Spin polarization in heavy-ion collisions

Matteo Buzzegoli
IOWA STATE UNIVERSITY
OF SCIENCE AND TECHNOLOGY

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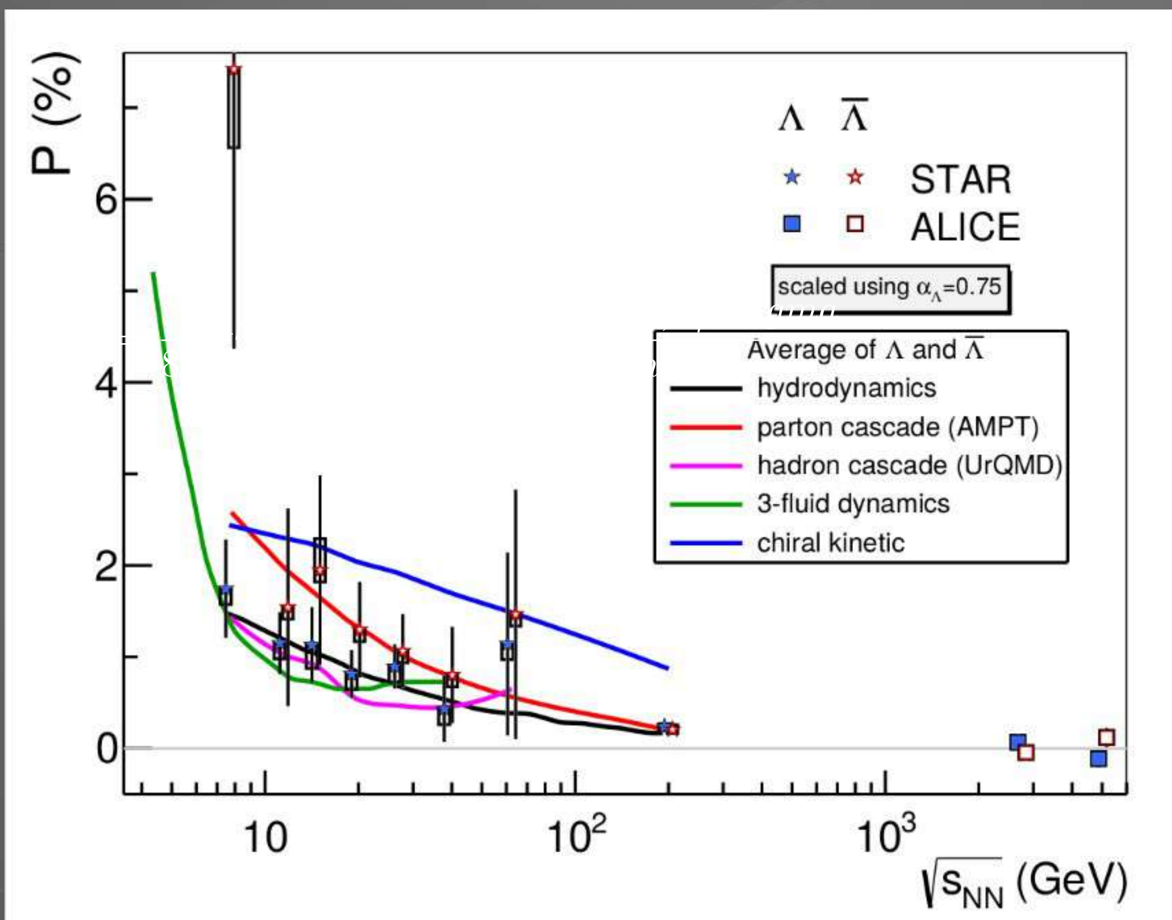
Outline

- Global and Local Spin polarization (sign puzzle)
- QFT derivation of vorticity induced and shear induced polarization
- Spin hydrodynamics and pseudo-gauges
- Spin polarization and in-medium form factors

Discovery of global Λ polarization

STAR Collaboration, Global Lambda hyperon polarization in nuclear collisions, Nature 548 6265, 2017



Different models of the collision, same formula for polarization

F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338:32 (2013)

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F (1 - n_F) \partial_\rho \beta_\sigma}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$

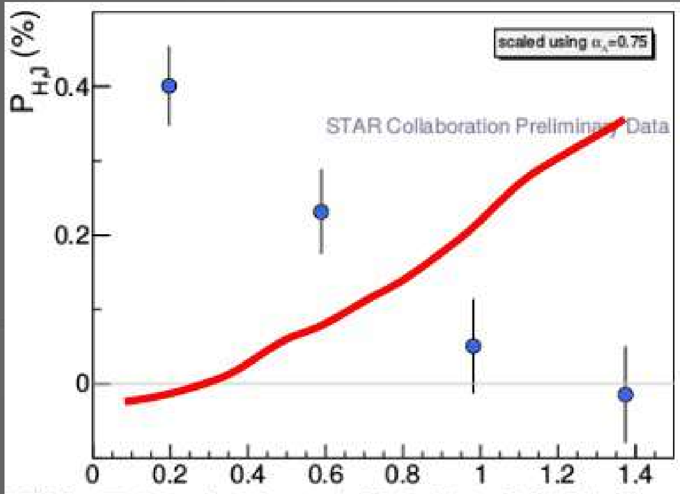
$$n_F = \left(\mathrm{e}^{\beta \cdot p - \zeta} + 1\right)^{-1}$$

$$\beta = \frac{1}{T} u$$

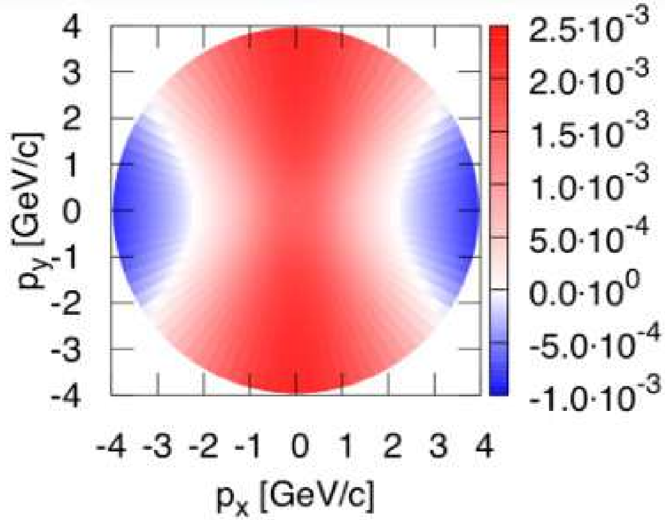
F. Becattini, M. Lisa, Polarization and vorticity in the QGP, Ann. Rev. Part, Nucl. Sc. 70, 395 (2020)

Puzzles: sign of local spin polarization

Spin polarization in the direction of the angular momentum



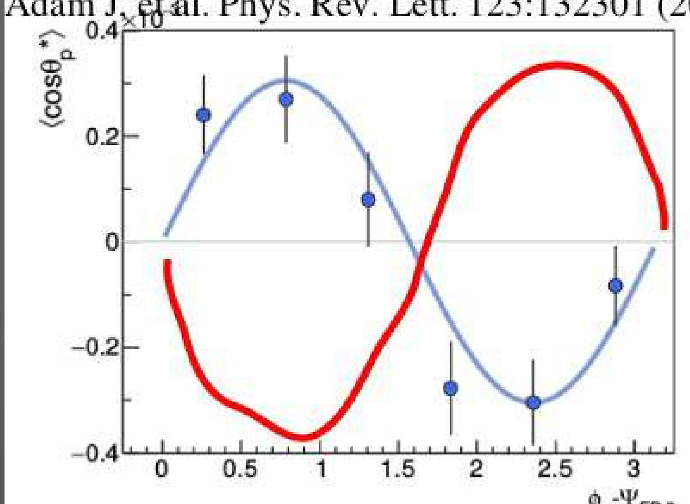
Niida T. Nucl. Phys. A982:511 (2019)

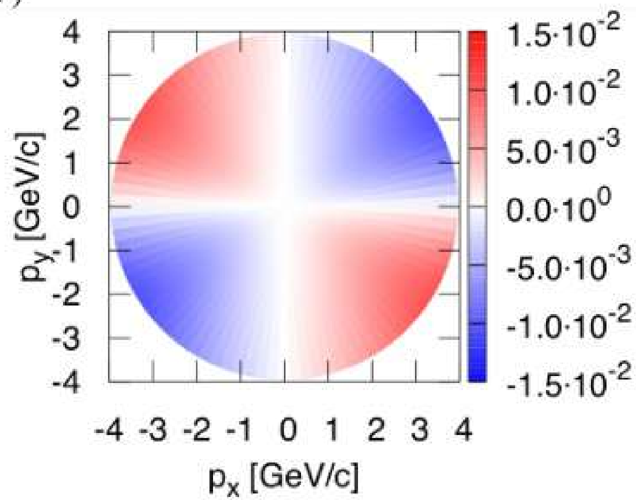


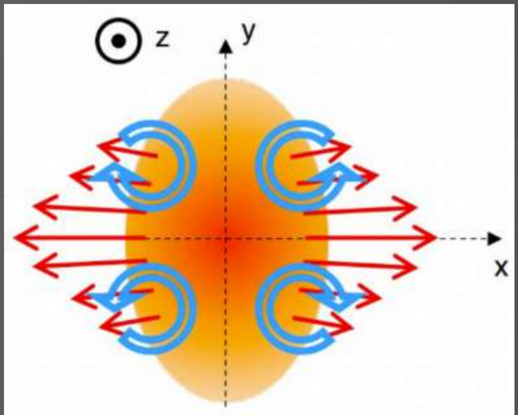
Theory prediction

Spin polarization in the direction of the beam axis

Adam J. et al. Phys. Rev. Lett. 123:132301 (2019)







Theory

The spin polarization vector for spin ½ particles:

$$S^\mu(p) = \frac{1}{2}\frac{\int \mathrm{d}\Sigma \cdot p \, \mathrm{tr}_4 \left[\gamma^\mu \gamma^5 W_+(x,p)\right]}{\int \mathrm{d}\Sigma \cdot p \, \mathrm{tr}_4 W_+(x,p)}$$

Wigner function is an expectation value of an integrated two point function of the Dirac field

$$W(x,k) = \mathrm{tr}\left(\widehat{\rho}\,\widehat{W}(x,k)\right)$$

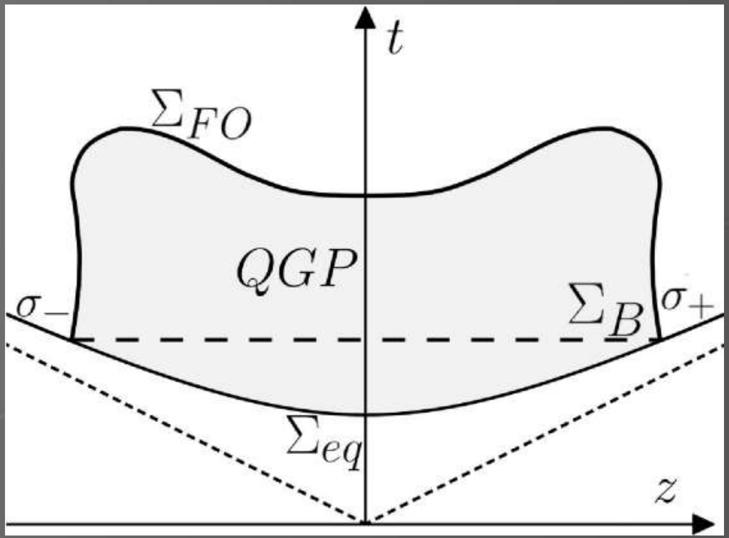
One needs to know the statistical operator to calculate mean values

$$\langle \widehat{X} \rangle = \mathrm{tr}\left(\widehat{\rho}\,\widehat{X}\right)$$

F. Becattini, Lect. Notes Phys. 987 (2021) 15-52.

The statistical operator from thermalization

F. Becattini, M. B., E. Grossi, *Reworking the Zubarev's approach to nonequilibrium quantum statistical mechanics,* Particles 2 (2019) 2, 197-207; 1902.01089



In the covariant Zubarev theory, this is the LTE at some initial “time”:

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_{\Sigma(\tau_0)} \mathrm{d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \, \widehat{j}^\mu \right) \right]$$

With the Gauss theorem:

NOTE: T_B stands for the symmetrized Belinfante stress-energy tensor

$$\widehat{\rho} = \frac{1}{Z} \exp\left[- \int_{\Sigma(\tau)} \mathrm{d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu} \beta_\nu - \zeta \, \widehat{j}^\mu \right) + \int_\Theta \mathrm{d}\Theta \left(\widehat{T}_B^{\mu\nu} \nabla_\mu \beta_\nu - \widehat{j}^\mu \nabla_\mu \zeta \right) \right]$$

Local equilibrium, non-dissipative terms

Dissipative terms

To evaluate the spin polarization we neglect the dissipative part.

Hydrodynamic limit: Taylor expansion

Expand the β and ζ fields from the point x where the Wigner operator is to be evaluated

$$\beta_\nu(y) \simeq \beta_\nu(x) + \partial_\lambda \beta_\nu(x)(y-x)^\lambda + \cdots$$

$$\widehat{\rho}_{LE} \simeq \frac{1}{Z} \exp\left[-\beta_\nu(x)\widehat{P}^\nu + \zeta(x)\widehat{Q} + \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}_x^{\mu\nu} - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}_x^{\mu\nu} + \cdots \right]$$

$$\widehat{J}_x^{\mu\nu} = \int \mathrm{d}\Sigma_\lambda \left[(y-x)^\mu \widehat{T}^{\lambda\nu}(y) - (y-x)^\nu \widehat{T}^{\lambda\mu}(y) \right] \qquad \widehat{Q}_x^{\mu\nu} = \int \mathrm{d}\Sigma_\lambda \left[(y-x)^\mu \widehat{T}^{\lambda\nu}(y) + (y-x)^\nu \widehat{T}^{\lambda\mu}(y) \right]$$

$$\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu\right)$$

Thermal vorticity

Adimensional in natural units

Equilibrium

$$\xi_{\mu\nu} = \frac{1}{2}\left(\partial_\mu \beta_\nu + \partial_\nu \beta_\mu\right)$$

Thermal shear

Adimensional in natural units

Non-equilibrium

At global equilibrium the thermal shear vanishes because of the Killing equation

Spin polarization at local thermal equilibrium

Linear response theory ⇨ $S^\mu(p) = S^\mu_\varpi + S^\mu_\xi + \cdots$

Vorticity:

$$S^\mu_\varpi(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\varpi_{\rho\sigma}}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}$$

Shear:

$$S^\mu_\xi(p) = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}\frac{p_\tau p^\rho}{\varepsilon}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F(1-n_F)\hat{t}_\nu \xi_{\sigma\rho}}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_F}$$

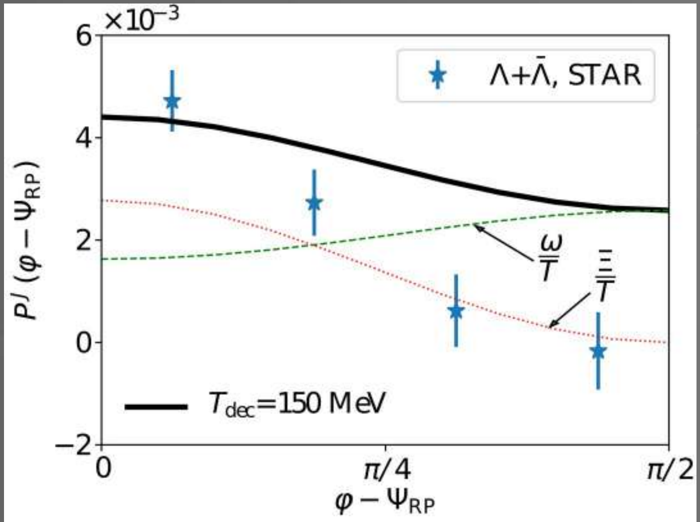
F. Becattini, MB, A. Palermo, Phys. Lett. B 820 (2021) 136519

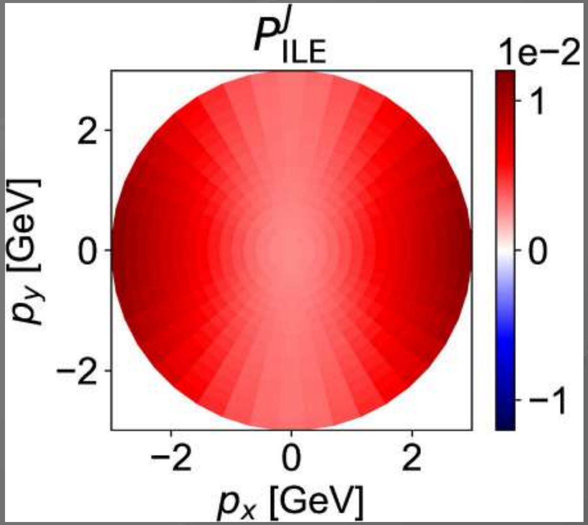
Same (not precisely the same) formula obtained by Liu and Yin with a different method:
S. Liu, Y. Yin, JHEP 07 (2021) 188

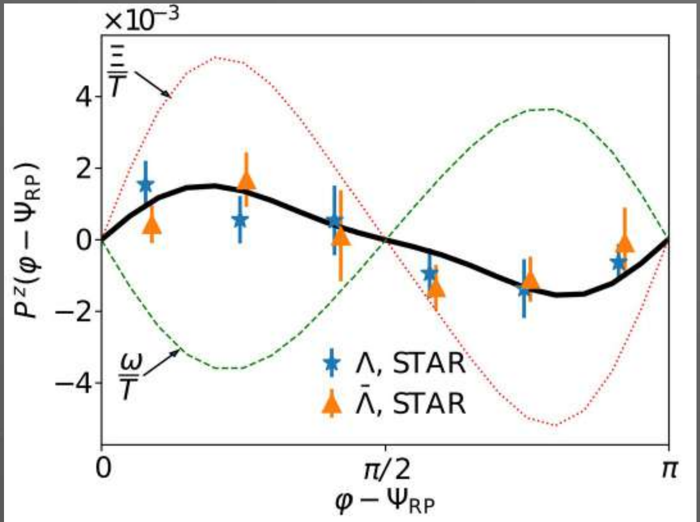
Solution of the puzzle

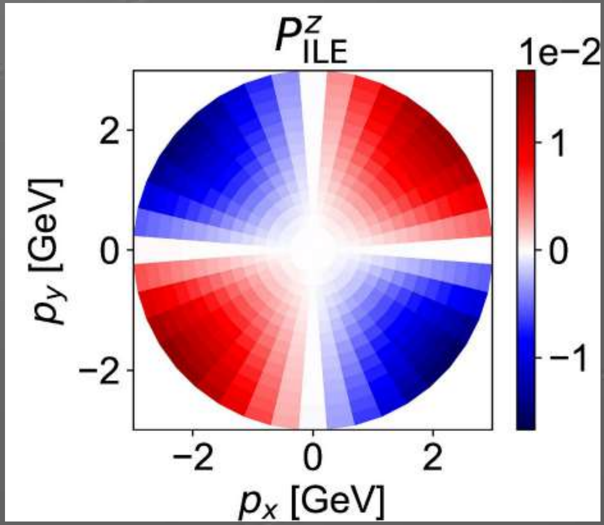
Isothermal local equilibrium (ILE)

At high energy, Σ_{FO} expected to be T_{FO}= constant! ⇨ Improved approximation









Kinematic vorticity

$$\omega_{\rho\sigma} = \frac{1}{2}\left(\partial_\sigma u_\rho - \partial_\rho u_\sigma\right)$$

$$S^\mu_{\omega\,\mathrm{ILE}} = -\epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F)\omega_{\rho\sigma}}{8mT_{\mathrm{FO}} \int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$

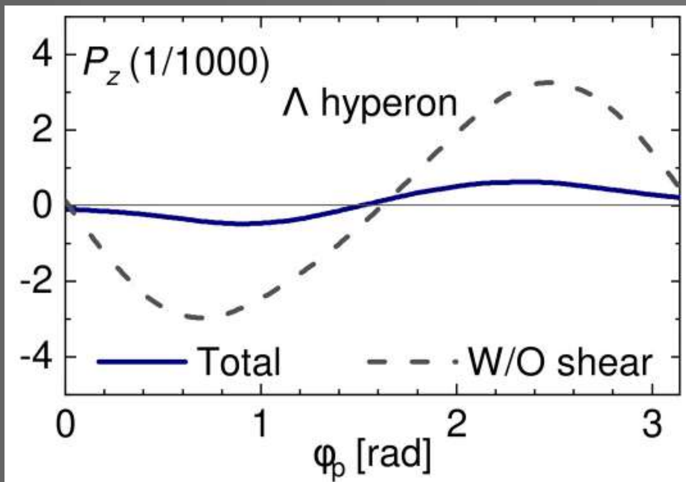
Kinematic shear

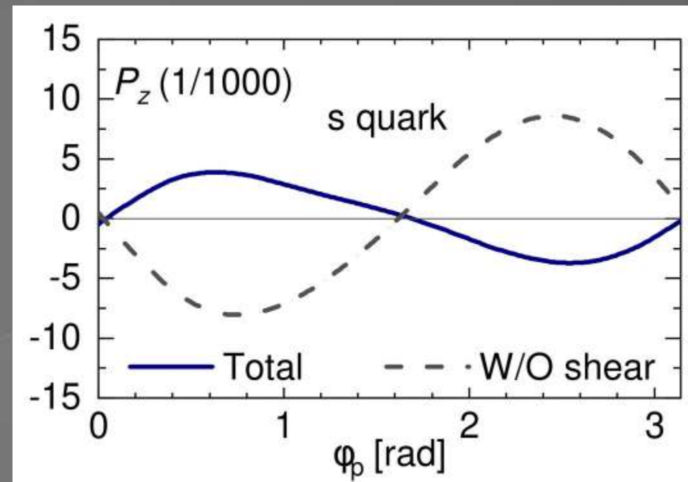
$$\Xi_{\rho\sigma} = \frac{1}{2}\left(\partial_\sigma u_\rho + \partial_\rho u_\sigma\right)$$

$$S^\mu_{\Xi\,\mathrm{ILE}}(p) = -\epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F(1-n_F) 2\,\hat{t}_\rho \frac{p^\lambda}{\varepsilon} \Xi_{\lambda\sigma}}{8mT_{\mathrm{FO}} \int_\Sigma \mathrm{d}\Sigma \cdot p\, n_F}$$

F. Becattini, M.B., A. Palermo, G. Inghirami and I. Karpenko, Phys. Rev. Lett. 127 (2021) 27, 272302

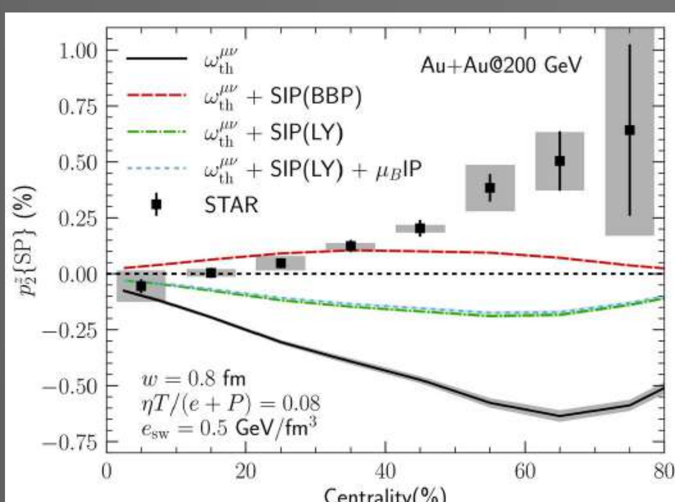
Comparison between different calculations

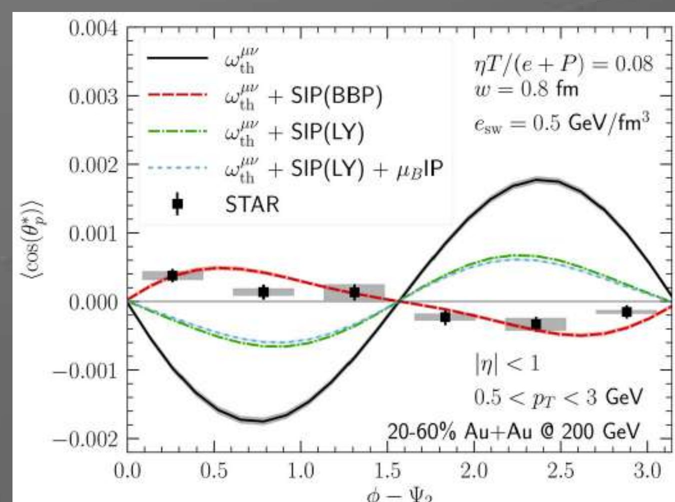




B. Fu, S. Liu, L. Pang, H. Song and Y. Yin, PRL 127 (2021) 14, 142301

Qualitative agreement found in the “strange memory” scenario





S. Alzhrani, S. Ryu, C. Shen, PRC 106 (2022) 1, 014905

Agreement with the BBP formula WITH thermal gradients!

Attributed to different initial conditions...

Polarization has a great potential to pin down the initial conditions and the QGP evolution which is yet unexploited to a large extent.

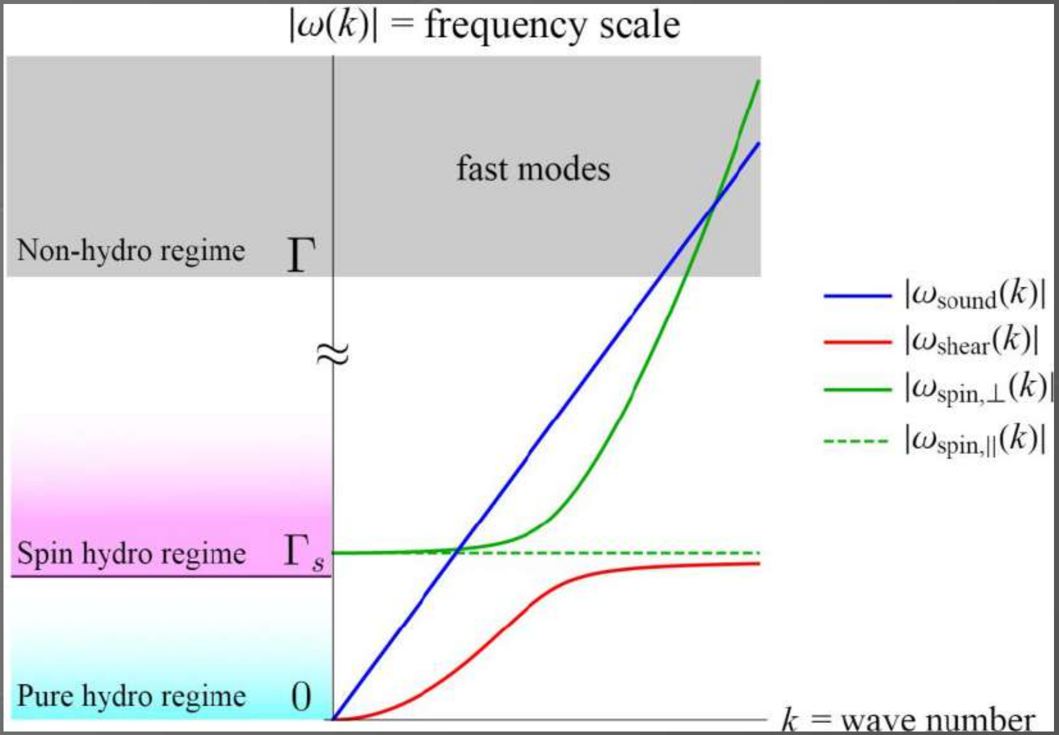
Spin hydrodynamics

Spin hydrodynamics is necessary if the spin relaxation time scale is much longer than the time scale of e.g. kinetic equilibration

Spin tensor $\widehat{S}^{\lambda,\mu\nu}$

Spin potential $\Omega^{\mu\nu}$

(This is not thermal vorticity)



M. Hongo, X. G. Huang, M. Kaminski, M. Stephanov and H. U. Yee, JHEP 11 (2021) 150

What spin tensor should one use?

Pseudo-gauge transformations

F. W. Hehl, Rept. Math. Phys. 9 (1976) 55

$$\widehat{T}'^{\mu\nu} = \widehat{T}^{\mu\nu} + \frac{1}{2}\nabla_\lambda\left(\widehat{\Phi}^{\lambda,\mu\nu} - \widehat{\Phi}^{\mu,\lambda\nu} - \widehat{\Phi}^{\nu,\lambda\mu}\right)$$

$$\widehat{S}'^{\lambda,\mu\nu} = \widehat{S}^{\lambda,\mu\nu} - \widehat{\Phi}^{\lambda,\mu\nu} + \nabla_\rho \widehat{Z}^{\mu\nu,\lambda\rho}$$

$$\widehat{\Phi}^{\lambda,\mu\nu} = -\widehat{\Phi}^{\lambda,\nu\mu}, \quad \widehat{Z}^{\mu\nu,\lambda\rho} = -\widehat{Z}^{\nu\mu,\lambda\rho} = -\widehat{Z}^{\mu\nu,\rho\lambda}$$

Arbitrary pseudo-gauge

Spin polarization predictions are pseudo-gauge dependent MB, PRC 105, 044907 (2022)

$$\widehat{\rho}^{\Phi}_{\rm LTE} = \frac{1}{\mathcal{Z}} \exp\left\{-\int {\rm d}\Sigma_\mu \left[\widehat{T}^{\mu\nu}_{\rm B}\beta_\nu - \frac{1}{2}\left(\varpi_{\lambda\nu}-\Omega_{\lambda\nu}\right)\widehat{\Phi}^{\mu,\lambda\nu} + \xi_{\lambda\nu}\widehat{\Phi}^{\lambda,\mu\nu} - \frac{1}{2}\Omega_{\lambda\nu}\nabla_\rho \widehat{Z}^{\lambda\nu,\mu\rho} - \widehat{j}^\mu\zeta\right]\right\}$$

Thermal shear Contribution

$$S^\mu_{B/C,\,\xi}(k) = -\frac{1}{4m}\epsilon^{\mu\lambda\sigma\tau}\frac{k_\tau k^\rho}{\varepsilon_k}\frac{\int_\Sigma \Sigma\cdot k\, n_{\rm F}\left(1-n_{\rm F}\right)\hat{t}_\lambda \xi_{\rho\sigma}}{\int_\Sigma \Sigma\cdot k\, n_{\rm F}}$$

$$S^\mu_{{\rm GLW/HW},\,\xi}(k) = 0$$

$$\Delta^{\partial\Sigma} S^\mu_\xi(k) = \mp 2K\frac{1}{4m}\epsilon^{\mu\lambda\sigma\tau}\frac{k_\tau k^\rho}{\varepsilon_k}\frac{\int_\Sigma {\rm d}\Sigma\cdot k\, n_{\rm F}\left(1-n_{\rm F}\right)\hat{t}_\lambda \xi_{\rho\sigma}}{\int_\Sigma {\rm d}\Sigma\cdot k\, n_{\rm F}}$$

- Choose a pseudo-gauge with $\widehat{\Phi}^{\lambda,\mu\nu}_{\partial\Sigma} = \frac{\rm i}{m}K\bar{\Psi}\overleftrightarrow{\partial}^\lambda\sigma^{\mu\nu}\Psi$

  where K is an arbitrary real parameter.
  You can **tune** the shear contribution by choosing K!

Spin rotation coupling and gravitomagnetic moment

Spin polarization along rotation is ultimately a consequence of the coupling between total angular momentum (spin) and rotation (similar to Zeeman coupling)

$$H = H_0 - \vec{J}\cdot\vec{\Omega}$$

C. G. de Oliveira and J. Tiomno, Nuovo Cim. (1962)
F. W. Hehl and W. T. Ni, PRD (1990)

The form factor related to this coupling is known as
gravitomagnetic moment g_Ω

$$\mathcal{L}_{\rm int} = \frac{1}{2}\left(g_{\mu\nu} - \eta_{\mu\nu}\right) T^{\mu\nu}$$

$$\langle p', s'|\widehat{T}^{\mu\nu}(0)|p, s\rangle = \bar{u}(p', s')\left[f_1(q^2)\frac{P^\mu P^\nu}{m} + g_\Omega(q^2)\,\frac{\sigma^{(\mu\alpha}q_\alpha P^{\nu)}}{2m} + O(q^2)\right]u(p, s)$$

The Einstein Equivalence Principle (EEP) forbids the appearance of an anomalous spin-rotation coupling.

Cho and Dass, PRD 14 (1976)
Kobzarev and Okun, Zh. Eksperim. i Teor. Fiz. (1962)
Teryaev, Front. Phys. (2016)

$$g_\Omega \equiv 1$$

In medium form factors

MB, D. Kharzeev, PRD (2021), S. Lin, J. Tian, 2306.14811

We can relate the spin polarization to form factors.
Radiative corrections beyond weakly interacting fields

$$\vec{S} \propto g_\Omega \, \vec{\Omega}$$

Donoghue, Holstein, and Robinett, PRD 30 (1984) and Gen. Rel. Grav. 17, 207 (1985)

EEP premises do not hold in the presence of a medium
The presence of a thermal bath breaks the Lorentz invariance of the vacuum
⇒ Breaking of EEP is possible at finite temperature
⇒ Gravitomagnetic moment can receive radiative corrections

MB, D. Kharzeev, PRD (2021)

$$\langle p', s'|\widehat{T}_{\mu\nu}(0)|p, s\rangle = \bar{u}(p', s')\Big\{ I_{P\gamma}(P, q)\,(P_\mu\gamma_\nu + P_\nu\gamma_\mu) + I_{u\gamma}(P, q)\,(u_\mu\gamma_\nu + u_\nu\gamma_\mu) + \\ + I_{Pl}(P, q)\hat{\not{l}}\left(P_\mu\hat{l}_\nu + P_\nu\hat{l}_\mu\right) + I_{ul}(P, q)\hat{\not{l}}\left(u_\mu\hat{l}_\nu + u_\nu\hat{l}_\mu\right)\Big\}u(p, s) + \cdots$$

$$g_\Omega = \lim_{\substack{q\to 0 \\ P_s\to 0}} 4\left(I_{P\gamma}(P, q) + \frac{I_{u\gamma}(P, q)}{\omega_P} - I_{Pl}(P, q) - \frac{I_{ul}(P, q)}{\omega_P}\right)$$

$$= 1 - \frac{N_C^2 - 1}{2}\begin{cases} \dfrac{1}{6}\dfrac{g^2T^2}{m^2} & T \ll m \\ \dfrac{5}{36}\dfrac{g^2T^2}{m^2} & T \gg m,\ m > gT \end{cases}$$

Summary and outlook

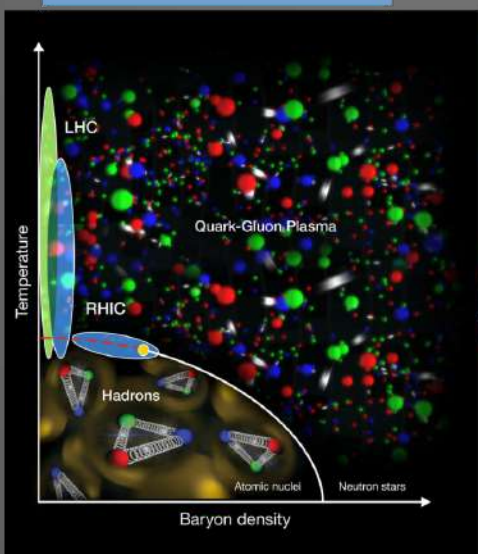
- Spin polarization open a new chapter in spin physic
- Spin-thermal shear coupling: new unexpected, non-dissipative phenomenon.
- Polarization has a great potential to pin down the initial conditions and the QGP evolution which is yet unexploited to a large extent.

- Spin hydrodynamics
- What is the role of pseudo-gauge transformations?

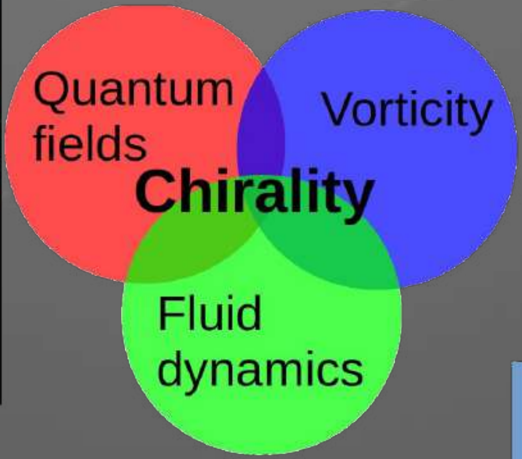
- Relation to hydrodynamics and form factors

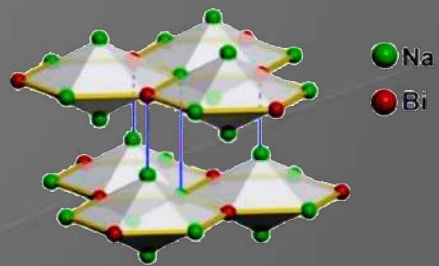
Thank you!

Nuclear physic

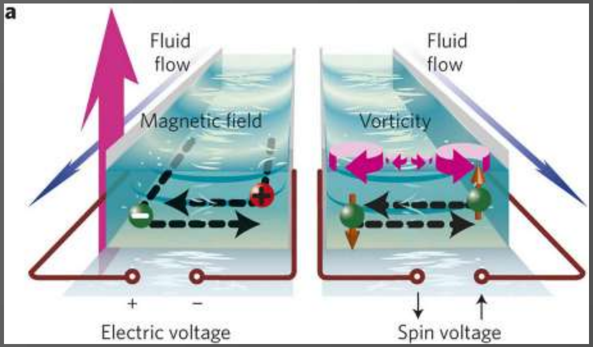


Particle physic





Condensed matter physic



Takahashi et al, Nature Physics 12, 52–56 (2016)

Applications

Cosmology

Astrophysic

Backup

What is this new term?

Does it have a non-relativistic limit? Let us decompose it

$$\xi_{\sigma\rho} = \frac{1}{2}\partial_\sigma\left(\frac{1}{T}\right)u_\rho + \frac{1}{2}\partial_\rho\left(\frac{1}{T}\right)u_\sigma + \frac{1}{2T}\left(A_\rho u_\sigma + A_\sigma u_\rho\right) + \frac{1}{T}\sigma_{\rho\sigma} + \frac{1}{3T}\theta\Delta_{\rho\sigma}$$

A is the acceleration field

$$\sigma_{\mu\nu} = \frac{1}{2}(\nabla_\mu u_\nu + \nabla_\nu u_\mu) - \frac{1}{3}\Delta_{\mu\nu}\theta$$

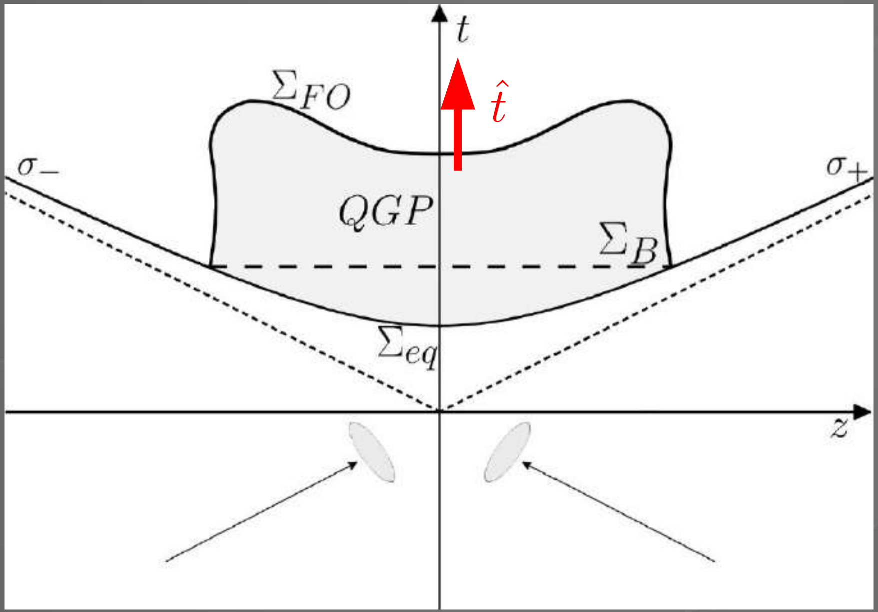
$$\theta = \nabla\cdot u \qquad \Delta_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$$

All terms are relativistic (they vanish in the infinite c limit)
EXCEPT grad T terms, which give rise to

$$\boxed{\mathbf{S}_\xi = \frac{1}{8}\mathbf{v}\times\frac{\int \mathrm{d}^3\mathrm{x}\, n_F(1-n_F)\nabla\left(\frac{1}{T}\right)}{\int \mathrm{d}^3\mathrm{x}\, n_F}}$$

There is an equal contribution in the NR limit from thermal vorticity

Why do we have a dependence on Σ?



The thermal shear term depends on the correlator

$$\langle \widehat{Q}_x^{\mu\nu} \widehat{W}(x,k) \rangle$$

$$\widehat{J}_x^{\mu\nu} = \int \mathrm{d}\Sigma_\lambda \left[(y-x)^\mu \widehat{T}^{\lambda\nu}(y) - (y-x)^\nu \widehat{T}^{\lambda\mu}(y) \right]$$

$$\widehat{Q}_x^{\mu\nu} = \int \mathrm{d}\Sigma_\lambda \left[(y-x)^\mu \widehat{T}^{\lambda\nu}(y) + (y-x)^\nu \widehat{T}^{\lambda\mu}(y) \right]$$

The divergence of the integrand of $J^{\mu\nu}$ vanishes, therefore it does not depend on the integration hypersurface (it is a constant of motion) and

$$\widehat{\Lambda} \widehat{J}_x^{\mu\nu} \widehat{\Lambda}^{-1} = \Lambda_\alpha^{-1\mu} \Lambda_\beta^{-1\nu} \widehat{J}_x^{\alpha\beta}$$

The divergence of the integrand of $Q^{\mu\nu}$ does not vanish, therefore it does depend on the integration hypersurface and

$$\widehat{\Lambda} \widehat{Q}_x^{\mu\nu} \widehat{\Lambda}^{-1} \neq \Lambda_\alpha^{-1\mu} \Lambda_\beta^{-1\nu} \widehat{Q}_x^{\alpha\beta}$$

Is it the best Approximation?

The formulas we have derived are based on a Taylor expansion of the density operator

$$W(x,k)_{LE} = \frac{1}{Z}\mathrm{tr}\left(\exp\left[-\int_{\Sigma_{FO}} \mathrm{d}\Sigma_\mu(y)\left(\widehat{T}_B^{\mu\nu}(y)\beta_\nu(y) - \zeta(y)\,\widehat{j}^\mu(y)\right)\right]\widehat{W}(x,k)\right)$$

$$\beta_\nu(y) \simeq \beta_\nu(x) + \partial_\lambda\beta_\nu(x)(y-x)^\lambda + \cdots$$

$$\widehat{\rho}_{LE} \simeq \frac{1}{Z}\exp\left[-\beta_\nu(x)\widehat{P}^\nu - \frac{1}{2}(\partial_\mu\beta_\nu(x) - \partial_\nu\beta_\mu(x))\widehat{J}_x^{\mu\nu} - \frac{1}{2}(\partial_\mu\beta_\nu(x) + \partial_\nu\beta_\mu(x))\widehat{Q}_x^{\mu\nu} + \cdots\right]$$

This is generally correct, but it is an approximation after all.

Can we find a better approximation for our specific case?

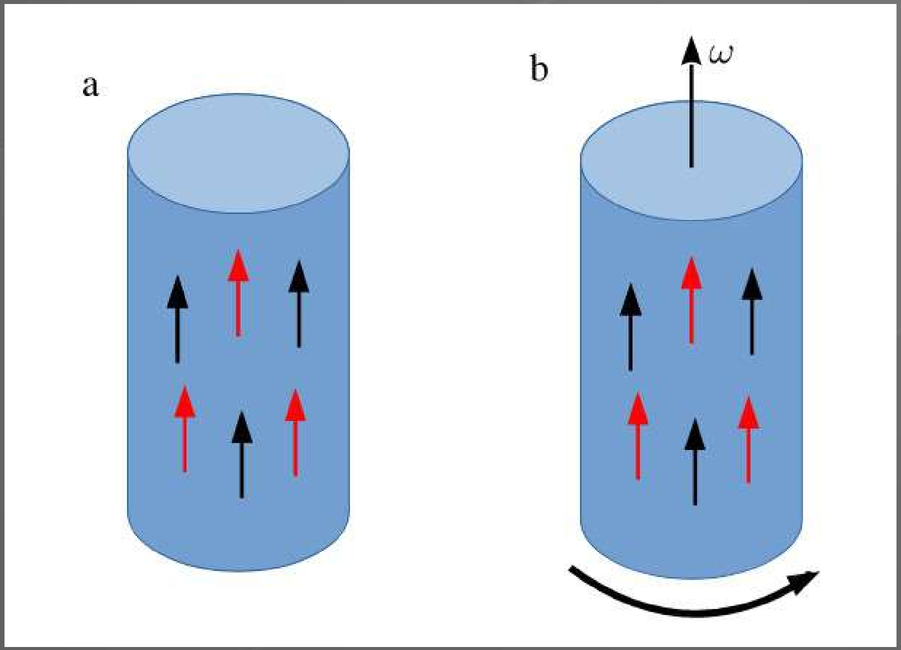
Experimental data for local polarization:

STAR, Au+Au at 200 GeV [PRL 123, 132301 (2019), Nucl. Phys. A 982, 511 (2019)]
Alice, Pb+Pb at 5 TeV S. [PRL 128, 172005 (2022)]

Different descriptions

F. Becattini, W. Florkowski, and E. Speranza, Phys. Lett. B 789, 419 (2019)

Consider a fluid temporarily at rest with a constant temperature T, hence β=(1/T)(1,0,0,0) , wherein both particles and anti-particles are polarized in the same direction.



a) Zero thermal vorticity.
Impossible with Belinfante decomposition
Possible with spin tensor

b) With thermal vorticity.
Possible with Belinfante decomposition

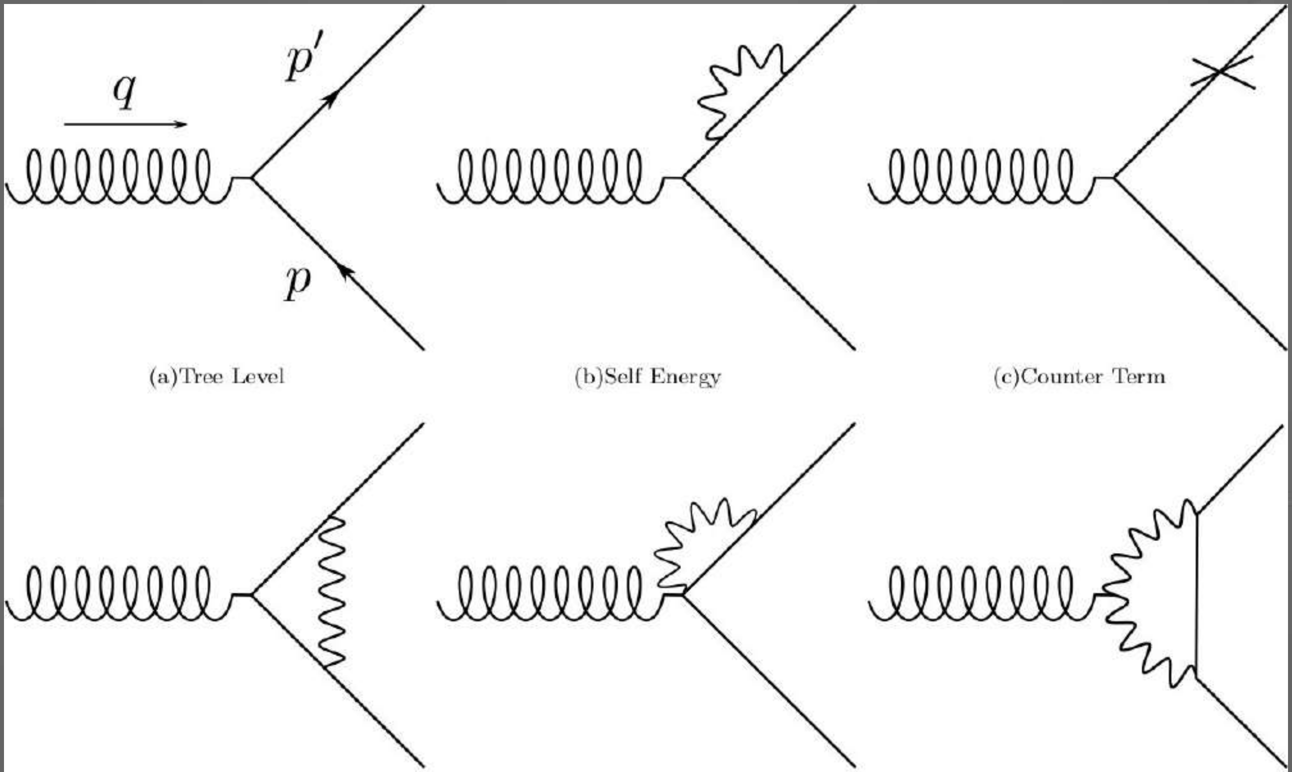
$$\widehat{\mathcal{S}}^{\lambda,\mu\nu} \neq 0$$

"Slow" evolution of spin
Weak spin-rotation coupling

$$\widehat{\mathcal{S}}^{\lambda,\mu\nu} = 0$$

"Fast" evolution of spin
Strong spin-rotation coupling

Renormalization at finite T



(a)Tree Level (b)Self Energy (c)Counter Term

(d)Electromagnetic Vertex (e)Contact (f)Photon Polarization

$$\langle p', s'|\widehat{T}_{\mu\nu}(0)|p, s\rangle = \bar{u}(p', s')M_{\mu\nu}(P, q)u(p, s)$$

The spin polarization induced by vorticity is given by the following correlator: