September 26th, 2023
Durham

25th International Spin Symposium
SPIN-2023

Modeling and simulation of quark spin effects in e^+e^- annihilation to hadrons

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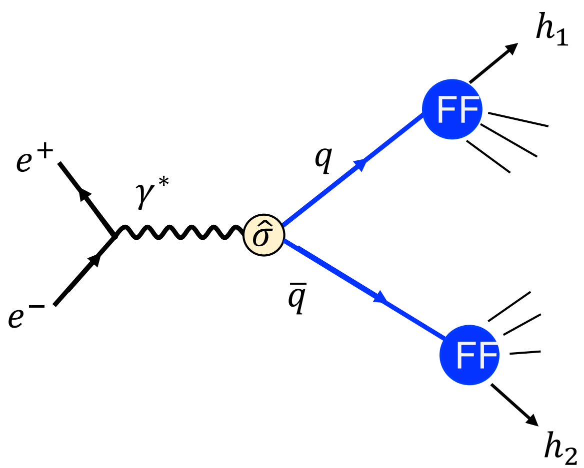
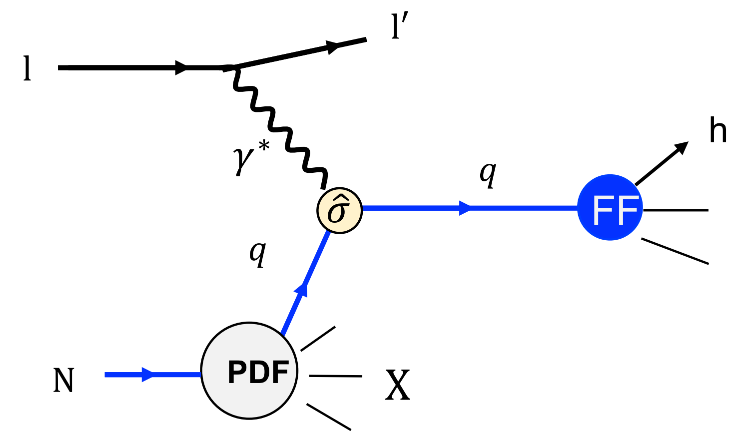
in collaboration with Xavier Artru, Leif Lönnblad and Anna Martin

POLFRAG

Studying hadronization





- ❑ e^+e^- annihilation to hadrons
  access to FFs x FFs

- ❑ Semi Inclusive DIS
  access to PDFs x FFs

Possibilities for studying hadronization

- ❑ Phenomenological fits — parametrize FFs (and PDFs), extract from data
  e.g. extraction of Collins FF (or IFF) and transversity

- ❑ Modeling — analytic calculations, or
  Monte Carlo event generators (need a sound model)
  develop a model, implement in a program, compare the results with the data, make predictions

Modeling hadronization: the string+$^3P_0$ model

- We have developed a model for the simulation of the fragmentation polarized quarks

→ **string+$^3P_0$ model:** extension of the Lund string fragmentation model to include the quark spin

AK, Artru, Belghobsi, Bradamante, Martin, PRD 97, 074010 (2018)	2018	PS mesons
AK, Artru, Belghobsi, Martin, PRD 100, 014003 (2019)	2019	PS mesons
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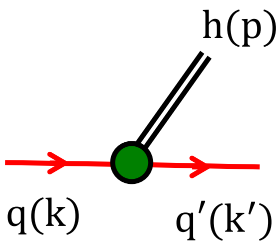

$\mathbf{k}_T = \mathbf{p}_T + \mathbf{k}'_T$
$Z_+ = p^+/k^+$

Quark splitting described by a 2x2 splitting amplitude

$$T_{q',h,q} \propto \left[F^{\text{Lund}}_{q',h,q}(Z_+, \mathbf{p}_T; \mathbf{k}_T)\right]^{1/2} \, [\mu + \sigma_z \boldsymbol{\sigma}_T \cdot \mathbf{k}'_T] \; \Gamma_{h,s_h}$$

$^3P_0$ mechanism
μ complex mass paramter

Coupling
e.g.
$\Gamma_{h=PS} = \sigma_z$

$\text{Im}(\mu)$ → T spin effects (Collins, dihadron)
$\text{Re}(\mu)$ → L spin effects ($G_1^{\perp}$..)

Coupling of quarks to h

PS mesons	no free parameters
Vector Mesons	f_L fraction of L polarized mesons
	θ_{LT} oblique polarization

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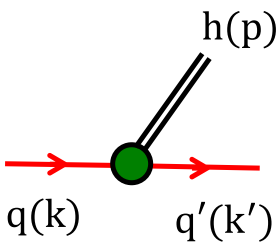

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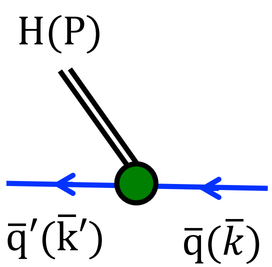

$$\bar{\mathbf{k}}_T = \mathbf{P}_T + \bar{\mathbf{k}}'_T$$
$$Z_- = P^-/\bar{k}^-$$

For anti-quark splitting

$$\{q, h, q'\} \to \{\bar{q}, H, \bar{q}'\}, \;\; Z_+ \to Z_-, \;\; \{\mathbf{k}_T, \mathbf{p}_T, \mathbf{k}'_T\} \to \{\bar{\mathbf{k}}_T, \mathbf{P}_T, \bar{\mathbf{k}}'_T\}$$

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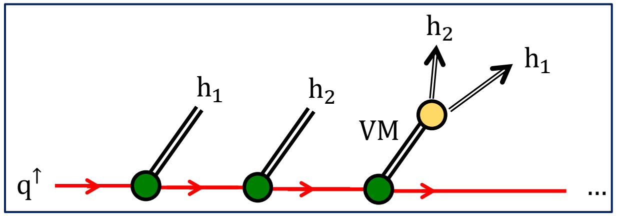
- Applied to the description of
  - SIDIS: polarized fragmentation quarks of struck quarks
    polarization of remnant neglected
    implemented in Pythia via StringSpinner (public)

AK, L. Lönnblad, CPC **272** (2022) 108234;
CPC **292** (2023) 108886



→ promising description of transverse-spin asymmetry data

*see most recent version including PS + VM production CPC **292** (2023) 108886*

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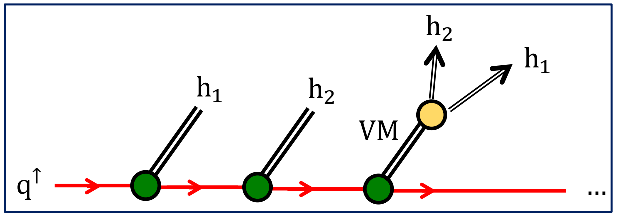

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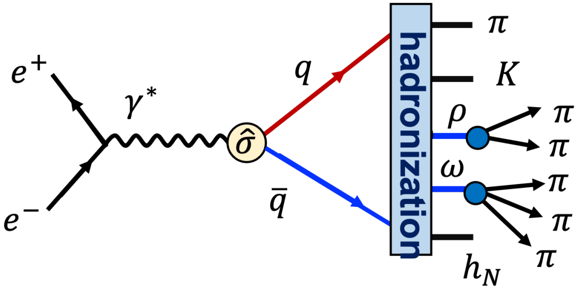
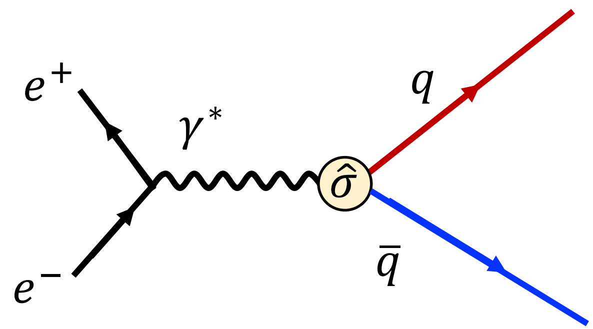
- e^+e^- annihilation to hadrons

hadronize $q\bar{q}$ using the string+$^3P_0$ model accounting for
correlated spin states of q and $\bar{q}$
quantum mechanical spin-correlations in fragmentation

in collaboration with X. Artru



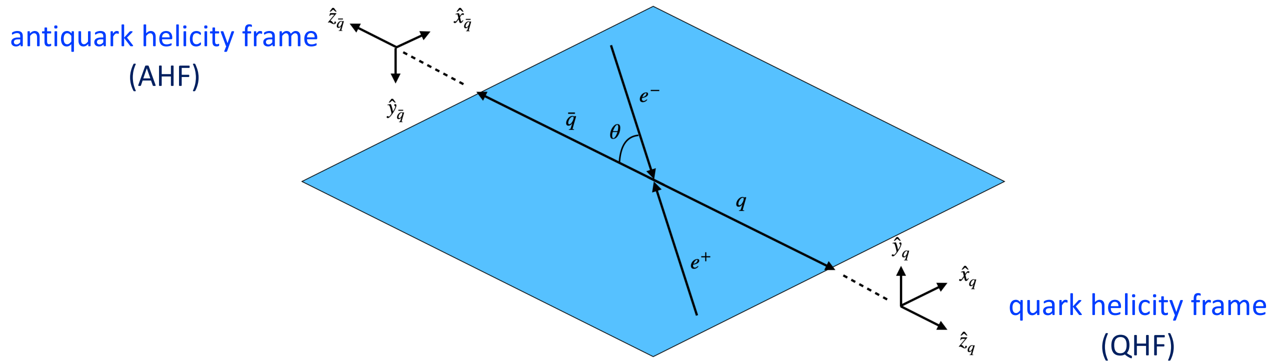
The recursive recipe for simulating e^+e^- annihilation



Steps:
1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from q
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. Exit condition

Set up the scattering $e^+e^- \rightarrow q\bar{q}$ in the c.m.s

generate the quark flavors and kinematics using differential cross section



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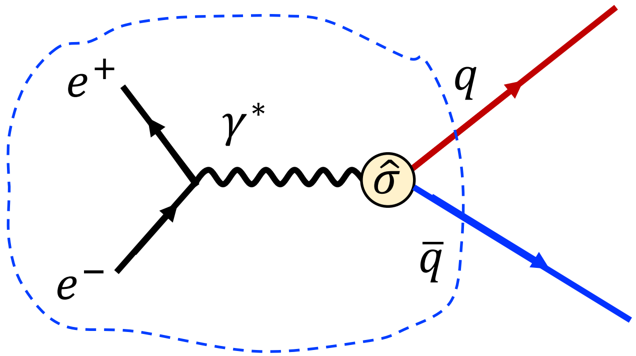


$\rho(q,\bar{q})$

- Set up the joint spin density matrix of the $q\bar{q}$ pair

$$\rho(q,\bar{q}) = C^{q\bar{q}}_{\alpha\beta} \quad \sigma^\alpha_q \otimes \sigma^\beta_{\bar{q}}$$

correlation coefficients

Pauli matrices along QHF and AHF

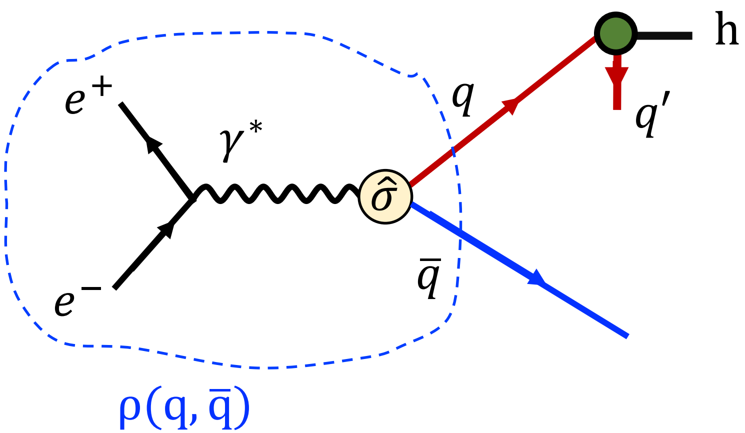
$\alpha = 0, x_q, y_q, z_q$

$\beta = 0, x_{\bar{q}}, y_{\bar{q}}, z_{\bar{q}}$

For γ^* exchange

$$\rho(q,\bar{q}) \propto 1_q \otimes 1_{\bar{q}} - \sigma^z_q \otimes \sigma^z_{\bar{q}} + \frac{\sin^2\theta}{1+\cos^2\theta}\left[\sigma^x_q \otimes \sigma^x_{\bar{q}} + \sigma^y_q \otimes \sigma^y_{\bar{q}}\right]$$

The recursive recipe for simulating e^+e^- annihilation



Steps:
1. Hard scattering
2. Joint spin density matrix
3. **Hadron emission from q**
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. Exit condition

- ❑ Emit the first hadron using the splitting function (emission probability density)

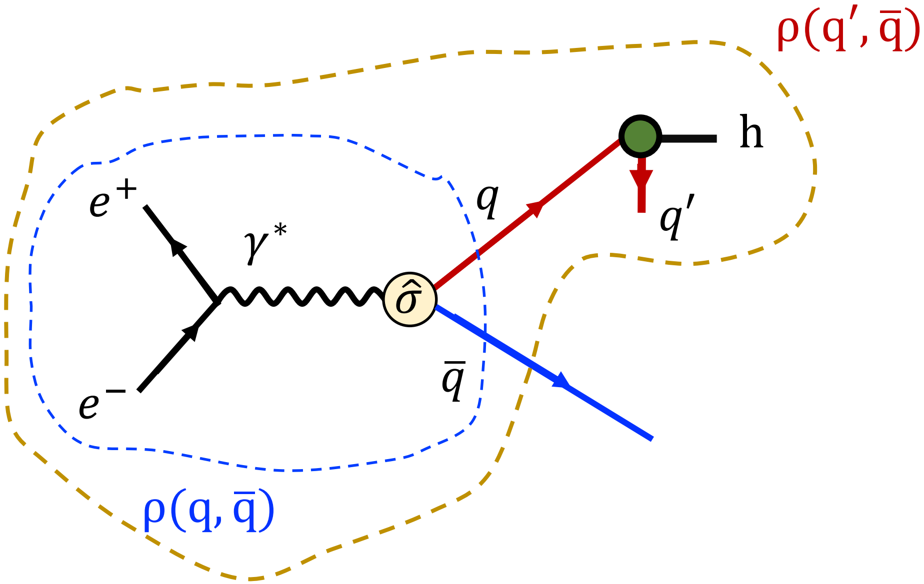
$$\frac{dP(q \to h + q'; q\bar{q})}{dZ_+ Z_+^{-1} d^2 p_T} = \mathrm{Tr}_{q'\bar{q}} \mathbf{T}_{q',h,q}\, \rho(q,\bar{q})\, \mathbf{T}^{\dagger}_{q',h,q} = F_{q',h,q}(Z_+, \mathbf{p}_T; \mathbf{k}_T, C^{q\bar{q}})$$

$$\mathbf{T}_{q',h,q} \equiv T_{q',h,q} \otimes 1_{\bar{q}}$$

in the QHF

- ❑ VM emission → backup

The recursive recipe for simulating e^+e^- annihilation



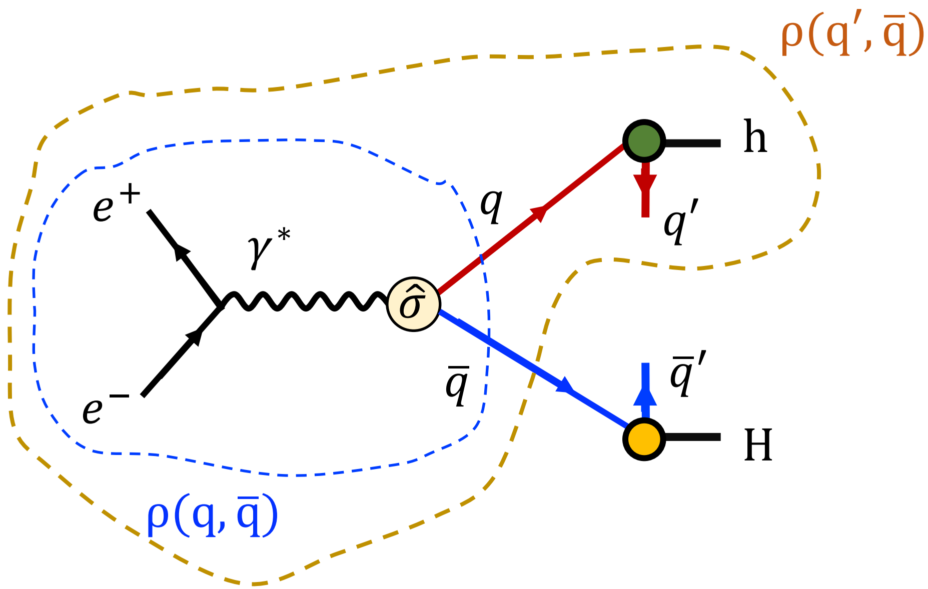
Steps:
1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from q
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. Exit condition

❑ Evaluate the spin density matrix $\rho(q'\bar{q})$

$$\rho(q', \bar{q}) = \mathbf{T}_{q',h,q} \; \rho(q, \bar{q}) \; \mathbf{T}^{\dagger}_{q',h,q}$$

includes the information on the emission of h

The recursive recipe for simulating e^+e^- annihilation



Steps:
1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from q
4. Update density matrix
5. **Hadron emission from $\bar{q}$**
6. Exit condition

- ❑ Emit a hadron from the $\bar{q}$ side using the splitting function

Depend on the azimuthal angle h

$$\frac{dP(\bar{q} \to H + \bar{q}'; q'\bar{q})}{dZ_- Z_-^{-1} d^2 P_T} = \mathrm{Tr}_{q'\bar{q}'} \mathbf{T}_{\bar{q}',H,\bar{q}}\, \rho(q', \bar{q})\, \mathbf{T}^{\dagger}_{\bar{q}',H,\bar{q}} = F_{\bar{q}',H,\bar{q}}(Z_-, P_T; \bar{k}_T, C^{q'\bar{q}})$$

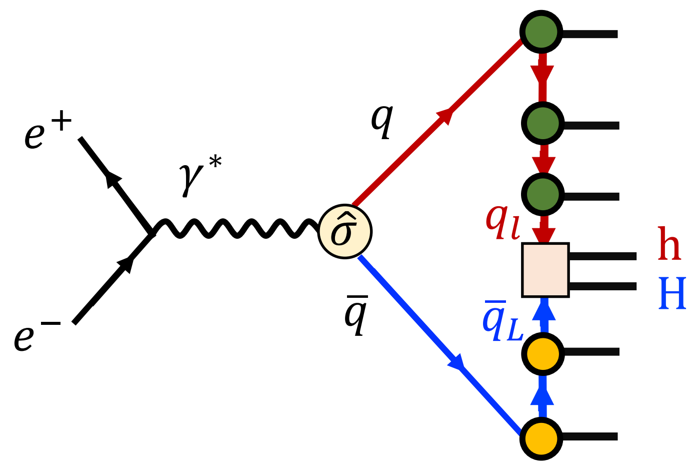
Expressed in the AHF

conditional probability of emitting H, having emitted h

→ correlations between the transverse momenta

[Collins NPB, 304:794–804, 1988, Knowles NPB, 310:571–588, 1988]

The recursive recipe for simulating e^+e^- annihilation



Steps:
1. Hard scattering
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3. Hadron emission from q
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. **Exit condition**

- ❑ Iterate until the exit condition is called and the last quark pair is hadronized

  more details in backup

Simulations of e^+e^- with spin effects

- Now possible in Pythia 8.3 by an extension of the StringSpinner package
- Free parameters as in [AK and L. Lönnblad CPC 292 (2023) 108886], except $f_L = 0.33$
  and $\theta_{LT} = -\pi/6$ (by «eye» tunning)
  OK for both e^+e^- and SIDIS

- Next slides → Collins asymmetries for back-to-back hadrons
  comparison with BELLE data (work ongoing for BaBar and BESIII)

in collaboration with L. Lönnblad and A. Martin

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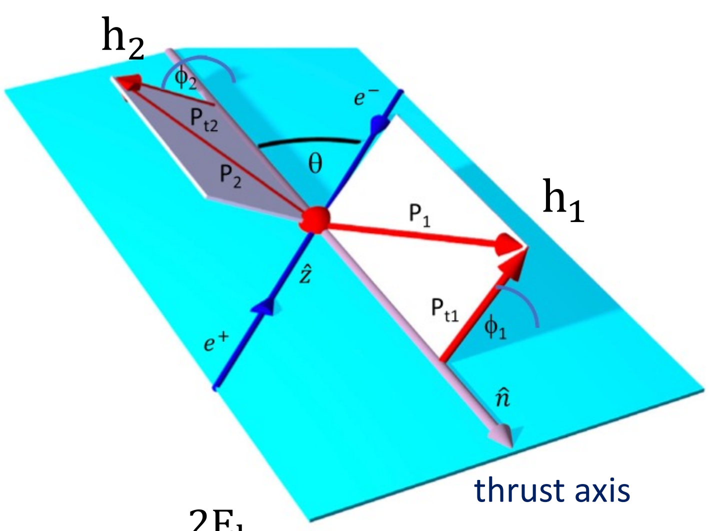
$$d\sigma^{e^+e^-\to h_1h_2X} \propto 1 + \frac{\sin^2\theta}{1+\cos^2\theta} A_{12}\cos(\phi_1+\phi_2)$$

Boer, NPB, 806:23–67, 2009
D'Alesio et al., JHEP 10 (2021) 078

$$A_{12}(z_1,z_2,P_{T1},P_{T2}) = \frac{\sum_q e_q^2\; H_{1q}^{\perp h_1}(z_1,P_{T1}) H_{1\bar{q}}^{\perp h_2}(z_2,P_{T2})}{\sum_q e_q^2\; D_{1q}^{h_1}(z_1,P_{T1}) D_{1\bar{q}}^{h_2}(z_2,P_{T2})}$$



The measured Collins asymmetry is obtained from ratios of normalized yields $R_{12}^{U,L}$ for Unlike-charge (U) and Like-charge (L) back-to-back pion pairs

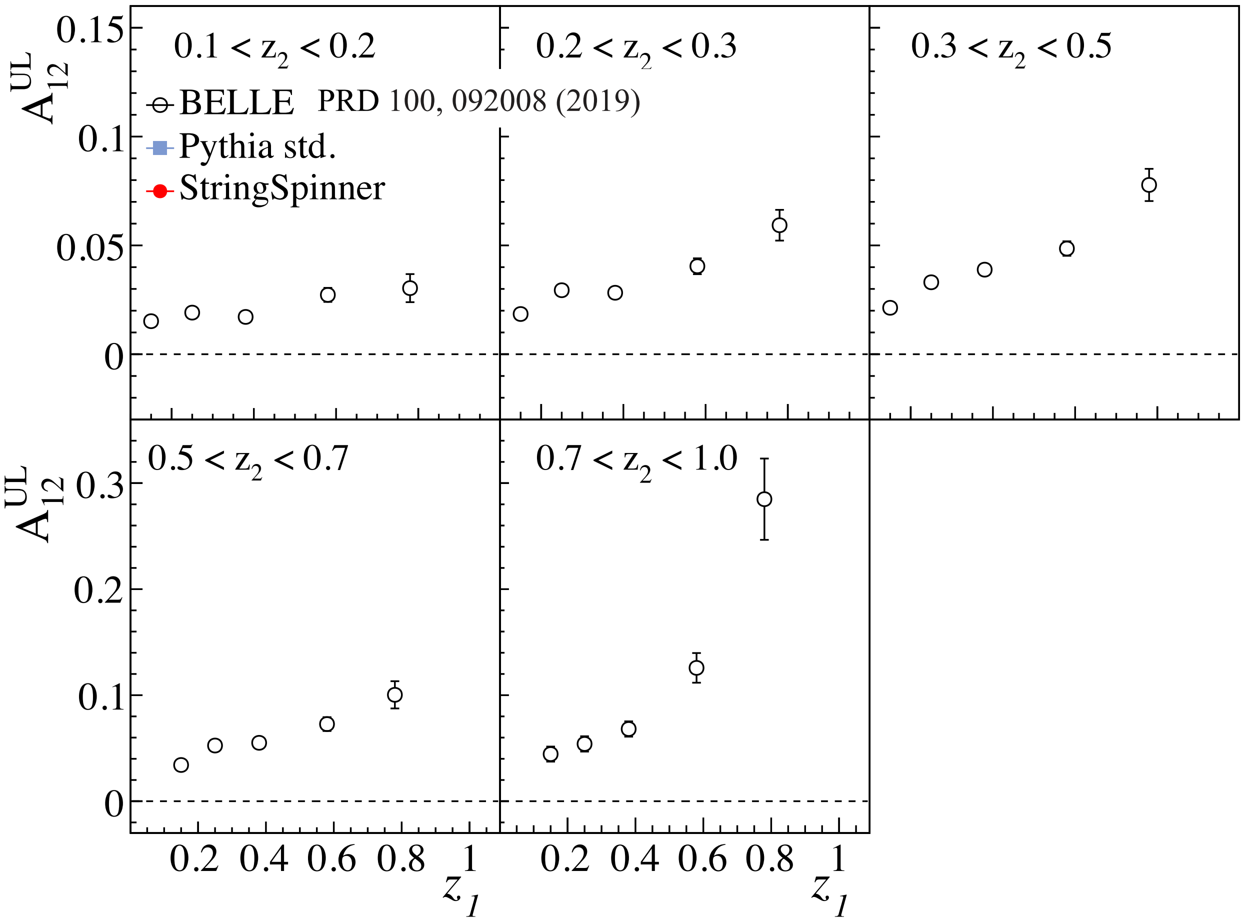
$$R_{12}^{UL} = \frac{R_{12}^{U}}{R_{12}^{L}} \approx 1 + \frac{\sin^2\theta}{1+\cos^2\theta} A_{12}^{UL}\cos(\phi_1+\phi_2)$$

$$\to A_{12}^{UL} = A_{12}^{U} - A_{12}^{L}$$

$$z_i = \frac{2E_{h_i}}{\sqrt{s}}$$

A_{12}^{UL} asymmetry for back-to-back $\pi^{\pm} - \pi^{\mp}$

$z_1 \times z_2$ - dependence



Asymmetries w.r.t thrust axis (not $q\bar{q}$ axis)

$T > 0.8$

$z > 0.1, P_T < 3.0$ GeV/c

$\alpha_O < 0.3$

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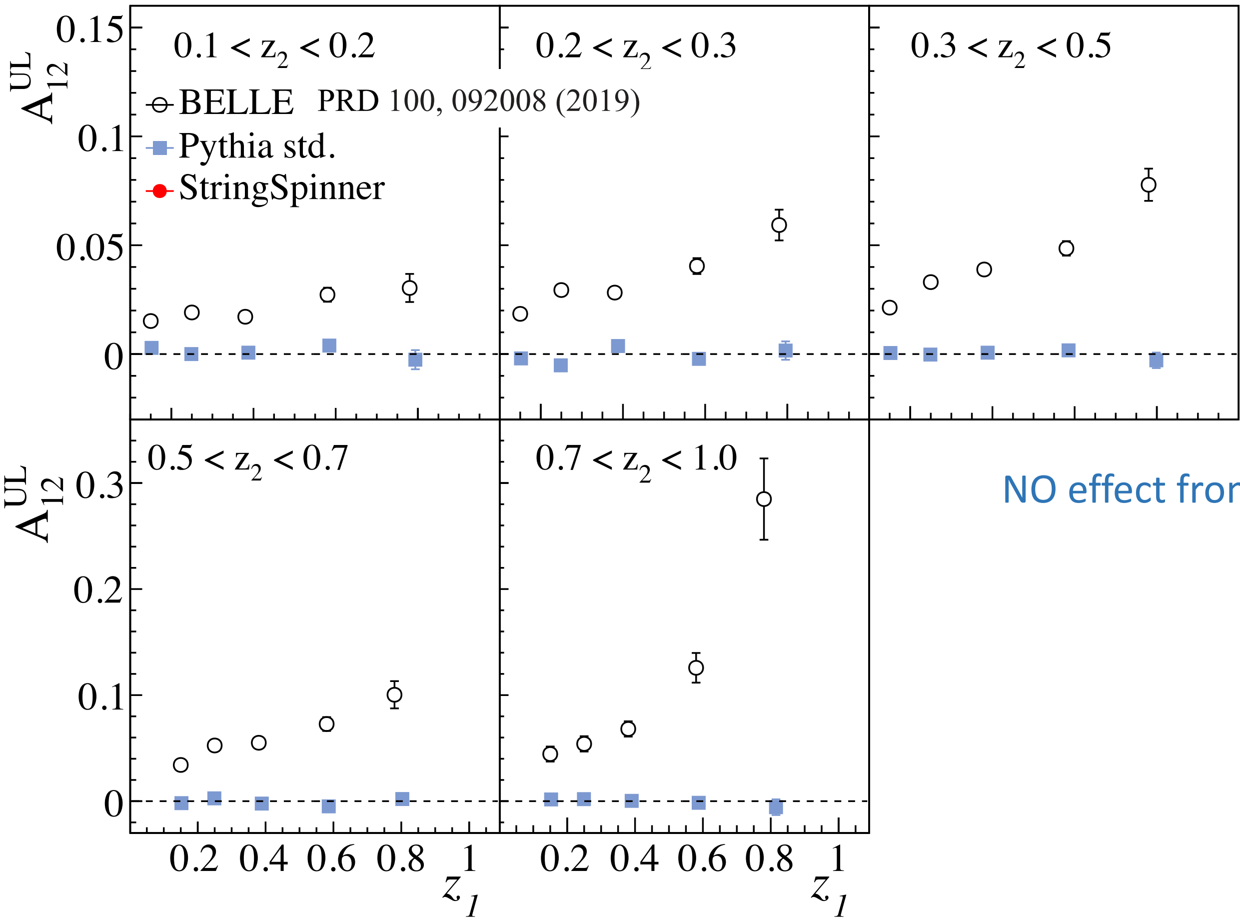

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NO effect from Pythia

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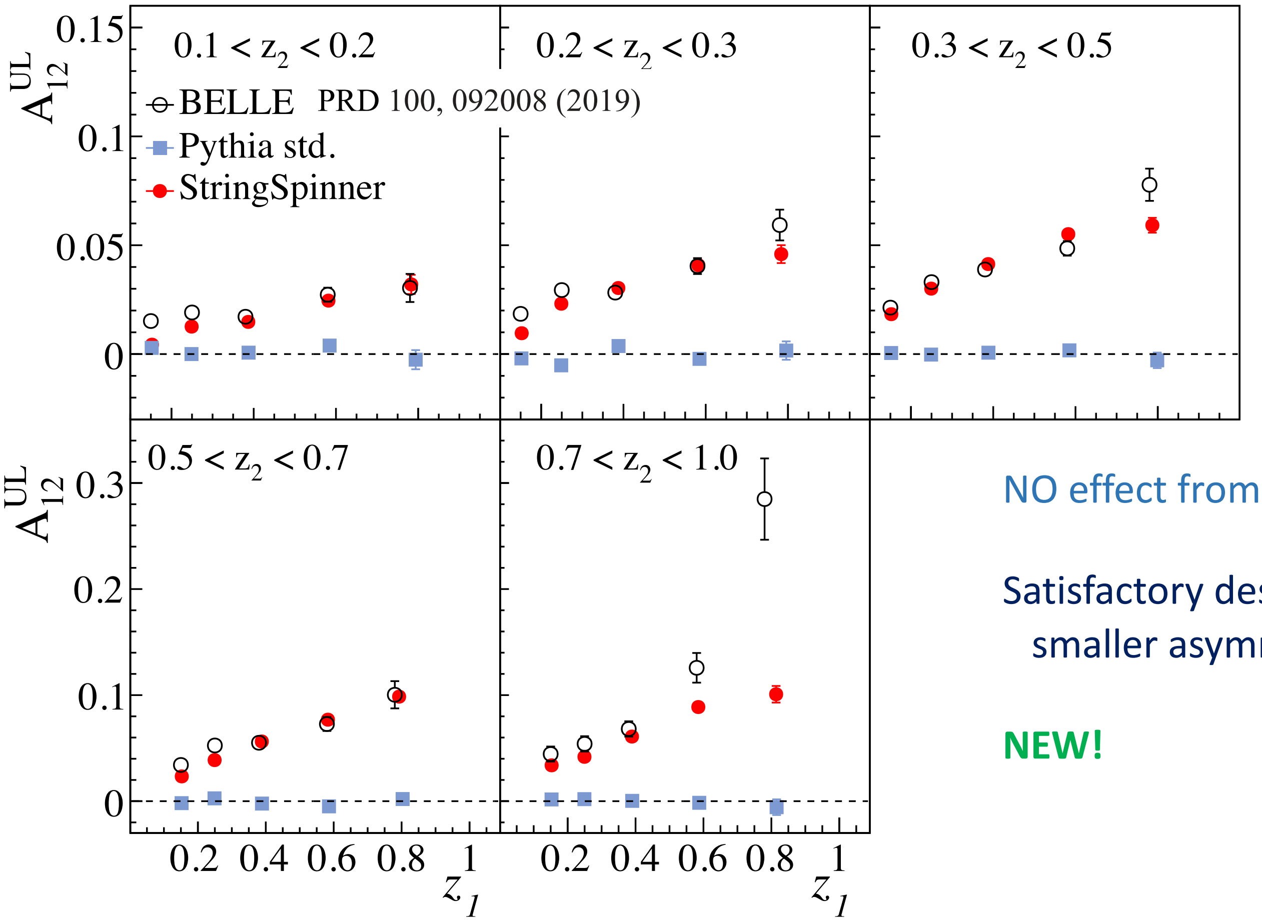

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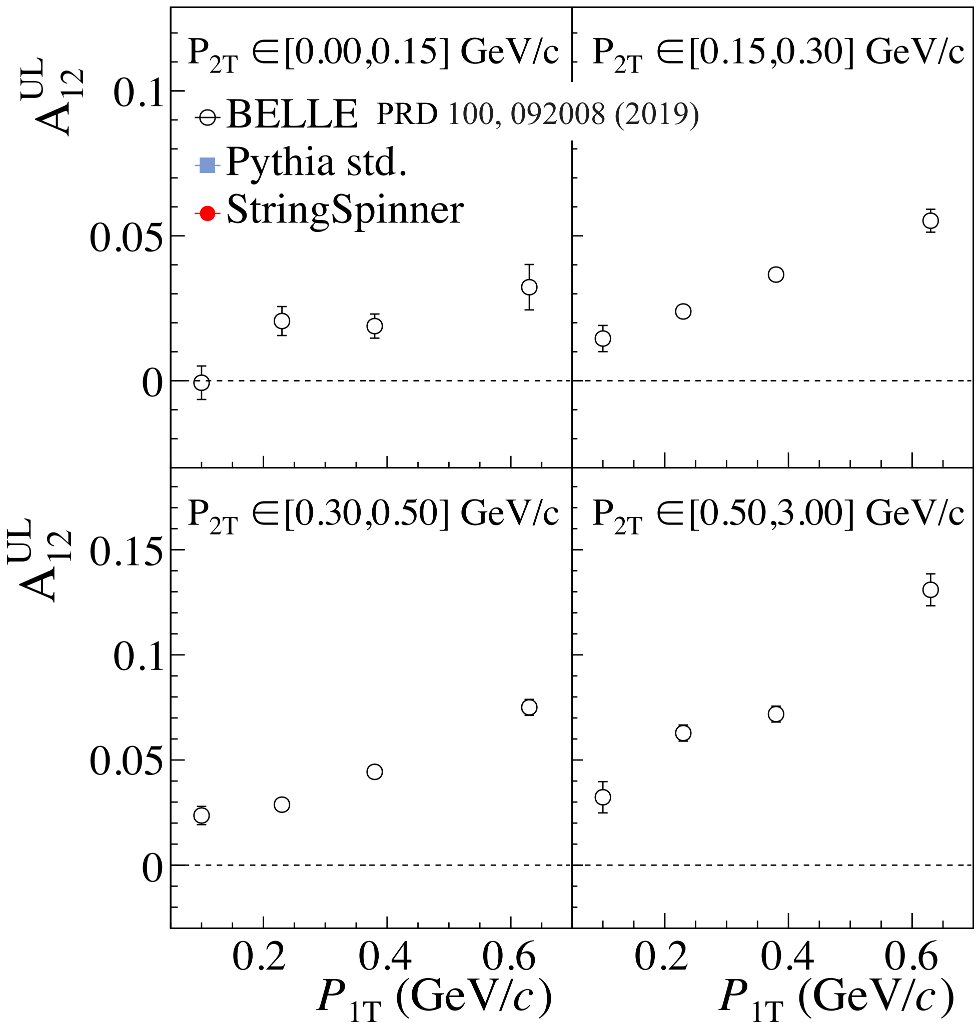
Satisfactory description from StringSpinner
smaller asymmetries at large z

NEW!

A_{12}^{UL} asymmetry for back-to-back $\pi^{\pm} - \pi^{\mp}$

$P_{T1} \times P_{T2}$ - dependence



Asymmetries w.r.t thrust axis (not $q\bar{q}$ axis)

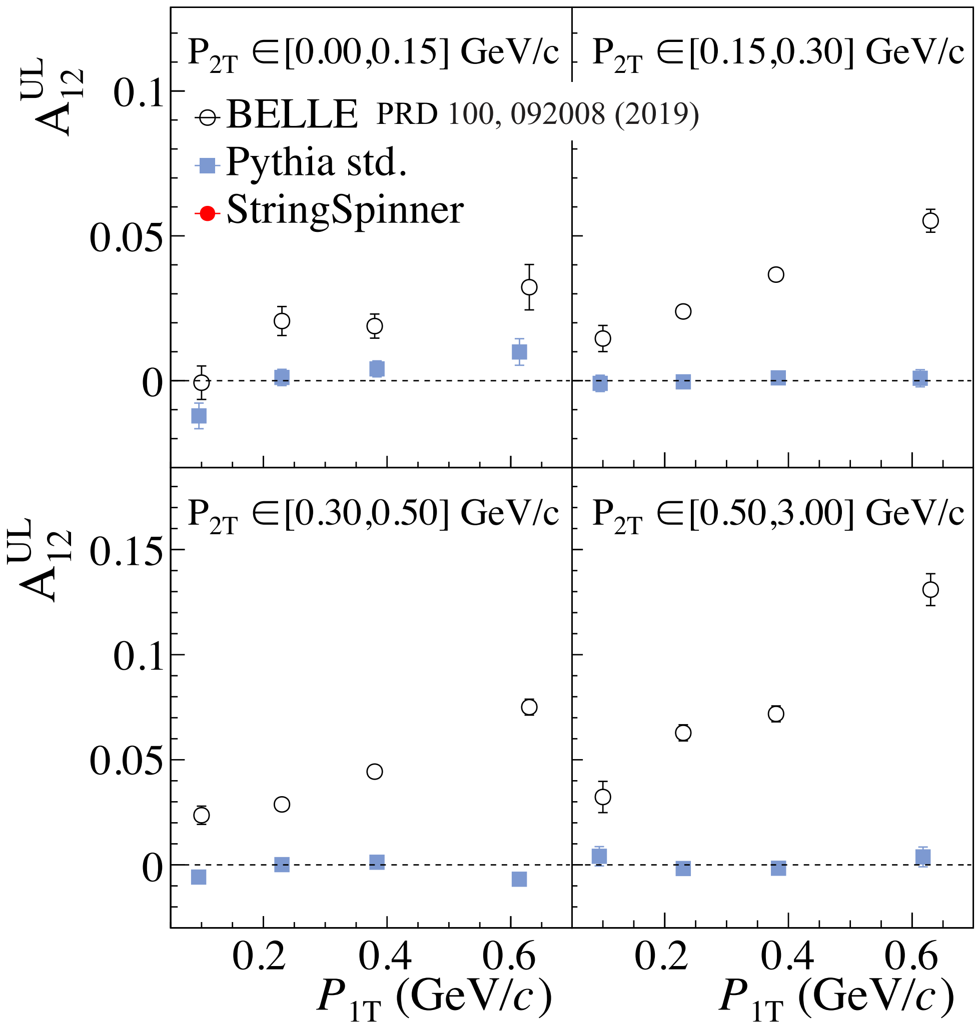
$T > 0.8$

$z > 0.2, P_T < 3.0$ GeV/c

$\alpha_O < 0.3$

A_{12}^{UL} asymmetry for back-to-back $\pi^{\pm} - \pi^{\mp}$

$P_{T1} \times P_{T2}$ - dependence



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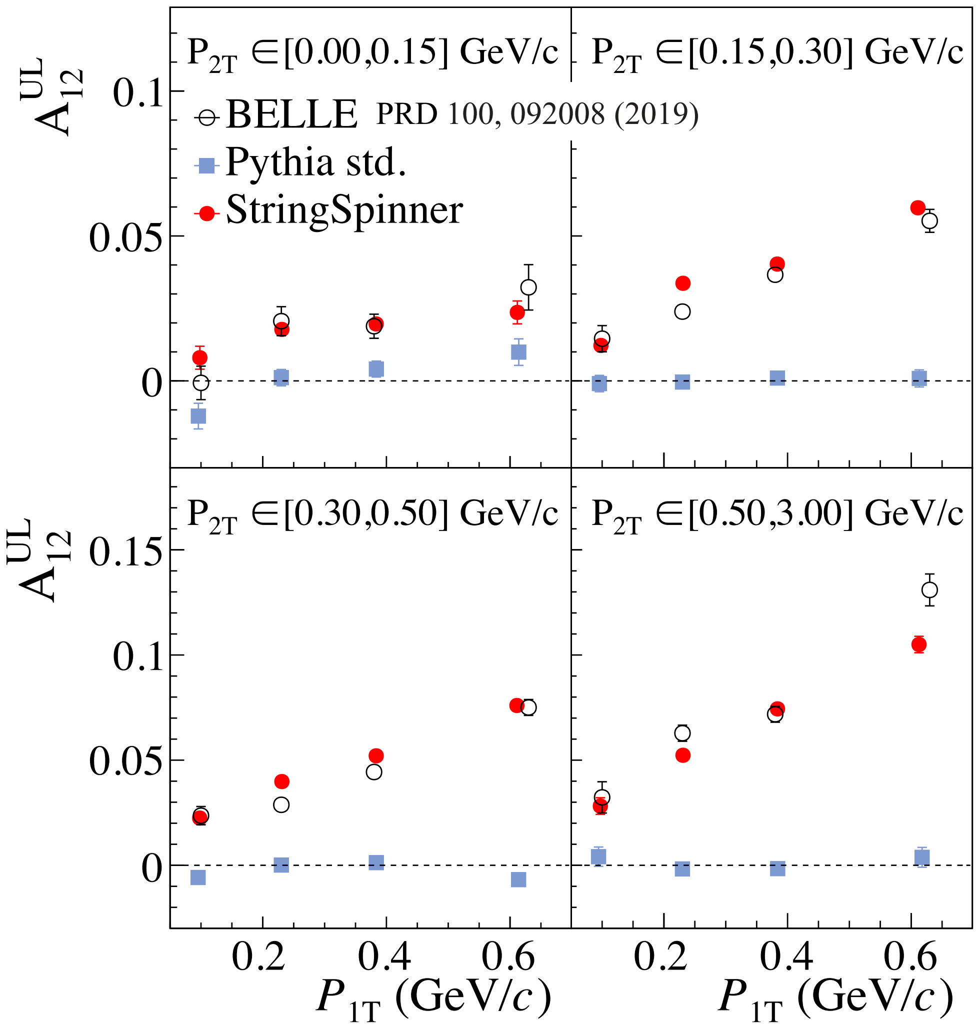
Pythia results consistent with zero

A_{12}^{UL} asymmetry for back-to-back $\pi^{\pm} - \pi^{\mp}$

$P_{T1} \times P_{T2}$ - dependence



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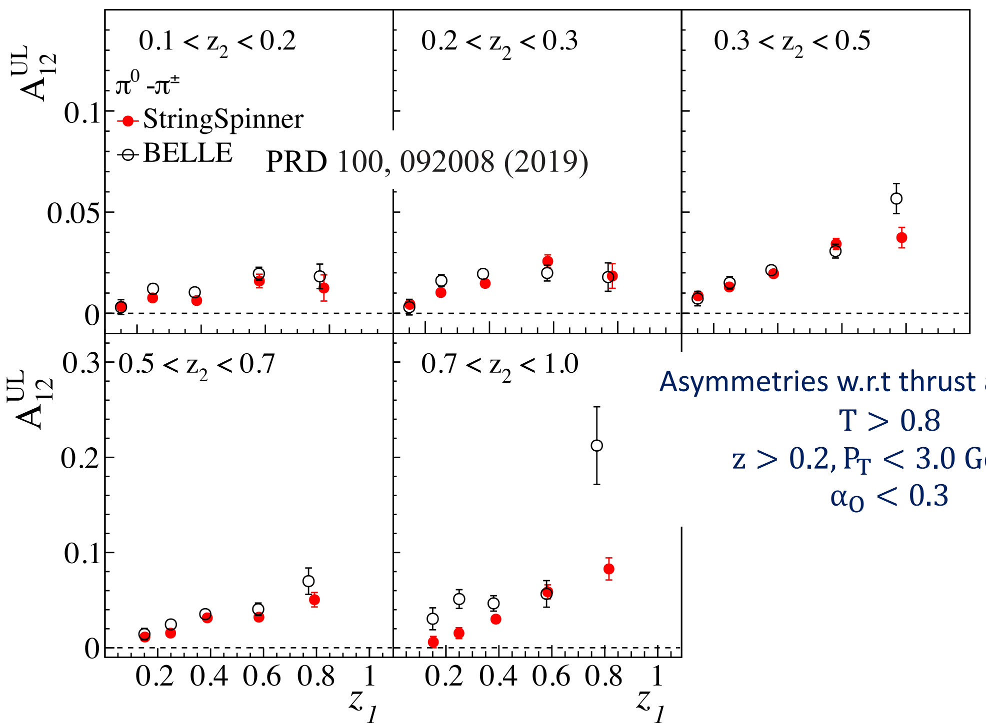
Pythia results consistent with zero

Satisfactory description from StringSpinner!

~linear trend in simulations shows up as an effect of the misalignment of thrust axis compared to $q\bar{q}$ axis (backup)

A_{12}^{UL} asymmetry for back-to-back $\pi^0 - \pi^{\mp}$



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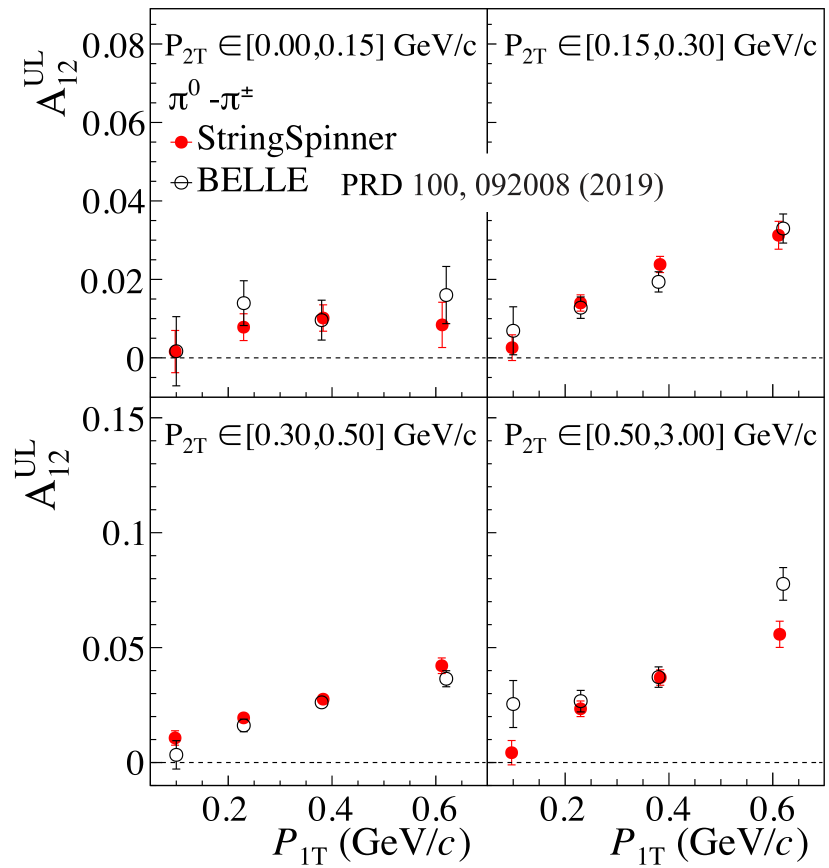


Satisfactory description

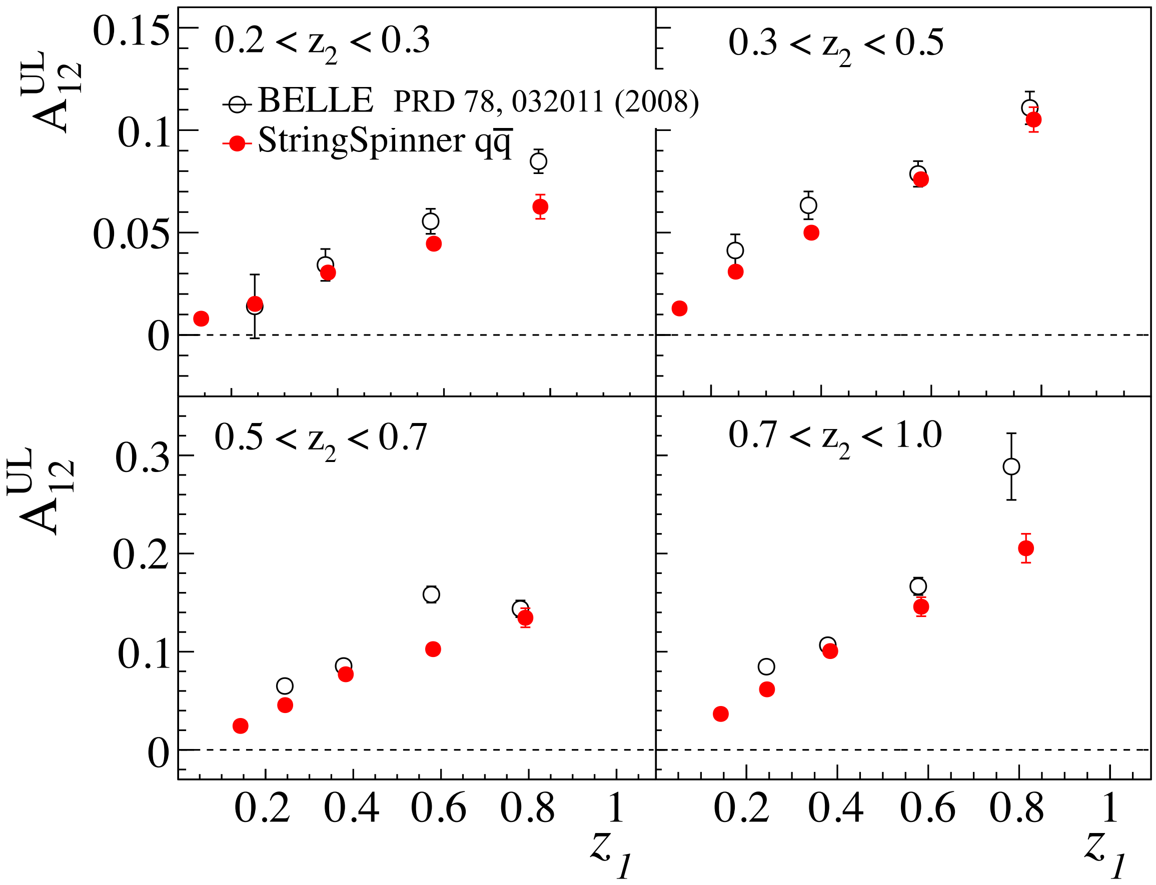
simulated asymmetries at large z lower

Also for the $\eta - \pi^{\mp}$ asymmetries (see backup)

Asymmetries measured w.r.t the thrust axis difficult to describe

A_{12}^{UL} asymmetry for back-to-back $\pi^{\pm} - \pi^{\mp}$ w.r.t $q\bar{q}$ axis

$z_1 \times z_2$ - dependence



Belle asymmetries measured using the thrust axis, then rescaled to $q\bar{q}$ axis

Integrated over P_T

$T > 0.8, z > 0.2$

StringSpinner gives a satisfactory description!

Conclusions

- We generalized the string+$^3P_0$ model of hadronization to $e^+e^- \rightarrow q\bar{q} \rightarrow$ hadrons
  general recipe, can be applied to other production channels of the $q\bar{q}$ pair

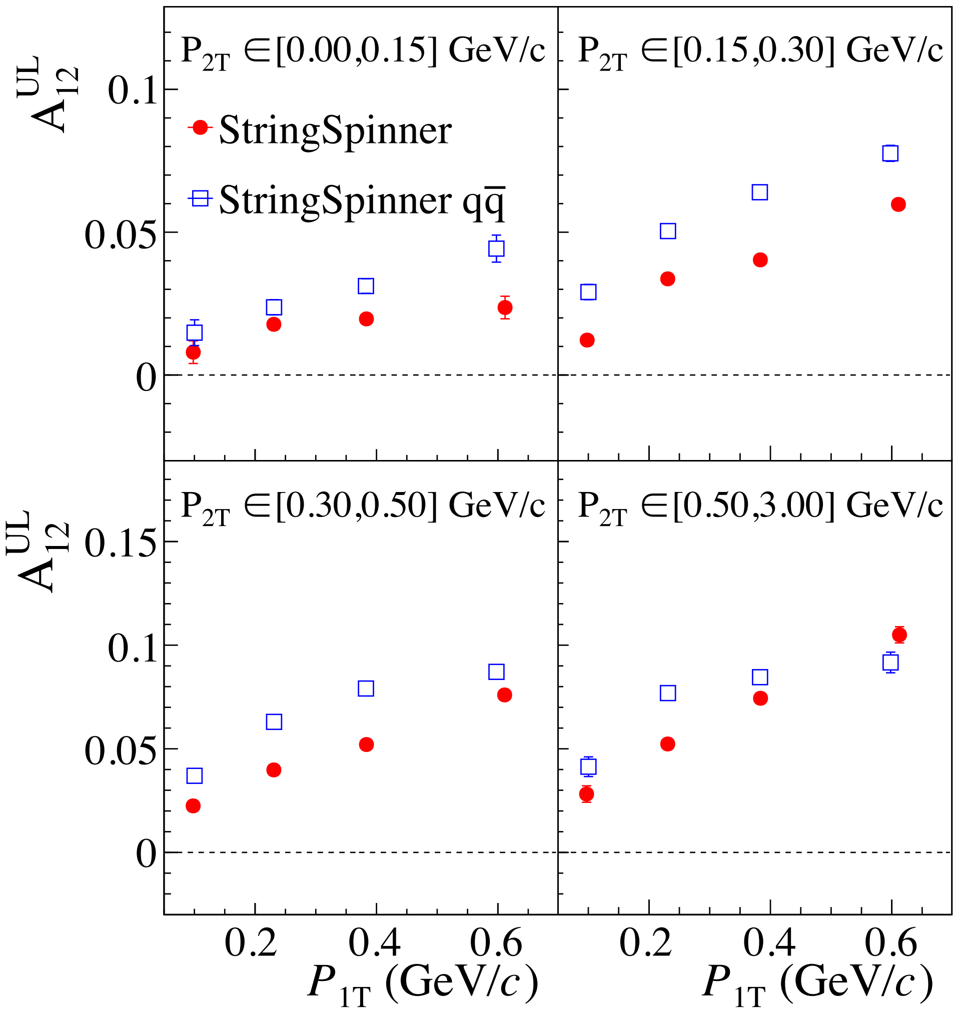
- The model is implemented in Pythia 8.3 by an extension of StringSpinner
  expected to be published soon

- Promising results on the Collins asymmetries for back-to-back hadrons in e^+e^-

- More phenomenological studies ongoing
  comparisons with BaBar and BESIII,
  calculation of the Artru-Collins asymmetries …

Backup

Effect of the thrust axis on the $P_{T1} \times P_{T2}$ - dependence

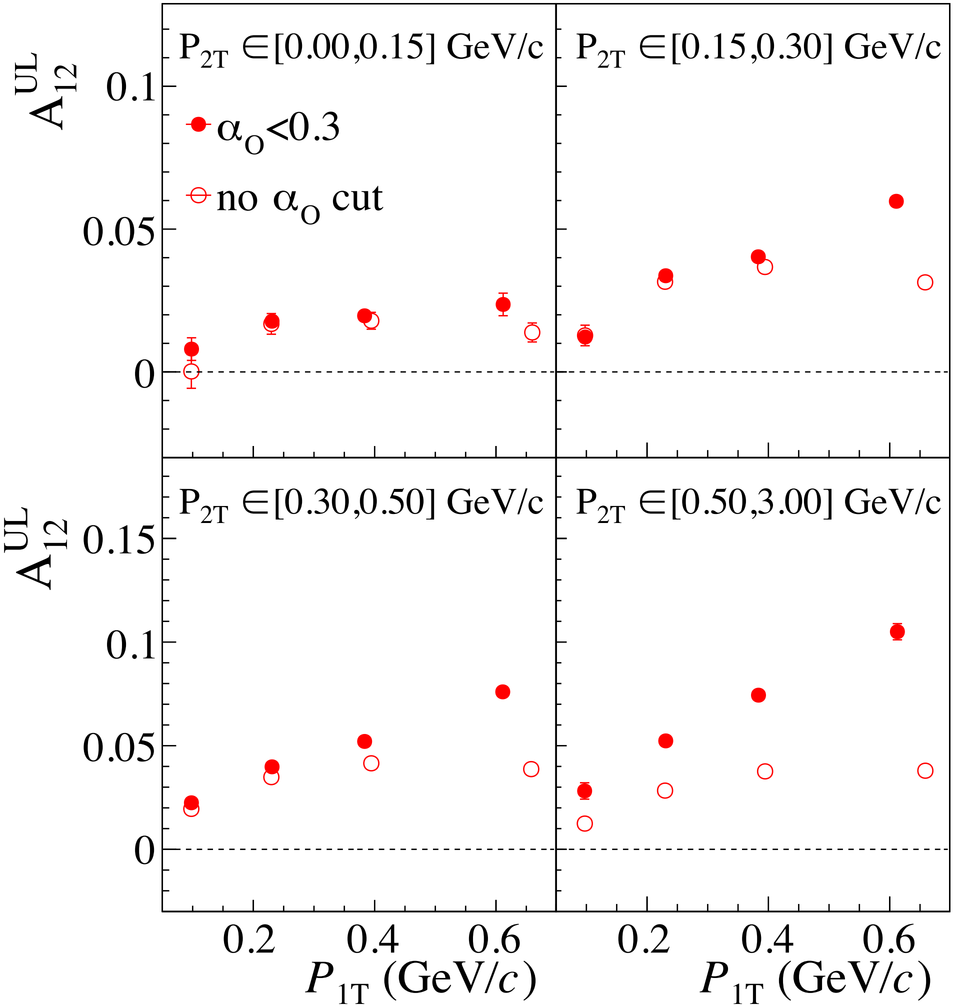


(circles) A_{12}^{UL} asymmetry evaluated using the thrust axis

(squares) A_{12}^{UL} asymmetry evaluated using the $q\bar{q}$ axis

Dilution and shape change when using the thrust axis..

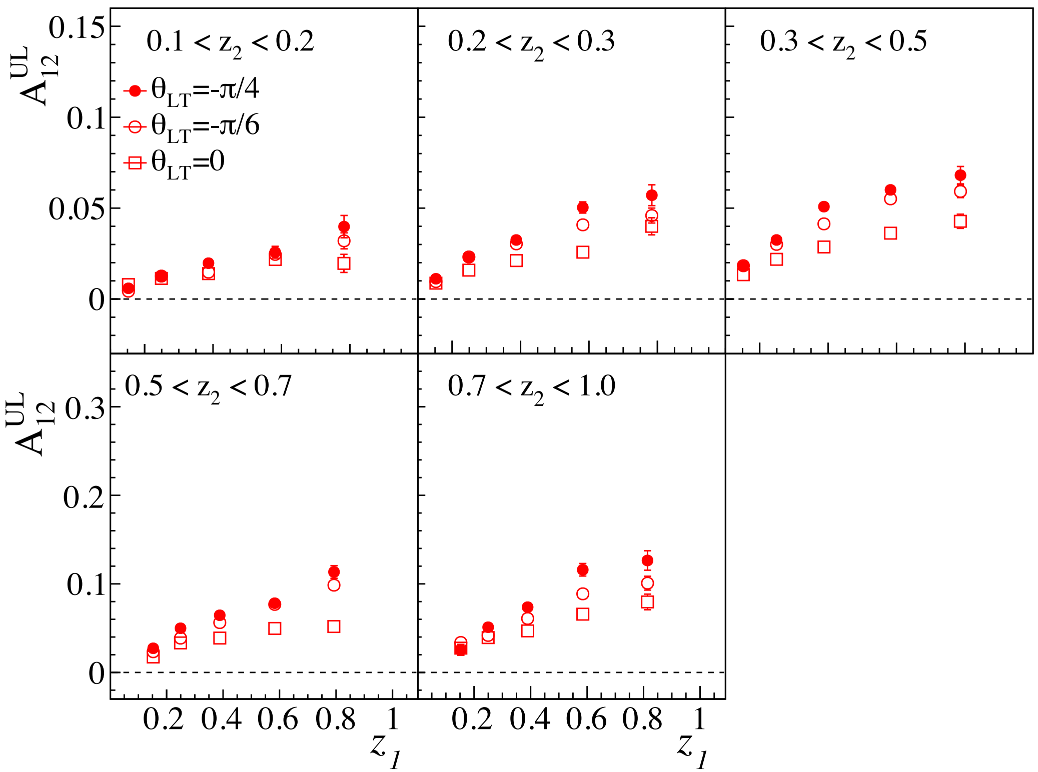
Effect of the opening angle cut on the $P_{T1} \times P_{T2}$ - dependence

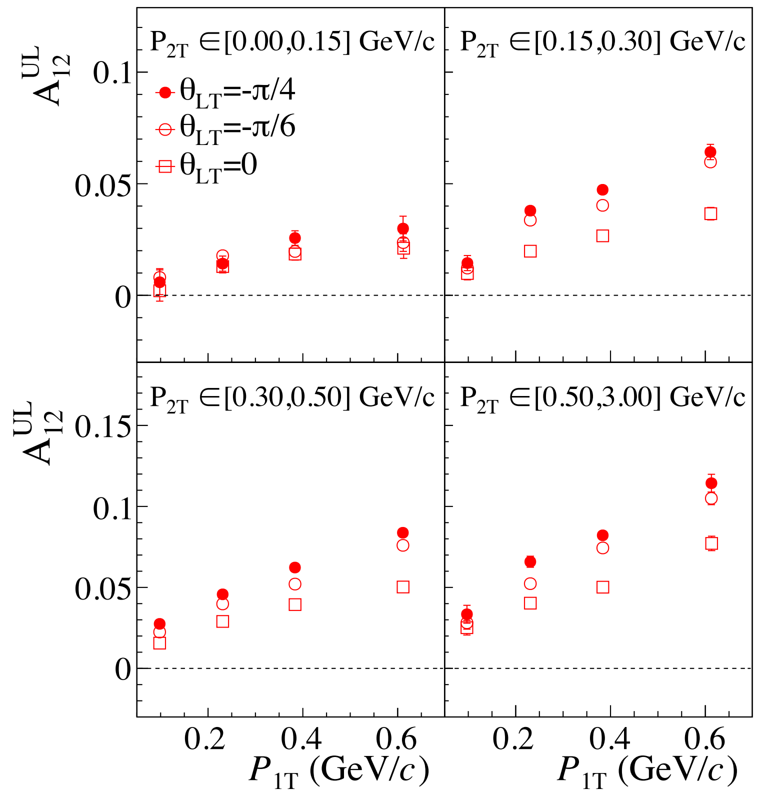


Sizeable effect at large P_T!

Smaller effect when looking as a function of z

Sensitivity of asymmetries to free parameters

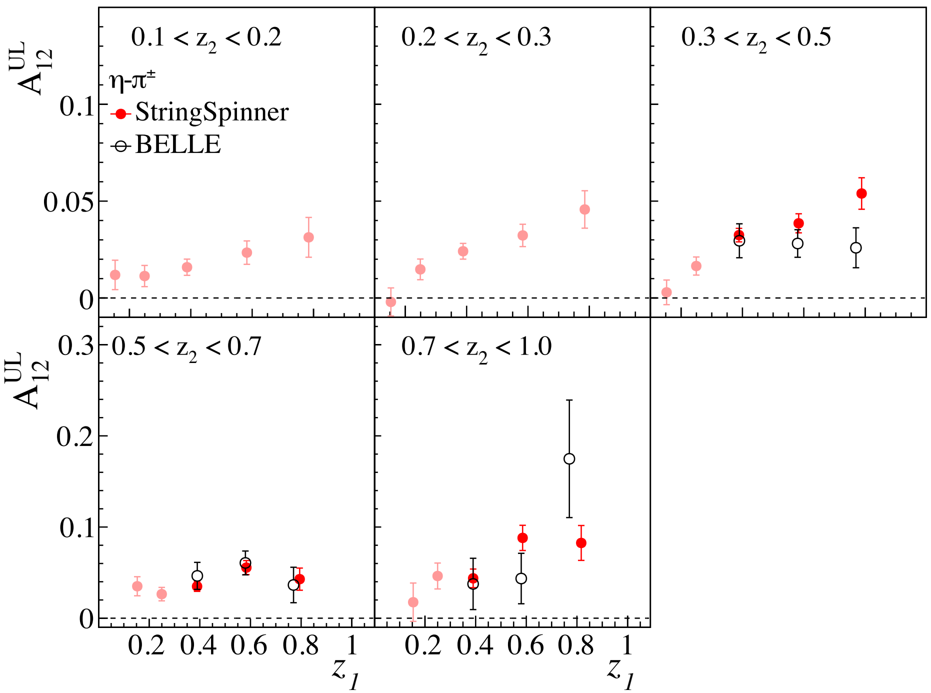


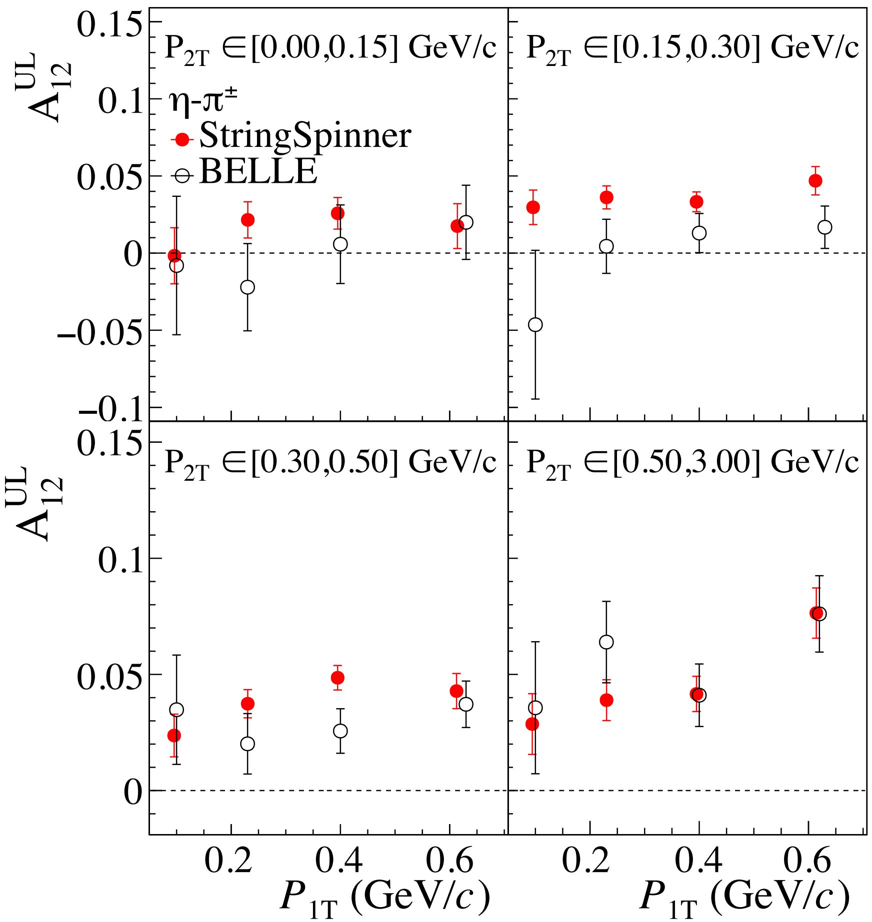


Asymmetries evaluated using the thrust axis

The oblique polarization θ_{LT} is varied, while all other parameters fixed

A_{12}^{UL} asymmetry for $\eta - \pi^{\pm}$





Relevant free parameters for string fragmentation used in simulations

(see AK, L. Lönnblad, arXiv: 2305.05058)

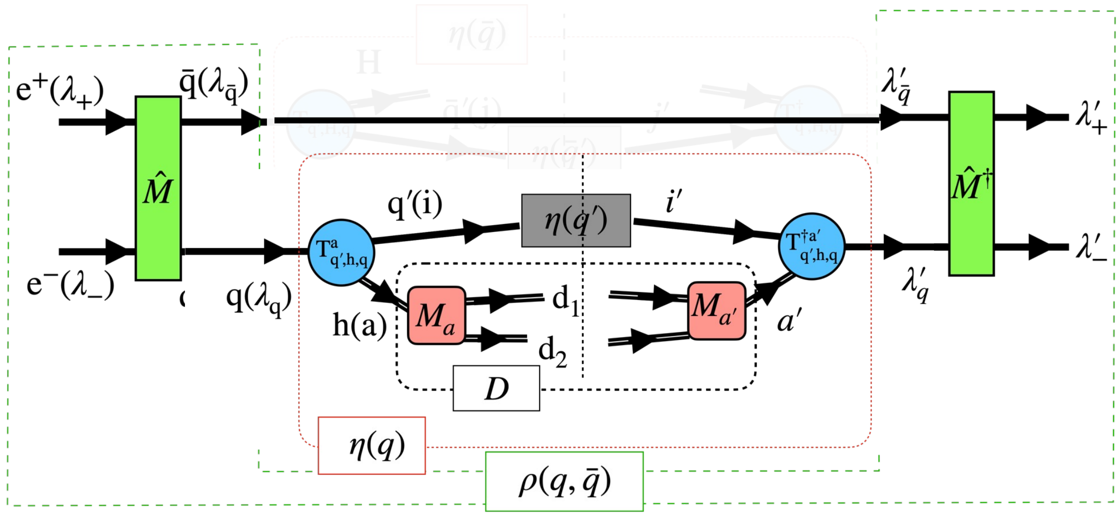
Pythia parameters

StringZ:aLund	default	
StringZ:bLund	default	
StringPT:sigma	default	
StringPT:enhancedFraction	0.0	
StringPT:enhancedWidth	0.0	GeV/c

String+$^3P_0$ parameters

$\mathrm{Re}(\mu)$	0.42	GeV/c^2
$\mathrm{Im}(\mu)$	0.76	GeV/c^2
f_L	0.33	
θ_{LT}	$-\pi/6$	

The recursive recipe for simulating e^+e^- annihilation: VM emission



For a vector meson h=VM

$$\rightarrow \eta(q) = \mathbf{T}^{a'\dagger}_{q',h=VM,q}\, \eta(q')\, \mathbf{T}^{a}_{q',h=VM,q} D_{a'a},\ \eta(q') = 1_{q'} \text{ and } \eta(\bar{q}) = 1_{\bar{q}}$$

Steps:

i) Emission probability density (summing over decay information, i.e. $D_{a'a} = \delta_{a'a}$)

$$\frac{dP(q \to h = VM + q'; q\bar{q})}{dM^2 dZ_+ Z_+^{-1} d^2p_T} = \mathrm{Tr}_{q'\bar{q}}\, \mathbf{T}^a_{q',h,q}\ \rho(q,\bar{q})\ \mathbf{T}^{a\dagger}_{q',h,q} = F_{q',h,q}(M^2, Z_+, p_T; k_T, C^{q\bar{q}})$$

ii) Calculate the spin density matrix of h=VM, and decay the meson

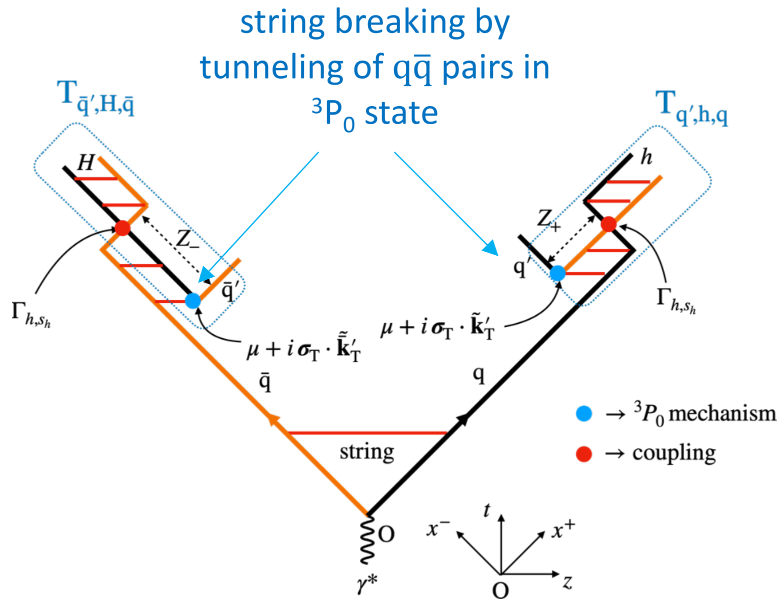
$$\rho_{aa'}(h) = \mathrm{Tr}_{q'\bar{q}}\, \mathbf{T}^a_{q',h,q}\, \rho(q,\bar{q})\, \mathbf{T}^{a'\dagger}_{q',h,q}$$

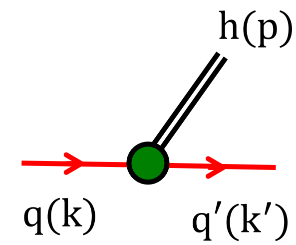
iii) Decay the meson $p \to p_1\, p_2..$

$$dN(p_1, p_2..)/d\Omega \propto M^a_{dec.}(p \to p_1 p_2..)\ \rho_{aa'}(h) M^{\dagger a'}_{dec.}(p \to p_1 p_2..)$$

iv) Build the decay matrix $D_{a'a}(p_1, p_2, ..) = M^{\dagger a'}_{dec.}(p \to p_1 p_2..)\ M^a_{dec.}(p \to p_1 p_2..)$

The hadronization model: **string+$^3P_0$**





quark splitting $q \to h + q'$

Relevant variables:
$\mathbf{k}_T = \mathbf{p}_T + \mathbf{k}'_T$
$Z_+ = p^+/k^+$
$\varepsilon_h^2 = M^2 + p_T^2$

Transverse vectors defined w.r.t. string axis

Quark splitting amplitude in the string+$^3P_0$ model

$$T_{q',h,q} \propto C_{q',h,q}\; D_h(M^2) \left(\frac{1-Z_+}{\varepsilon_h^2}\right)^{\frac{a}{2}} \exp\left[-\frac{b_L\varepsilon_h^2}{2Z_+}\right] N_a^{-\frac{1}{2}}(\varepsilon_h^2)\; e^{-\frac{b_T k'^2_T}{2}} \quad [\mu + \sigma_z \boldsymbol{\sigma}_T \cdot \mathbf{k'}_T] \quad \Gamma_{h,s_h}$$

flavor — mass — longitudinal momentum — transverse momentum (w.r.t string axis) — $^3P_0$ mechanism [μ complex mass paramter] — Coupling e.g. $\Gamma_{h=PS} = \sigma_z$

Free param. Lund

Free param. string+$^3P_0$

AK, Artru, Martin, PRD 104, 114038 (2021)

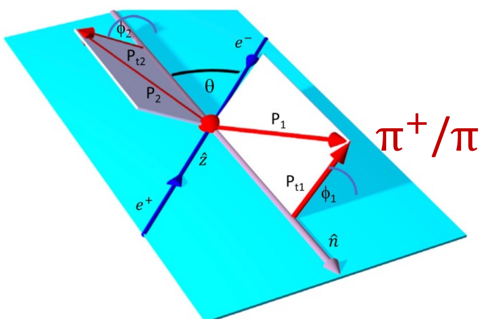
Steps for the extraction of Collins asymmetries

Example of $e^+e^- \rightarrow \pi^+\pi^- X$

i) Evaluate normalized yields for $\pi^\pm - \pi^\mp$ "Unlike pairs"

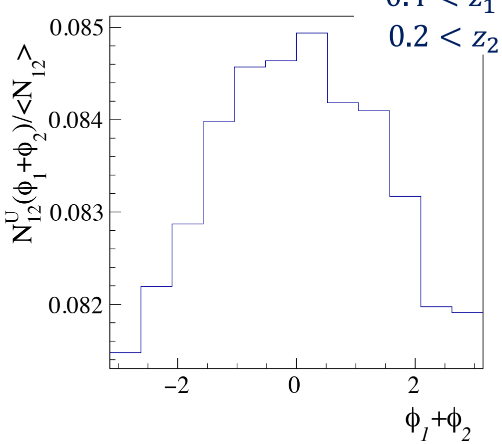
$$R^U_{12} = \frac{N^U_{12}(\phi_1+\phi_2)}{\langle N_{12}\rangle}$$

π^-/π^+



π^+/π^-

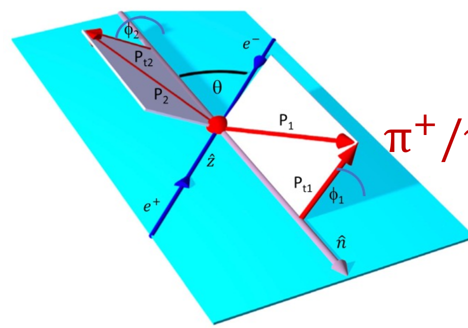
$0.4 < z_1 < 0.5$
$0.2 < z_2 < 1.0$



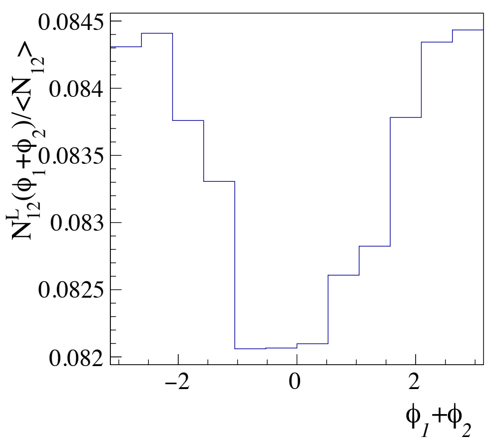
ii) Evaluate normalized yields for $\pi^\pm - \pi^\pm$ "Like pairs"

$$R^L_{12} = \frac{N^L_{12}(\phi_1+\phi_2)}{\langle N_{12}\rangle}$$

π^+/π^-



π^+/π^-

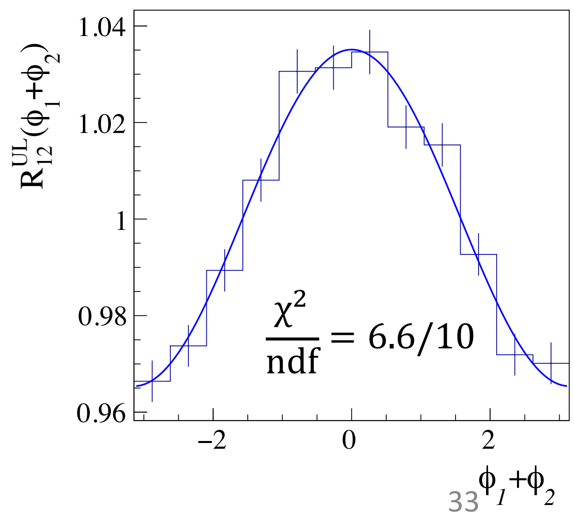


ii) Evaluate the ratio $\frac{R^U_{12}}{R^L_{12}}$ and fit the asymmetry

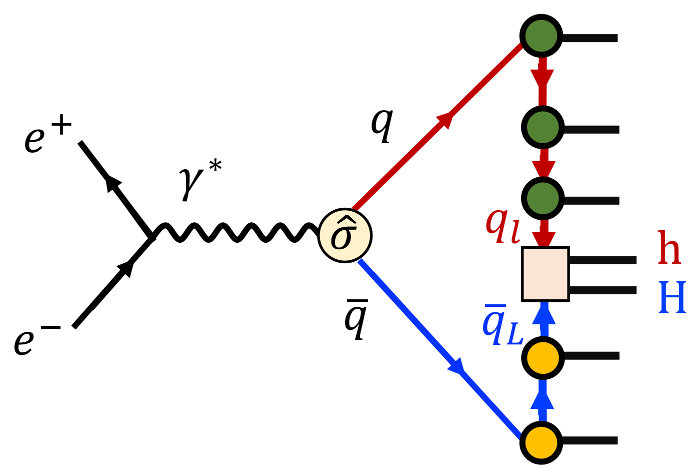
$$R^{UL}_{12} = \frac{R^U_{12}}{R^L_{12}} \approx 1 + A^{UL}_{12}\cos(\phi_1+\phi_2)$$

Fit function

$$f(\phi_1+\phi_2) = p_0 + p_1\cos(\phi_1+\phi_2)$$



The recursive recipe for simulating e^+e^- annihilation



Steps:
1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from q
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. **Exit condition**

- (after several emissions) Hadronize the last pair $q_l\bar{q}_L$
  - emit $h = q_l\bar{q}'$ from q_l and project $\bar{q}_L q'$ to the state H

$$dP(q_l \to h + q'; q_l\bar{q}_L) = \mathrm{Tr}_{q'\bar{q}_L}\left[T_{q',h,q_l} \otimes \Gamma_{H,s_H}\right] \quad \rho(q_l, \bar{q}_L) \quad \left[T^\dagger_{q',h,q_l} \otimes \Gamma^\dagger_{H,s_H}\right]$$

  - or emit $H = q'\bar{q}_L$ from $\bar{q}_L$ and project $q_l\bar{q}'$ to the state h