First lattice QCD exploration of chiral-even axial twist-3 GPDs of the proton

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Outline:

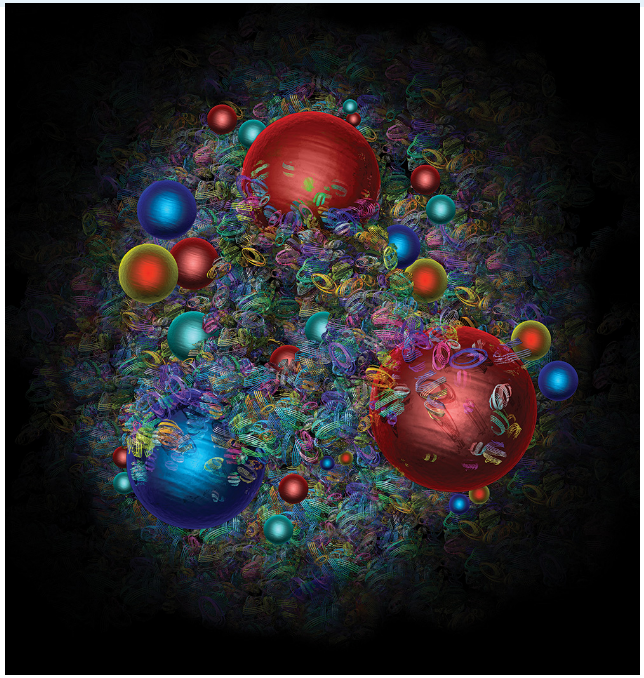
Introduction
Quasi-GPDs
Twist-3 axial GPDs
Prospects/conclusion

Many thanks to our "twist-3 team":

S. Bhattacharya, M. Constantinou, J. Dodson,
A. Metz, A. Scapellato, F. Steffens

Special thanks to J. Miller for help in the presentation of results

Nucleon structure and GPDs

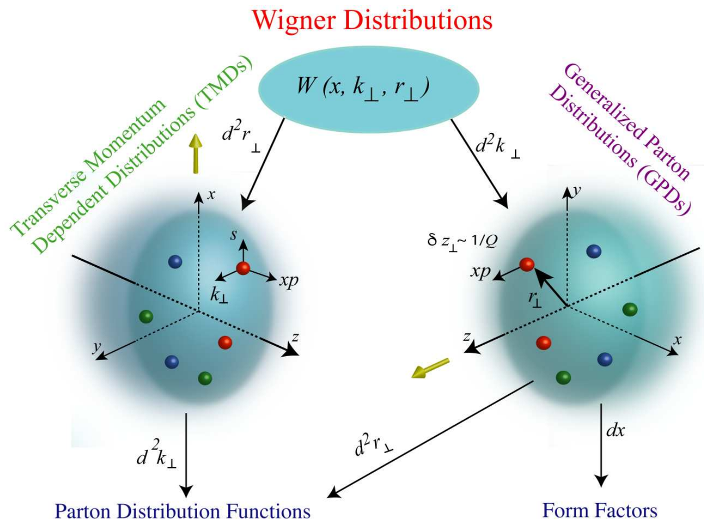


One of the central aims of hadron physics: to understand better nucleon's 3D structure.

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise? NAS report 2018
- What are the emergent properties of dense systems of gluons?
- Answering these questions is one of the crucial expectations for the upcoming years!
- For this, we need to probe the 3D structure.
- Transverse position of quarks: GPDs.
- Twist-2 GPDs as first aim, **but higher-twist of growing importance**.
- Both theoretical and experimental input needed.

Generalized parton distributions (GPDs):

- much more difficult to extract than PDFs,
- but they provide a wealth of information:
  - ⋆ spatial distribution of partons in the transverse plane,
  - ⋆ mechanical properties of hadrons,
  - ⋆ hadron's spin decomposition,
- reduce to PDFs in the forward limit, e.g. $H(x, 0, 0) = f_1(x)$,
- their moments are form factors, e.g. $\int dx\, H(x, \xi, t) = F_1(t)$.



Twist-3 – growing importance

Based on discussion by V. Braun, EPJ Web Conf. 274 (2022) 01012 (Confinement XV, Stavanger 2022 conf. proceedings)

Operator **twist = dimension − spin** determines the importance of an operator near the light-cone. D. Gross, S. Treiman, Phys. Rev. D4 (1971) 1059

Leading twist – dominant contribution to light-cone dominated processes,
Higher twist – contributions are power-suppressed.

Why important?

- one obvious reason – very high accuracy of present (JLab, LHC, KEK) and future data (EIC),
- more fundamental – insights on quark-gluon correlations and quantum interference effects in hadrons.

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Twist-3 GPDs – several interesting relations to:

- quark OAM,
- transverse force on a quark in a polarized nucleon,
- spin-orbit correlations,
- Wigner functions,
- . . . .

Two pioneering directions of our group: K.C., M.Constantinou, A.Scapellato, F.Steffens

+ S.Bhattacharya, A.Metz – matching and first extraction of twist-3 PDFs:
matching g_T, Phys. Rev. D102 (2020) 034005
matching h_L, e, Phys. Rev. D102 (2020) 114025
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Follow-up work for twist-2 GPDs:
S.Bhattacharya, K.C., M.Constantinou, A.Metz, A.Scapellato, F.Steffens
additional inputs from Temple Ph.D. students (J.Dodson, J.Miller)
and ANL/BNL (X.Gao, S.Mukherjee, P.Petreczky, Y.Zhao)
x-dependent GPDs from asymmetric frames of reference, Phys. Rev. D106 (2022) 114512
moments from OPE of non-local operators, Phys. Rev. D108 (2023) 014507
helicity, in preparation

S.Bhattacharya plenary Fri 8:30

M.Constantinou Tue 10:30

X.Gao today 11:22

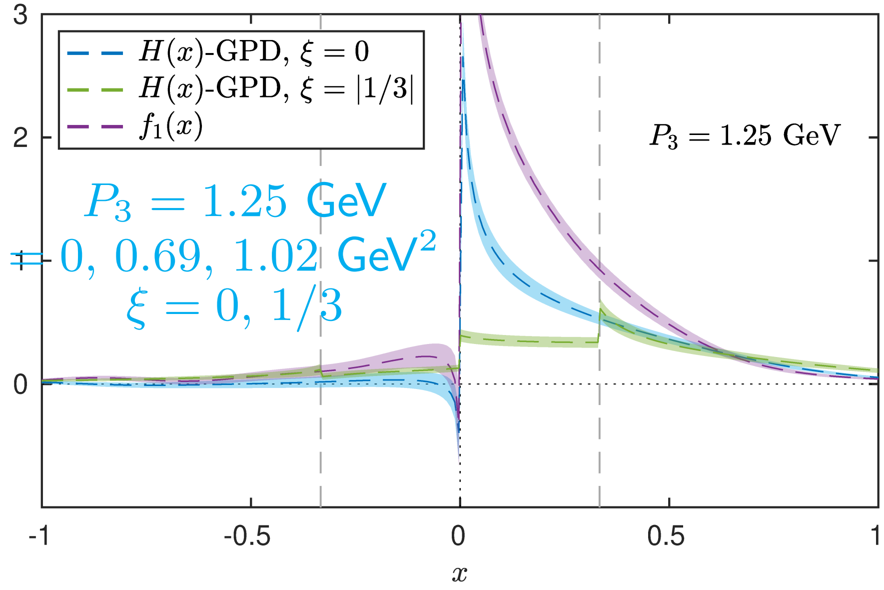
Another thread: twist-2 GPDs from pseudo-distributions
S.Bhattacharya, K.C., M.Constantinou, A.Metz, F.Steffens
+ Adam Mickiewicz Univ. Ph.D. student (N.Nurminen) and postdoc (W.Chomicki)

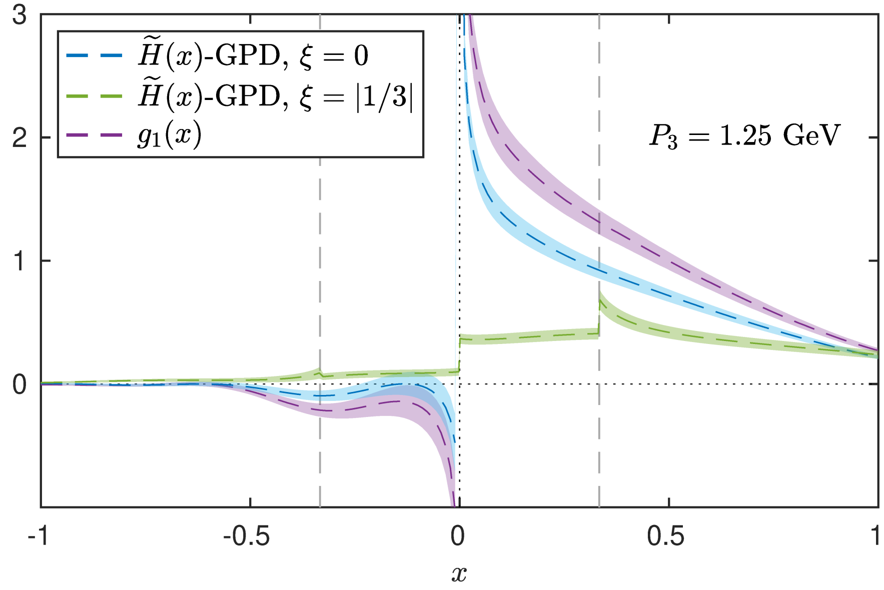
First extractions of x-dependent GPDs

unpolarized

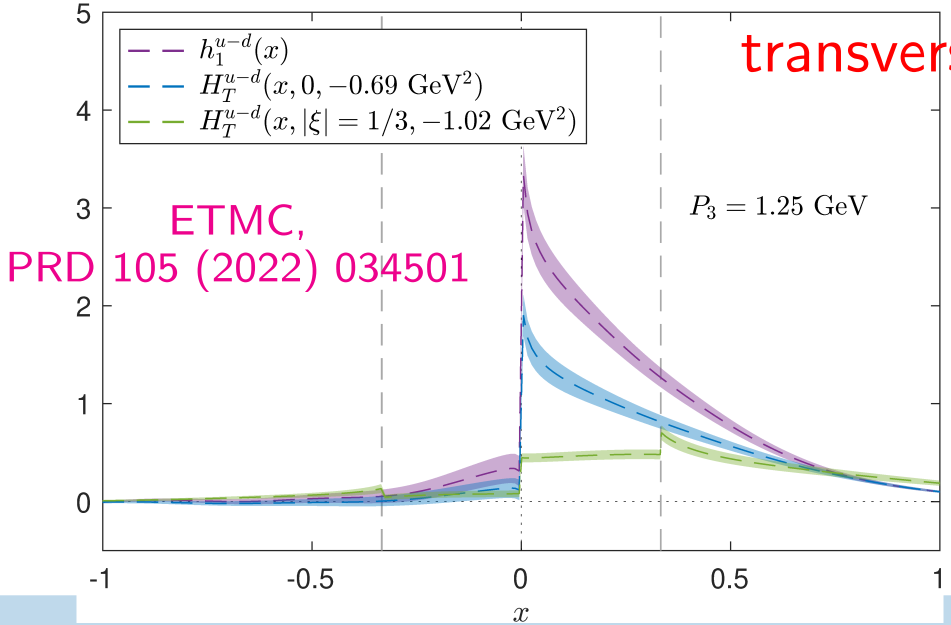
ETMC, Phys. Rev. Lett. 125 (2020) 262001

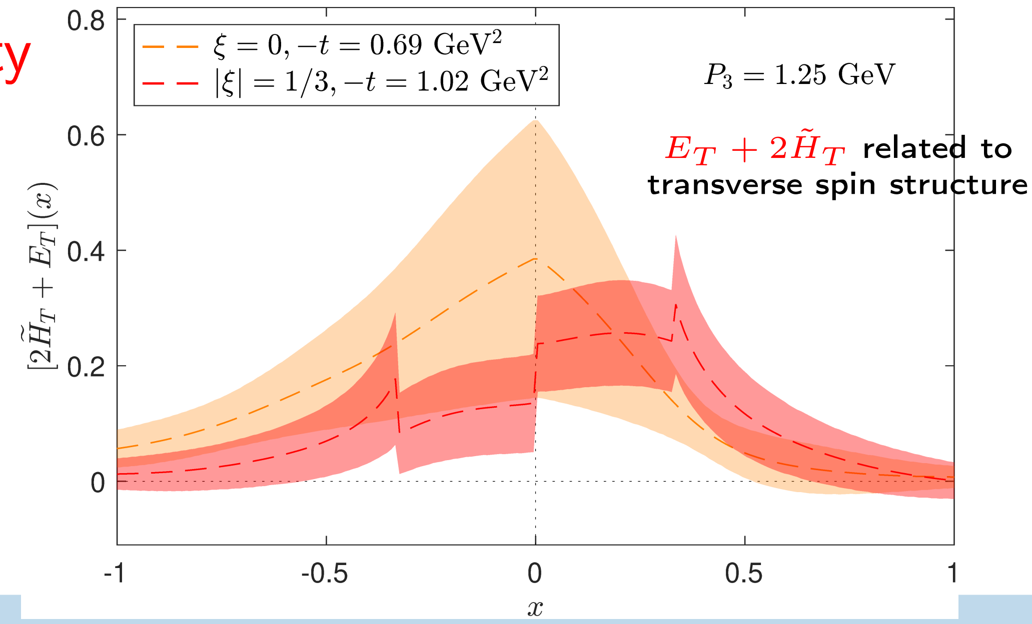
helicity





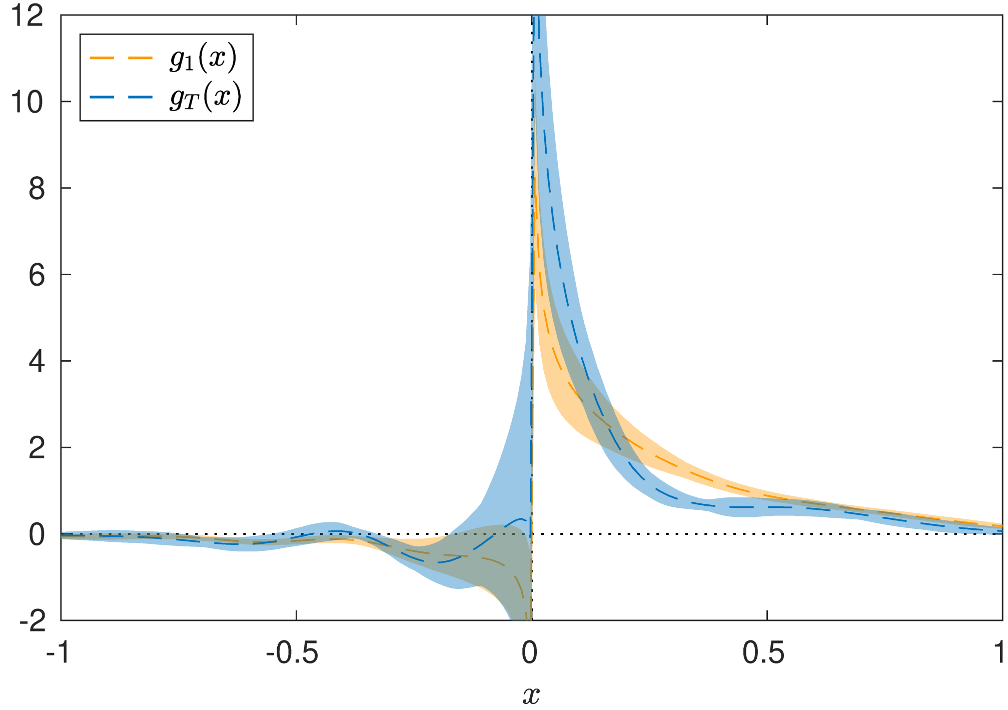
transversity





First extraction of twist-3 PDF g_T

Twist-2 g_1 vs. twist-3 g_T
(at the largest boost)



Burkhardt-Cottingham sum rule:

$$\int_{-1}^{1} dx g_T(x) = \int_{-1}^{1} dx g_1(x)$$

satisfied in our data.

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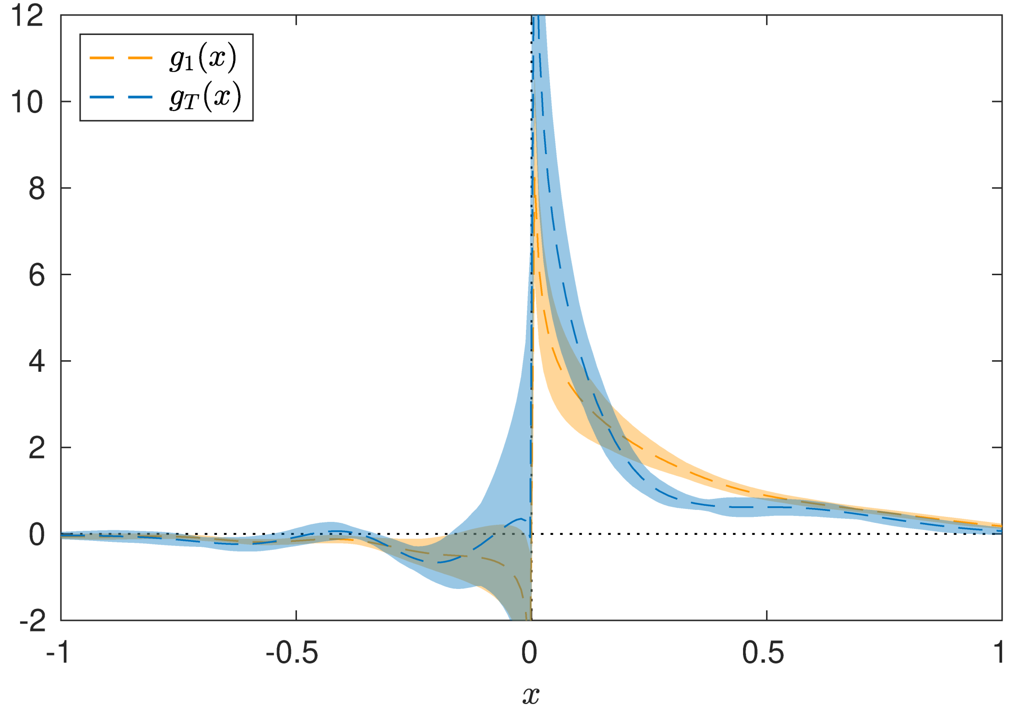

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Note: neglected qgq correlations
see also: V. Braun, Y. Ji, A. Vladimirov, JHEP 05(2021)086, 11(2021)087

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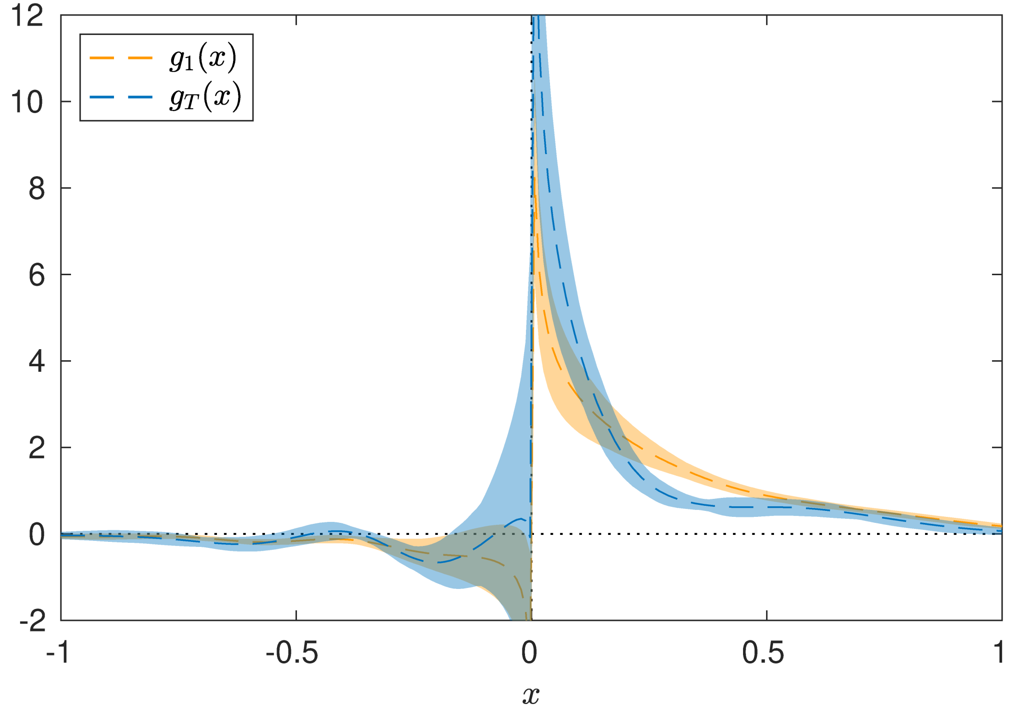
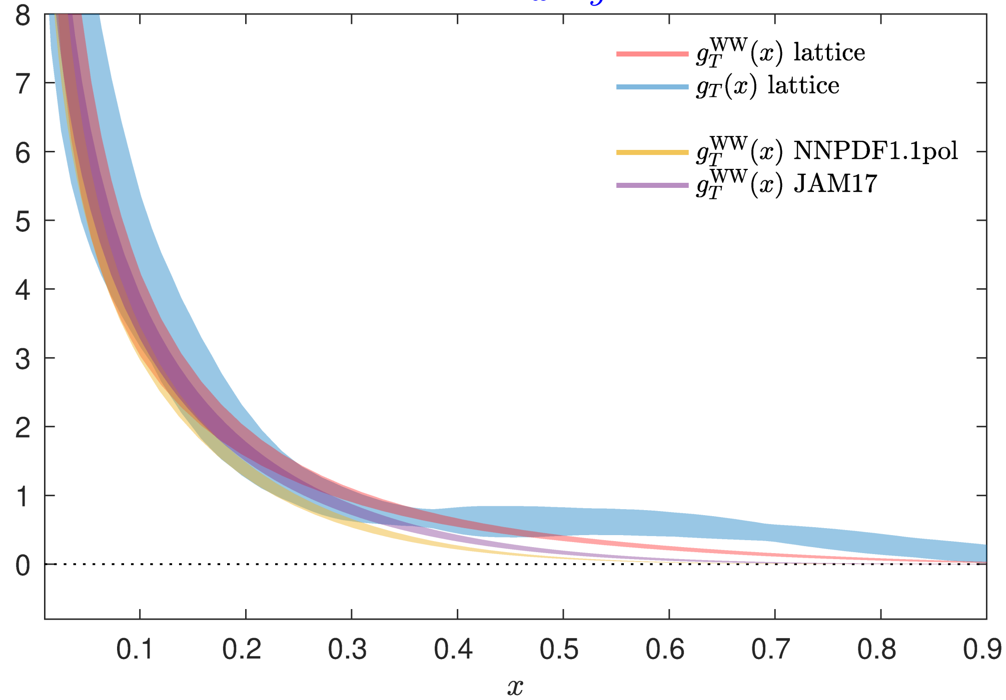


WW approx.: $g_T(x)$ fully determined by $g_1(x)$

$$g_T^{\mathrm{WW}}(x) = \int_x^1 \frac{dy}{y} g_1(y)$$



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agreement between $g_T(x)$ and $g_T^{\mathrm{WW}}(x)$
for $x \lesssim 0.5$ within uncertainties
still: possible violation up to 30-40%
interestingly, similar possible violation (15-40%)
in experimental data analysis by JLab:
A. Accardi, A. Bacchetta, W. Melnitchouk, M. Schlegel, JHEP 11 (2009) 093

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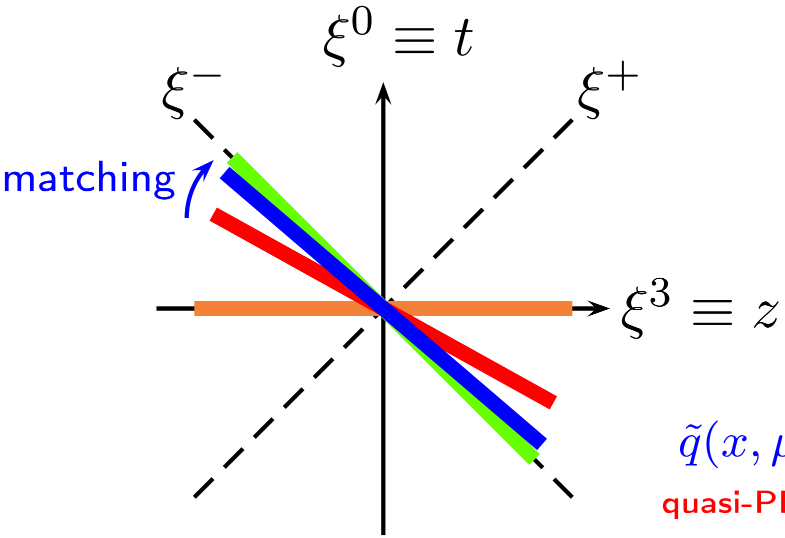
Quasi-distributions

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002



Euclidean matrix element:
$\langle P_f|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P_i\rangle$

$P_i = P_f$ – PDF
$P_i \neq P_f$ – GPD

Its Fourier transform (quasi-distribution)
can be matched onto the light-cone distribution:
(Large Momentum Effective Theory (LaMET))

$$\tilde{q}(x,\mu,P_3) = \int_{-1}^{1} \frac{dy}{|y|}\, C\left(\frac{x}{y}, \frac{\mu}{P_3}\right) q(y,\mu) + \mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, M_N^2/P_3^2\right)$$

quasi-PDF/GPD — pert.kernel — PDF/GPD — higher-twist effects

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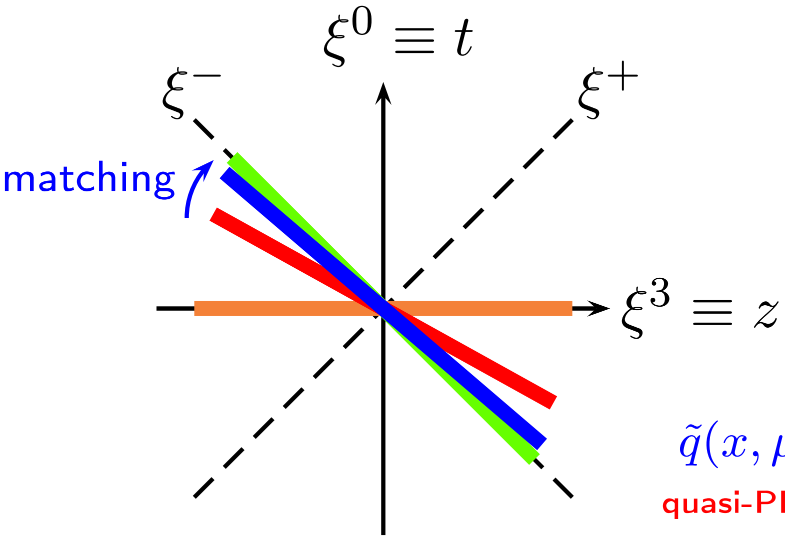

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quasi-PDF/GPD   pert.kernel   PDF/GPD   higher-twist effects

Dirac structures Γ for different PDFs/GPDs:

VECTOR: γ_0, γ_3: f_1, H, E (unpolarized twist-2),
γ_1, γ_2: G_1, G_2, G_3, G_4 (vector twist-3).

AXIAL VECTOR: $\gamma_5\gamma_0, \gamma_5\gamma_3$: $g_1, \tilde{H}, \tilde{E}$ (helicity twist-2),
$\gamma_5\gamma_1, \gamma_5\gamma_2$: $g_T, \tilde{G}_1, \tilde{G}_2, \tilde{G}_3, \tilde{G}_4$ (axial vector twist-3).

TENSOR: $\gamma_1\gamma_3, \gamma_2\gamma_3$: $h_1, H_T, E_T, \tilde{H}_T, \tilde{E}_T$ (transversity twist-2),
$\gamma_1\gamma_2$: h_L, H_2', E_2' (tensor twist-3).

Need different projectors to disentangle GPDs

UNPOL: $\mathcal{P} = \frac{1+\gamma_0}{4}$

POL-k: $\mathcal{P} = \frac{1+\gamma_0}{4} i\gamma_5\gamma_k$

Quasi-GPDs lattice procedure

spatial correlation in a boosted nucleon

$\langle P_f|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P_i\rangle$

$\vec{P}_f = \vec{P}_i + \vec{\Delta}$, $\vec{\Delta}$ – momentum transfer

lattice computation of bare ME

↓

disentanglement of GPDs

ME decomposition formulas

↓

renormalization

intermediate RI scheme

conversion to $\mathrm{M\overline{MS}}$ scheme

(incl. evolution to $\mu = 2$ GeV)

↓

reconstruction of x-dependence

z-space→x-space

Backus-Gilbert

↓

matching to light cone

$\mathrm{M\overline{MS}} \to \overline{\mathrm{MS}}$

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light-cone GPD

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several $\vec{\Delta}$ vectors

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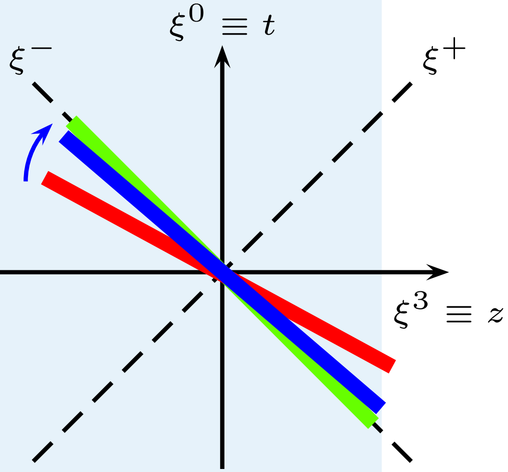

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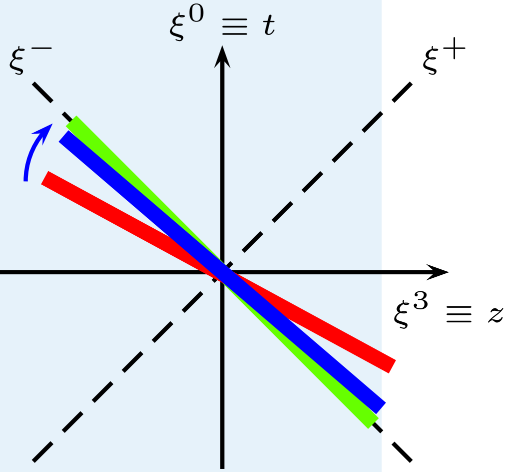
light-cone GPD

the final desired object!



Setup

Lattice setup:

- fermions: $N_f = 2$ twisted mass fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 1.778$
- gauge field configurations generated by ETMC
- lattice spacing $a \approx 0.093$ fm,
- $32^3 \times 64 \Rightarrow L \approx 3$ fm,
- $m_\pi \approx 260$ MeV.

Kinematics:

- three nucleon boosts: $P_3 = 0.83, 1.25, 1.67$ GeV,
- momentum transfers: $-t = 0.69, 1.38, 2.76$ GeV^2,
- zero skewness.

Twist-2 unpolarized+helicity GPDs C. Alexandrou et al. (ETMC), PRL 125(2020)262001

Twist-2 transversity GPDs C. Alexandrou et al. (ETMC), PRD 105(2022)034501

Twist-2 unpolarized GPDs S. Bhattacharya et al. (ETMC/BNL/ANL) PRD 106(2022)114512

Twist-2 unpolarized GPDs (OPE) S. Bhattacharya et al. (ETMC/BNL/ANL) PRD 108(2023)014507

Twist-3 axial GPDs S. Bhattacharya et al. (ETMC/Temple), PRD 108(2023)054501

Twist-2 helicity GPDs S. Bhattacharya et al. (ETMC/BNL/ANL) in preparation

Twist-3 axial PDF g_T S. Bhattacharya et al. (ETMC/Temple) PRD 102(2020)111501(R)

Twist-3 tensor PDF h_L S. Bhattacharya et al. (ETMC/Temple) PRD 104(2021)114510

Twist-3 axial GPDs

S. Bhattacharya et al., Phys.Rev. D108 (2023) 054501

Parametrization of $\gamma_5\gamma_\mu$ MEs (up to twist-3):

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$
$$+ \Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3)$$
$$+ \Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)\,.$$

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$$+\Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3)$$
$$+\Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)\,.$$

$\mu=0,3$ – twist-2 GPDs $\widetilde{H}$, $\widetilde{E}$,

$\mu=1,2$ – combinations of twist-2 & 3 $\widetilde{E}+\widetilde{G}_1$, $\widetilde{H}+\widetilde{G}_2$ and twist-3 $\widetilde{G}_3$, $\widetilde{G}_4$.

Symmetry properties in coordinate space:

- hermiticity $\Rightarrow \widetilde{G}_{1,2,4}(P^3z) = \widetilde{G}^*_{1,2,4}(-P^3z)$, $\widetilde{G}_3(P^3z) = -\widetilde{G}^*_3(-P^3z)$.
- + time reversal $\Rightarrow \widetilde{G}_{1,2,4}(\xi) = \widetilde{G}_{1,2,4}(-\xi)$, $\widetilde{G}_3(\xi) = -\widetilde{G}_3(-\xi)$.

Contributing MEs

$$\Pi^1(\Gamma_0) = C\left(- F_{\widetilde{H}+\widetilde{G}_2} \frac{P_3 \Delta_y}{4m^2} - F_{\widetilde{G}_4} \frac{\text{sign}[P_3]\, \Delta_y (E+m)}{2m^2} \right)$$

$$\Pi^1(\Gamma_1) = i\,C\left(F_{\widetilde{H}+\widetilde{G}_2} \frac{(4m(E+m) + \Delta_y^2)}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1} \frac{\Delta_x^2 (E+m)}{8m^3} + F_{\widetilde{G}_4} \frac{\text{sign}[P_3]\, \Delta_y^2 (E+m)}{4m^2 P_3} \right)$$

$$\Pi^1(\Gamma_2) = i\,C\left(- F_{\widetilde{H}+\widetilde{G}_2} \frac{\Delta_x \Delta_y}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1} \frac{\Delta_x \Delta_y (E+m)}{8m^3} - F_{\widetilde{G}_4} \frac{\text{sign}[P_3]\, \Delta_x \Delta_y (E+m)}{4m^2 P_3} \right)$$

$$\Pi^1(\Gamma_3) = C\left(- F_{\widetilde{G}_3} \frac{E \Delta_x (E+m)}{2m^2 P_3} \right)$$

$$\Pi^2(\Gamma_0) = C\left(F_{\widetilde{H}+\widetilde{G}_2} \frac{P_3 \Delta_x}{4m^2} + F_{\widetilde{G}_4} \frac{\text{sign}[P_3]\, \Delta_x (E+m)}{2m^2} \right)$$

$$\Pi^2(\Gamma_1) = i\,C\left(- F_{\widetilde{H}+\widetilde{G}_2} \frac{\Delta_x \Delta_y}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1} \frac{\Delta_x \Delta_y (E+m)}{8m^3} - F_{\widetilde{G}_4} \frac{\text{sign}[P_3]\, \Delta_x \Delta_y (E+m)}{4m^2 P_3} \right)$$

$$\Pi^2(\Gamma_2) = i\,C\left(F_{\widetilde{H}+\widetilde{G}_2} \frac{(4m(E+m) + \Delta_x^2)}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1} \frac{\Delta_y^2 (E+m)}{8m^3} + F_{\widetilde{G}_4} \frac{\text{sign}[P_3]\, \Delta_x^2 (E+m)}{4m^2 P_3} \right)$$

$$\Pi^2(\Gamma_3) = C\left(- F_{\widetilde{G}_3} \frac{E \Delta_y (E+m)}{2m^2 P_3} \right)$$

Bare MEs

$\Pi^j(\Gamma_0)$

$\widetilde{H}+\widetilde{G}_2$ & $\widetilde{G}_4$

$\Pi^j(\Gamma_j)$ with $\Delta_j = 0$

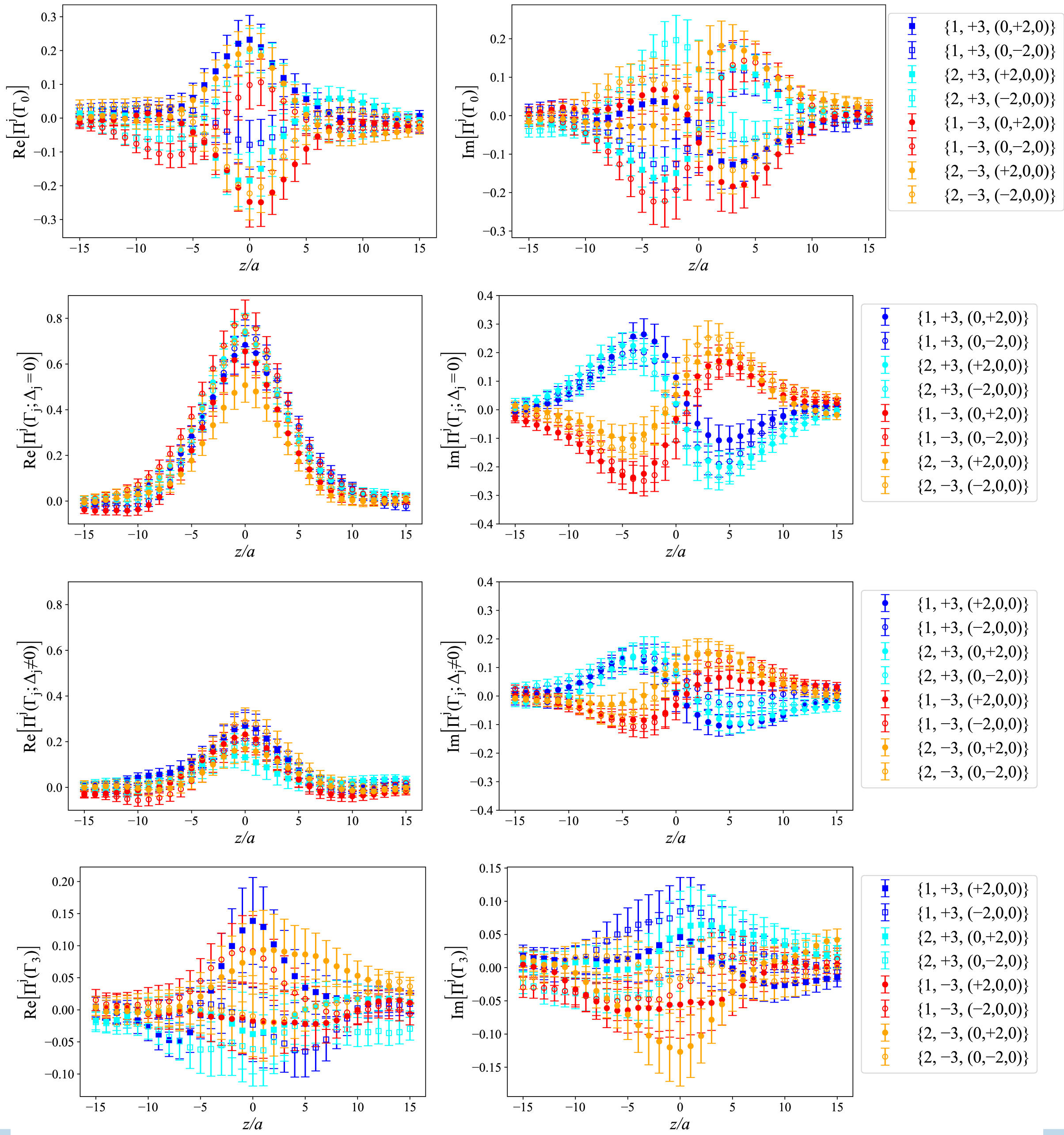
$\widetilde{H}+\widetilde{G}_2$ & $\widetilde{G}_4$

$\Pi^j(\Gamma_j)$ with $\Delta_j \neq 0$

$\widetilde{H}+\widetilde{G}_2$ & $\widetilde{E}+\widetilde{G}_1$

$\Pi^j(\Gamma_3)$

$\widetilde{G}_3$



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Disentangled GPDs in coordinate space

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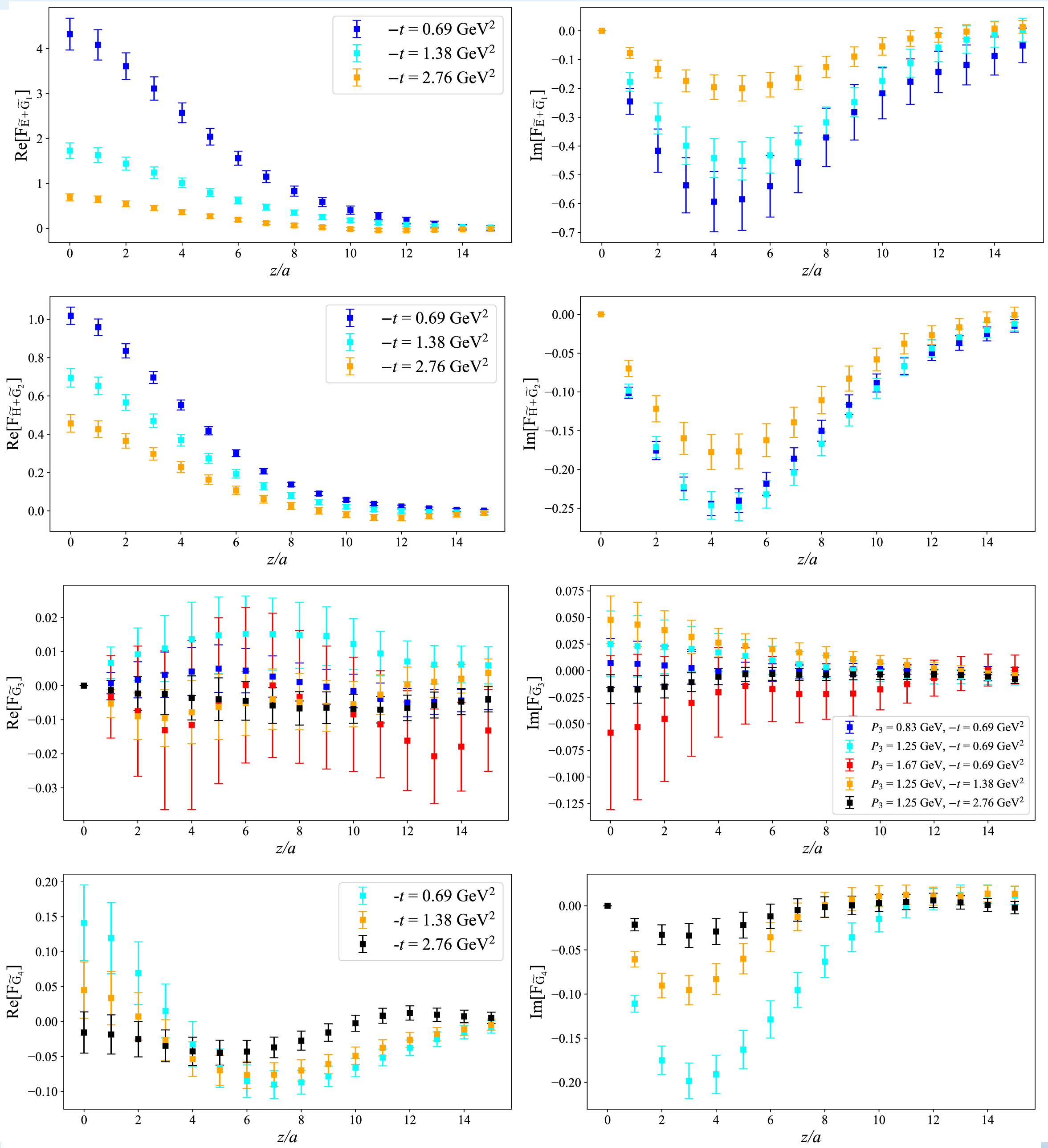
$\widetilde{E} + \widetilde{G}_1$

$\widetilde{H} + \widetilde{G}_2$

$\widetilde{G}_3$

$\widetilde{G}_4$



x-dependence reconstruction and matching

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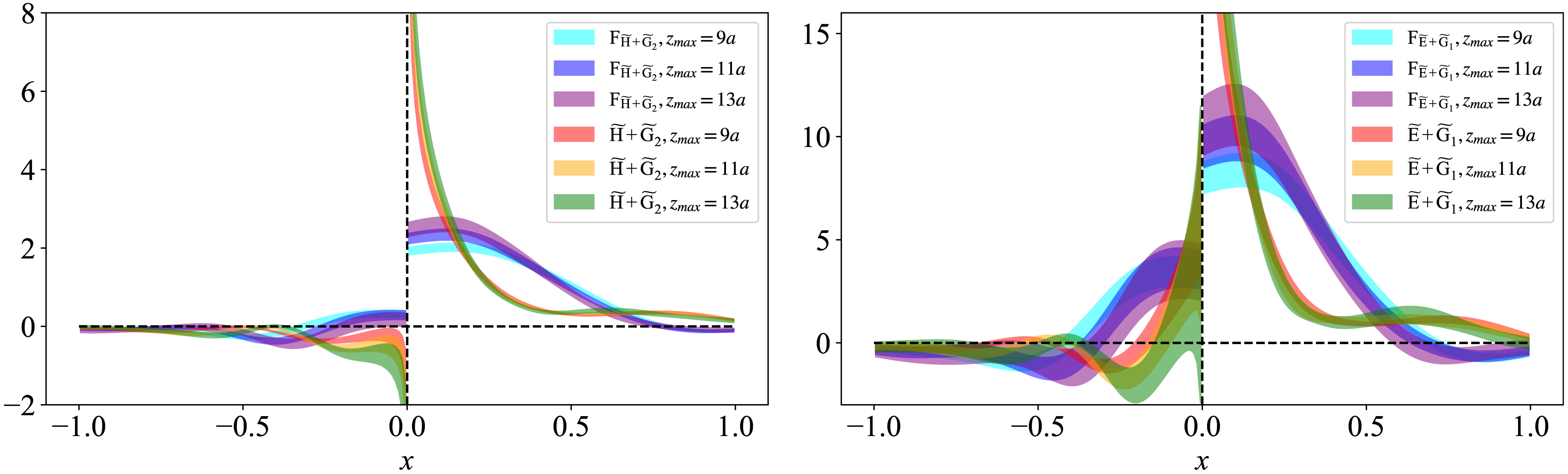
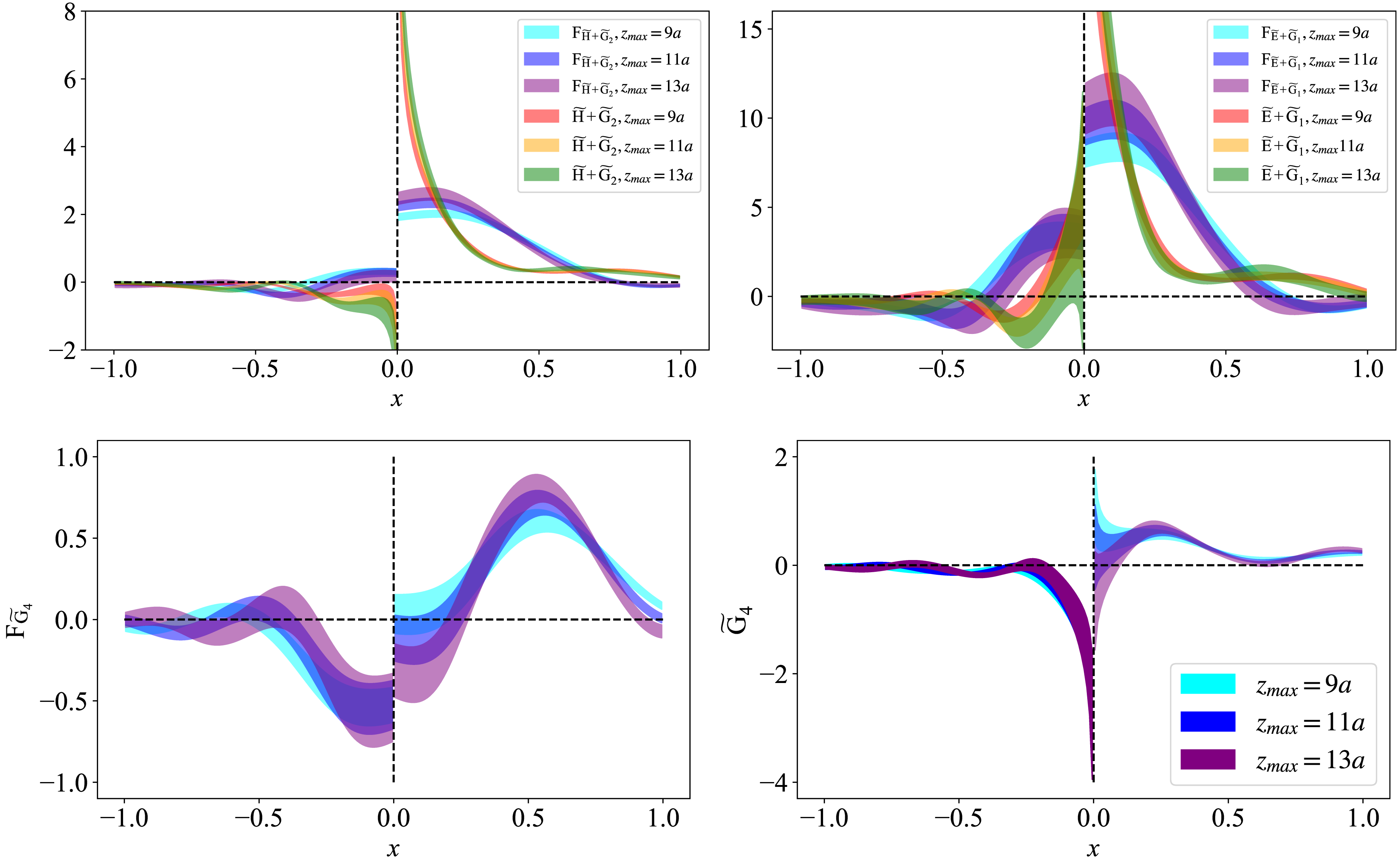
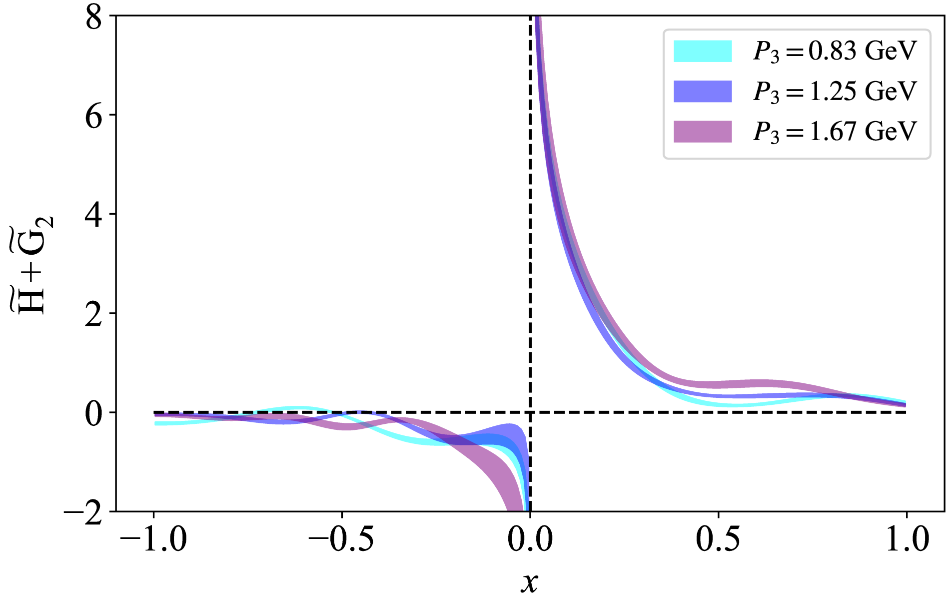


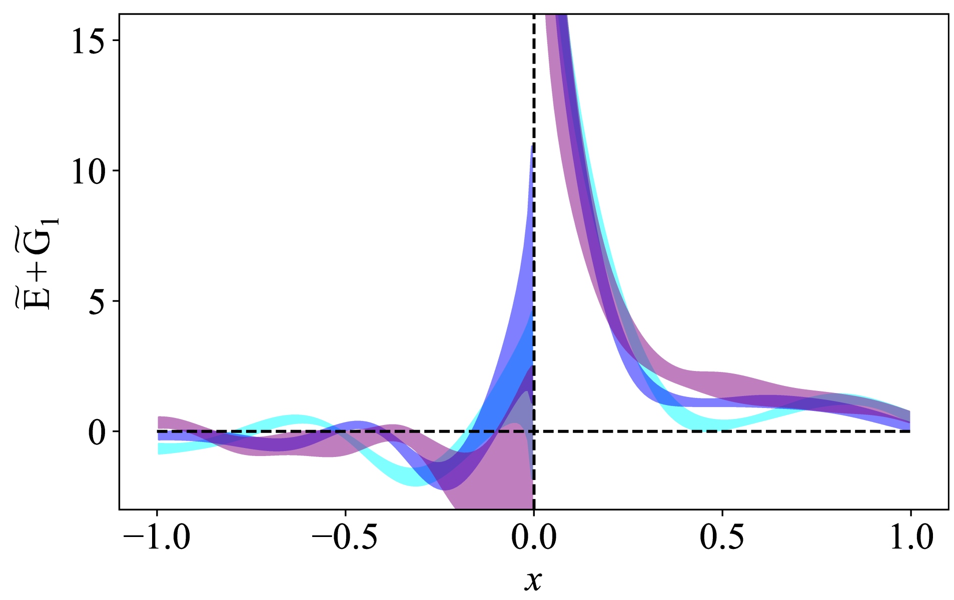
x-dependence reconstruction and matching

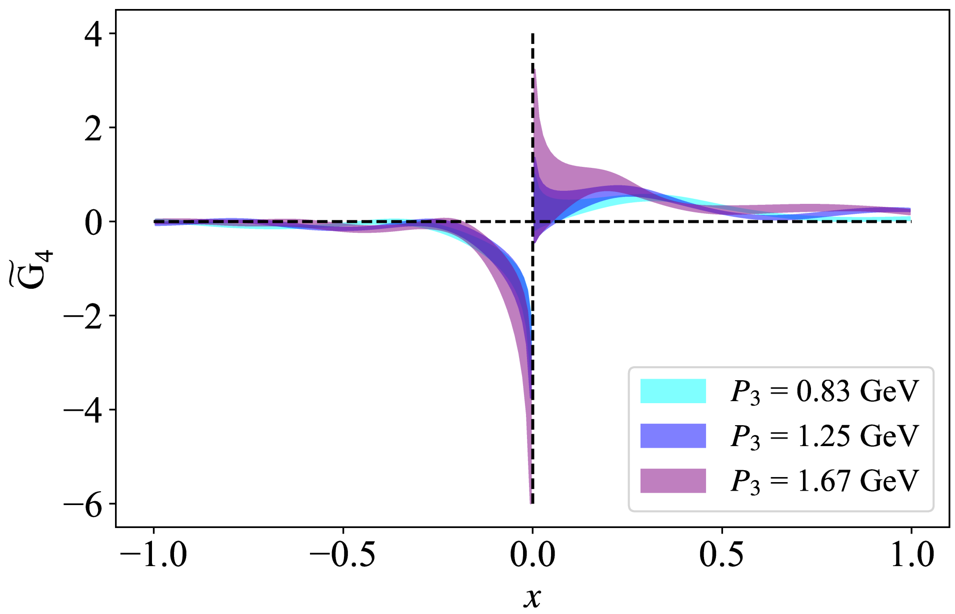
S. Bhattacharya et al., Phys.Rev. D108 (2023) 054501



Nucleon boost dependence







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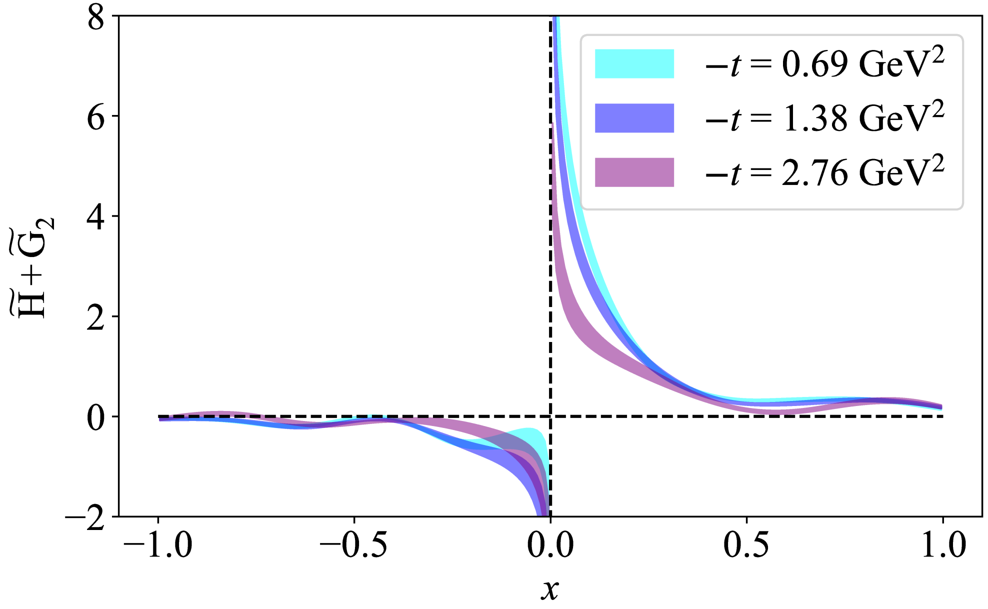
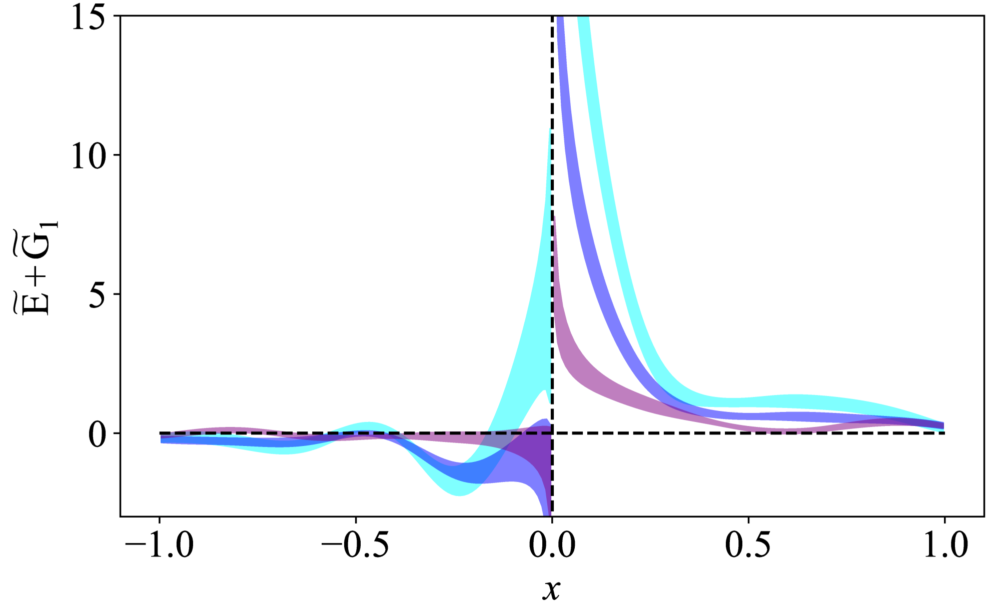
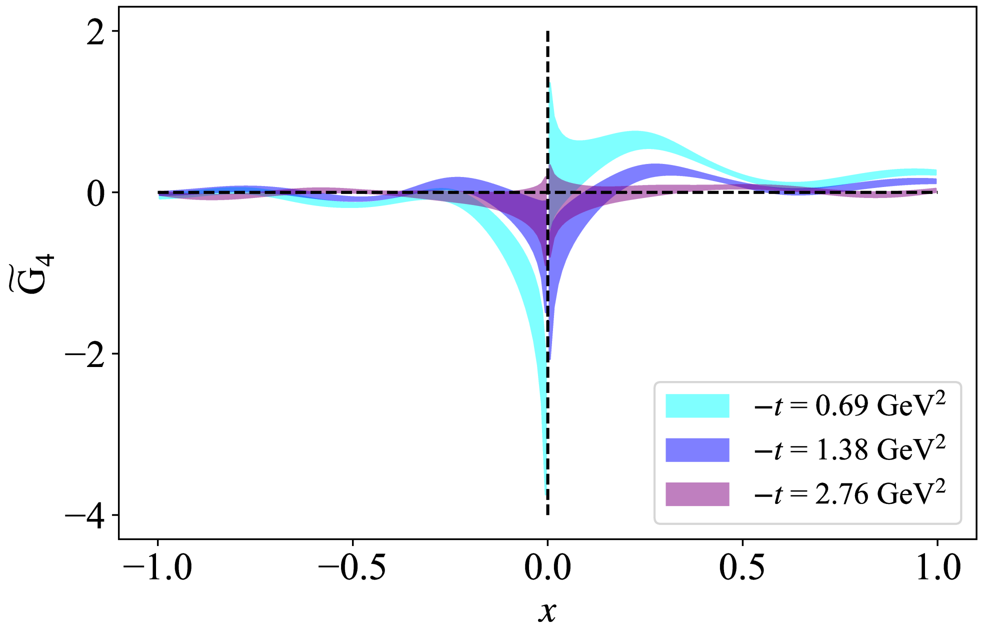
t-dependence







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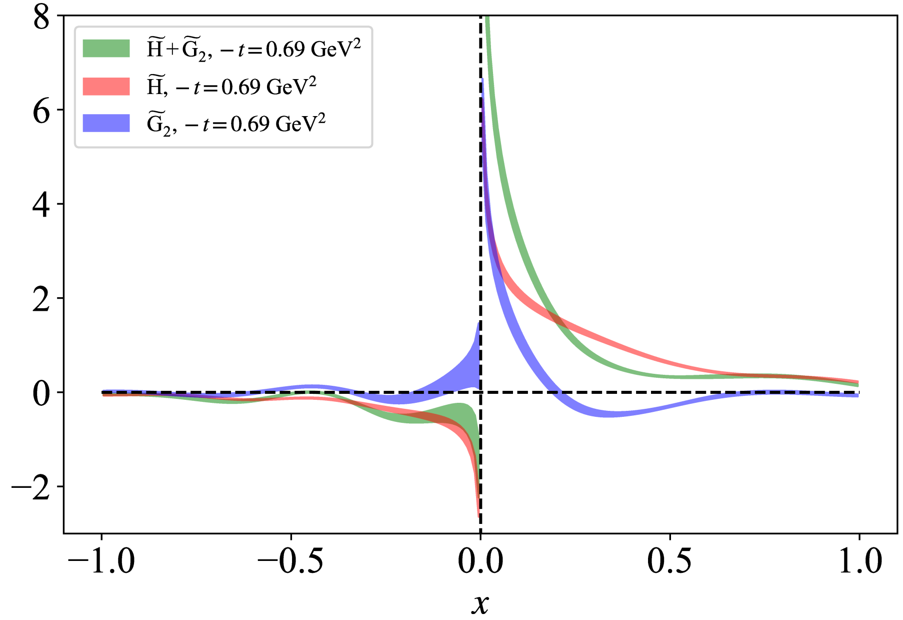
Isolating $\widetilde{G}_2$



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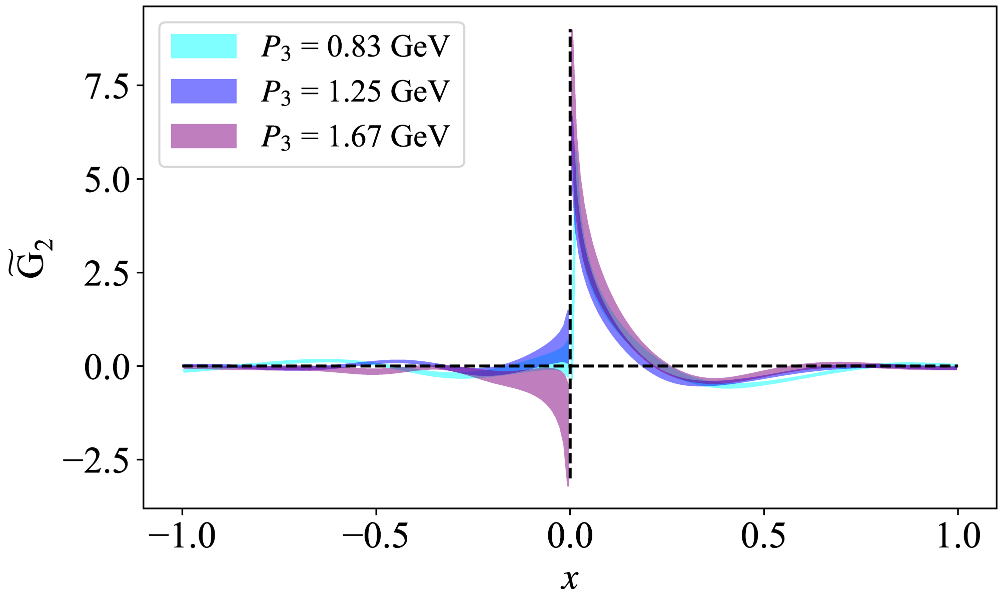

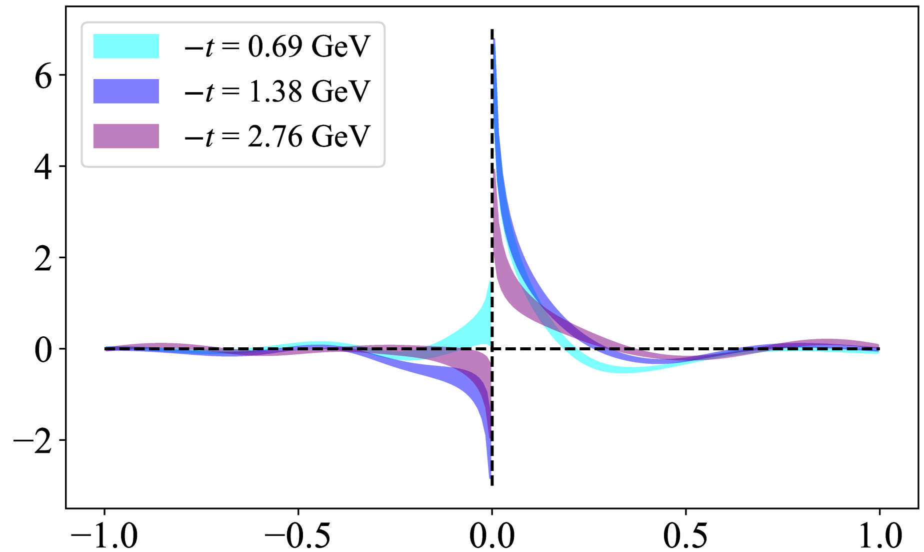

Consistency checks

Burkhardt-Cottingham-type sum rules:

$$G_P(t) = \int_{-1}^{1} dx \left(\widetilde{E}(x,\xi,t) + \widetilde{G}_1(x,\xi,t)\right) = \int_{-1}^{1} dx \, \widetilde{E}(x,\xi,t)$$

$$G_A(t) = \int_{-1}^{1} dx \left(\widetilde{H}(x,\xi,t) + \widetilde{G}_2(x,\xi,t)\right) = \int_{-1}^{1} dx \, \widetilde{H}(x,\xi,t)$$

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GPD	$P_3 = 0.83$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.67$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 1.38$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 2.76$ [GeV^2]
$\widetilde{H}$	0.741(21)	0.712(27)	0.802(48)	0.499(21)	0.281(18)
$\widetilde{H} + \widetilde{G}_2$	0.719(25)	0.750(33)	0.788(70)	0.511(36)	0.336(34)

- satisfied for $\widetilde{H} + \widetilde{G}_2$ – same local limit and norm as $\widetilde{H}$,
- cannot be tested for $\widetilde{E} + \widetilde{G}_1$ – $\widetilde{E}$ inaccessible at $\xi = 0$.
- norms of $\widetilde{G}_2$ and $\widetilde{G}_4$ close to vanishing.

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$$\int dx\, x\, \widetilde{G}_3(x,\xi,t) = \frac{\xi}{4} G_E(t)\,, \qquad \int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,\xi,t) = \frac{1}{4} G_E(t)\,.$$

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- $\widetilde{G}_3$ indeed vanishes at $\xi = 0$,
- $\widetilde{G}_4$ non-vanishing and small.

Conclusions and prospects

- First calculation of axial twist-3 GPDs, $\widetilde{H}+\widetilde{G}_2$, $\widetilde{E}+\widetilde{G}_1$, $\widetilde{G}_3$, $\widetilde{G}_4$, using the quasi-GPD method.
- Good signal for $\widetilde{H}+\widetilde{G}_2$ and $\widetilde{E}+\widetilde{G}_1$.
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  - ⋆ quark-gluon-quark 3-parton correlations,
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  - ⋆ wider range of momentum transfers,
  - ⋆ non-zero skewness,
  - ⋆ various systematics,
  - ⋆ other twist-3 GPDs.

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Thank you for your attention!