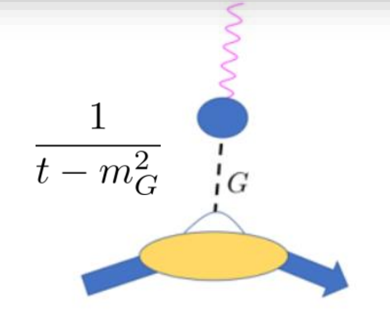
- **Unraveled profound & previously undiscovered connections between chiral/trace anomalies & GPDs**

Mechanic



Eta meson mass generation:

$$\boxed{\tilde{E}(x) \sim \frac{1}{t-m_{\eta'}^2}}$$



Glueball mass generation:

$$\boxed{H(x), E(x) \sim \frac{1}{t-m_G^2}}$$

Motivation for studying GPDs

3D imaging (Ji, 9603249)

4) Mass generations & chiral symmetry breaking

(SB, Hatta, Vogelsang, 2210.13419, 2305.09431)

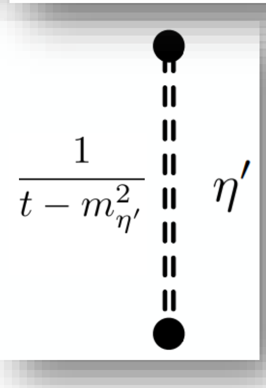
Chiral and trace anomalies in Deeply Virtual Compton Scattering:
QCD factorization and beyond

acharya,[1,*] Yoshitaka Hatta,[2,1,†] and Werner Vogelsang[3,‡]

Novel avenue of GPD research

Profound physical implication of anomaly poles:
Touches questions on mass generations, Chiral symmetry breaking, ...

chiral/trace anomalies & GPDs



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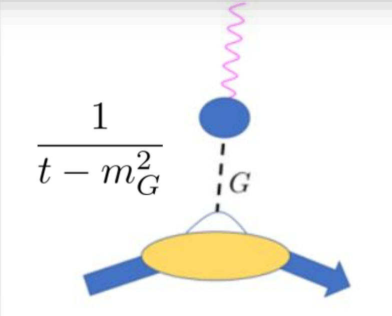

Glueball mass generation:

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Motivation for studying GPDs

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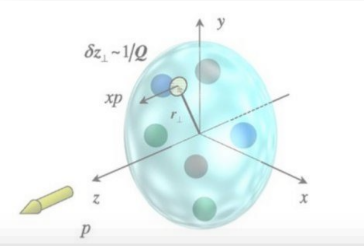


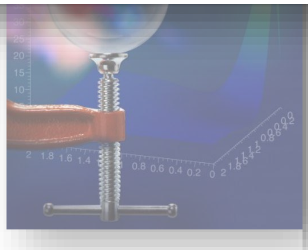
2) **Spin sum rule & orbital angular momentum** (Ji, 9603249)

Example:

$$\boxed{J_q = \int_{-1}^{1} dx\, x(H_q + E_q)\big|_{t=0}}$$

We have numerous compelling reasons to engage in GPD studies!

3) **Mechanical properties (pressure/shear) inside nucleon** (Polyakov, Shuvaev, 0207153 ...)



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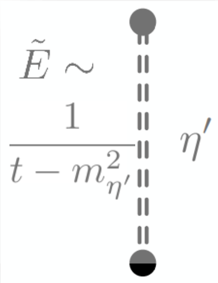
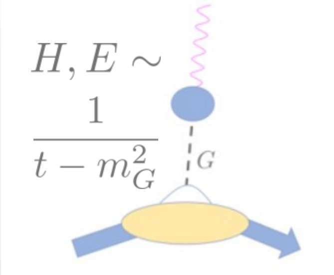
(SB, Hatta, Vogelsang, 2210.13419, 2305.09431)



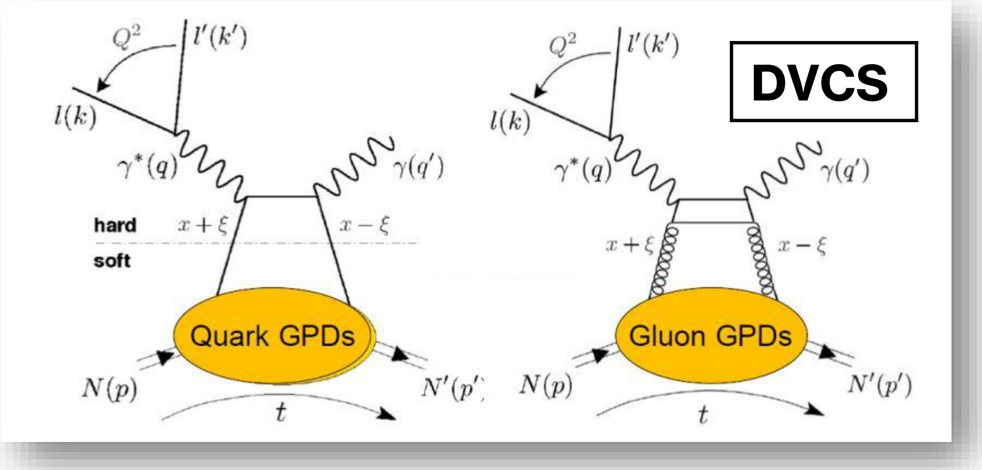
$$\tilde{E} \sim \frac{1}{t - m_{\eta'}^2}$$

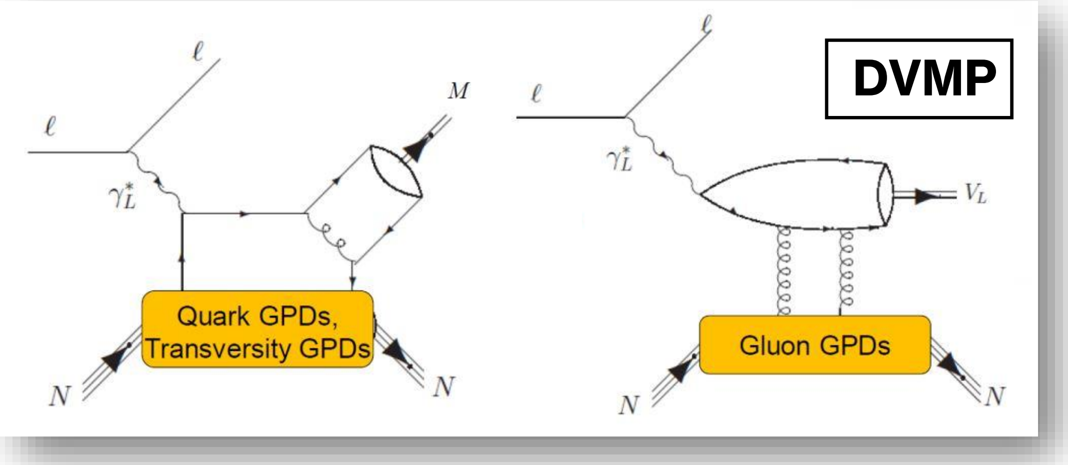


$$H, E \sim \frac{1}{t - m_G^2}$$

Physical processes sensitive to GPDs

See talks by Silvia, Spencer, Wim



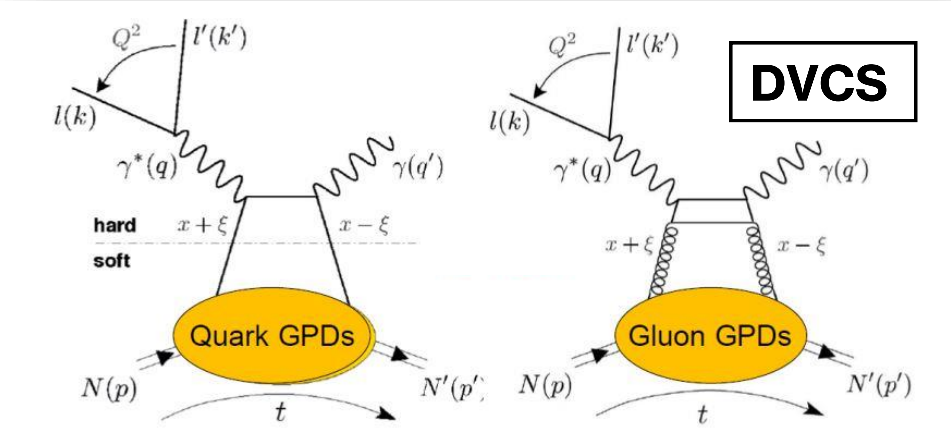


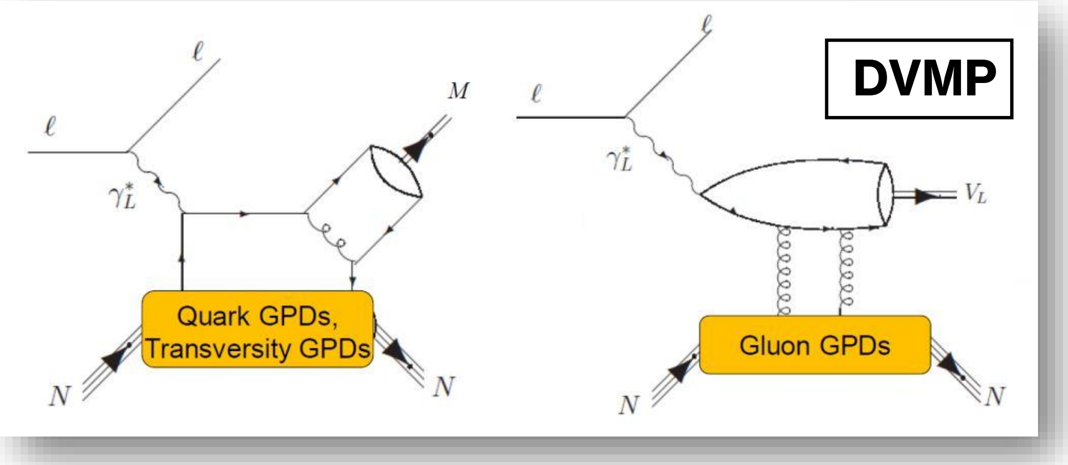
Courtesy: Hyon-Suk Jo, KPS Meeting

No access to x-dependence of GPDs

Physical processes sensitive to GPDs

See talks by Silvia, Spencer, Wim





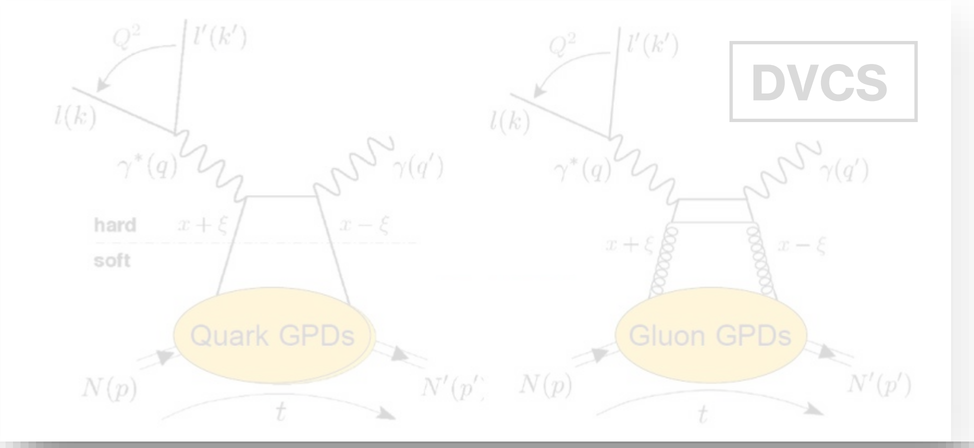
Courtesy: Hyon-Suk Jo, KPS Meeting

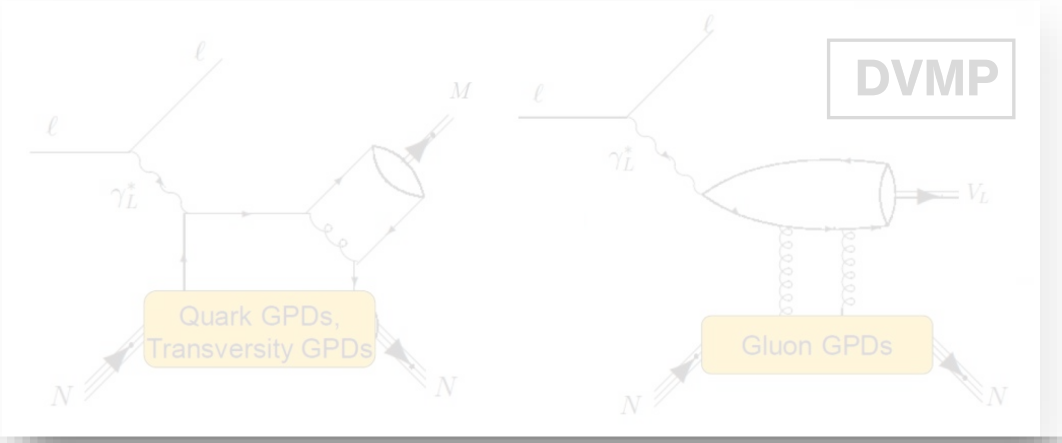
No access to x-dependence of GPDs

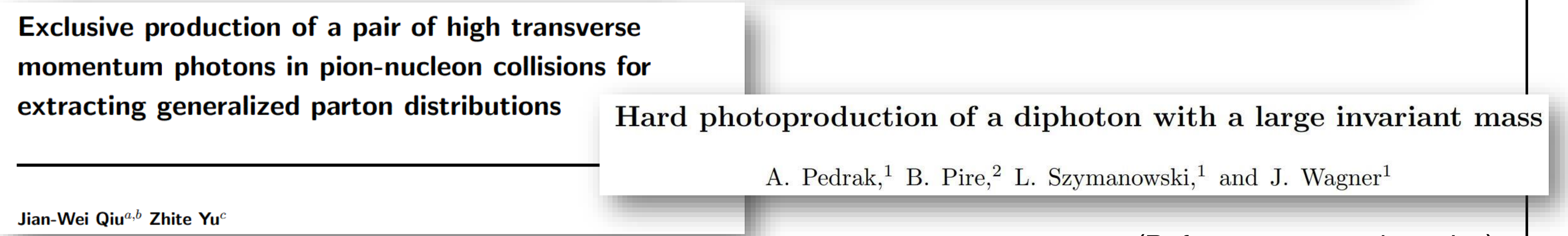
Complementarity: Lattice results can be integrated into global analysis of experimental data

Physical processes sensitive to GPDs

See talks by Silvia, Spencer, Wim







Exclusive production of a pair of high transverse momentum photons in pion-nucleon collisions for extracting generalized parton distributions

Jian-Wei Qiu^{a,b} Zhite Yu^{c}

Hard photoproduction of a diphoton with a large invariant mass

A. Pedrak,^{1} B. Pire,^{2} L. Szymanowski,^{1} and J. Wagner^1

(References not exhaustive)

Access to x-dependence of GPDs

Physical processes sensitive to GPDs

See talks by Silvia, Spencer, Wim

We require complementary measurements of the GPDs using Lattice QCD

In recent years, significant breakthroughs have been made in our ability to access the x-dependence of GPDs

Ex
m
extracting generalized parton distributions

Jian-Wei Qiu[a,b] Zhite Yu[c]

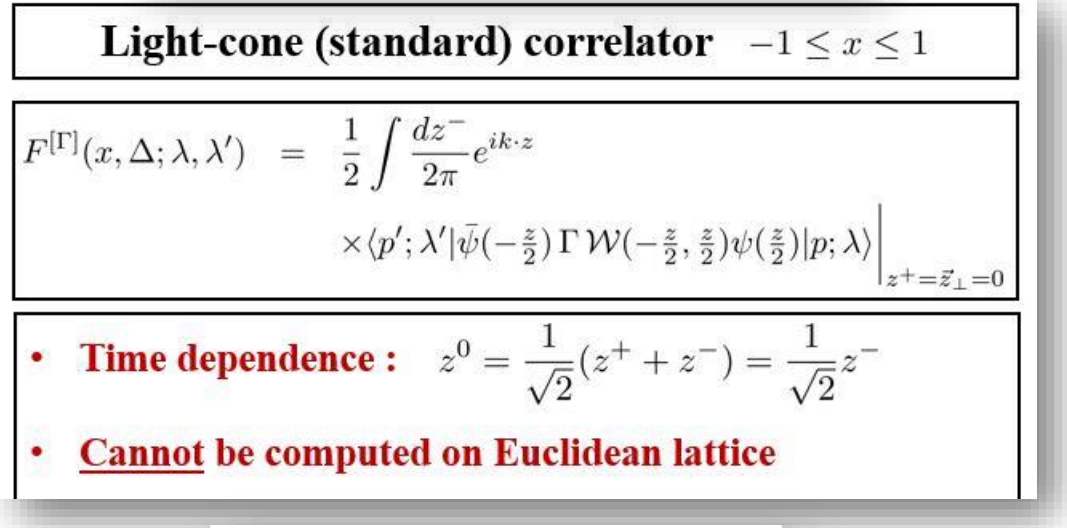
Hard photoproduction of a diphoton with a large invariant mass

A. Pedrak,[1] B. Pire,[2] L. Szymanowski,[1] and J. Wagner[1]

Access to x-dependence of GPDs

Calculating Parton Distributions in Lattice QCD

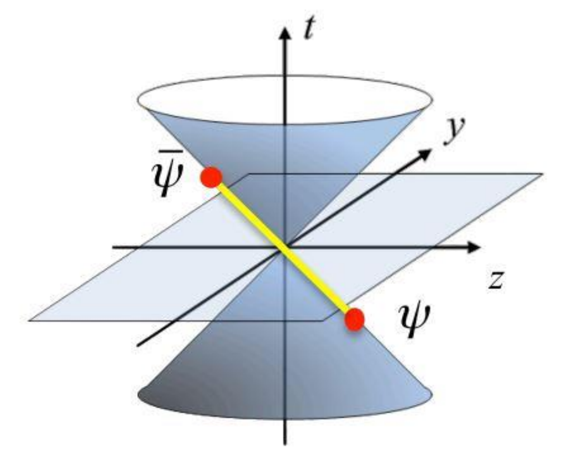
“Physical” distributions



Light-cone (standard) correlator $-1 \leq x \leq 1$

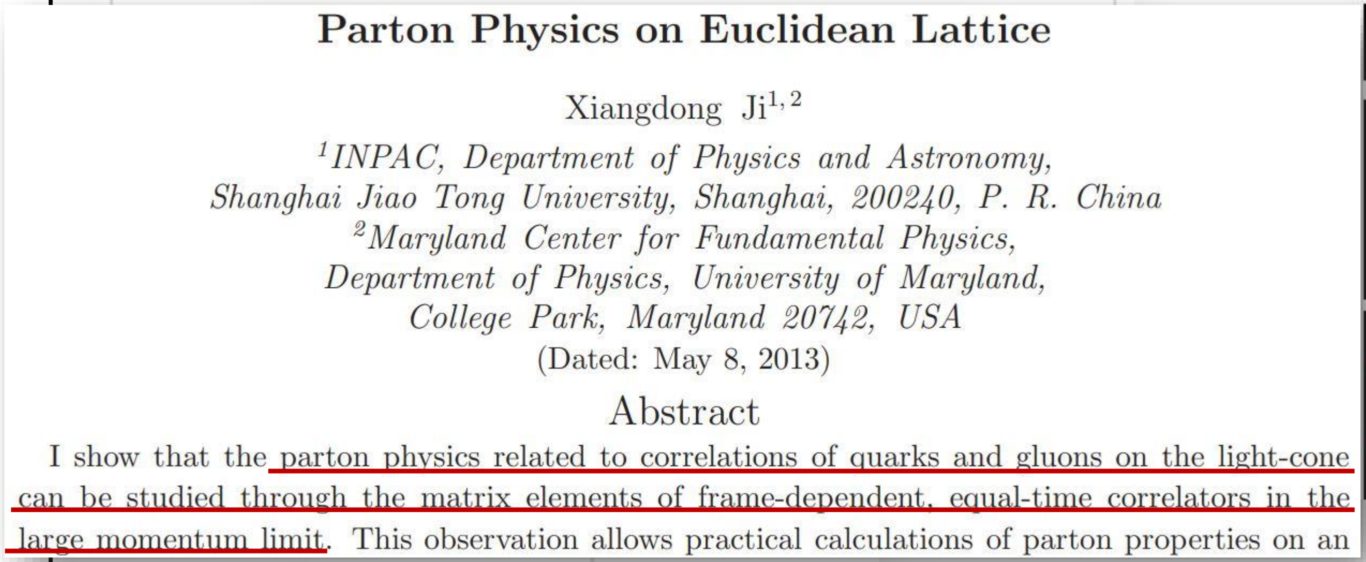
$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \times \langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=\vec{z}_\perp=0}$$

- **Time dependence :** $z^0 = \frac{1}{\sqrt{2}}(z^+ + z^-) = \frac{1}{\sqrt{2}} z^-$
- **Cannot be computed on Euclidean lattice**



Calculating Parton Distributions in Lattice QCD

“Physical” distributions



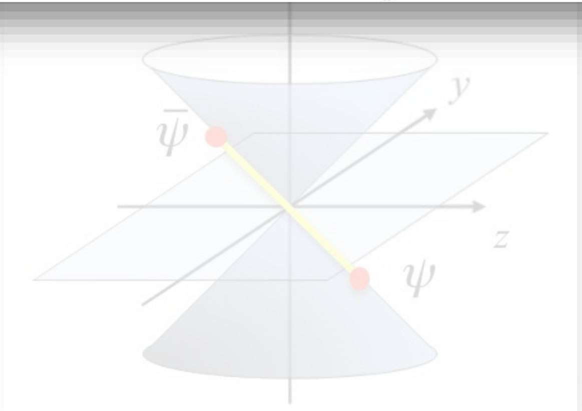
Parton Physics on Euclidean Lattice

Xiangdong Ji[1,2]

[1]*INPAC, Department of Physics and Astronomy,*
Shanghai Jiao Tong University, Shanghai, 200240, P. R. China
[2]*Maryland Center for Fundamental Physics,*
Department of Physics, University of Maryland,
College Park, Maryland 20742, USA
(Dated: May 8, 2013)

Abstract

I show that the parton physics related to correlations of quarks and gluons on the light-cone can be studied through the matrix elements of frame-dependent, equal-time correlators in the large momentum limit. This observation allows practical calculations of parton properties on an

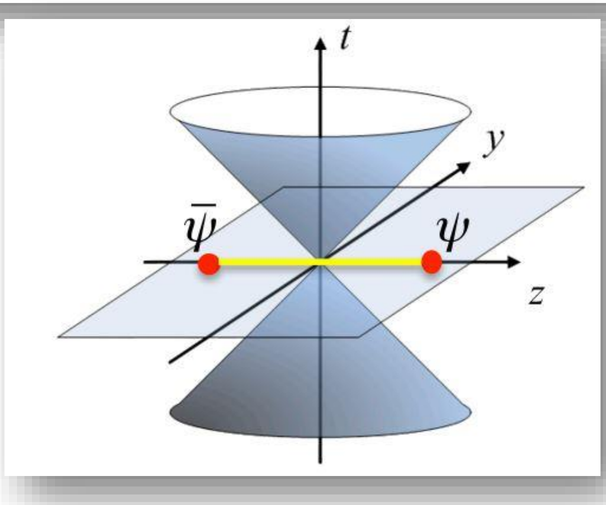


“Auxiliary” distributions

Correlator for quasi-GPDs (Ji, 2013) $-\infty \leq x \leq \infty$

$$F_{\mathrm{Q}}^{[\Gamma]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2}\int\frac{dz^3}{2\pi}e^{ik\cdot z} \times\langle p',\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}_{\mathrm{Q}}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p,\lambda\rangle\Big|_{z^0=\vec{z}_\perp=0}$$

- **Non-local correlator depending on position** z^3
- **Can be computed on Euclidean lattice**



Calculating Parton Distributions in Lattice QCD

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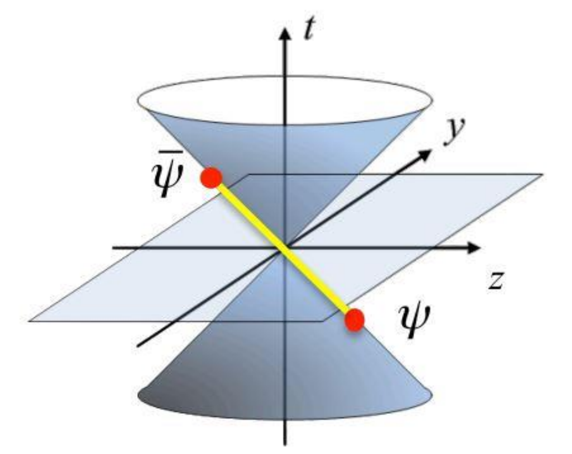

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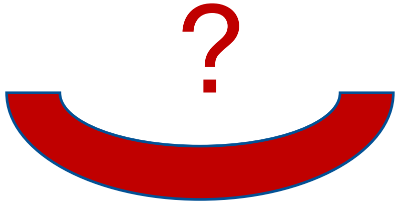
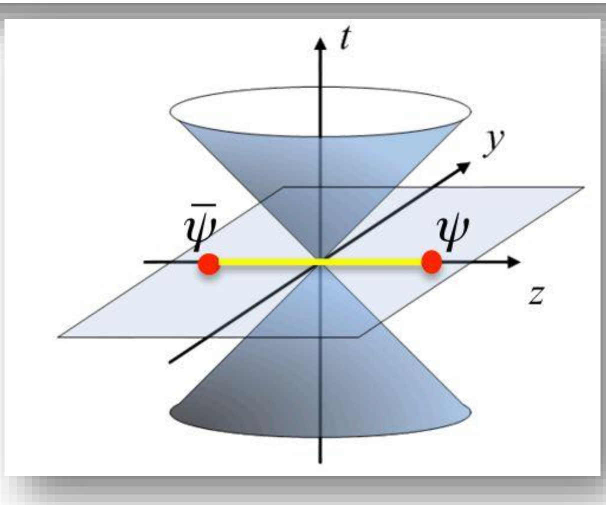
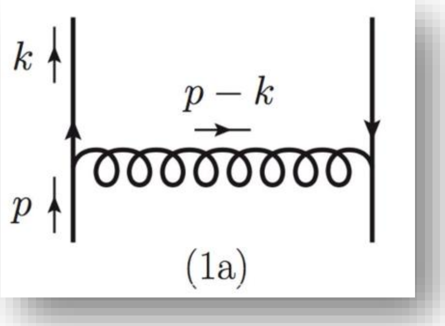




Calculating Parton Distributions in Lattice QCD

Essence of the quasi-distribution approach (Example: PDF)



(1a)

Light-cone PDF:

$$\boxed{f_1(x) = \frac{\alpha_s C_F}{2\pi}(1-x)\left(\mathcal{P}_{\text{UV}} + \ln\frac{\mu^2}{x m_g^2} - 2\right)}$$

$0 < x < 1$

$$\mathcal{P}_{\text{UV}} = \frac{1}{\epsilon_{\text{UV}}} + \ln 4\pi - \gamma_E$$

$$\int_0^{\infty} dk_\perp$$

Calculating Parton Distributions in Lattice QCD

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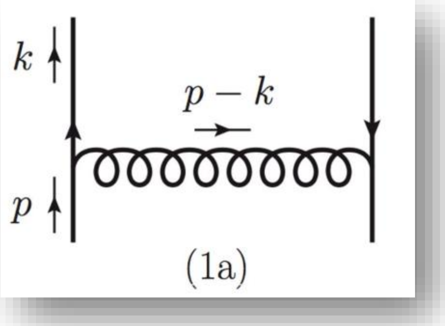

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$$f_1(x, p^3) = \frac{\alpha_s C_F}{2\pi}\begin{cases} (1-x)\ \ln\dfrac{x}{x-1} + 1 & x > 1 \\ (1-x)\ \ln\dfrac{4(1-x)p_3^2}{m_g^2} + x & 0 < x < 1 \\ (1-x)\ \ln\dfrac{x-1}{x} - 1 & x < 0 \end{cases}$$

Calculating Parton Distributions in Lattice QCD

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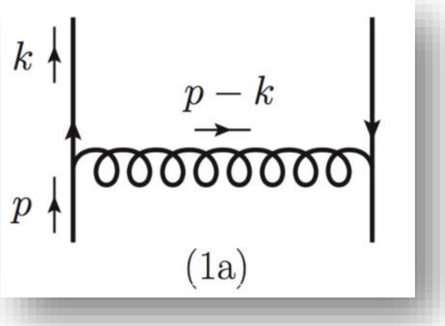

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Quasi PDF:

Support outside physical region $0 < x < 1$

$$f_1(x,p^3) \;=\; \frac{\alpha_s C_F}{2\pi}\begin{cases} (1-x)\,\ln\dfrac{x}{x-1} + 1 & x > 1 \\ (1-x)\,\ln\dfrac{4(1-x)p_3^2}{m_g^2} + x & 0 < x < 1 \\ (1-x)\,\ln\dfrac{x-1}{x} - 1 & x < 0 \end{cases}$$

Calculating Parton Distributions in Lattice QCD

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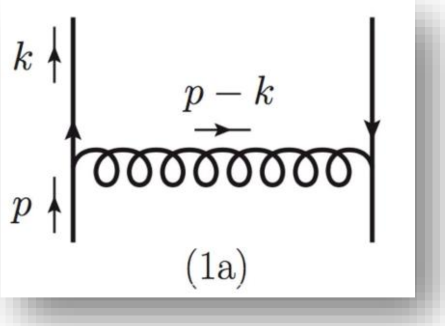

Light-cone PDF:

$$f_1(x) = \frac{\alpha_s C_F}{2\pi}(1-x)\left(\mathcal{P}_{\rm UV} + \ln\frac{\mu^2}{x\boldsymbol{m_g^2}} - 2\right)$$

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$\int_{\mathbf{0}}^{\infty} dk_\perp$

Absence of UV divergence: They manifest only after $\int dx$

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Calculating Parton Distributions in Lattice QCD

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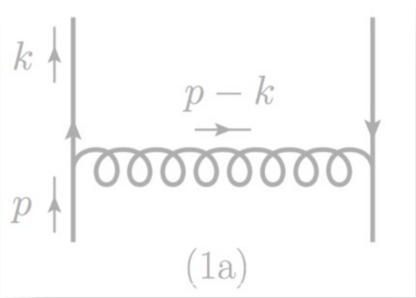

(1a)

Light-cone PDF:

$$f_1(x) = \frac{\alpha_s C_F}{}(1-x)\left(\mathcal{P}_{\mathrm{UV}} + \ln\frac{\mu^2}{} - 2\right) \qquad 0 < x < 1$$

By construction, if one boosts the quasi-observable to infinite-momentum frame, then it reduces to the light-cone observable

$$\int_0^\infty dk_\perp$$

Absence of UV divergence: They manifest only after $\int dx$

Quasi PDF:

Support outside physical region $0 < x < 1$

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Calculating Parton Distributions in Lattice QCD

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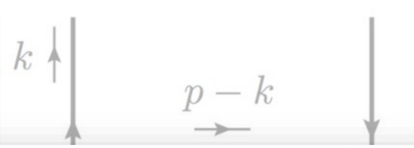
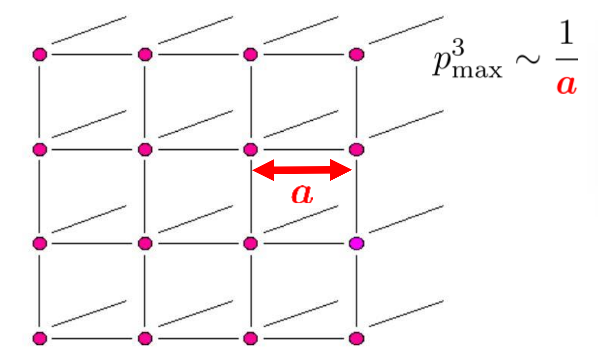




Light-cone PDF:

$$f_1(x) = \frac{\alpha_s C_F}{2}(1-x)\left(\mathcal{P}_{\rm UV} + \ln\frac{\mu^2}{2} - 2\right) \qquad 0 < x < 1$$

By construction, if one boosts the quasi-observable to infinite-momentum frame, then it reduces to the light-cone observable

$$\int_0^\infty dk_\perp$$

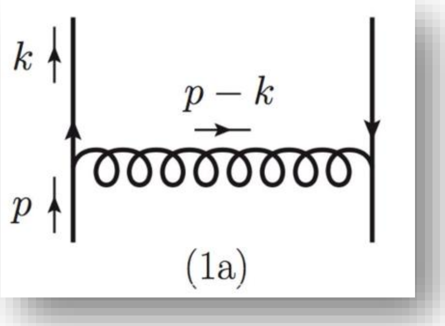
Absence of UV divergence: They manifest only after $\int dx$

In lattice computations, UV cut-offs (Λ) are given by the finite lattice spacing a ($\Lambda \sim a^{-1}$), and one (naturally) deals with UV renormalization before taking the limit $P^3 \to \infty$. The limits $\Lambda \to \infty$ and $P^3 \to \infty$ do not commute, which leads to non-trivial differences in the UV behavior of the quasi-PDFs and light-cone PDFs.

$$f_1(x, p^3) = \frac{\alpha_s C_F}{2\pi}\begin{cases} (1-x)\ln\frac{4(1-x)p_3^2}{m_g^2} + x & 0 < x < 1 \\ (1-x)\ln\frac{x-1}{x} - 1 & x < 0 \end{cases}$$

Calculating Parton Distributions in Lattice QCD

Essence of the quasi-distribution approach (Example: PDF)



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Support outside physical region $0 < x < 1$

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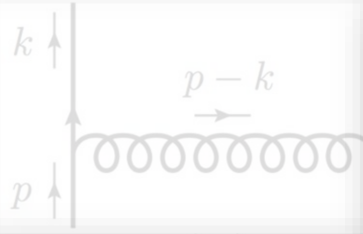
IR pole structure of light-cone & quasi-PDFs are same

Calculating Parton Distributions in Lattice QCD

Matching formula: (PDF)

Matching coefficient



$$\tilde{q}(x,\mu,P^3) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{P^3}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(P^3)^2}, \frac{M_N^2}{(P^3)^2}\right)$$

Xiong, Ji, Zhang, Zhao/ Stewart, Zhao/ Izubuchi, Ji, Jin, Stewart, Zhao ...

Essence of the quasi-PDF approach

IR pole structure of light-cone & quasi-PDFs are same

Quasi PDF:

Absence of UV divergence: They manifest only after $\int dx$

Support outside physical region $0 < x < 1$

$$f_1(x,p^3) = \frac{\alpha_s C_F}{2\pi} \begin{cases} (1-x)\ln\frac{x}{x-1}+1 & x>1 \\ (1-x)\ln\frac{4(1-x)p_3^2}{m_g^2}+x & 0<x<1 \\ (1-x)\ln\frac{x-1}{x} - & \end{cases}$$

IR pole structure of light-cone & quasi-PDFs are same

Calculating Parton Distributions in Lattice QCD

Matching formula: **(GPD)**

Matching coefficient

$$\tilde{q}(x,\xi,t,\mu,P^3) = \int_{-1}^{1}\frac{dy}{|y|}\, C\left(\frac{x}{y},\frac{\xi}{y},\frac{\mu}{P^3}\right) q(y,\xi,t,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(P^3)^2},\frac{M_N^2}{(P^3)^2},\frac{t}{(P^3)^2}\right)$$

GPD matching known up to one-loop order (non-singlet & singlet)

Absence of UV divergence: They manifest only after $\int dx$

References: (not exhaustive)

Connecting Euclidean to light-cone correlations: From flavor nonsinglet in forward kinematics to flavor singlet in non-forward kinematics

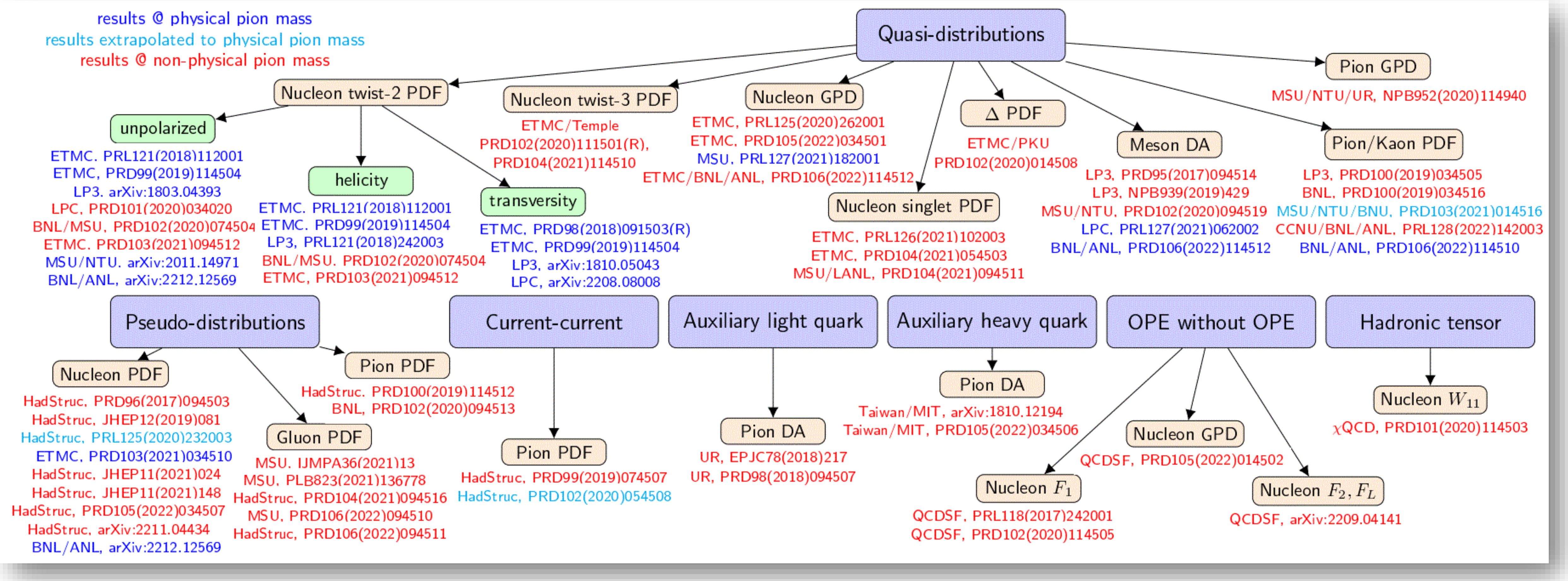
Yao Ji,[a] Fei Yao[b] and Jian-Hui Zhang[c,b]

One-Loop Matching for Generalized Parton Distributions

Xiangdong Ji,[1,2,3] Andreas Schäfer,[4] Xiaonu Xiong,[5,6] and Jian-Hui Zhang[1,4]

Dynamical Progress of Lattice QCD calculations of PDFs/GPDs

Lattice QCD calculations of x-dependence of PDFs & related quantities using Euclidean correlators:



Compilation by Cichy, 2110.07440

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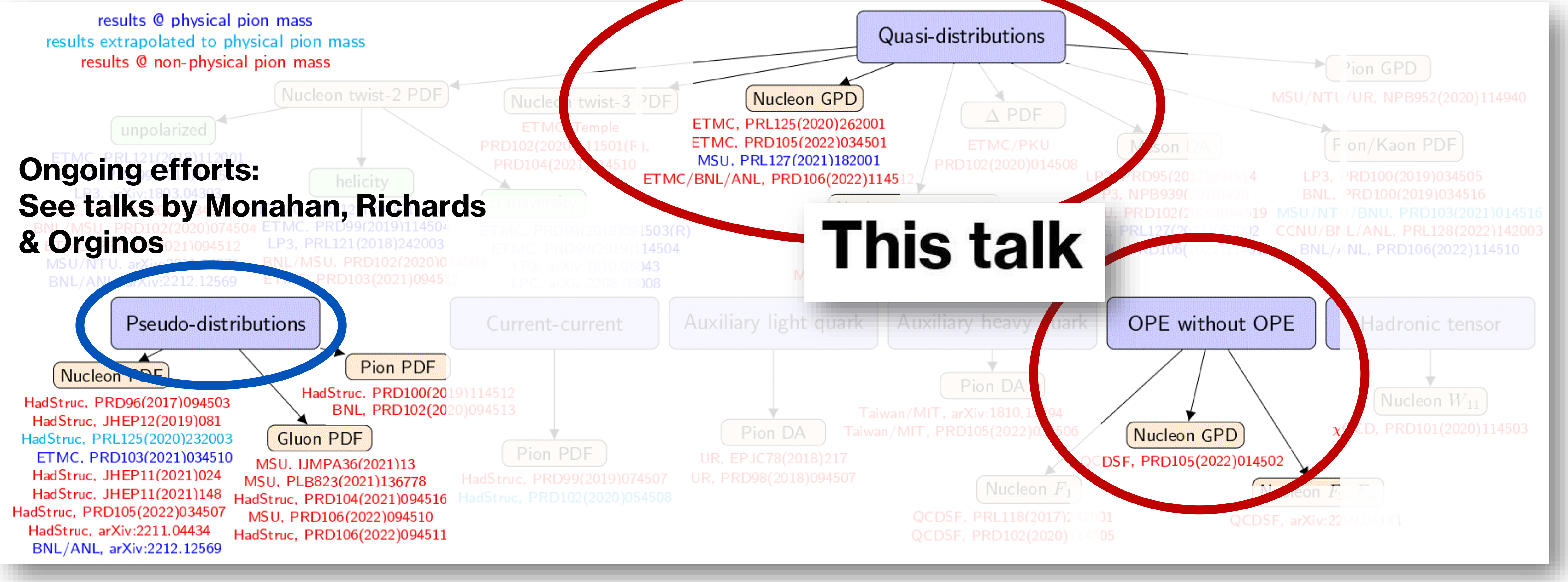

Ongoing efforts:
See talks by Monahan, Richards & Orginos

This talk

Compilation by Cichy, 2110.07440

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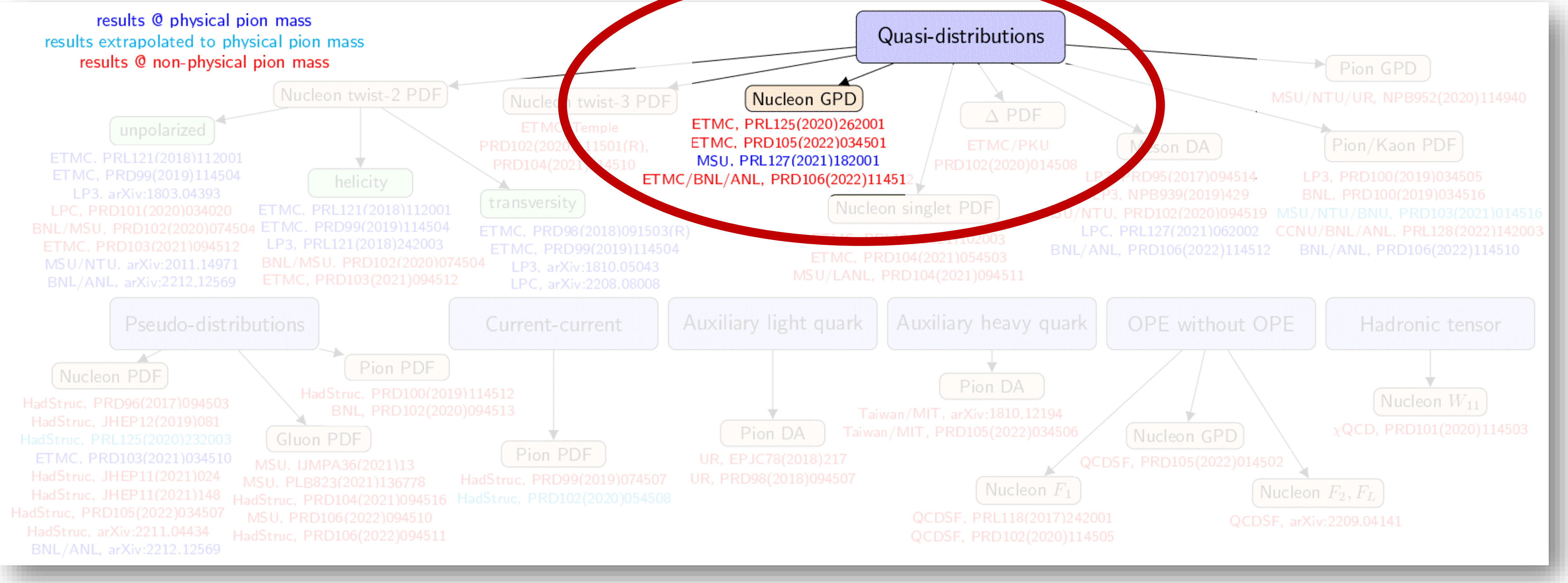
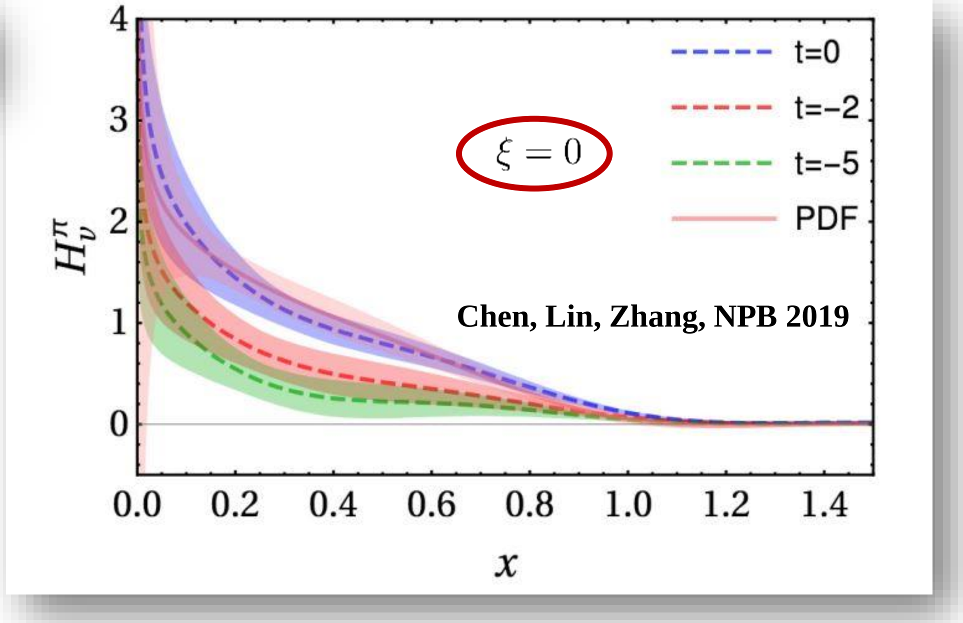


Compilation by Cichy, 2110.07440

First Lattice QCD results of the x-dependent GPDs

pion

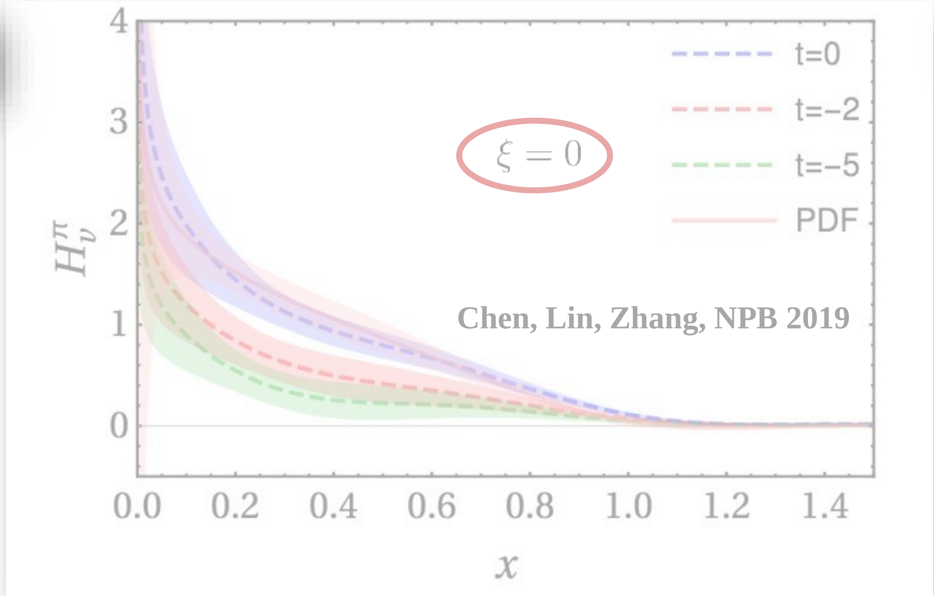


Chen, Lin, Zhang, NPB 2019

As t increases, the distribution flattens

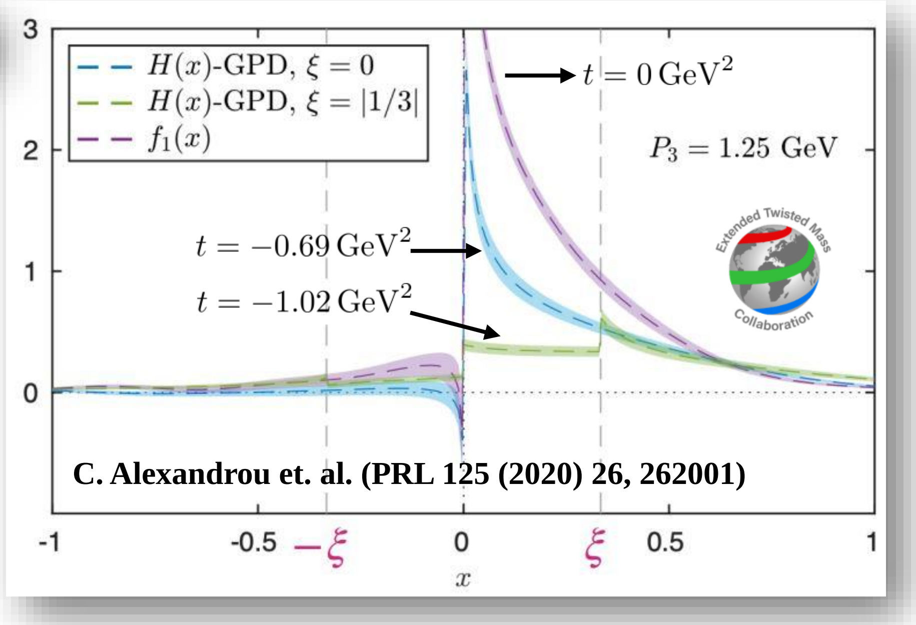
First Lattice QCD results of the x-dependent GPDs

pion



Chen, Lin, Zhang, NPB 2019

proton



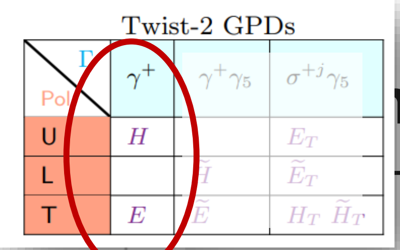
C. Alexandrou et. al. (PRL 125 (2020) 26, 262001)

ERBL/DGLAP: Qualitative differences

As $x \to 1$, qualitative behavior in agreement with power counting analysis

(F. Yuan, 0311288)

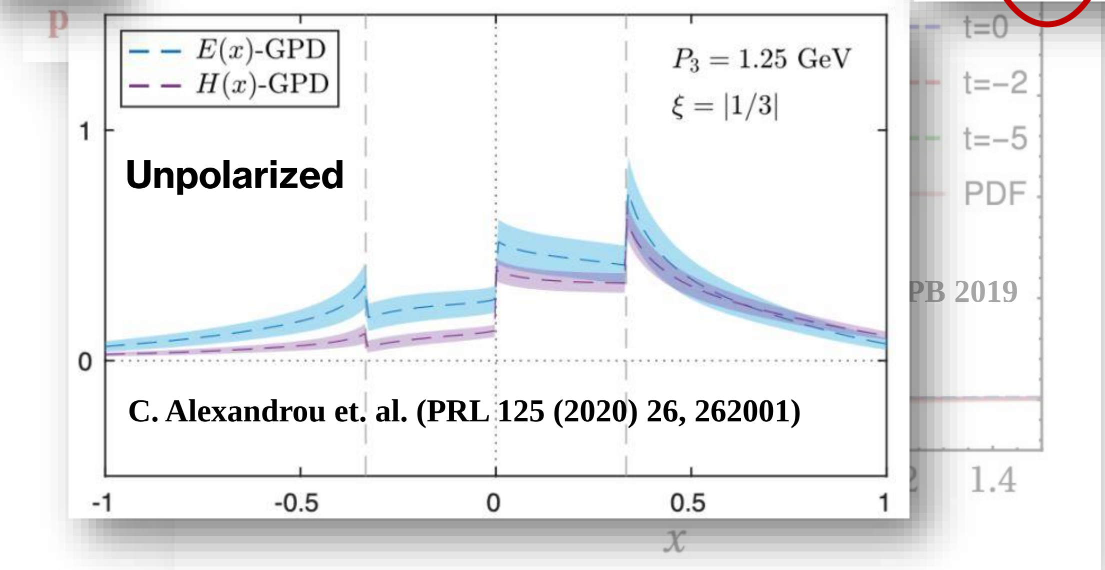
First Lattice QCD he x-dependent GPDs



Twist-2 GPDs

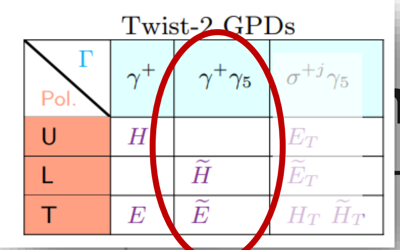
Pol \ Γ	γ^+	$\gamma^+\gamma_5$	$\sigma^{+j}\gamma_5$
U	H		E_T
L		$\widetilde{H}$	$\widetilde{E}_T$
T	E	$\widetilde{E}$	H_T $\widetilde{H}_T$

proton



C. Alexandrou et. al. (PRL 125 (2020) 26, 262001)

First Lattice QCD he x-dependent GPDs



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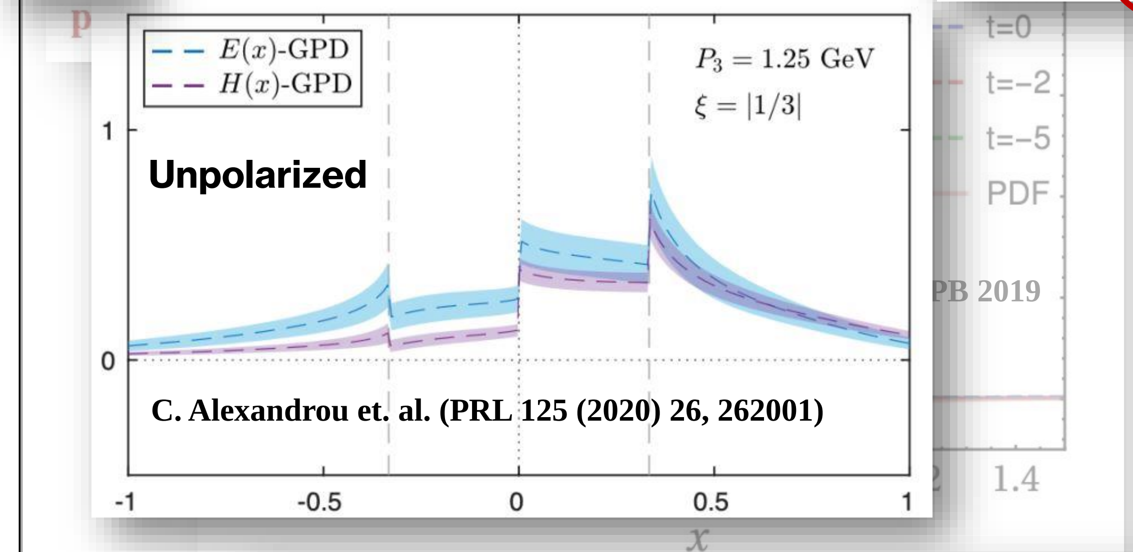
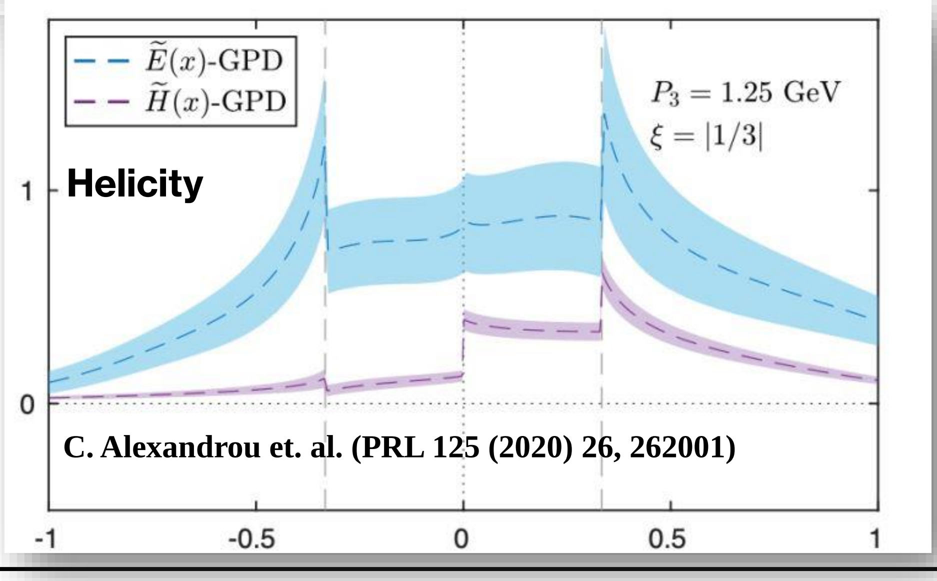
proton



Unpolarized

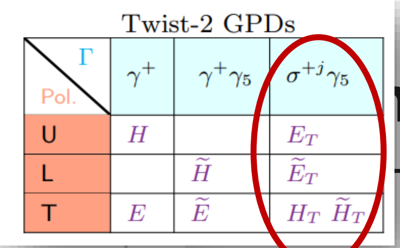
C. Alexandrou et. al. (PRL 125 (2020) 26, 262001)



Helicity

C. Alexandrou et. al. (PRL 125 (2020) 26, 262001)

First Lattice QCD he x-dependent GPDs



Twist-2 GPDs

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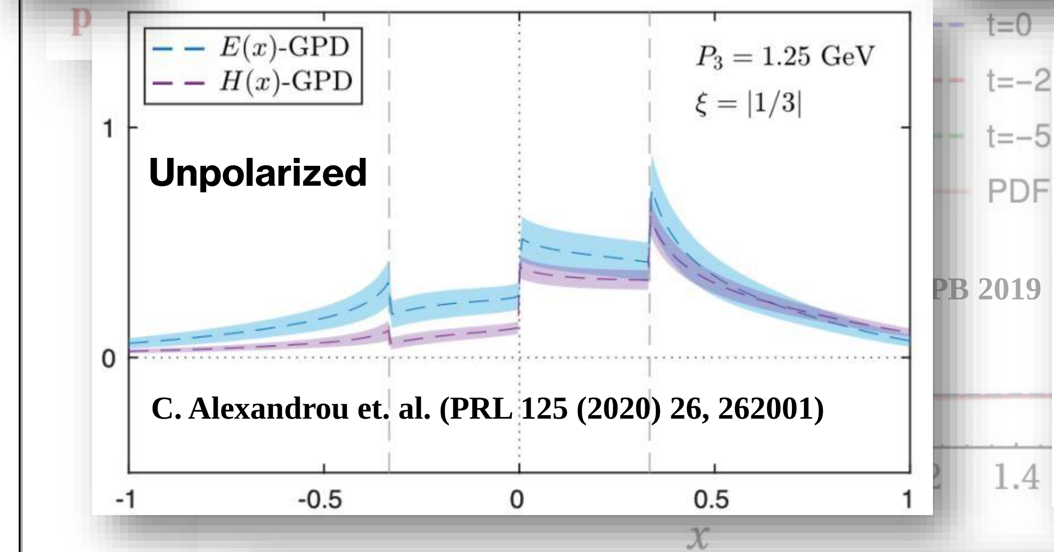
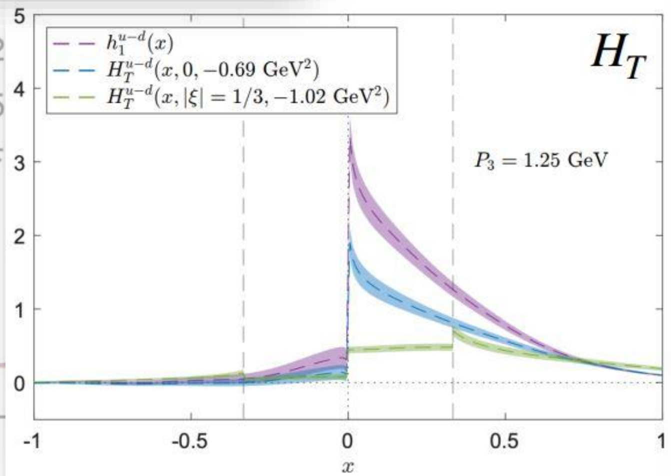
proton

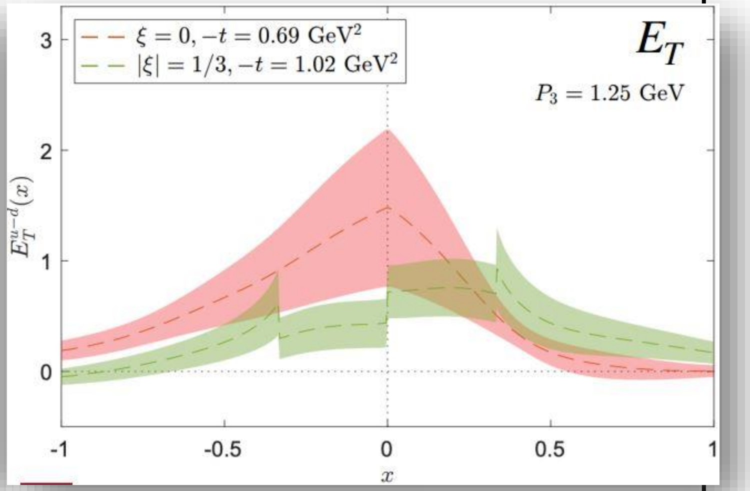


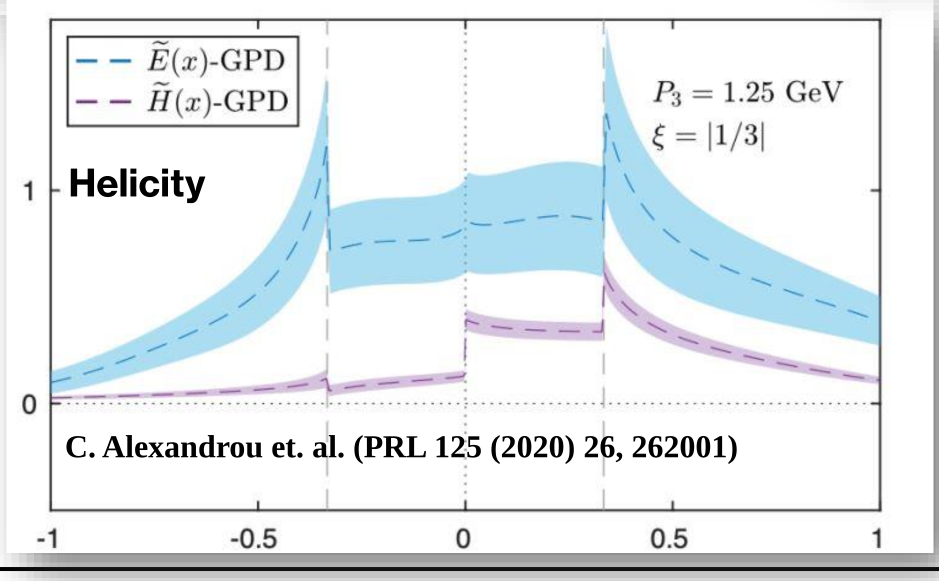
Unpolarized

C. Alexandrou et. al. (PRL 125 (2020) 26, 262001)

Transversity

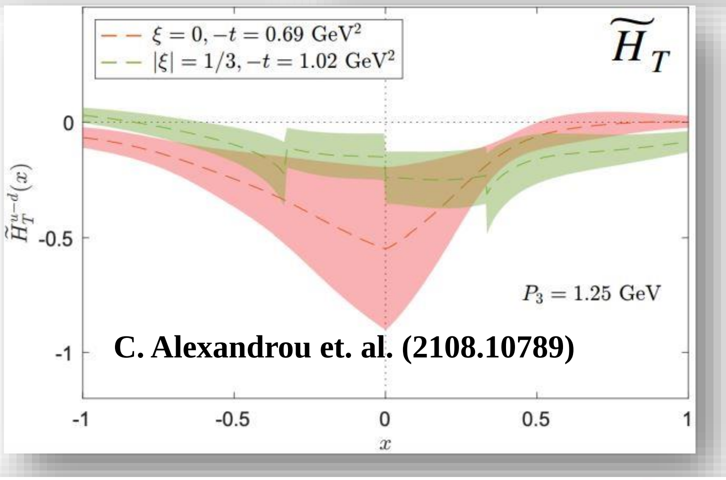






Helicity

C. Alexandrou et. al. (PRL 125 (2020) 26, 262001)



C. Alexandrou et. al. (2108.10789)

GPD $\widetilde{E}_T$ is small/zero within uncertainties (not shown)

Cichy’s talk

First exploration of twist-3 GPDs

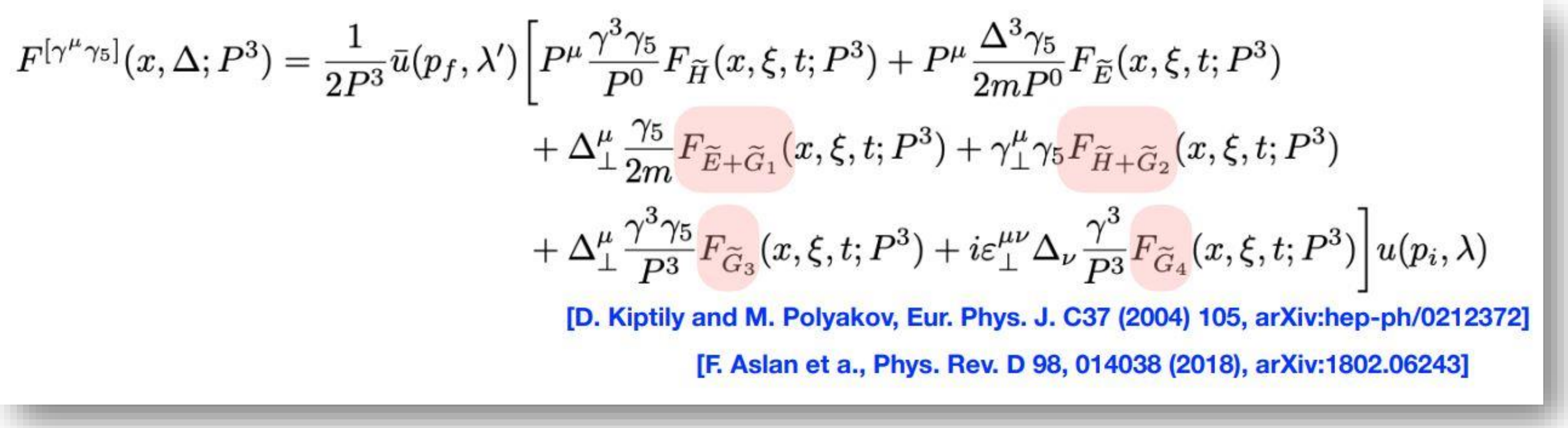
Why twist 3?

- As sizeable as twist 2
- Contain information about quark-gluon-quark correlations inside hadrons ...

Cichy's talk

First exploration of twist-3 GPDs

Definition:



$$
\begin{aligned}
F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[&P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)\\
&+\Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3)\\
&+\Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)
\end{aligned}
$$

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105, arXiv:hep-ph/0212372]

[F. Aslan et a., Phys. Rev. D 98, 014038 (2018), arXiv:1802.06243]

Cichy’s talk

First exploration of twist-3 GPDs

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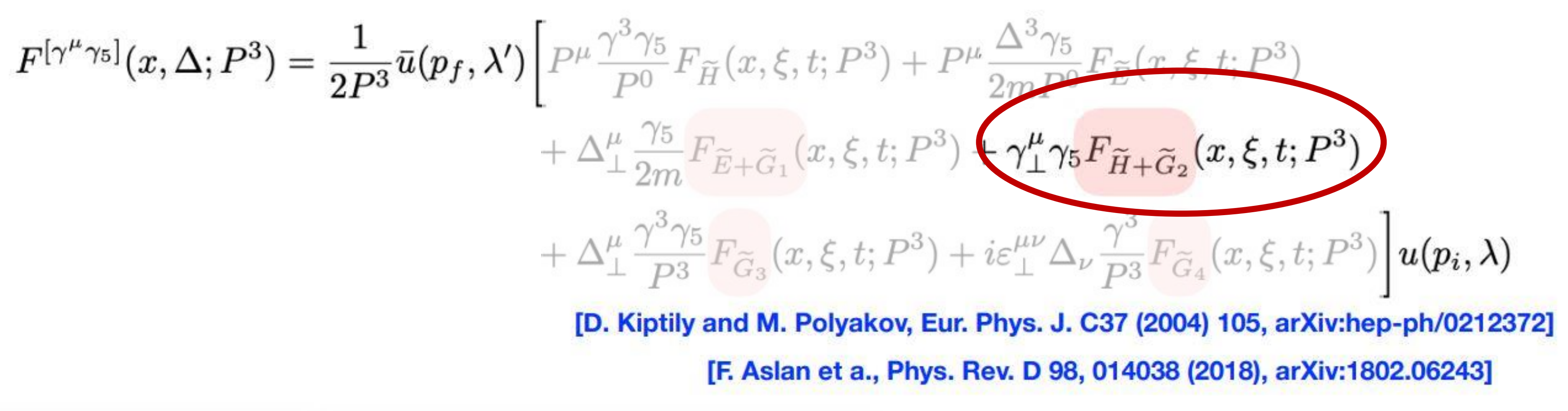

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[P^\mu \frac{\gamma^3\gamma_5}{P^0} F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3) + \Delta_\perp^\mu \frac{\gamma_5}{2m} F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3) + \Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)$$

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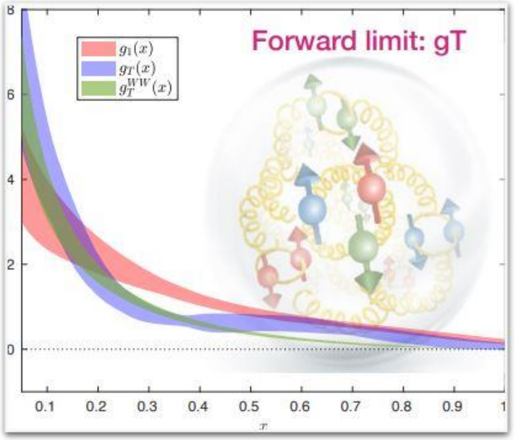
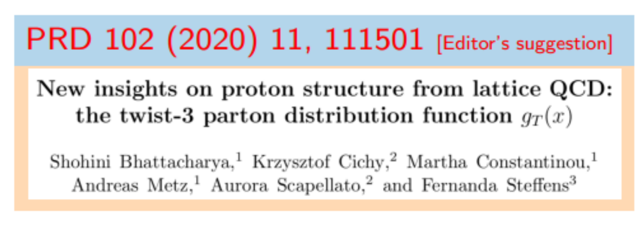


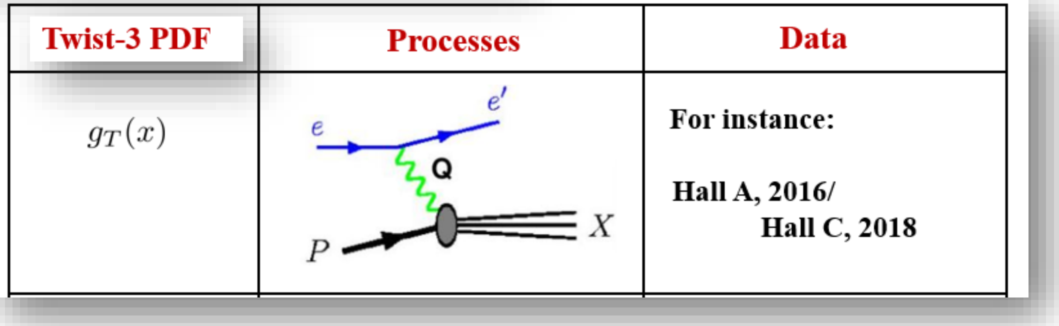
[S. Bhattacharya et al., PRD 102 (2020) 11]



PRD 102 (2020) 11, 111501 [Editor’s suggestion]

New insights on proton structure from lattice QCD: the twist-3 parton distribution function $g_T(x)$

Shohini Bhattacharya,[1] Krzysztof Cichy,[2] Martha Constantinou,[1] Andreas Metz,[1] Aurora Scapellato,[2] and Fernanda Steffens[3]

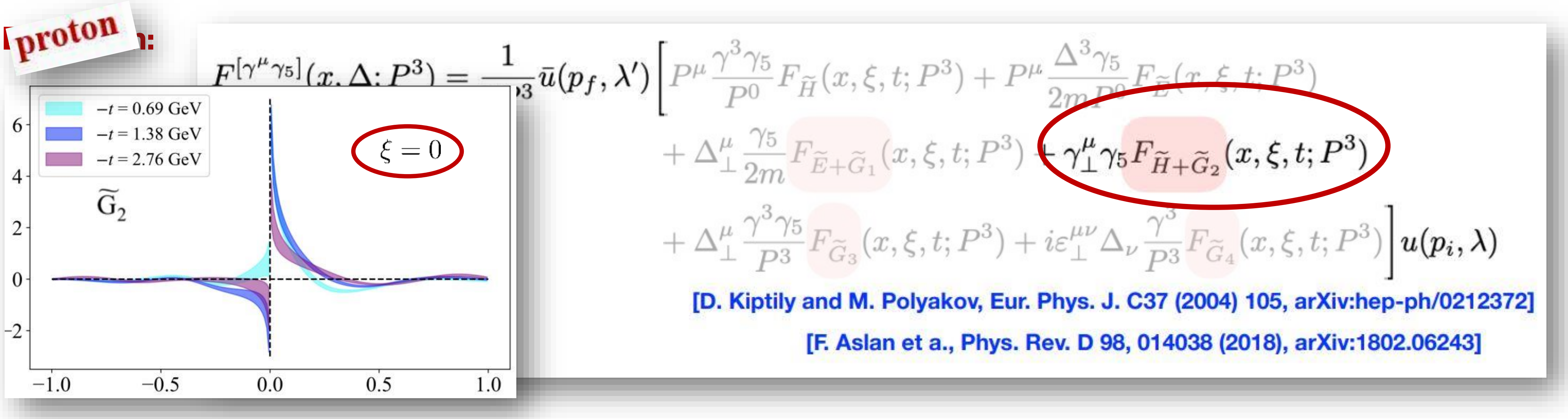


Twist-3 PDF	Processes	Data
$g_T(x)$	e e' Q P X	For instance: Hall A, 2016/ Hall C, 2018

Cichy's talk

First exploration of twist-3 GPDs

proton



$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$
$$+ \Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3)$$
$$+ \Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)$$

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Cichy's talk

First exploration of twist-3 GPDs

proton

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$$+\Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t$$

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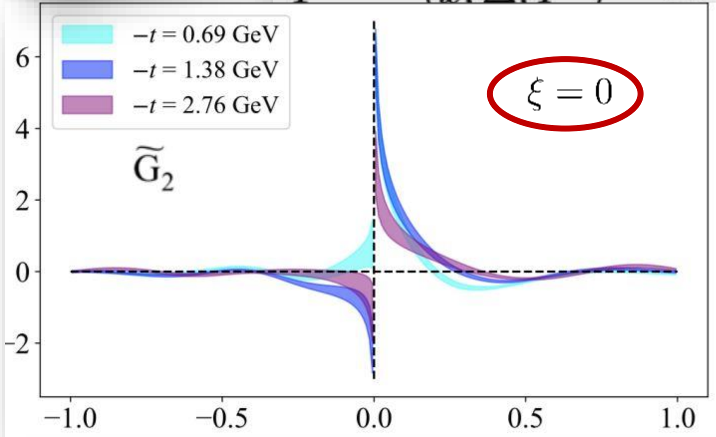
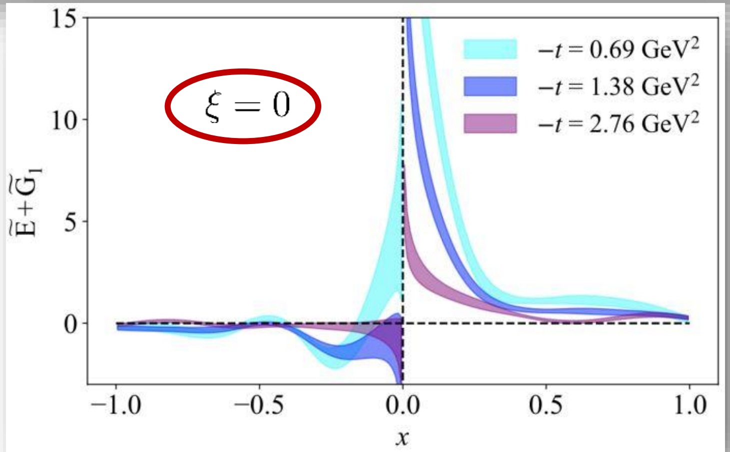


GPD $\widetilde{E}$ can not be accessed at zero skewness because it simply does not contribute to the matrix element at this point

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105, arXiv:hep-ph/0212372]

[F. Aslan et a., Phys. Rev. D 98, 014038 (2018), arXiv:1802.06243]

Glimpse into GPD $\widetilde{E}$ through twist 3 at zero skewness:



Cichy’s talk

First exploration of twist-3 GPDs

proton

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$

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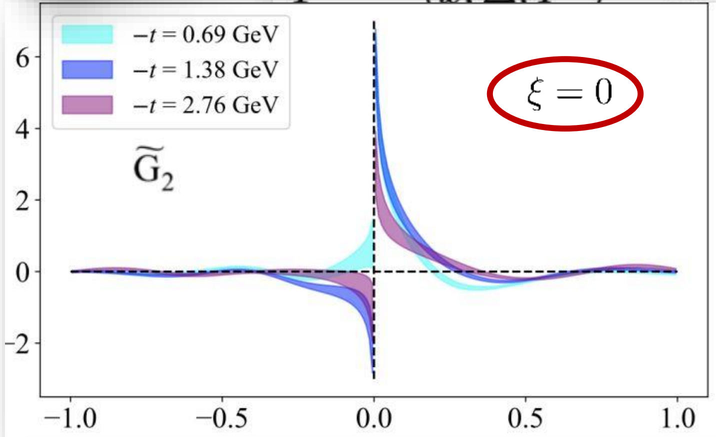

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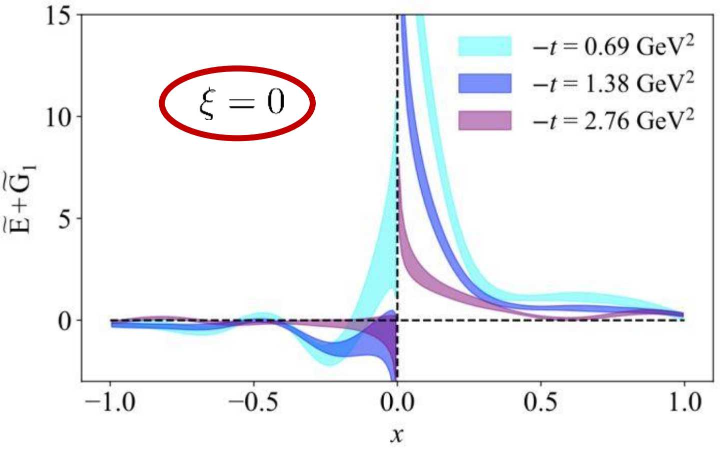

First indication of pion pole from Lattice QCD!

$$\tilde{E}_u - \tilde{E}_d \sim \frac{1}{l^2 - m_\pi^2}$$

(Penttinen, Polyakov, Goeke)

Cichy’s talk

First exploration of twist-3 GPDs

proton n:

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3) + \Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3) + \Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)$$

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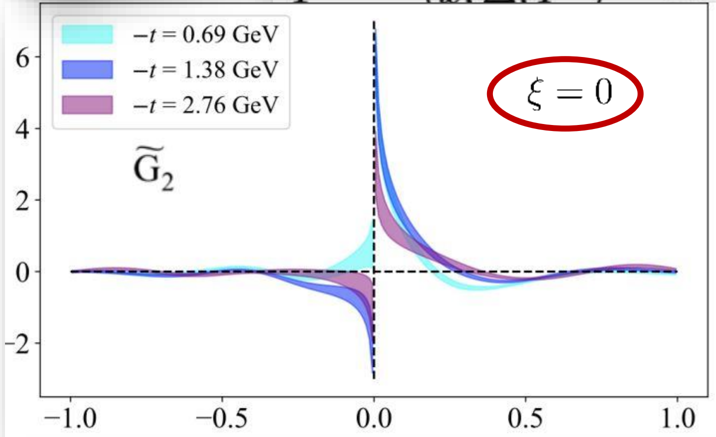
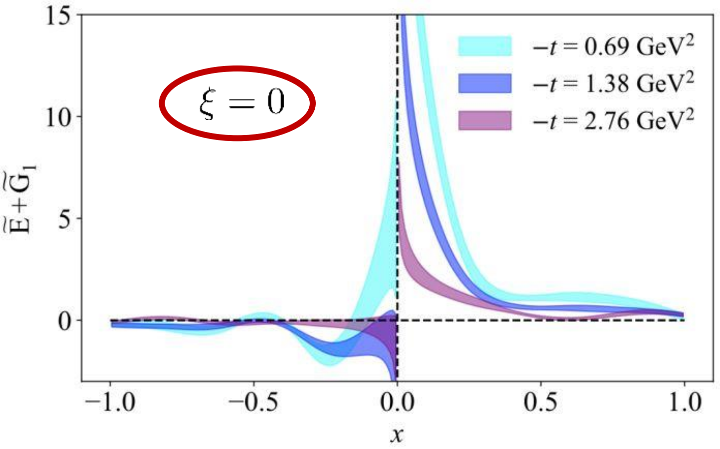
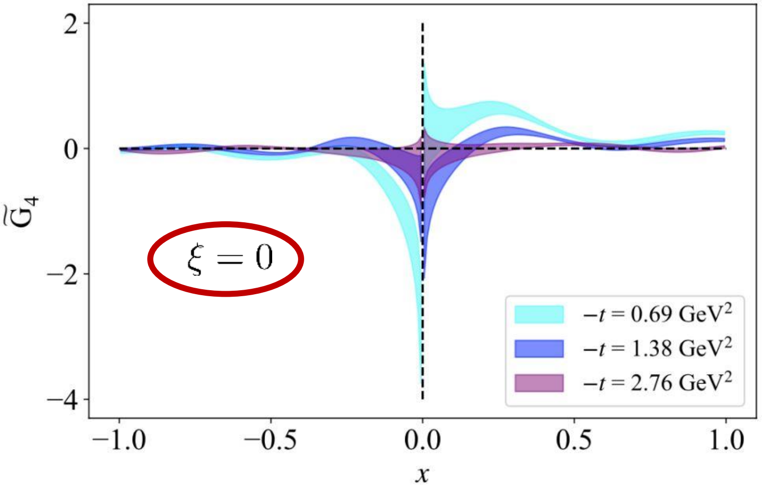


Glimpse into GPD $\widetilde{E}$ through twist 3 at zero skewness:



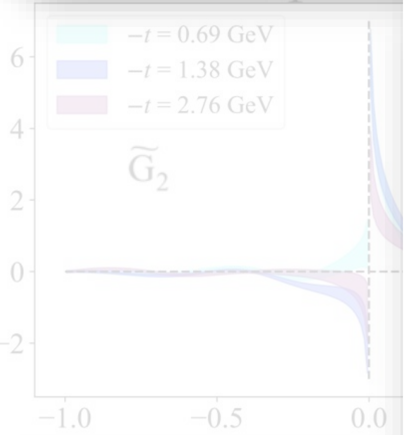


GPD $\widetilde{G}_3$ is zero within uncertainties (not shown)

But little hiccup ...

GPDs from asymmetric frames

Traditionally, GPDs have been calculated from “symmetric frames”



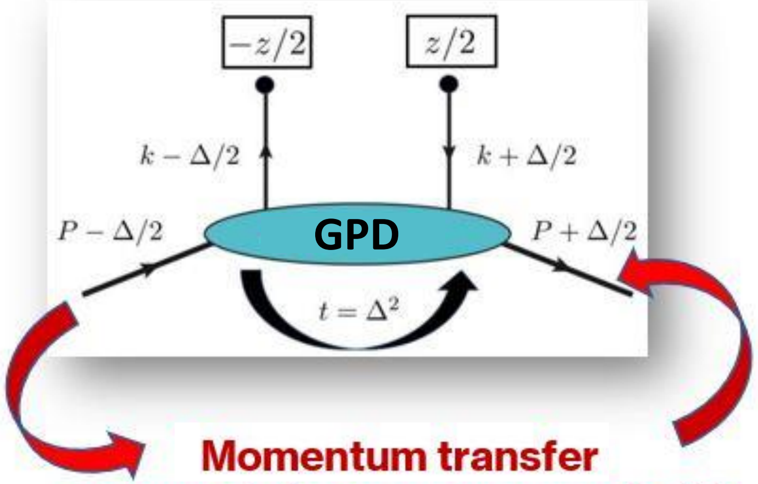
$u(p_i, \lambda)$

Xiv:hep-ph/0212372]

Xiv:1802.06243]

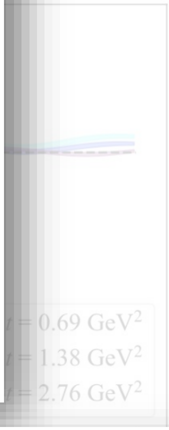
Glimpse into GPD

Practical drawback



Momentum transfer symmetric between source & sink

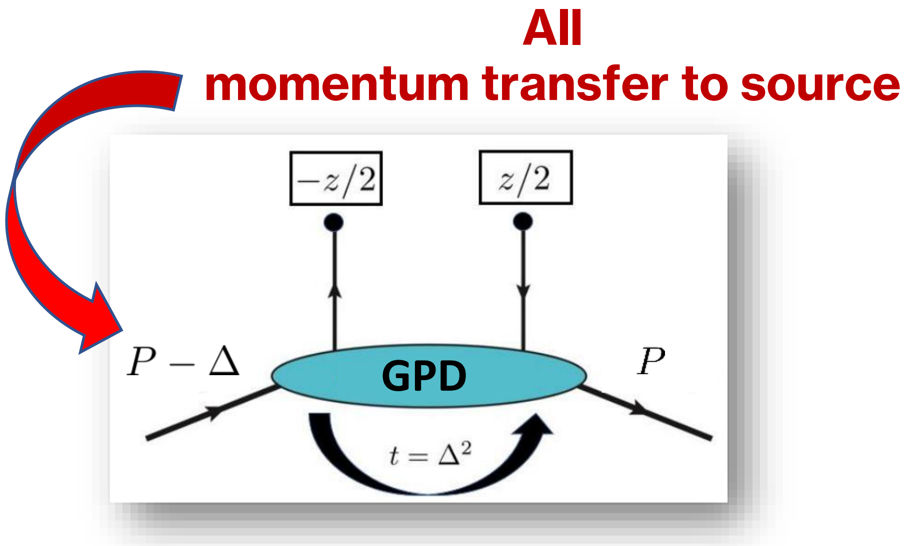
Lattice QCD calculations of GPDs in symmetric frames are expensive



In symmetric frame, full new calculation required for each momentum transfer (△)

GPDs from asymmetric frames

Resolution:

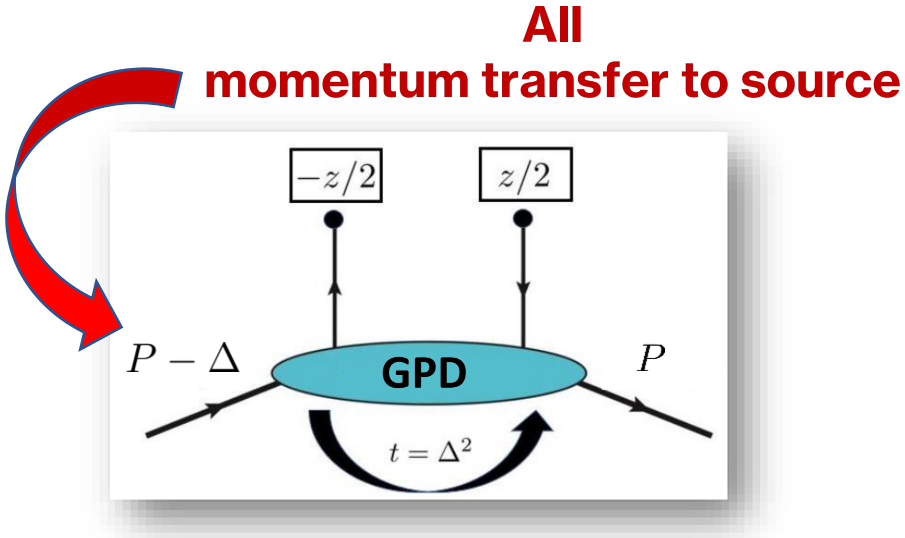


Perform Lattice QCD calculations of GPDs in asymmetric frames: Constantinou's talk

- **Reduction in computational cost**
- **Access to broad range of t (enabling creation of high-resolution partonic maps)**

GPDs from asymmetric frames

Resolution:



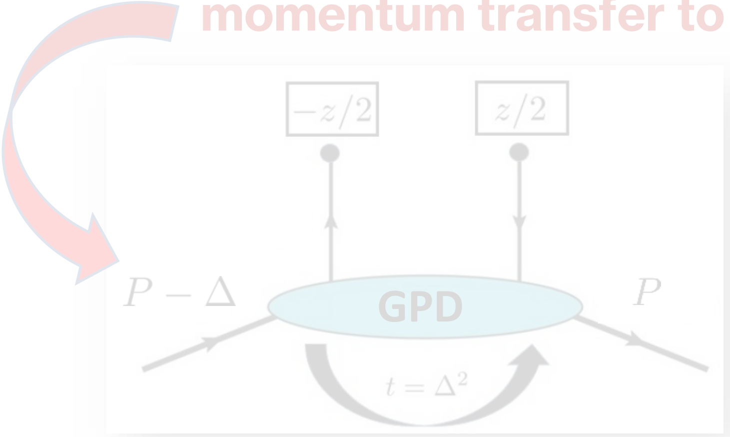
Major theoretical advances (2209.05373):

- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**
- **Elimination of power corrections potentially allowing faster convergence to light-cone GPDs**

GPDs from asymmetric frames

Resolution:

All
momentum transfer to source



Major theoretical advances:

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GPDs from asymmetric frames

Example

Lorentz covariant formalism

Novel parameterization of position-space matrix element: (Inspired from Meissner, Metz, Schlegel, 2009)

$$F^\mu(z,P,\Delta) = \bar{u}(p_f,\lambda')\left[\frac{P^\mu}{m}\boldsymbol{A_1} + mz^\mu\boldsymbol{A_2} + \frac{\Delta^\mu}{m}\boldsymbol{A_3} + im\sigma^{\mu z}\boldsymbol{A_4} + \frac{i\sigma^{\mu\Delta}}{m}\boldsymbol{A_5} + \frac{P^\mu i\sigma^{z\Delta}}{m}\boldsymbol{A_6} + mz^\mu i\sigma^{z\Delta}\boldsymbol{A_7} + \frac{\Delta^\mu i\sigma^{z\Delta}}{m}\boldsymbol{A_8}\right]u(p_i,\lambda)$$

Vector operator $F^\mu_{\lambda,\lambda'} = \langle p',\lambda'|\bar{q}(-z/2)\gamma^\mu q(z/2)|p,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$

Features:

- **8** linearly-independent Dirac structures
- **8** Lorentz-invariant (frame-independent) amplitudes $\boldsymbol{A_i \equiv A_i(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2)}$

GPDs from asymmetric frames

Example

Lorentz covariant formalism

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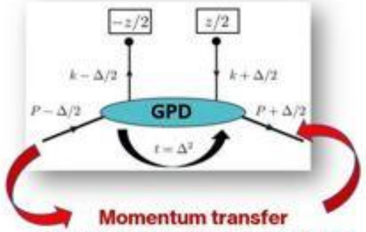
Vect

Featu

Traditional definition (symmetric frame):



Momentum transfer symmetric between source & sink

$$\begin{aligned}F^{0}_{\lambda,\lambda'}\Big|_s &= \langle p'_s,\lambda'|\bar{q}(-z/2)\gamma^0 q(z/2)|p_s,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}\\ &=\bar{u}_s(p'_s,\lambda')\left[\gamma^0 H_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\Big|_s+\frac{i\sigma^{0\mu}\Delta_{\mu,s}}{2M}E_{\mathrm{Q}(0)}(z,P_s,\Delta_s)\Big|_s\right]u_s(p_s,\lambda)\end{aligned}$$

Quasi-GPDs are intrinsically frame-dependent

GPDs from asymmetric frames

Lorentz covariant formalism

Novel para

Main point:

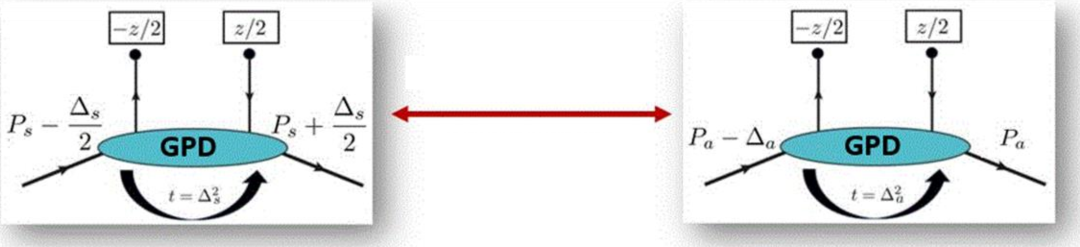
n space matrix element: (Inspired from Meissner, Metz, Schlegel, 2009)

$$H^{s}_{\mathrm{Q}(0)} = \sum_i A_i$$

$F^\mu(z,P,\Delta) = \bar{u}(p_f,\lambda')\Big[\frac{P^\mu}{m}A_1 + mz^\mu$

$\frac{P^\mu i\sigma^{z\Delta}}{m}A_6 + mz^\mu i\sigma^{z\Delta}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{m}A_8\Big]u(p_i,\lambda)$

Calculate quasi-GPD in symmetric frame through matrix elements of asymmetric frame



Symmetric frame

Asymmetric frame

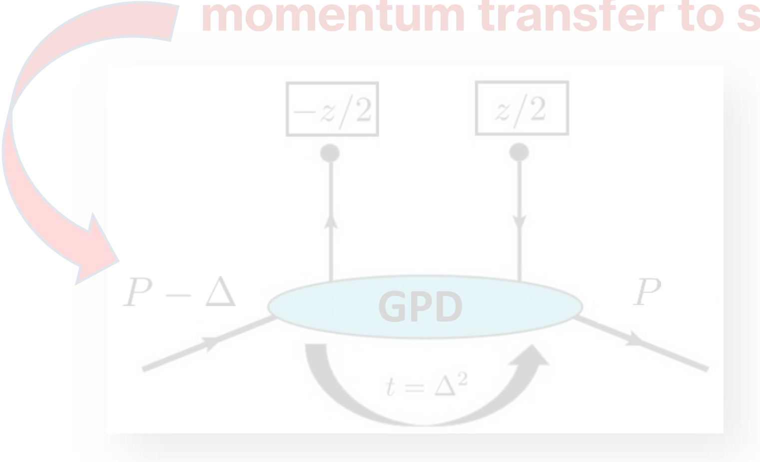
- **8** Lorentz

$A_i(z \cdot P, z \cdot \Delta, t = \Delta^2, z^2)$

GPDs from asymmetric frames

Resolution:

All
momentum transfer to source



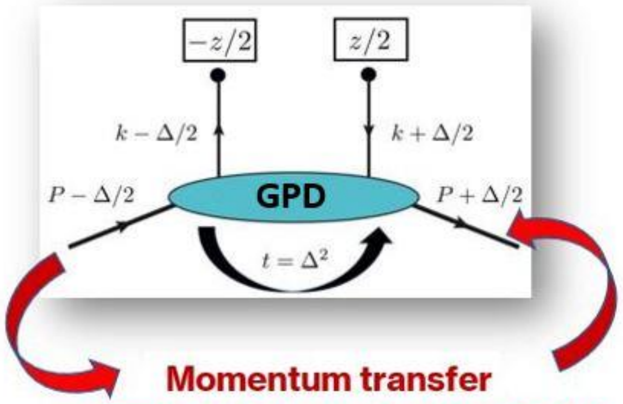
Major theoretical advances:

- Lorentz covariant formalism for calculating quasi-GPDs in any frame
- **Elimination of power corrections potentially allowing faster convergence to light-cone GPDs**

GPDs from asymmetric frames

Relations between GPDs & amplitudes

Example: Symmetric frame



Momentum transfer
symmetric between source & sink

Quasi-GPD:

$$H^s_{\mathrm{Q}(0)}(z, P^s, \Delta^s) = A_1 + \frac{\Delta^{0,s}}{P^{0,s}} A_3 - \frac{m^2 \Delta^{0,s} z^3}{2P^{0,s} P^{3,s}} A_4 + \left[\frac{(\Delta^{0,s})^2 z^3}{2P^{3,s}} - \frac{\Delta^{0,s} \Delta^{3,s} z^3 P^{0,s}}{2(P^{3,s})^2} - \frac{z^3 (\Delta^s_\perp)^2}{2P^{3,s}} \right] A_6$$
$$+ \left[\frac{(\Delta^{0,s})^3 z^3}{2P^{0,s} P^{3,s}} - \frac{(\Delta^{0,s})^2 \Delta^{3,s} z^3}{2(P^{3,s})^2} - \frac{\Delta^{0,s} z^3 (\Delta^s_\perp)^2}{2P^{0,s} P^{3,s}} \right] A_8 \,,$$

GPDs from asymmetric frames

Relations between GPDs & amplitudes

Light-cone GPD: (Lorentz-invariant)

$$H(z\cdot P^{s/a}, z\cdot\Delta^{s/a}, (\Delta^{s/a})^2) = A_1 + \frac{\Delta^{s/a}\cdot z}{P^{s/a}\cdot z}A_3$$

Quasi-GPD: (Symmetric frame)

$$H^{s}_{\mathrm{Q}(0)}(z, P^s, \Delta^s) = A_1 + \frac{\Delta^{0,s}}{P^{0,s}}A_3 - \frac{m^2\Delta^{0,s}z^3}{2P^{0,s}P^{3,s}}A_4 + \left[\frac{(\Delta^{0,s})^2 z^3}{2P^{3,s}} - \frac{\Delta^{0,s}\Delta^{3,s}z^3 P^{0,s}}{2(P^{3,s})^2} - \frac{z^3(\Delta^s_\perp)^2}{2P^{3,s}}\right]A_6$$
$$+\left[\frac{(\Delta^{0,s})^3 z^3}{2P^{0,s}P^{3,s}} - \frac{(\Delta^{0,s})^2\Delta^{3,s}z^3}{2(P^{3,s})^2} - \frac{\Delta^{0,s}z^3(\Delta^s_\perp)^2}{2P^{0,s}P^{3,s}}\right]A_8\,,$$

GPDs from asymmetric frames

Relations between GPDs & amplitudes

Light-cone GPD: (Lorentz-invariant)

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Contamination from additional amplitudes or explicit power corrections

Quasi-GPD: (Symmetric frame)

$$H^s_{\mathrm{Q}(0)}(z, P^s, \Delta^s) = A_1 + \frac{\Delta^{0,s}}{P^{0,s}} A_3 - \frac{m^2 \Delta^{0,s} z^3}{2P^{0,s} P^{3,s}} A_4 + \left[\frac{(\Delta^{0,s})^2 z^3}{2P^{3,s}} - \frac{\Delta^{0,s} \Delta^{3,s} z^3 P^{0,s}}{2(P^{3,s})^2} - \frac{z^3 (\Delta^s_\perp)^2}{2P^{3,s}} \right] A_6$$

$$+ \left[\frac{(\Delta^{0,s})^3 z^3}{2P^{0,s} P^{3,s}} - \frac{(\Delta^{0,s})^2 \Delta^{3,s} z^3}{2(P^{3,s})^2} - \frac{\Delta^{0,s} z^3 (\Delta^s_\perp)^2}{2P^{0,s} P^{3,s}} \right] A_8 \,,$$

GPDs from asymmetric frames

Relations between GPDs & amplitudes

Light-cone GI

You can think of eliminating additional amplitudes by the addition of other operators

(γ^1, γ^2)

Contamination from additional amplitudes or explicit power corrections

Quasi-GPD: (Symmetric frame)

$$H^s_{\mathrm{Q}(0)}(z, P^s, \Delta^s) = A_1 + \frac{\Delta^{0,s}}{P^{0,s}}A_3 - \frac{m^2\Delta^{0,s}z^3}{2P^{0,s}P^{3,s}}A_4 + \left[\frac{(\Delta^{0,s})^2 z^3}{2P^{3,s}} - \frac{\Delta^{0,s}\Delta^{3,s}z^3P^{0,s}}{2(P^{3,s})^2} - \frac{z^3(\Delta^s_\perp)^2}{2P^{3,s}}\right]A_6$$

$$+ \left[\frac{(\Delta^{0,s})^3 z^3}{2P^{0,s}P^{3,s}} - \frac{(\Delta^{0,s})^2\Delta^{3,s}z^3}{2(P^{3,s})^2} - \frac{\Delta^{0,s}z^3(\Delta^s_\perp)^2}{2P^{0,s}P^{3,s}}\right]A_8\,,$$

GPDs from asymmetric frames

Relations between GPDs & amplitudes

Lig

Main finding

Schematic structure of (operator-level) Lorentz-invariant definition of quasi-GPD:

$$\boxed{H_{\mathrm{Q}} \to c_0\langle\bar{\psi}\gamma^0\psi\rangle + c_1\langle\bar{\psi}\gamma^1\psi\rangle + c_2\langle\bar{\psi}\gamma^2\psi\rangle}$$

Here, c's are frame-dependent kinematic factors that cancel additional amplitudes such that quasi-GPD has the same functional form as light-cone GPD (Lorentz invariant)

Qua

$$H^s_{\mathrm{Q}(0)}(z, P^s, \Delta^s) = A_1 + \frac{\Delta^{0,s}}{P^{0,s}}A_3 - \frac{m^2\Delta^{0,s}z^3}{2P^{0,s}P^{3,s}}A_4 + \left[\frac{(\Delta^{0,s})^2z^3}{2P^{3,s}} - \frac{\Delta^{0,s}\Delta^{3,s}z^3P^{0,s}}{2(P^{3,s})^2} - \frac{z^3(\Delta^s_\perp)^2}{2P^{3,s}}\right]A_6$$

$$+ \left[\frac{(\Delta^{0,s})^3z^3}{2P^{0,s}P^{3,s}} - \frac{(\Delta^{0,s})^2\Delta^{3,s}z^3}{2(P^{3,s})^2} - \frac{\Delta^{0,s}z^3(\Delta^s_\perp)^2}{2P^{0,s}P^{3,s}}\right]A_8\,,$$

GPDs from asymmetric frames

New definition of quasi-GPDs

Light-cone GPD:

$$\boxed{H(z \cdot P^{s/a}, z \cdot \Delta^{s/a}, (\Delta^{s/a})^2) = A_1 + \frac{\Delta^{s/a} \cdot z}{P^{s/a} \cdot z} A_3}$$

$$A_i \equiv A_i(z^2 = 0)$$

Lorentz-invariant definition of quasi-GPD:

$$\boxed{\mathcal{H}(z \cdot P^{s/a}, z \cdot \Delta^{s/a}, (\Delta^{s/a})^2, z^2) = A_1 + \frac{\Delta^{s/a} \cdot z}{P^{s/a} \cdot z} A_3}$$

$$A_i \equiv A_i(z^2 \neq 0)$$

Same functional forms

GPDs from asymmetric frames

New definition of quasi-GPDs

Light-cone GPD:

$$H(z\cdot P^{s/a}, z\cdot\Delta^{s/a}, (\Delta^{s/a})^2) = A_1 + \frac{\Delta^{s/a}\cdot z}{P^{s/a}\cdot z}A_3$$

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Lorentz-invariant definition of quasi-GPD:

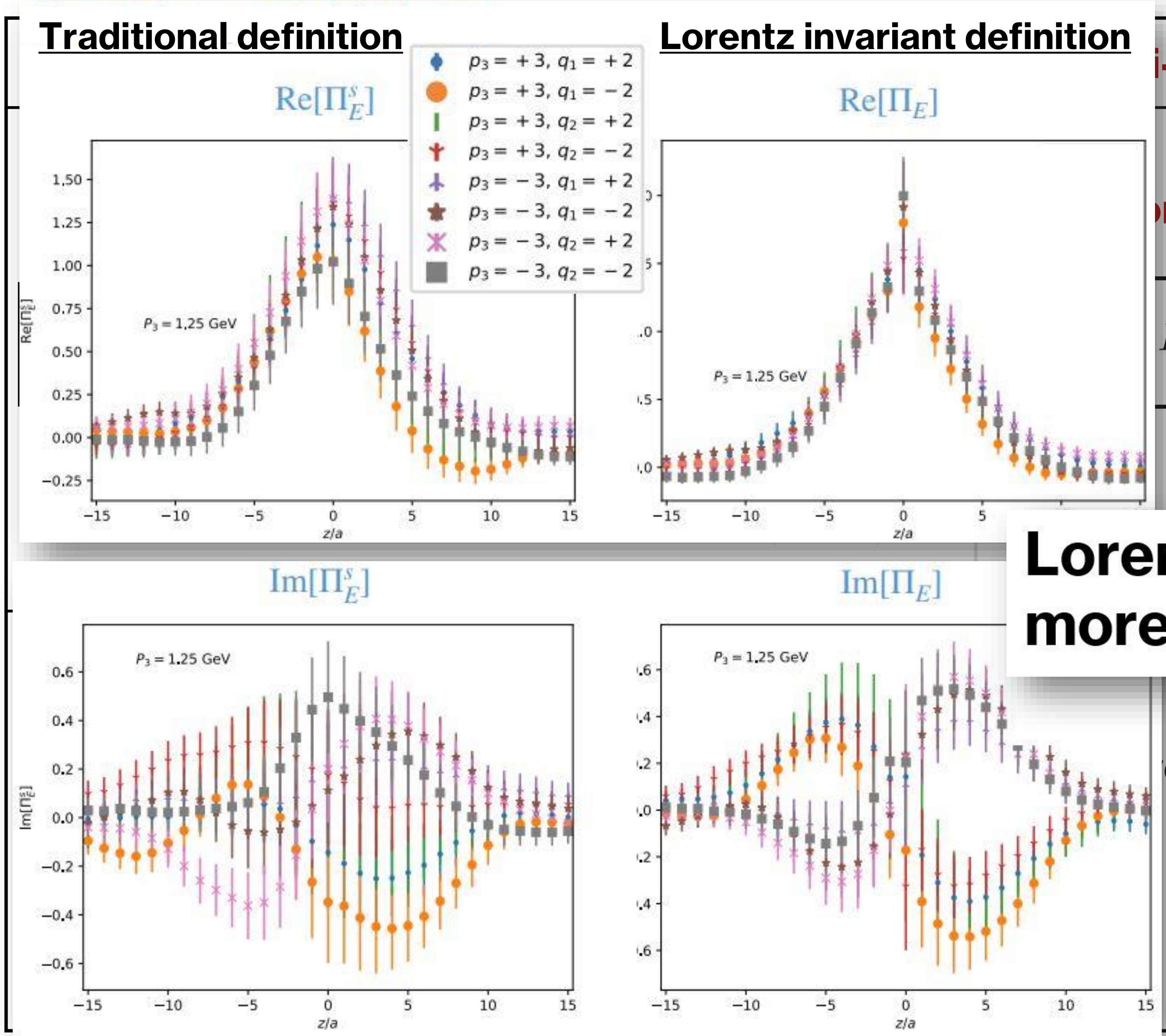
$$\mathcal{H}(z\cdot P^{s/a}, z\cdot\Delta^{s/a}, (\Delta^{s/a})^2, z^2) = A_1 + \frac{\Delta^{s/a}\cdot z}{P^{s/a}\cdot z}A_3$$

$$A_i \equiv A_i(z^2 \neq 0)$$

Feature:

- **Lorentz-invariant definition of quasi-GPDs may converge faster**

GPDs from asymmetric frames



i-GPDs

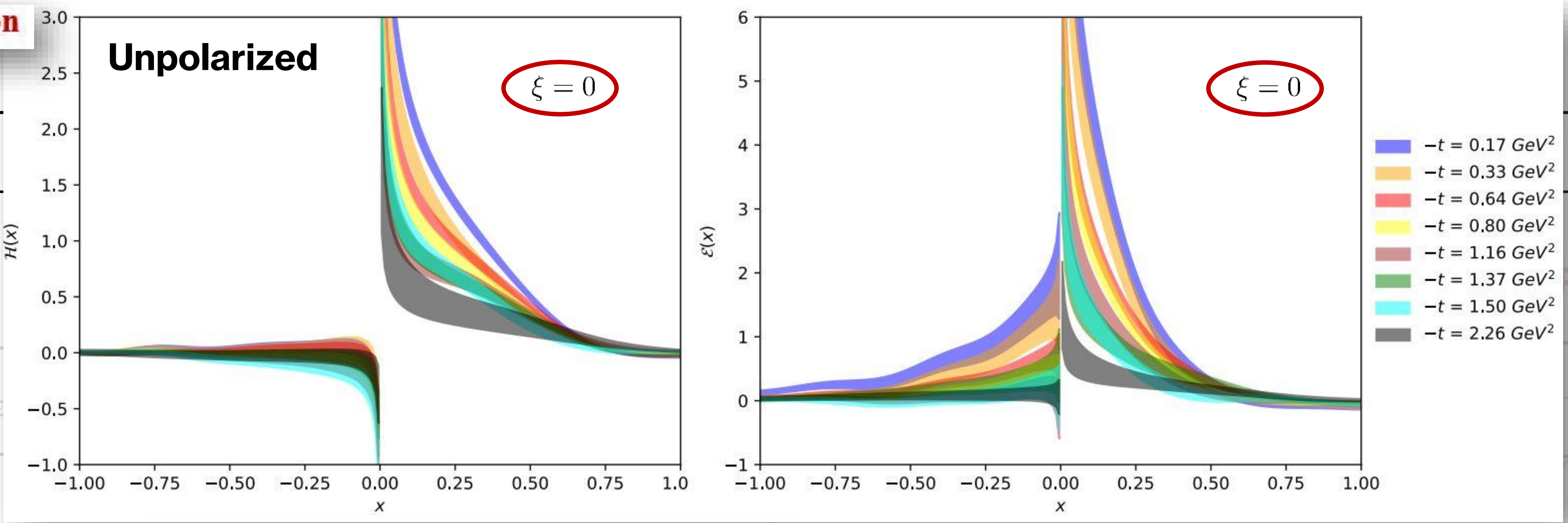
rentz-invariant definition of quasi-GPD:

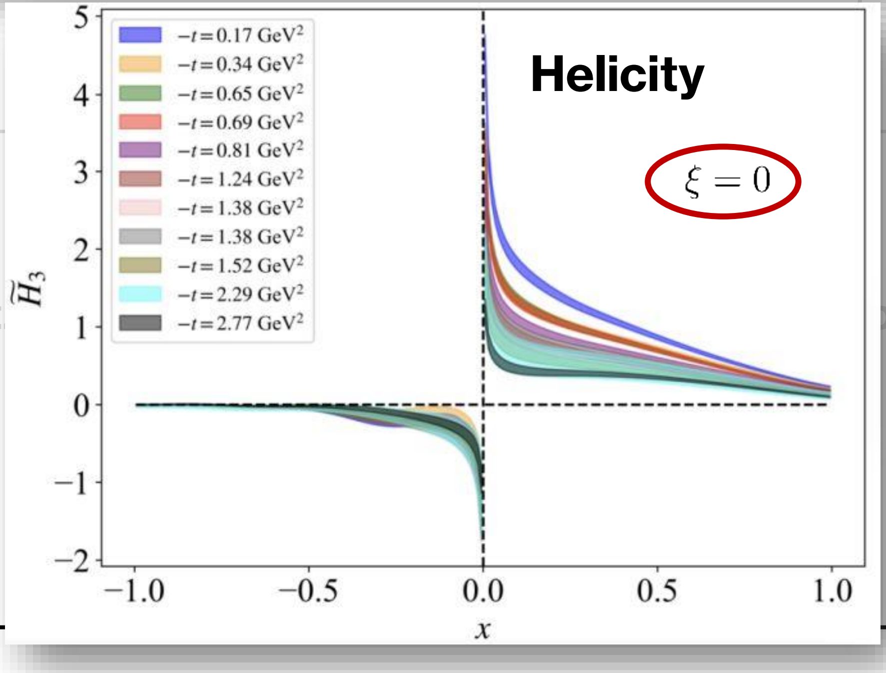
$$P^{s/a}, z\cdot\Delta^{s/a}, (\Delta^{s/a})^2, z^2) = A_1 + \frac{\Delta^{s/a}\cdot z}{P^{s/a}\cdot z}A_3$$

Lorentz invariant definition leads to more precise results for GPD E

erge faster

proton





$A_i \equiv A_i(z^2 \neq 0)$

Feature:

- Lorent

Constantinou’s talk

GPDs derived from asymmetric frames within the amplitude formalism

GPDs from asymmetric frames

New definition of quasi-GPDs

Shi's talk

Light-co GPD:

Lorentz-invariant definition of quasi-GPD:

$H(z\cdot P^{s/a}, z\cdot\Delta^{s/a}, (\Delta$

$)^2, z^2) = A_1 + \frac{\Delta^{s/a}\cdot z}{P^{s/a}\cdot z} A_3$

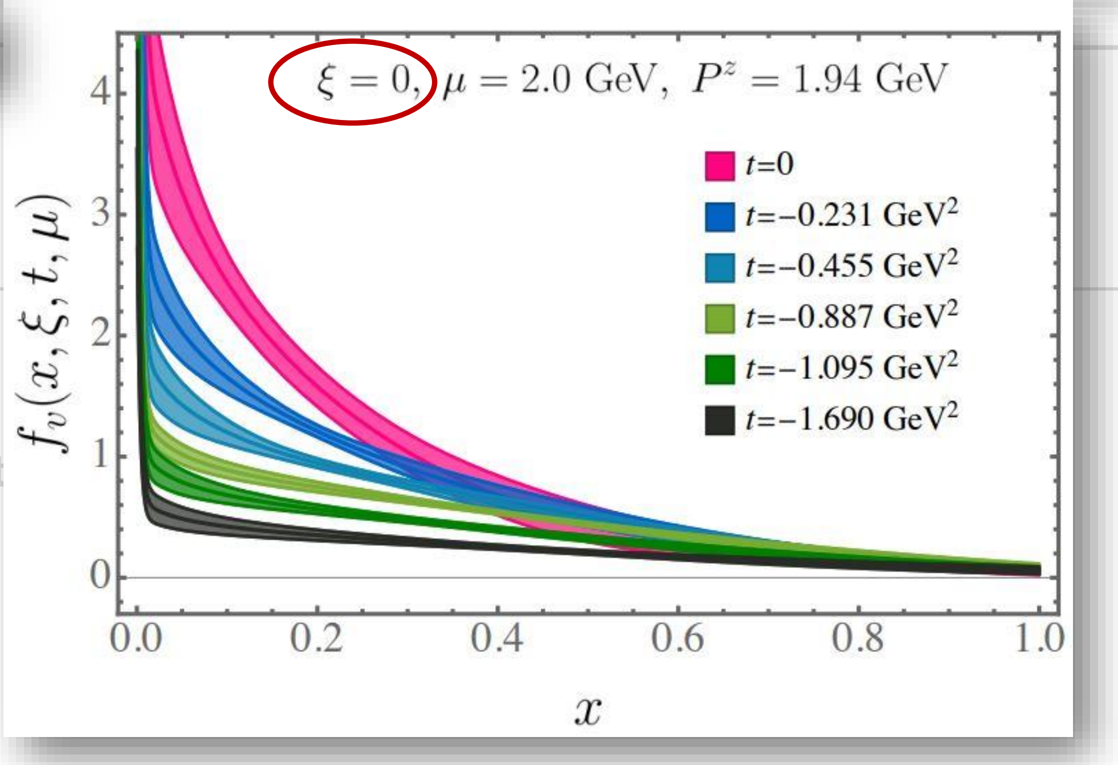
$A_i \equiv A_i(z^2 \neq 0)$

Feature:

- Lorentz-invariant definit

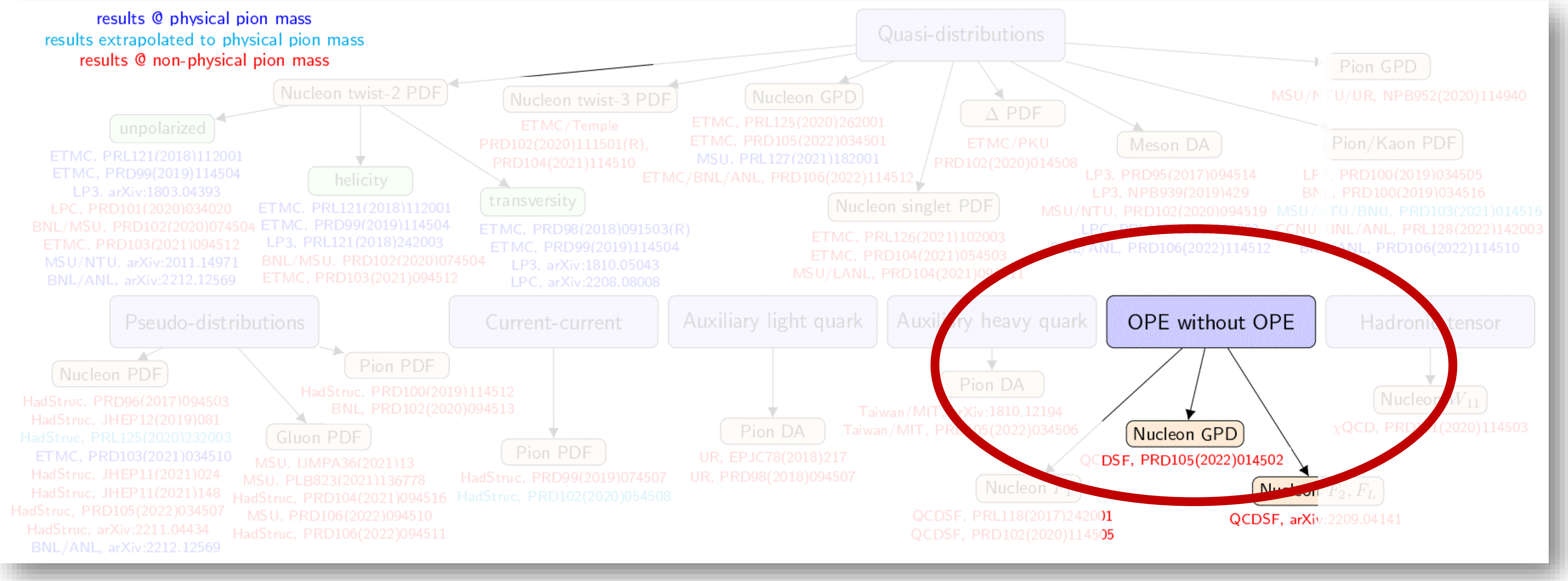
GPDs derived from asymmetric frames within the amplitude formalism

pion



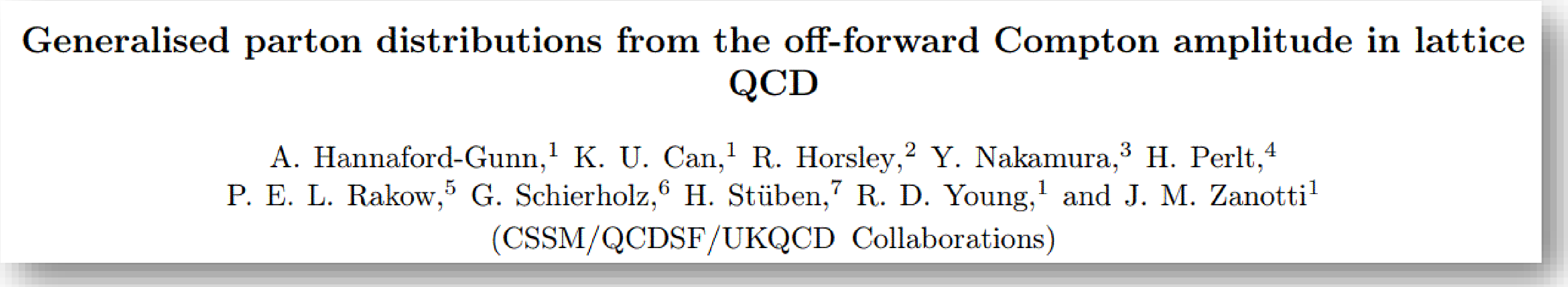
Dynamical Progress of Lattice QCD calculations of PDFs/GPDs

Lattice QCD calculations of x-dependence of PDFs & related quantities using Euclidean correlators:



Compilation by Cichy, 2110.07440

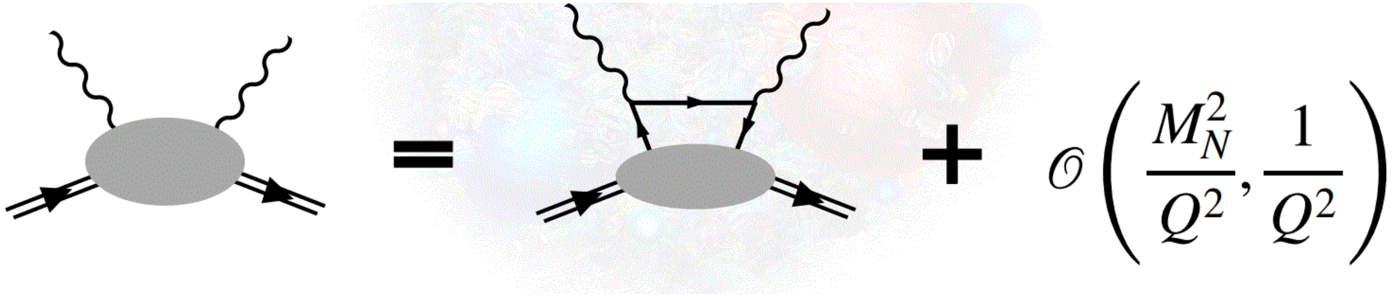
Compton amplitude in Lattices



Generalised parton distributions from the off-forward Compton amplitude in lattice QCD

A. Hannaford-Gunn,[1] K. U. Can,[1] R. Horsley,[2] Y. Nakamura,[3] H. Perlt,[4]
P. E. L. Rakow,[5] G. Schierholz,[6] H. Stüben,[7] R. D. Young,[1] and J. M. Zanotti[1]
(CSSM/QCDSF/UKQCD Collaborations)

Example: **Forward Compton amplitude**

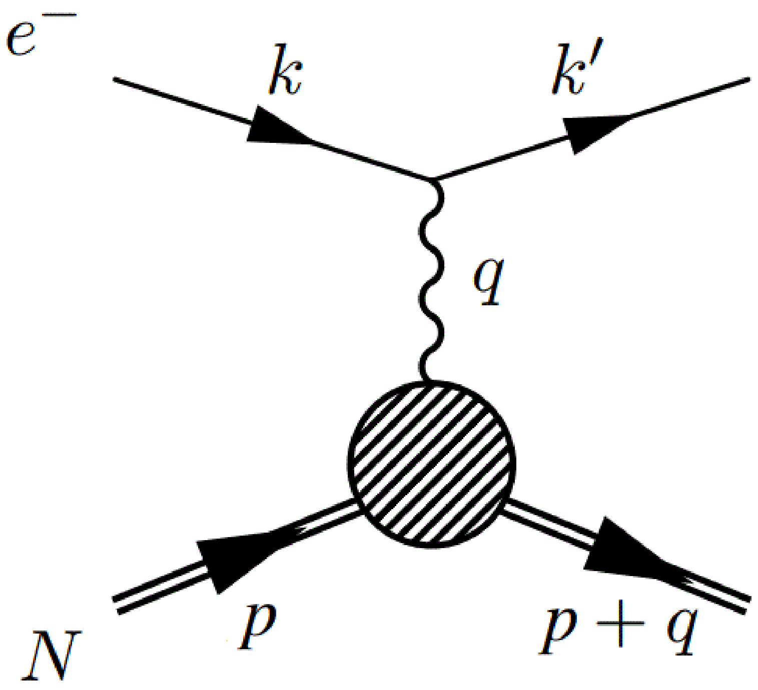


$$= \quad + \ \mathcal{O}\left(\frac{M_N^2}{Q^2}, \frac{1}{Q^2}\right)$$

Courtesy: Utku Can

Compton amplitude in Lattices

Deep Inelastic Scattering (DIS)



DIS & Hadronic Tensor:

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x, Q^2) + \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p \cdot q}{q^2} q_\nu\right) \frac{F_2(x, Q^2)}{p \cdot q}$$

Structure Functions

Compton amplitude in Lattices

Forward Compton amplitude:

$$\begin{aligned}T_{\mu\nu}(p,q) &= i\int d^4z\, e^{iq\cdot z}\rho_{ss'}\langle p,s'|\mathscr{T}\{J_\mu(z)J_\nu(0)\}\,|p,s\rangle \\ &= \left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)\mathscr{F}_1(\omega,Q^2)+\left(p_\mu-\frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu-\frac{p\cdot q}{q^2}q_\nu\right)\frac{\mathscr{F}_2(\omega,Q^2)}{p\cdot q}\end{aligned}$$

Compton Structure Functions (SF)

Same Lorentz decomposition as the Hadronic tensor

Compton amplitude in Lattices

Forward Compton amplitude:

$$T_{\mu\nu}(p,q) = i\int d^4z\, e^{iq\cdot z}\rho_{ss'}\langle p,s'|\mathcal{T}\{J_\mu(z)J_\nu(0)\}|p,s\rangle$$

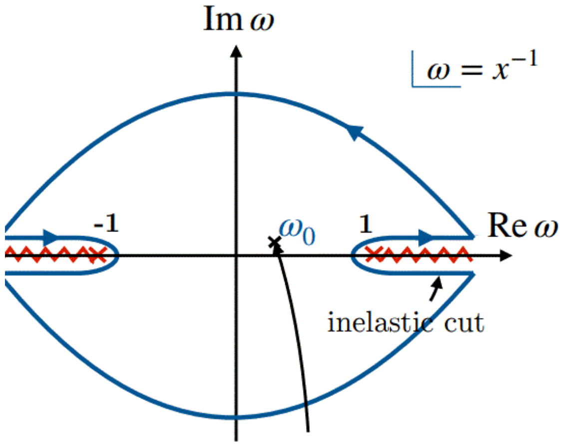
$$= \left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)\mathcal{F}_1(\omega,Q^2) + \left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right)\frac{\mathcal{F}_2(\omega,Q^2)}{p\cdot q}$$

Compton Structure Functions (SF)

Dispersion relations connecting Compton SFs to DIS SFs:

$$\underbrace{\mathcal{F}_1(\omega,Q^2)-\mathcal{F}_1(0,Q^2)}_{\equiv\overline{\mathcal{F}}_1(\omega,Q^2)} = 2\omega^2\int_0^1 dx\frac{2x\,F_1(x,Q^2)}{1-x^2\omega^2-i\epsilon}$$

$$\mathcal{F}_2(\omega,Q^2) = 4\omega\int_0^1 dx\frac{F_2(x,Q^2)}{1-x^2\omega^2-i\epsilon}$$



Compton Amplitude is an analytic function in the unphysical region $|\omega_0| < 1$

Courtesy: Utku Can

Compton amplitude in Lattices

Forward Compton amplitude:

$$T_{\mu\nu}(p,q) = i\int d^4z\, e^{iq\cdot z}\rho_{ss'}\langle p,s'|\mathscr{T}\{J_\mu(z)J_\nu(0)\}|p,s\rangle$$
$$= \left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)\mathscr{F}_1(\omega,Q^2) + \left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu-\frac{p\cdot q}{q^2}q_\nu\right)\frac{\mathscr{F}_2(\omega,Q^2)}{p\cdot q}$$

Compton Structure Functions (SF)

Compton amplitude approach gives access to moments of DIS SFs:

Example:

$$\mathscr{F}_2(\omega,Q^2) = \sum_{n=1}^{\infty} 4\omega^{2n-1}M_{2n}^{(2)}(Q^2), \text{ where } M_{2n}^{(2,L)}(Q^2) = \int_0^1 dx\, x^{2n-2}F_{2,L}(x,Q^2)$$

Compton amplitude in Lattices

Off-forward is very similar

$$T_{\mu\nu} = \frac{1}{2\bar{P}\cdot\bar{q}}\left[-\left(h\cdot\bar{q}\mathcal{H}_1 + e\cdot\bar{q}\mathcal{E}_1\right)g_{\mu\nu} + \frac{1}{\bar{P}\cdot\bar{q}}\left(h\cdot\bar{q}\mathcal{H}_2 + e\cdot\bar{q}\mathcal{E}_2\right)\bar{P}_\mu\bar{P}_\nu + \mathcal{H}_3 h_{\{\mu}\bar{P}_{\nu\}}\right] + \dots$$

Compton amplitude:

This approach gives access to moments GPDs:

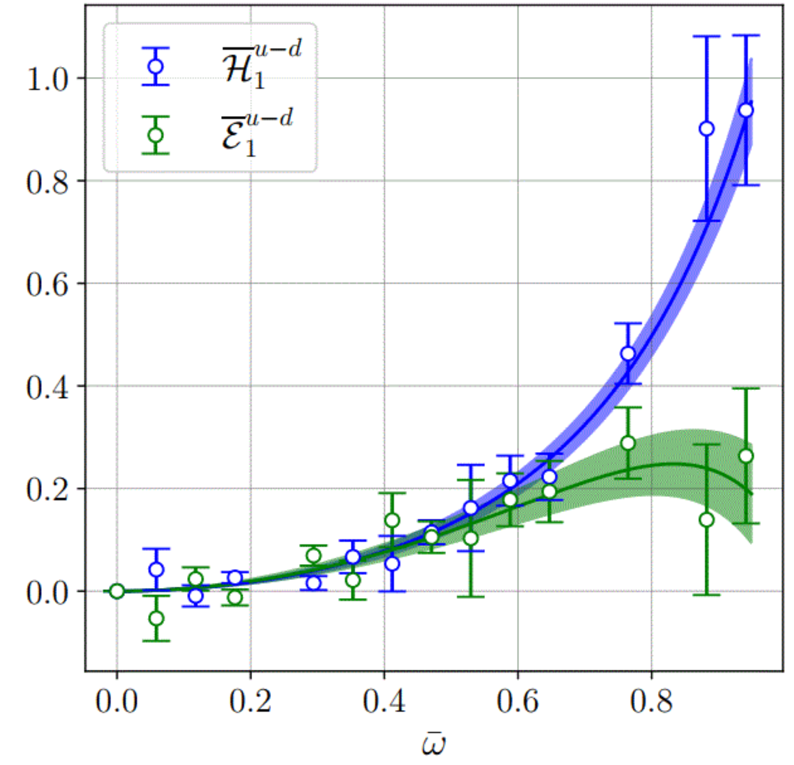
Compton amplitude approac

Fs:

Example:

$$\mathscr{F}_2(\omega, Q^2) = \sum_{n=1}^{\infty} 4\omega^{2n-1}$$

(x, Q^2)



Isovector results for $t = -0.57\ \text{GeV}^2$.

Summary

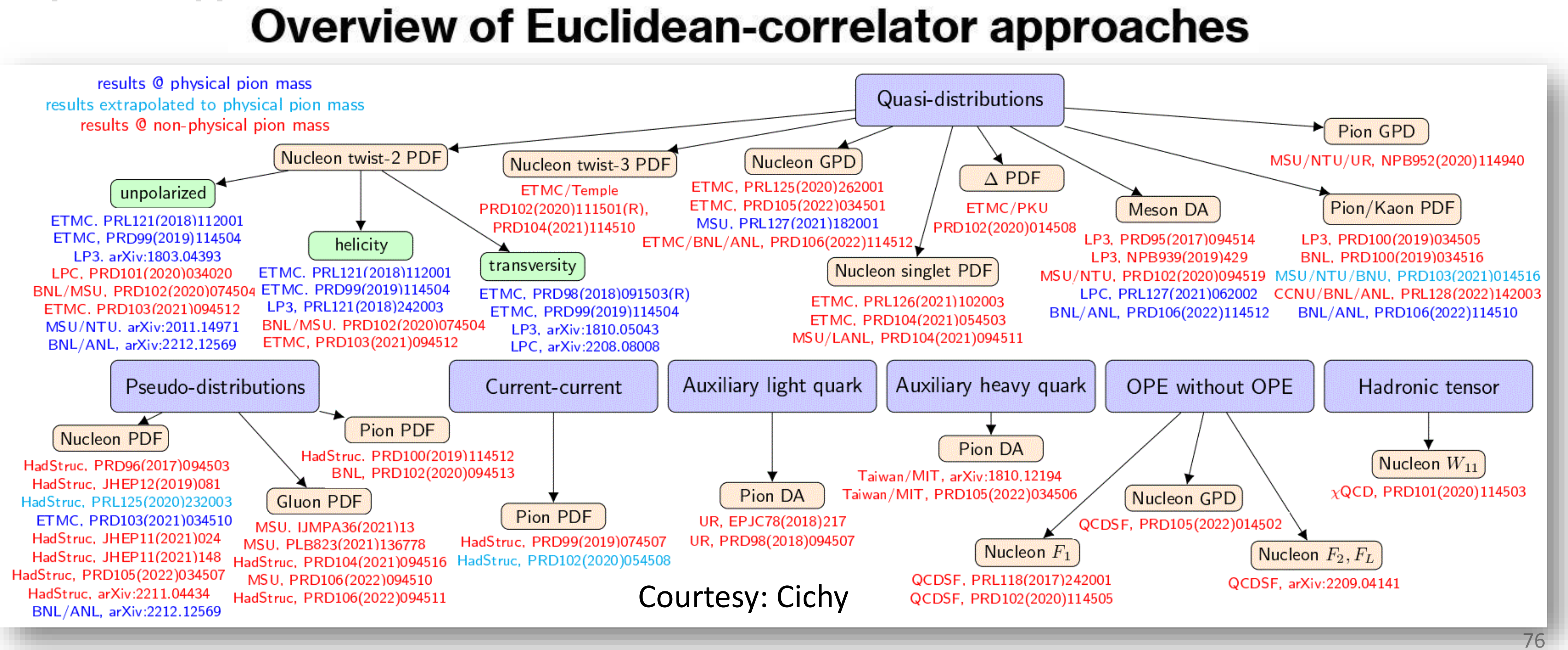
- **Tremendous recent activity in studying parton structure of hadrons in lattice QCD through Euclidean correlators**

- **Impact of approach(es) largest where experiments are difficult → GPDs**

Summary

- Tremendous recent activity in studying parton structure of hadrons in lattice QCD through Euclidean correlators

- Impact of approach(es) largest where experiments are difficult → GPDs



Overview of Euclidean-correlator approaches

results @ physical pion mass
results extrapolated to physical pion mass
results @ non-physical pion mass

Quasi-distributions

Nucleon twist-2 PDF

unpolarized
ETMC, PRL121(2018)112001
ETMC, PRD99(2019)114504
LP3, arXiv:1803.04393
LPC, PRD101(2020)034020
BNL/MSU, PRD102(2020)074504
ETMC, PRD103(2021)094512
MSU/NTU, arXiv:2011.14971
BNL/ANL, arXiv:2212.12569

helicity
ETMC, PRL121(2018)112001
ETMC, PRD99(2019)114504
LP3, PRL121(2018)242003
BNL/MSU, PRD102(2020)074504
ETMC, PRD103(2021)094512

transversity
ETMC, PRD98(2018)091503(R)
ETMC, PRD99(2019)114504
LP3, arXiv:1810.05043
LPC, arXiv:2208.08008

Nucleon twist-3 PDF
ETMC/Temple
PRD102(2020)111501(R),
PRD104(2021)114510

Nucleon GPD
ETMC, PRL125(2020)262001
ETMC, PRD105(2022)034501
MSU, PRL127(2021)182001
ETMC/BNL/ANL, PRD106(2022)114512

Nucleon singlet PDF
ETMC, PRL126(2021)102003
ETMC, PRD104(2021)054503
MSU/LANL, PRD104(2021)094511

Δ PDF
ETMC/PKU
PRD102(2020)014508

Meson DA
LP3, PRD95(2017)094514
LP3, NPB939(2019)429
MSU/NTU, PRD102(2020)094519
LPC, PRL127(2021)062002
BNL/ANL, PRD106(2022)114512

Pion GPD
MSU/NTU/UR, NPB952(2020)114940

Pion/Kaon PDF
LP3, PRD100(2019)034505
BNL, PRD100(2019)034516
MSU/NTU/BNU, PRD103(2021)014516
CCNU/BNL/ANL, PRL128(2022)142003
BNL/ANL, PRD106(2022)114510

Pseudo-distributions

Nucleon PDF
HadStruc, PRD96(2017)094503
HadStruc, JHEP12(2019)081
HadStruc, PRL125(2020)232003
ETMC, PRD103(2021)034510
HadStruc, JHEP11(2021)024
HadStruc, JHEP11(2021)148
HadStruc, PRD105(2022)034507
HadStruc, arXiv:2211.04434
BNL/ANL, arXiv:2212.12569

Pion PDF
HadStruc, PRD100(2019)114512
BNL, PRD102(2020)094513

Gluon PDF
MSU, IJMPA36(2021)13
MSU, PLB823(2021)136778
HadStruc, PRD104(2021)094516
MSU, PRD106(2022)094510
HadStruc, PRD106(2022)094511

Current-current

Pion PDF
HadStruc, PRD99(2019)074507
HadStruc, PRD102(2020)054508

Auxiliary light quark

Pion DA
UR, EPJC78(2018)217
UR, PRD98(2018)094507

Auxiliary heavy quark

Pion DA
Taiwan/MIT, arXiv:1810.12194
Taiwan/MIT, PRD105(2022)034506

OPE without OPE

Nucleon GPD
QCDSF, PRD105(2022)014502

Nucleon F_1
QCDSF, PRL118(2017)242001
QCDSF, PRD102(2020)114505

Nucleon F_2, F_L
QCDSF, arXiv:2209.04141

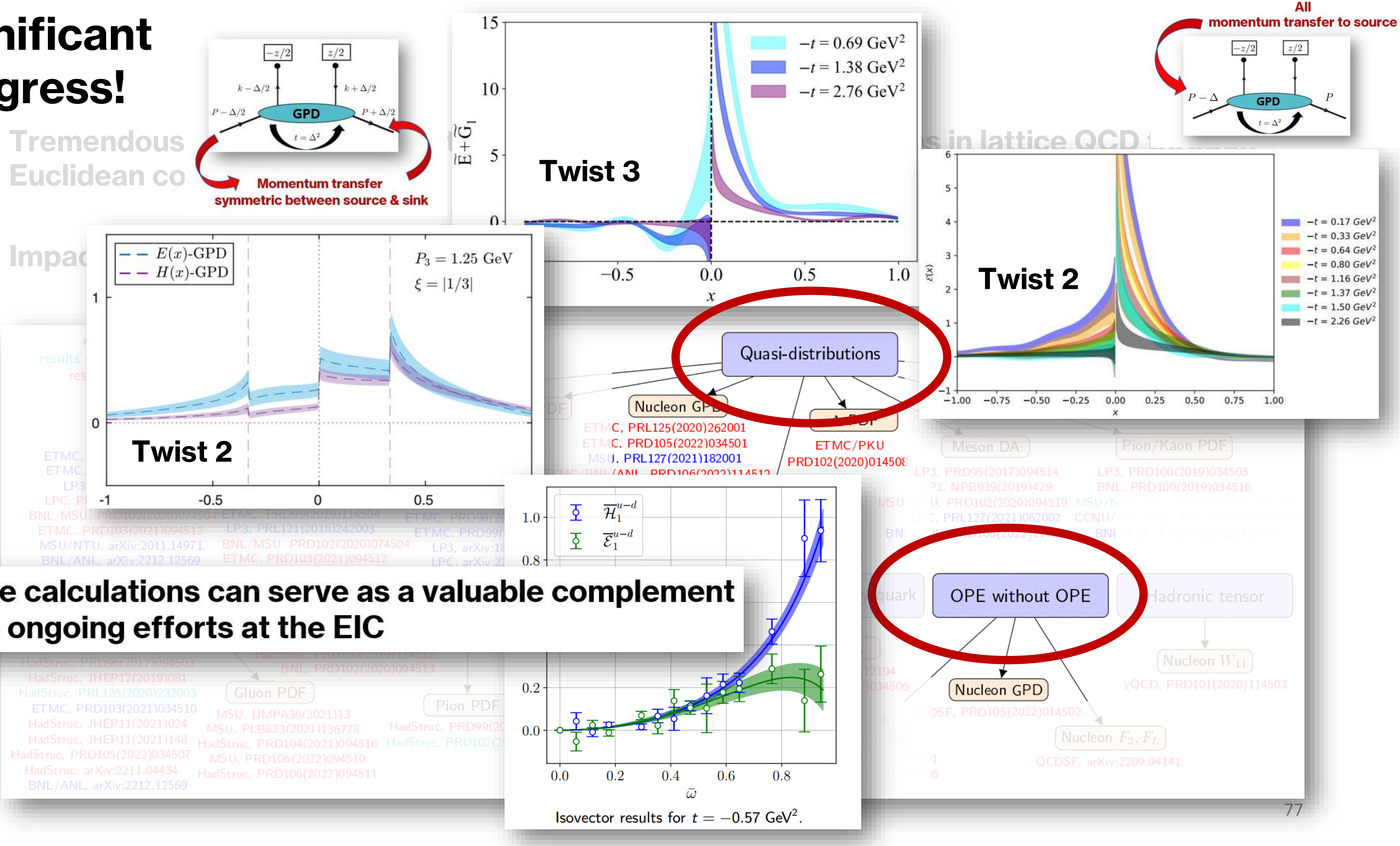
Hadronic tensor

Nucleon W_{11}
χQCD, PRD101(2020)114503

Courtesy: Cichy

Significant progress!

- Tremendous Euclidean co
- Impac



Lattice calculations can serve as a valuable complement to the ongoing efforts at the EIC

Outlook

- **Improving perturbative calculations**

- **Better understanding of power corrections**

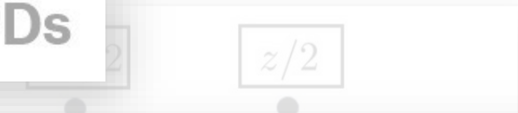
- **Synergy with phenomenology ...**

Backup slides

What are Generalized Parton Distributions?

Example:

At twist 2 there are 8 GPDs



Twist-2 GPDs

Γ / Pol.	γ^+	$\gamma^+\gamma_5$	$\sigma^{+j}\gamma_5$
T	E	$\widetilde{E}$	$H_T\ \widetilde{H}_T$

$F(x,\xi,t)$

GPD c

There are a total of 32 GPDs

Definition of GPD correlator:

$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int\frac{dz^-}{2\pi}e^{ik\cdot z}\langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

Physical processes sensitive to GPDs

(list not exhaustive)

Shadow GPDs

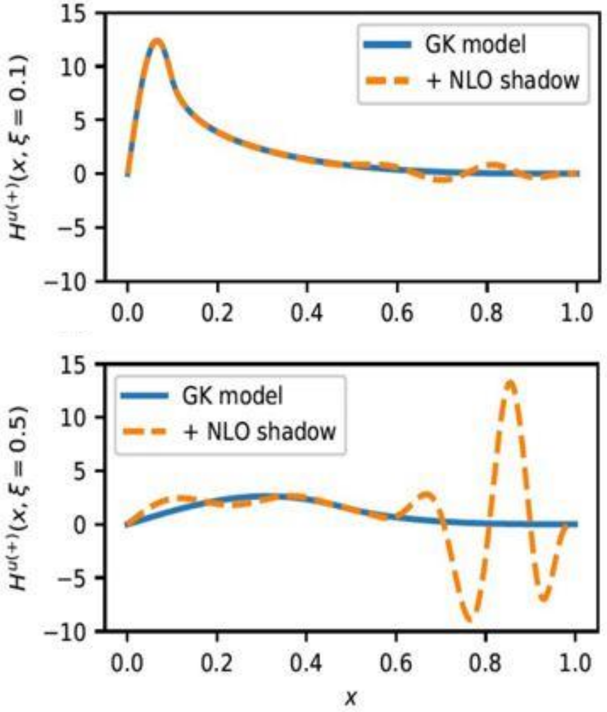
Deconvolution problem of deeply virtual Compton scattering

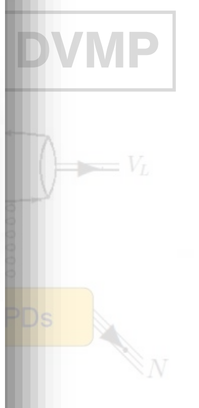
V. Bertone,[1,*] H. Dutrieux,[1,†] C. Mezrag,[1,‡] H. Moutarde,[1,§] and P. Sznajder[2,‖]

$$F(x,\xi,t) \to F(x,\xi,t) + S(x,\xi,t)$$

$$\text{with} \quad \int_{-1}^{1} \mathrm{d}x \, \frac{S(x,\xi,t)}{x-\xi+i\varepsilon} = 0$$

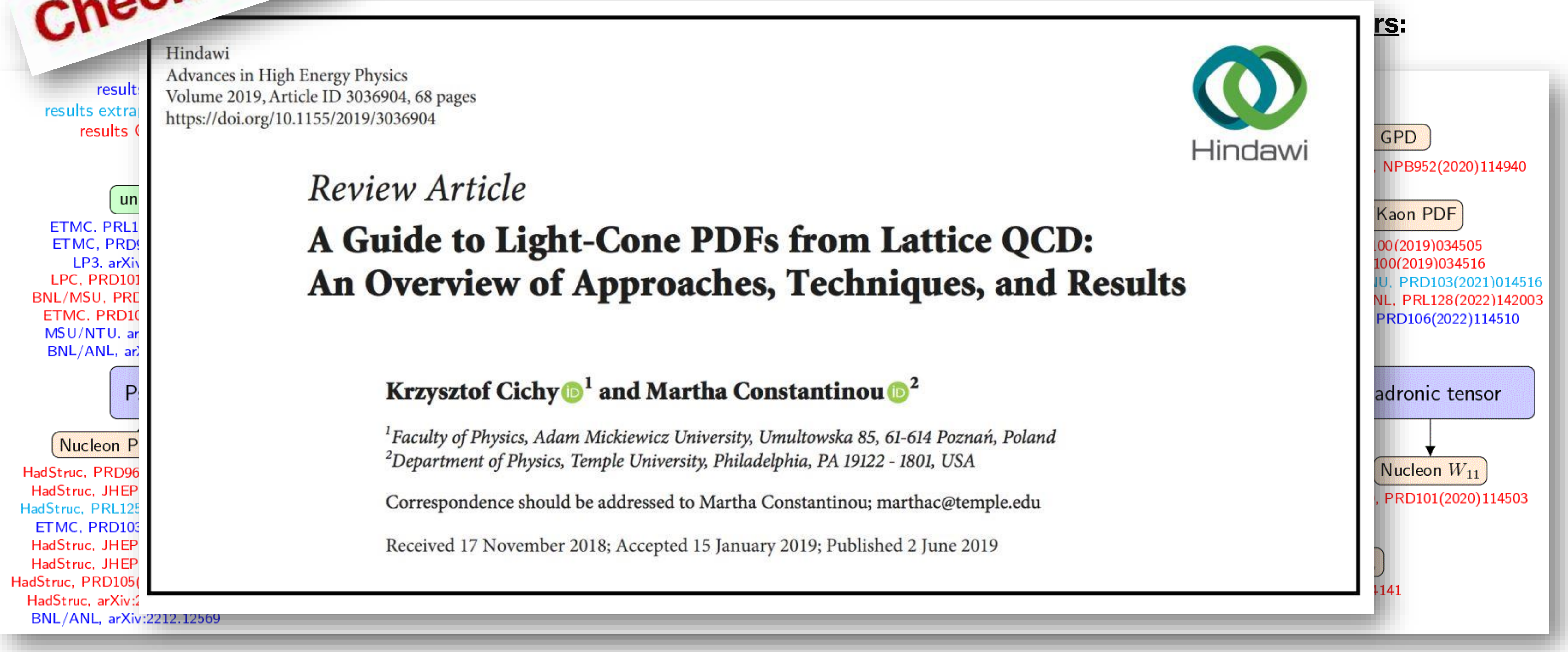
Blue and dashed
Fit the same CFFs!





Dyna Progress of Lattice QCD calculations of PDFs/GPDs

Check out!



Hindawi
Advances in High Energy Physics
Volume 2019, Article ID 3036904, 68 pages
https://doi.org/10.1155/2019/3036904

Hindawi

Review Article

A Guide to Light-Cone PDFs from Lattice QCD:
An Overview of Approaches, Techniques, and Results

Krzysztof Cichy[1] and Martha Constantinou[2]

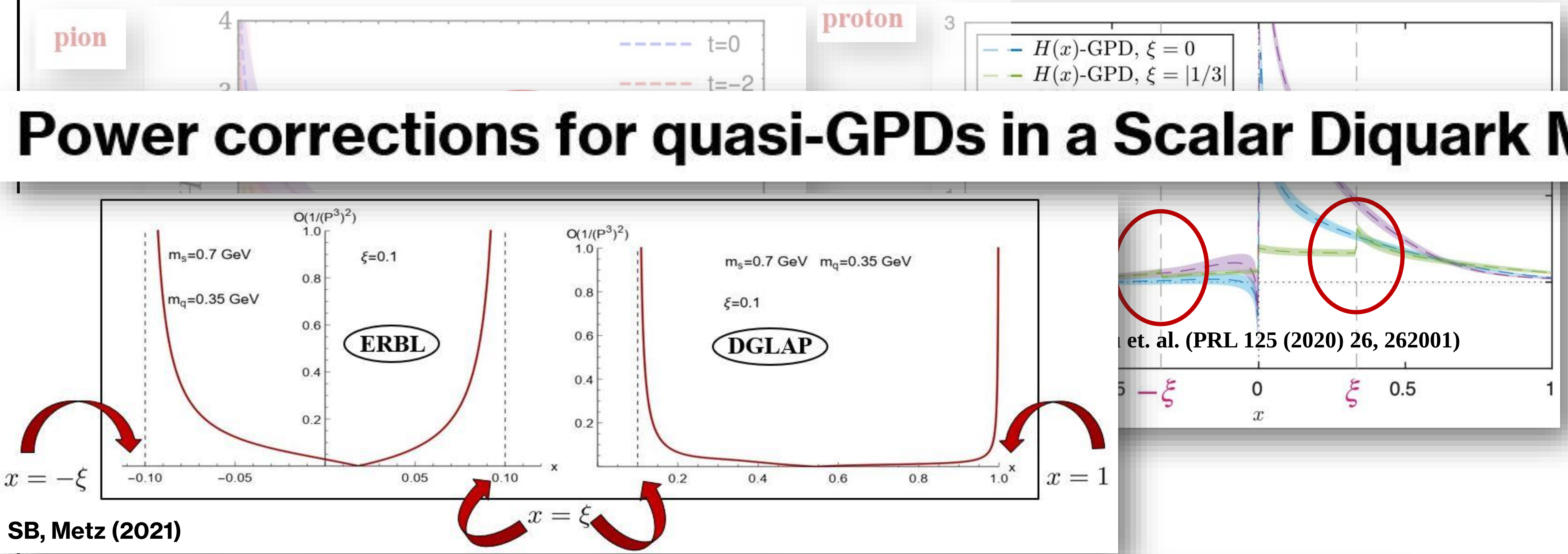
[1] *Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland*
[2] *Department of Physics, Temple University, Philadelphia, PA 19122 - 1801, USA*

Correspondence should be addressed to Martha Constantinou; marthac@temple.edu

Received 17 November 2018; Accepted 15 January 2019; Published 2 June 2019

Compilation by Cichy, 2110.07440

First Lattice QCD results of the x-dependent GPDs



Power corrections for quasi-GPDs in a Scalar Diquark Model

et. al. (PRL 125 (2020) 26, 262001)

SB, Metz (2021)

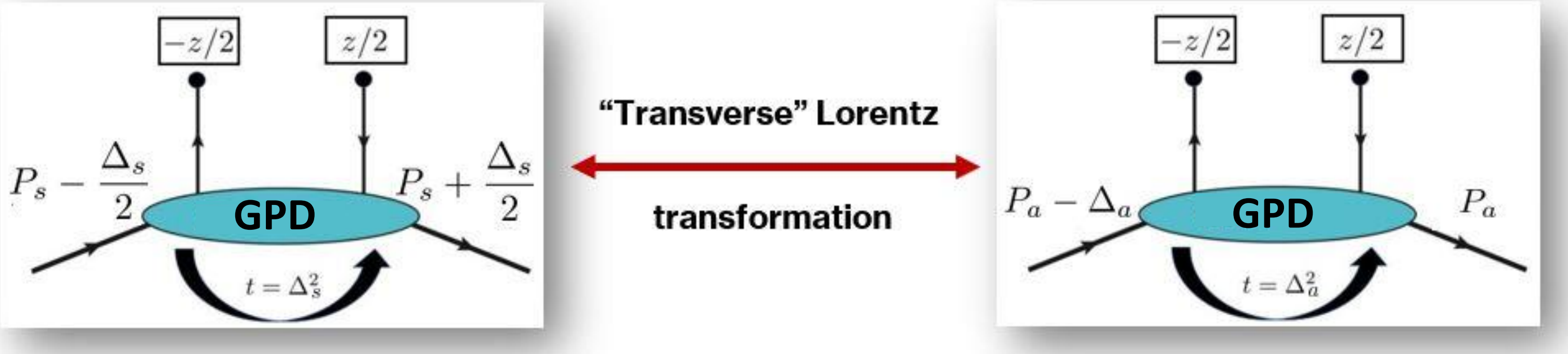
Our prediction regarding the structure of divergence: $q_{\mathrm{Q}}(x) \approx \mathcal{O}\left(\frac{1}{(x+\xi)(x-\xi)(1-x)P_3^2}\right)$

GPDs from asymmetric frames

Historic definitions of quasi-GPDs H & E are not manisfestly Lorentz invariant

Think about how γ^0 transforms under Lorentz transformation

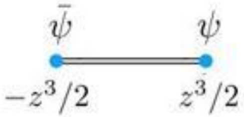
Tra
(s



"Transverse" Lorentz transformation

Symmetric frame

Asymmetric frame



"Transverse" with respect to Wilson Line

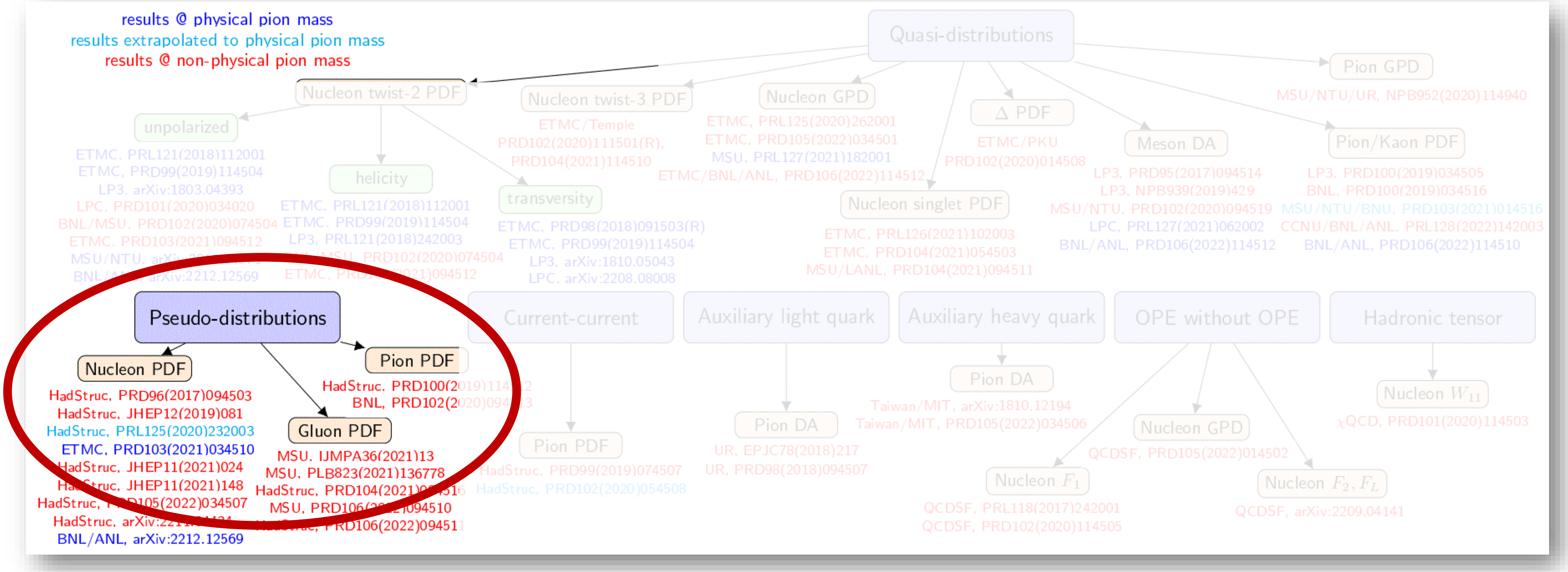
$$\boxed{F_s^0 = \gamma F_0^a - \gamma\beta F_\perp^a}$$

$$\beta = -\sqrt{\frac{E_i^a - E_f^a}{E_i^a + E_f^a}} < 0$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

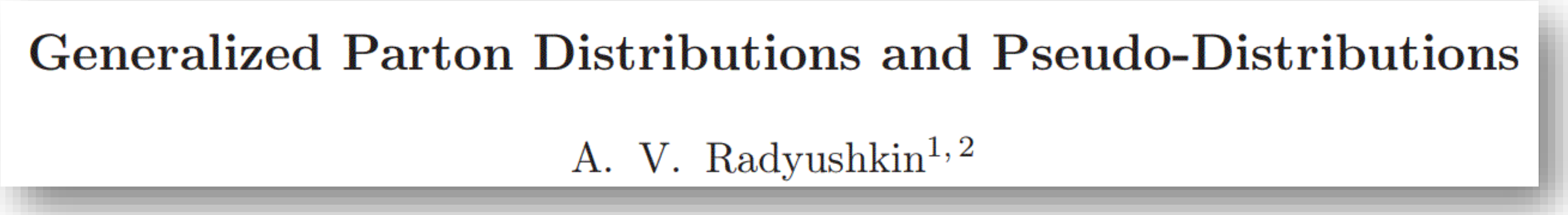
Dynamical Progress of Lattice QCD calculations of PDFs/GPDs

Lattice QCD calculations of x-dependence of PDFs & related quantities using Euclidean correlators:



Compilation by Cichy, 2110.07440

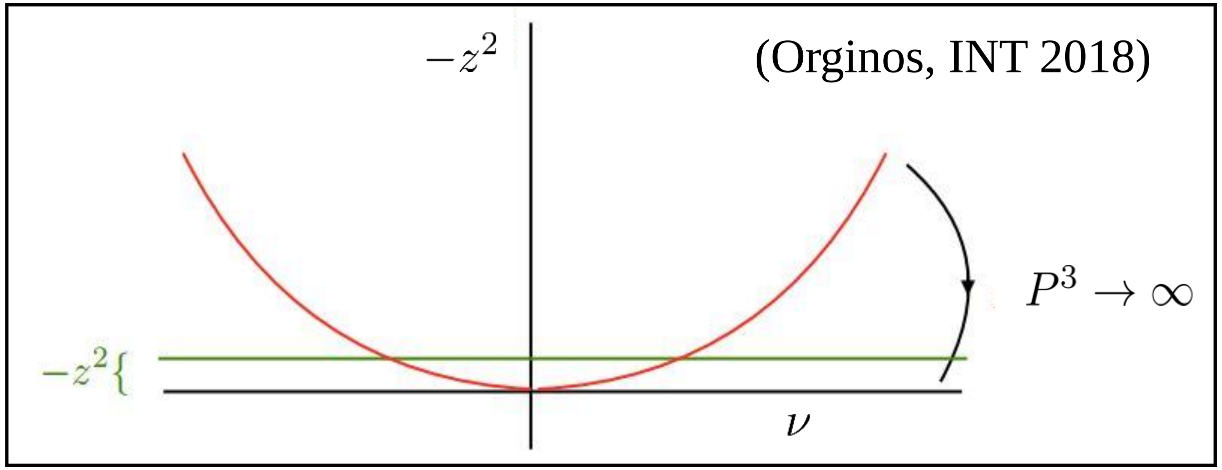
Pseudo-GPD approach



Generalized Parton Distributions and Pseudo-Distributions

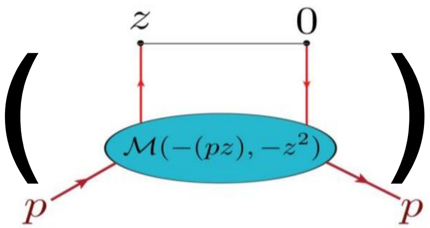
A. V. Radyushkin[1, 2]

Sketch of the approach:



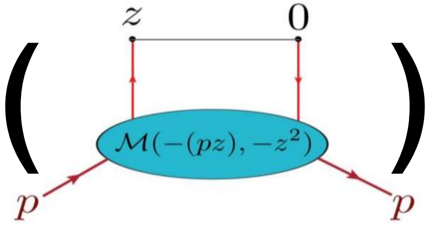
Quasi-PDF : Fixed P^3

$$Q(x, P^3) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\, e^{-ix\nu} \left(\quad \right)$$



Pseudo-PDF : Fixed z^2

$$P(x, -z^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\, e^{-ix\nu} \left(\quad \right)$$

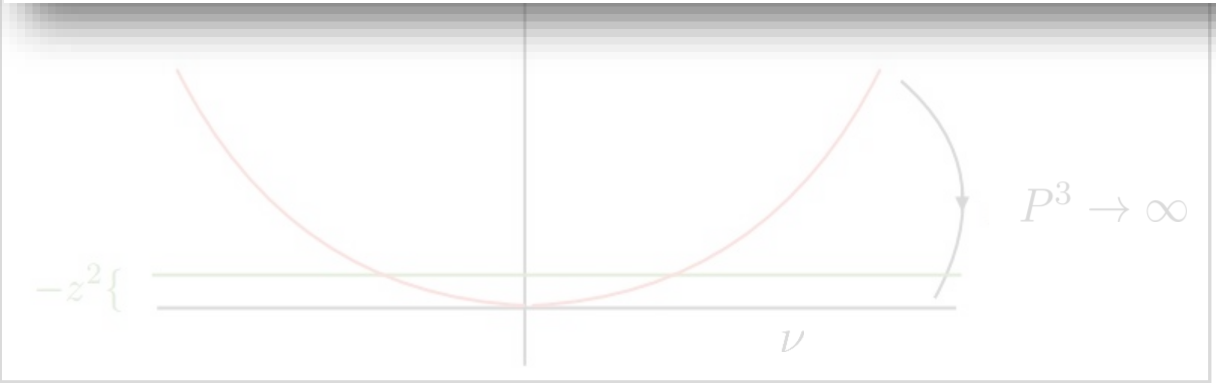


Pseudo-GPD approach

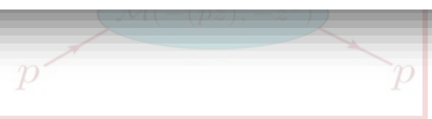
Generalized Parton Distributions and Pseudo-Distributions

A. V. Radyushkin[1, 2]

Progress is steadily advancing
& we anticipate forthcoming results regarding GPDs from the pseudo-GPD approach



Quasi-PDF : (fixed P^3)



Pseudo-PDF : (fixed z^2)

$$P(x,-z^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\, e^{-ix\nu}$$