Progress TMDs & factorization at sub-leading power

Leonard Gamberg
w/ Zhongbo Kang, Ding-Yu Shao, John Terry, Fany Zhao

arXiv: e-Print:221.13209

SPIN 2023

25TH INTERNATIONAL SPIN PHYSICS SYMPOSIUM

September 24 – 29, 2023
Durham Convention Center
Durham, NC, USA

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Summary/Outline

We explore sub-leading power Λ_{QCD}/Q TMDs in the context of factorization theorem

- NLP factorization based on "*TMD formalism*"

  —extend the tree level Amsterdam formalism and beyond leading order

  CSS, Ji Ma Yuan, Abyat Rogers, framework vs. SCET and Background Field Methods
- Revisit "Cahn effect" & matching related to early picture of importance intrinsic $\boldsymbol{k}_T$
  - *"Intrinsic"NLP TMDs related thru EOM in terms "kinematic" & "dynamical"*
- Consider RG consistency of matching to collinear factorization
  - Bacchetta, Boer, Diehl, Mulders JHEP 2008, Bacchetta et al. PLB 2019
- Report progress in this *necessary condition* NLP factorization (but not sufficient)

Importance of N*LP* TMDs & Factorization

- Importance of NLP TMD *observables* underscored by observation that while they are suppressed by *M/Q* wrt *LP* observables:

  ◉ NLP/SLP TMDs can be as sizable as leading-power TMDs in some situations, particularly when *Q* is not that large … not small in the kinematics of fixed-target experiments

- Their understanding is required for a complete description of "*benchmark processes*" SIDIS, DY & e^+e^-…

  ◉ They may are relevant for a proper extraction of the *leading-power* effects from data.

- Are of interest offer a mechanism to probe physics of quark-gluon-quark correlations, provide novel information about the partonic structure of hadrons, and are largely unexplored.
  ◉ Such correlations may be considered quantum interference effects, and they could be related to average transverse forces acting on partons inside (polarized) hadrons as well as other phenomena.

- Also, experimental information from SIDIS on effects related to subleading TMDs is & has been available
  ◉ In the future, the EIC with its *large* kinematical coverage will be ideal for making further groundbreaking progress in this area.

  ◉*NB*: Iff factorization can be established beyond "tree level" leading order

Challenges of *SLP/NLP* TMDs

SLP TMD observables challenging in comparison to the current state-of-the-art of leading power observables.
Treatments in the literature are mostly limited to a tree-level formalism until recently early studies beyond tree level
Bacchetta et al. JHEP 2008, Chen et al. PLB 2017

More recently results beyond *LO*
MIT group, Gao, Ebert, Stewart JHEP 2022
Vladimirov, Rodini, Scimemi, Moos, JHEP 2021, 2022, arXiv 2023
Balitsky 2023 rapidity only TMD evolution
Gamberg, Kang, Shao, Terry, Zhao arXiv: e-Print:221.13209
See also Ch. 10 TMD handbook, e-Print:2304.03302 [hep-ph]

Various sources for power suppressed terms have been identified and discussed in the literature from
Tree level Studies, Mulders, Tangerman (1996), Bacchetta et al. JHEP (2007)

- This includes corrections associated to kinematic prefactors involving contractions between the leptonic and hadronic tensors, referred to as ***kinematic power corrections***.
- Another involve subleading terms in quark-quark correlators involving Dirac structures that differ from LP ones called ***intrinsic power corrections***—*most familiar e.g. Cahn function* $f^{\perp}(x, k_T)$
- Another from hadronic matrix elements of (interaction dependent) quark-gluon-quark operators, referred to *qgq* correlators for short referred to as ***dynamic power corrections***.
- *In arXiv: e-Print:221.13209 we* present a systematic procedure for stress testing TMD factorization for DY & SIDIS at NLP using CSS formalism which addresses disagreements in the literature

Preprints: JLAB-THY-23-3780, LA-UR-21-20798, MIT-CTP/5386

TMD Handbook

Renaud Boussarie[1], Matthias Burkardt[2], Martha Constantinou[3], William Detmold[4], Markus Ebert[4,5], Michael Engelhardt[2], Sean Fleming[6], Leonard Gamberg[7], Xiangdong Ji[8], Zhong-Bo Kang[9], Christopher Lee[10], Keh-Fei Liu[11], Simonetta Liuti[12], Thomas Mehen[13], Andreas Metz[3], John Negele[4], Daniel Pitonyak[14], Alexei Prokudin[7,16], Jian-Wei Qiu[16,17], Abha Rajan[12,18], Marc Schlegel[2,19], Phiala Shanahan[4], Peter Schweitzer[20], Iain W. Stewart[4], Andrey Tarasov[21,22], Raju Venugopalan[18], Ivan Vitev[10], Feng Yuan[23], Yong Zhao[24,4,18]

Abstract

This handbook provides a comprehensive review of transverse-momentum-dependent parton distribution functions and fragmentation functions, commonly referred to as transverse momentum distributions (TMDs). TMDs describe the distribution of partons inside the proton and other hadrons with respect to both their longitudinal and transverse momenta. They provide unique insight into the internal momentum and spin structure of hadrons, and are a key ingredient in the description of many collider physics cross sections. Understanding TMDs requires a combination of theoretical techniques from quantum field theory, nonperturbative calculations using lattice QCD, and phenomenological analysis of experimental data. The handbook covers a wide range of topics, from theoretical foundations to experimental analyses, as well as recent developments and future directions. It is intended to provide an essential reference for researchers and graduate students interested in understanding the structure of hadrons and the dynamics of partons in high energy collisions.

10 - Subleading TMDs

L. Gamberg, A. Metz, I. Stewart

10 - Subleading TMDs

10.1 Introduction

From a historical perspective it is very interesting that the subleading-power $\cos\phi_h$ azimuthal modulation of the unpolarized SIDIS cross section was important for the development of the TMD field, since one of the earliest discussions of transverse parton momenta in DIS is related to this observable [1240, 293, 294]; see also Sec. 5.1 for more details. Generally, although suppressed by Λ/Q with respect to leading-power observables, subleading TMD observables are typically not small, especially in the kinematics of fixed-target experiments. In fact, the first-ever observed SSA in SIDIS was a sizeable power-suppressed longitudinal target SSA for pion production from the HERMES Collaboration [483]. Those measurements, which triggered many theoretical studies and preceded the first measurements of the (leading-power) Sivers and Collins SSAs, were critical for the growth of TMD-related research.

[483] HERMES collaboration, A. Airapetian et al., *Observation of a single spin azimuthal asymmetry in semiinclusive pion electro production*, *Phys. Rev. Lett.* **84** (2000) 4047 [hep-ex/9910062].

Important papers re: TMD fact at NLP(incomplete)

F. Rivindal PLB 1973
Georgi Politzer PRL 1978
Cahn PLB 1978 (response to Georgi Politzer PRL 1978)
R. Tangerman, P. Mulders hep-ph/9408305 [hep-ph] (1994)
P.Mulders, R. Tangerman, NPB 461(1996)
L. Gamberg, D Hwang, A Metz, M. Schlegel, Phys.Lett.B 639 (2006), rapidity div. @tw3-factorization endangered
D. Boer, P. Mulders, Phys.Rev.D 57 (1998)
A.Bacchetta, D. Boer, M. Diehl, P. Mulders JHEP (2008) factorization at NLP consistency checks on matching
A.P. Chen, J.P. Ma, Phys. Lett. B 768 (2017)
I. Feige, D.W. Kolodrubetz, I. Moult, I.W. Stewart, J. High Energy Phys. 11 (2017)
I. Balitsky, A. Tarasov, J. High Energy Phys. 07 (2017)
I. Balitsky, A. Tarasov, J. High Energy Phys. 05 (2018)
M.A. Ebert, I. Moult, I.W. Stewart, F.J. Tackmann, G. Vita, H.X. Zhu, J. High Energy Phys. 12 (2018)
M.A. Ebert, I. Moult, I.W. Stewart, F.J. Tackmann, G. Vita, H.X. Zhu, J. High Energy Phys. 04 (2019)
Moult, I.W. Stewart, G. Vita, arXiv:1905 .07411, 201
A. Bacchetta Bozzi, Echevarria, Pisano, Prokudin, Radici, Physics Letters B 797 (2019)
A.Vladimirov Moos, Scimemi, & S.Rodini JHEP 2022
M. Ebert A. Gao I. Stewart JHEP 06 (2022)
S. Rodini, A. Vladimirov JHEP 08 (2022)
L.Gamberg, Z.Kang, D.Shao, J.Terry, F.Zhao arXiv: e-Print:221.13209
I.Balitsky, JHEP 03 (2023)

Also …
Qui Sterman collinear higher twist 1991 NLB
Boer Vogelsang DY PRD 2006
Koike Nagashima Vogelsang SIDIS NPB 2006 Large P_T

Why TMDs @ "twist-3 " NLP

Some historical-context

- **Georgi Politzer, PRL 1978**
  Performed QCD analysis of *hard gluon* radiation in SIDIS to predict absolute value of final state hadron's $\boldsymbol{P}_T$, and the angular distribution relative to lepton scattering plane $\langle \cos \phi \rangle$
- ***~12-15% …effects would not arise as a result of the nonperturbative effects due to limited transverse momentum associated with confined quarks***
- **"Measurement of $\langle \cos \phi \rangle$ provide very *clean test of the perturbative predictions of QCD"***

- **Cahn, PLB 1978, (& earlier paper by Ravndal, PLB 1972)**
  Critique of the QCD calculation of azimuthal dependence in leptoproduction; emphasize importance *intrinsic* $\boldsymbol{k}_T$ …
- "We conclude that the azimuthal dependence in vector exchange interactions is inevitable since the partons have transverse momentum as a consequence of being confined and such dependence certainly does not require a special mechanism like gluon bremstrahlung"
  "…The results **(G&P78)** cast doubt on the utility of such azimuthal asymmetry as a clean test of quantum chromodynamics"

Clean tests of QCD

PHYSICAL REVIEW LETTERS

VOLUME 40 | 2 JANUARY 1978 | NUMBER 1

Clean Tests of Quantum Chromodynamics in μp Scattering

Howard Georgi
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

and

H. David Politzer
California Institute of Technology, Pasadena, California 91125

(Received 25 October 1977)

Hard gluon bremsstrahlung in μp scattering produces final-state hadrons with a large component of momentum transverse to the virtual-photon direction. Quantum chromodynamics can be used to predict not only the absolute value of the transverse momentum, but also its angular distribution relative to the muon scattering plane. The angular correlations should be insensitive to nonperturbative effects.

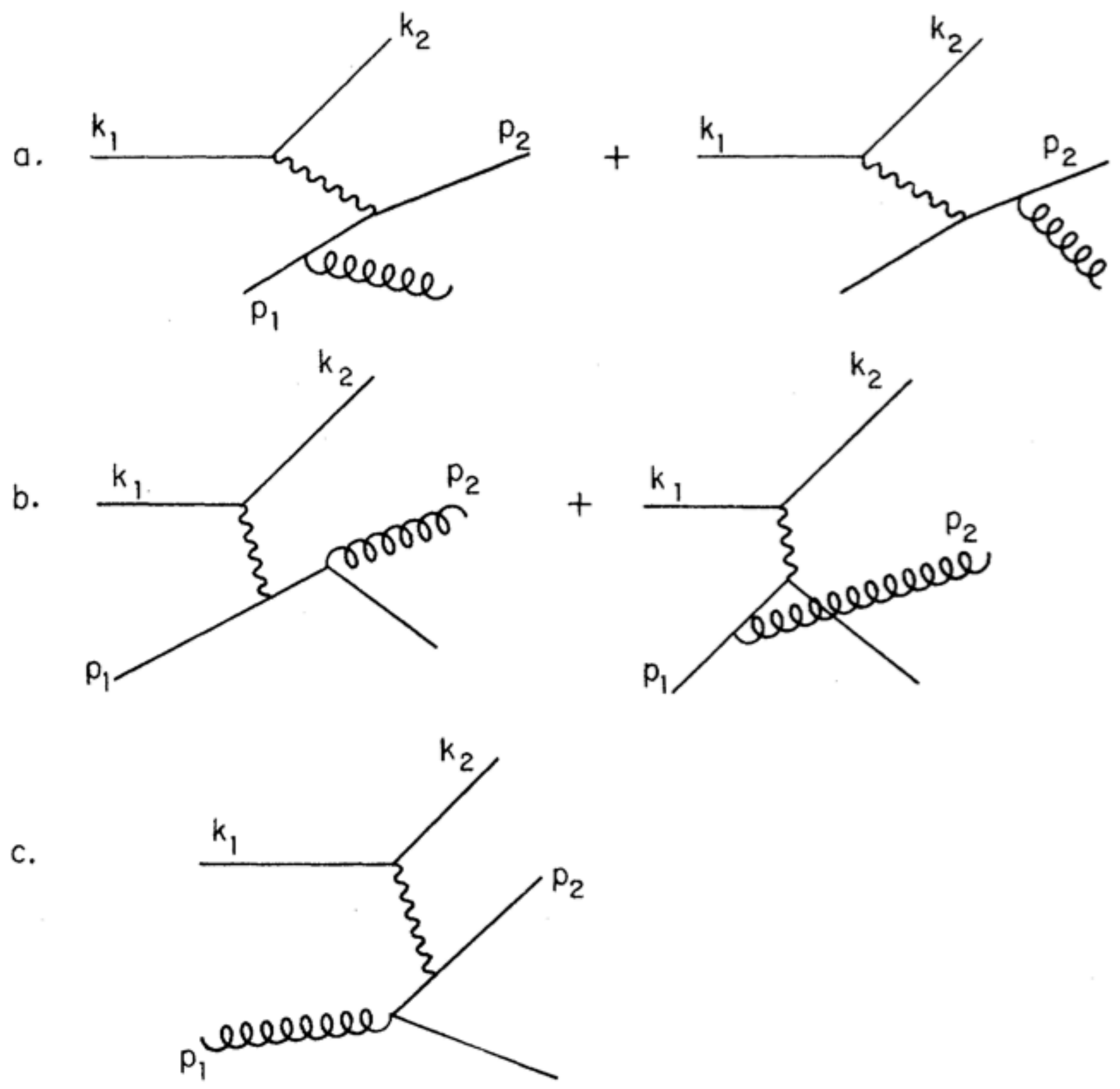


FIG. 1. Diagrams contributing to semi-inclusive μ-parton scattering to first order in α_s. k (p) denotes muon (parton) momentum. The wavy line is a virtual photon. The curly line is a gluon.

$$\langle \cos\varphi \rangle_{ep} = -\frac{\alpha_s}{2} z\sqrt{1-z}\,\frac{(2-y)\sqrt{1-y}}{1+(1-y)^2}$$

Cahn intrinsic k_T

Volume 78B, number 2,3 PHYSICS LETTERS 25 September 1978

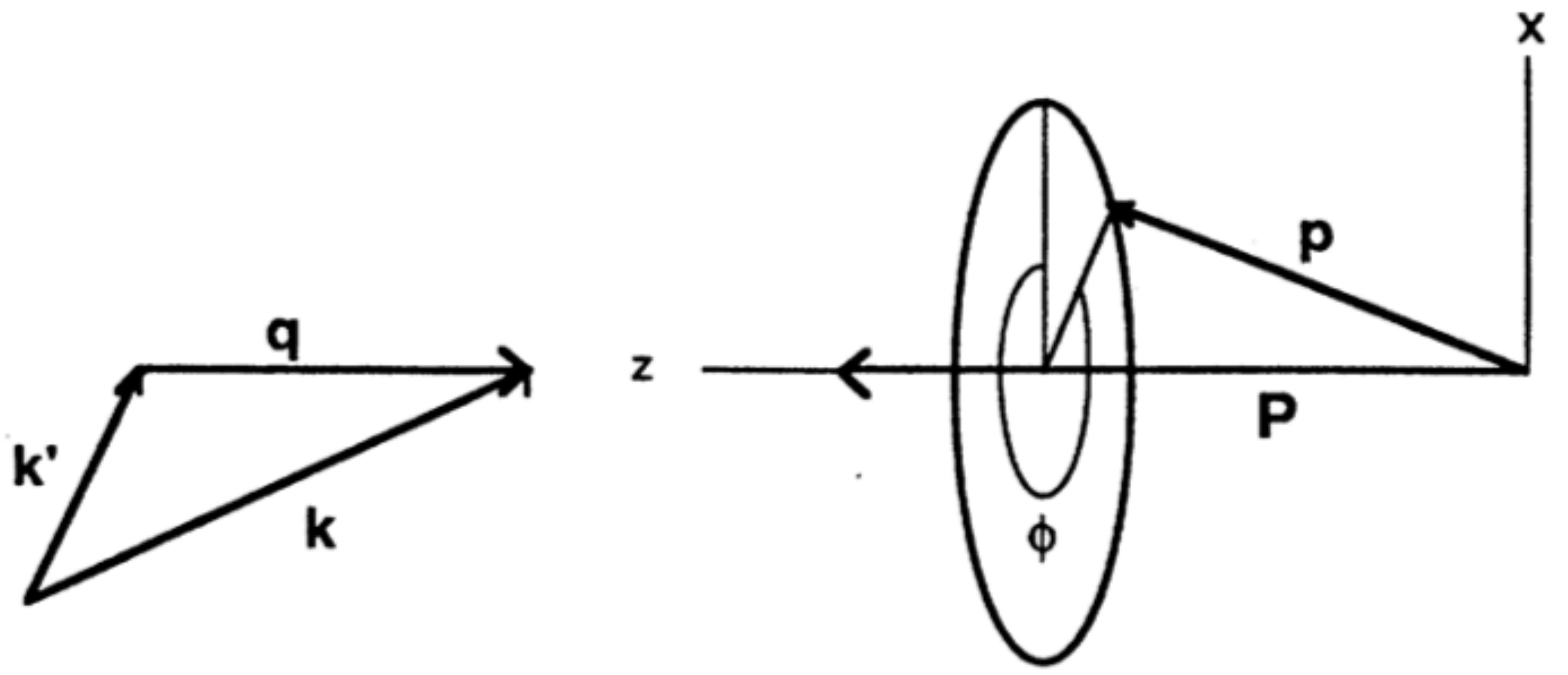
AZIMUTHAL DEPENDENCE IN LEPTOPRODUCTION: A SIMPLE PARTON MODEL CALCULATION☆

Robert N. CAHN
Department of Physics, University of Michigan, Ann Arbor, MI 48109, USA

Received 5 June 1978

parton model argument allowing for transverse momentum in Mandelstam variables...

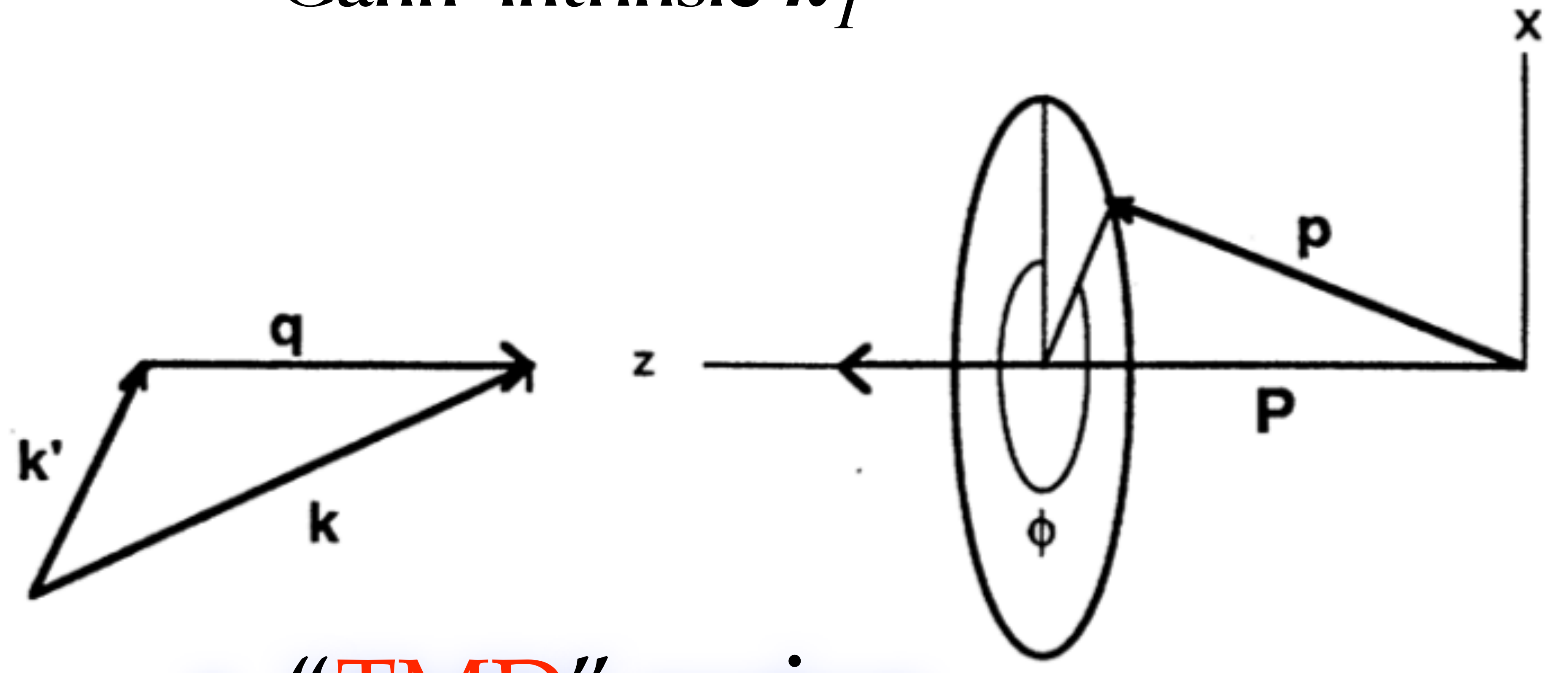
Semi-inclusive leptoproduction, $\ell + p \to \ell' + h + X$, is considered in the naive parton model. The scattered parton shows an azimuthal asymmetry about the momentum transfer direction. Simple derivations for the effects in ep, νp and $\bar{\nu}$p scattering are given. Reduction of the asymmetry due to fragmentation of partons into hadrons is estimated. The results cast doubt on the utility of such azimuthal asymmetry as a clean test of quantum chromodynamics.



$$\langle \cos\phi \rangle_{ep} = -\left[\frac{2p_\perp}{Q}\right]\frac{(2-y)\sqrt{1-y}}{1+(1-y)^2}$$

Two mechanisms ? Collinear Factorization

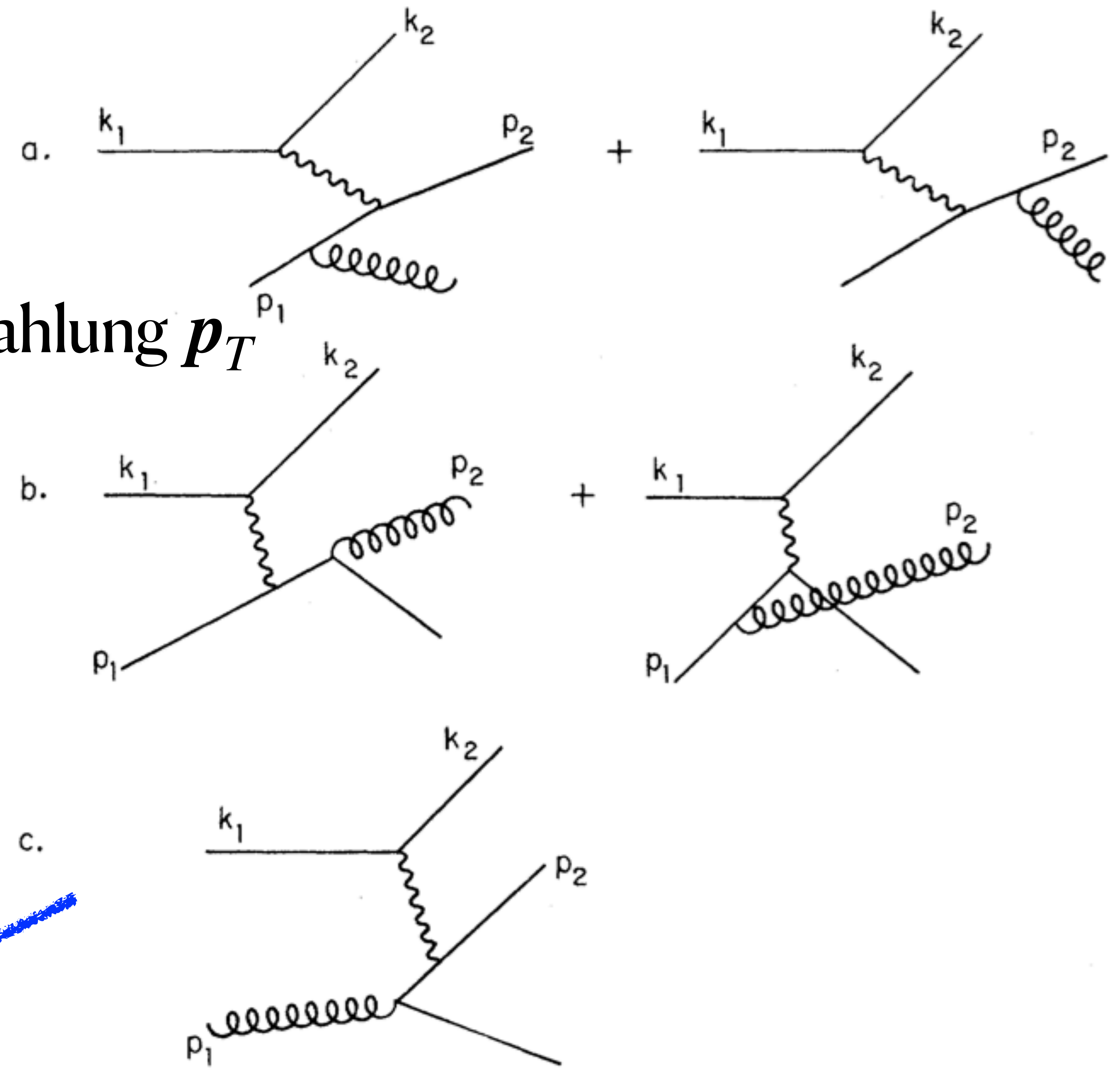
Cahn intrinsic $\boldsymbol{k}_T$



- "TMD" region

$(p_T \sim k_T) \sim q_T \ll Q$

Georgi & Politzer
hard gluon bremsstrahlung $\boldsymbol{p}_T$



$$\frac{d^5\sigma}{dx_{bj}\,dQ^2\,dz_f\,dq_T^2\,d\phi} = \frac{\alpha_e^2\alpha_s}{8\pi x_{bj}^2 S_{ep}^2 Q^2}\sum_k \mathcal{A}_k \int_{x_{\min}}^{1} \frac{dx}{x}\int_{z_f}^{1} \frac{dz}{z}\,[f\otimes D\otimes\hat{\sigma}_k \times \delta\left(\frac{q_T^2}{Q^2} - \left(\frac{1}{\hat{x}}-1\right)\left(\frac{1}{\hat{z}}-1\right)\right)$$

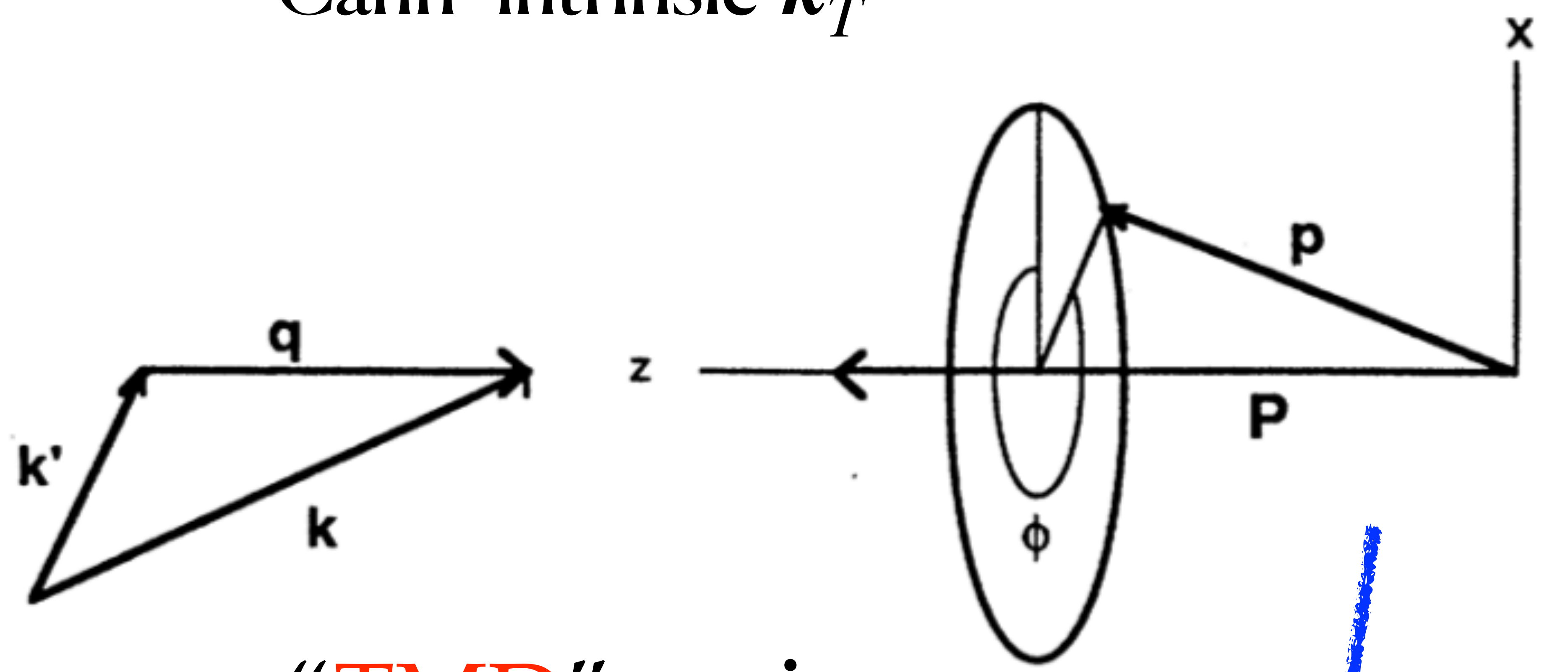
- "Collinear" region

$\Lambda_{qcd} \ll q_T \sim Q$

See e.g. Mendez NPB 1978 Koike, Vogelsang, Nagashima NPB 2006

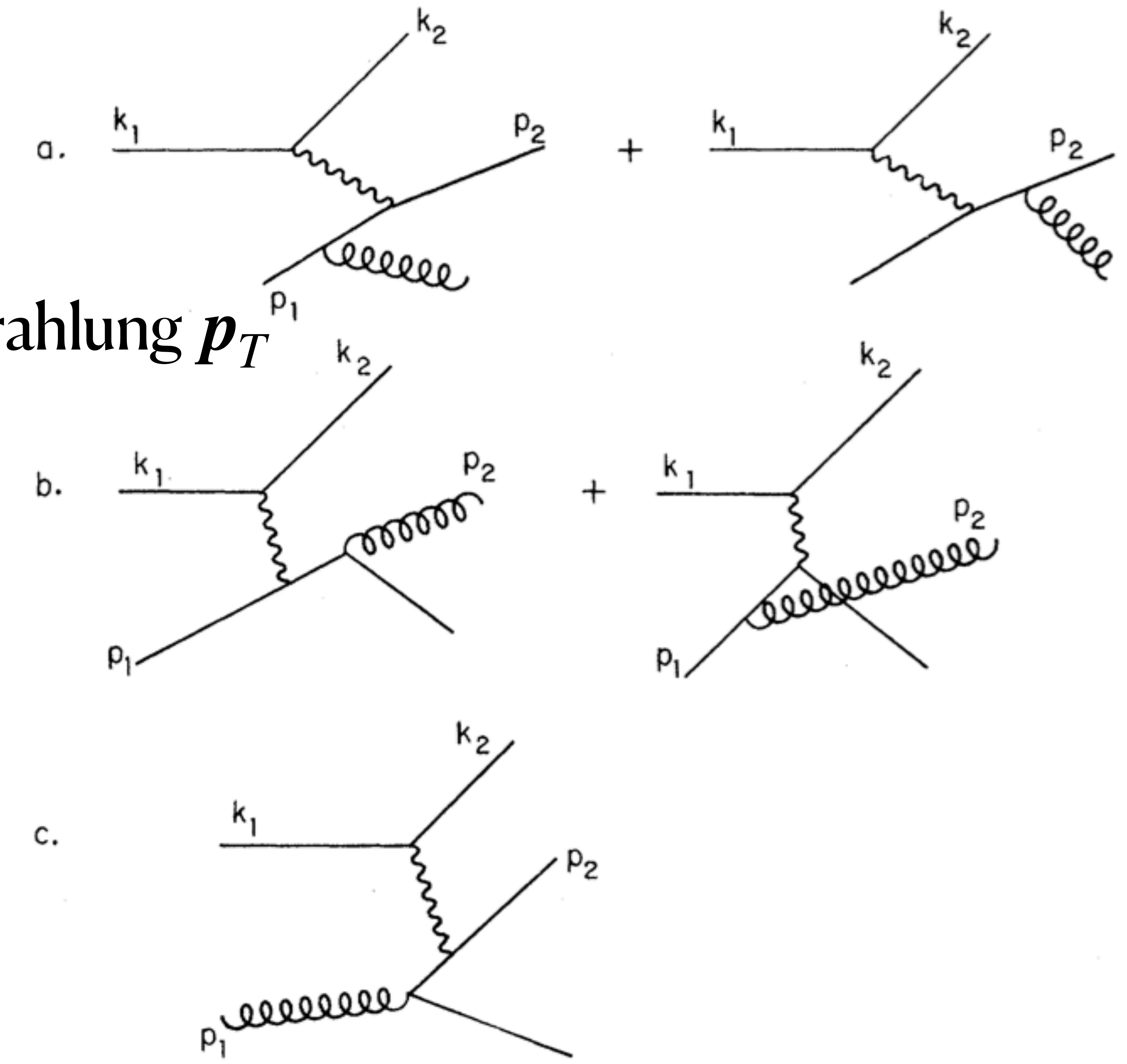
Two mechanisms ? TMD Factorization

Cahn intrinsic k_T



Georgi & Politzer

hard gluon bremsstrahlung p_T



- "TMD" region

$(p_T \sim k_T) \sim q_T \ll Q$

$$\frac{d\sigma}{dx\, dy\, d\psi\, dz\, d\phi_h\, dP_{h\perp}^2} =$$

$$\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}$$

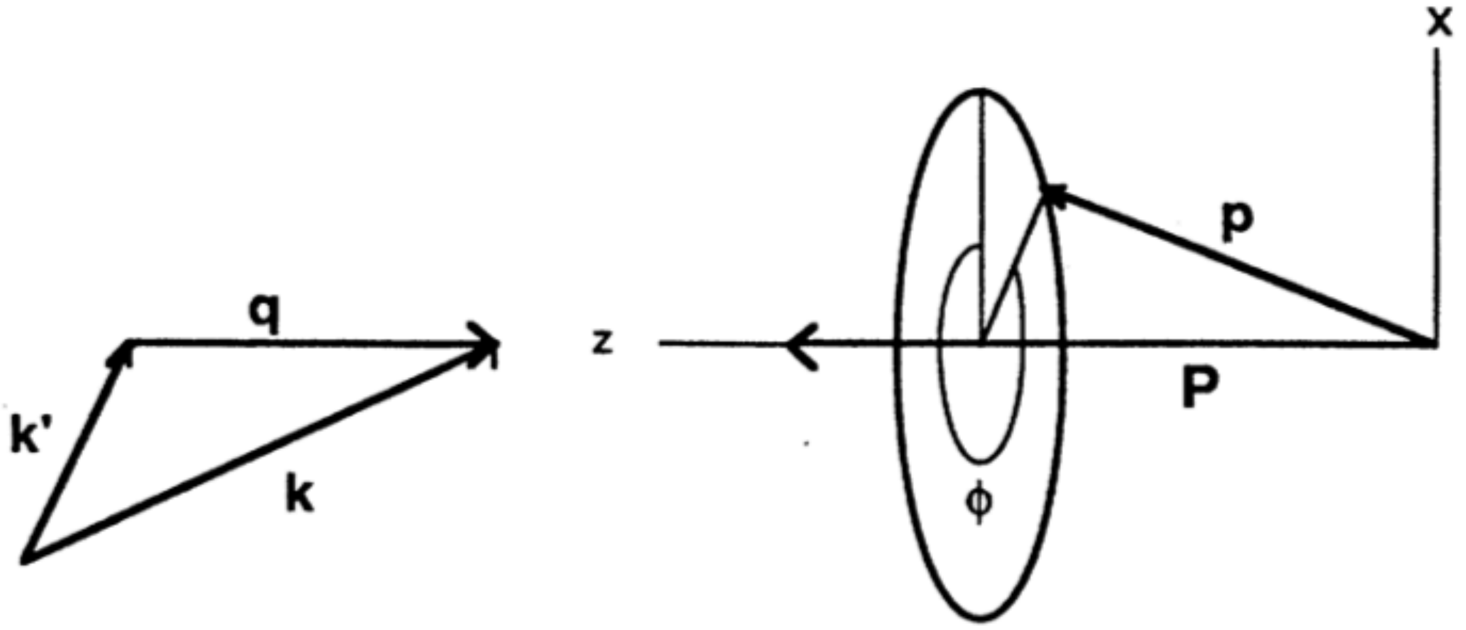
$$+\,\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}\Big\}$$

e.g. $F_{UU}^{\cos\phi_h} \approx \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\,f_1 D_1\Big]$

P.Mulders, R. Tangerman, NPB (1996), Bacchetta et al. JHEP 2007

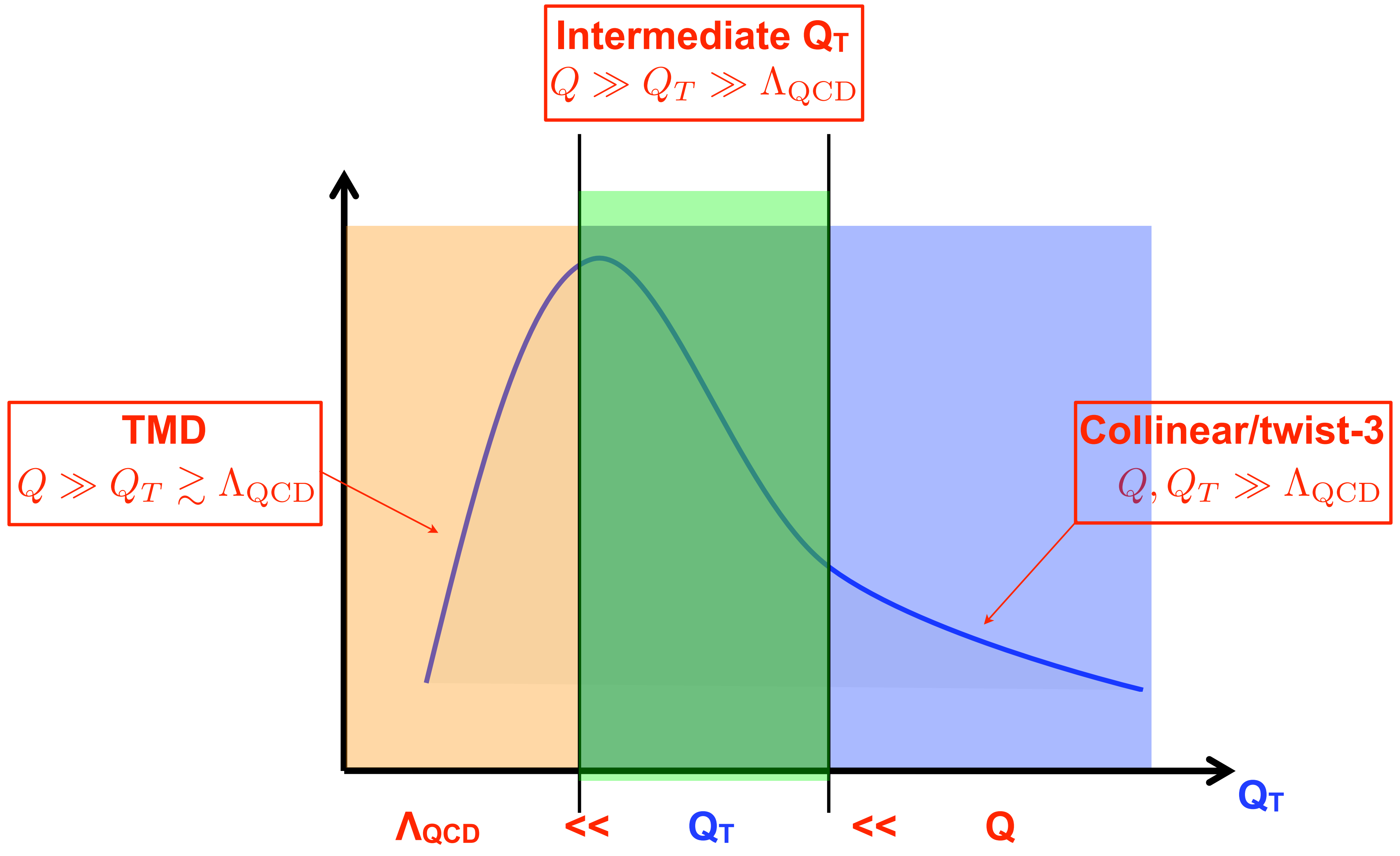
Matches Factorization @ sub-leading power

Cahn intrinsic k_T

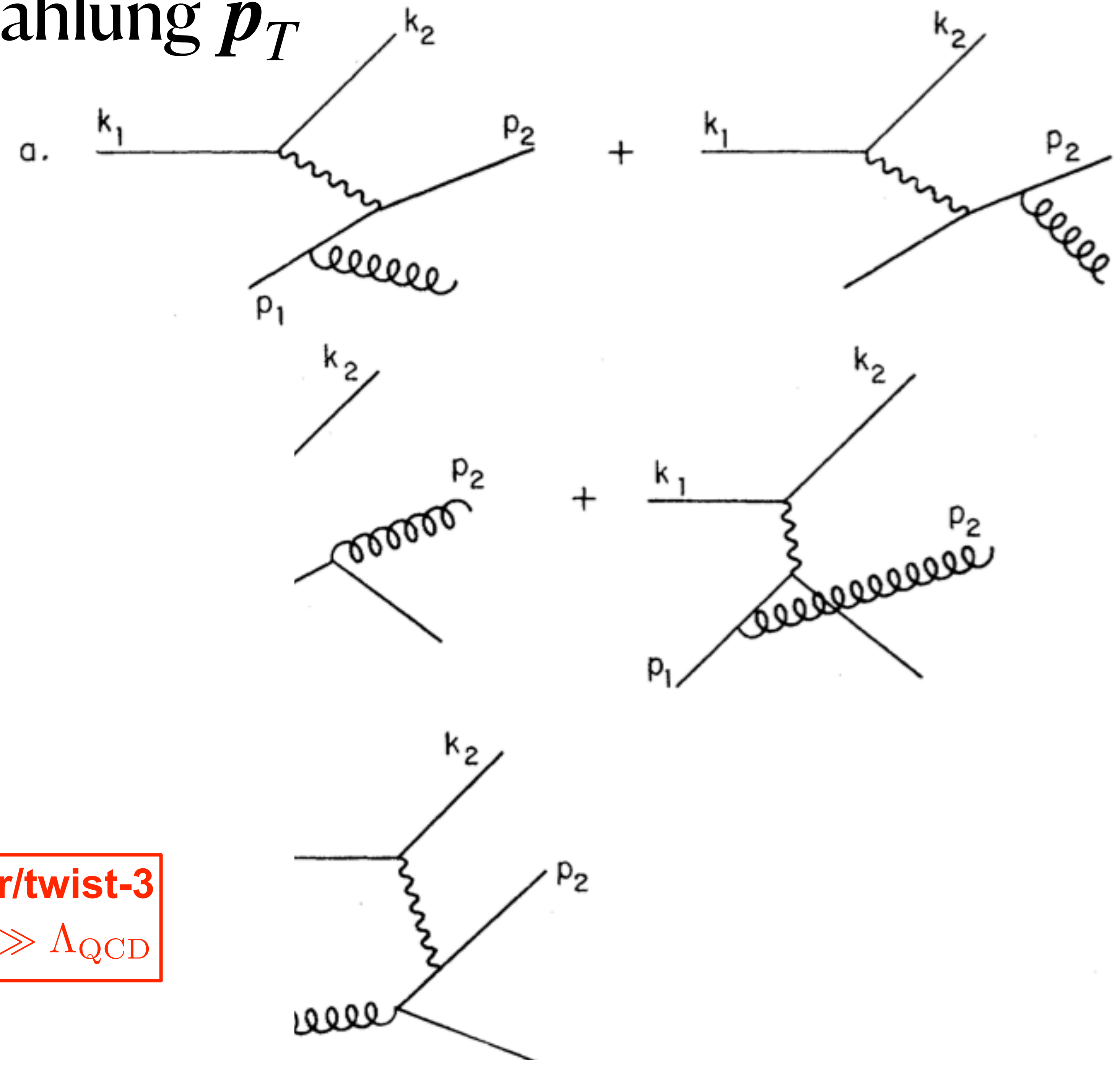


- "TMD" region

$(p_T \sim k_T) \sim q_T \ll Q$



Georgi & Politzer
hard gluon bremsstrahlung p_T



"Collinear" region

$\Lambda_{qcd} \ll q_T \sim Q$

A comprehensive study of matching the hi & low Q_T in the overlap region in SIDIS was carried out by JHEP (2008) Bacchetta et al.
where attention was given to azimuthal and polarization dependence

$$\frac{d\sigma^{ep\to ehX}}{d\phi} = \mathcal{A} + \mathcal{B}\cos\phi + \mathcal{C}\cos 2\phi + \mathcal{D}\sin\phi + \mathcal{E}\sin 2\phi$$

EMC collaboration Phys. Lett. B 130 (1983) 118, & Z. Phys. C 34 (1987) 277

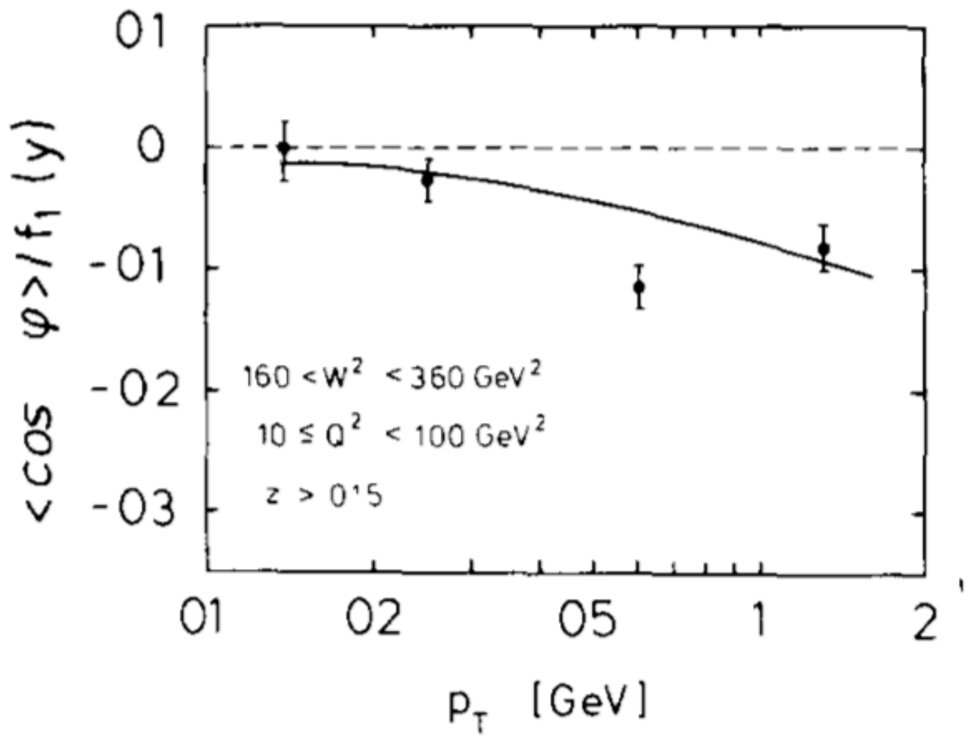
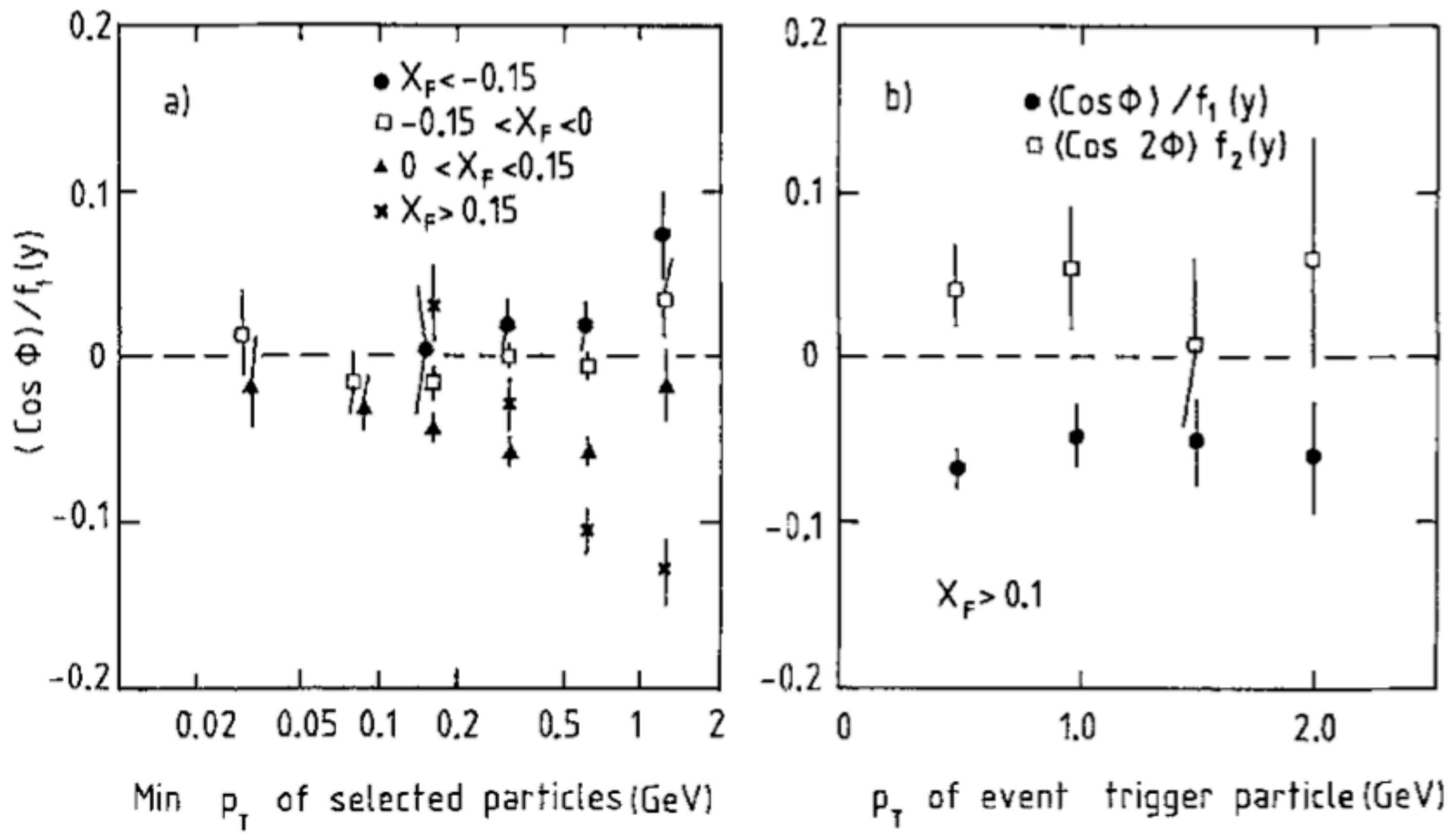
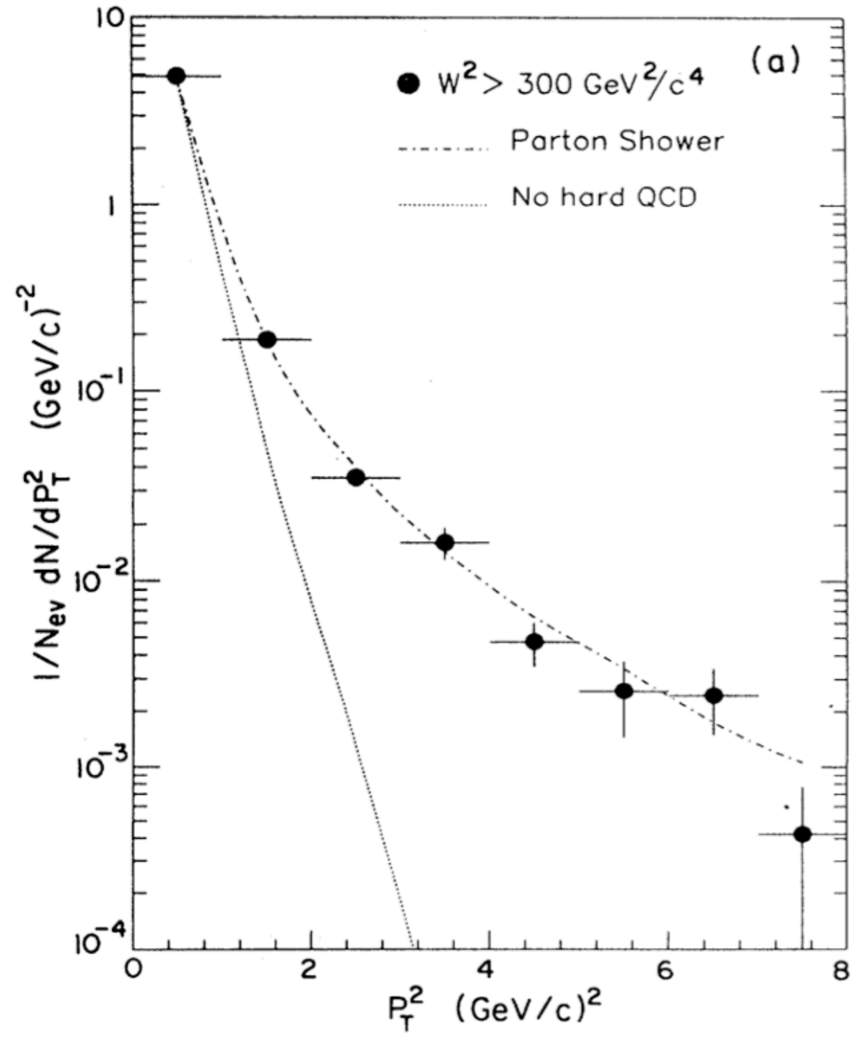


Fig 4 p_T dependence ($p_T > 50$ MeV) of cos φ moment for $160 \leqslant W^2 < 360$ GeV^2, $Q^2 > 10$ GeV^2 and $z > 0.15$ compared with model calculations described in ref [8] (statistical errors on model curve from Monte Carlo ±0 03 not shown)

M. Arneodo et al.: Measurement of Hadron Azimuthal Distributions

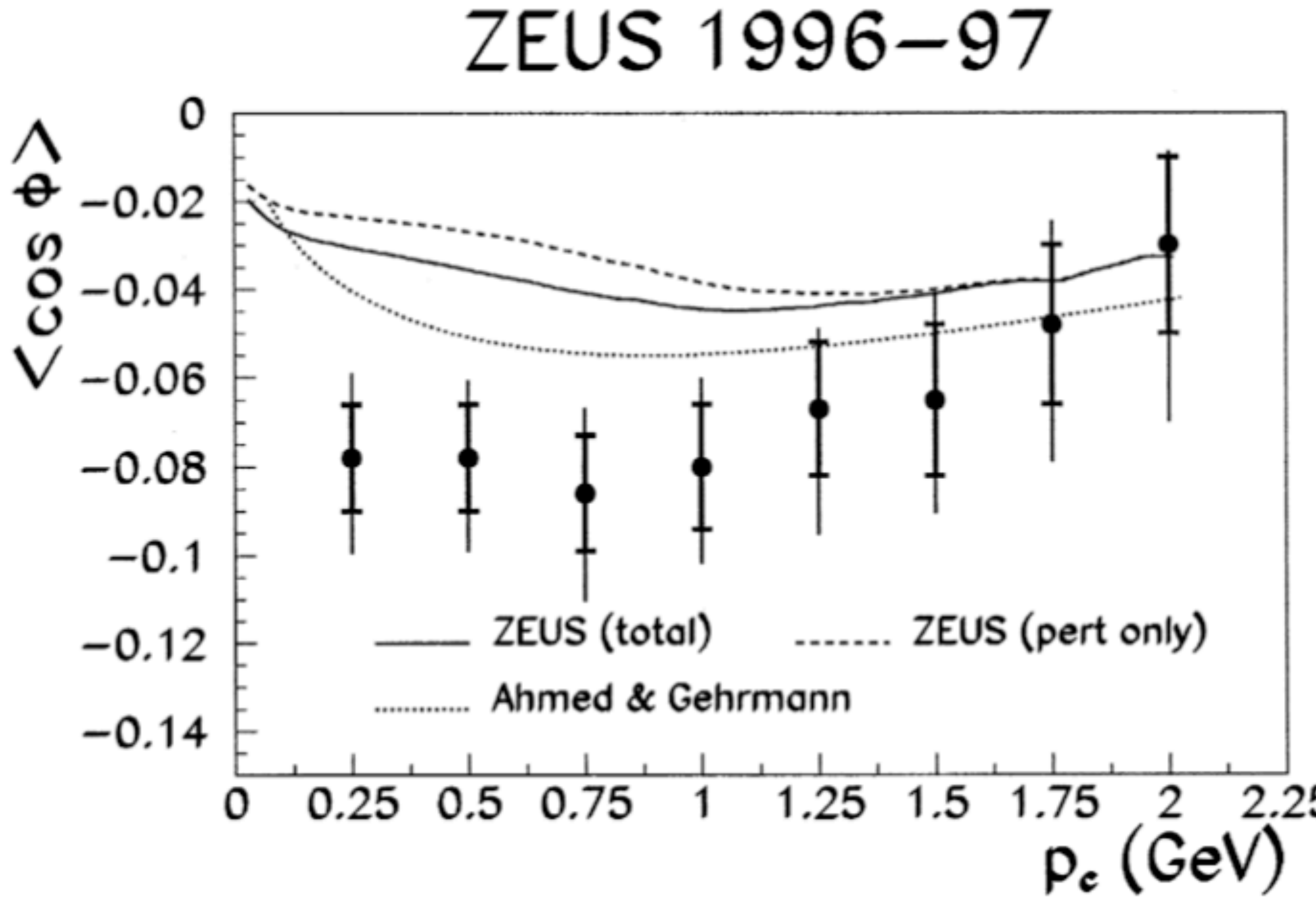


E665 Phys. Rev. D 48 (1993) 5057



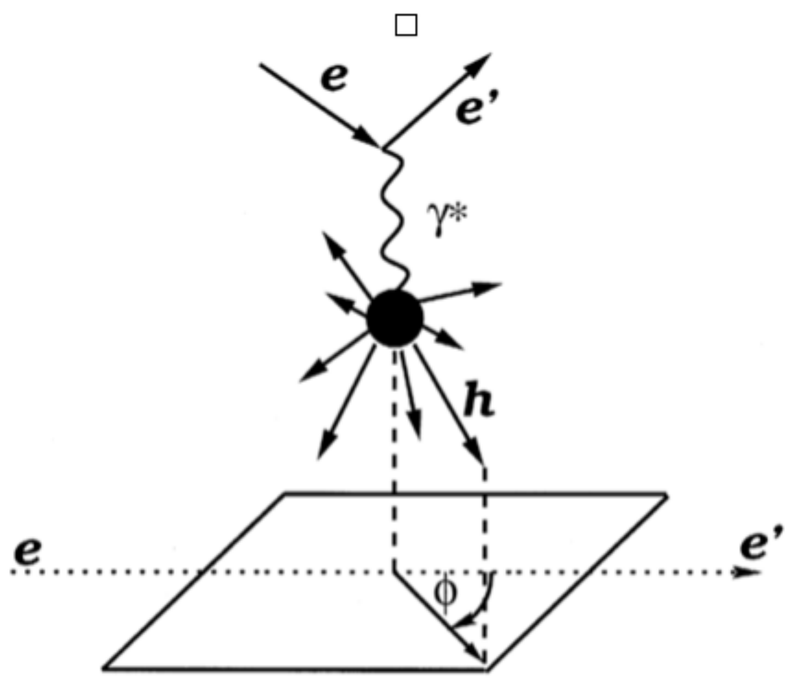
DATA

$\Lambda_{qcd} \ll q_T \sim Q$



Phys. Lett. B 481 (2000)

Fig. 4. The values of $\langle\cos\phi\rangle$ and $\langle\cos 2\phi\rangle$ are shown as a function of p_c in the kinematic region $0.01 < x < 0.1$ and $0.2 < y < 0.8$ for charged hadrons with $0.2 < z_h < 1.0$. The inner error bars represent the statistical errors, the outer are statistical and systematic errors added in quadrature. The lines are the LO predictions from ZEUS with perturbative and non-perturbative contributions (full line), ZEUS with the perturbative contribution only (dashed line) and Ahmed & Gehrmann (dotted line – see text



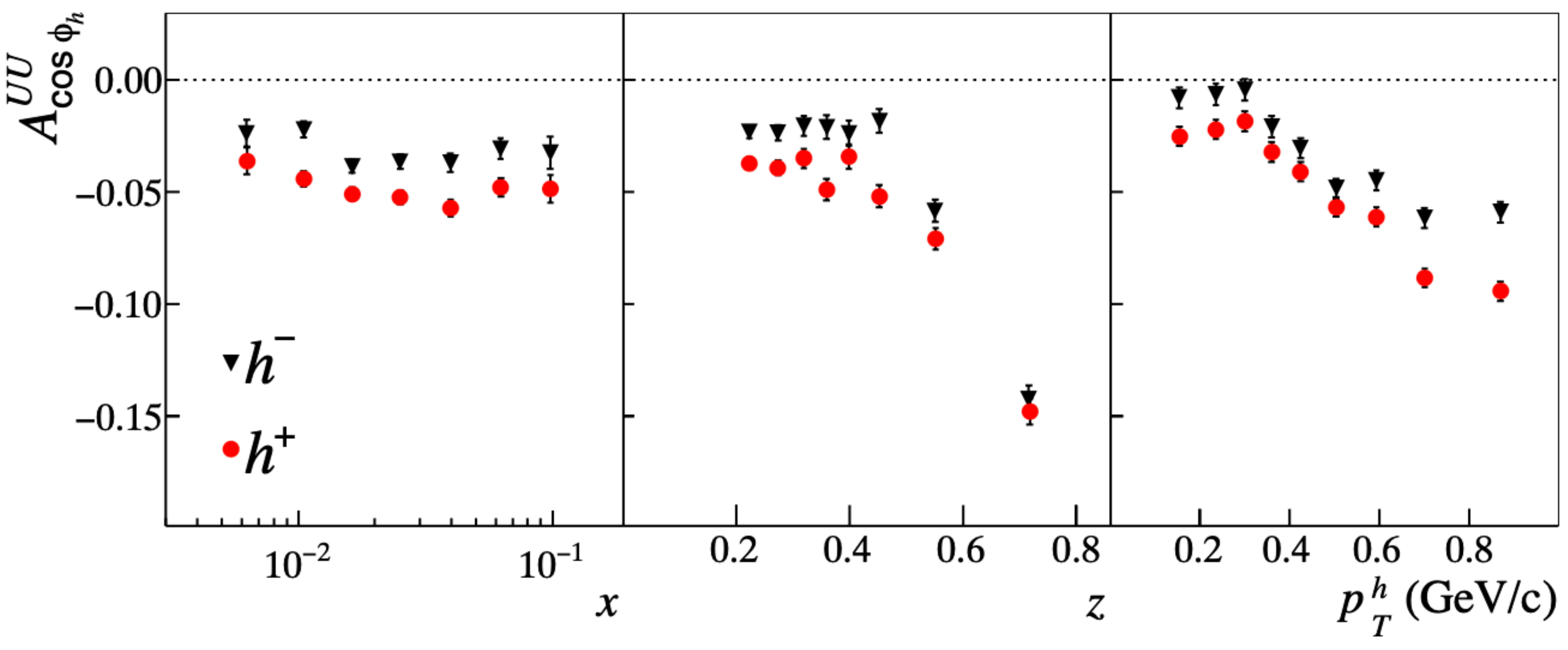
$$\frac{d\sigma^{ep\to ehX}}{d\phi} = \mathcal{A} + \mathcal{B}\cos\phi + \mathcal{C}\cos 2\phi + \mathcal{D}\sin\phi + \mathcal{E}\sin 2\phi$$

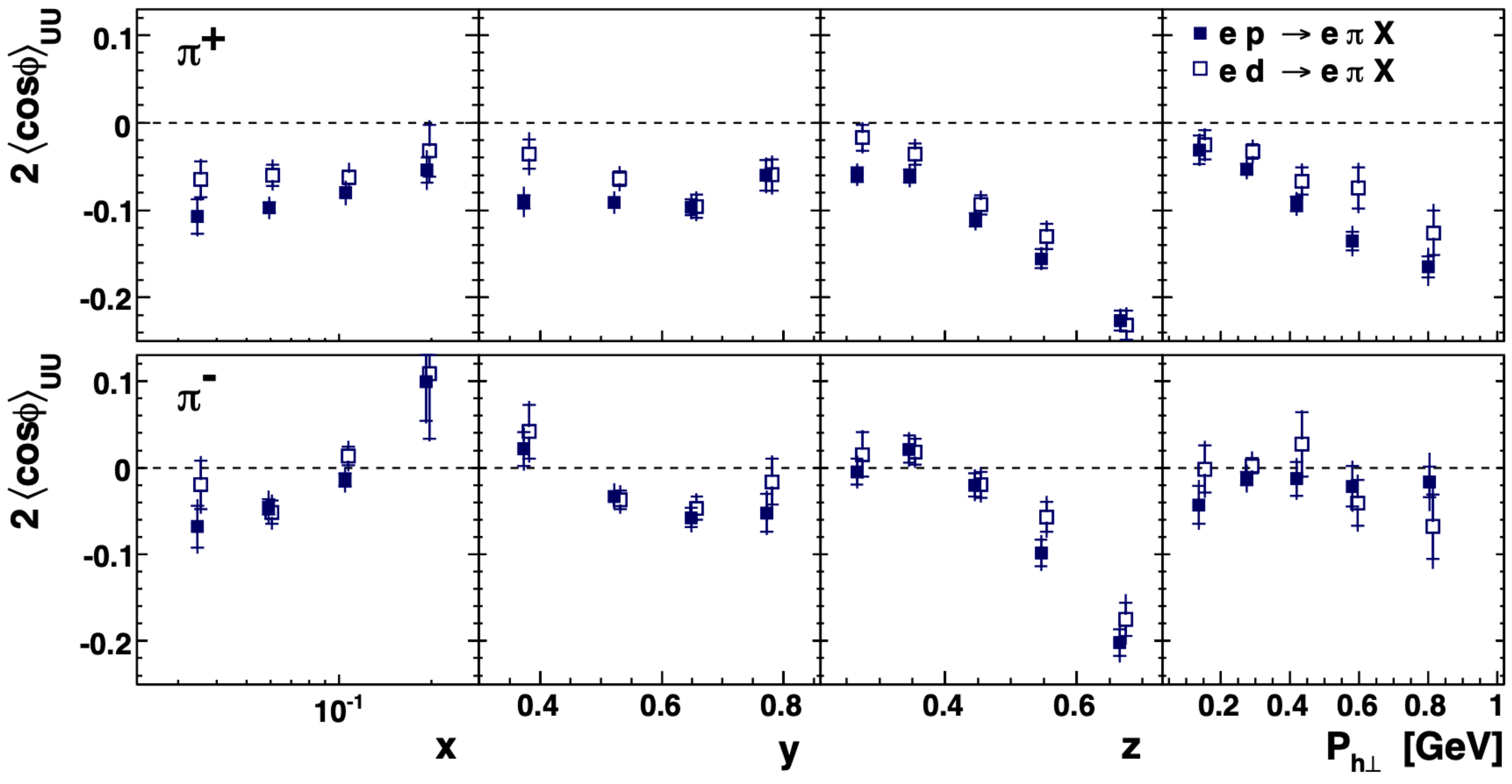
DATA

$(p_T \sim k_T) \sim q_T \ll Q$

HERMES, Phys. Rev. D 87 (2013) 012010

COMPASS, Nucl. Phys. B 886 (2014) 1046





Pheno studies

$$\langle \cos\phi \rangle = \frac{\int d\sigma^{(0)}\,\cos\phi + \int d\sigma^{(1)}\,\cos\phi}{\int d\sigma^{(0)} + \int d\sigma^{(1)}}$$

Chay, S.D. Ellis, Stirling, Phys. Lett. B (1991)

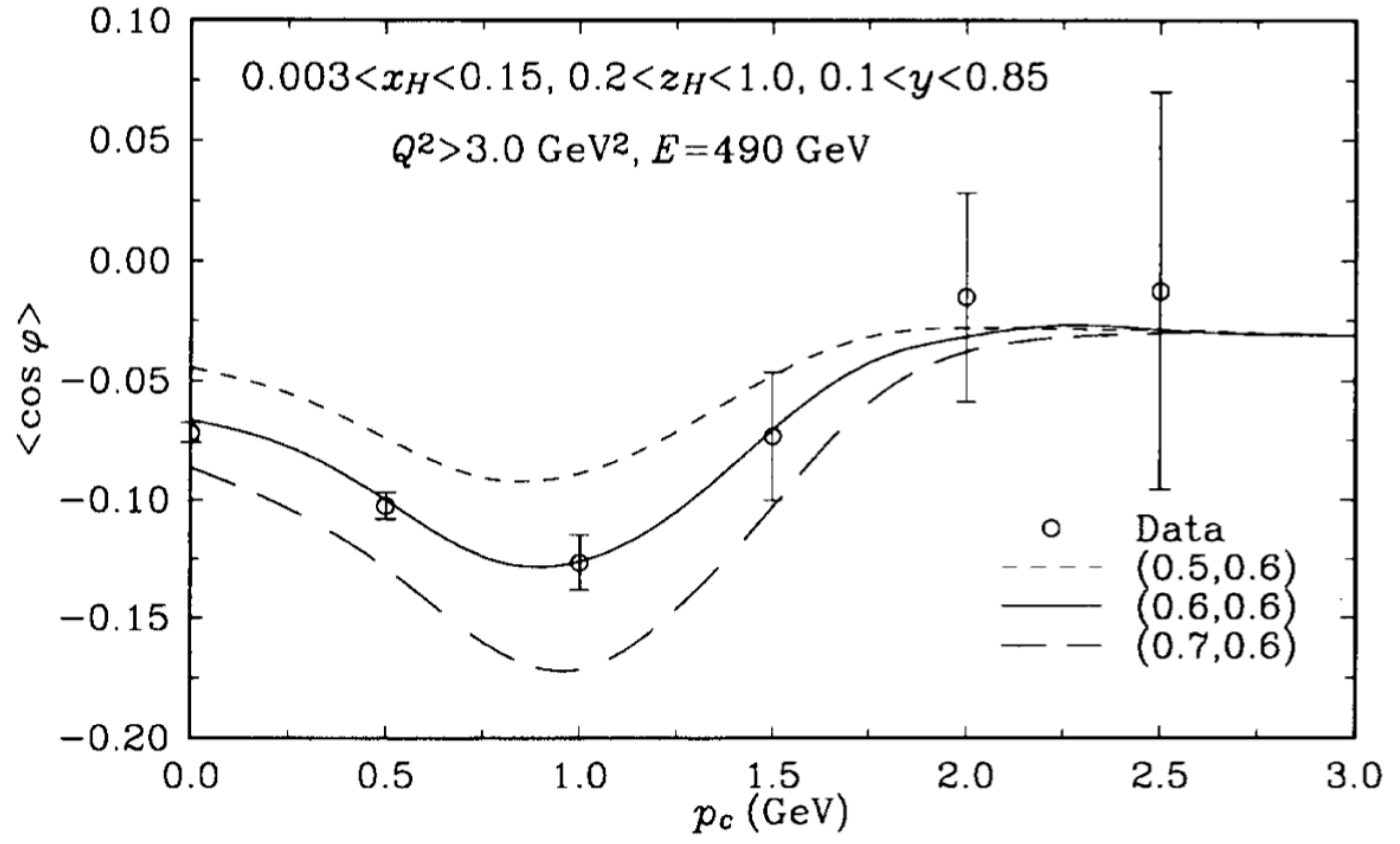
Oganessyan, Avakian, Bianchi, EPJC (1998)

$$\int d\sigma^{(0)} = 2\pi\frac{\alpha^2}{Q^2}\sum_j Q_j^2 F_j(x_H) D_j(z_H)\exp\left(-\frac{p_c^2}{b^2+z_H^2a^2}\right)$$
$$\times\left\{\frac{1+(1-y)^2}{y} + 4\frac{1-y}{yQ^2}\left[\frac{a^2b^2}{b^2+z_H^2a^2} + \left(\frac{z_H a^2}{b^2+z_H^2a^2}\right)^2 (p_c^2+b^2+z_H^2a^2)\right]\right\}$$

$$\int d\sigma^{(1)}\,\cos\phi = \int d^2P_T\,\cos\phi\frac{d\sigma}{dx_H\,dy\,dz_H\,d^2P_T}$$
$$= \frac{8}{3}\frac{\alpha_s\alpha^2}{Q^2}\frac{(2-y)\sqrt{1-y}}{y}\int_{x_H}^1\frac{dx}{x}\int_{z_H}^1\frac{dz}{z}\sum_j Q_j^2(A_j+B_j+C_j)$$

$$A_j = -\sqrt{\frac{xz}{(1-x)(1-z)}}\,[xz+(1-x)(1-z)]\,F_j\left(\frac{x_H}{x},Q^2\right)D_j\left(\frac{z_H}{z},Q^2\right)$$

Simple addition … “double counting”

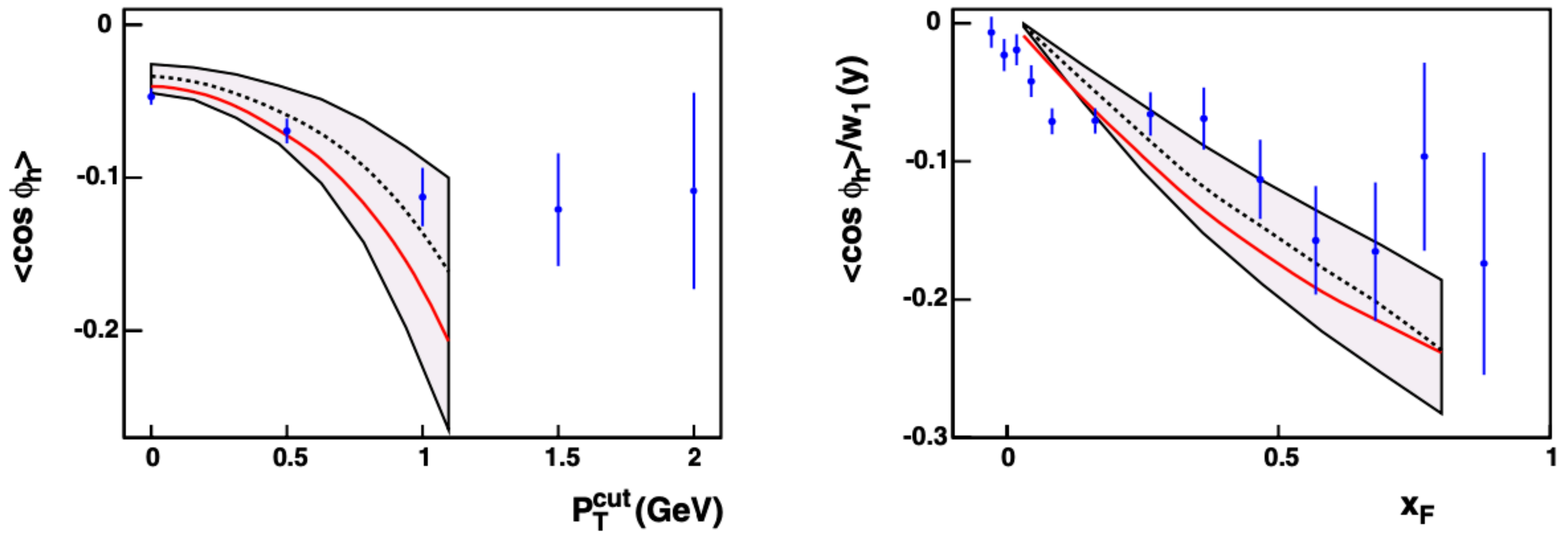


$\langle\cos\phi\rangle$ as a function of transverse momentum cutoff non-perturbative Cahn-like effect negligible at large values of p_c —- intrinsic transverse momentum in distribution and fragmentation functions are too small to produce $P_T > p_c$ (data E665 Fermi-lab).

One of the "first" TMD analysis
Role of Cahn effect in SIDIS from TMD framework
Modeling tree level result comparing w/ E665 data

Anselmino, Boglione, D'Alesio, Kotzinian, Murgia, Prokudin

PHYSICAL REVIEW D **71,** 074006 (2005)



$$F_{UU}^{\cos\phi_h} \approx \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\,f_1 D_1\right].$$

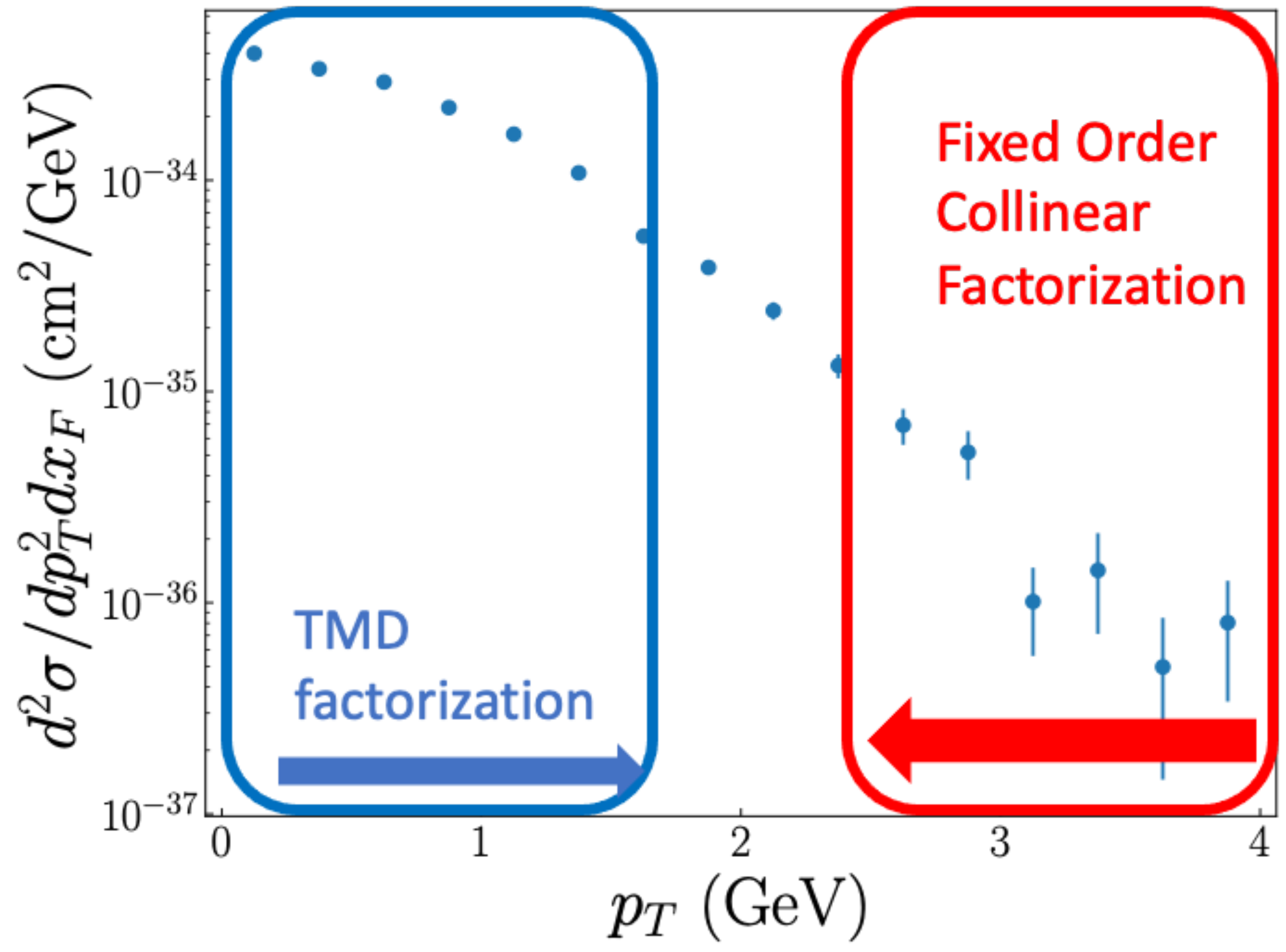
Wandzura Wilzeck approx from TMD fact—
Bacchetta et al. JHEP 2007

$$\frac{d^5\sigma^{\ell p\to\ell hX}}{dx_B dQ^2 dz_h d^2\boldsymbol{P}_T} \simeq \sum_q \frac{2\pi\alpha^2 e_q^2}{Q^4} f_q(x_B) D_q^h(z_h)\left[1+(1-y)^2 - 4\frac{(2-y)\sqrt{1-y}\langle k_\perp^2\rangle z_h P_T}{\langle P_T^2\rangle Q}\cos\phi_h\right]\frac{1}{\pi\langle P_T^2\rangle} e^{-P_T^2/\langle P_T^2\rangle},$$

Regions and matching

NPB Collins & Soper(1982), & Sterman 1985

Requires systematic factorization approach



- Goal to use p_T (q_T) data over full range &
- simultaneous fit of pdfs & TMDs

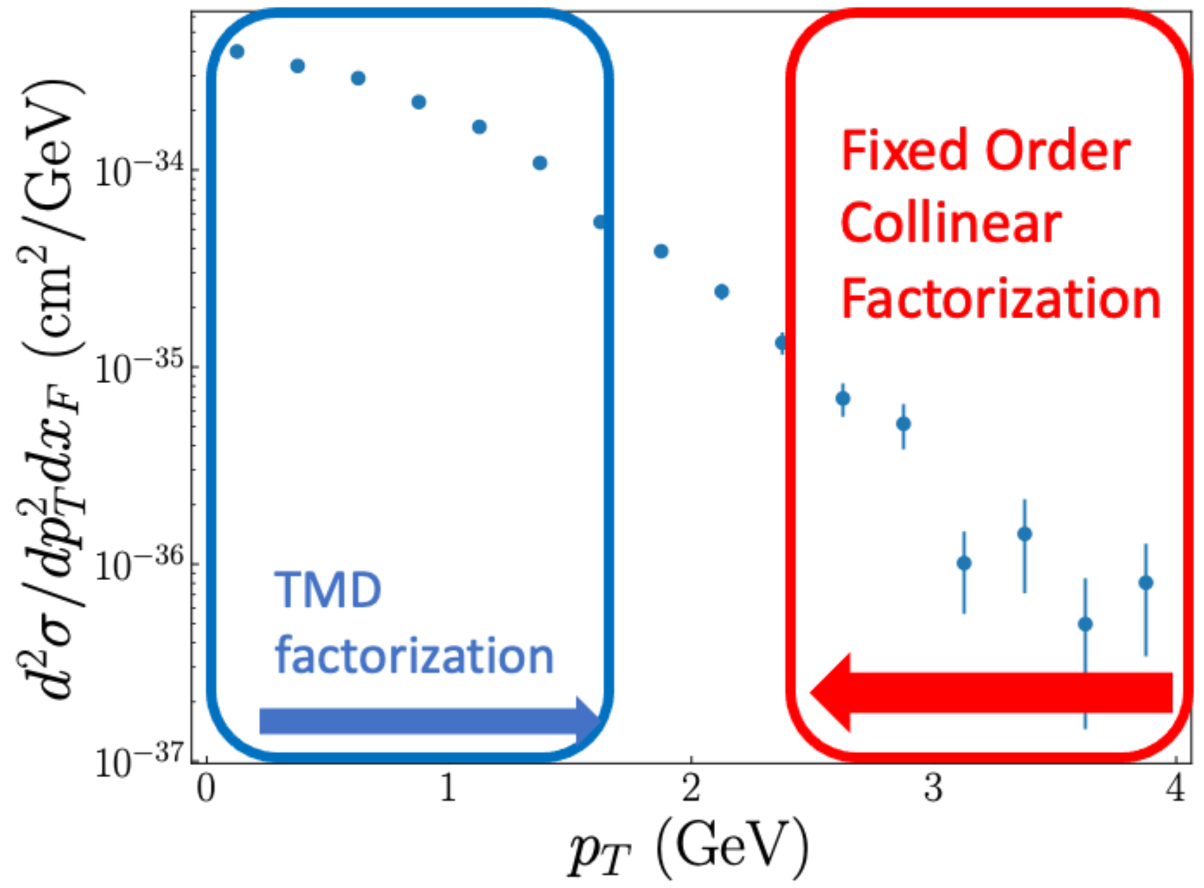
- **Cross section different "regions"-"two scales"**

- W valid for $\Lambda_{QCD} \sim p_T \ll Q$ TMD factorization
- FO valid for $\Lambda_{QCD} \ll p_T \sim Q$ Collinear factorization

$$\frac{d\sigma(m \lesssim q_T \lesssim Q, Q)}{dy dq^2 dp_T^2} = \left.\frac{d\sigma^W(q_T, Q)}{dy dq^2 dp_T^2}\right|_{m \lesssim q_T \ll Q} + \left.\frac{d\sigma^{FO}(q_T, Q)}{dy dq^2 dp_T^2}\right|_{m \ll q_T \lesssim Q} - \left.\frac{d\sigma^{ASY}(q_T, Q)}{dy dq^2 dp_T^2}\right|_{m \lesssim q_T \ll Q}$$

$$\equiv W(p_T, Q) + FO(p_T, Q) - AY(p_T, Q) + O\left(\frac{m}{Q}\right)^c$$

"mis"-Matches Factorization @ sub-leading power



$$\frac{d\sigma(m \lesssim q_T \lesssim Q, Q)}{dy\,dq^2\,dp_T^2} = W(p_T, Q) + FO(p_T, Q) - AY(p_T, Q) + O\left(\frac{m}{Q}\right)^c$$

$$\langle \cos\phi \rangle = \frac{\int d\sigma^{(1)} \cos\phi}{\int d\sigma^{(1)}}$$

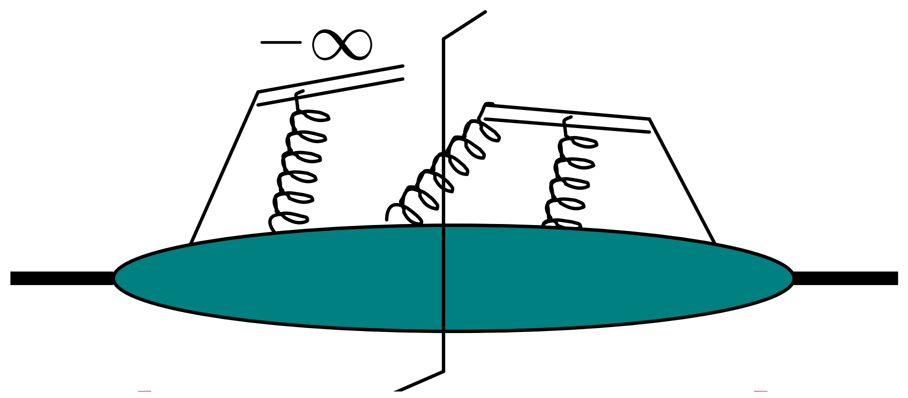
- Bacchetta, Boer, Diehl, Mulders JHEP (2008)
  mis-match/inconsistency breakdown of factorization at NLP?

- Bacchetta Bozzi, Echevarria, Pisano, Prokudin, Radici, PLB (2019)
  "… the requirement to match the high-q_T result (4.25) for $F_{UU}^{\cos\phi_h}$ at intermediate q_T can be used as a consistency check for any framework that extends Collins-Soper factorization to the twist-three sector."

“mis”-Matches Factorization @ sub-leading power

$$\langle \cos\phi \rangle = \frac{\int d\sigma^{(1)} \cos\phi}{\int d\sigma^{(1)}}$$

Solution Bacchetta et al. is to introduce soft factor subtraction on TMD at leading power as in Collins 2011. Assume same as @LP



$$\tilde{f}^{\rm sub}_{j/H}(x, b_T; \mu, y_n) = \lim_{\substack{y_A \to +\infty \\ y_B \to -\infty}} \underbrace{\tilde{f}^{\rm unsub}_{j/H}(x, b_T; \mu, y_P - y_B)} \sqrt{\frac{\tilde{S}(b_T; y_A, y_n)}{\tilde{S}(b_T; y_A, y_B)\tilde{S}(b_T; y_n, y_B)}} \times UV_{renorm}$$

$$\Updownarrow$$

$$\tilde{f}^{\rm unsub}_{j/H}(x, b_T; \mu, y_P - y_B) = \int \frac{db^-}{2\pi} e^{-ixP^+b^-} \langle P|\bar{\psi}(0)\gamma^+ \mathcal{U}_{[0,b]}\psi(b)|P\rangle|_{b^+=0}$$

JCC Soft factor further “repartitioned”
This is done to

1) cancel LC divergences in “unsubtracted” TMDs
2) separate “right & left” movers i.e. full factorization
3) remove double counting of momentum regions

To understand appreciate the subtleties review
Tree level TMD & LP factorization

In reviewing will remind you about the utility of using
the good and bad LC quark fields

Then onto Factorization at NLO

See John Terry's talk...

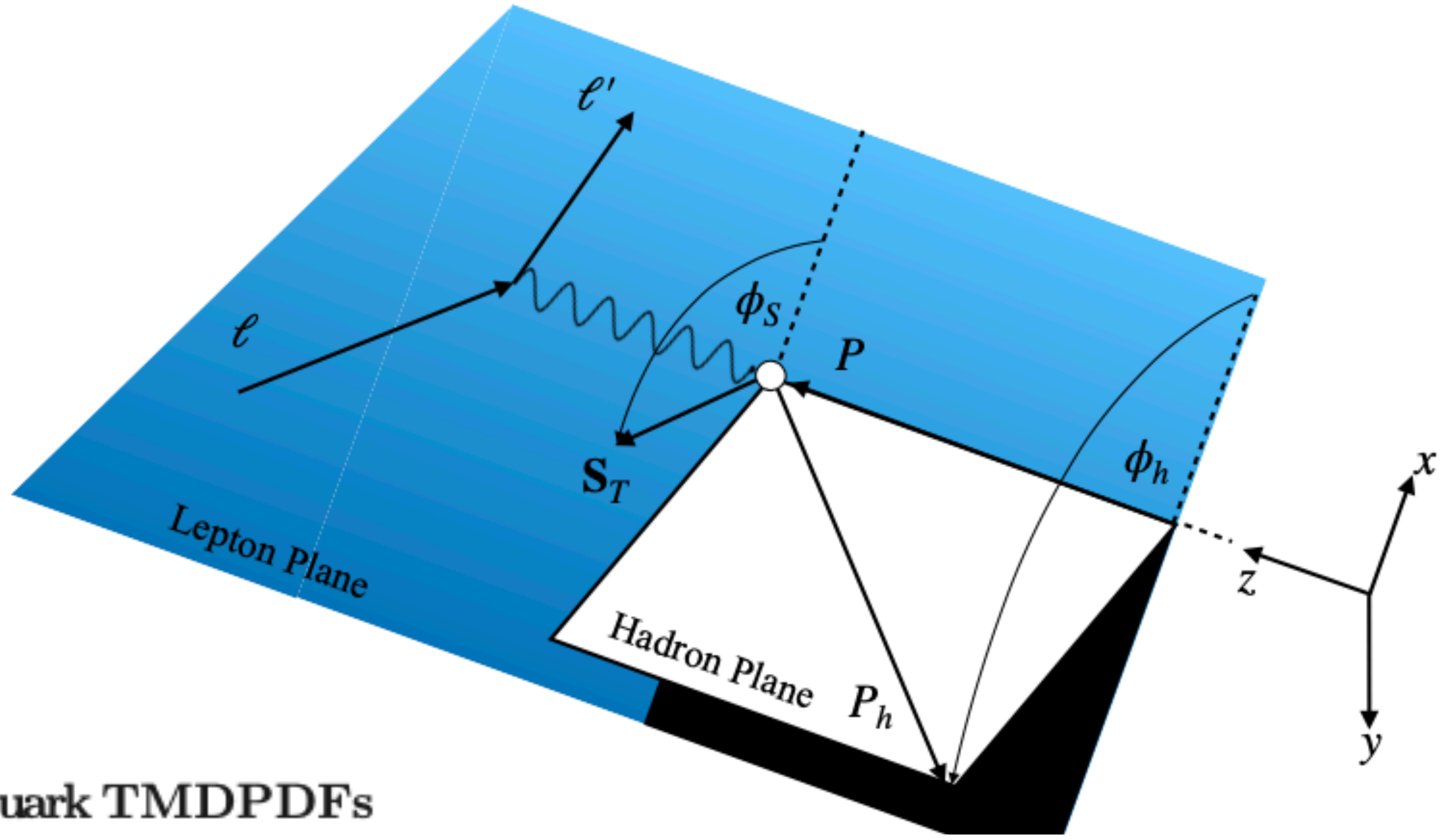
Factorization at sub-leading power … revisit Tree level

- "TMD" region $(p_T \sim k_T) \sim q_T \ll Q$

- Consider factorization beyond LO and LP
  via Ji Ma Yuan 2004,Collins, Aybat & Rogers 2011
- To do this at sub-leading power—revisited tree level build RG consistency
- Develop RG and rapidity renormalization group (CS equation)

arXiv: e-Print:221.13209

- Consider SIDIS cross section in the hadronic Breit frame



$$\frac{d\sigma}{dx\, dy\, d\Psi\, dz\, d^2 P_{h\perp}} = \kappa \frac{\alpha_{\rm em}^2}{4Q^4} \frac{y}{z} L_{\mu\nu} W^{\mu\nu}$$

Subleading Quark TMDPDFs

		Quark Chirality	
		Chiral Even	Chiral Odd
Nucleon Polarization	U	$f^\perp, g^\perp$	$e\,, h$
	L	$f_L^\perp, g_L^\perp$	e_L, h_L
	T	$f_T, f_T^\perp, g_T, g_T^\perp$	$e_T, e_T^\perp, h_T, h_T^\perp$

Subleading Quark-Gluon-Quark TMDPDFs

		Quark Chirality	
		Chiral Even	Chiral Odd
Nucleon Polarization	U	$\tilde{f}^\perp, \tilde{g}^\perp$	$\tilde{e}\,, \tilde{h}$
	L	$\tilde{f}_L^\perp, \tilde{g}_L^\perp$	$\tilde{e}_L, \tilde{h}_L$
	T	$\tilde{f}_T, \tilde{f}_T^\perp, \tilde{g}_T, \tilde{g}_T^\perp$	$\tilde{e}_T, \tilde{e}_T^\perp, \tilde{h}_T, \tilde{h}_T^\perp$

Subleading fields and correlator(s) Summary

Three possible sub-leading field configurations. They are related through the QCD EOM

$$\varphi^c(x) = \frac{\not{n}\bar{\not{n}}}{4}\psi^c(x) \qquad \chi^c(x) = \frac{\bar{\not{n}}\not{n}}{4}\psi^c(x) \qquad \varphi^c(x) = -\frac{\not{n}}{2}\frac{\not{D}_\perp}{n\cdot D}\chi^c(x)$$

Using properties of the Wilson lines, the relevant collinear functions are given by

$$\Phi^{\text{int}}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) = \int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta\left(\xi^+\right)\left[\langle P,\mathbf{S}\,|\bar{\chi}^c_{j'}(0)\,\mathcal{U}^{\bar{n}}_{\llcorner}(0)\,\mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\xi)\,\varphi^c_j(\xi)|\,P,\mathbf{S}\rangle + \text{h.c.}\right]$$

$$\Phi^{\text{dyn}}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) = \frac{ig}{k^+}\int d\eta^-\int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta\left(\xi^+\right)$$
$$\times\left[\left\langle P,\mathbf{S}\left|\bar{\chi}^c(0)\,\mathcal{U}^{\bar{n}}_{\llcorner}(0)\,\Gamma^a\,\mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\eta)\,F^{i+}(\eta)\,\mathcal{U}^{\bar{n}}\left(\eta^-,\xi^-;\xi^+,\xi_\perp\right)\,\gamma_i\frac{\not{n}}{2}\chi^c(\xi)\right|P,\mathbf{S}\right\rangle + \text{h.c.}\right]$$

$$\Phi^{\text{kin}}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) = \int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta\left(\xi^+\right)\left[\left\langle P,\mathbf{S}\left|\bar{\chi}^c_{j'}(0)\,\mathcal{U}^{\bar{n}}_{\llcorner}(0)\,\frac{i\partial^i_\perp}{in\cdot D}\,\mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\xi)\,\frac{\bar{\not{n}}}{2}\gamma^\perp_i\chi^c_{\text{kin}\,j}(\xi)\right|P,\mathbf{S}\right\rangle + \text{h.c.}\right]$$

All three distributions are not required to span the NLP cross section due to EOM

$$\Phi^{\text{int}}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) = \Phi^{\text{kin}}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) + \Phi^{\text{dyn}}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S})$$

[8] Gamberg, Kang, Shao, Terry, Zhao 2022

Subleading fields and correlator(s) Alternative

Three possible sub-leading field configurations. They are related through the QCD EOM

$$\varphi^c(x) = \frac{\not{n}\not{\bar{n}}}{4}\psi^c(x) \qquad \chi^c(x) = \frac{\not{\bar{n}}\not{n}}{4}\psi^c(x) \qquad \varphi^c(x) = -\frac{\not{n}}{2}\frac{\not{D}_\perp}{n\cdot D}\chi^c(x)$$

Using properties of the Wilson lines, the relevant collinear functions are given by

$$\Phi^{\rm int}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) = \int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta\left(\xi^+\right)\left[\langle P,\mathbf{S}\left|\bar{\chi}^c_{j'}(0)\,\mathcal{U}^{\bar{n}}_{\llcorner}(0)\,\mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\xi)\,\varphi^c_j(\xi)\right|P,\mathbf{S}\rangle + \text{h.c.}\right]$$

$$\Phi^{\rm dyn}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) = \frac{ig}{k^+}\int d\eta^-\int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta\left(\xi^+\right)$$

$$\times\left[\left\langle P,\mathbf{S}\left|\bar{\chi}^c(0)\,\mathcal{U}^{\bar{n}}_{\llcorner}(0)\,\Gamma^a\,\mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\eta)\,F^{i+}(\eta)\,\mathcal{U}^{\bar{n}}\left(\eta^-,\xi^-;\xi^+,\xi_\perp\right)\,\gamma_i\frac{\not{n}}{2}\chi^c(\xi)\right|P,\mathbf{S}\right\rangle + \text{h.c.}\right]$$

$$\Phi^{\rm kin}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) = \int \frac{d^4\xi}{(2\pi)^3}\, e^{ik\cdot\xi}\,\delta\left(\xi^+\right)\left[\left\langle P,\mathbf{S}\left|\bar{\chi}^c_{j'}(0)\,\mathcal{U}^{\bar{n}}_{\llcorner}(0)\,\frac{i\partial^i_\perp}{in\cdot D}\,\mathcal{U}^{\bar{n}\,\dagger}_{\llcorner}(\xi)\,\frac{\not{\bar{n}}}{2}\gamma^\perp_i\chi^c_{\rm kin\,j}(\xi)\right|P,\mathbf{S}\right\rangle + \text{h.c.}\right]$$ [9]

All three distributions are not required to span the NLP cross section due to EOM

$$\Phi^{\rm int}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) = \Phi^{\rm kin}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S}) + \Phi^{\rm dyn}_{q/P\,jj'}(x,\mathbf{k}_\perp,\mathbf{S})$$

[9] Ebert, Gao, Stewart 2021

Tree level factorization sub-leading power

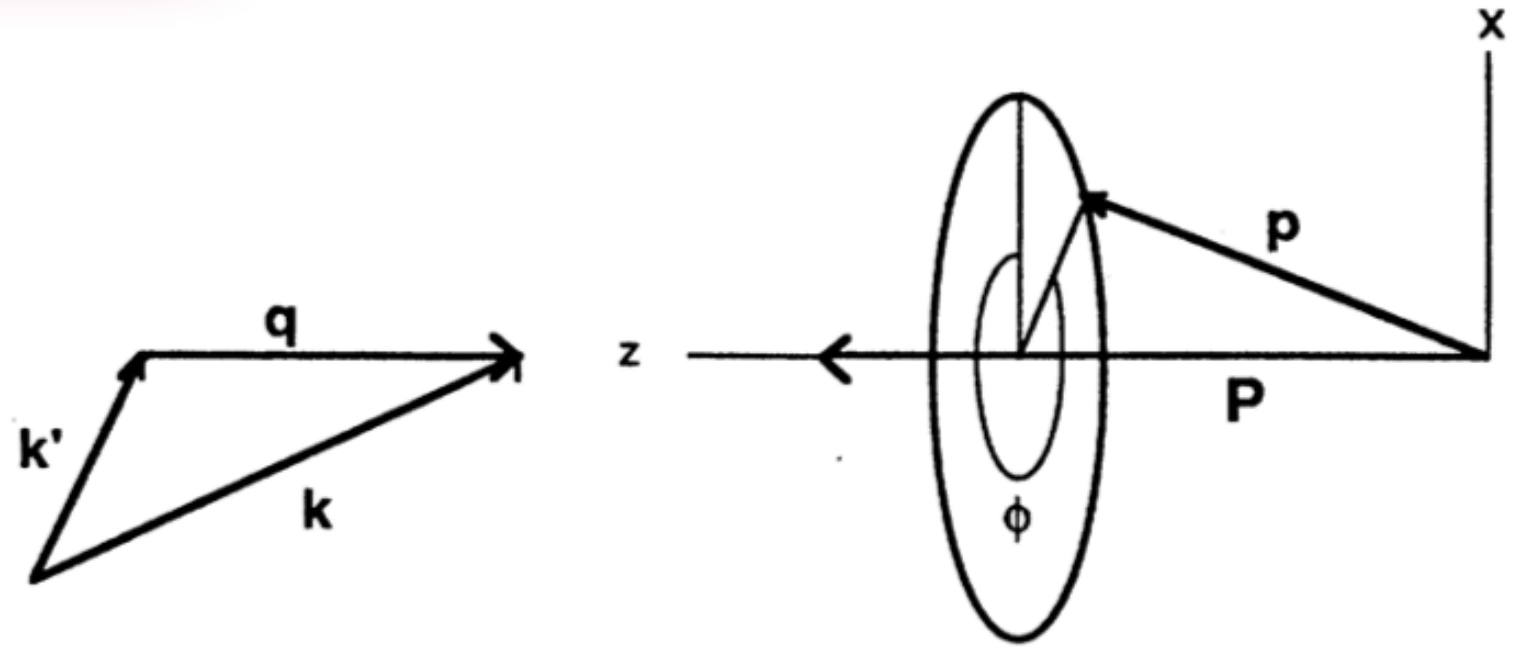
Combining these contributions and multiplying by leptonic tensor
get factorized Cahn and more …. Includes dynamical "tilde" contributions
Using "intrinsic & dynamical" basis

✦Mulders Tangerman NPB1995
✦Goeke Metz Schlegel PLB 2005
✦Bacchetta et al 2007 JHEP

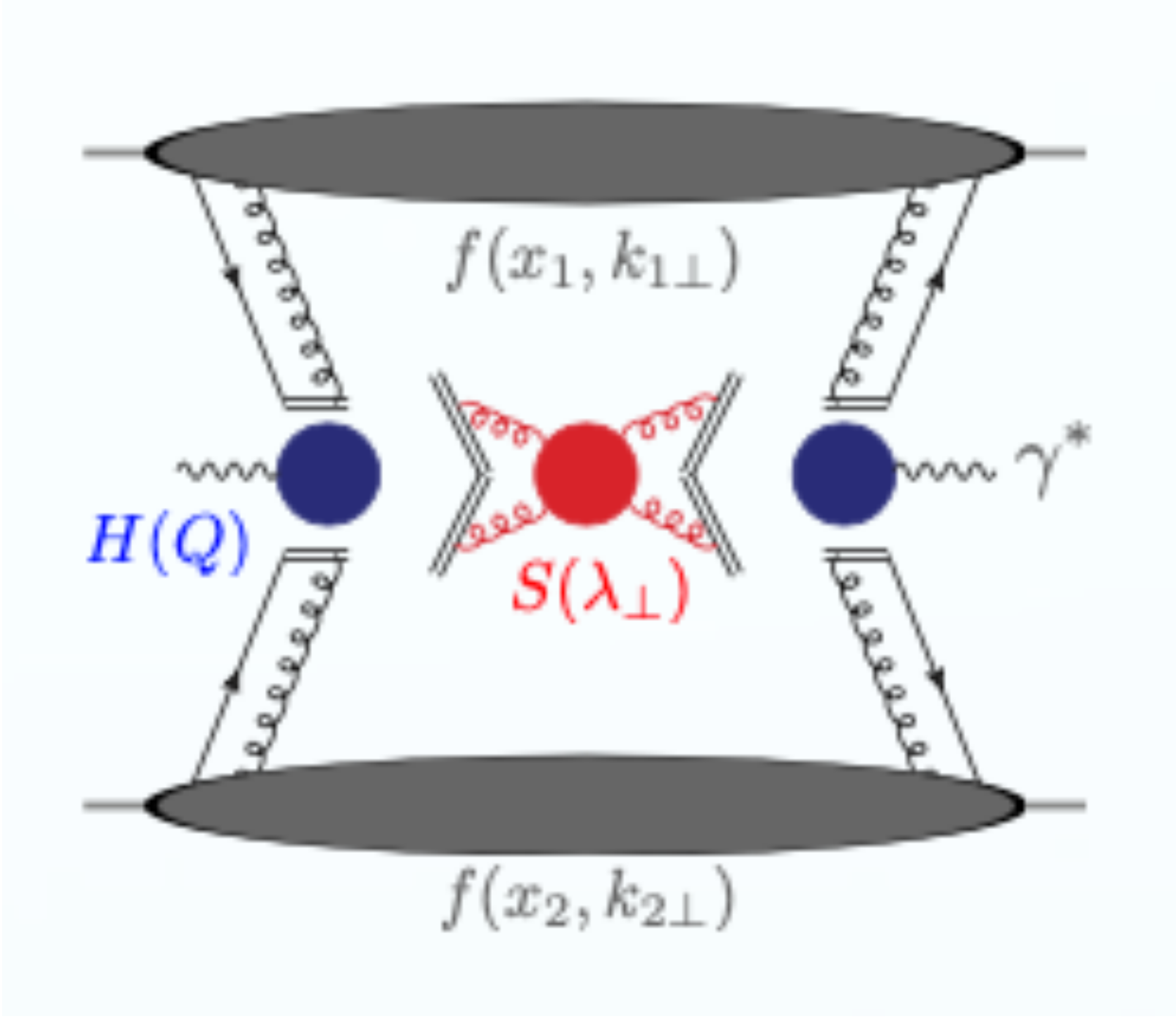
$$F^3_{\rm DIS}(x,z,\boldsymbol{P}_{h\perp}) = \mathcal{C}^{\rm DIS}\left[\frac{q_\perp}{Q} f_1\, D_1\right] - \mathcal{C}^{\rm DIS}\left[\left(x\,\frac{\boldsymbol{k}_\perp\cdot\hat{x}}{Q} f^\perp\right) D_1 - f_1\left(\frac{\boldsymbol{p}_\perp\cdot\hat{x}}{zQ} D^\perp\right)\right]$$
$$- \int \frac{dx_g}{x_g}\, \mathcal{C}^{\rm DIS}_{\rm dyn\, x_g}\left[\left(x\,\frac{\boldsymbol{k}_\perp\cdot\hat{x}}{Q}\tilde{f}^\perp\right) D_1\right] + \int \frac{dz_g}{z_g}\, \mathcal{C}^{\rm DIS}_{\rm dyn\, z_g}\left[f_1\left(\frac{\boldsymbol{p}_\perp\cdot\hat{x}}{zQ}\tilde{D}^\perp\right)\right],$$

Cahn and more intrinsic k_T

Slightly different setup allows us to check RG consistency
Gamberg, Kang, Shao, Terry, Zhao arXiv: e-Print:221.13209



TMD factorization at NLO and NLP



$q_T \sim k_T \ll Q$

TMD Factorization

- ✦*Collins Soper Sterman NPB 1985*
- ✦*Ji Ma Yuan PRD PLB …2004, 2005*
- ✦*Aybat Rogers PRD 2011*
- ✦*Collins 2011 Cambridge Press*
- ✦*Echevarria, Idilbi, Scimemi JHEP 2012, …*
- ✦*SCET Becher & Neubert, 2011 EJPC*

$$\frac{d\sigma^W}{dQ^2\, dx_F\, dp_T^2} = \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2}\, e^{i\boldsymbol{p}_T\cdot\boldsymbol{b}_T}\,\tilde{W}(x_F, b_T, Q)$$

$$\tilde{W}(x_F, b_T, Q) = \sum_j H^{\rm DY}_{j\bar{j}}(Q, \mu, a_s(\mu))\,\tilde{f}_{j/A}(x_A, b_T; \zeta_A, \mu)\,\tilde{f}_{\bar{j}/B}(x_B, b_T; \zeta_B, \mu)$$

….“with resummation”….

Factorization & resummation at NLO and NLP

Clarify how these terms in factorization ansatz are calculated beyond tree level

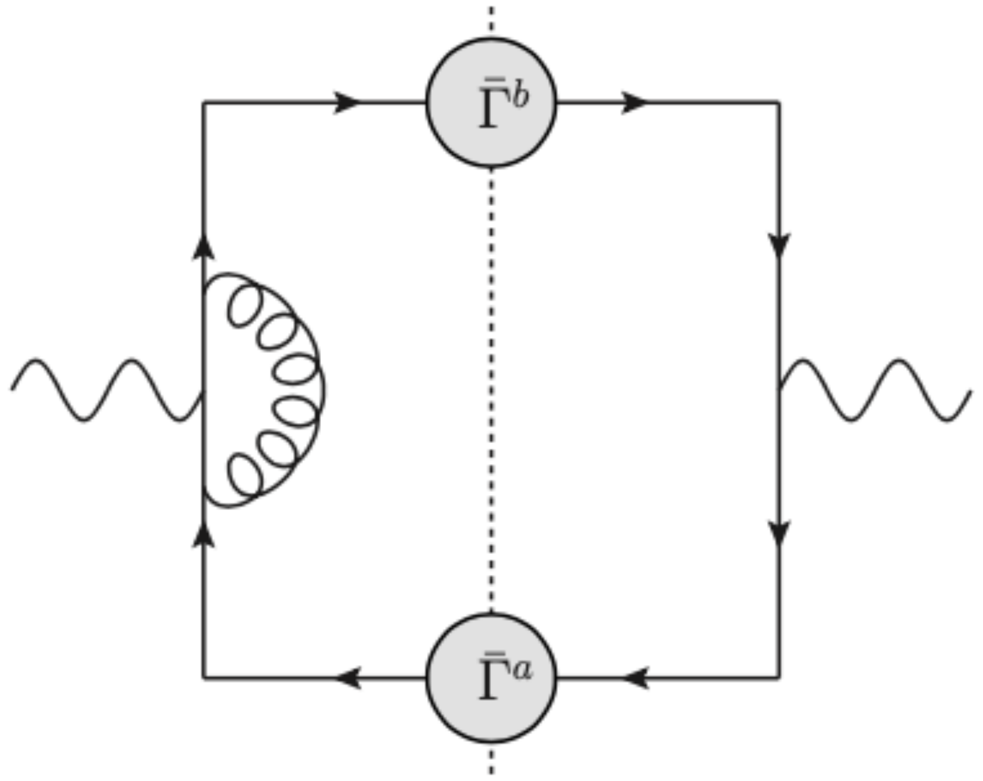
Note first attempt Bacchetta Boer Diehl Mulders JHEP 2008

- We perform one loop calculation,
- attempt to establish renormalization group consistency
- perform resummation for terms that enter into the factorized cross section

$$
\begin{aligned}
F^{3}_{\rm DIS}(x,z,\boldsymbol{P}_{h\perp}) = {} & H^{\rm LP}_{\rm DIS}(Q;\mu)\,\mathcal{C}^{\rm DIS}\left[\frac{q_\perp}{Q} f_1\, D_1\, \mathcal{S}^{\rm LP}\right] \\
& - H^{\rm int}_{\rm DIS}(Q;\mu)\,\mathcal{C}^{\rm DIS}\left[\left(x\,\frac{\boldsymbol{k}_\perp\cdot\hat{x}}{Q} f^\perp D_1 - \frac{\boldsymbol{p}_\perp\cdot\hat{x}}{zQ} f_1\, D^\perp\right)\mathcal{S}^{\rm int}\right] \\
& - \int \frac{dx_g}{x_g} H^{\rm dyn}_{\rm DIS}(x_g,Q;\mu)\,\mathcal{C}^{\rm DIS}\left[x\,\frac{\boldsymbol{k}_\perp\cdot\hat{x}}{Q}\tilde{f}^\perp D_1\, \mathcal{S}^{\rm dyn}\right] \\
& + \int \frac{dz_g}{z_g} H^{\rm dyn}_{\rm DIS}(z_g,Q;\mu)\,\mathcal{C}^{\rm DIS}\left[\frac{\boldsymbol{p}_\perp\cdot\hat{x}}{zQ} f_1\, \tilde{D}^\perp \mathcal{S}^{\rm dyn}\right] .
\end{aligned}
$$

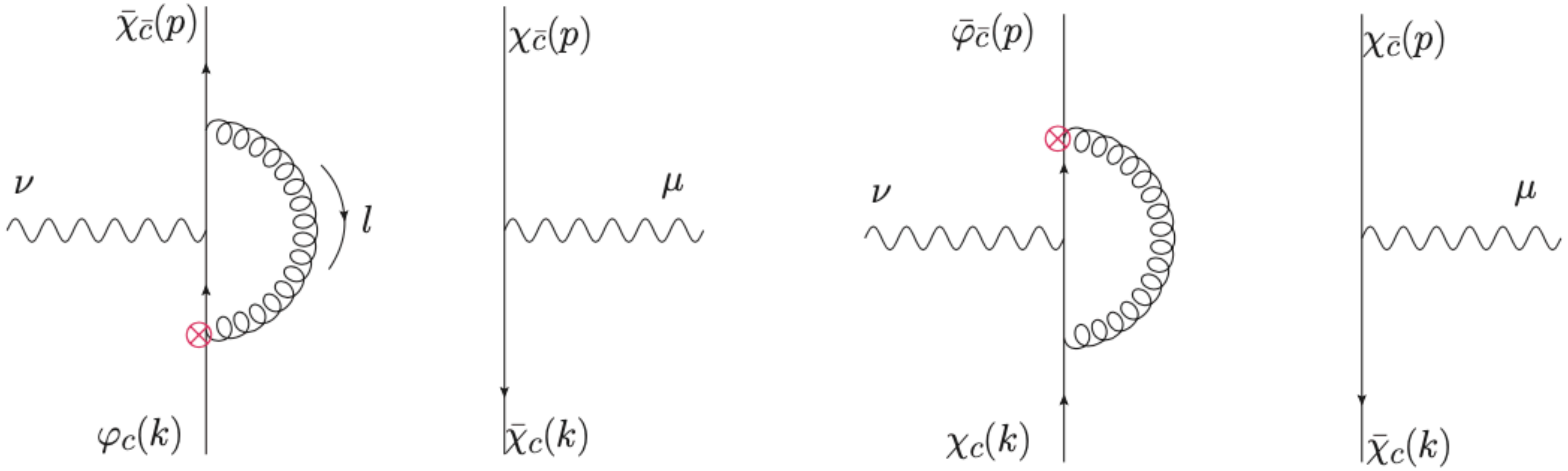
- H^{LP}, H^{int} and H^{dynam} represent LP, intrinsic NLP, and dynamic NLP hard functions.
- S^{LP}, S^{int} and S^{dyn} denote the LP, intrinsic sub-leading power, and dynamic sub-leading power soft function
- We have also introduced the more general shorthand for the convolution integrals

NLO Ingredients hard factor



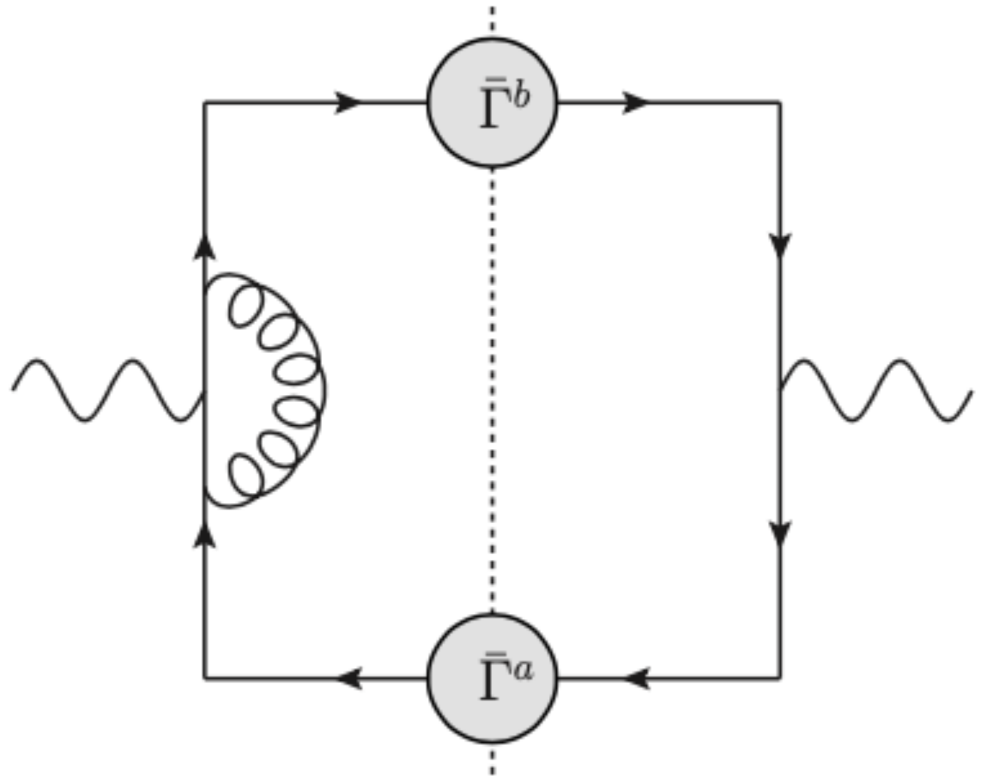
$+$h.c.

$$\gamma^\nu \to \gamma^\nu + \frac{\alpha_s C_F}{2\pi} F^\nu_{\mathrm{DIS}}\left(Q;\mu\right) + \mathcal{O}\left(\alpha_s^2\right)$$



$$\mathcal{M}^{\nu\,(1)}_{\mathrm{NLP}}\left(k,p;\mu\right) = \bar{\varphi}_{\bar{c}}(p) F^\nu_{\mathrm{DIS\,LP}}\left(Q;\mu\right)\chi_c(k) + \bar{\chi}_{\bar{c}}(p) F^\nu_{\mathrm{DIS\,LP}}\left(Q;\mu\right)\varphi_c(k)$$

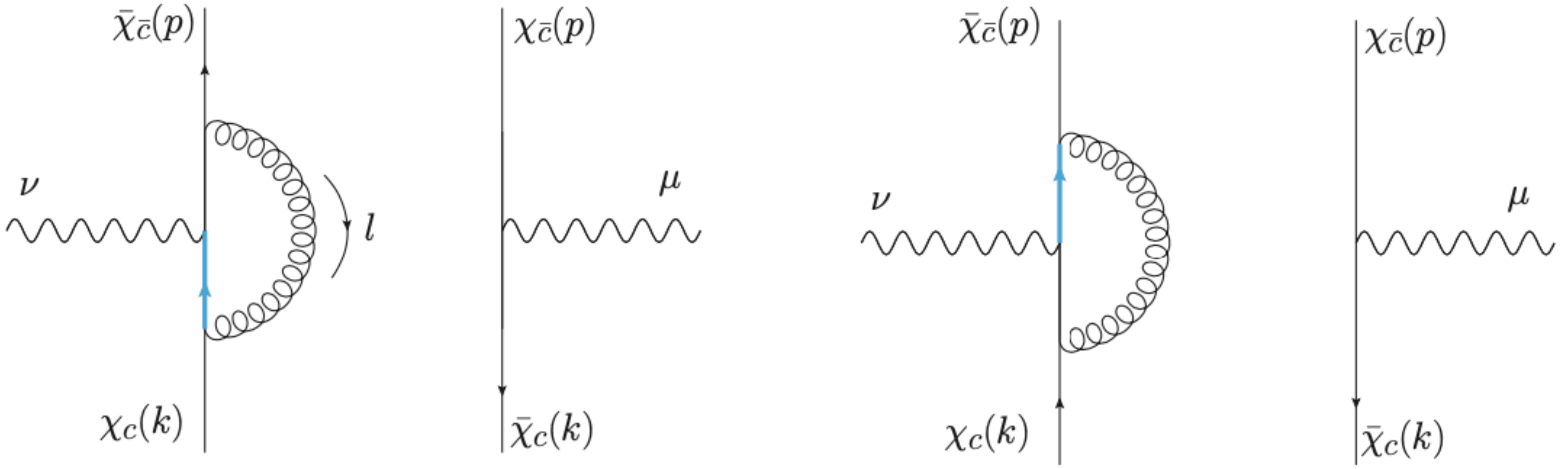
NLO Ingredients hard factor



+h.c.

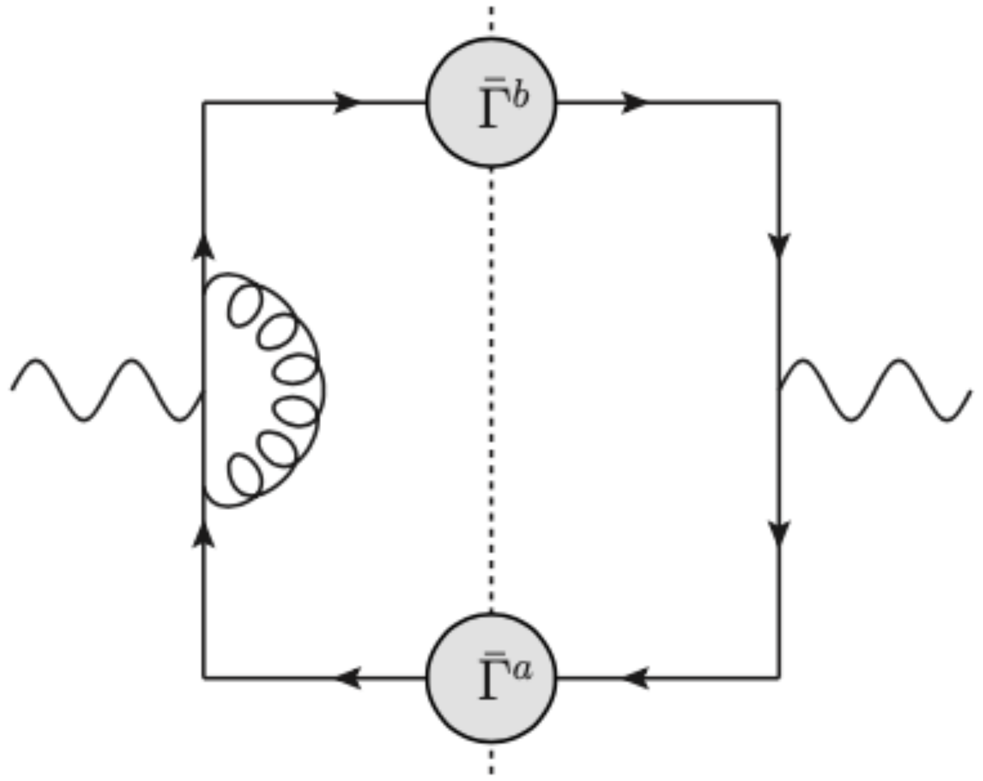
$$\gamma^\nu \to \gamma^\nu + \frac{\alpha_s C_F}{2\pi} F^\nu_{\mathrm{DIS}}\left(Q;\mu\right) + \mathcal{O}\left(\alpha_s^2\right)$$

We have found also addition, we must also consider sub-leading contributions entering from the transverse momentum of the quark propagators.



$$\begin{aligned}\mathcal{M}^{\nu\,(1)}_{\mathrm{NLP}}\left(k,p;\mu\right) = {} & \bar{\varphi}_{\bar{c}}(p) F^\nu_{\mathrm{DIS\,LP}}\left(Q;\mu\right)\chi_c(k) + \bar{\chi}_{\bar{c}}(p) F^\nu_{\mathrm{DIS\,LP}}\left(Q;\mu\right)\varphi_c(k) \\ & + \bar{\chi}_{\bar{c}}(p) F^\nu_{\mathrm{DIS\,k}}\left(k_\perp, Q;\mu\right)\chi_c(k) + \bar{\chi}_{\bar{c}}(p) F^\nu_{\mathrm{DIS\,p}}\left(p_\perp, Q;\mu\right)\chi_c(k)\end{aligned}$$

NLO Ingredients hard factor



$+$h.c.

$$H^{(1)\,i}_{\rm DIS}(Q;\mu) = \bar{\mathcal{V}}^i_{\mu\nu}\left[\mathcal{M}^{\mu\,(1)}_{\rm LP}\mathcal{M}^{\dagger\,\nu\,(0)}_{\rm LP} + \mathcal{M}^{\mu\,(0)}_{\rm LP}\mathcal{M}^{\dagger\,\nu\,(1)}_{\rm LP}\right].$$

LP

$$\mathcal{M}^{\nu\,(1)}_{\rm LP}(k,p;\mu) = \frac{\alpha_s C_F}{2\pi}\bar{\chi}_{\bar{c}}(p)\gamma^\nu\chi_c(k)\left[\frac{3}{2\epsilon}+\frac{1}{\epsilon^2}+2L_Q^2-\frac{2L_Q}{\epsilon}-3L_Q-\frac{\pi^2}{12}+4\right]$$

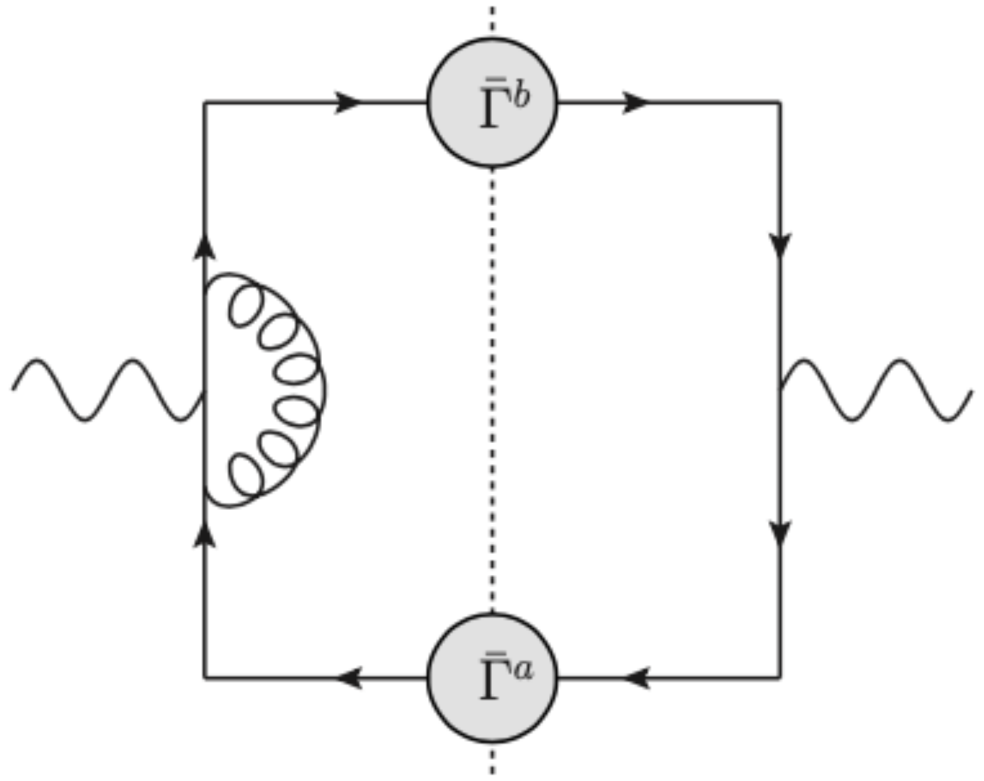
$$\hat{H}^{\rm LP}_{\rm DIS}(Q;\mu) = 1+\frac{\alpha_s C_F}{2\pi}\left[-\frac{2}{\epsilon^2}-\frac{3}{\epsilon}-4L_Q^2+\frac{4L_Q}{\epsilon}+6L_Q+\frac{\pi^2}{6}-8\right]$$

NLP

$$\mathcal{M}^{\nu\,(1)}_{\rm NLP}(k,p;\mu) = \left(\frac{1}{\epsilon}+\frac{1}{\epsilon^2}+2L_Q^2-\frac{2L_Q}{\epsilon}-2L_Q-\frac{\pi^2}{12}+\frac{7}{2}\right)\bar{\chi}_{\bar{c}}(p)\frac{\not{n}}{2}\hat{t}^\nu\varphi_c(k)$$
$$+\left(\frac{1}{\epsilon}+\frac{1}{\epsilon^2}+2L_Q^2-\frac{2L_Q}{\epsilon}-2L_Q-\frac{\pi^2}{12}+\frac{7}{2}\right)\bar{\varphi}_{\bar{c}}(p)\frac{\not{\bar{n}}}{2}\hat{t}^\nu\chi_c(k)+\text{dyn}\,.$$

$$\hat{H}^{\rm NLP}_{\rm DIS}(Q;\mu) = 1+\frac{\alpha_s C_F}{2\pi}\left[-\frac{2}{\epsilon^2}-\frac{5}{2\epsilon}+\frac{4}{\epsilon}L_Q-4L_Q^2+5L_Q+\frac{\pi^2}{6}-\frac{15}{2}\right]$$

NLO Ingredients hard factor



Using the definition of the unsubtracted (UV divergent) hard function, we obtain the subtracted hard function through multiplicative renormalization as

$$H(Q;\mu) = Z(Q;\mu)\hat{H}(Q;\mu),$$

$$H_{\rm DIS}^{\rm LP}(Q;\mu) = 1 + \frac{\alpha_s C_F}{2\pi}\left[-4L_Q^2 + 6L_Q + \frac{\pi^2}{6} - 8\right]$$

$$H_{\rm DIS}^{\rm NLP}(Q;\mu) = 1 + \frac{\alpha_s C_F}{2\pi}\left[-4L_Q^2 + 5L_Q + \frac{\pi^2}{6} - \frac{15}{2}\right]$$

$$Z_{\rm DIS}^{\rm LP}(Q;\mu) = 1 + \frac{\alpha_s C_F}{2\pi}\left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} + \frac{4L_Q}{\epsilon}\right],$$

$$Z_{\rm DIS}^{\rm NLP}(Q;\mu) = 1 + \frac{\alpha_s C_F}{2\pi}\left[-\frac{2}{\epsilon^2} - \frac{5}{2\epsilon} + \frac{4L_Q}{\epsilon}\right].$$

Since the bare operator H is RG invariant, the RG equation of H yields the hard anomalous dimension

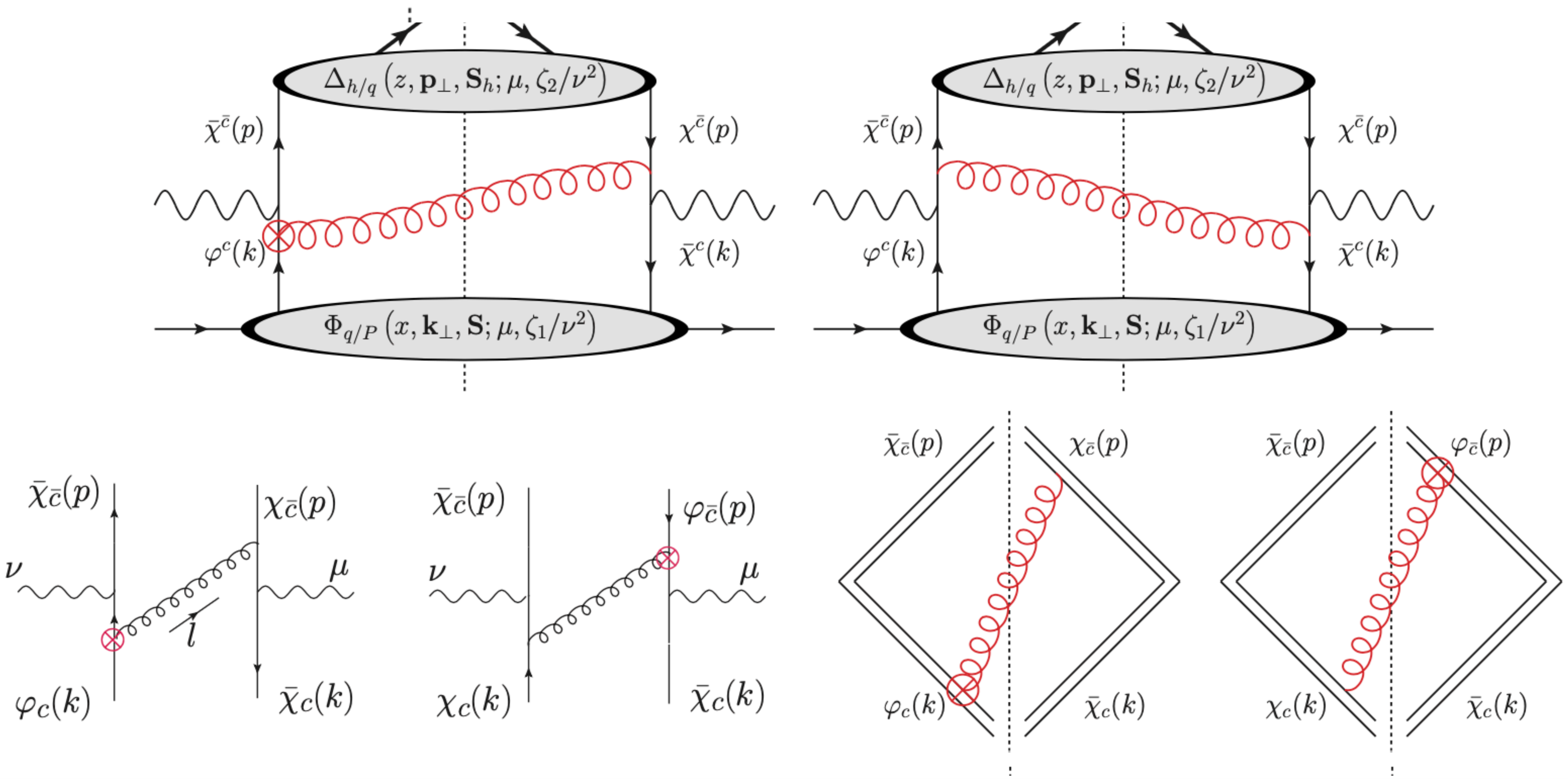
$$\Gamma_H = -\frac{\partial}{\partial \ln\mu} Z(Q;\mu),$$

$$\Gamma^{\mu}_{\rm H\,LP}(Q;\mu) = \frac{\alpha_s C_F}{\pi}\left(4L_Q - 3\right), \qquad \Gamma^{\mu}_{\rm H\,NLP}(Q;\mu) = \frac{\alpha_s C_F}{\pi}\left(4L_Q - \frac{5}{2}\right)$$

NLO Ingredients soft factor

The soft region

The soft function is generated through the emissions of soft gluons in the partonic cross section



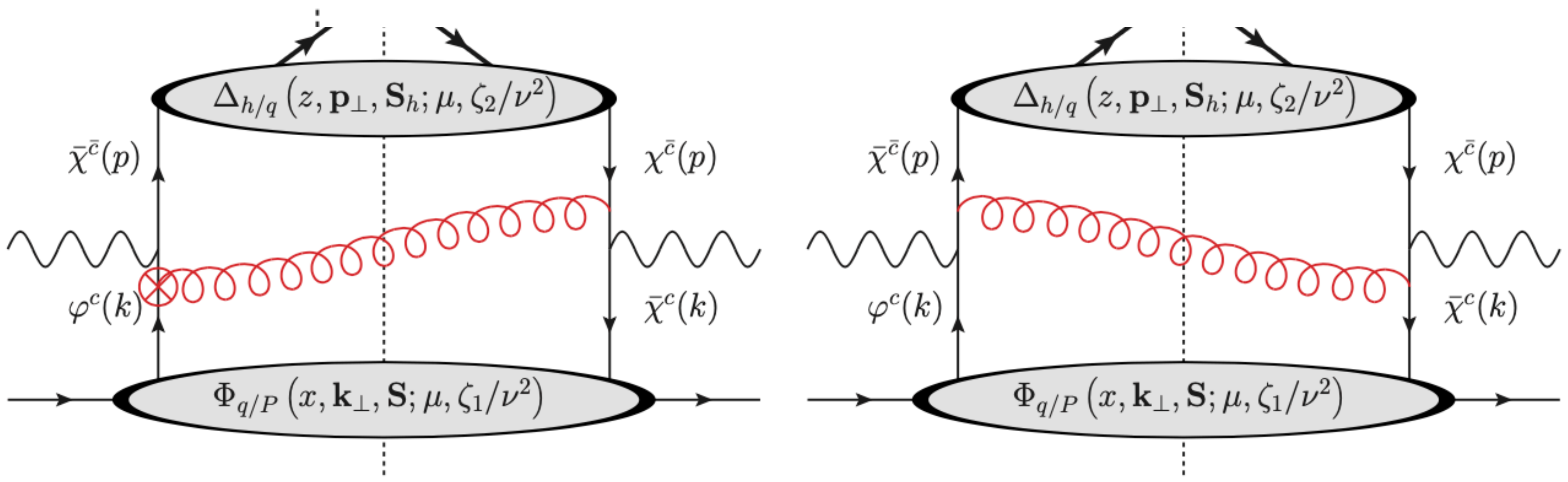
Soft emission from the sub-leading fields vanishes. NLO+NLP soft function is half the LP one

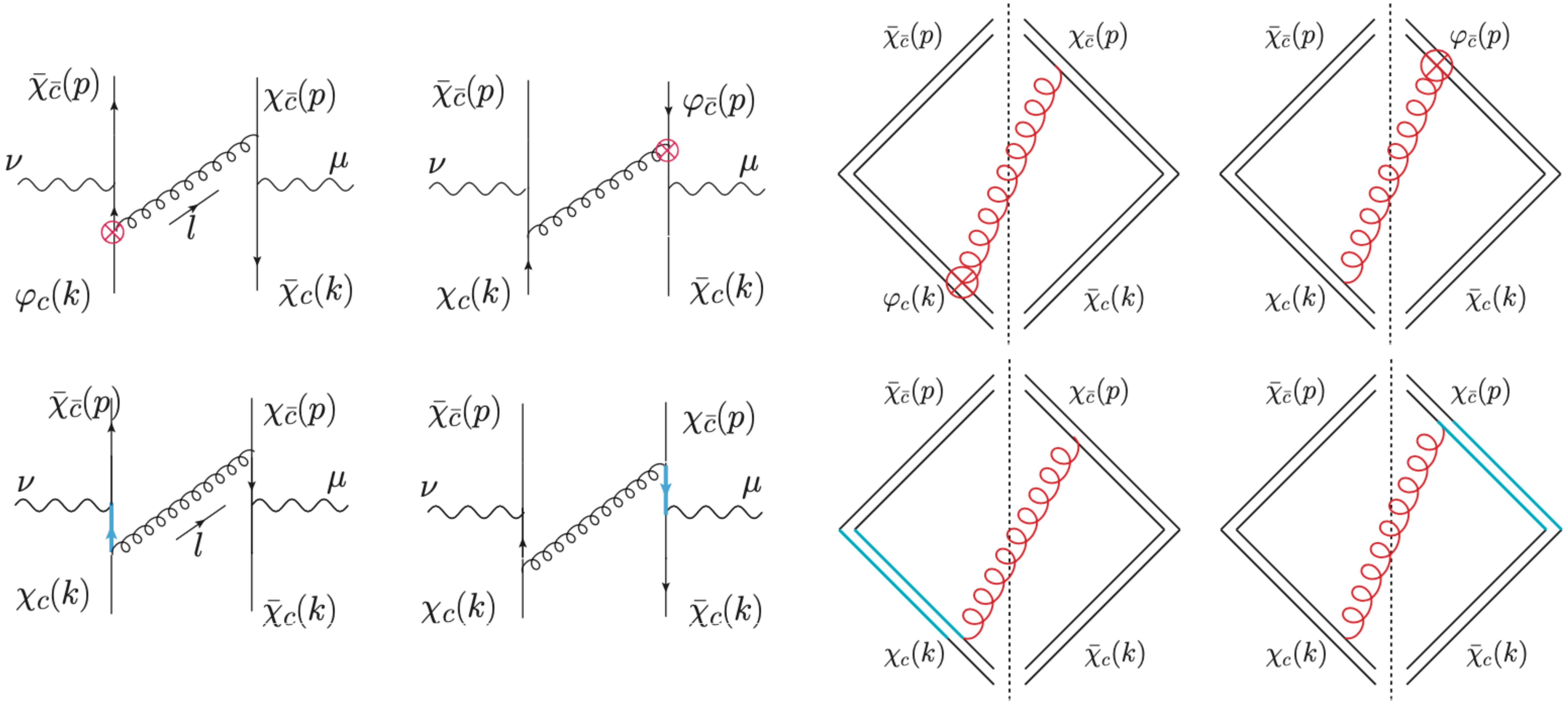
$$\Gamma^{\mu}_{\mathcal{S}\,\text{int}} = \frac{1}{2}\Gamma^{\mu}_{\mathcal{S}\,\text{LP}}\,, \qquad \Gamma^{\nu}_{\mathcal{S}\,\text{int}} = \frac{1}{2}\Gamma^{\nu}_{\mathcal{S}\,\text{LP}}$$

NLO Ingredients soft factor

The soft region

The soft function is generated through the emissions of soft gluons in the partonic cross section





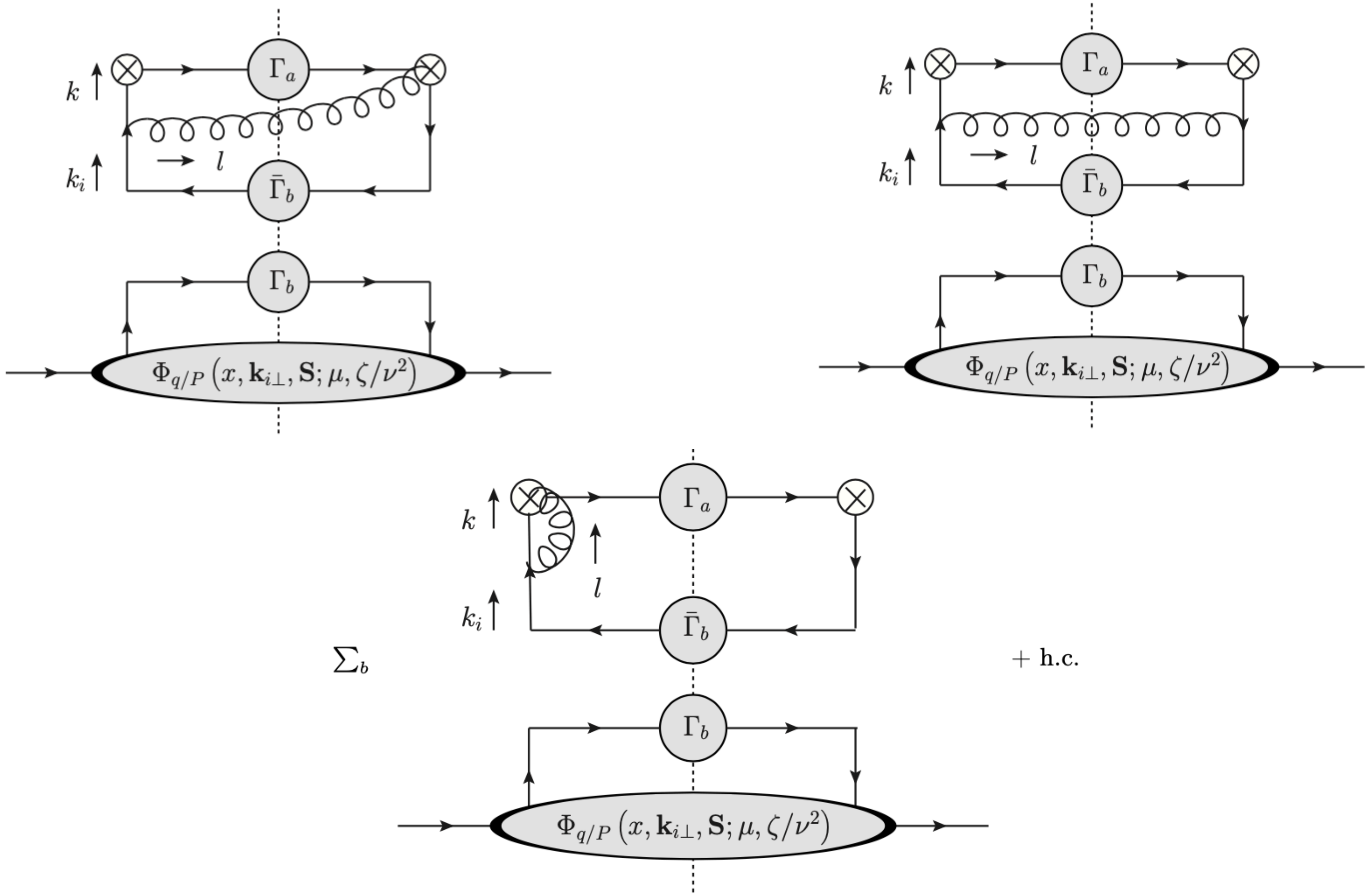
Progress Report
Stay tuned ...

Contributions to the soft factor after applying the eikonal approximation and including the effect from the transverse momentum contributions from the quark propagators.

NLO Ingredients collinear factor

The collinear region

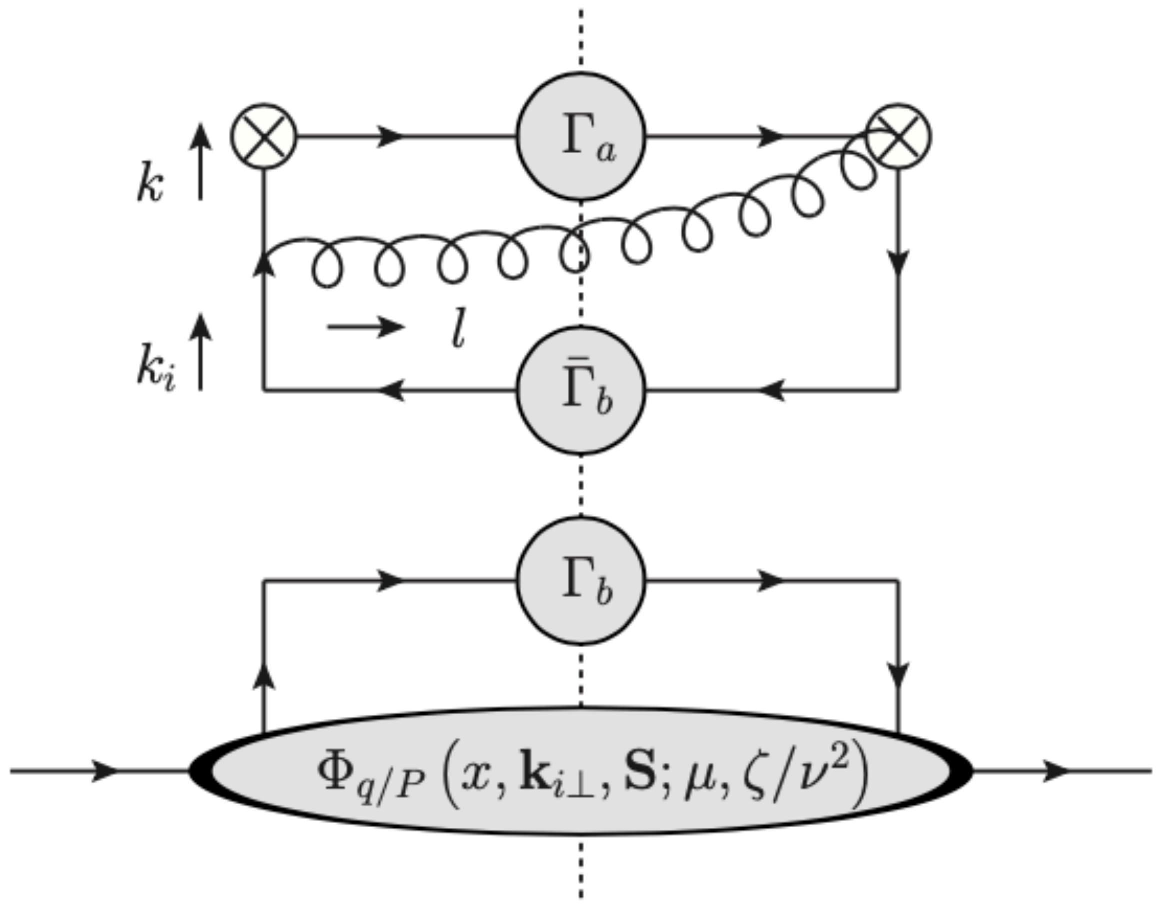
Diagrams associated with the evolution of the collinear distributions

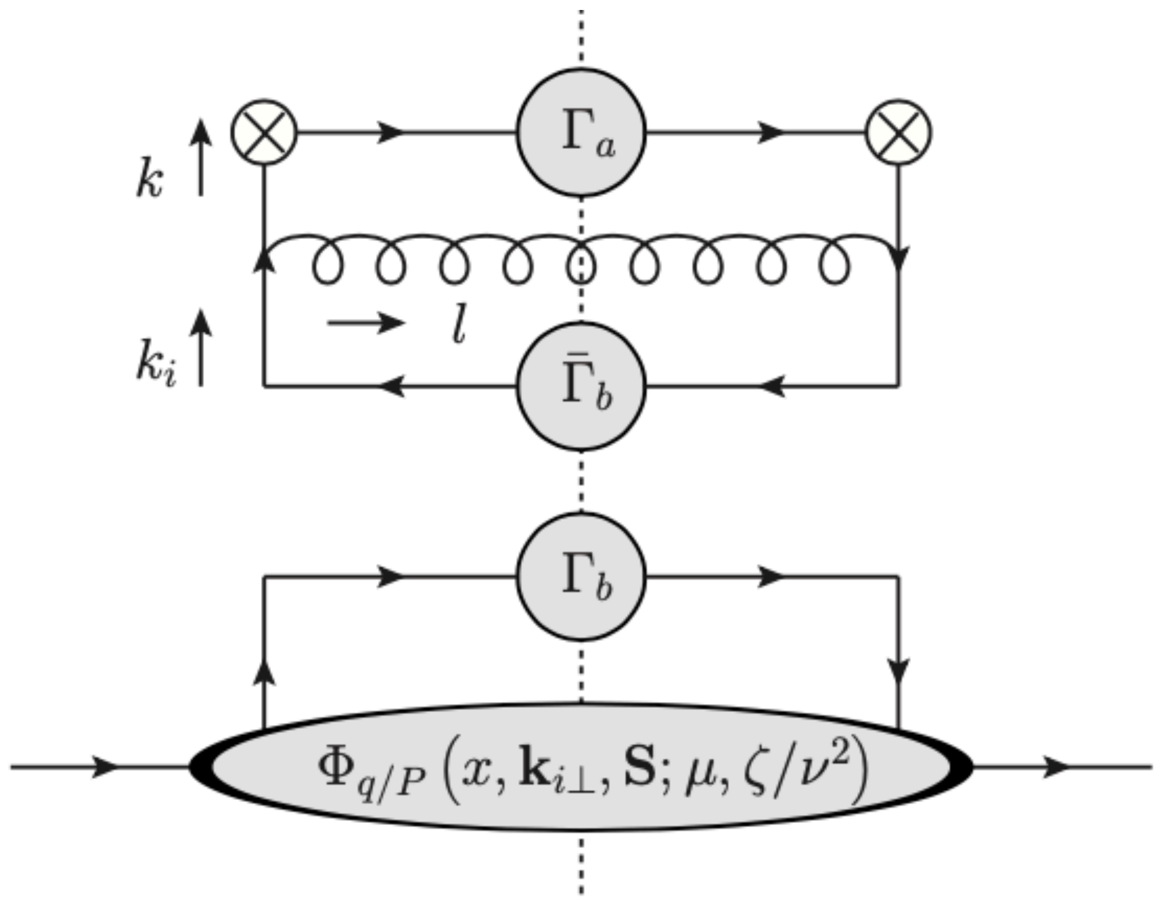


$$\hat{\Phi}^{[\Gamma^a]}\left(x, \boldsymbol{b}, \boldsymbol{S}; \mu, \zeta/\nu^2\right) = Z_{\Gamma^a \Gamma^b}\left(b, \mu, \zeta/\nu^2\right) \Phi^{[\Gamma^b]0}\left(x, \boldsymbol{b}, \boldsymbol{S}; xP^+\right)$$

Renormalize TMDs soft and UV subtraction

As a consequence rapidity and UV subtracted TMDs obey Collins-Soper-equations & we can determine rapidity and UV anomalous dimensions





Renormalization and TMD Evolution- $\{\zeta, \mu\}$

- Collins Soper Eq. $$\frac{\partial \ln \tilde{f}_{j/H}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu)$$

$$\tilde{K}(b_T, \mu) \equiv \frac{1}{2} \frac{\partial}{\partial y_n} \ln \frac{S(b_T, y_n, -\infty)}{S(b_T, y_n, -\infty)}$$

- RGE for C.S. kernel

$$\frac{d\tilde{K}(b_T; \mu)}{d \ln \mu} = -\gamma_k(\alpha_s(\mu))$$

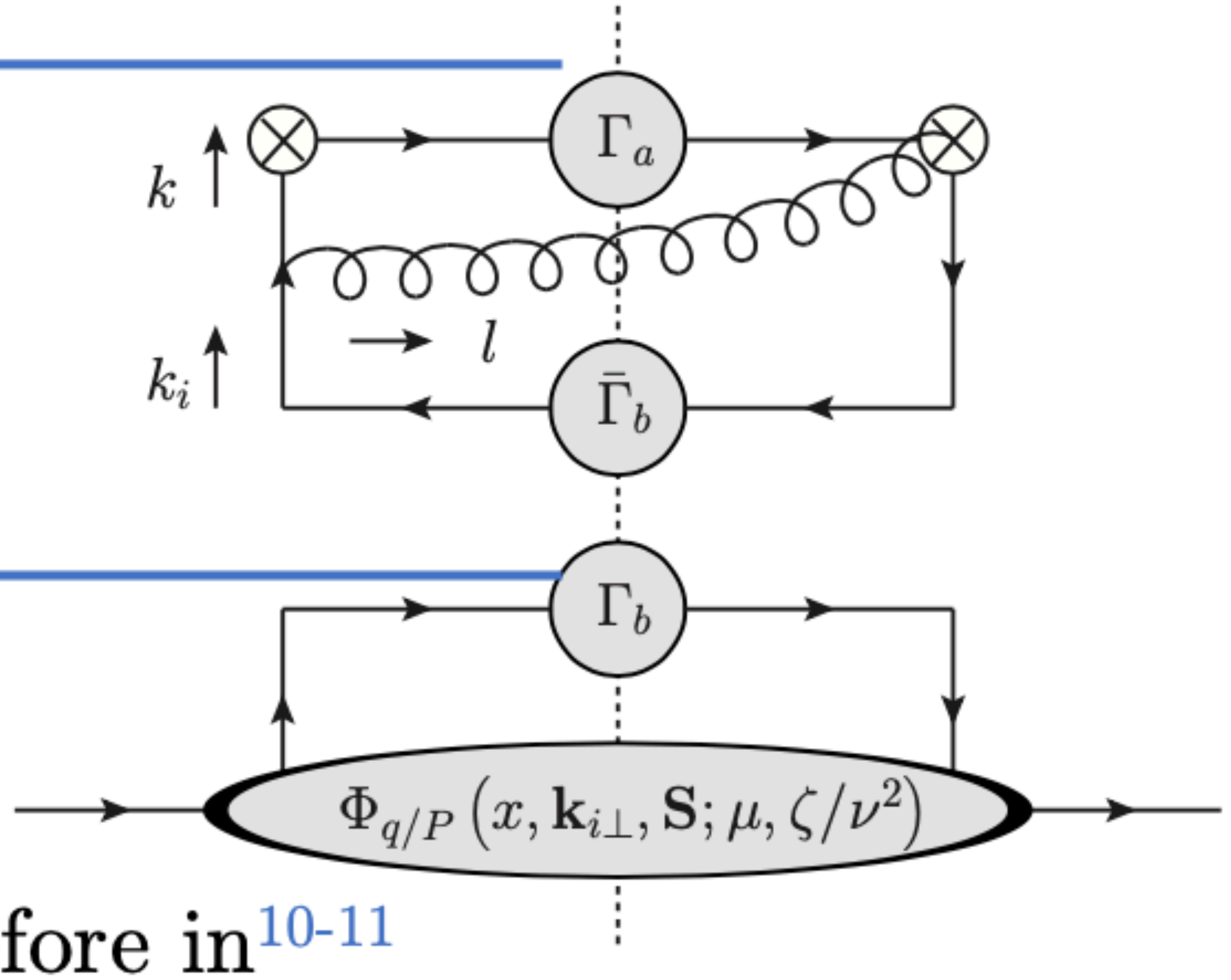
- RGE for TMD $$\frac{d \ln \tilde{f}_{j/H}(x, b_T; \mu, \zeta)}{d \ln \mu} = -\gamma_F(\alpha_s(\mu), \zeta/\mu)$$

Solve simultaneously and get evolved renormalized TMD $\rightarrow \zeta = Q^2, \quad \mu = \mu_Q \sim Q$

Anomalous dimension matrices

Evolution equations naturally enter as matrices due to mixing

$$\frac{\partial}{\partial \ln \mu}\begin{bmatrix} \Phi^{[\not n]} \\ \Phi^{[\not n\gamma^5]} \\ \Phi^{[i\sigma^{i+}\gamma^5]} \\ \Phi^{[1]} \\ \Phi^{[\gamma^5]} \\ \Phi^{[\gamma^i]} \\ \Phi^{[\gamma^i\gamma^5]} \\ \Phi^{[i\sigma^{ij}\gamma^5]} \\ \Phi^{[i\sigma^{+-}\gamma^5]} \end{bmatrix} = \mathbf{\Gamma}^\mu \begin{bmatrix} \Phi^{[\not n]} \\ \Phi^{[\not n\gamma^5]} \\ \Phi^{[i\sigma^{l+}\gamma^5]} \\ \Phi^{[1]} \\ \Phi^{[\gamma^5]} \\ \Phi^{[\gamma^l]} \\ \Phi^{[\gamma^l\gamma^5]} \\ \Phi^{[i\sigma^{lm}\gamma^5]} \\ \Phi^{[i\sigma^{+-}\gamma^5]} \end{bmatrix} \qquad \frac{\partial}{\partial \ln \nu}\begin{bmatrix} \Phi^{[\not n]} \\ \Phi^{[\not n\gamma^5]} \\ \Phi^{[i\sigma^{i+}\gamma^5]} \\ \Phi^{[1]} \\ \Phi^{[\gamma^5]} \\ \Phi^{[\gamma^i]} \\ \Phi^{[\gamma^i\gamma^5]} \\ \Phi^{[i\sigma^{ij}\gamma^5]} \\ \Phi^{[i\sigma^{+-}\gamma^5]} \end{bmatrix} = \mathbf{\Gamma}^\nu \begin{bmatrix} \Phi^{[\not n]} \\ \Phi^{[\not n\gamma^5]} \\ \Phi^{[i\sigma^{l+}\gamma^5]} \\ \Phi^{[1]} \\ \Phi^{[\gamma^5]} \\ \Phi^{[\gamma^l]} \\ \Phi^{[\gamma^l\gamma^5]} \\ \Phi^{[i\sigma^{lm}\gamma^5]} \\ \Phi^{[i\sigma^{+-}\gamma^5]} \end{bmatrix}.$$



We find operator mixing in the Collins-Soper equation. Seen before in[10-11]

$$\mathbf{\Gamma}^\mu = \begin{bmatrix} \Gamma_2^\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Gamma_2^\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Gamma_2^\mu\delta_l^i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Gamma_3^\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Gamma_3^\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Gamma_3^\mu\delta_l^i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_3^\mu\delta_l^i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4}\Gamma_3^\mu\left(\delta_l^i\delta_m^j - \delta_l^j\delta_m^i\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_3^\mu \end{bmatrix}$$

$$\mathbf{\Gamma}^\nu = \frac{\alpha_s C_F}{2\pi}\begin{bmatrix} 2L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2L & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2L\delta_l^i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2ib_l}{xP^+}\frac{\partial L}{\partial b^2} & 0 & L & 0 & 0 & 0 & 0 \\ \frac{2ib^i}{xP^+}\frac{\partial L}{\partial b^2} & 0 & 0 & 0 & 0 & L\delta_l^i & 0 & 0 & 0 \\ 0 & \frac{2ib^i}{xP^+}\frac{\partial L}{\partial b^2} & 0 & 0 & 0 & 0 & L\delta_l^i & 0 & 0 \\ 0 & 0 & \frac{i}{xP^+}\frac{\partial L}{\partial b^2}\left(b^j\delta_l^i - b^i\delta_l^j\right) & 0 & 0 & 0 & 0 & L\left(\delta_l^i\delta_m^j - \delta_l^j\delta_m^i\right) & 0 \\ 0 & 0 & \frac{2ib_l}{xP^+}\frac{\partial L}{\partial b^2} & 0 & 0 & 0 & 0 & 0 & L \end{bmatrix}$$

LP to LP

LP to NLP

NLP to NLP

Necessary condition rapidity RG Consistency

$$\frac{d\sigma}{d\ln\nu} = 0 \quad \& \quad \frac{d\sigma}{d\ln\mu} = 0$$

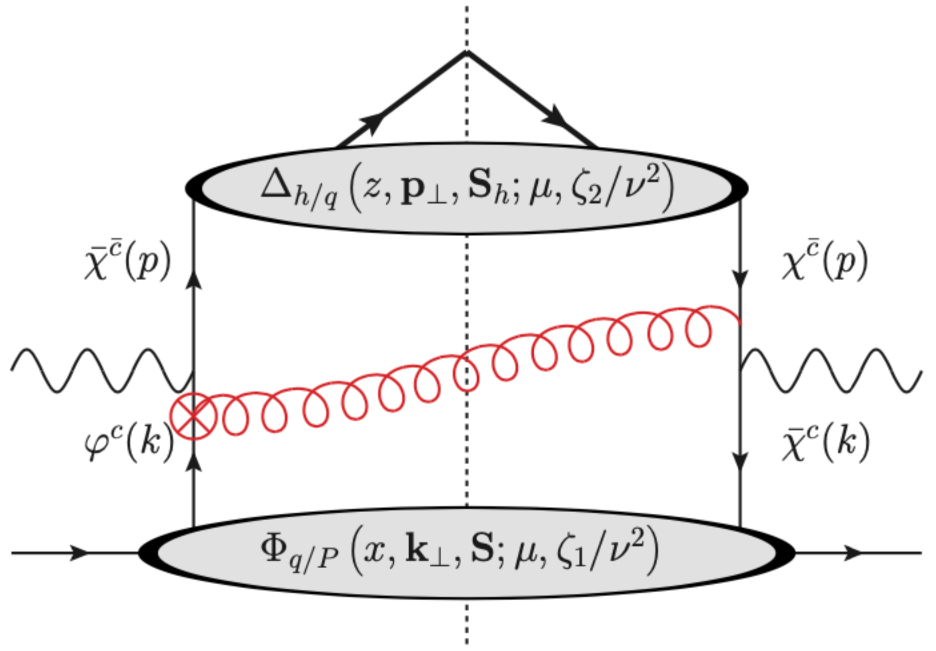
Leading power

$$f_1(x, b; \mu, \zeta_1) = f_1(x, b; \mu, \zeta_1/\nu^2)\sqrt{\mathcal{S}^{\rm LP}(b; \mu, \nu)}$$

$$D_1(z, b; \mu, \zeta_2) = D_1(z, b; \mu, \zeta_2/\nu^2)\sqrt{\mathcal{S}^{\rm LP}(b; \mu, \nu)}$$

$$\Gamma^\nu_2 + \frac{1}{2}\Gamma^\nu_S = 0\,, \qquad \Gamma^\nu_2 + \frac{1}{2}\Gamma^\nu_S = 0$$

$$\gamma^\nu_{S_{NLP}} + \gamma^\nu_{f_1^{kin}} + \gamma^\nu_{f^\perp} = 0$$



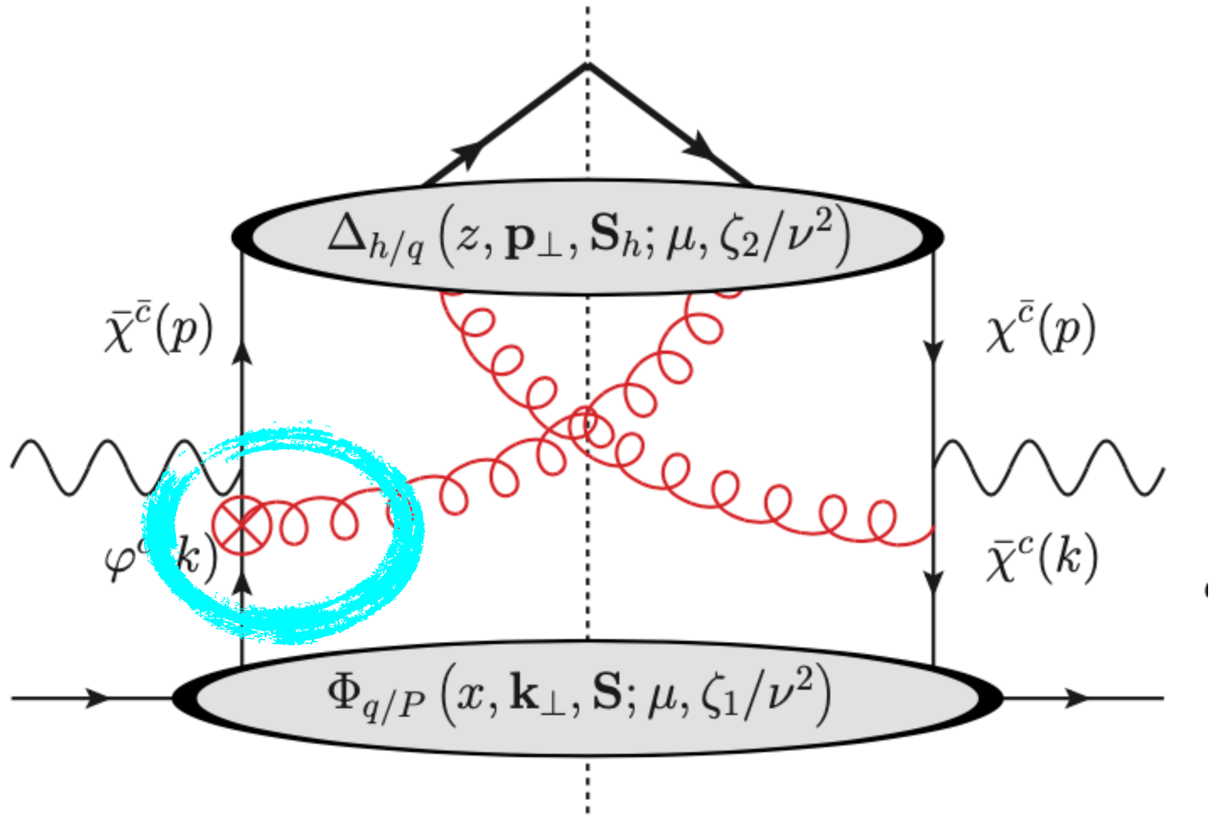
Recall: **Next to leading power**

$$-\,H^{\rm int}_{\rm DIS}(Q;\mu)\,\mathcal{C}^{\rm DIS}\left[\left(x\frac{\boldsymbol{k}_\perp\cdot\hat{x}}{Q}f^\perp D_1 - \frac{\boldsymbol{p}_\perp\cdot\hat{x}}{zQ}f_1 D^\perp\right)\mathcal{S}^{\rm int}\right]$$

$$ib^\mu M^2 f^{\perp\,(1)}(x, b; \mu, \zeta_1) = ib^\mu M^2 f^{\perp\,(1)}(x, b; \mu, \zeta_1/\nu^2)\sqrt{\mathcal{S}^{\rm int}(b; \mu, \nu)}$$

$$\Gamma^\nu_{3\,\rm int} + \frac{1}{2}\Gamma^\nu_{S\,\rm int} = 0$$

Non-trivial result



However,

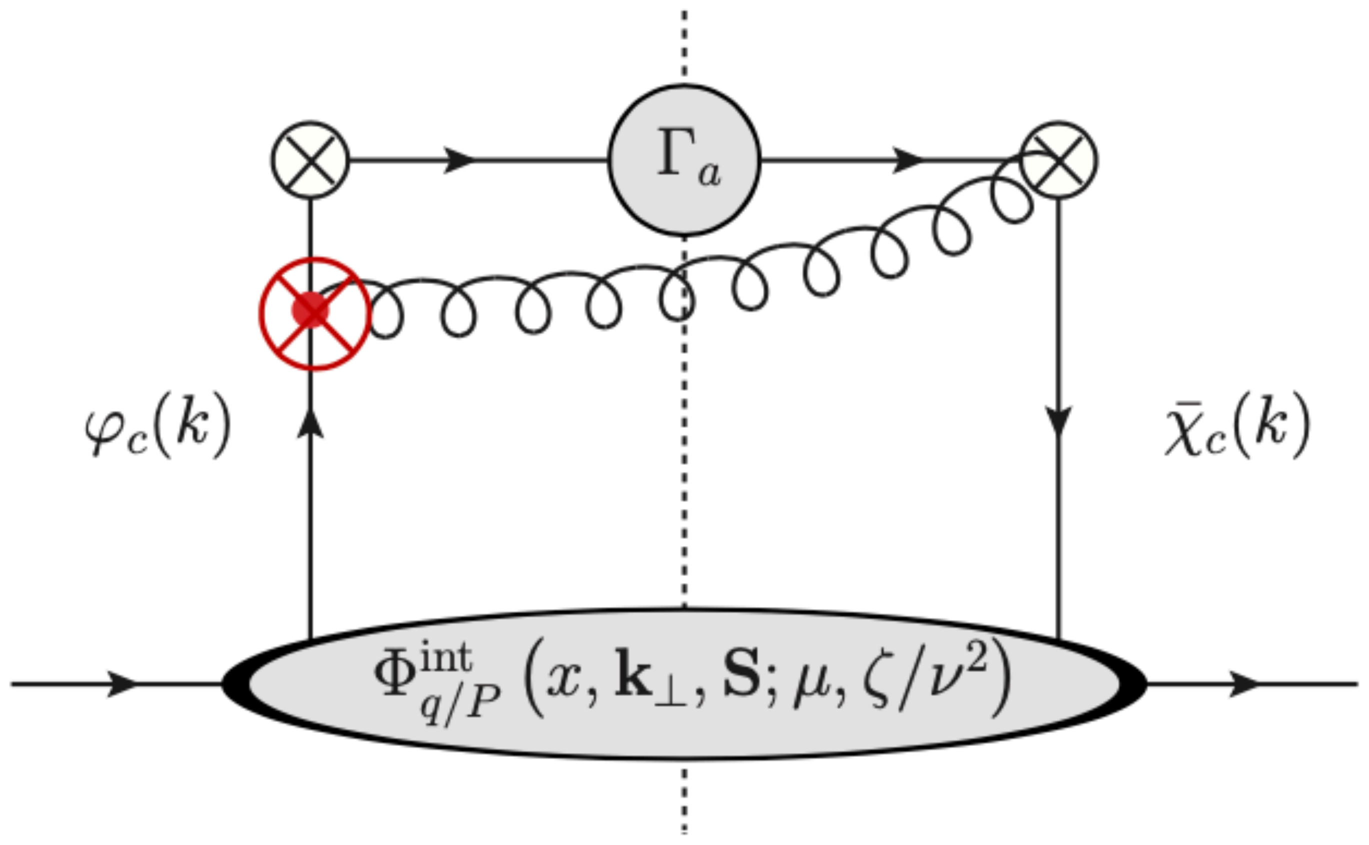
$$D_1(z, b; \mu, \zeta_2) = D_1(z, b; \mu, \zeta_2/\nu^2)\sqrt{\mathcal{S}^{\rm LP}(b; \mu, \nu)} \quad \longleftrightarrow \quad \Gamma^\nu_2 + \frac{1}{2}\Gamma^\nu_{S\,\rm int} \neq 0 \quad !!$$

$$D_1(z, b; \mu, \zeta_2) = D_1(z, b; \mu, \zeta_2/\nu)\sqrt{S^{int}}\,?? \quad \longleftrightarrow \quad \Gamma^\nu_{2\,\rm mod} + \frac{1}{2}\Gamma^\nu_{S\,\rm int} = 0 \quad !!$$

NLO Ingredients collinear factor

Differences from LP TMDs

Study the interaction of the sub-leading fields with the Wilson lines



$$\frac{\not{n}}{2}\varphi_c(k) = 0$$

Can show that these interactions vanish trivially

[8]Gamberg, Kang, Shao, Terry, Zhao 2022

Necessary condition RG Consistency

$$\frac{d\sigma}{d\ln\nu} = 0 \quad \& \quad \frac{d\sigma}{d\ln\mu} = 0$$

Taking into account this aforementioned modification of leading distribution by the presence of the sub- leading field, we thus also demonstrate renormalization group consistency at one loop both RRG and RG

$$\Gamma^{\nu}_{\mathcal{S}\,\mathrm{int}} + \Gamma^{\nu}_{3\,\mathrm{int}} + \Gamma^{\nu}_{2\,\mathrm{mod}} = 0$$

Similarly

$$\Gamma^{\mu}_{H\,\mathrm{int}} + \Gamma^{\mu}_{\mathcal{S}\,\mathrm{int}} + \Gamma^{\mu}_{3\,\mathrm{int}} + \Gamma^{\mu}_{2\,\mathrm{mod}} = 0$$

$$\gamma^{\nu}_{S_{NLP}} + \gamma^{\nu}_{f_1^{kin}} + \gamma^{\nu}_{f^{\perp}} = 0$$

Necessary condition RG Consistency

$$\frac{d\sigma}{d\ln\nu} = 0 \quad \& \quad \frac{d\sigma}{d\ln\mu} = 0$$

... other power corrections such that

$$\Gamma^{\nu}_{S_{NLP}} + \Gamma^{\nu}_{f^{\perp}_{int}} + ? = 0$$

Progress Report
Stay tuned ...

Summary

- We have revisited TMD factorization beyond leading power and beyond leading order in terms of intrinsic TMDs
- We are examining RG consistency and consistency of the EOM beyond leading order
- In doing so, we provide the basis for improved phenomenology of one the earliest observables used to study the intrinsic 3-D momentum structure of the nucleon—important observables for EIC study of nucleon
- Comparison of the work of Bacchetta et al. 2008 & 2019 & Chen 2017 and others Vladimirov et al. and Gao/Stewart
- … stay tuned…