RELATIVISTIC MAGNETOHYDRODYNAMICS WITH SPIN

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based on:
Phys. Lett. B 814 136096 (2021); *Phys. Rev. D* 103, 014030 (2021); *Phys. Rev. Lett* 129, 192301 (2022)

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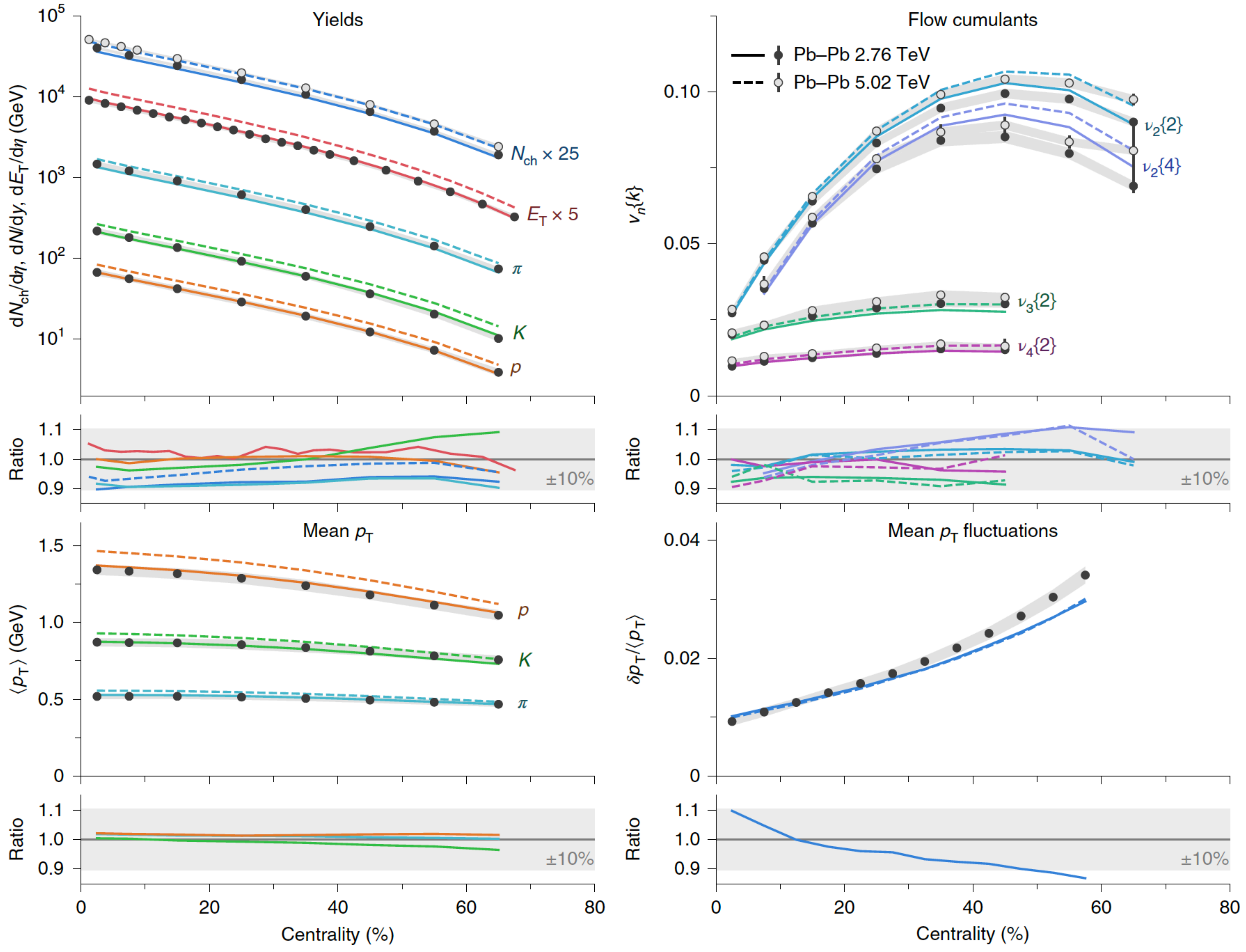
Motivation

QGP evolves hydrodynamically



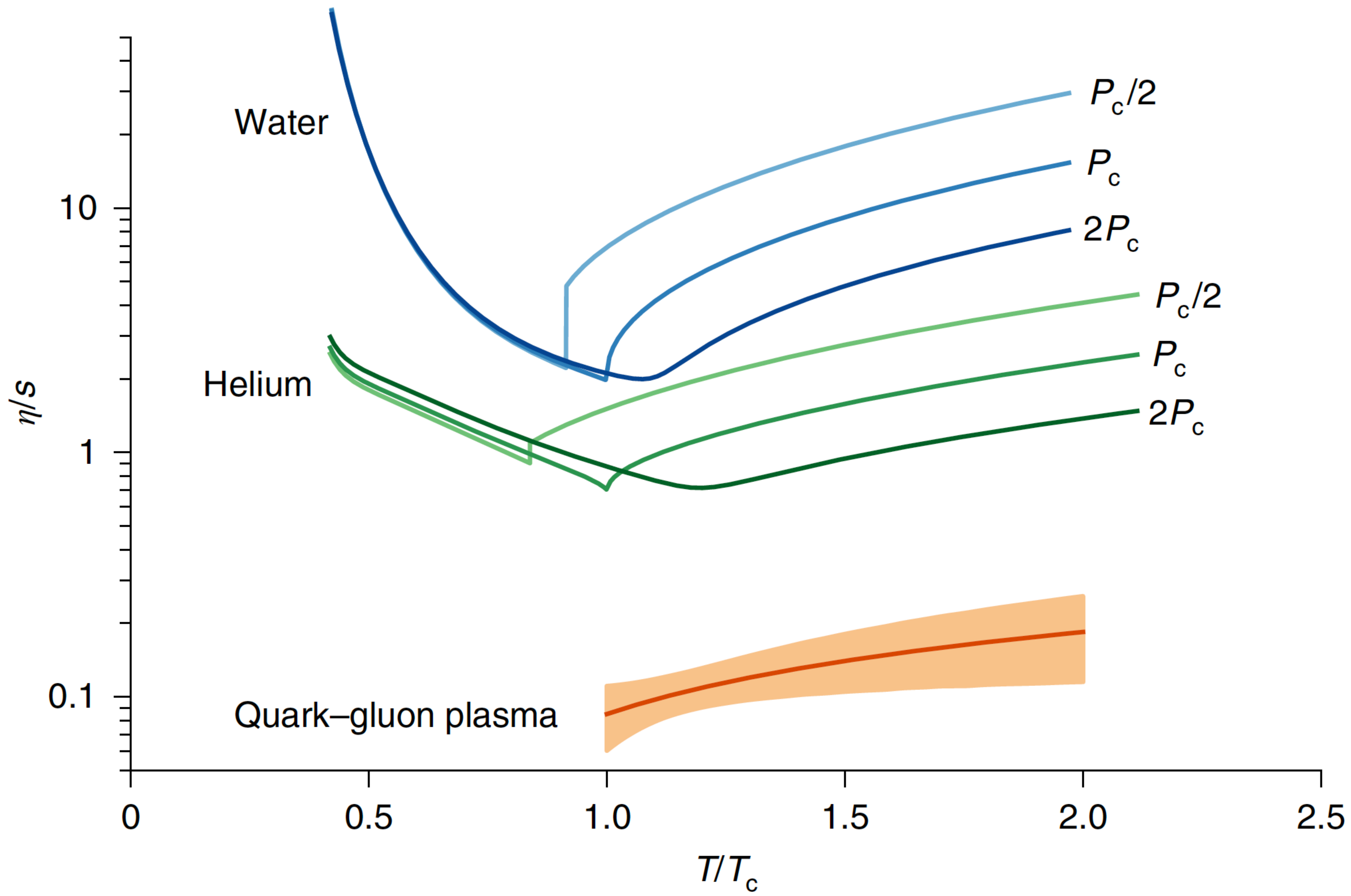
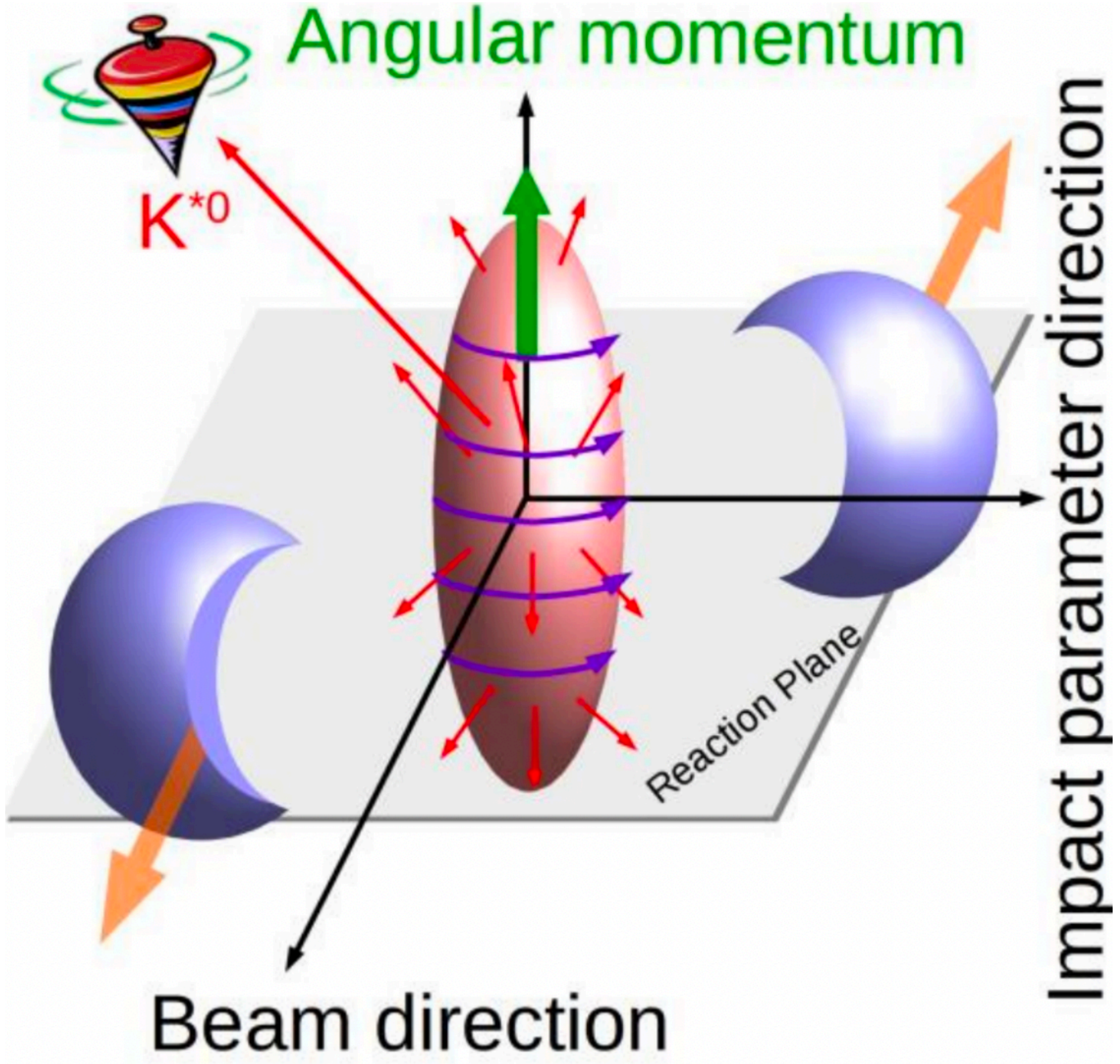


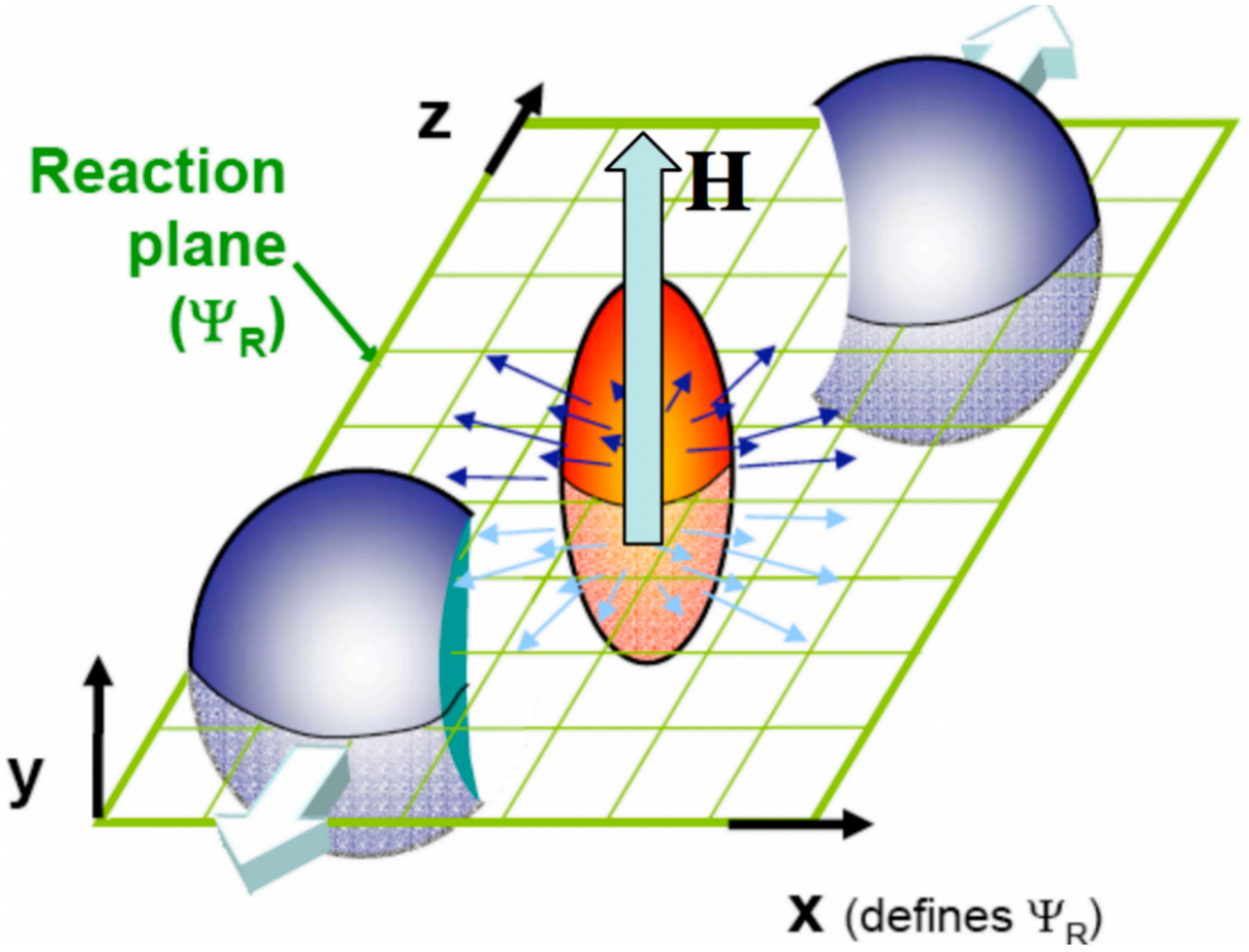
figure: J. Bernhard, J. Moreland, S. Bass, Nature Phys. 15, 1113–1117 (2019)

Well established properties of the produced QCD matter:

- Behaves like a fluid ➡ hydrodynamics applicable
- Low viscosity but important for observables ➡ inclusion of dissipative effects required

NON-CENTRAL HEAVY-ION COLLISIONS





Non-central collisions are interesting:

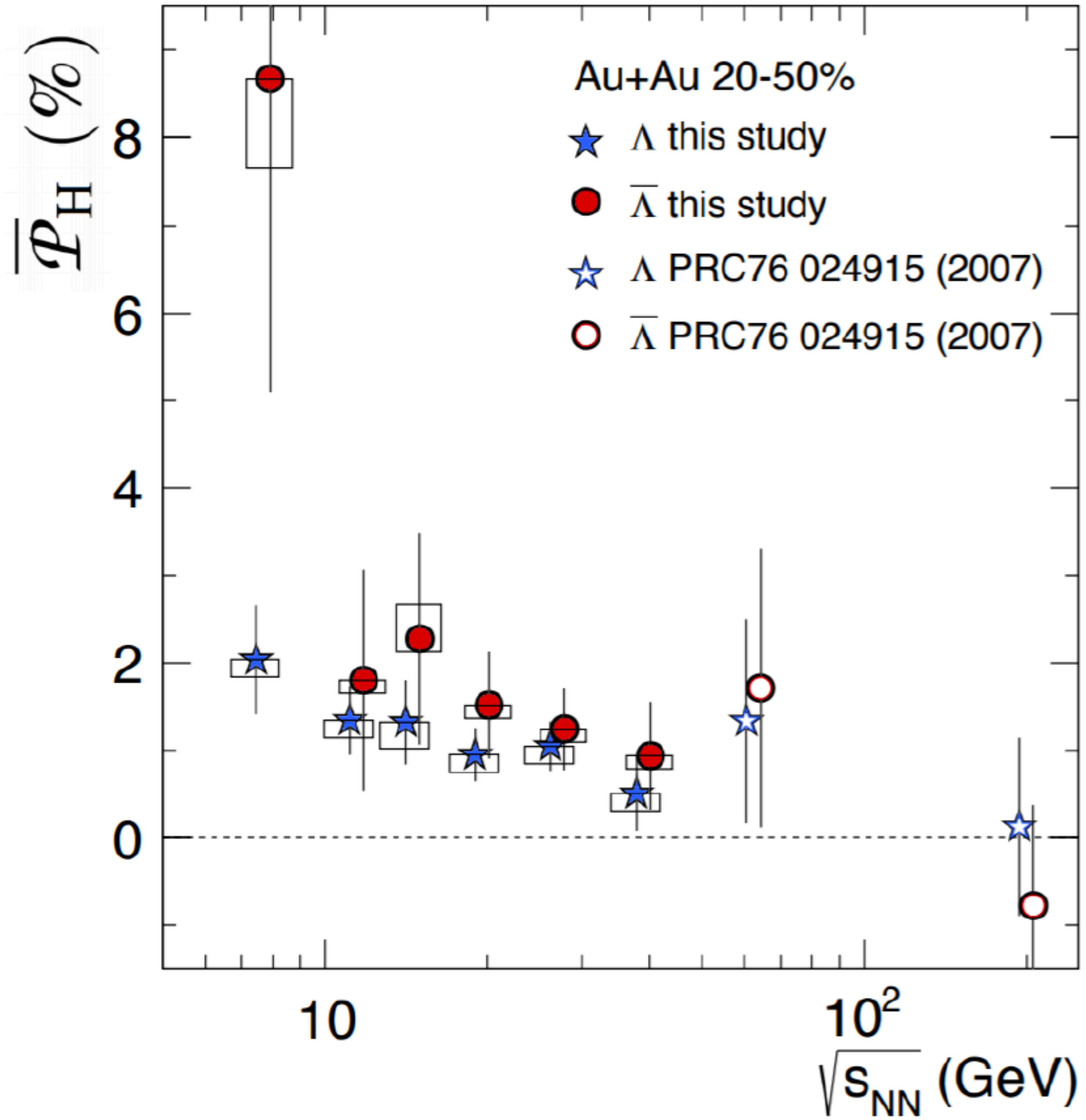
- Large initial orbital angular momentum

F. Becattini, F. Piccinini, J. Rizzo, PRC 77 (2008) 024906

- Large magnetic field

A. Bzdak and, V. Skokov, PLB 710 (2012) 171-174

Measurement of Λ and $\bar{\Lambda}$ global spin polarization



L. Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65

Spin polarization in equilibrated QGP — spin-thermal approach

In thermodynamic equilibrium one can establish a link between spin and vorticity

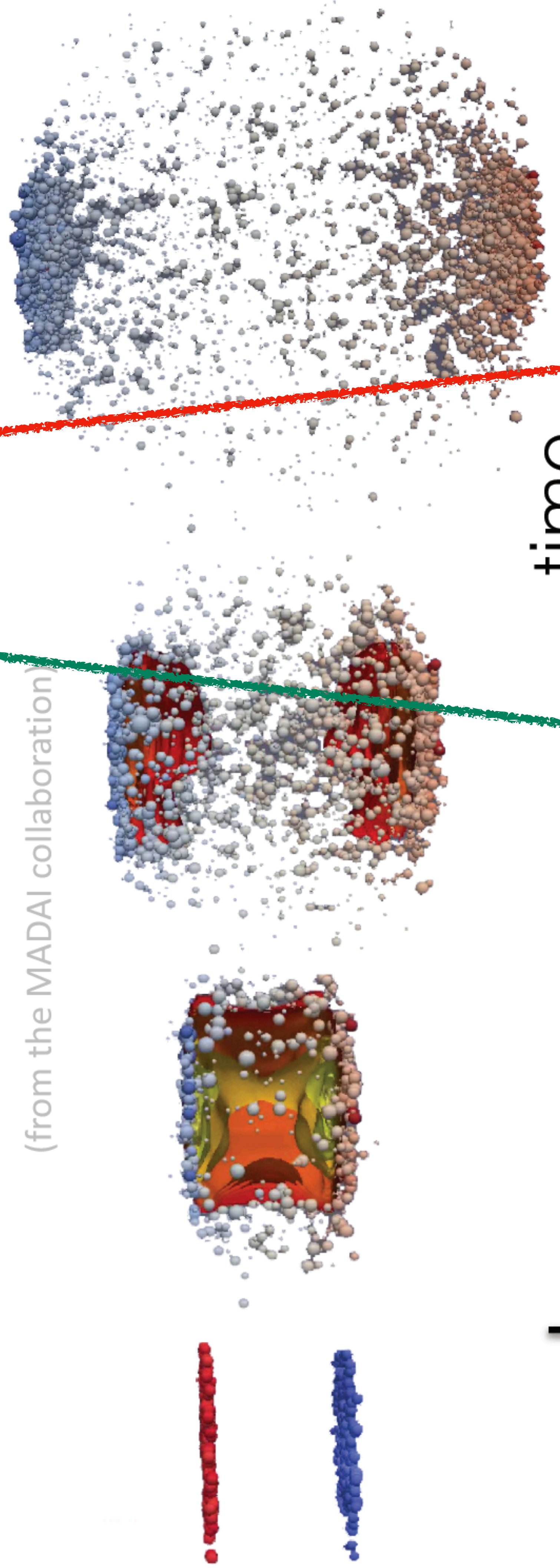
Becattini F, Chandra V, Del Zanna L, Grossi E. AP 338:32 (2013)
F. Becattini, L. Csernai, and D. J. Wang, PRC 88, 034905 (2013)
Fang R, Pang L,Wang Q,Wang X. PRC 94:024904 (2016)
F. Becattini, I. Karpenko, M. Lisa, I. Upsal, and S. Voloshin PRC 95, 054902 (2017)

$$S^\mu(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int d\Sigma_\lambda p^\lambda n_F(1-n_F)\varpi_{\rho\sigma}}{\int d\Sigma_\lambda p^\lambda n_F}$$

$$\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu\right) \qquad \beta^\mu = \frac{u^\mu}{T}$$

Spin is enslaved to thermal vorticity

Very attractive: Allows to extract polarization at the freeze-out hypersurface in any model which provides u^μ, T and μ.



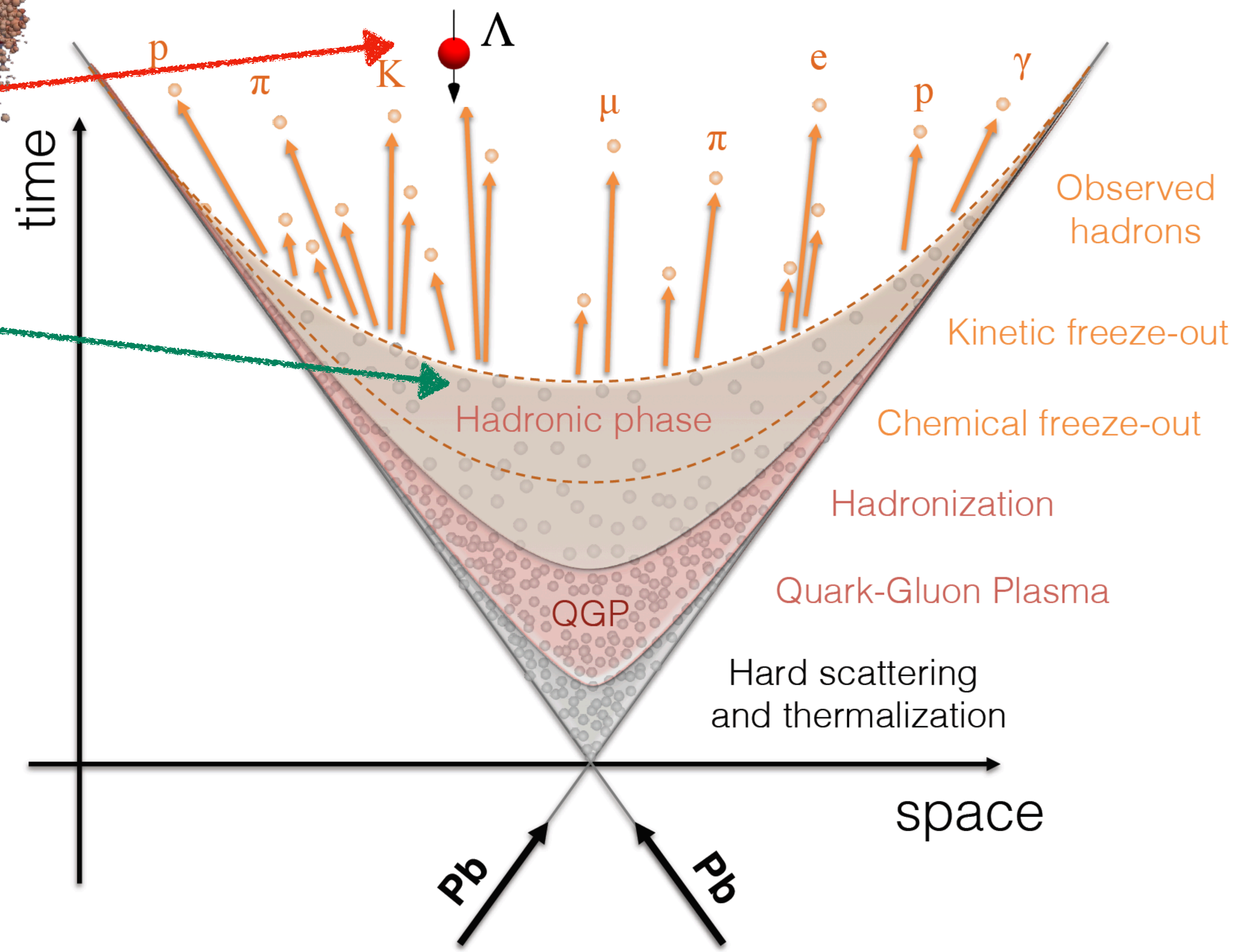


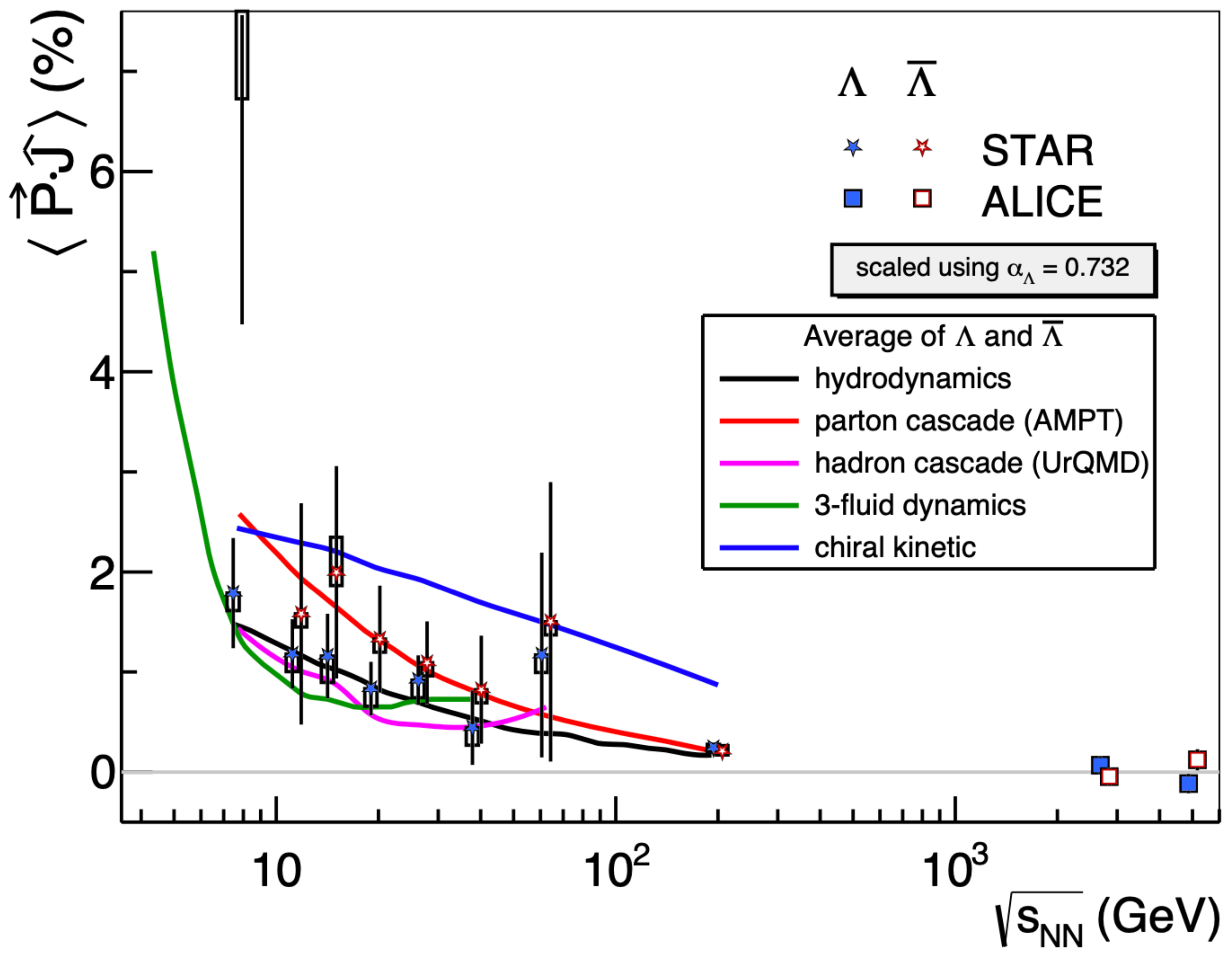
figure: D.D. Chinellato

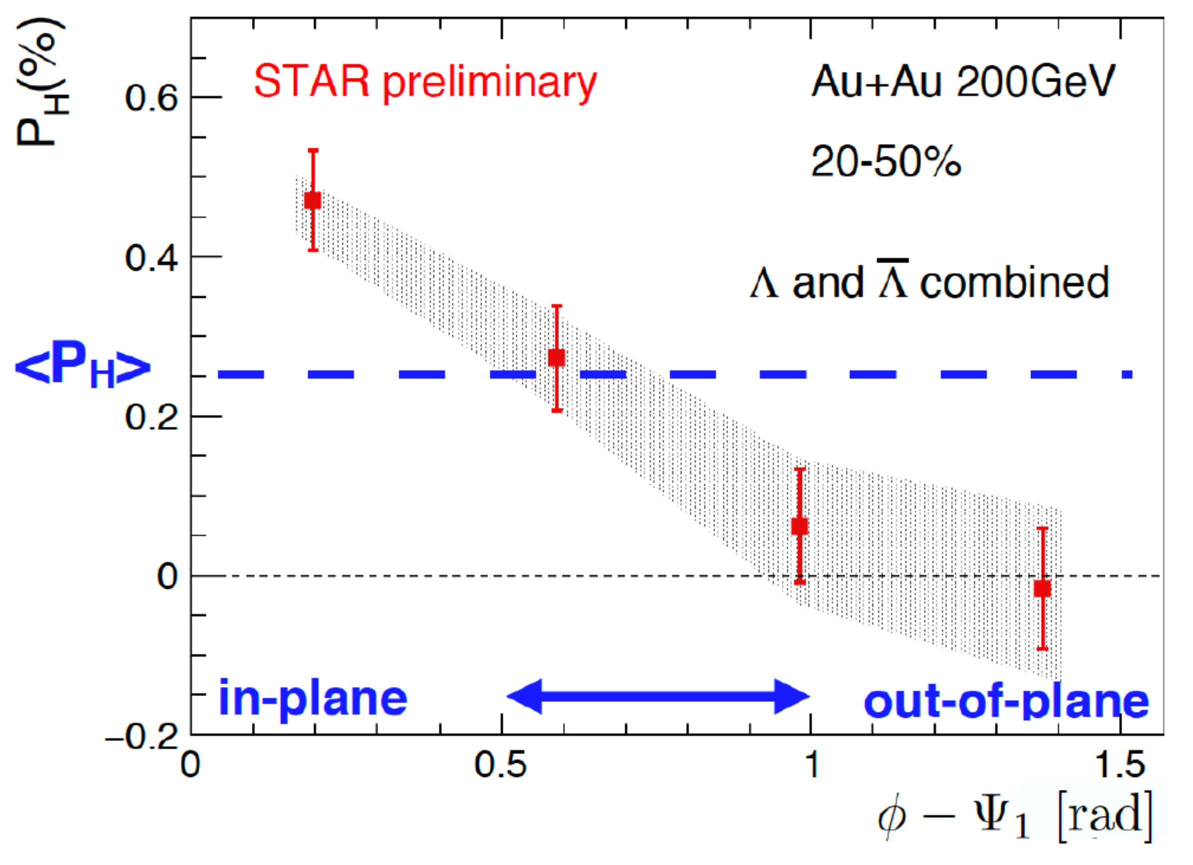
Measurement vs spin-thermal approach: global polarization

Global polarization data supports the spin-thermal approach

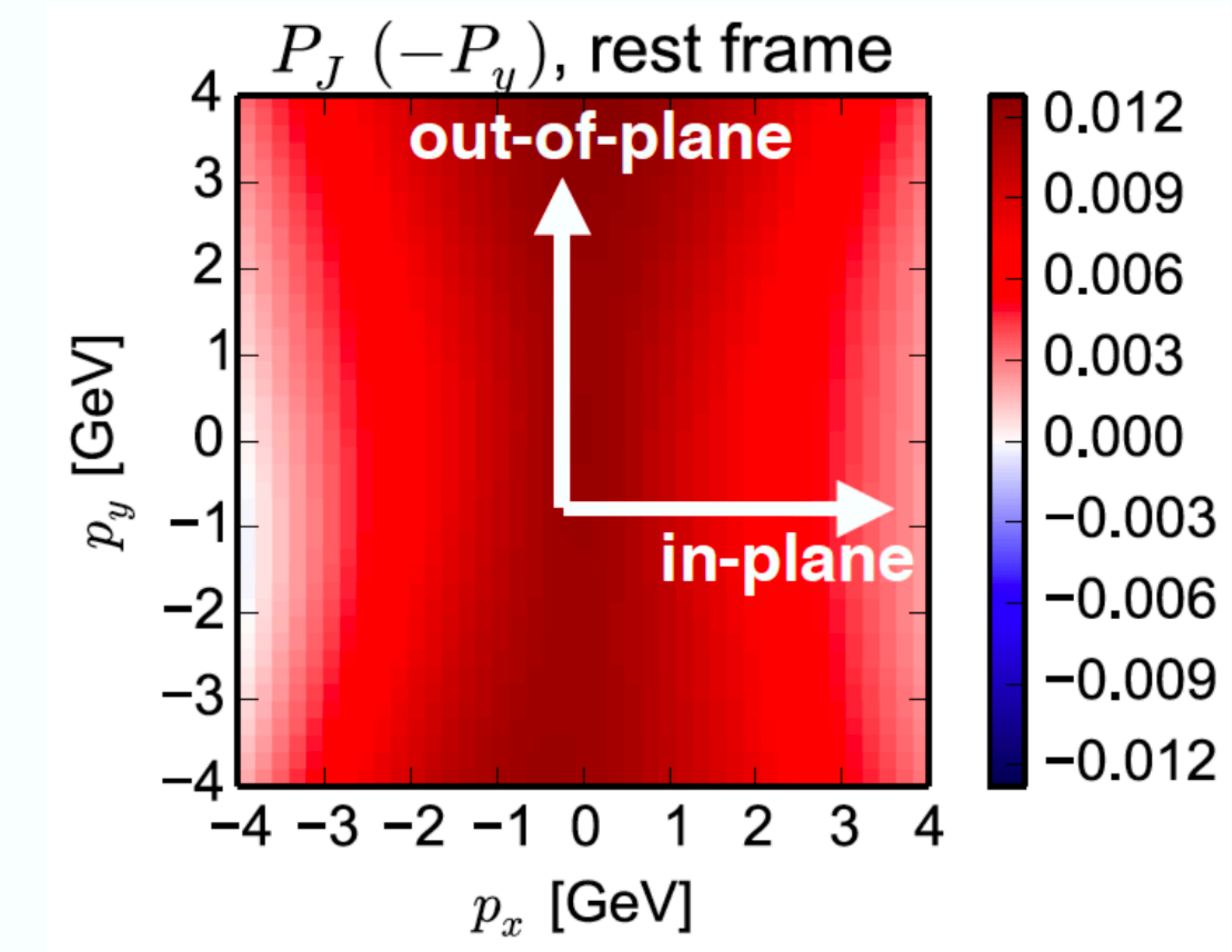
Azimuthal modulation is not captured

F. Becattini, J. Liao, M. Lisa Lect.Notes Phys. 987 (2021) 1-14



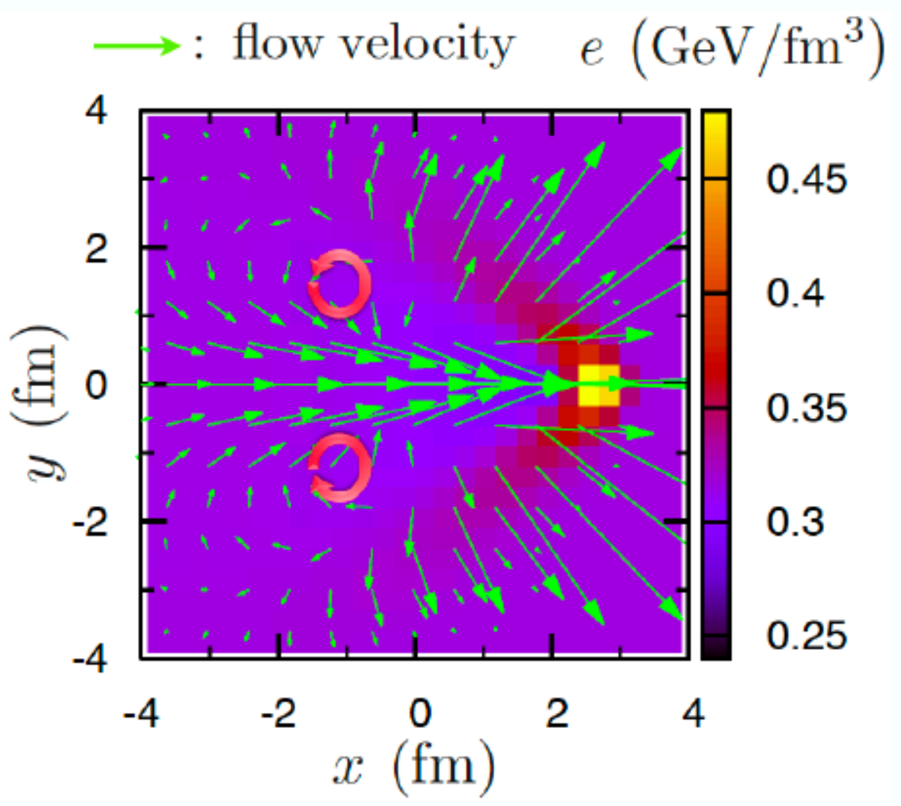


Credit: T.Niida, The 5th Workshop on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions, 2019

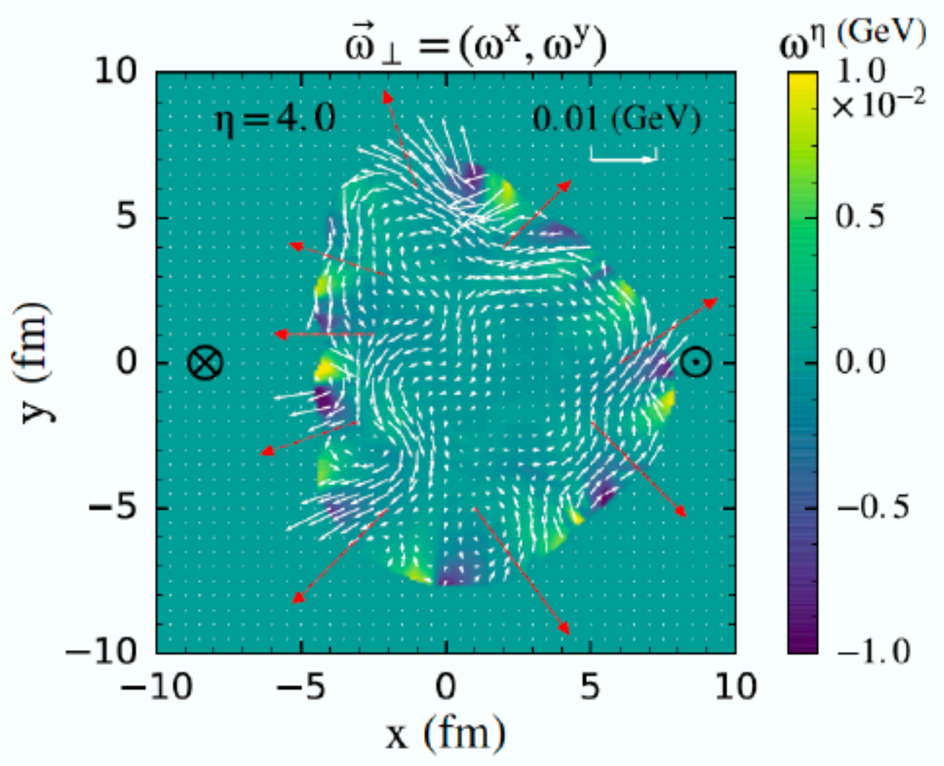


I. Karpenko, F. Becattini, EPJC 77, 213 (2017)

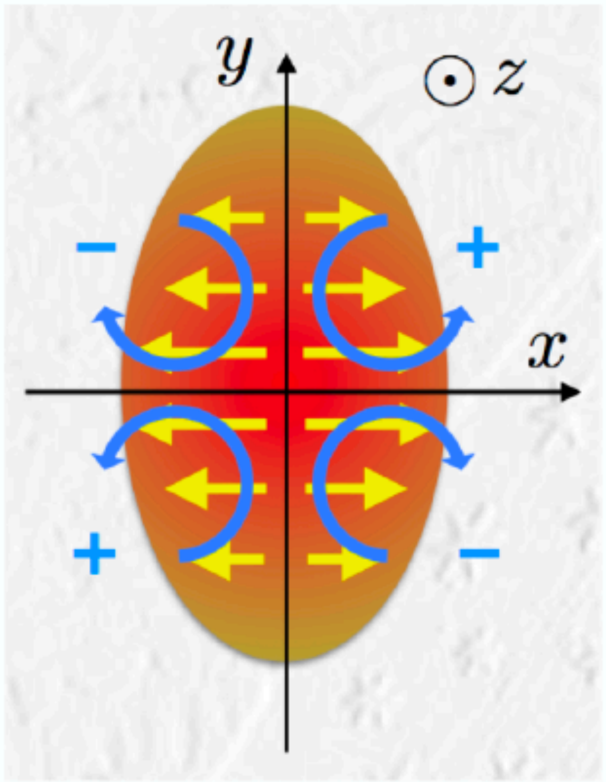
Longitudinal (beam-axis) polarization



Y. Tachibana and T. Hirano, NPA904-905 (2013) 1023

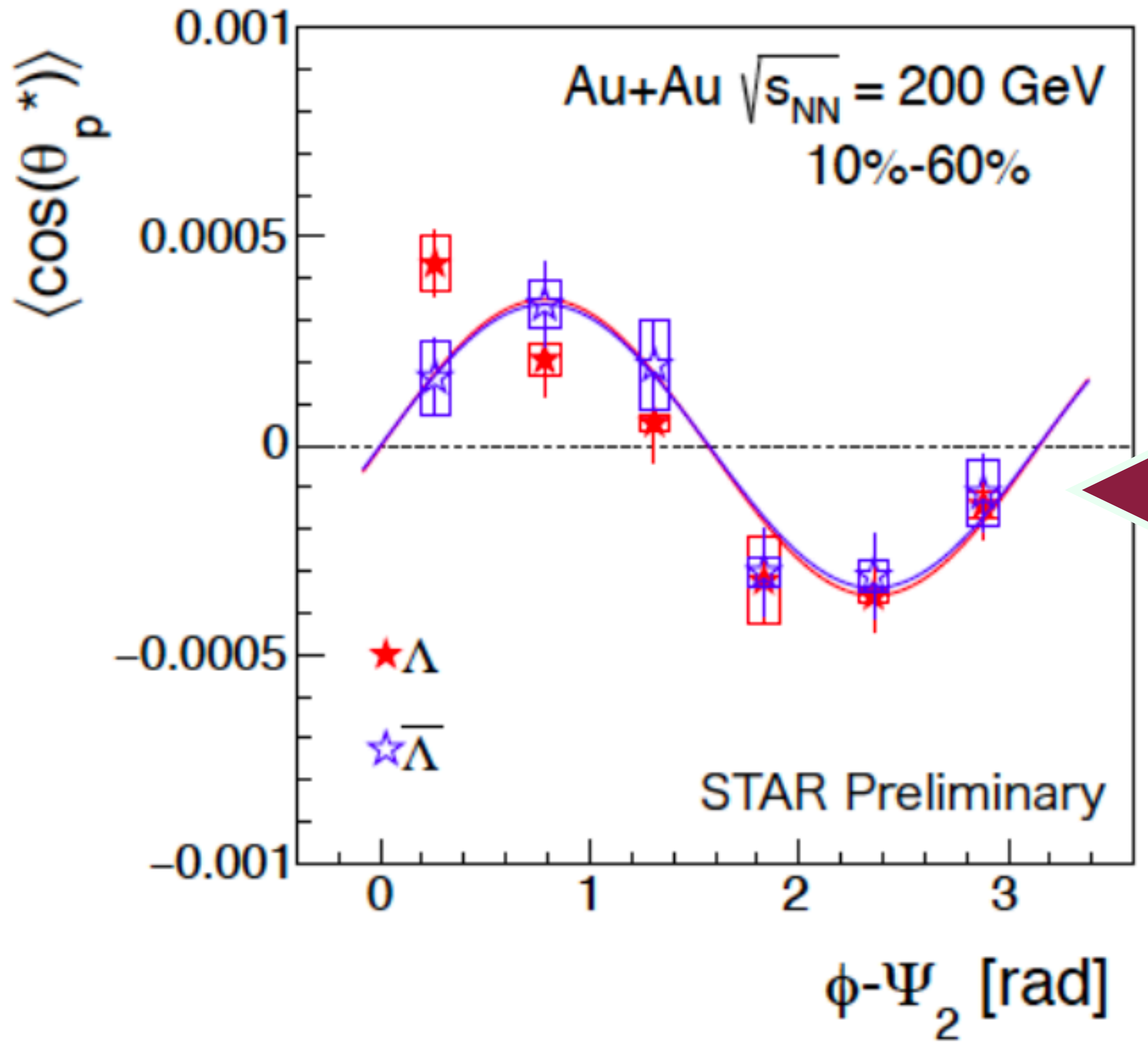


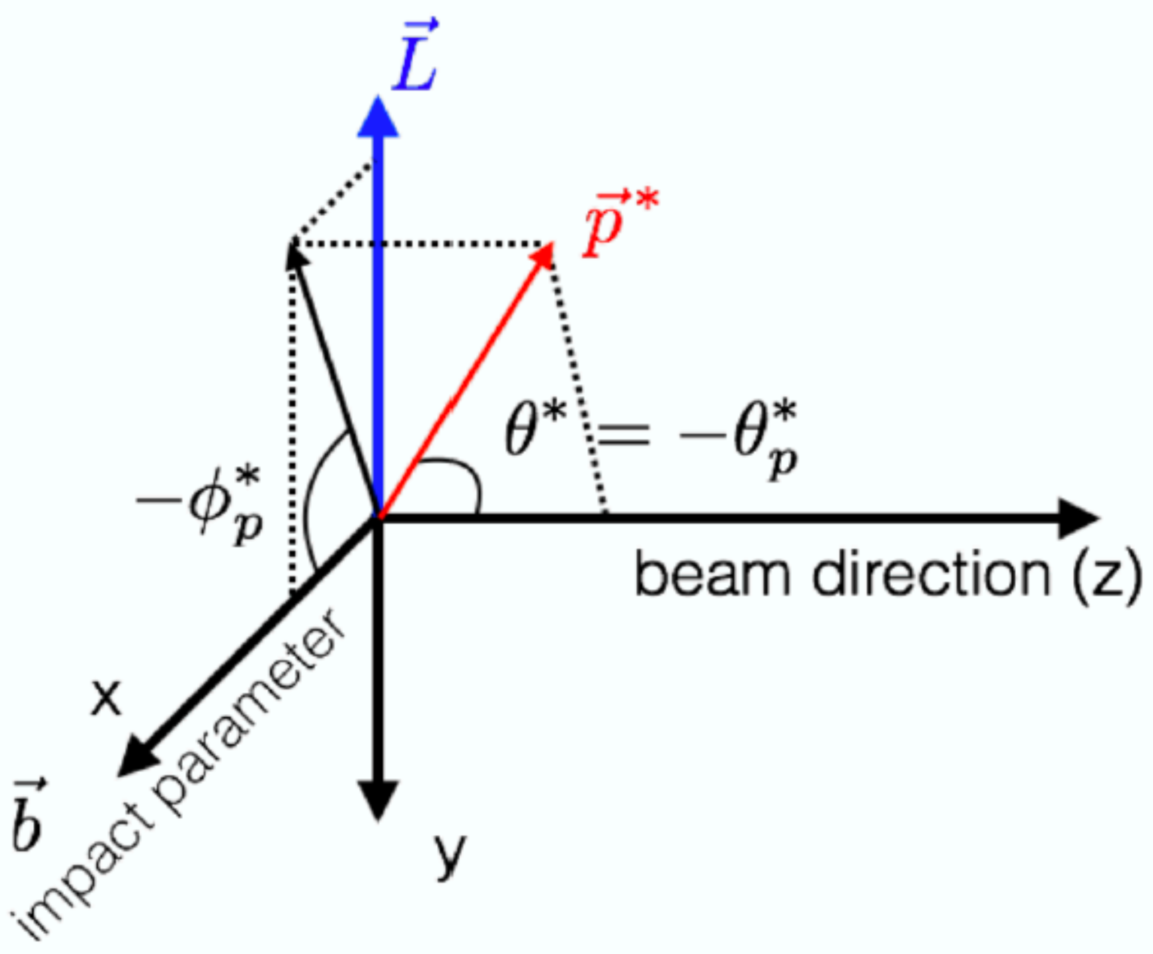
L.-G. Pang, H. Peterson, Q. Wang, and X.-N. Wang PRL117, 192301 (2016)



F. Becattini and I. Karpenko, PRL120.012302 (2018)
S. Voloshin, EPJ Web Conf.171, 07002 (2018)

Flow structures in the plane transverse to beam (jet, ebe fluctuations etc.) may generate longitudinal polarization





$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha_H \mathbf{P_H} \cdot \mathbf{p}_p^*)$$

$$\langle\cos\theta_p^*\rangle = \int \frac{dN}{d\Omega^*}\cos\theta_p^* d\Omega^*$$

$$= \alpha_H P_z \langle(\cos\theta_p^*)^2\rangle$$

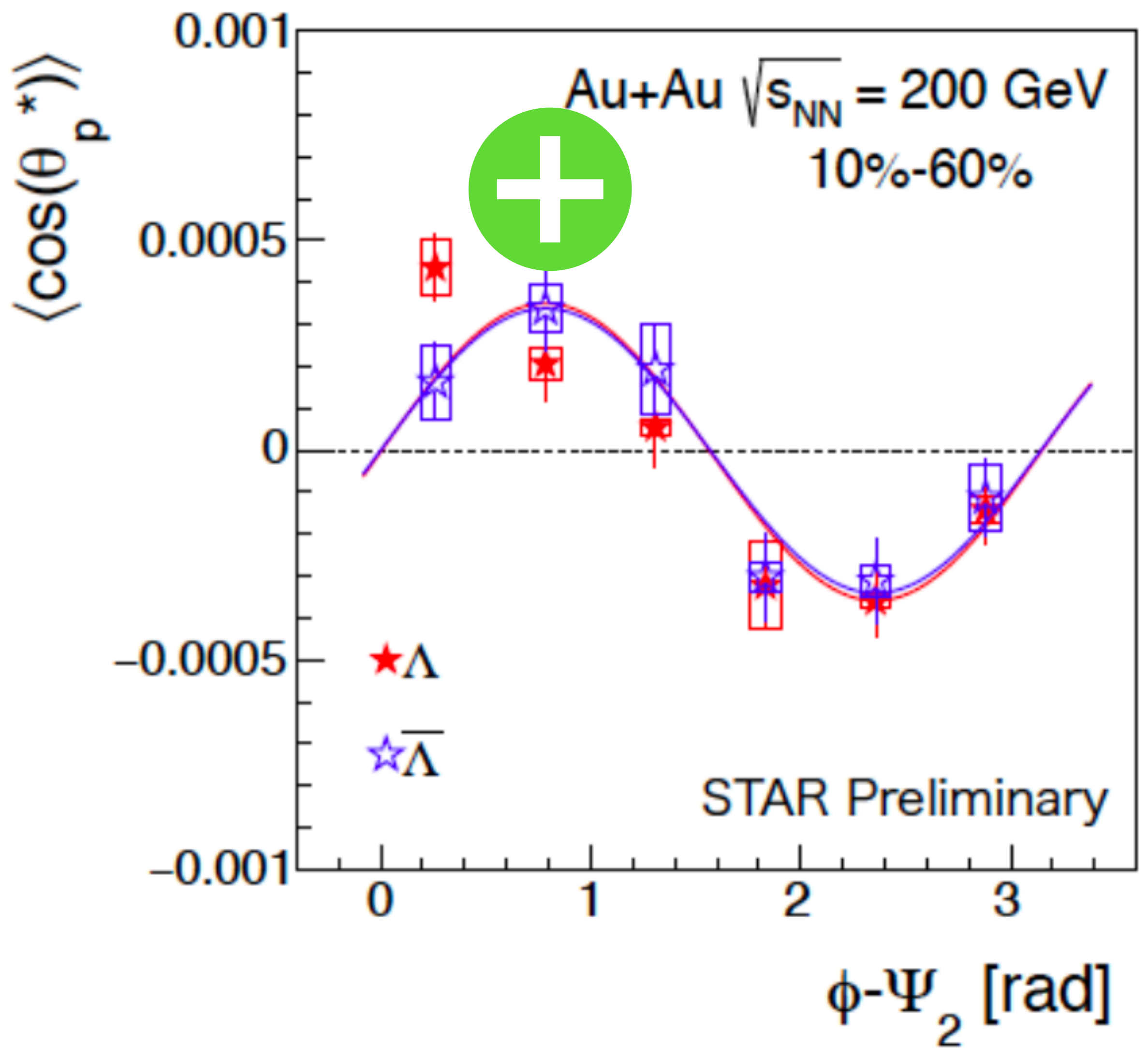
$$\therefore\ P_z = \frac{\langle\cos\theta_p^*\rangle}{\alpha_H\langle(\cos\theta_p^*)^2\rangle}$$

$$= \frac{3\langle\cos\theta_p^*\rangle}{\alpha_H} \quad \text{(if perfect detector)}$$

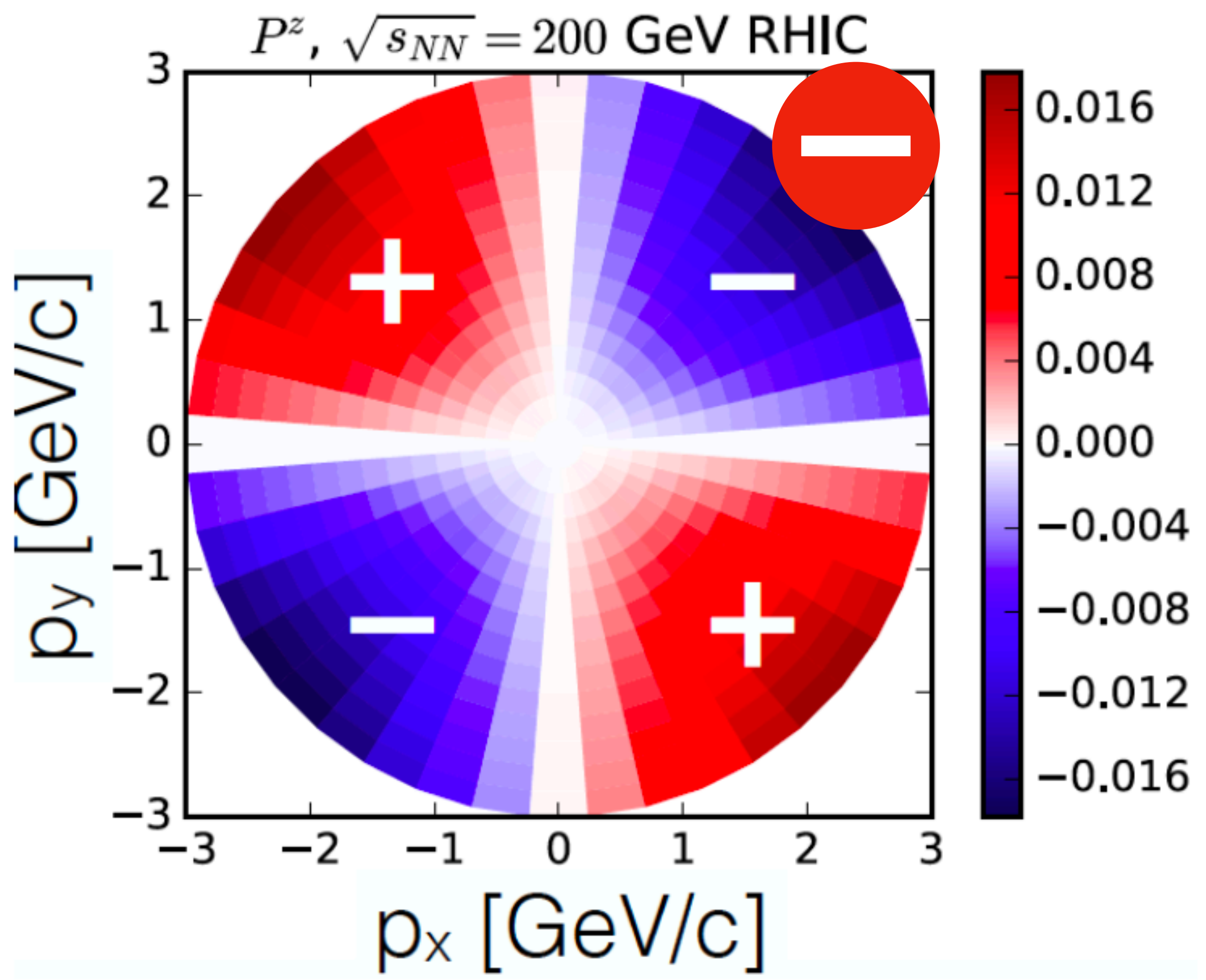
α_H: hyperon decay parameter

θ_p^*: θ of daughter proton in Λ rest frame

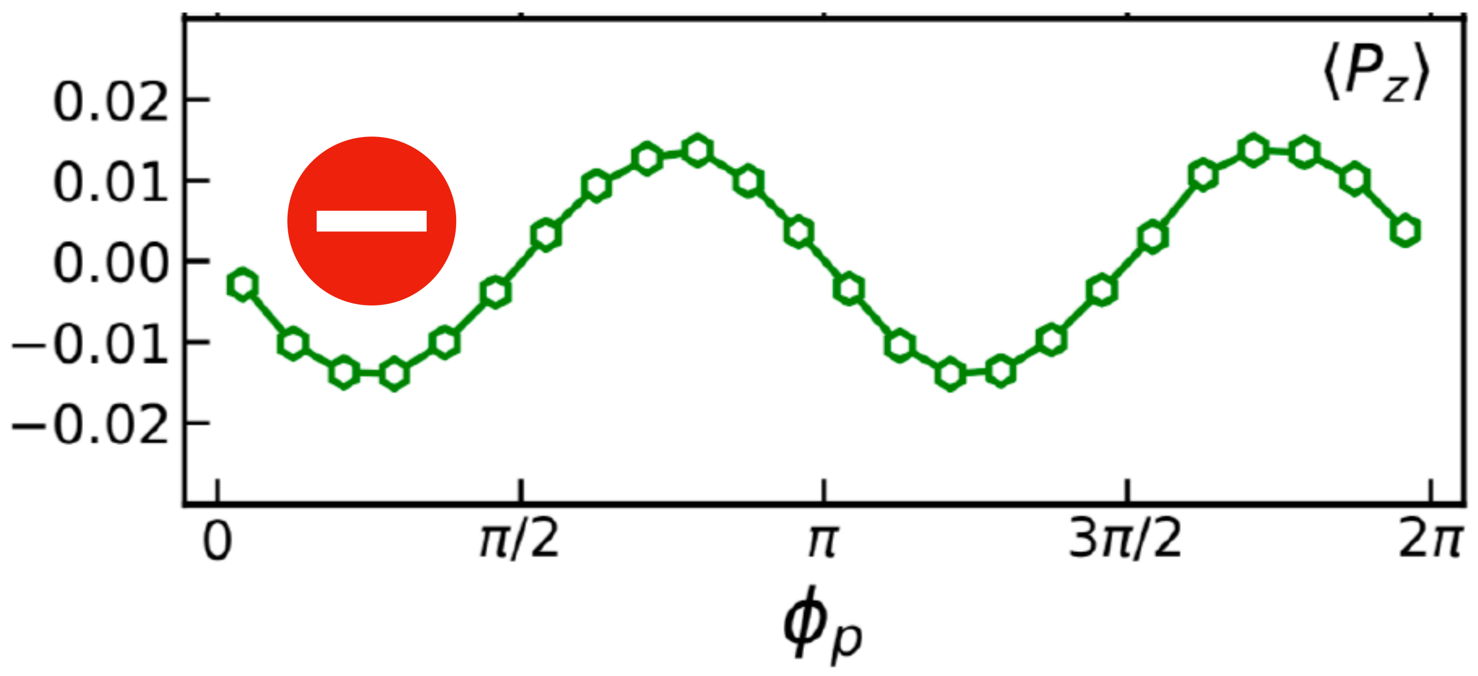
Longitudinal polarization — 'Spin sign' puzzle



T. Niida, NPA 982 (2019) 511514

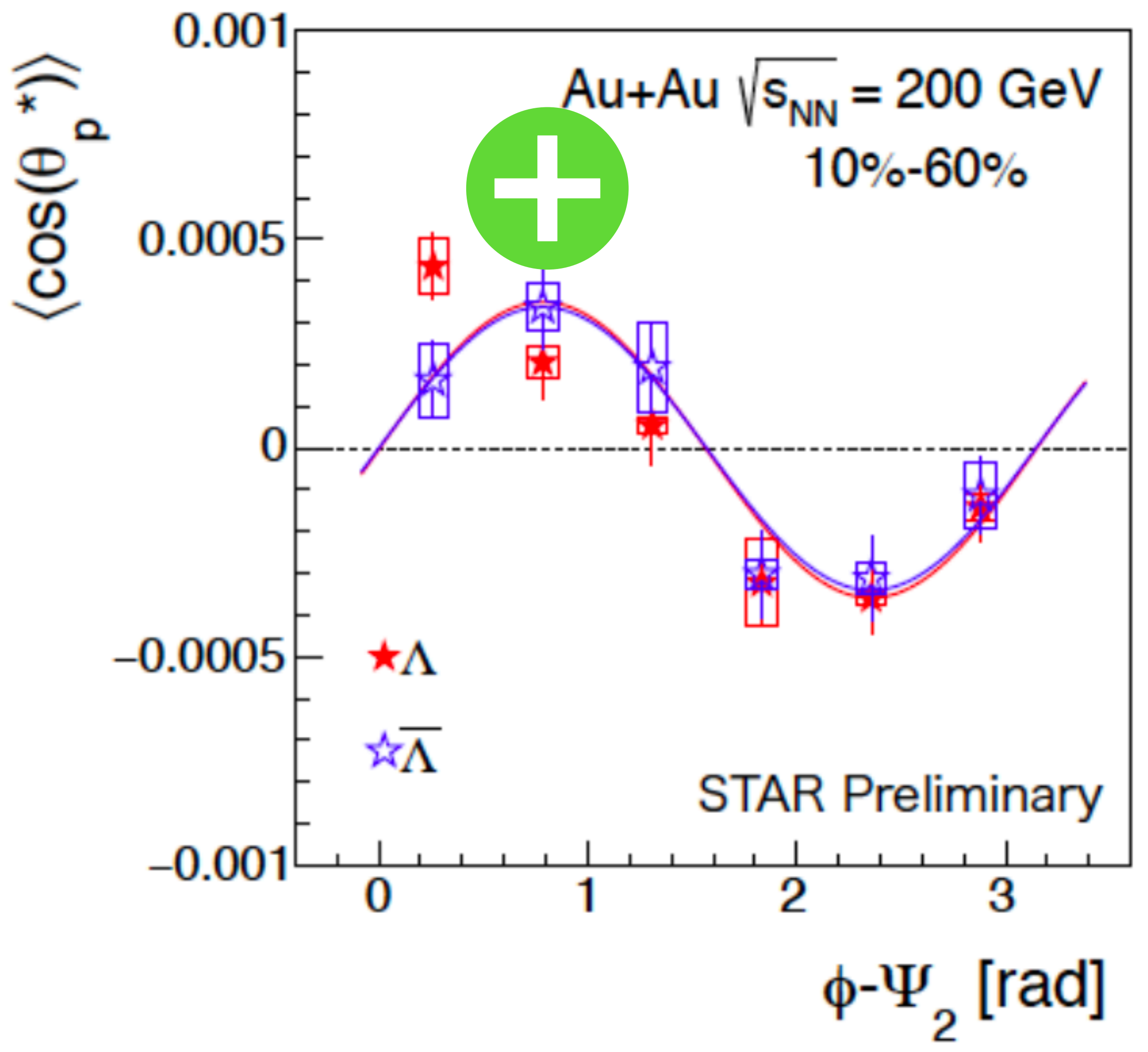


UrQMD+vHLLE: F. Becattini, I. Karpenko, PRL 120 (2018) no.1, 012302,

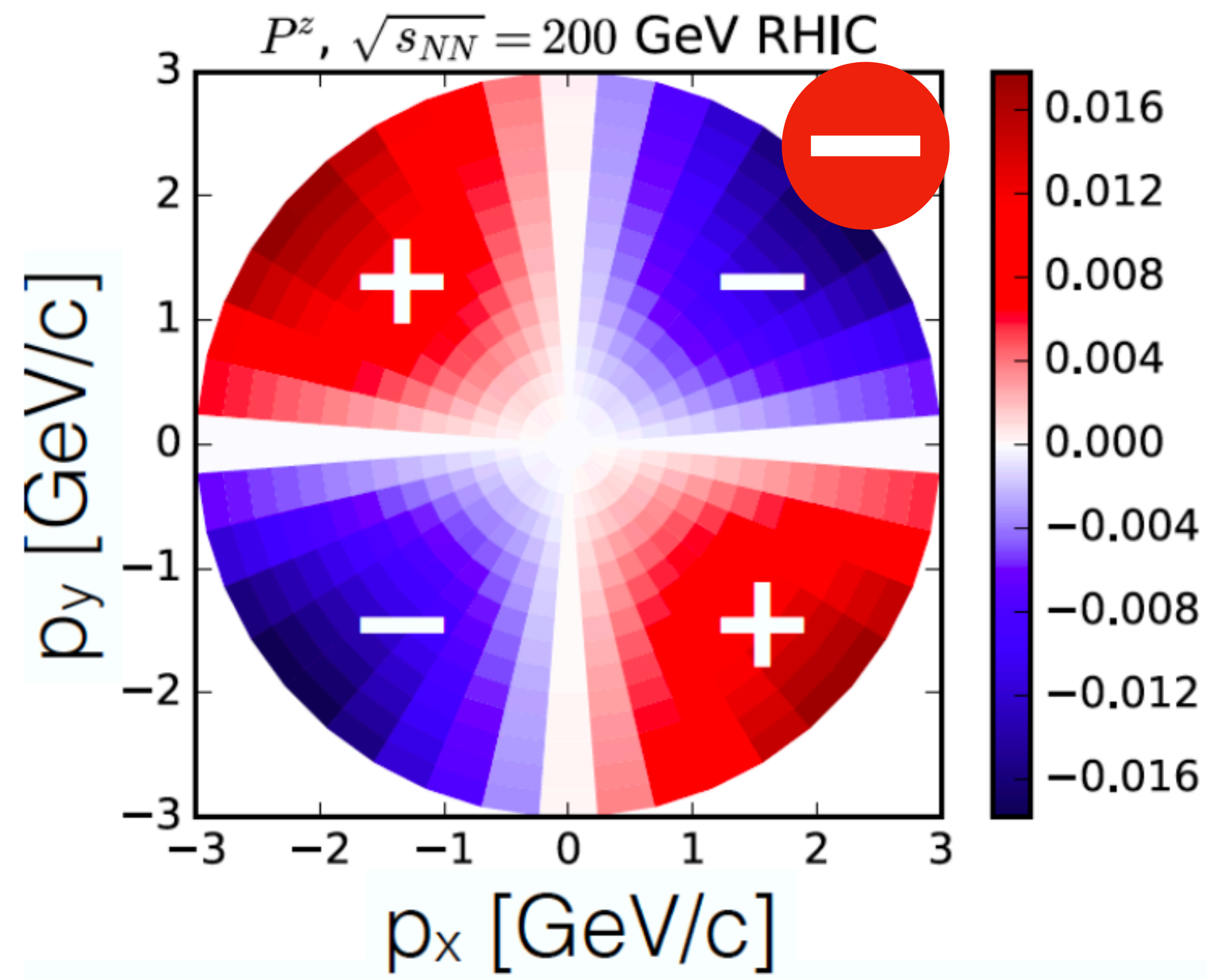


AMPT: X. Xia, H. Li, Z. Tang, Q. Wang, PRC98.024905 (2018)

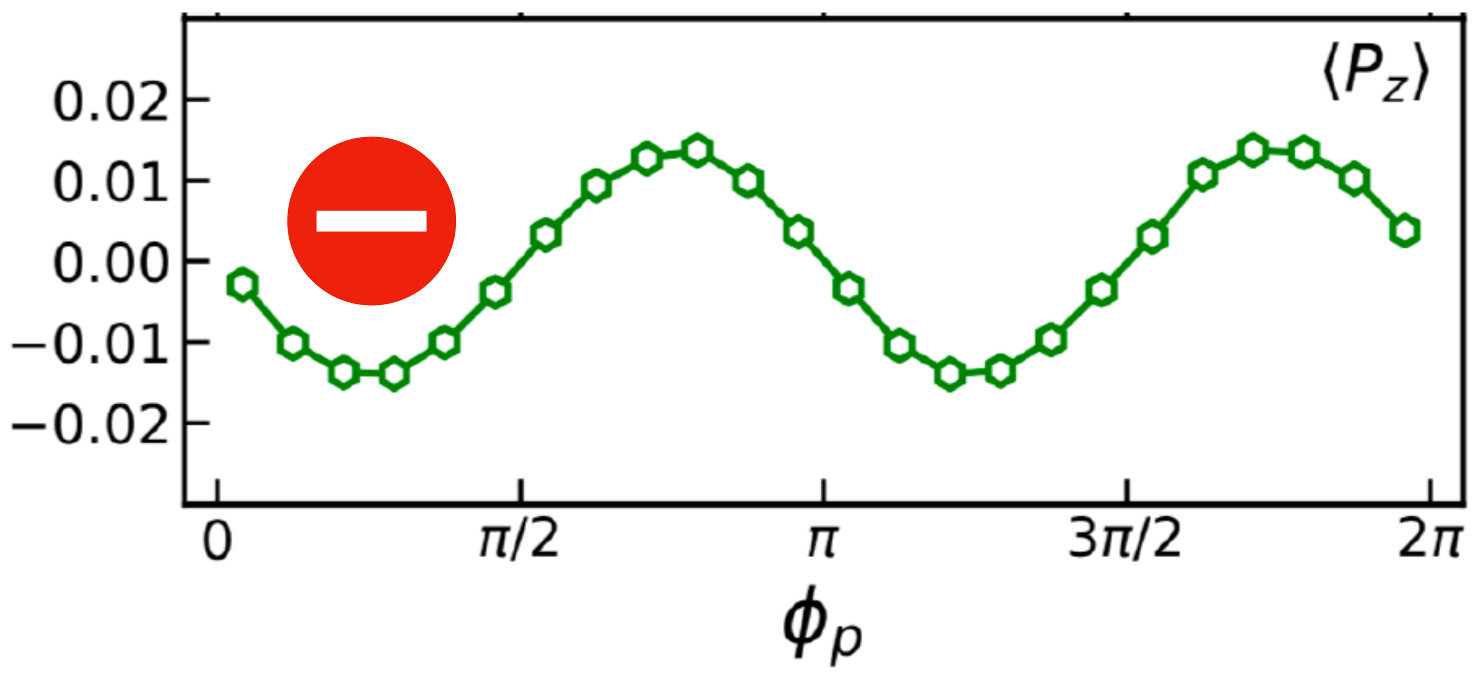
LONGITUDINAL POLARIZATION — 'SPIN SIGN' PUZZLE



T. Niida, NPA 982 (2019) 511514



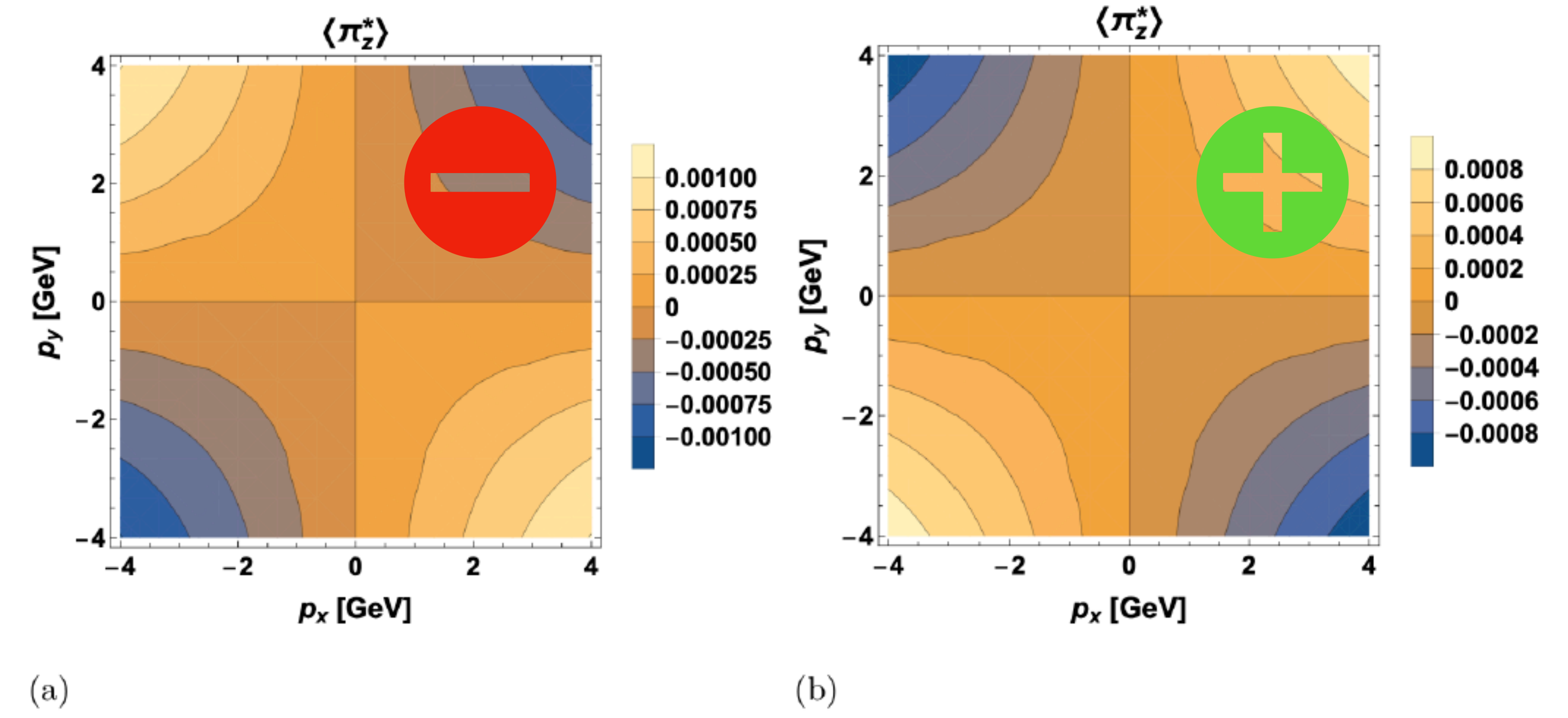
UrQMD+vHLLE: F. Becattini, I. Karpenko, PRL 120 (2018) no.1, 012302,



AMPT: X. Xia, H. Li, Z. Tang, Q. Wang, PRC98.024905 (2018)

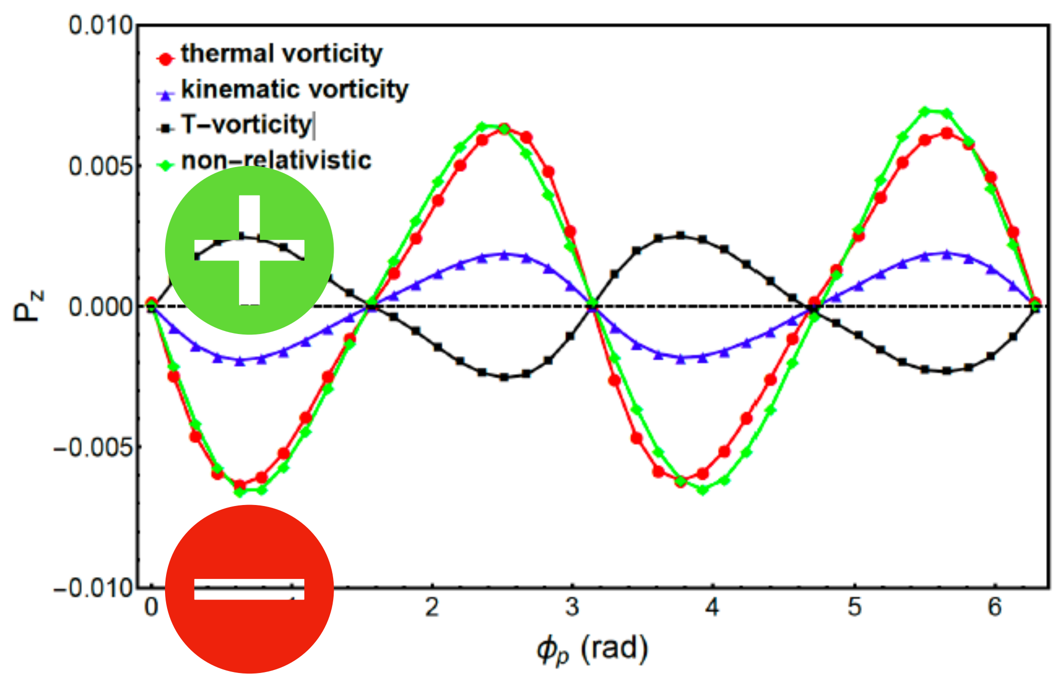
thermal model with projected vorticity $\omega_{\mu\nu} = \varpi_{\alpha\beta}\overline{\Delta}^{\alpha}_{\mu}\overline{\Delta}^{\beta}_{\nu}$

W.Florkowski, A. Kumar, A. Mazeliauskas, R.R., [1904.00002]

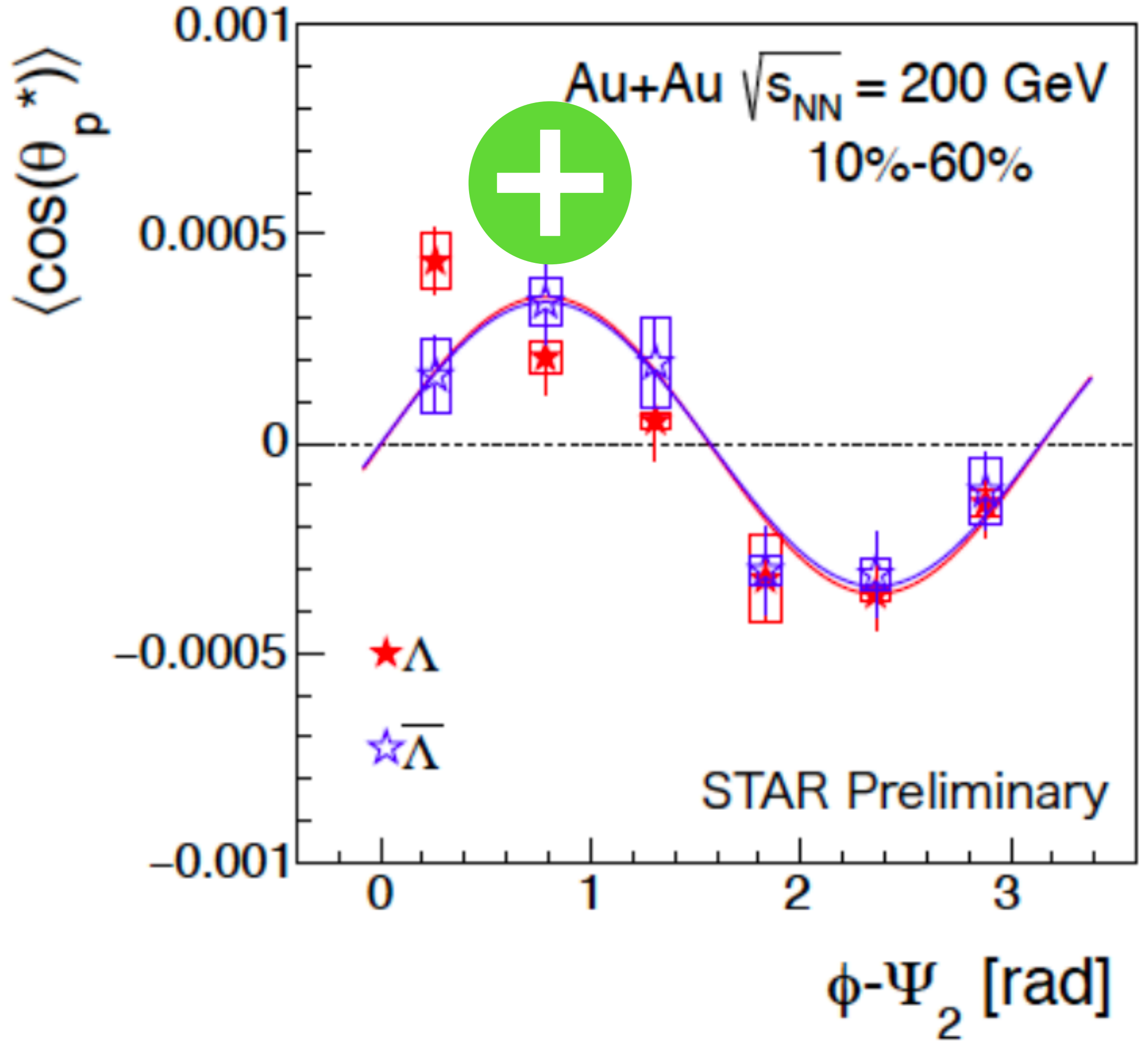


3D VH + AMPT IC with T-vorticity $\omega^{(T)}_{\mu\nu} = -\frac{1}{2}\left[\partial_\mu (Tu_\nu) - \partial_\nu \left(Tu_\mu\right)\right]$

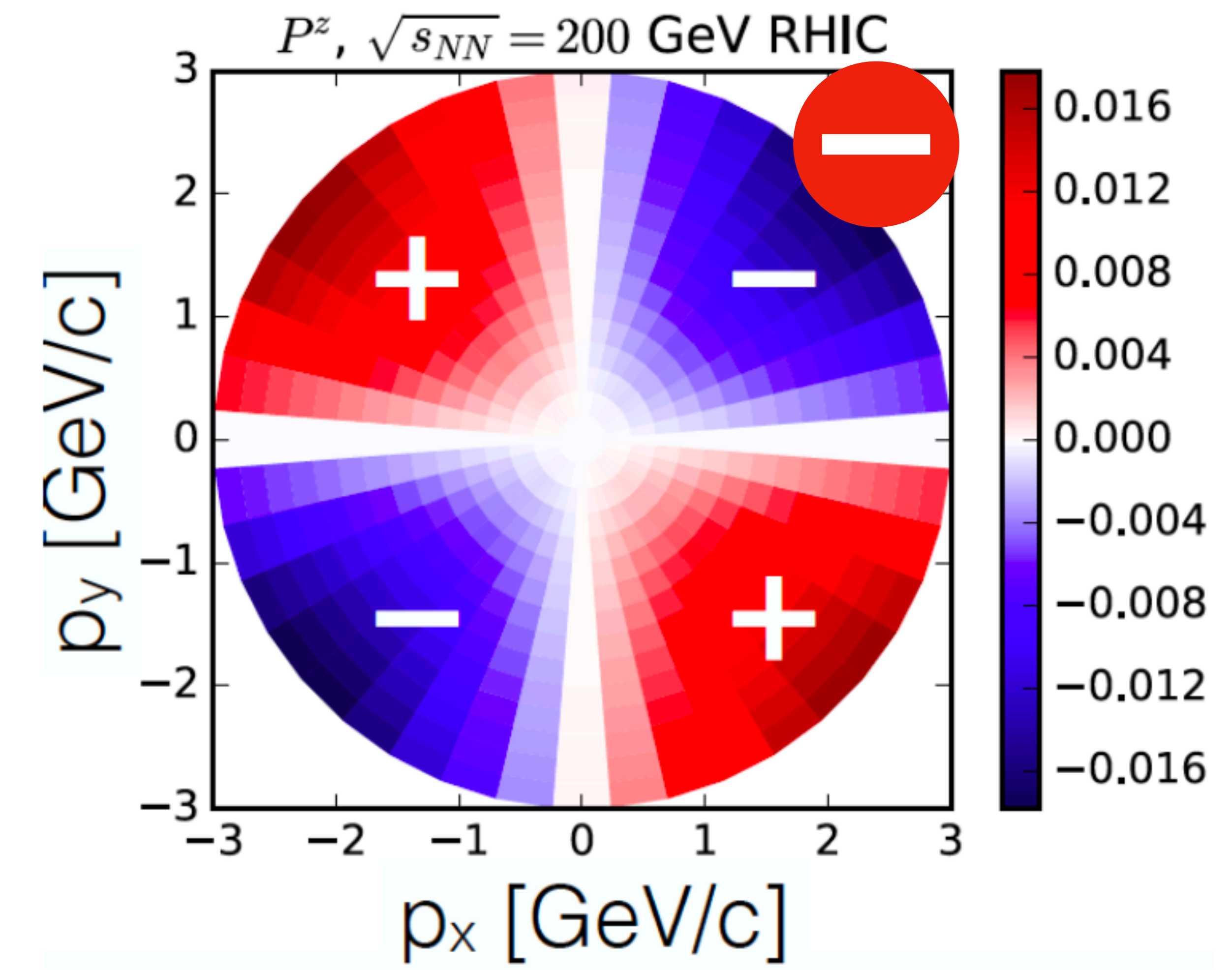
H-Z Wu, L-G Pang, X-G Huang, Q. Wang [1906.09385]



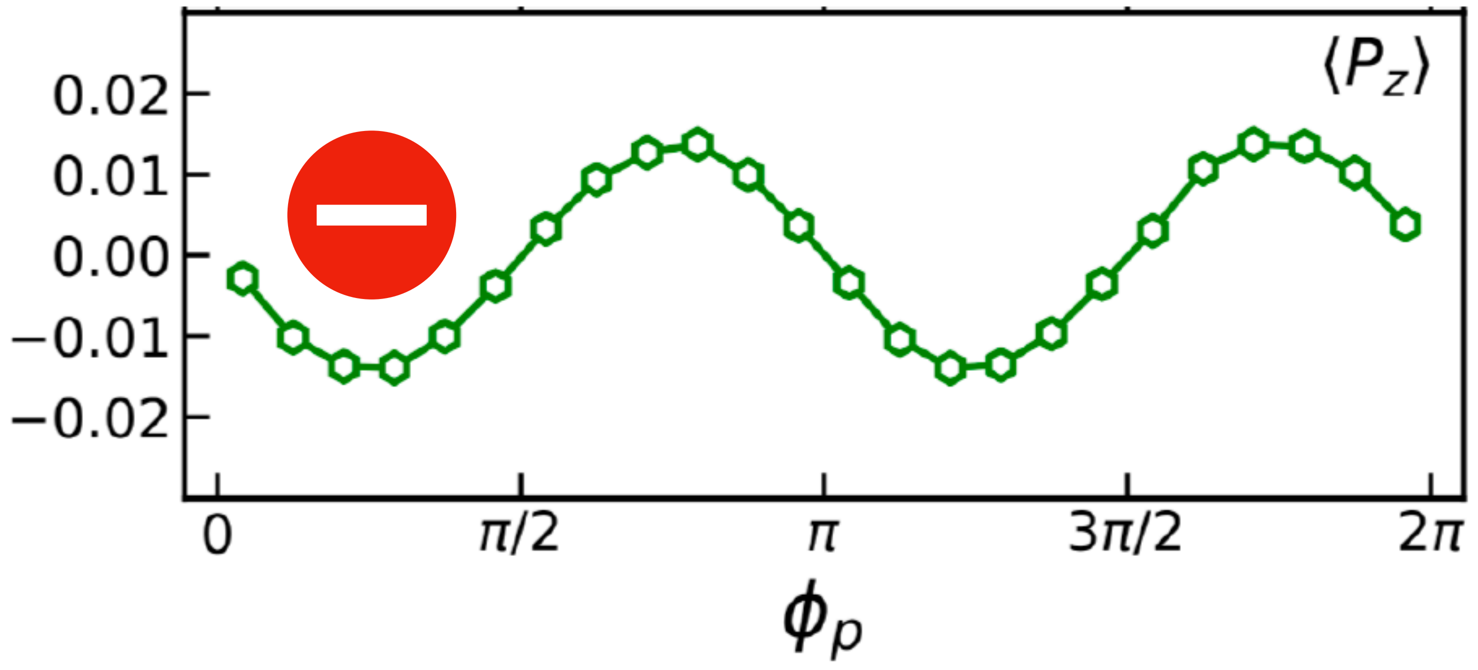
LONGITUDINAL POLARIZATION — 'SPIN SIGN' PUZZLE



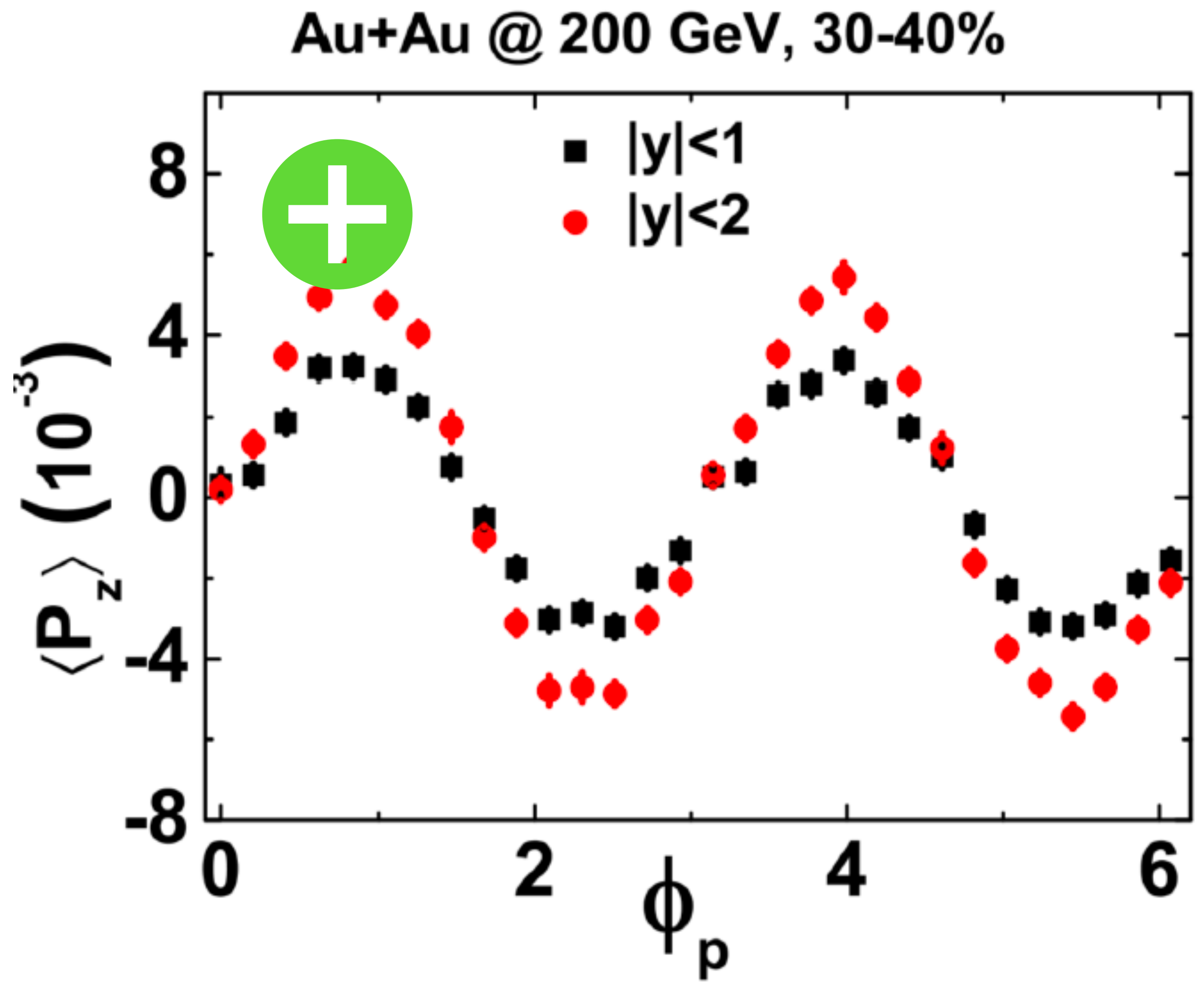
T. Niida, NPA 982 (2019) 511514

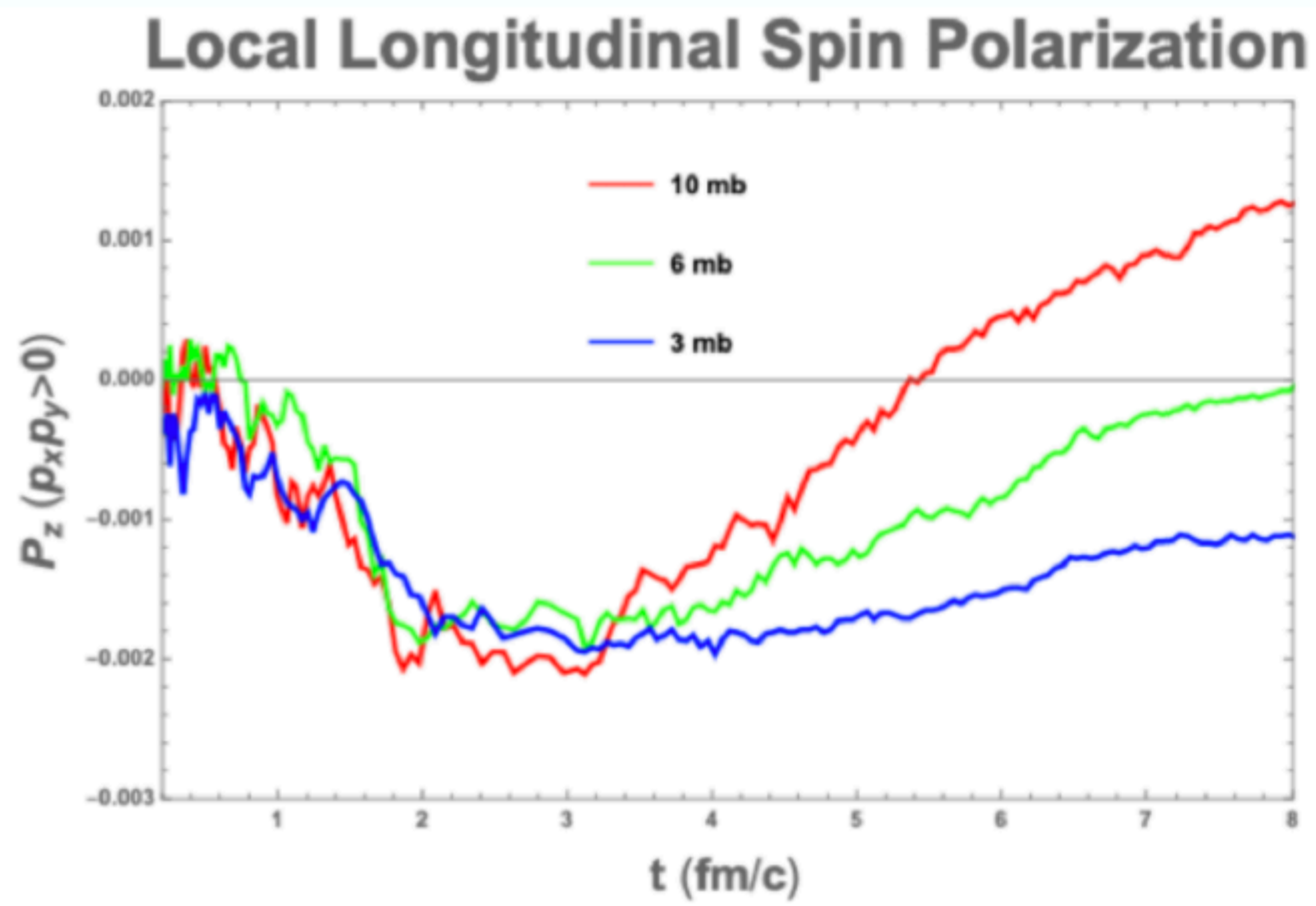


UrQMD+vHLLE: F. Becattini, I. Karpenko, PRL 120 (2018) no.1, 012302,



AMPT: X. Xia, H. Li, Z. Tang, Q. Wang, PRC98.024905 (2018)





Y. Sun, C-M. Ko, Phys.Rev. C99 (2019) no.1, 011903

Fluid dynamics of spin?!

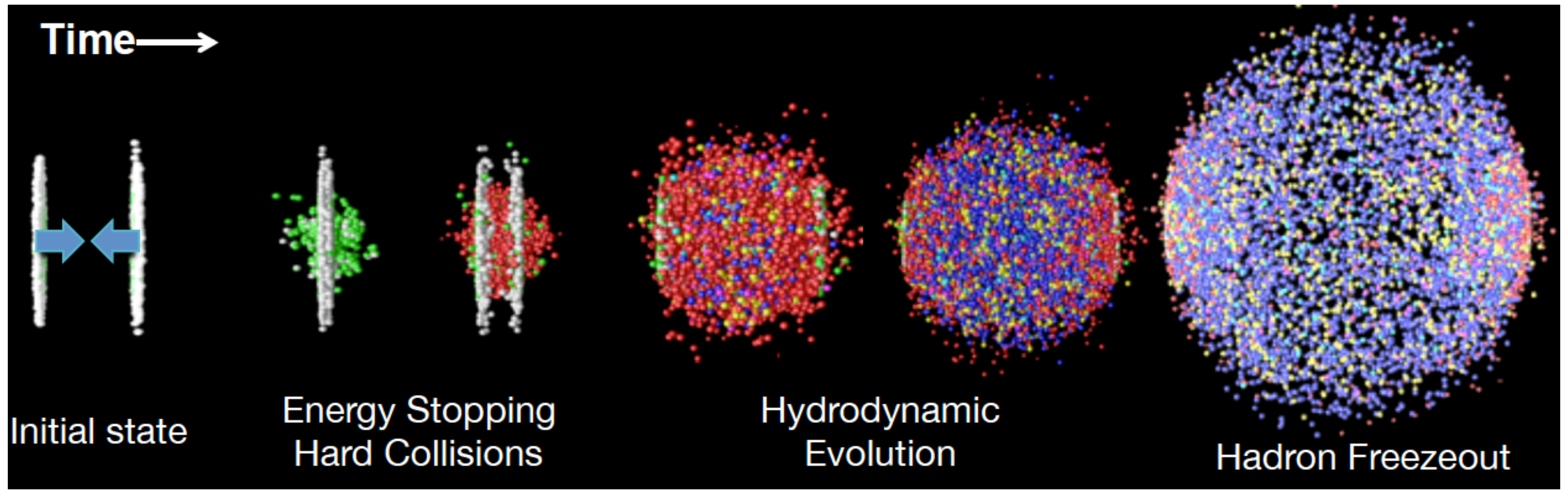
Spin-thermal approach does not capture properly phenomena seen in experiment.

Nonequilibrium dynamics of spin is suggested.

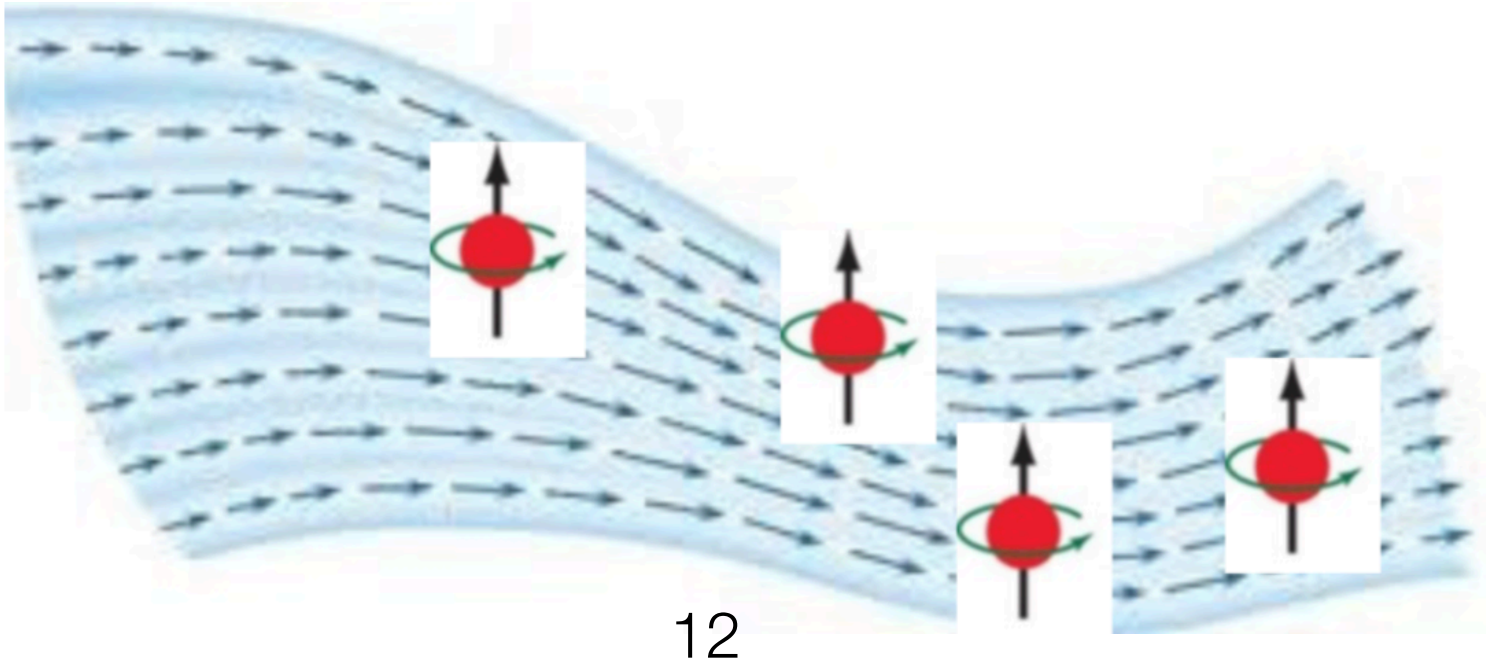
If spin polarization is trully hydrodynamic quantity it should not be enslaved to thermal vorticity.

W. Florkowski, B. Friman, A. Jaiswal, E. Speranza, PRC 97 (4) (2018) 041901

Relativistic fluid dynamics forms the core of HIC models



Fluid dynamics with spin



INCORPORATING SPIN IN HYDRODYNAMICS

Conservation of angular momentum and spin chemical potential

Conservation of charge (baryon number, electric charge, ...)

$$\partial_\mu \widehat{N}^\mu(x) = 0 \qquad \text{(1 equation/charge)} \quad \rightarrow \quad \mu \equiv \xi T$$

Conservation of energy and momentum

$$\widehat{J}_C^{\mu,\alpha\beta}(x) = \underbrace{x^\alpha \widehat{T}_C^{\mu\beta}(x) - x^\beta \widehat{T}_C^{\mu\alpha}(x)}_{\widehat{L}_C^{\mu,\alpha\beta}(x)} + \widehat{S}_C^{\mu,\alpha\beta}(x)$$

$$\partial_\mu \widehat{T}_C^{\mu\alpha}(x) = 0 \qquad \text{(4 equations)} \quad \rightarrow \quad T, u^\nu$$

The conservation of angular momentum implies introduction of new hydrodynamic variables — spin chemical potential

Conservation of total angular momentum

$$\partial_\mu \widehat{J}_C^{\mu,\alpha\beta}(x) = 0 \quad \Rightarrow \quad \partial_\mu \widehat{S}_C^{\mu,\alpha\beta}(x) = \widehat{T}_C^{\beta\alpha}(x) - \widehat{T}_C^{\alpha\beta}(x) \quad \rightarrow \quad \Omega_{\mu\nu} \equiv T\omega_{\mu\nu}$$

Conservation of angular momentum and spin chemical potential

If the **energy-momentum tensor is symmetric** the spin tensor is conserved

W. Florkowski, B. Friman, A. Jaiswal, E. Speranza, PRC 97 (4) (2018) 041901
W. Florkowski, B. Friman, A. Jaiswal, R. R., E. Speranza, PRD 97 (2018) 116017
F. Becattini, W. Florkowski, E. Speranza, PLB 789 (2019) 419-425
W. Florkowski, A. Kumar, R. R., PPNP 108 (2019) 103709

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\lambda S^{\lambda,\mu\nu} = 0, \quad \partial_\mu N^\mu = 0$$

What are the **constitutive relations** which enter **equations of motion**?

$$T^{\mu\nu} = T^{\mu v}[\beta, \omega, \xi], \quad S^{\mu,\lambda v} = S^{\mu,\lambda v}[\beta, \omega, \xi], \quad N^\mu = N^\mu[\beta, \omega, \xi]$$

Coarse-graining of underlying microscopic theory is required!
Relativistic kinetic theory (RKT) is commonly used.

Relativistic kinetic theory with spin

To include spin in RKT, we start from the **Wigner function (WF)** that bridges the gap between QFT and RKT

$$\mathcal{W}_{\alpha\beta} = \frac{1}{4}\left(\mathcal{F} + i\gamma^5\mathcal{P} + \gamma^\mu\mathcal{V}_\mu + \gamma^5\gamma^\mu\mathcal{A}_\mu + \frac{1}{2}\Sigma^{\mu\nu}\mathcal{S}_{\mu\nu}\right)_{\alpha\beta} \qquad \Sigma^{\mu\nu} = i\gamma^{[\mu}\gamma^{\nu]}$$

For spin-1/2 particles the WF satisfies the **quantum kinetic equation**

$$\left[\gamma\cdot\left(p+\frac{i}{2}\partial\right)-m\right]\mathcal{W}_{\alpha\beta} = \mathcal{C}\left[\mathcal{W}_{\alpha\beta}\right]$$

D. Vasak, M. Gyulassy, H.T. Elze, Ann. Phys. 173 (1987) 462–492,
P. Zhuang, U.W. Heinz, Ann. Phys. 245 (1996) 311–338,
H.T. Elze, M. Gyulassy, D. Vasak, Phys. Lett. B 177 (1986) 402–408; Nucl. Phys. B 276 (1986) 706–728
W. Florkowski, J. Hufner, S.P. Klevansky, L. Neise, Ann. Phys. 245 (1996) 445–463
N. Weickgenannt, X.L. Sheng, E. Speranza, Q. Wang, D.H. Rischke, Phys. Rev. D 100 (5) (2019) 056018;

Relativistic kinetic theory with spin

From the LO and NLO of the **semi-classical expansion of the WF** in powers of Planck's constant, one obtains **two independent kinetic equations**

	scalar	axial vector
Kin. Eq.	$k^\mu \partial_\mu \mathcal{F}(x,k) = \mathcal{C}_\mathcal{F}$	$k^\mu \partial_\mu \mathcal{A}^\nu(x,k) = \mathcal{C}^\nu_\mathcal{A}$
RTA	$C_\mathcal{F} = \frac{(k\cdot u)}{\tau_{\rm eq}}\left[\mathcal{F}_{\rm eq}(x,k) - \mathcal{F}(x,k)\right]$	$C^\nu_\mathcal{A} = \frac{(k\cdot u)}{\tau_{\rm eq}}\left[\mathcal{A}^\nu_{\rm eq}(x,k) - \mathcal{A}^\nu(x,k)\right]$
Dist. fn.	$\mathcal{F}^\pm(x,k) = 2m \int_{p,s} f^\pm(x,p,s)\ \delta^{(4)}(k \mp p)$	$\mathcal{A}^\mu_\pm(x,k) = 2m \int_{p,s} s^\mu f^\pm(x,p,s)\ \delta^{(4)}(k \mp p)$

Momentum measure $\rightarrow \quad \int_p(\cdots) \rightarrow \int d^3p/\,(2\pi)^3\, p^0.$

Spin measure $\rightarrow \quad \int_s(\cdots) \rightarrow (m/\pi\mathfrak{s}) \int d^4s\,\delta(s\cdot s + \mathfrak{s}^2)\delta(p\cdot s).$

$$\mathfrak{s}^2 = \tfrac{1}{2}(1+\tfrac{1}{2})$$

CLASSICAL APPROACH TO SPIN HYDRODYNAMICS

In the classical treatments of particles with spin-1/2 one introduces internal angular momentum tensor of particles

[M. Mathisson, APPB 6 (1937) 163-2900]

$$s^{\alpha\beta} = \frac{1}{m}\epsilon^{\alpha\beta\gamma\delta}p_\gamma s_\delta$$

Satisfies Frenkel (or Weyssenhoff)

$$p_\alpha s^{\alpha\beta} = 0$$

In particle rest frame (PRF)

$$p^\mu = (m,0,0,0),\, s^\alpha = (0,\mathbf{s}_*)$$

$$-s^2 = -s^\alpha s_\alpha = |\mathbf{s}_*|^2 = \mathfrak{z}^2 = \tfrac{1}{2}\left(1+\tfrac{1}{2}\right) = \tfrac{3}{4}$$

M.Mathisson

J. Weyssenhoff

Relativistic kinetic theory with spin

The **distribution function** in the **extended phase-space** is a function of **spacetime**, **momentum**, and **internal angular momentum** of the particles

$$f^{\pm}(x, p, s) \qquad \boldsymbol{x} \equiv x^{\mu} \quad \boldsymbol{p} \equiv p^{\mu} \quad \boldsymbol{s} \equiv s^{\mu\nu}$$

The **kinetic equation (KE)** governing the evolution of the distribution function can be written as

W. G. Dixon, Nuovo Cimento (1955–1965) 34, 317 (1964).
L. Suttorp and S. De Groot, Il Nuovo Cimento A (1965–1970) 65, 245 (1970)
C.G. van Weert, thesis, The University of Amsterdam, 1970.

$$p^{\mu}\partial_{\mu}^{(x)} f^{\pm} + m\mathcal{F}^{\mu}\partial_{\mu}^{(p)} f^{\pm} + m\mathcal{S}^{\mu\nu}\partial_{\mu\nu}^{(s)} f^{\pm} = \mathcal{C}[f^{\pm}]$$

where

$$\partial_{\mu}^{(x)} \equiv \frac{\partial}{\partial x^{\mu}}, \qquad \partial_{\mu}^{(p)} \equiv \frac{\partial}{\partial p^{\mu}}, \qquad \partial_{\mu\nu}^{(s)} \equiv \frac{\partial}{\partial s^{\mu\nu}}, \qquad \mathcal{F}^{\alpha} \equiv \frac{dp^{\alpha}}{d\tau} \qquad \mathcal{S}^{\alpha\beta} \equiv \frac{ds^{\alpha\beta}}{d\tau}$$

Using the Frenkel condition, one can derive the **force (Lorentz and Mathisson)** and **torque**

I. Bailey and W. Israel, Commun. Math. Phys. 42, 65 (1975).

$$\mathcal{F}^{\alpha} = \boxed{\frac{\mathfrak{q}}{m} F^{\alpha\beta} p_{\beta}} + \boxed{\frac{1}{2}\left(\partial^{\alpha} F^{\beta\gamma}\right) m_{\beta\gamma}} \qquad \mathcal{S}^{\alpha\beta} = 2\, F^{\gamma[\alpha}\, m^{\beta]}{}_{\gamma} - \frac{1}{m^2}\left(\chi - \frac{\mathfrak{q}}{m}\right) F_{\phi\gamma}\, s^{\phi[\alpha}\, p^{\beta]} p^{\gamma}$$

where **magnetic dipole moment** is $m^{\alpha\beta} = \chi\, s^{\alpha\beta}$

Infinite conductivity limit

In the limit of **infinite conductivity**, field strength tensor is

$$F^{\mu\nu} \to B^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} u_\alpha B_\beta$$

$$u_\mu B^\mu = 0 \qquad\qquad B_\mu B^\mu \leq 0$$

If the medium if magnetizable, then the **Maxwell's equations** are given by

$$\partial_\mu H^{\mu\nu} = J^\nu, \qquad\qquad \partial_\mu \widetilde{F}^{\mu\nu} = 0,$$

$$\left(\widetilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}\right)$$

$$H^{\mu\nu} = F^{\mu\nu} + M^{\mu\nu}$$

From KT to Spin MHD

The **particle current**, **energy-momentum tensor**, and **spin tensor** of the fluid can be expressed as

S. Bhadury, W. Florkowski, A. Jaiswal, A. Kumar, and R. R., Phys. Lett. B 814, 136096 (2021); Phys. Rev. D 103, 014030 (2021).

$$N^{\mu} = \int_{p,s} p^{\mu} \left(f^{+} - f^{-}\right),$$

$$T_f^{\mu\nu} = \int_{p,s} p^{\mu} p^{\nu} \left(f^{+} + f^{-}\right),$$

$$S^{\lambda,\mu\nu} = \int_{p,s} p^{\lambda} s^{\mu\nu} \left(f^{+} + f^{-}\right)$$

while the **polarization-magnetization tensor** is

$$M^{\alpha\beta} = m \int_{p,s} m^{\alpha\beta} \left(f^{+} - f^{-}\right)$$

From KT to Spin MHD

Assuming that the microscopic interactions **preserve fundamental conservation laws** one requires

S. Bhadury, W. Florkowski, A. Jaiswal, A. Kumar, and R. R., Phys. Lett. B 814, 136096 (2021); Phys. Rev. D 103, 014030 (2021).

$$\int_{p,s} \mathcal{C}\left[f\right] = 0,$$

$$\int_{p,s} p^{\mu}\mathcal{C}\left[f\right] = 0,$$

$$\int_{p,s} s^{\mu\nu}\mathcal{C}\left[f\right] = 0$$

Zeroth, first, and 'spin' moment of the KE (**in absence of the torque**) then lead to equations defining relativistic magnetohydrodynamics for fluid with spin

$$p^{\mu}\partial_{\mu}^{(x)} f^{\pm} + m\mathcal{F}^{\mu}\partial_{\mu}^{(p)} f^{\pm} = \mathcal{C}[f^{\pm}] \quad \Rightarrow$$

$$\partial_{\mu} N^{\mu} = 0$$

$$\partial_{\nu} T_{f}^{\mu\nu} = F^{\mu}{}_{\alpha} J_{f}^{\alpha} + \frac{1}{2}\left(\partial^{\mu} F^{\nu\alpha}\right) M_{\nu\alpha}$$

$$\partial_{\lambda} S^{\lambda,\mu\nu} = 0$$

$$J_{\mathrm{f}}^{\mu} = \mathfrak{q} N^{\mu}$$

Relativistic MHD with spin

Kinetic equation with collision kernel in the **relaxation-time approximation (RTA)** reads

J. L. Anderson and H. Witting, Physica (Utrecht) 74, 466 (1974)

$$p^\mu \partial_\mu^{(x)} f^\pm + m\mathcal{F}^\mu \partial_\mu^{(p)} f^\pm = -\frac{(u\cdot p)}{\tau_\mathrm{R}} \delta f^\pm$$

$$\delta f^\pm(x,p,s) = f^\pm(x,p,s) - f^\pm_{\mathrm{eq}}(x,p,s)$$

Using RTA kinetic equation we can write the **first-order gradient correction** as

$$p^\mu \partial_\mu^{(x)} f^\pm + m\mathcal{F}^\mu \partial_\mu^{(p)} f^\pm = -\frac{(u\cdot p)}{\tau_\mathrm{R}} \delta f^\pm \quad \Rightarrow \quad \delta f^\pm_{(1)} = -\mathcal{D} f^\pm_{\mathrm{eq}},$$

$$\mathcal{D} = \frac{\tau_R}{(u\cdot p)}\left(p^\alpha \frac{\partial}{\partial x^\alpha} + \mathcal{F}^\alpha \frac{\partial}{\partial p^\alpha}\right)$$

The **equilbrium distribution function** has the form

$$f^\pm_{\mathrm{eq}} = \frac{1}{1+\exp\left[\beta(u\cdot p) \mp \xi - \frac{1}{2}\omega : s\right]}$$

small polarization limit $\Rightarrow$

$$f_{\mathrm{eq}} = f_0 + \frac{1}{2}(\omega : s)\, f_0 \tilde{f}_0,$$

$$f_0 \equiv \{1+\exp[\beta(u\cdot p) - \xi]\}^{-1} \qquad \tilde{f}_0 \equiv 1 - f_0$$

Relativistic MHD with spin

The expressions for **dissipative currents** in terms of the nonequilibrium correction to the distribution function are

$$N^\mu = nu^\mu + n^\mu$$

$$T_{\mathrm{f}}^{\mu\nu} = \epsilon u^\mu u^\nu - (P + \Pi)\,\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$S^{\lambda,\mu\nu} = S_{\mathbf{eq}}^{\lambda,\mu\nu} + \delta S^{\lambda,\mu\nu}$$

$$\Pi = -\frac{\Delta_{\alpha\beta}}{3}\int_{p,s} p^\alpha p^\beta \left(\delta f^+ + \delta f^-\right)$$

$$\pi^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta}\int_{p,s} p^\alpha p^\beta \left(\delta f^+ + \delta f^-\right)$$

$$n^\mu = \Delta^\mu_\alpha \int_{p,s} p^\alpha \left(\delta f^+ - \delta f^-\right)$$

$$\delta S^{\lambda,\mu\nu} = \int_{p,s} p^\lambda s^{\mu\nu} \left(\delta f^+ + \delta f^-\right)$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

$$\Delta^{\mu\nu}_{\alpha\beta} \equiv \frac{1}{2}\left(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\mu_\beta \Delta^\nu_\alpha\right) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}$$

Barnett effect

Equilibrium polarization-magnetization tensor is

$$M^{\mu\nu}_{\mathrm{eq}} = a_1(T,\mu)\,\omega^{\mu\nu} + a_2(T,\mu)\,u^{[\mu}u_\gamma\omega^{\nu]\gamma}$$

In **global equilibrium**, spin chemical potential corresponds to rotation of the fluid

F. Becattini and F. Piccinini, Ann. Phys. (Amsterdam) 323, 2452 (2008)
F. Becattini, W. Florkowski, and E. Speranza, Phys. Lett. B 789, 419 (2019).

$$\omega^{\mu\nu}\big|_{\mathrm{geq}} \propto \varpi^{\mu\nu} = \left(\partial^\mu\beta^\nu - \partial^\nu\beta^\mu\right)/2$$

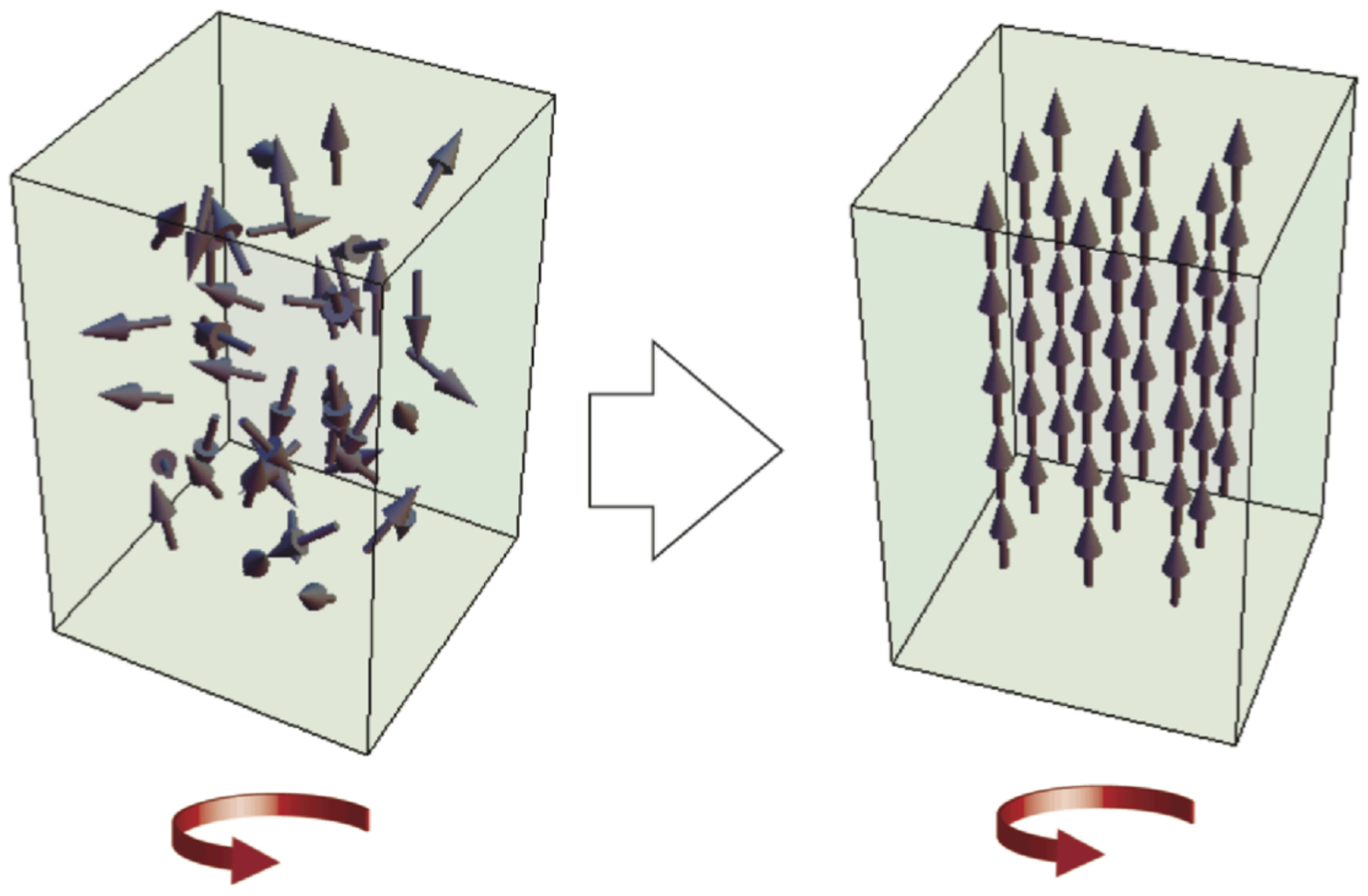


figure: Journal of the Physical Society of Japan 90, 081003 (2021)

We conclude that **rotation of the fluid produces magnetization**, which is precisely the physics of **Barnett effect**.

S. J. Barnett, Rev. Mod. Phys. 7, 129 (1935)
A. Einstein and W. de Haas, Deutsch. Phys. Ges., Verh. 17, 152 (1915)

Conversion between vorticity and spin

Using the **spin matching condition** we obtain the **evolution equation for the spin polarization tensor**

$$\dot{\omega}^{\mu\nu} = \mathcal{D}_{\Pi}^{\mu\nu}\,\theta + \mathcal{D}_{\mathrm{n}}^{\mu\nu\gamma}\,(\nabla_{\gamma}\xi) + \mathcal{D}_{\mathrm{a}}^{\mu\nu\gamma}\,\dot{u}_{\gamma} + \mathcal{D}_{\pi}^{\mu\nu\rho\kappa}\,\sigma_{\rho\kappa} + \mathcal{D}_{\Omega}^{\mu\nu\rho\kappa}\,\Omega_{\rho\kappa} + \mathcal{D}_{\Sigma}^{\mu\nu\phi\rho\kappa}\,(\nabla_{\phi}\omega_{\rho\kappa})$$

$$\Omega_{\mu\nu} \equiv (\partial_{\mu}u_{\nu} - \partial_{\nu}u_{\mu})/2$$

fluid vorticity

We observe that the above equation contains information about the **connection between evolution of spin polarization tensor** and **fluid vorticity.**

$\mathcal{D}_{\Omega}^{\mu\nu\rho\kappa}$ vanishes in absence of electromagnetic field which leads us to conclusion that the **conversion between spin-polarization and vorticity proceeds via coupling with electromagnetic field**.

First-order dissipative currents in SMHD

The expressions for **dissipative currents** in terms of the nonequilibrium corrections to the distribution function are

$$X = \tau_{\text{eq}} \Big[\beta_{X\Pi}\, \theta + \beta^{\alpha}_{Xn} \left(\nabla_\alpha \xi\right) + \beta^{\alpha}_{Xa} \dot{u}_\alpha + \beta^{\alpha\beta}_{X\pi} \sigma_{\alpha\beta} + \beta^{\alpha\beta}_{X\Omega} \Omega_{\alpha\beta} + \beta^{\alpha\beta}_{XF} \left(\nabla_\alpha B_\beta\right) + \beta^{\alpha\beta\gamma}_{X\Sigma} \left(\nabla_\alpha \omega_{\beta\gamma}\right) \Big],$$

where

$$X \equiv n^\mu,\ \Pi,\ \pi^{\mu\nu},\ \delta S^{\lambda,\mu\nu}$$

These expressions contain gradients of magnetic field.

Demanding that the divergence of the above entropy current is positive definite we identify **first-order dissipative gradient terms**

$$\Pi = -\zeta\theta, \quad n^\mu = \kappa^{\mu\alpha} \left(\nabla_\alpha \xi\right), \quad \pi^{\mu\nu} = \eta^{\mu\nu\alpha\beta} \sigma_{\alpha\beta},$$
$$\delta S^{\mu,\alpha\beta} = \Sigma^{\mu\alpha\beta\lambda\gamma\rho} \left(\nabla_\lambda \omega_{\gamma\rho}\right).$$

Conclusions

We presented the first kinetic theory formulation of **relativistic dissipative nonresistive magnetohydrodynamics with spin** in the limit of small polarization.

We demonstrated that multiple transport coefficients, dissipative as well as non-dissipative, are present.

We showed that our framework naturally leads to the **emergence of the relativistic analog of Barnett effect**.

We show that the **coupling between the magnetic field and spin polarization appears at gradient order**.

Simulation based on our unified framework has the potential of **explaining the difference of Λ and anti-Λ polarization.**

Many more interesting theory developments: see talks by **Buzzegoli, Liao, Palermo, Singh, …**

THANK YOU FOR YOUR ATTENTION.