Los Alamos NATIONAL LABORATORY

Neutron Electric Dipole Moment from Isovector Quark Chromo-Electric Dipole Moment

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Introduction

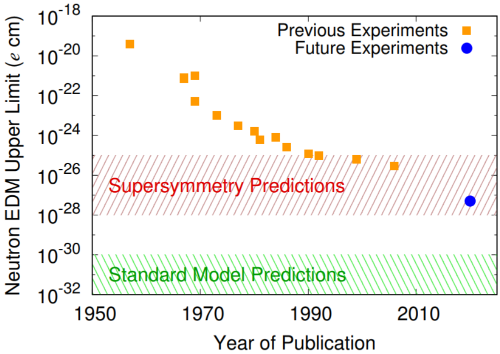


Figure: The timeline of the upper bound on the neutron EDM from previous and future experiments. BY, TB, and RG (2018)

Possible outcome of nEDM:

1. Non-zero nEDM: new source of $\not{CP}$
   $\rightarrow$ Sakharov's condition
2. Constraints on BSM theories

Introduction

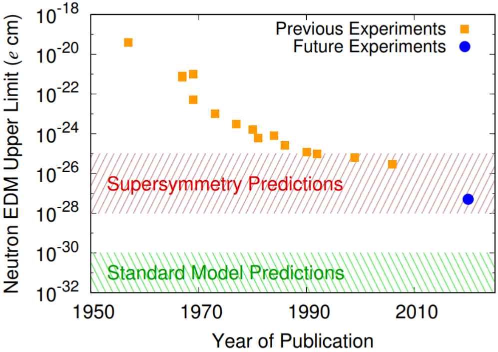


Figure: The timeline of the upper bound on the neutron EDM from previous and future experiments. BY, TB, and RG (2018)

- Current experimental bound $d_n < 1.8 \times 10^{-26}$e-cm
- Planned experiment aims $d_n \sim 3 \times 10^{-28}$e-cm

Effective CPV Lagrangian at Hadronic scale

$$
\begin{aligned}
\mathcal{L}_{\mathrm{CPV}}^{d\leq 6} &= -\frac{g_s^2}{32\pi^2}\bar{\theta}G\tilde{G}; && d = 4 \text{ QCD } \theta\text{-term} \\
&- \frac{i}{2}\sum_{q=u,d,s} d_q\bar{q}(\sigma\cdot F)\gamma_5 q; && d = 5 \text{ quark EDM} \\
&- \frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q g_s\bar{q}(\sigma\cdot G)\gamma_5 q; && d = 5 \text{ quark chromo EDM} \\
&+ d_w\frac{g_s}{6}G\tilde{G}G; && d = 6 \text{ Weinberg's 3g operator} \\
&+ \sum_i C_i^{(4q)}O_i^{(4q)}; && d = 6 \text{ Four-quark operators}
\end{aligned}
\qquad (1)
$$

- $\bar{\theta} \leq \mathcal{O}(10^{-8} - 10^{-11})$: strong CP problem
- qEDM, qcEDM suppressed by $\sim \frac{v_{EW}}{\Lambda^2_{BSM}}$
- dim-6 operators are suppressed by $\frac{1}{\Lambda^2_{BSM}}$

QCD θ-term

$$\mathcal{L}^{d=4}_{\rm CPV} = -\frac{g_s^2}{32\pi^2}\bar{\theta} G\tilde{G} \tag{2}$$

- External electric field method: $\langle N\bar{N}\rangle_{\bar{\theta}}(\vec{\mathcal{E}}, t) = \langle N(t)\bar{N}(0)e^{i\bar{\theta}Q}\rangle_{\vec{\mathcal{E}}}$
  Aoki and Gocksch (1989), Aoki,Gocksch,Manohar, and Sharpe(1990), CP-PACS Collaboration (2006), Abramzyk et al(2017)
- Simulation with imaginary $\bar{\theta}$: $\bar{\theta} = i\tilde{\theta}$, $S^q_{\bar{\theta}} = \tilde{\theta}\frac{m_l m_s}{2m_s+m_l}\sum_x \bar{q}\gamma_5 q$
  Horsley, et al. (2008), Guo, et al. (2005)
- Expansion in $\bar{\theta}$:

$$\begin{aligned}\langle O(x)\rangle_{\bar{\theta}} &= \frac{1}{Z_{\bar{\theta}}}\int d[U,q,\bar{q}]O(x)e^{-S_{QCD}-i\bar{\theta}Q} \\ &= \langle O(x)\rangle_{\bar{\theta}=0} - i\bar{\theta}\langle O(x)Q\rangle_{\bar{\theta}=0} + \mathcal{O}\left(\bar{\theta}^2\right)\end{aligned} \tag{3}$$

Shintani, et al.(2005), Berruto, Blum, Originos, and Soni(2006)
Shindler, T. Luu, J. de Vries (2015), Shintani, Blum, Izubuchi, and Soni (2016)
Alexandrou, et al., (2016), Abramczyk, et al. (2017), Dragos, et al. (2019)
Bhattacharya, et al.(2021)

Quark Chromo Electric Dipole Moment

$$-\frac{i}{2}\sum_{q=u,d,s}\tilde{d}_q g_s \bar{q}(\sigma\cdot G)\gamma_5 q \tag{4}$$

- Dimension-5 operator arising from Dimension-6 operator beyon electroweak symmetry breaking
- SU(3) color analog of quark EDM: $d_q\bar{q}(\sigma\cdot F)\gamma_5 q$
- breaks P, CP symmetry
- Fermion bilinear, can computed by Schwinger source trick

$$\mathcal{P}=\left[\not{D}+m-\frac{r}{2}D^2+c_{\mathrm{SW}}\Sigma\cdot G\right]^{-1}\rightarrow\left[\not{D}+m-\frac{r}{2}D^2+\Sigma\cdot\left(c_{\mathrm{SW}}G+i\epsilon\tau\widetilde{G}\right)\right]^{-1} \tag{5}$$

- ϵ/a needs to be small

Lattice setup

- 2+1+1 highly improved staggered quark(HISQ) by MILC is used (Phys. Rev. D87, 054505 (2013))
- Mixed action: Clover-on-HISQ
- $M_\pi L > 3.9$

ensID	a(fm)	M_π^{sea}(MeV)	M_π^{val}(MeV)	$L^3 \times T$	N_{conf}	ϵ	ϵ_5
a12m310	0.1207(11)	305.3(4)	310.2(2.8)	$24^3 \times 64$	1013	0.008	0.0024
a12m220L	0.1189(09)	217.0(2)	227.6(1.7)	$40^3 \times 64$	475	0.001	0.0003
a09m310	0.0888(08)	312.7(6)	313.0(2.8)	$32^3 \times 96$	447	0.008	0.0024
a06m310	0.0582(04)	319.3(5)	319.3(0.5)	$48^3 \times 144$	72	0.009	0.0012

Nucleon 3point function

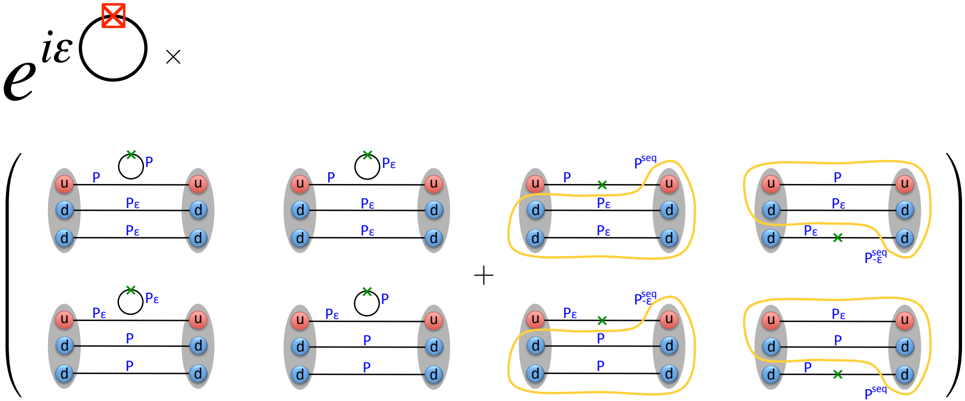


Figure: arXiv:2301.08161

- All disconnected contributions are neglected or cancel for isovector cEDM

Nucleon wave function

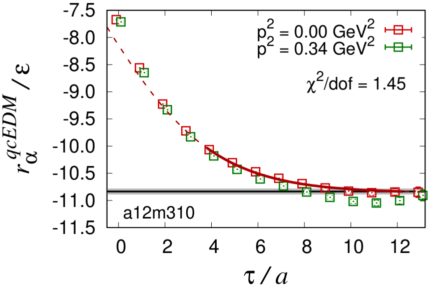
The wave function of the nucleon in the standard basis:

$$N_\alpha = e^{-i\alpha_N} \epsilon^{abc} \left[\psi_d^{aT} C\gamma_5 \frac{1\pm\gamma_4}{2} \psi_u^b \right] \psi_d^c$$

$$\alpha_N = \lim_{\tau\to\pm\infty} \frac{\Im \mathrm{Tr}\, \gamma_5 \,(1\pm\gamma_4) \left\langle N_0(0)\bar{N}_0(\tau)\right\rangle}{\Re \mathrm{Tr}\,(1\pm\gamma_4) \left\langle N_0(0)\bar{N}_0(\tau)\right\rangle}$$

$$\approx \tan\alpha_0 \times \frac{1 + \frac{\sin(2\alpha_1)}{\sin(2\alpha_0)} |\tilde{\mathcal{A}}_1|^2 \, e^{-(M_1-M_0)\tau}}{1 + \frac{\cos^2(\alpha_1)}{\cos^2(\alpha_0)} |\tilde{\mathcal{A}}_1|^2 \, e^{-(M_1-M_0)\tau}}$$

α_N linear in ϵ and momentum-independent.



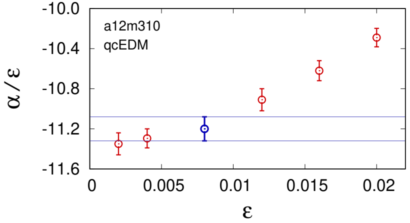


Figure:

Power divergent mixing

- $C \equiv \bar{\psi}\Sigma \cdot \tilde{G}\tau_3\psi$ has power divergent mixing with $P \equiv \bar{\psi}\gamma_5\tau_3\psi$.
- Allowed even with good chiral symmetry
- Does not mix with $G\tilde{G}$ due to isospin invariance even when chiral symmetry is broken

Isovector CPV mass term P can be rotated away by nonsingle axial rotation

AWI for Wilson-like fermions

$$Z_A(m)\partial_\mu A_3^\mu + iac_A\partial^2 P_3 + 2imP_3 \\ = iaK\tilde{C}_3 + O\left(a^2\right) \tag{6}$$

where

- $\tilde{C} \equiv C - a^{-2}AP_3^2$ is free of power divergence
- K comes from c_{sw} mistuning

Apart from field redefinition,

$$\frac{2am}{K}\frac{P_3}{a} \sim \frac{2am}{2am+K}aC_3 + O\left(a^2\right) \tag{7}$$

and is power divergence free.

Mixing: determination of nonperturbative parameters

Ensemble	$\tilde{F}_3^{\gamma_5}/\tilde{F}_3^{\mathrm{qcEDM}}$					$\frac{K_{X1}}{2am+AK_{X1}}$
	$Q^2=1$	$Q^2=2$	$Q^2=3$	$Q^2=4$	$Q^2=5$	
a12m310	0.879(17)	0.863(14)	0.867(18)	0.844(23)	0.864(13)	0.694(48)
a12m220L	0.81(10)	0.769(77)	0.869(75)	0.98(18)	0.94(11)	0.7807(70)
a09m310	1.063(35)	1.042(40)	1.078(45)	1.006(58)	1.039(44)	0.740(61)
a06m310						0.859(64)

Table: The ratio $\tilde{F}_3^{\gamma_5}/\tilde{F}_3^{\mathrm{qcEDM}}$ for the γ_5 and qcEDM unsubtracted lattice operators for the five smallest values of Q^2. The data for $\tilde{F}_3$ are obtained using the "standard" method

- independent of Q^2 and the quark mass
- close to the $K_{X1}/(2am+AK_{X1})$ obtained from the pion correlators
- No significant signal in $a06m310$.

Renormalization

- All power-law mixing subtracted
- Mixing with only dimension-5 operators
- dim-5 qEDM: $\bar{\psi}\Sigma \cdot \tilde{F}\tau_3\psi$ has mixing $\sim\ 1\%$ at $O(\alpha_{EM})$.
- $\int d^4x \tilde{C}_3 J_\mu^{EM} A^\mu$ has mixing with qEDM at $O(\alpha_s)$.
- Tree-level matching and 1-loop running

$$\vec{O}_{\overline{\text{MS}}}(\mu) = U \begin{pmatrix} \left(\frac{\alpha_s(\mu)}{\alpha_s(a^{-1})}\right)^{-\gamma_{11}/\beta_0} & 0 \\ 0 & \left(\frac{\alpha_s(\mu)}{\alpha_s(a^{-1})}\right)^{-\gamma_{22}/\beta_0} \end{pmatrix} U^{-1}\vec{O}(a) \quad (8)$$

where $\vec{O}(a) = \begin{pmatrix} \tilde{C}^{(3)}(a) \\ E^{(3)}(a) \end{pmatrix}$, $\vec{O}_{\overline{\text{MS}}}(\mu) = \begin{pmatrix} C^{(3)}_{\overline{\text{MS}}}(\mu) \\ E^{(3)}_{\overline{\text{MS}}}(\mu) \end{pmatrix}$, $U = \begin{pmatrix} 1 & -\frac{\gamma_{12}}{\gamma_{11}-\gamma_{22}} \\ 0 & 1 \end{pmatrix}$

Extrapolation

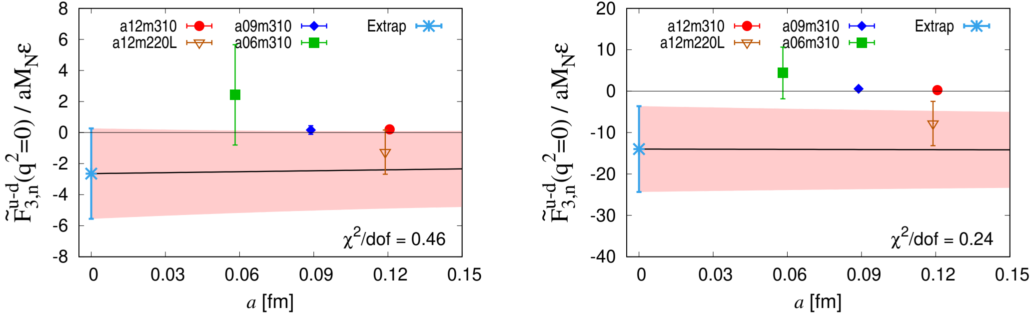


Figure: Extrapolation to the continuum and physical pion mass limit using the fit ansatz $c_1 + c_2 M_\pi^2 + c_3 a$ where ES fit with (Left) Standard excited state (Right) $N\pi$-excited state

Extrapolation

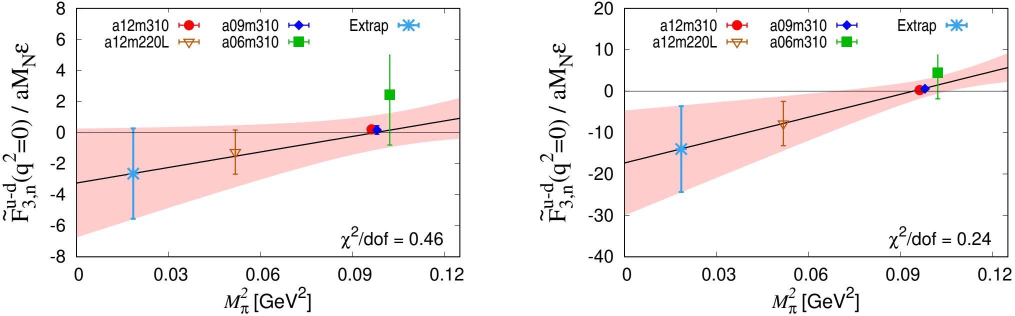


Figure: Extrapolation to the continuum and physical pion mass limit using the fit ansatz $c_1 + c_2 M_\pi^2 + c_3 a$ where ES fit with (Left) Standard excited state (Right) $N\pi$-excited state

Final Result

Standard excited fit $X_c \equiv -\frac{F_3(0)}{aM_N\epsilon} = 2.6(2.9)$

$N\pi$ **excited fit** $X_c \equiv -\frac{F_3(0)}{aM_N\epsilon} = 14(10)$

Current Challenges and Conclusion

- Possibly large $O(a^2)$ effects
- difference in estimates between removing ESC with and without $N\pi$ excited states
- Power divergence in isovector cEDM present even with good chiral symmetry
- The power-divergent mixing is with P_3 which implements chiral rotation, but no CP-violation in the continuum
- Any lattice artifact in it is enhanced by $1/ma$. Important to demonstrate control
- Perturbative O(a)-improved Wilson fermions still have large uncertainty, though chiral rotation agrees with χPT at 10%.
- Control over Excited State Contamination needs to be demonstrated
- Disconnected diagrams and possible chiral+isospin breaking mixing with $\theta G\tilde{G}$ need to be considered

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