ALESSANDRO BACCHETTA, PAVIA U. AND INFN

RECENT RESULTS ON TMD EXTRACTIONS FROM THE MAP COLLABORATION

BY THE MAP COLLABORATION

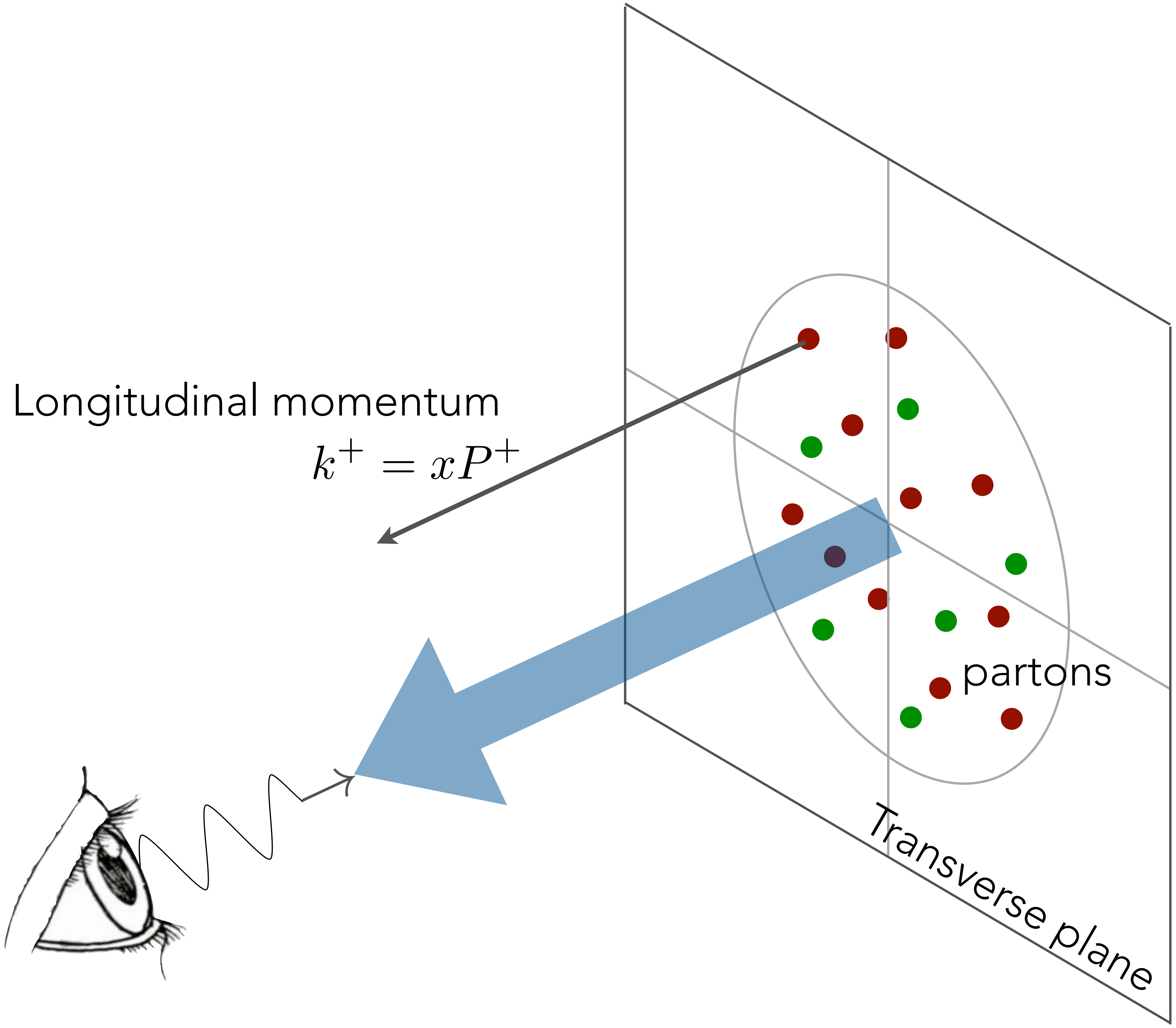
https://github.com/MapCollaboration

PARTICULAR THANKS TO

Matteo Cerutti

Lorenzo Rossi

INTRODUCTION

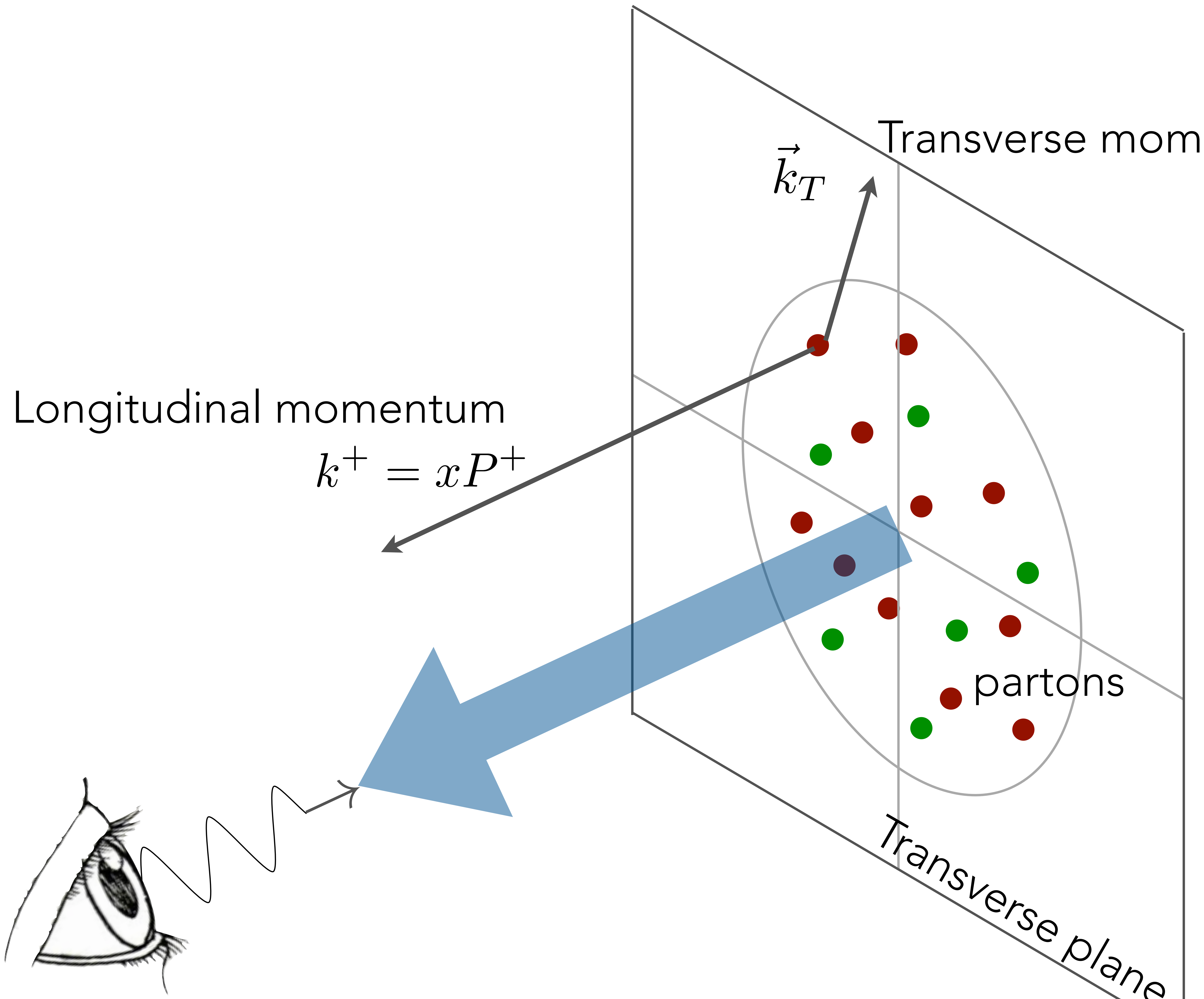


Longitudinal momentum

$k^+ = xP^+$

partons

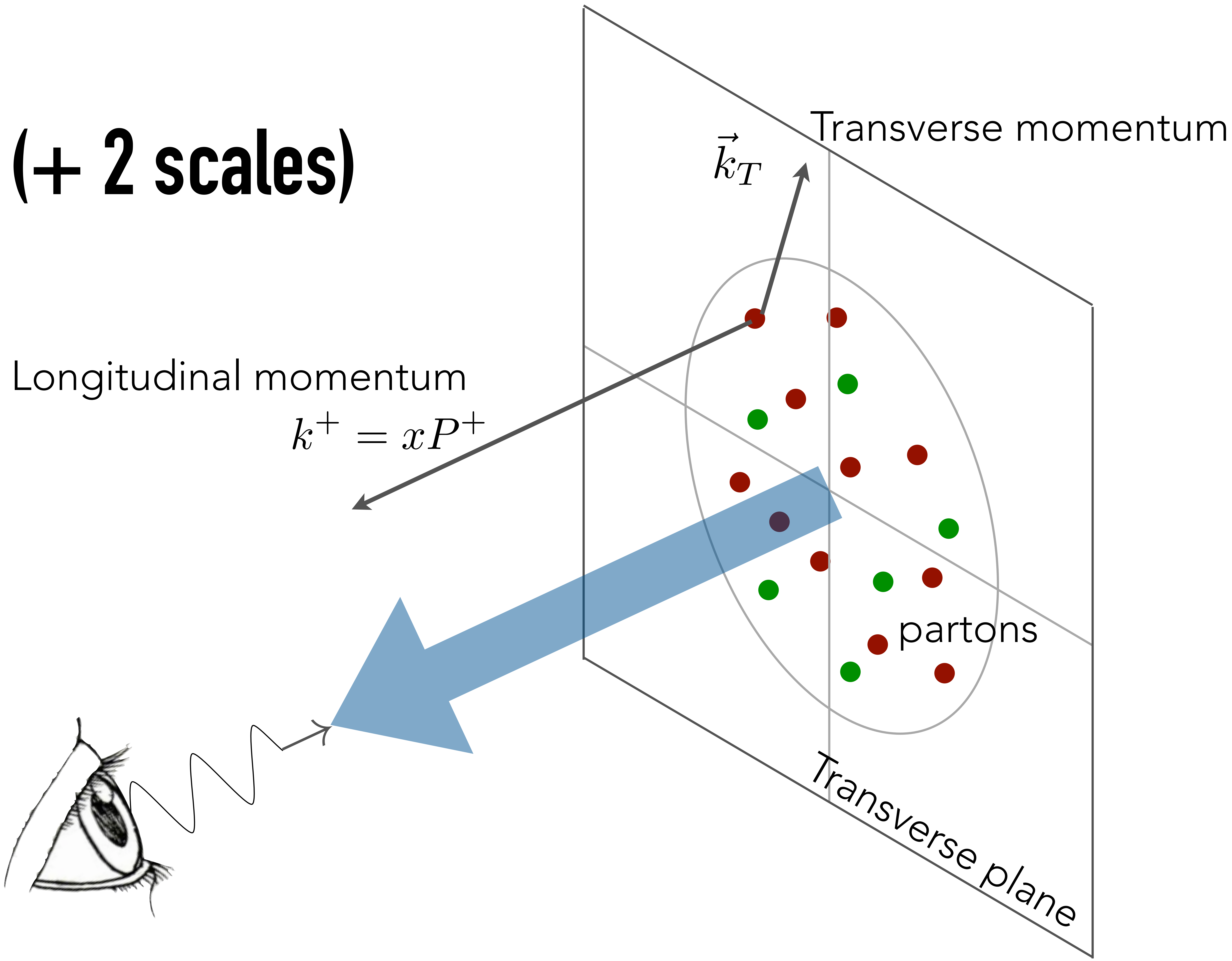
Transverse plane


Transverse momentum
$\vec{k}_T$
Longitudinal momentum
$k^+ = xP^+$
partons
Transverse plane

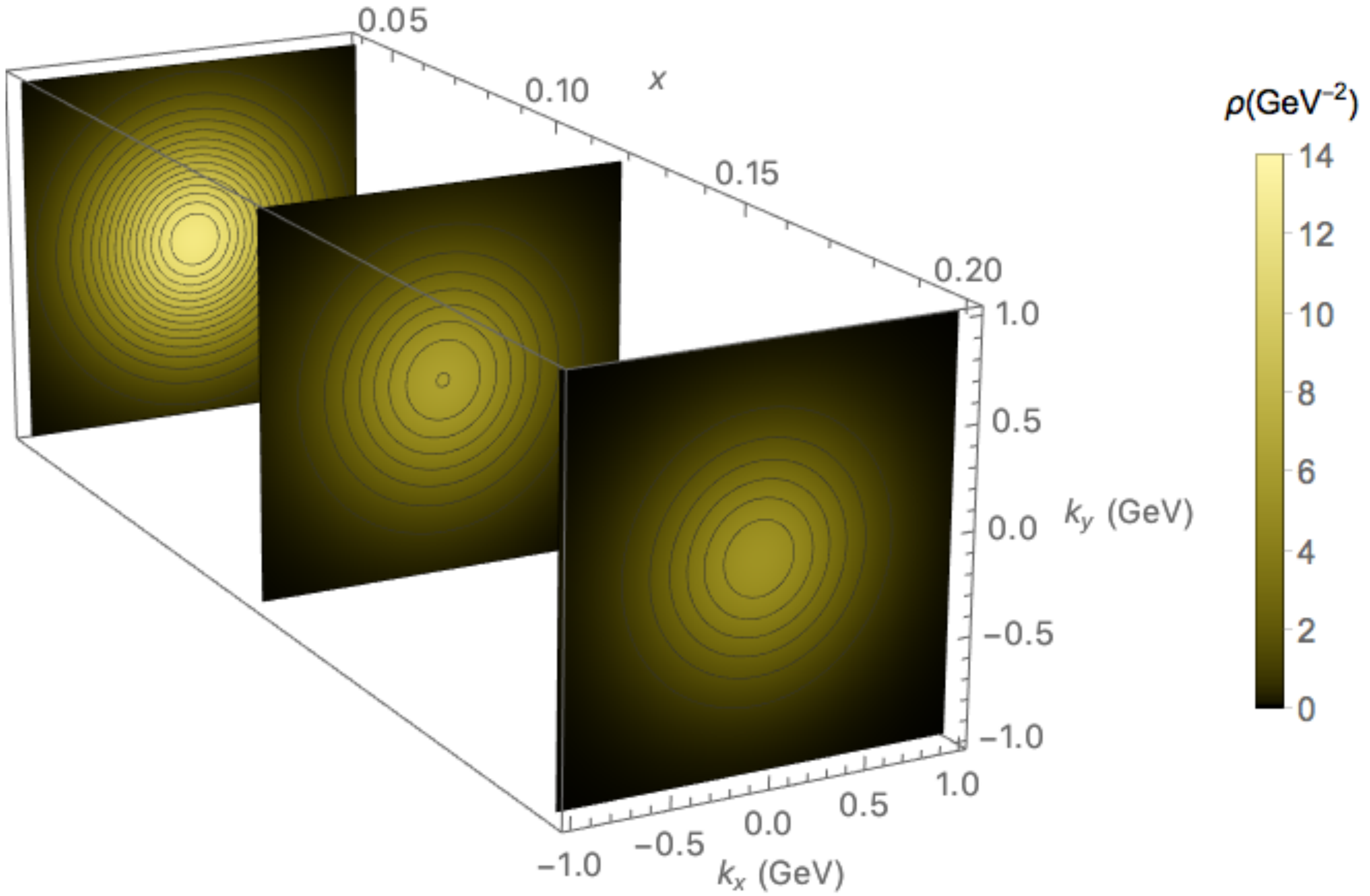
Transverse-Momentum Distributions

$f(x, \vec{k}_T)$

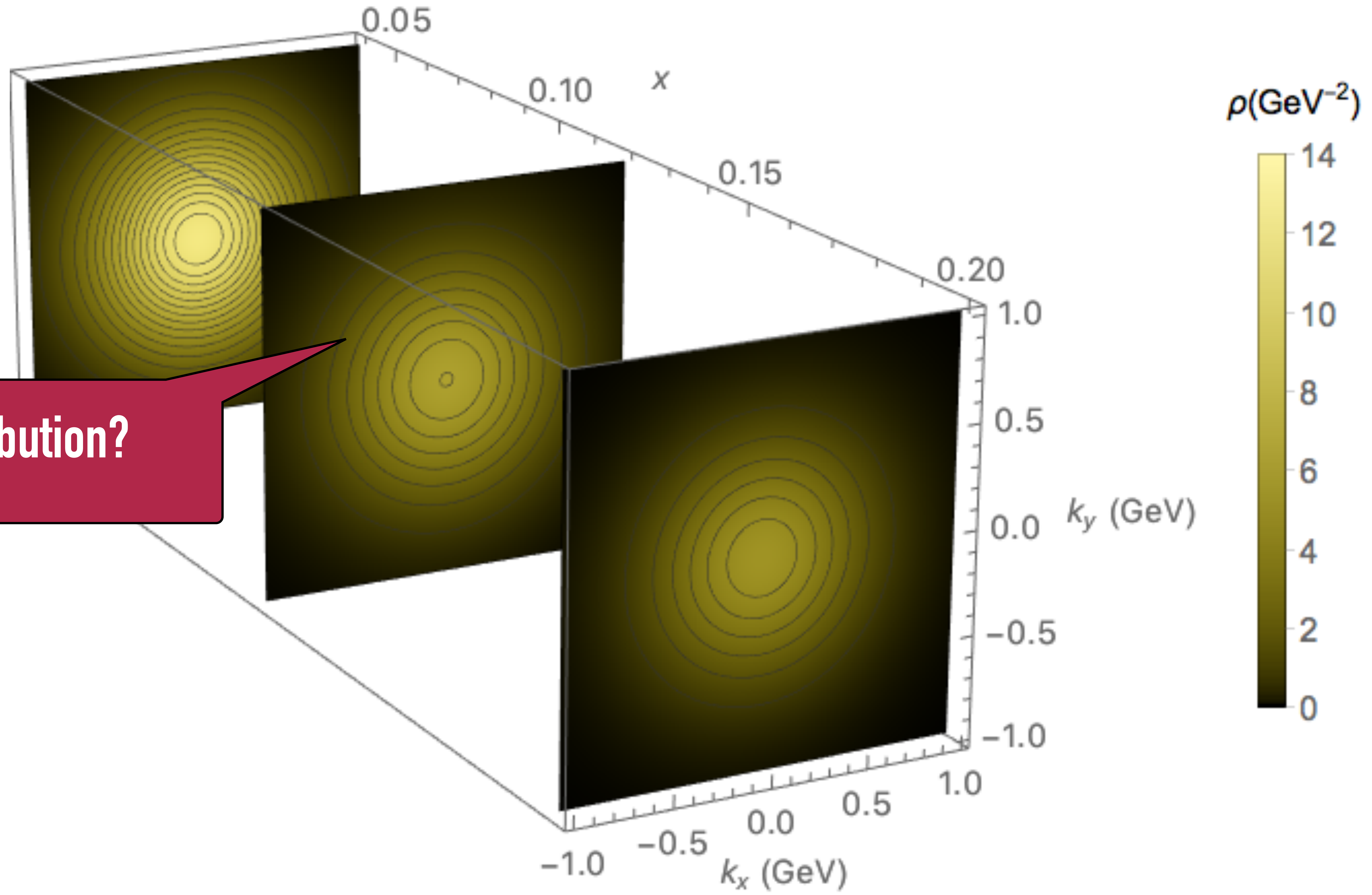
3 dimensional (+ 2 scales)



QUESTIONS

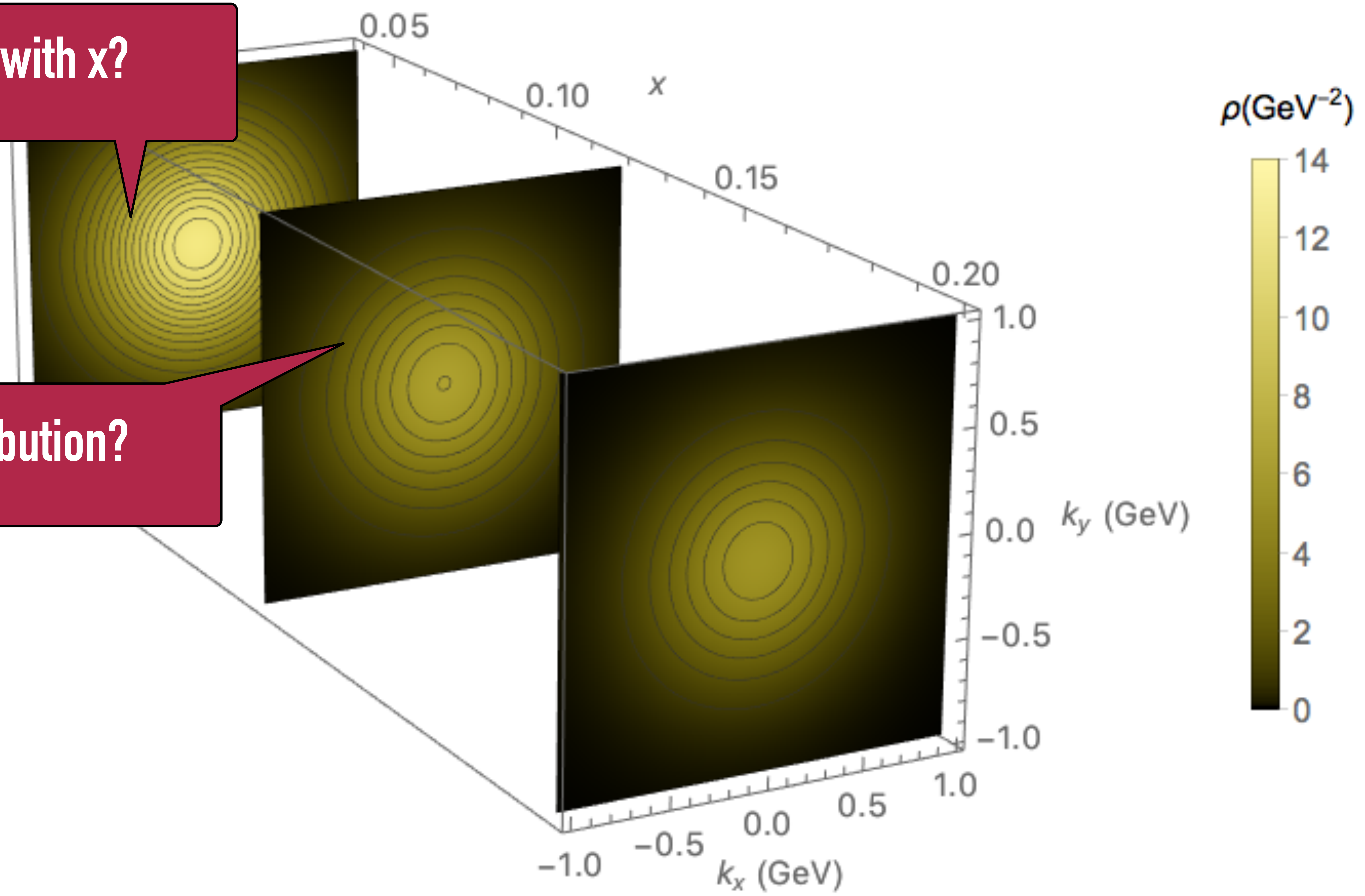

0.05
0.10
x
0.15
0.20
1.0
0.5
0.0
k_y (GeV)
−0.5
−1.0
1.0
0.5
0.0
−0.5
−1.0
k_x (GeV)
$\rho(\mathrm{GeV}^{-2})$
14
12
10
8
6
4
2
0

QUESTIONS



How “wide” is the distribution?

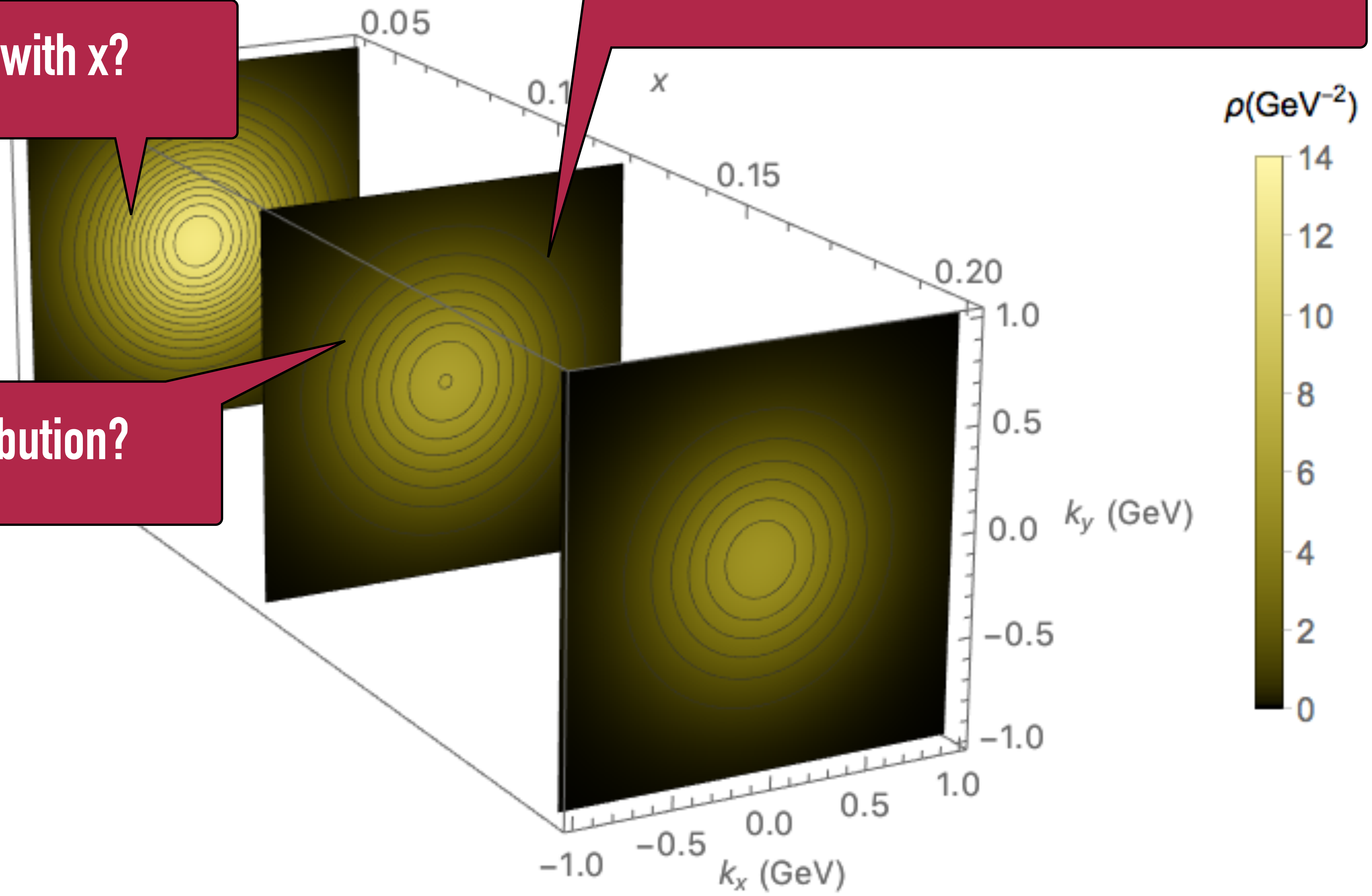
QUESTIONS



How does it change with x?

How “wide” is the distribution?

QUESTIONS

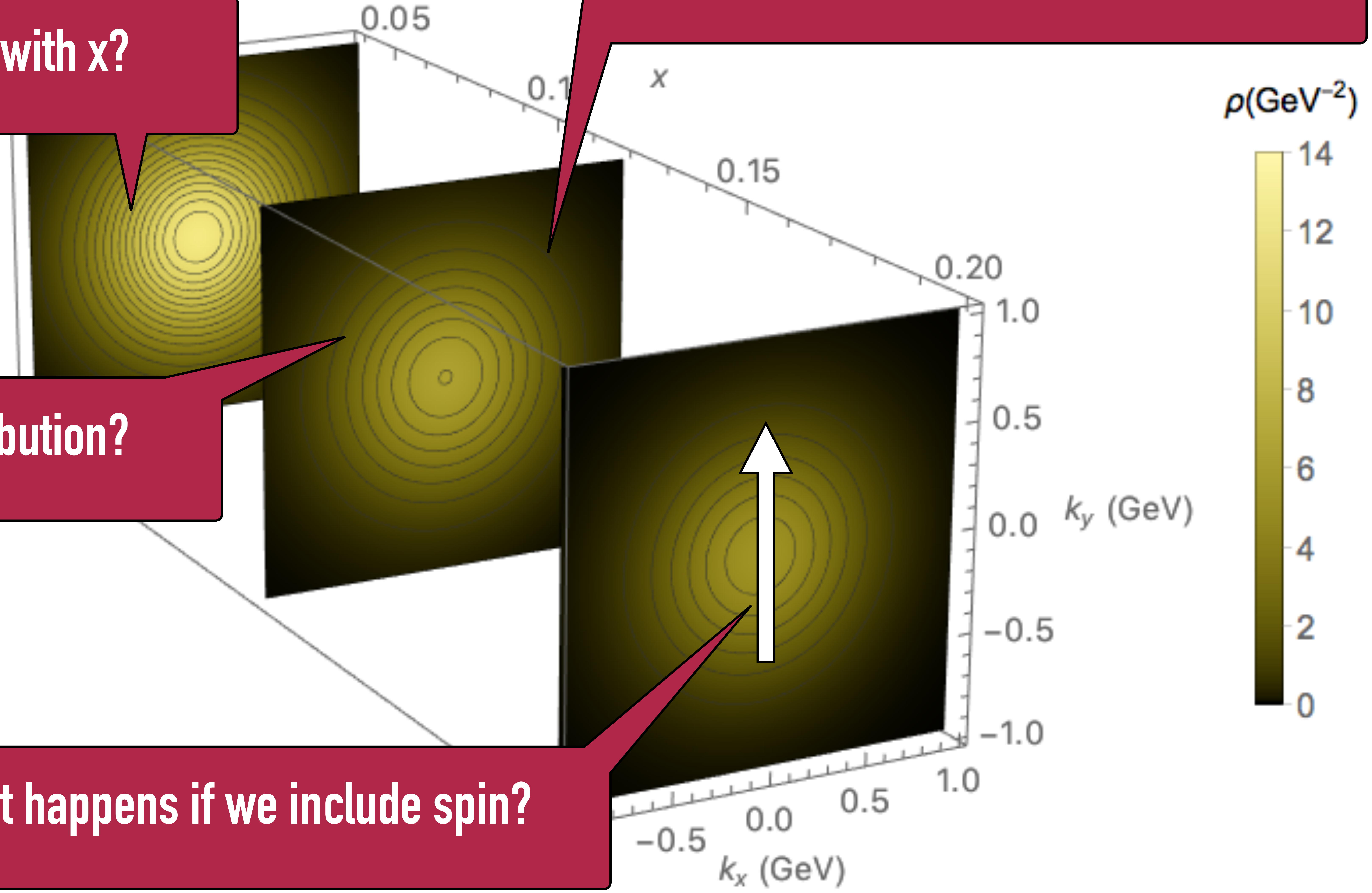


How does it change with x?

Is there a difference between flavors?

How “wide” is the distribution?

QUESTIONS



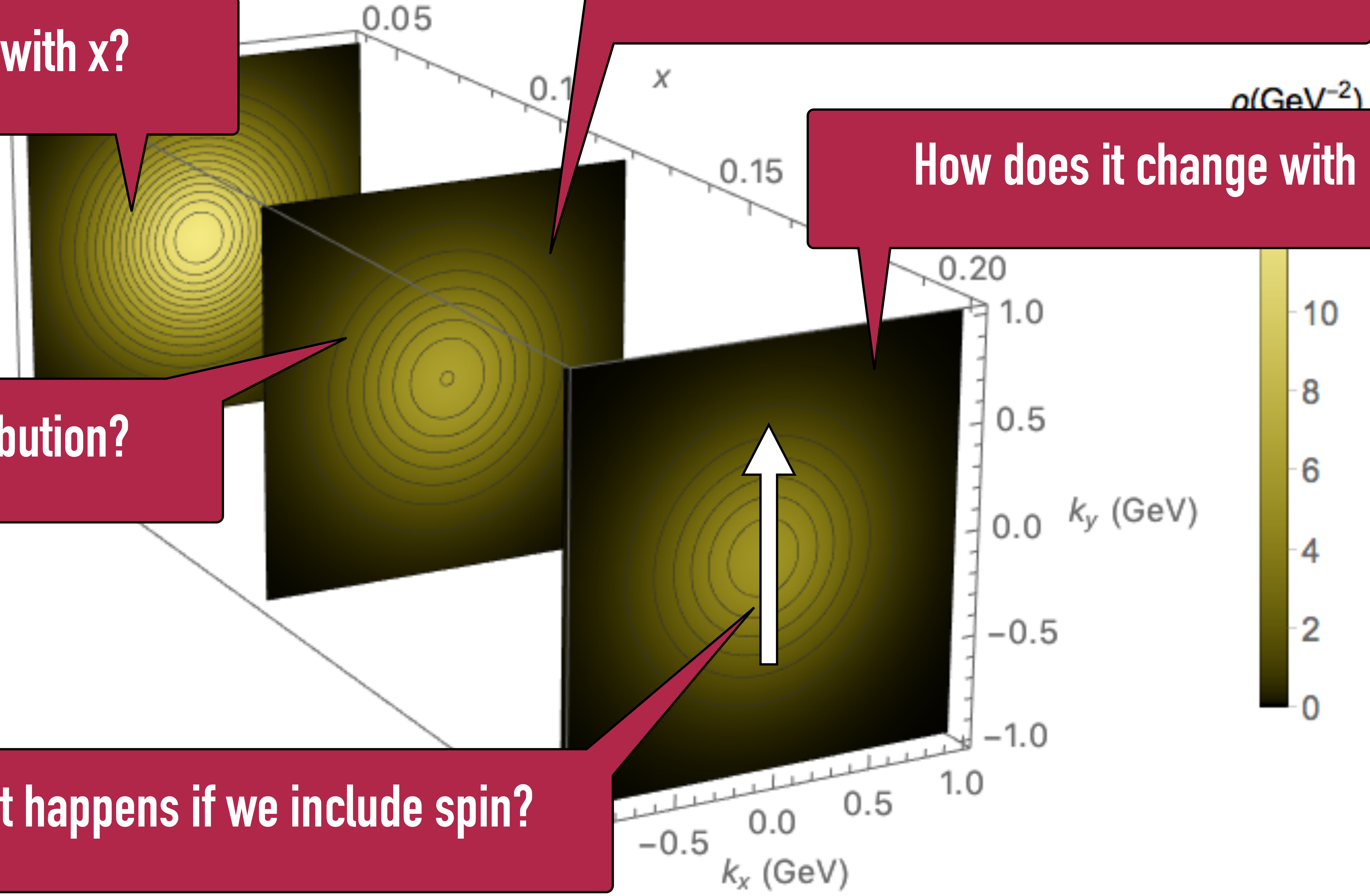
Is there a difference between flavors?

How does it change with x?

How “wide” is the distribution?

What happens if we include spin?

QUESTIONS



How does it change with x?

Is there a difference between flavors?

How does it change with Q?

How “wide” is the distribution?

What happens if we include spin?

TMD TABLES: QUARK, LEADING TWIST

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)

quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

TMDs in **black** survive integration over transverse momentum
TMDs in **red** are time-reversal odd

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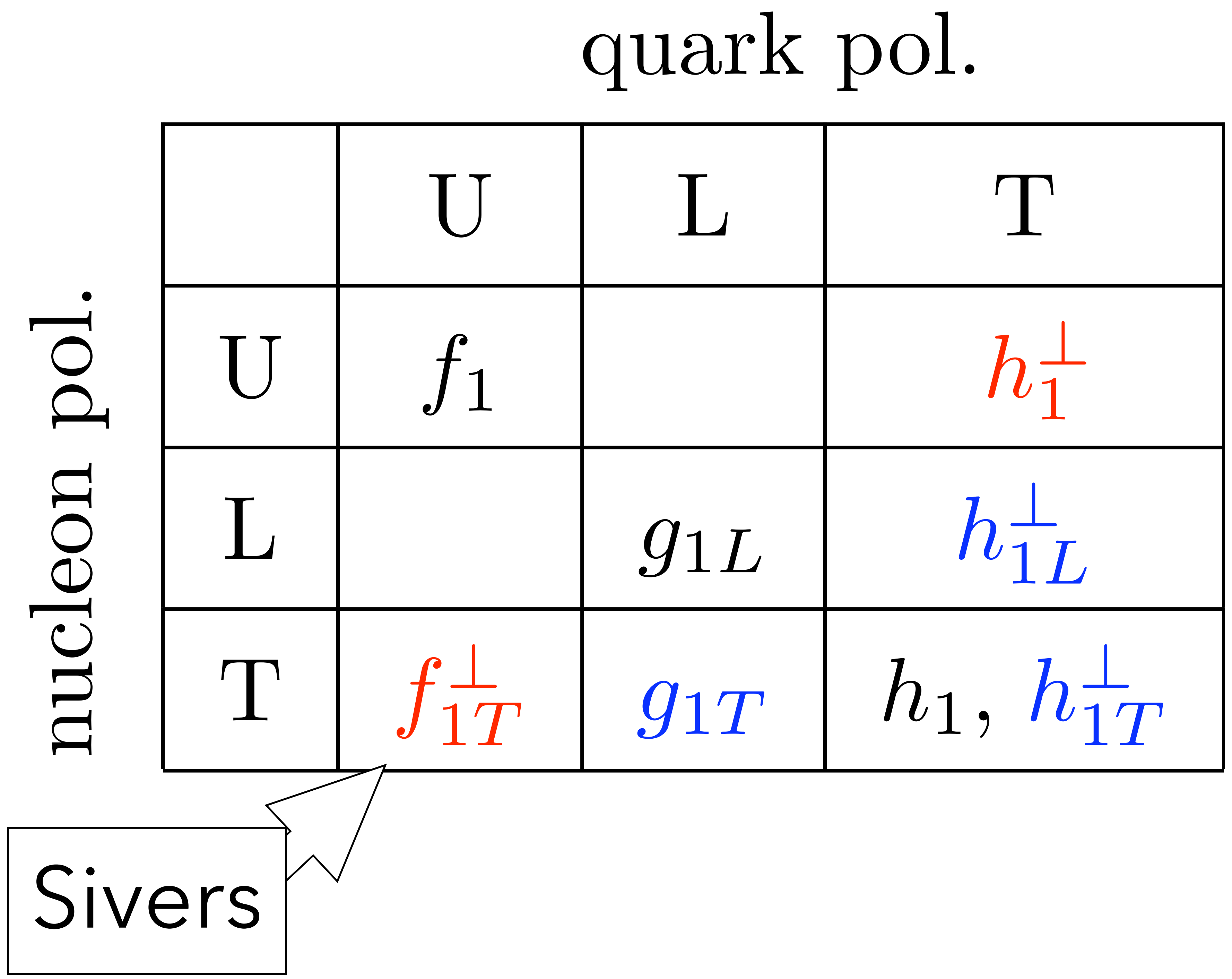

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Sivers

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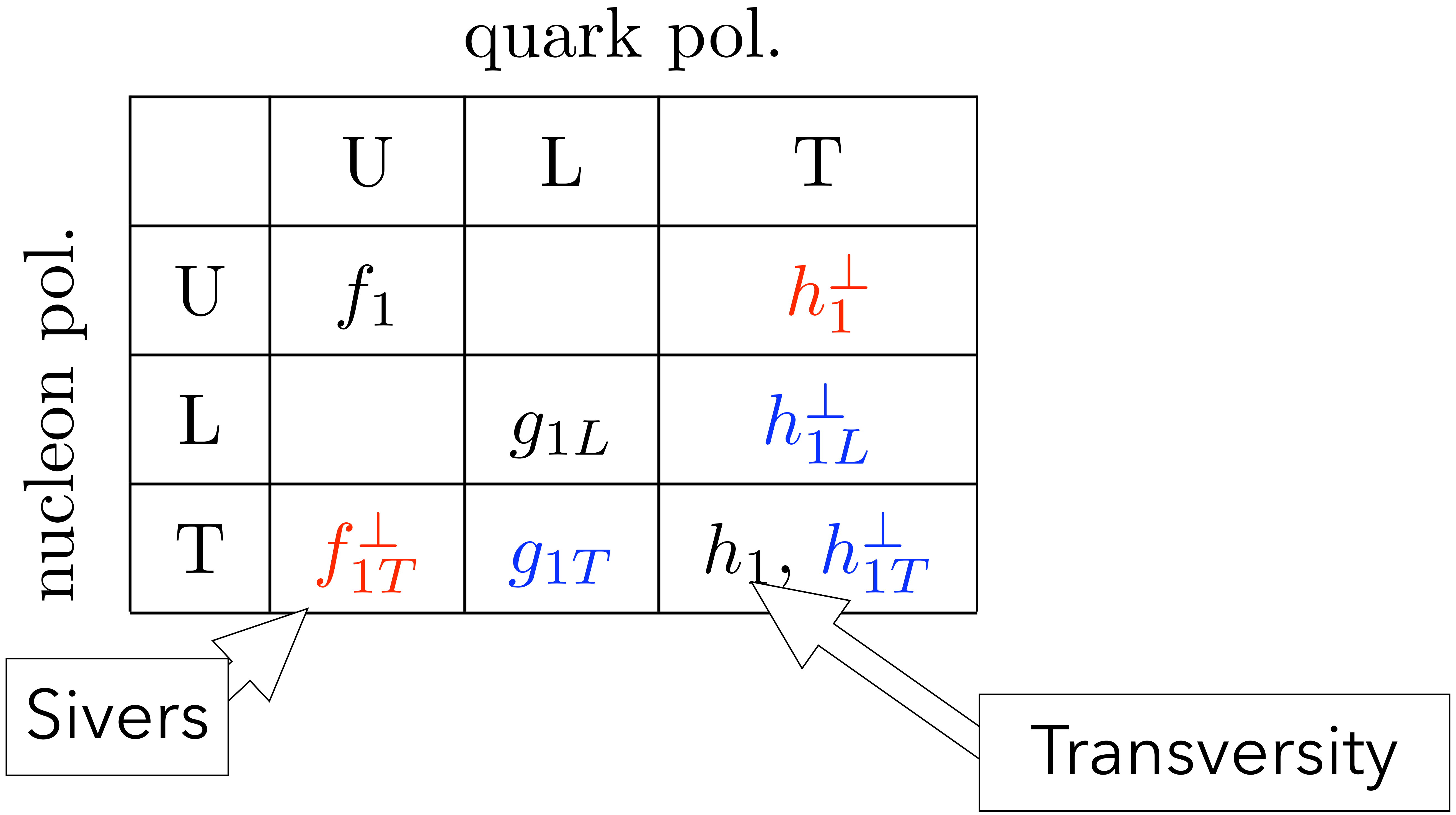

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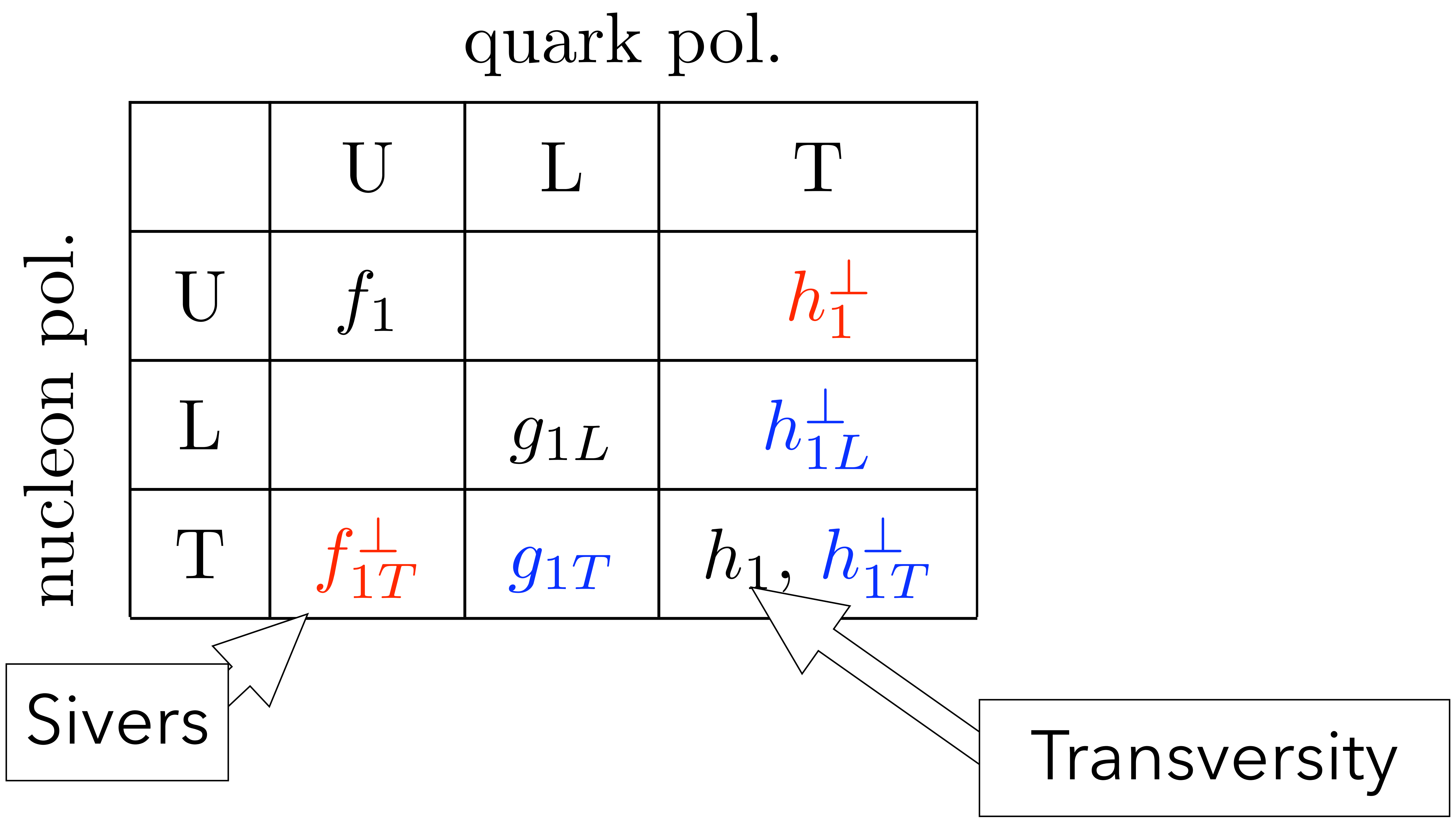

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Sivers

Transversity

- Very good knowledge of x dependence of f_1 and g_{1L}

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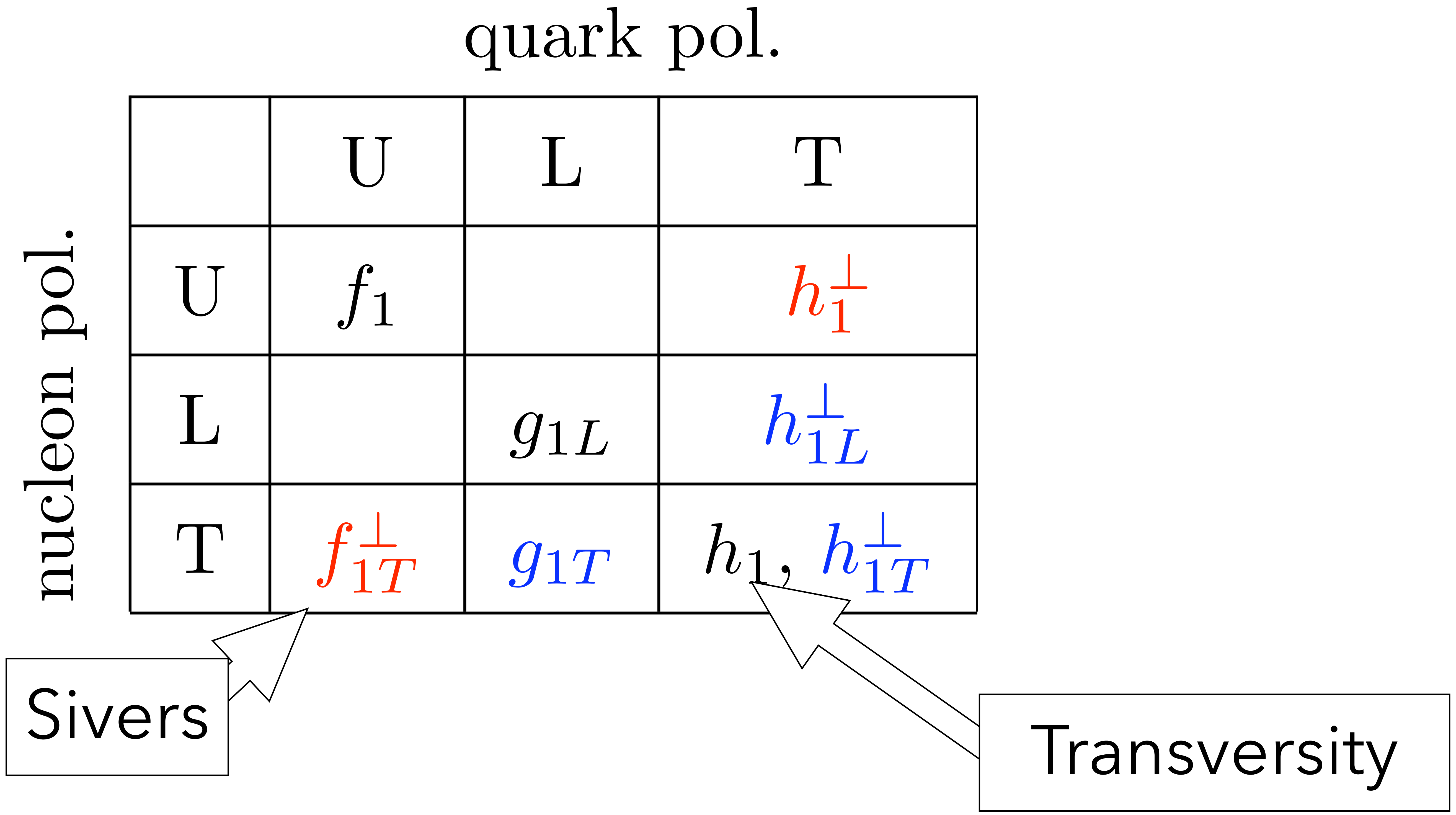

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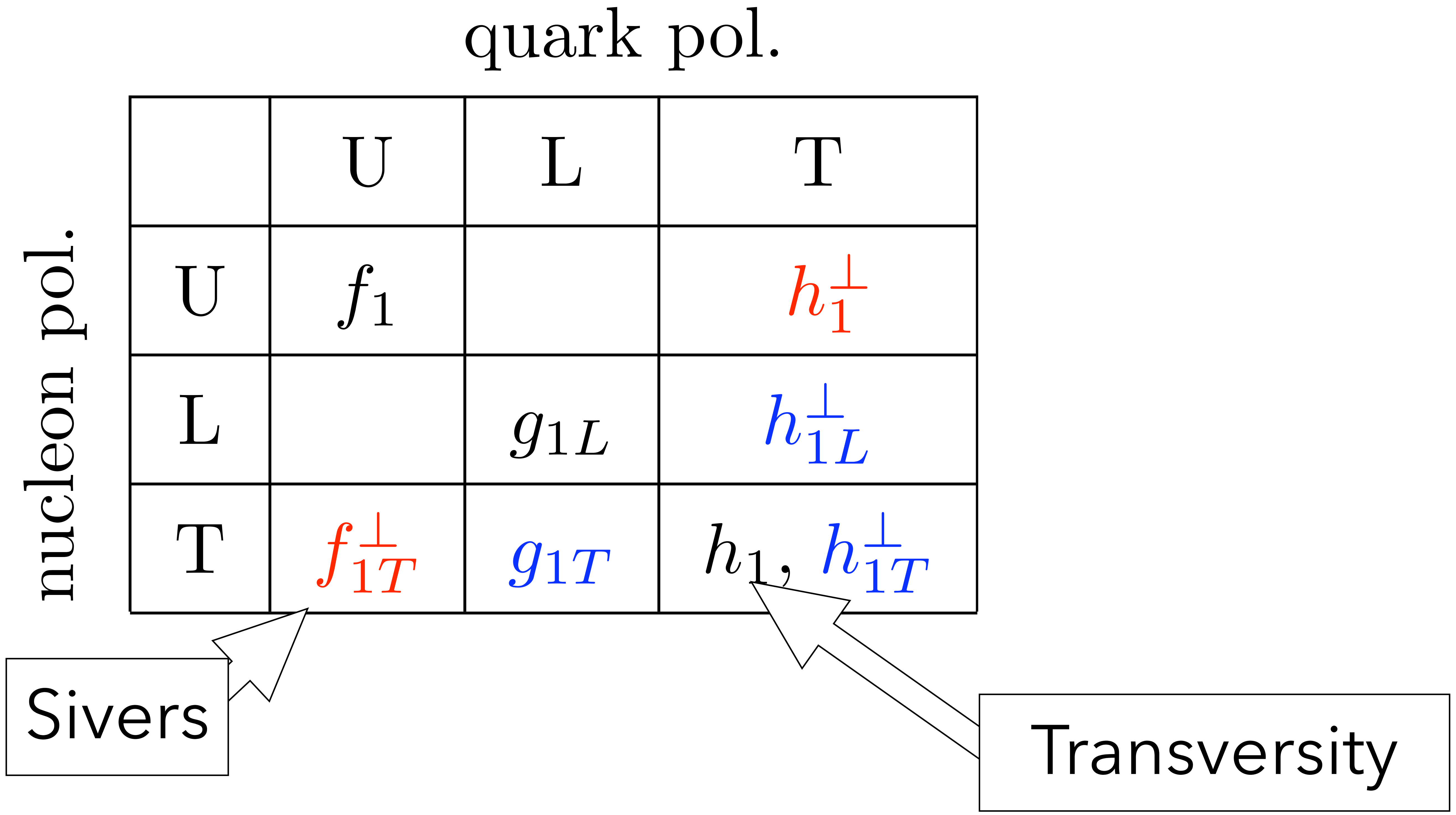

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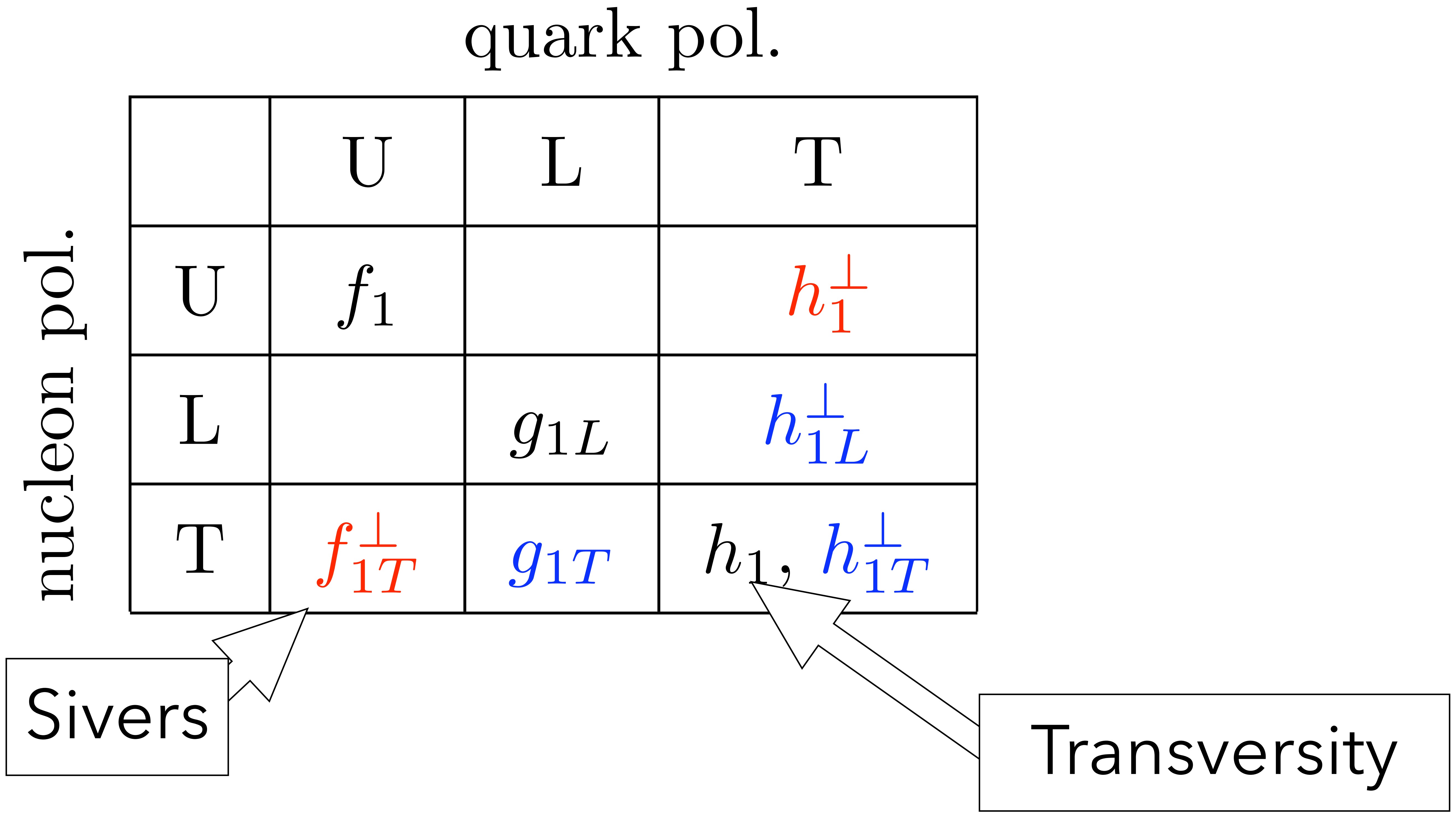

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nucleon pol.

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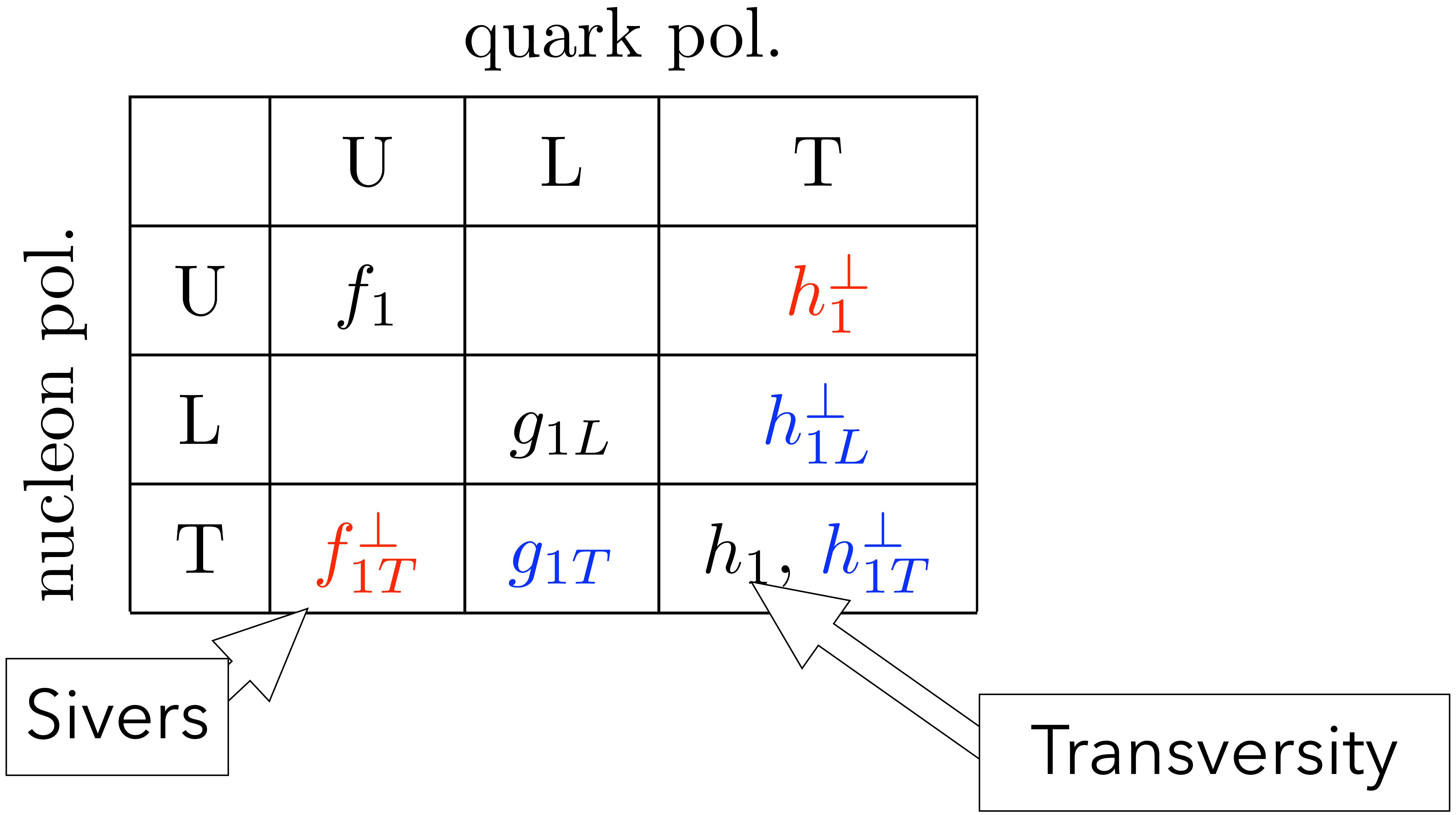

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- Fair knowledge of Sivers and transversity (mainly x dependence)

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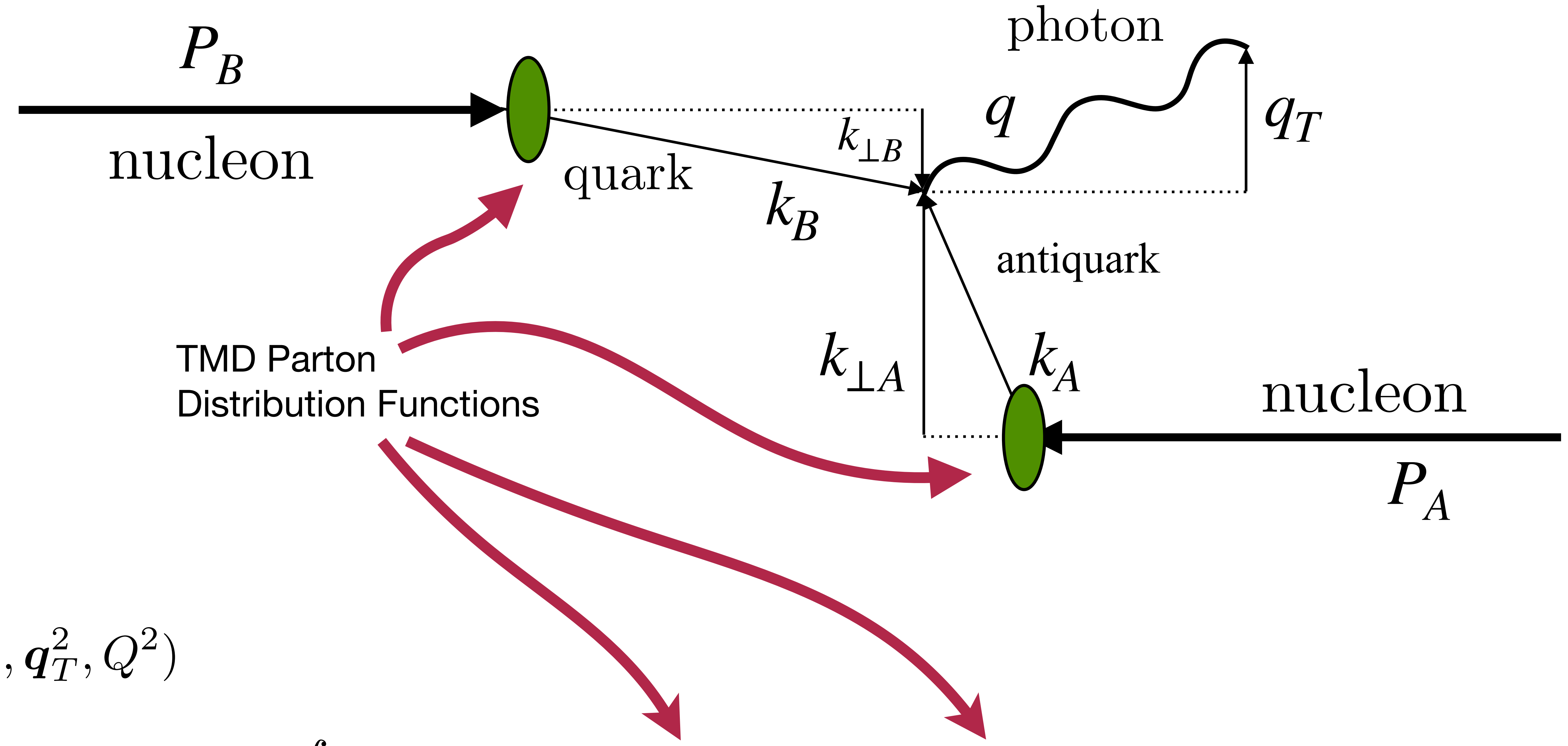
- Some hints about all others

TMDS IN DRELL-YAN PROCESSES



TMD Parton
Distribution Functions

$$F_{UU}^{1}(x_A, x_B, \boldsymbol{q}_T^2, Q^2)$$

$$= \sum_a \mathcal{H}_{UU}^{1a}(Q^2, \mu^2) \int d^2\boldsymbol{k}_{\perp A}\, d^2\boldsymbol{k}_{\perp B}\, f_1^a(x_A, \boldsymbol{k}_{\perp A}^2; \mu^2)\, f_1^{\bar{a}}(x_B, \boldsymbol{k}_{\perp B}^2; \mu^2)\, \delta^{(2)}(\boldsymbol{k}_{\perp A} - \boldsymbol{q}_T + \boldsymbol{k}_{\perp B})$$

$$+ Y_{UU}^{1}(Q^2, \boldsymbol{q}_T^2) + \mathcal{O}(M^2/Q^2)$$

Collins, Soper, Sterman, NPB250 (85)
see also talk Tommaso Rainaldi

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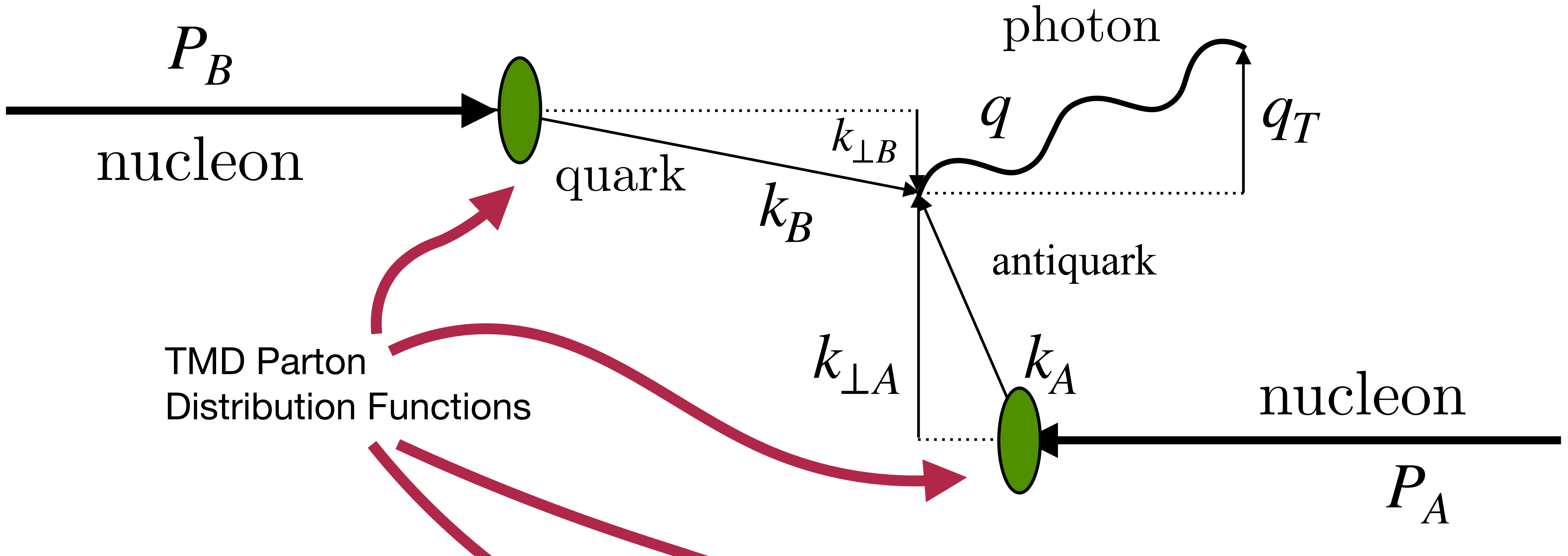

TMD Parton
Distribution Functions

$$F^1_{UU}(x_A, x_B, \boldsymbol{q}_T^2, Q^2)$$

$$= \overset{\text{W term}}{\boxed{\sum_a \mathcal{H}^{1a}_{UU}(Q^2,\mu^2) \int d^2\boldsymbol{k}_{\perp A}\, d^2\boldsymbol{k}_{\perp B}\, f_1^a(x_A, \boldsymbol{k}^2_{\perp A}; \mu^2)\, f_1^{\bar{a}}(x_B, \boldsymbol{k}^2_{\perp B}; \mu^2)\, \delta^{(2)}(\boldsymbol{k}_{\perp A} - \boldsymbol{q}_T + \boldsymbol{k}_{\perp B})}}$$

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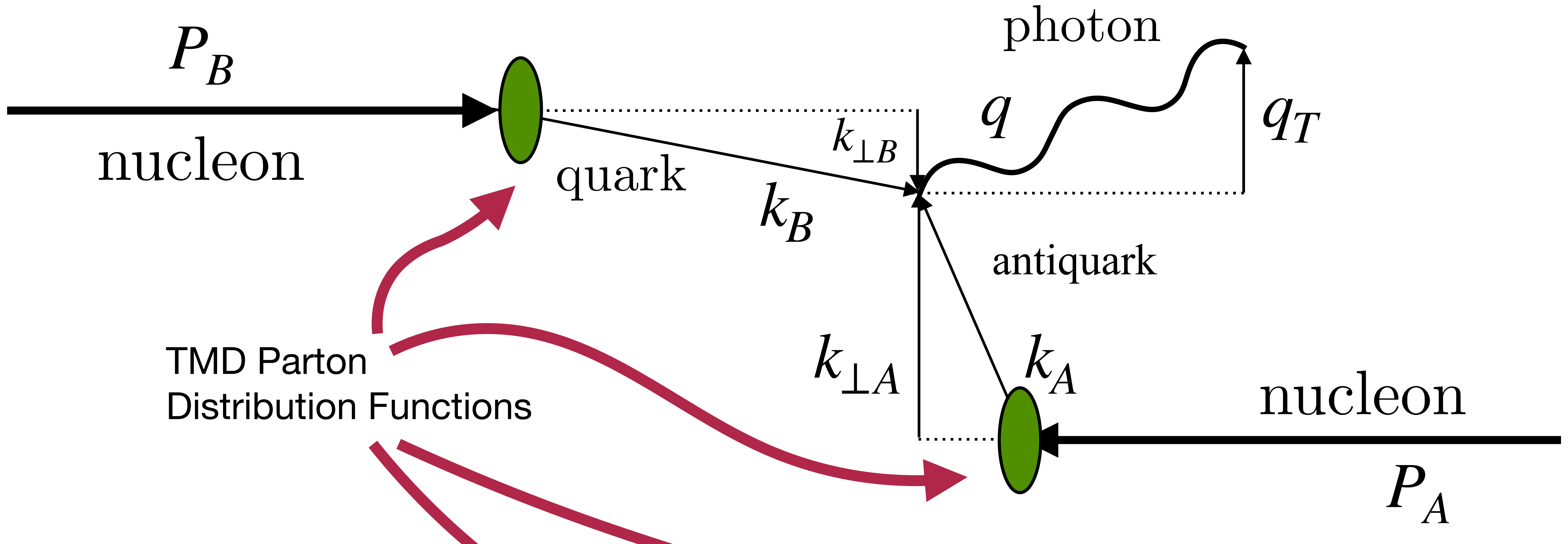

TMD Parton Distribution Functions

W term

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hard factor

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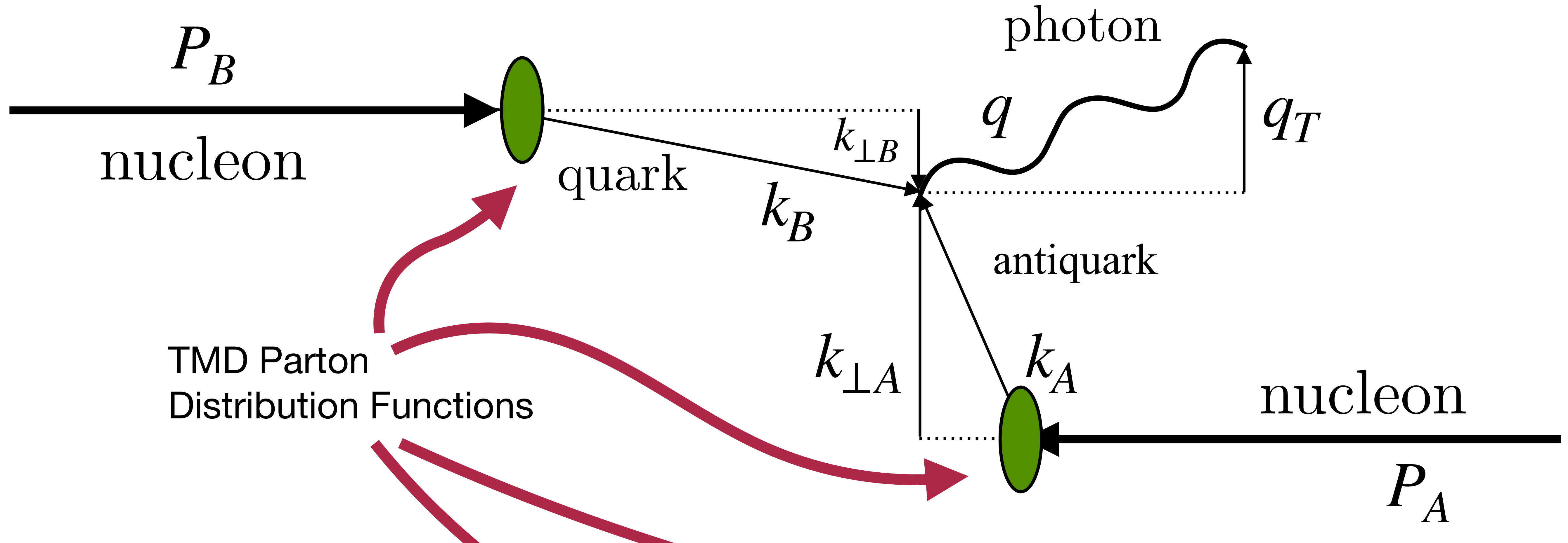
TMDS IN DRELL-YAN PROCESSES



TMD Parton
Distribution Functions

$$F^1_{UU}(x_A, x_B, \boldsymbol{q}_T^2, Q^2) = \overbrace{\sum_a \mathcal{H}^{1a}_{UU}(Q^2,\mu^2)\int d^2\boldsymbol{k}_{\perp A}\, d^2\boldsymbol{k}_{\perp B}\, f_1^a(x_A,\boldsymbol{k}^2_{\perp A};\mu^2)\, f_1^{\bar a}(x_B,\boldsymbol{k}^2_{\perp B};\mu^2)\,\delta^{(2)}(\boldsymbol{k}_{\perp A}-\boldsymbol{q}_T+\boldsymbol{k}_{\perp B})}^{\text{W term}} + Y^1_{UU}(Q^2,\boldsymbol{q}_T^2)+\mathcal{O}(M^2/Q^2)$$

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Y term

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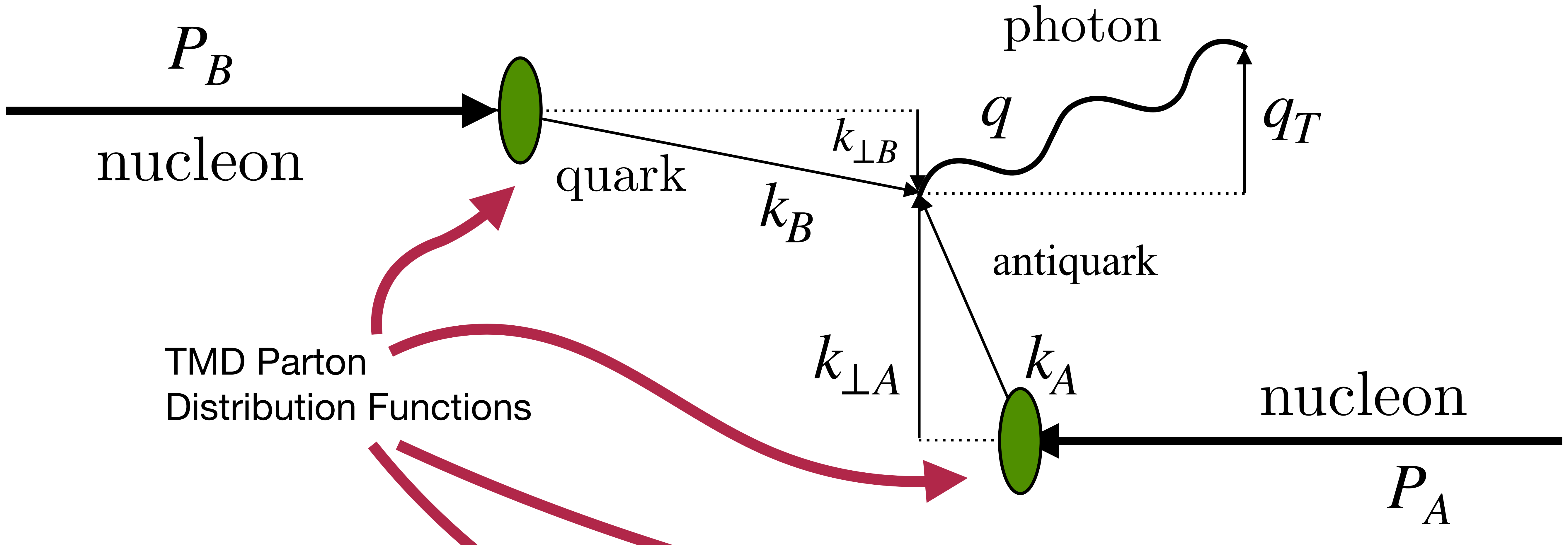

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higher twist contributions

Collins, Soper, Sterman, NPB250 (85)
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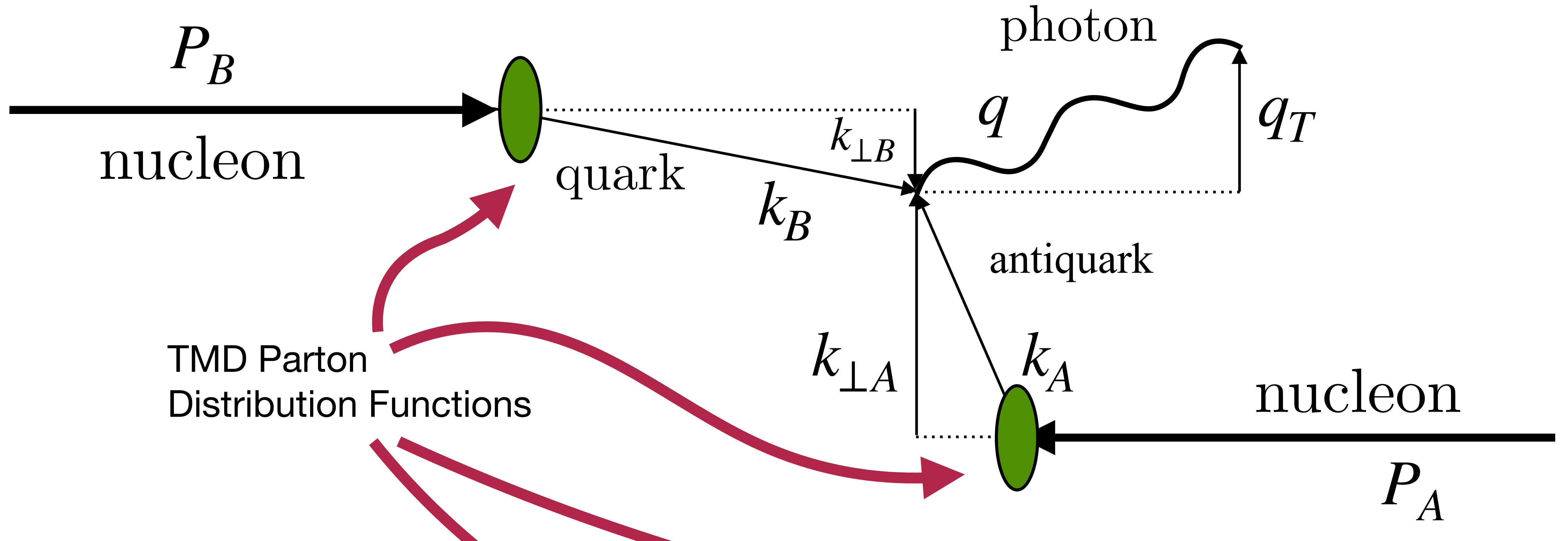

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$$+\, Y^1_{UU}(Q^2, \boldsymbol{q}_T^2) + \mathcal{O}(M^2/Q^2)$$

hard factor

Y term

higher twist contributions

The W term dominates at low transverse momentum ($q_T \ll Q$) and contains the TMDs

Collins, Soper, Sterman, NPB250 (85)

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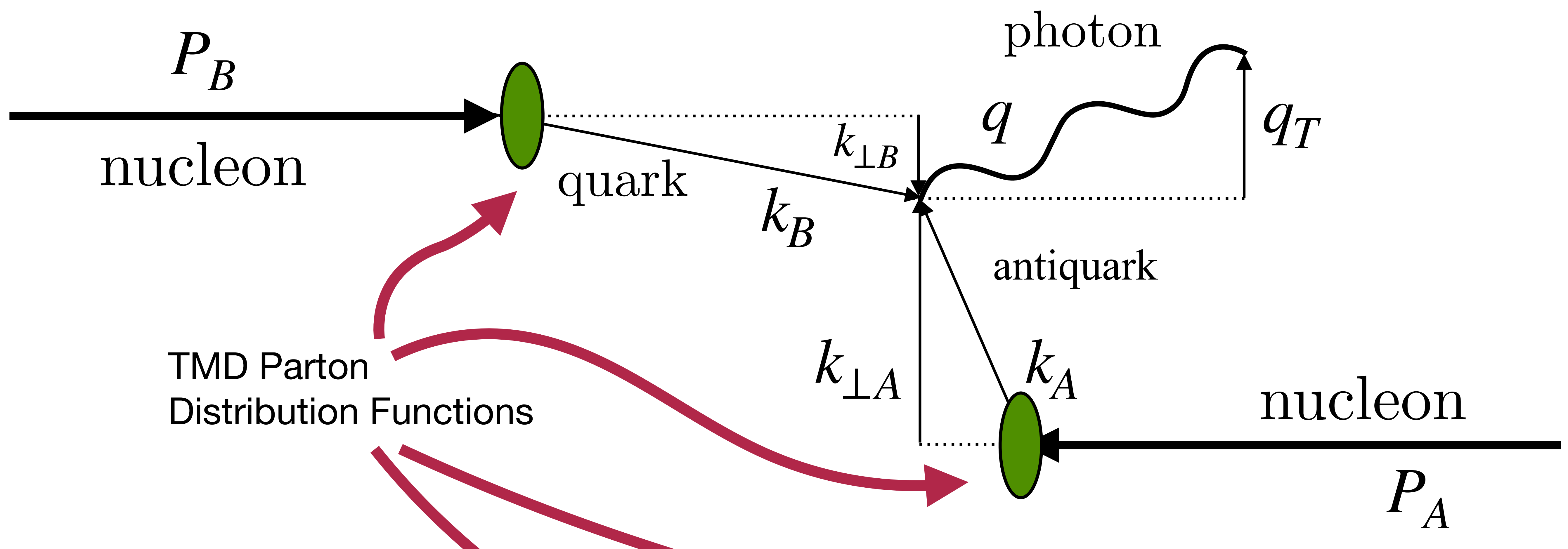

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So far, the Y term has been neglected in TMD extractions

Collins, Soper, Sterman, NPB250 (85)

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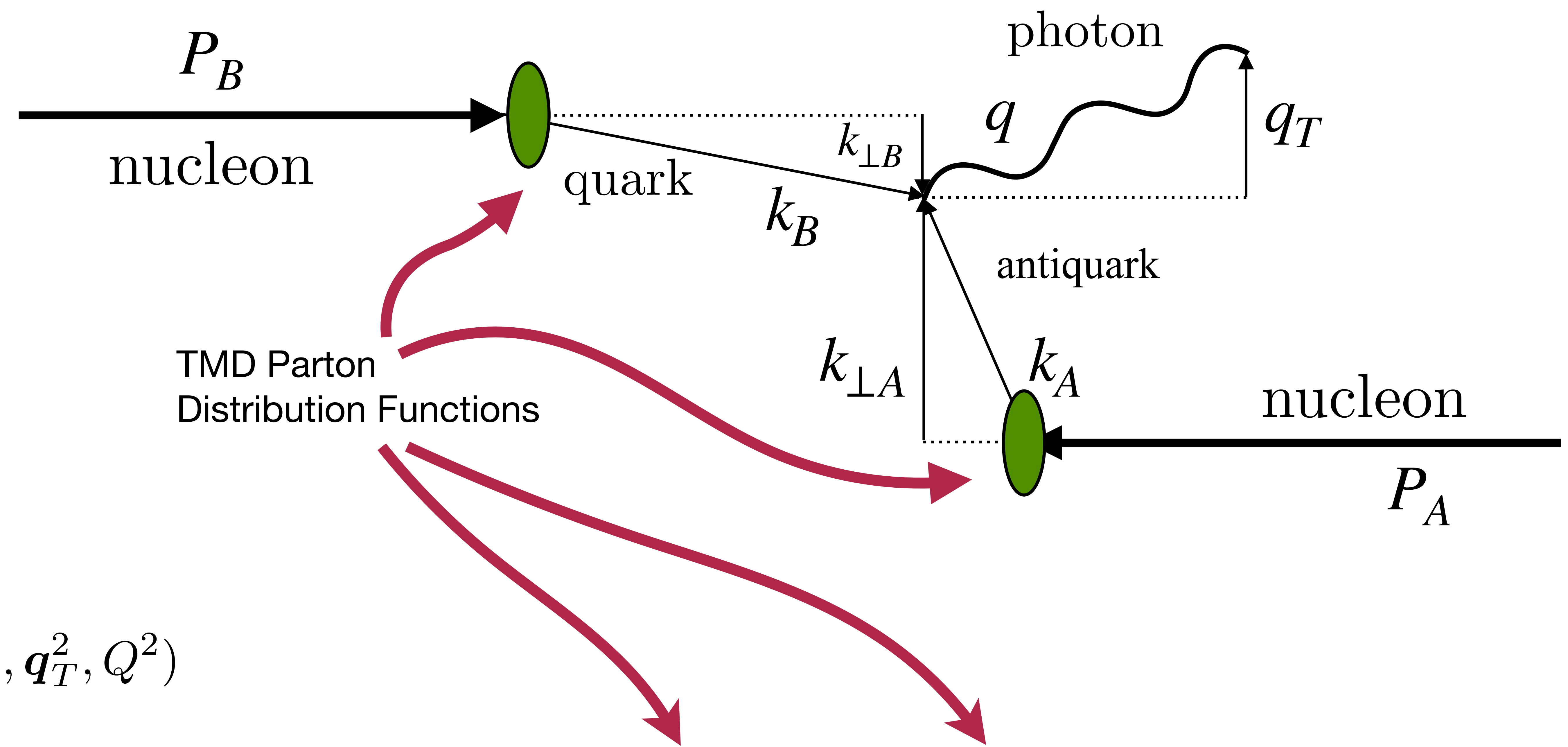

TMD Parton
Distribution Functions

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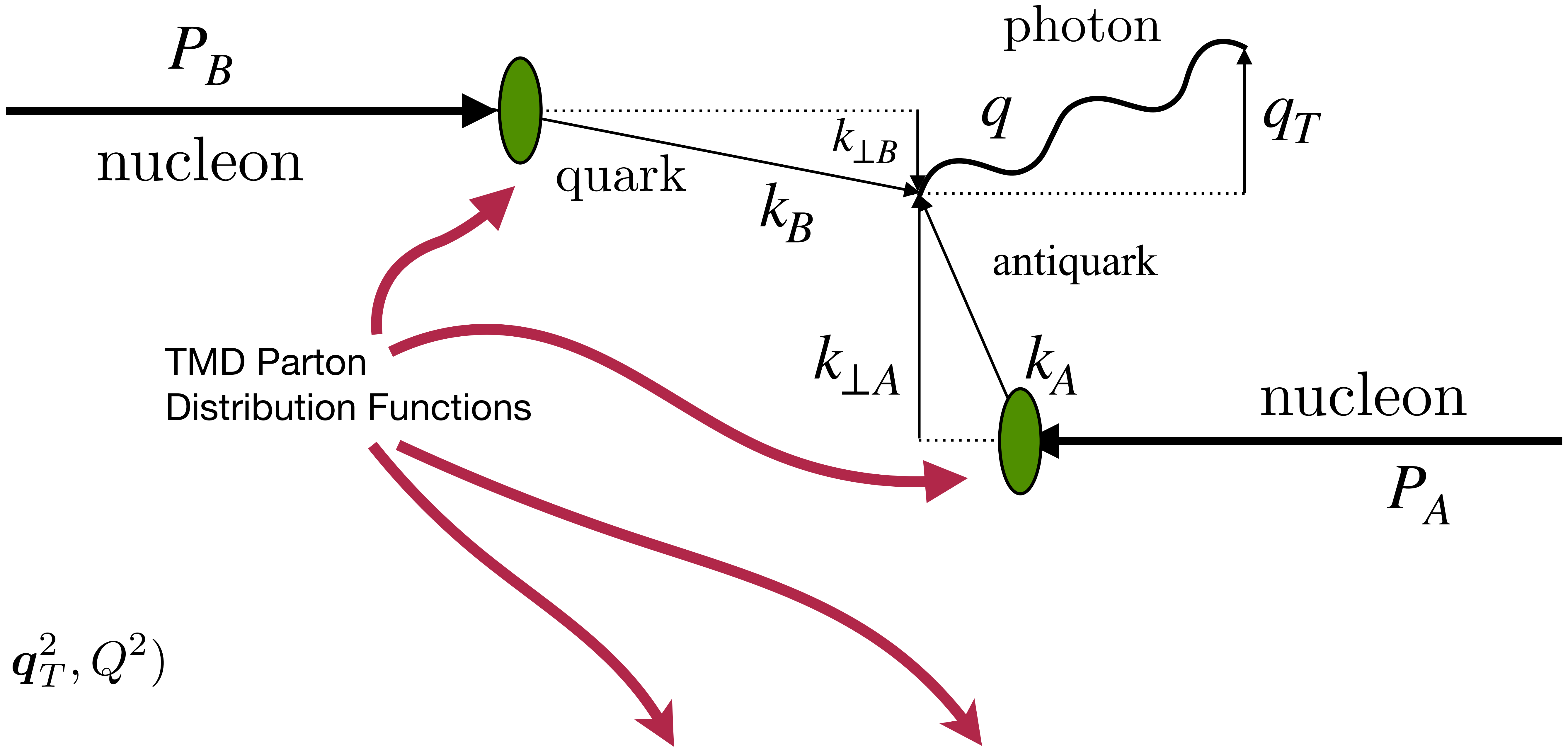
$$\approx \sum_q \mathcal{H}^{1q}_{UU}(Q^2, \mu^2) \int d^2\boldsymbol{k}_{\perp A}\, d^2\boldsymbol{k}_{\perp B}\, f_1^q\left(x_A, \boldsymbol{k}^2_{\perp A}; \mu^2\right)\, f_1^{\bar{q}}\left(x_B, \boldsymbol{k}^2_{\perp B}; \mu^2\right)\, \delta^{(2)}\left(\boldsymbol{k}_{\perp A} - \boldsymbol{q}_T + \boldsymbol{k}_{\perp B}\right)$$

Collins, Soper, Sterman, NPB250 (85)

TMDS IN DRELL-YAN PROCESSES



TMD Parton
Distribution Functions

$$F_{UU}^{1}(x_A, x_B, \boldsymbol{q}_T^2, Q^2)$$

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$$= \sum_q \mathcal{H}_{UU}^{1q}(Q^2, \mu^2) \int db_T b_T J_0(b_T |\boldsymbol{q}_T|) \hat{f}_1^q(x_A, b_T^2; \mu^2)\, \hat{f}_1^{\bar{q}}(x_B, b_T^2; \mu^2)$$

The analysis is usually done in Fourier-transformed space

Collins, Soper, Sterman, NPB250 (85)

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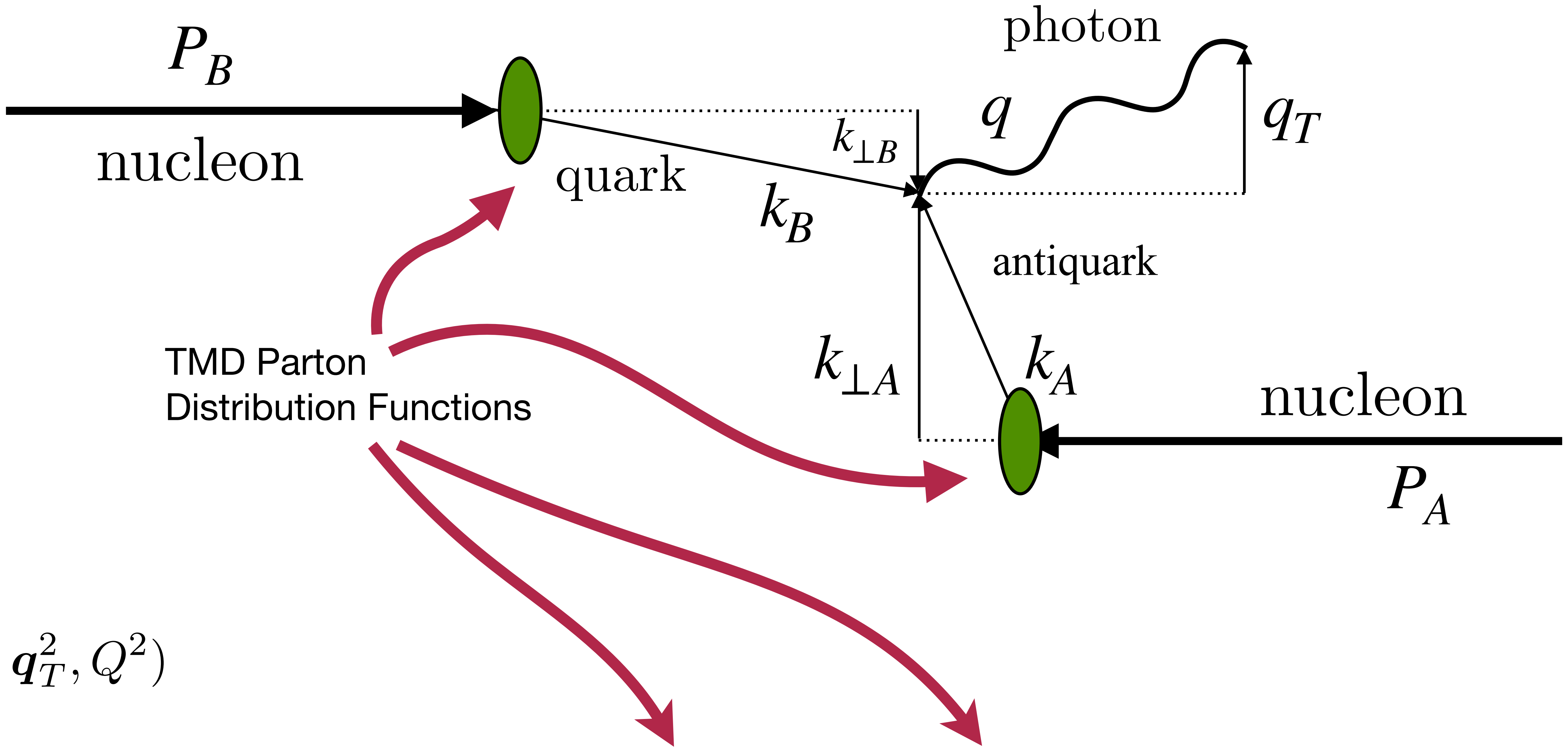

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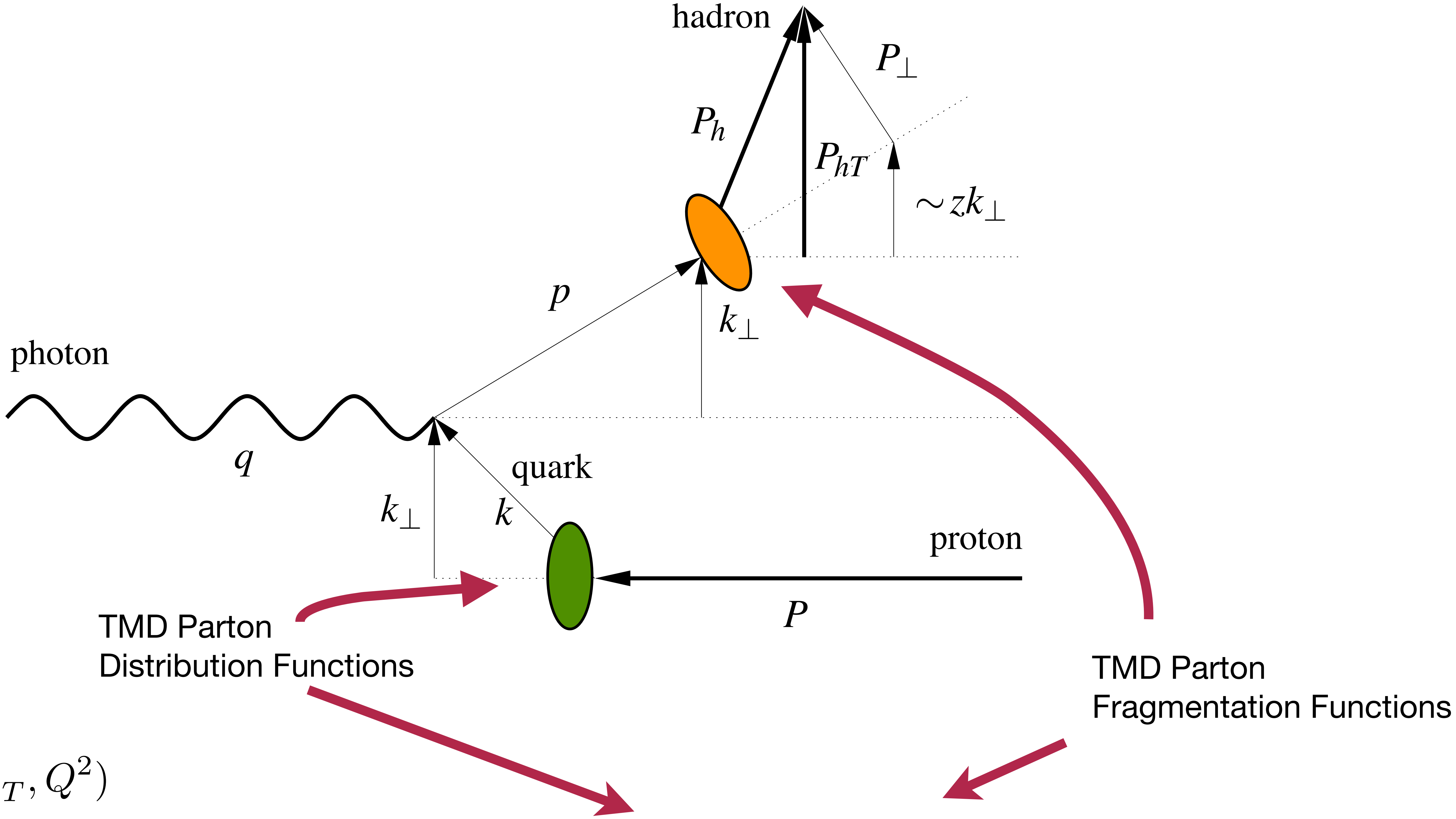
TMDs formally depend on two scales, but we set them equal.

Collins, Soper, Sterman, NPB250 (85)

TMDS IN SEMI-INCLUSIVE DIS (SIDIS)



TMD Parton
Distribution Functions

TMD Parton
Fragmentation Functions

$$
\begin{aligned}
&F_{UU,T}(x,z,\boldsymbol{P}_{hT}^2,Q^2)\\
&\quad= x\sum_q \mathcal{H}^q_{UU,T}(Q^2,\mu^2)\int d^2\boldsymbol{k}_\perp\, d^2\boldsymbol{P}_\perp\, f_1^a(x,\boldsymbol{k}_\perp^2;\mu^2)\, D_1^{a\to h}(z,\boldsymbol{P}_\perp^2;\mu^2)\,\delta(z\boldsymbol{k}_\perp-\boldsymbol{P}_{hT}+\boldsymbol{P}_\perp)\\
&\quad= x\sum_a \mathcal{H}^q_{UU,T}(Q^2,\mu^2)\int db_T b_T J_0(b_T|\boldsymbol{P}_{h\perp}|)\hat{f}_1^q(x,z^2b_\perp^2;\mu^2)\,\hat{D}_1^{a\to h}(z,b_\perp^2;\mu^2)
\end{aligned}
$$

TMD STRUCTURE

$$\hat{f}_1^a(x, |\boldsymbol{b}_T|; \mu, \zeta) = \int d^2\boldsymbol{k}_\perp \, e^{i\boldsymbol{b}_T \cdot \boldsymbol{k}_\perp} \, f_1^a(x, \boldsymbol{k}_\perp^2; \mu, \zeta)$$

see, e.g., Collins, "Foundations of Perturbative QCD" (11)
TMD collaboration, "TMD Handbook," arXiv:2304.03302

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see, e.g., Collins, "Foundations of Perturbative QCD" (11)
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perturbative Sudakov
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collinear PDF

Collins-Soper kernel
(perturbative and
nonperturbative)

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matching coefficients
(perturbative)

see, e.g., Collins, "Foundations of Perturbative QCD" (11)
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TMD STRUCTURE

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$$\mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

$$\Big\Downarrow$$

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collinear PDF

matching coefficients
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Collins-Soper kernel
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nonperturbative)

nonperturbative part
of TMD

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

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nonperturbative part of TMD

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

↓

$$\mu_{b*} = \frac{2e^{-\gamma_E}}{\bar{b}_*}$$

Some choices can be different in different extractions, but the overall results for the TMD should be compatible

f Perturbative QCD" (11)

ook," arXiv:2304.03302

LATEST PUBLISHED RESULTS (2022)

TMD GLOBAL FITS

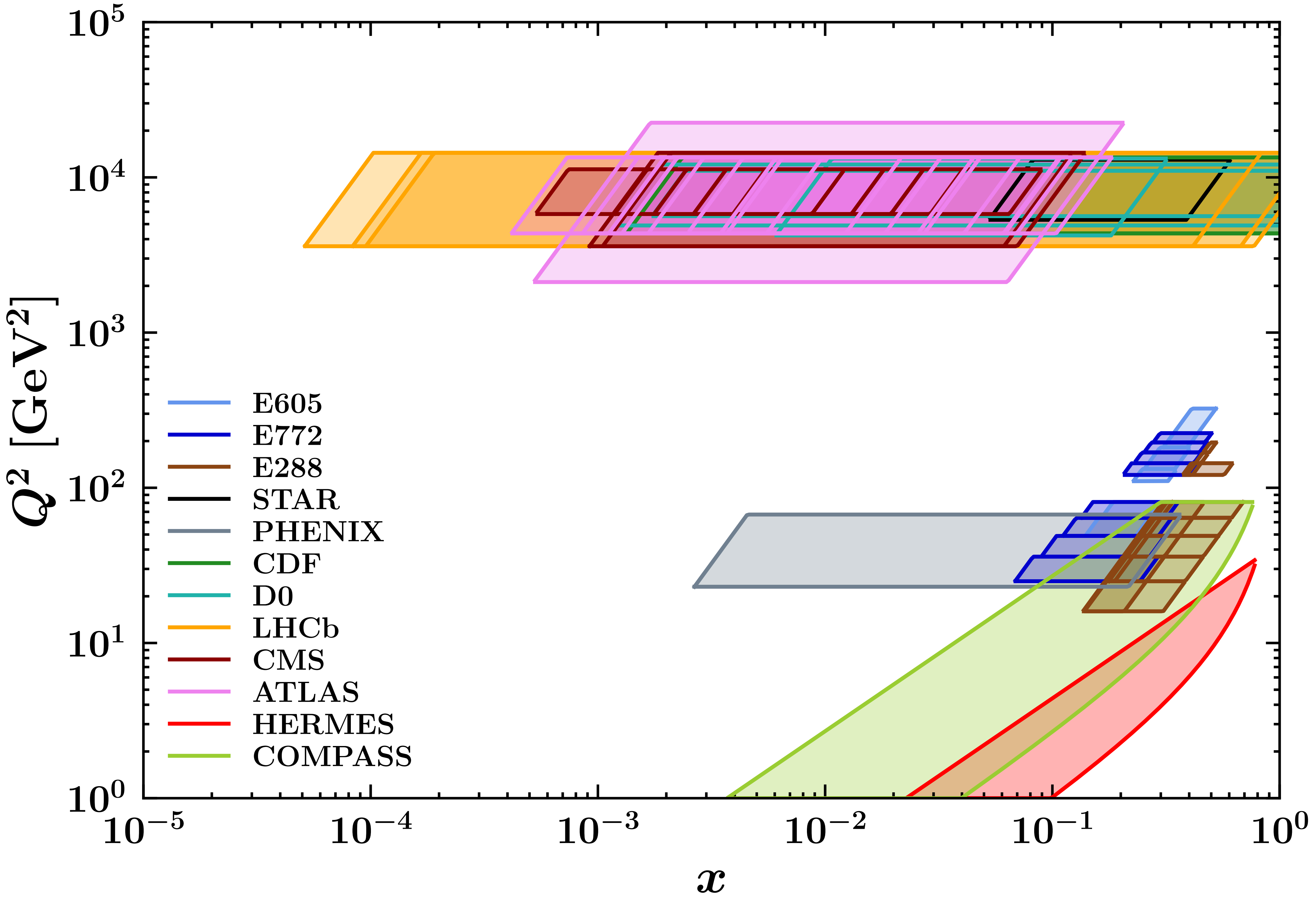
	Accuracy	SIDIS HERMES	SIDIS COMPASS	DY fixed target	DY collider	N of points	χ^2/N_{points}
Pavia 2017 arXiv:1703.10157	NLL	✓	✓	✓	✓	8059	1.55
SV 2019 arXiv:1912.06532	N^3LL^-	✓	✓	✓	✓	1039	1.06
MAP22 arXiv:2206.07598	N^3LL^-	✓	✓	✓	✓	2031	1.06

MAP22: the "Monte Barone" fit

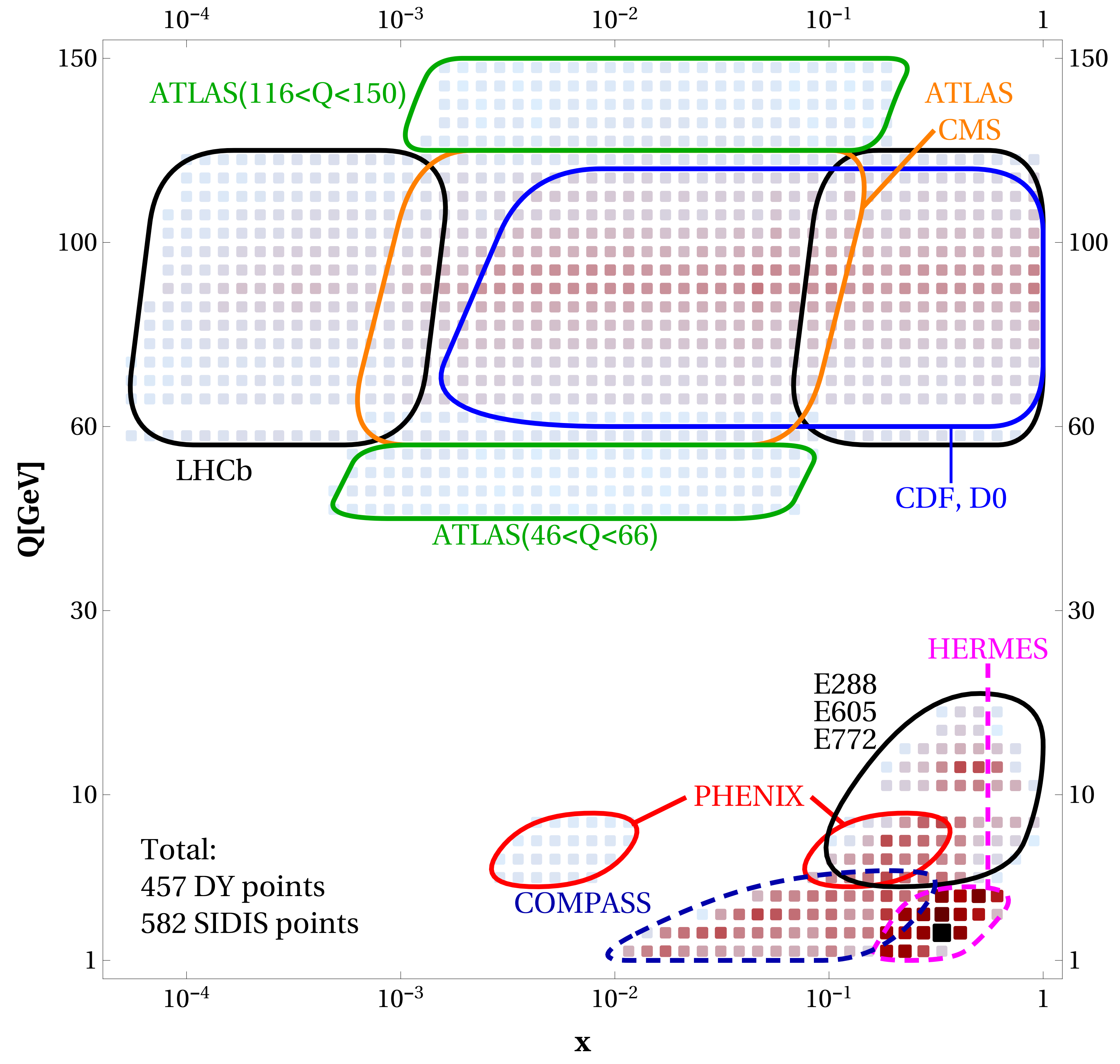
Monte Barone, Piemonte, 2044 m

Summit of Monte Barone, June 2023

x-Q^2 COVERAGE



MAP Collaboration
Bacchetta, Bertone, Bissolotti, Bozzi, Cerutti, Piacenza, Radici, Signori, arXiv:2206.07598



Scimemi, Vladimirov, arXiv:1912.06532

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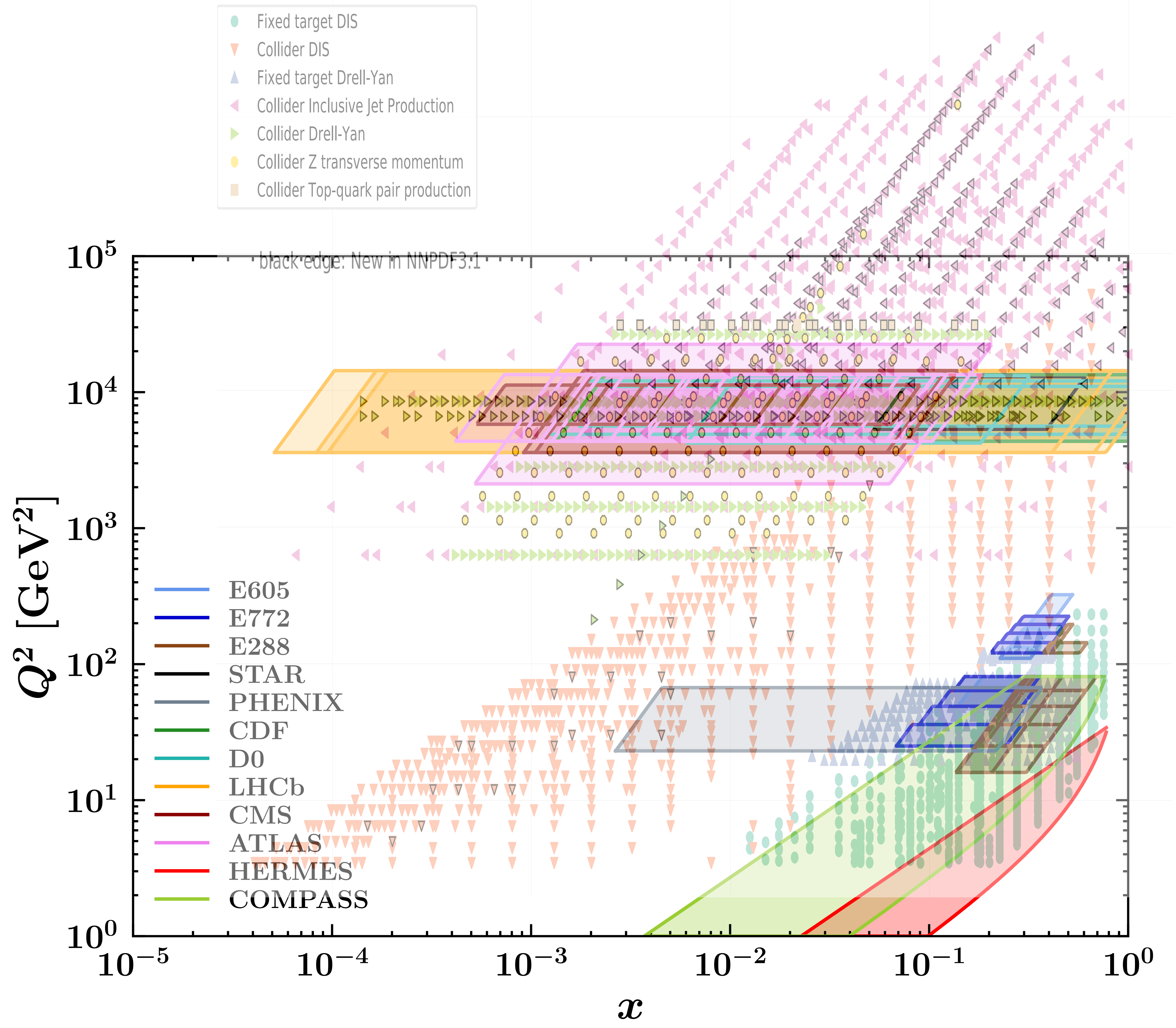

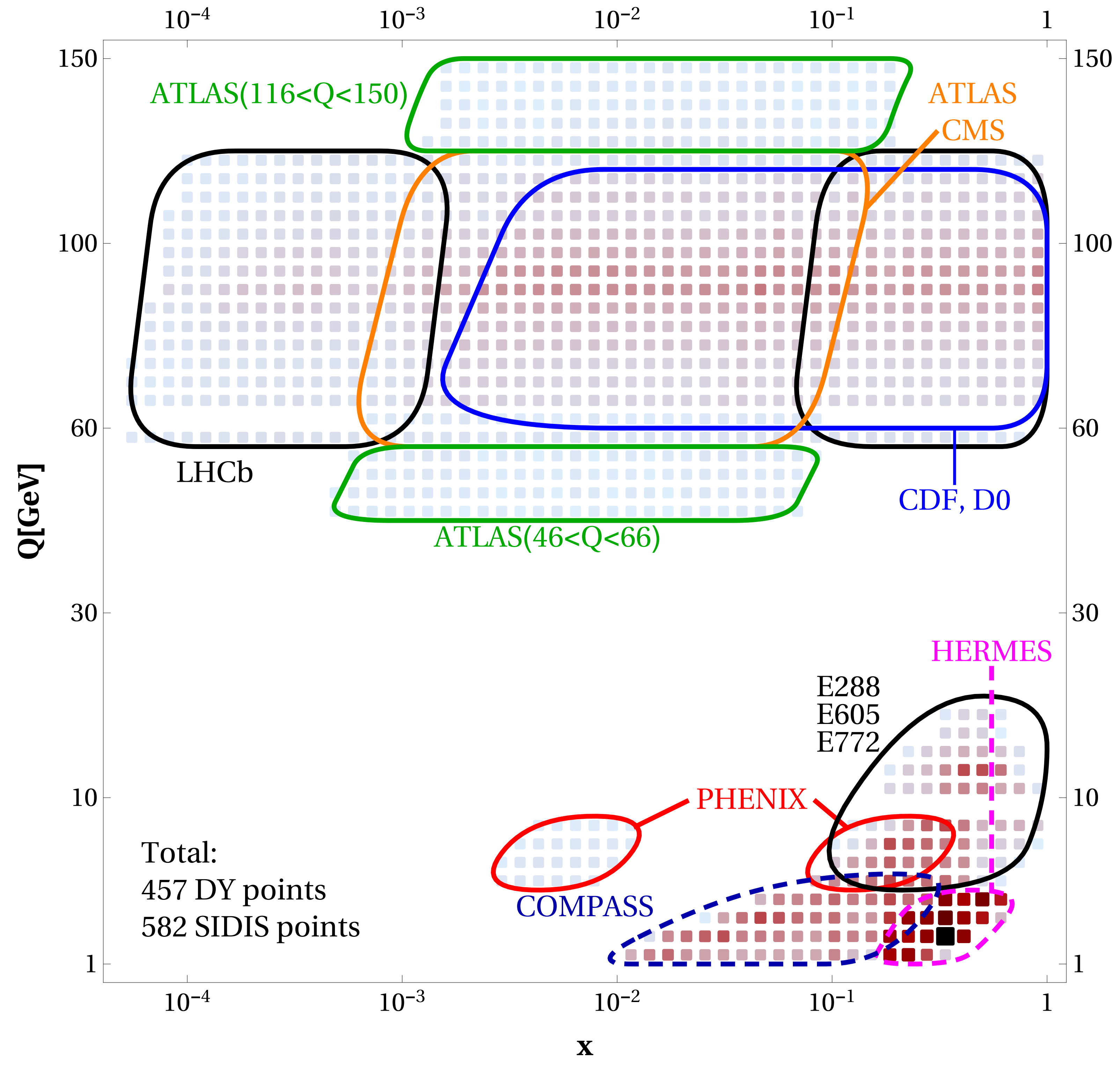

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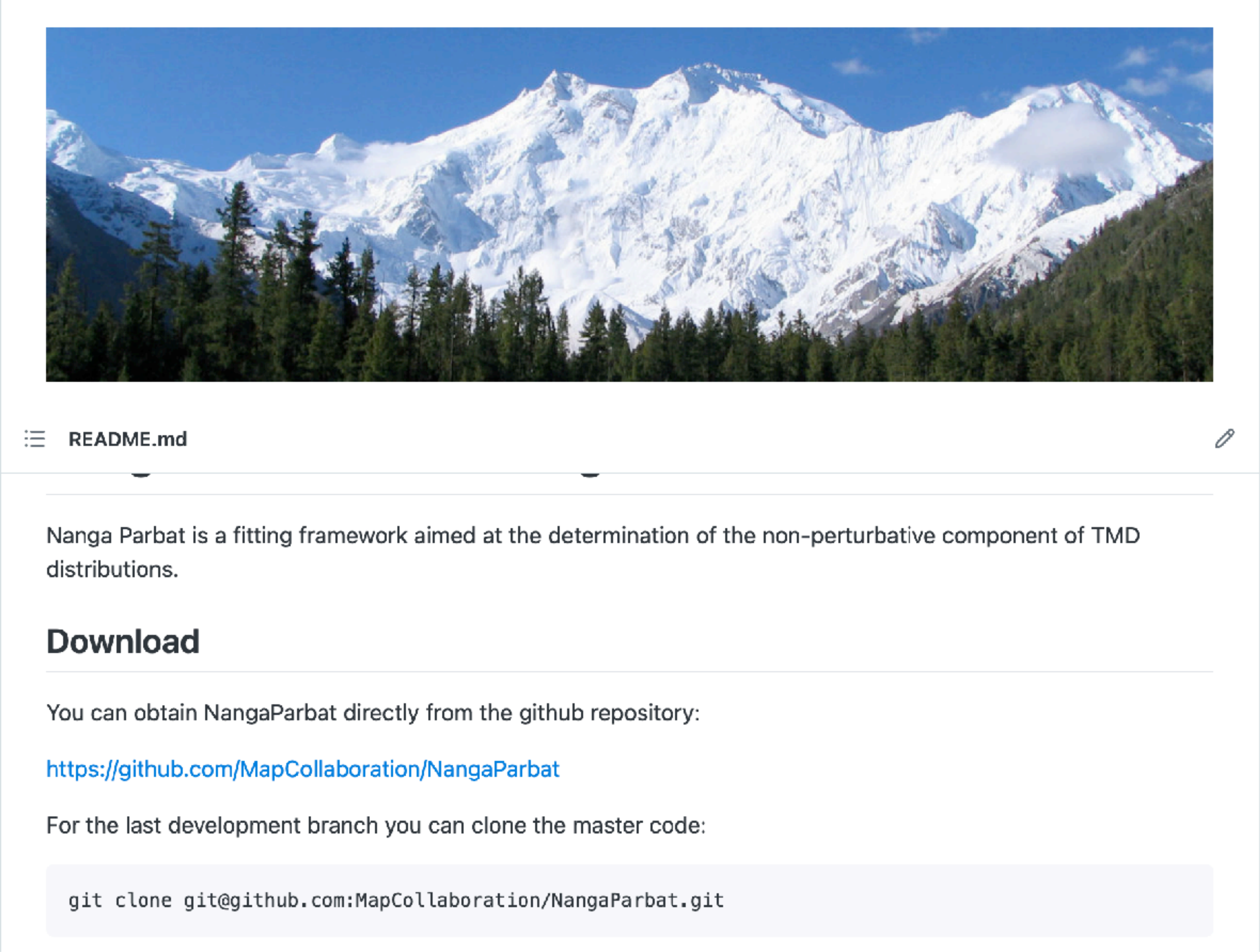
Bacchetta, Bertone, Bissolotti, Bozzi, Cerutti, Piacenza, Radici, Signori, arXiv:2206.07598

Scimemi, Vladimirov, arXiv:1912.06532

AVAILABLE TOOLS: NANGA PARBAT

https://github.com/MapCollaboration/NangaParbat



Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

Download

You can obtain NangaParbat directly from the github repository:

https://github.com/MapCollaboration/NangaParbat

For the last development branch you can clone the master code:

```
git clone git@github.com:MapCollaboration/NangaParbat.git
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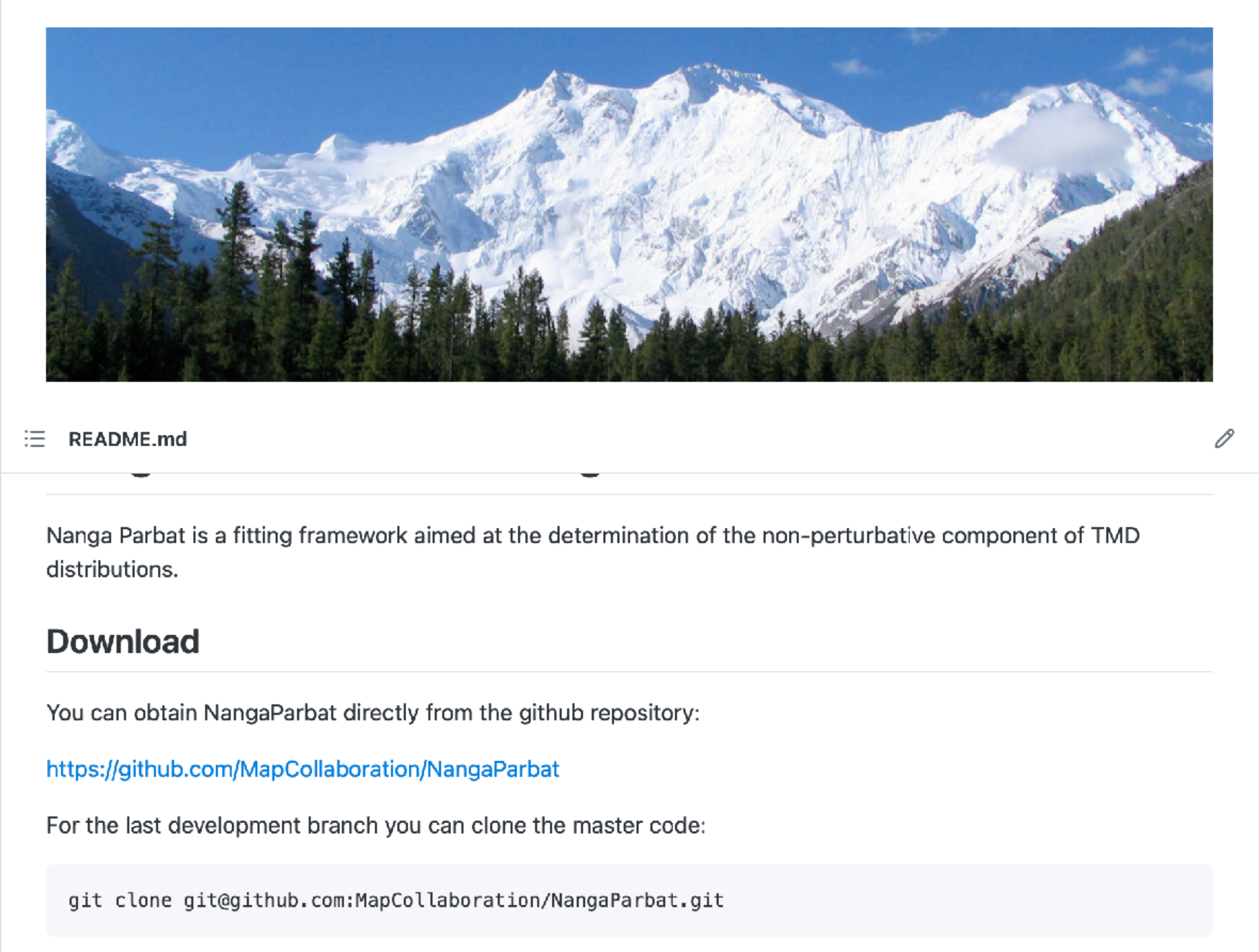

README.md

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Also:

ARTEMIDE

https://teorica.fis.ucm.es/artemide/

TMDLIB

https://tmdlib.hepforge.org/

MAP22 FUNCTIONAL FORM

$$f_{1\mathrm{NP}}(x, b_T^2) \propto \mathrm{F.T.\ of} \left(e^{-\frac{k_T^2}{g_1}} + \lambda^2 k_T^2 e^{-\frac{k_T^2}{g_{1B}}} + \lambda_2^2 e^{-\frac{k_T^2}{g_{1C}}} \right)$$

$$g_1(x) = N_1 \ \frac{(1-x)^\alpha \ x^\sigma}{(1-\hat{x})^\alpha \ \hat{x}^\sigma}$$

$$g_K(b_T^2) = -\frac{g_2^2}{2} b_T^2$$

11 parameters for TMD PDF
+ 1 for NP evolution +9 for FF
= 21 free parameters

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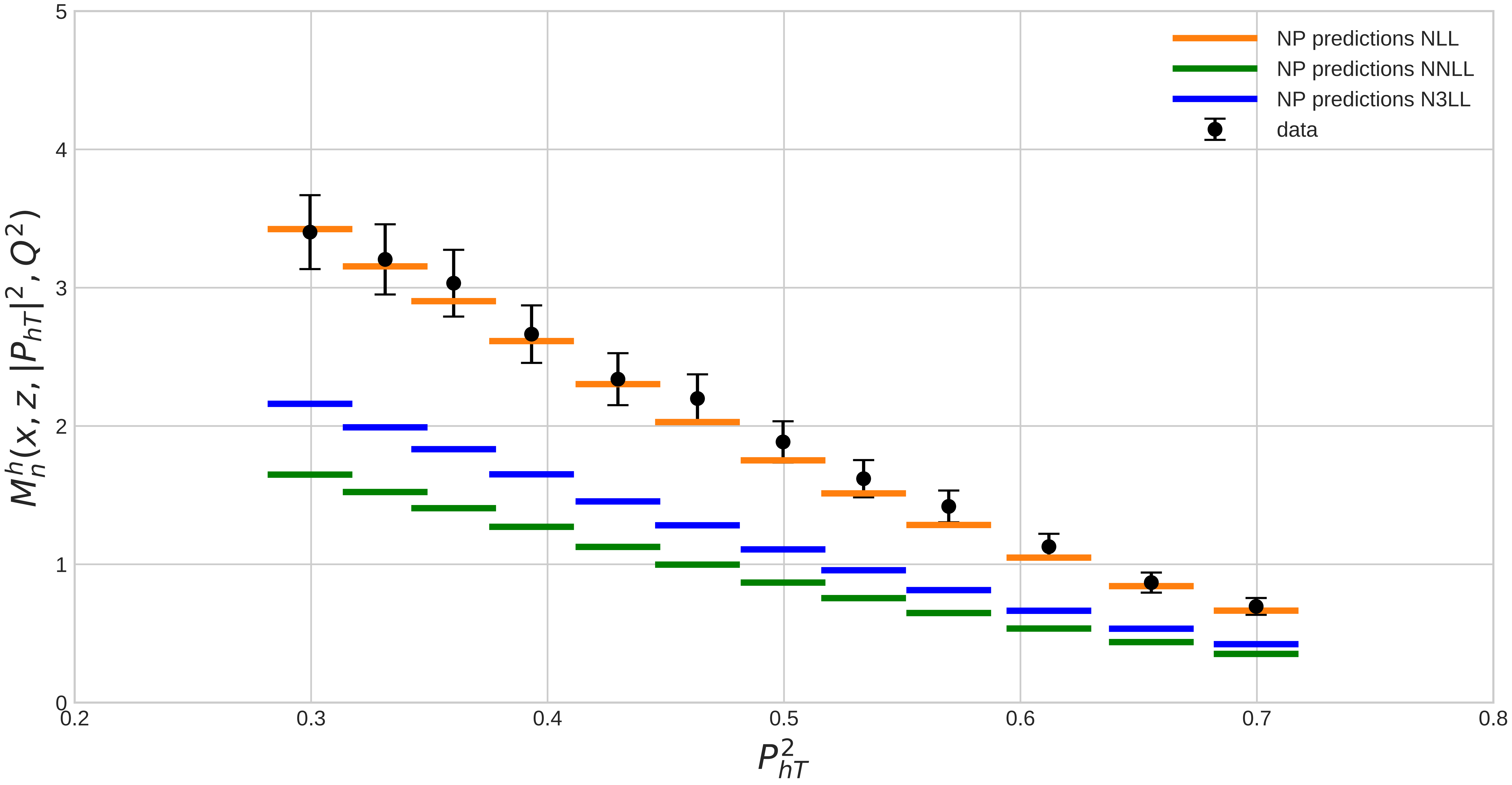
$$g_K(b_T^2) = -\frac{g_2^2}{2} b_T^2$$

Is is too flexible?

11 parameters for TMD PDF
+ 1 for NP evolution +9 for FF
= 21 free parameters

CHALLENGE: SIDIS NORMALIZATION VS ACCURACY

COMPASS multiplicities (one of many bins)



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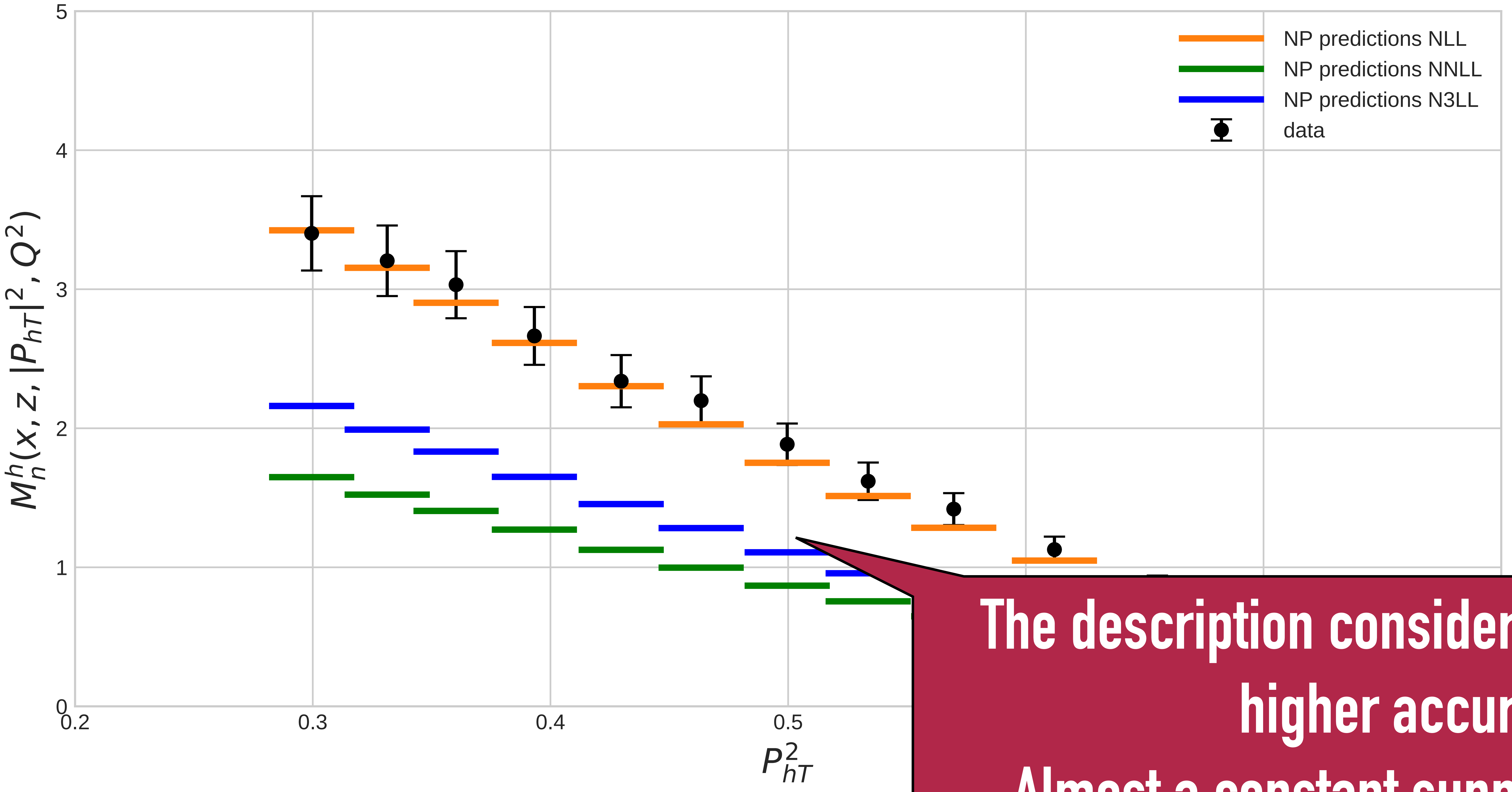

The description considerably worsens at higher accuracy.
Almost a constant suppression factor.

CHALLENGE: MAP22 TENTATIVE SOLUTION

ENHANCEMENT PREFACTOR

$$= \frac{\left.\frac{d\sigma}{dxdzdQ^2}\right|_{\text{nonmix.}}}{\int \text{TMD}\ d^2P_{hT}}$$

CHALLENGE: MAP22 TENTATIVE SOLUTION

$$\text{ENHANCEMENT PREFACTOR} = \frac{\left.\dfrac{d\sigma}{dxdzdQ^2}\right|_{\text{nonmix.}}}{\int \text{TMD}\ d^2P_{hT}}$$

The prefactor is independent of the fitting parameters

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The prefactor is independent of the fitting parameters

Higher-order corrections decrease the role of the TMD region.
We need to enhance it with a prefactor.

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The prefactor is independent of the fitting parameters

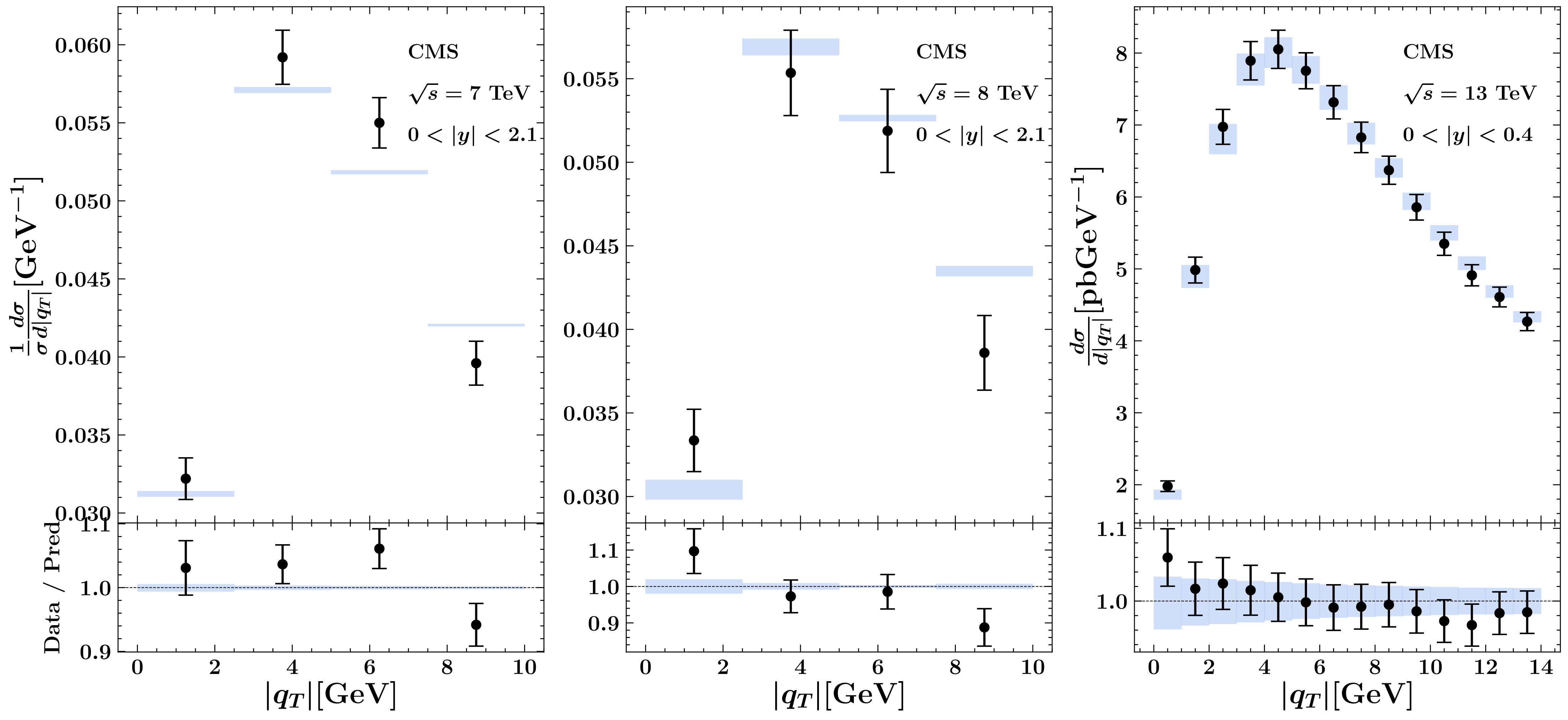
Higher-order corrections decrease the role of the TMD region.
We need to enhance it with a prefactor.

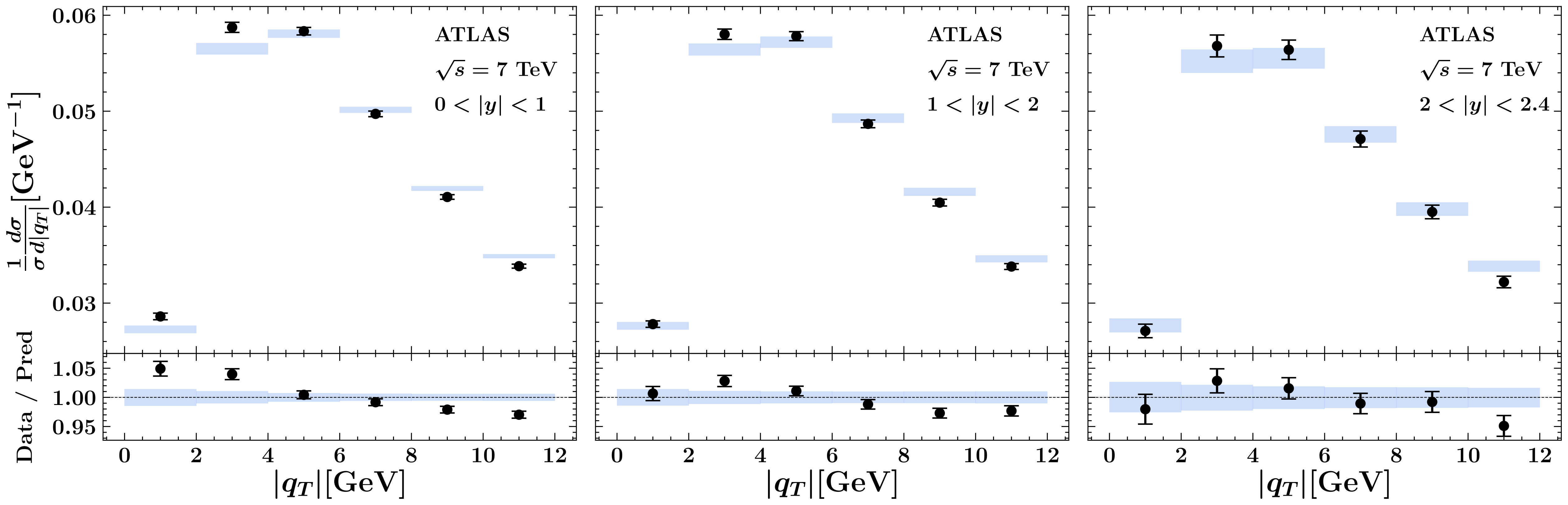
Possible justification in terms of power-suppressed corrections?

Vladimirov, arXiv:2307.13054

for TMD at NLP, see also talks by J. Terry and L. Gamberg

EXAMPLE OF AGREEMENT WITH DATA: DRELL-YAN





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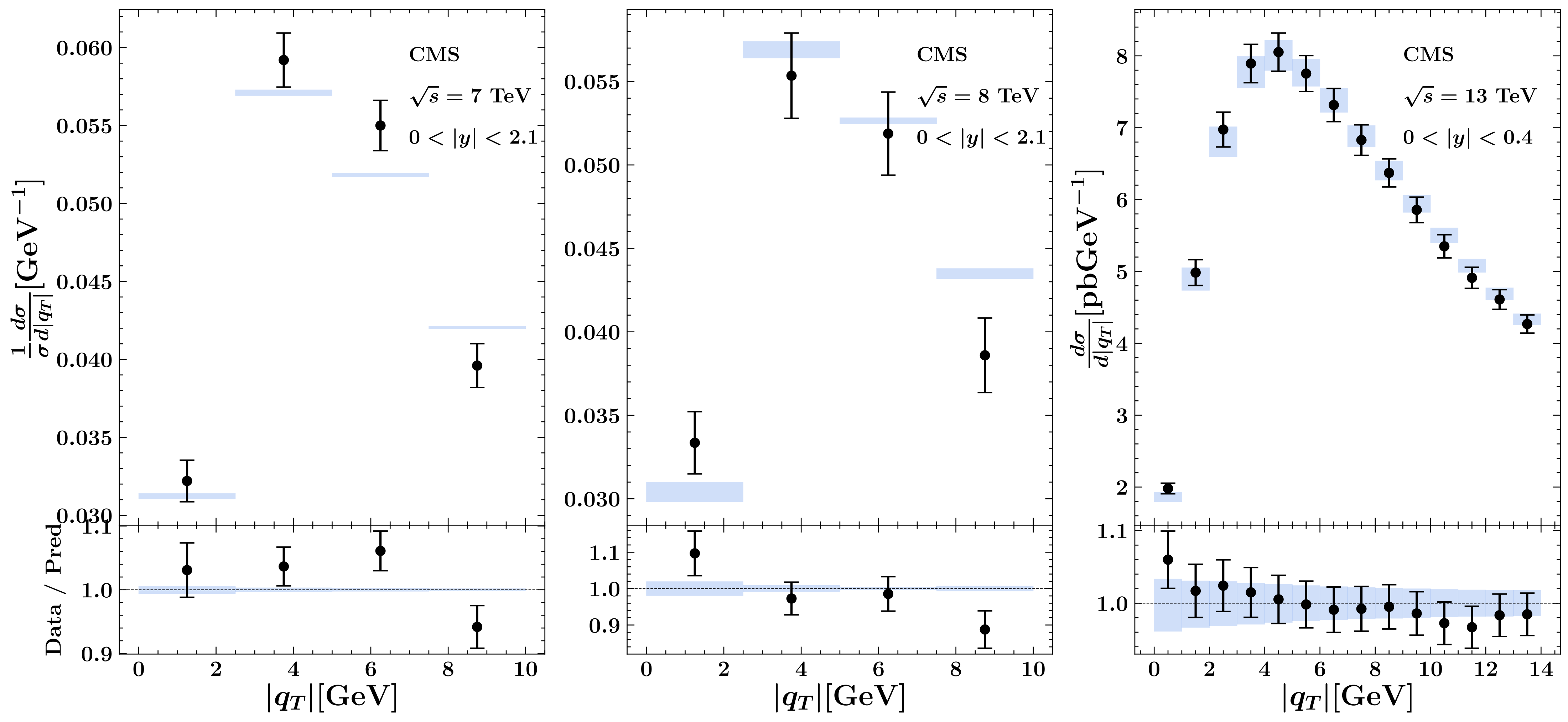
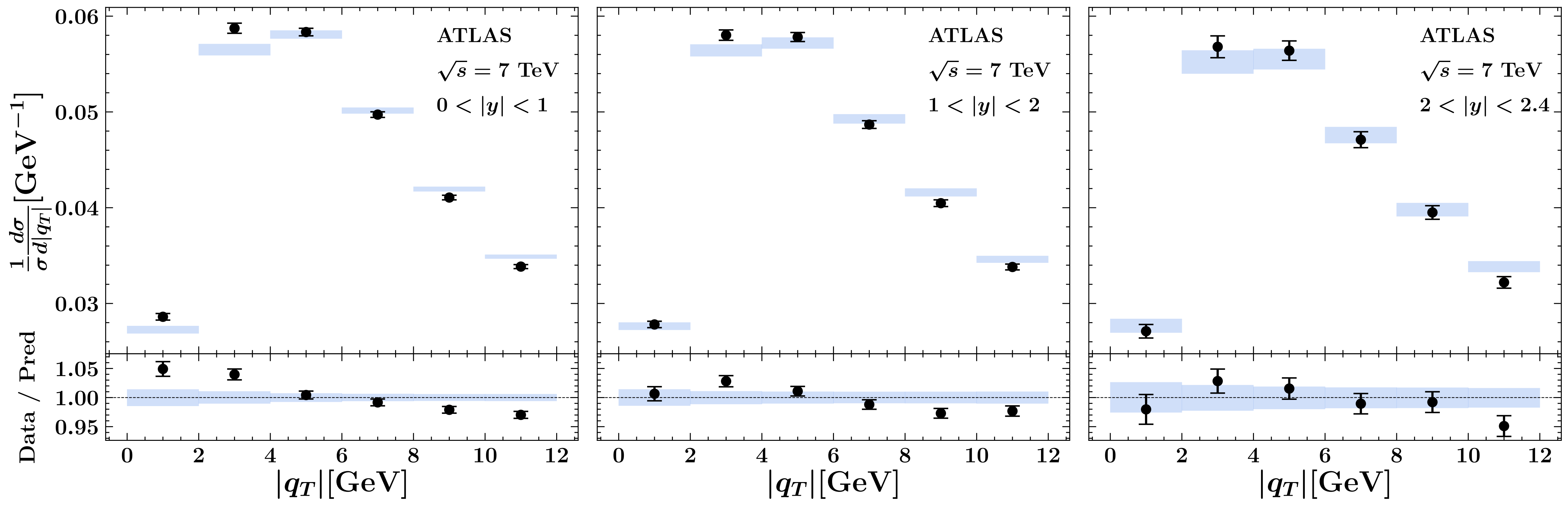
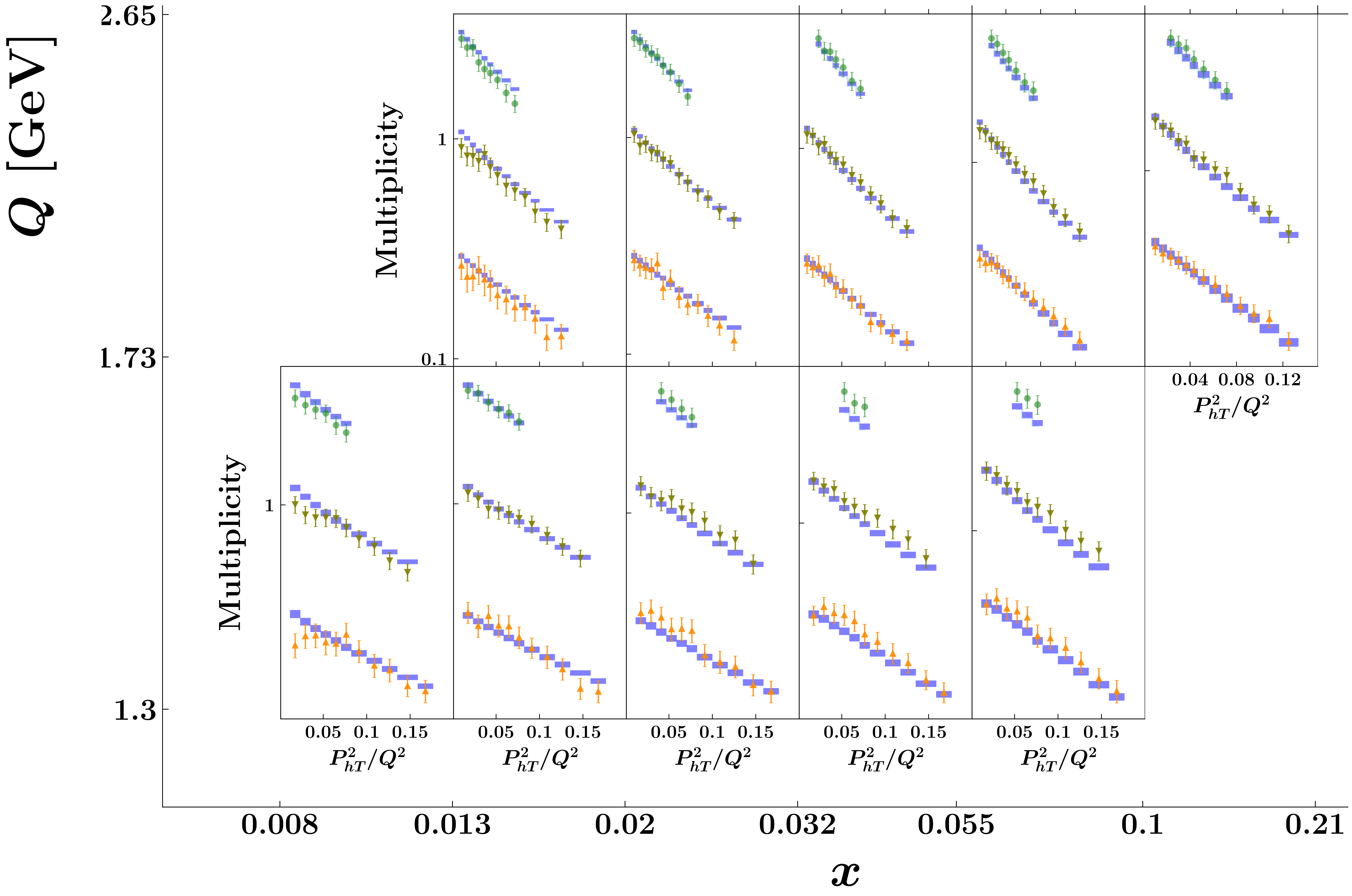




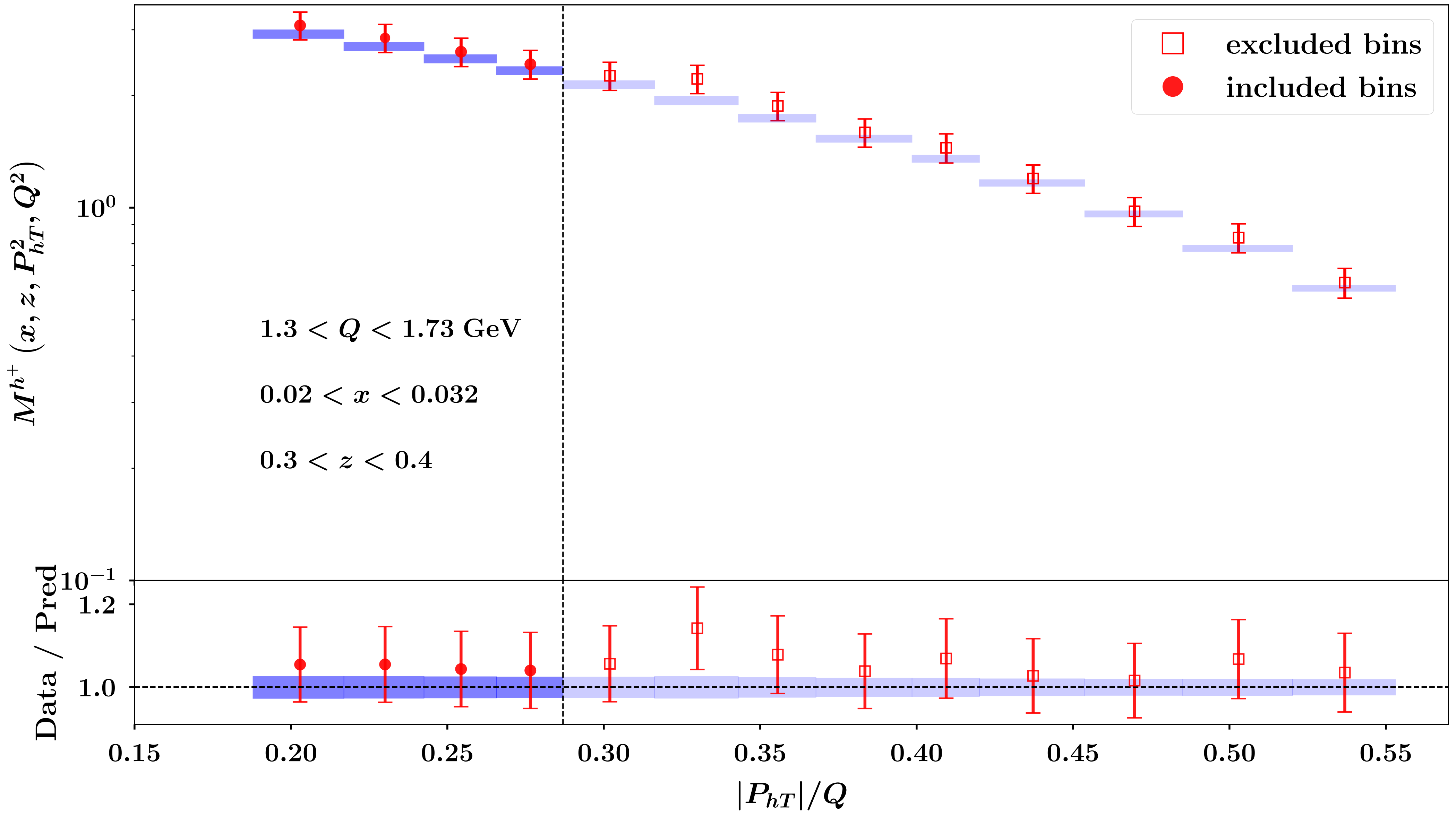
Maybe it looks good, but unfortunately it is not

EXAMPLE OF AGREEMENT WITH DATA: SIDIS



PUZZLE: REGION OF VALIDITY OF TMD FORMALISM

$$|q_T| = |P_{hT}|/z \ll Q$$



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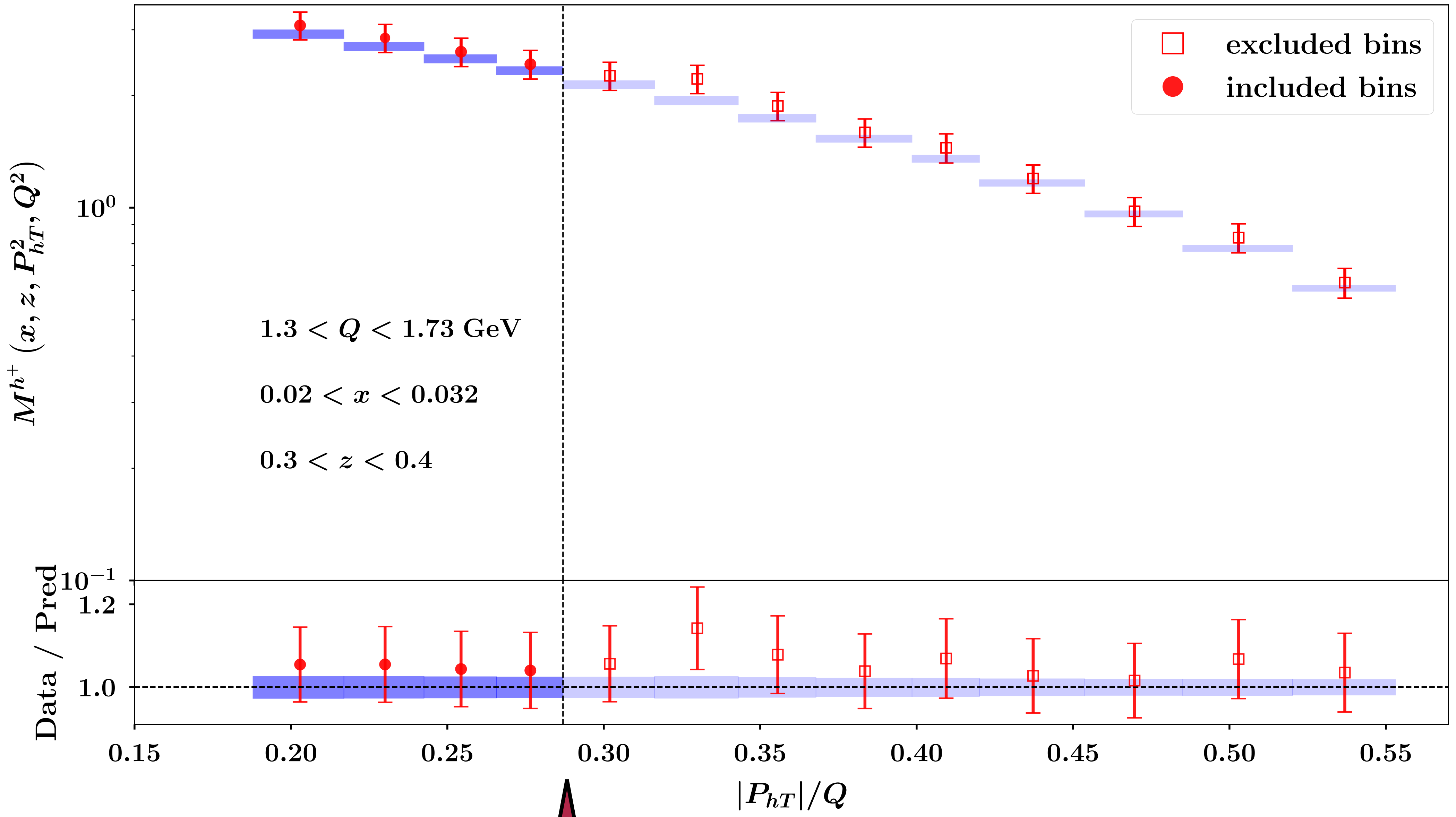

MAP22 cut

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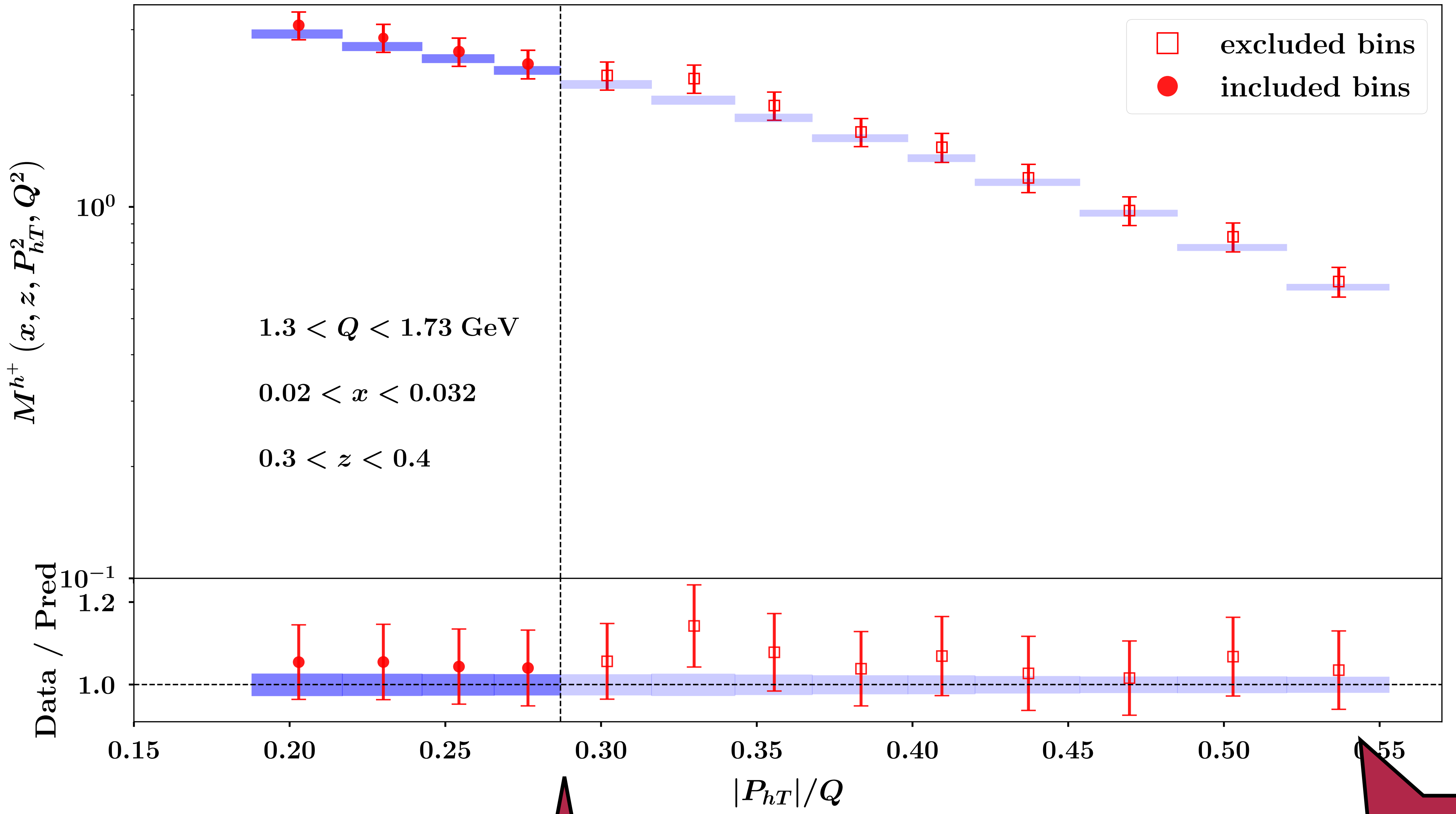

MAP22 cut

MAP22 extrapolation

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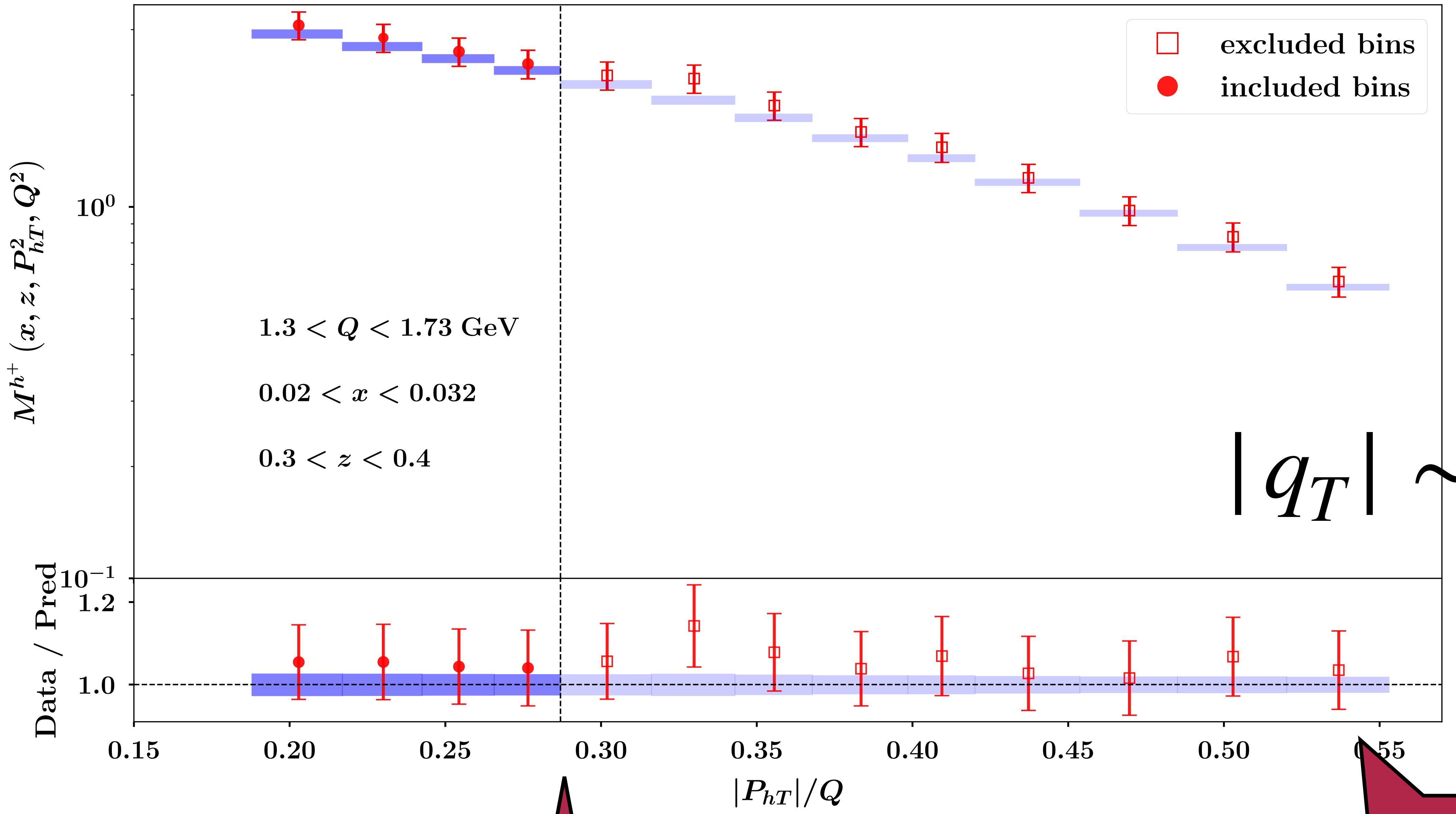

$$|q_T| \sim 1.5\ Q$$!

MAP22 cut

MAP22 extrapolation

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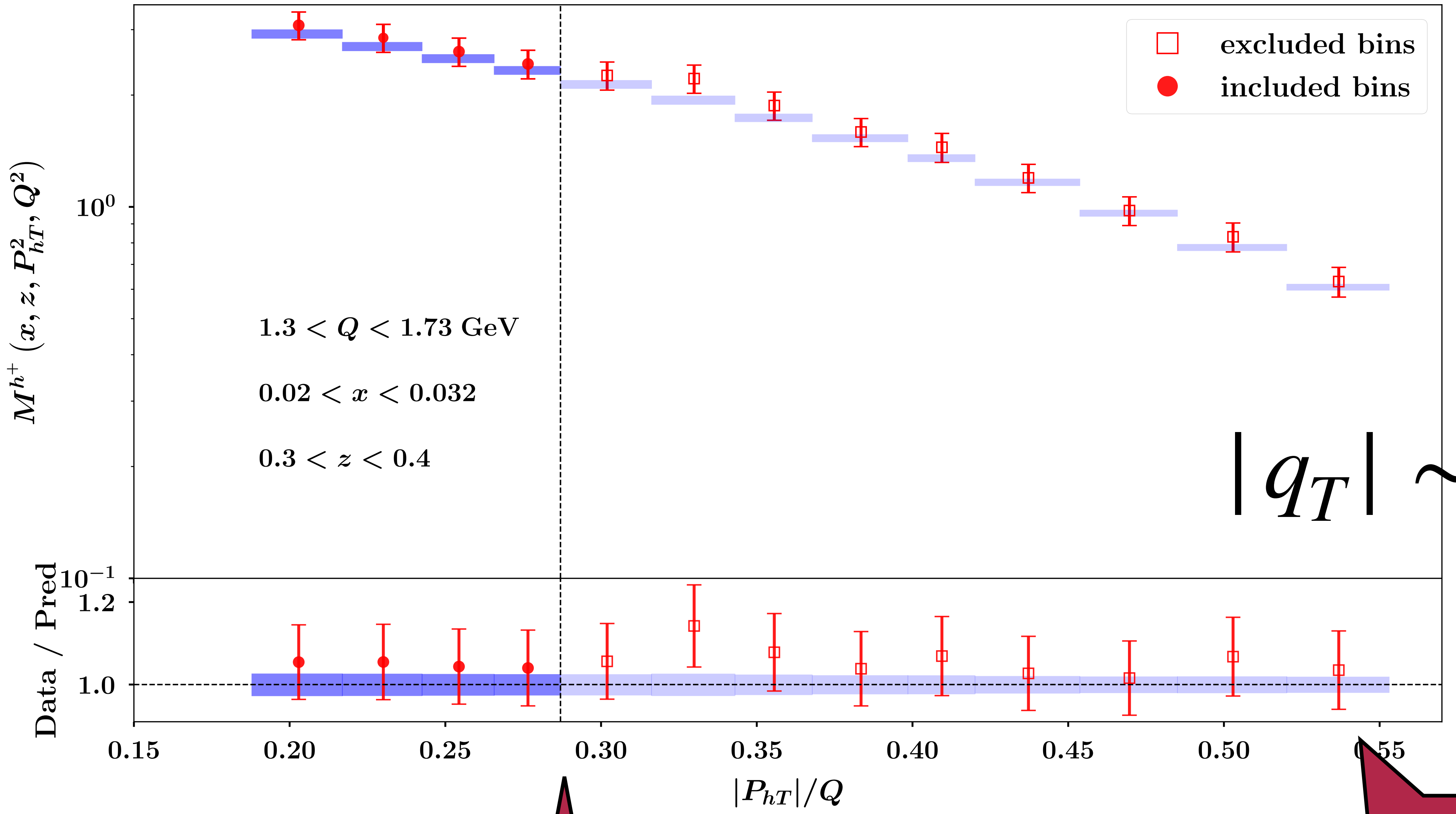

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MAP22 cut

MAP22 extrapolation

The MAP22 cut is already considered to be “generous”, but the physics seems to be the same for a much wider transverse momentum

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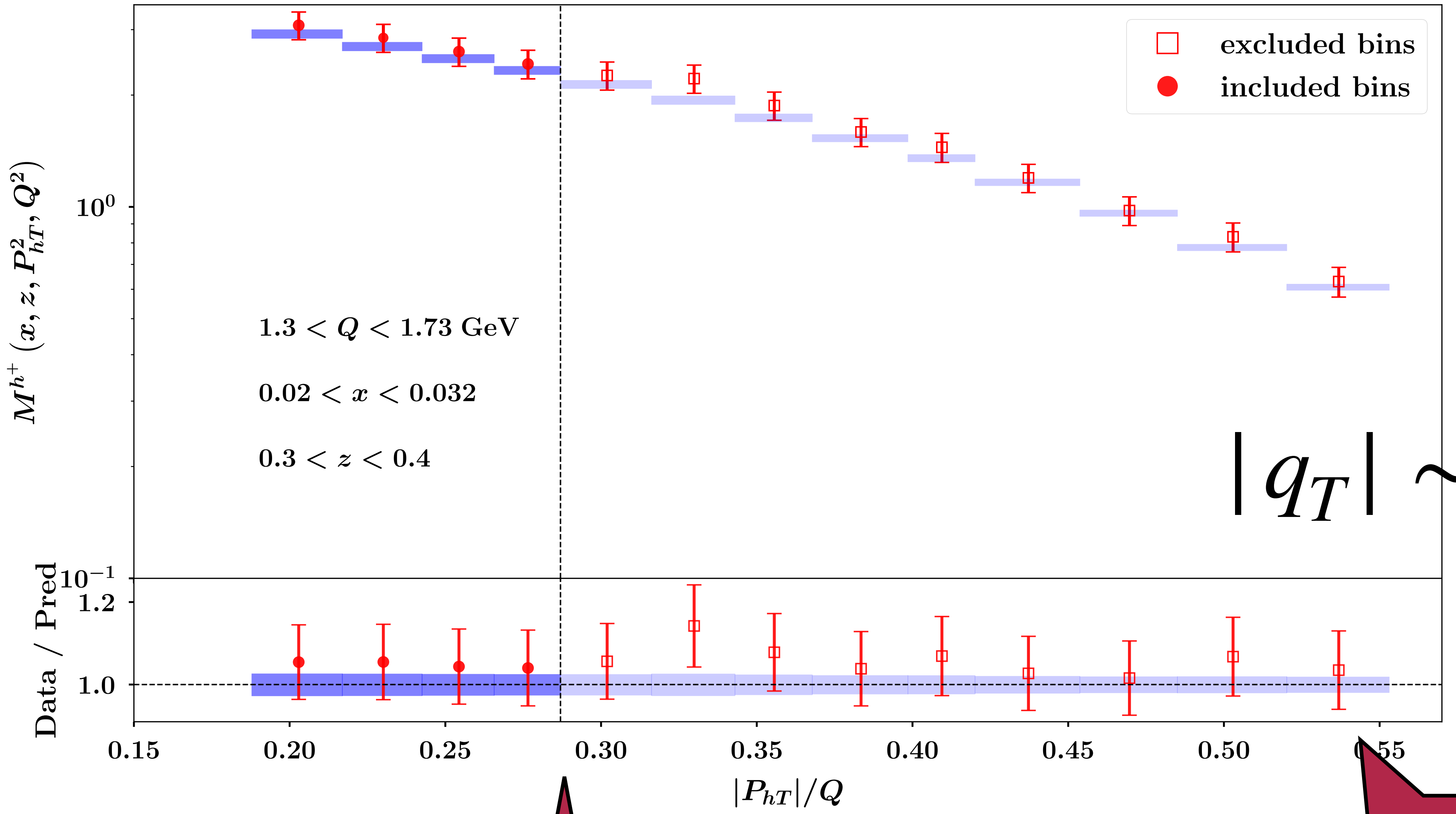
$$|q_T| = |P_{hT}|/z \ll Q$$

?



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Data set	N_{dat}	$\chi_D^2/N_{\mathrm{dat}}$	$\chi_\lambda^2/N_{\mathrm{dat}}$	$\chi_0^2/N_{\mathrm{dat}}$
Tevatron total	71	0.87	0.06	0.93
LHCb total	21	1.15	0.3	1.45
ATLAS total	72	4.56	0.48	5.05
CMS total	78	0.53	0.02	0.55
PHENIX 200	2	2.21	0.88	3.08
STAR 510	7	1.05	0.10	1.15
DY collider total	251	1.86	0.2	2.06
DY fixed-target total	233	0.85	0.4	1.24
HERMES total	344	0.48	0.23	0.71
COMPASS total	1203	0.62	0.3	0.92
SIDIS total	1547	0.59	0.28	0.87
Total	**2031**	**0.77**	**0.29**	**1.06**

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Is this dataset compatible?

RESULTING TMDS

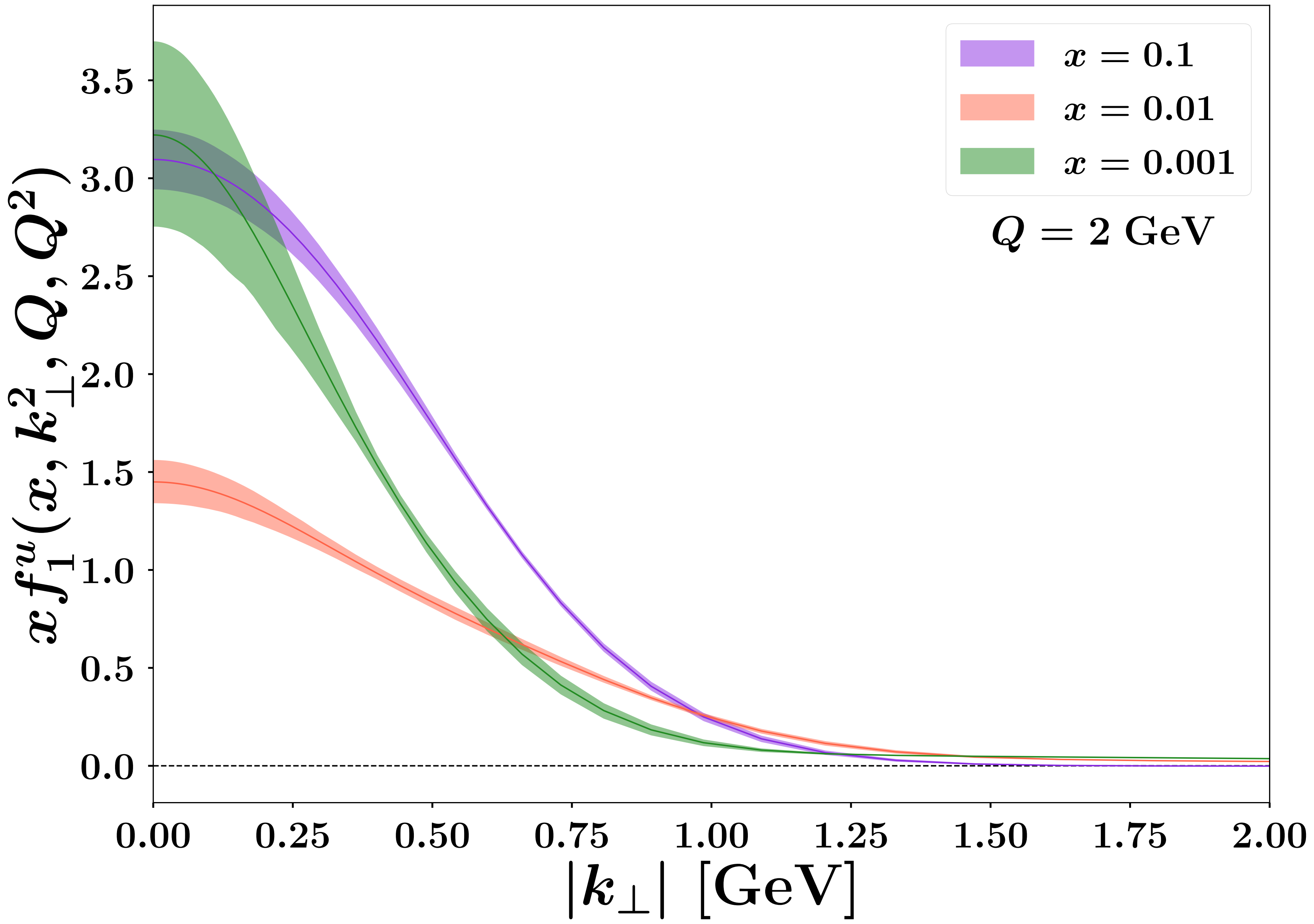
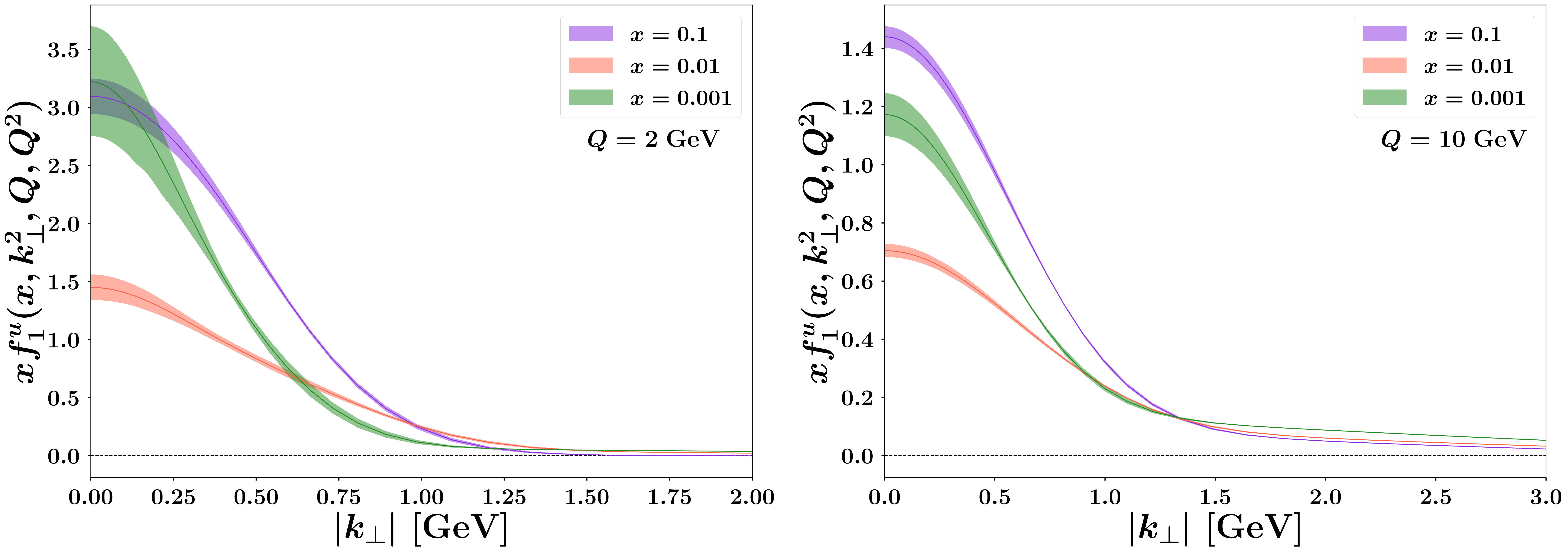


FIG. 13: The TMD PDF of the up quark in a proton at $\mu = \sqrt{\zeta} = Q = 2$ GeV (left panel) and 10 GeV (right panel) as a function of the partonic transverse momentum $|\boldsymbol{k}_\perp|$ for $x = 0.001$, 0.01 and 0.1. The uncertainty bands represent the 68% CL.

RESULTING TMDS



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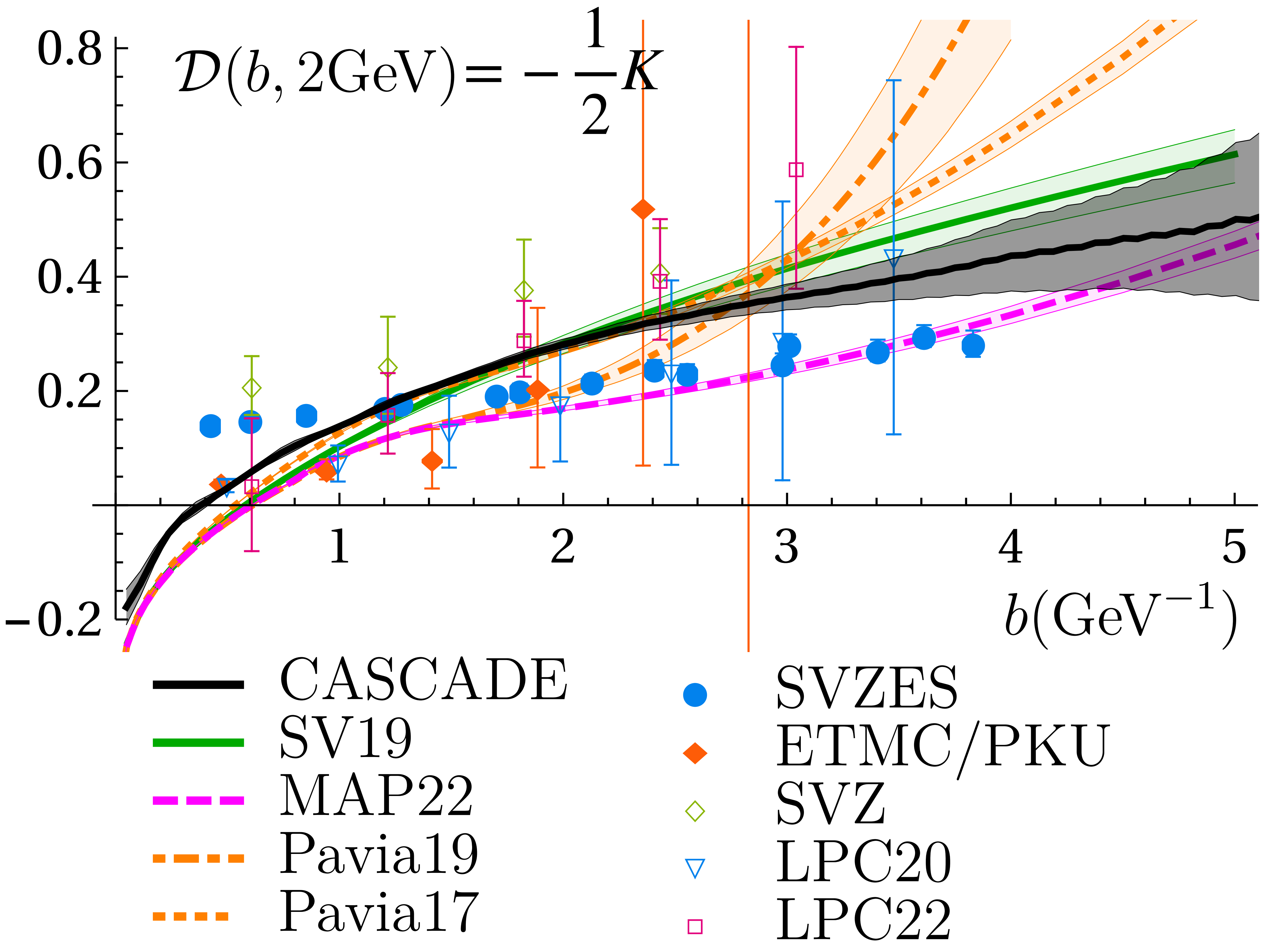
CONNECTIONS WITH LATTICE QCD: COLLINS–SOPER KERNEL

[Bermudez Martinez, Vladimirov, arXiv:2206.01105]



see also talks by Y. Zhao

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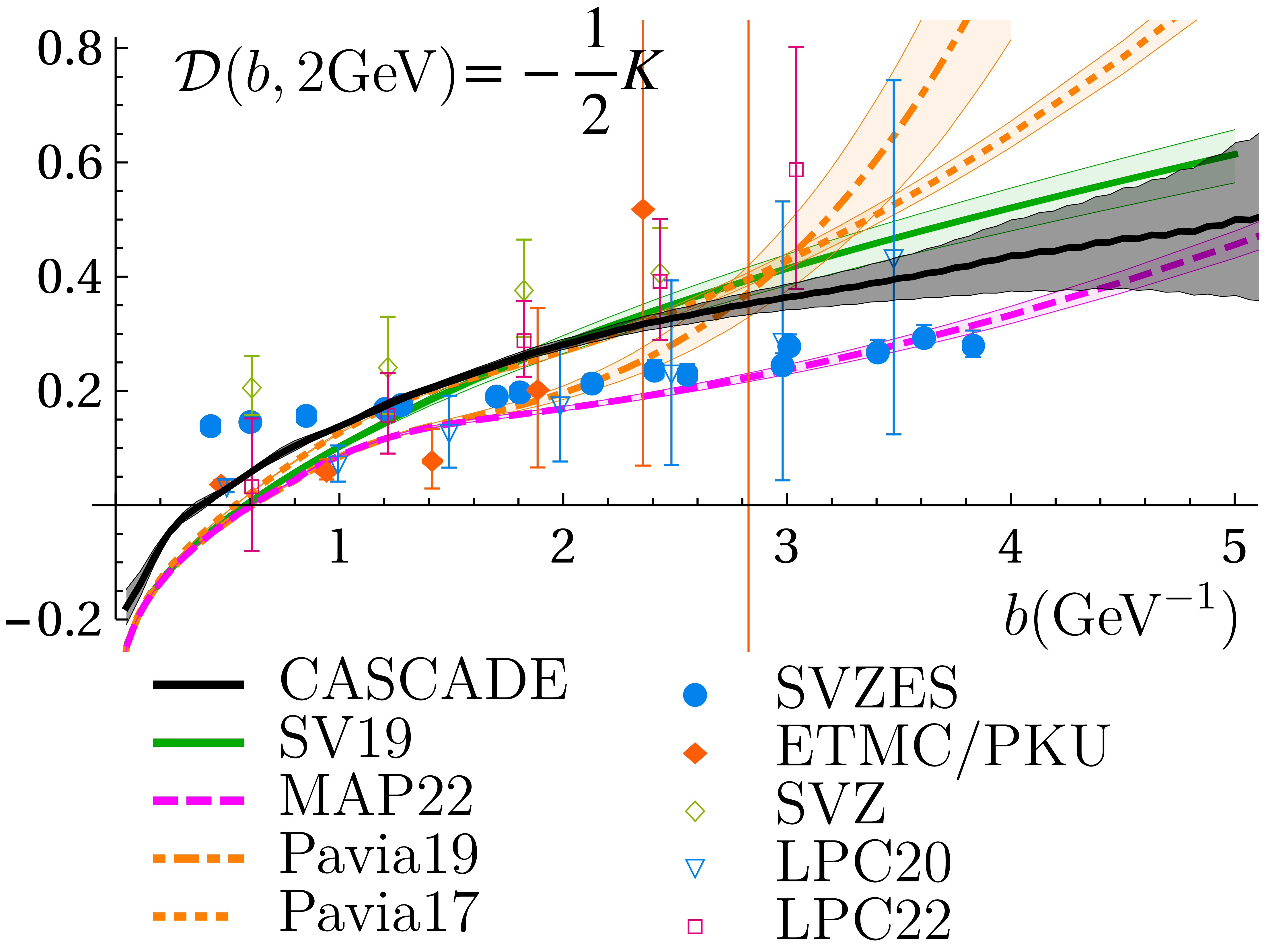
Bermudez Martinez, Vladimirov, arXiv:2206.01105



TMD phenomenology

see also talks by Y. Zhao

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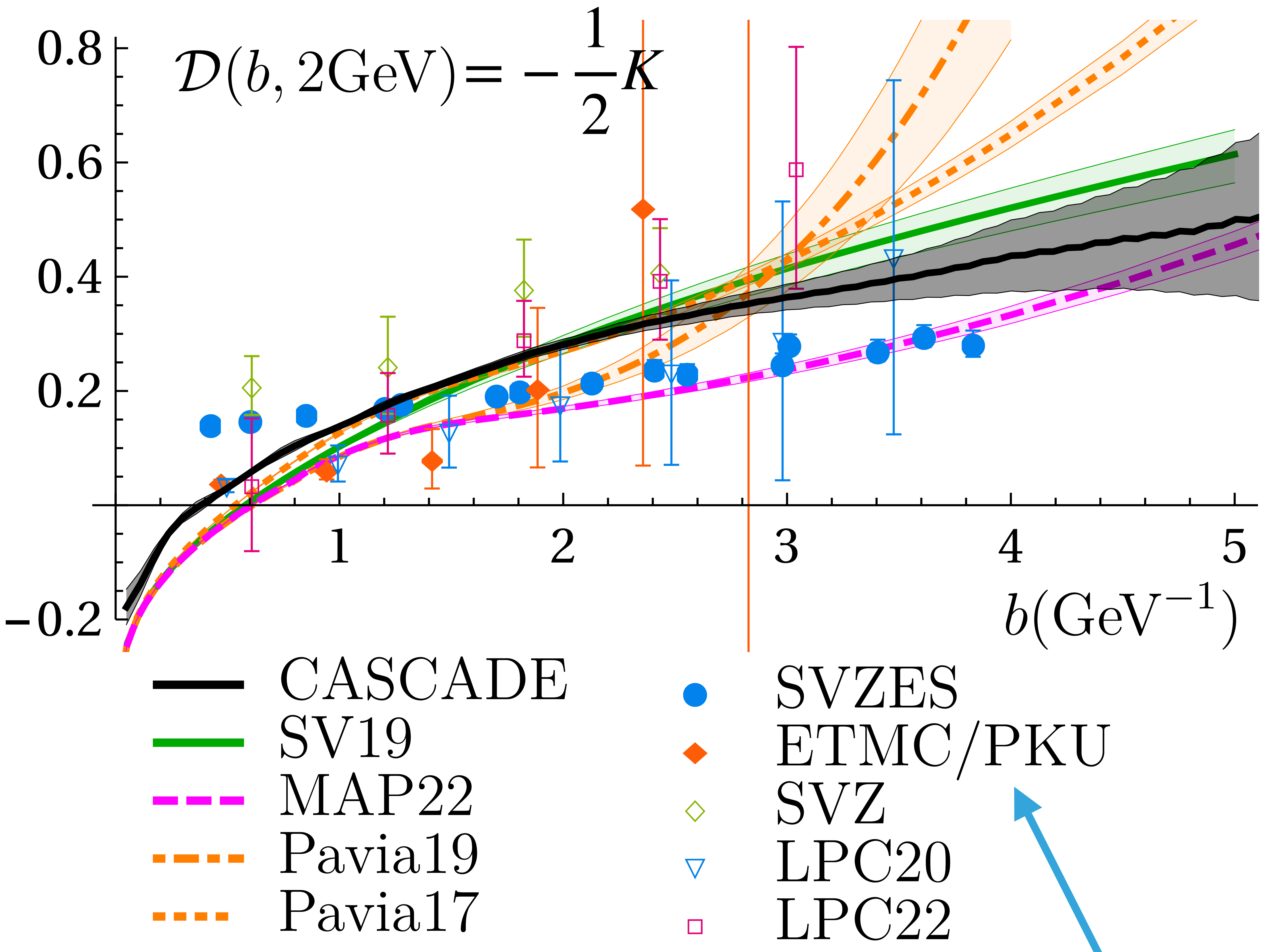

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Lattice QCD

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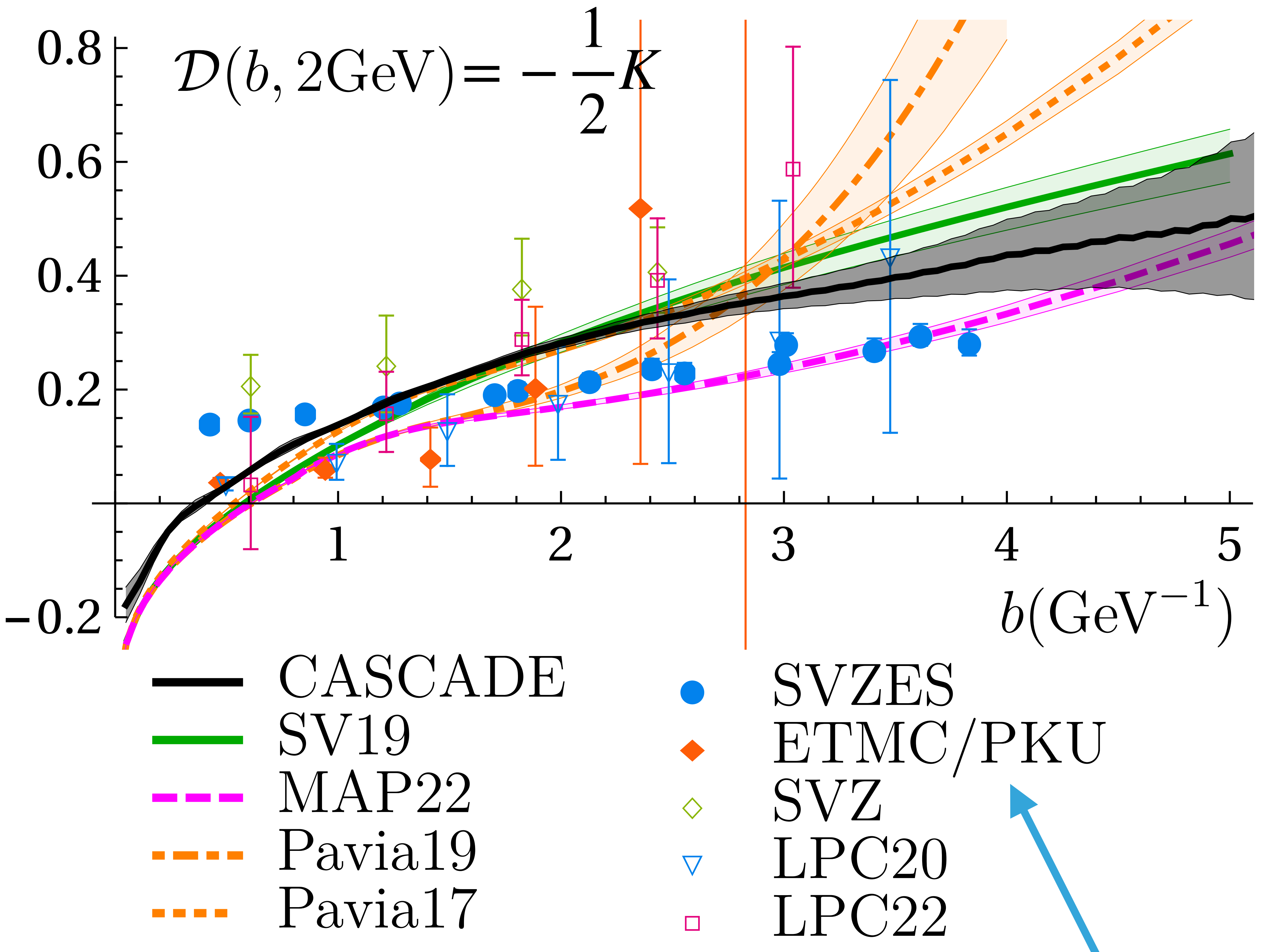

TMD phenomenology

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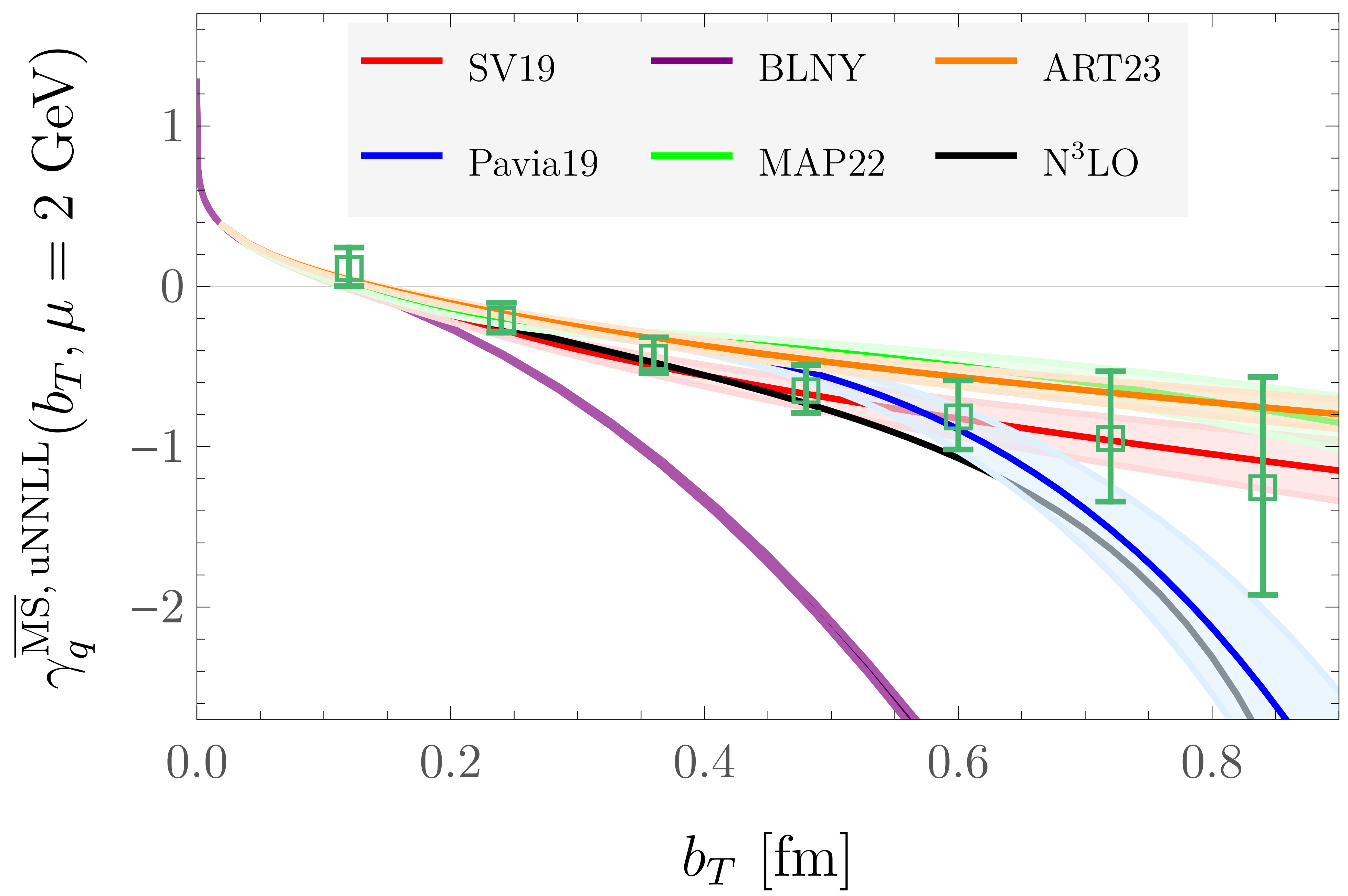
Avkhadiev, Shanahan, Wagman, Zhao, sarXiv:2307.12359



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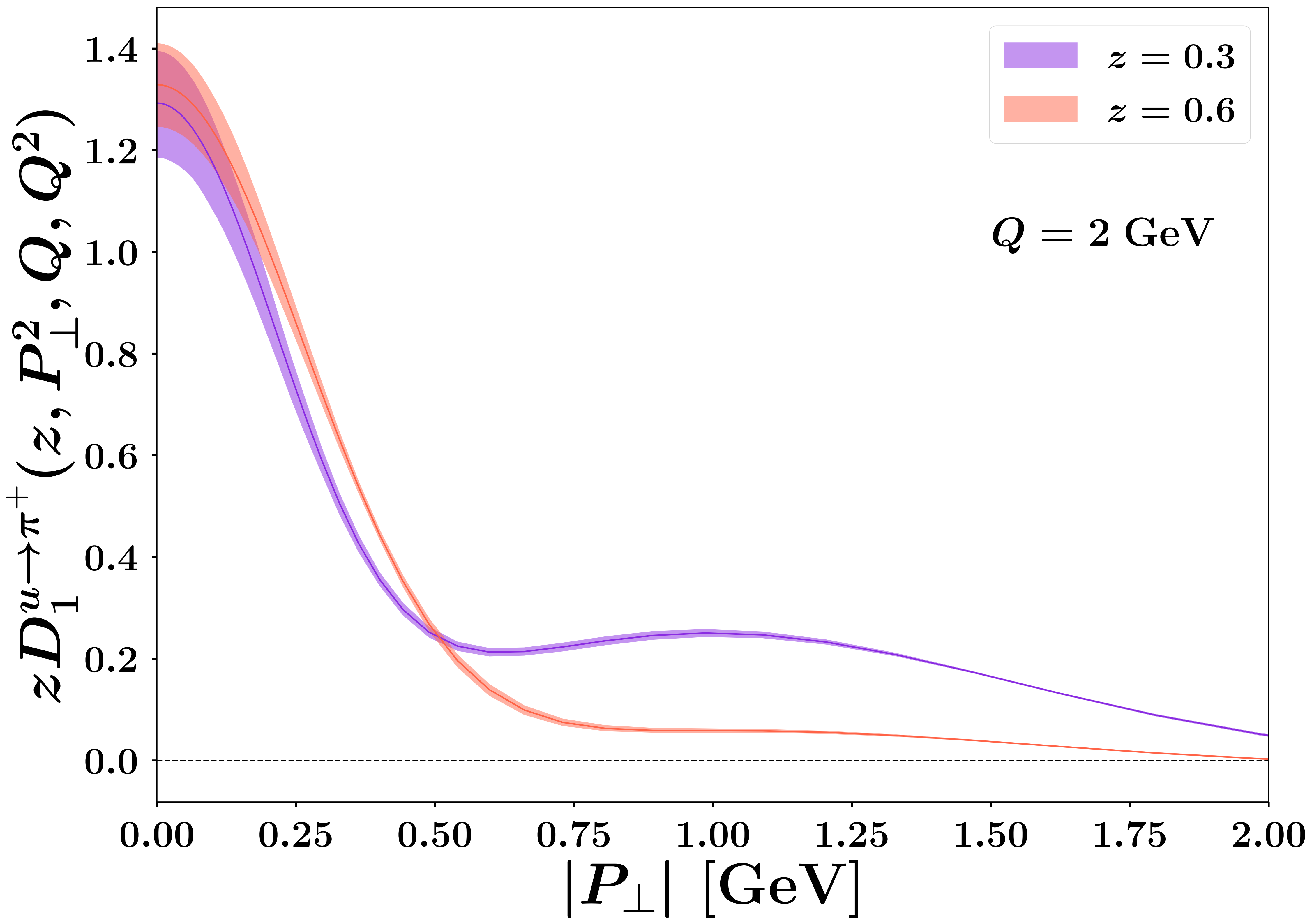
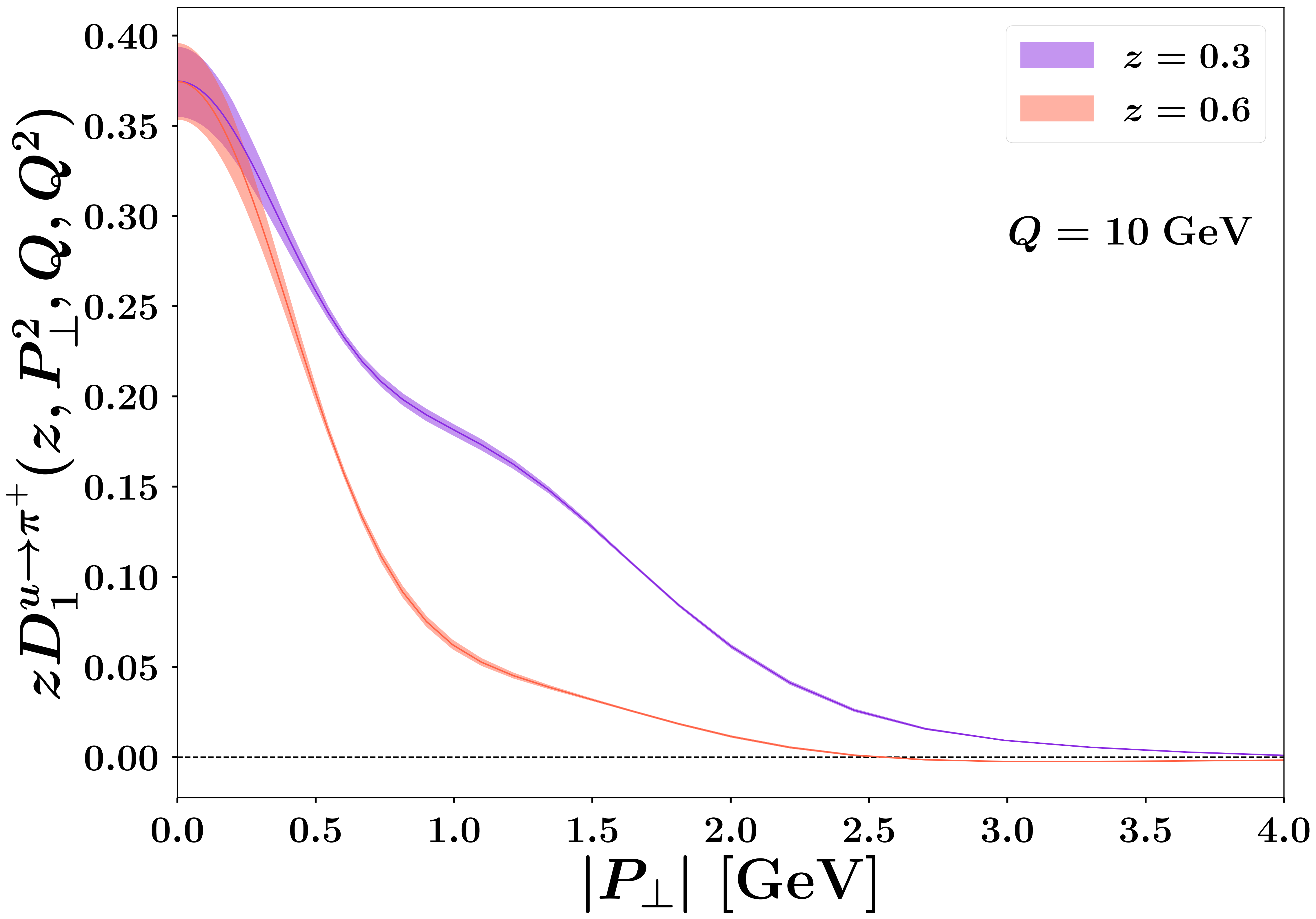
RESULTING TMD FRAGMENTATION FUNCTIONS

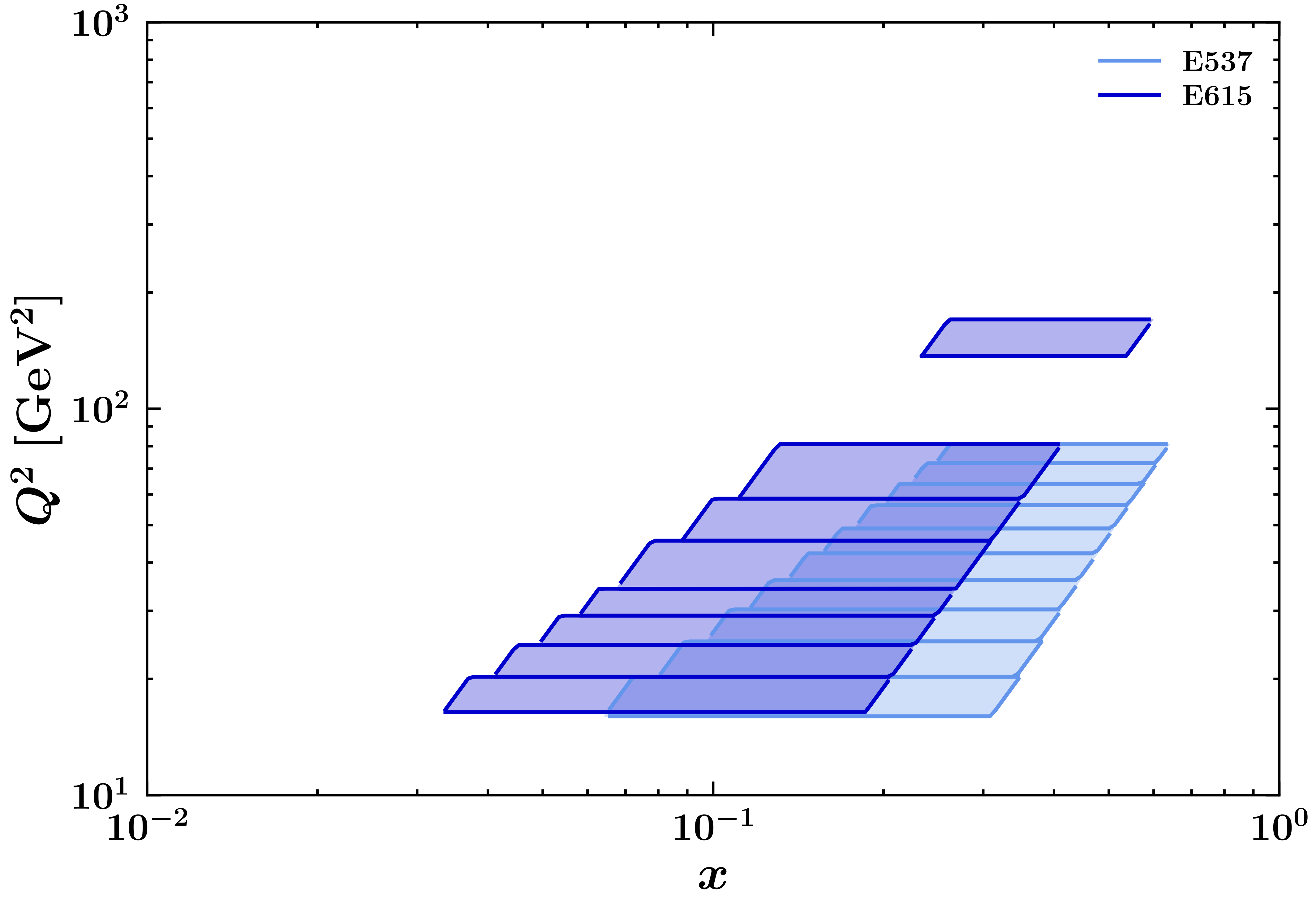




PION TMDS

MAP collaboration, arXiv:2210.01733

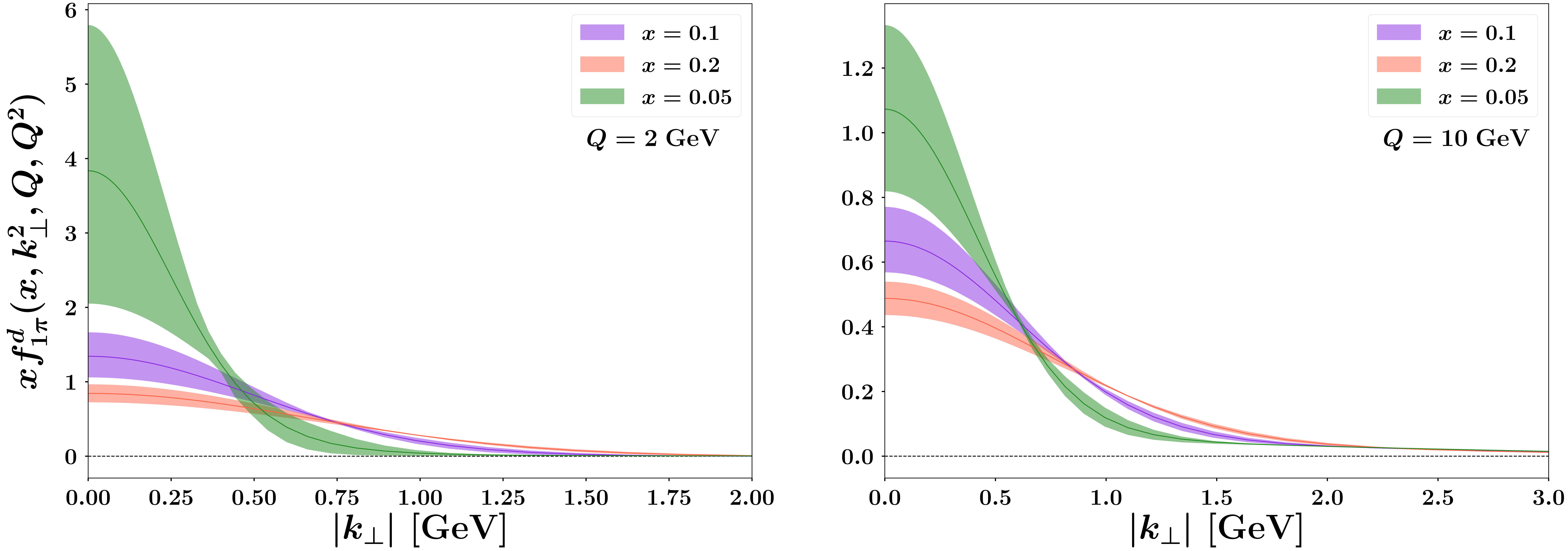
Experiments	N_{cut}	χ^2_D/N_{cut}	$\chi^2_\lambda/N_{\text{cut}}$	χ^2_0/N_{cut}
E537	64	1.00	0.57	1.57
E615	74	0.31	1.22	1.53
Total	138	0.63	0.92	1.55



PION TMDS

MAP collaboration, arXiv:2210.01733

$$f_{1\mathrm{NP}}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{k_T^2}{g_{1\pi}}} + \lambda_\pi^2 k_T^2 e^{-\frac{k_T^2}{g_{1B\pi}}} + \lambda_{2\pi}^2 e^{-\frac{k_T^2}{g_{1C\pi}}} \right)$$

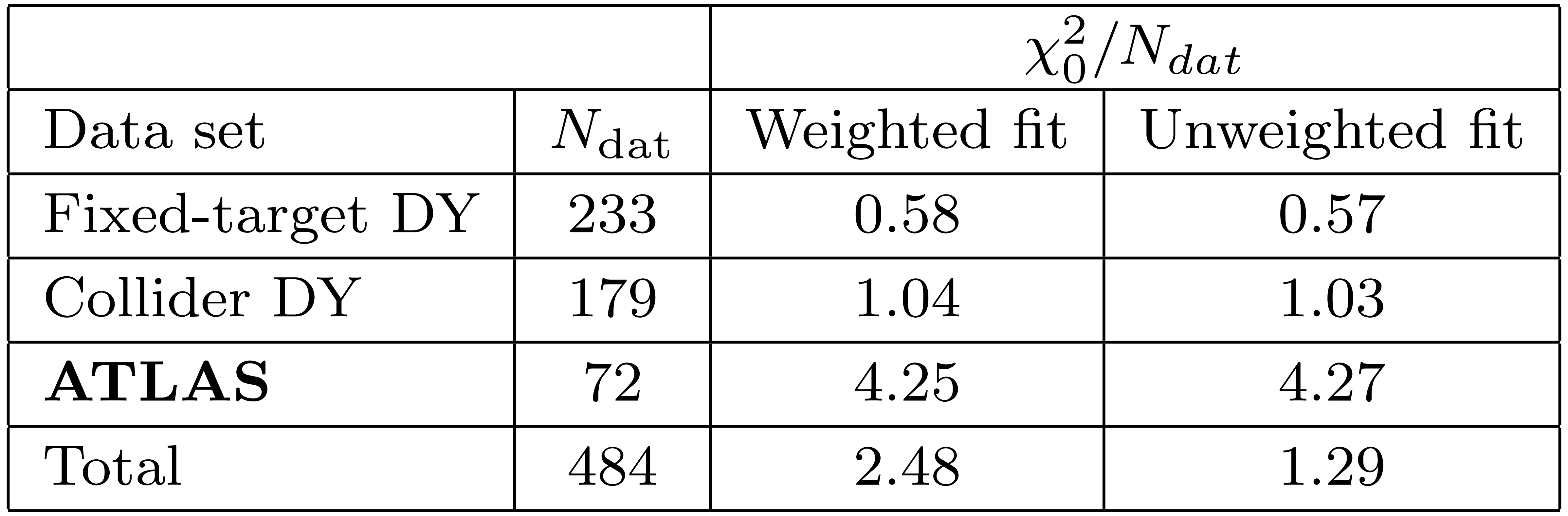


see also talk Patrick Barry

WORK IN PROGRESS

COMPATIBILITY STUDIES

PhD thesis of M. Cerutti, in preparation



		χ_0^2/N_{dat}	
Data set	$N_{\rm dat}$	Weighted fit	Unweighted fit
Fixed-target DY	233	0.58	0.57
Collider DY	179	1.04	1.03
ATLAS	72	4.25	4.27
Total	484	2.48	1.29

COMPATIBILITY STUDIES

PhD thesis of M. Cerutti, in preparation

Fit where the contribution of the ATLAS data is enhanced by a factor (412/72 in this case)

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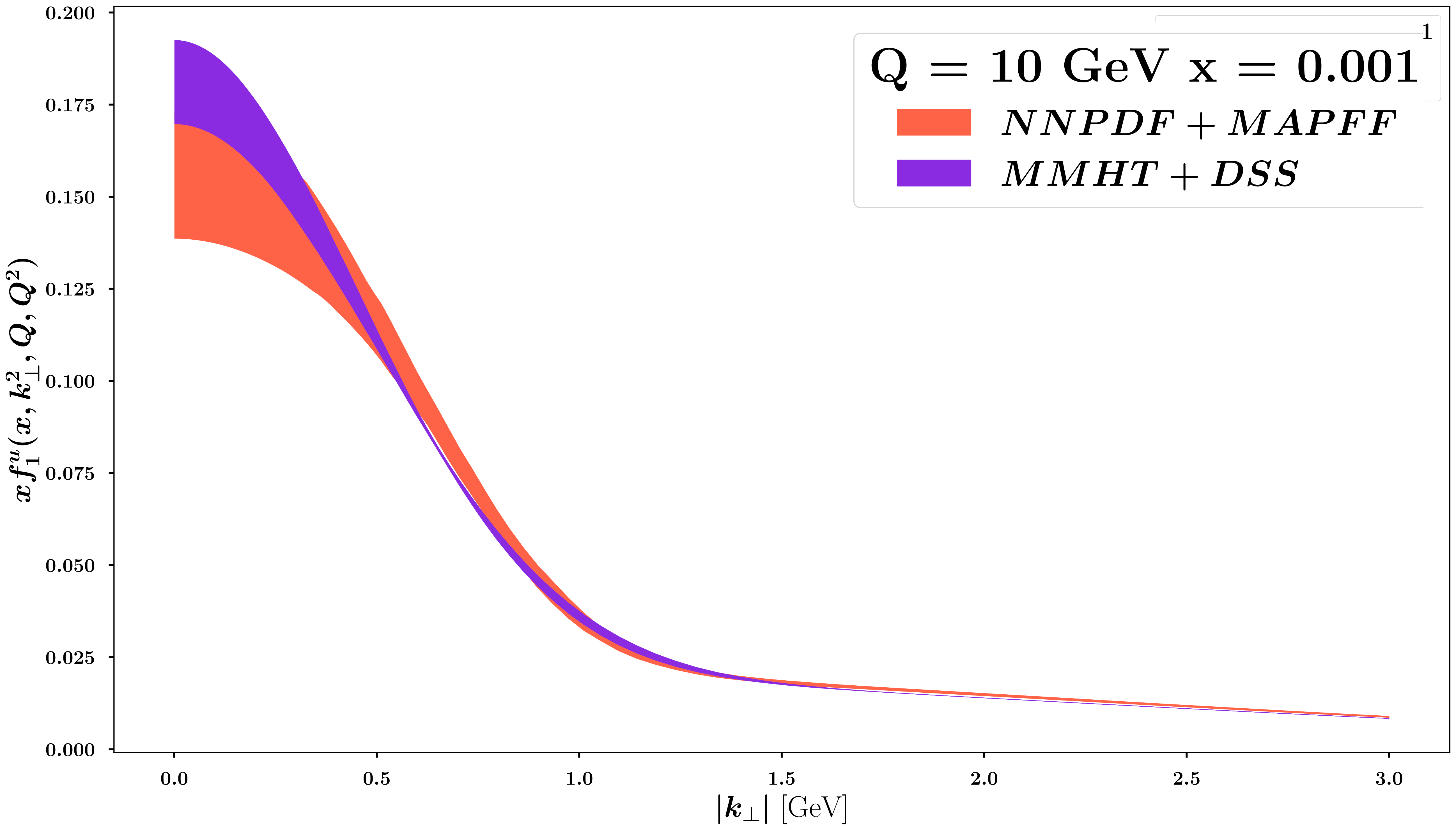
ATLAS data are incompatible with our model, including or excluding them does not affect the agreement with the rest of the data

INCLUSION OF COLLINEAR PDF UNCERTAINTIES

PDFS

MMHT2014 → NNPDF3.1

Hessian set → Monte Carlo set



INCLUSION OF COLLINEAR PDF UNCERTAINTIES

PDFS

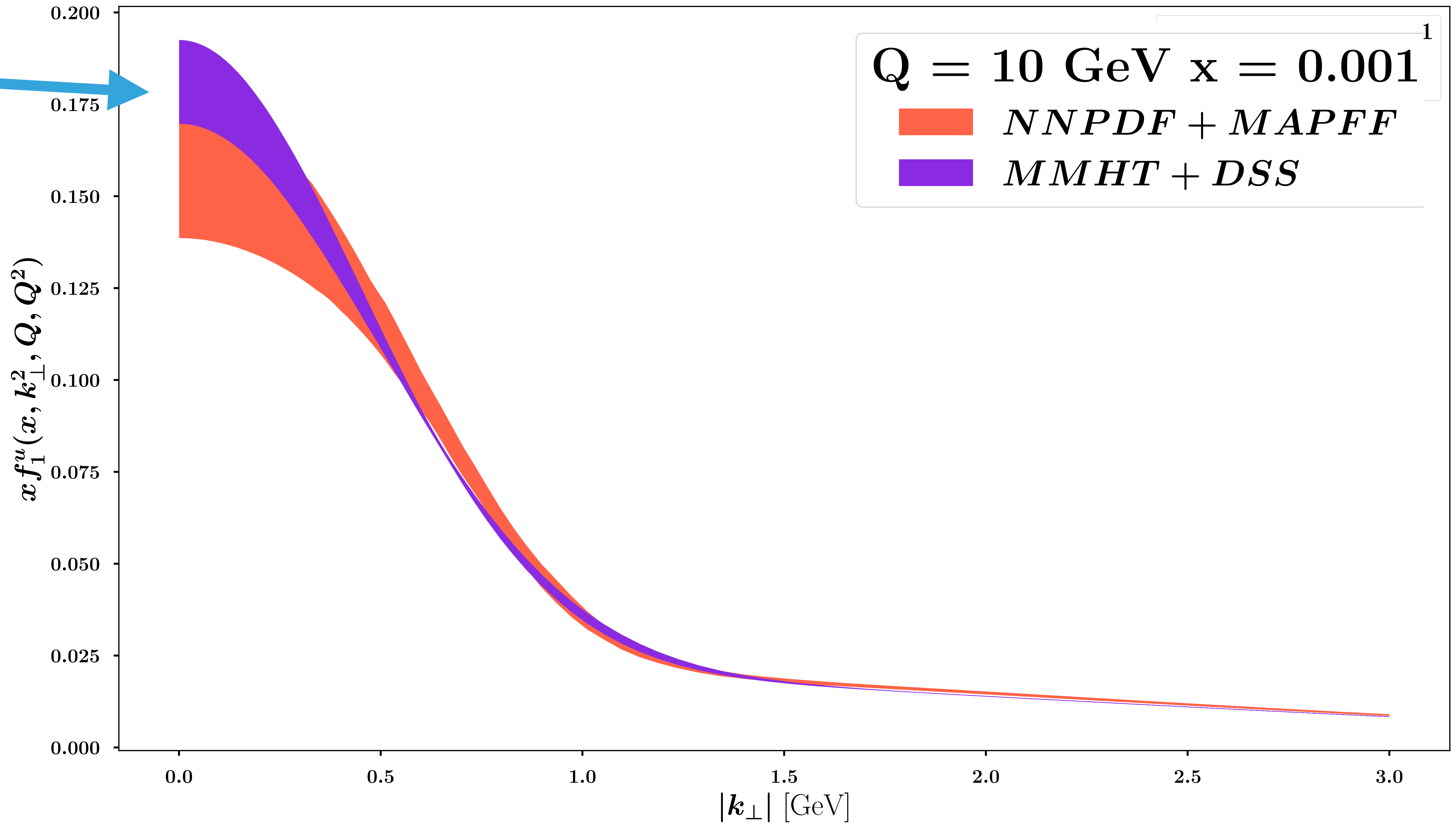
MMHT2014

Hessian set

NNPDF3.1

Monte Carlo set

MAP22 fit



INCLUSION OF COLLINEAR PDF UNCERTAINTIES

PDFS

MMHT2014

Hessian set

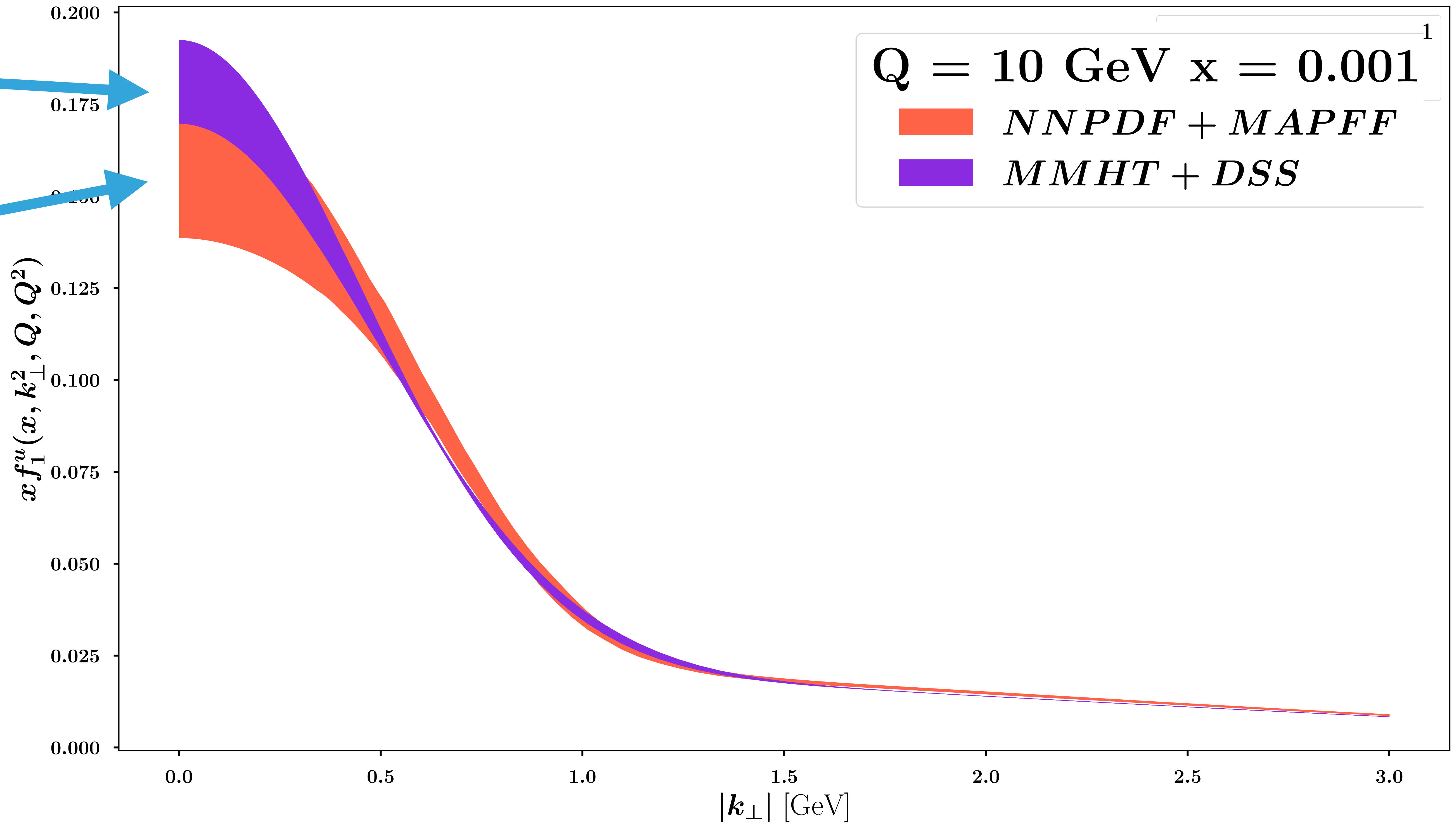
→

NNPDF3.1

Monte Carlo set

MAP22 fit

Fit with NNPDF set



INCLUSION OF COLLINEAR PDF UNCERTAINTIES

PDFS

MMHT2014

Hessian set

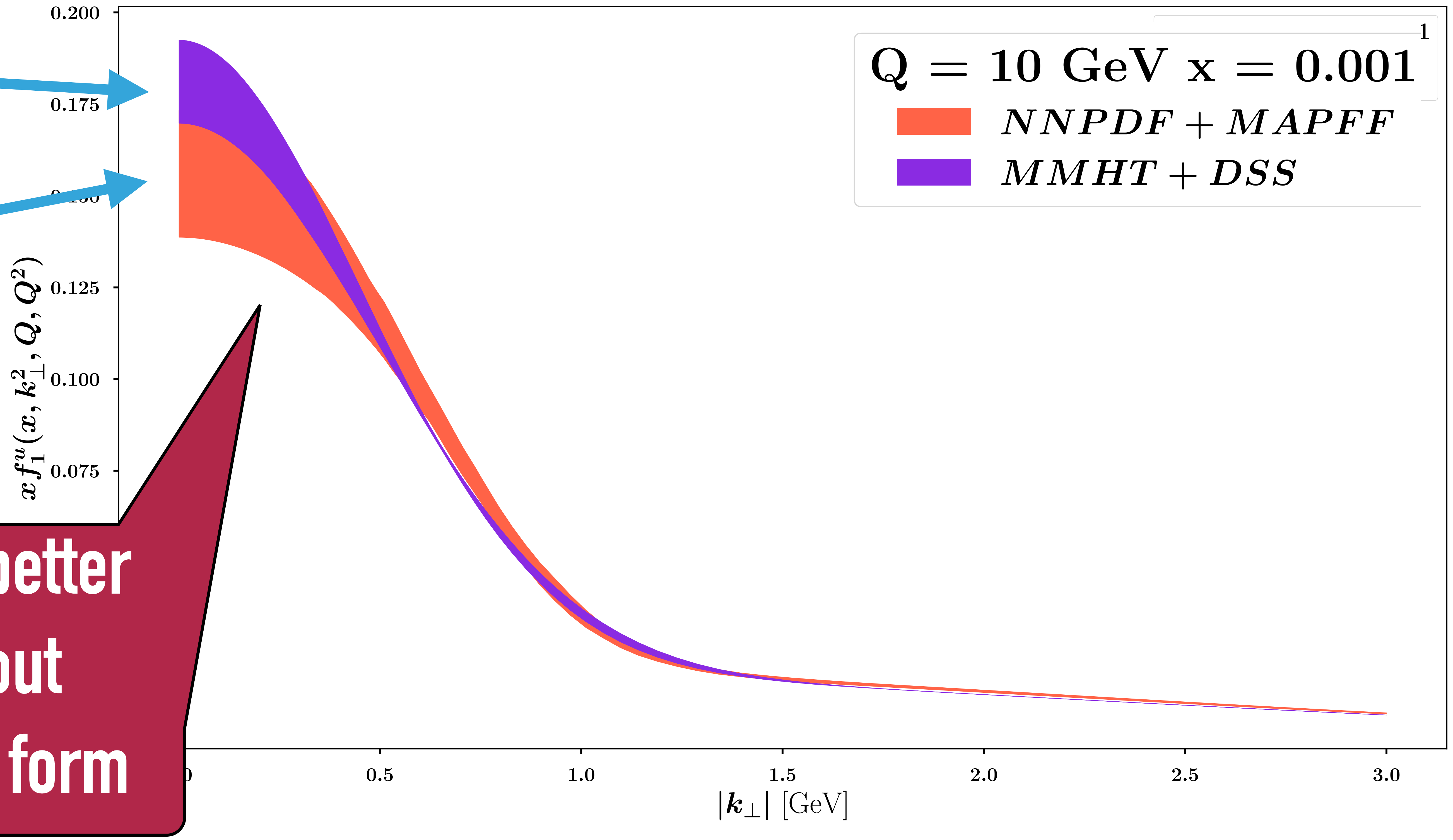
→

NNPDF3.1

Monte Carlo set

MAP22 fit

Fit with NNPDF set



The use of a Monte Carlo PDF set gives a better idea of the full TMD uncertainties, without dramatically changing the TMD functional form

FULL N^{3}LL WITH NEW MAPFF

FFS

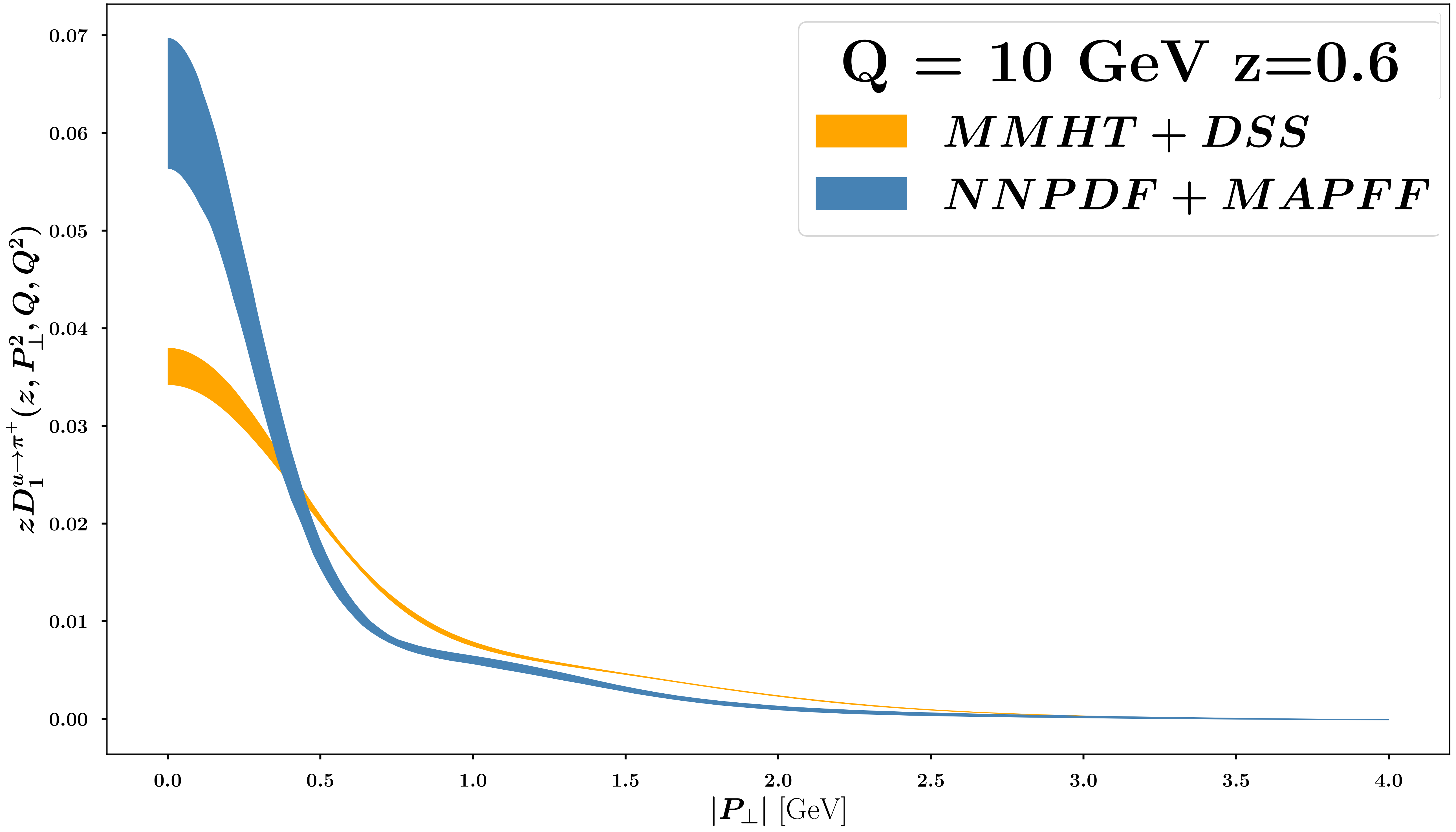
DEHSS NLO

NLO Hessian set

MAPFF

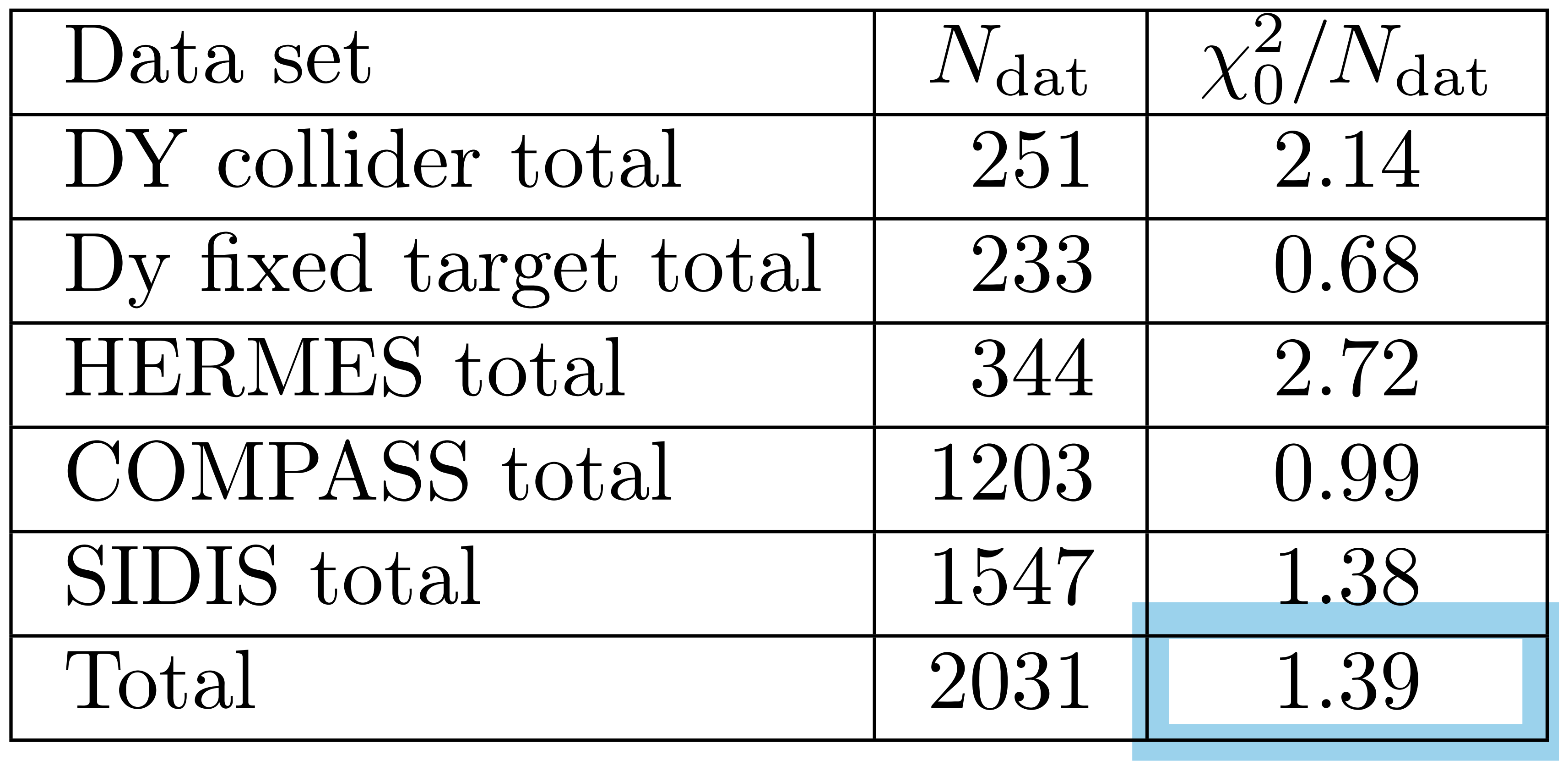
N^{2}LO Monte Carlo set

MAP collaboration, arXiv:2204.10331



INCLUSION OF HADRON DEPENDENCE IN TMD FF

Nonpert. TMD components of FF
equal for pions and kaons



Data set	N_{dat}	$\chi_0^2/N_{\mathrm{dat}}$
DY collider total	251	2.14
Dy fixed target total	233	0.68
HERMES total	344	2.72
COMPASS total	1203	0.99
SIDIS total	1547	1.38
Total	2031	1.39

Flavor blind

INCLUSION OF HADRON DEPENDENCE IN TMD FF

Nonpert. TMD components of FF equal for pions and kaons

→

Distinction between pions and kaons

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Flavor blind

Data set	N_{dat}	$\chi_0^2/N_{\mathrm{dat}}$
DY collider total	251	2.19
Dy fixed target total	233	0.72
HERMES total	344	1.61
COMPASS total	1203	0.82
SIDIS total	1547	1.00
Total	2031	1.11

PRELIMINARY

Hadron dependent

INCLUSION OF HADRON DEPENDENCE IN TMD FF

Nonpert. TMD components of FF equal for pions and kaons → Distinction between pions and kaons

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Total	2031	1.11

PRELIMINARY

Hadron dependent

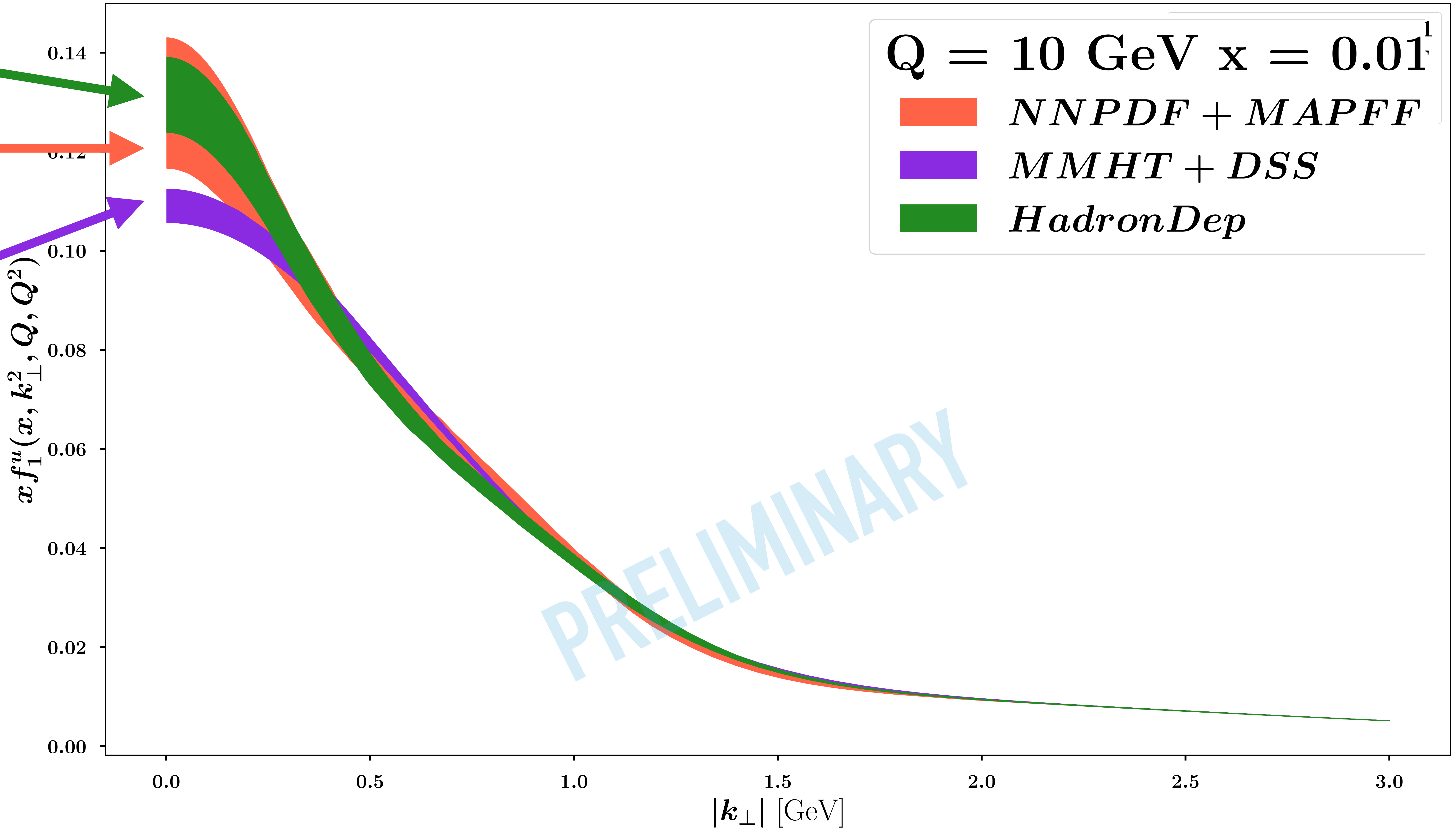
A good description of data requires different TMD FF for pions and kaons

PRELIMINARY RESULTS FOR TMD PDFS

new fit w/ hadron dependence
(χ^2 = 1.11)

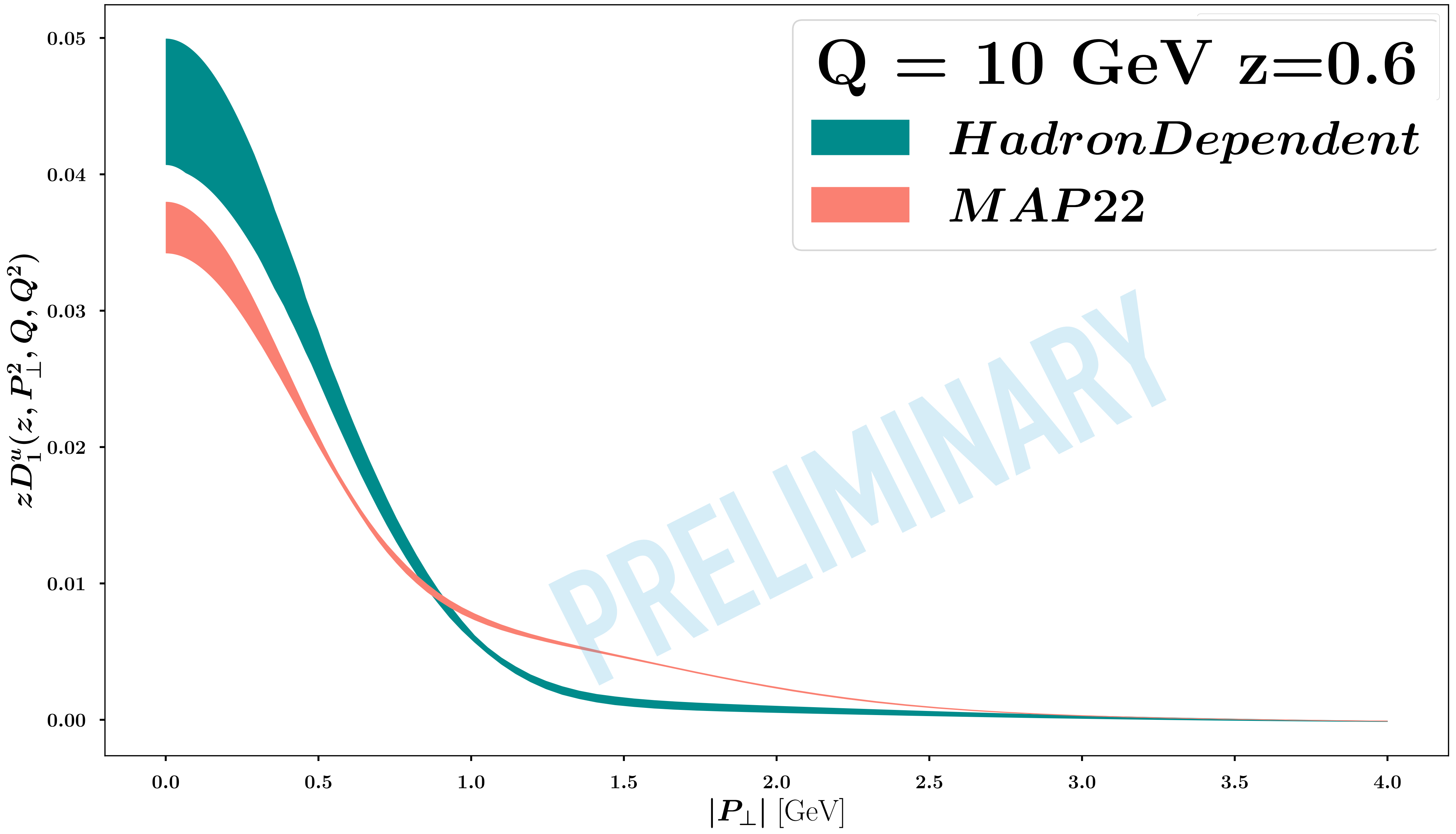
new fit (χ^2 = 1.39)

MAP22 fit (χ^2 = 1.06)



PRELIMINARY RESULTS FOR TMD FFS

comparison between different fits



PRELIMINARY RESULTS FOR TMD FFS

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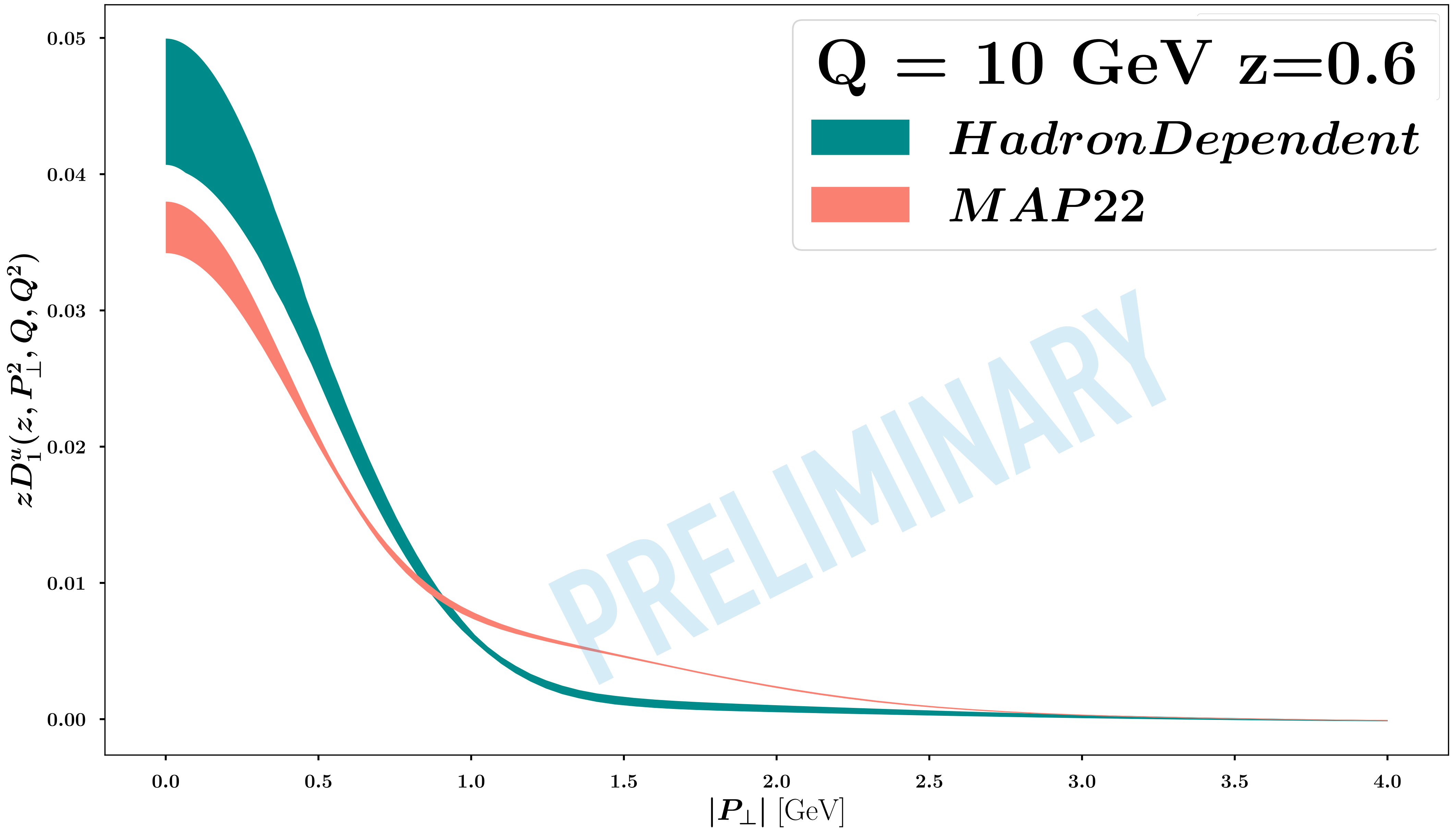
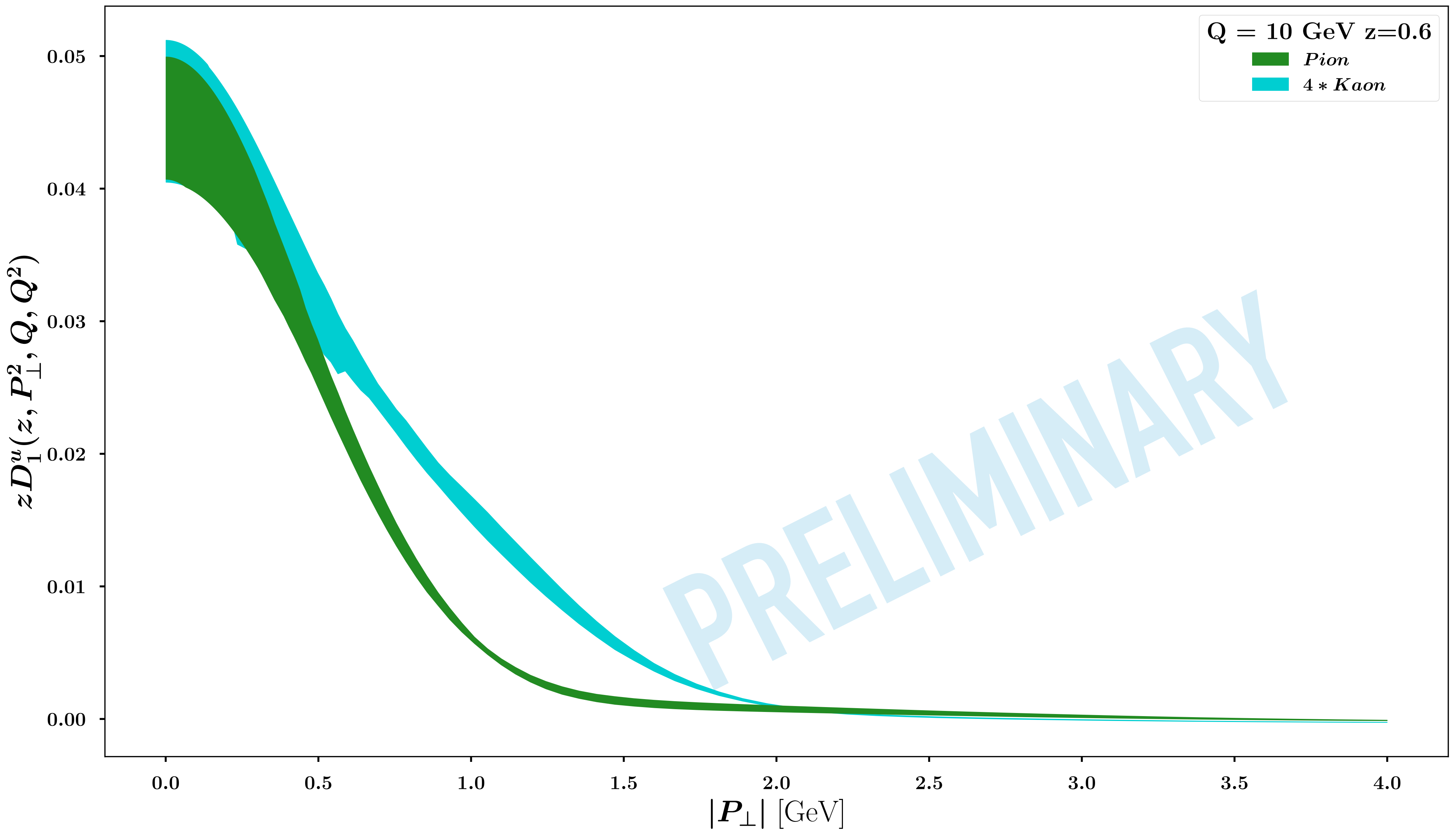


comparison between pions and kaons



CONCLUSIONS

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- The MAP22 TMD fit is currently the most advanced global TMD fit and tentatively addresses the problem of the SIDIS normalization
- We are working toward an updated global fit at full N^3LL with Monte Carlo PDF and FF replicas
- To achieve a good description of data with this updated conditions, it is necessary to introduce different TMD FF for pions and kaons