Meson and Baryon spin-dependent GPDs via Quantum Computers

Spin 2023

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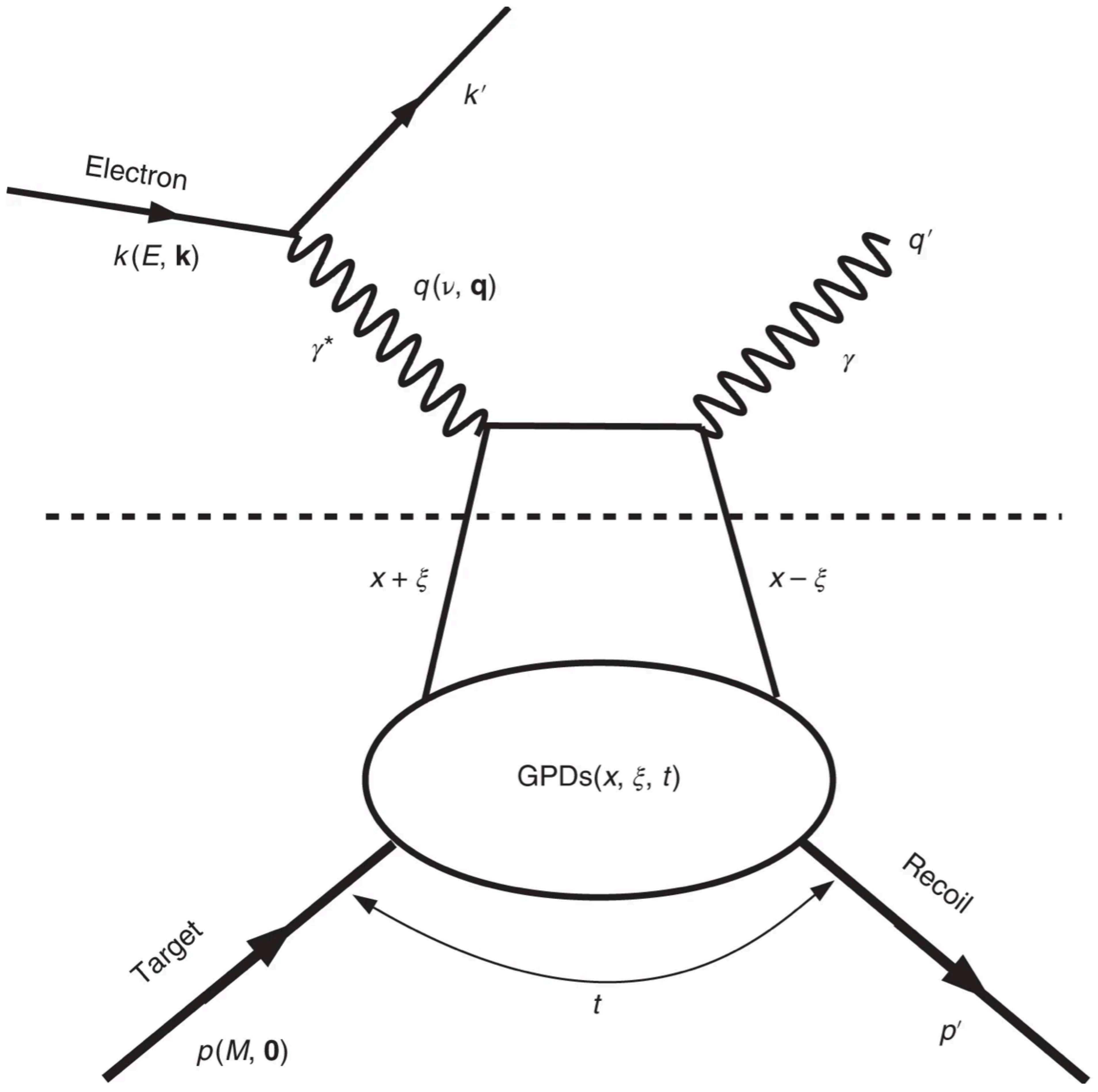
Overview

- Motivation
- Basics of Quantum Computation
- Discretized Light Cone Quantization (DLCQ) for Mesons
- Basis Lightfront Quantization (BLFQ) for Baryons

Motivation

- Calculating GPDs → 3D nuclear image
- This is usually done via Lattice QCD
- We aim to do this on a quantum computer in lightfront (LF) coordinates

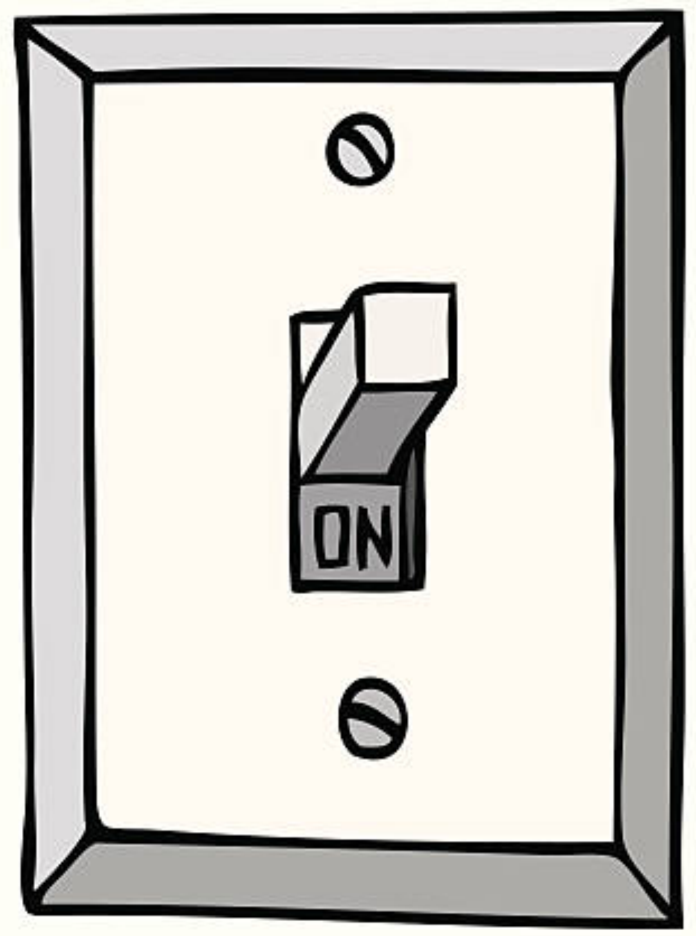
$$F_q = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P' | \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right)|P\rangle|_{z^+=\mathbf{z}=0}$$



Deeply Virtual Compton Scattering off the Neutron (Nature), *Benali et al.*

Classical Computation

0



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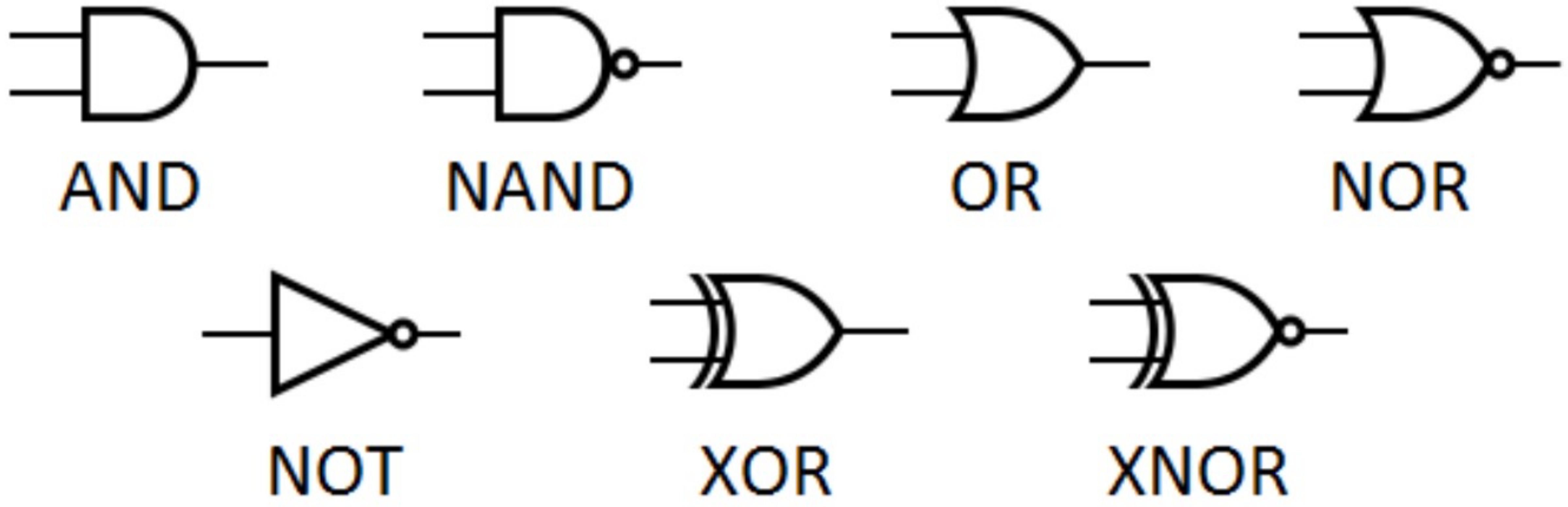
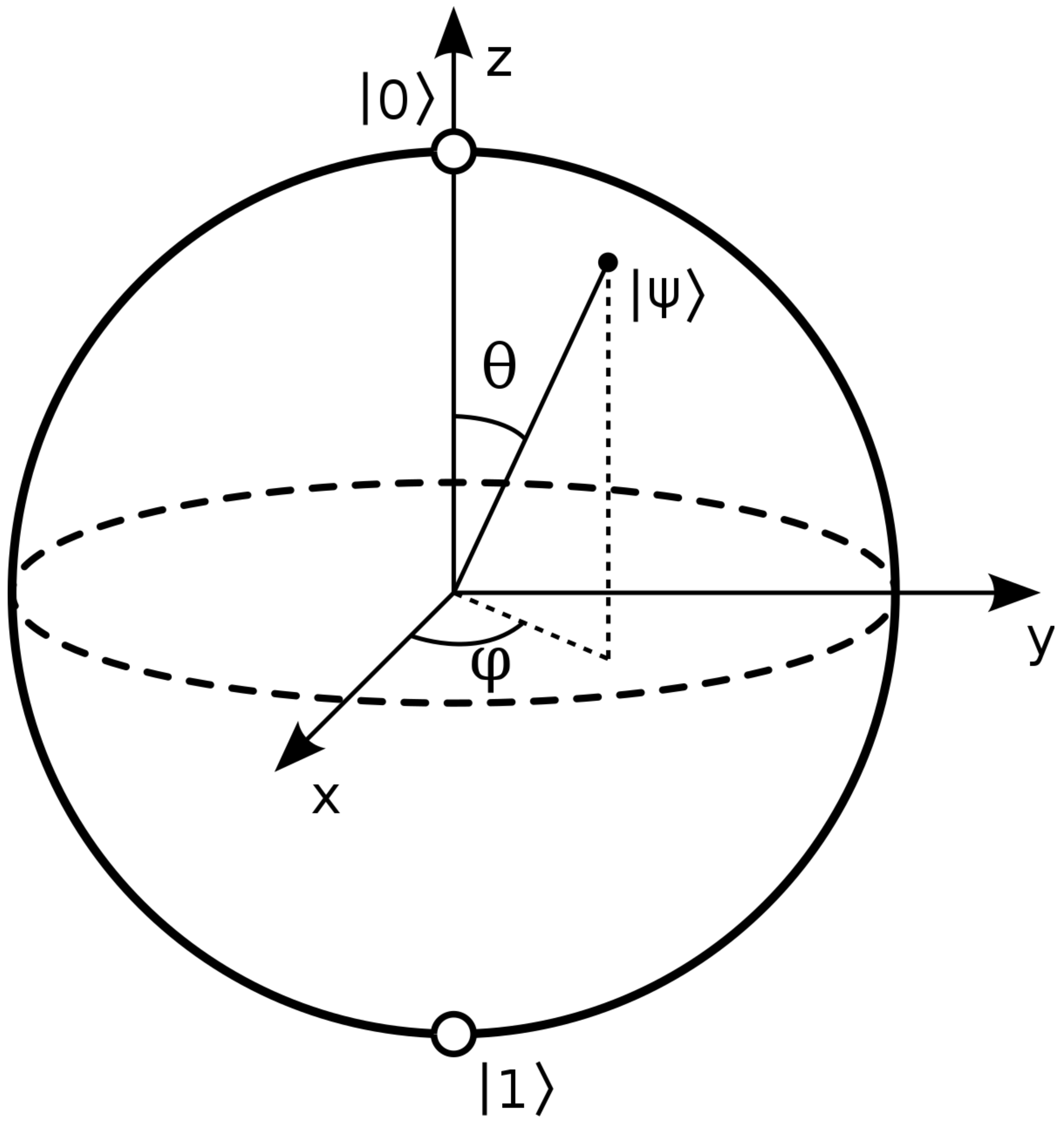


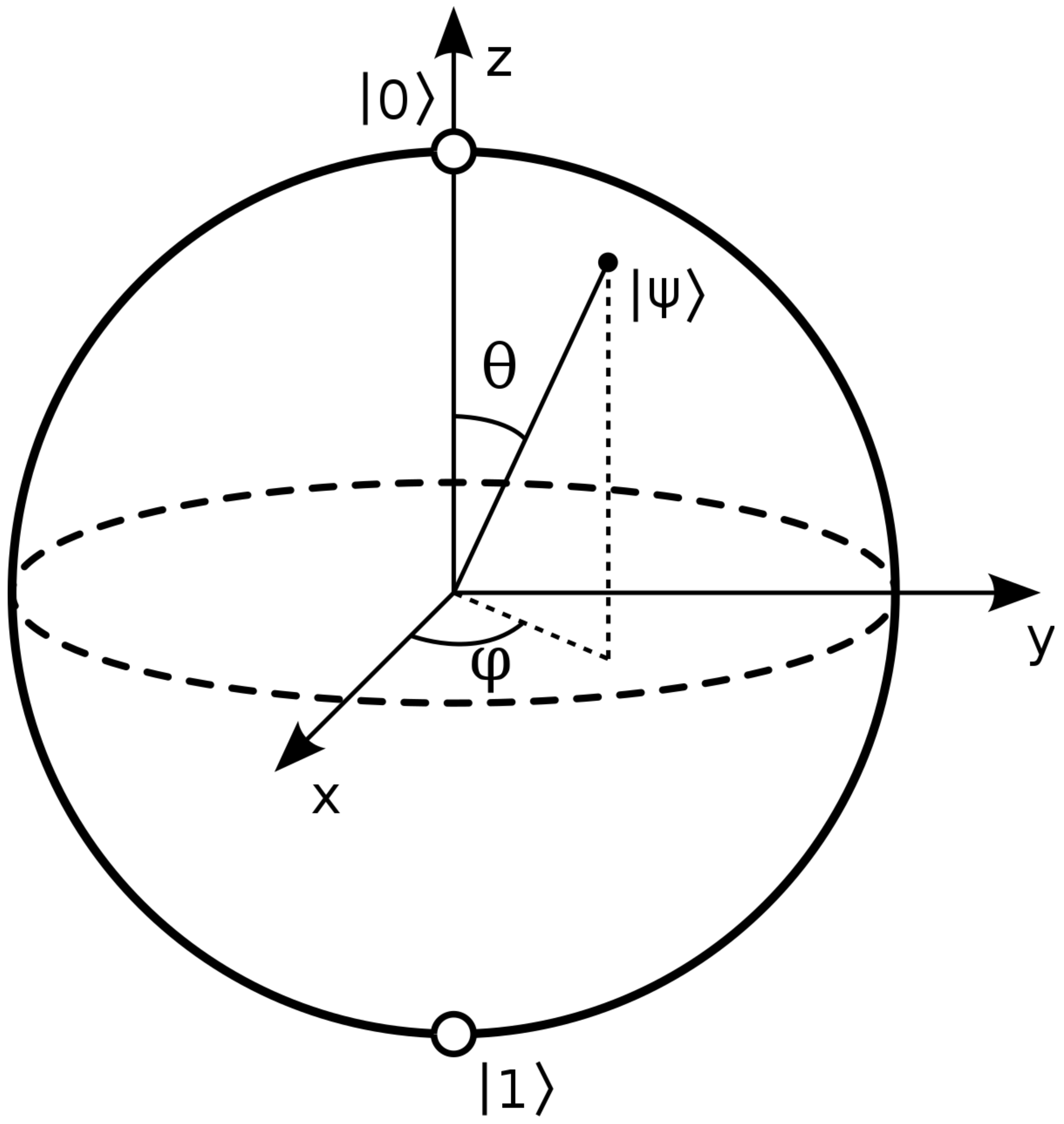
State space: $\{0,1\}$

Quantum Computation



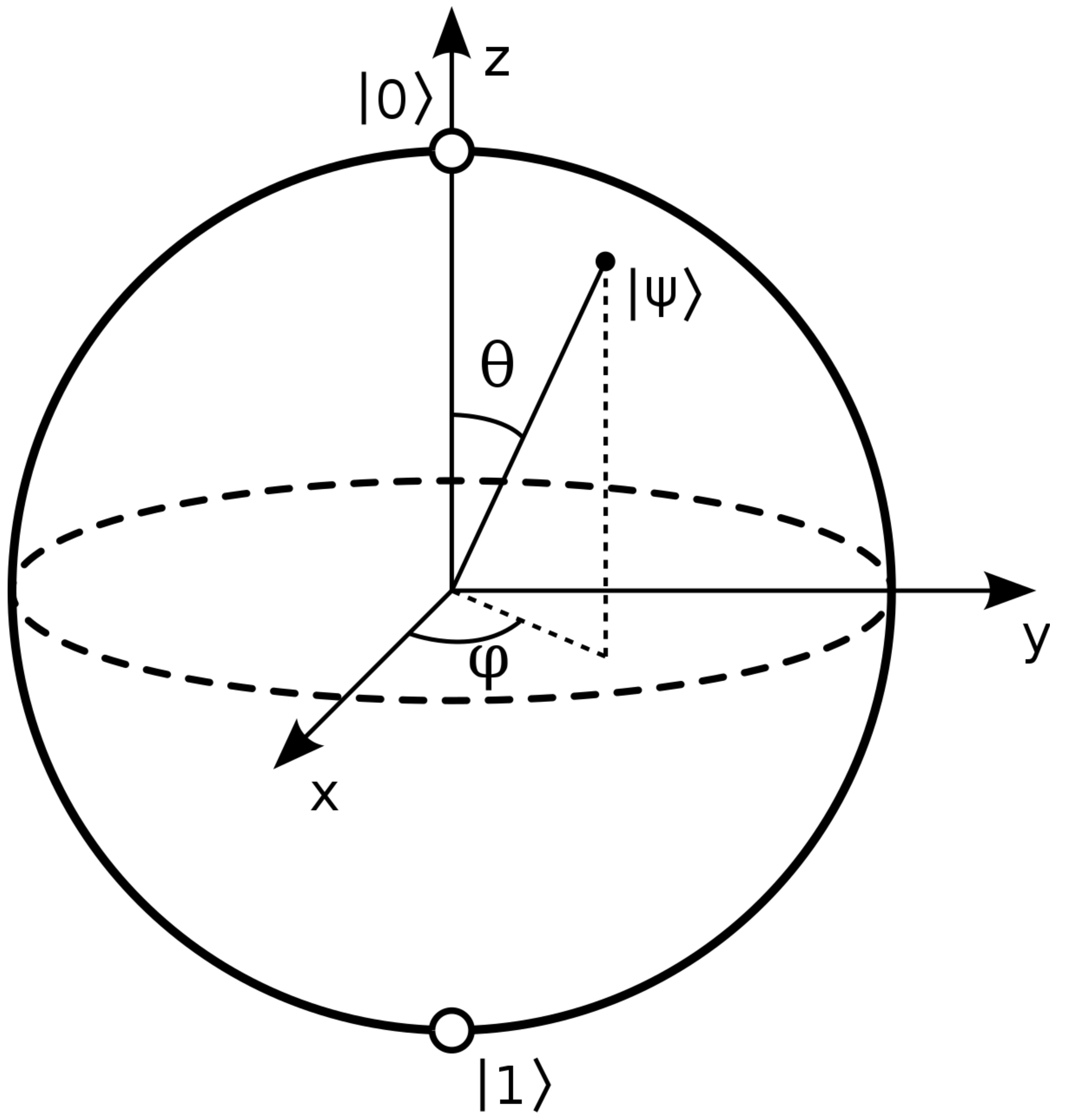
N	$\|\psi\rangle$
1	$\alpha\|0\rangle + \beta\|1\rangle$

Quantum Computation



N	$\lvert\psi\rangle$
1	$\alpha\lvert 0\rangle + \beta\lvert 1\rangle$
2	$\alpha\lvert 00\rangle + \beta\lvert 01\rangle + \gamma\lvert 10\rangle + \delta\lvert 11\rangle$

Quantum Computation



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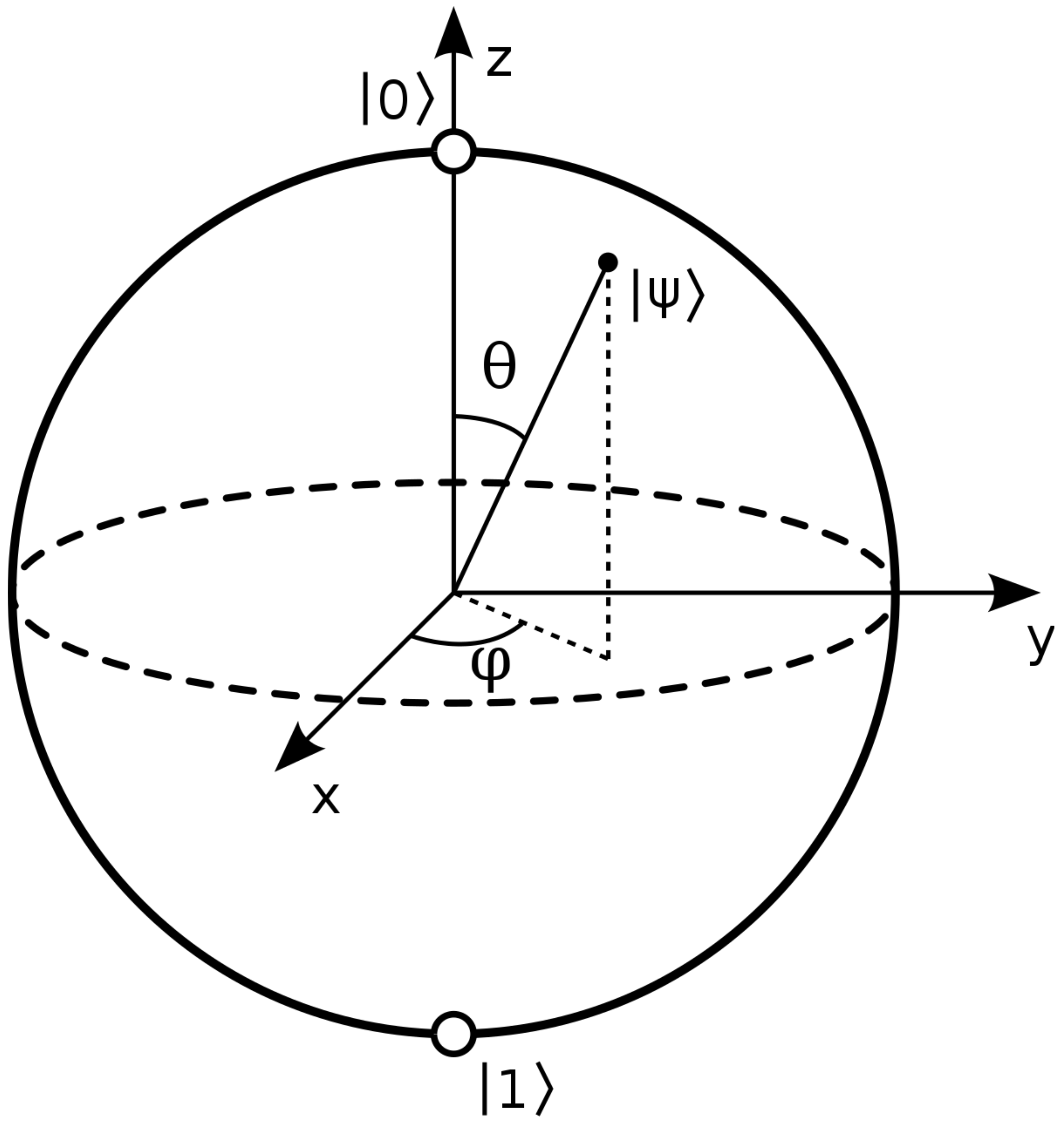
3	$\alpha\|000\rangle + \beta\|001\rangle + \gamma\|010\rangle + \delta\|011\rangle + \gamma\|100\rangle + \delta\|101\rangle + \epsilon\|110\rangle + \tau\|111\rangle$

Quantum Computation



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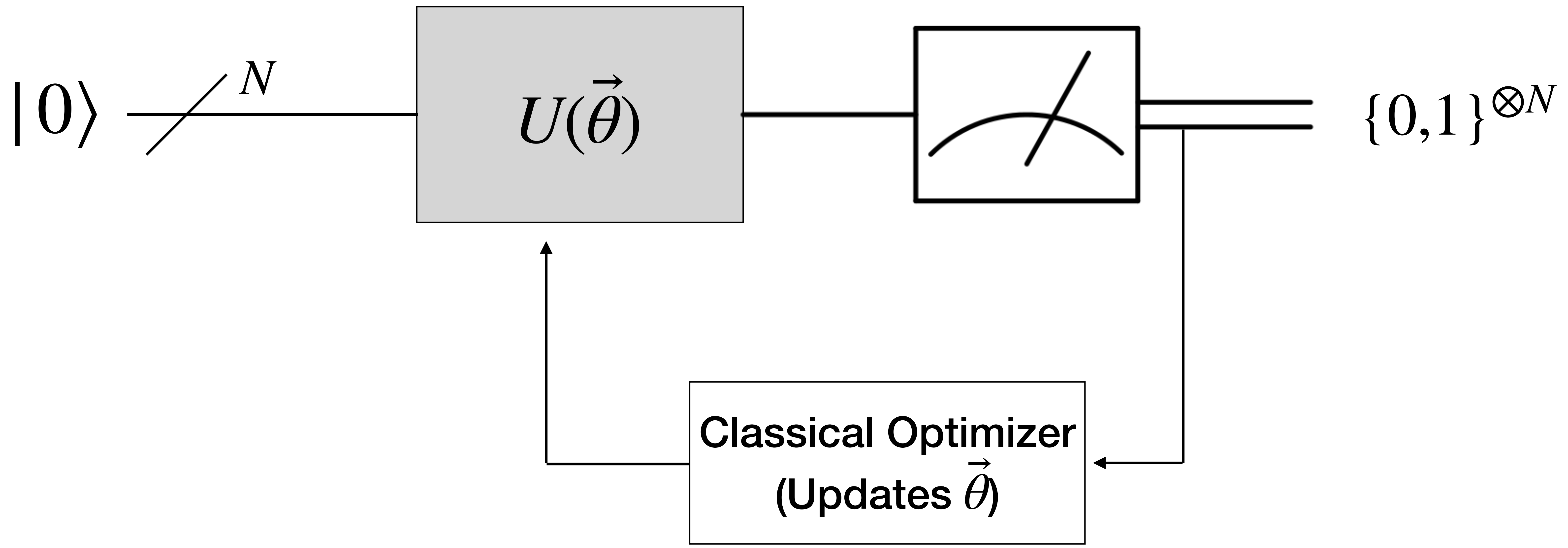
⋮	$\vert\psi\rangle \in \mathbb{C}^{2^N}$

Variational Quantum Eigensolver (VQE)

An Optimization Algorithm



A Variational eigenvalue solver on a protonic quantum processor, *Peruzzo et al.*

Variational Quantum Eigensolver (VQE)

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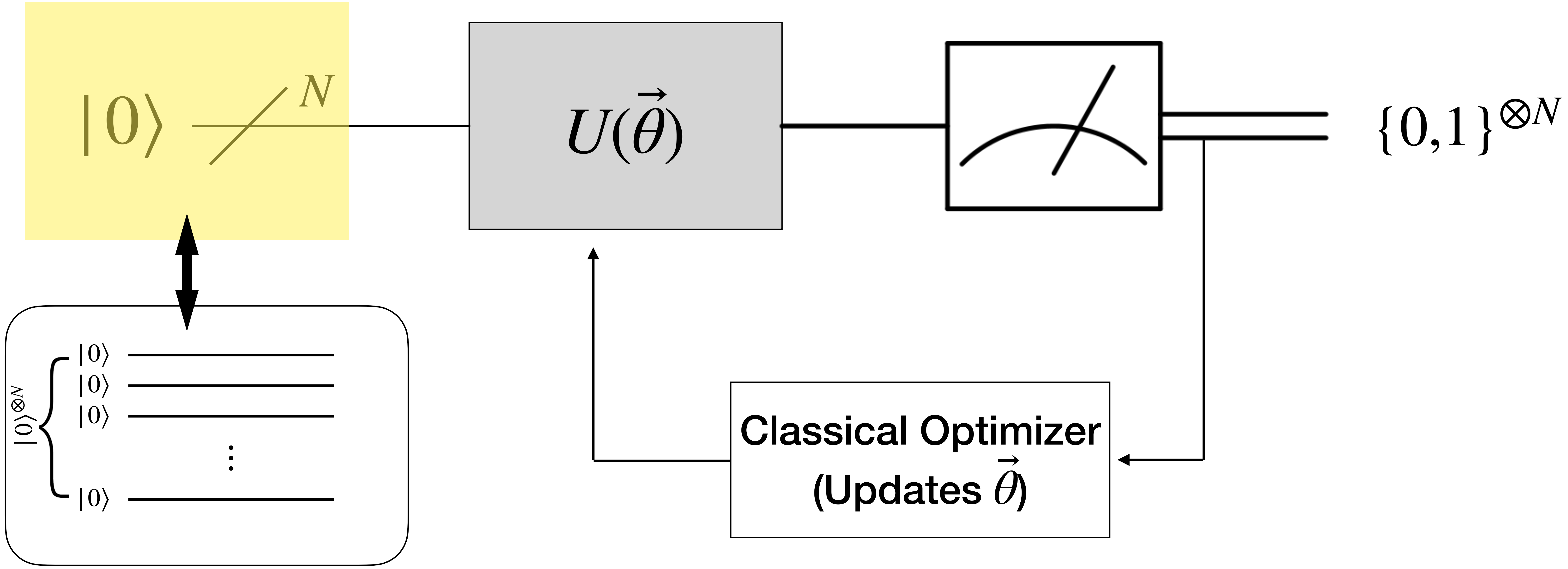

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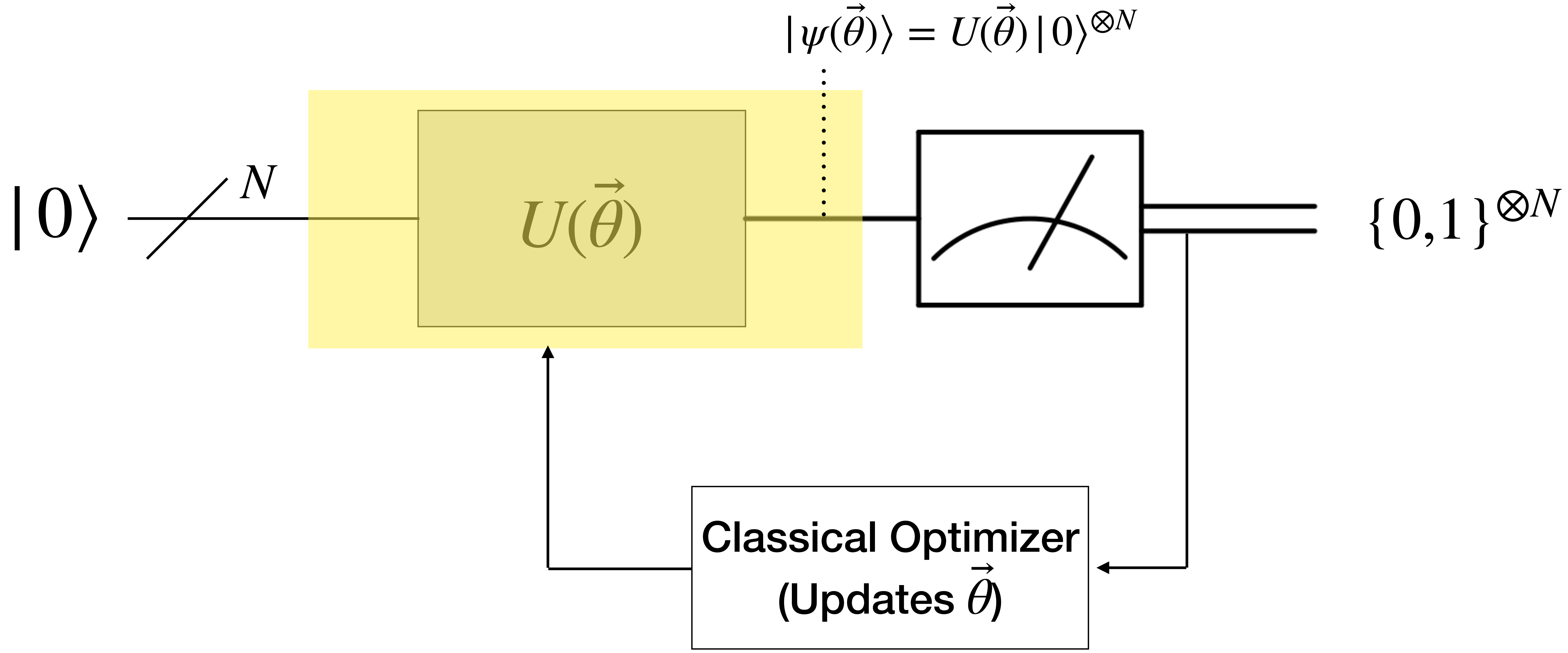

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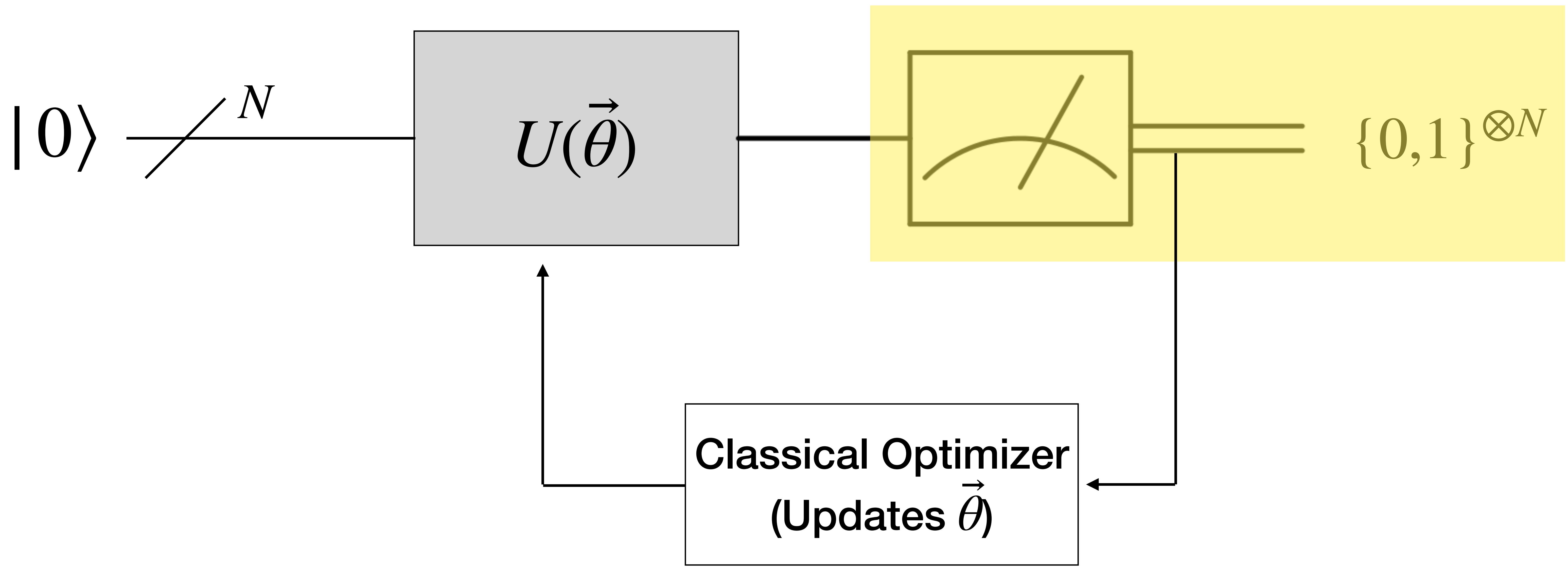

$$\langle H \rangle = \frac{1}{N} \sum_{i}^{N} \langle H_i(\theta_i) \rangle$$

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Variational Quantum Eigensolver (VQE)

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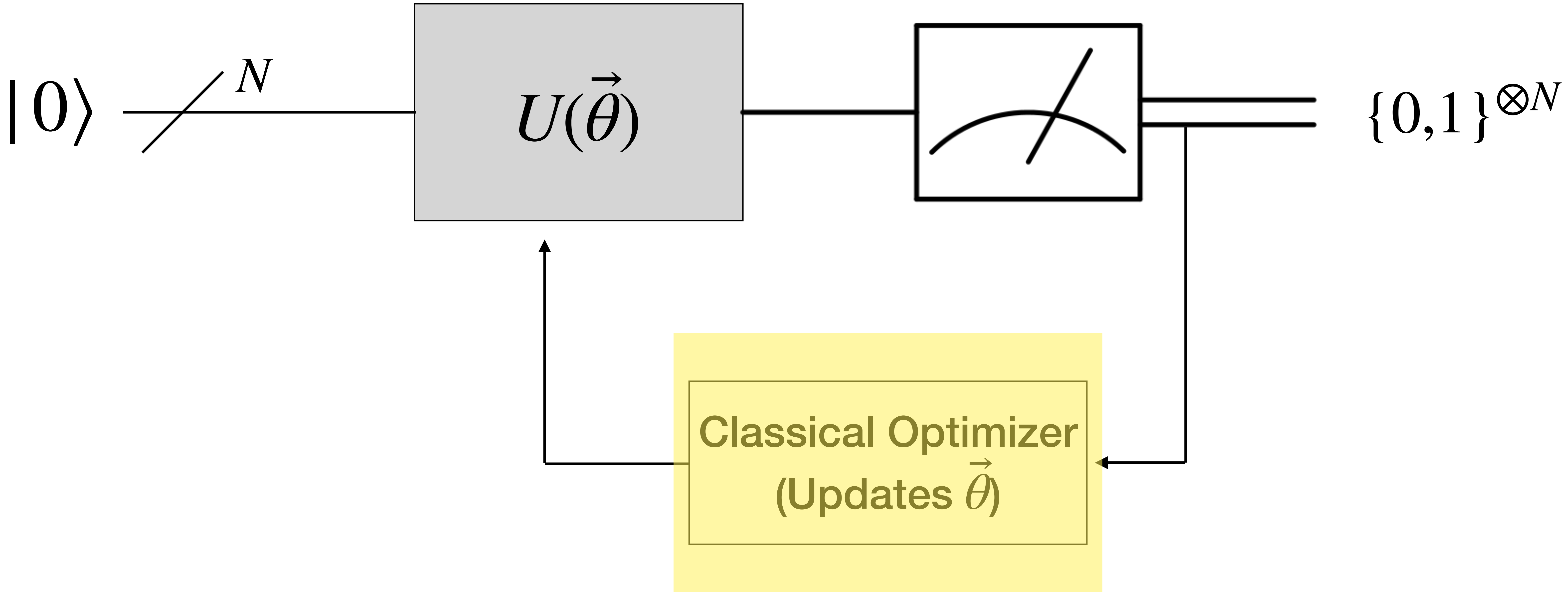

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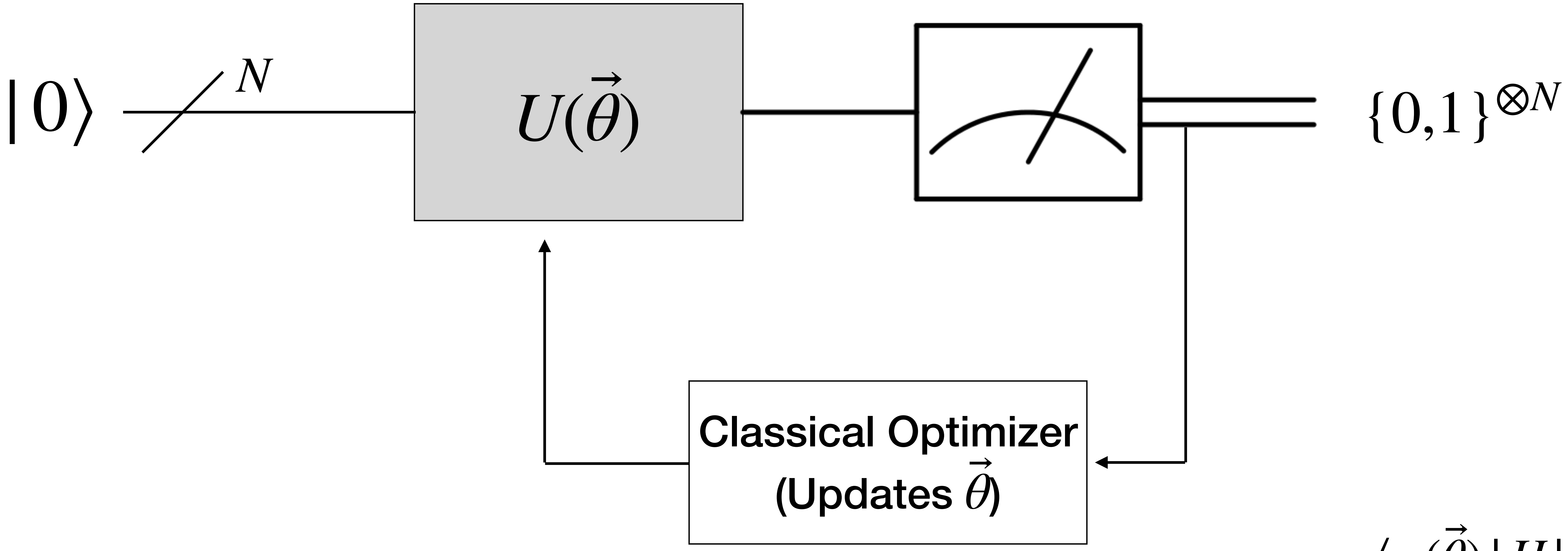

$$\langle H \rangle = \frac{1}{N} \sum_{i}^{N} \langle H_i(\theta_i) \rangle$$

$$\langle \psi(\vec{\theta}) | H | \psi(\vec{\theta}) \rangle \equiv E(\vec{\theta}) \geq E_0$$

A Variational eigenvalue solver on a protonic quantum processor, *Peruzzo et al.*

Discretized Light Cone Quantization
(DLCQ)

- In DLCQ we quantize (in a box) by momentum states
- Fock states:

$$|i\rangle = |\underbrace{n_1, \ldots, n_N}_{\text{Fermions}}; \underbrace{\bar{n}_1, \ldots \bar{n}_{\bar{N}}}_{\text{Antifermions}}; \underbrace{\tilde{n}_1^{\tilde{m}_1}, \ldots \tilde{n}_{\tilde{N}}^{\tilde{m}_{\tilde{N}}}}_{\text{Bosons}}\rangle$$

$$n_j, \bar{n}_j \in \{0,1\}$$

$$\tilde{m}_j \in \{0, \ldots \Lambda\}$$

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- Total hadronic momentum: $$K = \sum_n n(a_n^\dagger a_n + b_n^\dagger b_n + d_n^\dagger d_n)$$

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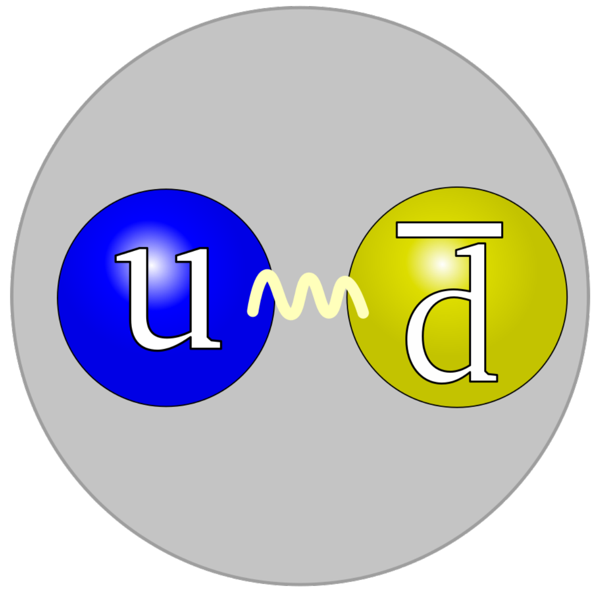
- Example:



$$K = 4$$

$$|u\bar{d}\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle$$

$$\{|i\rangle\} = \{|1; \bar{1}; \tilde{1}^2\rangle, |2; \bar{1}; \tilde{1}^1\rangle, |1; \bar{2}; \tilde{1}^1\rangle\}$$

DLCQ Hamiltonian

- QCD Hamiltonian in Lightfront (LF) coordinates:

$$H = \underset{\text{Kinetic}}{\underset{\uparrow}{T}} + \underbrace{V + F + S + C}_{\text{Interaction}}$$

Quantum Chromodynamics and Other Field Theories on the Lightcone, *Brodsky, Pinsky, Pauli*

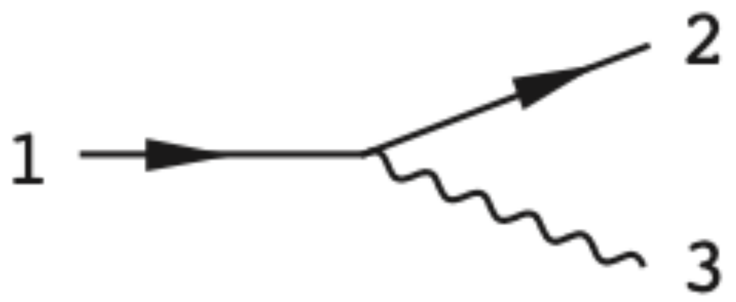
DLCQ Hamiltonian

- QCD Hamiltonian in Lightfront (LF) coordinates:

$$H = \underbrace{T}_{\text{Kinetic}} + \underbrace{V + F + S + C}_{\text{Interaction}}$$

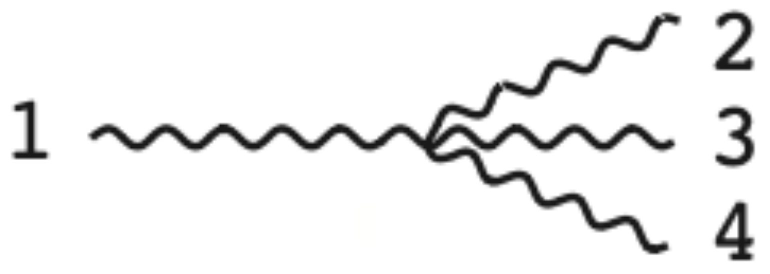
Vertex (V):

$q \rightarrow qg$



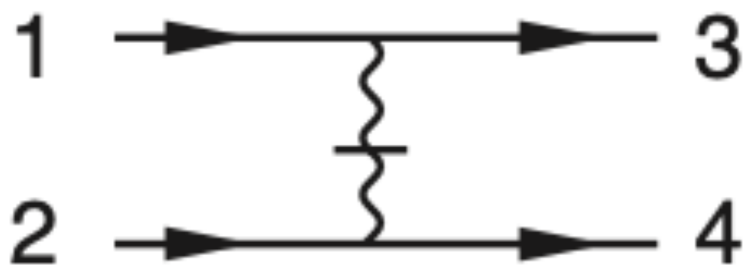
Fork (F):

$g \rightarrow ggg$



Seagull (S):

$qq \rightarrow qq$



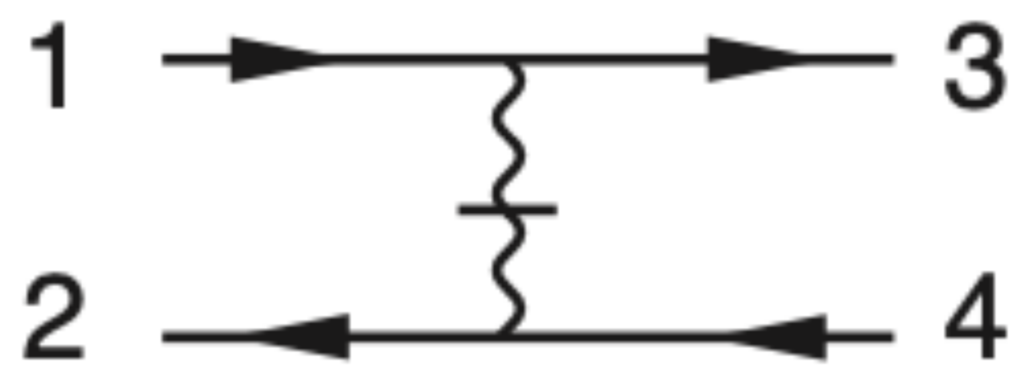
Self-induced Inertia (C):

$g \rightarrow g$

Quantum Chromodynamics and Other Field Theories on the Lightcone, *Brodsky, Pinsky, Pauli*

π^0 DLCQ

- Valence Fock state: $|\pi^0\rangle = |q\bar{q}\rangle$
- Only 1 nontrivial interaction



- In 2 + 1D LF coordinates, $n_i = \{x_i, \vec{k}_i^{\perp}\}$

$$\vec{k}_i^{\perp} \in \{0, \ldots, \Lambda^{\perp}\}$$

$$\sum_i \vec{k}_i^{\perp} = P^{\perp}$$

Calculating the GPD in LF Form

$$H^q(x,\xi) = \frac{1}{2\bar{P}^+}\int \frac{d^2k_T}{2\sqrt{|x^2-\xi^2|}(2\pi)^3}$$

$$\sum_\lambda [\langle P'|b^\dagger_\lambda((x-\xi)\bar{P}^+, k_T-\Delta_T)b_\lambda((x+\xi)\bar{P}^+, k_T)|P\rangle\theta(x\geq\xi)$$
$$+\langle P'|d^\dagger_\lambda((-x+\xi)\bar{P}^+, -k_T+\Delta_T)b_{-\lambda}((x+\xi)\bar{P}^+, k_T)|P\rangle\theta(-\xi<x<\xi)$$
$$-\langle P'|d^\dagger_\lambda((-x-\xi)\bar{P}^+, k_T-\Delta_T)d_\lambda((-x+\xi)\bar{P}^+, k_T)|P\rangle\theta(x\leq\xi)$$

Off-diagonal parton distributions and their evolution, *Golec-Biernant and Martin*

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$$+\langle P'|d_\lambda^\dagger((-x+\xi)\bar{P}^+, -k_T+\Delta_T)b_{-\lambda}((x+\xi)\bar{P}^+, k_T)|P\rangle\theta(-\xi<x<\xi)$$ (ERBL)

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- How can we calculate $\langle P'|\hat{O}(x)|P\rangle$ on a quantum computer?

Off-diagonal parton distributions and their evolution, *Golec-Biernant and Martin*

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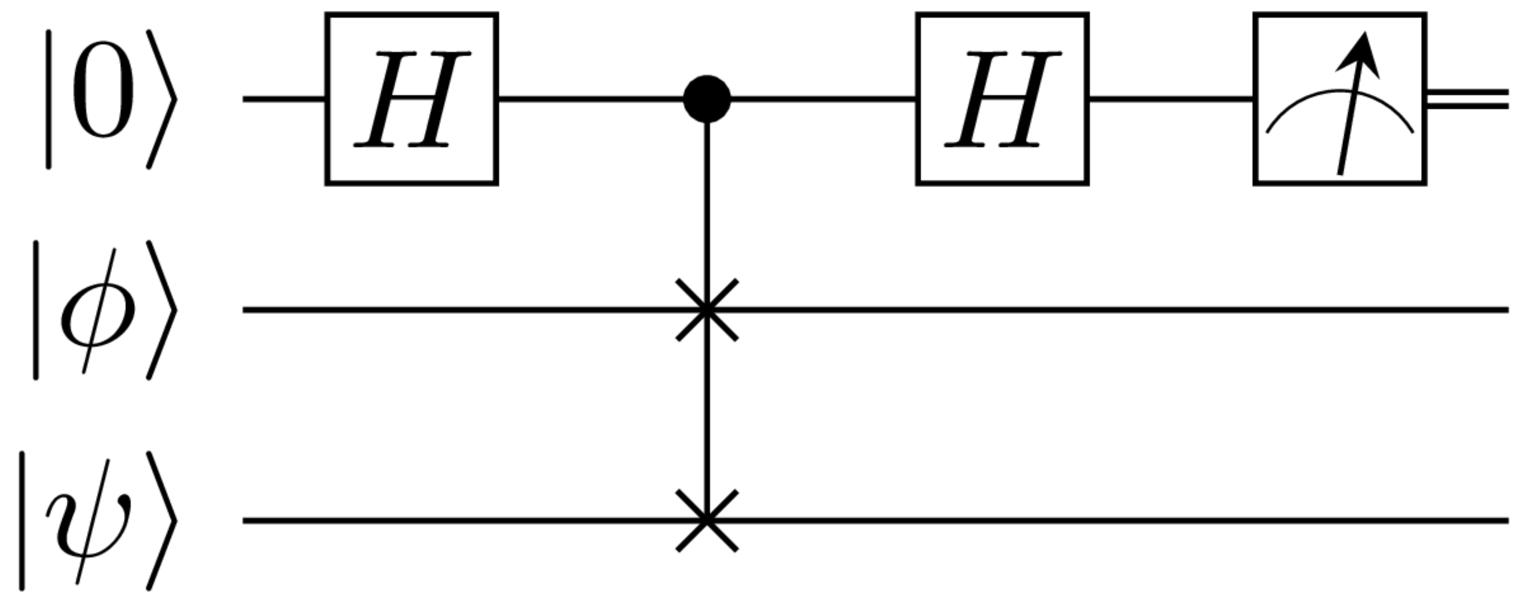
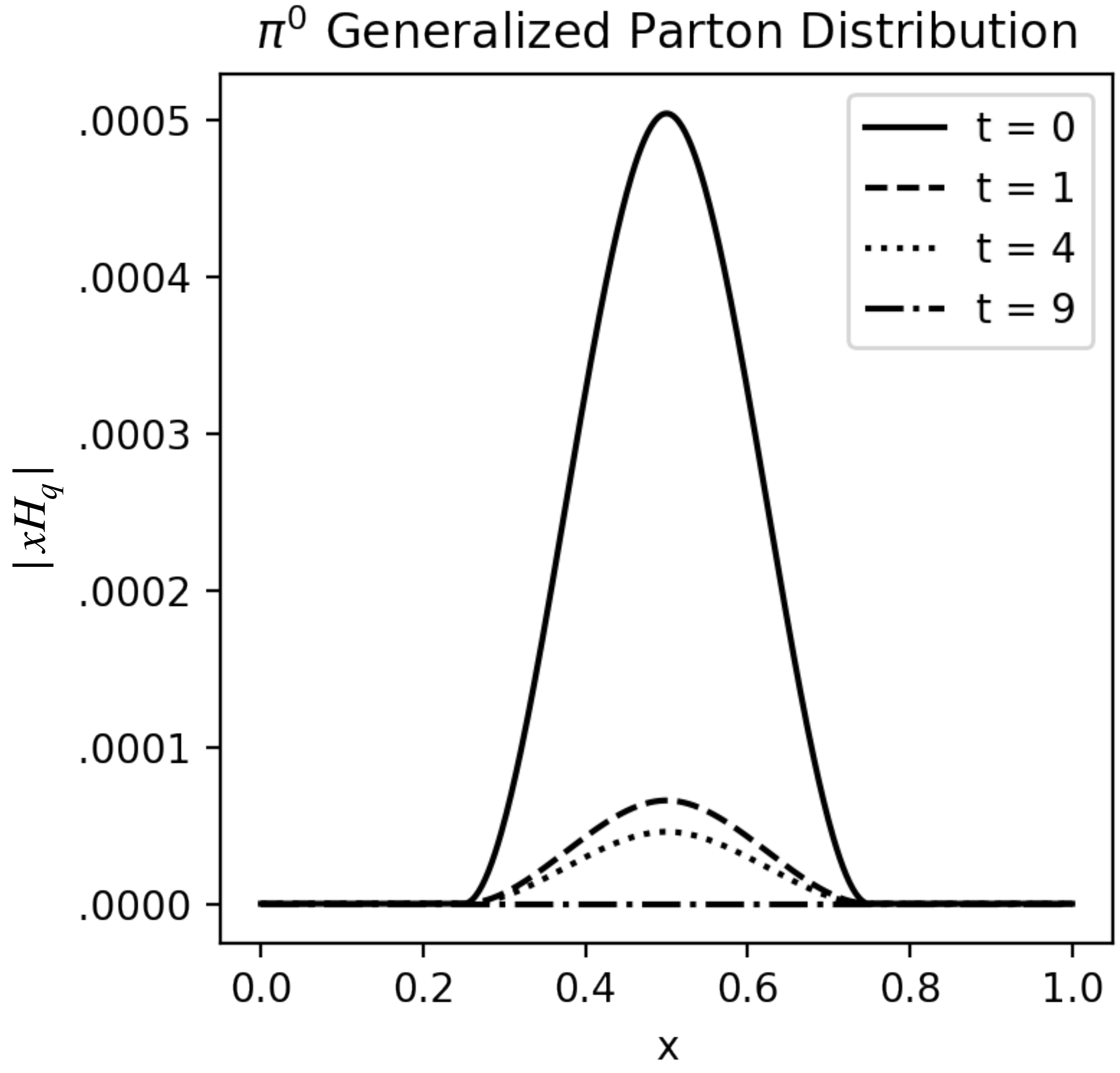
- How can we calculate $\langle P'|\hat{O}(x)|P\rangle$ on a quantum computer?

- SWAP Test estimates $|\langle\phi|\psi\rangle|^2$



Off-diagonal parton distributions and their evolution, *Golec-Biernant and Martin*

π^0 GPD



π^0 Generalized Parton Distribution
t = 0
t = 1
t = 4
t = 9
.0005
.0004
.0003
.0002
.0001
.0000
$|xH_q|$
0.0
0.2
0.4
0.6
0.8
1.0
x

Basis Light-front Quantization (BLFQ)

- DLCQ can become *very* expensive *very* quickly

$$\mathrm{Q} \leq \underbrace{2K}_{\substack{\text{number of}\\\text{occupied}\\\text{fermion/antifermion}\\\text{modes}}} \overbrace{\Big[\underbrace{\lceil \log_2 K \rceil + 2\lceil \log_2 \Lambda_\perp \rceil}_{\text{momentum}} + \underbrace{1}_{\text{helicity}} + \underbrace{\lceil \log_2 n_f \rceil}_{\text{flavors}} + \underbrace{\lceil \log_2 n_c \rceil}_{\text{colors}}\Big]}^{\text{fermion/antifermion modes}}$$

$$+ \underbrace{K}_{\substack{\text{number of}\\\text{occupied}\\\text{boson modes}}} \overbrace{\Big[\underbrace{\lceil \log_2 K \rceil + 2\lceil \log_2 \Lambda_\perp \rceil}_{\text{momentum}} + \underbrace{\lceil \log_2 K \rceil}_{\text{occupancy}} + \underbrace{1}_{\text{helicity}} + \underbrace{\lceil \log_2 (n_c^2 - 1) \rceil}_{\text{colors}}\Big]}^{\text{boson modes}} \approx 1360 \text{ Qubits}$$

- DLCQ uses momentum orbitals as basis (plane waves). BLFQ exploits symmetry and uses ‘smart’ basis choice and works in terms of relative momentum

Kreshchuk et. al

Basis Light Front Quantization

Basis Function Representation

- Bound states in BLFQ are found via light front wavefunctions, $|\Psi\rangle$ such that $H_{eff}|\Psi\rangle = M^2|\Psi\rangle$ (where $H_{eff} = P^+P^- - P_\perp^2$).
  - $H_{eff} = H_0 + H_{eff}^{int}$

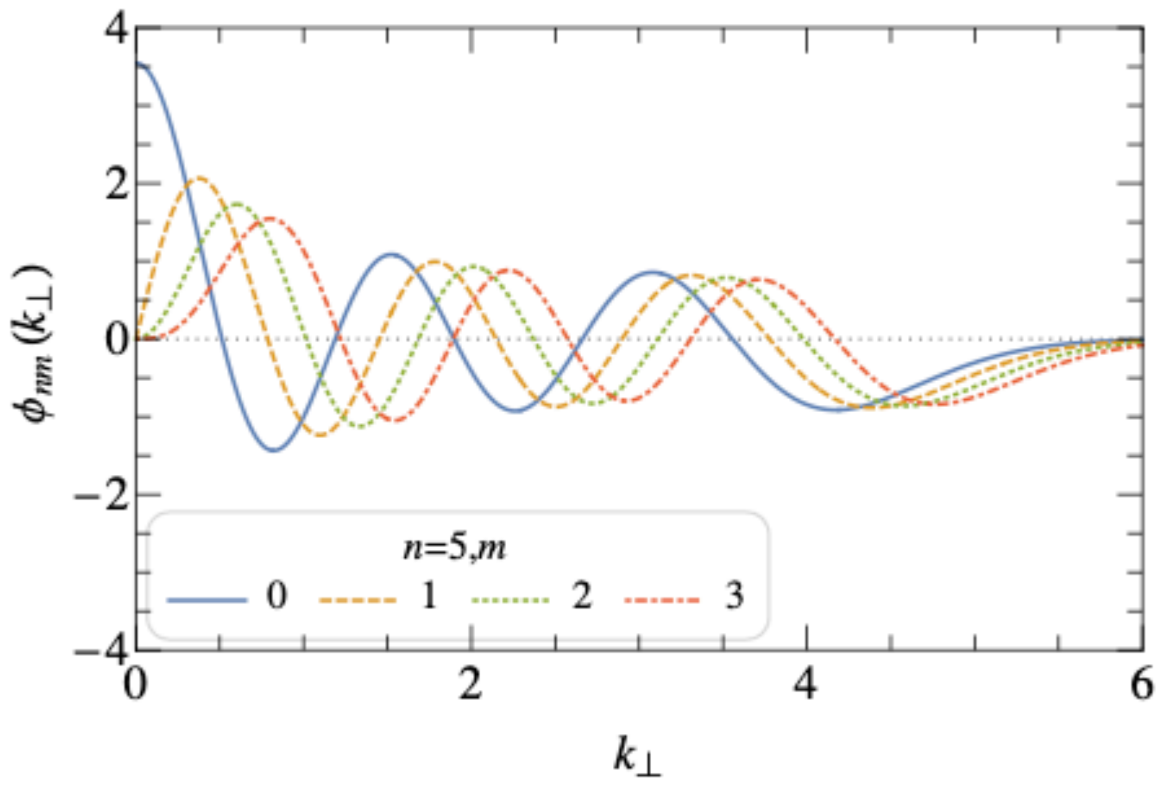
The interaction Hamiltonian is the instantaneous gluon exchange that comes from $\mathscr{L}_{qd}^{int}$

H_0 consists of the holographic AdS/QCD SW Hamiltonian + quark kinetic energy + longitudinal confinement

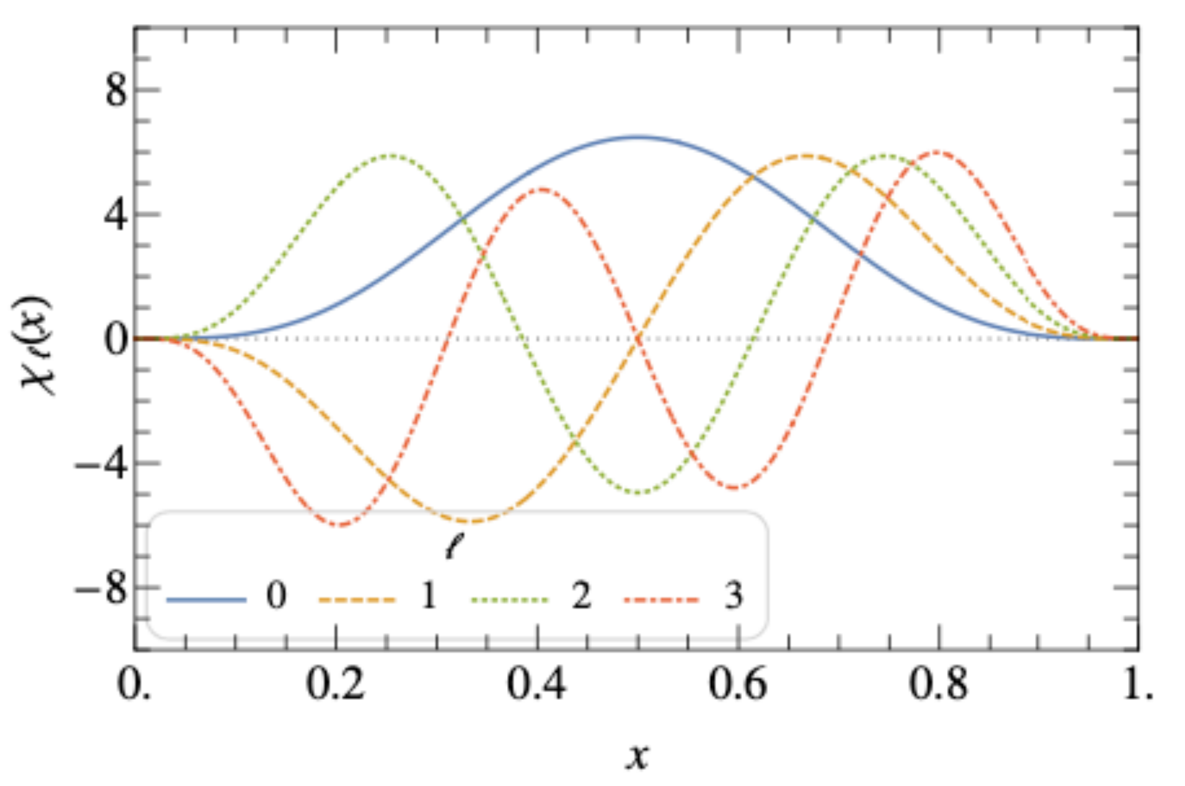
GPDs are diagonal in this basis representation:

We can use the Hadamard test

Transverse



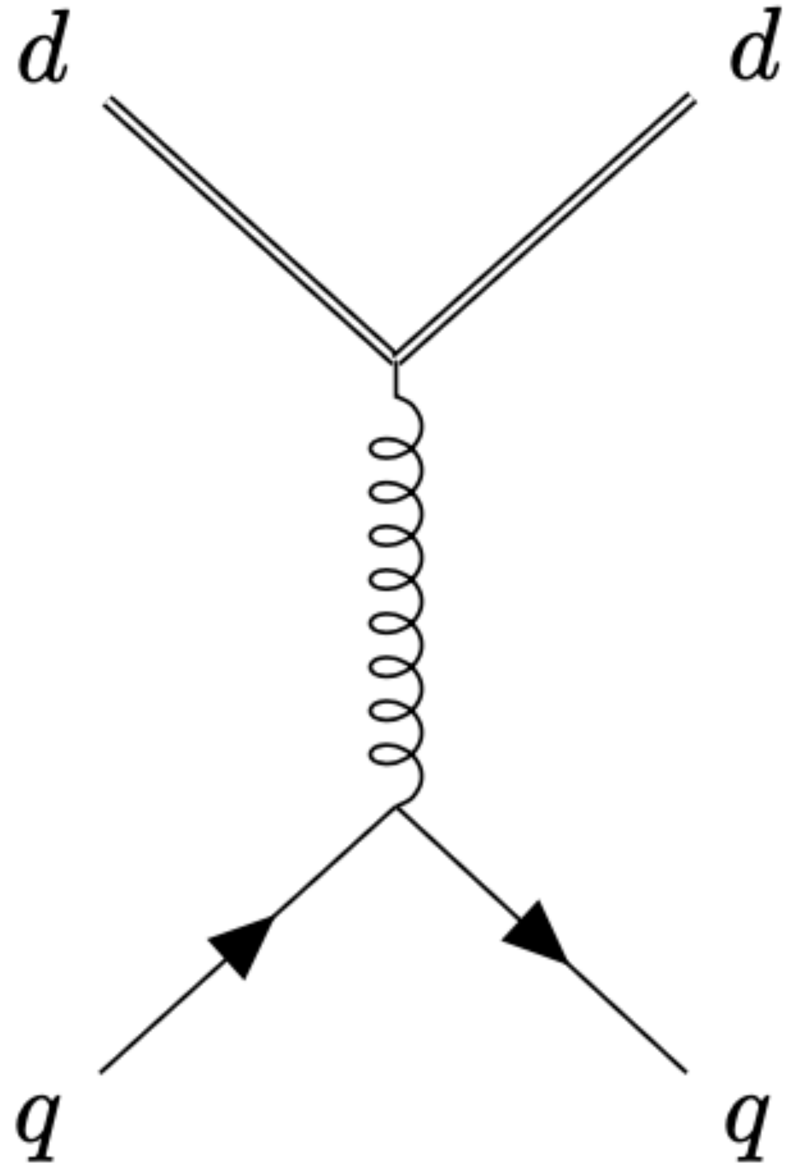
Longitudinal



Quarkonium light-front wave functions, *Yang Li*

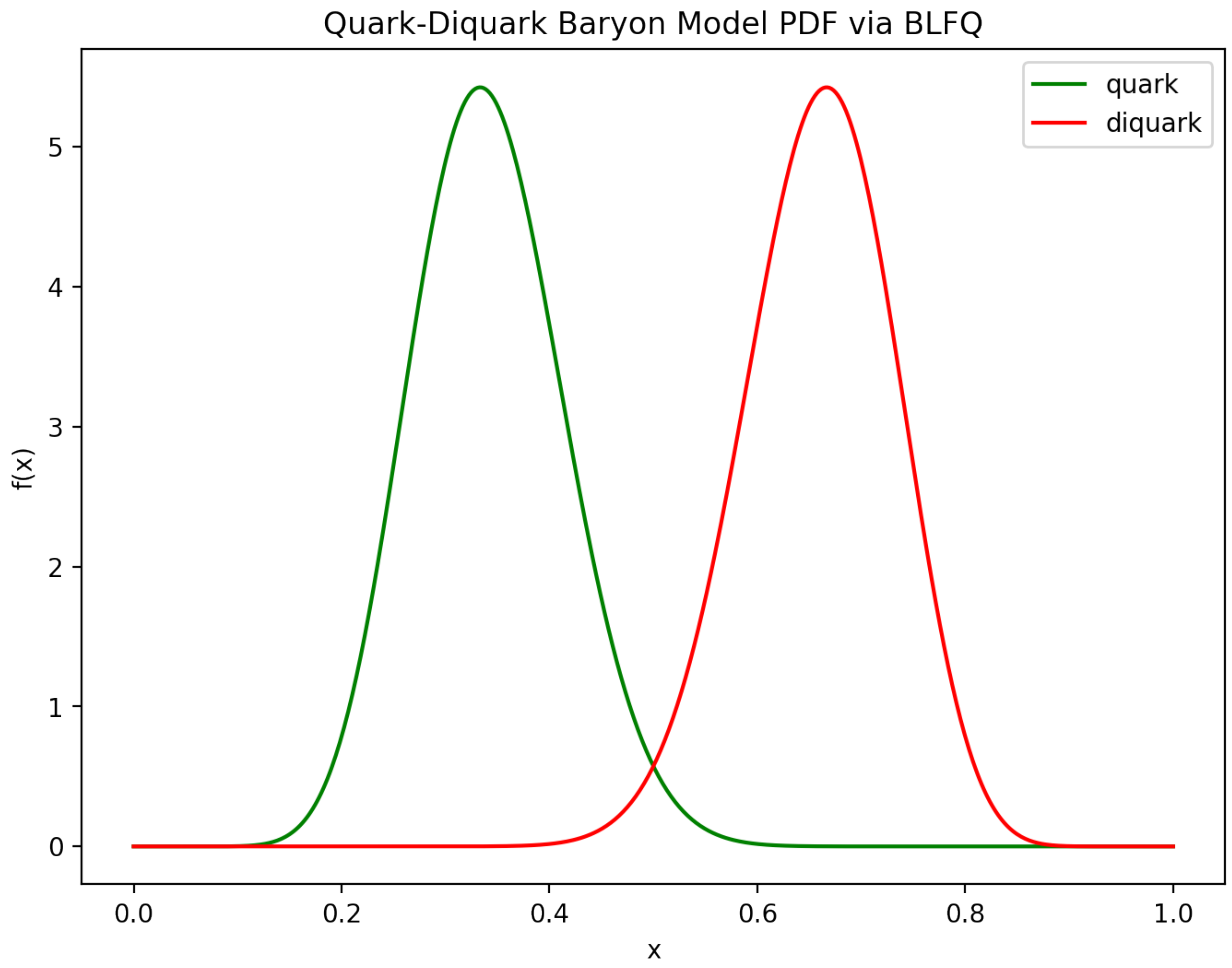
Quark-Diquark Baryon Model

The Lagrangian

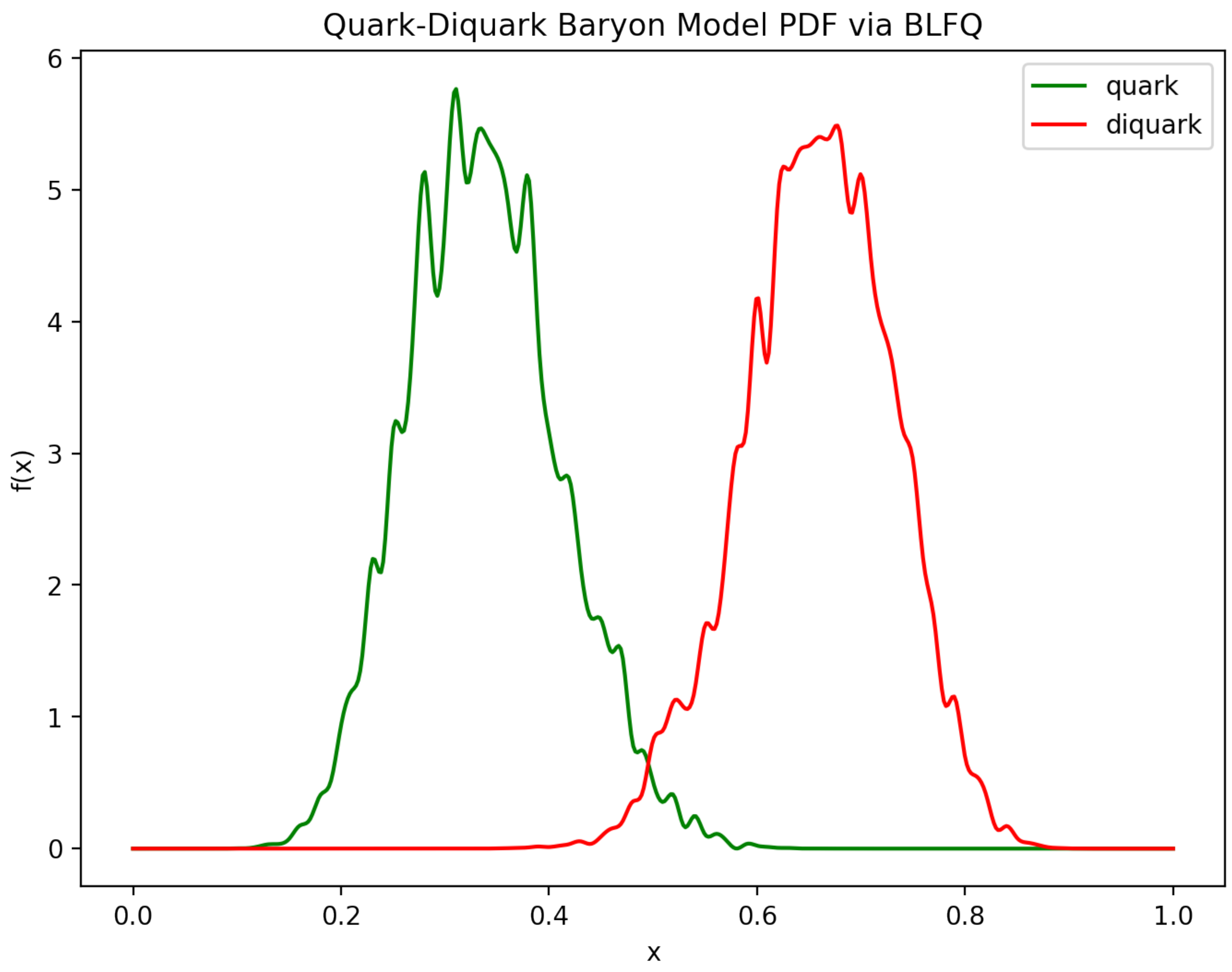


- Our representation for the baryon is the quark-diquark model:
  - A quark field, ψ, interacting with a scalar (spin 0) diquark field, φ, via a $SU(1)$ color gauge gluon field, A_μ.
- The corresponding Lagrangian for this model is:
  - $\mathscr{L}_{qd} = \underbrace{\frac{1}{2}\bar{\psi}(i\gamma^\mu D_\mu - m_q)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}}_{\text{QCD Lagrangian}} + \underbrace{(D_\mu\varphi)^\dagger(D^\mu\varphi) - m_d^2\varphi^\dagger\varphi}_{\text{Scalar QCD Lagrangian}}$

$|qd\rangle$ PDF Results

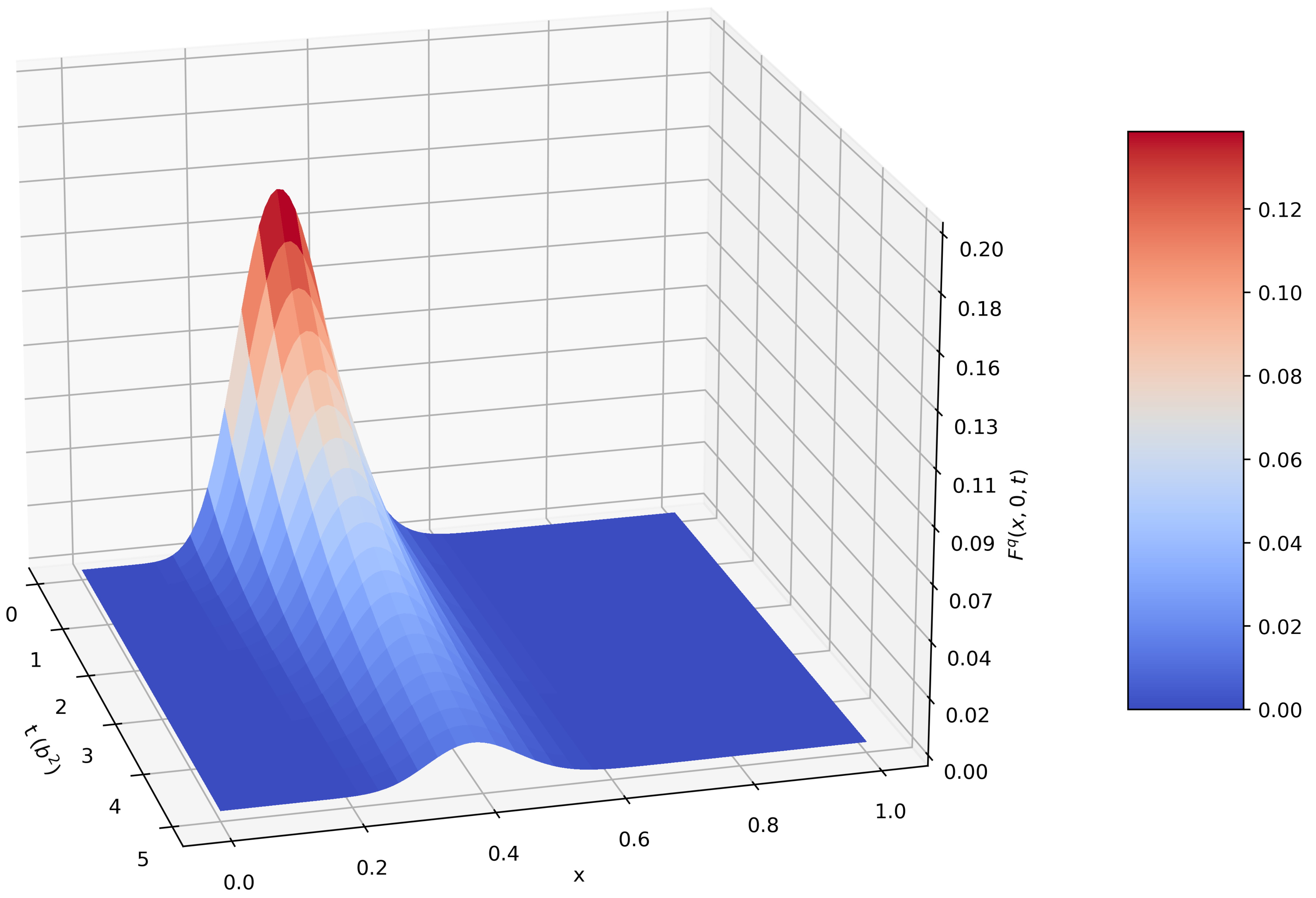


Statevector Solution



Shot-based “noisy” Solution

$|qd\rangle$ Baryon GPD



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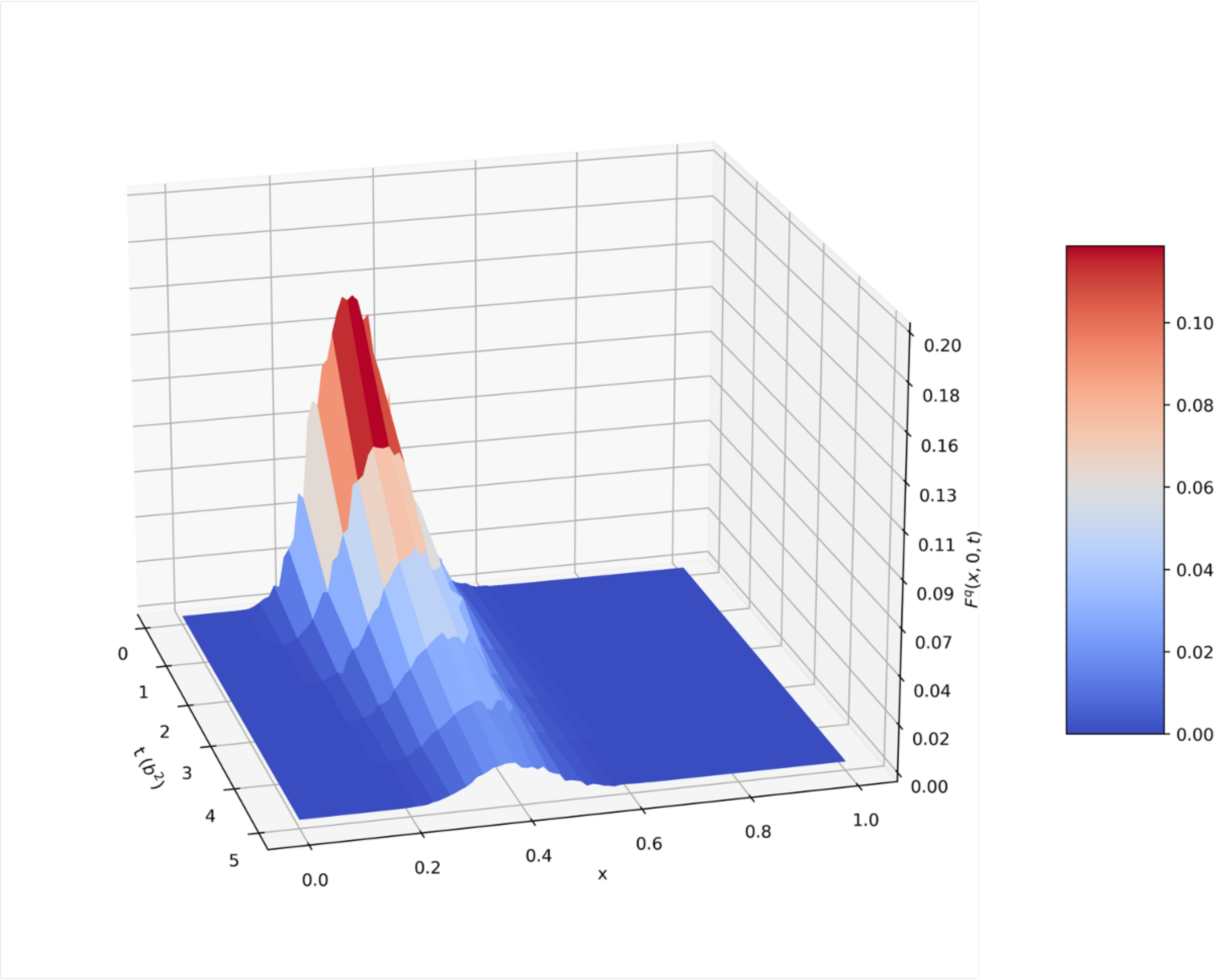


Shot-based “noisy” Solution

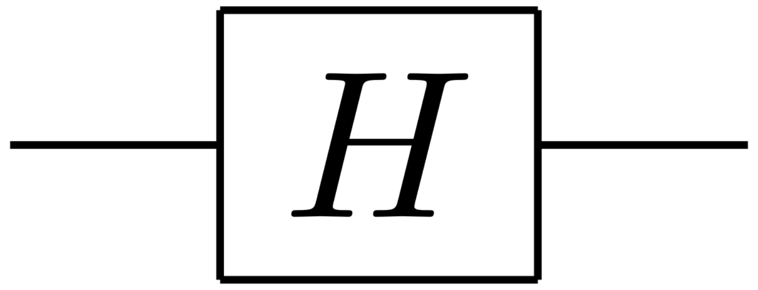
Summary

- PDFs & GPDs are generally calculated on classical computers via Lagrangian, lattice based methods
- We present an alternative approach following the Hamiltonian formulation on quantum computers
- We show two quantization methods: DLCQ and BLFQ to model mesons and baryons respectively

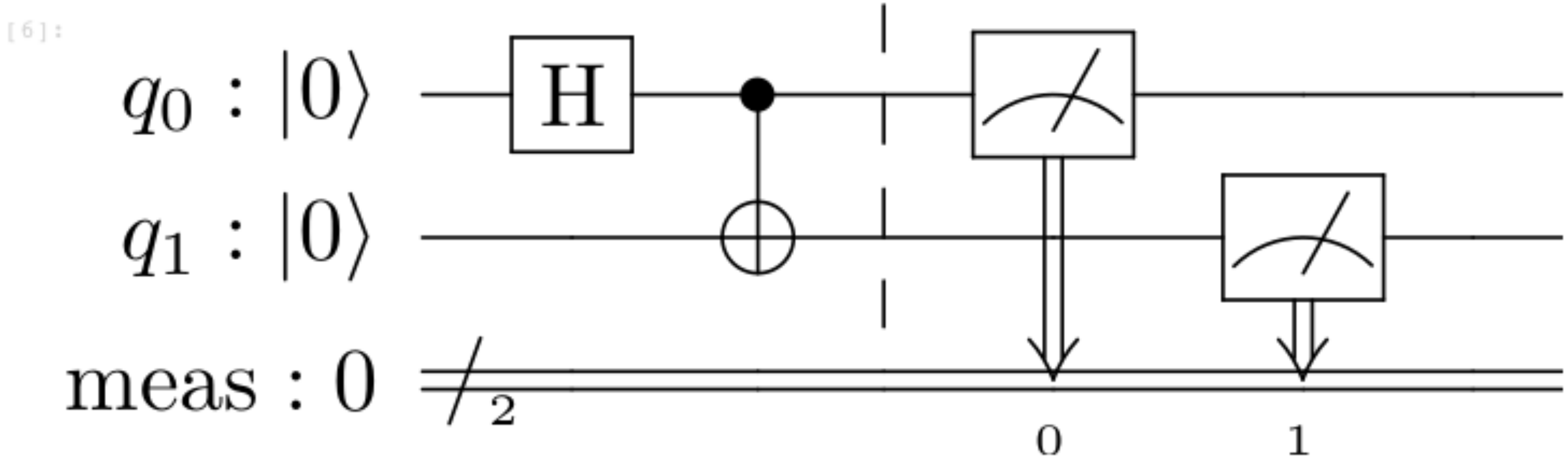
Supplemental Material

Quantum Computing Basics

Example



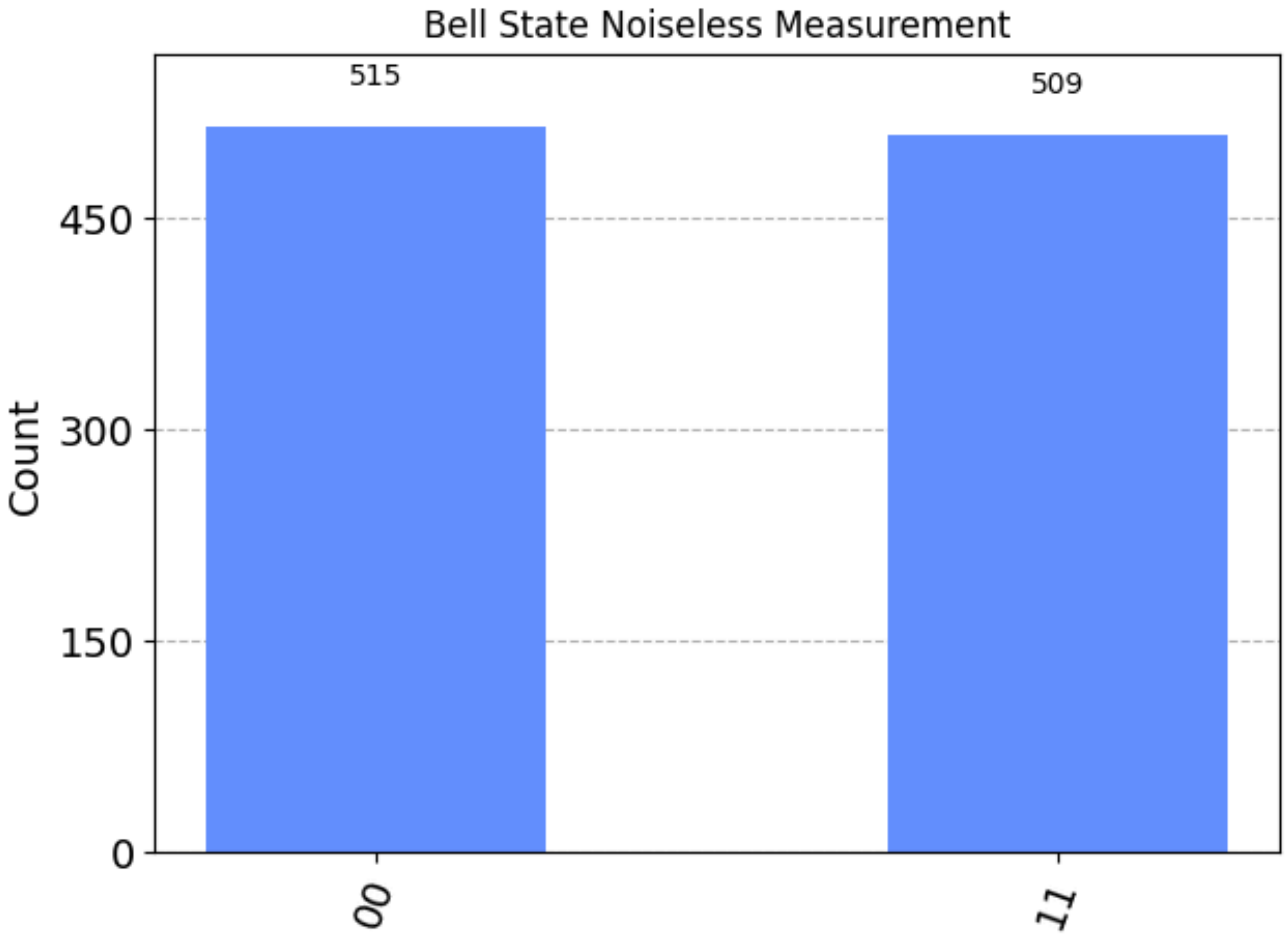
$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

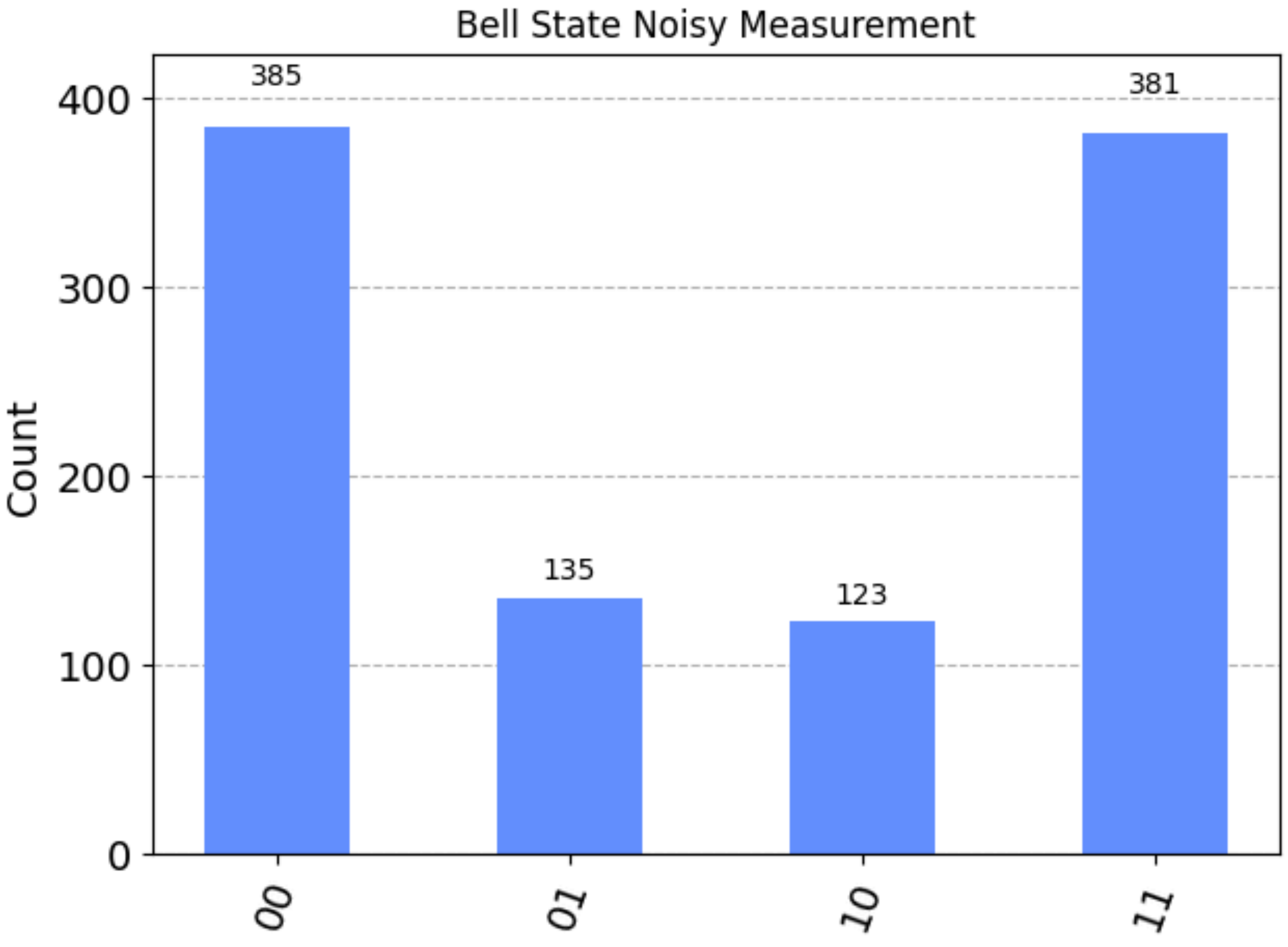


$$|00\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|00\rangle + |10\rangle\right) \rightarrow \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

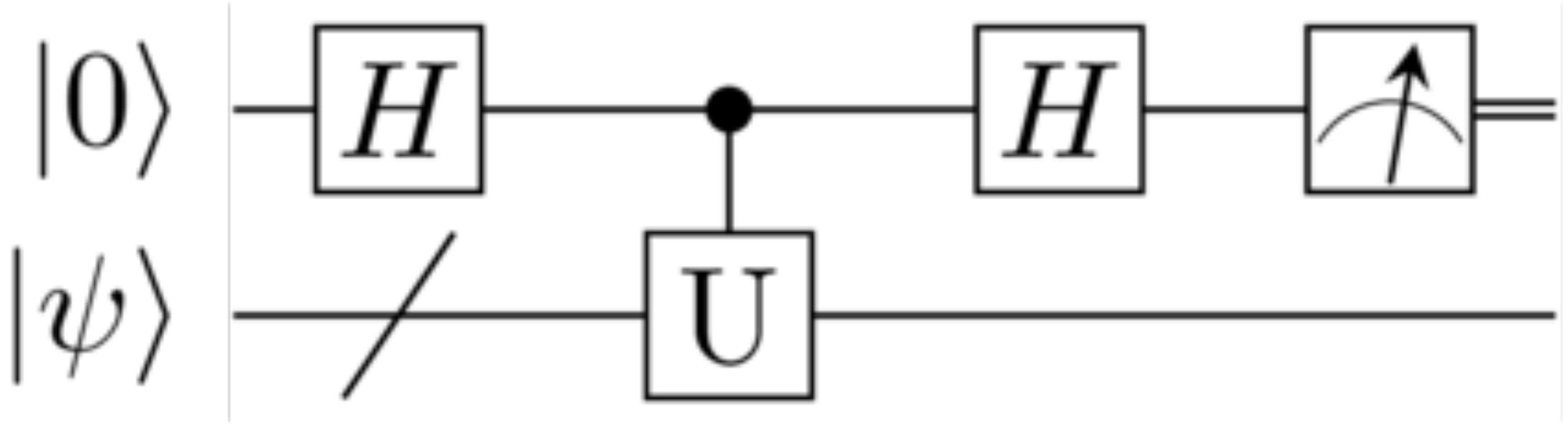
Quantum Circuit Example

Measurement





The Hadamard Test



- The Hadamard Test (shown above) is a method used to approximate expectation values, specifically $\Re(\langle\psi|U|\psi\rangle)$.
- This circuit above leads to determining the real part of the expectation value $\langle\psi|U|\psi\rangle$ by taking the difference between probabilities that the ancillary qubit is in state 0 minus the probability it is in state 1.
- Is this useful for calculating PDFs which are diagonal expectation values i.e. $f_q(x) = \langle P|O(x)|P\rangle$?

https://en.wikipedia.org/wiki/Hadamard_test_(quantum_computation)

Encoding

Cutoffs

$$\begin{cases} n \in [0, N_{max}], \\ m \in [-M_{max}, M_{max}] \\ l \in [0, L_{max}] \end{cases}$$

- Cutoffs on the basis functions $\chi_l(x)$ & $\phi_{nm}(\vec{k}_\perp)$ are imposed as with $N_{max} = 0, M_{max} = 2, L_{max} = 1$
- The LF Hamiltonian conserves m_j: diagonalize the Hamiltonian in blocks of fixed $m_j = m + s_q + s_d$. Here, we look at the $m_j = \frac{1}{2}$ block of the Hamiltonian

Label						Direct	Compact
1	0	0	0	+	0	$\vert 0001\rangle$	$\vert 00\rangle$
2	0	1	0	-	0	$\vert 0010\rangle$	$\vert 01\rangle$
3	0	0	1	+	0	$\vert 0100\rangle$	$\vert 10\rangle$
4	0	1	1	-	0	$\vert 1000\rangle$	$\vert 11\rangle$

Basis Light Front Quantization

The Instantaneous Gluon Interaction

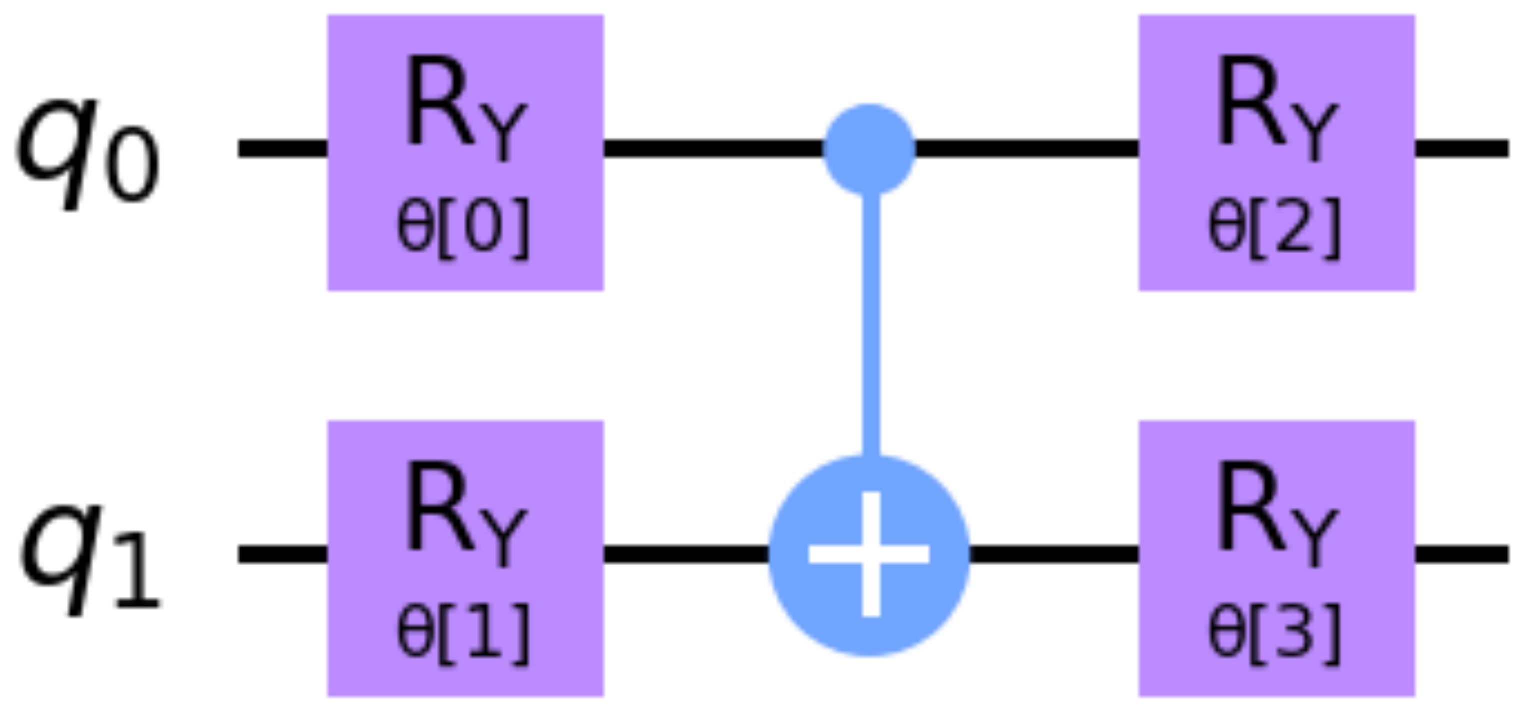
- Light front coordinates put constraints on the quark and gluon fields
  - This gives rise to instantaneous interactions unique to LF coordinates.
  - $\mathscr{L}_{qd}$ leads to an instantaneous gluon interaction between ψ & φ

$$P^-_{qd\to qd} = \frac{1}{2} g^2 \int dx^- d\vec{x}^\perp \bar{\psi}\gamma^+\psi \frac{1}{(i\partial^+)^2} \left(\partial^+\varphi\right)^\dagger \varphi$$

VQE

Circuit

- We use a 2-qubit hardware efficient ansatz (HEA) to find the ground state of H.
  - The HEA used is qiskit's Real Amplitudes circuit:



  - The reference state used is $|00\rangle$

Qubit PDF
In BLFQ

- In our basis function representation, the qubit PDF is given as

$$f_q(x) = \frac{1}{4\pi} \sum_{n,m,l',l,s,\bar{s}} \langle \psi | n, m, l', s, \bar{s} \rangle \langle n, m, l, s, \bar{s} | \psi \rangle \chi'_l(x) \chi_l(x)$$

  - $|n, m, l', s, \bar{s}\rangle\langle n, m, l, s, \bar{s}|$ is a density matrix

- For each momentum fraction, x, this equation gives us a linear combination of Pauli operators:

$$\frac{1}{4\pi} \sum_{n,m,l',l,s,\bar{s}} |n, m, l', s, \bar{s}\rangle\langle n, m, l, s, \bar{s}| \chi'_l(x) \chi_l(x) \rightarrow \sum_i c_i P_i$$

- Calculating the expectation value of this Pauli term w.r.t. $|\psi\rangle$ gives $f_q(x)$

Qubit GPD
In BLFQ

- Again, in our basis representation, the GPD is given as:

$$H^q(x) = \sum_{n,n',m,l',l,s,\bar{s}} \langle \psi | n, m, l', s, \bar{s} \rangle \langle n, m, l, s, \bar{s} | \psi \rangle \chi_l'(x) \chi_l(x) \int d^2\vec{\kappa} \varphi^*_{n',m} \left(\frac{\overrightarrow{\kappa'}}{\sqrt{x(1-x}} \right) \varphi_{n,m} \left(\frac{\vec{\kappa}}{\sqrt{x(1-x}} \right)$$

- The same procedure applies here: we turn the outer product into a linear combination of Paulis at some momentum fraction x, and some value of t, and take an expectation value with a quantum computer