Single spin asymmetries in electron nucleon scattering at low and intermediate energies

Jose L. Goity

Hampton University and Jefferson Lab

In collaboration with Christian Weiss and Cintia Willemyns

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SINGLE SPIN ASYMMETRIES IN ELECTRON-NUCLEON SCATTERING

- Polarized beam or target
- Longitudinally polarized electron beam: test of Parity Violation (HERMES, HAPPEx, SAMPLE, QWeak)
- Transversely polarized target or beam (NSSA): test of spin effects at hadronic and partonic level: e.g. two photon exchange, Sivers function. (A1, PREX, JLab, COMPASS, HERMES, COMPASS)

$$d\sigma_{SSA} \propto \vec{k} \cdot \vec{S}$$
$$\not{P} \quad EW$$

$$d\sigma_{SSA} \propto (\vec{k} \wedge \vec{k}') \cdot \vec{S}$$
$$QED \;\&\; QCD$$

- SSAs in exclusive, semi-inclusive and inclusive processes
- historical works that initiated the theory of SSAs: Barut & Fronsdal (1960); Leroy & Piketty; De Rujula, Kaplan and De Rafael; Gunther and Rodenberg; etc
- numerous works since the 2000's on **TPE** and SSA's: Guichon & Vanderhaeghen; Blunden, et al; Gorchtein et al; Afanasev et al.; Myhrer et al; in **DIS**: Metz, et al; etc
- NSSAs measured in high energy regime (**Pol target**: COMPASS, HERMES, JLab Hall A) (**Pol beam**: A1 at MAMI, JLab Hall C, SAMPLE). Lmited kinematic ranges

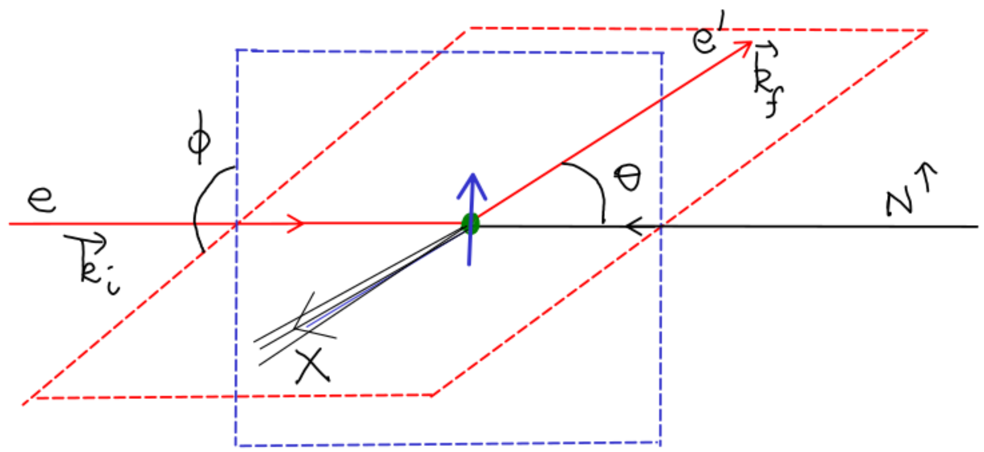
OUTLINE

- Normal single spin asymmetries in electron scattering and two photon exchange - generalities
- Target normal SSA (TNSSA) - the 1/Nc expansion framework
- Results
- Comments and summary

NORMAL SINGLE SPIN ASYMMETRIES - GENERALITIES

NSSA cross section linear in spin vector

Parity invariance requires two momentum vectors



$$d\sigma = d\sigma_U + 2\vec{e}_N \cdot \vec{S}\, d\sigma_N$$

$$\vec{e}_N = \frac{\vec{k}_f \times \vec{k}_i}{|\vec{k}_f \times \vec{k}_i|}$$

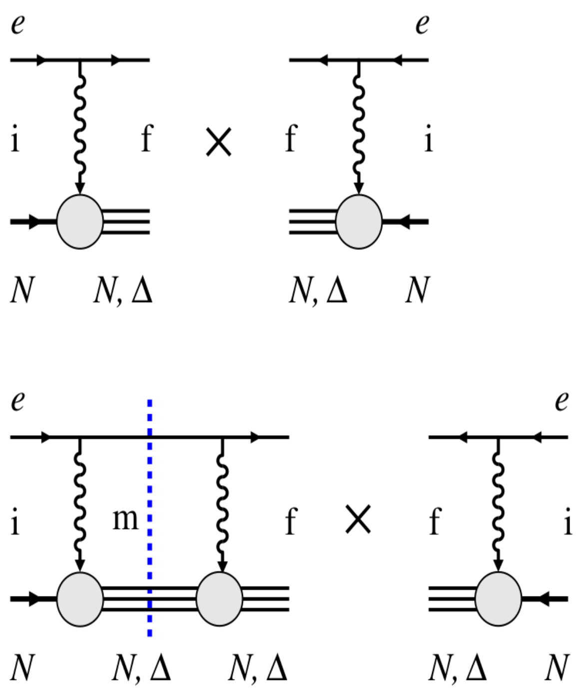
$$A_N = d\sigma_N / d\sigma_U$$

Cases with more momenta (eg semi-inclusive scattering) will present more combinations of this type (e.g. for accessing Sivers function)

QED+QCD: **T reversal** sym forbid tree level SSAs (Barut-Fronsdal (1960))

SM: has CP violation = T violation: tree level SSA possible but too small to be observable (Christ-Lee (1966))

T reversal symmetry allows SSA in the presence of absorptive part of scattering amplitude (Barut-Fronsdal (1960))



Absorptive part of amplitude results naturally from **FSI**s

In **inclusive case**, the FSI is EM: the absorptive part then starts at order $\mathbf{e^4}$

Driven by **TPE**: absorptive part of a box diagram

- Only states with invariant mass below $\sqrt{s}$ can contribute
- $d\sigma_N$ **free of collinear and IR divergencies**: key role played by gauge invariance
- NSSA can be selected for exclusive and inclusive final states
- In principle electroproduction helicity amplitudes determine **elastic TNSSA** – limited by range of experimentally established amplitudes. Not possible in inelastic or inclusive cases!.
- **Energy regimes**:

  1) low/intermediate up to onset of second resonance region (this work)

  2) resonance region: 1.5 to few GeV

  3) High energy region: e.g., partonic regime

1/Nc EXPANSION

- fundamental expansion of QCD: many phenomenological successes
- additional recent tests in LQCD with Nc>3
- expansion can be implemented at hadronic level
- particularly powerful for baryons
- for large Nc baryon sector develops dynamical spin-flavor SU(4) or SU(6) symmetry
- N and Δ belong to a SU(4) multiplet $\Rightarrow$ unification of their properties
- breaking of SU(4) by subleading orders in 1/Nc: well established formalism
- Nc scaling of basic observables:

$$m_{N,\Delta} = O(N_c) \qquad m_\Delta - m_N = O(1/N_c)$$

$$\Gamma_\Delta = O(1/N_c^2) \qquad g_{\pi N} = O(\sqrt{N_c})$$

$$F_\pi = O(\sqrt{N_c})$$

Kinematic ranges for SSA and ordering in 1/Nc

	Energy regime	$1/N_c$ expansion regime	Channels open	Final states possible
I	$m_N < \sqrt{s} < m_\Delta$	$\sqrt{s} - m_N \sim N_c^{-1}, \quad k \sim N_c^{-1}$	N	elastic
II	$m_\Delta < \sqrt{s} \ll m_{N*}$	$\sqrt{s} - m_N \sim N_c^{-1}, \quad k \sim N_c^{-1}$	N, Δ	elastic or inelastic
III	$m_\Delta < \sqrt{s} \lesssim m_{N*}$	$\sqrt{s} - m_N \sim N_c^{0}, \quad k \sim N_c^{0}$	N, Δ, N^*(suppr)	elastic or inelastic

SSA calculation will cover the three regimes

Expansions

I & II - Low energy combined with $1/N_c$:

$O(p) = O(1/N_c) = O(\xi)$: BChPT x $1/N_c$

III - $1/N_c$ expansion only

NR expansion $(1/m_N)$ is part of $1/\mathrm{Nc}$ expansion

SYSTEMATIC EXPANSION IN 1/Nc: LO AND NLO FOR NSSA

In regime of interest 1/Nc expansion implies NR expansion:
baryons have small velocities O(1/Nc) in CM frame

Expand the Baryon EM current to first subleading order in 1/Nc: O(Nc) & O(Nc^0)

$$J_{EM\,\mu} = \frac{1}{2} J^S_\mu + J^V_\mu$$

$$J^S_\mu = g_{\mu 0} G^S_E(t) - \frac{i}{\Lambda} G^S_M(t) \epsilon_{0\mu ij} q^i \hat{S}^j$$

$$J^V_\mu = g_{\mu 0} G^V_E(t) \hat{I}^3 - \frac{i}{\Lambda} \frac{6}{5} G^V_M(t) \epsilon_{0\mu ij} q^i \hat{G}^{j3}$$

$$\{\hat{S}^i, \hat{I}^a, \hat{G}^{ia}\} \quad \text{generators of SU(4)}$$

$$\Lambda = m_N^{\text{phys}} :$$

$$G^S_E = G^V_E = G^p_E$$

$$G^S_M = G^p_M + G^n_M$$

$$G^V_M = G^p_M - G^n_M$$

G_{ia} spin-flavor generators of SU(4)- connect baryons of different spin

$$< B' | G_{ia} | B >= O(N_c)$$

Hierarchy of magnetic moments determined by Nc

EM current has O(Nc) term: isotriplet magnetic

Nc hierarchy in isosinglet vs isotriplet N magnetic moments

$$\mu_0 = \frac{1}{2}(\mu_p + \mu_n) = 0.44\mu_N; \quad \mu_3 = \frac{1}{2}(\mu_p - \mu_n) = 2.35\mu_N$$

$$\mathcal{O}(N_c^0) \qquad\qquad \mathcal{O}(N_c)$$

Subleading terms in current

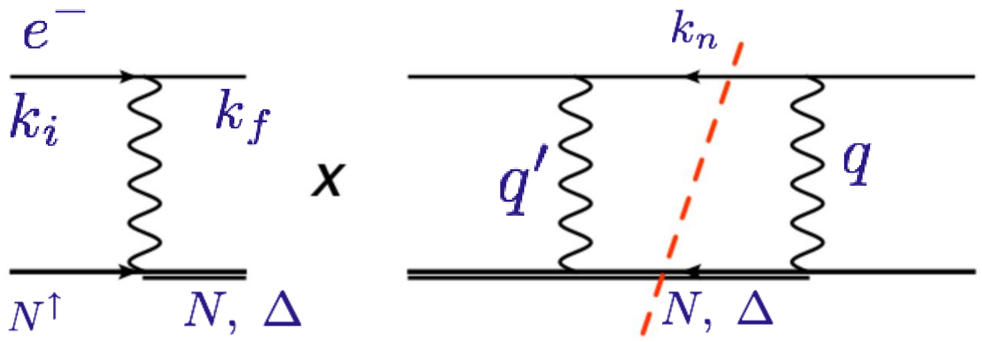
convection current: $(\vec{p}_i + \vec{p}_f)/2m_N = O(1/N_c)$

electric quadrupole current: $O(1/N_c^2)$

neglected term: $\frac{1}{m_N \Lambda} g^{\mu 0} \epsilon^{0ijk} q^i (p_i + p_f)^j G^{ka} = O(N_c^0)$

$$= O(\xi^3) \text{ for } q = O(1/N_c)$$

CALCULATION OF THE TARGET NSSA



$$e_N^{\mu} a_{\mu} \frac{d\sigma_{N_{fi,n}}}{d\Omega} = \frac{\alpha^3}{16\pi} \frac{k_f}{k_i} \frac{m_N m_{B_f} m_n}{t s^{3/2} k_i k_f k_n} \operatorname{Im}\Big(\int d\Omega_{\hat{\mathbf{k}}_\mathbf{n}} \frac{L_{\mu\nu\rho}(k_\mathrm{i}, k_\mathrm{f}, k_n) H^{\mu\nu\rho}_{\mathrm{fi,n}}(k_\mathrm{i}, k_\mathrm{f}, k_n)}{(1 - \hat{\mathbf{k}}_\mathrm{i} \cdot \hat{\mathbf{k}}_\mathbf{n})(\mathbf{1} - \hat{\mathbf{k}}_\mathrm{f} \cdot \hat{\mathbf{k}}_\mathbf{n})} \Big)$$

Leptonic tensor

$$L^{\mu\nu\rho}(k_\mathrm{i}, k_\mathrm{f}, k_n) = Tr(\not{k}_\mathrm{i} \gamma^\mu \not{k}_\mathrm{f} \gamma^\nu \not{k}_n \gamma^\rho)$$

Hadronic tensor

$$\begin{aligned} H^{\mu\nu\rho}_{\mathrm{fi},n}(k_\mathrm{i}, k_\mathrm{f}, k_n) = {} & \langle B_\mathrm{i} \mid (J^\mu_{\mathrm{EM}}(k_\mathrm{i} - k_\mathrm{f}))^\dagger \mid B_\mathrm{f} \rangle \\ & \times \langle B_\mathrm{f} \mid J^\nu_{\mathrm{EM}}(k_n - k_\mathrm{f}) \mathcal{P}_n J^\rho_{\mathrm{EM}}(k_\mathrm{i} - k_n) \mid B_\mathrm{i} \rangle \end{aligned}$$

- SSA needs t-channel J=1 components of hadronic tensor, I=0, 1
- Box has t-channel J_{Box}=0,1 for B_f=N, and J_{Box}=1,2 for B_f=Δ
- Large Nc limit: only isotriplet spin current contributes: LO given by

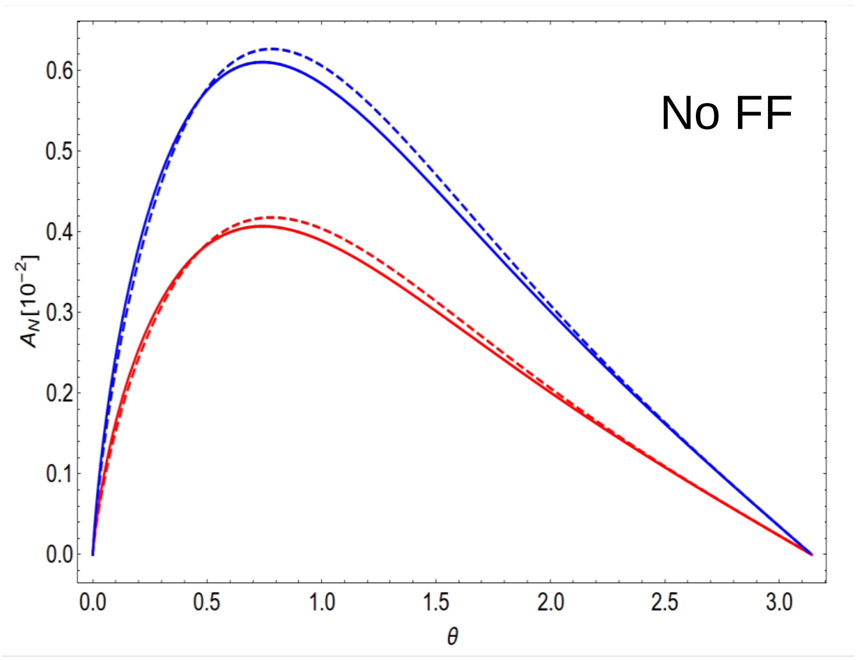
  J=I and $J_{Box}=I_{Box}$ (I=J rule of large Nc)
- Subleading corrections: I≠J with LO current and subleading currents
- Q^2 range up to $4E_e^2$ - need to include FFs in calculation: common dipole form for E and M components
- Integrals of box have IR and/or collinear divergencies: they cancel for each gauge invariant combination of terms of the hadronic current, for each J_{Box} and I_{Box} , and for different B_f and B_n – key check for calculation
- Calculation checked with known purely elastic case calculated relativistically and by taking the corresponding limits

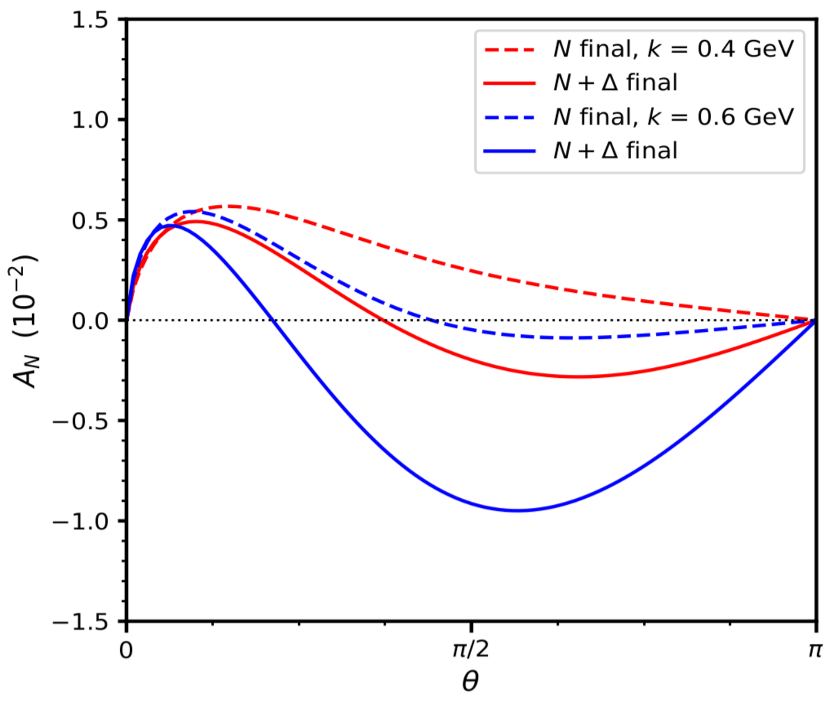
FIRST CALCULATION: LO IN 1/Nc – LARGE Nc LIMIT

JLG, Weiss, Willemyns
Phys Lett B835 (2022) 137580

- given in terms of the isovector magnetic current
- match magnetic coupling at Nc=3 to physical one
- pick LO terms in hadronic tensor: LO given by I=J t-channel rule
- simple results without t-dependence of FF – shows dominance of the elastic SSA
- with intermediate state contribution of N equal ½ of the Δ
- Inclusion of FF makes major difference: inelastic channel, Bf=Δ, becomes

large and of opposite sign to elastic – very significant FF interplay in box!





NLO CALCULATION

JLG, Weiss, Willemyns,
Phys Rev D 107 (2023) 9, 094026

- decompose hadronic tensor into t-channel irreducible angular momentum and isospin components: SU(4) algebra organizes contributions in powers of 1/Nc
- separate effects by N and Δ channels in the box and final state
- check cancellations of IR and collinear divergences for each channel and for gauge invariant combinations of EM current components
- validated with NR expansion of the known purely elastic case
- calculation without t-dependence in FFs
- inclusion of FFs and Δ width

Results without FFs

$$\frac{d\sigma_N}{d\Omega}(N_{\rm i},N,N) = \frac{1}{4}k^2 m_N^2\Big(k(G_M^{I_3}(N_c+7)-G_M^{-I_3}(N_c-3))-10G_E^{I_3}\Lambda\Big)$$
$$\times\Big(G_M^{I_3}(N_c+7)-G_M^{-I_3}(N_c-3)\Big)^2\Big((3+x)\log\frac{1-x}{2}-2(1+x)\Big)\,\mathbf{X}$$

$$\frac{d\sigma_N}{d\Omega}(N_{\rm i},N,\Delta) = \theta(k_\Delta)k_\Delta m_\Delta m_N^3(N_c+5)(N_c-1)(G_M^p-G_M^n)^2$$
$$\times\Big(k(G_M^{I_3}(N_c+7)-G_M^{-I_3}(N_c-3))+5G_E^{I_3}\Lambda\Big)$$
$$\times\Big(\Big(\frac{\mathbf{Y}}{2m_N^4}-\frac{k}{m_N^2}(1+x)\Big)\log\frac{1-x}{2}-\frac{k^2}{\mathbf{Y}}(1+x)\Big)\,\mathbf{X}$$

$$\frac{d\sigma_N}{d\Omega}(N_{\rm i},\Delta,N) = \theta(k_\Delta)\frac{1}{32}\frac{m_\Delta}{m_N}(N_c+5)(N_c-1)(G_M^p-G_M^n)^2\mathbf{Y}$$
$$\times\Big((1+x)\Big((G_M^{I_3}(N_c+7)-G_M^{-I_3}(N_c-3))(11k^2-kk_\Delta+4k_\Delta^2)+4G_E^{I_3}(k-k_\Delta)\Lambda\Big)$$
$$+2\Big(\Big(G_M^{I_3}(N_c+7)-G_M^{-I_3}(N_c-3)\Big)(2k^2+2(2-x)k_\Delta^2+3(1-x)kk_\Delta)$$
$$+20G_E^{I_3}(2k-(1+x)k_\Delta)\Lambda\Big)\log\frac{1-x}{2}\Big)\mathbf{X}$$

$$\frac{d\sigma_N}{d\Omega}(N_{\rm i},\Delta,\Delta) = \theta(k_\Delta)\frac{1}{160}\frac{k_\Delta m_\Delta^2}{k\;m_N^2}(N_c+5)(N_c-1)\mathbf{Y}(G_M^p-G_M^n)^2$$
$$\times\Big((1+x)\Big((23k_\Delta-9k)(G_M^{I_3}(N_c+27)-G_M^{-I_3}(N_c-23))+200G_E^{I_3}(k-k_\Delta)\Lambda\Big)$$
$$+2\Big((G_M^{I_3}(N_c+27)-G_M^{-I_3}(N_c-23))(6(k^2+k_\Delta^2)-(3+5x)kk_\Delta)$$
$$+100G_E^{I_3}((1+x)k-2k_\Delta)\Lambda\Big)\log\frac{1-x}{2}\Big)\,\mathbf{X}$$

$$x=\cos\theta$$

$$\frac{1-x}{2}=\sin^2\frac{\theta}{2}$$

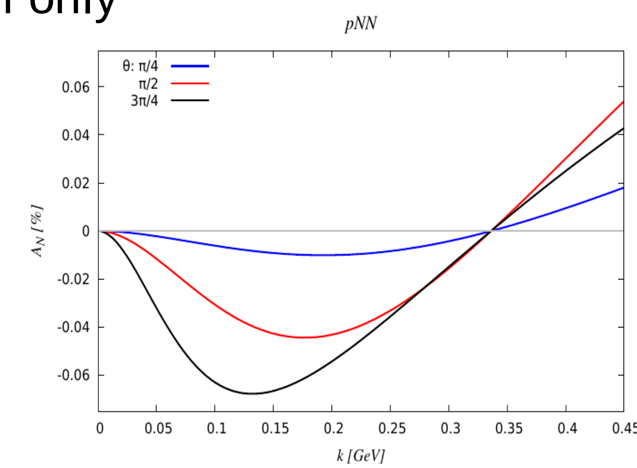
$$\mathbf{X}=\frac{m_N\alpha^3}{1000\;s^{3/2}t(1+x)\Lambda^3}\hat{\mathbf{k}}_{\rm f}\times\hat{\mathbf{k}}_{\rm i}\cdot\mathbf{S}$$
$$\mathbf{Y}=\sqrt{k^2+m_N^2}\;(m_N^2-m_\Delta^2)+k\,(m_N^2+m_\Delta^2)$$

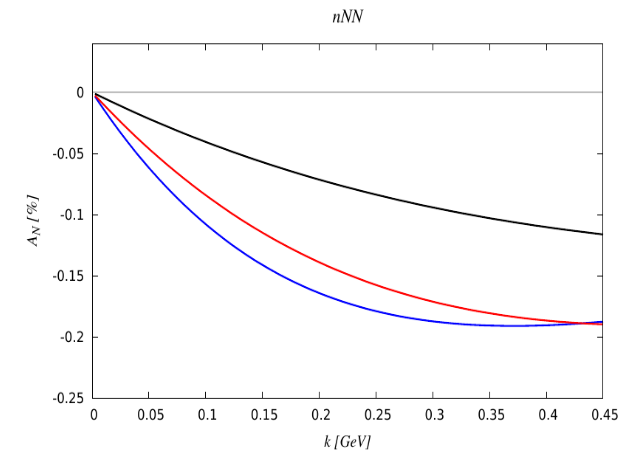
$$A_N = \frac{d\sigma_N}{d\sigma_{U\,\mathrm{elastic}}}$$

no FF, vanishing Γ_Δ

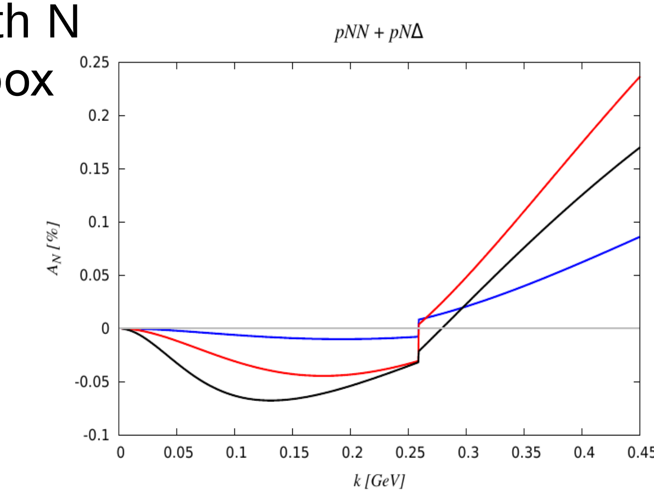
- Electric component of current only appears linearly
- Large Nc limit checked
- Dominance of elastic asymmetry
- Very small inelastic asymmetry
- Kinematics taken exactly
- Threshold enhancement at Δ mass
- Δ in box gives important contribution ~ twice as N

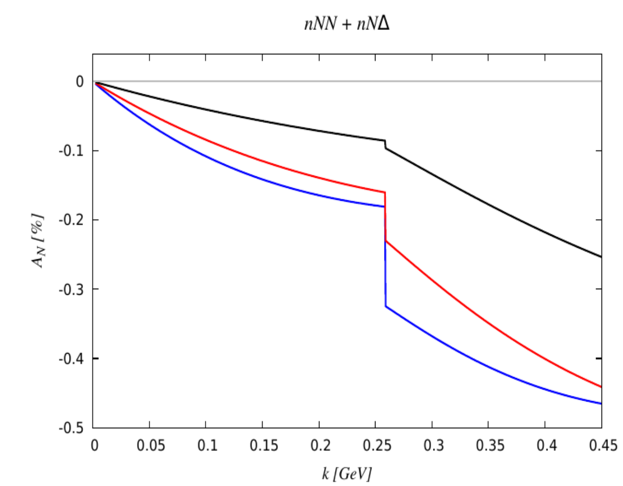
Elastic with only N in box



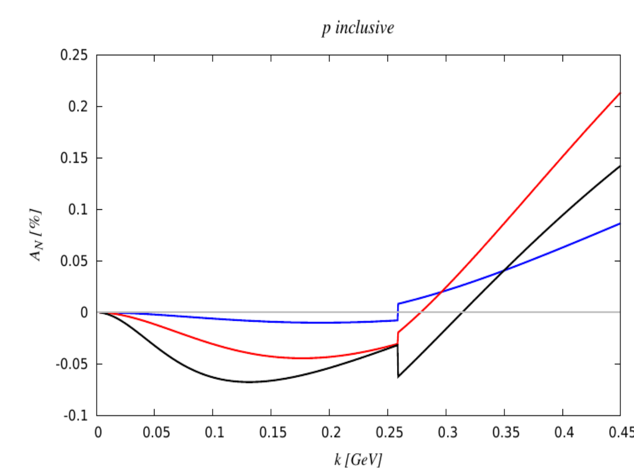


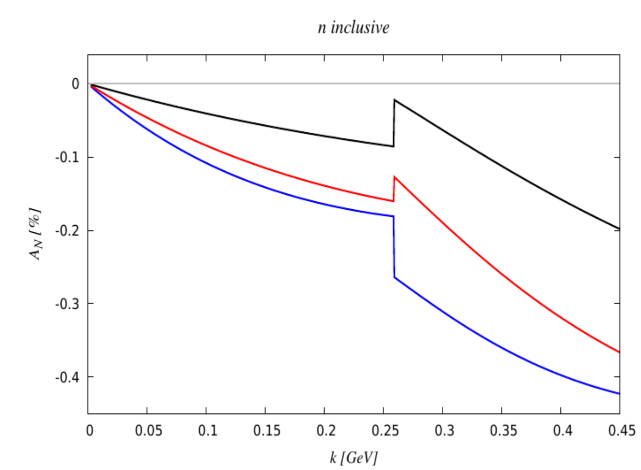
Elastic with N and Δ in box





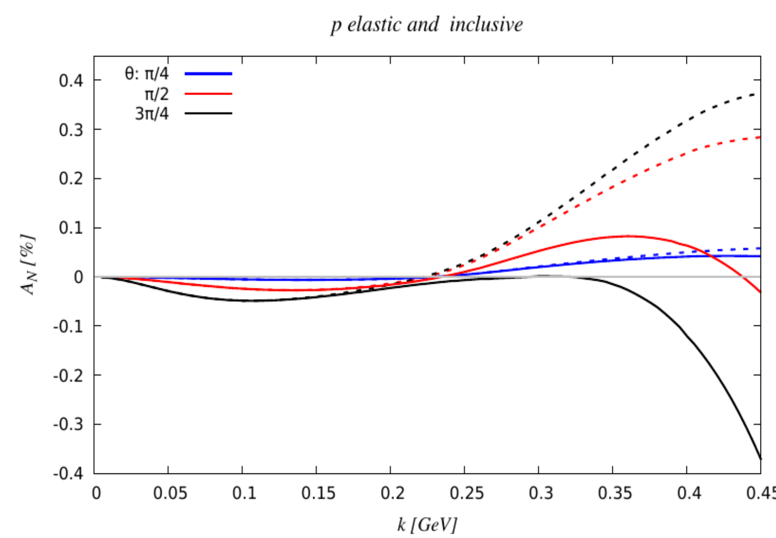
Inclusive

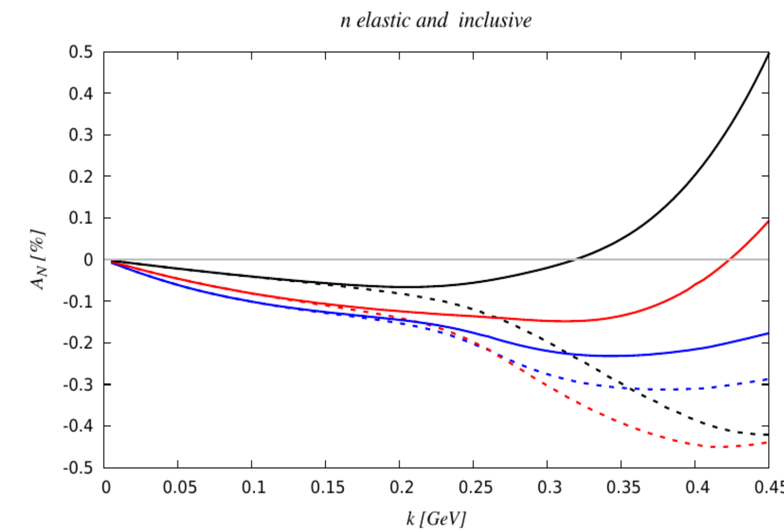


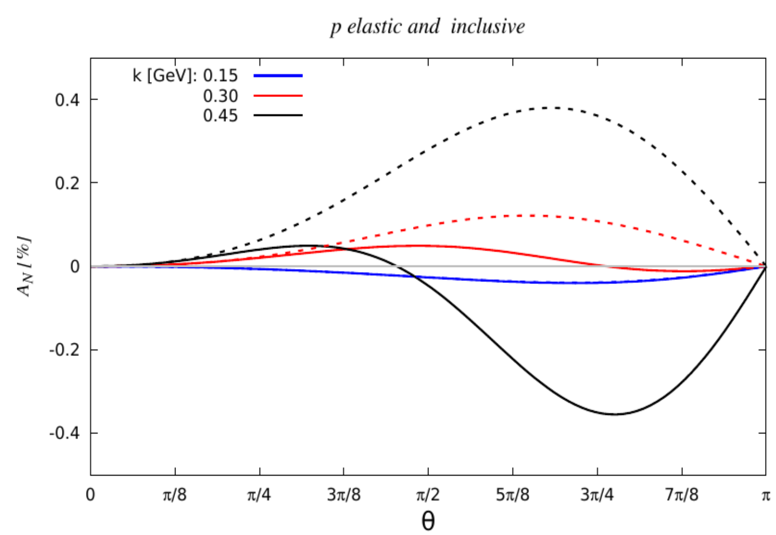


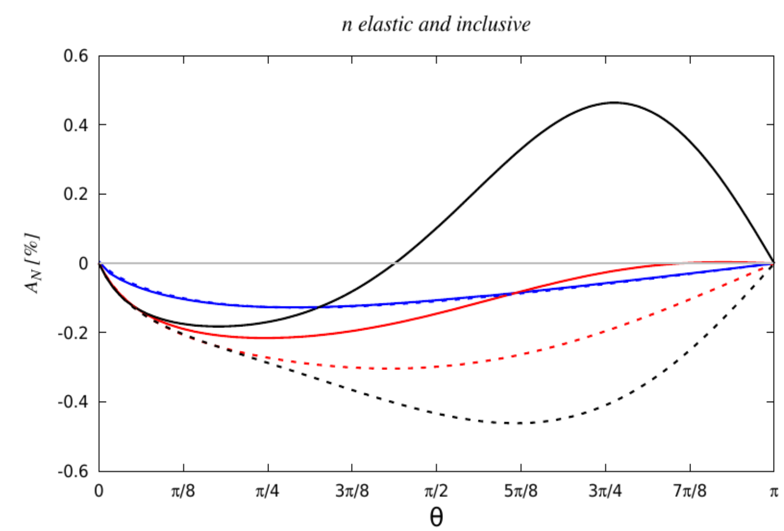
$\mathrm{A_N}$ with common dipole FF and $\Gamma_\Delta = 125\mathrm{MeV}$

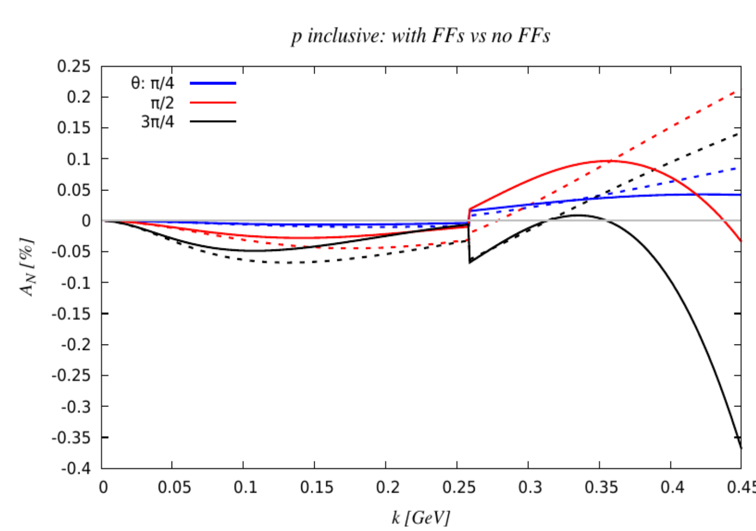
- FFs play crucial role – enhances inelastic channel
- inelastic channel gives opposite sign asymmetry to elastic one: changes sign of inclusive asymmetry
- sensitive energy dependence of asymmetry – we keep kinematic factors exact





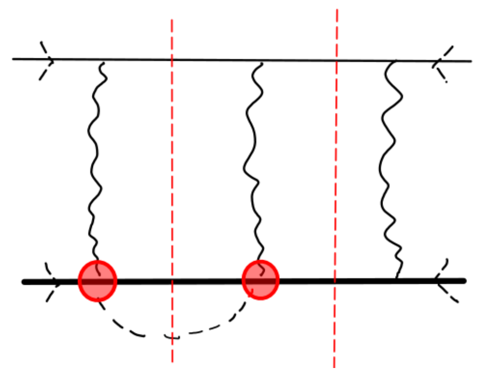






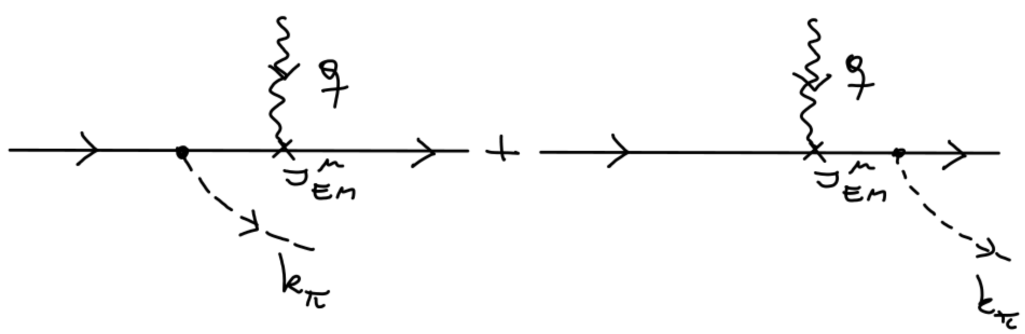
Other TSSA contributions

- πN Continuum



$$\pi \text{ baryon coupling:} \;\; \propto \frac{g_A}{F_\pi} k_\pi^i G^{ia} = O(\sqrt{N_c})$$

$$[G^{ia}, G^{ib}] = O(N_c^0); \;\; [G^{ia}, S^i] = O(N_c); \;\; [G^{ia}, I^b] = O(N_c)$$



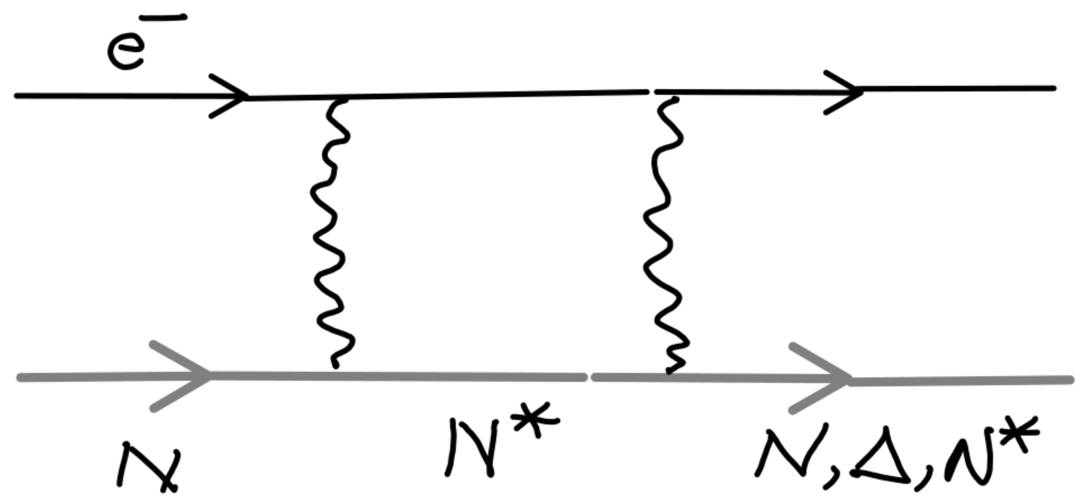
$$\propto \frac{g_A}{F_\pi} k_\pi^i \left[G^{ia}, J^\mu_{EM}\right]$$

$$\mathrm{O}(1/N_c) \text{ wrt LO, and } \mathrm{O}(\xi^3) \text{ wrt LO term in } \xi \text{ expansion domain}$$

- N* resonances

EM N-N* couplings carry extra factor $1/\sqrt{N_c}$

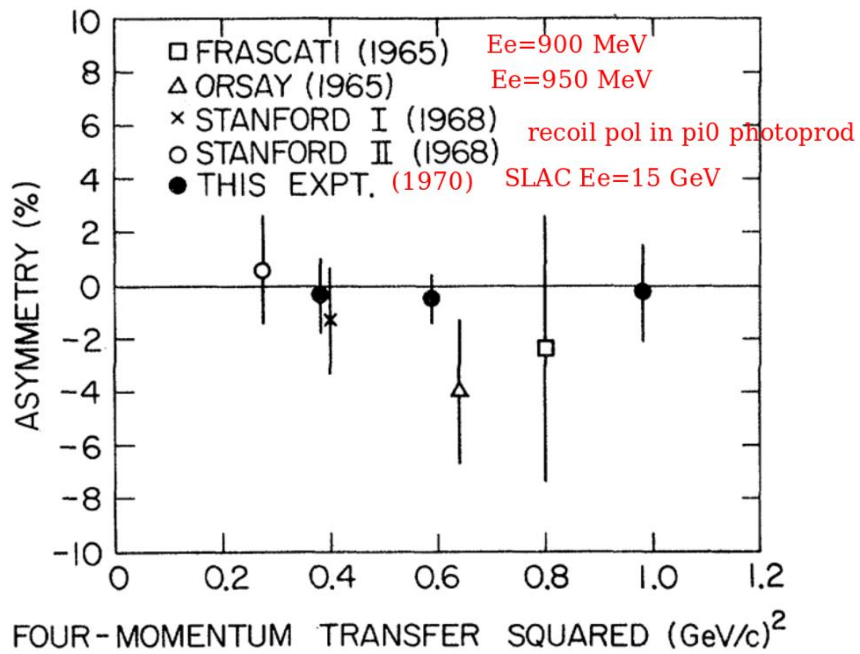
Individual resonances contribute $\mathrm{O}(1/N_c)$ wrt N and Δ

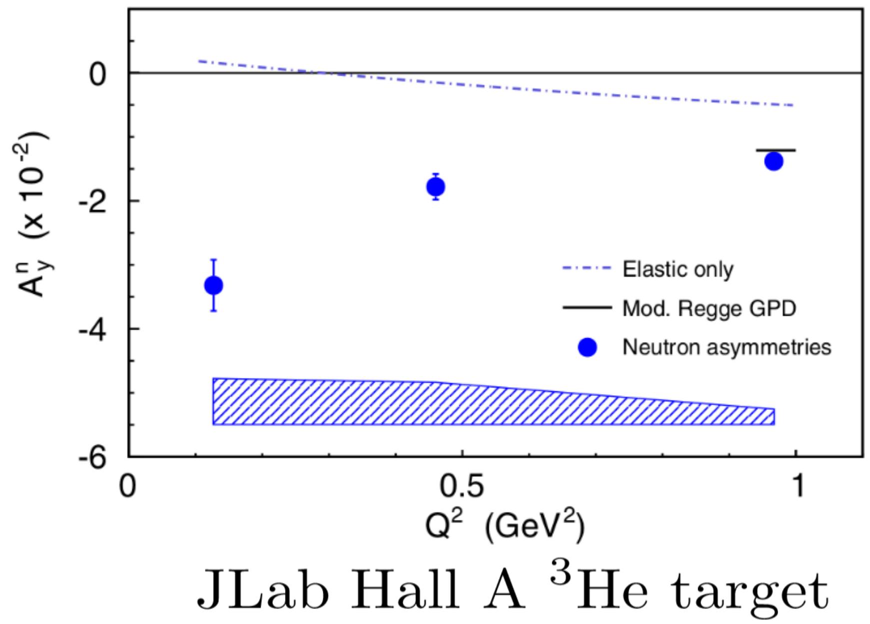


Case of a few resonances was studied (Vanderhaeghen et al) - still a lot to be done

New developments with the beam SSA (previous talk by Peter Blunden)

THE EXPERIMENTAL SIDE





- Very old expts at relatively low energy - errors too large, no definite signal of asymmetry
- JLab Hall A SIDIS (^{3}He target): E_e=1.2, 2.4 & 3.6 GeV - energy a bit high for direct comparison consistent with magnitude – more complicated target
- HERMES on proton: SIDIS with e- and e+ beam – E_e=27.6 GeV
- new proposal for JLab Hall A (Grauvogel et al): E_e>2.2 GeV (may use e+)
- only e- beam energy in region of our interest is MAMI @ Mainz: A4 experiment

  measures normal beam SSAs on various targets
- important evolution of target NSSA from low to intermediate energies: first significant measurements would be very important and interesting!

COMMENTS & SUMMARY

- SSAs are important tool to study baryon structure in all energy regimes
- 1/Nc expansion provides one systematic approach in the energy range below second resonance region with N and Δ as effective dof
- it organizes the contributions by ordering in powers of 1/Nc; helps sort out the physics in more detail
- LO qualitatively OK, but NLO is important to describe the transition from purely elastic to inclusive
- very important role played by form factors: big enhancement of Δ final state asymmetry - changes of inclusive asymmetry
- The resonance region needs further theoretical exploration – interesting and important problem
- Lack of experimental results from low energy to onset of partonic regime: need for experiments eN from ~.3 to few GeV -
- SSAs can provide important insights on spin physics in baryon resonances, an area still open for exploration by experiment and theory