An extraction of the Sivers, and the Boer-Mulders functions in SU(3) with DNNs

Ishara Fernando & Dustin Keller

September 25-29, 2023

25TH INTERNATIONAL SPIN PHYSICS SYMPOSIUM

September 24 – 29, 2023
Durham Convention Center
Durham, NC, USA

Based on the recent publication (September 2023)
https://doi.org/10.1103/PhysRevD.108.054007

UVA - Spin PHYSICS GROUP

UNIVERSITY of VIRGINIA

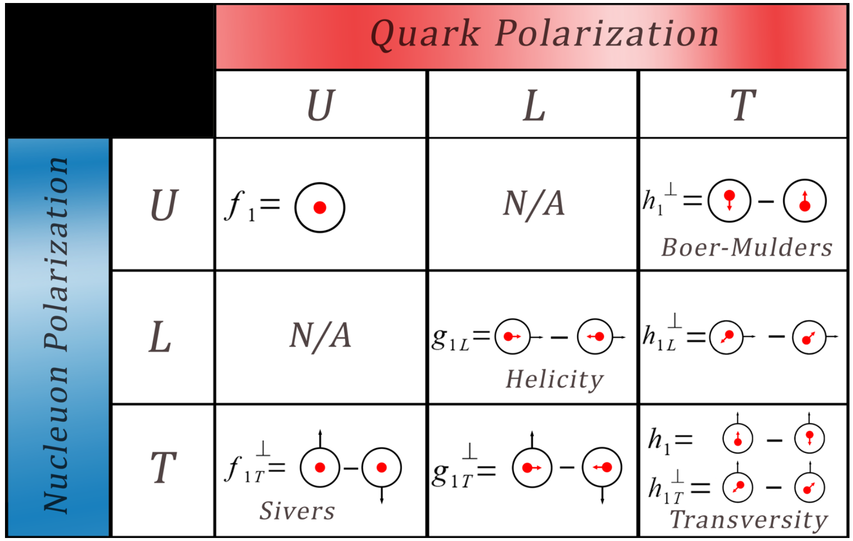
U.S. DEPARTMENT OF ENERGY | Office of Science

This work is supported by DOE contract DE-FG02-96ER40950

Outline

- A brief introduction to TMDs
- Sivers asymmetry from SIDIS
- Generalization of $\mathcal{N}_q(x)$ for $SU(3)_{\text{flavor}}$
- Deep Neural Network (DNN) method
- Testing with pseudo-data
- DNN Fits & Results for Sivers function
- DNN Boer-Mulders function extraction
- Summary and Outlook

TMD PDFs



$$\Phi(x, k_T; S) = \int \frac{d\xi^- d\xi_T}{(2\pi)^3} e^{ik.\xi} \langle P, S|\bar{\psi}(0)\, \mathcal{U}_{[0,\xi]}\, \psi(\xi)|P, S\rangle|_{\xi^+=0}$$

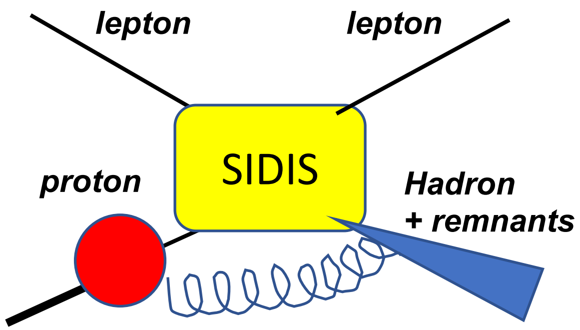
At leading-twist, the Quark correlator can be decomposed into 8 components (6 T - even and 2 T -odd terms)

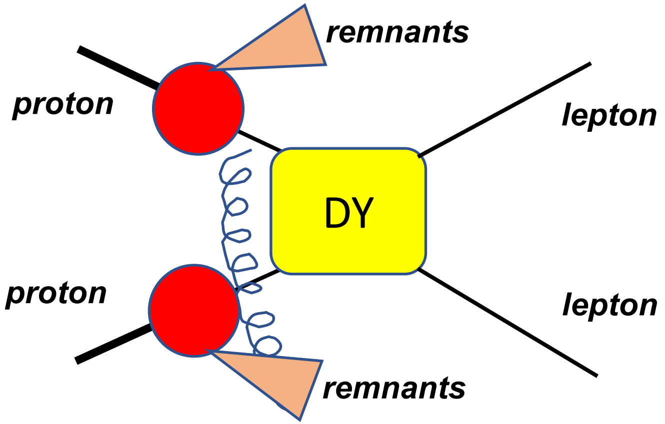
$$\Phi(x, k_T, P, S) = f_1(x, k_T^2)\frac{\not{P}}{2} + \frac{h_{1T}(x, k_T^2)}{4}\gamma_5[\not{S}_T, \not{P}] + \frac{S_L}{2} g_{1L}(x, k_T^2)\gamma_5 \not{P} + \frac{k_T.S_T}{2M} g_{1T}(x, k_T^2)\gamma_5 \not{P}$$
$$+ S_L h_{1L}^\perp(x, k_T^2)\gamma_5 \frac{[\not{k}_T, \not{P}]}{4M} + \frac{k_T.S_T}{2M} h_{1T}^\perp(x, k_T^2)\gamma_5 \frac{[\not{k}_T, \not{P}]}{4M}$$

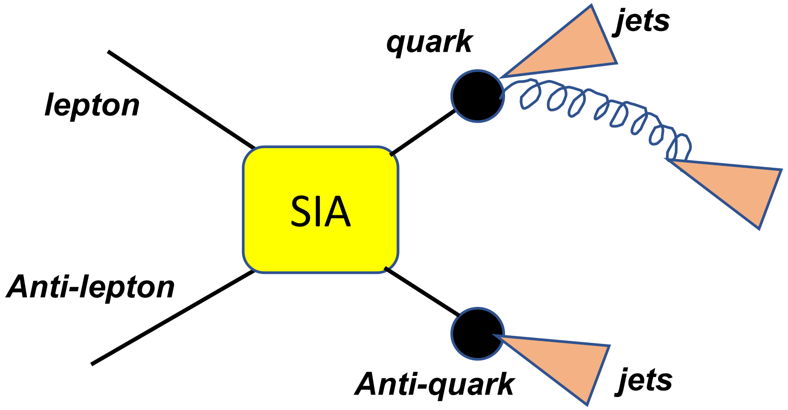
T-even

$$+ i h_1^\perp(x, k_T^2)\frac{[\not{k}_T, \not{P}]}{4M} - \frac{\epsilon_T^{k_T S_T}}{4M} f_{1T}^\perp(x, k_T^2)\not{P}$$

T-odd



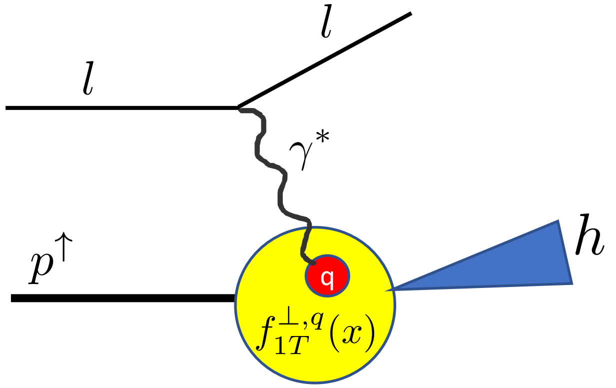




TMD PDFs

		Quark Polarization		
		U	L	T
Nucleon Polarization	U	$f_1 =$	N/A	$h_1^\perp =$ − (Boer-Mulders)
	L	N/A	$g_{1L} =$ − (Helicity)	$h_{1L}^\perp =$ −
	T	$f_{1T}^\perp =$ − (Sivers)	$g_{1T}^\perp =$ −	$h_1 =$ −, $h_{1T}^\perp =$ − (Transversity)

Polarized Semi Inclusive DIS



$$\frac{d\sigma^{LO}_{SIDIS}}{dxdydzdp_T^2 d\phi_h d\psi} = \left[\frac{\alpha}{xyQ^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{y^2}{2x}\right)\right]$$

$$\times (F_{UU,T}+\epsilon F_{UU,L})\left\{1+\cos 2\phi_h\left(\epsilon A_{UU}^{\cos 2\phi_h}\right)\right.$$

$$+S_T\left[\sin(\phi_h-\phi_s)\left(A_{UT}^{\sin(\phi_h-\phi_s)}\right)+\sin(\phi_h+\phi_s)\left(\epsilon A_{UT}^{\sin(\phi_h+\phi_s)}\right)\right.$$

$$\left.+\sin(3\phi_h-\phi_s)\left(\epsilon A_{UT}^{\sin(3\phi_h-\phi_s)}\right)\right]$$

$A_{UU}^{\cos 2\phi_h} \propto h_1^{\perp q} \circledast H_{1q}^{\perp h}$ BM ⊛ CF

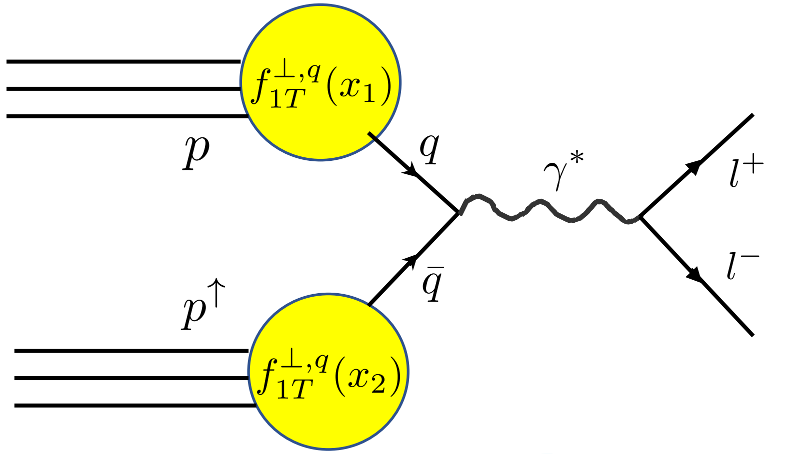
$A_{UT}^{\sin(\phi_h-\phi_s)} \propto f_{1T}^{\perp q} \circledast D_{1q}^{h}$ Sivers ⊛ FF

$A_{UT}^{\sin(\phi_h+\phi_s)} \propto h_1^{q} \circledast H_{1q}^{\perp h}$ Transv ⊛ CF

$A_{UT}^{\sin(3\phi_h-\phi_s)} \propto h_{1T}^{\perp q} \circledast H_{1q}^{\perp h}$ Pretz ⊛ CF

* For these two processes TMD factorization is proven

Polarized DY



$$\frac{d\sigma^{LO}}{d\Omega} = \frac{\alpha_{em}^2}{Fq}F_v^1\left\{1+\cos^2\theta + sin^2\theta cos2\phi_{CS} A_U^{cos2\phi_{CS}}\right.$$

$$+S_T\left[(1+\cos^2\theta)\sin\phi_s A_T^{\sin\phi_s} + \sin^2\theta\left(\sin(2\phi_{CS}+\phi_s)A_T^{\sin(2\phi_{CS}+\phi_s)}\right.\right.$$

$$\left.\left.+\sin(2\phi_{CS}-\phi_s)A_T^{\sin(2\phi_{CS}-\phi_s)}\right)\right]$$

$A_T^{\cos 2\phi_{CS}} \propto h_1^{\perp q} \circledast h_1^{\perp q}$ BM ⊛ BM

$A_T^{\sin \phi_s} \propto f_1^{q} \circledast f_{1T}^{\perp q}$ PDF ⊛ Sivers

$A_T^{\sin(2\phi_{CS}-\phi_S)} \propto h_1^{\perp q} \circledast h_1^{q}$ BM ⊛ Transv

$A_T^{\sin(2\phi_{CS}+\phi_S)} \propto h_1^{\perp q} \circledast h_{1T}^{\perp q}$ BM ⊛ Pretz

$h_1^{\perp q}\big|_{SIDIS} = -h_1^{\perp q}\big|_{DY}$

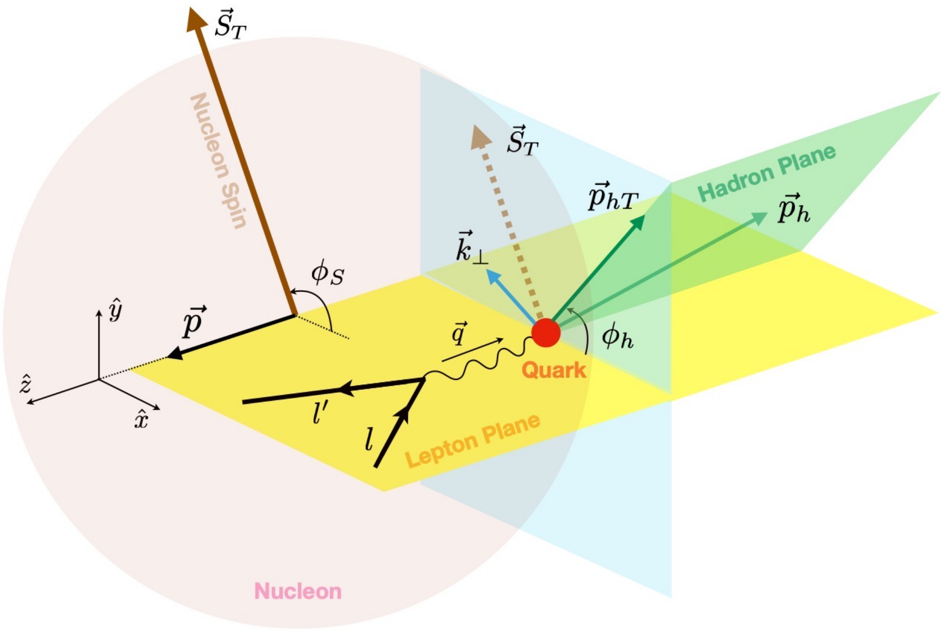
$f_{1T}^{\perp q}\big|_{SIDIS} = -f_{1T}^{\perp q}\big|_{DY}$

$h_1^{q}\big|_{SIDIS} = h_1^{q}\big|_{DY}$

$h_{1T}^{\perp q}\big|_{SIDIS} = h_{1T}^{\perp q}\big|_{DY}$

Sivers Asymmetry from SIDIS

$$\frac{d^5\sigma^{lp\to lhX}}{dx\,dQ^2\,dz\,d^2p_\perp} = \sum_q e_q^2 \int d^2\mathbf{k}_\perp \left(\frac{2\pi\alpha^2}{x^2 s^2}\frac{\hat{s}^2+\hat{u}^2}{Q^4}\right) \times \hat{f}_{q/p^\uparrow}(x,k_\perp) D_{h/q}(z,p_\perp) + \mathcal{O}(k_\perp/Q)$$



$$\hat{f}_{q/p^\uparrow}(x,k_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\vec{S}_T\cdot(\hat{p}\times\hat{k}_\perp)$$

$$\Delta^N f_{q/p^\uparrow}(x,k_\perp) = 2\mathcal{N}_q(x)h(k_\perp)f_{q/p}(x,k_\perp)$$

Anselmino et al. (2017)

Single Spin Asymmetry (Sivers Asymmetry)

$$\langle p_\perp^2\rangle = 0.12 \pm 0.01 \text{ GeV}^2$$
$$\langle k_\perp^2\rangle = 0.57 \pm 0.08 \text{ GeV}^2$$

$$A_{UT}^{\sin(\phi_h-\phi_S)}(x,y,z,p_{hT}) = \frac{d\sigma^{l\uparrow p\to hlX} - d\sigma^{l\downarrow p\to lhX}}{d\sigma^{l\uparrow p\to hlX} + d\sigma^{l\downarrow p\to hlX}} \equiv \frac{d\sigma\uparrow - d\sigma\downarrow}{d\sigma\uparrow + d\sigma\downarrow}$$

$$\mathcal{A}_0(z,p_{hT},m_1) = \frac{\sqrt{2e}zp_{hT}}{m_1}\frac{[z^2\langle k_\perp^2\rangle + \langle p_\perp^2\rangle]\langle k_S^2\rangle^2}{[z^2\langle k_S^2\rangle + \langle p_\perp^2\rangle]^2\langle k_\perp^2\rangle} \times \exp\left[-\frac{p_{hT}^2 z^2\left(\langle k_S^2\rangle - \langle k_\perp^2\rangle\right)}{\left(z^2\langle k_S^2\rangle + \langle p_\perp^2\rangle\right)\left(z^2\langle k_\perp^2\rangle + \langle p_\perp^2\rangle\right)}\right]$$

$$\langle k_S^2\rangle = \frac{m_1\langle k_\perp^2\rangle}{m_1^2 + \langle k_\perp^2\rangle}$$

$$A_{UT}^{\sin(\phi_h-\phi_S)}(x,z,p_{hT}) = \mathcal{A}_0(z,p_{hT},m_1)\left(\frac{\sum_q \mathcal{N}_q(x) e_q^2 f_q(x) D_{h/q}(z)}{\sum_q e_q^2 f_q(x) D_{h/q}(z)}\right)$$

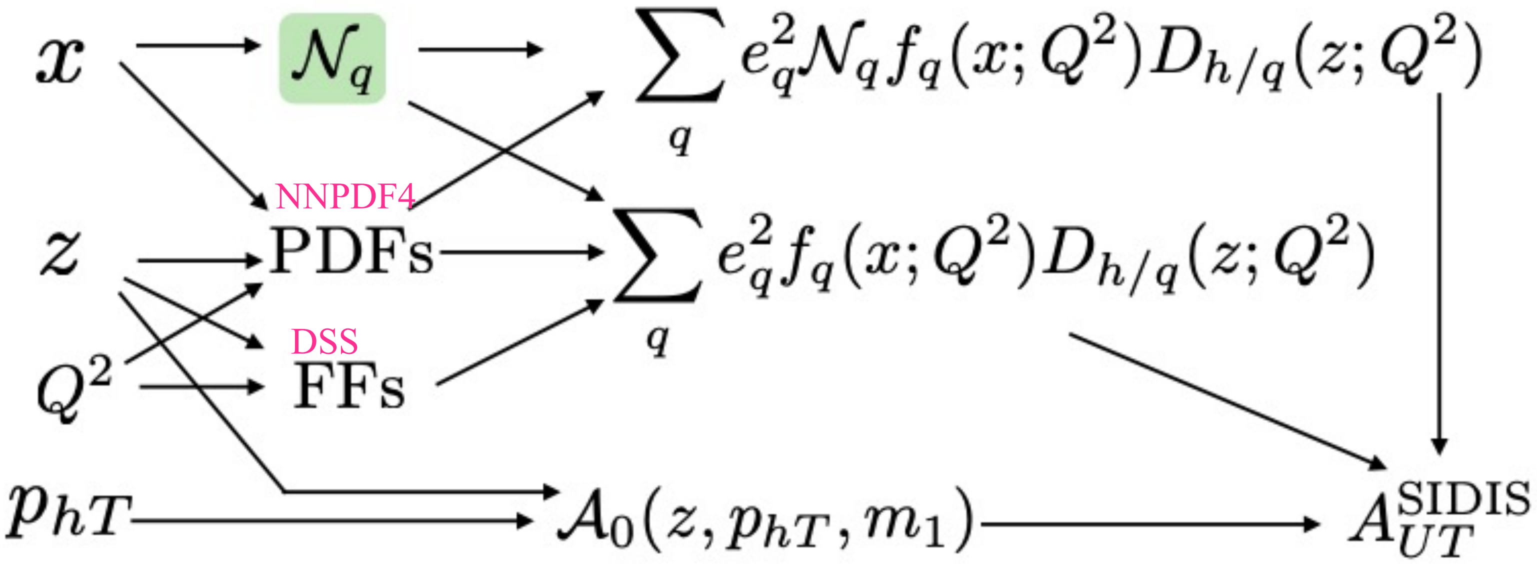
$$\mathcal{N}_q(x) = N_q x^{\alpha_q}(1-x)^{\beta_q}\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}$$

$$\mathcal{N}_{\bar{q}}(x) = N_{\bar{q}}$$

DNN Approach

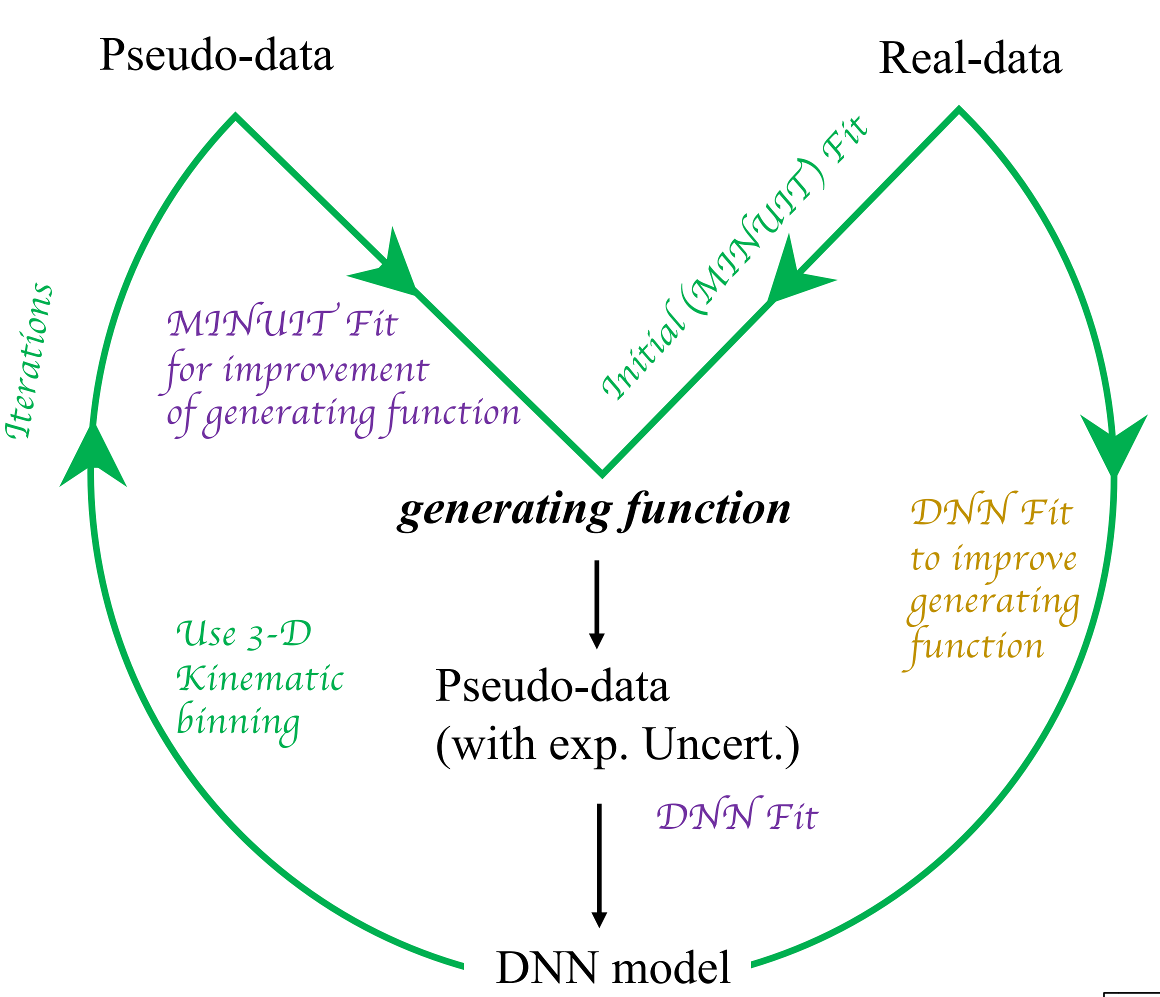
$$A_{UT}^{\sin(\phi_h - \phi_S)}(x, z, p_{hT}) = \mathcal{A}_0(z, p_{hT}, m_1) \left(\frac{\sum_q \mathcal{N}_q(x) e_q^2 f_q(x) D_{h/q}(z)}{\sum_q e_q^2 f_q(x) D_{h/q}(z)} \right)$$

- The exceptional capacity of DNN to be ideal for function approximation (Universal Approximation Theorem).
- Each quark flavor q is independently handled by a separate $\mathcal{N}_q(x)$.
- The only input to to each $\mathcal{N}_q(x)$ is x.
- Statistical & Systematic uncertainties from the experimental data are combined in quadrature; then propagated using bootstrap method by generating replicas.
- Systematic uncertainty in method is evaluated with variations in generating function.



The "DNN Method" for extracting TMDs

I. P. Fernando and D. Keller
Phys. Rev. D.108.054007 (2023)



- Systematic study for both DNN models were performed separately using various generating functions.

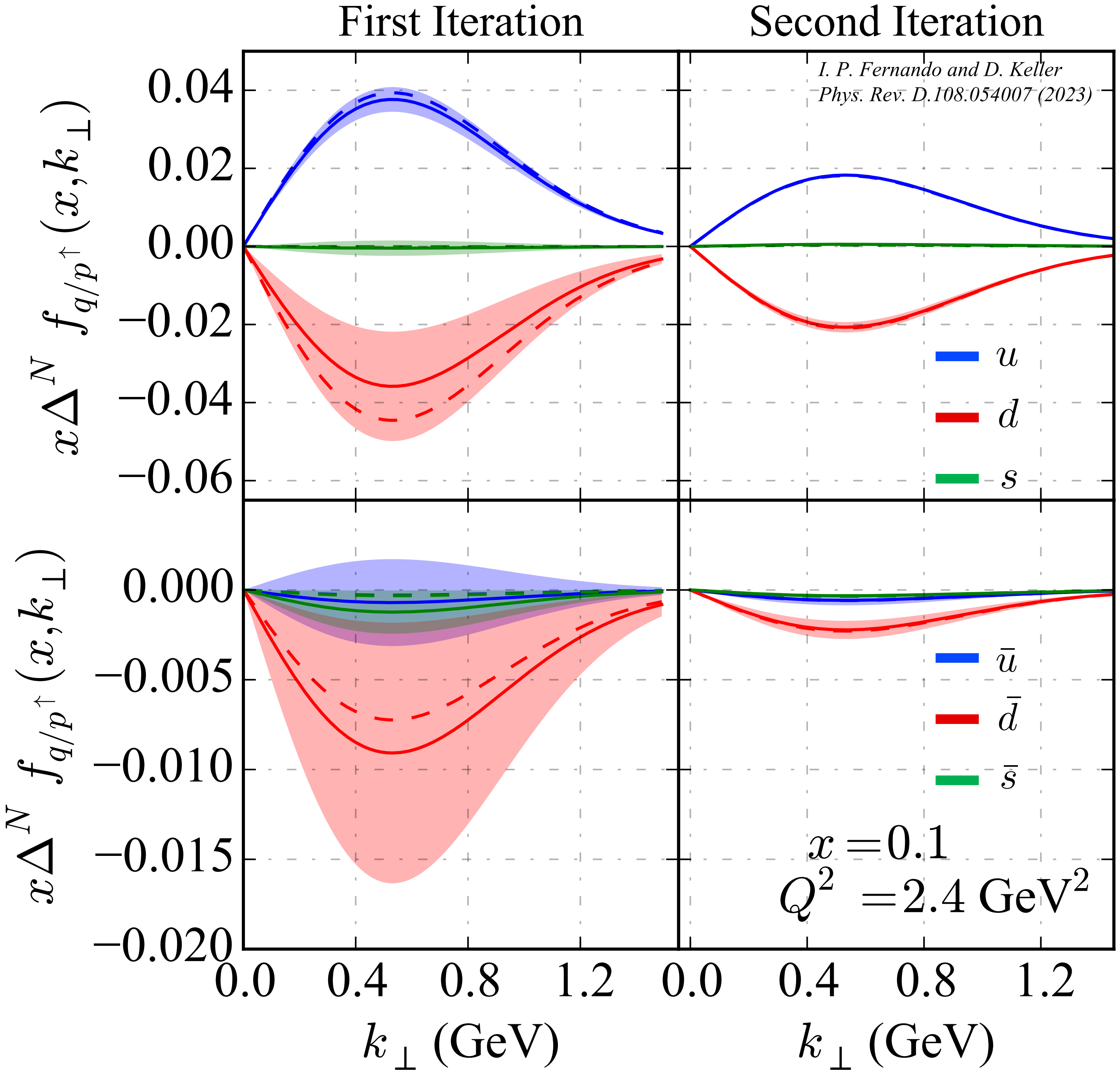
- We trained two separate models for "proton" and "neutron" (deuteron)
- To take full advantage of the information provided by the model testing in the previous slide, the steps from method testing with pseudo-data are performed again separately for proton and deuteron SIDIS data.

$$\epsilon_q(x, k_\perp) = \left(1 - \frac{|\Delta^N f_{q/p^\uparrow}^{(\text{true})} - \Delta^N f_{q/p^\uparrow}^{(\text{mean})}|}{\Delta^N f_{q/p^\uparrow}^{(\text{true})}}\right) \times 100\%$$

$$\sigma_q(x, k_\perp) = \sqrt{\frac{\sum_i \left(\Delta^N f_{q/p^\uparrow}^{(i)} - \Delta^N f_{q/p^\uparrow}^{(\text{mean})}\right)^2}{N}}$$

DNN Method testing with Pseudo data

- Dashed lines represent the ***generating function*** in each iteration.
- A comparison:
  Improving the ***generating functio***n
  Fine-tuning the hyperparameters
- Solid-lines and the band represent the mean and 68% CL with 1000 replicas of the DNN model.



Data Selection

Dataset	Kinematic coverage	Reaction	Data points
HERMES2009	$0.023 < x < 0.4$	$p^{\uparrow} + \gamma^* \to \pi^+$	21
(SIDIS)	$0.2 < z < 0.7$	$p^{\uparrow} + \gamma^* \to \pi^-$	21
[53]	$0.1 < p_{hT} < 0.9$	$p^{\uparrow} + \gamma^* \to \pi^0$	21
	$Q^2 > 1$ GeV^2	$p^{\uparrow} + \gamma^* \to K^+$	21
		$p^{\uparrow} + \gamma^* \to K^-$	21
HERMES2020	$0.023 < x < 0.6$	$p^{\uparrow} + \gamma^* \to \pi^+$	27, **64**
(SIDIS)	$0.2 < z < 0.7$	$p^{\uparrow} + \gamma^* \to \pi^-$	27, **64**
[55]	$0.1 < p_{hT} < 0.9$	$p^{\uparrow} + \gamma^* \to \pi^0$	27
	$Q^2 > 1$ GeV^2	$p^{\uparrow} + \gamma^* \to K^+$	27, **64**
		$p^{\uparrow} + \gamma^* \to K^-$	27, **64**
COMPASS2015	$0.006 < x < 0.28$	$p^{\uparrow} + \gamma^* \to \pi^+$	26
(SIDIS)	$0.2 < z < 0.8$	$p^{\uparrow} + \gamma^* \to \pi^-$	26
[54]	$0.15 < p_{hT} < 1.5$	$p^{\uparrow} + \gamma^* \to K^+$	26
	$Q^2 > 1$ GeV^2	$p^{\uparrow} + \gamma^* \to K^-$	26
COMPASS2009	$0.006 < x < 0.28$	$d^{\uparrow} + \gamma^* \to \pi^+$	26
(SIDIS)	$0.2 < z < 0.8$	$d^{\uparrow} + \gamma^* \to \pi^-$	26
[49]	$0.15 < p_{hT} < 1.5$	$d^{\uparrow} + \gamma^* \to K^+$	26
	$Q^2 > 1$ GeV^2	$d^{\uparrow} + \gamma^* \to K^-$	26
JLAB2011	$0.156 < x < 0.396$	$^3He^{\uparrow} + \gamma^* \to \pi^+$	4
(SIDIS) [52]	$0.50 < z < 0.58$	$^3He^{\uparrow} + \gamma^* \to \pi^-$	4
	$0.24 < p_{hT} < 0.43$		
	$1.3 < Q^2 < 2.7$		
COMPASS2017	$0.1 < x_N < 0.25$	$p^{\uparrow} + \pi^- \to l^+ l^- X$	15
(DY) [50]	$0.3 < x_\pi < 0.7$		
	$4.3 < Q_M < 8.5$		
	$0.6 < q_T < 1.9$		

Proton DNN model

Deuteron DNN model

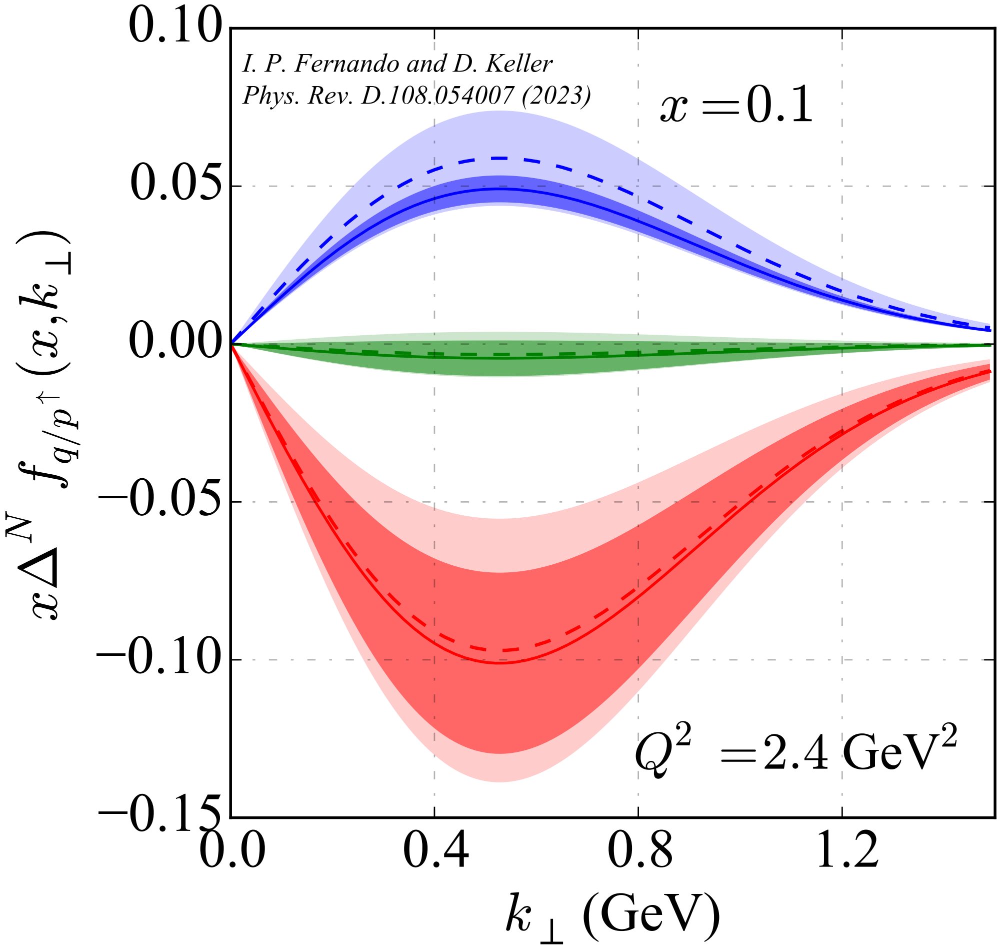
HERMES2020 3D binned data

Projections from Proton DNN model

Projections from Deuteron DNN model

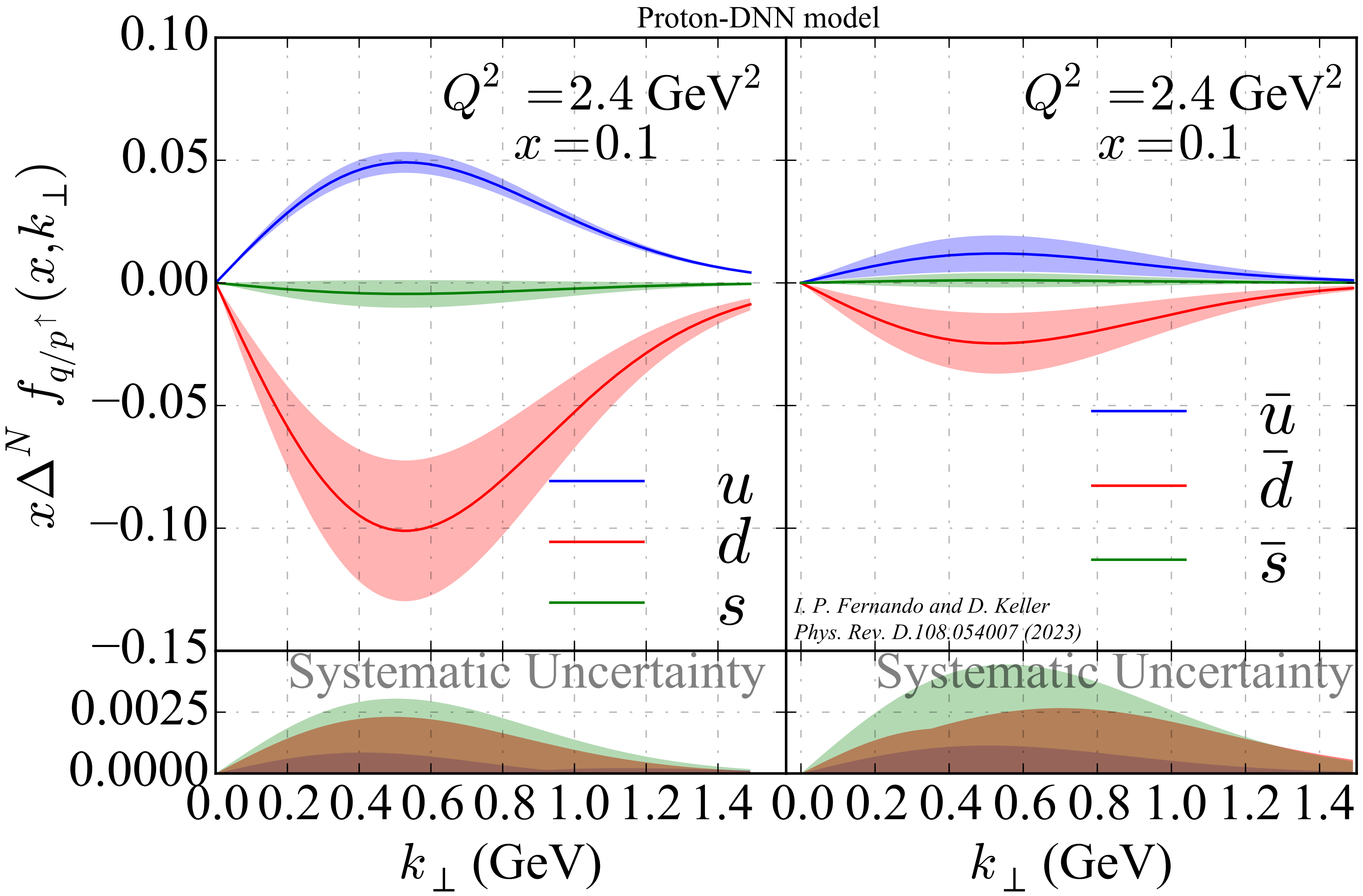
$$\Delta^N f_{q/p^{\uparrow}}(x, k_\perp)\big|_{\text{SIDIS}} = -\,\Delta^N f_{q/p^{\uparrow}}(x, k_\perp)\big|_{\text{DY}}$$

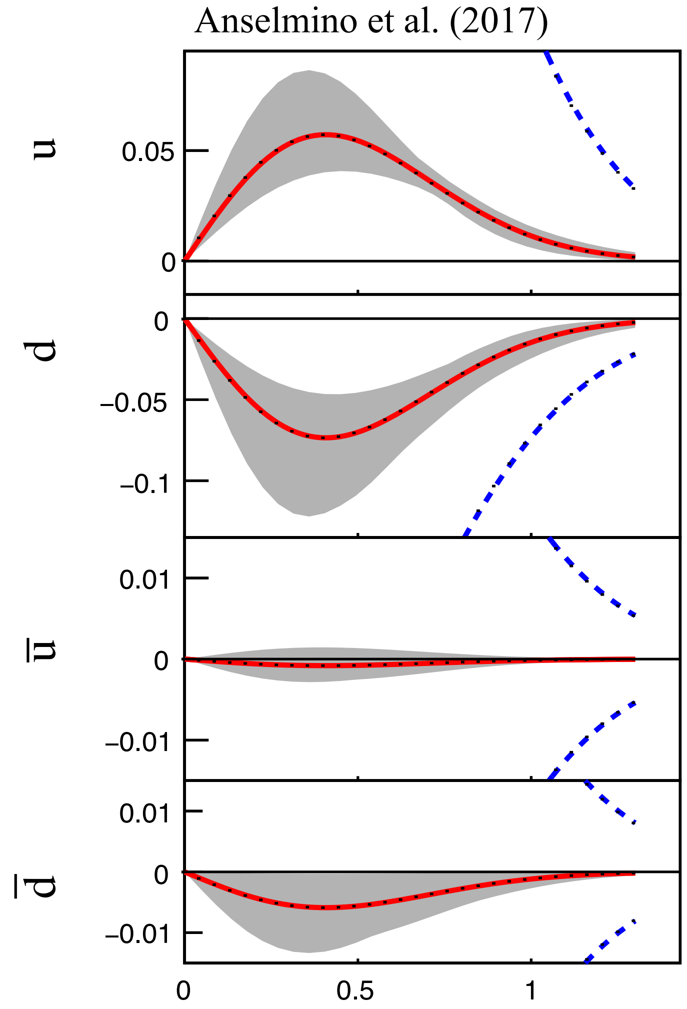
DNN Method: With Real data (Quality of the extraction)



The qualitative improvement of the extracted Sivers functions for u (blue), d (red), and s (green) quarks at x = 0.1 and Q^2=2.4 GeV^2 using the optimized proton-DNN model at the Second Iteration (solid-lines with dark-colored error bands with 68% CL), compared to the First Iteration (dashed-lines with light-colored error bands with 68% CL)

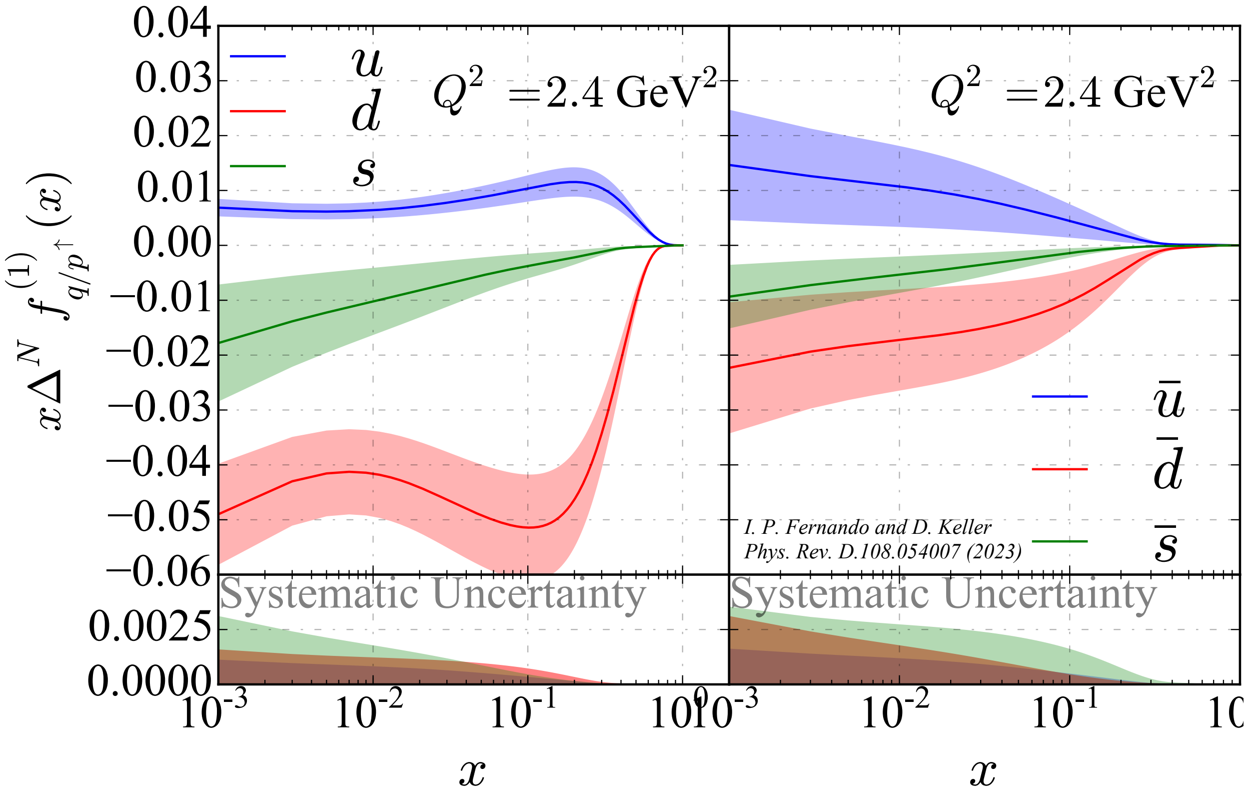
Sivers functions from the "Proton" DNN Model

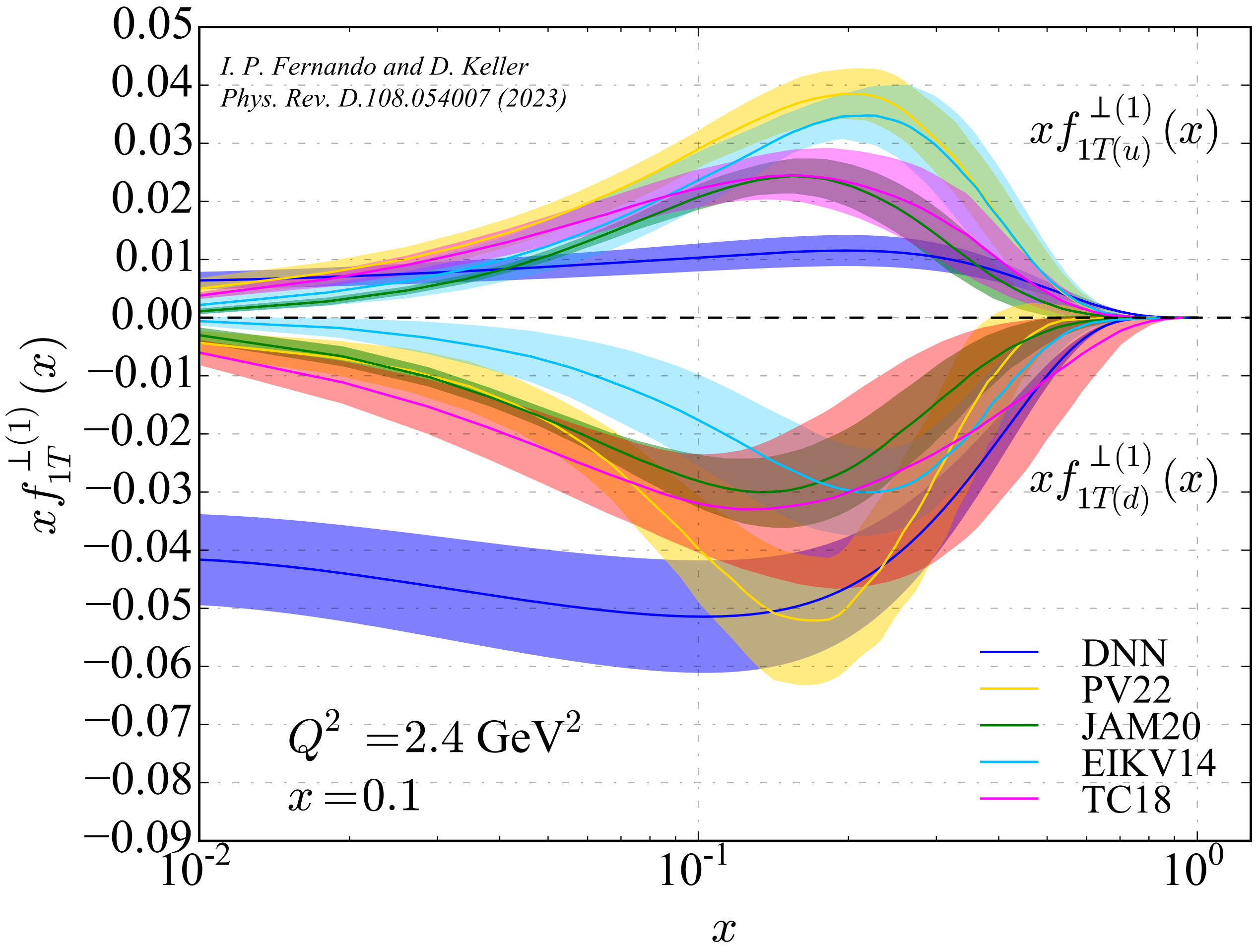




Sivers 1st moments from the "Proton" Model

$$\Delta^N f^{(1)}_{q/p^\uparrow}(x) = \int d^2k_\perp \frac{k_\perp}{4m_p}\Delta^N f_{q/p^\uparrow}(x,k_\perp) = -f^{\perp(1)q}_{1T}(x)$$



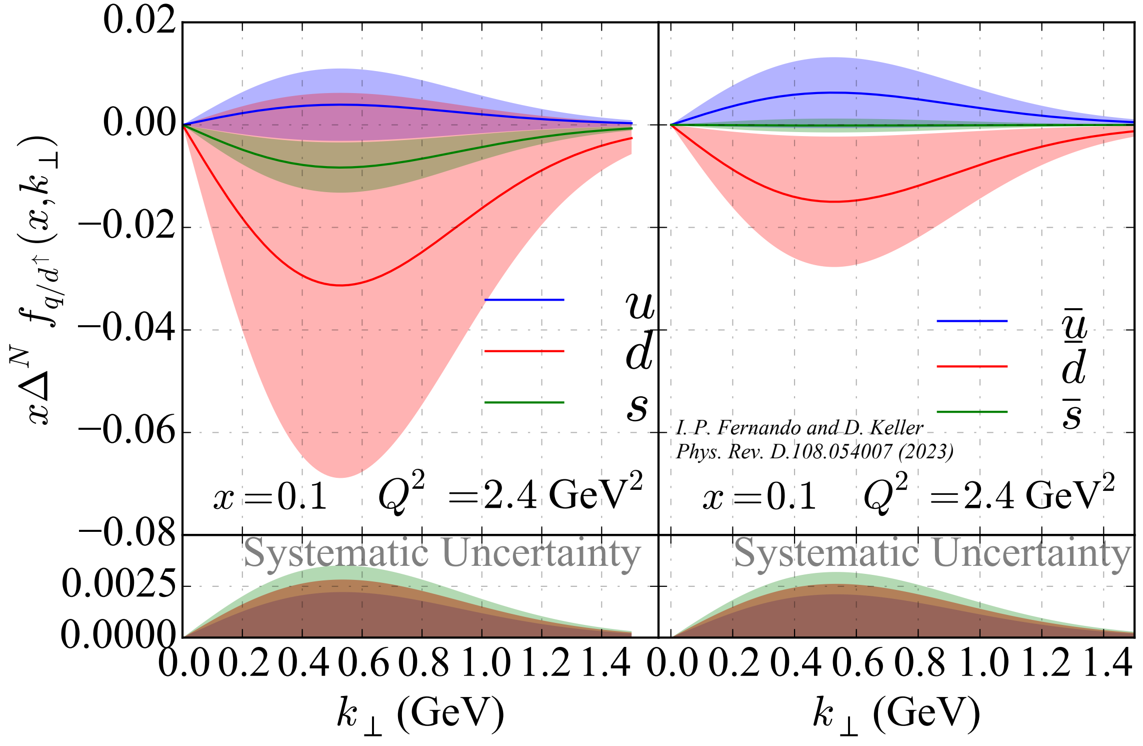


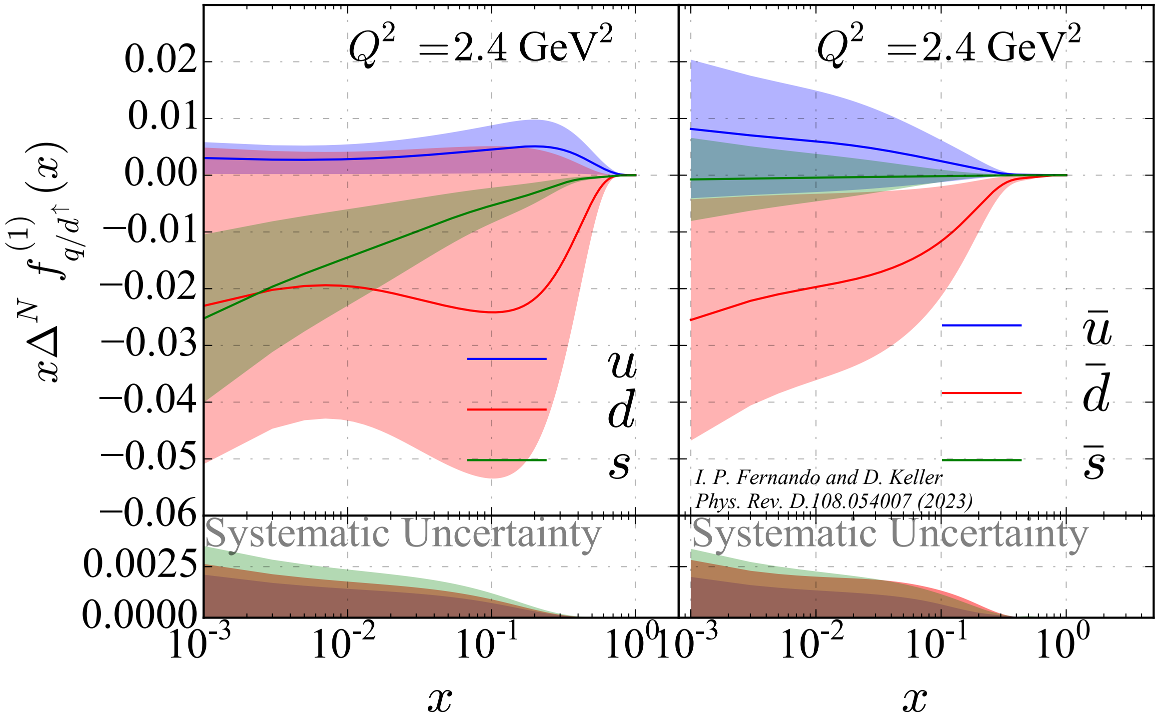
DNN Method: Results from the "Deuteron" Model

- ➢ Trained on COMPASS 2009 SIDIS data with Deuteron target.
- ➢ Did not imposed iso-spin symmetric conditions, or data cuts.

$$f^{\perp}_{1T,u\leftarrow d} = f^{\perp}_{1T,d\leftarrow d} = \frac{f^{\perp}_{1T,u\leftarrow p} + f^{\perp}_{1T,d\leftarrow p}}{2}$$

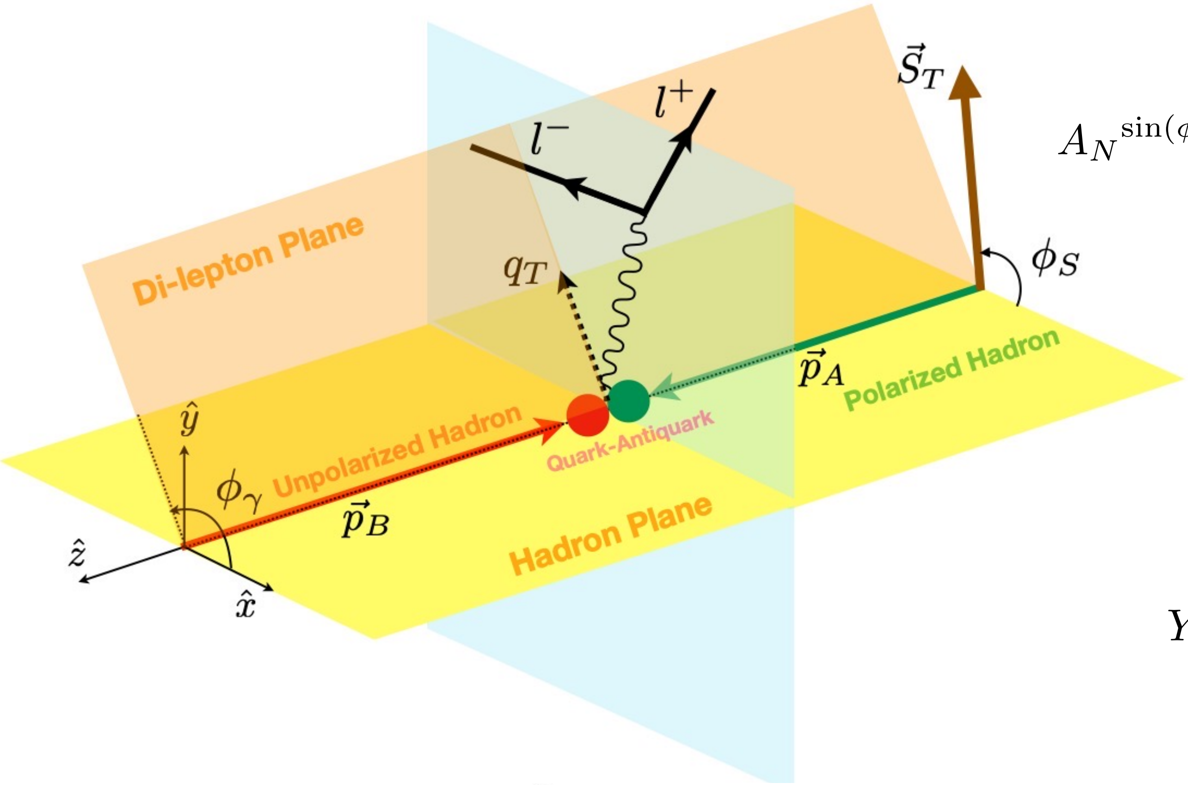
$$\Delta^N f^{(1)}_{q/p^\uparrow}(x) = \int d^2k_\perp \frac{k_\perp}{4m_p} \Delta^N f_{q/p^\uparrow}(x, k_\perp) = -f^{\perp(1)q}_{1T}(x)$$





DNN Model Projections: DY

Based on Anselmino et al. (2017)



$$A_N^{\sin(\phi_\gamma - \phi_S)}(x_F, M, q_T) \quad = \mathcal{B}_0(q_T, m_1) \frac{\sum_q \frac{e_q^2}{x_1+x_2} \mathcal{N}_q(x_1) f_{q/A}(x_1) f_{\bar{q}/B}(x_2)}{\sum_q \frac{e_q^2}{x_1+x_2} f_{q/A}(x_1) f_{\bar{q}/B}(x_2)}$$

$$\mathcal{B}_0(q_T, m_1) = \frac{q_T\sqrt{2e}}{m_1} \frac{Y_1(q_T, k_S, k_{\perp 2})}{Y_2(q_T, k_{\perp 1}, k_{\perp 2})}$$

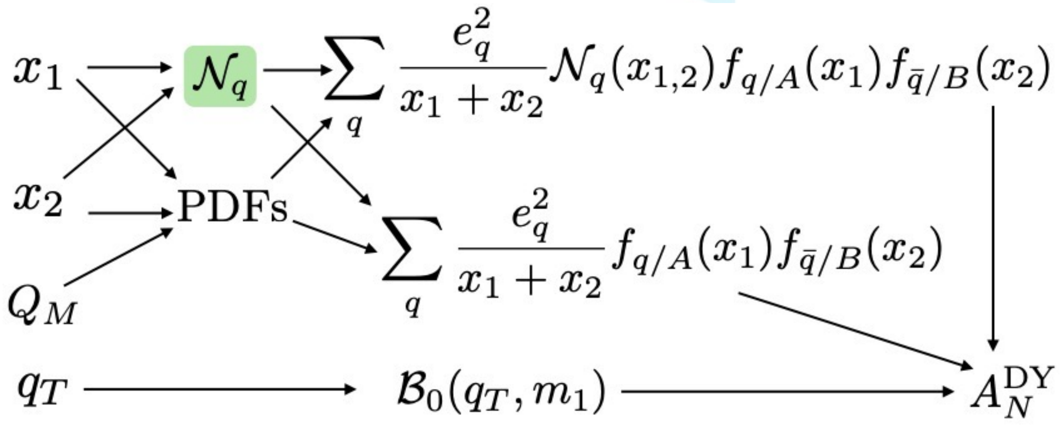
$$Y_1(q_T, k_S, k_{\perp 2}) = \left(\frac{\langle k_S^2 \rangle^2}{\langle k_{\perp 2}^2 \rangle \left(\langle k_S^2 \rangle + \langle k_{\perp 2}^2 \rangle \right)^2} \right) \times \exp\left(\frac{-q_T^2}{\langle k_S^2 \rangle + \langle k_{\perp 2}^2 \rangle} \right)$$

$$Y_2(q_T, k_{\perp 1}, k_{\perp 2}) = \left(\frac{1}{\langle k_{\perp 1}^2 \rangle + \langle k_{\perp 2}^2 \rangle} \right) \times \exp\left(\frac{-q_T^2}{\langle k_{\perp 1}^2 \rangle + \langle k_{\perp 2}^2 \rangle} \right)$$

$$\frac{1}{\langle k_S^2 \rangle} = \frac{1}{m_1^2} + \frac{1}{\langle k_{\perp 1}^2 \rangle}$$

$$\langle k_{\perp 1}^2 \rangle = \langle k_{\perp 2}^2 \rangle = \langle k_{\perp}^2 \rangle = 0.25 \text{ GeV}^2$$

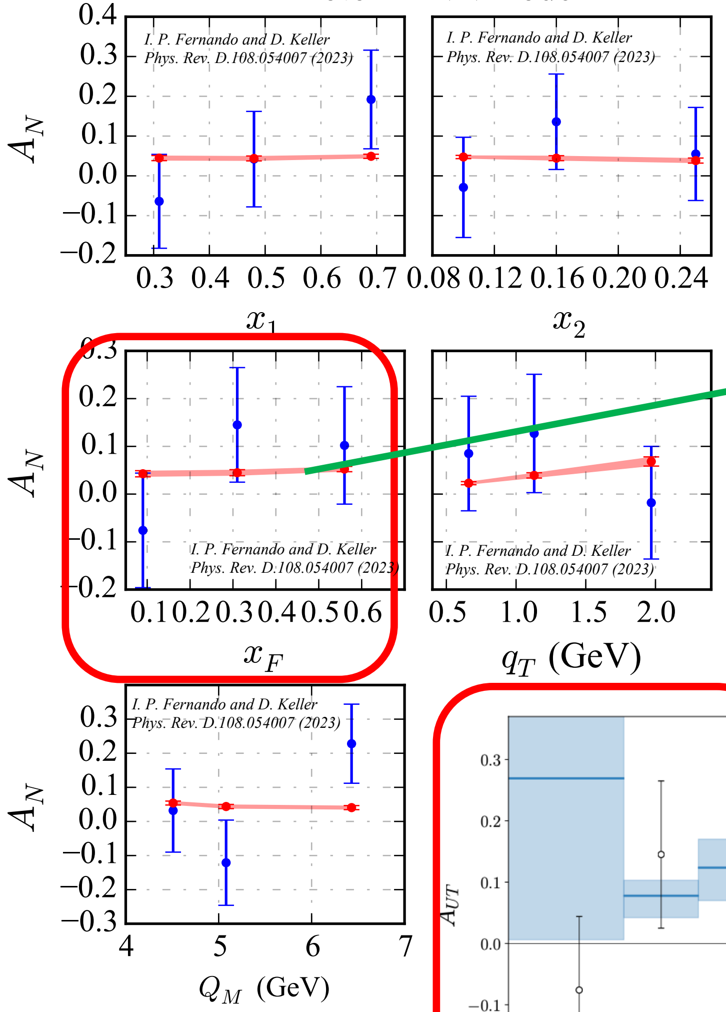
$$\Delta^N f_{q/p^\uparrow}(x, k_\perp)\big|_{\text{SIDIS}} = -\, \Delta^N f_{q/p^\uparrow}(x, k_\perp)\big|_{\text{DY}}$$

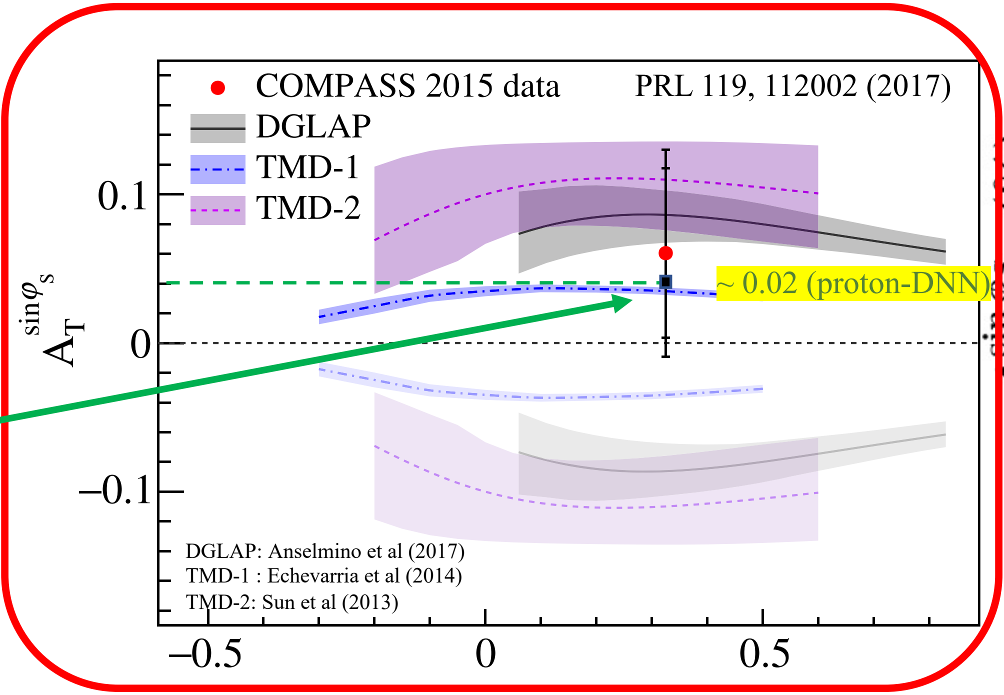


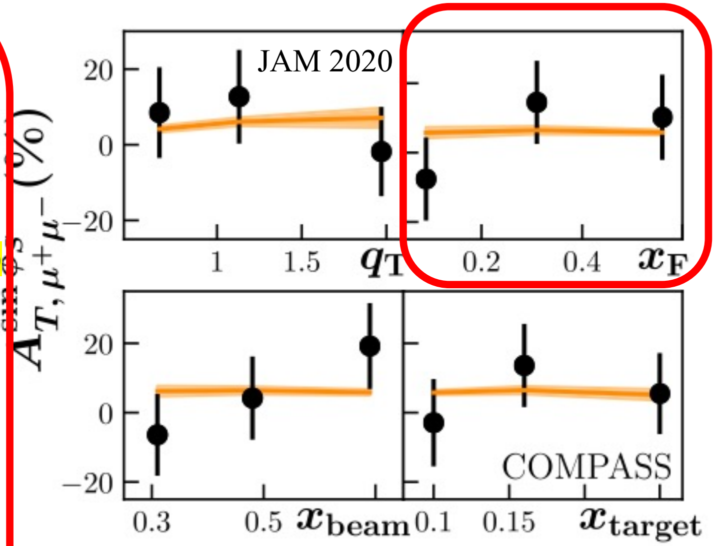
DNN Model Projections: DY

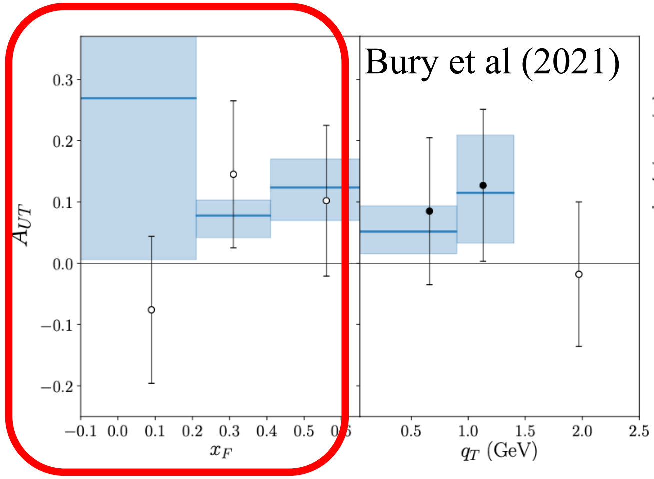
COMPASS 2017 DY Projections

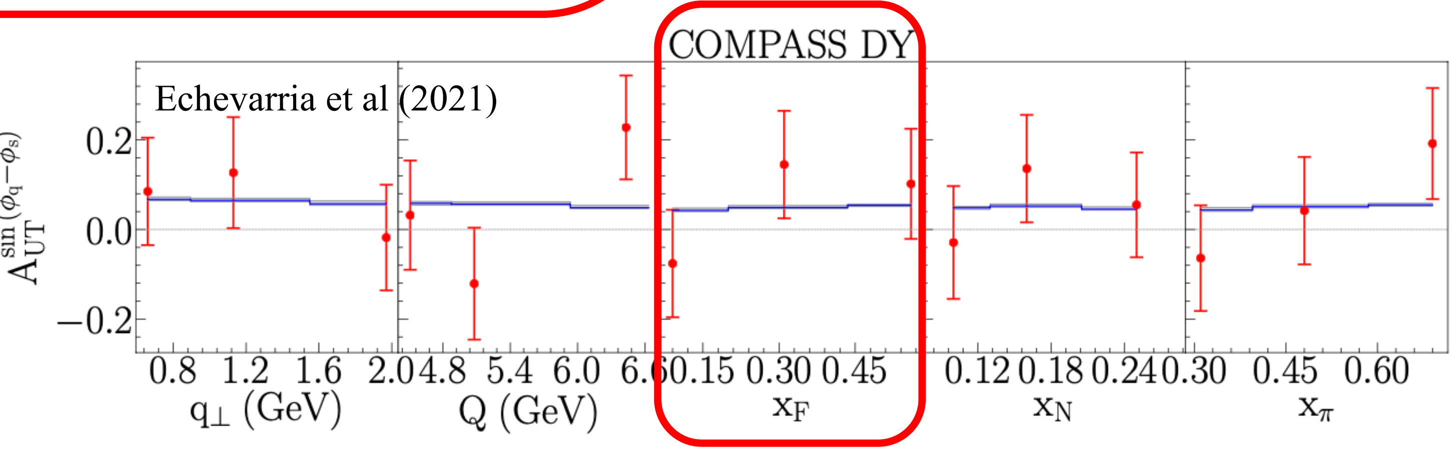
Proton-DNN model



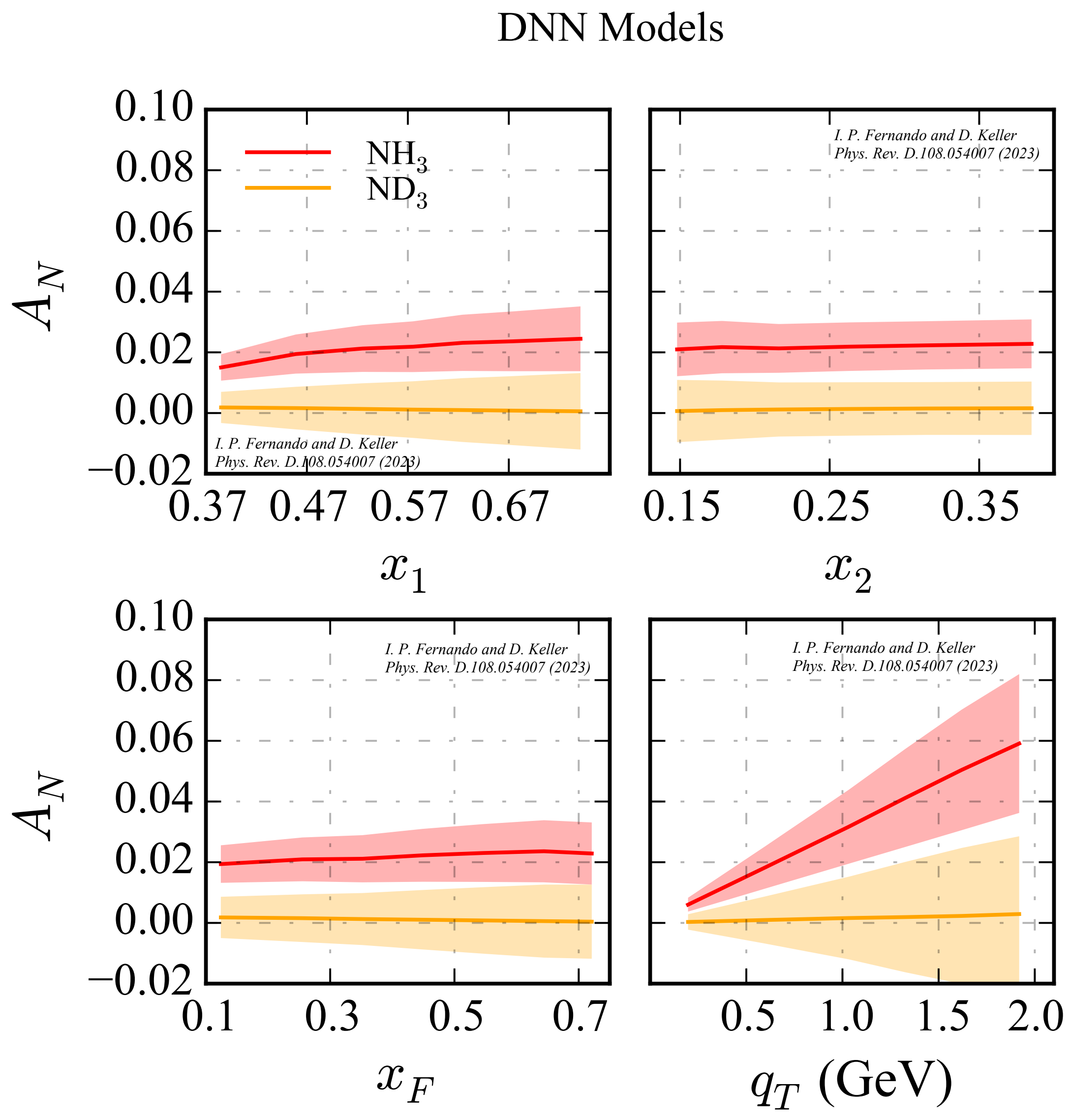






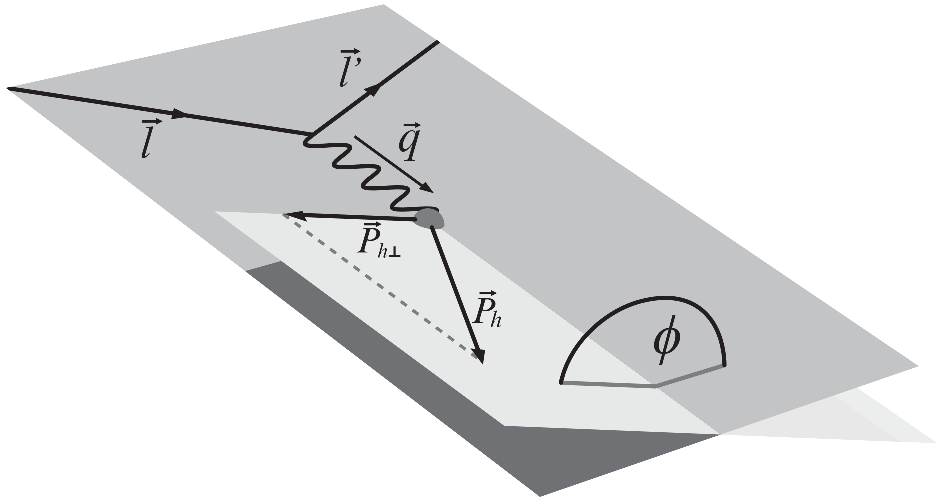


DNN Model Projections: DY @ SpinQuest



- SpinQuest (E1039) experiment at Fermilab is aiming to extract the Sivers function for the light-sea quarks.
- Unpolarized 120 GeV proton beam with polarized proton and deuteron targets (separately).
- Proton-DNN model predictions (Red)
  Deuteron-DNN model predictions (Orange)

Boer-Mulders (BM) function



Boer Mulders function describes the polarized quark distributions inside an unpolarized hadron.

Based on the framework by Barone et al., PRD 91, 074019 (2015)

$$\frac{d\sigma}{dx_B dy dz_h dp_{hT} d\phi} = \frac{\pi\alpha^2}{Q^2 x_B y}\left((1+(1-y)^2)F_{UU} + 2(2-y)\sqrt{1-y}F_{UU}^{\cos\phi}\cos\phi + 2(1-y)F_{UU}^{\cos 2\phi}\cos 2\phi\right)$$

Twist-3

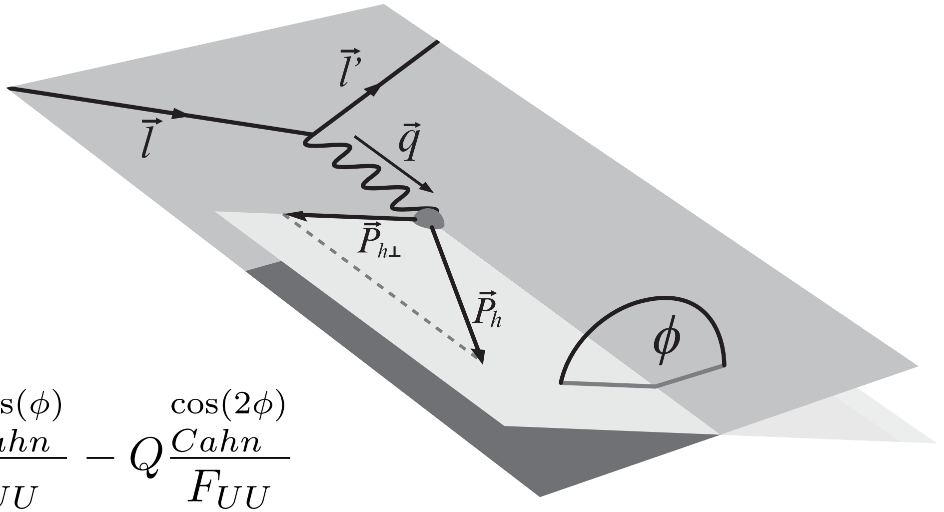
$$F_{UU}^{\cos\phi,h}|_{\mathrm{BM}} = \sum_q e_q^2 x \int d^2\mathbf{k}_\perp \frac{k_\perp p_{hT}^2 - z(\mathbf{k}_\perp\cdot\mathbf{p_{hT}})}{Q p_{hT}^2}\frac{k_\perp}{m_p} h_1^\perp(x,k_\perp)\frac{2p_\perp}{z m_h} H_1^\perp(z,p_\perp) + F_{UU}^{\cos\phi}|_{\mathrm{Cahn}} = -2\sum_q e_q^2 x \int d^2\mathbf{k}_\perp \frac{(\mathbf{k}_\perp\cdot\mathbf{p_{hT}})}{Q p_{hT}} f_q(x,k_\perp) D_q(z,p_\perp)$$

Twist-4

$$F_{UU}^{\cos 2\phi,h}|_{\mathrm{BM}} = -\sum_q e_q^2 x \int d^2\mathbf{k}_\perp \frac{p_{hT}(\mathbf{k}_\perp\cdot\mathbf{p_{hT}}) + \mathbf{z}(\mathbf{k}_\perp^2\mathbf{p_{hT}} - \mathbf{2}(\mathbf{k}_\perp\cdot\mathbf{p_{hT}}))}{2k_\perp p_{hT}^2}\frac{k_\perp}{m_p} h_1^\perp(x,k_\perp)\frac{2p_\perp}{z m_h} H_1^\perp(z,p_\perp)$$

$$+ F_{UU}^{\cos\phi}|_{\mathrm{Cahn}} = 2\sum_q e_q^2 x \int d^2\mathbf{k}_\perp \frac{2(\mathbf{k}_\perp\cdot\mathbf{p_{hT}}) - \mathbf{k}_\perp^2\mathbf{p}_{\mathbf{hT}}^2}{Q^2 p_{hT}^2} f_q(x,k_\perp) D_q(z,p_\perp)$$

Boer-Mulders (BM) function



$$A^{\cos(\phi)} = A^{\cos(\phi)}_{BM} + A^{\cos(\phi)}_{Cahn} \qquad A^{\cos(2\phi)} = A^{\cos(2\phi)}_{BM} + A^{\cos(2\phi)}_{Cahn}$$

An exploratory rearrangement

$$A^{\cos(\phi)} - QA^{\cos(2\phi)} = \frac{F^{\cos(\phi)}_{BM}}{F_{UU}} - Q\frac{F^{\cos(2\phi)}_{BM}}{F_{UU}} + \frac{F^{\cos(\phi)}_{Cahn}}{F_{UU}} - Q\frac{{}^{\cos(2\phi)}_{Cahn}}{F_{UU}}$$

$$h_1^{\perp}(x,k_\perp) = \mathcal{N}_q(x) h(k_\perp) f_q(x,k_\perp;Q^2)$$

$$\mathcal{N}_q(x) = N_q x^\alpha (1-x)^\beta \frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha \beta^\beta}$$

$$f_q(x,k_\perp;Q^2) = f_q(x;Q^2)\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}$$

$$h(k_\perp) = \sqrt{2e}\frac{k_\perp}{m_{BM}} e^{-k_\perp^2/m_{BM}^2}$$

$$\Delta D_{h/q\uparrow}(z,p_\perp) = \mathcal{N}_q^C(z) h^C(k_\perp) D_{h/q}(z,p_\perp;Q^2)$$

$$\mathcal{N}_q^C(x) = N_q^C z^\gamma (1-z)^\delta \frac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^\gamma \delta^\delta}$$

$$D_{h/q}(z,p_\perp;Q^2) = D_{h/q}(z;Q^2)\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}$$

$$h^C(p_\perp) = \sqrt{2e}\frac{p_\perp}{m_C} e^{-p_\perp^2/m_C^2}$$

From the framework by Barone et al., PRD 91, 074019 (2015)

$$\frac{F^{\cos(\phi)}_{BM}}{F_{UU}} - Q\frac{F^{\cos(2\phi)}_{BM}}{F_{UU}} = \frac{2(2-y)}{[1+(1-y)^2]}\left(\frac{e p_{hT}}{m_{BM} m_c Q}\right)\frac{\langle p_{hT}^2\rangle}{\langle p_{hT}^2\rangle^3_{BM}}\left(\frac{\sum_q e^q \mathcal{N}_q(x) f_{q/p}(x)\mathcal{N}_q^c(x) D_{h/q}(z)}{\sum_q e^q f_{q/p}(x) D_{h/q}(z)}\right)\exp\left(\frac{p_{hT}^2}{\langle p_{hT}^2\rangle} - \frac{p_{hT}^2}{\langle p_{hT}^2\rangle}_{BM}\right)$$

$$\left(\frac{2\sqrt{1-y}}{\langle p_{hT}^2\rangle_{BM}}\frac{\langle k_\perp^2\rangle^2_{BM}\langle p_\perp^2\rangle^2_C}{\langle k_\perp^2\rangle^2\langle p_\perp^2\rangle^2}\left(z^2\langle k_\perp^2\rangle^2_{BM}\langle(p_{hT}^2 - \langle p_{hT}^2\rangle^2_{BM}) + \langle p_\perp^2\rangle^2_C\langle p_{hT}^2\rangle^2_{BM}\right) + Q^2 p_{hT}\frac{z\langle k_\perp^2\rangle^2_{BM}\langle p_\perp^2\rangle^2_C}{\langle k_\perp^2\rangle^2\langle p_\perp^2\rangle^2}\right)$$

The “DNN Method” for extracting BM

Method Testing

$$\langle k_\perp^2\rangle_{BM} = \frac{m_{BM}^2\langle k_\perp^2\rangle}{\langle k_\perp^2\rangle + m_{BM}^2}$$

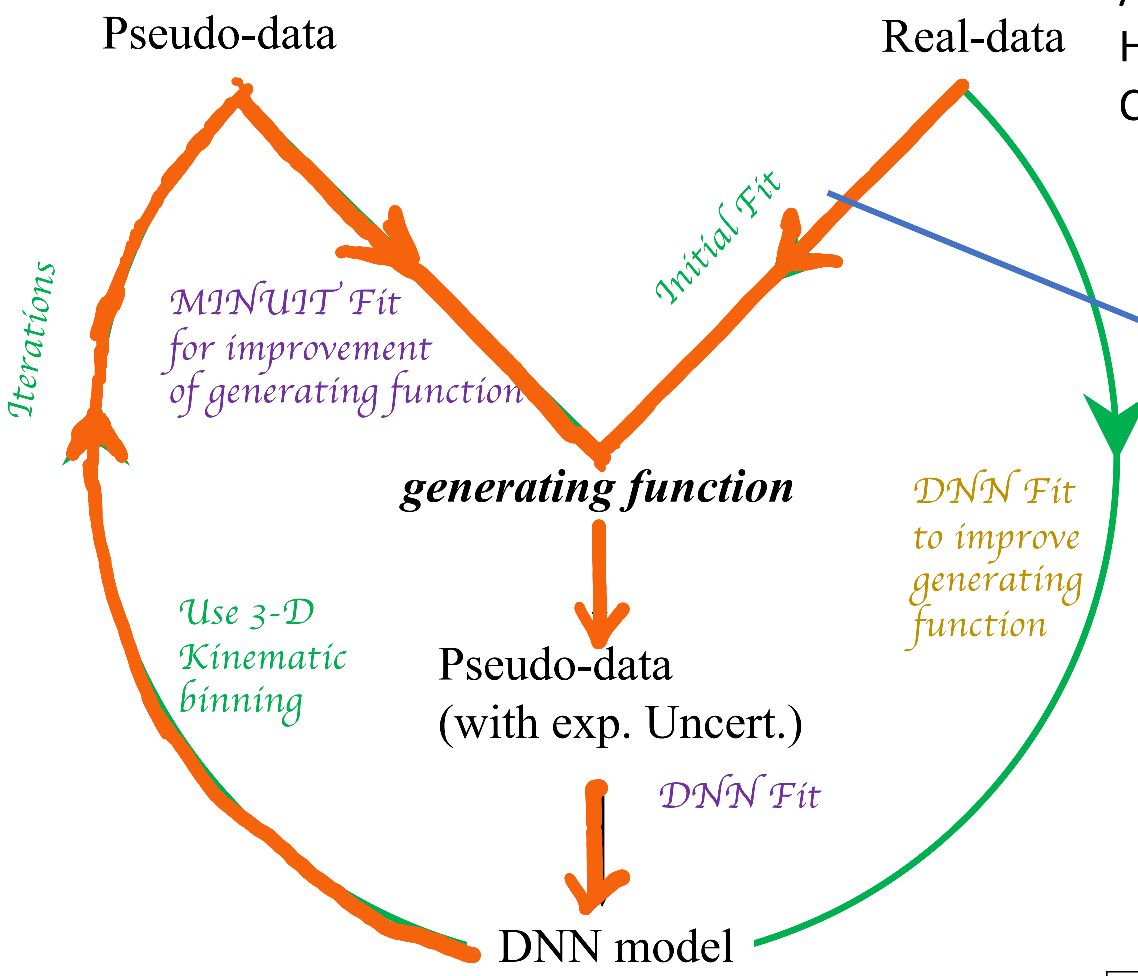
$$\langle p_\perp^2\rangle_C = \frac{m_C^2\langle p_\perp^2\rangle}{\langle p_\perp^2\rangle + m_C^2}$$

$$\langle P_{hT}^2\rangle = z^2\langle k_\perp^2\rangle + \langle p_\perp^2\rangle$$

$$\langle p_\perp^2\rangle = A + z^2 B$$

$$\langle P_{hT}^2\rangle = A + z^2(B+C)$$

$$\langle k_\perp^2\rangle = C$$



Available data:
HERMES (2013) [identified hadrons]
COMPASS (2014) [unidentified hadrons]

Since this is exploratory, we use the fit parameters from Barone et al for the initial MINUIT fit

Barone et al., PRD 91, 074019 (2015)

Parameter	Value
m_{BM}^2	0.09 GeV^2
A	0.20
B	0.57
C	0.03
N_d	-1.00
N_u	-0.49
α_q, β_q	0
m_C^2	1.5 GeV^2
$N_{q(fav)}^C$	0.49
$N_{q(disf)}^C$	-1.00
γ	1.06
δ	0.07

$$\epsilon_q(x,k_\perp) = \left(1 - \frac{|\Delta^N f_{q/p^\uparrow}^{(\text{true})} - \Delta^N f_{q/p^\uparrow}^{(\text{mean})}|}{\Delta^N f_{q/p^\uparrow}^{(\text{true})}}\right) \times 100\%$$

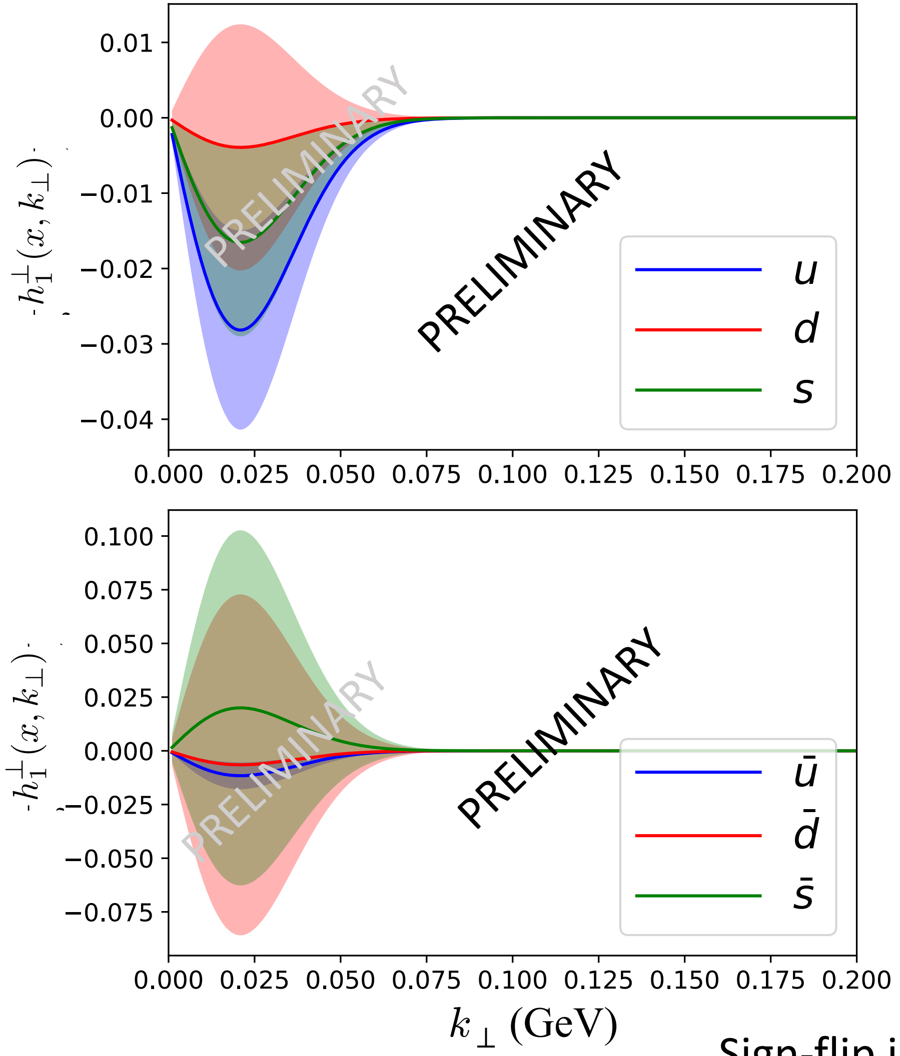
$$\sigma_q(x,k_\perp) = \sqrt{\frac{\sum_i \left(\Delta^N f_{q/p^\uparrow}^{(i)} - \Delta^N f_{q/p^\uparrow}^{(\text{mean})}\right)^2}{N}}$$

The “DNN Method” for extracting BM

- All quark flavors in SU(3) are considered in a more generic fashion.
- As this is exploratory, the fit results for the coefficients not related to $\mathcal{N}_q(x)$ are fixed by the Barone et al. (2015)fit results, for the purpose of extracting the “generating function”.
- Twist-4 Cahn term was not ignored.
- A linear combination of $\cos(\phi)$ and $\cos(2\phi)$ asymmetries were assumed.
- Training in-progress for two separate DNN models: proton target and deuteron target
- Prospective plan: use inputs x, Q^2 (for evolution), and target type

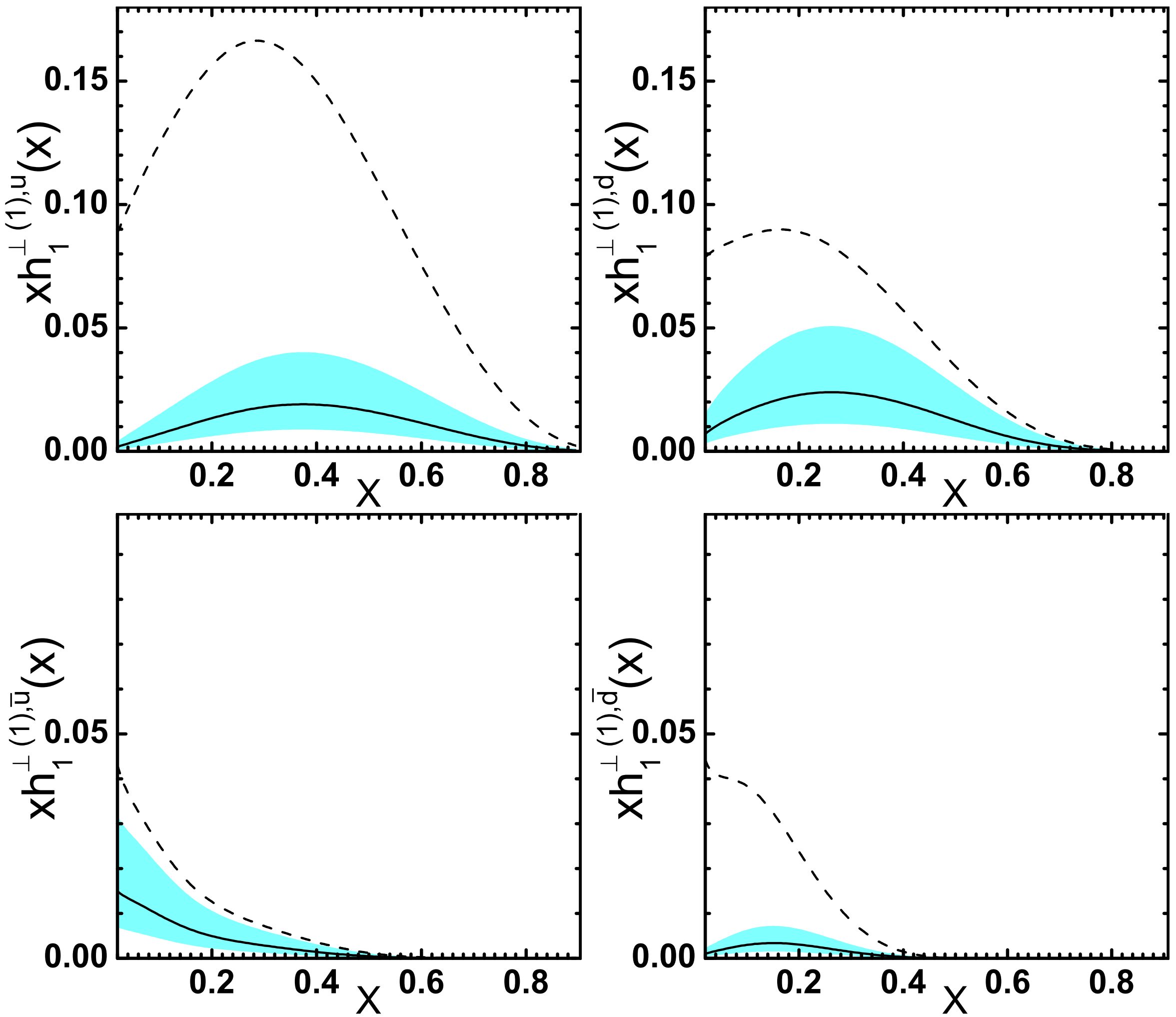
Boer-Mulders (BM) function

<u>Very very</u> preliminary DNN fits
to HERMES 'proton' SIDIS data



Zhun Lu and Ivan Schmidt PhysRevD.81.034023 (2010)

From the fits to DY data



Sign-flip is expected from
QCD gauge invariance as BM is T-odd

Summary & Outlook

- We proposed a new method for performing global fits to extract TMDs.
- Our method is based on the integration of AI and the use of generating function to ensure the accuracy and precision.
- We have successfully tested our method with pseudo-data, also a dedicated systematic study.
- We chose Deep Neural Net (DNN) to incorporate all x-dependent features of $\mathcal{N}_q(x)$.
- We performed global fit with experimental data: Separately on polarized SIDIS with Proton target and Deuteron target and obtained reasonably well description and extracted the Sivers functions for all light quark flavors in SU(3).
- We projected SIDIS and DY Sivers asymmetries: for existing (as a validation check) and upcoming experiments (such as SpinQuest).
- Currently working on Boer-Mulders function extraction…

Next:

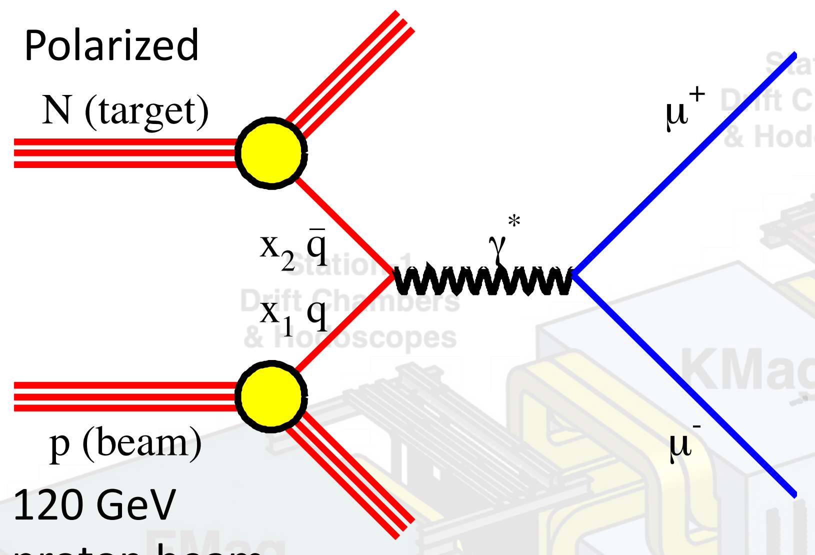
- Performing simultaneous global fit with SIDIS, DY, W^+/W^- data including a study of TMD evolution with DNN techniques to extract the unpolarized TMD using DNN.
- Applying the "DNN method" to extract other TMDs such as Transversity, Boer-Mulders function, as well as Spin-1 TMDs.

SpinQuest (E1039) Experiment at Fermilab

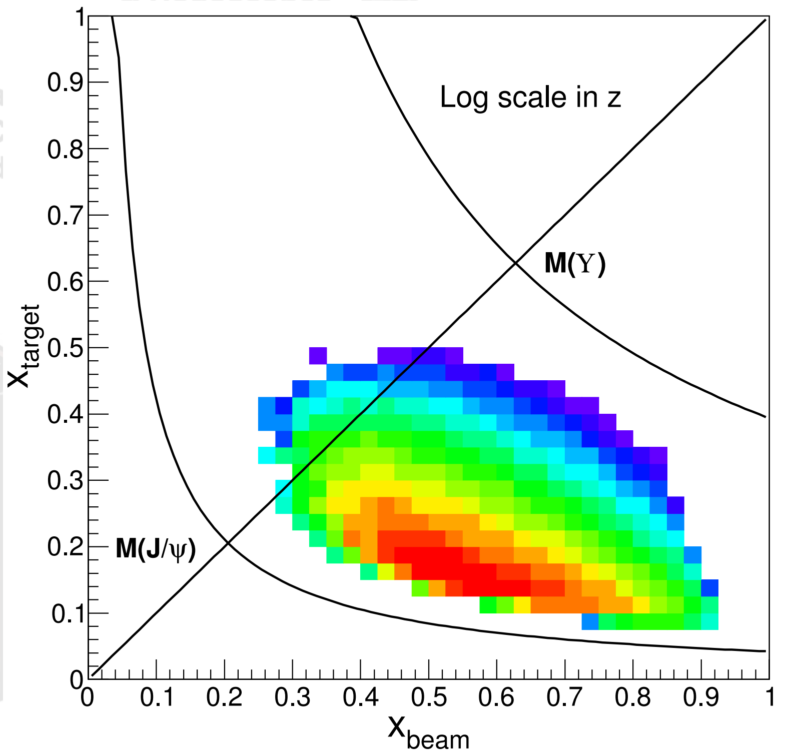
- Measurement of 'sea' quark Sivers function

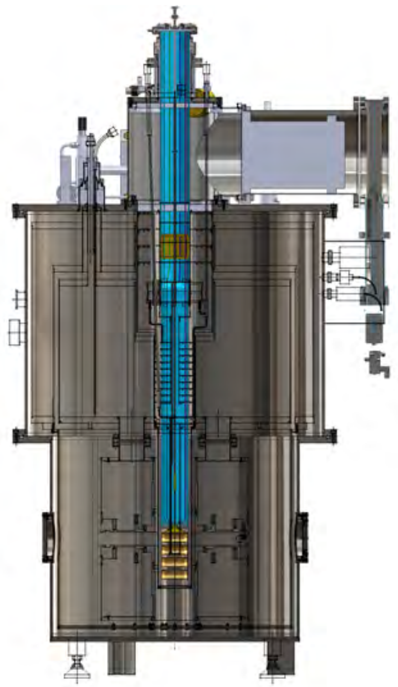
Stay Tuned!

$pp^{\uparrow}(d^{\uparrow}) \rightarrow \mu^{+}\mu^{-}X,\ 4 < M_{\mu\mu} < 9\ GeV$



$$\frac{d\sigma}{dx_1 dx_2} = \frac{4\pi\alpha^2}{9 s x_1 x_2} \sum_i e_i^2 (q_i^B(x_1, Q^2)\bar{q}_i^T(x_2, Q^2) + \bar{q}_i^B(x_1, Q^2) q_i^T(x_2, Q^2))$$





LANL-UVA Polarized Target

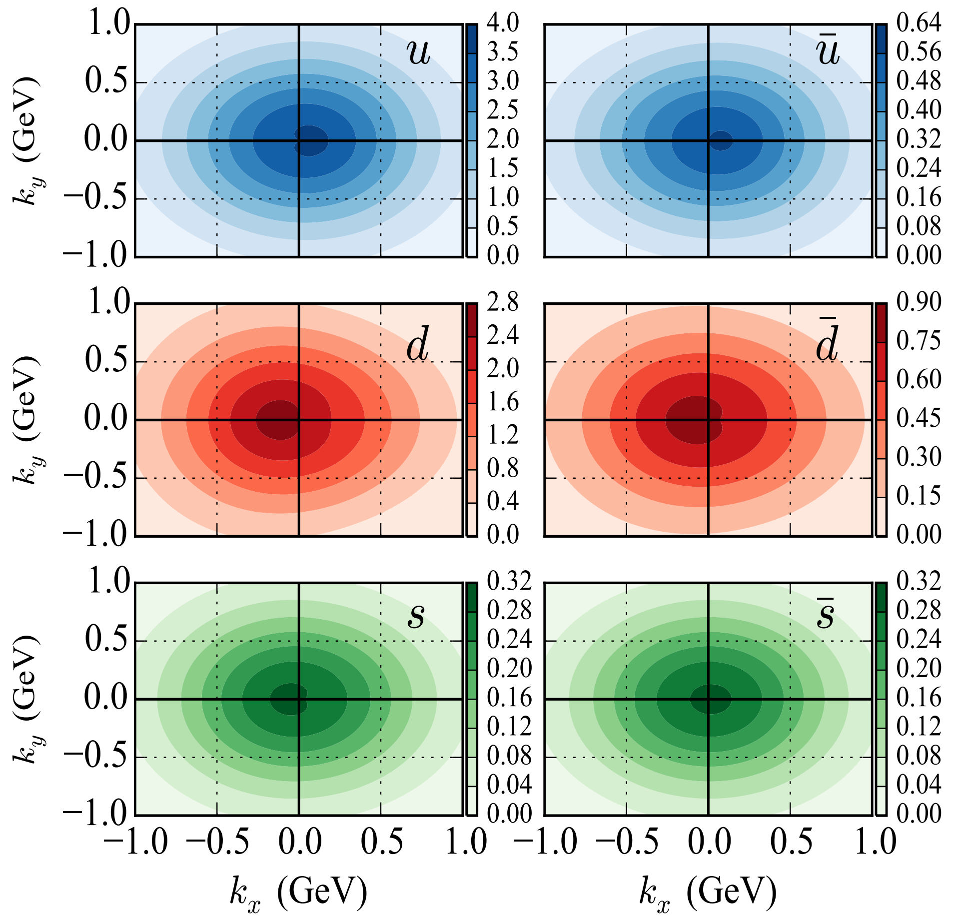
https://spinquest.fnal.gov/
http://twist.phys.virginia.edu/E1039/

Please Join The Effort

Dustin Keller (dustin@virginia.edu)[Spokesperson]
Kun Liu (liuk@lanl.gov)[Spokesperson])

Highest beam intensity on a polarized target ever!

Thank you



University of Virginia

UVA - Spin PHYSICS GROUP

U.S. DEPARTMENT OF ENERGY | Office of Science

This work is supported by DOE contract DE-FG02-96ER40950

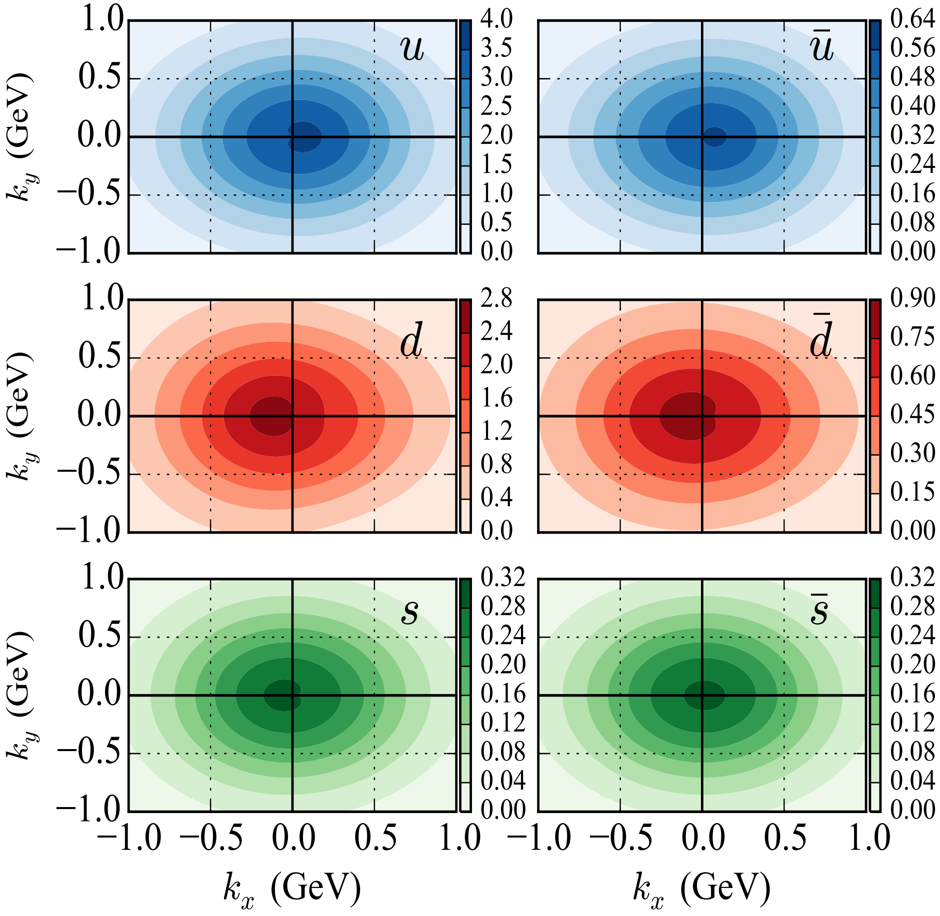
Backup Slides

3D Tomography from the "Proton" DNN Model

$$\rho^{a}_{p\uparrow}(x, k_x, k_y; Q^2) = f_1^a(x, k_\perp^2; Q^2) - \frac{k_x}{m_p} f_{1T}^{\perp a}(x, k_\perp^2; Q^2)$$

I. P. Fernando and D. Keller
Phys. Rev. D.108.054007 (2023)

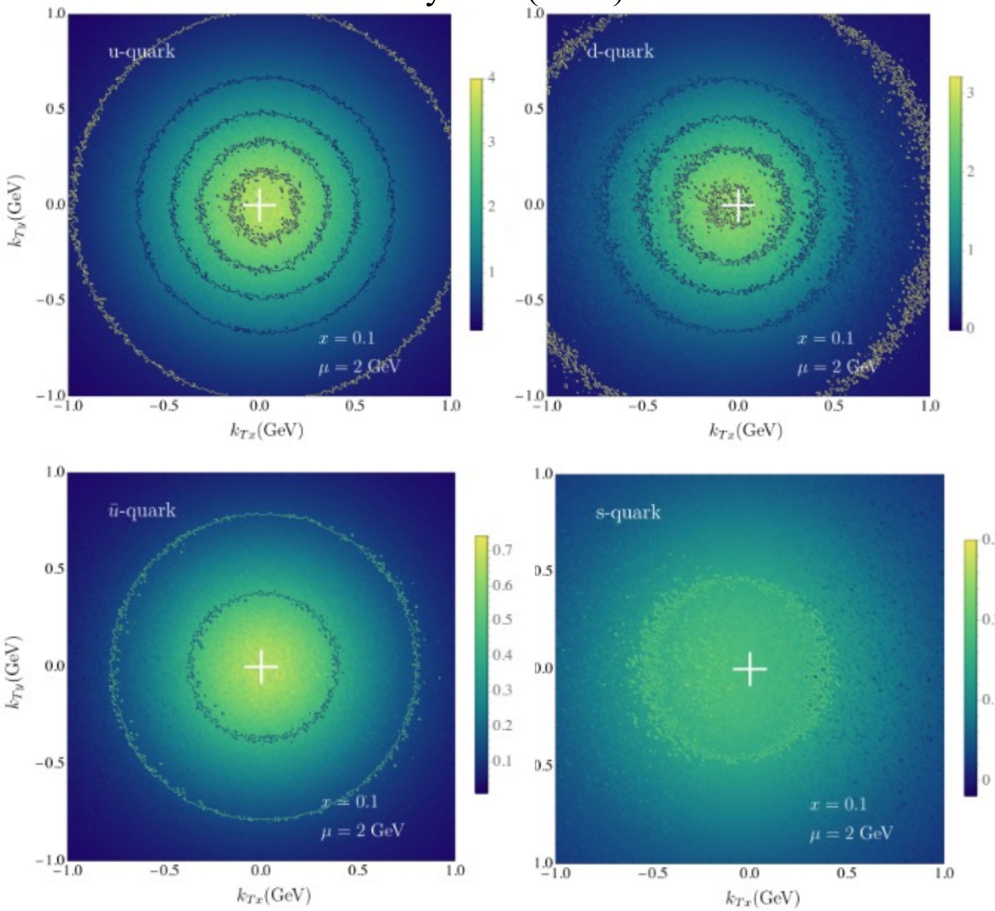
Proton-DNN model



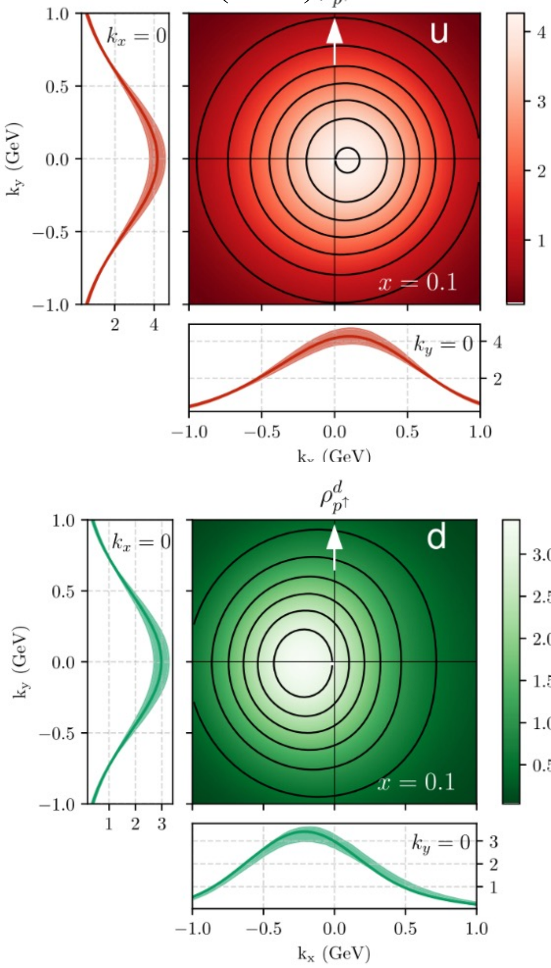
$Q^2 = 2.4\ \mathrm{GeV}^2$

$x = 0.1$

Bury et al (2021)

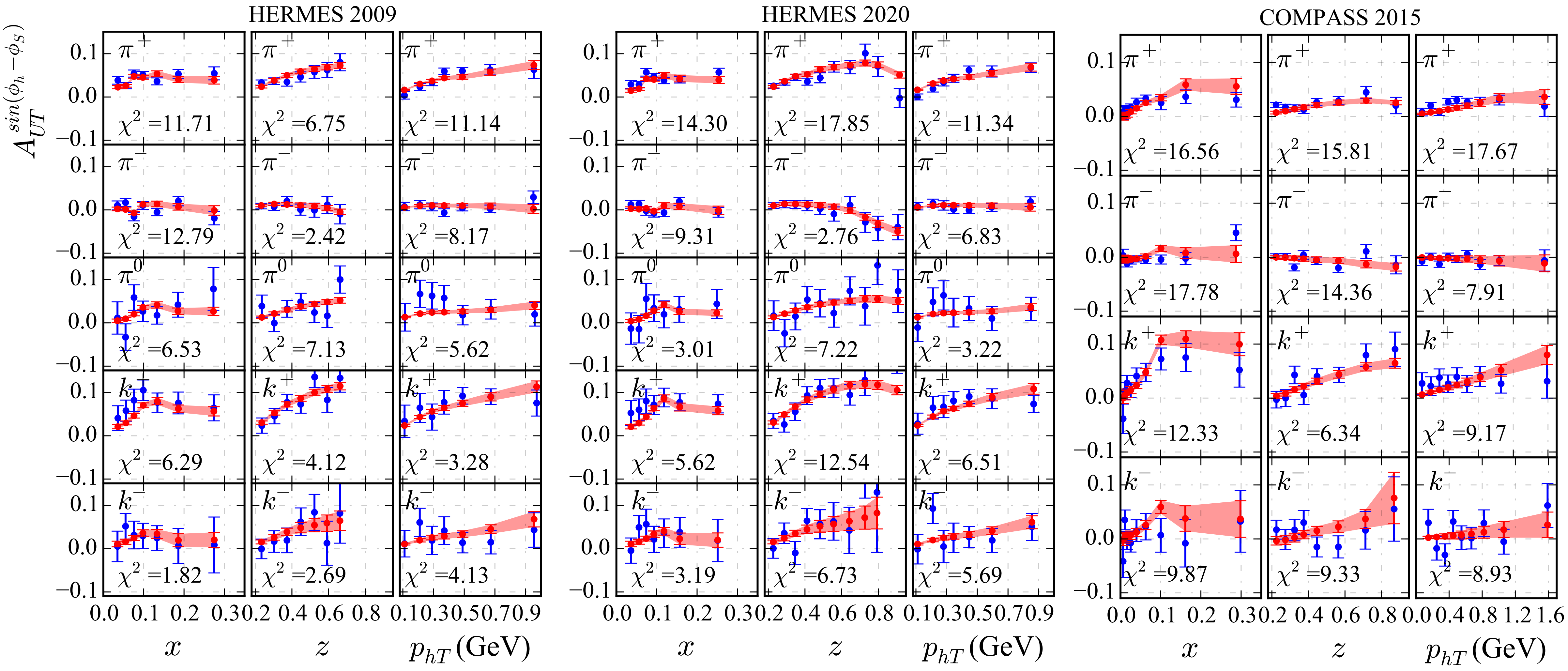


A. Bacchetta et al (2021) $\rho^u_{p\uparrow}$



Proton DNN Fit Results

I. P. Fernando and D. Keller
Phys. Rev. D.108.054007 (2023)



- ➢ All data points are well-described by the proton-DNN model.
- ➢ No kinematic cuts were implemented.

Calculated χ_{total}^2/N_{pt} = 1.04

Deuteron DNN Fit Results

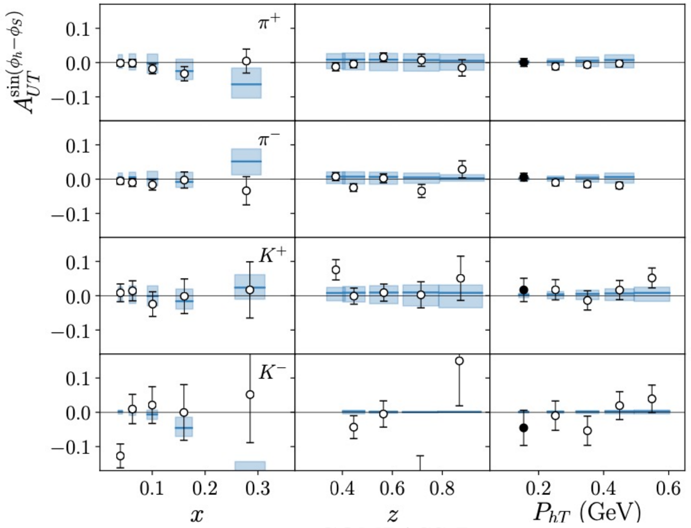
Backup

- No kinematic cuts are applied
  Deuteron-DNN model can describe data reasonably well
- No iso-spin symmetry conditions are applied

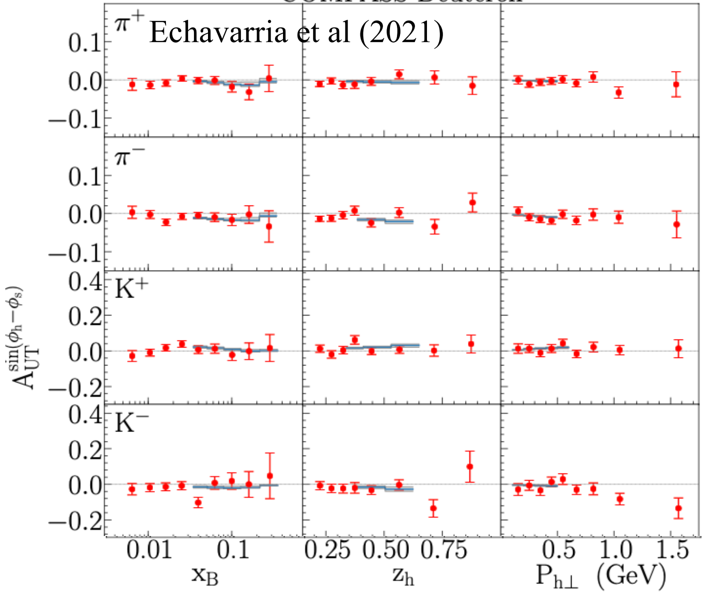
~~$f^{\perp}_{1T,u\leftarrow d} = f^{\perp}_{1T,d\leftarrow d} = \frac{f^{\perp}_{1T,u\leftarrow p} + f^{\perp}_{1T,d\leftarrow p}}{2}$~~

$\chi^2/N_{pt} = 0.81$

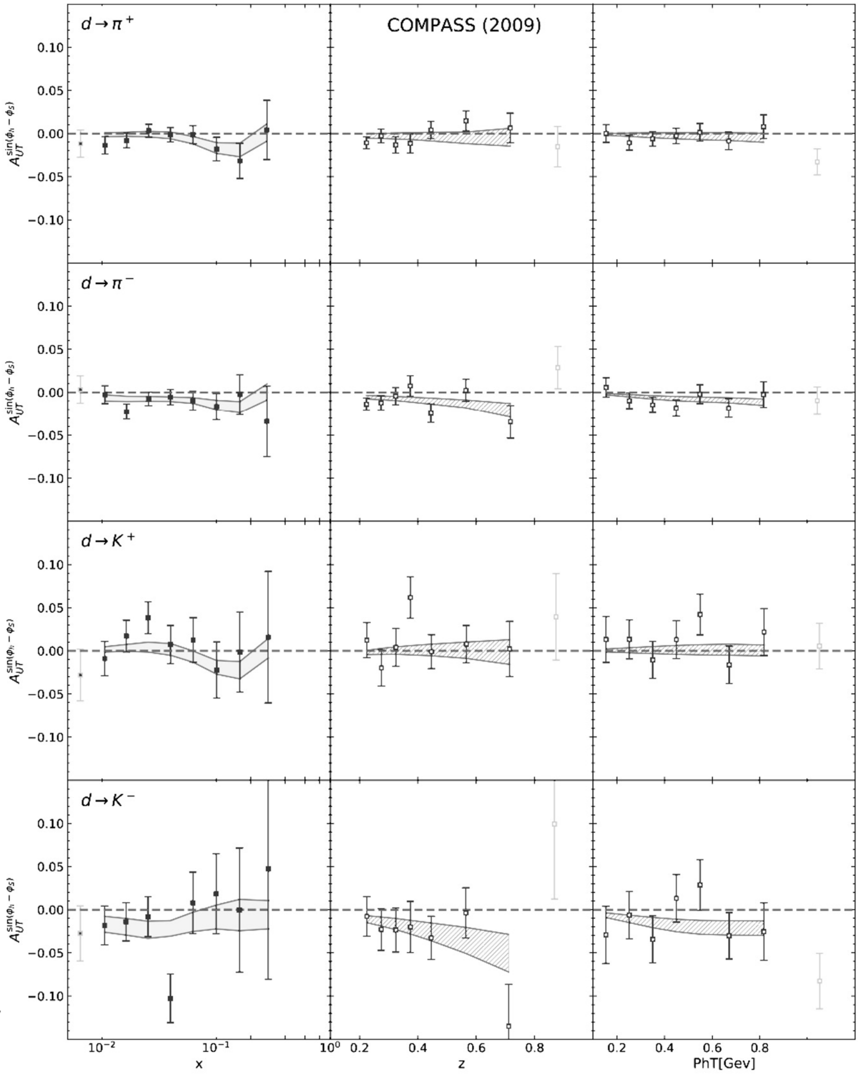
Bury et al (2021)



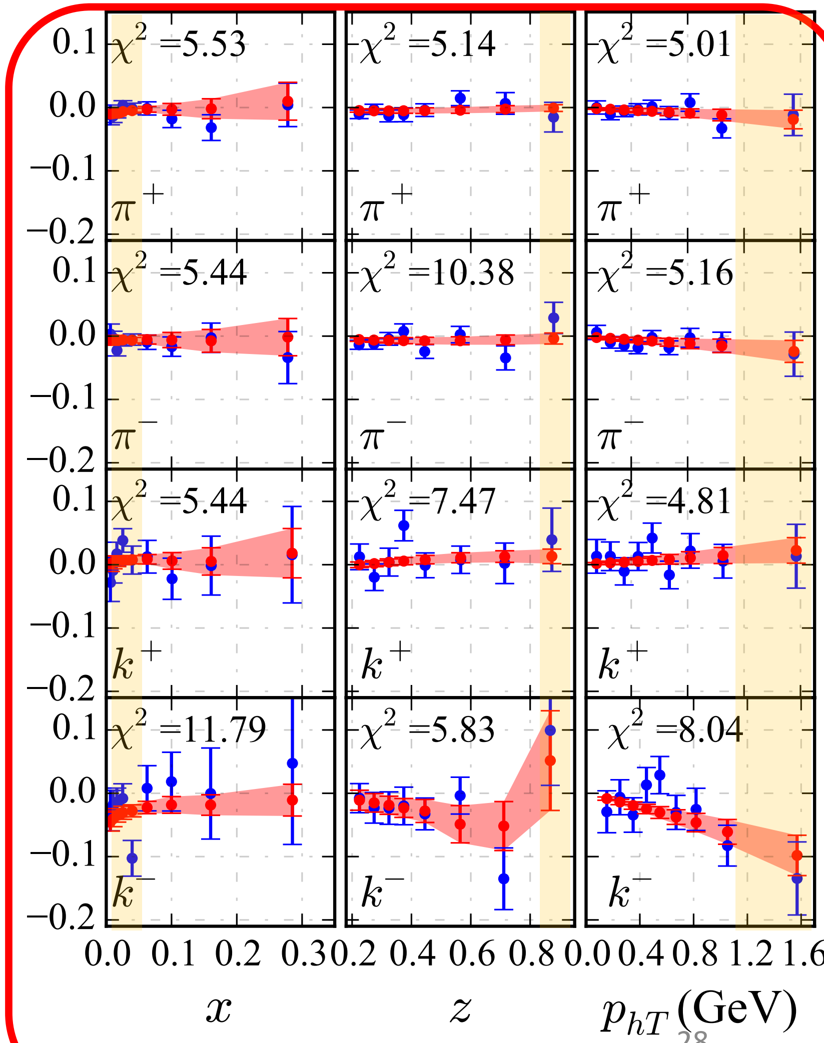
COMPASS Deuteron



Bacchetta et al (2022)



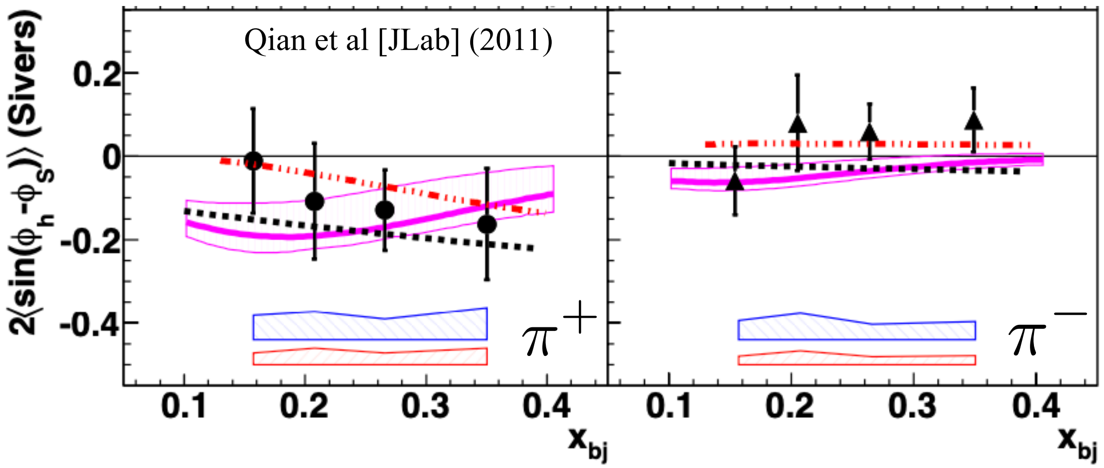
COMPASS 2009



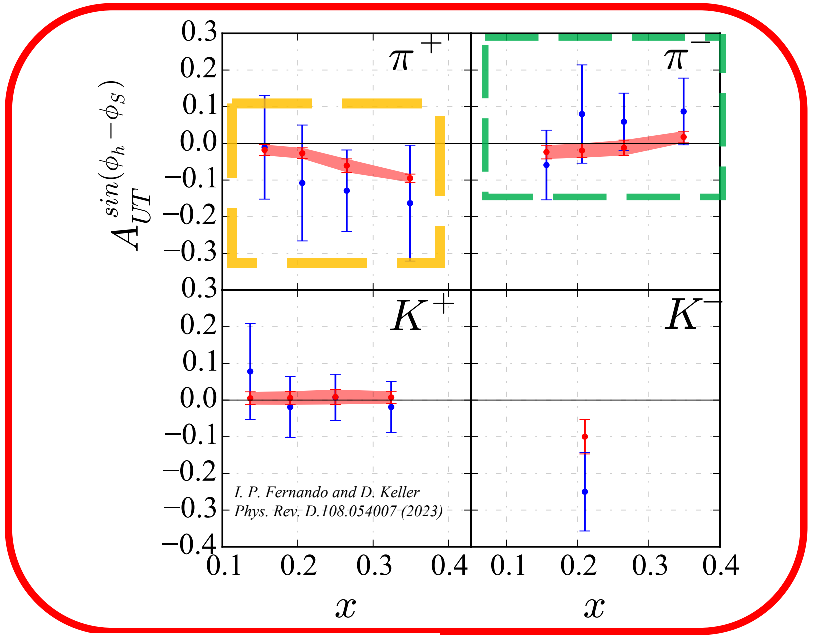
I. P. Fernando and D. Keller
Phys. Rev. D.108.054007 (2023)

Deuteron DNN Projections for JLab Kinematics

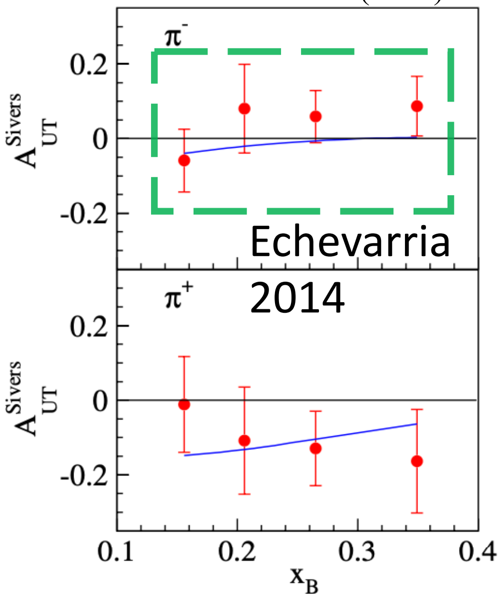
Backup



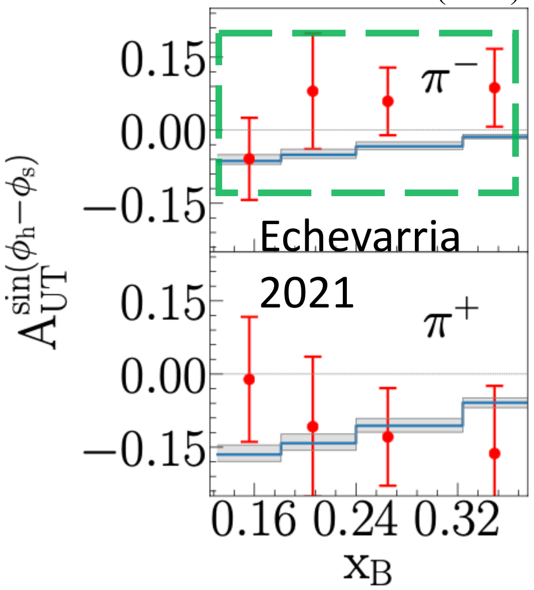
Deuteron-DNN

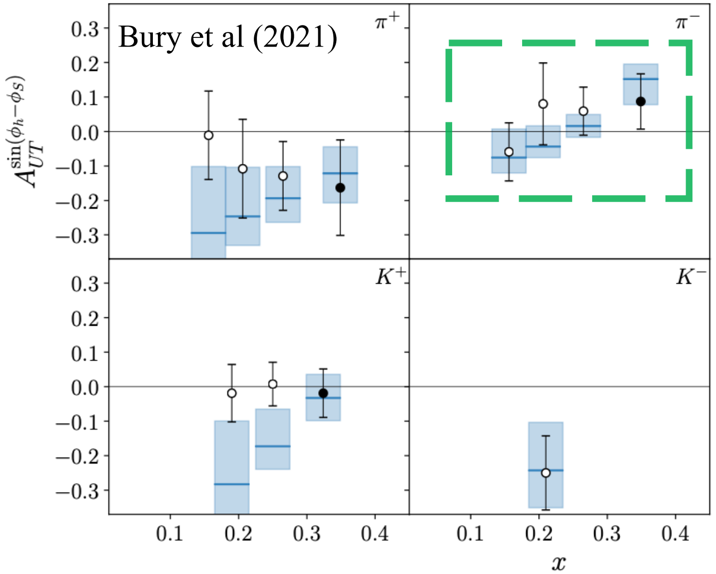


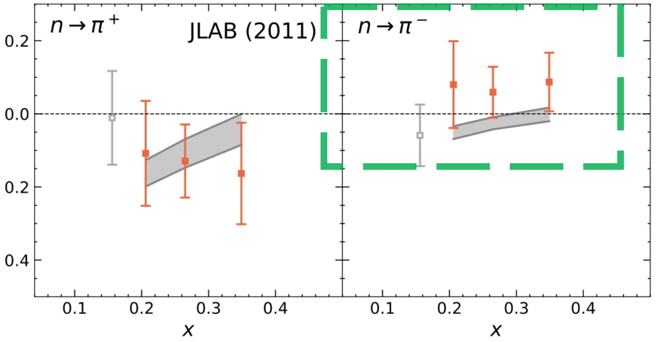
Echevarria et al (2014)



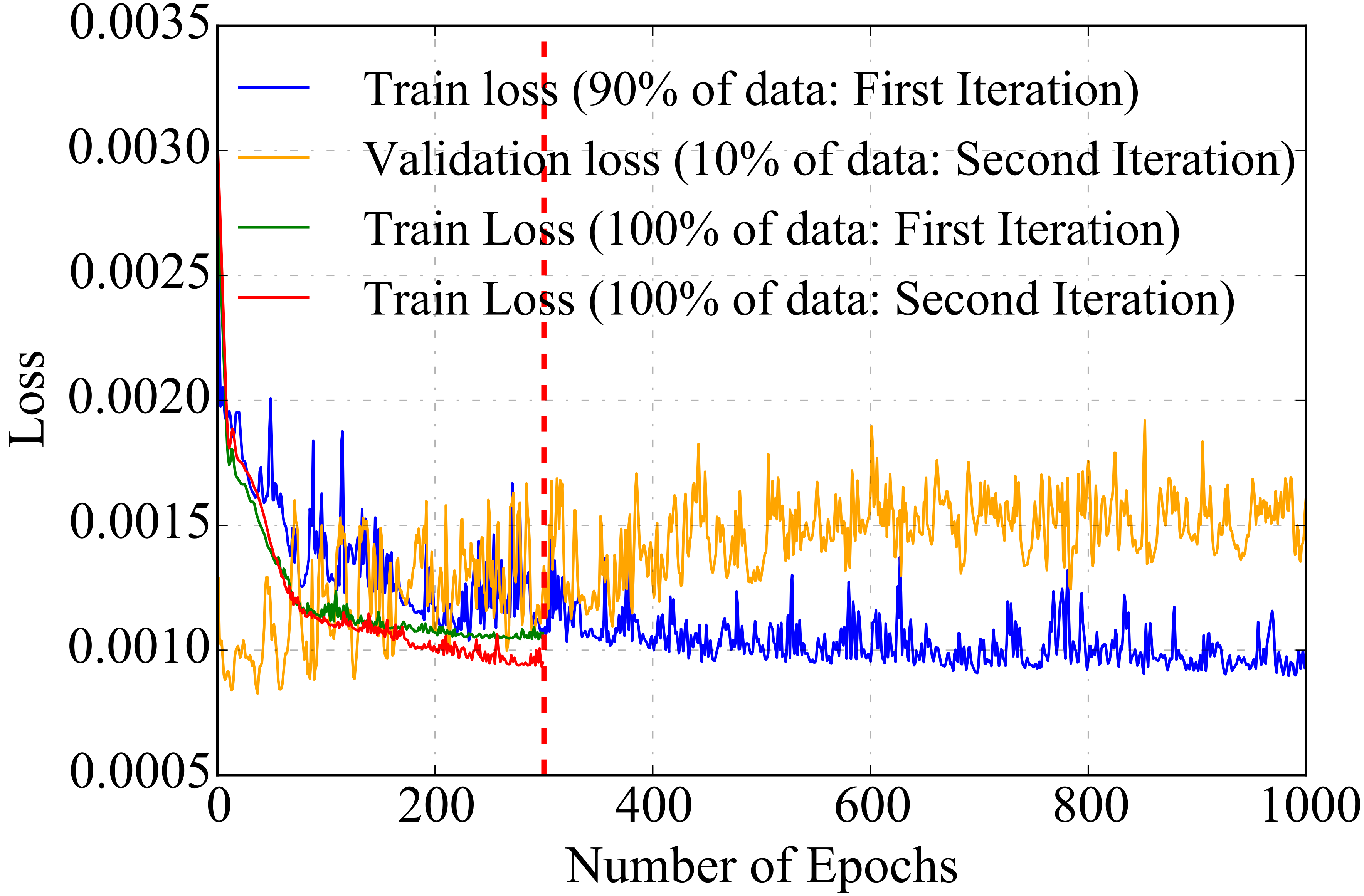
Echevarria et al (2021)







Bacchetta et al (2022)



Train loss (90% of data: First Iteration)
Validation loss (10% of data: Second Iteration)
Train Loss (100% of data: First Iteration)
Train Loss (100% of data: Second Iteration)
Loss
0.0035
0.0030
0.0025
0.0020
0.0015
0.0010
0.0005
0
200
400
600
800
1000
Number of Epochs

TABLE III. The summary of the optimized sets of hyperparameters: The indications in the table are $\mathcal{C}_0^i$ and $\mathcal{C}_0^f$ for results from the pseudodata from the generating function, $\mathcal{C}_p^i$, and $\mathcal{C}_p^f$ for results from SIDIS data from experiments associated with the polarized-proton target, and $\mathcal{C}_d^i$ and $\mathcal{C}_d^f$ for results from SIDIS data from experiments associated with the polarized-deuterium target, where i and f indicate the *First Iteration* and *Second Iteration* respectively. The initial learning rate is also listed ($\times 10^{-4}$) as is the final training loss ($\times 10^{-3}$). The accuracy and precision in each case are the maxima over the phase space.

Hyperparameter	$\mathcal{C}_0^i$	$\mathcal{C}_0^f$	$\mathcal{C}_p^i$	$\mathcal{C}_p^f$	$\mathcal{C}_d^i$	$\mathcal{C}_d^f$
Hidden layers	5	7	5	7	5	8
Nodes/layer	256	256	550	550	256	256
Learning rate	1	0.125	5	1	10	1
Batch size	200	256	300	300	100	20
Number of epochs	1000	1000	300	300	200	200
Training loss	0.6	0.05	1.5	1	2	1
$\varepsilon_u^{\max}$	95.67	99.27	55.21	94.04	56.80	93.02
$\varepsilon_{\bar{u}}^{\max}$	42.62	98.09	52.57	96.70	34.83	91.40
$\varepsilon_d^{\max}$	80.46	98.89	55.69	93.13	52.44	89.27
$\varepsilon_{\bar{d}}^{\max}$	74.59	97.08	55.37	95.04	46.60	92.58
$\varepsilon_s^{\max}$	45.53	79.27	49.54	90.64	36.34	93.41
$\varepsilon_{\bar{s}}^{\max}$	59.27	91.13	33.89	82.51	65.57	91.45
$\sigma_u^{\max}$	3	0.1	5	2	2	0.4
$\sigma_{\bar{u}}^{\max}$	2	0.2	6	2	8	2
$\sigma_d^{\max}$	10	1	20	6	2	1
$\sigma_{\bar{d}}^{\max}$	7	4	20	8	7	1
$\sigma_s^{\max}$	2	0.2	4	1	6	2
$\sigma_{\bar{s}}^{\max}$	1	0.1	4	2	6	3

Systematic Studies: data cuts

Backup

The applicability of TMD factorization was ensured by applying cuts to SIDIS data based on various criteria in the literature.

$$W^{\mu\nu} = \sum_f |\mathcal{H}_f(Q^2,\mu)|^{\mu\nu}$$
$$\times \int d^2k_\perp d^2p_\perp \delta^{(2)}(z_h k_\perp + p_\perp - p_{hT})$$
$$\times F_{f/N^\uparrow}(x, z_h k_\perp, S; \mu, \zeta_F) D_{h/f}(z_h, p_\perp; \mu, \zeta_D)$$
$$+ Y(p_{hT}, Q^2),$$

In this exploratory effort with DNNs, the power corrections are not directly imposed. In addition to the basic data cut $Q^2 > 1$ GeV^2 we performed $Q^2 > 2$ GeV^2 and $p_{hT} < zQ$ cuts separately with the proton-DNN model to understand the impact on the extracted Sivers functions.

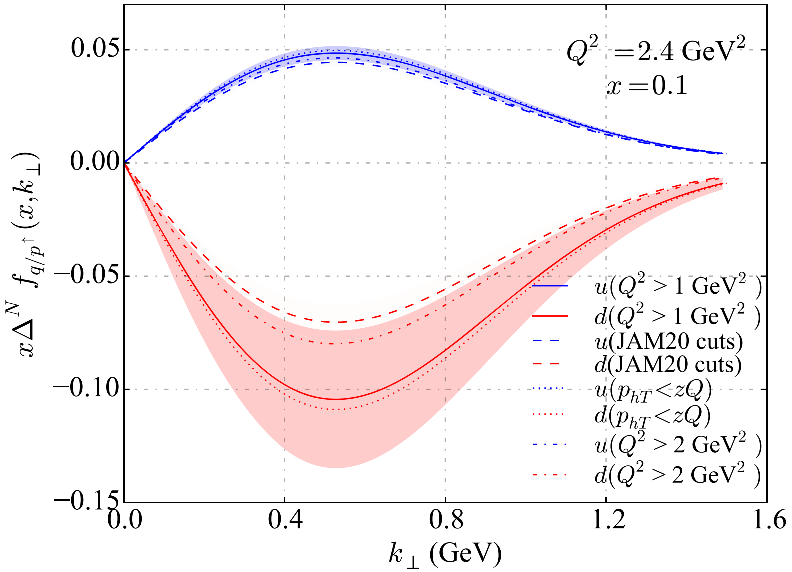


FIG. 17. Solid lines with light band represent the u (in blue), d (in red) Sivers functions using the cut $Q^2 > 1$ GeV^2. These resulting DNN models made from the cuts from all tests are also shown.

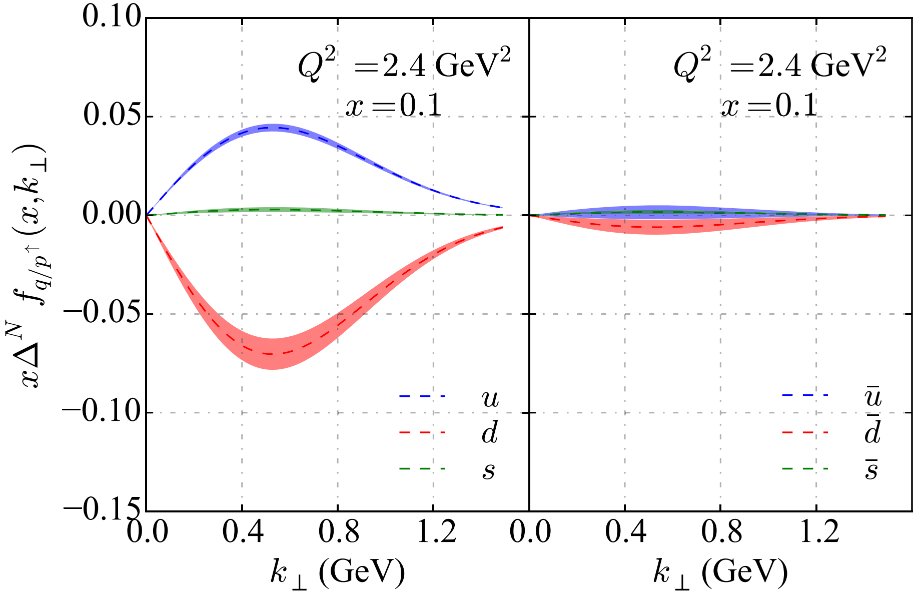


FIG. 18. Sivers functions from a retrained DNN model using the cuts [65] to the data demonstrating that being selective with the data can reduce the error bands of the fit but may also add an unintentional bias.

Systematic Studies: Choice of h(k)

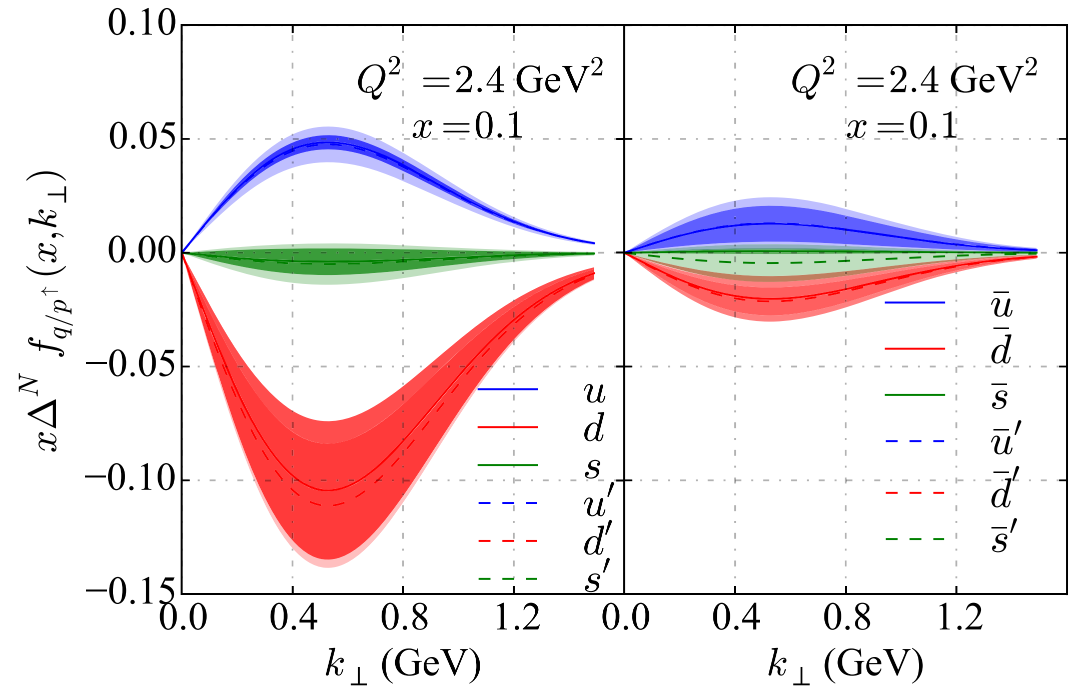


FIG. 19. Using two different $h(k_\perp)$. Solid line with dark band represents the Sivers functions with $h(k_\perp) = \sqrt{2e}\frac{k_\perp}{m_1}e^{-k_\perp^2/m_1^2}$, whereas the dashed line with light band represents the Sivers functions with $h(k_\perp) = \frac{2k_\perp m_1}{m_1^2+k_\perp^2}$.

$$h(k_\perp) = \sqrt{2e}\,\frac{k_\perp}{m_1}\,e^{-k_\perp^2/m_1^2}$$

$$h(k_\perp) = \frac{2k_\perp m_1}{m_1^2 + k_\perp^2}$$

Systematic Studies : TMD Evolution

Backup

$$\mathcal{N}_q(x) \longrightarrow \mathcal{N}_q(x, Q^2)$$

The solution of the TMD evolution equations

$$\mu^2 \frac{dF(x,b;\mu,\zeta)}{d\mu^2} = \frac{\gamma_F(\mu,\zeta)}{2} F(x,b;\mu,\zeta)$$

$$\zeta \frac{F(x,b;\mu,\zeta)}{d\zeta} = -\mathcal{D}(b,\mu) F(x,b;\mu,\zeta),$$

$$F(x,b;\mu,\zeta) = \left(\frac{\zeta}{\zeta_\mu(b)}\right)^{-\mathcal{D}(b,\mu)} F(x,b)$$

$$\mu \sim Q, \qquad \zeta_F \zeta_D \sim Q^4, \qquad \mu^2 = \zeta^2 = Q^2$$

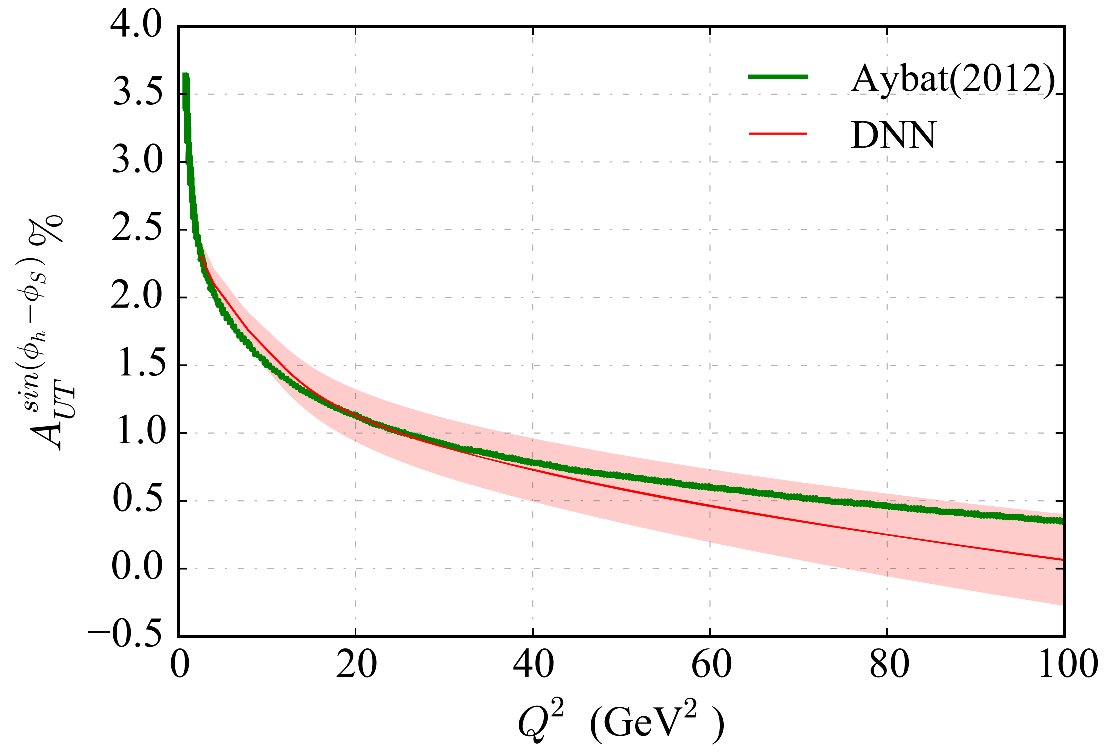


FIG. 21. The Sivers asymmetry evolution in Q^2 compared to the result from [6]. The red-colored solid line and the band represent the mean and standard deviation of the $A_{UT}^{\sin(\phi_h-\phi_S)}$ from 1000 replica models of the proton DNN at $x = 0.12$, $z = 0.32$, $p_{hT} = 0.14$ GeV.