Polarized J/ψ Production in NRQCD

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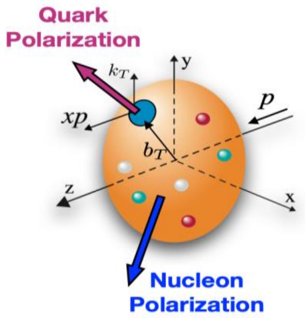
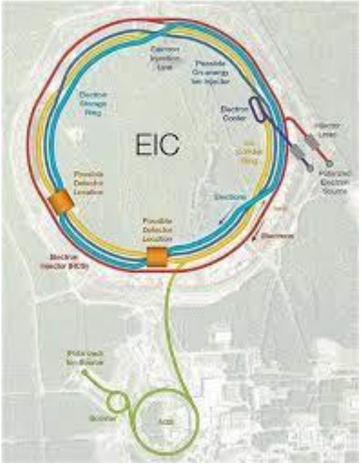
Motivation

- Transverse Momentum Dependent (TMD) PDFs and FFs probe the 3D structure of hadrons.
  - They provide correlations between hadron spin and parton polarization, in addition to the motion of the parton.

- Semi-Inclusive Deep Inelastic Scattering (SIDIS) will be a major focus of the upcoming Electron-Ion Collider.

- Gluon content in the proton is poorly constrained.





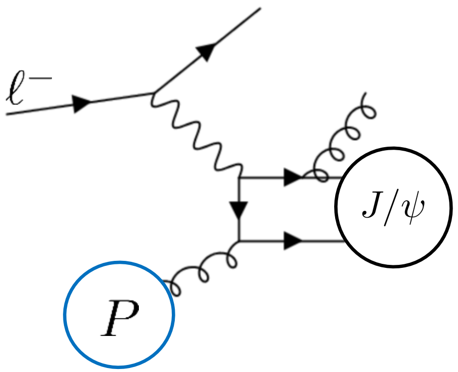
[1]

Why J/ψ Production?

- The J/ψ promises to be one of the most direct probes of gluon TMD PDFs in the proton.

- TMDs benefit from clean observables.
  - J/ψ is easy to identify – decays into $\ell^+\ell^-$ ~ 15% of the time.

- Something we can actually calculate!
  - The large heavy quark masses allow for the non-perturbative dynamics to be studied using Non-Relativistic QCD (NRQCD).



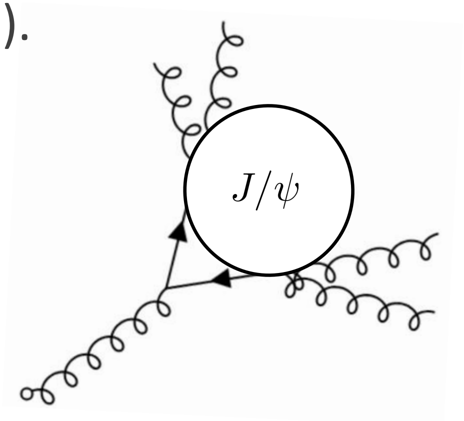
NRQCD

- Non-Relativistic QCD is an effective field theory of QCD where heavy quarks are treated as non-relativistic.

- Calculation involves a double expansion in α_S and in v (small relative velocity of $c\bar{c}$).

- NRQCD factorization theorem separates quarkonium production into short distance coefficients ($d_{i\to c\bar{c}}$) and a NRQCD TMD Fragmentation function ($D_{c\bar{c}\to J/\psi}$).

$$\Delta_{i\to J/\psi}(z,\boldsymbol{k}_\perp) \to \int d^2\boldsymbol{p}_\perp d^2\boldsymbol{q}_\perp d_{i\to c\bar{c}}(z,\boldsymbol{q}_\perp) D_{c\bar{c}\to J/\psi}(\boldsymbol{p}_\perp)\delta^{(2)}(\mathbf{q}_\perp - \mathbf{k}_\perp + \mathbf{p}_\perp)$$



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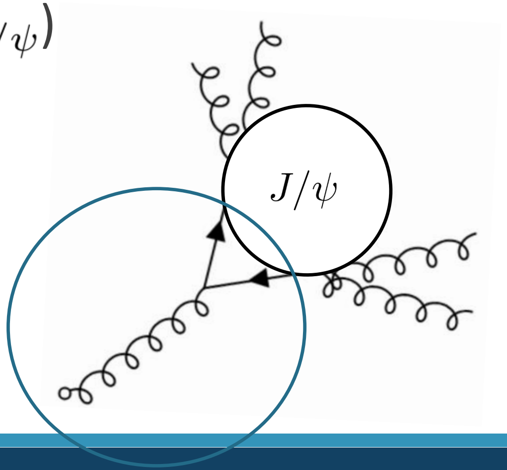

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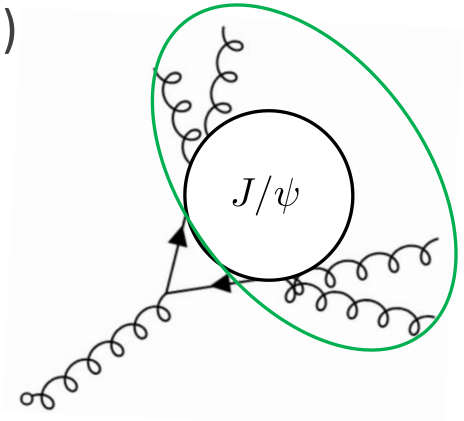
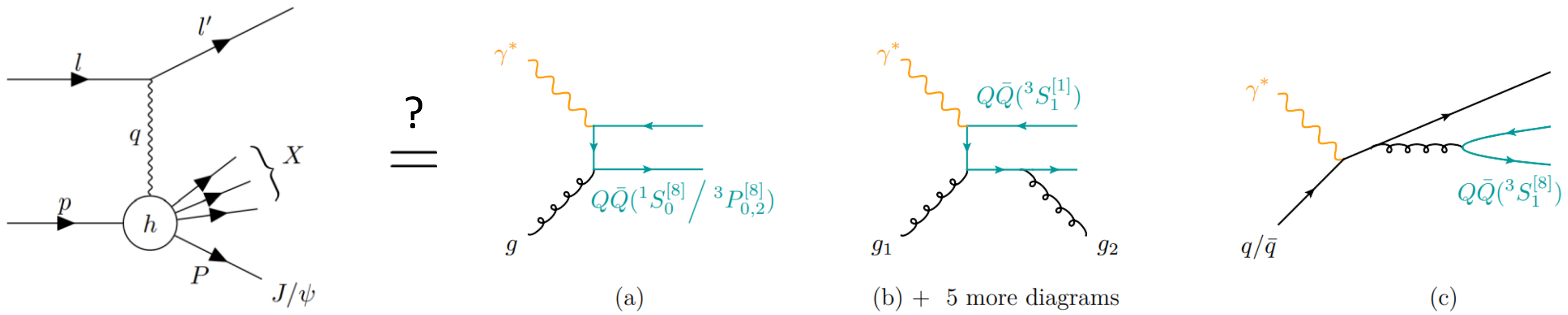


Production Mechanisms

- In SIDIS there are two ways to produce a J/ψ at leading order.

- Photon-gluon fusion or quark fragmentation.
  - Mechanisms compete at different kinematic regions - need to be disentangled.

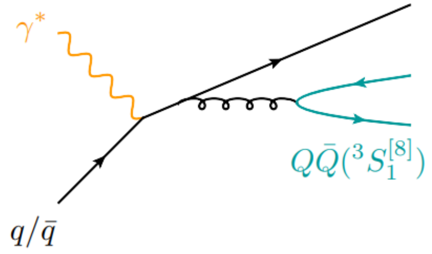
- Photon-gluon fusion is a probe of gluon structure at leading twist!



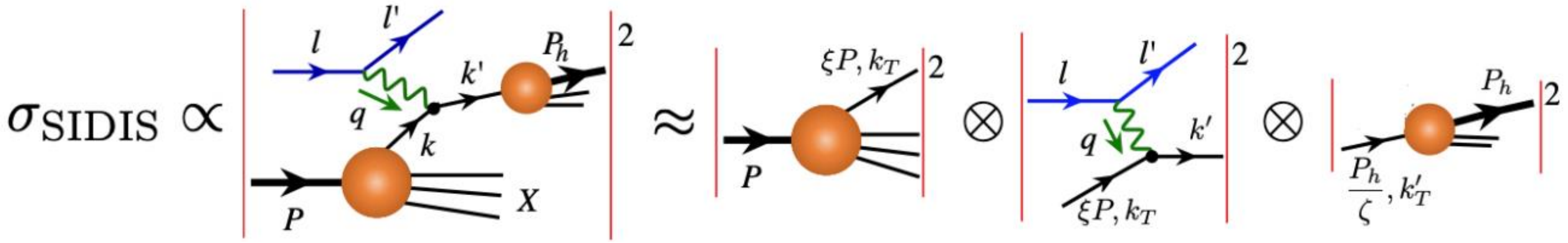
[2]

Fragmentation Functions

- Fragmentation functions tell us how a parton “i” hadronizes into “H” + other stuff “X”.
- Provide information on how hadrons emerge from energetic quarks and gluons.



- Like PDFs, defined by non-perturbative matrix elements, are used in factorization theorems, and are universal.

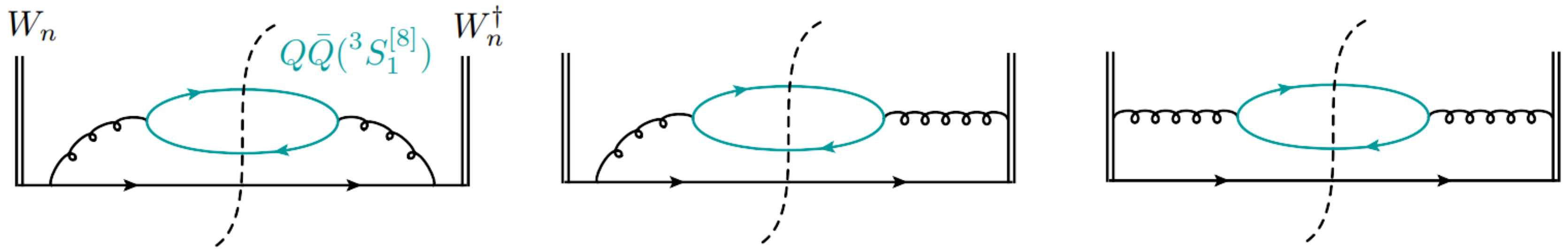


Quark Fragmentation

- Production of a $c\bar{c}$ pair fragmenting from a light quark gives 3 possible diagrams (plus mirrors) at lowest order.

$$\Delta_{q\to J/\Psi} = \frac{1}{2N_C z}\mathrm{Tr}\left[\int \frac{\mathrm{d}b^-}{2\pi} e^{ib^- P^+/z} \sum_X \Gamma^{\alpha\alpha'} \langle 0|W_n^\dagger(b)\psi_i^{\alpha 0}|J/\psi, X\rangle\langle J/\psi, X|\overline{\psi}_i^{0\alpha'} W_n(0)|0\rangle\right]$$

- Only unpolarized quark to unpolarized J/ψ TMD FF has been studied before [2].



A

B1 + mirror diagram (B2)

C

Quark Polarizations

- Quark can be unpolarized ($\frac{\gamma^+}{2}$), longitudinally polarized ($\frac{\gamma^+\gamma_5}{2}$), or and transversely polarized ($\frac{1}{2}\sigma^{\alpha+}\gamma_5$).

- Project out these states by completing spin trace in definition.

$$\Delta_{q\to J/\Psi} = \frac{1}{2N_C z}\mathrm{Tr}\left[\int \frac{\mathrm{db}^-}{2\pi}\mathrm{e}^{\mathrm{ib^-P^+/z}}\sum_{\mathrm{X}} \Gamma^{\alpha\alpha'}\langle 0|\mathrm{W}_\mathrm{n}^\dagger(\mathrm{b})\psi_\mathrm{i}^{\alpha 0}|\mathrm{J}/\psi,\mathrm{X}\rangle\langle \mathrm{J}/\psi,\mathrm{X}|\overline{\psi}_\mathrm{i}^{0\alpha'}\mathrm{W}_\mathrm{n}(0)|0\rangle\right]$$

$$\Gamma \in \frac{\gamma^+}{2}, \frac{\gamma^+\gamma_5}{2}, \frac{1}{2}\sigma^{\alpha+}\gamma_5$$

J/ψ Polarizations

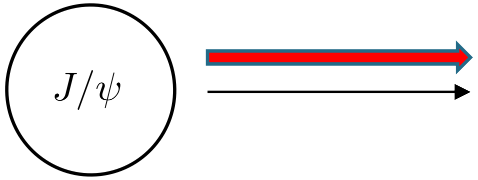
- Project out J/ψ polarization by parameterizing polarization vectors (like tensor decomposition).

$$\epsilon^i \epsilon^{*j} = \frac{1}{3}\delta^{ij} - \frac{i}{2}\epsilon^{ijk} S_k - T^{ij}$$

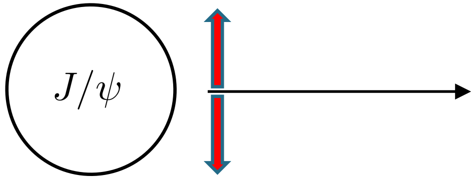
- J/ψ can be unpolarized, longitudinally, or transversely polarized.
  - Determined by values for spin parameters [4].

$$\vec{S} = (S_T^x, S_T^y, S_L) \qquad T_{ij} = \frac{1}{2}\begin{pmatrix} -\frac{2}{3}S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^{x} \\ S_{TT}^{yx} & -\frac{2}{3}S_{LL} - S_{TT}^{xx} & S_{LT}^{y} \\ S_{LT}^{x} & S_{LT}^{y} & \frac{4}{3}S_{LL} \end{pmatrix}.$$

- Polarized fragmentation functions are defined as the objects proportional to these spin parameters.



$S_{LL} = -1$



$S_{LL} = 1/2$

Quark Fragmentation Functions

- There are 18 polarized quark to J/ψ TMD fragmentation functions. [3]
  - At leading order in the strong coupling, only six FFs survive!

- Unpolarized quark:

$$D_1(z,\mathbf{k}_T;\mu) = \frac{2\alpha_s^2(\mu)}{9\pi N_c M^3 z}\frac{\mathbf{k}_T^2 z^2(z^2-2z+2)+2M^2(z-1)^2}{[z^2\mathbf{k}_T^2+M^2(1-z)]^2}\left\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\right\rangle ,$$

$$D_{1LL}(z,\mathbf{k}_T;\mu) = \frac{2\alpha_s^2(\mu)}{9\pi N_c M^3 z}\frac{\mathbf{k}_T^2 z^2(z^2-2z+2)-4M^2(z-1)^2}{[z^2\mathbf{k}_T^2+M^2(1-z)]^2}\left\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\right\rangle ,$$

$$D_{1LT}(z,\mathbf{k}_T;\mu) = \frac{2\alpha_s^2(\mu)}{3\pi N_c M}\frac{(2-z)(1-z)}{[z^2\mathbf{k}_T^2+M^2(1-z)]^2}\left\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\right\rangle ,$$

$$D_{1TT}(z,\mathbf{k}_T;\mu) = \frac{2\alpha_s^2(\mu)}{3\pi N_c M}\frac{z(z-1)}{[z^2\mathbf{k}_T^2+M^2(1-z)]^2}\left\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\right\rangle .$$

		Quark polarization		
		Unpolarized	Longitudinal	Transverse
Hadron polarization	Unpolarized	D_1		$H_1^\perp$
	Longitudinal		G_1	$H_{1L}^\perp$
	Transverse	$D_{1T}^\perp$	$G_{1T}^\perp$	$H_1,\ H_{1T}^\perp$
	LL	D_{1LL}		$H_{1LL}^\perp$
	LT	D_{1LT}	G_{1LT}	$H_{1LT}^\perp,\ H_{1LT}'$
	TT	D_{1TT}	G_{1TT}	$H_{1TT}^\perp,\ H_{1TT}'$

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$$D_{1LT}(z,\mathbf{k}_T;\mu) = \frac{2\alpha_s^2(\mu)}{3\pi N_c M}\frac{(2-z)(1-z)}{[z^2\mathbf{k}_T^2+M^2(1-z)]^2}\left\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\right\rangle ,$$

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	Transverse	$D_{1T}^\perp$	$G_{1T}^\perp$	H_1, $H_{1T}^\perp$
	LL	D_{1LL}		$H_{1LL}^\perp$
	LT	D_{1LT}	G_{1LT}	$H_{1LT}^\perp$, H'_{1LT}
	TT	D_{1TT}	G_{1TT}	$H_{1TT}^\perp$, H'_{1TT}

Quark Fragmentation Functions

- Longitudinally polarized quark: [3]

$$G_{1L}(z,\mathbf{k}_T;\mu) = \frac{\alpha_s^2(\mu)}{3\pi N_c M^3}\frac{\mathbf{k}_T^2 z^2(2-z)}{[z^2\mathbf{k}_T^2+M^2(1-z)]^2}\left\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\right\rangle ,$$

$$G_{1T}^{\perp}(z,\mathbf{k}_T;\mu) = \frac{2\alpha_s^2(\mu)}{3\pi N_c M}\frac{z(z-1)}{[z^2\mathbf{k}_T^2+M^2(1-z)]^2}\left\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\right\rangle .$$

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Hadron polarization	Unpolarized	D_1		$H_1^{\perp}$
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	LL	D_{1LL}		$H_{1LL}^{\perp}$
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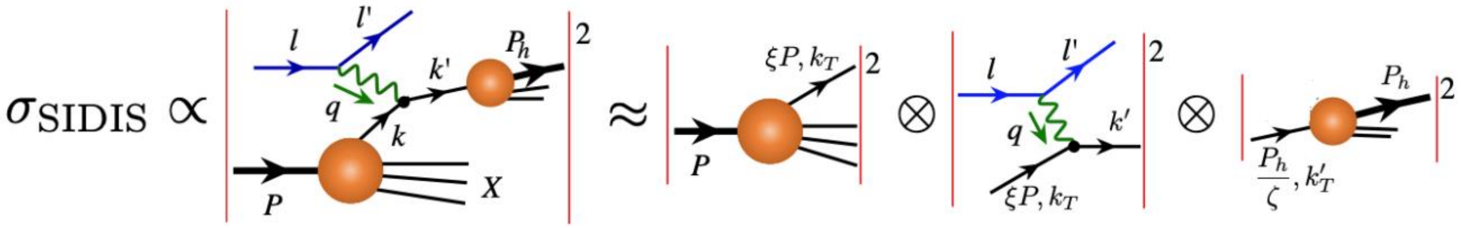
SIDIS Cross Sections from Fragmentation

- With unpolarized J/ψ, unpolarized beam, and unpolarized target, there is only one contribution to the cross section at leading twist.

$$\frac{d\sigma_{UU}(l+H\to l'+J/\psi+X)}{dx\ dz\ dy\ d^2\mathbf{P}_\perp}=\frac{4\pi\alpha^2 s}{Q^4}\left(1-y+\frac{y^2}{2}\right)\mathbf{I}[f_1(3D_1)]$$

where

$$\mathbf{I}[f_1D_1]=2z\int d^2\mathbf{k_T}d^2\mathbf{p_T}f_1(x,\mathbf{p_T})D_1(z,\mathbf{k_T})\delta^{(2)}(\mathbf{k_T}-\mathbf{p_T}-\mathbf{q_T})$$

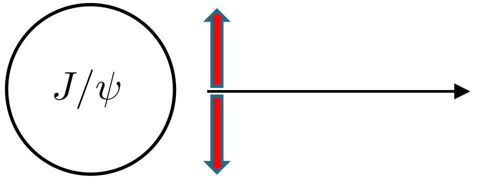


Polarized SIDIS cross section

- Polarized J/ψ production is a richer test of QCD!

$$\frac{d\sigma_{UU}(l+H\to l'+J/\psi+X)}{dx\ dz\ dy\ d^2\mathbf{P}_\perp}=\frac{4\pi\alpha^2 s}{Q^4}\left(1-y+\frac{y^2}{2}\right)\left\{\mathbf{I}[f_1D_1]+S_{LL}\mathbf{I}[f_1D_{LL}]\right\}$$

$$\frac{d\sigma_{LL}(l+H\to l'+J/\psi+X)}{dx\ dz\ dy\ d^2\mathbf{P}_\perp}=\frac{4\pi\alpha^2 s}{Q^4}2\lambda_c S_{qL}\,y\left(1-\frac{y}{2}\right)x\left\{\mathbf{I}[g_{1L}D_1]+S_{LL}\mathbf{I}[g_{1L}D_{1LL}]\right\}.$$



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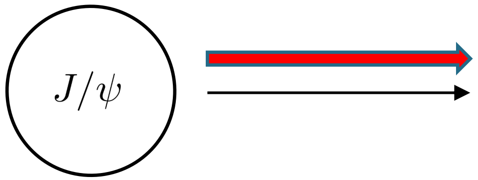


$S_{LL}=-1$

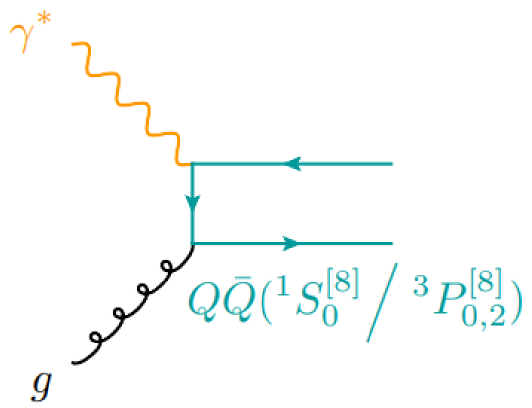
Direct Production

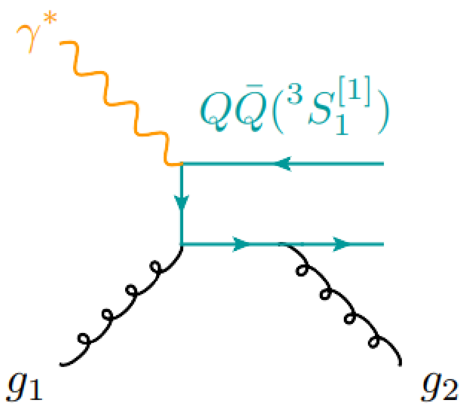
- Photon-gluon fusion accesses the gluon TMD PDFs in the proton.

- Two ways J/ψ can be produced through photon-gluon fusion.

- J/ψ can either be in a color singlet or a color octet (dominate in different regions of z).
  - Color octet is leading order in α_s .
  - Color singlet is leading order in “v”.

- Introduce TMD shape functions to model nonperturbative effects.





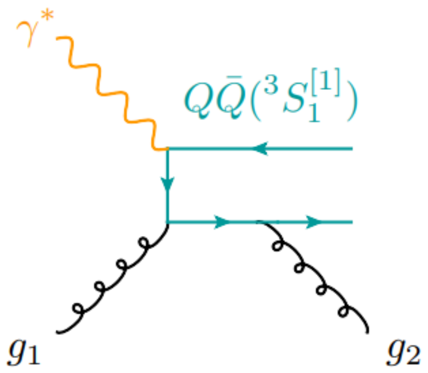
Color Singlet Photon-Gluon Fusion

- 6 diagrams – J/ψ is in the $^3S_1^{[1]}$ state.

$$z = \frac{p_{g_1} \cdot P_{J/\psi}}{p_{g_1} \cdot q}$$

- Expected to contribute more for z << 1.
  - Radiated gluon steals momentum from the initial parton.

$$\frac{d\sigma}{dxdzdQd\mathbf{P_T}^2 d\phi} \propto (A_T + \epsilon A_L + \sqrt{\epsilon(1+\epsilon)} \cos\phi A_\phi + \epsilon \cos 2\phi A_{2\phi}) \circledast f_g(x, \mathbf{p}_T, \mu^2)$$



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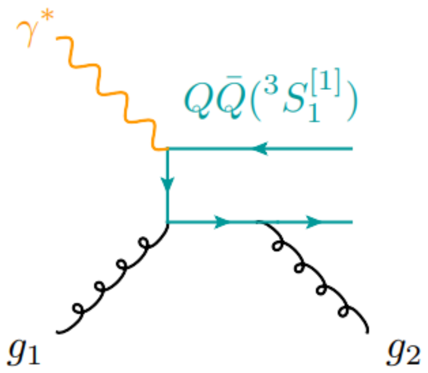


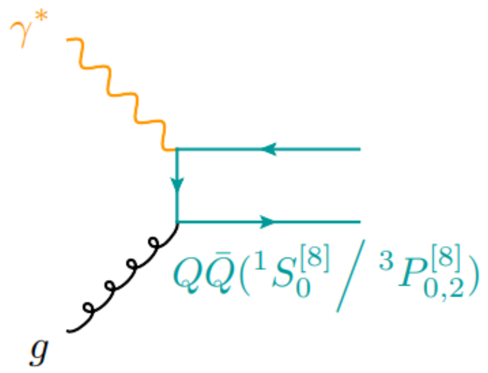
$$f_g(x, \mathbf{p}_T, \mu^2) \sim f_g(x, \mu^2)\left[\delta^2(\mathbf{p}_T) + \mathcal{O}\left(\frac{\mathbf{p}_T^2}{\Lambda}\right)\right]$$

Color Octet Photon-Gluon Fusion

- At leading order in α_s , J/ψ is either in ${}^1S_0^{[8]}$ or ${}^3P_J^{[8]}$.

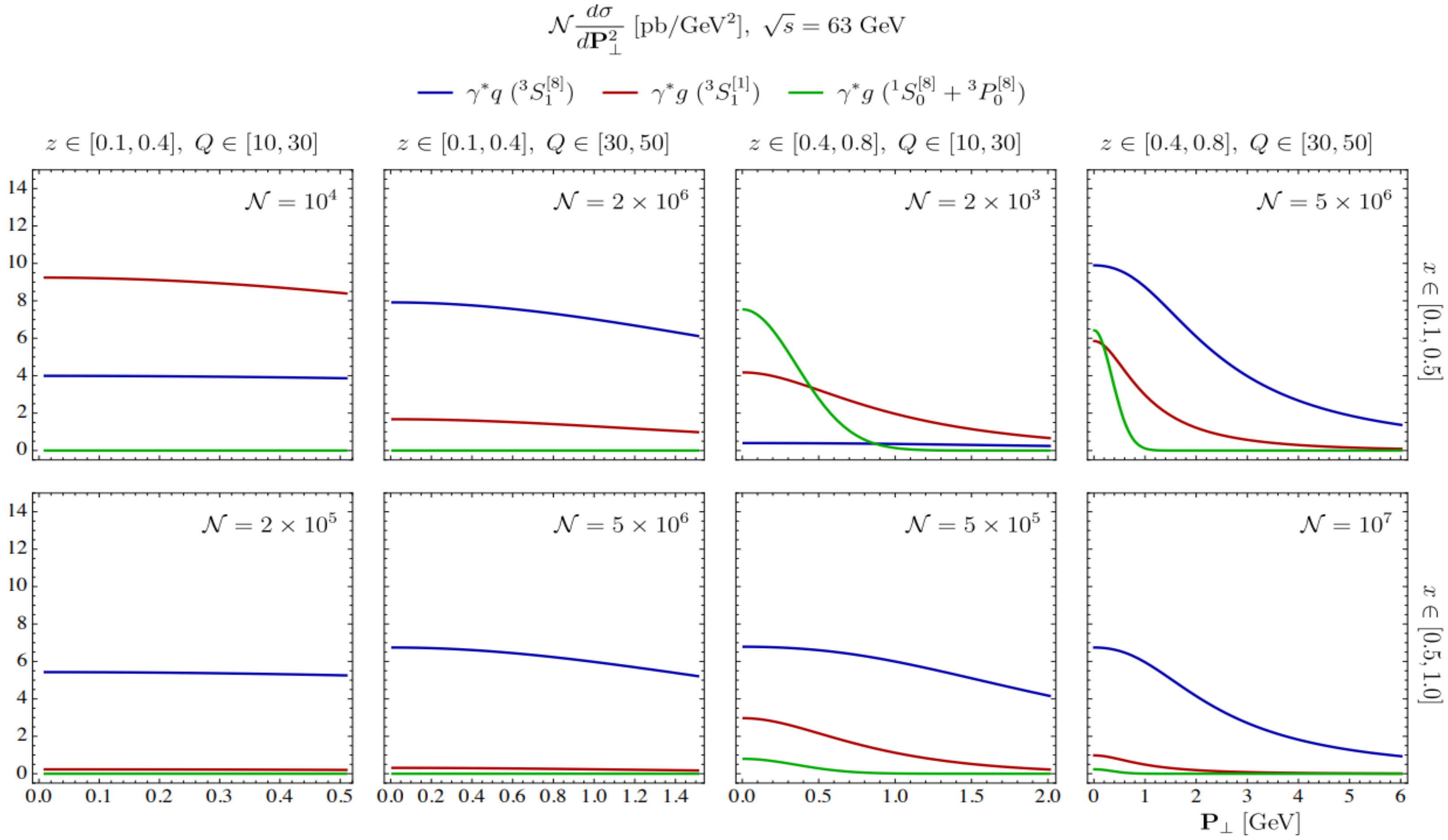
- No gluon is radiated. The J/ψ carries away all of the initial parton momentum.
  - This process dominates as z approaches 1.

$$z = \frac{p_{g_1} \cdot P_{J/\psi}}{p_{g_1} \cdot q}$$

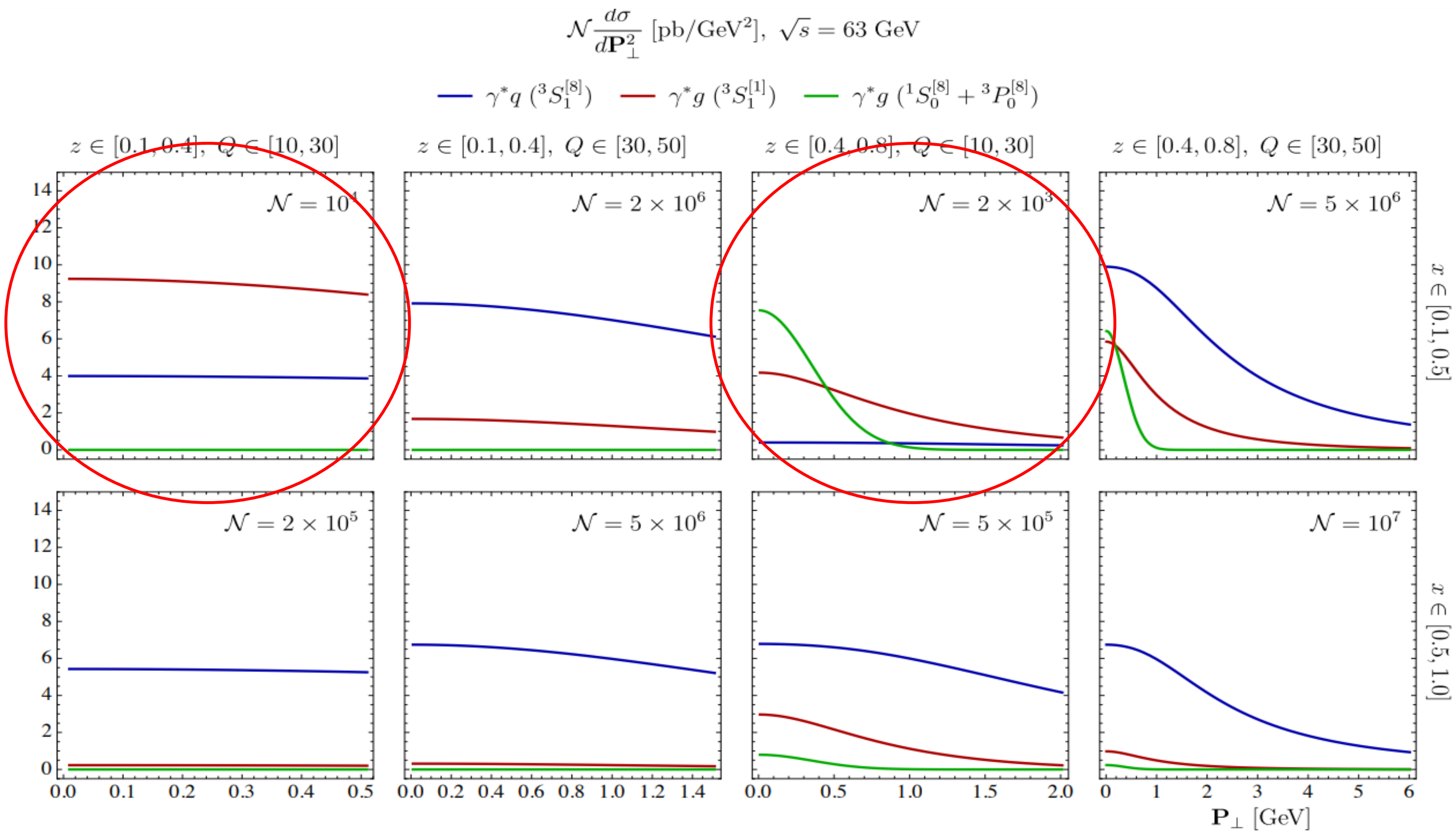


$$d\sigma \propto \delta(1-z)\delta^{(2)}(\mathbf{P}_\perp) \to \frac{1}{\sqrt{\pi\langle z_0\rangle}} e^{-(1-z)^2/\langle z_0\rangle} \frac{1}{\pi\langle \mathbf{P}_\perp\rangle} e^{-\mathbf{P}_\perp^2/\langle \mathbf{P}_\perp\rangle}$$

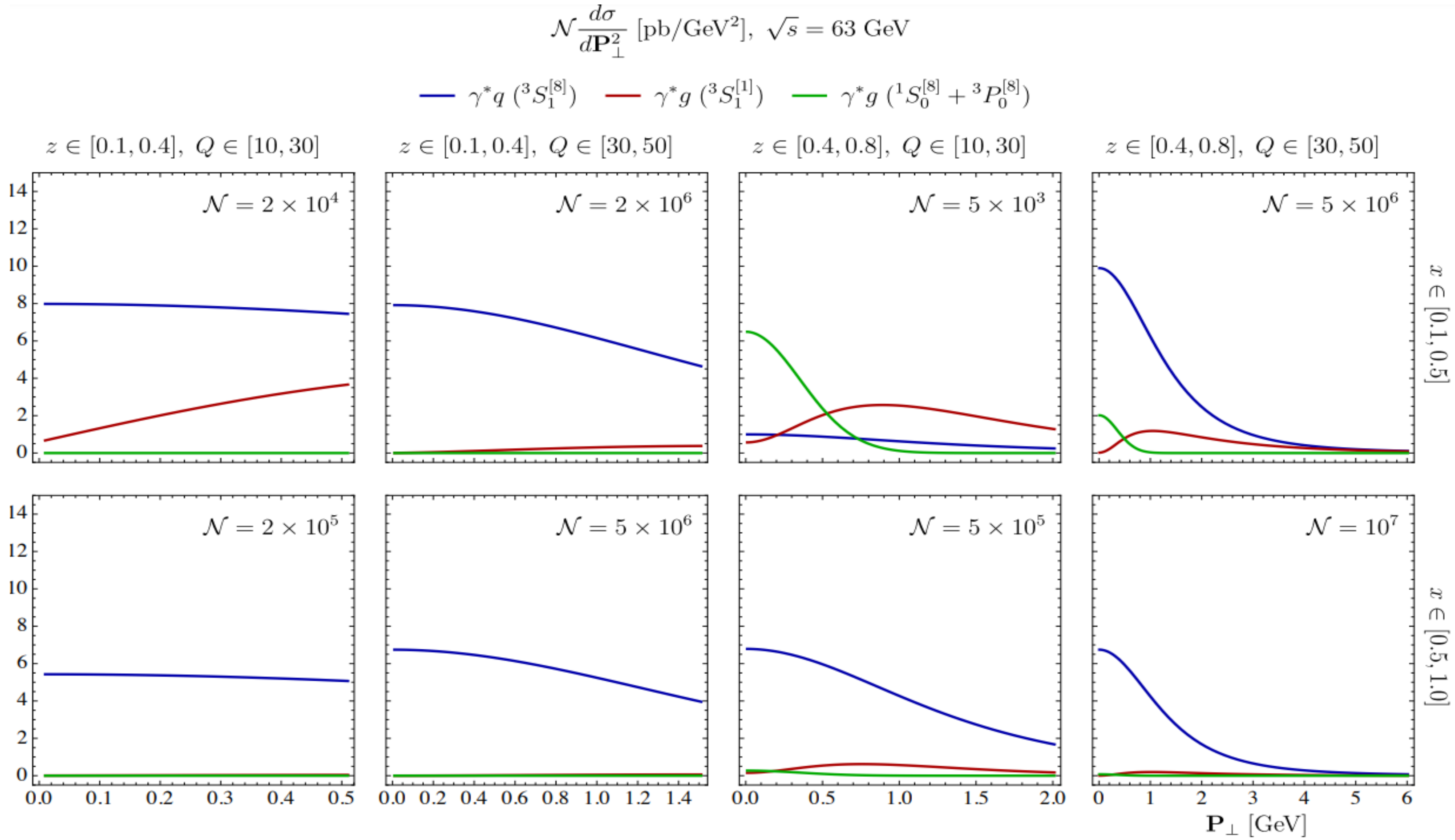
Comparing production mechanisms



Comparing production mechanisms



Comparing production mechanisms (L)



Future Work

- Calculate soft gluon corrections to NRQCD factorization theorems.
  - Important for very small transverse momenta.

- More phenomenological studies (beam asymmetries, identify other interesting observables, etc.)
  - Extract NRQCD long distance matrix elements from fitting to world data.

- Calculate polarized gluon fragmentation at NLO and study quarkonium production in jets.

- Calculate TMD fragmentation functions for other hadrons.
  - Other quarkonia
  - Kaons and pions.

References

- [1] TMD Handbook. Arxiv:2304.03302 (2023)
- [2] Echevarria, Makris, Scimemi. JHEP 10, 164 (2020).
- [3] Copeland, Fleming, Gupta, Hodges, Mehen. Arxiv:2308.08605 (2023)
- [4] Bachetta, Mulders. Phys. Rev. D 57, 5780 (1998).

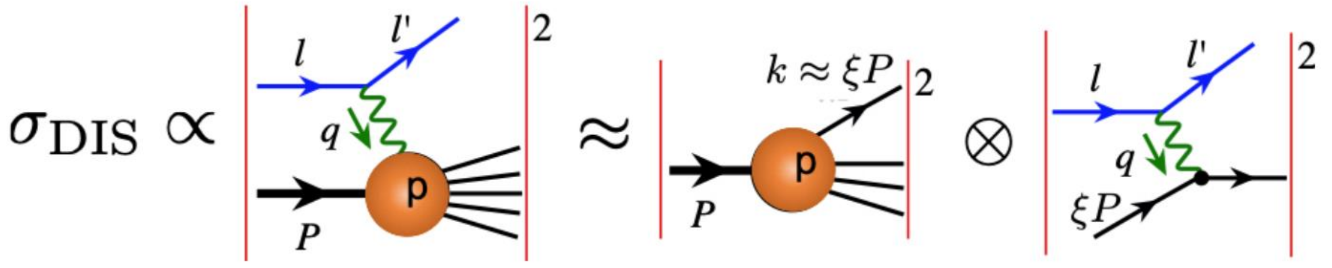
Back up slides

QCD Factorization

- The parton model allows for “factorization” of cross sections.

- . Separate physics taking place at different scales.
  - (a) Process independent non-perturbative contributions given by PDFs and FFs.
  - (b) Perturbatively calculable short-distance partonic cross sections ($\hat{\sigma}$).

- (a) can be extracted from experimental data, calculated on the lattice, or computed using effective field theories.



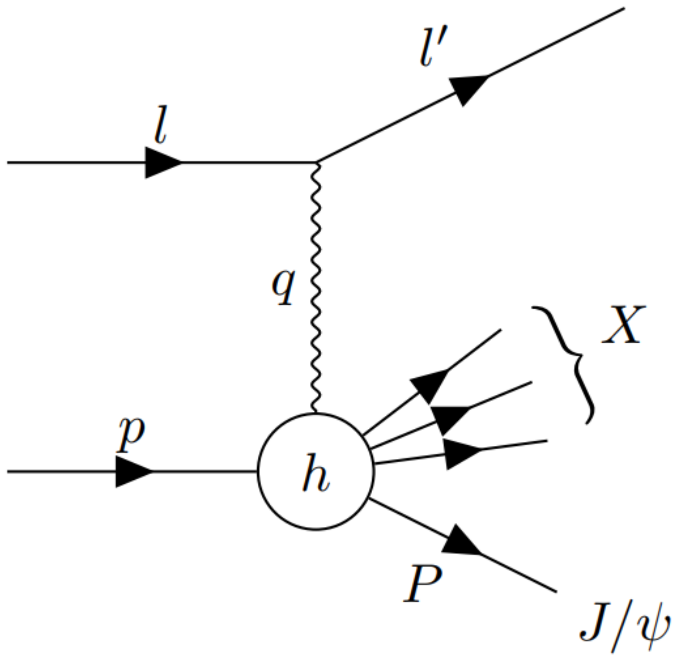
Kinematics

- Process can be written in terms of standard DIS variables.

$$x_B = \frac{Q^2}{2p \cdot q} \qquad q^2 = -Q^2$$

$$y = \frac{p \cdot q}{p \cdot l} \qquad z = \frac{p \cdot P_{J/\psi}}{p \cdot q}$$

$$l + p \to l' + P + X$$



NRQCD

- Non-Relativistic QCD is an effective field theory of QCD where heavy quarks are treated as non-relativistic, but gluons and light-quarks are left as the fully relativistic fields.

- Can be derived by making a non-relativistic expansion of the spinors in powers of small relative velocity, of $Q\overline{Q}$ pair, v.

- Calculation involves a double expansion in α_S and in v.

$$\mathcal{L}_{NRQCD} = \sum \overline{q} i \gamma^{\mu} D_{\mu} q - \frac{1}{2} \mathrm{Tr}\, \mathrm{G}_{\mu\nu} \mathrm{G}^{\mu\nu} + \psi^{\dagger} \left(\mathrm{iD_t} + \frac{\mathbf{D}}{\mathrm{2M}} \right) \psi + \chi^{\dagger} \left(\mathrm{iD_t} - \frac{\mathbf{D}}{\mathrm{2M}} \right) \chi$$

$$D^{\mu} = \partial^{\mu} + igA^{\mu}$$

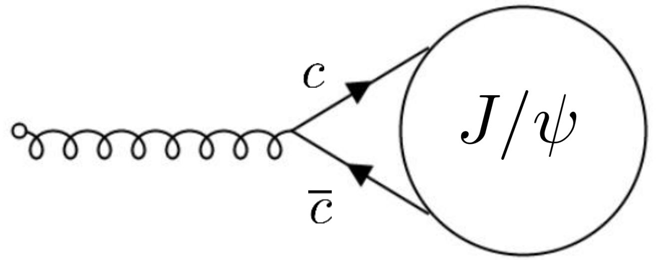
NRQCD Factorization (collinear)

- NRQCD factorization theorem separates quarkonium TMDFF into short distance coefficients ($d_{i\to c\bar{c}}$) and NRQCD long distance matrix elements ($\langle\mathcal{O}^{J/\psi}\rangle$).

- Short distance coefficients $d_{i\to c\bar{c}}$ describe production of $c\bar{c}$ from a parton, "i."
  - Perturbatively calculable though NRQCD matching.

- NRQCD LDMEs describe the hadronization of a $c\bar{c}$ with specific quantum numbers into a J/ψ .
  - Formally, a NRQCD double parton fragmentation function.
  - In practice, a constant extracted from experiment.

$$\Delta_{i\to J/\psi}(z) \to \sum_{L,s,c} d_{i\to c\bar{c}}^{L,s,c}(z)\langle\mathcal{O}^{J/\psi}(^{2s+1}L_J^{[c]})\rangle$$

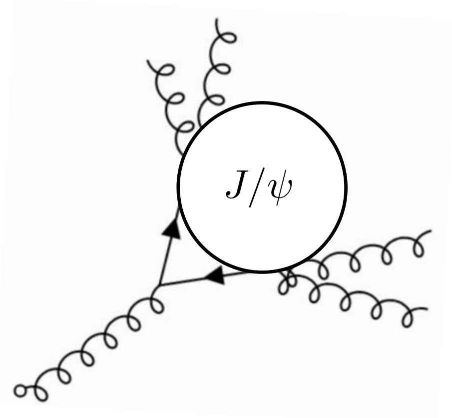


NRQCD Factorization (TMD)

- TMD NRQCD factorization theorem applies the same principles, but now both $d_{i\to c\bar{c}}$ and $\langle \mathcal{O}^{J/\psi} \rangle$ have transverse momentum dependence.

- We calculate the TMD short distance matching coefficients, $d_{i\to c\bar{c}}(\mathbf{k}_\perp)$.

- NRQCD TMDFF $(D_{c\bar{c}\to J/\psi}(\mathbf{p}_\perp))$ can have additional transverse momentum dependence due to soft gluon radiation.



$$\Delta_{i\to J/\psi}(z,\boldsymbol{k}_\perp) \to \int d^2\boldsymbol{p}_\perp d^2\boldsymbol{q}_\perp d_{i\to c\bar{c}}(z,\boldsymbol{q}_\perp) D_{c\bar{c}\to J/\psi}(\boldsymbol{p}_\perp)\delta^{(2)}(\mathbf{q}_\perp - \mathbf{k}_\perp + \mathbf{p}_\perp)$$

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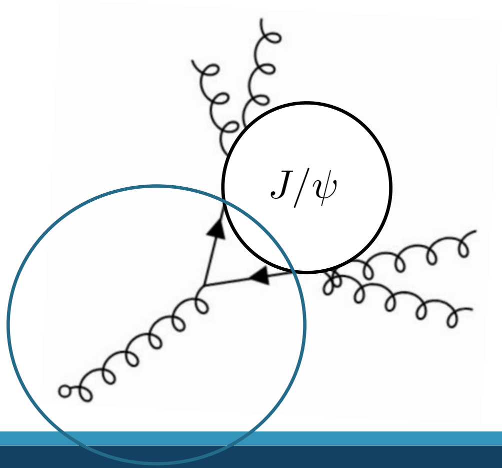

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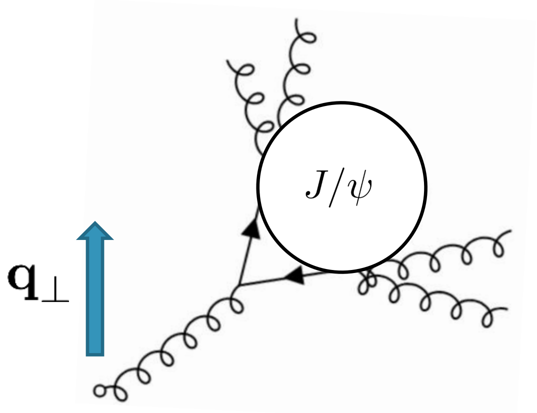

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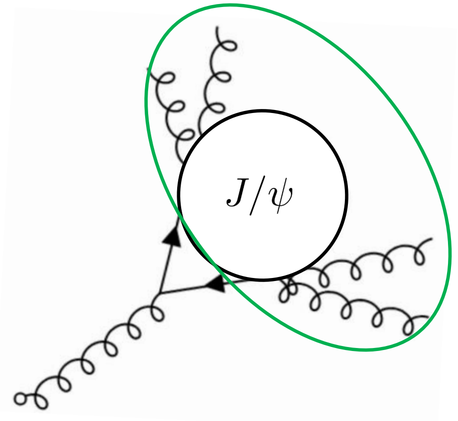

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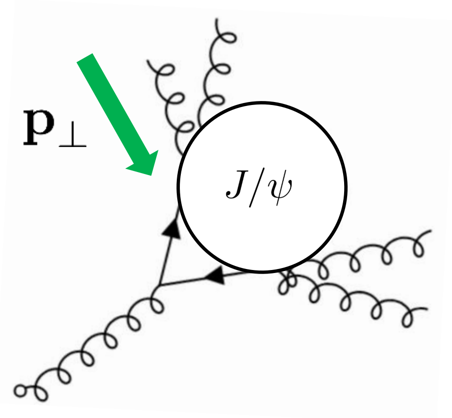


NRQCD Factorization (TMD)

- If the transverse momentum is sufficiently greater than $\Lambda_s \sim m_c v^2$, then we can expand the NRQCD TMDFF.

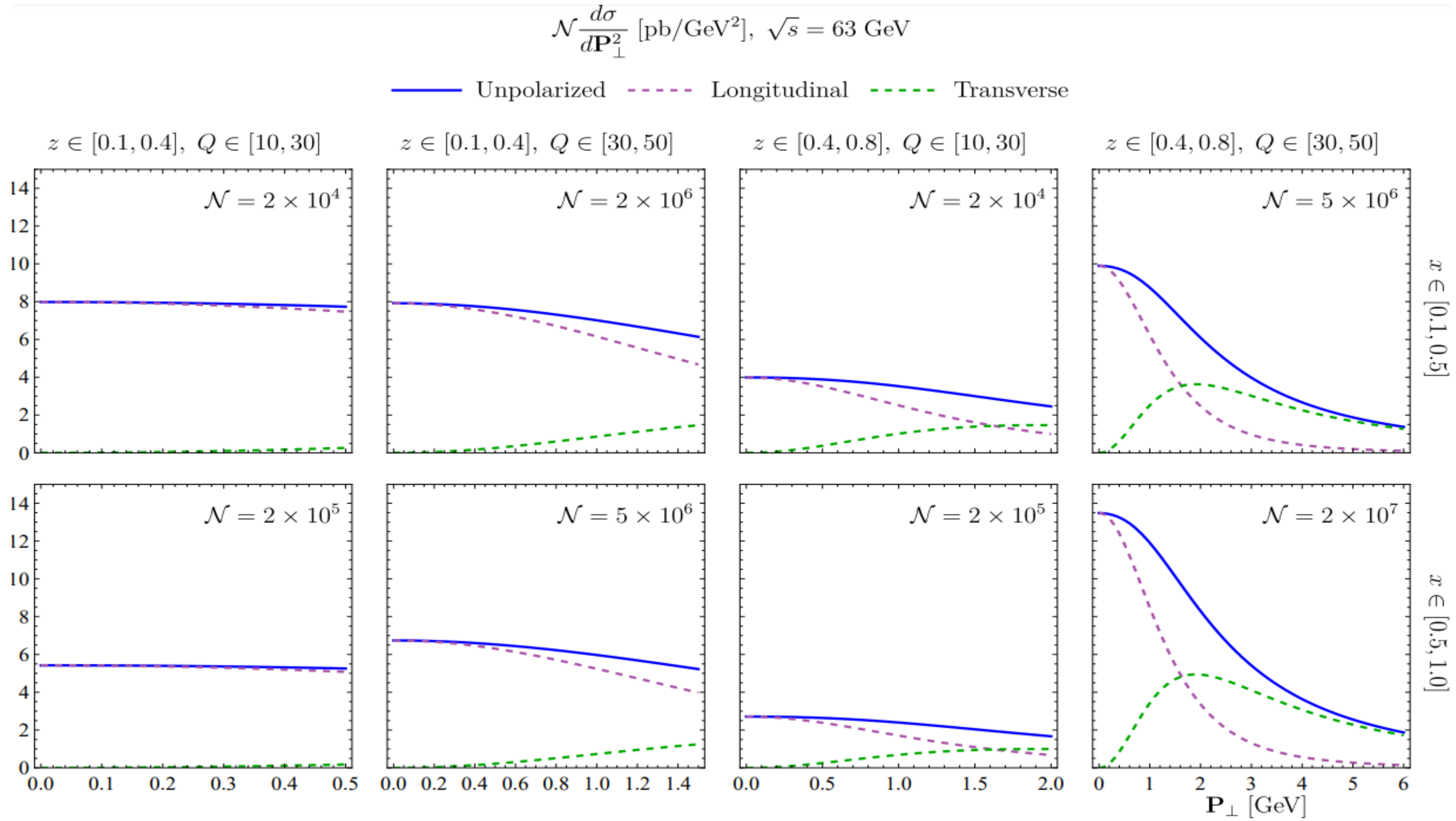
$$D_{c\bar{c}\to J/\psi}(\boldsymbol{p}_\perp) \sim \sum_{L,s,c} \langle \mathcal{O}^{J/\psi}(^{2s+1}L_J^{[c]})\rangle H_{(L,s;c)}(\boldsymbol{p}_\perp),$$

$$H_{(L,s;c)}(\boldsymbol{p}_\perp) = \delta^{(2)}(\boldsymbol{p}_\perp) + \sum_{n=1}^{\infty} \left(\frac{\alpha_s(\boldsymbol{p}_\perp)}{4\pi}\right)^n h^{(n)}_{(L,s;c)}(\boldsymbol{p}_\perp)$$

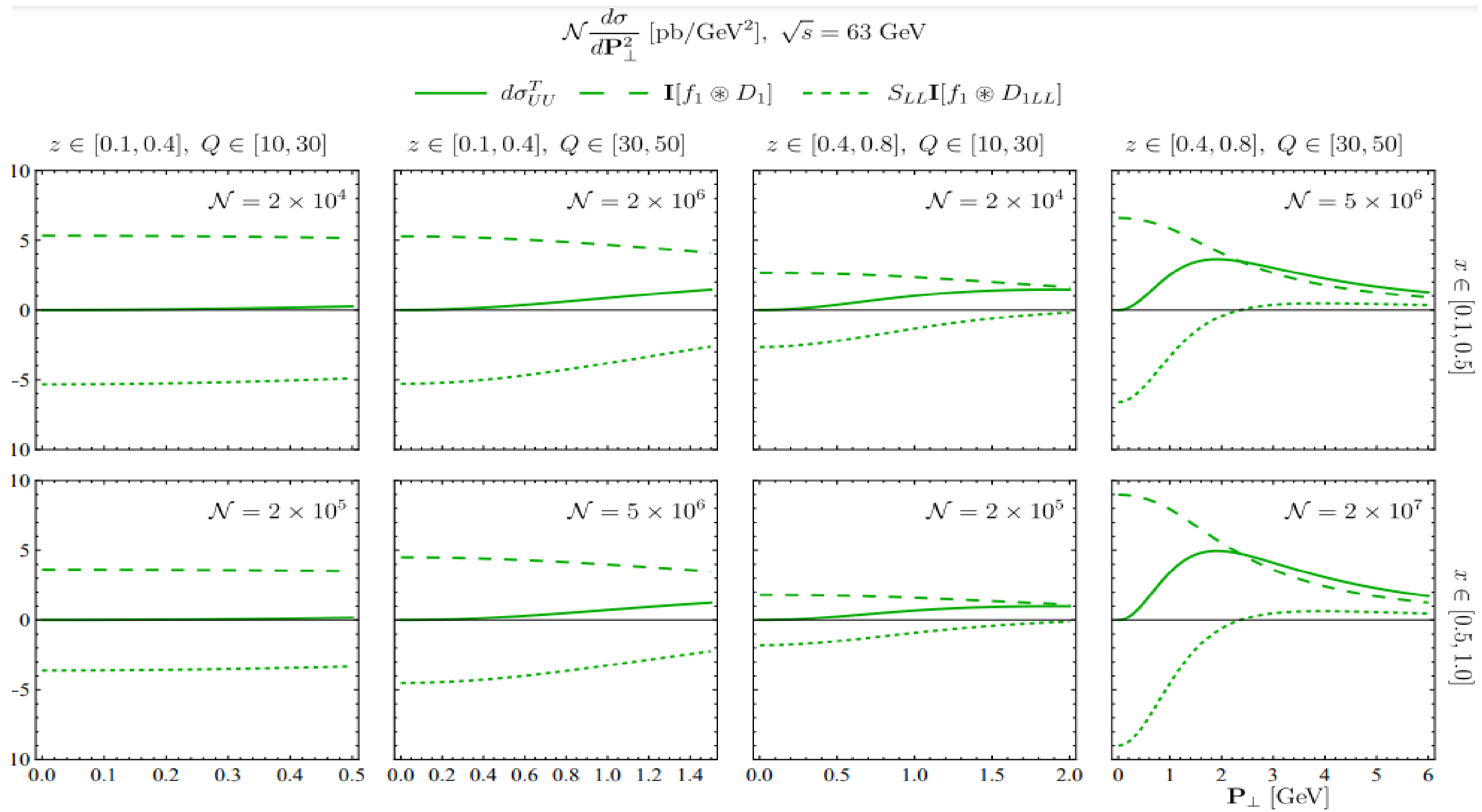
- This puts all of the transverse momentum dependence in the perturbative matching coefficient!

$$\Delta_{i\to J/\psi}(z, \boldsymbol{k}_\perp) \to \sum_{L,s,c} d_{i\to c\bar{c}}(z, \boldsymbol{k}_\perp) \langle \mathcal{O}^{J/\psi}(^{2s+1}L_J^{[c]})\rangle$$

Fragmentation contribution to $d\sigma_{UU}$



Transversely Polarized J/ψ production



Fragmentation contribution to $d\sigma_{LL}$