Exploring fracture functions with semi-inclusive target- and double-spin asymmetries in the target fragmentation region with CLAS12

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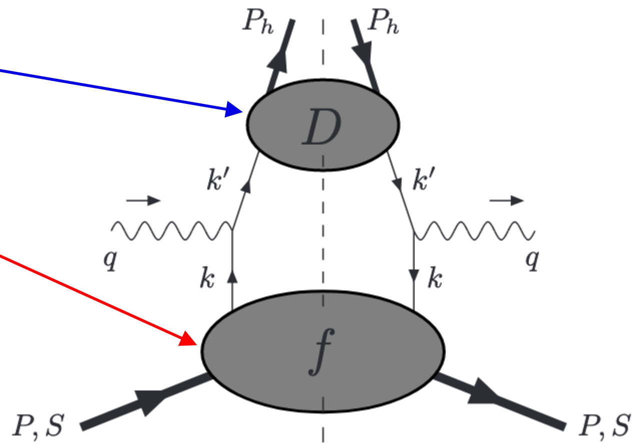
Traditional SIDIS measurements

- Decades of study have led to detailed mappings of the momentum distribution of partons in the nucleon in terms of 1-D and 3-D (TMD) parton distribution functions (PDFs).
- SIDIS measurements rely on the assumption that measured hadrons are produced in the CFR.
- Cross section factorized as a convolution of PDFs and Fragmentation Functions (FFs)[1].

$$\frac{\mathrm{d}\sigma^{\mathrm{CFR}}}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}z_h} = \sum_a e_a^2 \boxed{f_a(x_B)} \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}y} \boxed{D_a(z_h)}$$

- PDFs
  - Confined motion of quarks and gluons inside the nucleus
  - Orbital motion of quarks, correlations between quarks and gluons
- Fragmentation Functions
  - Probability for a quark to form particular final state particles
  - Insight into transverse momenta and polarization



M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

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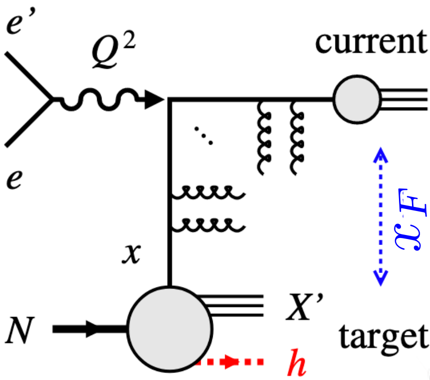
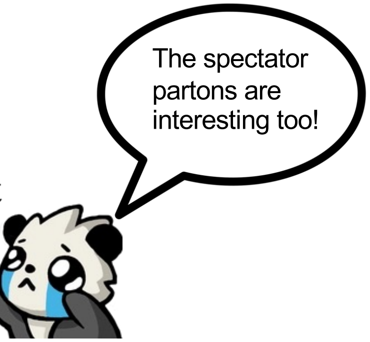
[1] A. Bacchetta et al., JHEP 02 (2007) 093 [hep-ph] 0611265,

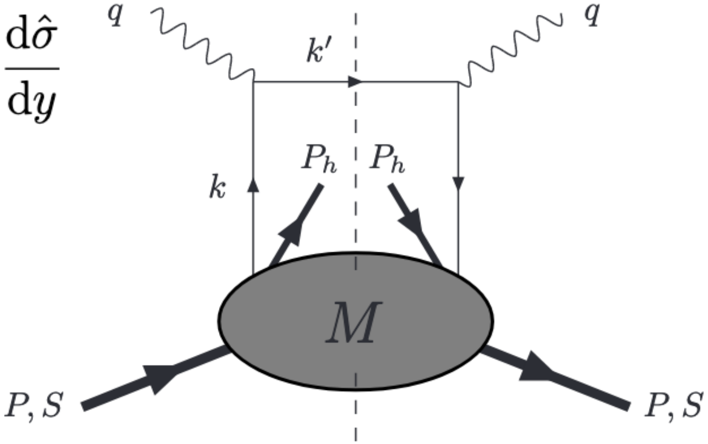
The Neglected Hemisphere – Target Fragmentation

- Final state hadrons also form from the left-over target remnant (TFR) whose partonic structure is defined by “fracture functions”[1,2]: the probability for the target remnant to form a certain hadron given a particular ejected quark.
- In the TFR, factorization into x and z does not hold because it is not possible to separate quark emission from hadron production.

$$\frac{d\sigma^{\mathrm{TFR}}}{dx_B\,dy\,dz} = \sum_a e_a^2\,(1-x_B)\boxed{M_a(x_B,(1-x_B)z)}\,\frac{d\hat{\sigma}}{dy}$$







M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

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1. L. Trentadue and G. Veneziano, Phys. Lett. B323 (1994) 201,
2. M. Anselmino et al., Phys. Lett. B. 699 (2011), 108-118, [hep-ph] 1102.4214
3. TFR/CFR Fig. from EIC Yellow Report, (2021) [physics.ins-det] 2103.05419

Analog to PDFs; Momentum Sum Rules

- A direct relationship exists to the eight leading twist PDFs after the fracture functions are integrated over the fractional longitudinal nucleon momentum, ζ.

$$\sum_h \int_0^{1-x} d\zeta \zeta M_a(x,\zeta) = (1-x) f_a(x)$$

M. Anselmino et al., Phys. Lett. B. 699 (2011), 108, [hep-ph] 1102.4214

Quark polarization

Nucleon polarization

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

Unpolarized PDF analog

helicity PDF analog

etc. etc.

Quark polarization

Nucleon polarization

	U	L	T
U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$	$\hat{t}_1^h, \hat{t}_1^\perp$
L	$\hat{u}_{1L}^{\perp h}$	$\hat{l}_{1L}$	$\hat{t}_{1L}^h, \hat{t}_{1L}^\perp$
T	$\hat{u}_{1T}^h, \hat{u}_{1T}^\perp$	$\hat{l}_{1T}^h, \hat{l}_{1T}^\perp$	$\hat{t}_{1T}, \hat{t}_{1T}^{hh}$ $\hat{t}_{1T}^{\perp\perp}, \hat{t}_{1T}^{\perp h}$

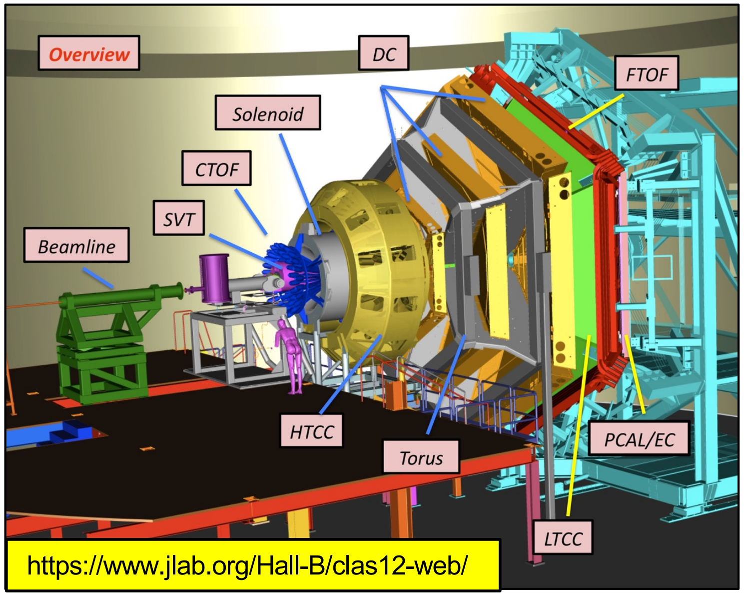
M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

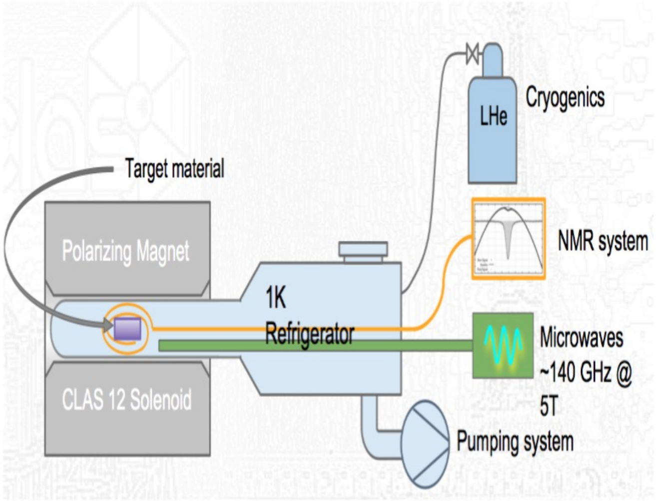
CFR ⟷ TFR

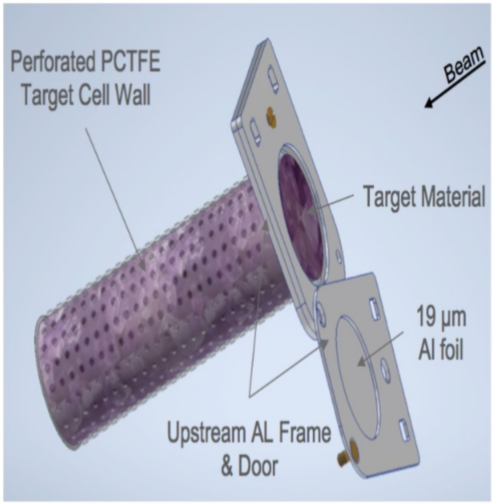
Why fracture functions?

- Sometimes possible to kinematically separate CFR and TFR (some jets, high energy DY, etc) … but not always clear (fixed target experiments).
- Without an understanding of the signals that we expect from target fragmentation we may misinterpret results that we expect are from the current.
- Interpretation of TFR structure functions is often simpler due to the lack of Collins mechanism reducing the number of available tensor structures (one fracture function per structure function!).
- Studying the TFR tests our complete understanding of the SIDIS production mechanism while also providing access to information not available in the CFR.
- Access to more familiar TMD/PDFs through momentum sum rules, but with different systematics.

CLAS12 and the Polarized Target



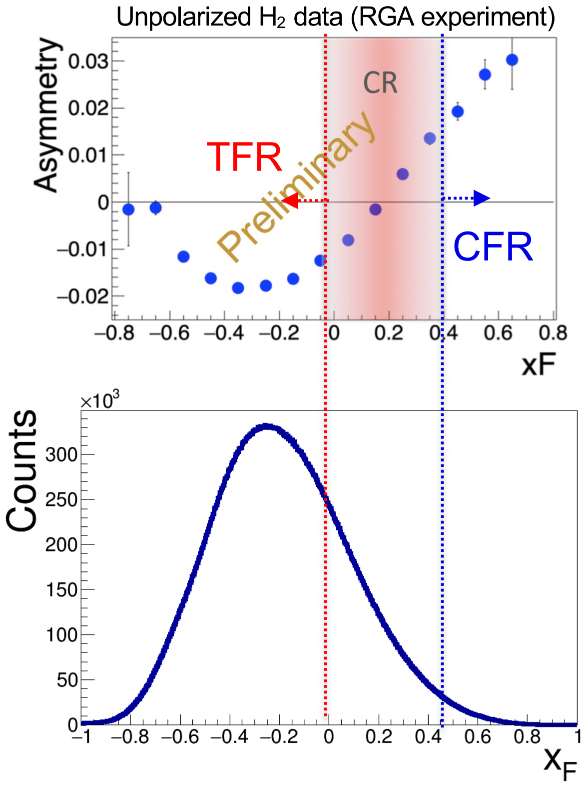




- CLAS12: very high luminosity and wide acceptance (ideal for multiparticle final states).
- Preliminary data (~5% of total experiment) available from the summer-2022 run period.
- 10.55 GeV electron beam, longitudinally polarized beam (~83%).
- Nuclear target (NH_3) dynamically polarized (~70%) using the CLAS12 solenoid. First polarized target experiment in Hall-B during the 12 GeV era. **Preliminary data; preliminary analysis.**
- Select semi-inclusive DIS proton final state.

Can We Separate Target and Current?

ep → e'pX



Feynman variable

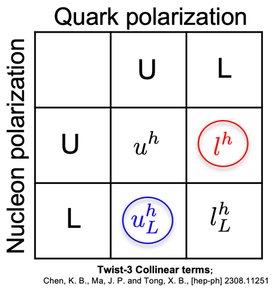
$$x_F = \frac{p_h^z}{p_h^z\,(\text{max})} \quad \text{in CM frame } \mathbf{p} = -\,\mathbf{q}, \qquad -1 < x_F < 1$$

Rapidity

$$y_h = \frac{1}{2}\log\frac{p_h^+}{p_h^-} = \frac{1}{2}\log\frac{E_h + p_h^z}{E_h - p_h^z}$$

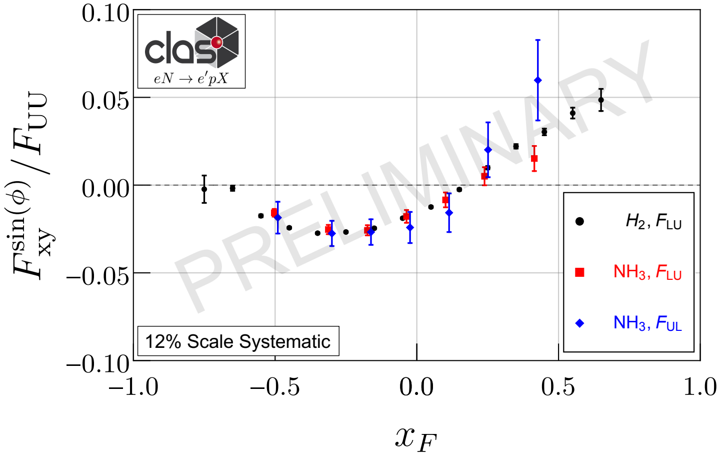
- No clear *experimental* definition of what constitutes current production versus target production.
- Odd structure functions, with different production mechanisms in both regions, give a possible clue.
- Protons (as opposed to mesons) at CLAS12 kinematics give a unique opportunity because they have extensive coverage in both regions.

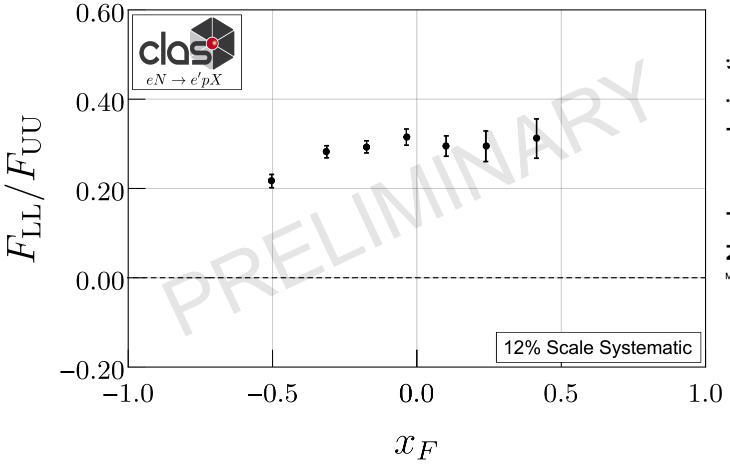
Current and Target Separation

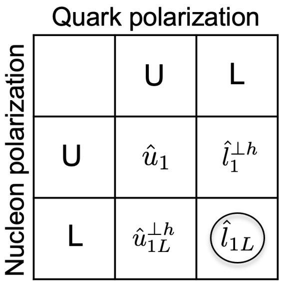


		Quark polarization	
		U	L
Nucleon polarization	U	u^h	l^h
	L	u^h_L	l^h_L

Twist-3 Collinear terms;
Chen, K. B., Ma, J. P. and Tong, X. B., [hep-ph] 2308.11251





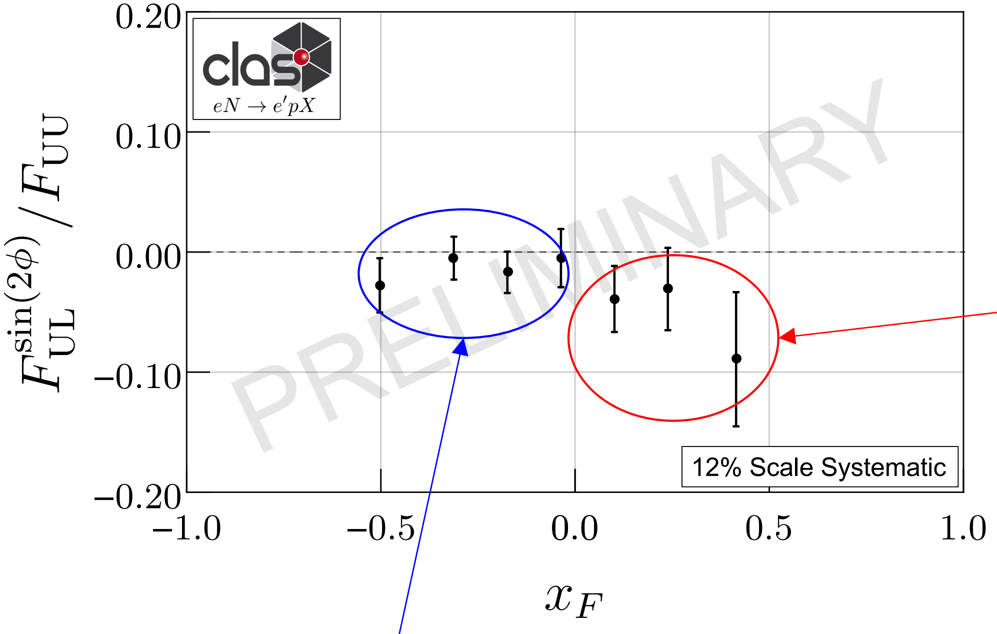


		Quark polarization	
		U	L
Nucleon polarization	U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$
	L	$\hat{u}_{1L}^{\perp h}$	$\hat{l}_{1L}$

M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

- Odd-function (sine) modulations exhibit a sign flip around the transition from target to current fragmentation. Interestingly, we observe $F_{LU} \sim F_{UL}$.
- Even-function (cosine) behavior of double-spin asymmetry does not show a sign flip; possible signs decreasing F_{LL} as $x_F \rightarrow \pm 1$ (x_B decreasing but likely not the only cause).
- Consistent beam-spin asymmetries in unpolarized H_2 and polarized NH_3 indicates minimal nuclear medium modification.

Kotzinian-Mulders Asymmetry



- No Collins mechanism in the TFR so $F_{UL}^{\sin 2\varphi}$ (and $F_{UU}^{\cos 2\varphi}$) are pure twist-4. We would expect small magnitude at $-x_F$.

$$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\left[-\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{MM_h} h_{1L}^{\perp} H_1^{\perp}\right]$$

- The $F_{UL}^{\sin 2\varphi}$ asymmetry is purely generated by the Collins mechanism – whereby a transversely polarized quark flips orientation during hadronization and produces an asymmetric distribution in the transverse plane.
- Hadronization in the TFR is more isotropic – there is no additional chiral-odd quantity like the Collins function to pair with the Kotzinian-Mulders TMD because factorization into separate soft and hard scale processes does not hold.

Early signs give a *possible* hint but need more statistics!

A_{LL} – The Best of Both Worlds

$$\frac{d\sigma}{dx\,dy\,d\zeta\,dP_T^2} = 2\pi\hat{\sigma}_U \sum_q e_q^2 \left[F_{UU,T} + \lambda S_L \sqrt{1-\varepsilon^2} F_{LL}\right]$$

M. Anselmino et al., Phys. Lett. B. 699 (2011), 108, [hep-ph] 1102.4214

Quark polarization

Nucleon polarization

	U	L
U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$
L	$\hat{u}_{1L}^{\perp h}$	$\hat{l}_{1L}$

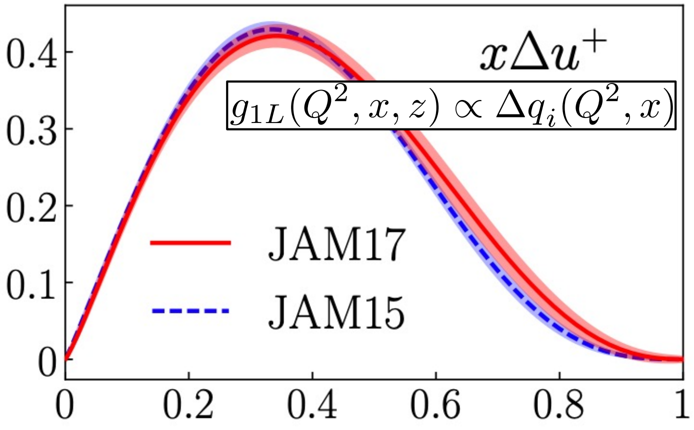
M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

At leading twist for the case of a longitudinally polarized target and a single hadron produced in the TFR, only two terms appear:

$$F_{UU,T} \propto \tilde{u}_1(x,\zeta,P_T^2) = \int d^2k_T \hat{u}_1$$

$$F_{LL} \propto \tilde{l}_{1L}(x,\zeta,P_T^2) = \int d^2k_T \hat{l}_{1L}$$

J.J. Ethier et al., *Phys. Rev. Lett.*, 119, (2017), [hep:ph 1705.05889]



Double Spin Asymmetry: $A_{LL} = \lambda_\ell S_L \dfrac{\sqrt{1-\epsilon^2} F_{LL}}{F_{UU,T}}$

1. Single hadron → Highest statistics
2. Leading twist → Simple interpretation
3. Linked to g_1 → easiest test of FrF prediction

$$\sum_h \int \zeta d\zeta \int d^2P_T \underbrace{\hat{l}_{1L} = (1-x)g_{1L}}_{}(x,k_T^2)$$

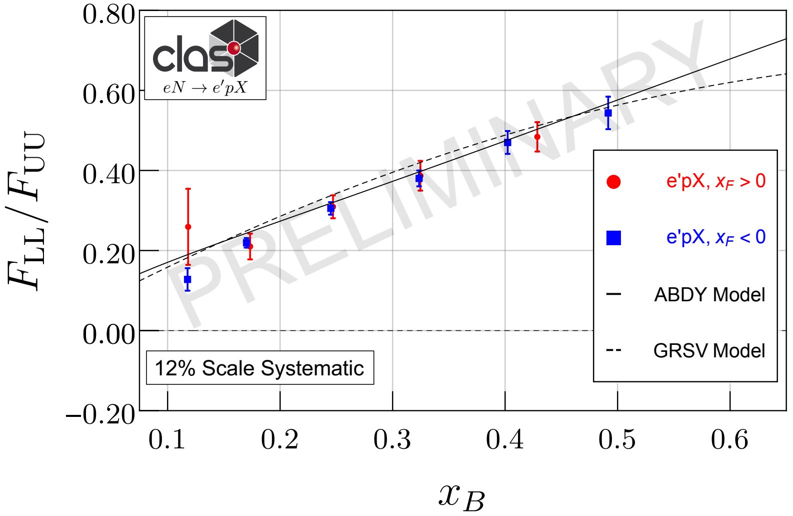
Double-spin results

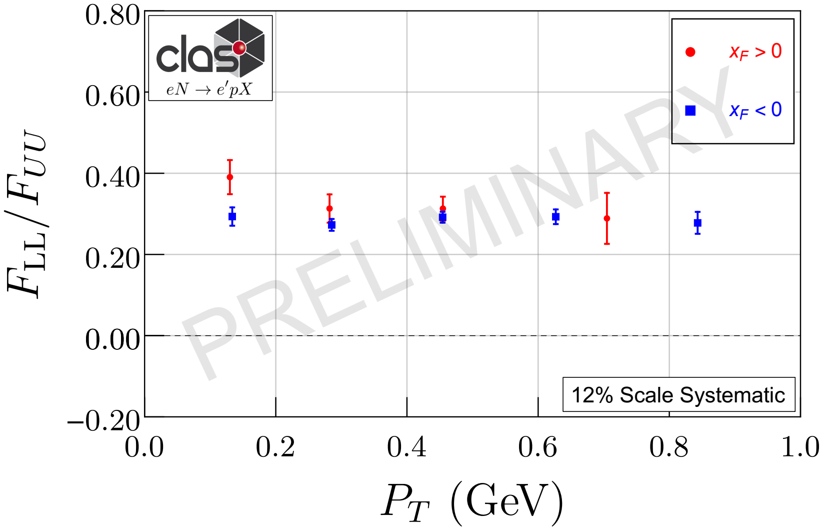
Quark polarization

Nucleon polarization

	U	L
U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$
L	$\hat{u}_{1L}^{\perp h}$	$\hat{l}_{1L}$

M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132



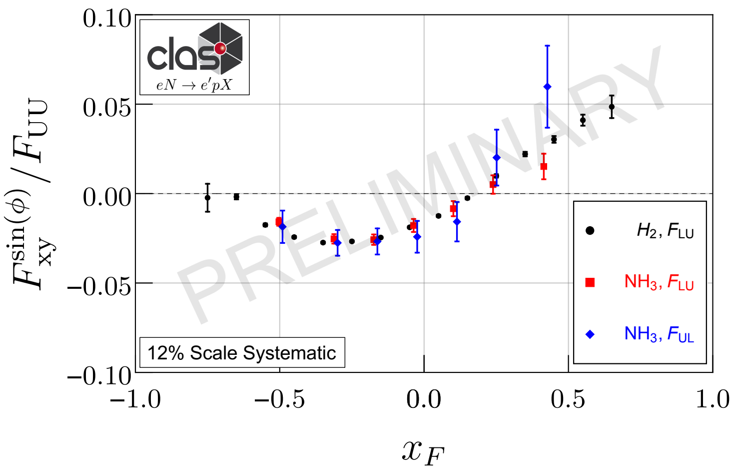


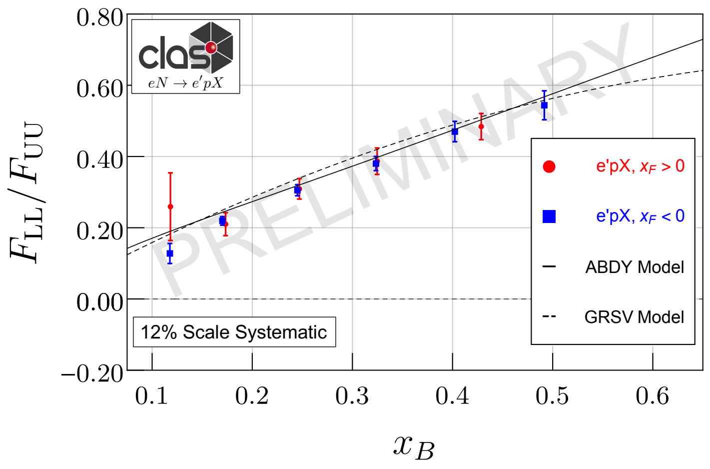
- ABDY model: analysis at large x_B in pQCD considering orbital angular momentum from valence Fock states that provide large logarithmic enhancement to ratio of polarized to unpolarized quark distributions; model terms constrained by fits to SLAC, HERMES, Hall-A and CLAS DIS data.
- GRSV model: Model incorporating totally flavor-asymmetric light sea densities $(\overline{d} > \overline{u})$ with a Pauli–blocking ansatz at the low radiative/dynamical input scales.
- Good agreement between the inclusive proton and the DIS helicity distribution models. The P_T dependence will allow for high-statistics constraints on the helicity TMD.

1. ABDY Model: H. Avakian et al., Phys.Rev.Lett. 99 (2007) 082001, [hep-ph] 0705.1553
2. GRSV Model: M. Glück et al., Phys. Rev. D. 63 (2000) 094005, [hep-ph] 0011215v1
3. Fragmentation function from DSS: Phys.Rev.D 75 (2007) 114010 [hep-ph] 0703242

Conclusions

- Protons produced in SIDIS at CLAS12 provide a unique opportunity to study both target- and current-fragmentation as well as the transition between them.
- The RGC polarized target provides the first opportunity to study leading-twist TFR measurements that have a direct PDF-analog as well as investigate the (assumed) lack of Collins mechanism in the TFR.
- Double-spin asymmetry is linked to the helicity distribution and can be used in global collinear fits or for studying the TMD.

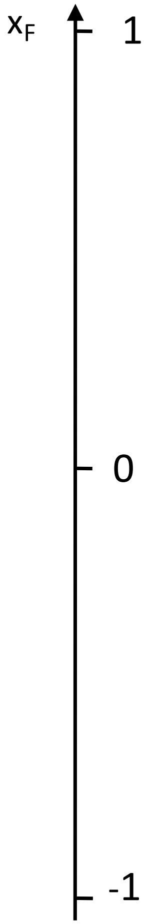


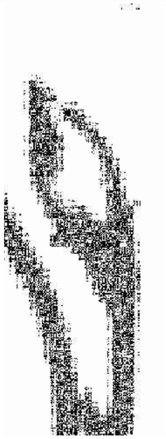


Back Up Slides

Completing the Picture

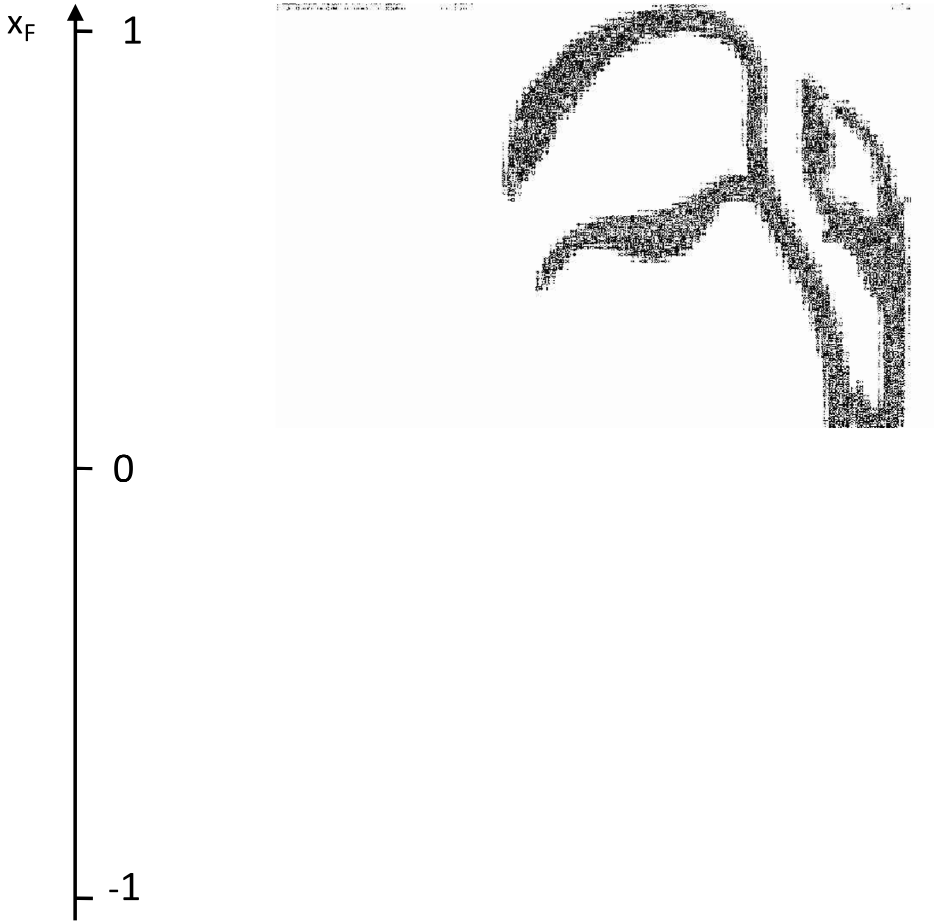
CFR





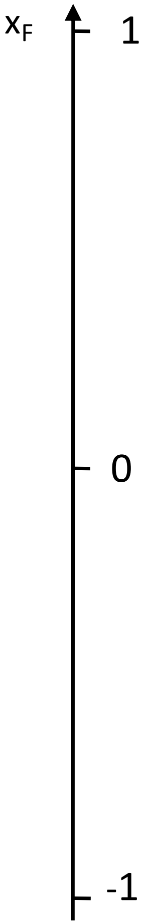
Completing the Picture

Larger phase space

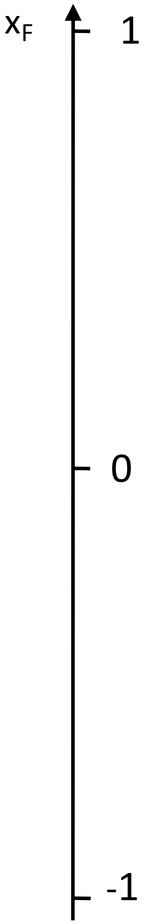


Completing the Picture

Better statistics



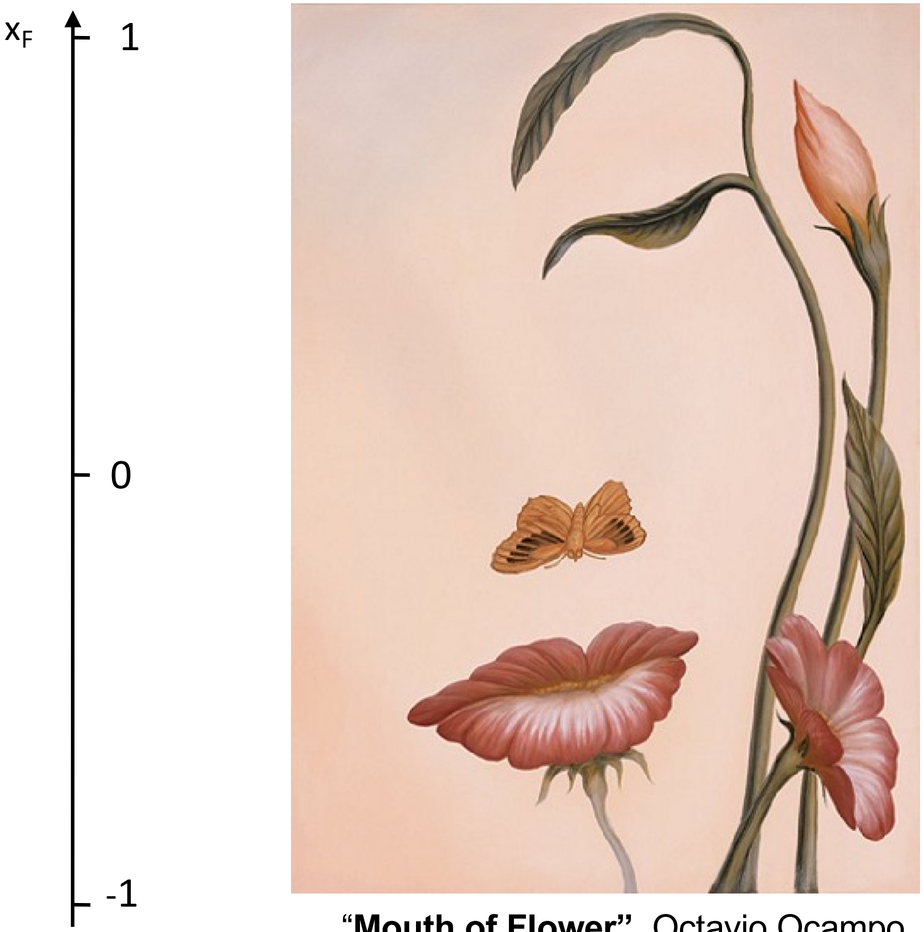
Completing the Picture



TFR with good statistics

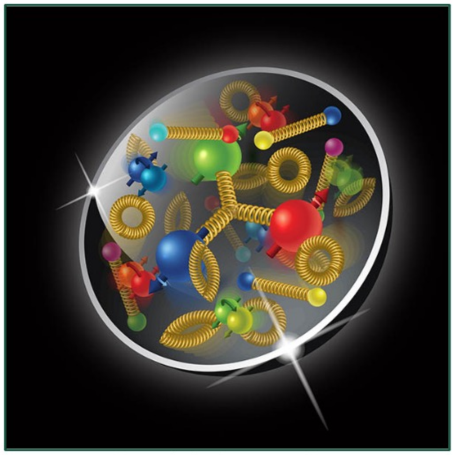
Completing the Picture

Full picture!

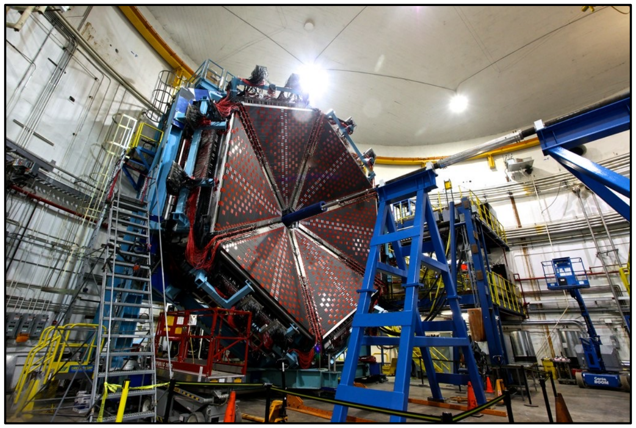


"Mouth of Flower", Octavio Ocampo

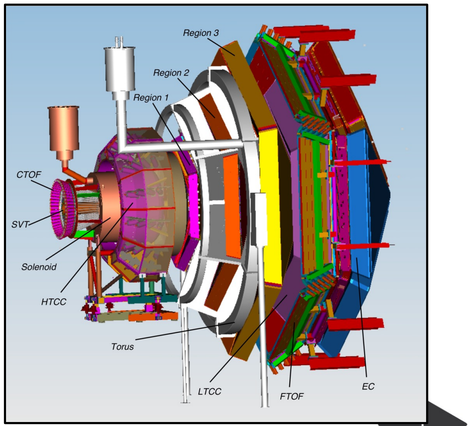
CLAS12 (Hall B) Physics Program


Stony Brook University News, August 2017

- International collaboration with more than 40 member institutions and 200 full members.
- CLAS(12) is the world’s only large acceptance and high luminosity spectrometer for fixed target lepton scattering experiments.



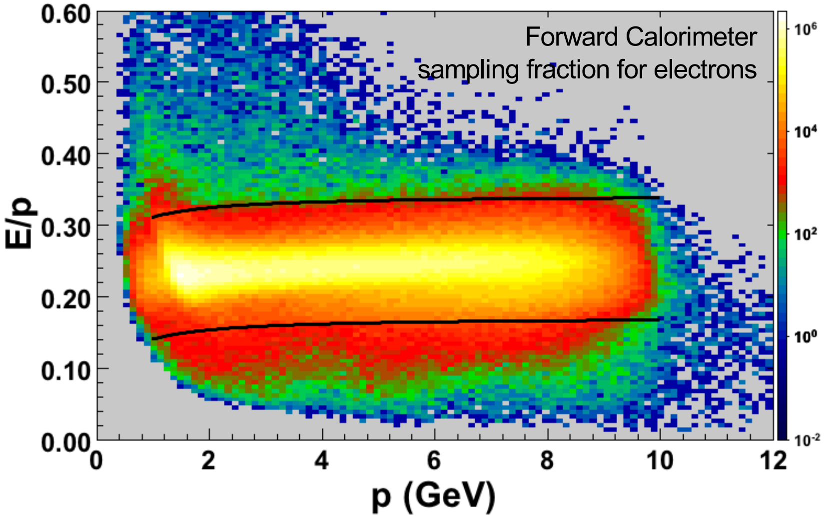
1. Study of the nucleon resonance structure at photon virtualities from 2.0 to 12 GeV^2
2. Study of Generalized Parton Distributions (GPDs), (2 +1) D imaging of the proton and the study of its gravitational and mechanical structure.
3. Study of the Transverse Momentum Dependence (TMDs) and the of 3D structure in momentum space.
4. Study of J/ψ Photoproduction, LHCb Pentaquarks and Timelike Compton Scattering.
5. Study of meson spectroscopy in search of hybrid mesons
6. Much more!

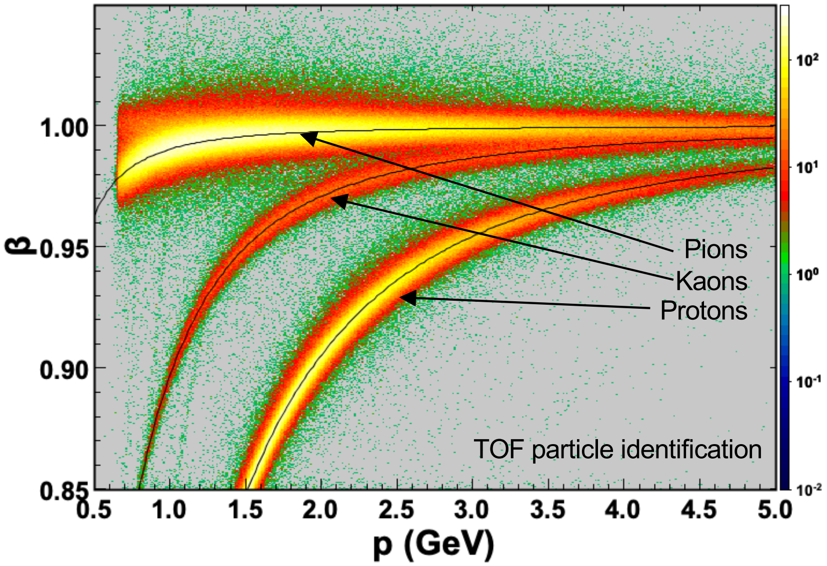


Particle ID

- Electron
  - Electromagnetic calorimeter.
  - Cherenkov detector.
  - Vertex and fiducial cuts.

- Hadron
  - β vs p comparison between vertex timing and event start time.
  - Vertex and fiducial cuts..





Extracting structure functions

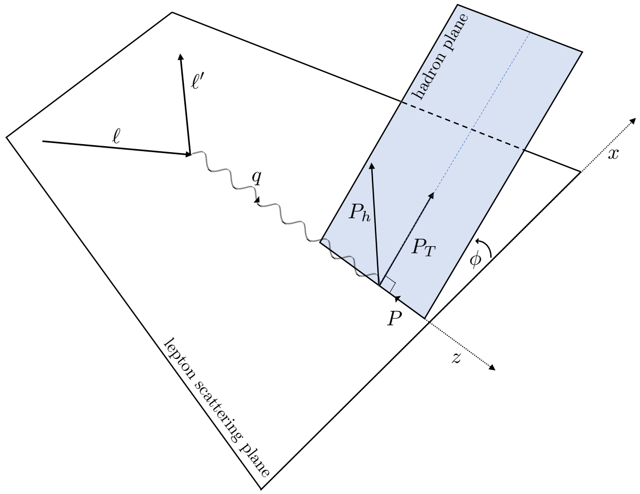
- Select $ep \rightarrow e'pX$. (X could be any additional particles)
- Cuts on the missing mass to avoid exclusive events and resonances (ρ^0, ϕ, f_2-meson etc).
- Unbinned maximum likelihood method:

$$\Sigma^{bt} = \sum_i^{N^{bt}} \Big[1 + \frac{V(\epsilon,y)}{A(\epsilon,y)} \frac{F_{UU}^{\cos\phi}}{F_{UU}} \cos\phi + \frac{B(\epsilon,y)}{A(\epsilon,y)} \frac{F_{UU}^{\cos(2\phi)}}{F_{UU}} \cos(2\phi) + P_b h_b \frac{W(\epsilon,y)}{A(\epsilon,y)} \frac{F_{LU}^{\sin\phi}}{F_{UU}} \sin\phi +$$

$$P_t h_t D_f \frac{V(\epsilon,y)}{A(\epsilon,y)} \frac{F_{UL}^{\sin\phi}}{F_{UU}} \sin\phi + P_t h_t D_f \frac{B(\epsilon,y)}{A(\epsilon,y)} \frac{F_{UL}^{\sin(2\phi)}}{F_{UU}} \sin(2\phi) +$$

$$P_b h_b P_t h_t D_f \frac{C(\epsilon,y)}{A(\epsilon,y)} \frac{F_{LL}^{1}}{F_{UU}} + P_b h_b P_t h_t D_f \frac{W(\epsilon,y)}{A(\epsilon,y)} \frac{F_{LL}^{\cos\phi}}{F_{UU}} \Big].$$

- Use MINUIT to minimize the -log likelihood.
- Include relevant beam polarization (~83% at JLab), target polarizations (~70%) and depolarization factors on an event-by-event basis.



Channel selection
- Q^2>1.0 GeV^2
- W>2.0 GeV
- y<0.75
- M_x>1.40 GeV

Potential Ambiguities

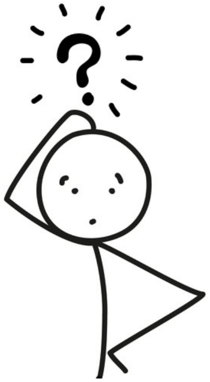
$$\frac{\mathrm{d}\sigma^{\mathrm{TFR}}}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}\zeta\,\mathrm{d}^2\boldsymbol{P}_{h\perp}\,\mathrm{d}\phi_S} = \frac{2\alpha_{\mathrm{em}}^2}{Q^2 y}\left\{\left(1-y+\frac{y^2}{2}\right)\right. \times \sum_a e_a^2\left[\hat{u}_1(x_B,\zeta,\boldsymbol{P}_{h\perp}^2) - |\boldsymbol{S}_\perp|\frac{|\boldsymbol{P}_{h\perp}|}{m_h}\,\hat{u}_{1T}^{\perp}(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)\boxed{\sin(\phi_h-\phi_S)}\right] + \lambda_l\, y\left(1-\frac{y}{2}\right)\sum_a e_a^2\left[S_\parallel\ \hat{l}_{1L}\ (x_B,\zeta,\boldsymbol{P}_{h\perp}^2) + \left.|\boldsymbol{S}_\perp|\frac{|\boldsymbol{P}_{h\perp}|}{m_h}\ \hat{l}_{1T}^{\perp}\ (x_B,\zeta,\boldsymbol{P}_{h\perp}^2)\boxed{\cos(\phi_h-\phi_S)}\right]\right\}.$$

M. Anselmino et al., Phys. Lett. B. 699 (2011), 108-118, [hep-ph] 1102.4214

$$\boxed{\begin{aligned}\left[F_{UT,T}^{\sin(\phi_h-\phi_S)}\right]_{\mathrm{TFR}} &= -\sum_a e_a^2\, x_B\,\frac{|\boldsymbol{P}_{h\perp}|}{m_h}\hat{u}_{1T}^{\perp}(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)\\ \left[F_{UT,T}^{\sin(\phi_h-\phi_S)}\right]_{\mathrm{CFR}} &= \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp}{m_N}\,f_{1T}^{\perp}\,D_1\right]\end{aligned}}$$

$$\boxed{\begin{aligned}\left[F_{LT}^{\cos(\phi_h-\phi_S)}\right]_{\mathrm{TFR}} &= \sum_a e_a^2\, x_B\,\frac{|\boldsymbol{P}_{h\perp}|}{m_h}\ \hat{l}_{1T}^{\perp}\ (x_B,\zeta,\boldsymbol{P}_{h\perp}^2)\\ \left[F_{LT}^{\cos(\phi_h-\phi_S)}\right]_{\mathrm{CFR}} &= \mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp}{m_N}\,g_{1T}\,D_1\right]\end{aligned}}$$

The same azimuthal asymmetries can appear in both the CFR and TFR, complicating their interpretation…



… six more azimuthal asymmetries appear in the CFR at leading twist which are absent in the TFR.

Cross Section and *Key* Observables

$$\begin{aligned}\frac{d\sigma}{dxdyd\zeta dP_T^2 d\phi_h} =\hat{\sigma}_U \Big[& F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} F_{UU}^{\cos\phi_h}\cos\phi_h + \varepsilon F_{UU}^{\cos 2\phi_h}\cos(2\phi_h) + \\ & \lambda_\ell\sqrt{2\varepsilon(1-\varepsilon)}\boxed{F_{LU}^{\sin\phi_h}}\sin\phi_h + S_L\sqrt{2\varepsilon(1+\varepsilon)}\boxed{F_{UL}^{\sin\phi_h}}\sin\phi_h + \\ & \lambda_\ell S_L\sqrt{1-\varepsilon^2}\boxed{F_{LL}} + \lambda_\ell S_L\sqrt{2\varepsilon(1-\varepsilon)}F_{LL}^{\cos\phi_h}\cos\phi_h + S_L\varepsilon\boxed{F_{UL}^{\sin 2\phi_h}}\sin(2\phi_h)\Big]\end{aligned}$$

1. $F_{LU}^{\sin\phi}$ - qualitative cross check with H_2, most precise observable detailing CFR/TFR split
2. $F_{UL}^{\sin\phi}$ - complimentary information to twist-3 beam-spin asymmetry
3. $F_{UL}^{\sin(2\phi)}$ - opportunity to test the lack of Collins mechanism in the TFR
4. F_{LL}^{1} - link to the relatively well-known helicity TMD-PDF; contribution to global collinear fits
5. $F_{LU}^{\sin\phi}/F_{LL}^{1}$ - test of twist-3 Q^2 dependence without F_{UU}

Tables

Twist-2

Quark polarization

Nucleon polarization

	U	L	T
U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$	$\hat{t}_1^h, \hat{t}_1^\perp$
L	$\hat{u}_{1L}^{\perp h}$	$\hat{l}_{1L}$	$\hat{t}_{1L}^h, \hat{t}_{1L}^\perp$
T	$\hat{u}_{1T}^h, \hat{u}_{1T}^\perp$	$\hat{l}_{1T}^h, \hat{l}_{1T}^\perp$	$\hat{t}_{1T}, \hat{t}_{1T}^{hh}$ $\hat{t}_{1T}^{\perp\perp}, \hat{t}_{1T}^{\perp h}$

M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

Twist-3 Collinear

Quark polarization

Nucleon polarization

	U	L	T
U	u^h	l^h	u_T
L	u_L^h	l_L^h	l_T, l_T^h
T	u_T^h		

Collinear terms; Chen, K. B., Ma, J. P. and Tong, X. B., [hep-ph] 2308.11251

Tables

Twist-2

Quark polarization

Nucleon polarization

	U	L
U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$
L	$\hat{u}_{1L}^{\perp h}$	$\hat{l}_{1L}$

M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

Twist-3 Collinear

Quark polarization

Nucleon polarization

	U	L
U	u^h	l^h
L	u_L^h	l_L^h

Collinear terms; Chen, K. B., Ma, J. P. and Tong, X. B., [hep-ph] 2308.11251

Tables

Twist-2

Quark polarization

Nucleon polarization

	U	L
U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$
L	$\hat{u}_{1L}^{\perp h}$	$\hat{l}_{1L}$

M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

Twist-3 Collinear

Quark polarization

Nucleon polarization

	U	L
U	u^h	l^h
L	u_L^h	l_L^h

Twist-3 Collinear terms;
Chen, K. B., Ma, J. P. and Tong, X. B., [hep-ph] 2308.11251

Single hadron limitations

- FrFs describing transversely polarized quarks are chiral odd and inaccessible in TFR single hadron production where there is no access to a chiral odd FF.
- Functions with double superscripts containing h and ⊥ have give the unique possibility of measuring longitudinal polarized quarks in unpolarized nucleons (and vice versa) but disappear after integration over either momentum.

Quark polarization

Nucleon polarization

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$

CFR ⟷ TFR

Quark polarization

Nucleon polarization

	U	L	T
U	$\hat{u}_1$	$\hat{l}_1^{\perp h}$	$\hat{t}_1^h, \hat{t}_1^\perp$
L	$\hat{u}_{1L}^{\perp h}$	$\hat{l}_{1L}$	$\hat{t}_{1L}^h, \hat{t}_{1L}^\perp$
T	$\hat{u}_{1T}^h, \hat{u}_{1T}^\perp$	$\hat{l}_{1T}^h, \hat{l}_{1T}^\perp$	$\hat{t}_{1T}, \hat{t}_{1T}^{hh}$ $\hat{t}_{1T}^{\perp\perp}, \hat{t}_{1T}^{\perp h}$

M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132

Depolarization Factors

$$\frac{d\sigma}{dx\,dy\,d\zeta\,dP_T^2\,d\phi_h} = \hat{\sigma}_U \Big[F_{UU,T} + \varepsilon F_{UU,L} + \boxed{\sqrt{2\varepsilon(1+\varepsilon)}}\, F_{UU}^{\cos\phi_h} \cos\phi_h + \boxed{\varepsilon}\, F_{UU}^{\cos 2\phi_h} \cos(2\phi_h) + \lambda_\ell \boxed{\sqrt{2\varepsilon(1-\varepsilon)}}\, F_{LU}^{\sin\phi_h} \sin\phi_h + S_L \boxed{\sqrt{2\varepsilon(1+\varepsilon)}}\, F_{UL}^{\sin\phi_h} \sin\phi_h + \lambda_\ell \boxed{S_L\sqrt{1-\varepsilon^2}}\, F_{LL} + \lambda_\ell S_L \boxed{\sqrt{2\varepsilon(1-\varepsilon)}}\, F_{LL}^{\cos\phi_h} \cos\phi_h + S_L \boxed{\varepsilon}\, F_{UL}^{\sin 2\phi_h} \sin(2\phi_h) \Big].$$

$$A(\epsilon, y) \equiv \frac{y^2}{2(1-\epsilon)} = \frac{1}{1+\gamma^2}\left(1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2\right) \approx \left(1 - y + \frac{1}{2}y^2\right), \tag{1.15}$$

$$B(\epsilon, y) \equiv \frac{y^2}{2(1-\epsilon)}\epsilon = \frac{1}{1+\gamma^2}\left(1 - y - \frac{1}{4}\gamma^2 y^2\right) \approx (1-y), \tag{1.16}$$

$$C(\epsilon, y) \equiv \frac{y^2}{2\,(1-\epsilon)}\sqrt{1-\epsilon^2} = \frac{y}{\sqrt{1+\gamma^2}}\left(1 - \frac{1}{2}y\right) \approx y\left(1 - \frac{1}{2}y\right), \tag{1.17}$$

$$V(\epsilon, y) \equiv \frac{y^2}{2\,(1-\epsilon)}\sqrt{2\epsilon(1+\epsilon)} = \frac{2-y}{1+\gamma^2}\sqrt{1 - y - \frac{1}{4}\gamma^2 y^2} \approx (2-y)\sqrt{1-y}, \tag{1.18}$$

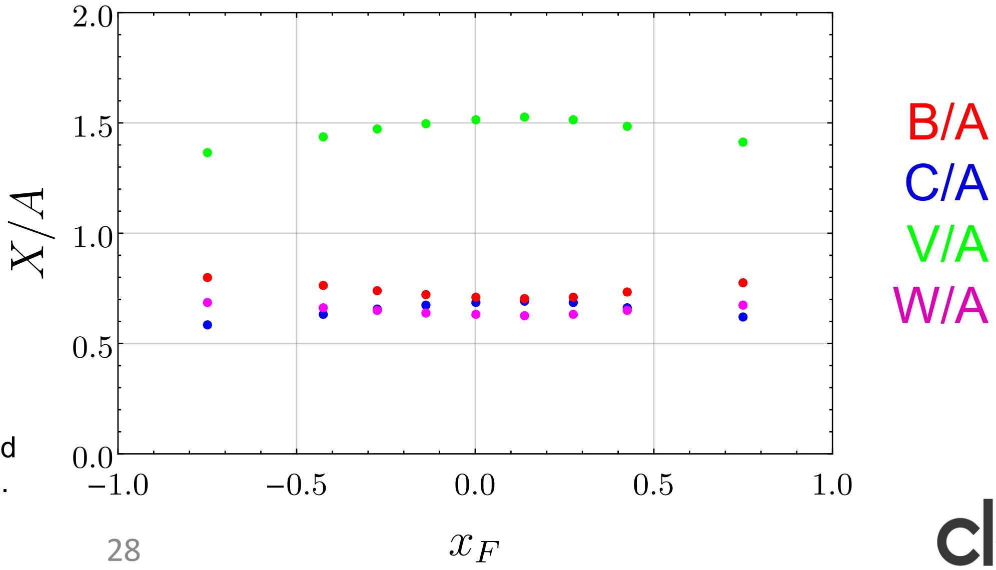
$$W(\epsilon, y) \equiv \frac{y^2}{2\,(1-\epsilon)}\sqrt{2\epsilon(1-\epsilon)} = \frac{y}{\sqrt{1+\gamma^2}}\sqrt{1 - y - \frac{1}{4}\gamma^2 y^2} \approx y\sqrt{1-y}, \tag{1.19}$$

$$A_{LU}^{\sin\phi} = \left(\frac{W}{A}\right)\left(\frac{F_{LU}^{\sin\phi}}{F_{UU}}\right) \qquad A_{LL} = \left(\frac{C}{A}\right)\left(\frac{F_{LL}^{0}}{F_{UU}}\right)$$

$$A_{UL}^{\sin\phi} = \left(\frac{V}{A}\right)\left(\frac{F_{UL}^{\sin\phi}}{F_{UU}}\right) \qquad A_{LL}^{\cos\phi} = \left(\frac{W}{A}\right)\left(\frac{F_{LL}^{\cos(\phi)}}{F_{UU}}\right)$$

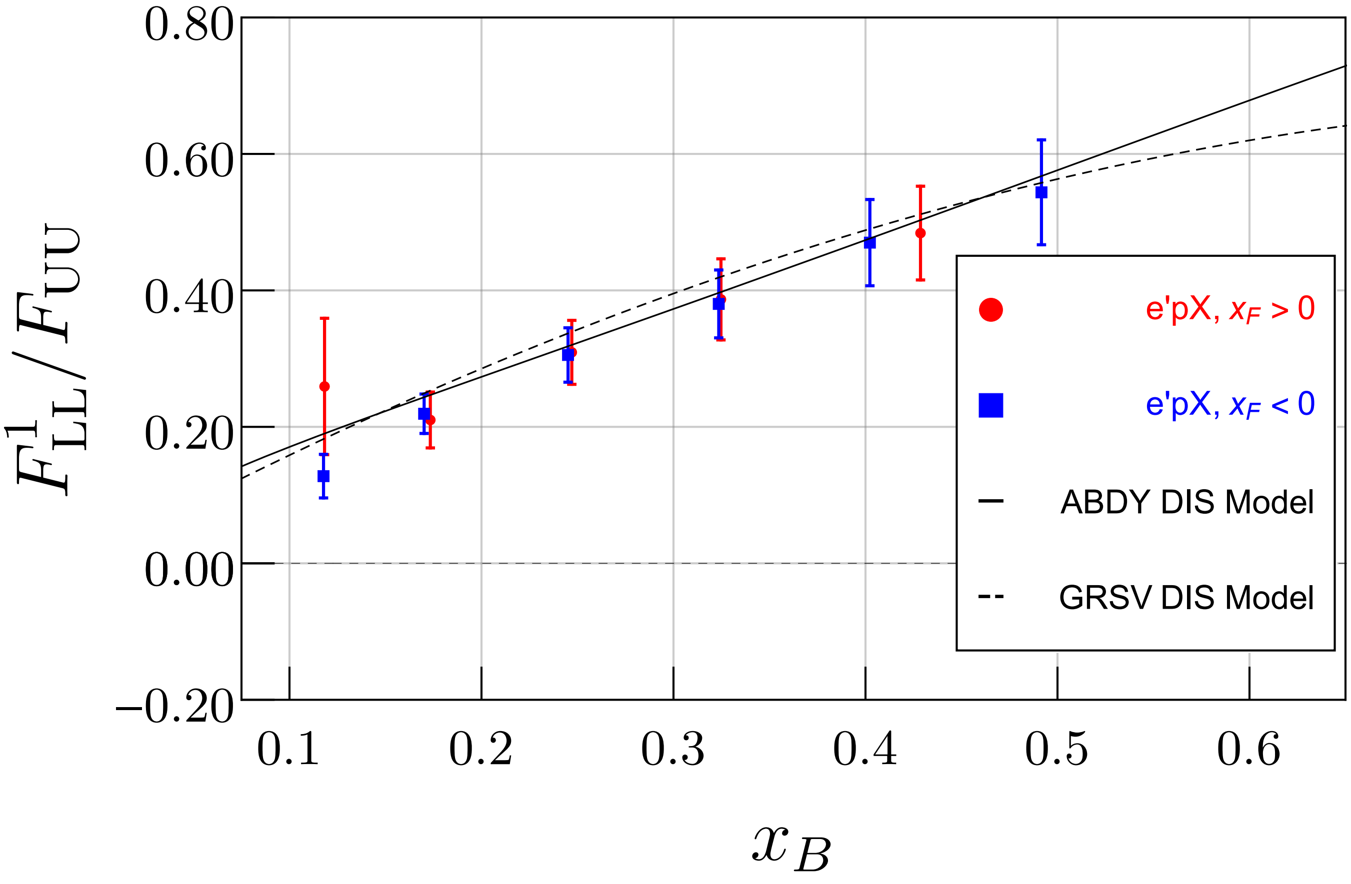
$$A_{UL}^{\sin(2\phi)} = \left(\frac{B}{A}\right)\left(\frac{F_{UL}^{\sin(2\phi)}}{F_{UU}}\right)$$

- Terms added event-by-event in likelihood function.
- Relatively small difference between event-by-event and weighted average; more appropriate for F_{UL} and F_{LL} (that have two terms).



Consistent with both models

Statistical and systematic uncertainties added in quadrature.



Pion Release

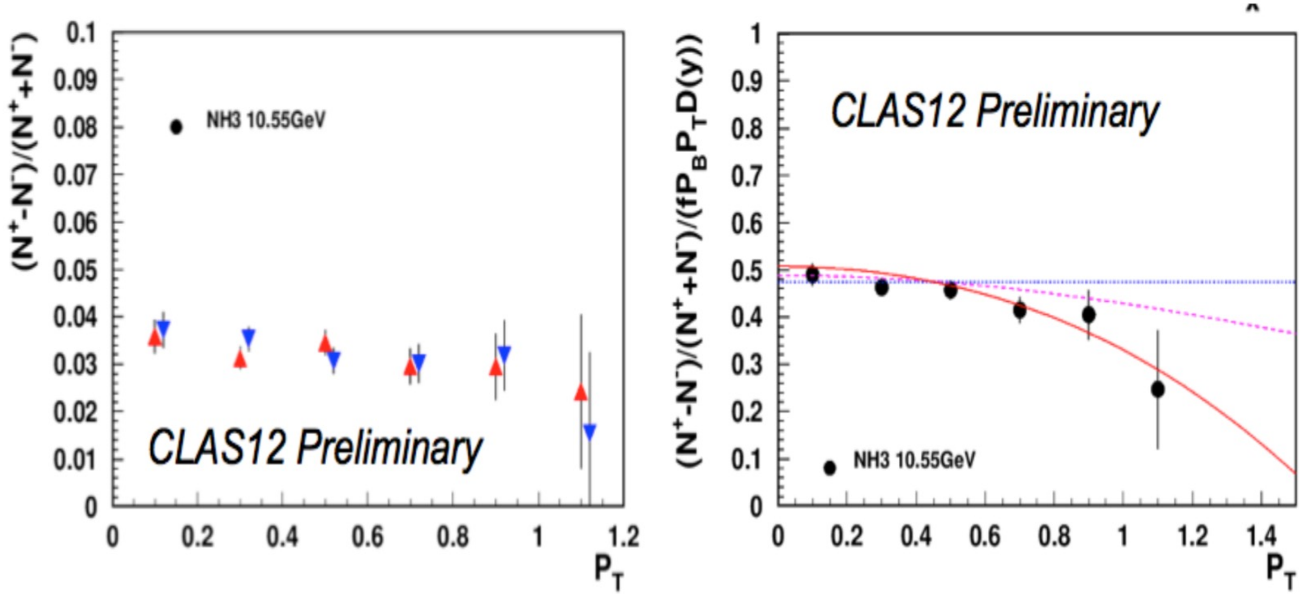


Figure 11: The P_T-dependence of the raw asymmetry in a bin in (left) and the double spin asymmetry corrected for polarization, dilution,and depolarization factors (right). Dotted line is for the equal widths of $f_1(x, k_T)$, dashed magenta line for 10% difference in Gaussian widths, and the red curve corresponds to widths predicted by lattice [Mus05] (see APPENDIX:A)

Twist-3 TFR Observables

$$\int \frac{d\lambda}{2\pi} e^{-ix\lambda P^+} \sum_X \langle h_A \,| \bar{\psi}_j(\lambda n) \mathcal{L}_n^\dagger(\lambda n) | h_B(P_h), X \rangle \langle X, h_B(P_h) | \mathcal{L}_n(0) \psi_i(0) |\, h_A \rangle \Big|_{\text{twist}-3,\ U,L-\text{target}}$$

$$= \frac{1}{2N_c P^+} \left[\boxed{(\gamma_\perp \cdot P_{h\perp})_{ij} \hat{u}_2^{\perp h}(x, \xi, P_{h\perp})} + \boxed{(\gamma_5 \gamma_\perp \cdot \tilde{P}_{h\perp})_{ij} \hat{l}_2^{\perp h}(x, \xi, P_{h\perp})} \right]$$

$$+ \frac{S_L}{2N_c P^+} \left[\boxed{(\gamma_\perp \cdot \tilde{P}_{h\perp})_{ij} \hat{u}_{2L}^{\perp h}(x, \xi, P_{h\perp})} + \boxed{(\gamma_5 \gamma_\perp \cdot P_{h\perp})_{ij} \hat{l}_{2L}^{\perp h}(x, \xi, P_{h\perp})} \right] + \cdots,$$

K.B. Chen, J.B. Ma, X.B. Tong (Private correspondence)
c.f. JHEP, vol 11 (2021), [hep-ph] 2108.13582

$$\frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}$$

Target-spin Asymmetry

$$\boxed{A_{UL} \propto S_L \sqrt{2\epsilon(1+\epsilon^2)} F_{UL}^{\sin\phi_h} \sin\phi_h}$$

$$F_{UU}^{\cos\phi_h} \sim \tilde{u}_2^{\perp h} + \cdots\,, \quad F_{LU}^{\sin\phi_h} \sim \tilde{l}_2^{\perp h} + \cdots\,, \quad F_{UL}^{\sin\phi_h} \sim \tilde{u}_{2L}^{\perp h} + \cdots\,, \quad F_{LL}^{\cos\phi_h} \sim \tilde{l}_{2L}^{\perp h} + \cdots$$

Beam-spin Asymmetry

$$\boxed{A_{LU} \propto S_L \sqrt{2\epsilon(1-\epsilon^2)} F_{LU}^{\sin\phi_h} \sin\phi_h}$$

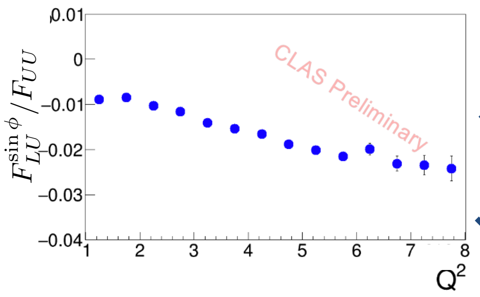
$$\frac{d\sigma}{dx dy d\zeta dP_T^2 d\phi_h} = \hat{\sigma}_U \Big[F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \boxed{F_{UU}^{\cos\phi_h} \cos\phi_h} + \varepsilon F_{UU}^{\cos 2\phi_h} \cos(2\phi_h) +$$
$$\lambda_\ell \sqrt{2\varepsilon(1-\varepsilon)} \boxed{F_{LU}^{\sin\phi_h} \sin\phi_h} + S_L \sqrt{2\varepsilon(1+\varepsilon)} \boxed{F_{UL}^{\sin\phi_h} \sin\phi_h} +$$
$$\lambda_\ell S_L \sqrt{1-\varepsilon^2} F_{LL} + \lambda_\ell S_L \sqrt{2\varepsilon(1-\varepsilon)} \boxed{F_{LL}^{\cos\phi_h} \cos\phi_h} + S_L \varepsilon F_{UL}^{\sin 2\phi_h} \sin(2\phi_h) \Big]$$

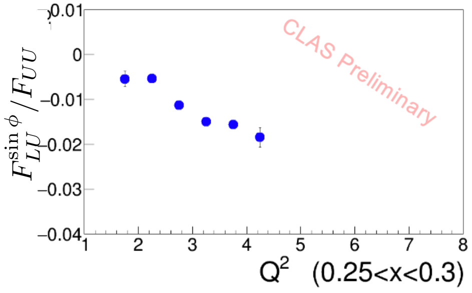
In the TFR the sin(2ϕ) and cos(2ϕ) modulations appear at twist-4 because there are no appropriate FrFs to generate the correct tensor structure.

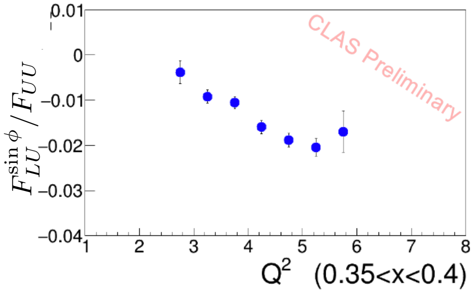
Mapping the Q^2 dependence

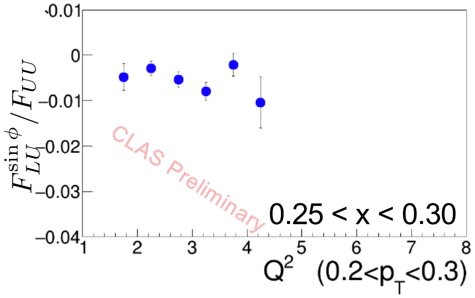
- SSAs in single hadron production are twist-3 (M/Q suppression).
- ”Twist 3” asymmetries not behaving as expected (EMC, COMPASS, CLAS12 etc!). Possible $F_{UU,L}$ contributions.
- Proper interpretation of the Q^2 dependence is crucial for our understanding of the underlying dynamics.

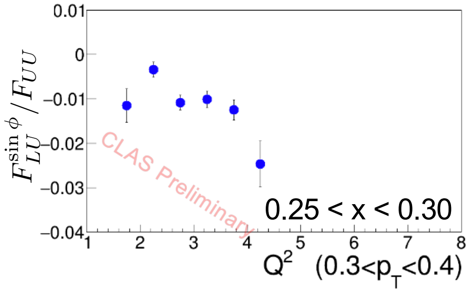
RGA Data

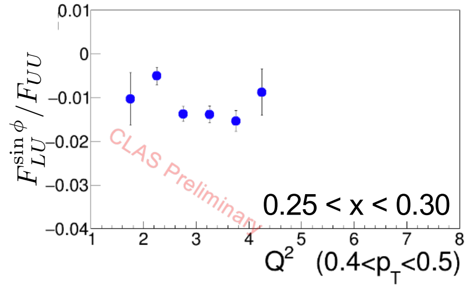












All plots $x_F < 0$.

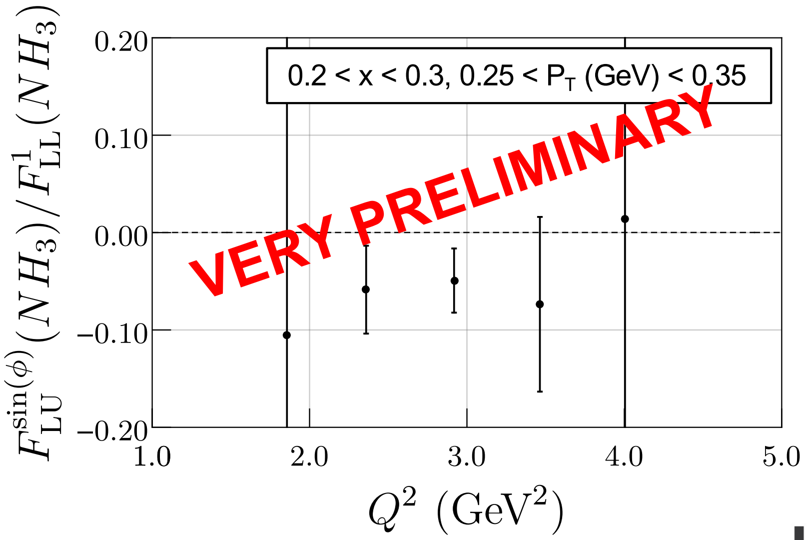
Onward to 4D

Observing the expected Q^2 behavior for subleading twist objects may require high precision multidimensional bins and care around F_{UU}.

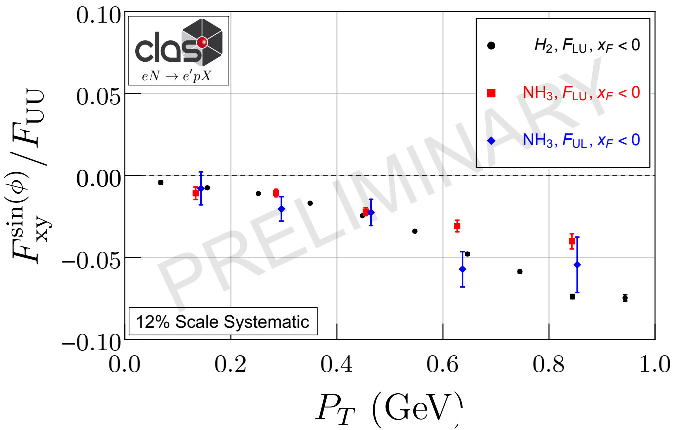
Double Ratio

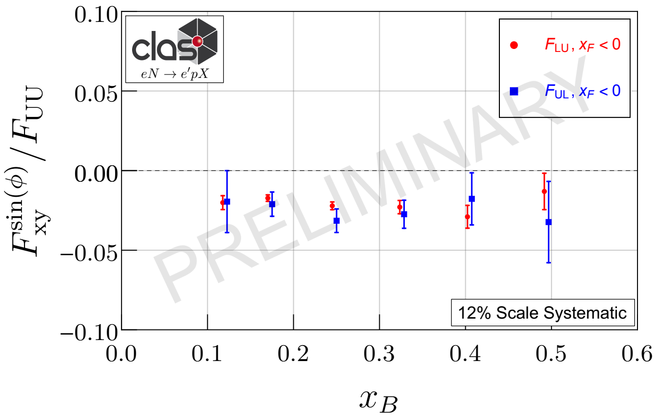
$$\frac{d\sigma}{dx\,dy\,d\zeta\,dP_T^2\,d\phi_h} = \hat{\sigma}_U \Big[F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\,F_{UU}^{\cos\phi_h}\cos\phi_h + \varepsilon F_{UU}^{\cos 2\phi_h}\cos(2\phi_h) +$$
$$\lambda_\ell \sqrt{2\varepsilon(1-\varepsilon)}\,\boxed{F_{LU}^{\sin\phi_h}}\sin\phi_h + S_L\sqrt{2\varepsilon(1+\varepsilon)}\,F_{UL}^{\sin\phi_h}\sin\phi_h +$$
$$\lambda_\ell S_L \sqrt{1-\varepsilon^2}\,\boxed{F_{LL}} + \lambda_\ell S_L\sqrt{2\varepsilon(1-\varepsilon)}\,F_{LL}^{\cos\phi_h}\cos\phi_h + S_L\varepsilon F_{UL}^{\sin 2\phi_h}\sin(2\phi_h)\Big]$$

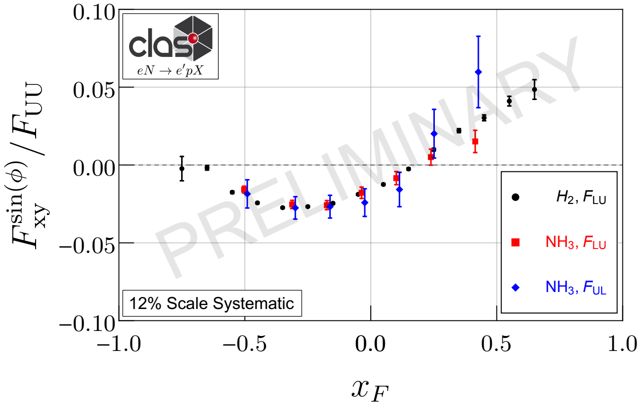
- The “double ratio” F_{LU}/F_{LL} should behave as a very clear twist-3/twist-2 ratio without the possible unconstrained contributions from $F_{UU,L}$ that have been present in previous F_{LU}/F_{UU} studies.
- Important to do very precise multidimensional bins… defer this study until the full statistics are available.



$F_{LU} \sim F_{UL}$ in epX, $x_F < 0$





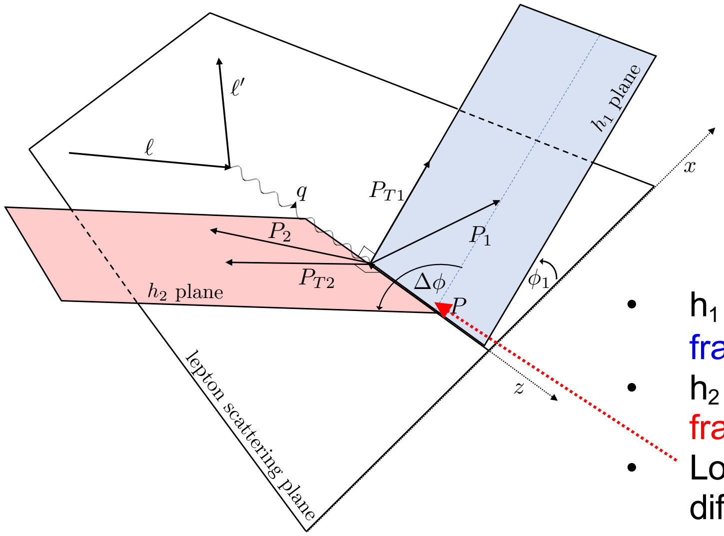


Back-to-back (dSIDIS) Formalism

- When two hadrons are produced “back-to-back”[1,2] with one in the CFR and one in the TFR the structure function contains a convolution of a fracture function and a fragmentation function.
- Leading twist beam(target)-spin asymmetry.

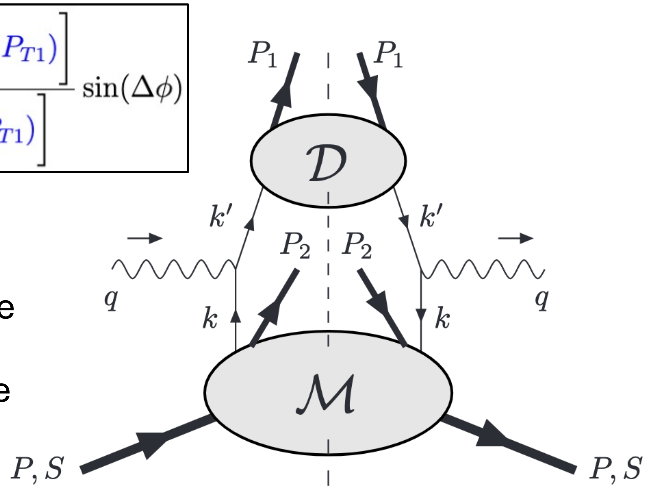
$\hat{l}_1^{\perp h}$ Unique access to longitudinally polarized quarks in unpolarized nucleon… no corresponding PDF!

$$A_{LU} = -k(\epsilon)\frac{P_{T1}P_{T2}}{m_1 m_2}\frac{\mathcal{C}\left[w_5\hat{l}_1^{\perp h}(x,\zeta_2,P_{T2})D_1(z_1,P_{T1})\right]}{\mathcal{C}\left[\hat{u}_1(x,\zeta_2,P_{T2})D_1(z_1,P_{T1})\right]}\sin(\Delta\phi)$$



Kinematic plane for b2b dihadron production.

- h_1 in the CFR with production dictated by the fragmentation function
- h_2 in the TFR with production dictated by the fracture function
- Long range correlation depends on the difference in azimuthal angles of both hadrons



Handbag diagram for dihadron production; lower blob contains FrFs and upper blob contains the FFs.

[1] M. Anselmino et al., Phys. Lett. B. 706 (2011), 46-52, [hep-ph] 1109.1132
[2] M. Anselmino et al., Phys. Lett. B. 713 (2012), 317-320, [hep-ph] 1112.2604

Access to unmeasured fracture functions

- x-dependence increases in magnitude in the valance quark region.
- ζ_2-dependence shows decreasing amplitude with increasing momenta. Possibly due to correlations with x.
- Relatively flat as a function of z_1, possibly due to cancellation of fragmentation functions.
- First observation of TMD fracture functions and long-range correlations between current and target. Already working on follow up (negative pion, deuteron target, more statistics etc.)

ep → e'pπ^+X

$$A_{LU} \propto \frac{\mathcal{C}\left[w_5 \hat{l}_1^{\perp h}(x, \zeta_2, P_{T2}) D_1(z_1, P_{T1})\right]}{\mathcal{C}\left[\hat{u}_1(x, \zeta_2, P_{T2}) D_1(z_1, P_{T1})\right]}$$

H. Avakian, T.B. Hayward et al., Phys. Rev. Lett. 130 (2023), [hep-ex] 2208.05086