### Hyperon polarization measurements in heavy-ion collisions



Image from https://www.bnl.gov/newsroom/news.php?a=112068

### Takafumi Niida







25th International Symposium on Spin Physics Durham, Sep. 24-29/2023



## Heavy-ion collision experiment



Relativistic heavy-ion collisions at RHIC and the LHC

- Understand properties of quark-gluon plasma
- Explore the QCD phase structure: signatures of QCD critical point and 1st-order phase transition
- Connection to neutron star physics in the high baryon density



### Orbital angular momentum / Strong magnetic field



In the initial state of non-central HIC:

### Large orbital angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p}$

 $\sim bA\sqrt{s_{_{NN}}} \sim 10^6\hbar$ 

Z.-T. Liang and X.-N. Wang, PRL94, 102301 (2005)

Strong magnetic field

 $B \sim m_\pi^2/e$  $\sim 10^{14} \mathrm{T}$ 

D. Kharzeev et al., Nucl. Phys. A803, 227 (2008) L. McLerran and V. Skokov, Nucl. Phys. A929, 184 (2014)



typical magnet  $B \sim 0.1 - 0.5 \text{ T}$ 



magnetar  $B \sim 10^{11} \mathrm{T}$ 







### Orbital angular momentum / Strong magnetic field



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Rotating system under strong B-field produces:

- Particles "globally" polarized along *L or B* via spin-orbit/spin-magnetic coupling

Z.-T. Liang and X.-N. Wang, PRL94, 102301 (2005)

S. Voloshin, nucl-th/0410089 (2004)

F. Becattini, F. Piccinini, and J. Rizzo, PRC77, 024906 (2008)

- In case of the magnetic coupling, particles and antiparticles are oppositely aligned along **B** 

### Relativistic version of "Barnett effect"

S. Barnett, Phys. Rev. 6, 239 (1915)

i.e. magnetization of spinning matter (ferromagnet)





## **Polarization measurement**

### **Parity-violating weak decay of hyperons**

Daughter baryon is preferentially emitted in the direction of hyperon's spin (opposite for anti-particle)

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi} \left( 1 + \alpha_H \mathbf{P}_H^* \cdot \hat{\mathbf{p}}_B^* \right)$$

 $\mathbf{P}_{H}$ : hyperon polarization  $\hat{\mathbf{P}}_{B}$ : unit vector of daughter baryon momentum  $\alpha_{H}$ : hyperon decay parameter Asterisk\* denotes "in hyperon rest frame"

Any hyperons can be used but the sensitivity is different, depending on  $\alpha_{H_{-}}$ 







### Nuclear Theory Global polarization, measurement Hanzo Secondary Particles in unpolarized high energy hadr hadron collisions?



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## **Observation of A global polarization**



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- Increasing trend toward lower energies, described well by various theoretical models
  - I. Karpenko and F. Becattini, EPJC(2017)77:213, UrQMD+vHLLE H. Li et al., PRC96, 054908 (2017), AMPT Y. Sun and C.-M. Ko, PRC96, 024906 (2017), CKE Y. Xie et al., PRC95, 031901(R) (2017), PICR Y. B. Ivanov et al., PRC100, 014908 (2019), 3FD model
- Indication of thermal vorticity

 $P_{\Lambda(\bar{\Lambda})} \simeq \frac{1}{2} \frac{\omega}{T} \pm \frac{\mu_{\Lambda}B}{T}$ F. Becattini et al., PRC95, 054902 (2017)  $\omega = (P_{\Lambda} + P_{\bar{\Lambda}})k_B T/\hbar \sim 10^{22} \,\mathrm{s}^{-1}$  $\mu_{\Lambda}$ :  $\Lambda$  magnetic moment T: temperature at thermal equilibrium

• Possible difference between  $\Lambda$  and anti- $\Lambda$ 



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## **A global polarization at lower energies**

STAR, PRC104, L061901 (2021) HADES, PLB835(2022)137506



# New data from STAR/HADES at lower energies Continuous increase down to $\sqrt{s_{NN}} \sim 2.5$ GeV

![](_page_7_Picture_5.jpeg)

## **A global polarization at lower energies**

STAR, PRC104, L061901 (2021) HADES, PLB835(2022)137506

![](_page_8_Figure_2.jpeg)

- New data from STAR/HADES at lower energies Continuous increase down to  $\sqrt{s_{NN}}$ ~2.5 GeV
  - Predicted to have the maximum around  $\sqrt{s_{NN}} = 3 \text{ GeV}$ - Initial L & "stopping" to "transparency" at midrapidity

![](_page_8_Figure_6.jpeg)

![](_page_8_Figure_7.jpeg)

![](_page_9_Figure_1.jpeg)

- Lifetime of B-field would be very short (<0.5 fm/c) but could be sustained by QGP depending on its electric conductivity
- Polarization splitting provides an upper limit of the late-stage B-field

F. Becattini et al., PRC95.054902 (2017) B. Müller and A. Schäfer, PRD98, 071902(R) (2018) Y. Guo et al., PLB798(2019)134929

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![](_page_9_Picture_8.jpeg)

![](_page_10_Figure_1.jpeg)

- by QGP depending on its electric conductivity

B. Müller and A. Schäfer, PRD98, 071902(R) (2018) Y. Guo et al., PLB798(2019)134929

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![](_page_11_Figure_1.jpeg)

- by QGP depending on its electric conductivity

F. Becattini et al., PRC95.054902 (2017) B. Müller and A. Schäfer, PRD98, 071902(R) (2018) Y. Guo et al., PLB798(2019)134929

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### STAR, PRC108, 014910 (2023)

No significant difference in new STAR BES-II results. An upper limit of the late-stage B-field is  $B \le 10^{13} \text{ T}$ 

![](_page_11_Figure_8.jpeg)

![](_page_12_Figure_1.jpeg)

• Lifetime of B-field would be very short (<0.5 fm/c) but could be sustained by QGP depending on its electric conductivity.

Polarizat Caveat: There are other mechanisms to create the polarization difference...

- different emission time/position of  $\Lambda$  and  $\Lambda$ bar Vitiuk et al., PLB803(2020)135298 F. Becatti
- B. Müller Y. Guo et
- spin interaction with meson field Csernai et al., PRC99.021901(R) (2019)
- chemical potential Fang et al.,, PRC94, 024904 (2016)

![](_page_12_Figure_10.jpeg)

### **Global polarization in isobar collisions**

![](_page_13_Figure_1.jpeg)

- Smaller system, larger polarization?

$$P_{\Lambda}^{\mathrm{Au}} < P_{\Lambda}^{\mathrm{Ru}} \approx P_{\Lambda}^{\mathrm{Zr}} < P_{\Lambda}^{\mathrm{Cu}} < P_{\Lambda}^{\mathrm{O}} ?$$

S. Shi et al., PLB788(2019)409 S. Alzhrani et al., PRC106, 014905 (2022)

- Larger B-field, larger polarization splitting?

$$\frac{|B^{\mathrm{Ru}}|^2}{|B^{\mathrm{Zr}}|^2} \approx 15\%$$

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![](_page_13_Figure_8.jpeg)

No significant splitting nor system size dependence

See Xingrui Gou's talk

![](_page_13_Picture_11.jpeg)

![](_page_13_Picture_12.jpeg)

### **Global polarization of multistrangeness**

- $\Xi$  and  $\Omega$  hyperons
  - ✓ Different spin, decay parameter
  - ✓ Less feed-down
  - ✓ Likely different freeze-out in time
- Challenges: small  $\alpha_H$  and low production rate

![](_page_14_Figure_6.jpeg)

![](_page_14_Figure_8.jpeg)

hyperon	decay mode	aн	magnetic moment µн	spin
∧ (uds)	Λ→ρπ- (BR: 63.9%)	0.732	-0.613	1/2
∃- (dss)	Ξ-→Λπ- (BR: 99.9%)	-0.401	-0.6507	1/2
Ω- (sss)	Ω-→ΛK- (BR: 67.8%)	0.0157	-2.02	3/2

![](_page_14_Figure_11.jpeg)

\*  $\gamma_{\Omega}$  is unknown  $\alpha_{\Omega}, \beta_{\Omega} \ll 1 \rightarrow \gamma_{\Omega} \sim \pm 1$ Polarization transfer factor:  $C_{\Omega\Lambda} \approx +1 \text{ or } -0.6$ 

![](_page_14_Picture_13.jpeg)

### **Example 1 Example 1 Example 3 Example 3 Constant of a set and a global polarizations at \sqrt{s\_{NN}} = 200 GeV**

![](_page_15_Figure_1.jpeg)

![](_page_15_Figure_2.jpeg)

Confirmation of global vorticity picture!

T. Niida, SPIN2023

Hint of hierarchy in  $P_H$  but not significant yet

 $\langle P_{\Lambda} \rangle = 0.24 \pm 0.03 \text{ (stat)} \pm 0.03 \text{ (syst)} \%$  $\langle P_{\Xi} \rangle = 0.47 \pm 0.10 \text{ (stat)} \pm 0.23 \text{ (syst)} \%$ 

 $\langle P_{\Omega} \rangle = 1.11 \pm 0.87 \text{ (stat)} \pm 1.97 \text{ (syst)} \%$ 

![](_page_15_Picture_9.jpeg)

![](_page_15_Picture_10.jpeg)

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![](_page_16_Figure_1.jpeg)

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T. Niida, SPIN2023

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Thermal model:  $P_{\Lambda}=P_{\Xi}=3/5^*P_{\Omega}$ 

$$\mathbf{P} = rac{\langle \mathbf{s} \rangle}{s} pprox rac{(s+1)}{3} rac{\omega}{T}$$
 F. Becattini et al., PRC95.054902

![](_page_16_Picture_10.jpeg)

![](_page_16_Picture_11.jpeg)

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- Earlier freeze-out leads to larger PH O.Vitiuk, L.V.Bravina, and E.E.Zabrodin, PLB803(2020)135298
- Feed-down contribution
  - 10-15% reduction of primary  $\Lambda P_H$  F. Becattini, PRC95, 054902 (2017)
  - ~25% increase of primary  $\Xi P_H$  H. Li et al., PLB827(2022)136971

![](_page_17_Figure_14.jpeg)

### **Example 1 Example 1 Example 3 Example 3 Constant of a set and a set and a set of a**

![](_page_18_Figure_1.jpeg)

![](_page_18_Figure_2.jpeg)

Confirmation of global vorticity picture!

T. Niida, SPIN2023

Hint of hierarchy in  $P_H$  but not significant yet

 $\langle P_{\Lambda} \rangle = 0.24 \pm 0.03 \text{ (stat)} \pm 0.03 \text{ (syst)} \%$  $\langle P_{\Xi} \rangle = 0.47 \pm 0.10 \text{ (stat)} \pm 0.23 \text{ (syst)} \%$  $\langle P_{\Omega} \rangle = 1.11 \pm 0.87 \text{ (stat)} \pm 1.97 \text{ (syst)} \%$ 

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  - ~25% increase of primary  $\Xi P_H$  H. Li et al., PLB827(2022)136971
- Explore  $\Omega$ /anti $\Omega$  difference in the coming data for B-field effect  $\mu_{\Omega} \approx 3\mu_{\Lambda}$

![](_page_18_Figure_15.jpeg)

## Local vorticity

Vortex induced by jet or in asymmetric collisions

![](_page_19_Figure_2.jpeg)

 S. Voloshin, EPJ Web Conf. 171, 07002 (2018)

 Y. Tachibana and Ind Trading, NRA904:9954 (20513) 1302333 1023 M.026 Serenone et al., PLB820 (2021) 136500

 B. Betz et al., PRC76.044901 (2007)

 M. Lisa et al., PRC104, L011901 (2021)

### Local vorticity induced by collective expansion with density fluctuations

L.-G. Pang et al., PRL117, 192301 (2016) X.-L. Xia et al., PRC98.024905 (2018)

Complex vortical structures might be created

## **Polarization along the beam direction:** P<sub>z</sub>

![](_page_20_Picture_1.jpeg)

- Polarization along the beam direction expected from the "elliptic flow" F. Becattini and I. Karpenko, PRL120.012302 (2018) S. Voloshin, EPJ Web Conf.171, 07002 (2018)

![](_page_20_Picture_6.jpeg)

## **Polarization along the beam direction:** P<sub>z</sub>

![](_page_21_Picture_1.jpeg)

- Polarization along the beam direction expected from the "elliptic flow" F. Becattini and I. Karpenko, PRL120.012302 (2018) S. Voloshin, EPJ Web Conf.171, 07002 (2018)
- Data indeed show a quadrupole pattern; the sign of  $P_z$  depends on azimuthal angle (~sine function)

Anisotropic-flow-driven polarization!

![](_page_21_Figure_6.jpeg)

![](_page_21_Figure_7.jpeg)

![](_page_21_Picture_8.jpeg)

# Even due to higher harmonic flow

### STAR, arXiv:2303.09074

![](_page_22_Figure_2.jpeg)

- plane ( $\Psi_3$ ) dependence of the polarization

\*Not accounted for EP resolution and decay parameter

Recent isobar data (Ru+Ru&Zr+Zr) even show triangular flow

Indicating triangular-flow-driven sextupole pattern of vorticity

See Xingrui Gou's talk

![](_page_22_Figure_10.jpeg)

![](_page_22_Picture_11.jpeg)

![](_page_22_Picture_12.jpeg)

## **P**<sub>z</sub> sine modulation

![](_page_23_Figure_1.jpeg)

- - Hydrodynamic model with shear-induced polarization shows a bit different trend
- No significant collision system nor energy dependence
  - Scaling better with N<sub>part</sub>, suggesting the importance of system size in addition to the geometry?
- Sensitivity to specific shear and bulk viscosities, and the initial state

STAR, arXiv:2303.09074

Similar  $p_T$  dependence as  $v_n$ , further supporting flow-driven polarization

S. Alzharani et al., PRC106.014905 (2022)

A. Palermo et al., EPJ Web Conf. 276 (2023) 01026

![](_page_23_Picture_13.jpeg)

![](_page_23_Picture_14.jpeg)

![](_page_23_Picture_15.jpeg)

# Spin sign puzzle still remains?

![](_page_24_Figure_1.jpeg)

vorticity: 
$$\omega_{\rho\sigma} = \frac{1}{2} \left( \partial_{\sigma} u_{\rho} - \partial_{\rho} u_{\sigma} \right)$$
  
shear:  $\Xi_{\rho\sigma} = \frac{1}{2} \left( \partial_{\sigma} u_{\rho} + \partial_{\rho} u_{\sigma} \right)$ 

S. Liu, Y. Yin, JHEP07(2021)188 B. Fu et al., PRL127, 142301 (2021) F. Becattini et al., PLB820(2021)136519 F. Becattini et al., PRL127, 272302 (2021)

- "Shear tensor" seems to be needed to explain the data but the sign changes depending on the implementation detail
  - Large cancellation of the thermal vorticity and shear contributions

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The sign of  $P_z$  is not reproduced by models based on thermal vorticity, referred to as "spin sign puzzle"

Spin may not be in equilibrium?

![](_page_24_Picture_11.jpeg)

## Outlook

- Global polarization
  - Complete the energy dependence filling in  $\sqrt{s_{NN}} = 3-7.7$  GeV
  - Any chance to see  $P_H$  splitting? lower energy or  $\Omega$  hyperon?
  - $\triangleright$  More precise measurements of  $\Xi$  and  $\Omega$  polarization needed
  - Measurement at forward/backward rapidity where models predict differently
- Local polarization
  - $\blacktriangleright$   $\phi$ -polarization (toroidal vortex)
    - due to jet or in p+A, Cu+Au, O+Pb(?) collisions
  - Spin Hall Effect?

![](_page_25_Figure_10.jpeg)

![](_page_25_Figure_11.jpeg)

S. Voloshin, EPJ Web Conf. 171, 07002 (2018) X.-L. Xia et al., PRC98, 024905 (2018) W. M. Serenone et al., PLB820 (2021) 136500 M. Lisa et al., PRC104, L011901 (2021)

![](_page_25_Figure_13.jpeg)

![](_page_25_Figure_14.jpeg)

 $\mathbf{P} = \frac{\langle \mathbf{s} \rangle}{\mathbf{s}} \approx \frac{(s+1)}{2} \frac{\boldsymbol{\omega}}{T}$ 

![](_page_25_Figure_15.jpeg)

![](_page_25_Picture_16.jpeg)

# Summary

- QCD matter and spin dynamics in heavy-ion collisions
- - measurements, multi-strangeness hyperons
  - shear term to explain the data but how to implement it?

Observation of the hyperon polarization open new directions in the study of

 A lot of progress in the measurements but still some open questions remain Global polarization: energy dependence from a few GeV to TeV, differential

Polarization along the beam direction: extended to 3<sup>rd</sup>-order, importance of the

Other predicted phenomena to be explored: vortex ring and spin Hall effect

![](_page_26_Picture_11.jpeg)

## Backup

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![](_page_27_Picture_3.jpeg)

## Spin Hall effect

![](_page_28_Figure_1.jpeg)

![](_page_28_Figure_2.jpeg)

![](_page_28_Picture_3.jpeg)

### S. Liu and Y. Yin, PRD104, 054043 (2021) B. Fu et al., arXiv:2201.12970

![](_page_28_Picture_6.jpeg)

### Phase diagram of rotating nuclear matter

![](_page_29_Figure_1.jpeg)

Y. Jiang and J. Liao, PRL117.192302(2016)

 Vorticity ω acts like baryon cher transition temperature

![](_page_29_Figure_5.jpeg)

Y. Fujimoto, K. Fukushima, Y. Hidaka, PLB816(2021)136184

• Vorticity  $\omega$  acts like baryon chemical potential  $\mu_B$  and lower deconfinement

![](_page_29_Picture_8.jpeg)

### **Energy dependence of global polarization**

STAR, Nature 548, 62 (2017) STAR, PRC90, 014910 (2018)

![](_page_30_Figure_2.jpeg)

I.Karpenko, F. Becattini, EPJ(2017)77.213 Y. Xie, D. Wang, L. P. Csernai, PRC95, 031901(R) (2017)

![](_page_30_Figure_8.jpeg)

![](_page_30_Picture_9.jpeg)

### Feed-down effect

- $\square \sim 60\%$  of measured  $\land$  are feed-down from  $\Sigma^* \rightarrow \land \pi$ ,  $\Sigma^0 \rightarrow \land \gamma$ ,  $\Xi \rightarrow \land \pi$
- $\Box$  Polarization of parent particle R is transferred to its daughter A (Polarization transfer could be negative!)

$$\mathbf{S}_{\Lambda}^{*} = C\mathbf{S}_{R}^{*} \qquad \langle S_{y} \rangle \propto \frac{S(S+1)}{3} (\omega + \frac{\mu}{S}B) \qquad \begin{array}{l} \text{f}_{\Lambda R} : \text{fraction of } \Lambda \text{ originating from particle } R\\ \mu_{R} : \text{magnetic moment of particle } R \end{array}$$

$$\begin{pmatrix} \varpi_{c} \\ B_{c}/T \end{pmatrix} = \begin{bmatrix} \frac{2}{3} \sum_{R} \left( f_{\Lambda R} C_{\Lambda R} - \frac{1}{3} f_{\Sigma^{0} R} C_{\Sigma^{0} R} \right) S_{R}(S_{R}+1) & \frac{2}{3} \sum_{R} \left( f_{\Lambda R} C_{\Lambda R} - \frac{1}{3} f_{\Sigma^{0} R} C_{\Sigma^{0} R} \right) (S_{R}+1) \mu_{R} \\ \frac{2}{3} \sum_{R} \left( f_{\overline{\Lambda R}} C_{\overline{\Lambda R}} - \frac{1}{3} f_{\overline{\Sigma}^{0} \overline{R}} C_{\overline{\Sigma}^{0} \overline{R}} \right) S_{\overline{R}}(S_{\overline{R}}+1) & \frac{2}{3} \sum_{\overline{R}} \left( f_{\overline{\Lambda R}} C_{\overline{\Lambda R}} - \frac{1}{3} f_{\overline{\Sigma}^{0} \overline{R}} C_{\overline{\Sigma}^{0} \overline{R}} \right) (S_{\overline{R}}+1) \mu_{\overline{R}} \end{bmatrix}^{-1} \begin{pmatrix} P_{\Lambda}^{\text{meas}} \\ P_{\overline{\Lambda}}^{\text{meas}} \end{pmatrix}$$

Decay	С
Parity conserving: $1/2^+ \rightarrow 1/2^+ 0^-$	-1/3
Parity conserving: $1/2^- \rightarrow 1/2^+ 0^-$	1
Parity conserving: $3/2^+ \rightarrow 1/2^+ 0^-$	1/3
Parity-conserving: $3/2^- \rightarrow 1/2^+ 0^-$	-1/5
$\Xi^0  ightarrow \Lambda + \pi^0$	+0.900
$\Xi^-  ightarrow \Lambda + \pi^-$	+0.927
$\Sigma^0  ightarrow \Lambda + \gamma$	-1/3

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 $C_{\Lambda R}$  : coefficient of spin transfer from parent R to  $\Lambda$ 

S<sub>R</sub> : parent particle's spin

![](_page_31_Figure_11.jpeg)

Becattini, Karpenko, Lisa, Upsal, and Voloshin, PRC95.054902 (2017)

Primary  $\Lambda$  polarization will be diluted by 15%-20% (model-dependent)

This also suggests that the polarization of daughter particles can be used to measure their parent polarization! e.g.  $\Xi$ ,  $\Omega$ 

![](_page_31_Figure_15.jpeg)

![](_page_31_Picture_16.jpeg)

### **Polarization along the beam direction**

F. Becattini and I. Karpenko, PRL120.012302 (2018) S. Voloshin, SQM2017

![](_page_32_Picture_2.jpeg)

Stronger flow in in-plane than in out-of-plane, known as elliptic flow, makes local vorticity (thus polarization) along beam axis.

![](_page_32_Figure_6.jpeg)

![](_page_32_Figure_7.jpeg)

![](_page_32_Figure_8.jpeg)

![](_page_32_Picture_9.jpeg)