Lattice QCD Calculation of TMD Physics

25th International Spin Symposium (SPIN 2023)

Durham Convention Center, Durham, NC, USA

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SEP 28, 2023

Argonne NATIONAL LABORATORY

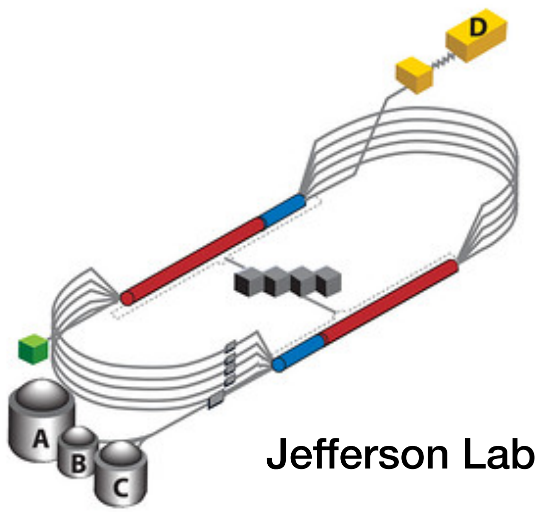
Outline

- **TMDs from experiments**

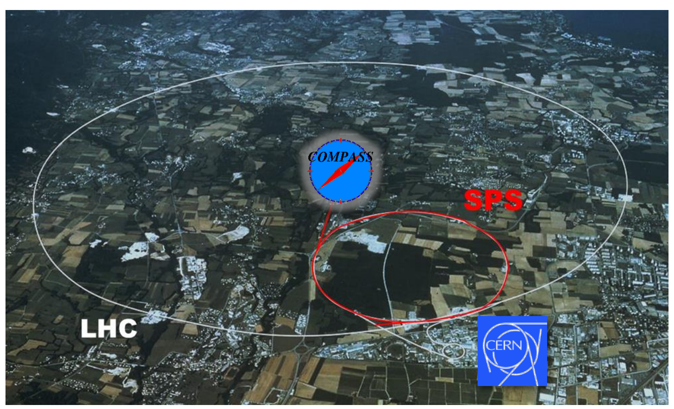
- **Lattice methods for TMD calculation**

- **Results from lattice QCD**

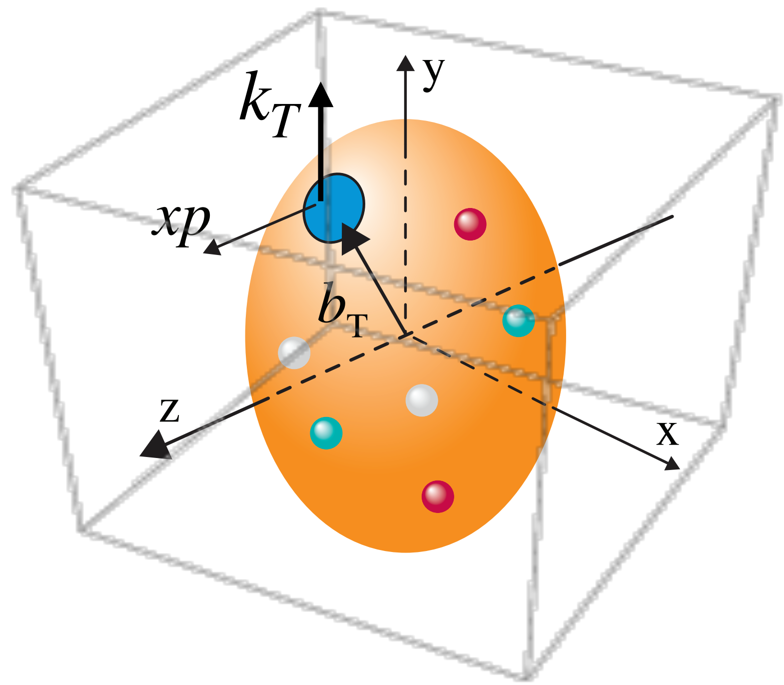
3D Imaging of the Proton

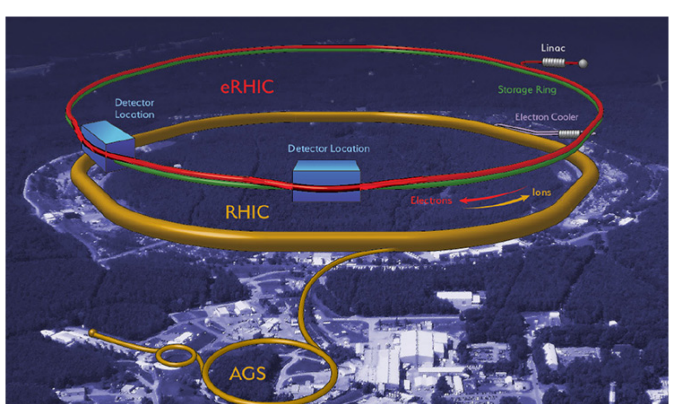


Jefferson Lab 12 GeV

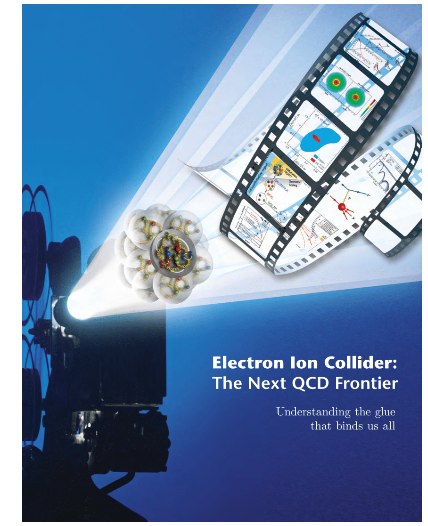


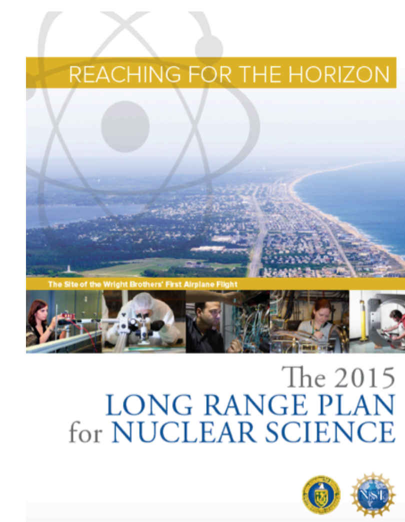
COMPASS, CERN

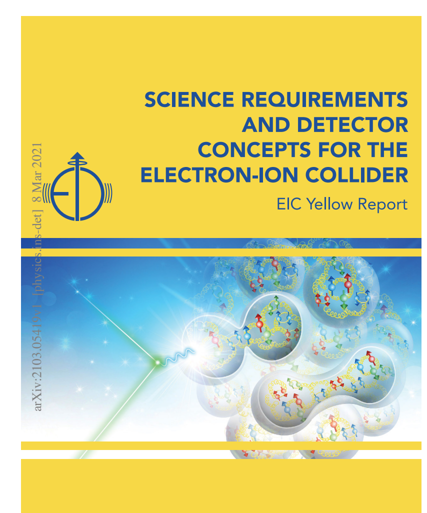




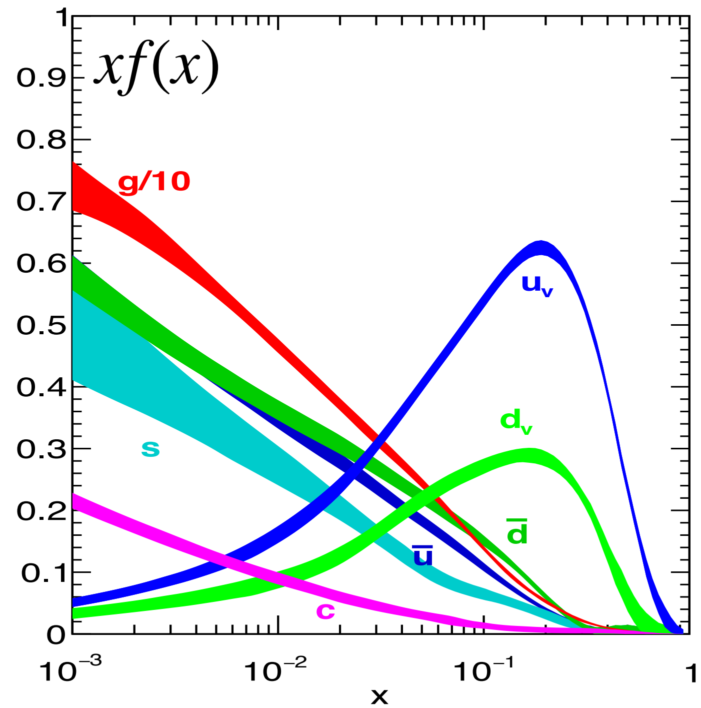
The Electron-Ion Collider, BNL







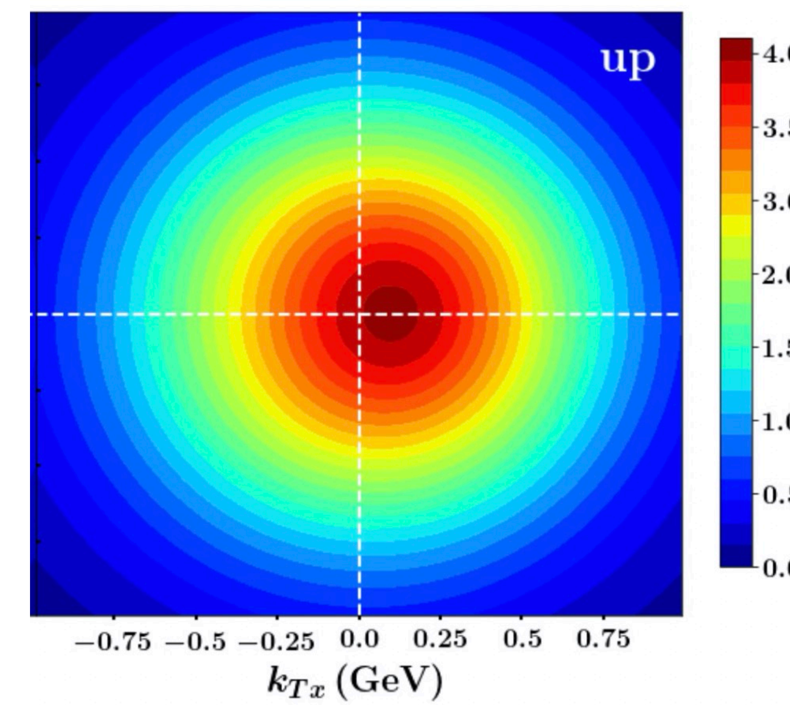
Parton Distribution Functions (PDFs)



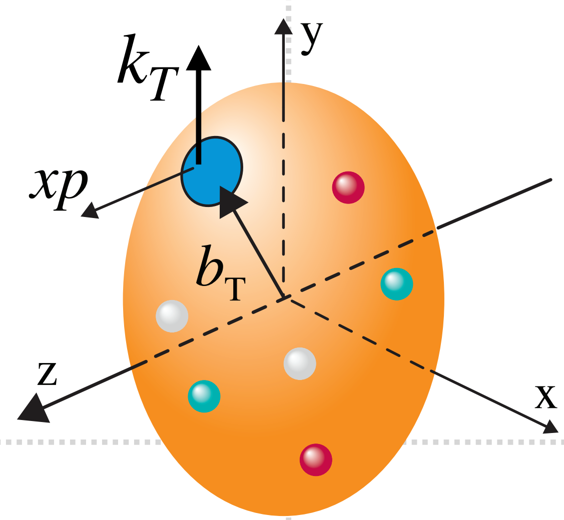
NNPDF, EPJ C77 (2017)

$$\int d^2\vec{k}_T$$

Transvers momentum distributions (TMDs)



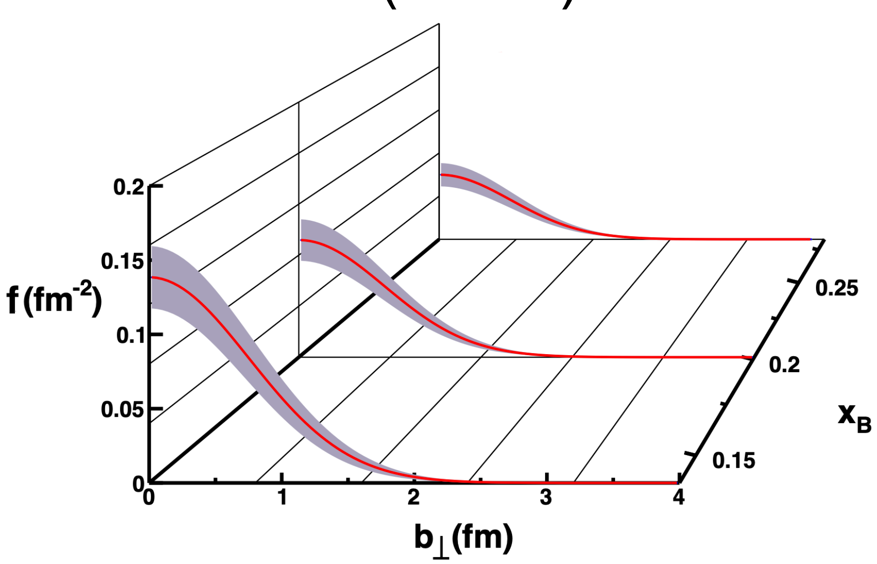
Cammarota, et al. (JAM), PRD **102** (2020).



$$\int d^2\vec{b}_T$$

$$\int d^2\vec{b}_T$$

Generalized parton distributions (GPDs)



W. Armstrong et al., arXiv: 1708.00888.

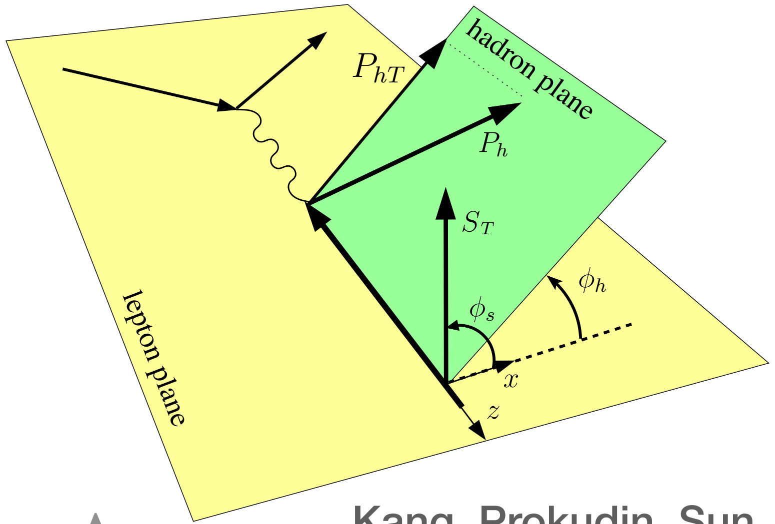
$$\int d^2\vec{k}_T$$

Wigner distributions/Generalized TMDs

$$W(x, \vec{k}_T, \vec{b}_T)$$

TMDs from global analyses

e.g., semi-inclusive DIS: $l + p \longrightarrow l + h(P_h) + X$



Kang, Prokudin, Sun and Yuan, PRD 93 (2016)

$$\frac{d\sigma}{dxdydz_h d^2\mathbf{P}_{hT}} \sim \int d^2\mathbf{b}_T \, e^{i\mathbf{b}_T\cdot\mathbf{P}_{hT}/z}$$

$$\times f_{i/p}(x, \mathbf{b}_T, Q, Q^2) \, D_{h/i}(z_h, \mathbf{b}_T, Q, Q^2) + Y(P_{hT}, Q) + \mathcal{O}(\frac{\Lambda_{\rm QCD}}{Q})$$

$$f_{i/p}(x, \mathbf{b}_T, \mu, \zeta) = f_{i/p}^{\rm pert}(x, b^*(b_T), \mu, \zeta)$$

$$\times \left(\frac{\zeta}{Q_0^2}\right)^{g_K(b_T)/2} f_{i/p}^{\rm NP}(x, b_T)$$

$g_K(b_T)/2$ → **Collins-Soper kernel (Non-perturbative part)**

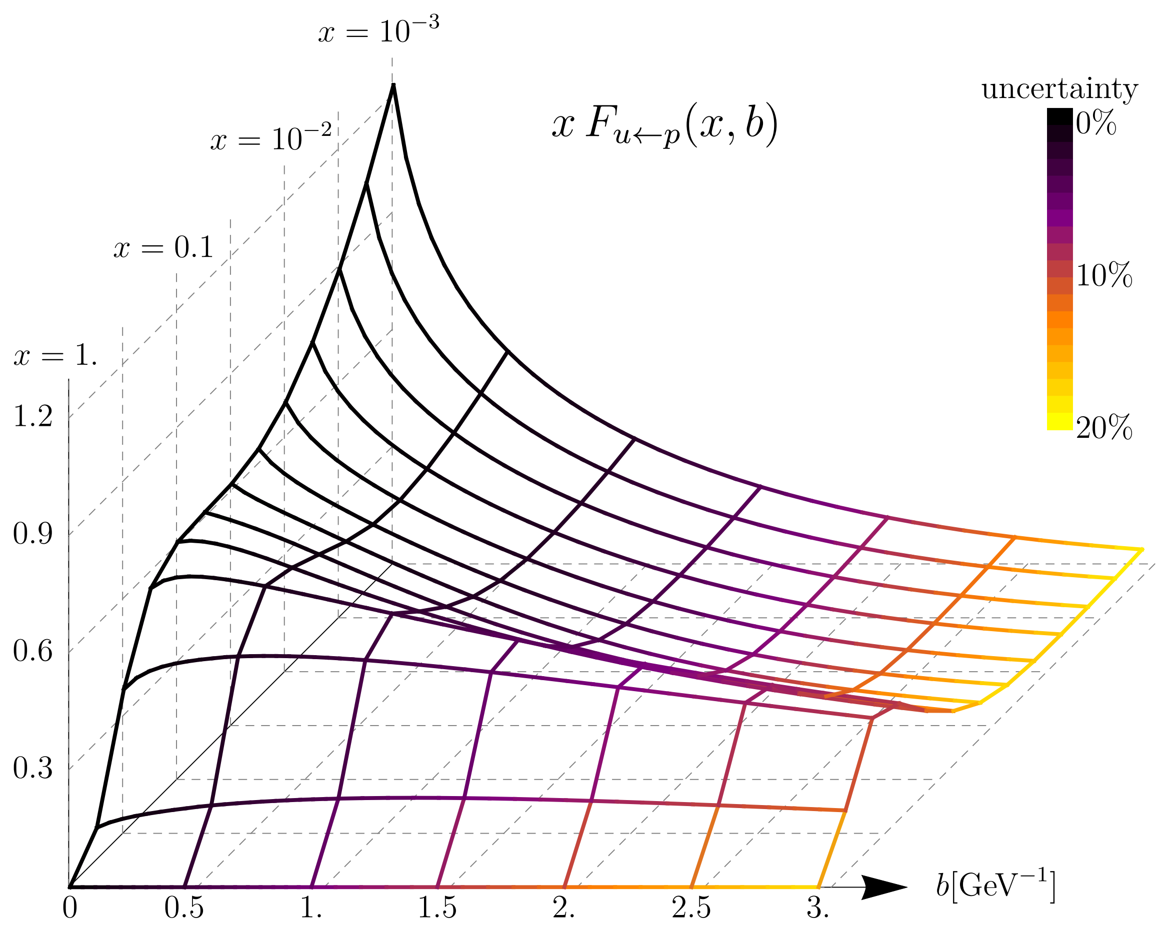
$f_{i/p}^{\rm NP}(x, b_T)$ → **Intrinsic TMD**

$Q_0 \sim 1$ GeV

Non-perturbative when $b_T \sim 1/\Lambda_{\rm QCD}$!

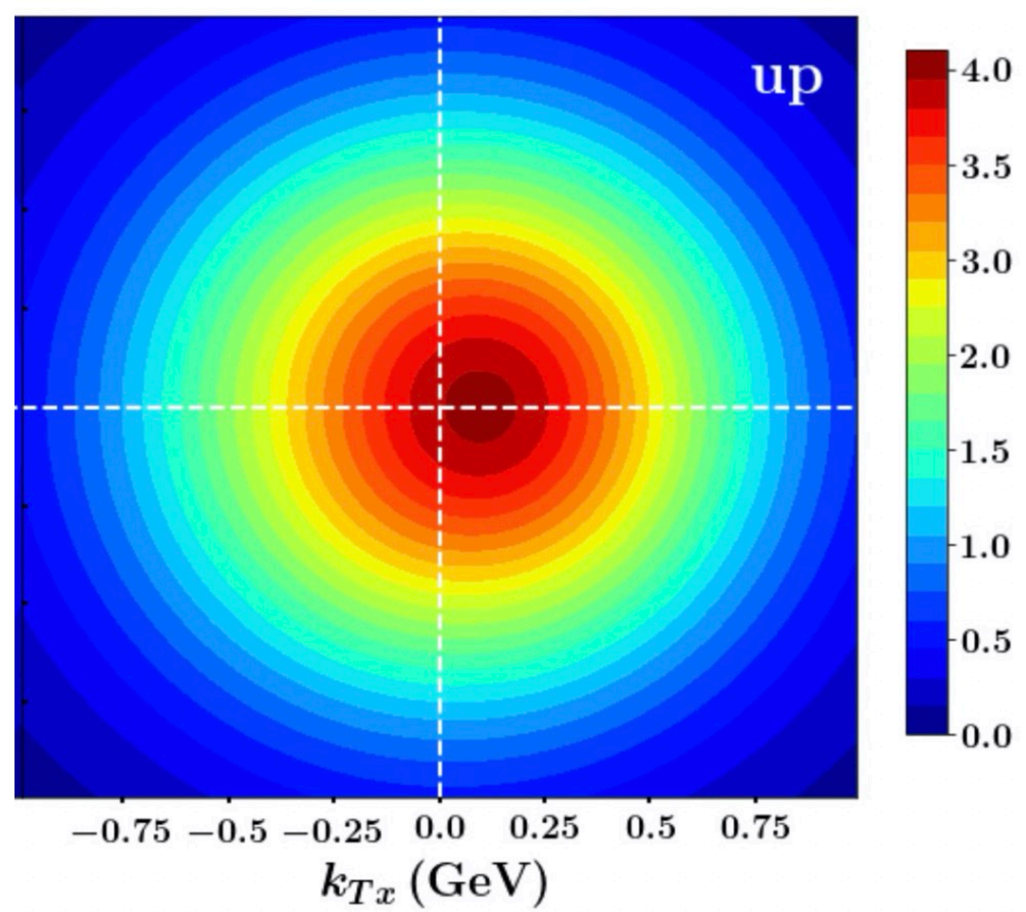
TMDs from global analyses

Unpolarized quark TMD



Scimemi and Vladimirov, JHEP **06** (2020).

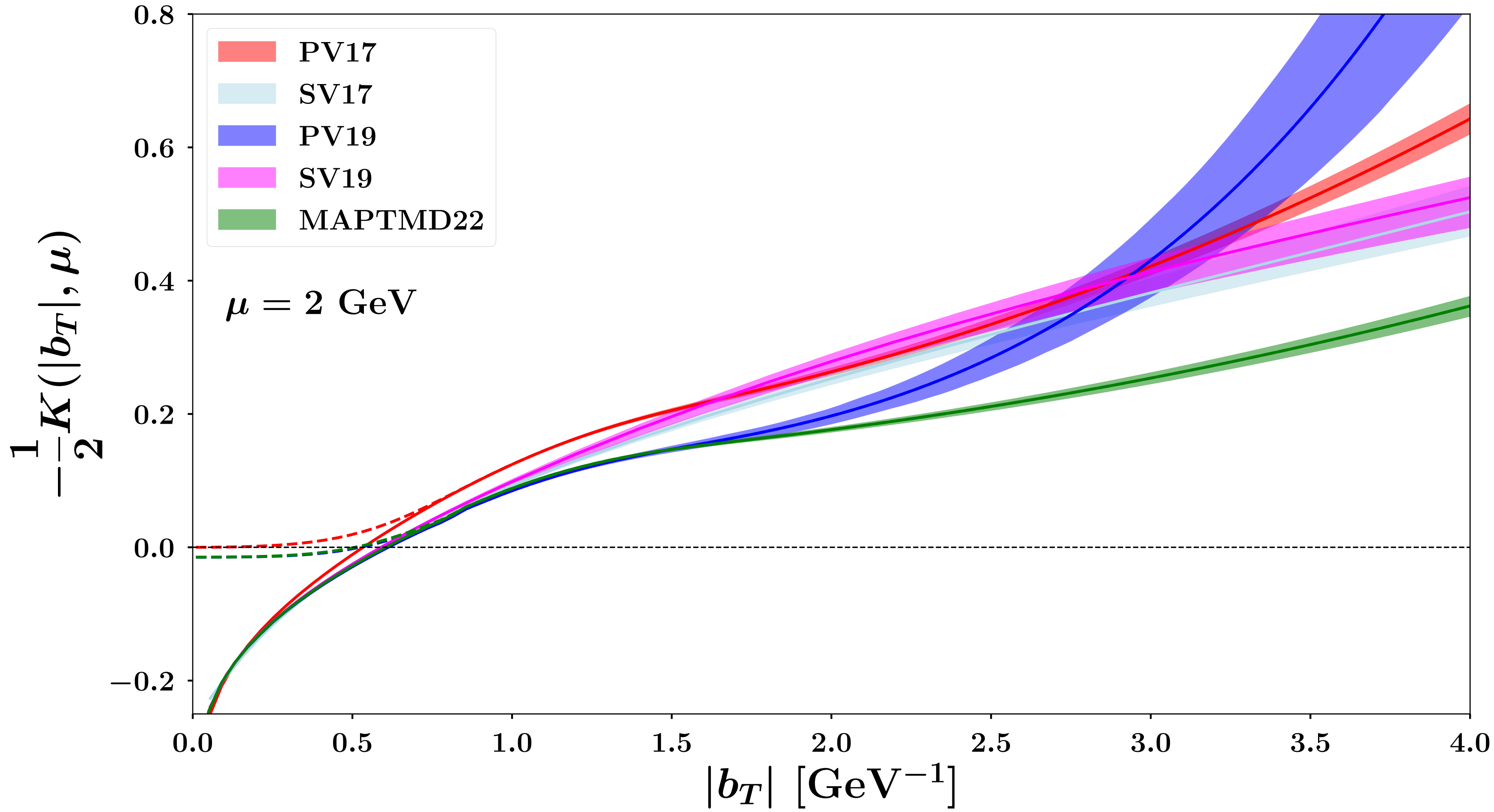
Quark Sivers function



Cammarota, Gamberg, Kang et al. (JAM Collaboration), PRD **102** (2020).

TMDs from global analyses

Collins-Soper Kernel $K(b_T, \mu)$ **or** $\gamma_\zeta(b_T, \mu)$ $K(b_T, \mu) = K^{\rm pert}(b_T, \mu) + g_K(b_T)$



Bacchetta, Bertone, Bissolotti, et al., MAP Collaboration, JHEP **10** (2022).

See A. Bacchetta's talk on Tue and A. Prokudin's talk on Thu.

Outline

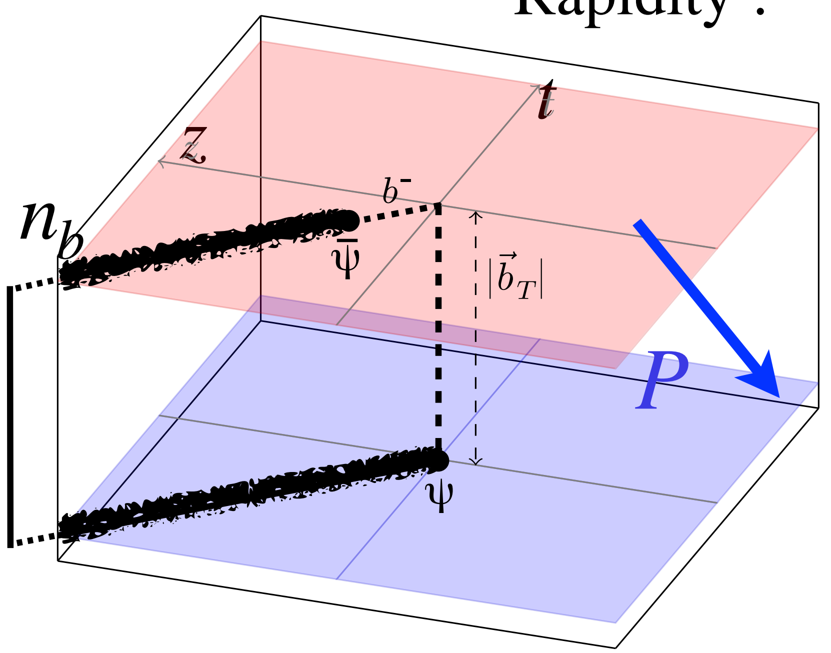
- TMDs from experiments
- Lattice methods for TMD calculation
- Results from lattice QCD

TMD definition

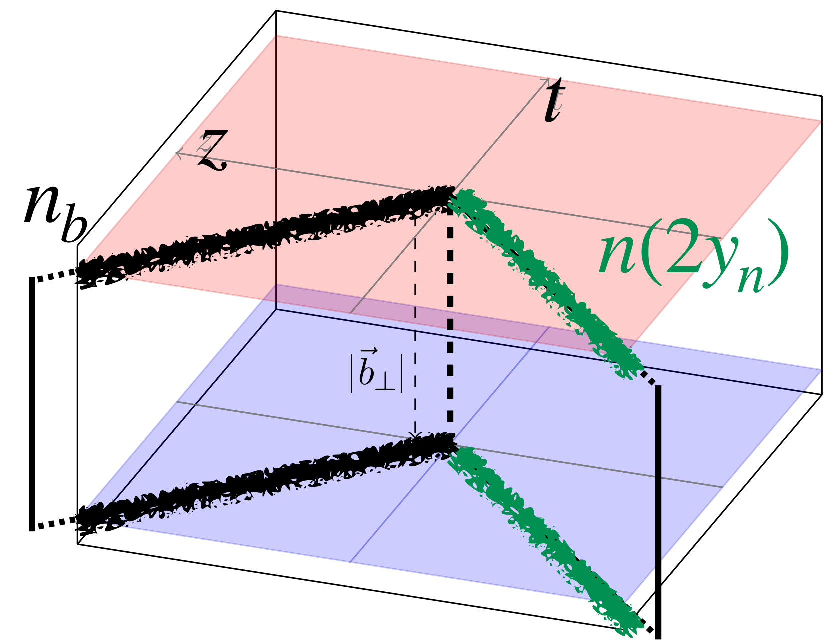
- Beam function:
- Soft function :

$$n_b^2 = 0$$

$$\text{Rapidity}: \quad y_B = \frac{1}{2}\ln\left|\frac{n_b^+}{n_b^-}\right| = -\infty$$



Hadronic matrix element



Vacuum matrix element

$$f_i(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\epsilon\to 0} Z_{\mathrm{UV}} \lim_{\tau\to 0} \frac{B_i}{\sqrt{S^q}}$$

Collins-Soper scale: $\zeta = 2(xP^+e^{-y_n})^2$

Rapidity divergence regulator

TMD definition

- Beam function:
- Soft function :

$$n_b^2 = 0$$

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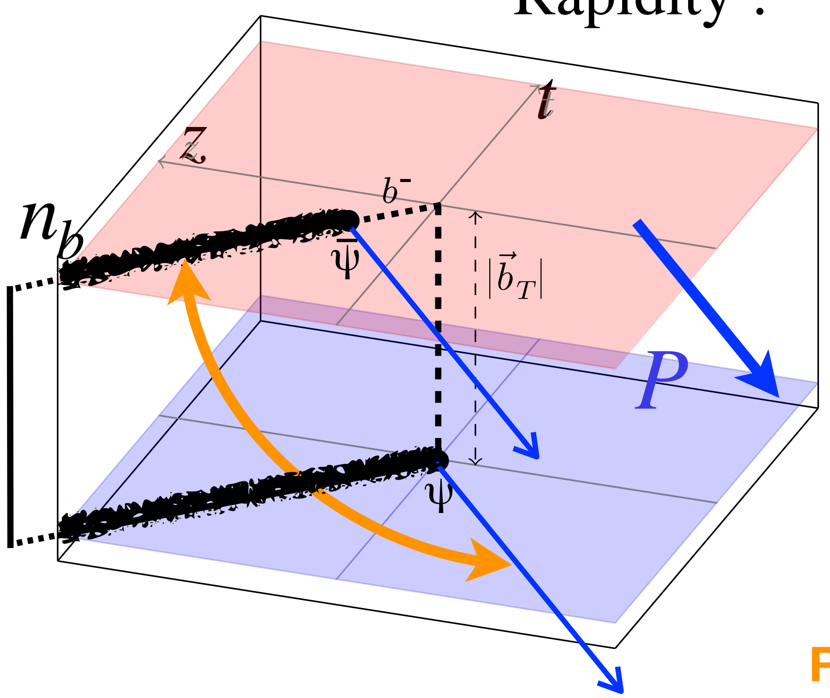
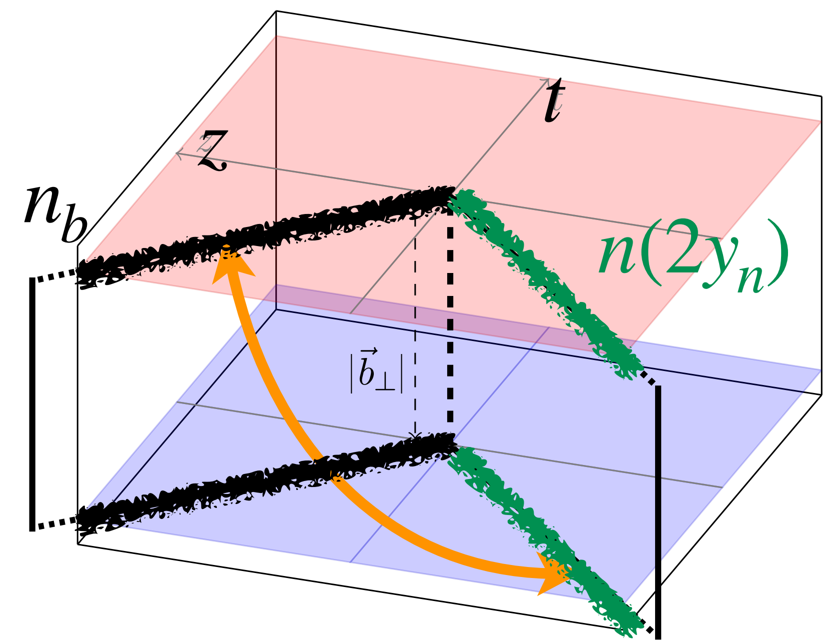


Hadronic matrix element

Rapidity divergences



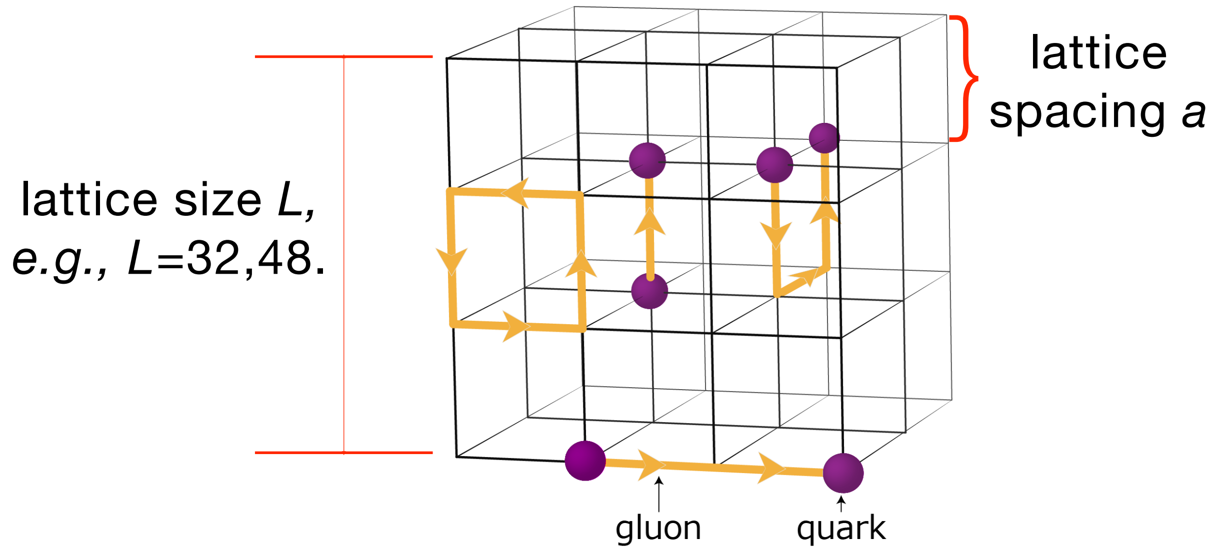
Vacuum matrix element

$$f_i(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\epsilon\to 0} Z_{\mathrm{UV}} \lim_{\tau\to 0} \frac{B_i}{\sqrt{S^q}}$$

Collins-Soper scale: $\zeta = 2(xP^+e^{-y_n})^2$

Rapidity divergence regulator

Simulating partons on the lattice



Imaginary time: $t \to i\tau$

- $P = \infty$ hadron state? ✘ $P \ll \frac{2\pi}{a}$!

  $t = 0$

- Light-cone correlations? ✘

  $z + ct = 0$

  $z - ct \neq 0$

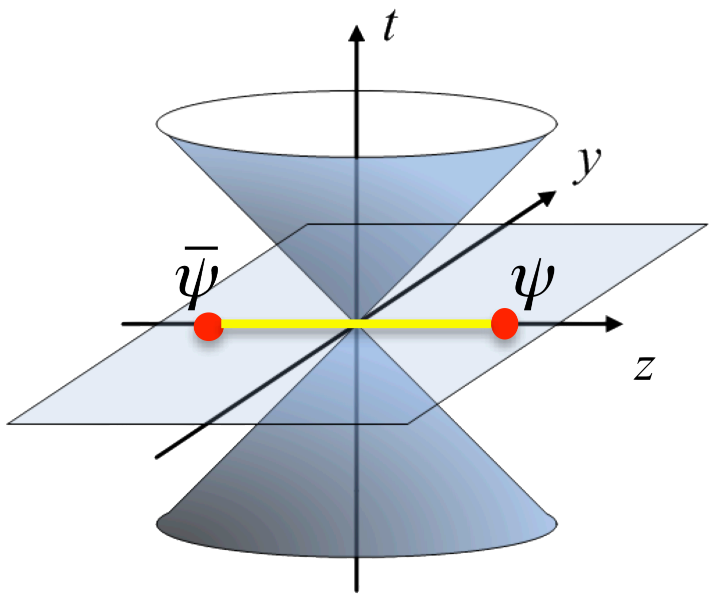
  Real-time sign problem 🙁

Nevertheless, it is possible to approach the Feynman partons by simulating a boosted hadron on the lattice 😇

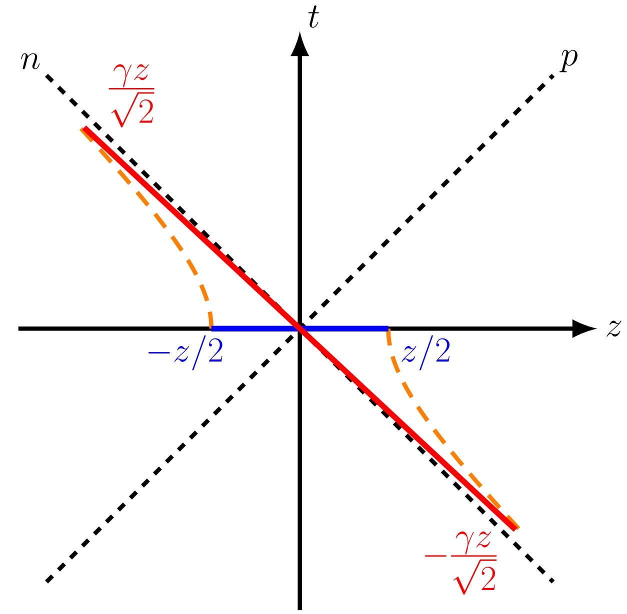
Large-Momentum Effective Theory (LaMET)

A quasi-PDF $\tilde{f}(x, P^z)$ to expand from:

- X. Ji, PRL **110** (2013); SCPMA **57** (2014).
- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, RMP **93** (2021).



Lorentz boost



Power expansion and effective theory matching:

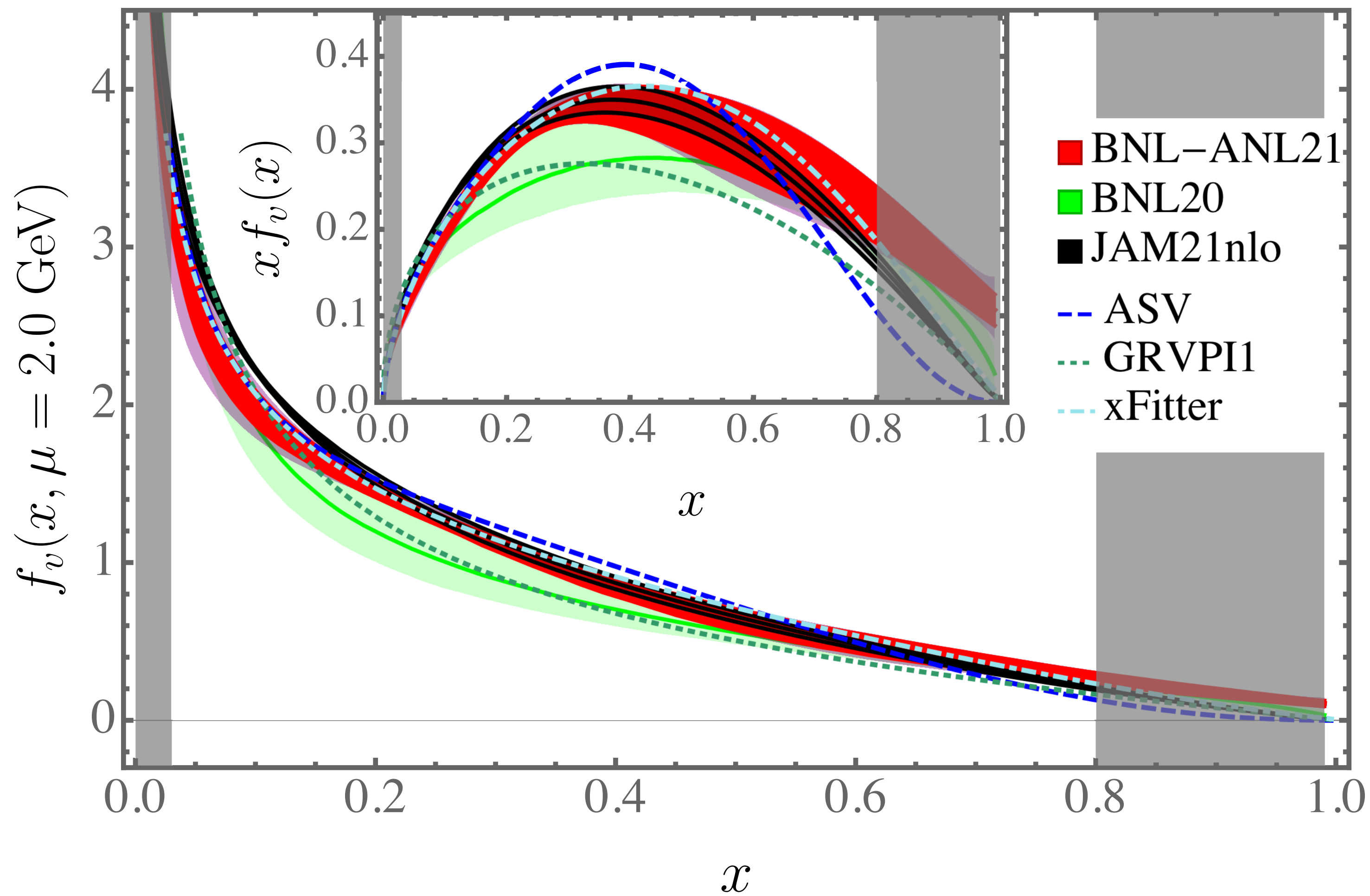
$$f(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} \bar{C}\left(\frac{x}{y}, \frac{\mu}{2xP^z}, \frac{\tilde{\mu}}{\mu}\right) \tilde{f}(y, P^z, \tilde{\mu}) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P^z)^2}\right)$$

- X. Xiong, X. Ji, J.-H. Zhang and YZ, PRD **90** (2014);
- Y. Ma and J. Qiu, PRD **98** (2018);
- T. Izubuchi, X. Ji, L. Jin, I. Stewart, and YZ, PRD **98** (2018)

Reliable prediction within [$x_{\rm min}$, $x_{\rm max}$] at a given finite P^z !

Lattice calculation of pion valence PDF at NNLO

BNL-ANL21, Gao, Hanlon, Mukherjee, Petreczky, Scior, Syritsyn and YZ, PRL**128** (2022).



- **JAM21nlo**, PRL **127** (2021);
- **xFitter (2020)**, PRD **102** (2020);
- **ASV**, PRL **105** (2010);
- **GRVPI1,** ZPC **53** (1992);
- **BNL20**, X. Gao, N. Karthik, YZ, et al., PRD **102** (2020).

Towards better perturbative and power precisions

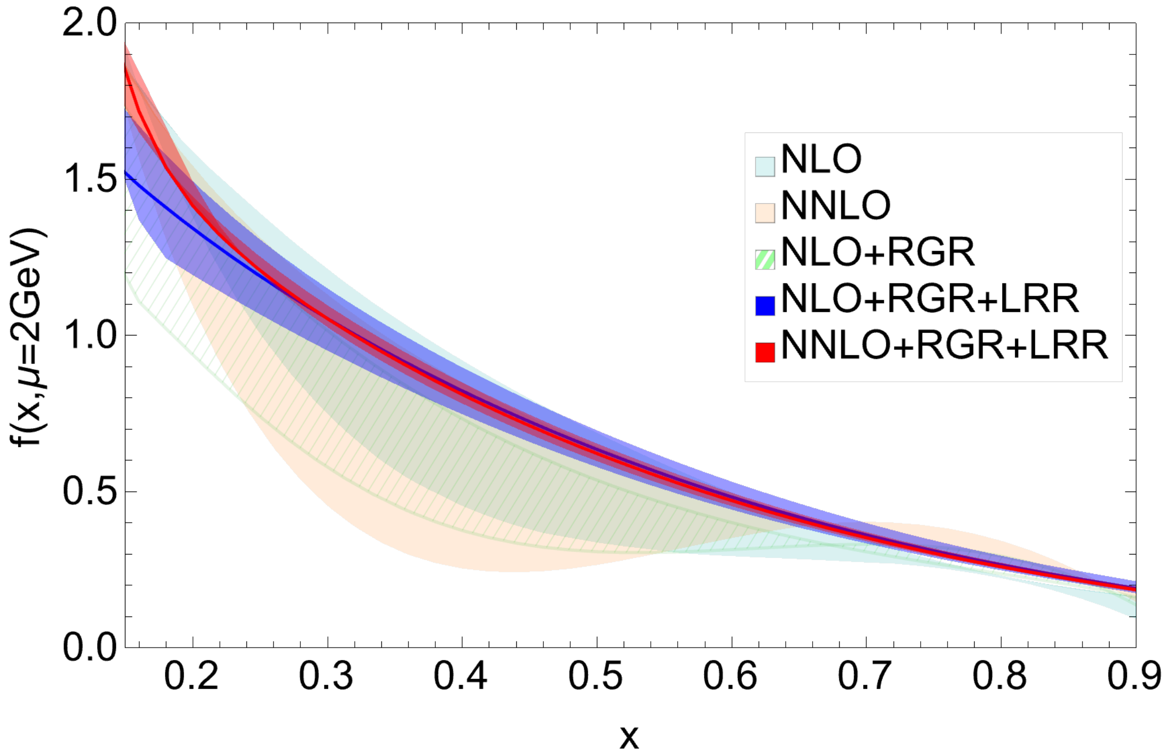
$$f(x,\mu) = U^{\rm RGR}(\mu, 2xP^z) \otimes \int_{-\infty}^{\infty} \frac{dy}{|y|} \bar{C}_{\rm LRR}\left(\alpha_s(2xP^z), \frac{x}{y}, 1, \frac{\tilde{\mu}}{2xP^z}\right) \tilde{f}(y, P^z, \tilde{\mu})$$

$$+ \mathcal{O}\left(\cancel{\frac{\Lambda_{\rm QCD}}{|xP^z|}}\right) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P^z)^2}\right)$$

- Holligan, Ji, Lin, Su and R. Zhang, NPB **993** (2023);
- R. Zhang, Ji, Holligan and Su, PLB **844** (2023);
- X. Gao, K. Lee, and YZ et al., PRD **103** (2021).

- **RGR:** renormalization group resummation, resuming small x logarithms.
- **LRR:** leading-renormalon resummation, summing the asymptotic series in the Wilson line self-energy, improving power accuracy to $1/P_z^2$.
- **THR: threshold resummation, resuming the large *x* logarithms.**
  - X. Gao, K. Lee, and YZ et al., PRD **103** (2021);
  - X. Ji, Y. Liu and Y. Su, JHEP **08** (2023).

Better perturbative convergence with LRR!

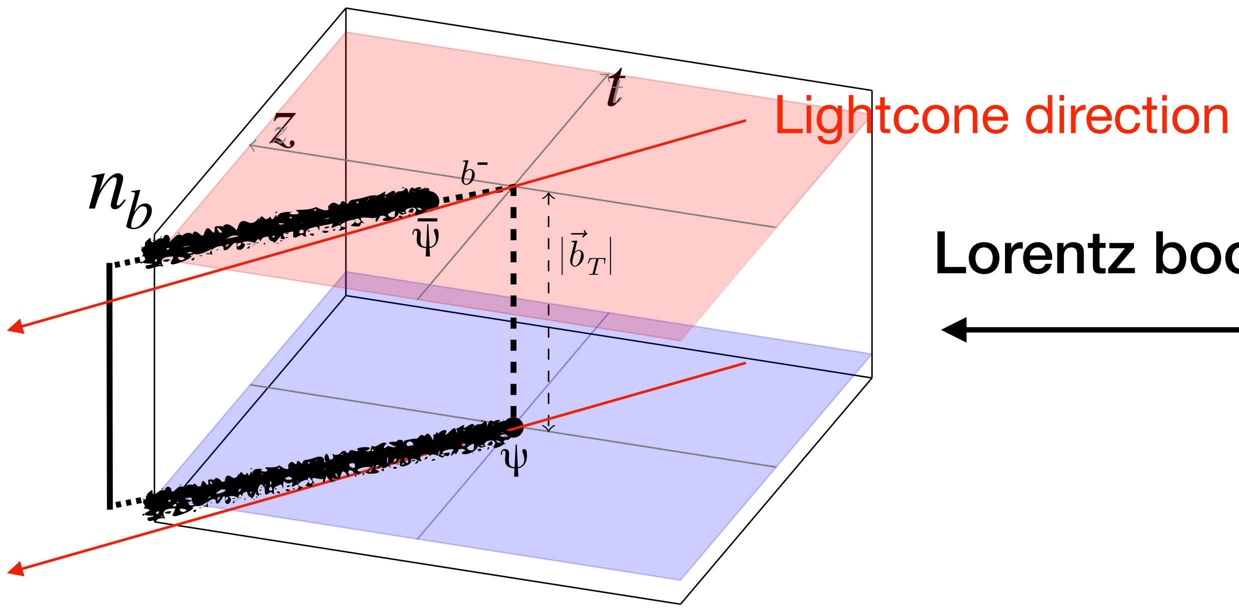


Zhang, Ji, Holligan and Su, PLB **844** (2023).

See Q. Shi's talk on Wed for on application to pion GPD.

Quasi TMD in LaMET

- **Beam function in Collins scheme:**

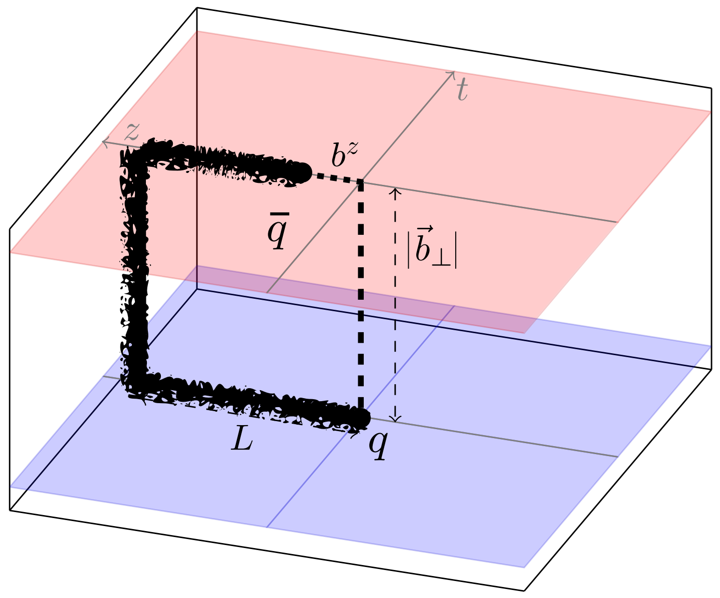


$$n_b^\mu(y_B) = (n_b^+, n_b^-, \vec{0}_\perp) = (-e^{2y_B}, 1, \vec{0}_\perp)$$

Spacelike but close-to-lightcone ($y_B \to -\infty$) Wilson lines, not calculable on the lattice ☹️

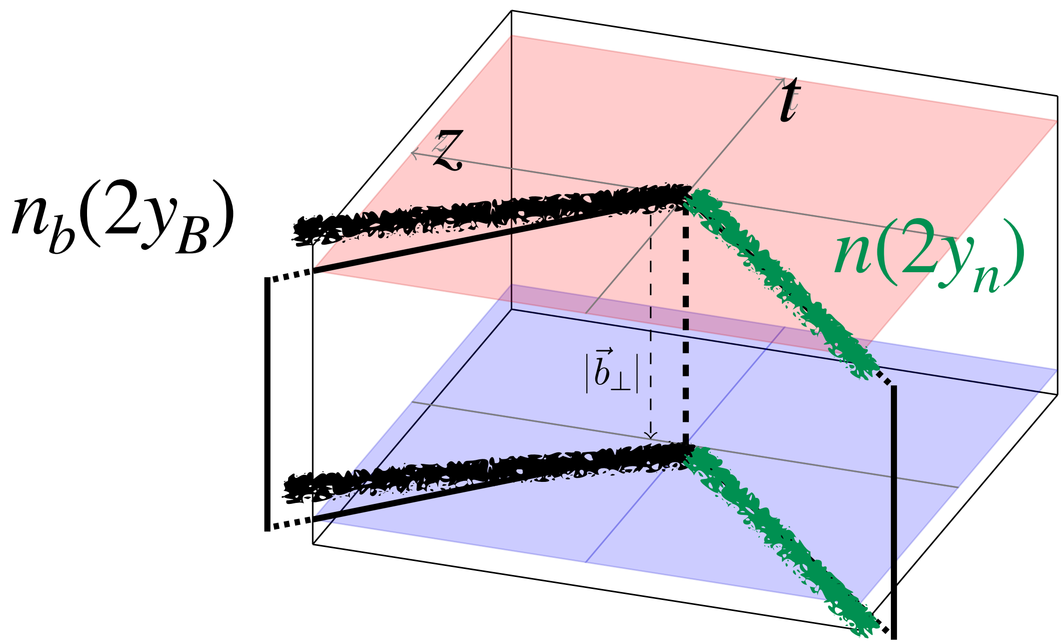
Lorentz boost and $L \to \infty$

- **Quasi beam function :**



Equal-time Wilson lines, directly calculable on the lattice🙂

Soft factor

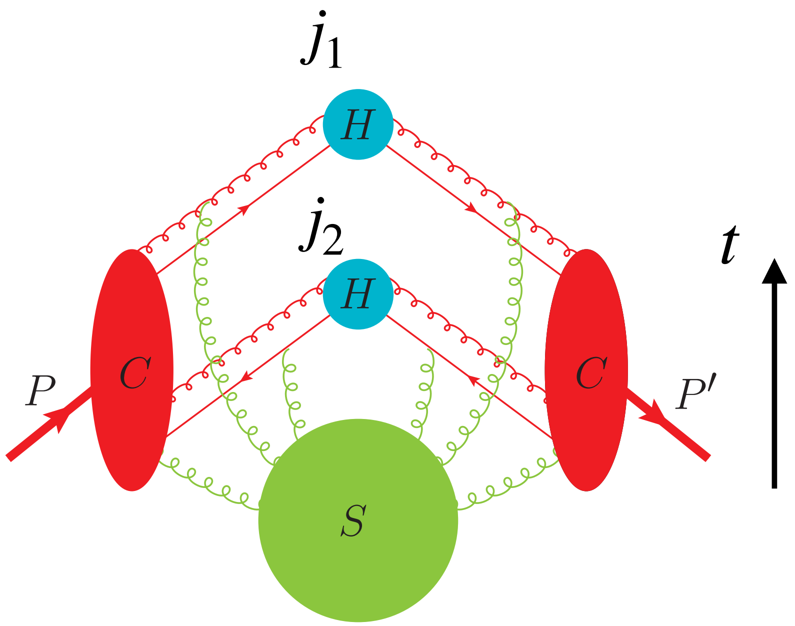


$$\xrightarrow{y_n - y_B \to \infty} S_r(b_T, \mu)\, e^{-2(y_n - y_B)\gamma_\zeta(b_T,\mu)}$$

Collins-Soper kernel

Reduced soft factor

Light-meson form factor:



$$F(b_T, P^z) = \langle \pi(-P) \,|\, j_1(b_T) j_2(0) \,|\, \pi(P) \rangle$$

$$\overset{P^z \gg m_N}{=} S_r(b_T, \mu) \int dx dx' \; H(x, x', \mu)$$

$$\times \, \Phi^\dagger(x, b_T, P^z, \mu) \Phi(x', b_T, P^z, \mu)$$

$\Phi(x, b_T, P^z, \mu)$**: quasi-TMD wave function**

- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- Ji and Liu, PRD **105** (2022);
- Deng, Wang and Zeng, JHEP **09** (2022).

Factorization formula for the quasi-TMDs

$$\frac{\tilde{f}_{i/p}^{\text{naive}[s]}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{\sqrt{S_r(b_T, \mu)}} = C\left(\mu, x\tilde{P}^z\right) \exp\left[\frac{1}{2}\gamma_\zeta(\mu, b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right]$$
$$\times f_{i/p}^{[s]}(x, \mathbf{b}_T, \mu, \zeta)\left\{1 + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda_{\text{QCD}}^2}{(x\tilde{P}^z)^2}\right]\right\}$$

- Ji, Sun, Xiong and Yuan, PRD91 (2015);
- Ji, Jin, Yuan, Zhang and YZ, PRD99 (2019);
- Ebert, Stewart, YZ, PRD99 (2019), JHEP09 (2019) 037;
- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- Ebert, Schindler, Stewart and YZ, JHEP 09 (2020);
- Vladimirov and Schäfer, PRD 101 (2020);
- Ji, Liu, Schäfer and Yuan, PRD 103 (2021);
- Ebert, Schindler, Stewart and YZ, JHEP 04, 178 (2022).

Matching coefficient:

- Independent of spin;
- No quark-gluon or flavor mixing, which makes gluon calculation much easier.

- Schindler, Stewart and YZ, JHEP 08 (2022);
- Zhu, Ji, Zhang and Zhao, JHEP 02 (2023).

Factorization formula for the quasi-TMDs

$$\frac{\tilde{f}_{i/p}^{\mathrm{naive}[s]}(x,\mathbf{b}_T,\mu,\tilde{P}^z)}{\sqrt{S_r(b_T,\mu)}} = C\left(\mu, x\tilde{P}^z\right) \exp\left[\frac{1}{2}\gamma_\zeta(\mu,b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right] \times f_{i/p}^{[s]}(x,\mathbf{b}_T,\mu,\zeta)\left\{1+\mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2},\frac{\Lambda_{\mathrm{QCD}}^2}{(x\tilde{P}^z)^2}\right]\right\}$$

✳ Collins-Soper kernel; $\gamma_\zeta(\mu,b_T) = \frac{d}{d\ln\tilde{P}^z}\ln\frac{\tilde{f}_{i/p}^{\mathrm{naive}[s]}(x,\mathbf{b}_T,\mu,\tilde{P}^z)}{C\left(\mu,x\tilde{P}^z\right)}$

✳ Flavor separation; $\frac{f_{i/p}^{[s]}(x,\mathbf{b}_T)}{f_{j/p}^{[s']}(x,\mathbf{b}_T)} = \frac{\tilde{f}_{i/p}^{\mathrm{naive}[s]}(x,\mathbf{b}_T)}{\tilde{f}_{j/p}^{\mathrm{naive}[s']}(x,\mathbf{b}_T)}$

✳ Spin-dependence, e.g., Sivers function (single-spin asymmetry);

✳ Full TMD and TMD wave function kinematic dependence.

✳ Twist-3 PDFs from small b_T expansion of TMDs. Ji, Liu, Schäfer and Yuan, PRD 103 (2021).

✳ Sub-leading power TMDs. Rodini and Vladimirov, JHEP 08 (2022).

Lorentz-invariant (LI) approach

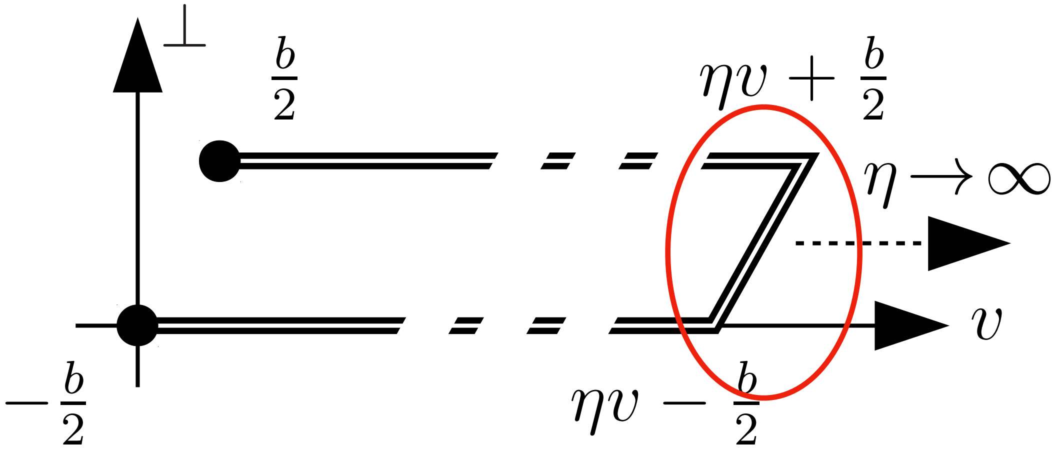
- Wilson line geometry ensuring maximal Lorentz symmetry.

- Lorentz-covariant decomposition of the lattice TMD correlator.

- Amplitudes related to the beam function by Lorentz invariance.

- Ratios of TMDs can be calculated at leading order in perturbation theory.

- Factorization theorem yet to be derived.



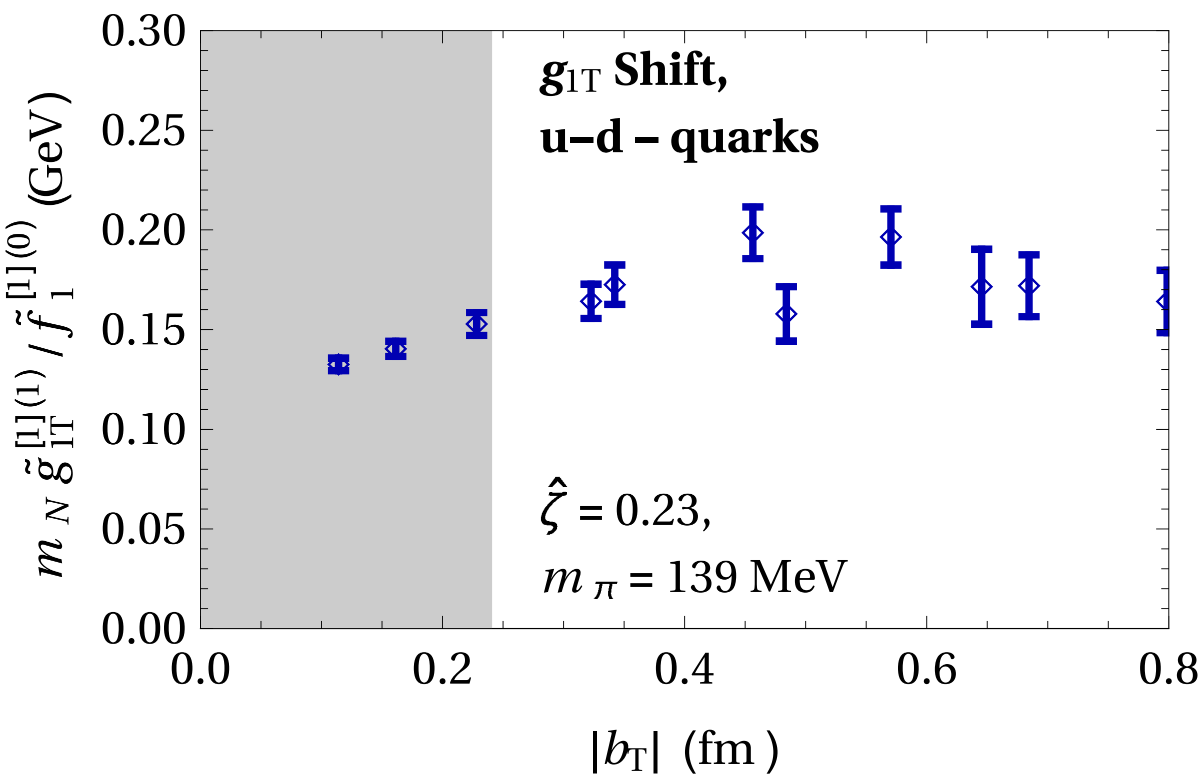
Hägler, Musch, Engelhardt, Negele, Schäfer, et al., EPL88 (2009), PRD83 (2011), PRD85 (2012), PRD93 (2016), 1601.05717, PRD96 (2017).

Outline

- TMDs from experiments
- Lattice methods for TMD calculation
- Results from lattice QCD

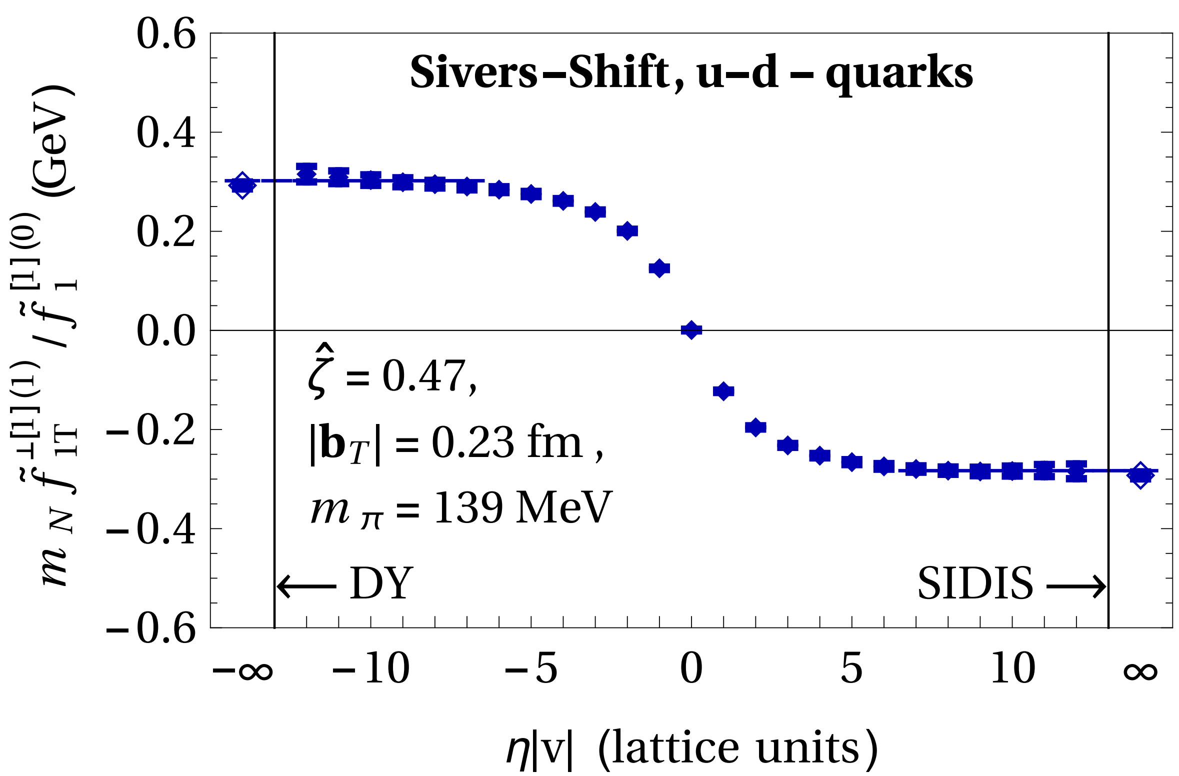
Ratio of TMD moments from the LI approach

$$\frac{m_N \tilde{g}_{1T}^{[1](1)}(b_T)}{\tilde{f}_1^{[1](0)}(b_T)}$$



M. Engelhardt, et al., *PoS* LATTICE2022 (2023)

$$\frac{m_N \tilde{f}_{1T}^{\perp[1](1)}(b_T)}{\tilde{f}_1^{[1](0)}(b_T)}$$



M. Engelhardt, et al., TMD Handbook, 2304.03302.

Collins-Soper kernel from LaMET

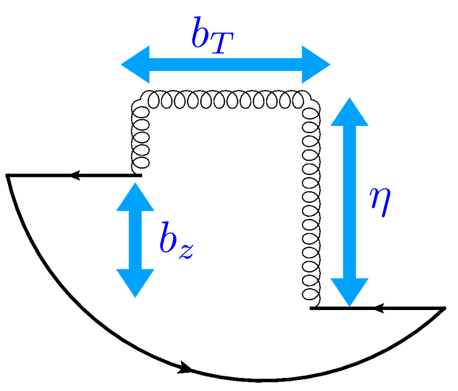
$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{C(\mu, xP_2^z) \int db^z\ e^{ib^z x P_1^z}\ \tilde{Z}'(b^z, \mu, \tilde{\mu}) \tilde{Z}_{\mathrm{UV}}(b^z, \tilde{\mu}, a) \tilde{W}(b^z, \mathbf{b}_T, a, \eta, P_1^z)}{C(\mu, xP_1^z) \int db^z\ e^{ib^z x P_2^z}\ \tilde{Z}'(b^z, \mu, \tilde{\mu}) \tilde{Z}_{\mathrm{UV}}(b^z, \tilde{\mu}, a) \tilde{W}(b^z, \mathbf{b}_T, a, \eta, P_2^z)}$$

Perturbative matching

Renormalization (and operator mixing)

$$\times \left\{ 1 + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(x\tilde{P}^z)^2}, \frac{1}{((1-x)\tilde{P}^z b_T)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{((1-x)\tilde{P}^z)^2} \right] \right\}$$

Power corrections



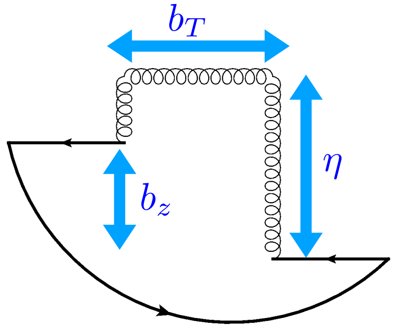
$$\bar{\psi}(b) \frac{\Gamma}{2} \mathscr{W}_{\mathrm{staple}}(b,0) \psi(0)$$

Current status for the Collins-Soper kernel

	Pion mass	Renormalization	Operator mixing	Fourier transform	Matching	*x*-plateau search
SWZ20 PRD 102 (2020) Quenched	$m_\pi = 1.2$ GeV	Yes	Yes	Yes	LO	Yes
LPC20 PRL 125 (2020)	$m_\pi = 547$ MeV	N/A	No	N/A	LO	N/A
SVZES JHEP08 (2021), 2302.06502	$m_\pi = 422$ MeV	N/A	No	N/A	NLO	N/A
PKU/ETMC 21 PRL 128 (2022)	$m_\pi = 827$ MeV	N/A	No	N/A	LO	N/A
SWZ21 PRD 106 (2022)	$m_\pi = 580$ MeV	Yes	Yes	Yes	NLO	Yes
LPC22 PRD 106 (2022)	$m_\pi = 670$ MeV	Yes	No	Yes	NLO	Yes
LPC23 JHEP 08 (2023)	$m_\pi = 220$ MeV	Yes	No	Yes	NLO	Yes
ASWZ23 2307.12359	$m_\pi = 148.8$ MeV	Yes	Yes	Yes	NNLL	Yes

Improved calculation at physical pion mass

Φ: Quasi-TMD wave function

$\tilde{\Phi} = \langle 0 |$  $| \pi(P) \rangle$

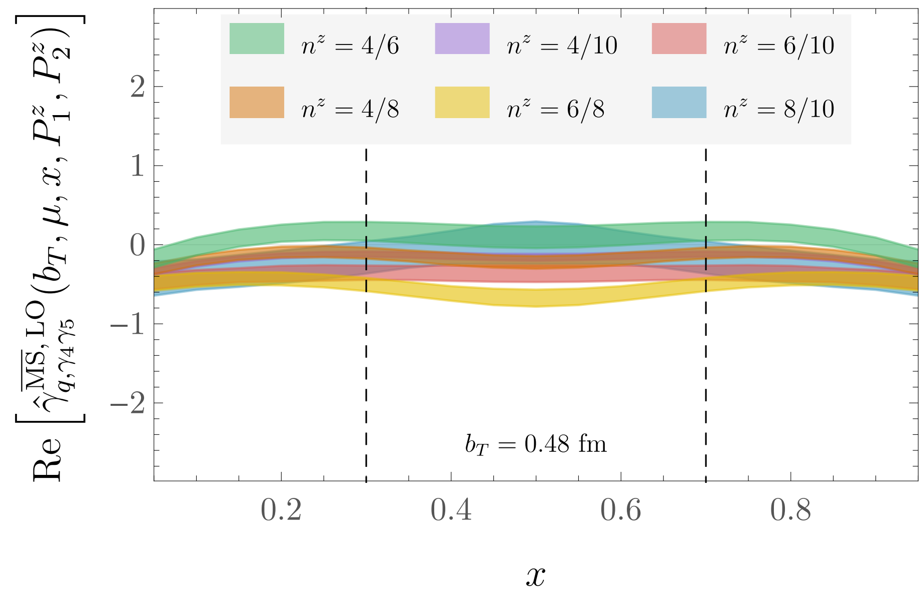
$a = 0.12$ fm,
$m_\pi = 148.8$ MeV,
$P^z_{\max} = 2.15$ GeV

- **Close-to-Physical pion mass**
  - Better suppressed power corrections
- **More stable Fourier transform**
- **Renormalization of nonlocal operator**
  - Systematic treatment of operator mixing using the RI-xMOM scheme

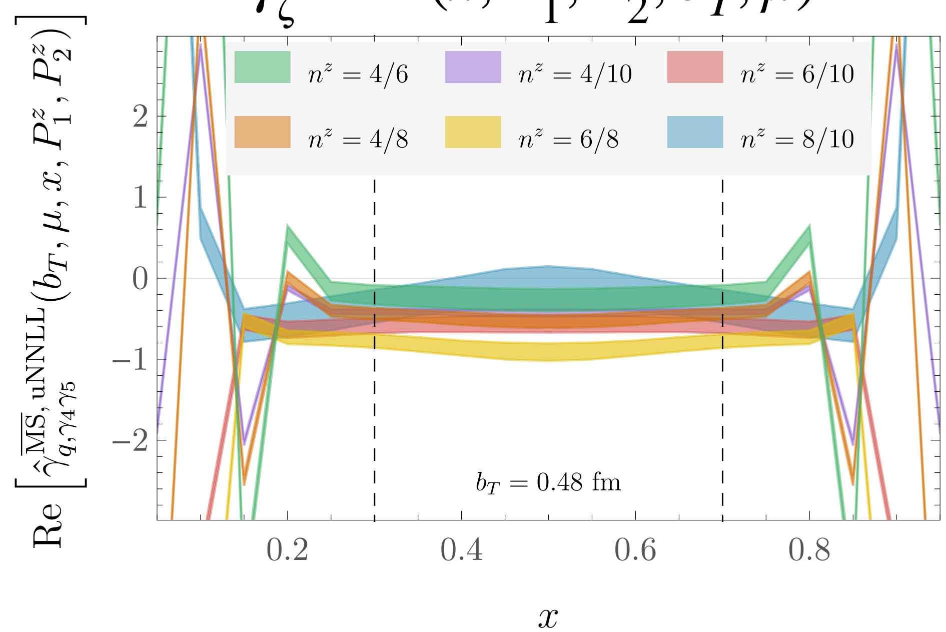
- Green, Jansen and Steffens, PRL **121** (2018) and PRD **101** (2020).
- Constantinou, Panagopoulos, and Spanoudes, PRD **99** (2019).

A. Avkhadiev, P. Shanahan, M. Wagman and YZ, 2307.12359.

$\gamma_\zeta^{\rm LO}(x, P_1^z, P_2^z, b_T, \mu)$



$\gamma_\zeta^{\rm uNNLL}(x, P_1^z, P_2^z, b_T, \mu)$



Matching and (perturbative) power corrections

- **Matching correction:**

$$\delta\gamma_q(x, P_1^z, P_2^z, \mu) = \frac{1}{\ln(P_1^z/P_2^z)}\left[\ln\frac{C(xP_2^z,\mu)}{C(xP_1^z,\mu)} + x \to \bar{x}\right], \quad C(p^z,\mu) = C(p^z, 2p^z)\ \exp\left[K(p^z, 2p^z)\right]$$

Up to NNLO   Up to N3LL

- del Río and Vladimirov, 2304.14440.
- Ji, Liu and Su, JHEP 08 (2023).
- Braun, Chetyrkin and Kniehl, JHEP 07 (2020).
- Stewart, Tackmann and Waalewijn, JHEP 09 (2010).

Accuracy	K_Γ	K_{γ_C}	K_{γ_μ}	η	C_ϕ
NLL	2	1	1	1	0
NNLL	3	2	2	2	1

- **Collinear v.s. TMD factorization:**
  - $p^z \gg \Lambda_{\rm QCD}$ so a factorization exists.
    - If $p^z b_T \gg 1$, TMD region.
    - If $p^z b_T \ll 1$, collinear region.
    - If $p^z b_T \sim 1$, collinear but with calculable power corrections.

e.g., p^z=2 GeV, b_T=0.2 fm, p^z*b_T=2

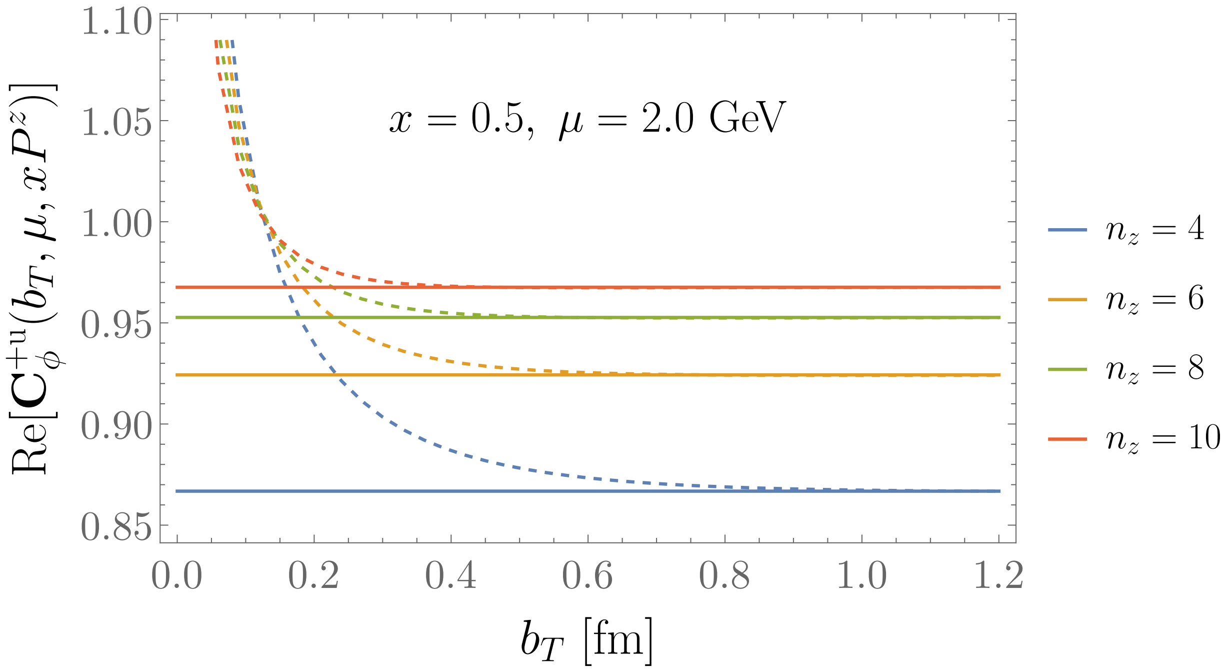
Matching and (perturbative) power corrections

- **Unexpanded matching coefficient:**

$$C^{\mathrm{uNLO}}(p^z, b_T, \mu) = C(p^z, \mu) + \delta C(p^z, b_T)$$

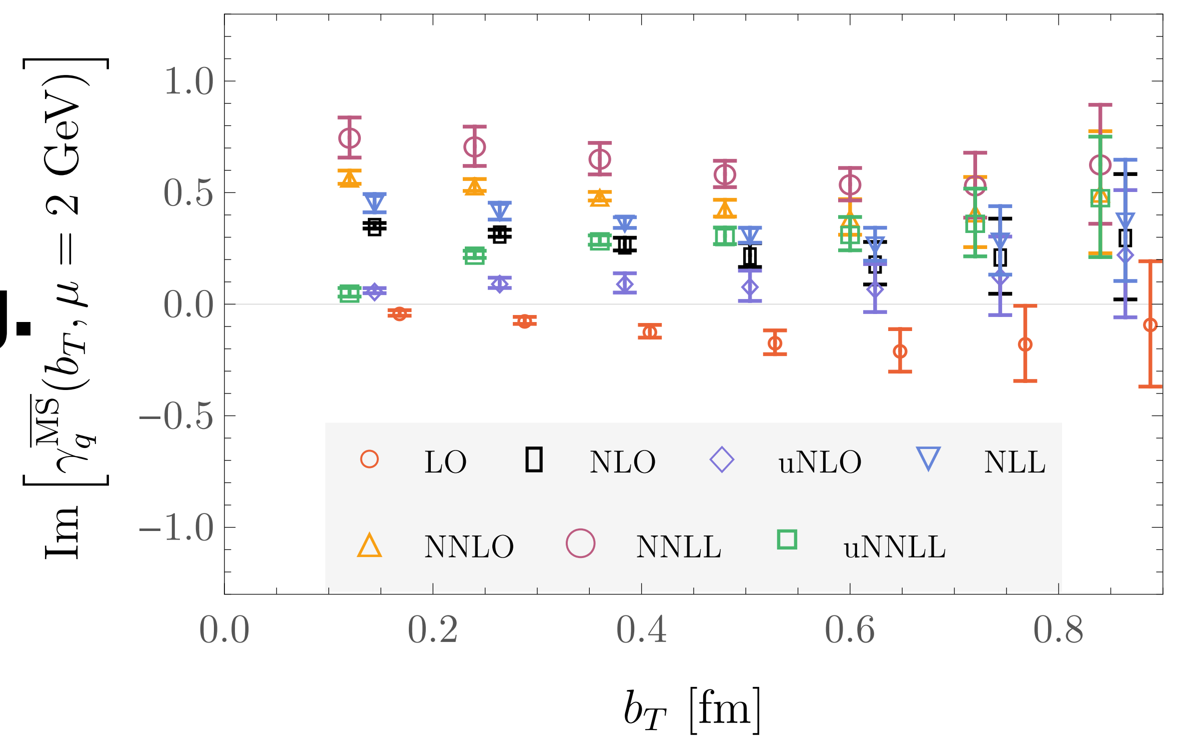
$$\lim_{p^z b_T \to \infty} \delta C(p^z, b_T) = 0$$

$$C^{\mathrm{uNNLL}}(p^z, b_T, \mu) = C^{\mathrm{uNLO}}(p^z, b_T 2p^z) \times \exp\left[K^{\mathrm{NNLL}}(p^z, 2p^z)\right]$$



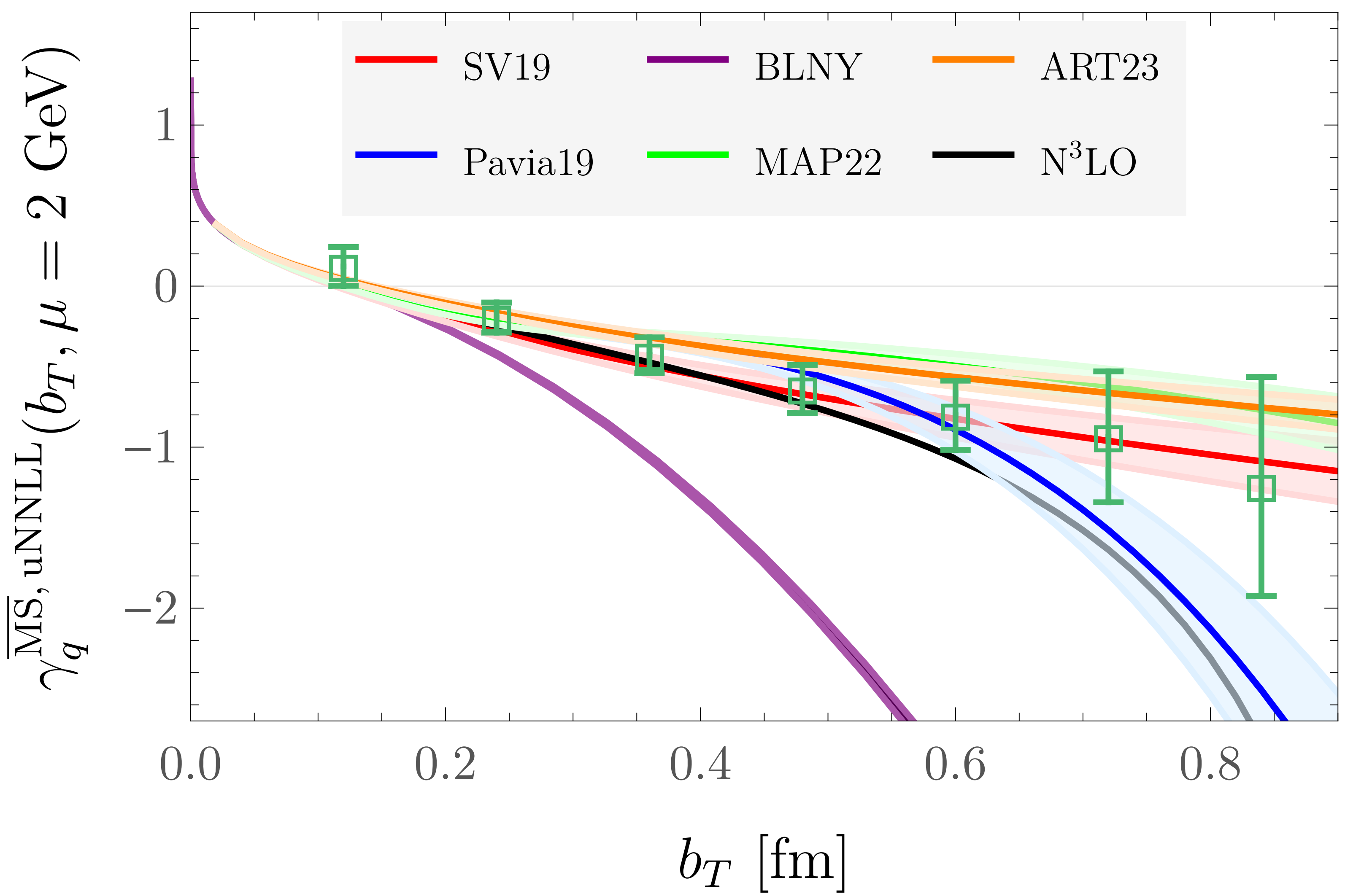
  - Ebert, Stewart, YZ, PRD99 (2019), JHEP09 (2019) 037;
  - Deng, Wang and Zeng, JHEP 09 (2022).

- **The CS kernel has a unphysical imaginary part which cannot be cancelled by NLO/NNLO matching.**
  - Significantly reduced with the unexpanded matching!
  - Convergence in P^z also improved.



Collins-Soper kernel from lattice at NNLL

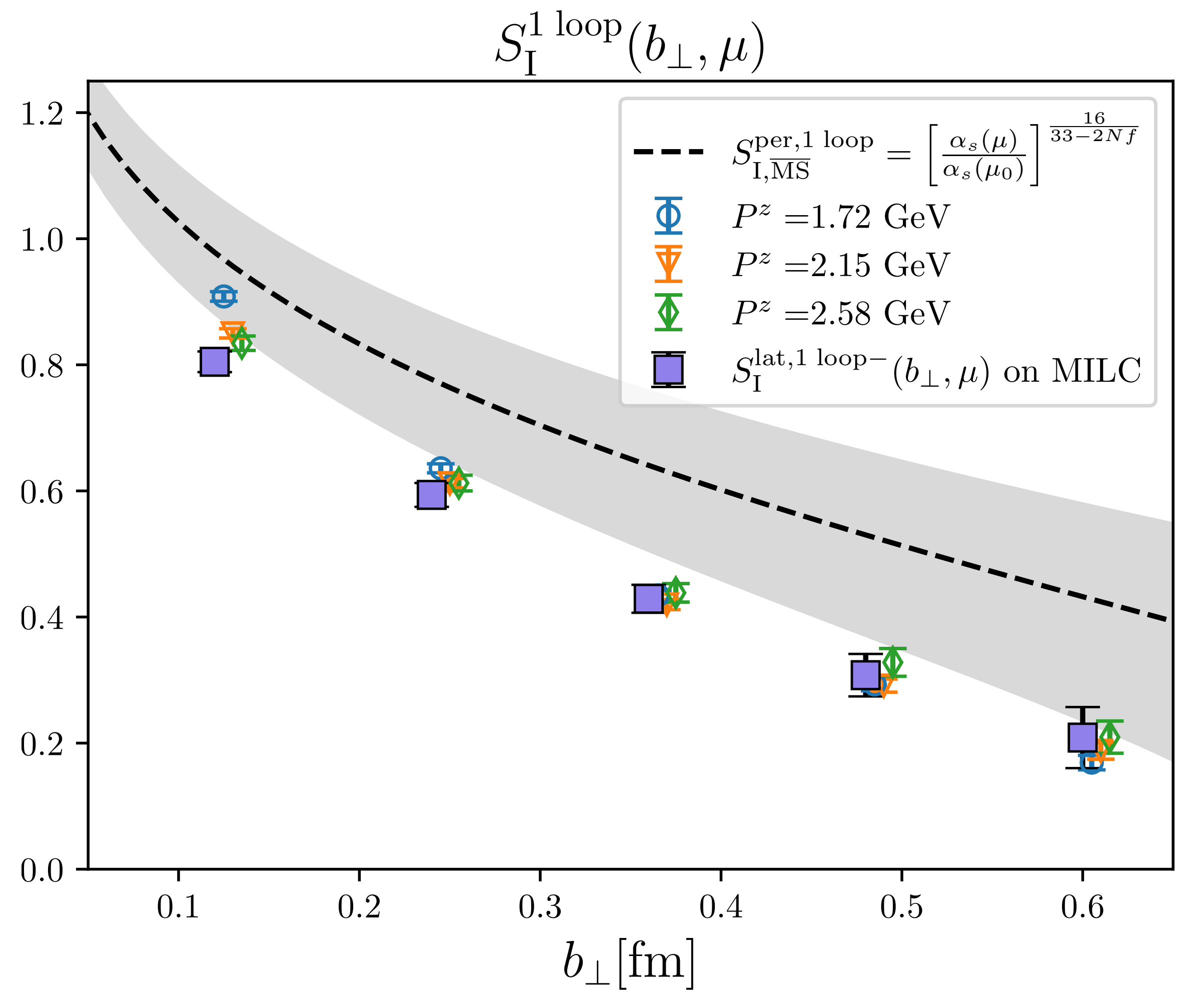
- Final result in comparison with global fits and perturbative QCD

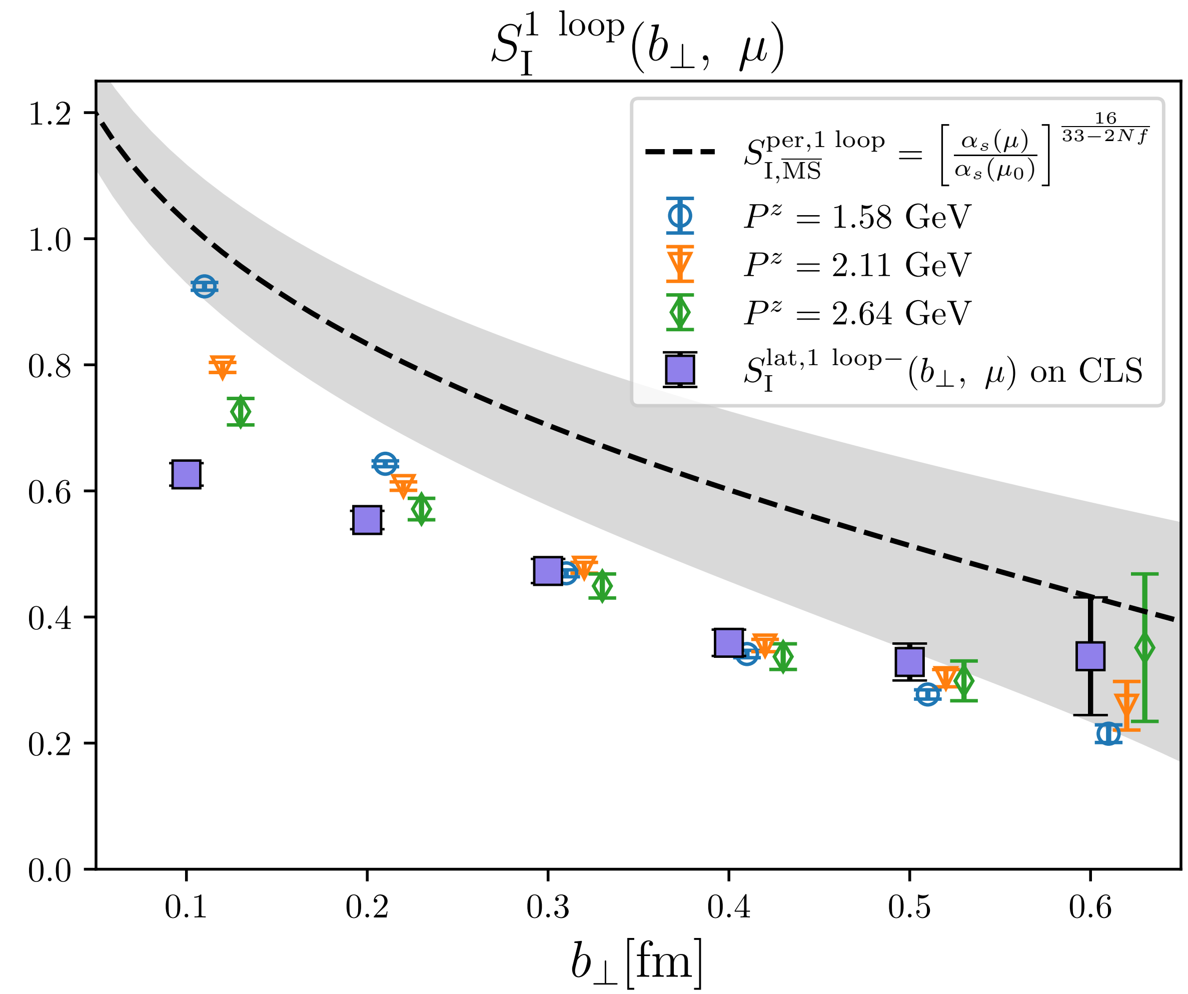


SV19: I. Scimemi and A. Vladimirov, JHEP **06** (2020)
Pavia19: A. Bacchetta et al., JHEP **07** (2020)
BLNY: Landry, Brock, Nadolsky and Yuan, PRD 67 (2003)

Reduced soft factor at NLO accuracy

Ensemble	a(fm)	$N_\sigma^3 \times N_\tau$	m_π^{sea}	m_π^{val}	Measure
X650	0.098	$48^3\times$ 48	333 MeV	662 MeV	911×4
A654	0.098	$24^3\times$ 48	333 MeV	662 MeV	4923×20
a12m130	0.121	$48^3\times$ 64	132 MeV	310 MeV	1000×4
				220 MeV	1000×16
a12m310	0.121	$24^3\times$ 64	305 MeV	670 MeV	1053×8

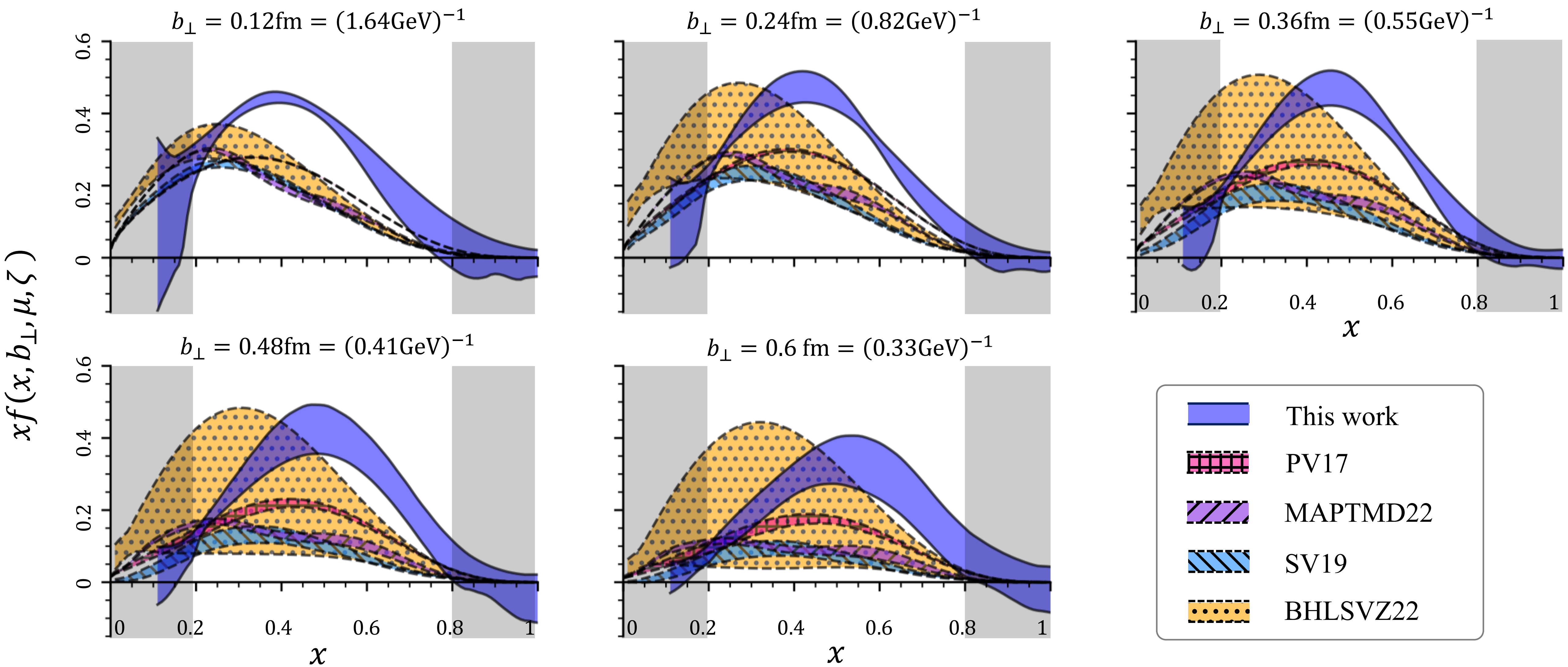




M.-H. Chu et al., LPC, JHEP 08 (2023).

(x, b_T) dependence of the unpolarized proton TMD

J.-C. He, M.-H. Chu, J. Hua et al., (LPC), arXiv: 2211.02340.



$a = 0.12$ fm, $m_\pi = \{310, 220\}$ MeV, $P^z_{max} = 2.58$ GeV

SV19: Scimemi and Vladimirov, JHEP **06** (2020)
Pavia19: Bacchetta et al., JHEP **07** (2020).
MAPTMD22: Bacchetta et al., JHEP **10** (2022).
BHLSVZ22: Bury et al., JHEP **10** (2022).

Pion TMD wave function also available in

M.-H. Chu, J.-C. He, J. Hua et al., (LPC), 2302.09961, and JHEP **08** (2023).

Conclusion

- The quark and gluon quasi TMDs can be factorized into the physical TMDs, without any mixing.
- Lattice calculation of the Collins-Soper kernel has made significant progress in reducing the systematics.
- First calculations of the soft function and TMD are available, but the systematics needs to be under control.
- Understanding the power corrections is important!

Outlook

Observables	Status
Non-perturbative Collins-Soper kernel	Better understanding of the systematics, with room for improvement (e.g., np power corrections, $a \to 0$)
Soft factor	to be under systematic control
Spin-dependent TMDs (in ratios)	In progress
Proton v.s. pion TMDs, (x, b_T) (in ratios)	In progress
Flavor dependence of TMDs, (x, b_T) (in ratios)	to be studied
TMDs and TMD wave functions, (x, b_T)	to be under systematic control
Gluon TMDs (x, b_T)	to be studied
Wigner distributions (x, b_T, Δ_T)	to be studied

Backup Slides

Factorization relation with the TMDs

Lattice

Quasi

New

Large-rapidity (LR) scheme

Collins scheme

Continuum

$$\tilde{f}_i(x, \mathbf{b}_T, \mu, \tilde{\zeta}, \tilde{P}^z) = \lim_{\tilde{P}^z \gg m_N} \lim_{a \to 0} \tilde{Z}_{\mathrm{UV}} \frac{\tilde{B}_i}{\sqrt{S^q}}$$

Lorentz boost $\qquad y_{\tilde{P}} = y_P - y_B$

$$f_i^{\mathrm{LR}}(x, \mathbf{b}_T, \mu, \zeta, y_P - y_B) = \lim_{-y_B \gg 1} \lim_{\epsilon \to 0} Z_{\mathrm{UV}}^{\mathrm{LR}} \frac{B_i}{\sqrt{S^q}}$$

Same matrix elements, but different orders of UV limits

Perturbative matching in LaMET!

$$f_i(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\epsilon \to 0} Z_{\mathrm{UV}} \lim_{y_B \to -\infty} \frac{B_i}{\sqrt{S^q}}$$

Ebert, Schindler, Stewart and YZ, JHEP **04**, 178 (2022).

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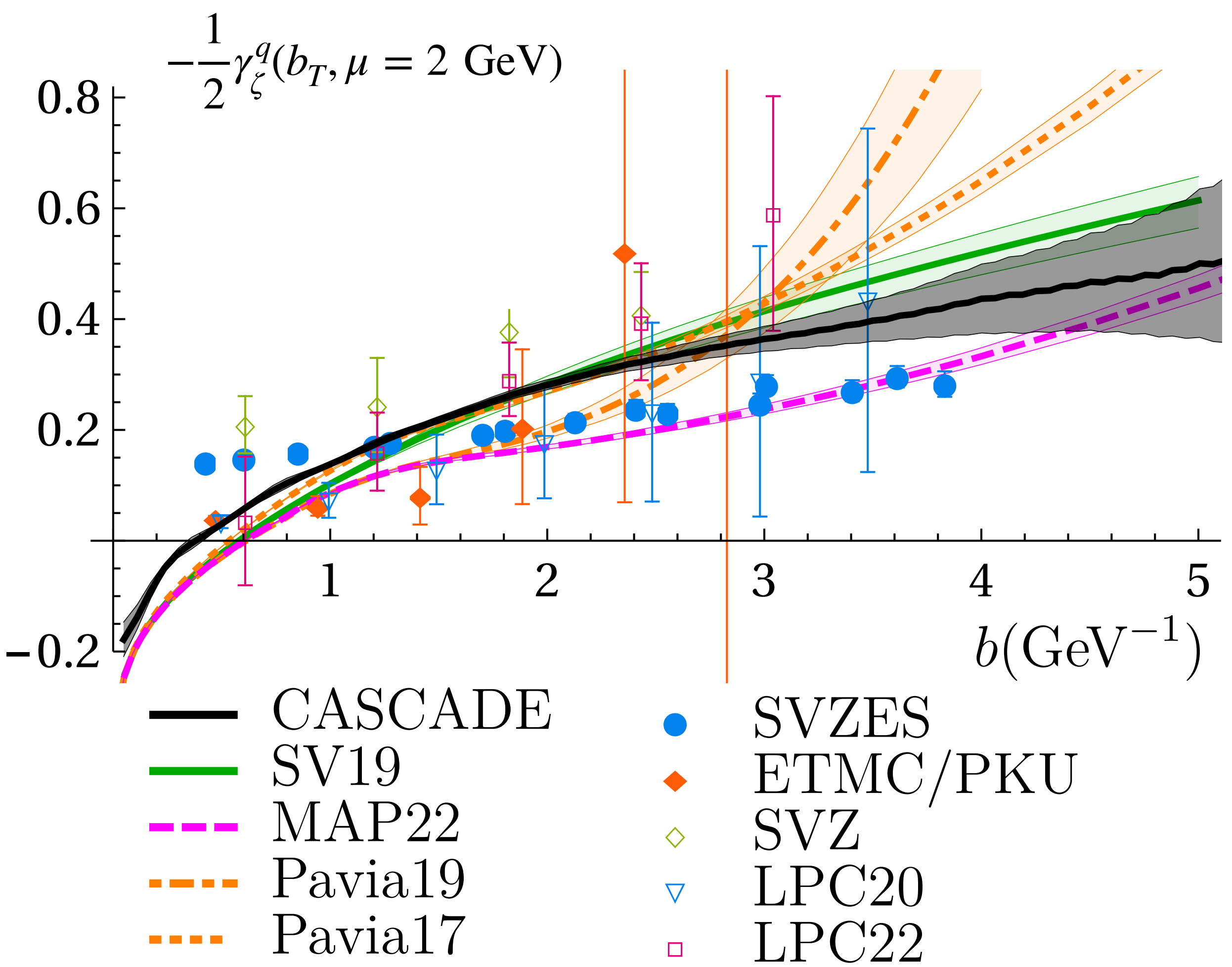
Collins Soper kernel from Lattice QCD

Comparison between lattice results and global fits



MAP22: Bacchetta, Bertone, Bissolotti, et al., JHEP **10** (2022)
SV19: I. Scimemi and A. Vladimirov, JHEP **06** (2020)
Pavia19: A. Bacchetta et al., JHEP **07** (2020)
Pavia 17: A. Bacchetta et al., JHEP **06** (2017)
CASCADE: Martinez and Vladimirov, PRD **106** (2022)

Approach	Collaboration
Quasi beam functions	P. Shanahan, M. Wagman and YZ (SWZ21), PRD **104** (2021)
Quasi TMD wavefunctions	Q.-A. Zhang, et al. (LPC20), PRL **125** (2020).
	Y. Li et al. (ETMC/PKU 21), PRL **128** (2022).
	M.-H. Chu et al. (LPC22), PRD 106 (2022) JHEP 08 (2023)
Moments of quasi TMDs	Schäfer, Vladmirov et al. (SVZES21), JHEP **08** (2021), 2302.06502

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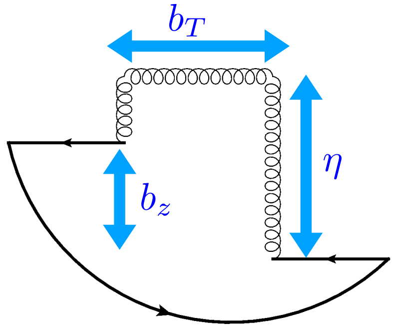
$\tilde{\Phi} = \langle 0|$  $|\pi(P)\rangle$

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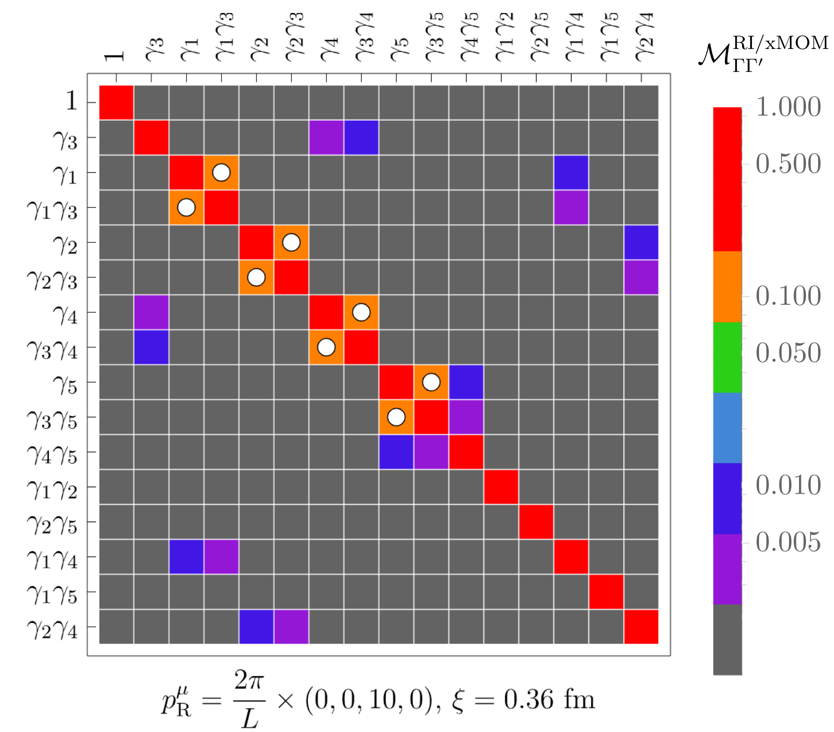
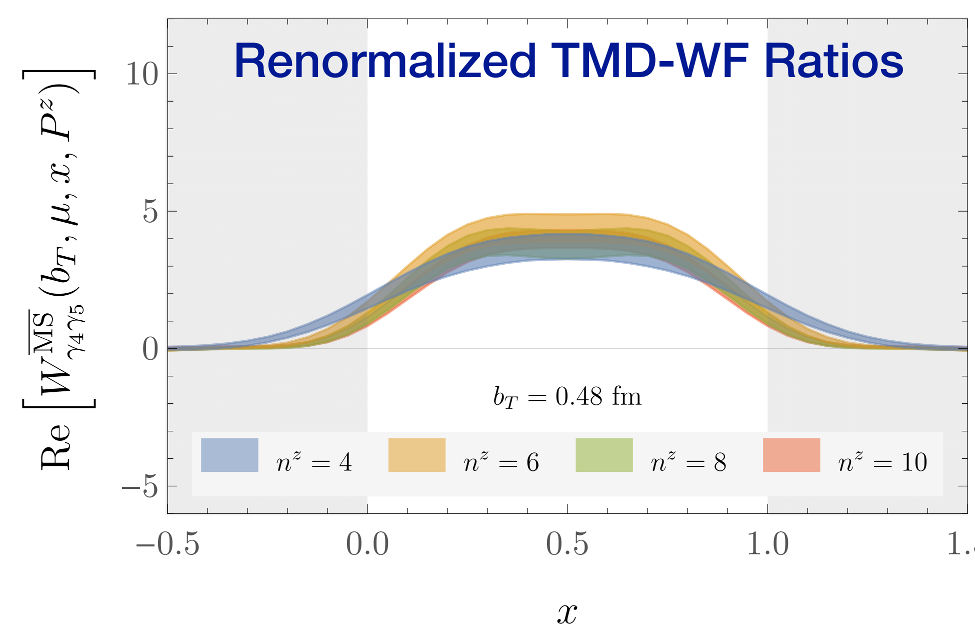
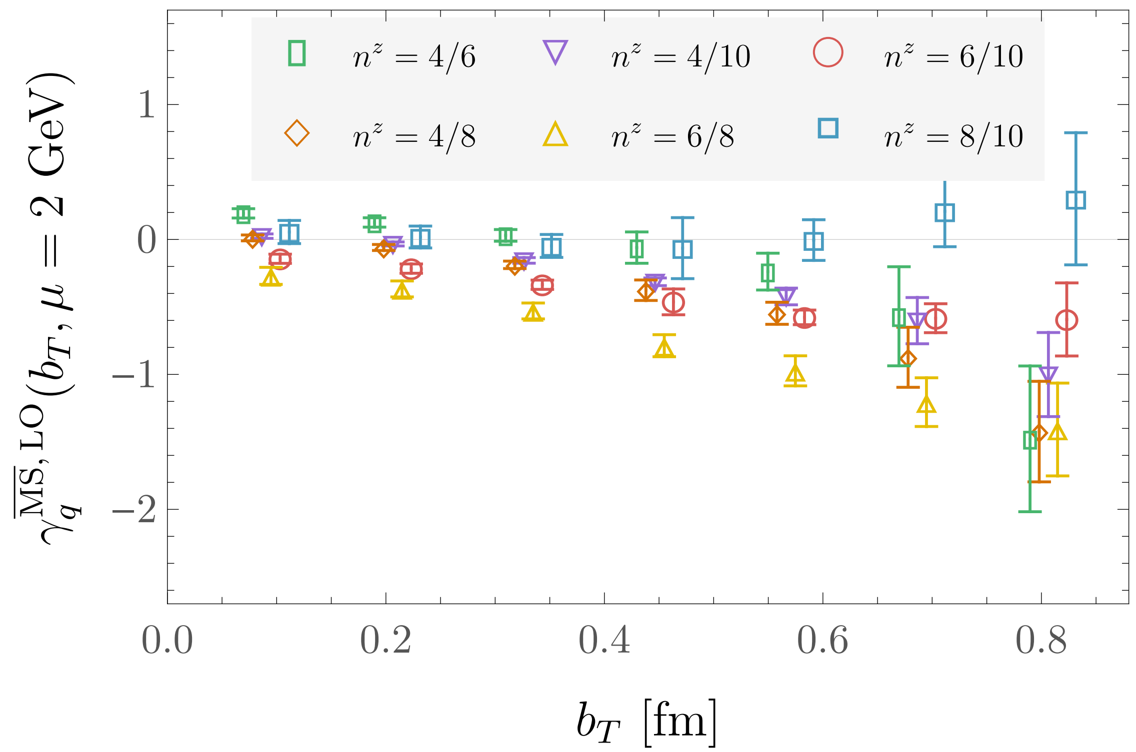
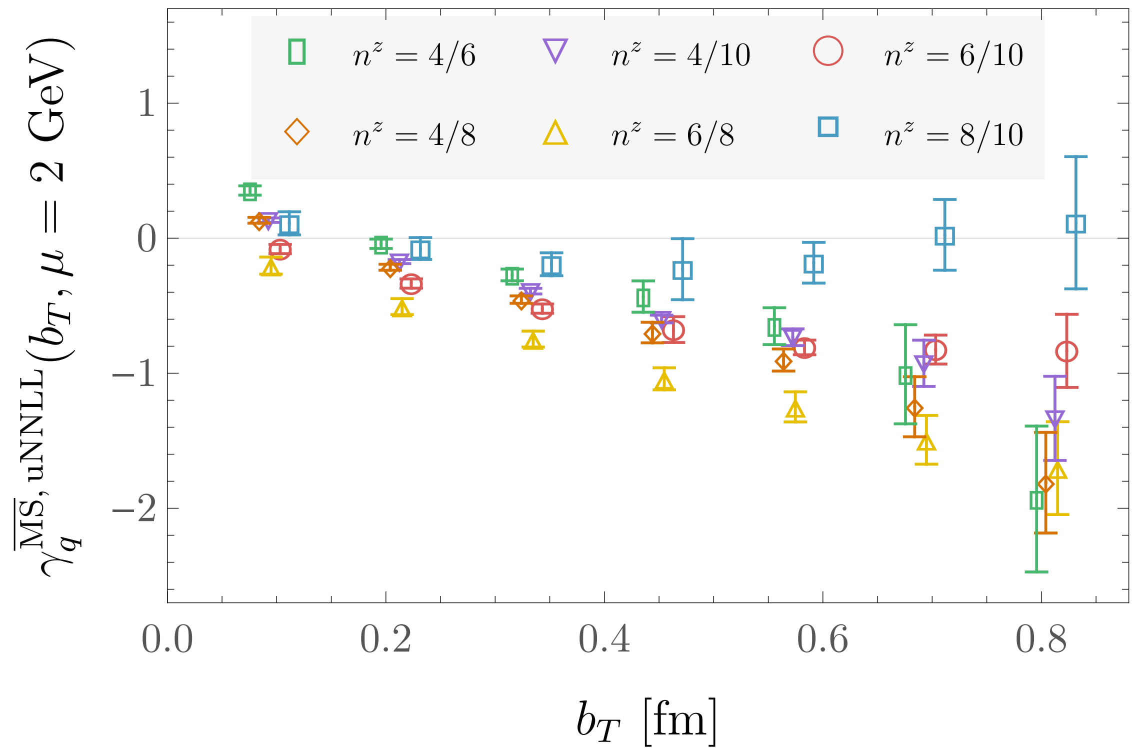
Operator mixing pattern







$n^z = 4/6$
$n^z = 4/10$
$n^z = 6/10$
$n^z = 4/8$
$n^z = 6/8$
$n^z = 8/10$
$\gamma_q^{\overline{\rm MS},\,{\rm LO}}(b_T, \mu = 2\ {\rm GeV})$
b_T [fm]



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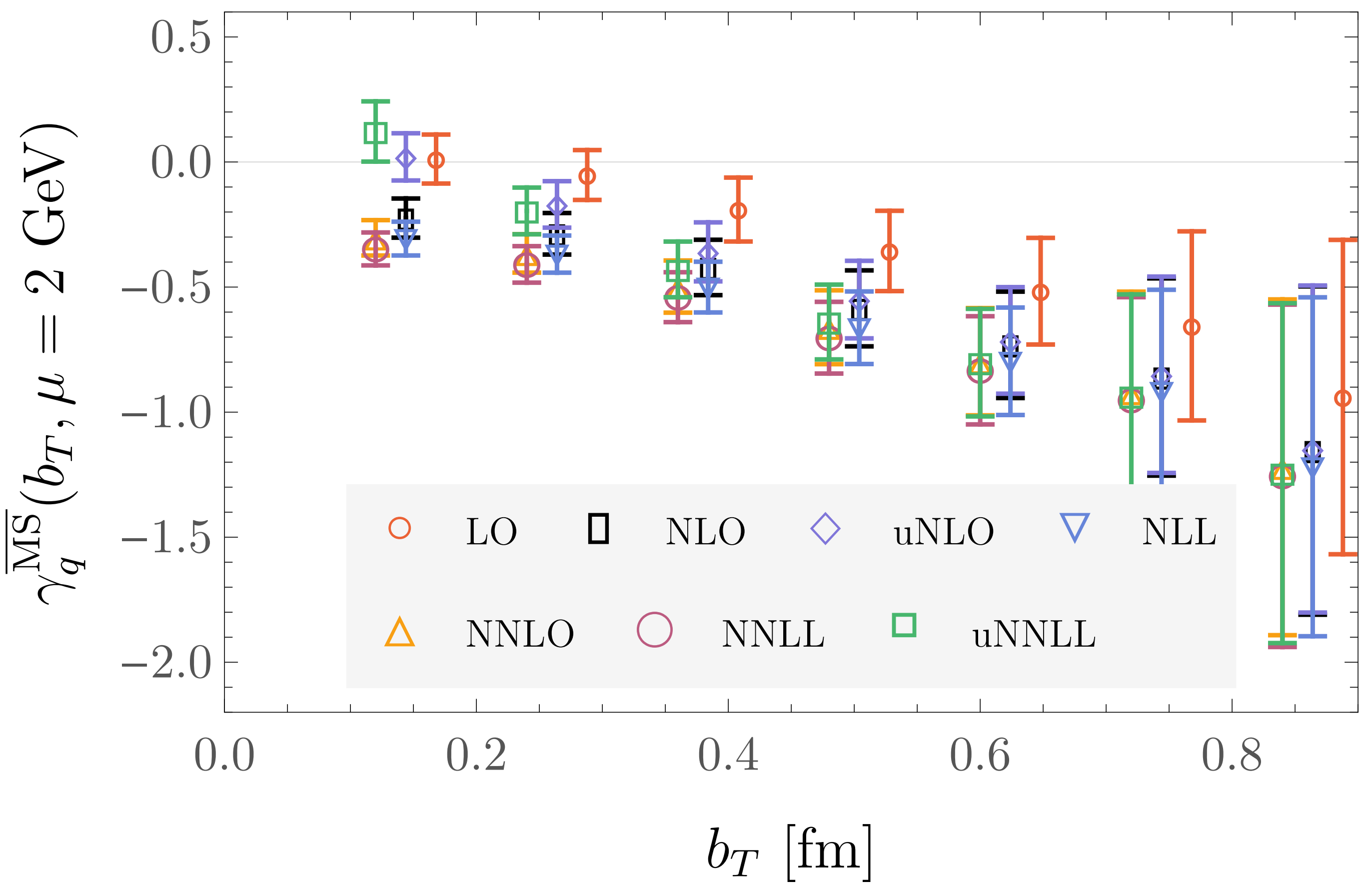
b_T [fm]



LO
NLO
uNLO
NLL
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$\gamma_q^{\overline{\rm MS}}(b_T, \mu = 2\ {\rm GeV})$
b_T [fm]