



Vector meson spin alignments in high energy reaction processes

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Outline

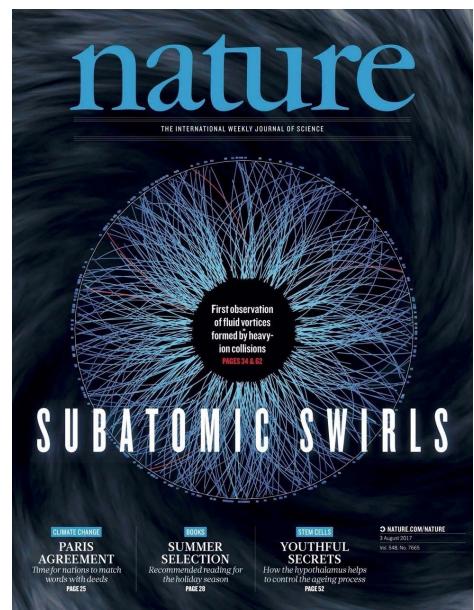


- Introduction
- The **global vector meson spin alignment vs the global hyperon polarization in HIC**
- Vector meson alignment in quark fragmentation
- Summary and outlook

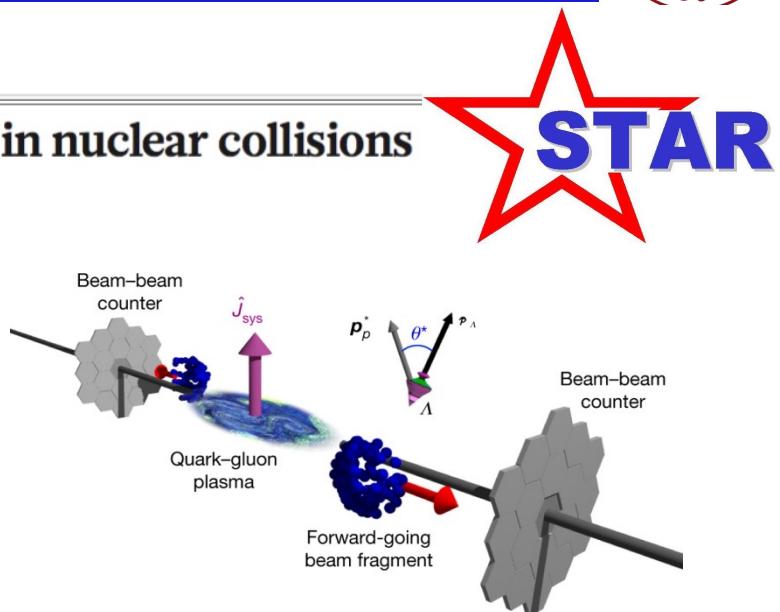
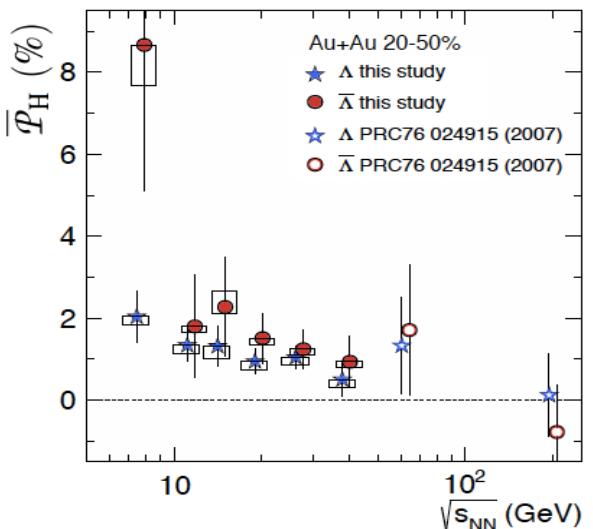


Introduction: Global Λ polarization in HIC has been observed

Nature 548, 62-65 (2017) LETTER



Global Λ hyperon polarization in nuclear collisions



PRL 94, 102301 (2005)

PHYSICAL REVIEW LETTERS

week ending
18 MARCH 2005

Globally Polarized Quark-Gluon Plasma in Noncentral $A + A$ Collisions

Zuo-Tang Liang¹ and Xin-Nian Wang^{2,1}

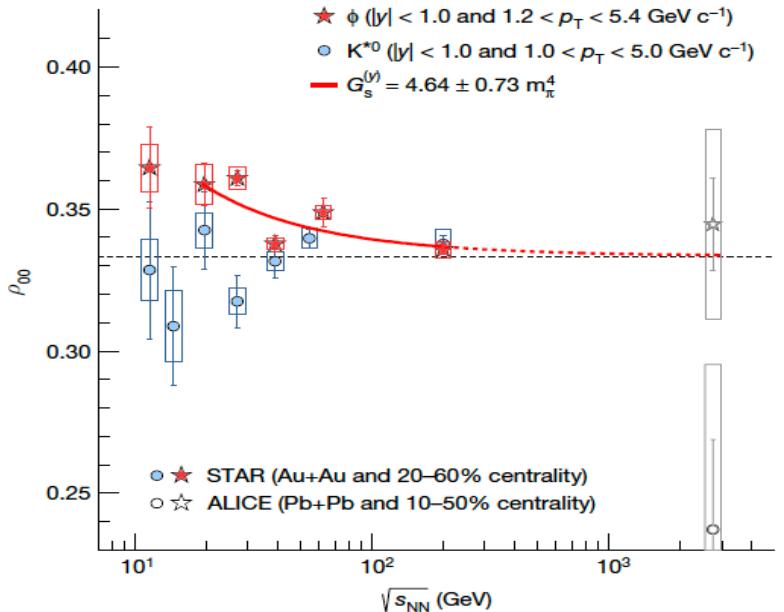
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Introduction: Global vector meson spin alignment in HIC

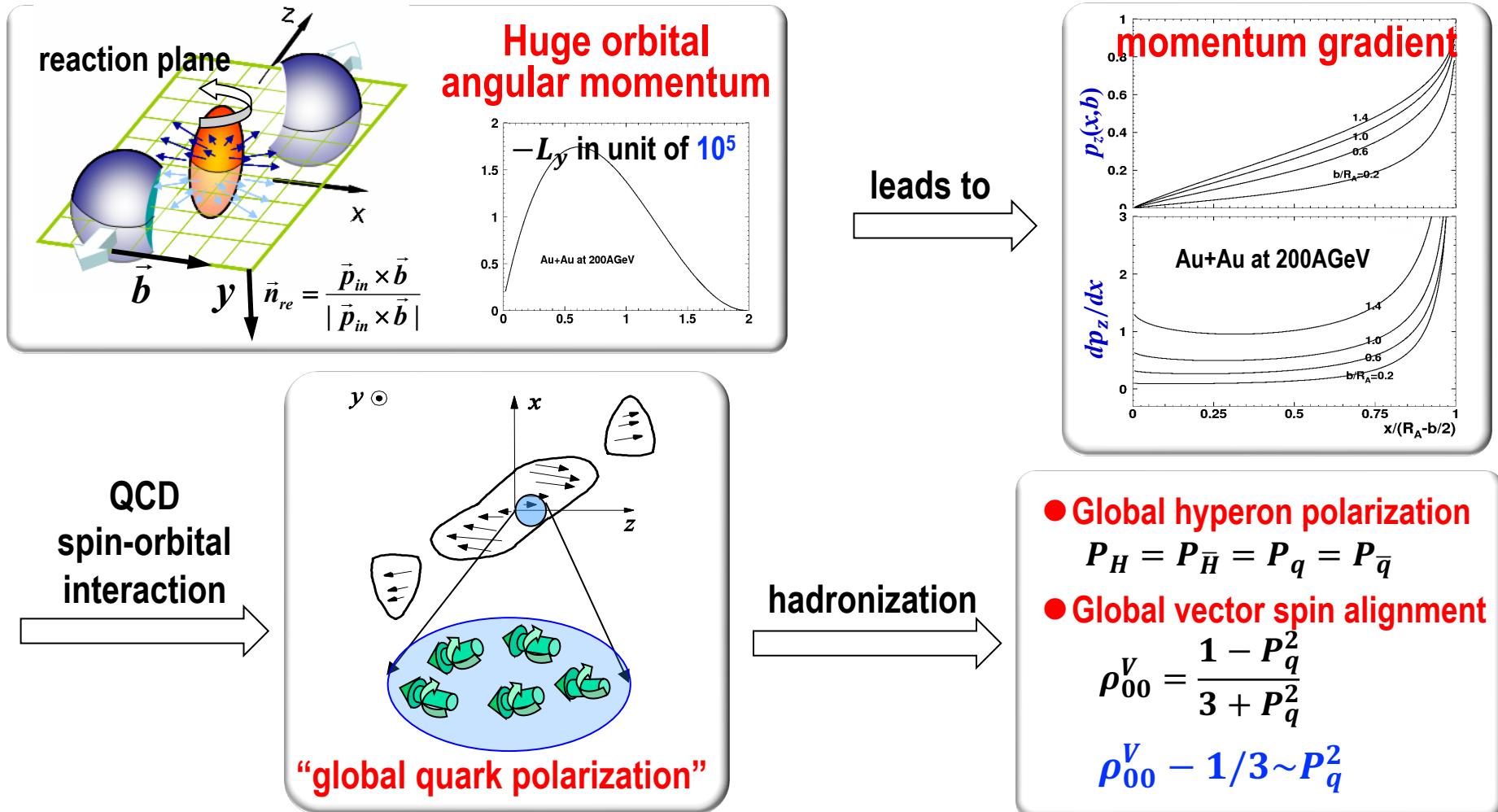
STAR, M.S. Abdallah et al., Nature 614, 244 (2023).



- Why the vector meson spin alignment so interesting?
- What can we learn?
- How about it in other high energy reactions?

Prediction of globally polarized QGP

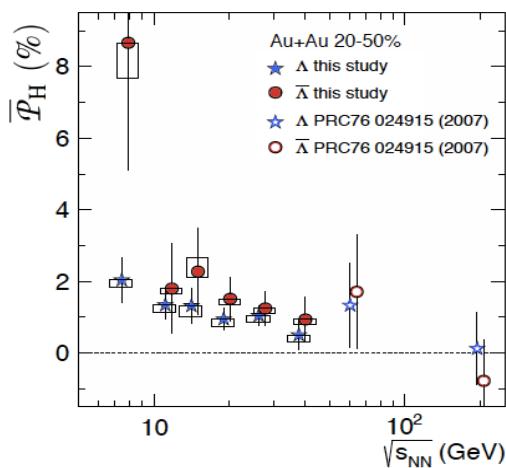
ZTL & Xin-Nian Wang, PRL 94, 102301(2005); PLB 629, 20 (2005)



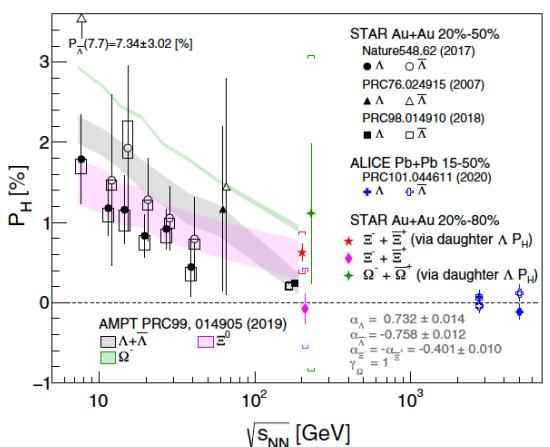
Global: the direction is fixed; the magnitude is approximately the same.

Great efforts of our experimental colleagues

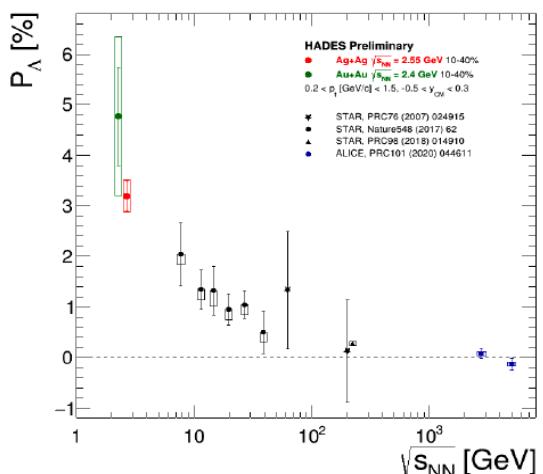
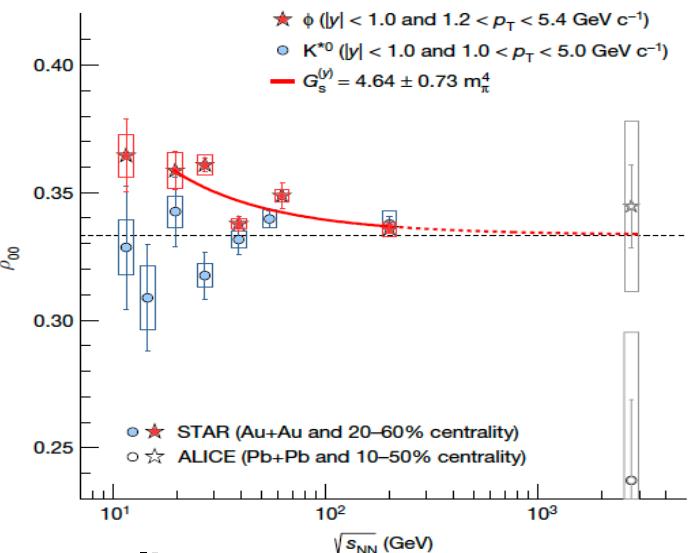
STAR, L. Adamczyk *et al.*,
Nature 548, 62 (2017).



STAR, J. Adam *et al.*,
PRL 126, 162301 (2021)



STAR, M.S. Abdallah *et al.*,
Nature 614, 244 (2023).



HADES, R. Yassine *et al.*, PLB 835, 137506 (2022)

Aihong Tang, plenary talk

$$\left| \rho_{00}^V - \frac{1}{3} \right| \gg P_\Lambda^2 \sim P_q^2$$

What does it tell us?
 How can we understand it?



Global hyperon polarization

ZTL & Xin-Nian Wang, PRL 94, 102301 (2005).

Quark combination scenario $q_1^\uparrow + q_2^\uparrow + q_3^\uparrow \rightarrow H$

dominates at small
and intermediate p_T

$$\hat{\rho}_{q_1 q_2 q_3} = \hat{\rho}_{q_1} \otimes \hat{\rho}_{q_2} \otimes \hat{\rho}_{q_3} \quad \hat{\rho}_q = \frac{1}{2} \begin{pmatrix} 1 + P_q & 0 \\ 0 & 1 - P_q \end{pmatrix}$$

$$\begin{aligned} \rho_H(\mathbf{m}, \mathbf{m}') &= \langle \mathbf{j}_H, \mathbf{m}' | \hat{\rho}_{q_1 q_2 q_3} | \mathbf{j}_H, \mathbf{m} \rangle \\ &= \sum_{\mathbf{m}_i, \mathbf{m}'_i} \rho_{q_1 q_2 q_3}(\mathbf{m}_i, \mathbf{m}'_i) \langle \mathbf{j}_H, \mathbf{m}' | \mathbf{m}'_1, \mathbf{m}'_2, \mathbf{m}'_3 \rangle \langle \mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3 | \mathbf{j}_H, \mathbf{m} \rangle \end{aligned}$$

Clebsch-Gordon coefficients

after normalization

$$\rho_H(\mathbf{m}, \mathbf{m}') = \frac{\sum_{\mathbf{m}_i, \mathbf{m}'_i} \rho_{q_1 q_2 q_3}(\mathbf{m}_i, \mathbf{m}'_i) \langle \mathbf{j}_H, \mathbf{m}' | \mathbf{m}'_1, \mathbf{m}'_2, \mathbf{m}'_3 \rangle \langle \mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3 | \mathbf{j}_H, \mathbf{m} \rangle}{\sum_{\mathbf{m}, \mathbf{m}_i, \mathbf{m}'_i} \rho_{q_1 q_2 q_3}(\mathbf{m}_i, \mathbf{m}'_i) \langle \mathbf{j}_H, \mathbf{m} | \mathbf{m}'_1, \mathbf{m}'_2, \mathbf{m}'_3 \rangle \langle \mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3 | \mathbf{j}_H, \mathbf{m} \rangle}$$



Global hyperon polarization

ZTL & Xin-Nian Wang, PRL 94, 102301 (2005).

Quark combination scenario $q_1^\uparrow + q_2^\uparrow + q_3^\uparrow \rightarrow H$

dominates at small
and intermediate p_T

$$P_H = \rho_H \left(\frac{1}{2}, \frac{1}{2} \right) - \rho_H \left(-\frac{1}{2}, -\frac{1}{2} \right)$$

$$P_H = c_1 P_{q_1} + c_2 P_{q_2} + c_3 P_{q_3}$$

c_i 's are constants
determined by C.G. coefficients.

hyperon	Λ	Σ^+	Σ^0	Σ^-	Ξ^0	Ξ^-
combination	P_s	$\frac{4P_u - P_s}{3}$	$\frac{2(P_u + P_d) - P_s}{3}$	$\frac{4P_d - P_s}{3}$	$\frac{4P_s - P_u}{3}$	$\frac{4P_s - P_d}{3}$

In the case that $P_u = P_d = P_s = P_{\bar{u}} = P_{\bar{d}} = P_{\bar{s}}$,

$P_H = P_{\bar{H}} = P_q$ for all H 's and \bar{H} 's (global polarization)

Global vector meson spin alignment



ZTL & Xin-Nian Wang, PLB 629, 20 (2005).

Quark combination scenario $q_1^\uparrow + \bar{q}_2^\uparrow \rightarrow V$

$$\hat{\rho}_{q_1\bar{q}_2} = \hat{\rho}_{q_1} \otimes \hat{\rho}_{\bar{q}_2} \quad \hat{\rho}_q = \frac{1}{2} \begin{pmatrix} 1 + P_q & 0 \\ 0 & 1 - P_q \end{pmatrix} \quad \hat{\rho}_{\bar{q}} = \frac{1}{2} \begin{pmatrix} 1 + P_{\bar{q}} & 0 \\ 0 & 1 - P_{\bar{q}} \end{pmatrix}$$

$$\rho_V(m, m') = \frac{\sum_{m_i, m'_i} \rho_{q_1\bar{q}_2}(m_i, m'_i) \langle j_V, m' | m'_1, m'_2 \rangle \langle m_1, m_2 | j_V, m \rangle}{\sum_{m, m_i, m'_i} \rho_{q_1\bar{q}_2}(m_i, m'_i) \langle j_V, m | m'_1, m'_2 \rangle \langle m_1, m_2 | j_V, m \rangle}$$

$$\rho_{00}^V = \frac{1 - P_{q_1} P_{\bar{q}_2}}{3 + P_{q_1} P_{\bar{q}_2}} = \frac{1 - P_q^2}{3 + P_q^2}$$

$$\hat{\rho}^V = \begin{pmatrix} \rho_{11}^V & \rho_{10}^V & \rho_{1-1}^V \\ \rho_{01}^V & \rho_{00}^V & \rho_{0-1}^V \\ \rho_{-11}^V & \rho_{-10}^V & \rho_{-1-1}^V \end{pmatrix}$$

In both calculations, we considered only the spin degree of freedom and took P_q as a constant, no fluctuation, no correlation etc.

What does it change if we take other degrees of freedom into account?



Take other degrees of freedom into account

We make a minimal step forward and consider other degree of freedom denoted by α

The basis state for a quark: $|m, \alpha_q\rangle$

The element of the spin density matrix: $\rho_{qm_q, m'_q}(\alpha_q, \alpha'_q) = \langle m'_q, \alpha'_q | \hat{\rho}_q | m_q, \alpha_q \rangle$

We consider the simple case:

For $q_1^\uparrow + \bar{q}_2^\uparrow \rightarrow V$

- $\rho_{qm_q, m'_q}(\alpha_q, \alpha'_q) = \rho_{qm_q, m'_q}(\alpha_q) \delta_{\alpha_q, \alpha'_q}$ diagonal w.r.t. α_q
- $\hat{\rho}_{q_1 \bar{q}_2} = \hat{\rho}_{q_1} \otimes \hat{\rho}_{\bar{q}_2}$ wave function of V with α_V
- factorized: $\langle \alpha_{q_1}, m_{q_1}; \alpha_{\bar{q}_2}, m_{\bar{q}_2} | j_V, m_V, \alpha_V \rangle = \langle \alpha_{q_1}, \alpha_{\bar{q}_2} | \alpha_V \rangle \langle m_{q_1}, m_{\bar{q}_2} | j_V, m_V \rangle$

$$\rho_{mm'}^V(\alpha_V) = \rho_{mm'}^V(\alpha_V, \alpha_V) = \sum_{\alpha_{q_1}, \alpha_{\bar{q}_2}} |\langle \alpha_{q_1}, \alpha_{\bar{q}_2} | \alpha_V \rangle|^2 \rho_{mm'}^{V(l)}(\alpha_{q_1}, \alpha_{\bar{q}_2}) \quad \text{average inside } V$$

$$\rho_{mm'}^{V(l)}(\alpha_{q_1}, \alpha_{\bar{q}_2}) = \sum_{m_{q_1}, m_{\bar{q}_2}, m'_{q_1}, m'_{\bar{q}_2}} \langle j_V m' | m'_{q_1} m'_{\bar{q}_2} \rangle \rho_{m_{q_1} m'_{q_1}}^q(\alpha_{q_1}) \rho_{m_{\bar{q}_2} m'_{\bar{q}_2}}^q(\alpha_{\bar{q}_2}) \langle m_{q_1}, m_{\bar{q}_2} | j_V m \rangle$$

similar to what we had when α -dependence were not considered.

We can also further average over α_V and obtain the α_V -averaged spin alignment.



Take other degrees of freedom into account

In this way, we obtain

average inside V

$$\rho_{00}^V(\alpha_V) = \frac{1 - \langle P_{q_1} P_{\bar{q}_2} \rangle_V}{3 + \langle P_{q_1} P_{\bar{q}_2} \rangle_V} \quad \langle P_{q_1} P_{\bar{q}_2} \rangle_V = \sum_{\alpha_{q_1}, \alpha_{\bar{q}_2}} |\langle \alpha_{q_1}, \alpha_{\bar{q}_2} | \alpha_V \rangle|^2 P_{q_1}(\alpha_{q_1}) P_{\bar{q}_2}(\alpha_{\bar{q}_2})$$

We further average over α_V and obtain the α_V -averaged spin alignment.

$$\langle \rho_{00}^V \rangle = \frac{1 - \langle P_{q_1} P_{\bar{q}_2} \rangle}{3 + \langle P_{q_1} P_{\bar{q}_2} \rangle} \quad \langle P_{q_1} P_{\bar{q}_2} \rangle = \sum_{\alpha_V} f_V(\alpha_V) \langle P_{q_1} P_{\bar{q}_2} \rangle_V$$

In general, α_V denotes a set of variables $(\alpha_{1V}, \alpha_{2V}, \dots, \alpha_{jV})$.

We can integrate over only part of them and study the dependence on the others, i.e.,

$$\langle \rho_{00}^V \rangle(\alpha_{1V}, \alpha_{2V}, \dots, \alpha_{kV}) = \frac{1 - \langle P_{q_1} P_{\bar{q}_2} \rangle_{\alpha_{k+1V}, \dots, \alpha_{jV}}}{3 + \langle P_{q_1} P_{\bar{q}_2} \rangle_{\alpha_{k+1V}, \dots, \alpha_{jV}}}$$

$$\langle P_{q_1} P_{\bar{q}_2} \rangle_{\alpha_{k+1V}, \dots, \alpha_{jV}} = \sum_{\alpha_{k+1V}, \dots, \alpha_{jV}} f(\alpha_{1V}, \alpha_{2V}, \dots, \alpha_{jV}) \langle P_{q_1} P_{\bar{q}_2} \rangle_V / \sum_{\alpha_{k+1V}, \dots, \alpha_{jV}} f(\alpha_{1V}, \alpha_{2V}, \dots, \alpha_{jV})$$

Take other degrees of freedom into account



The average is two folded:

$$\langle P_{q_1} P_{\bar{q}_2} \rangle = \left\langle \left\langle P_{q_1} P_{\bar{q}_2} \right\rangle_V \right\rangle_S$$

average inside the vector meson V

average over the whole system or a sub-system S

$$\left\langle P_{q_1} P_{\bar{q}_2} \right\rangle_V = \sum_{\alpha_{q_1}, \alpha_{\bar{q}_2}} |\langle \alpha_{q_1}, \alpha_{\bar{q}_2} | \alpha_V \rangle|^2 P_{q_1}(\alpha_{q_1}) P_{\bar{q}_2}(\alpha_{\bar{q}_2})$$

$$\left\langle P_{q_1} P_{\bar{q}_2} \right\rangle_{\alpha_{k+1V}, \dots, \alpha_{jV}} = \frac{\sum_{\alpha_{k+1V}, \dots, \alpha_{jV}} f_S(\alpha_{1V}, \alpha_{2V}, \dots, \alpha_{jV}) \left\langle P_{q_1} P_{\bar{q}_2} \right\rangle_V}{\sum_{\alpha_{k+1V}, \dots, \alpha_{jV}} f_S(\alpha_{1V}, \alpha_{2V}, \dots, \alpha_{jV})}$$

Hyperon polarization v.s. vector meson spin alignment



For $q_1^\uparrow + \bar{q}_2^\uparrow \rightarrow V$

$$\rho_{00}^V = \frac{1 - \langle P_{q_1} P_{\bar{q}_2} \rangle}{3 + \langle P_{q_1} P_{\bar{q}_2} \rangle}$$

For $q_1^\uparrow + q_2^\uparrow + q_3^\uparrow \rightarrow H$

$$\begin{aligned} P_H &= \left\langle \left\langle c_1 P_{q_1} + c_2 P_{q_2} + c_3 P_{q_3} \right\rangle_H \right\rangle_S = \left\langle c_1 \langle P_{q_1} \rangle_H + c_2 \langle P_{q_2} \rangle_H + c_3 \langle P_{q_3} \rangle_H \right\rangle_S \\ &= c_1 \left\langle \langle P_{q_1} \rangle_H \right\rangle_S + c_2 \left\langle \langle P_{q_2} \rangle_H \right\rangle_S + c_3 \left\langle \langle P_{q_3} \rangle_H \right\rangle_S = c_1 \langle P_{q_1} \rangle + c_2 \langle P_{q_2} \rangle + c_3 \langle P_{q_3} \rangle \end{aligned}$$

The STAR data show that: $\langle P_q P_{\bar{q}} \rangle \neq \langle P_q \rangle \langle P_{\bar{q}} \rangle$

One has to take fluctuations into account, so that: $\langle P_q P_{\bar{q}} \rangle \neq \langle P_q \rangle \langle P_{\bar{q}} \rangle$

By studying P_H , we study the average of quark polarization P_q ;
by studying ρ_{00}^V , we study the correlation between P_q and $P_{\bar{q}}$.

Local correlation or long range correlation



One has to take fluctuations into account, i.e.,: $\langle P_q P_{\bar{q}} \rangle \neq \langle P_q \rangle \langle P_{\bar{q}} \rangle$

$$\langle P_q P_{\bar{q}} \rangle = \left\langle \langle P_q P_{\bar{q}} \rangle_V \right\rangle_S$$

average inside the vector meson V

average over the whole system or a sub-system S

(1) local correlation: $\langle P_q P_{\bar{q}} \rangle_V \neq \langle P_q \rangle_V \langle P_{\bar{q}} \rangle_V$

(2) long range correlation: $\langle P_q P_{\bar{q}} \rangle_V = \langle P_q \rangle_V \langle P_{\bar{q}} \rangle_V$

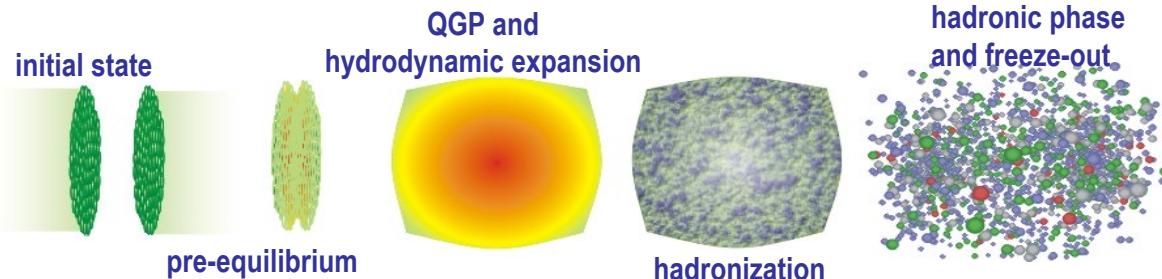
$$\left\langle \langle P_q \rangle_V \langle P_{\bar{q}} \rangle_V \right\rangle_S \neq \left\langle \langle P_q \rangle_V \right\rangle_S \left\langle \langle P_{\bar{q}} \rangle_V \right\rangle_S$$

Vector meson spin alignment contains both contributions.

Vector meson spin alignment — model



strong indication of phi-meson filed \longrightarrow strong local correlation



the only explanation yet

Qun Wang,
plenary talk

$$P_s^\mu(x, p) \approx \frac{1}{4m_s} \epsilon^{\mu\nu\rho\sigma} \left(\omega_{\rho\sigma} + \frac{g_\phi}{(u \cdot p) T_h} F_{\rho\sigma}^\phi \right) p_\nu$$
$$P_{\bar{s}}^\mu(x, p) \approx \frac{1}{4m_s} \epsilon^{\mu\nu\rho\sigma} \left(\omega_{\rho\sigma} - \frac{g_\phi}{(u \cdot p) T_h} F_{\rho\sigma}^\phi \right) p_\nu$$

very strong
local correlation!

- [1] Yang-guang Yang, Ren-hong Fang, Qun Wang, and Xin-Nian Wang, PRC 97, 034917 (2018).
- [2] Xin-Li Sheng, Lucia Oliva, and Qun Wang, PRD 101, 096005 (2020).
- [3] Xin-Li Sheng, Qun Wang and Xin-Nian Wang, PRD 102, 056013 (2020).
- [4] Xin-Li Sheng, Lucia Oliva, ZTL, Qun Wang and Xin-Nian Wang, PRL131, 042304 (2023).
- [5] Xin-Li Sheng, Lucia Oliva, ZTL, Qun Wang and Xin-Nian Wang, 2206.05868 [hep-ph].

Local correlation or long range correlation



Can we separate local or long range correlation experimentally?

Study $\Lambda - \bar{\Lambda}$ or $\Lambda - \Lambda$ spin correlations

ZTL & X.N. Wang

$$C_{NN}^{\Lambda\bar{\Lambda}} \equiv \frac{N_{\Lambda\bar{\Lambda}}^{\uparrow\uparrow} + N_{\Lambda\bar{\Lambda}}^{\downarrow\downarrow} - N_{\Lambda\bar{\Lambda}}^{\uparrow\downarrow} - N_{\Lambda\bar{\Lambda}}^{\downarrow\uparrow}}{N_{\Lambda\bar{\Lambda}}^{\uparrow\uparrow} + N_{\Lambda\bar{\Lambda}}^{\downarrow\downarrow} + N_{\Lambda\bar{\Lambda}}^{\uparrow\downarrow} + N_{\Lambda\bar{\Lambda}}^{\downarrow\uparrow}}$$

Even, in general, $H_i - \bar{H}_j$ or $H_i - H_j$ spin correlations

$$C_{NN}^{H_i\bar{H}_j} \equiv \frac{N_{H_i\bar{H}_j}^{\uparrow\uparrow} + N_{H_i\bar{H}_j}^{\downarrow\downarrow} - N_{H_i\bar{H}_j}^{\uparrow\downarrow} - N_{H_i\bar{H}_j}^{\downarrow\uparrow}}{N_{H_i\bar{H}_j}^{\uparrow\uparrow} + N_{H_i\bar{H}_j}^{\downarrow\downarrow} + N_{H_i\bar{H}_j}^{\uparrow\downarrow} + N_{H_i\bar{H}_j}^{\downarrow\uparrow}}$$

sensitive to the long range correlation

because H_i and \bar{H}_j come from different phase space points



Off-diagonal elements of $\hat{\rho}^V$?

- ZTL & Xin-Nian Wang, PRL 94, 102301 (2005); PLB 629, 20 (2005).

considered the average $\langle \hat{\rho}_q \rangle = \frac{1}{2} \begin{pmatrix} 1 + \langle P_q \rangle & 0 \\ 0 & 1 - \langle P_q \rangle \end{pmatrix}$

i.e., $\langle P_{qy} \rangle = \langle P_q \rangle$, $\langle P_{qz} \rangle = \langle P_{qx} \rangle = 0$, also $\langle P_{q_1 y} P_{\bar{q}_2 y} \rangle = \langle P_{q_1} \rangle \langle P_{\bar{q}_2} \rangle$

- The STAR data show that: $\langle P_q P_{\bar{q}} \rangle \neq \langle P_q \rangle \langle P_{\bar{q}} \rangle$ $\langle P_q P_{\bar{q}} \rangle \gg \langle P_q \rangle \langle P_{\bar{q}} \rangle$

indicates that the fluctuation $\Delta P_{qy}^2 \equiv \langle P_{qy}^2 \rangle - \langle P_{qy} \rangle^2 \sim \langle P_{qy}^2 \rangle \gg \langle P_{qy} \rangle^2$

i.e., compared to ΔP_{qy}^2 , we can even take $\langle P_{qy} \rangle \sim \langle P_{qz} \rangle = \langle P_{qx} \rangle = 0$

Similar fluctuations $\langle P_{qz}^2 \rangle$ and $\langle P_{qx}^2 \rangle$ for $\langle P_{qz} \rangle$ and $\langle P_{qx} \rangle$?

- take also the off-diagonal components into account

$$\hat{\rho}_q = \frac{1}{2} \begin{pmatrix} 1 + P_{qy} & P_{qz} - iP_{qx} \\ P_{qz} + iP_{qx} & 1 - P_{qy} \end{pmatrix} \quad \hat{\rho}_{\bar{q}} = \frac{1}{2} \begin{pmatrix} 1 + P_{\bar{q}y} & P_{\bar{q}z} - iP_{\bar{q}x} \\ P_{\bar{q}z} + iP_{\bar{q}x} & 1 - P_{\bar{q}y} \end{pmatrix}$$



Off-diagonal elements of $\hat{\rho}^V$?

In this case, we obtain

$$\rho_{00}^V = \frac{1 + \vec{P}_q \cdot \vec{P}_{\bar{q}} - 2P_{qy}P_{\bar{q}y}}{3 + \vec{P}_q \cdot \vec{P}_{\bar{q}}}$$

also the off-diagonal elements of $\hat{\rho}^V$

$$\rho_{10}^V = \frac{P_{qz}(1 + P_{\bar{q}y}) + (1 + P_{qy})P_{\bar{q}z} - iP_{qx}(1 + P_{\bar{q}y}) - i(1 + P_{qy})P_{\bar{q}x}}{\sqrt{2}(3 + \vec{P}_q \cdot \vec{P}_{\bar{q}})}$$

$$\rho_{0-1}^V = \frac{P_{qz}(1 - P_{\bar{q}y}) + (1 - P_{qy})P_{\bar{q}z} - iP_{qx}(1 - P_{\bar{q}y}) - i(1 - P_{qy})P_{\bar{q}x}}{\sqrt{2}(3 + \vec{P}_q \cdot \vec{P}_{\bar{q}})}$$

$$\rho_{1-1}^V = \frac{P_{qz}P_{\bar{q}z} - P_{qx}P_{\bar{q}x} + i(P_{qx}P_{\bar{q}y} + P_{qy}P_{\bar{q}x})}{3 + \vec{P}_q \cdot \vec{P}_{\bar{q}}}$$

They should be sensitive to the local correlations.



Off-diagonal elements of $\hat{\rho}^V$?

Take the average

$$\langle \rho_{00}^V \rangle = \frac{1 + \langle P_{qz}P_{\bar{q}z} \rangle + \langle P_{qx}P_{\bar{q}x} \rangle - \langle P_{qy}P_{\bar{q}y} \rangle}{3 + \langle P_{qz}P_{\bar{q}z} \rangle + \langle P_{qx}P_{\bar{q}x} \rangle + \langle P_{qy}P_{\bar{q}y} \rangle}$$

$$\langle \rho_{10}^V \rangle = \frac{\langle P_{qz}P_{\bar{q}y} \rangle + \langle P_{qy}P_{\bar{q}z} \rangle - i\langle P_{qx}P_{\bar{q}y} \rangle - i\langle P_{qy}P_{\bar{q}x} \rangle}{\sqrt{2}(3 + \langle \vec{P}_q \cdot \vec{P}_{\bar{q}} \rangle)}$$

$$\langle \rho_{0-1}^V \rangle = \frac{-\langle P_{qz}P_{\bar{q}y} \rangle - \langle P_{qy}P_{\bar{q}z} \rangle + i\langle P_{qx}P_{\bar{q}y} \rangle + i\langle P_{qy}P_{\bar{q}x} \rangle}{\sqrt{2}(3 + \langle \vec{P}_q \cdot \vec{P}_{\bar{q}} \rangle)}$$

$$\langle \rho_{1-1}^V \rangle = \frac{\langle P_{qz}P_{\bar{q}z} \rangle - \langle P_{qx}P_{\bar{q}x} \rangle + i(\langle P_{qx}P_{\bar{q}y} \rangle + \langle P_{qy}P_{\bar{q}x} \rangle)}{3 + \langle \vec{P}_q \cdot \vec{P}_{\bar{q}} \rangle}$$

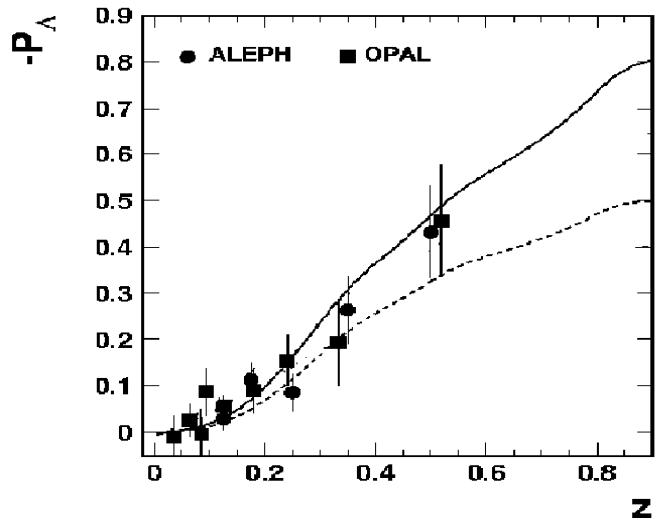
They should be sensitive to the local correlations.

Vector meson spin alignment in fragmentation processes



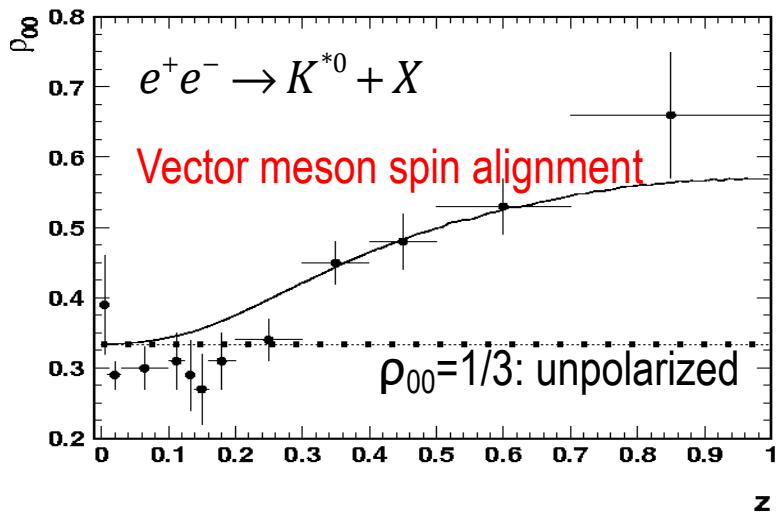
Hadron polarization in $e^+e^- \rightarrow Z^0 \rightarrow \vec{q} + \vec{\bar{q}} \rightarrow H \text{ (or } V\text{)} + X$ at LEP

ALEPH PLB 374, 319 (1996);
OPAL EPJC 2, 49 (1998)



spin transfer in $q \rightarrow \Lambda + X$

OPAL PLB412, 210 (1997);
DELPHI PLB 406, 271 (1997).



induced polarization?

Fragmentation is described by fragmentation functions (FFs)

FFs are defined via the quark-quark correlator



Three dimensional (i.e. transverse momentum dependent, TMD) FFs

The quark-quark correlator $\hat{\Xi}^{(0)}(k_F; p, S) = \frac{1}{2\pi} \sum_X \int d^4\xi e^{-ik_F\xi} \langle hX | \bar{\psi}(\xi) \mathcal{L}(\xi, \infty) | 0 \rangle \langle 0 | \mathcal{L}^\dagger(0, \infty) \psi(0) | hX \rangle$

integrate over k_F^- : $\hat{\Xi}^{(0)}(z, k_{F\perp}; p, S) = \frac{1}{2\pi} \sum_X \int p^+ d\xi^- d^2\xi_\perp e^{-i(p^+ \xi^- / z - \vec{k}_{F\perp} \cdot \vec{\xi}_\perp)} \langle hX | \bar{\psi}(\xi) \mathcal{L}(\xi, \infty) | 0 \rangle \langle 0 | \mathcal{L}^\dagger(0, \infty) \psi(0) | hX \rangle$

Expansion in terms of the Γ -matrices

$$\begin{aligned}
 \hat{\Xi}^{(0)}(z, k_{F\perp}; p, S) &= \Xi^{(0)}(z, k_{F\perp}; p, S) && \text{scalar} \\
 &+ i\gamma_5 \tilde{\Xi}^{(0)}(z, k_{F\perp}; p, S) && \text{pseudo-scalar} \\
 &+ \gamma^\alpha \Xi_\alpha^{(0)}(z, k_\perp; p, S) && \text{vector} \\
 &+ \gamma_5 \gamma^\alpha \tilde{\Xi}_\alpha^{(0)}(z, k_{F\perp}; p, S) && \text{axial vector} \\
 &+ i\gamma_5 \sigma^{\alpha\beta} \Xi_{\alpha\beta}^{(0)}(z, k_{F\perp}; p, S) && \text{tensor}
 \end{aligned}$$

$$\begin{aligned}
 \text{e.g.: } \Xi_\alpha^{(0)}(z, k_{F\perp}; p, S) &= \frac{1}{4} \text{Tr} \left[\gamma_\alpha \hat{\Xi}^{(0)}(z, k_{F\perp}; p, S) \right] \\
 &= \frac{1}{2\pi} \sum_X \int p^+ d\xi^- d^2\xi_\perp e^{-i(p^+ \xi^- / z - \vec{k}_{F\perp} \cdot \vec{\xi}_\perp)} \langle hX | \bar{\psi}(\xi) \mathcal{L}(\xi, \infty) \frac{\gamma_\alpha}{4} | 0 \rangle \langle 0 | \mathcal{L}^\dagger(0, \infty) \psi(0) | hX \rangle
 \end{aligned}$$

Description of polarization of particles with different spins

Spin 1/2 hadrons:

The spin density matrix is 2x2: $\rho = \begin{pmatrix} \rho_{++} & \rho_{+-} \\ \rho_{-+} & \rho_{--} \end{pmatrix} = \frac{1}{2}(1 + \vec{S} \cdot \vec{\sigma})$

Vector polarization: $S^\mu = (0, \vec{S})$

Spin 1 hadrons:

The spin density matrix is 3x3:

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1-1} \\ \rho_{01} & \rho_{00} & \rho_{0-1} \\ \rho_{-11} & \rho_{-10} & \rho_{-1-1} \end{pmatrix} = \frac{1}{3}(1 + \frac{3}{2}\vec{S} \cdot \vec{\Sigma} + 3T^{ij}\Sigma^{ij})$$

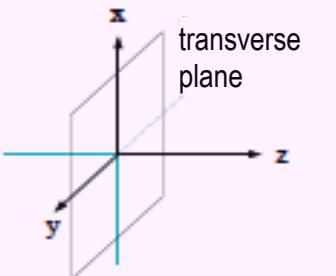
Vector polarization: $S^\mu = (0, \vec{S}) = (0, \vec{S}_T, S_L)$

Tensor polarization: $S_{LL}, \quad S_{LT}^\mu = (0, S_{LT}^x, S_{LT}^y, 0), \quad S_{TT}^{x\mu} = (0, S_{TT}^{xx}, S_{TT}^{xy}, 0)$

3 } 8 independent components
5 }

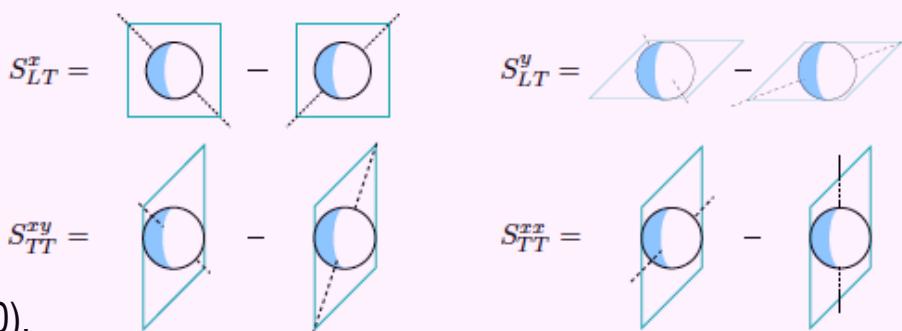
$$T = \frac{1}{2} \begin{pmatrix} -\frac{2}{3}S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^x \\ S_{TT}^{xx} & -\frac{2}{3}S_{LL} - S_{TT}^{xx} & S_{LT}^y \\ S_{LT}^x & S_{LT}^y & \frac{4}{3}S_{LL} \end{pmatrix}$$

$$\rho = \begin{pmatrix} \frac{1+S_{LL}}{3} + \frac{S_L}{2} & \frac{(S_{LT}^x - iS_{LT}^y) + (S_T^x - iS_T^y)}{2\sqrt{2}} & \frac{S_{TT}^{xx} - iS_{TT}^{xy}}{2} \\ \frac{(S_{LT}^x + iS_{LT}^y) + (S_T^x + iS_T^y)}{2\sqrt{2}} & \frac{1 - 2S_{LL}}{3} & \frac{-(S_{LT}^x - iS_{LT}^y) + (S_T^x - iS_T^y)}{2\sqrt{2}} \\ \frac{S_{TT}^{xx} + iS_{TT}^{xy}}{2} & \frac{-(S_{LT}^x + iS_{LT}^y) + (S_T^x + iS_T^y)}{2\sqrt{2}} & \frac{1 + S_{LL}}{3} - \frac{S_L}{2} \end{pmatrix}$$



$$S_{LL} = \frac{-\text{outward circle} + \text{inward circle}}{2} - \text{outward circle}$$

See e.g. A. Bacchetta, & P.J. Mulders, PRD62, 114004 (2000).





TMD FFs defined via the quark-quark correlator

The Lorentz decomposition

e.g. for spin-1/2 hadrons

totally 8(twist 2)+16(twist 3)+8(twist 4) components

$$\Xi_S^{(0)}(z, k_{F\perp}; p, \mathbf{S}) = M E(z, k_{F\perp}) + (\tilde{k}_{F\perp} \cdot \mathbf{S}_T) E^\perp(z, k_{F\perp})$$

← twist-3

$$\Xi_\alpha^{(0)}(z, k_{F\perp}; p, \mathbf{S}) = \frac{p^+}{M} \bar{n}_\alpha \left[M \mathbf{D}_1(z, k_{F\perp}) + (\tilde{k}_{F\perp} \cdot \mathbf{S}_T) \mathbf{D}_{1T}^\perp(z, k_{F\perp}) \right] \leftarrow$$

twist-2

$$+ k_{F\perp\alpha} \mathbf{D}^\perp(z, k_{F\perp}) + M \tilde{\mathbf{S}}_{T\alpha} \mathbf{D}_T(z, k_{F\perp}) + \frac{\tilde{k}_{F\perp\alpha}}{M} \left[\lambda M \mathbf{D}_L^\perp(z, k_{F\perp}) + (k_{F\perp} \cdot \mathbf{S}_T) \mathbf{D}_T^\perp(z, k_{F\perp}) \right]$$

$$+ \frac{M}{p^+} n_\alpha \left[M \mathbf{D}_3(z, k_{F\perp}) + (\tilde{k}_{F\perp} \cdot \mathbf{S}_T) \mathbf{D}_{3T}^\perp(z, k_{F\perp}) \right] \leftarrow$$

twist-4

$$p = p^+ \bar{n} + \frac{M^2}{2p^+} n, \quad \mathbf{S} = \lambda \frac{p^+}{M} \bar{n} + \mathbf{S}_T - \lambda \frac{M^2}{2p^+} n, \quad \tilde{k}_{\perp\alpha} \equiv \epsilon_{\perp\rho\alpha} k_\perp^\rho$$

$$\text{E.g.: } D_1(z, k_{F\perp}) = \frac{1}{2\pi} \sum \int d\xi^- d^2\xi_\perp e^{-i(p^+ \xi^- / z - \vec{k}_{F\perp} \cdot \vec{\xi}_\perp)} \langle hX | \bar{\psi}(\xi) \mathcal{L}(\xi, \infty) \frac{\gamma^+}{4} | 0 \rangle \langle 0 | \mathcal{L}^\dagger(0, \infty) \psi(0) | hX \rangle$$

See e.g., K.B. Chen, S.Y. Wei, W.H. Yang, & ZTL, PRD94, 034003 (2016).

Twist-2 TMD FFs defined via the quark-quark correlator (spin-1)



Quark pol	Hadron pol \rightarrow	TMD FFs (2+6+10=18)	integrated over $k_{F\perp}$	name
	U	$D_1(z, k_{F\perp})$	$D_1(z)$	number density
U	T	$D_{1T}^\perp(z, k_{F\perp})$	\times	Sivers-type function
(D)	LL	$D_{1LL}(z, k_{F\perp})$	$D_{1LL}(z)$	spin alignment
	LT	$D_{1LT}^\perp(z, k_{F\perp})$	\times	
	TT	$D_{1TT}^\perp(z, k_{F\perp})$	\times	
L	L	$G_{1L}(z, k_{F\perp})$	$G_{1L}(z)$	spin transfer (longitudinal)
	T	$G_{1T}^\perp(z, k_{F\perp})$	\times	
(G)	LT	$G_{1LT}^\perp(z, k_{F\perp})$	\times	
	TT	$G_{1TT}^\perp(z, k_{F\perp})$	\times	
T	U	$H_1^\perp(z, k_{F\perp})$	\times	Collins function
	$T(\parallel)$	$H_{1T}(z, k_{F\perp})$		spin transfer (transverse)
	$T(\perp)$	$H_{1T}^\perp(z, k_{F\perp})$	$H_{1T}(z)$	
\uparrow	L	$H_{1L}^\perp(z, k_{F\perp})$	\times	
(H)	LL	$H_{1LL}^\perp(z, k_{F\perp})$	\times	
	LT	$H_{1LT}^\perp(z, k_{F\perp}), H_{1LT}^\perp(z, k_{F\perp})$	$H_{1LT}(z)$	
	TT	$H_{1TT}^\perp(z, k_{F\perp}), H_{1TT}^\perp(z, k_{F\perp})$	\times, \times	

See e.g., K.B. Chen, S.Y. Wei, W.H. Yang, & ZTL, PRD94, 034003 (2016).

One dimensional FFs defined via the quark-quark correlator



The vector meson sin alignment v.s. the longitudinal spin transfer

$$\psi_{L/R} \equiv \frac{1}{2}(1 \pm \gamma_5)\psi$$

$$D_1(z) + S_{LL} D_{1LL}(z) = \frac{1}{8\pi} \sum_X \int zd\xi^- e^{-ip^+\xi^-/z} \sum_{\lambda_q=L,R} \langle hX | \bar{\psi}_{\lambda_q}(\xi) \gamma^+ | 0 \rangle \langle 0 | \psi_{\lambda_q}(0) | hX \rangle$$

the vector meson spin alignment

independent on the spin λ_q of the fragmenting quark!

$$S_L G_{1L}(z) = \frac{1}{8\pi} \sum_X \int zd\xi^- e^{-ip^+\xi^-/z} [\langle hX | \bar{\psi}_L(\xi) \gamma^+ | 0 \rangle \langle 0 | \psi_L(0) | hX \rangle - \langle hX | \bar{\psi}_R(\xi) \gamma^+ | 0 \rangle \langle 0 | \psi_R(0) | hX \rangle]$$

the longitudinal spin transfer

dependent on the spin λ_q of the fragmenting quark!

Hadron polarization in $e^+e^- \rightarrow hX$



Hyperon polarization:

$$P_{L\Lambda}(z, Q) = \frac{\sum_q P_q(Q) W_q(Q) G_{1Lq}(z, Q)}{\sum_q W_q(Q) D_{1q}(z, Q)}$$

$$W_q(Q) = \frac{2}{3} (e_q^2 + \chi c_1^e c_1^q + \chi_{int}^q c_V^e c_V^q)$$

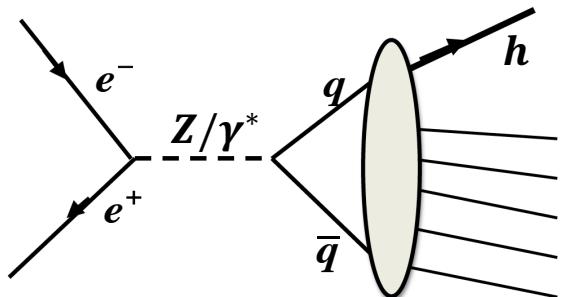
$$\chi = \frac{s^2}{[(s - M_Z^2)^2 + \Gamma_Z^2 M_Z^2] \sin^4 2\theta_W}$$

$$\chi_{int}^q = \frac{-2e_q s(s - M_Z^2)}{[(s - M_Z^2)^2 + \Gamma_Z^2 M_Z^2] \sin^2 2\theta_W}$$

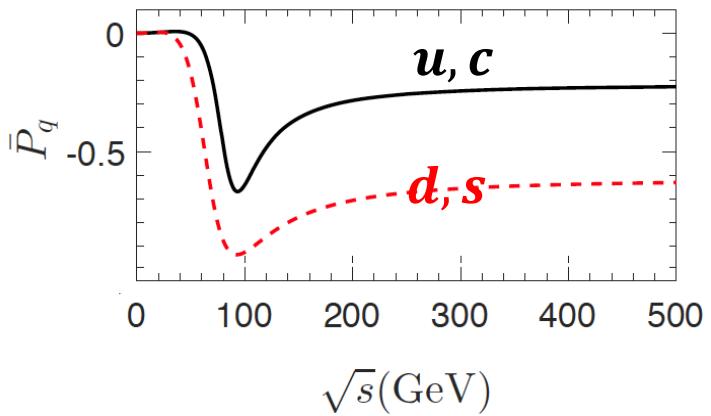
Vector meson spin alignment:

$$\langle S_{LL} \rangle(z, Q) = \frac{1}{2} \frac{\sum_q W_q(Q) D_{1LLq}(z, Q)}{\sum_q W_q(Q) D_{1q}(z, Q)}$$

K.B. Chen, S.Y. Wei, W.H. Yang and ZTL, PRD94, 034003 (2016);
 K.B. Chen, W.H. Yang, Y.J. Zhou and ZTL, PRD95, 034009 (2017).



$$\bar{P}_q = -\frac{\chi c_1^e c_3^q + \chi_{int}^q c_V^e c_A^q}{e_q^2 + \chi c_1^e c_1^q + \chi_{int}^q c_V^e c_V^q},$$



Hyperon polarization in $e^+e^- \rightarrow H + X$



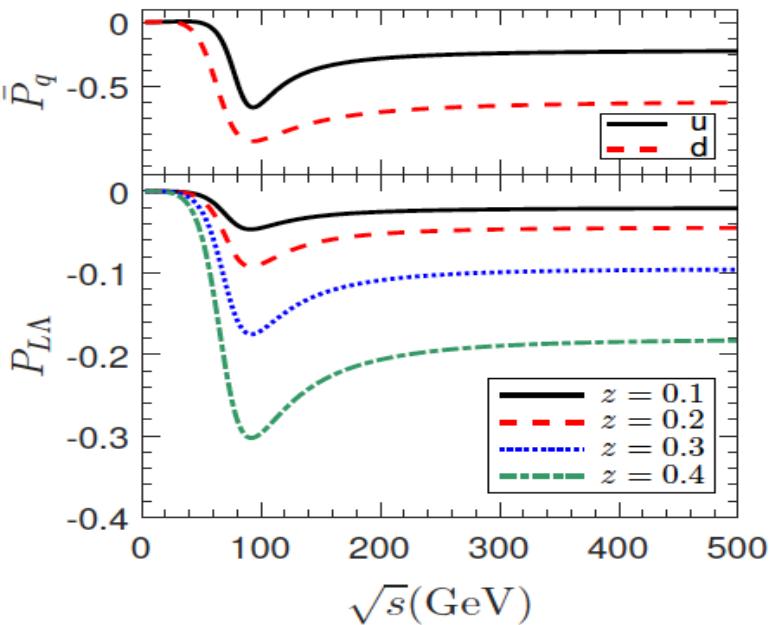
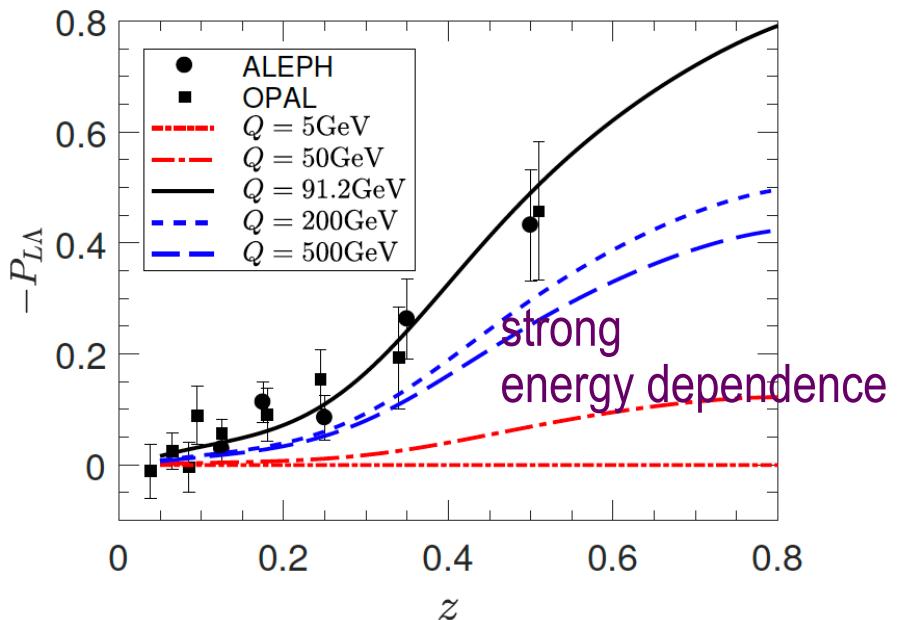
Parameterization at a initial scale:

$$G_{1L}^{s \rightarrow \Lambda}(z, \mu_0) = z^a D_1^{s \rightarrow \Lambda}(z, \mu_0)$$

$$G_{1L}^{u/d \rightarrow \Lambda}(z, \mu_0) = N z^a D_1^{u/d \rightarrow \Lambda}(z, \mu_0)$$

QCD Evolution:
(DGLAP equation)

$$\frac{\partial}{\partial \ln Q^2} G_{1L}^{i \rightarrow h}(z, Q^2) = \frac{\alpha_s}{2\pi} \sum_j \int_z^1 \frac{dy}{y} G_{1L}^{j \rightarrow h}\left(\frac{z}{y}, Q^2\right) \Delta P_{ij}(y, \alpha_s)$$



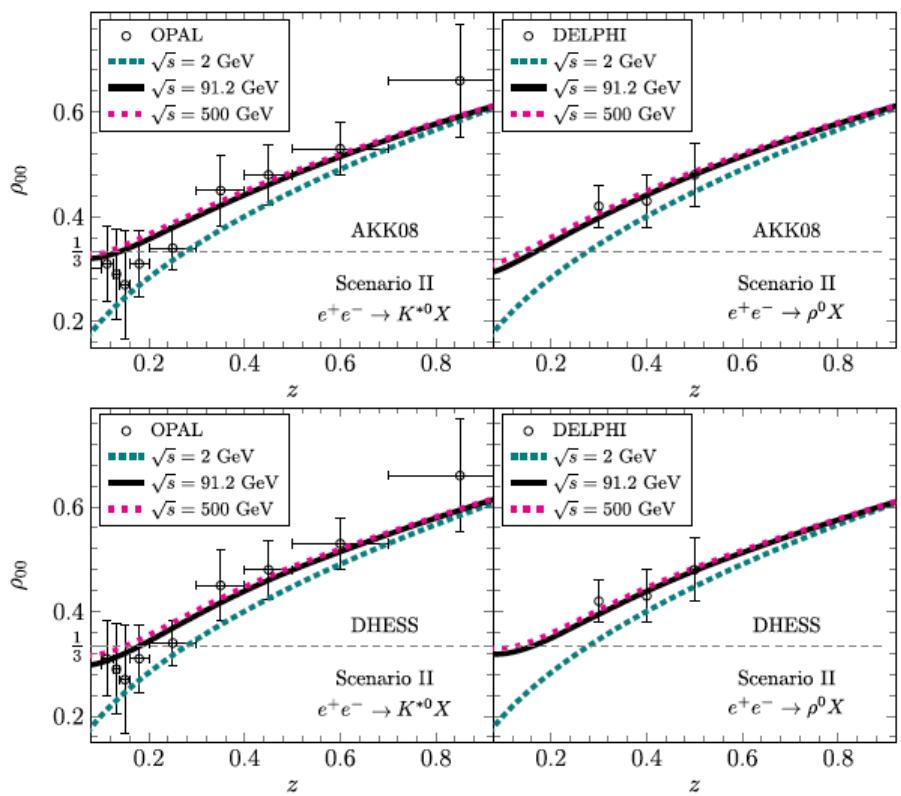
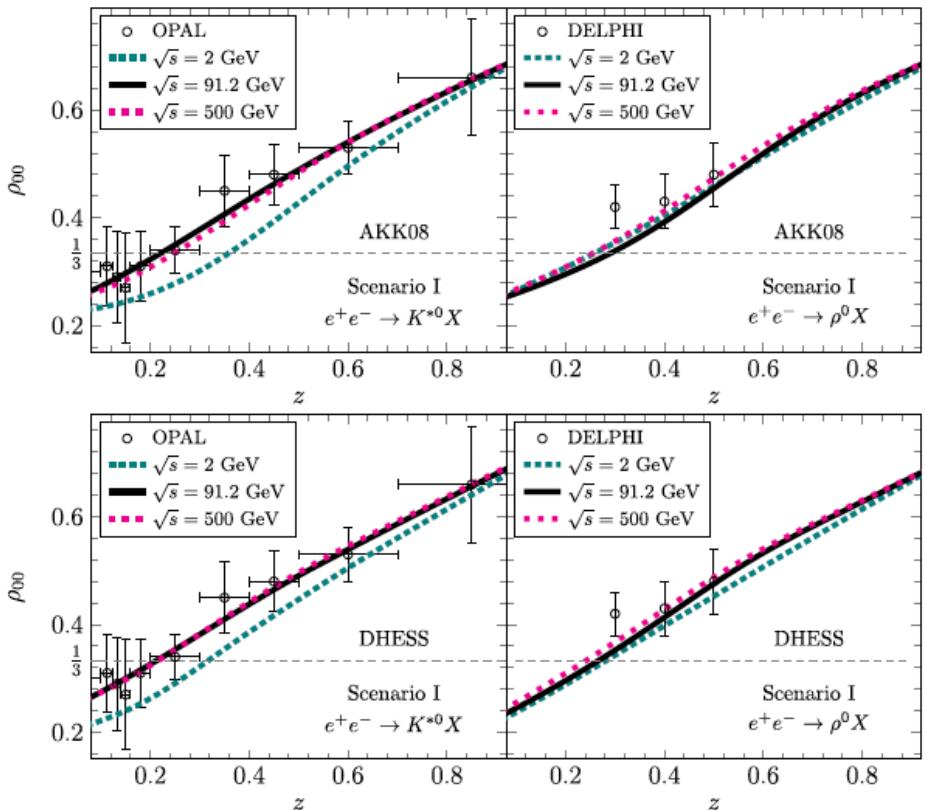
K.B. Chen, W.H. Yang, Y.J. Zhou and ZTL, PRD95, 034009 (2017).

Vector meson spin alignment in $e^+e^- \rightarrow V + X$



Spin alignment in $e^+e^- \rightarrow \rho$ or $K^* + X$

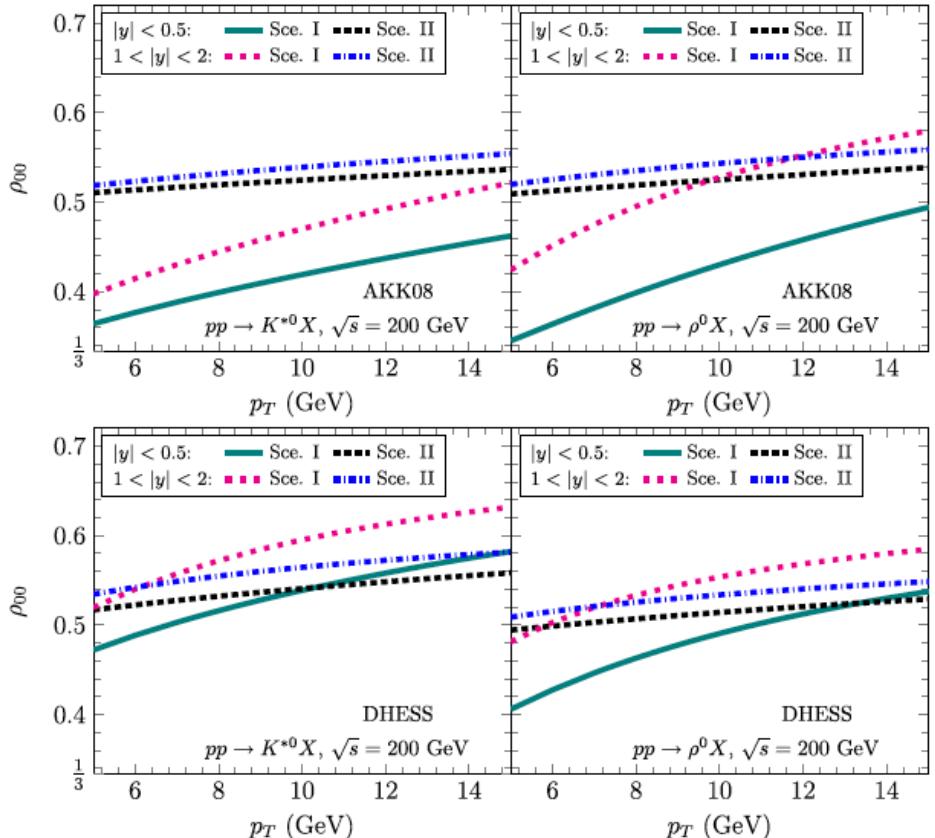
weak energy dependence



K.B. Chen, W.H. Yang, Y.J. Zhou and ZTL, PRD95, 034009 (2017).

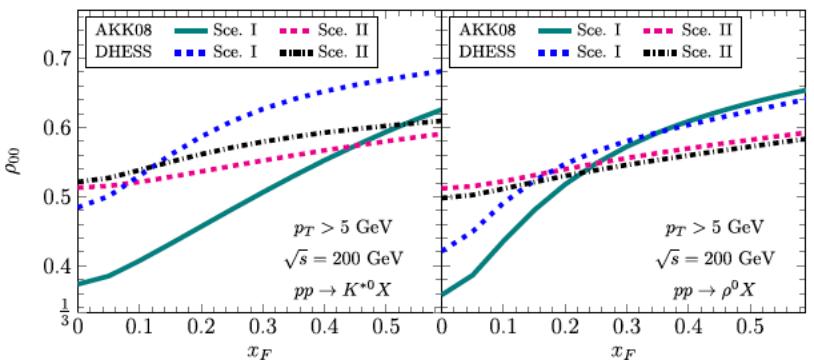
K.B. Chen, ZTL, Y.K. Song and S.Y. Wei, PRD102, 034001 (2020).

Spin alignment in $pp \rightarrow VX$



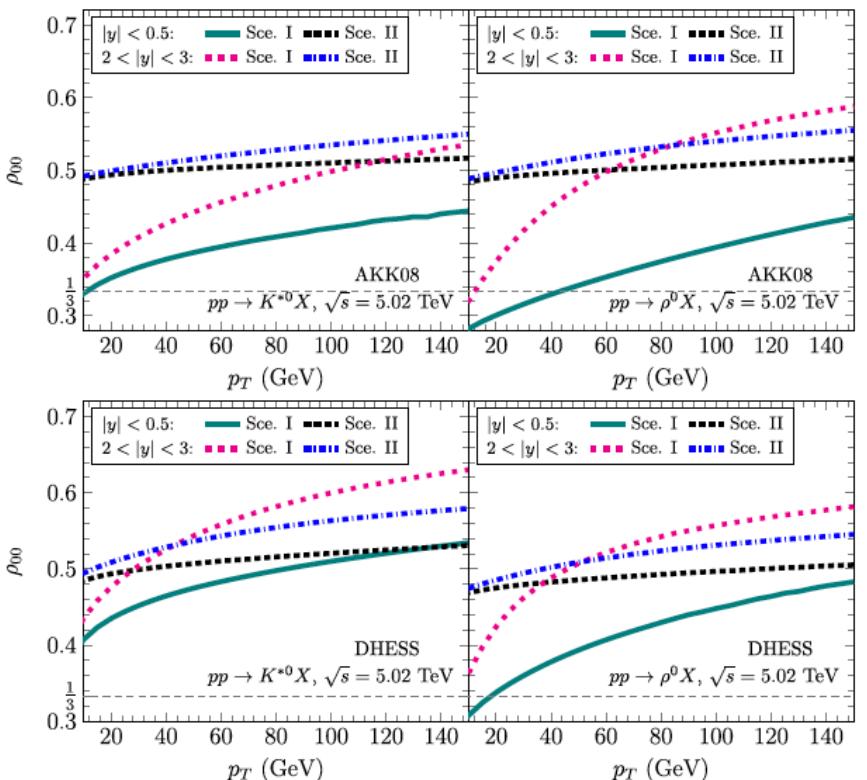
$\sqrt{s} = 200 \text{ GeV}$

$\rho_{00} > 1/3$ and
increase with increasing p_T or x_F



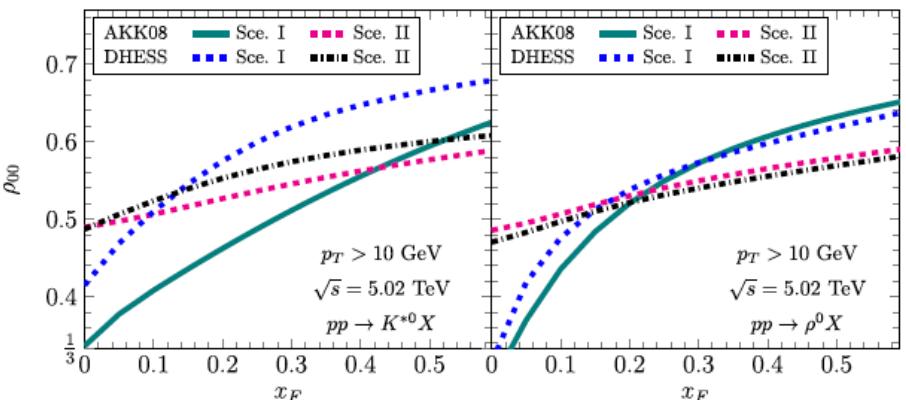
K.B. Chen, ZTL, Y.K. Song and S.Y. Wei, PRD102, 034001 (2020).

Spin alignment in $pp \rightarrow VX$



$$\sqrt{s} = 5.02 \text{ TeV}$$

$\rho_{00} > 1/3$ and
increase with increasing p_T or x_F



K.B. Chen, ZTL, Y.K. Song and S.Y. Wei, PRD102, 034001 (2020).

Summary and outlook



- Global hyperon polarization and global vector meson spin alignment have been observed experimentally.
- The global hyperon polarization is a measure of the average value of the global quark polarization in the system, while the global vector meson spin alignment measures the correlation between quark and anti-quark polarization.
- Correlation between the polarization of hyperon-hyperon or hyperon-antihyperon can be sensitive to the long range correlation while off-diagonal elements of vector meson spin density matrix may provide important information on the local correlation.
- Vector meson spin alignment in fragmentation mechanism is independent on the spin of the initial quark. Predictions have been made for different high energy reactions that can be tested by experiments.

Thank you for your attention!

One dimensional FFs defined via the quark-quark correlator



The one-dimensional quark-quark correlator

$$\widehat{\Xi}(z; p, S) = \frac{1}{2\pi} \sum_X \int p^+ d\xi^- e^{-ip^+\xi^-/z} \langle hX | \bar{\psi}(\xi) | 0 \rangle \langle 0 | \psi(0) | hX \rangle$$

Lorentz decomposition

$\widehat{\Xi}(z; p, S) = \Xi(z; p, S)$	_____	scalar
$+i\gamma_5 \widetilde{\Xi}(z; p, S)$	_____	pseudo-scalar
$+\gamma^\alpha \Xi_\alpha(z; p, S)$	_____	vector
$+\gamma_5 \gamma^\alpha \widetilde{\Xi}_\alpha(z; p, S)$	_____	axial vector
$+i\gamma_5 \sigma^{\alpha\beta} \Xi_{\alpha\beta}(z; p, S)$	_____	tensor

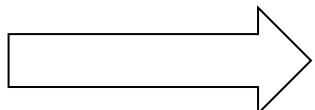
$$\begin{aligned} \Xi_\alpha(z; p, S) &= \frac{1}{4} \text{Tr} [\gamma_\alpha \widehat{\Xi}(z; p, S)] \\ &= \frac{1}{8\pi} \sum_X \int p^+ d\xi^- e^{-ip^+\xi^-/z} \langle hX | \bar{\psi}(\xi) \gamma_\alpha | 0 \rangle \langle 0 | \psi(0) | hX \rangle \end{aligned}$$

From the physical meaning of FF's

For the fragmentation process $q \rightarrow V + X$

Quark helicity: $\lambda_q = \frac{\vec{s} \cdot \vec{k}}{|\vec{k}|}$ \longleftrightarrow $-\lambda_q = -\frac{\vec{s} \cdot \vec{k}}{|\vec{k}|}$

Vector meson spin alignment ρ_{00} \longleftrightarrow ρ_{00}



ρ_{00} should be independent of the spin λ_q of quark!

The FF $D_{1LL}(z)$ should be independent of the spin of the fragmenting quark!

Quark polarization in $e^+e^- \rightarrow q\bar{q}$

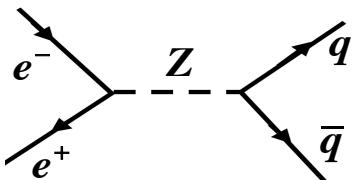


At the Z-pole: $e^+e^- \rightarrow Z \rightarrow q\bar{q}$

The cross section: $\frac{d\hat{\sigma}^{zz}}{d\Omega} = \frac{\alpha^2}{4s} \chi \left[c_1^e c_1^q (1 + \cos^2 \theta) + 2 c_3^e c_3^q \cos \theta \right]$

Longitudinal polarization of q or \bar{q} : $\bar{P}_q^{zz} = -\frac{c_3^q}{c_1^q}$

Correlation of transverse polarizations of q and \bar{q} : $\bar{c}_{nn}^{qzz} = \frac{c_2^q}{2c_1^q}$

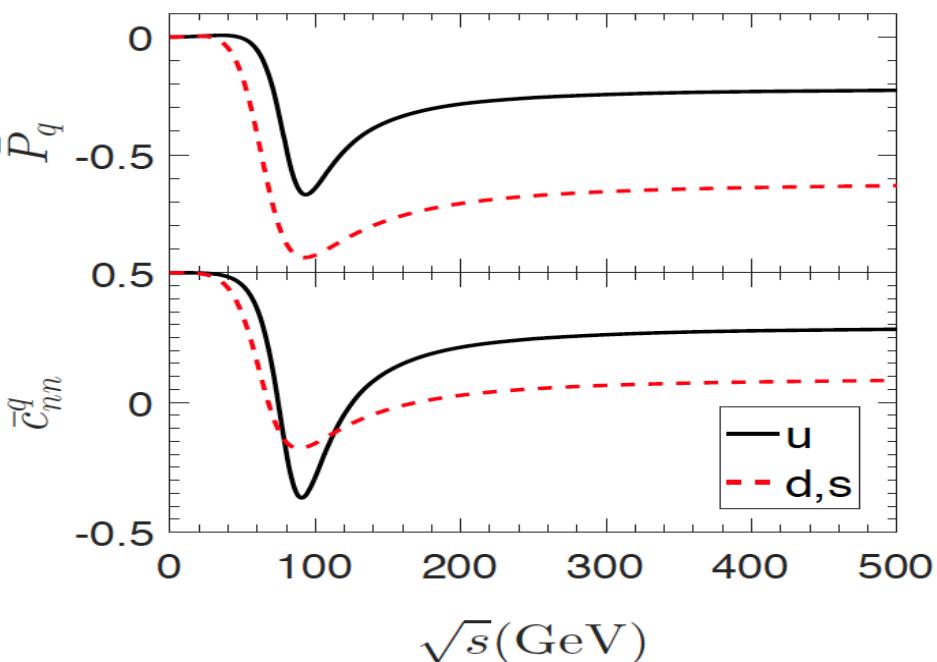


At any energy: $e^+e^- \rightarrow \gamma^*/Z \rightarrow q\bar{q}$

$$\frac{d\hat{\sigma}}{d\Omega} = \frac{d\hat{\sigma}^{zz}}{d\Omega} + \frac{d\hat{\sigma}^{z\gamma}}{d\Omega} + \frac{d\hat{\sigma}^{\gamma\gamma}}{d\Omega}$$

$$\bar{P}_q = -\frac{\chi c_1^e c_3^q + \chi_{int}^q c_V^e c_A^q}{e_q^2 + \chi c_1^e c_1^q + \chi_{int}^q c_V^e c_V^q},$$

$$\bar{c}_{nn}^q = \frac{e_q^2 + \chi c_1^e c_2^q + \chi_{int}^q c_V^e c_V^q}{2(e_q^2 + \chi c_1^e c_1^q + \chi_{int}^q c_V^e c_V^q)}$$



Hyperon polarization in $e^+e^- \rightarrow H + X$



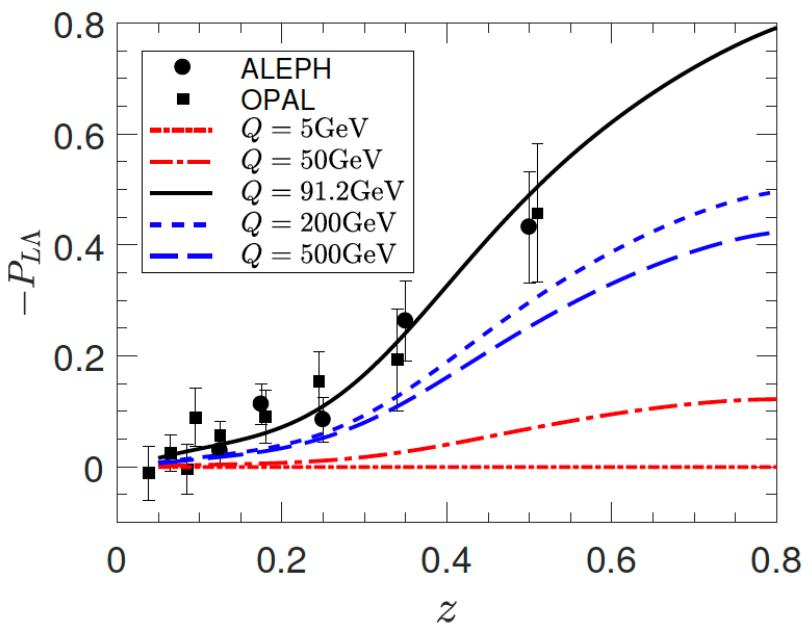
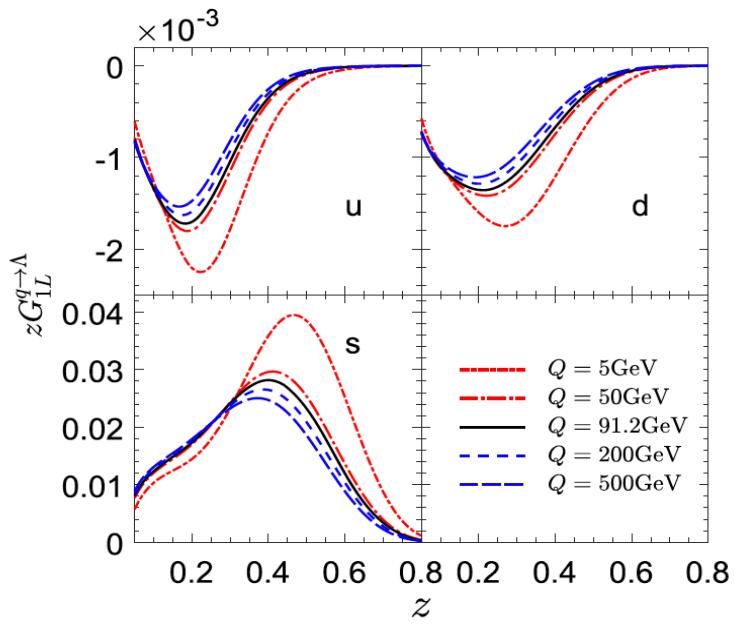
Parameterization at a initial scale:

$$G_{1L}^{s \rightarrow \Lambda}(z, \mu_0) = z^a D_1^{s \rightarrow \Lambda}(z, \mu_0)$$

$$G_{1L}^{u/d \rightarrow \Lambda}(z, \mu_0) = N z^a D_1^{u/d \rightarrow \Lambda}(z, \mu_0)$$

QCD Evolution:
(DGLAP equation)

$$\frac{\partial}{\partial \ln Q^2} G_{1L}^{i \rightarrow h}(z, Q^2) = \frac{\alpha_s}{2\pi} \sum_j \int_z^1 \frac{dy}{y} G_{1L}^{j \rightarrow h}\left(\frac{z}{y}, Q^2\right) \Delta P_{ij}(y, \alpha_s)$$



K.B. Chen, W.H. Yang, Y.J. Zhou and ZTL, PRD95, 034009 (2017).

Vector meson spin alignment in $e^+e^- \rightarrow V + X$



Two scenarios of parameterization at an initial scale.

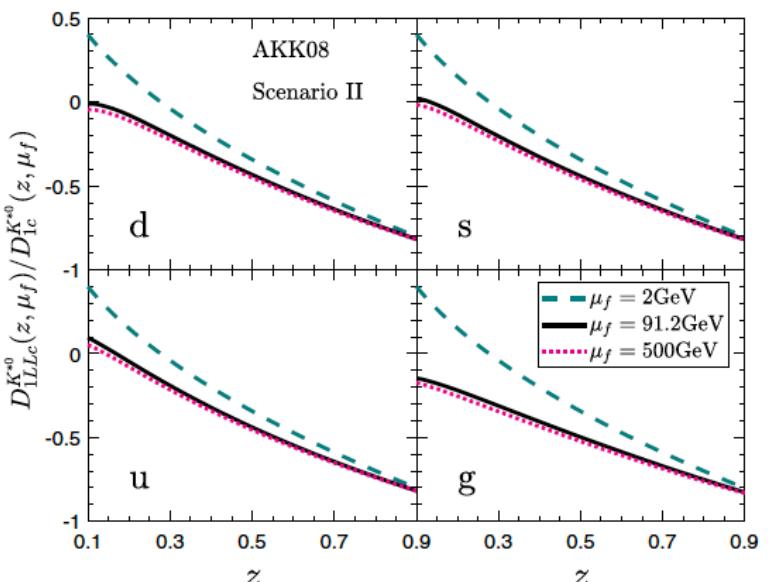
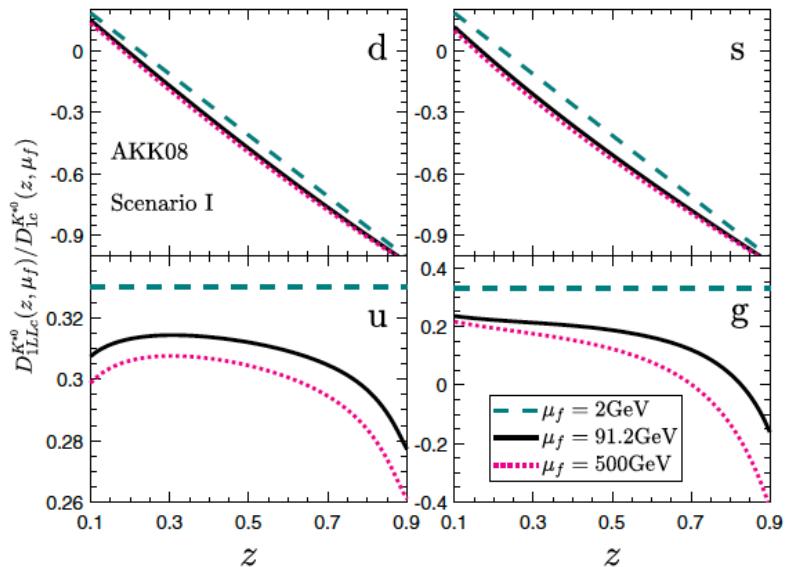
Scenario I: $D_{1LL}^{\text{favored}}(z, \mu_0) = c_1(a_1 z + 1) D_1^{\text{favored}}(z, \mu_0)$

$$D_{1LL}^{\text{unfavored}}(z, \mu_0) = c_1 D_1^{\text{unfavored}}(z, \mu_0)$$

Scenario II: $D_{1LL}(z, \mu_0) = c_2(a_2 z^{1/2} + 1) D_1(z, \mu_0)$

favored, e.g.:
 $d \rightarrow K^{*0}(d\bar{s}) + X$

unfavored, e.g.:
 $u \rightarrow K^{*0}(d\bar{s}) + X$

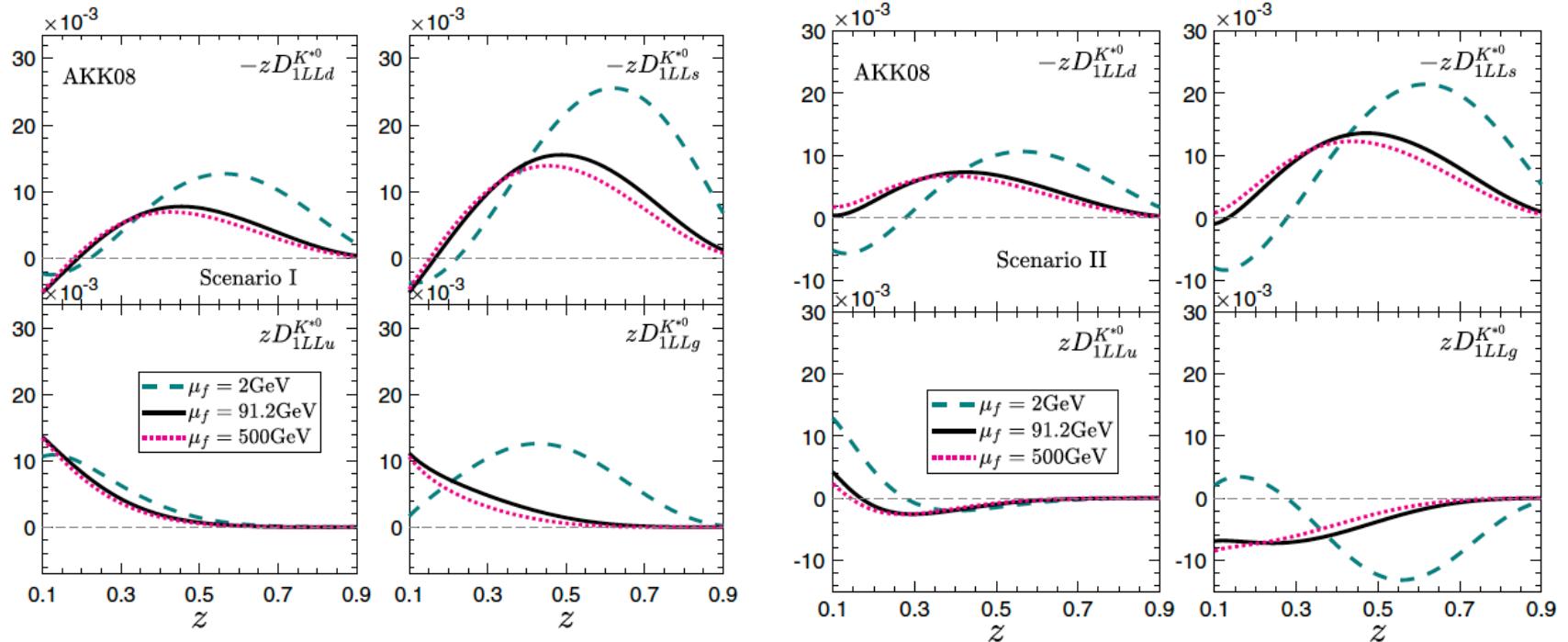


K.B. Chen, ZTL, Y.K. Song and S.Y. Wei, PRD102, 034001 (2020).

Vector meson spin alignment in $e^+e^- \rightarrow V + X$



The fragmentation functions $D_{1LLq}^{K^*}(z, \mu_f)$ in $q \rightarrow K^* + X$



different for different q or g in $q/g \rightarrow K^{*+} + X$

K.B. Chen, ZTL, Y.K. Song and S.Y. Wei, PRD102, 034001 (2020).