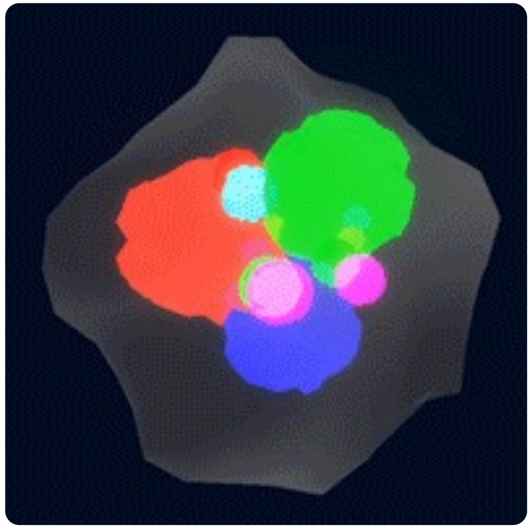
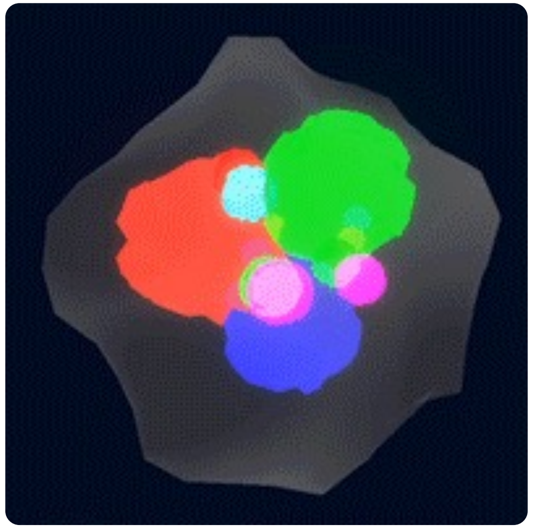
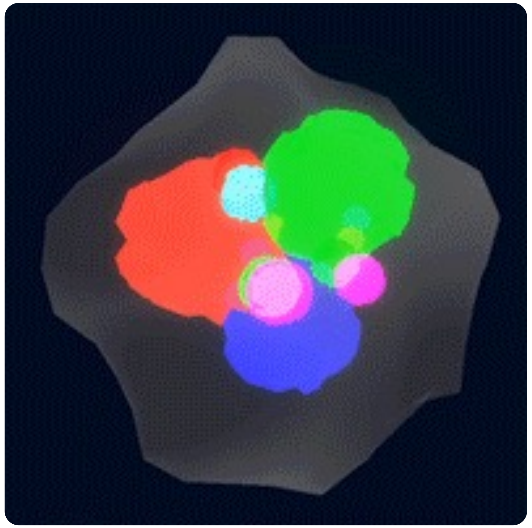




The Correlated Spatial Structure of the Proton

Two-body densities as a framework for dynamical imaging

Presenter: Zaki Panjsheeri*

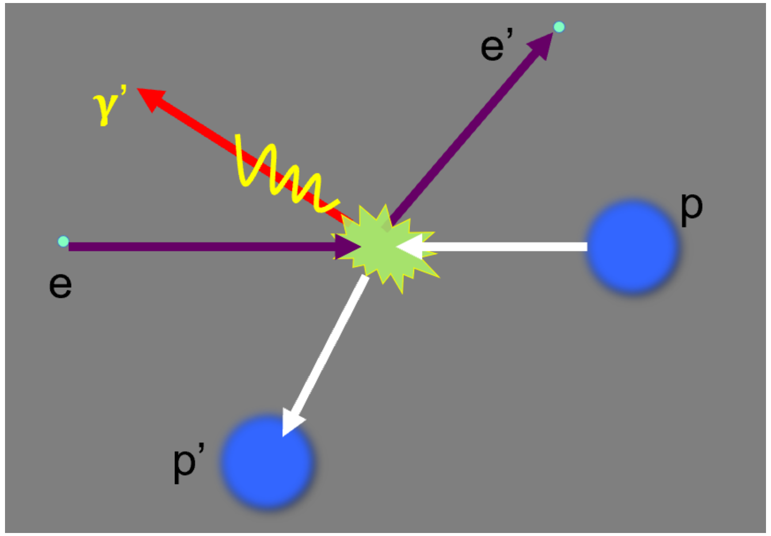
Joshua Bautista*, Simonetta Liuti*, Brandon Kriesten**

*University of Virginia

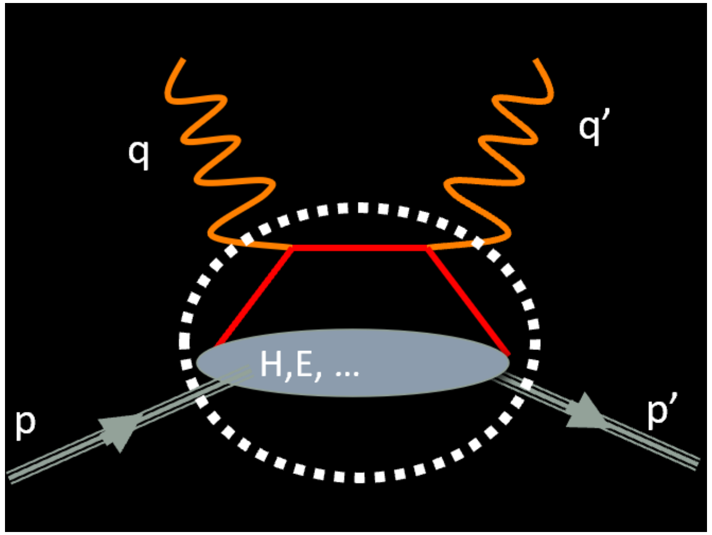
**Argonne National Laboratory

Generalized parton distributions (GPDs)

- Nuclear femtography has the GPDs at its disposal, which appear in the DVCS cross section through the Compton Form Factors (CFFs)



X.D. Ji, “Gauge-Invariant Decomposition of Nucleon Spin,” Phys. Rev. Lett. 78 (1997), 610-613



Figures: Simonetta Liuti, “Hadron Ion Tea (HIT@LBL) seminar” (2021).

3D Coordinate Space Representation

- The GPDs, through Fourier transform, give us spatial information on the charge, matter, and radial distributions of the quarks and gluons inside the nucleon
- The Fourier transform of GPD H^q gives the one-body parton density distribution in b_T

$$H_q(x,0,t) = \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p' \mid \bar{\psi}(0)\,\gamma^+\,\psi(z) \mid p\rangle\big|_{z^+=\mathbf{z}_T=0}$$

Definition of the GPD

$$\mathcal{H}^q(X,0,b_T) \;=\; \int \frac{d^2\Delta_T}{(2\pi)^2} H^q(X,0,\Delta_T)e^{-i\Delta_T\cdot b_T}$$

Fourier transform of the GPD

$$H_q(x,0,t) = \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p' \mid \bar{\psi}(0)\,\gamma^+\,\psi(z) \mid p\rangle\big|_{z^+=\mathbf{z}_T=0}$$

Unpolarized parton correlation function; trivial gauge link

$$H_q(x,0,t) = \int \frac{dz^-}{2\pi}\, e^{ixp^+z^-} \sum_X \langle p' \mid \bar{\psi}_+(0) \mid X\rangle\langle X \mid \psi_+(z) \mid p\rangle\big|_{z^+=\mathbf{z}_T=0}$$

Having projected out the "good components," introducing a complete set of states, and applying translational invariance

$$\begin{aligned} H_q(x,0,t) &= \int d^2\mathbf{k}_X dk_X^+\, \delta(k_X^+ - (1-x)P^+)\, \langle p' \mid \bar{\psi}_+(0) \mid X\rangle\, \langle X \mid \psi_+(0) \mid p\rangle \\ &= \int d^2\mathbf{k}\, \phi^*(x,\mathbf{k}-\boldsymbol{\Delta})\phi(x,\mathbf{k}), \end{aligned}$$

Having replaced the sum over final states X with an integral over the four momentum, k_X, of the final state

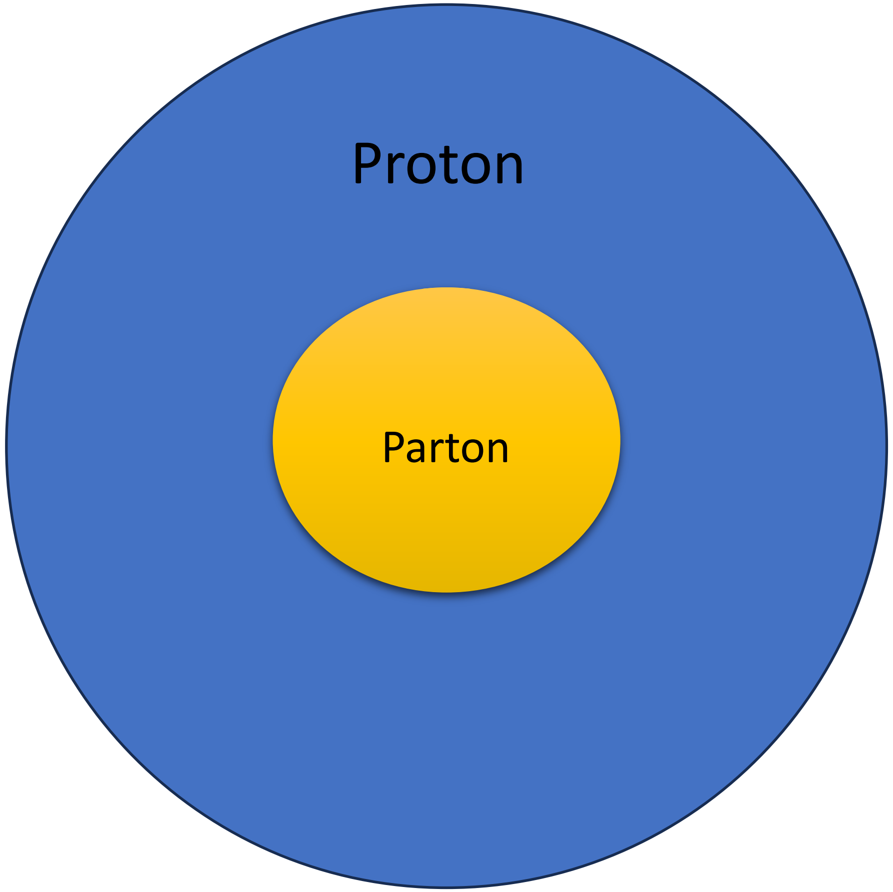
$$\phi(x,\mathbf{k}) = \langle p \mid \psi_+(0) \mid X\rangle.$$ Vertex function

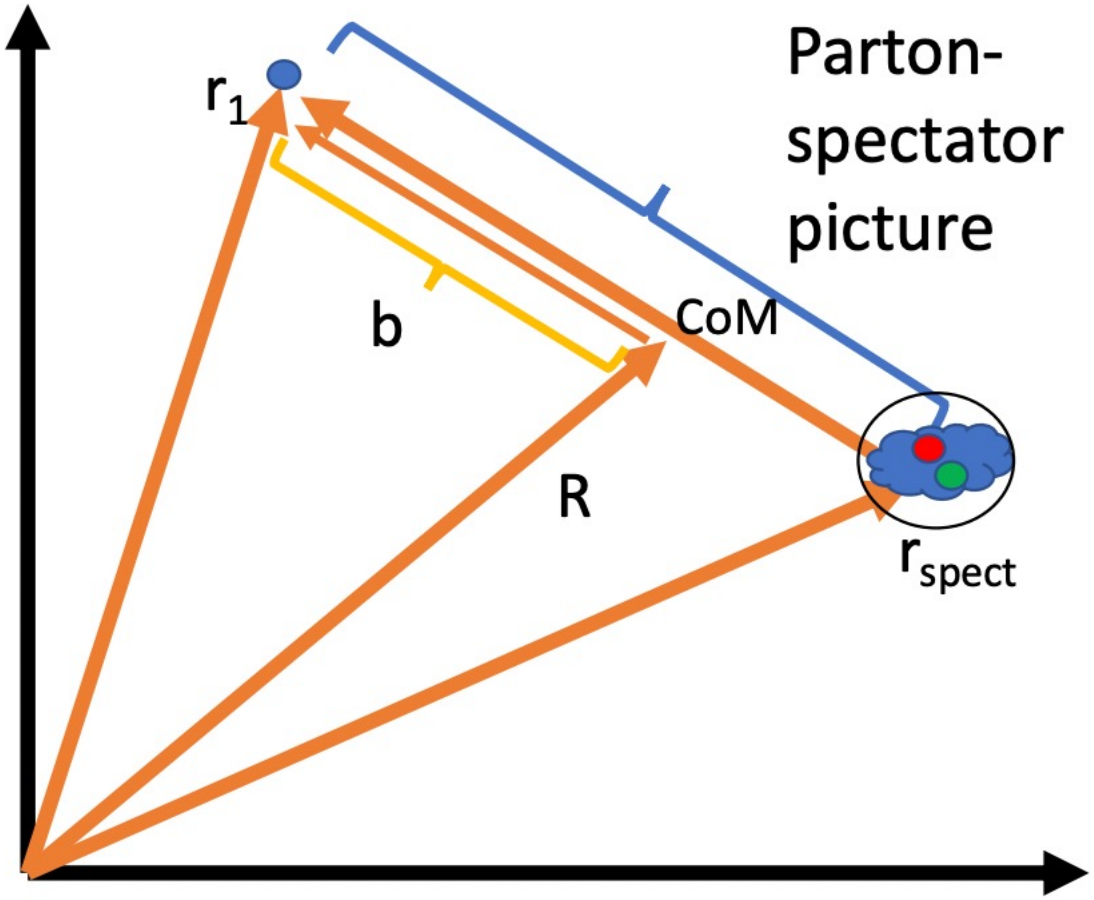
$$H_q(x,0,t) = \int d^2\mathbf{k} \int d^2\mathbf{z}_T\, d^2\mathbf{z}'_T\, e^{-i\mathbf{z}'_T\cdot(\mathbf{k}-\mathbf{\Delta})}\, e^{i\mathbf{z}_T\cdot\mathbf{k}}\, \tilde{\phi}^*(x,\mathbf{z}'_T)\, \tilde{\phi}(x,\mathbf{z}_T) =$$

$$= \int d^2\mathbf{k} \int d^2\mathbf{r}\, d^2\mathbf{b}\, e^{i\mathbf{r}\cdot\mathbf{k}}\;\; e^{i(\mathbf{b}-\mathbf{r}/2)\cdot\mathbf{\Delta}}\, \tilde{\phi}^*\left(x,\mathbf{b}-\frac{\mathbf{r}}{2}\right)\, \tilde{\phi}\left(x,\mathbf{b}+\frac{\mathbf{r}}{2}\right) = \int d^2\mathbf{b}\, e^{i\mathbf{b}\cdot\mathbf{\Delta}}\, \rho(x,\mathbf{b})$$

We obtain a one-body parton density distribution in the transverse plane, or the impact parameter dependent distribution (IPPDF)

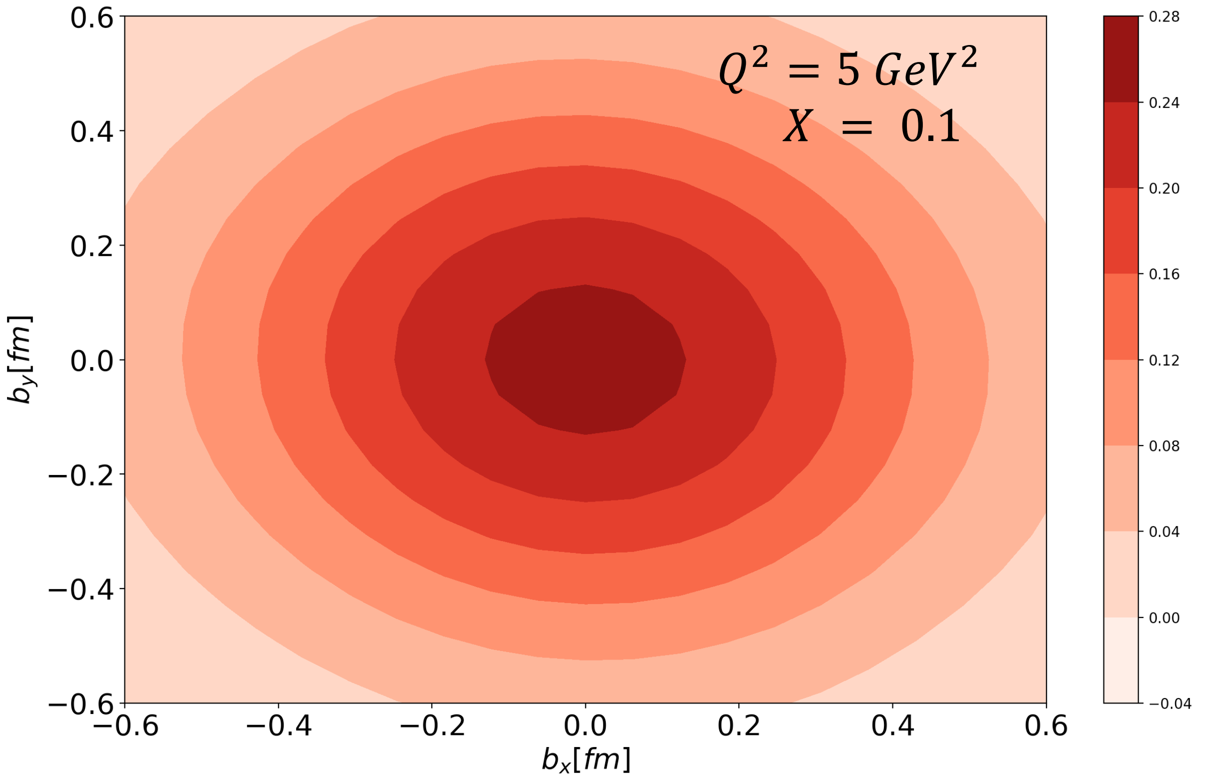
What do we see about the proton so far?





Hu distribution

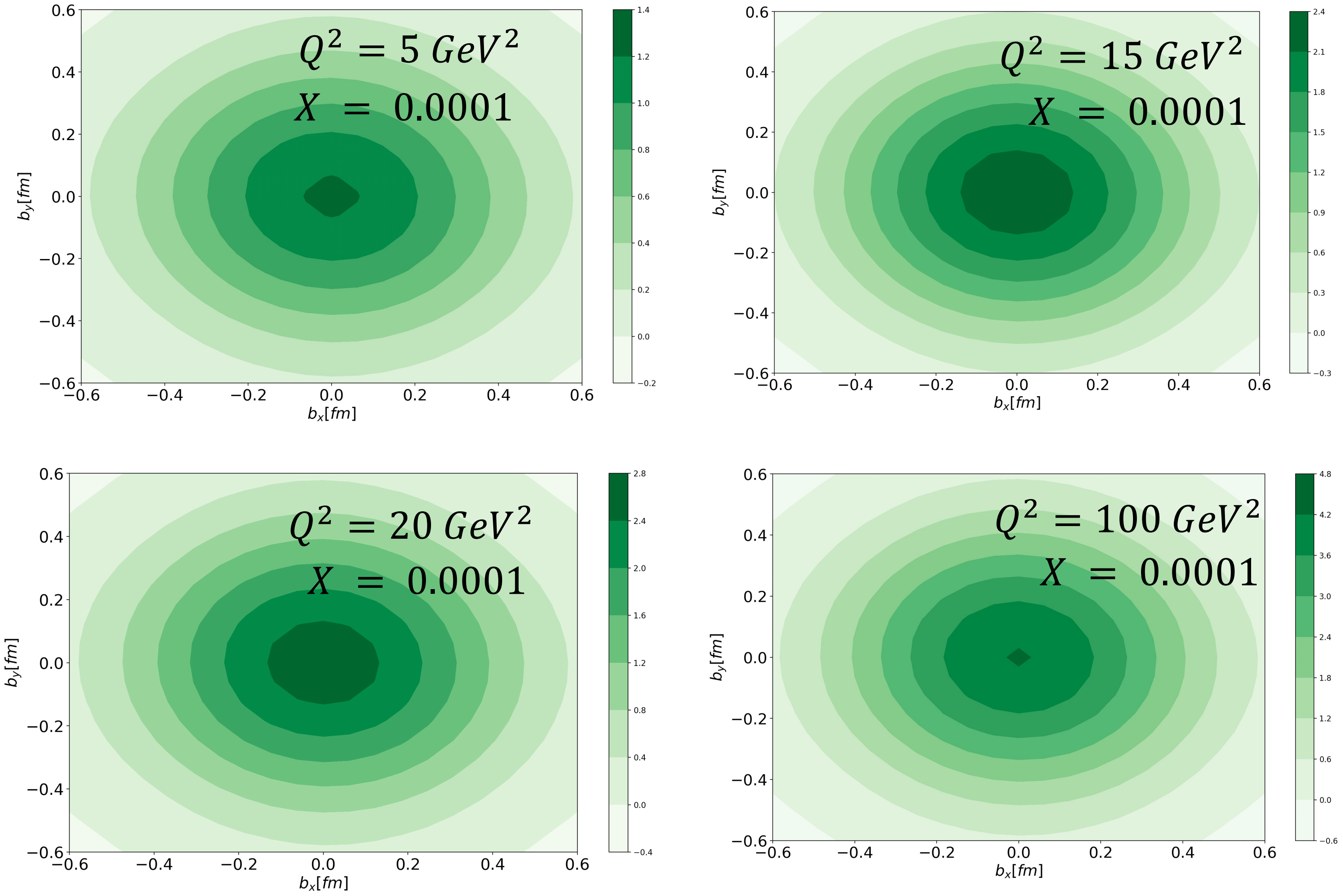
- The Fourier transform of the GPD Hu for fixed Q^2 at different values of X.
- We obtain these distributions by evolving and Fourier transform our parametrization, fitted to various data, within the spectator model.



$$H^{M_\Lambda}_{M_X,m} = 2\pi\mathcal{N}\left(1-\frac{\zeta}{2}\right)\int_0^\infty \frac{dk_\perp k_\perp}{1-X}\,\frac{a\left[(m+MX)(m+MX')+k_\perp^2\right]-b(1-X')k_\perp\Delta_\perp}{D^2\,(a^2-b^2)^{3/2}}$$

$$H_q^- = H_{q_v}(X,\zeta,t) = H^{M_\Lambda}_{M_X,m}(X,\zeta,t)\,R_p^{\alpha,\alpha'}(X,t)$$

B. Kriesten. P. Velie, E. Yeats, F. Y. Lopez, & S. Liuti, *Phys.Rev.D* 105 (2022) 5, 056022



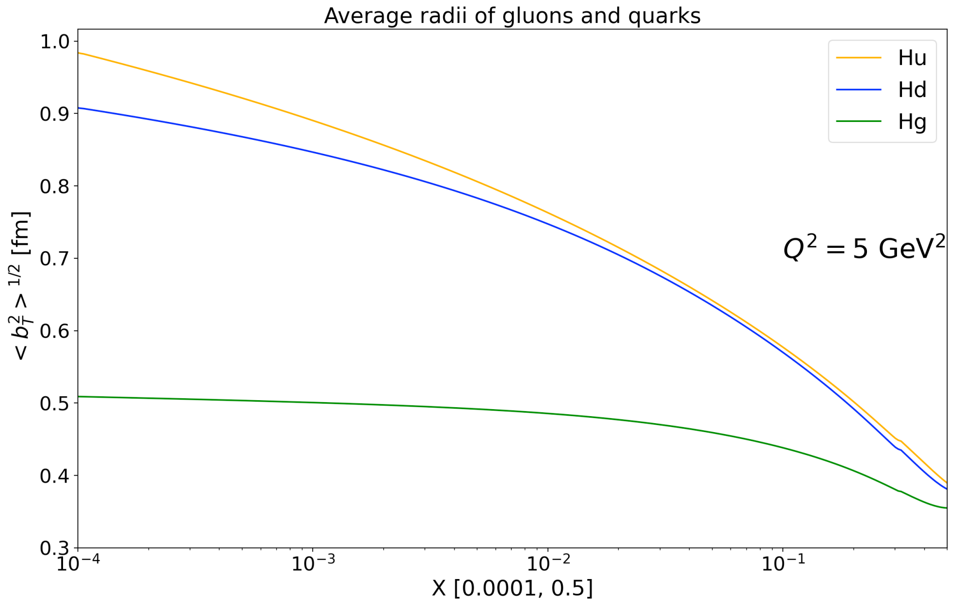
Gluon distribution

- Hg, which corresponds to the gluon momentum distribution.
- Fitted in Kriesten et. al. to lattice QCD moment calculations
- Varying values of Q^2

Average radii

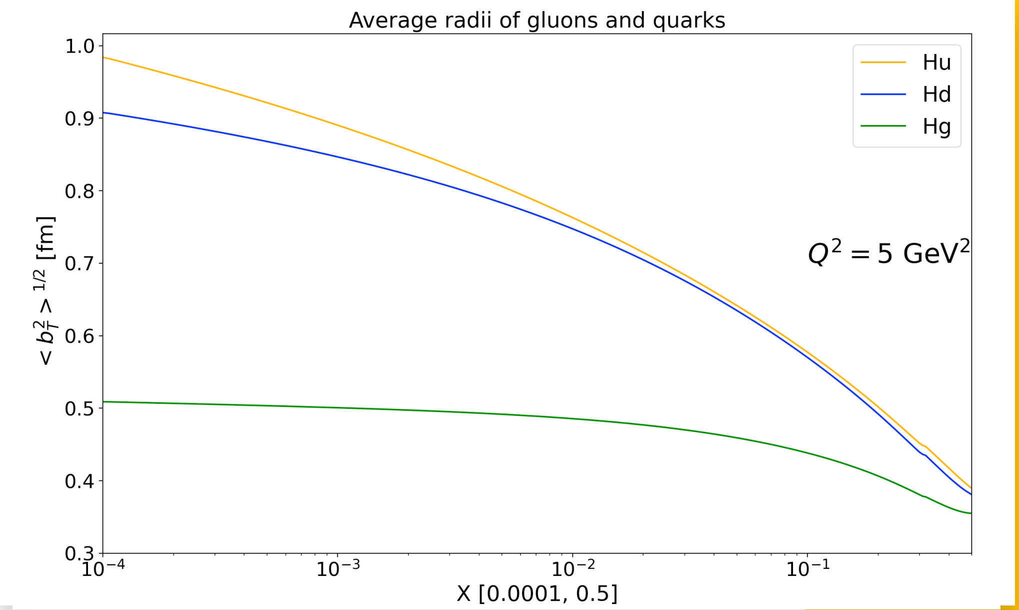
- Expectation value of the transverse impact parameter distance

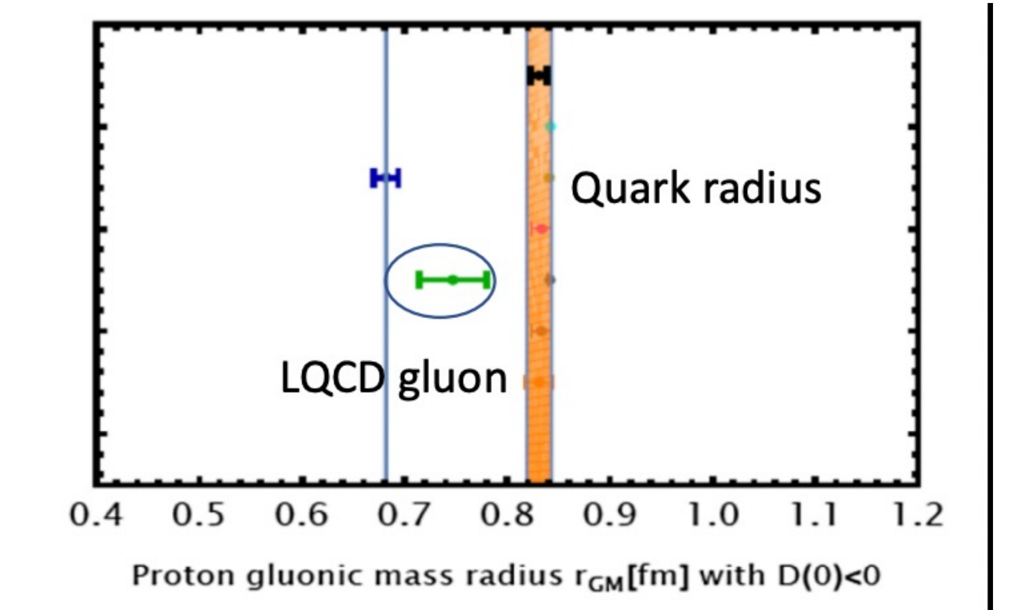
$$< b_T^2 >^q (X) = \frac{\int_0^\infty d^2 b_T b_T^2 \mathcal{H}^q(X,0,b_T)}{\int_0^\infty d^2 b_T \mathcal{H}^q(X,0,b_T)}$$



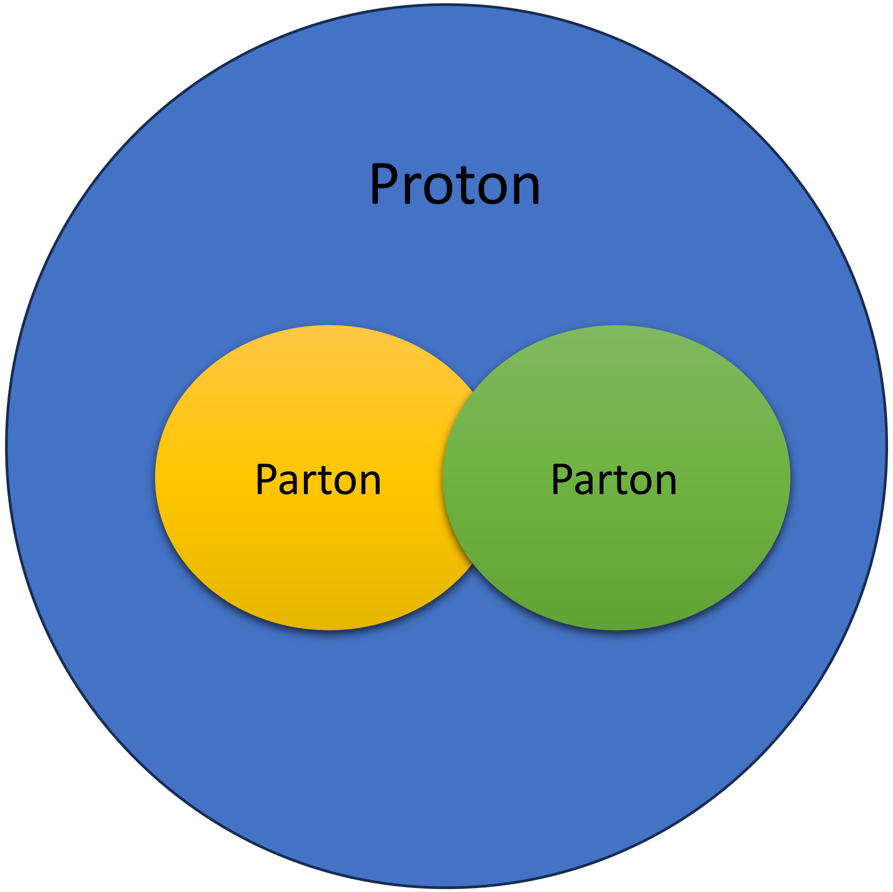
Average radii

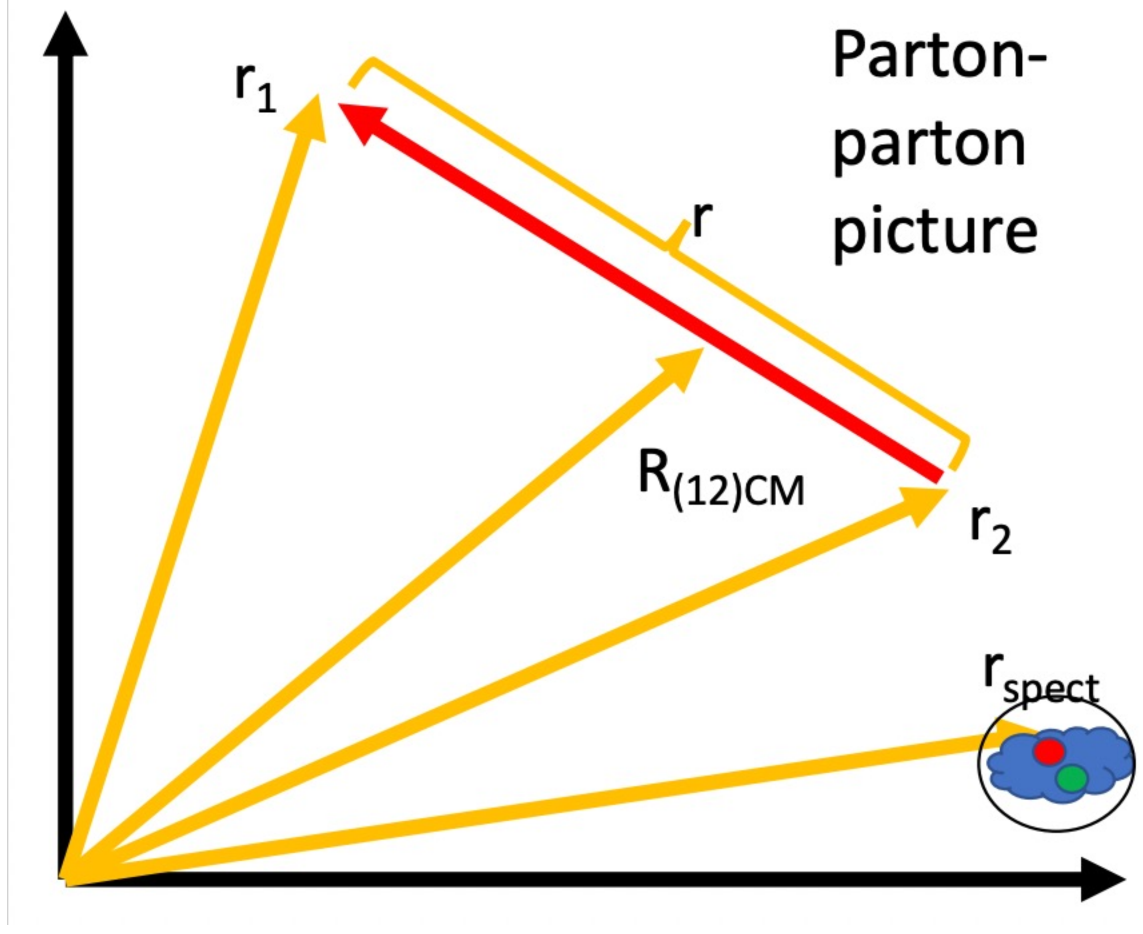
- Expectation value of the transverse impact parameter distance
- Compare to lattice and AdS/CFT results.
- Mamo and I. Zaeed PRD106, 086004 (2022)
- LQCD: Shanahan and Detmold Phys. Rev. Lett. 122, 072003 (2019)





Double-parton correlations





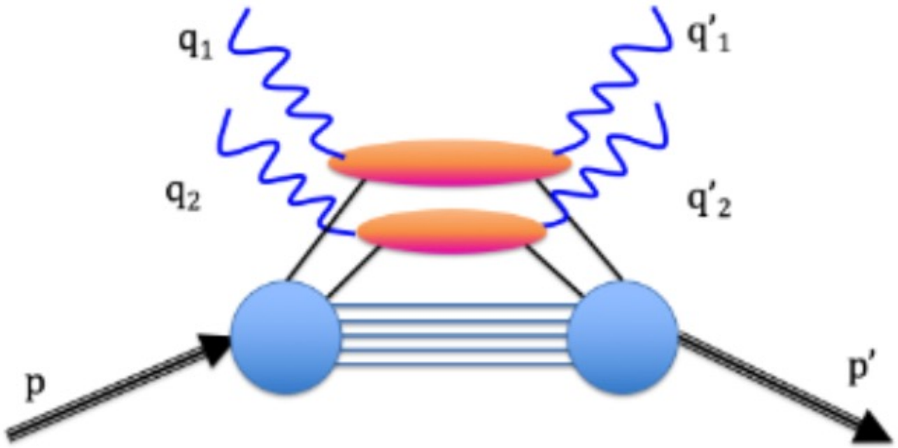
Double parton distributions (DPDs)

$$F_{ij}(x_1, x_2, \mathbf{k}_T) = \frac{p^+}{4} \int \frac{d\mathbf{y}_T}{(2\pi)^2} e^{-i\mathbf{k}_T \cdot \mathbf{y}_T} \int dz_1^- \int dz_2^- \, e^{ix_1 p^+ z_1^-} e^{ix_1 p^+ z_1^-}$$
$$\times \left\langle p, \Lambda \left| \bar{\psi}\left(-\frac{z_1}{2}\right) \gamma^+ \psi\left(y + \frac{z_1}{2}\right) \bar{\psi}\left(y - \frac{z_2}{2}\right) \gamma^+ \psi\left(y + \frac{z_2}{2}\right) \right| p, \Lambda \right\rangle \Bigg|_{z_1^+ = z_2^+ = y^+ = 0}^{\mathbf{z}_{1,T} = \mathbf{z}_{2,T} = 0}$$

DPD defined through its correlation function for parton type i,j = q, g

$$F_{ij}(x_i, x_j, \mathbf{r}) = \int d^2\mathbf{b} \rho_i(x_i, \mathbf{y} + \mathbf{b}) \, \rho_j(x_j, \mathbf{b})$$

Quark double parton distribution is related to the two-parton density through Fourier transform



See, e.g., Diehl, M., Ostermeier, D. & Schäfer, A. Elements of a theory for multiparton interactions in QCD. *J. High Energ. Phys.* **2012**, 89 (2012).

Two-body densities

$$\rho_2^{q,q}(x, \mathbf{b}_1, \mathbf{b}_2) = \frac{1}{2}\left[\rho(\mathbf{b}_1)\rho(\mathbf{b}_2) - \frac{1}{2}\rho(\mathbf{b}_1, \mathbf{b}_2)\right]$$

General two-body density

$$\rho_2^{q,g}(x, \mathbf{b}_1, \mathbf{b}_2) = \rho(\mathbf{b}_1)\rho(\mathbf{b}_2)$$

Assuming independent particle motion

- In the two-body density framework, the Fourier transform of the GPDs act as densities that allow us to define useful quantities

$$\langle r^2_{q_1,q_2}(x_1,x_2)\rangle = \frac{\int d^2r \int d^2R_{CM}\, r^2\, H_{q_1}(x_1, R_{CM}+\frac{r}{2})H_{q_2}(x_2, R_{CM}-\frac{r}{2})}{\int d^2r \int d^2R_{CM}\, H_{q_1}(x_1, R_{CM}+\frac{r}{2})H_{q_2}(x_2, R_{CM}-\frac{r}{2})}$$

Average relative distance

Two-body densities

$$\rho_2^{q,q}(x,\mathbf{b}_1,\mathbf{b}_2) = \frac{1}{2}\left[\rho(\mathbf{b}_1)\rho(\mathbf{b}_2) - \frac{1}{2}\rho(\mathbf{b}_1,\mathbf{b}_2)\right]$$

General two-body density

$$\rho_2^{q,g}(x,\mathbf{b}_1,\mathbf{b}_2) = \rho(\mathbf{b}_1)\rho(\mathbf{b}_2)$$

Assuming independent particle motion

- In the two-body density framework, the Fourier transform of the GPDs act as densities that allow us to define useful quantities

$$O_{q_1,q_2}(x_1,x_2,r) = \frac{\int d^2R_{CM}A_o(r)H_{q_1}(x_1,R_{CM}+\frac{r}{2})H_{q_2}(x_2,R_{CM}-\frac{r}{2})}{\int d^2R_{CM}H_{q_1}(x_1,R_{CM}+\frac{r}{2})H_{q_2}(x_2,R_{CM}-\frac{r}{2})}$$

Overlap between two partons

$$A_o(r) = R_1^2\cos^{-1}\left(\frac{r^2+R_1^2-R_2^2}{2rR_1}\right) + R_2^2\cos^{-1}\left(\frac{r^2+R_2^2-R_1^2}{2rR_2}\right) - \frac{1}{2}\sqrt{(-r+R_1+R_2)(r+R_1-R_2)(r-R_1+R_2)(r+R_1+R_2)}$$

Geometric overlap of two circles, where R_1, R_2 are the average radii of the partons q_1, q_2

Two-body densities – Examples

Suppose we fit the density distribution, which is obtained through the GPDs, to a Gaussian:

$$\rho_1(|\vec{b}_T|) = Ce^{-\vec{b}_T^2/a^2},$$

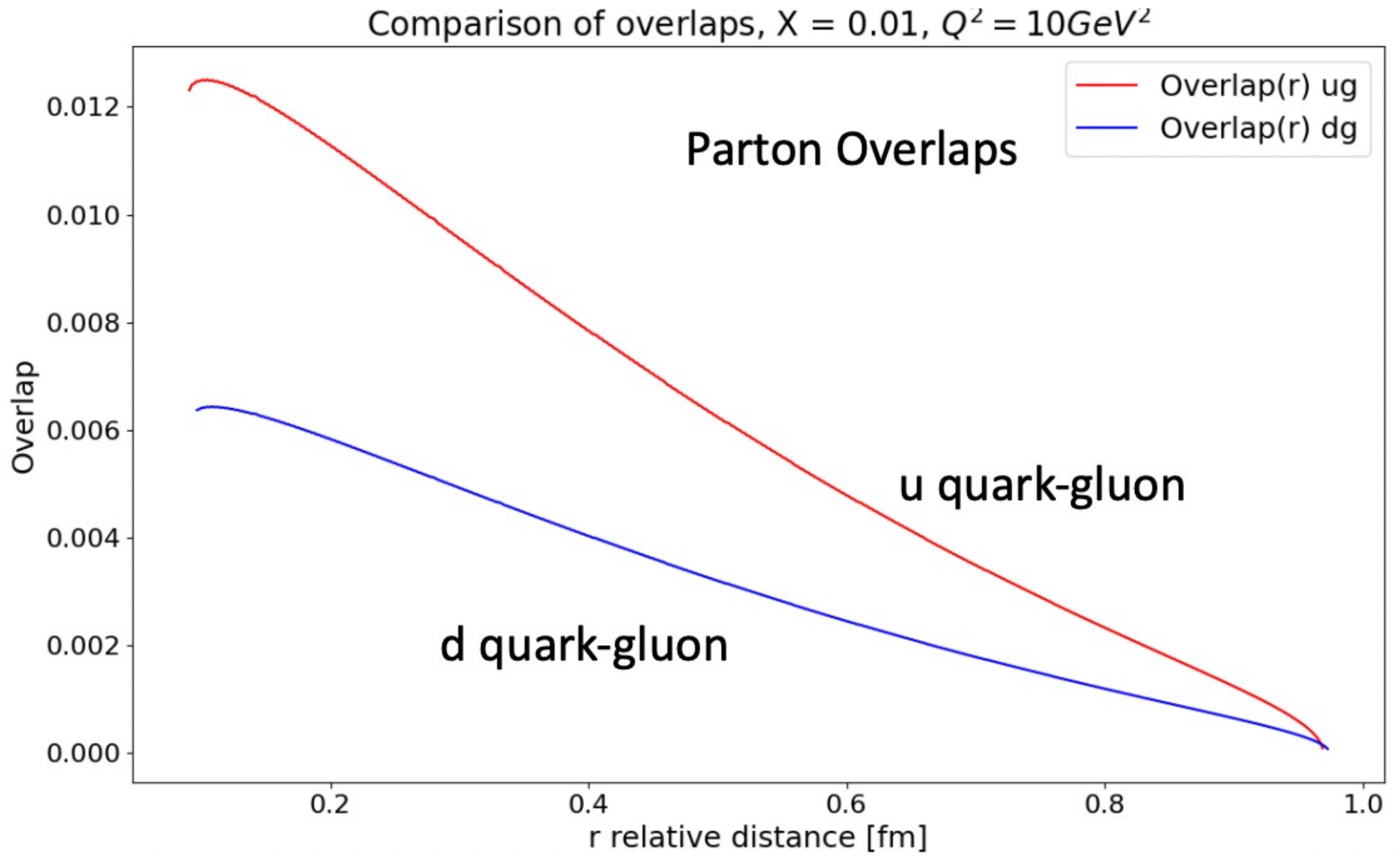
Taking two partons, say two gluon distributions, at the same X, we obtain the following two-body density:

$$\begin{aligned}\rho_2 &= \rho_1(|\vec{R}+\vec{r}/2|)\rho_1(|\vec{R}-\vec{r}/2|)\\ &= C^2 e^{-\frac{1}{a^2}(\vec{R}^2+\vec{r}^2/4)-\frac{1}{a^2}(2Rr\cos(\alpha))}\\ &\times e^{-\frac{1}{a^2}(\vec{R}^2+\vec{r}^2/4)+\frac{1}{a^2}(2Rr\cos(\alpha))}\\ &= C^2 e^{\frac{-\vec{r}^2}{2a^2}} e^{\frac{-2\vec{R}^2}{a^2}}.\end{aligned}$$

We obtain a simple relation for the average relative distance in such a scenario:

$$\langle r_{gg}^2(x)\rangle^{1/2} = \sqrt{2}a,$$

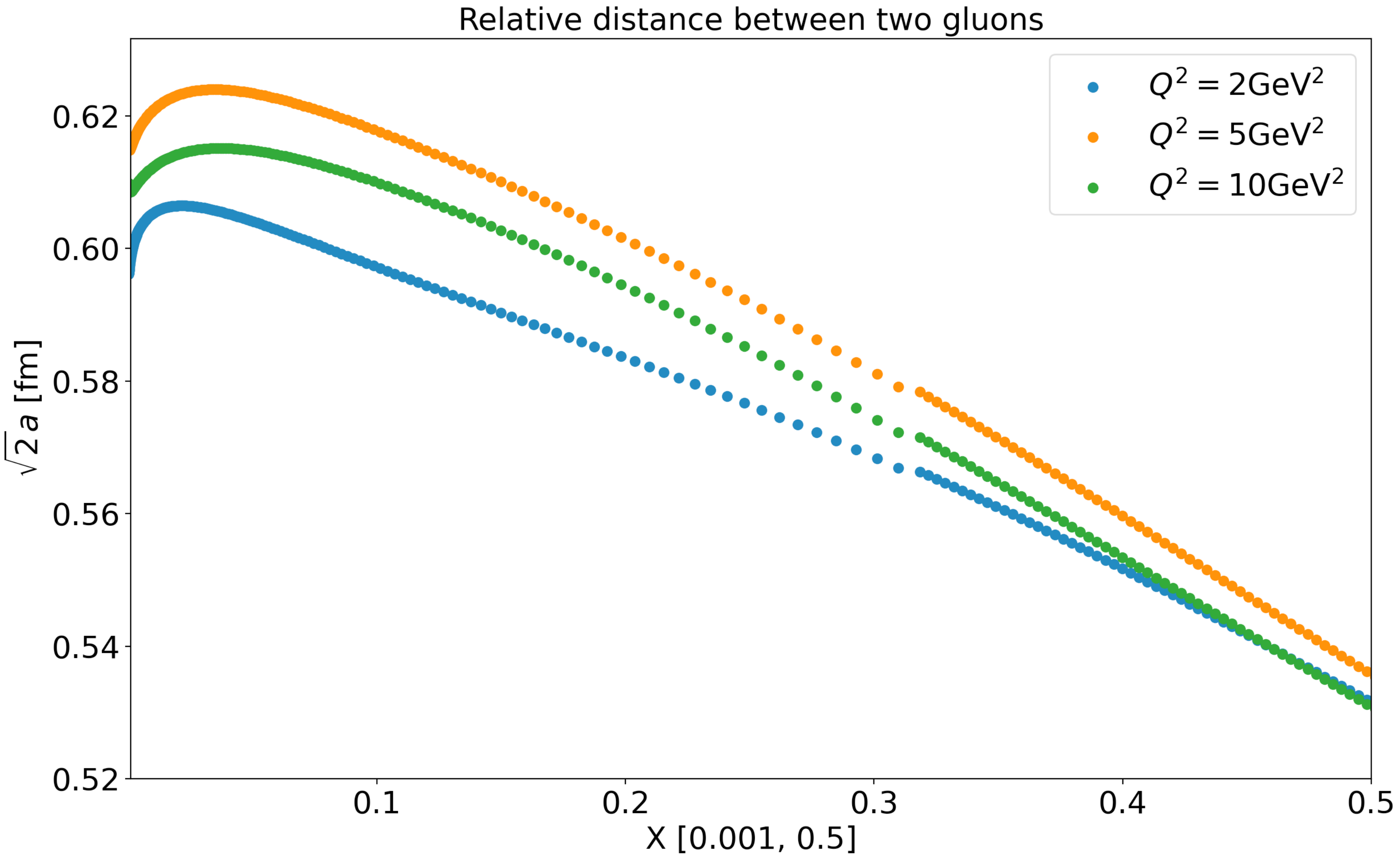
Two-body densities – Examples



Here, we fit the u, d and g distributions to Gaussians at some X and Q^2 to obtain overlaps between u and g and d and g.

*Numerical results here are preliminary

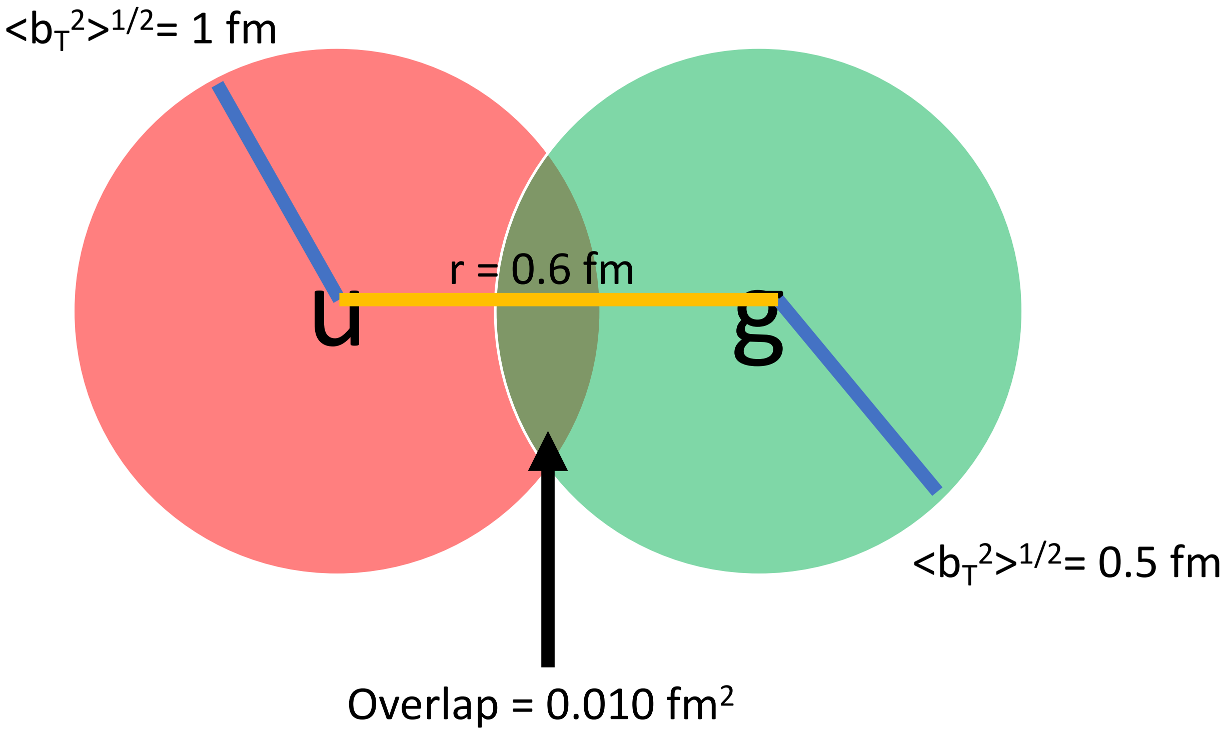
Two-body densities – Examples



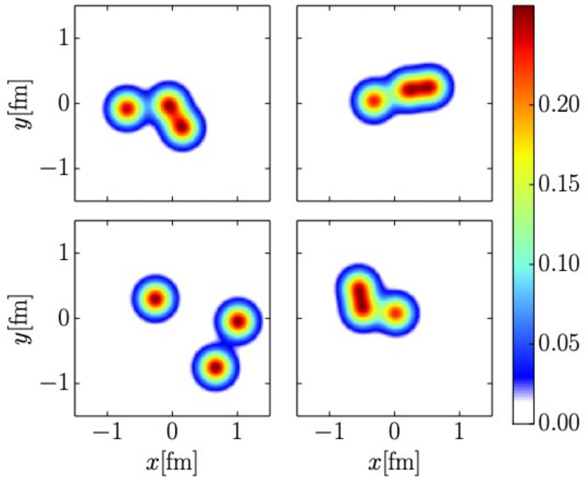
We fit the Fourier transform of Hg, the gluon distribution, to a Gaussian at different Q^2 for a limited range in X.

*Numerical results here are preliminary

	This work	Mantysaari and Schenke
Observable	DDVCS	J/ψ production
Picture	u, d, g have varying overlaps	u, d valence surrounded by g cloud
Parameter determining size	$\langle b_T^2 \rangle^{1/2}$ average radius	B_q hotspot size

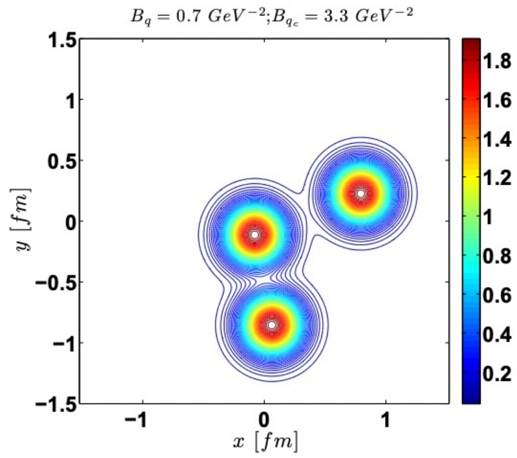


X = 0.01, Q^2 = 10 GeV^2



(a) $B_{qc} = 3.3$ GeV^{-2}, $B_q = 0.7$ GeV^{-2}

Heikki Mäntysaari and Björn Schenke Phys. Rev. D **94**, 034042



Traini, M.C., Blaizot, JP. *Eur. Phys. J. C* **79**, 327 (2019)

Conclusion

- The one-body density picture provides incredible insight into partonic structure
- Moving from a one-body density picture to a two-body density picture can greatly improve our understanding of the proton's internal structure
  - Differently from the hotspot formalism (Mantysaari and Schenke), we use GPDs to describe the relative motion of quarks and gluons